Determinant sheet solutions (For Leader)

Do-Yourself 1

(i) Find the Minors & Cofactors of Element '6', '5', '0' & '4' of the determinant

$$\begin{vmatrix} 2 & 1 & 3 \\ 6 & 5 & 7 \\ 3 & 0 & 4 \end{vmatrix}$$

Solution:- Minor for $6 = M_{21} = \begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = 4$

Cofactor for $6 = (-1)^{2+1} M_{21} = -4$

Minor for $5 = M_{22} = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1$

Cofactor for $5 = (-1)^{2+2} M_{22} = -1$

Minor for $0 = M_{32} = \begin{vmatrix} 2 & 3 \\ 6 & 7 \end{vmatrix} = -4$

Cofactor for $0 = (-1)^{3+2} M_{32} = 4$

Minor for $4 = M_{33} = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} = 4$

Cofactor for $4 = (-1)^{3+3} M_{33} = 4$

(ii) Calculate the value of the determinant $\begin{vmatrix} 5 & -3 & 7 \\ -2 & 4 & -8 \\ 9 & 3 & -10 \end{vmatrix}$

Solⁿ:

$$\begin{vmatrix} 5 & -3 & 7 \\ -2 & 4 & -8 \\ 9 & 3 & -10 \end{vmatrix}$$

$= 5[-40+24] + 3[20+72] + 7[-6-36]$

$= -80 + 276 - 294$

$= -98$

(iii) The value of the determinant $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix}$ is equal to -

Solⁿ: $a[a^2-0] - b[0-b^2] + 0$

$= a^3 + b^3$

(iv) Find the value of k if $\begin{vmatrix} 1 & 2 & 0 \\ 2 & 3 & 1 \\ 3 & k & 2 \end{vmatrix} = 4$

Solⁿ: $1[6-k] - 2[4-3] + 0 = 4$

$4 - k = 4$

$k = 0$

Do-Yourself :- 2

(i) Without Expanding the determinant prove that -

$$\begin{vmatrix} a & p & l \\ b & q & m \\ c & r & n \end{vmatrix} + \begin{vmatrix} r & n & c \\ q & m & b \\ p & l & a \end{vmatrix} = 0$$

Soln :- let $D_1 = \begin{vmatrix} a & p & l \\ b & q & m \\ c & r & n \end{vmatrix}$ & $D_2 = \begin{vmatrix} r & n & c \\ q & m & b \\ p & l & a \end{vmatrix}$

in $D_2 = \begin{vmatrix} r & n & c \\ q & m & b \\ p & l & a \end{vmatrix}$

$C_1 \leftrightarrow C_3$

$$D_2 = -\begin{vmatrix} c & n & r \\ b & m & q \\ a & l & p \end{vmatrix}$$

$C_2 \leftrightarrow C_3$

$$D_2 = \begin{vmatrix} c & r & n \\ b & q & m \\ a & p & l \end{vmatrix}$$

$R_1 \leftrightarrow R_3$

$$D_2 = -\begin{vmatrix} a & p & l \\ b & q & m \\ c & r & n \end{vmatrix} = -D_1$$

So $D_1 + D_2 = 0$

(ii) If $D = \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix}$, then $\begin{vmatrix} 2\alpha & 2\beta \\ 2\gamma & 2\delta \end{vmatrix}$ is equal to -

Solⁿ:-

$$D_1 = \begin{vmatrix} 2\alpha & 2\beta \\ 2\gamma & 2\delta \end{vmatrix}$$

$$D_1 = 2 \times 2 \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix}$$

$$D_1 = 4D$$

Date: __/__/____

MON TUE WED THU FRI SAT SUN

Subject

Do your self (3)

(i) Find the value of $\begin{vmatrix} 53 & 106 & 159 \\ 52 & 65 & 91 \\ 102 & 153 & 221 \end{vmatrix}$

$$53 \times 13 \times 17 \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 6 & 9 & 13 \end{vmatrix}$$

$R_3 \to R_3 - R_2$

$$53 \times 13 \times 17 \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 2 & 4 & 6 \end{vmatrix}$$

$$2 \times 13 \times 17 \times 53 \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 1 & 2 & 3 \end{vmatrix}$$

$R_1 = R_3$ ∴ $\boxed{\Delta = 0}$

Scanned with CamScanner

Date : __/__/____

MON TUE WED THU FRI SAT SUN

Subject ______

(ii) Solve for x : $\begin{vmatrix} x & 2 & 0 \\ 2+x & 5 & -1 \\ 5-x & 1 & 2 \end{vmatrix} = 0$

