IITM Remote Internship 2018

Project Report

Graph Partitioning

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Contents

1 Problem

Consider a graph G = (V, E), where V denotes the set of vertices and E the set of edges. For $1 < K < |V|/2$, partition V into K parts (subsets) V_1, V_2, ..., V_K such that the parts are disjoint and have (almost) equal size, and the number of edges with endpoints in different parts is minimized.

2 Kernighan Lin algorithm

The Kernighan Lin algorithm is a heuristic algorithm for finding partitions of graphs. The algorithm has important applications in the layout of digital circuits and components in VLSI. A slight variation of it has been used to find K partitions of a graph.

2.1 Pseudocode

Input : G(V, E) and K
Initially : P :- set of all nodes

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 1: do
 2: split P into A and B of size N - N/K and N/K
 3: compute D values for all a in A and b in B
 4: let gval, pos1 and pos2 be empty lists
 5:     for (n :- 1 to |B|/2) do
 6:         find a from A and b from B, such that g = D[a] + D[b] − 2*c(a, b) is
    maximal.
 7:         remove a and b from further consideration in this pass.
 8:         add g to gval, position of a in A to pos1 and b in B to pos2.
 9:         update D values for the elements of A = A \ a and B = B \ b
10:     end for
11: find k which maximizes maxg, the sum of gval[1],...,gval[k]
12: if (maxg > 0) then
13:     exchange the elements in A with positions pos1[1], pos1[2],..., pos1[k] and B
    with positions pos2[1], pos2[2],..., pos2[k]
14: permanently remove nodes in B from further consideration
15: K :- K-1, N :- N - N/K and consider A as P
16: while (K > 1)
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2.2 Time Complexity

Line 3: Initial computation of D: $O(N^2)$
Line 5: The for loop: $O(N/K)$
The body of the for loop: $O(N^2 \log E)$
Lines 2 - 15: Each pass of the do-while loop: $O(N^3 \log E/K)$.

The do-while loop terminates after K-1 passes.
The total running time: $O(N^3 \log E)$.

2.3 Results

The algorithm was applied on 4 datasets. The results are shown below with the following parameters :
N - No. of nodes
E - No. of edges
K - No. of desired partitions
T - Time taken only for partitioning (in seconds)
M - Mean size of a partition
V - Varience of partition size

1. **Small Graph** : A small undirected and unweighted graph with 8 nodes and 13 edges.

N	E	K	T	M	V
8	13	2	0.001	4.0	0.0
8	13	3	0.001	2.6	0.3
8	13	4	0.001	2.0	0.0

2. **Zachary's karate club** : A well-known social network of a university karate club described in the paper "An Information Flow Model for Conflict and Fission in Small Groups" by Wayne W. Zachary.

N	E	K	T	M	V
34	78	2	0.002	17.00	0.00
34	78	3	0.002	11.33	0.33
34	78	4	0.002	8.50	0.33
34	78	5	0.003	6.80	0.20
34	78	6	0.002	5.67	0.27
34	78	7	0.002	4.85	0.14
34	78	8	0.002	4.25	0.21
34	78	9	0.003	3.78	0.19
34	78	10	0.001	3.40	0.27
34	78	11	0.001	3.09	0.09
34	78	12	0.001	2.83	0.15
34	78	13	0.003	2.61	0.25
34	78	14	0.004	2.42	0.26
34	78	15	0.002	2.27	0.21
34	78	16	0.002	2.12	0.11
34	78	17	0.003	2.00	0.00

3. **Caenorhabditis elegans worm's neural network** : The the neural network of the Caenorhabditis elegans worm (C.elegans). It was studied by Watts and Strogatz (1998). The network contains 306 nodes that represent neurons. Two neurons are connected if at least one synapse or gap junction exist between them. The weight is the number of synapses and gap junctions.

N	E	K	T	M	V
306	2345	20	0.27	15.30	0.22
306	2345	40	0.25	7.65	0.23
306	2345	60	0.29	5.10	0.09
306	2345	80	0.35	3.82	0.14
306	2345	100	0.39	3.06	0.05
306	2345	120	0.48	2.55	0.24
306	2345	140	0.54	2.18	0.15

4. **US power grid** : The network is the high-voltage power grid in the Western States of the United States of America. The nodes are transformers, substations, and generators, and the ties are high-voltage transmission lines. This network was originally used in Watts and Strogatz (1998). Although the transmission lines can be directed and differentiated based on their capacity, this information is not available.

N	E	K	T	M	V
4941	13188	100	220.20	49.41	0.24
4941	13188	300	165.63	16.47	0.24
4941	13188	500	205.78	9.88	0.10
4941	13188	754	286.80	6.55	0.24
4941	13188	891	376.03	5.54	0.24
4941	13188	1101	404.31	4.48	0.25
4941	13188	1333	479.19	3.70	0.20
4941	13188	1535	563.05	3.21	0.17
4941	13188	1717	582.70	2.87	0.10
4941	13188	1999	726.42	2.47	0.24
4941	13188	2213	798.93	2.23	0.17

2.4 Optimization

The best way to speed performance of this implementation would be to eliminate the descriptive output. Most of the algorithm's running time is bound by I/O, and to eliminate intermediate (descriptive) I/O would be to cut running time dramatically.

3 References

https://goo.gl/8iszXp
https://goo.gl/g2Zs7x
https://goo.gl/6REyFH
https://goo.gl/9vAkYE