$x\,|10+1| - 2\,|4+2x+5-x| + 0 = 0$

$11x - 18 - 2x = 0$

$9x = 18$

$\boxed{x=2}$

Date: __/__/____

MON TUE WED THU FRI SAT SUN

Subject

(iii) If $D_r = \begin{vmatrix} 2r & 1 & n \\ 1 & -2 & 3 \\ 3 & 2 & 1 \end{vmatrix}$, then find the value of $\sum_{r=1}^{n} D_r$

$$\begin{vmatrix} \sum_{r=1}^{n} 2r & 1 & n \\ \sum_{r=1}^{n} 1 & -2 & 3 \\ \sum_{r=1}^{n} 3 & 2 & 1 \end{vmatrix}$$

$$\begin{vmatrix} \cancel{2}\left[\frac{n(n+1)}{\cancel{2}}\right] & 1 & n \\ n & -2 & 3 \\ 3n & 2 & 1 \end{vmatrix} =$$

$$n(n+1)(-2-6) - n(1-2n) + 3n(3+2n)$$

$$-8n^2 - 8n - n + 2n^2 + 9n + 6n^2$$

$$\cancel{-8n^2} + \cancel{8n^2} - \cancel{9n} + \cancel{9n}$$

$$\boxed{0}$$

Date : __/__/__

MON TUE WED THU FRI SAT SUN

Subject

Do your self (4)

(i) Without expanding the determinant prove that

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & c-a & ab-bc \end{vmatrix}$$

$$= 1\begin{vmatrix} b-a & ca-bc \\ c-a & ab-bc \end{vmatrix} + 0 + 0 -$$

$$= 1\left[(b-a)(ab-bc) - (c-a)(ca-bc)\right]$$

$$= 1\left[b(b-a)(a-c) - (c-a)c(a-b)\right]$$

$$= (a-b)(c-a)\left[b-c\right]$$

$$\boxed{(a-b)(b-c)(c-a)}$$

Date: __/__/____

MON TUE WED THU FRI SAT SUN

Subject ____________

(ii) Using factor theorem, Find the Solution set of the equation

$$\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 4 & 20 \\ 0 & -6 & -15 \\ 0 & 2x-4 & 5x^2-20 \end{vmatrix} = 0$$

$$1\begin{vmatrix} -6 & -15 \\ 2x-4 & 5x^2-20 \end{vmatrix} + 0 + 0 = 0$$

$-6(5x^2-20) + 15(2x-4) = 0$

$-30x^2 + 120 + 30x - 60 = 0$

$30x^2 - 30x - 60 = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$\boxed{x = -1, 2}$

Date : __/__/____

MON TUE WED THU FRI SAT SUN

Subject ____________________

Do your self (5)

(i) If the determinant $D = \begin{vmatrix} 1 & 1 & 1 \\ \alpha+\beta & \alpha^2+\beta^2 & 2\alpha\beta \\ \alpha+\beta & 2\alpha\beta & \alpha^2+\beta^2 \end{vmatrix}$ &

$D_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{vmatrix}$ then find the determinant D_2 such that $D_2 = \dfrac{D}{D_1}$.

$$D = \begin{vmatrix} 1 & 1 & 1 \\ \alpha+\beta & \alpha^2+\beta^2 & 2\alpha\beta \\ \alpha+\beta & 2\alpha\beta & \alpha^2+\beta^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \beta & \alpha \end{vmatrix}$$

$$\therefore D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \beta & \alpha \end{vmatrix}$$

Date: __/__/__

MON TUE WED THU FRI SAT SUN

Subject

(ii) If $D_1 = \begin{vmatrix} ab^2 - ac^2 & bc^2 - a^2b & a^2c - b^2c \\ ac - ab & ab - bc & bc - ac \\ c - b & a - c & b - a \end{vmatrix}$ &

$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ac & ab \end{vmatrix}$ then $D_1 D_2$ is equal to

$D_1 = \begin{vmatrix} (b+c)(b-c)a & b(c-a)(c+a) & (a+b)(a-b)c \\ (c-b)a & b(a-c) & (b-a)c \\ (c-b) & (a-c) & b-a \end{vmatrix}$

$(c-b)(a-c)(b-a) \begin{vmatrix} -a(b+c) & -b(c+a) & -c(a+b) \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

$C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$(c-b)\cdot(a-c)\cdot(b-a) \begin{vmatrix} -a(b+c) & -b(c+a)+a(b+c) & -c(a+b)+a(b+c) \\ a & b-a & c-a \\ 1 & 0 & 0 \end{vmatrix}$

$= (c-b)(a-c)(b-a)\cdot(a-b)(c-a)(c-b)$

$= (a-b)^2 (b-c)^2 \cdot (c-a)^2$

& $D_2 = (a-b)(b-c)(c-a)$ By cyclic

$\therefore \boxed{D_1 D_2 = D_2^3}$

Do-yourself 6

(i) The value of the determinant

$$\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix}$$

(A) $k(a+b)(b+c)(c+a)$ (B) $k\,abc\,(a^2+b^2+c^2)$

(C) $k(a-b)(b-c)(c-a)$ (D) $k(a+b-c)(b+c-a)(c+a-b)$

Sol^n

$$\Rightarrow k\begin{vmatrix} a & k^2+a^2 & 1 \\ b & k^2+b^2 & 1 \\ c & k^2+c^2 & 1 \end{vmatrix}$$

$$C_2 \longrightarrow C_2 - k^2C_3$$

$$\Rightarrow k\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \longrightarrow \text{We know this}$$

$$\Rightarrow k(a-b)(b-c)(c-a)$$ (C)

(2) Find the value of the determinant

$$\begin{vmatrix} a^2+b^2 & a^2-c^2 & a^2-c^2 \\ -a^2 & 0 & c^2-a^2 \\ b^2 & -c^2 & b^2 \end{vmatrix}$$

<u>Soln</u>

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 2b^2 & a^2-2c^2 & b^2 \\ -a^2 & 0 & c^2-a^2 \\ b^2 & -c^2 & b^2 \end{vmatrix}$$

$R_1 \to R_1 - 2R_3$

$$\begin{vmatrix} 0 & a^2 & -b^2 \\ -a^2 & 0 & c^2-a^2 \\ b^2 & -c^2 & b^2 \end{vmatrix}$$

open by R_1

$$0(\quad) - a^2\left(-a^2b^2 - b^2(c^2-a^2)\right)$$
$$+ b^2(+a^2c^2)$$

$$0 - a^2(\cancel{-b^2c^2}) - \cancel{a^2b^2c^2}$$

$$= \underline{0}$$

(iii) Prove that $\begin{vmatrix} a & b & c \\ bc & ca & ab \\ b+c & c+a & (a+b) \end{vmatrix}$

$= (a+b+c)(a-b)(b-c)(c-a)$

Sol^n $R_3 \to R_3 + R_1$

$$\Rightarrow \begin{vmatrix} a & b & c \\ bc & ca & ab \\ a+b+c & a+b+c & a+b+c \end{vmatrix}$$

$$\Rightarrow (a+b+c) \begin{vmatrix} a & b & c \\ bc & ca & ab \\ 1 & 1 & 1 \end{vmatrix}$$

$C_1 \to aC_1$, $C_2 \to bC_2$, $C_3 \to cC_3$

$$\Rightarrow \frac{(a+b+c)}{abc} \begin{vmatrix} a^2 & b^2 & c^2 \\ abc & abc & abc \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow (a+b+c) \frac{(abc)}{(abc)} \begin{vmatrix} a^2 & b^2 & c^2 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$$ $\to$ We know

$$\Rightarrow (a+b+c)(a-b)(b-c)(c-a)$$ H.P

Do-yourself (7)

(i) find nature of Solution for given System of equations

$2x+y+z=3$, $x+2y+z=4$, $3x+z=2$

Sol^n

$$\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 0 & 1 \end{vmatrix} = 2(2) - 1(-2) + 1(-6) = 0$$

$$\Delta_1 = \begin{vmatrix} 3 & 1 & 1 \\ 4 & 2 & 1 \\ 2 & 0 & 1 \end{vmatrix} = \cancel{3(2) - 1(+4) + 1(} = 3(2) - 1(4-2) + 1(-4) = 0$$

$$\Delta_2 = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 4 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 2(2) - 3(-2) + 1(-10) = 0$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & 4 \\ 3 & 0 & 2 \end{vmatrix} = 2(4) - 1(2-12) + 3(-6) \Rightarrow 0$$

$$\boxed{\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0}$$

Since None of two planes are parallel

Hence infinite Solutions

(ii) If the System of equations $x+y+z=2$ $2x+y-z=3$ & $3x+2y+kz=4$ has a unique soln, then

(A) $k \neq 0$ (B) $-1<k<1$ (C) $-2<k<1$

(D) $k=0$

Soln Unique soln $\boxed{\Delta \neq 0}$

$$\Rightarrow \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$

$$\Rightarrow 2(k-2) -1(k-3) -1(2-3) \neq 0$$

$$\Rightarrow 2k-4-k+3+1 \neq 0$$

$$\Rightarrow \boxed{k \neq 0}$$

(A)

Scanned with CamScanner

(iii) The System of equations $\lambda x + y + z = 0$, $-x + \lambda y + z = 0$ & $-x - y + \lambda z = 0$ has a non-trivial Solution, then possible value of λ are -

(A) 0 (B) 1 (C) -3 (D) $\sqrt{3}$

Soln for non-trivial Soln

$$\boxed{\Delta = 0}$$

$$\Rightarrow \begin{vmatrix} \lambda & 1 & 1 \\ -1 & \lambda & 1 \\ -1 & -1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda(\lambda^2 + 1) - 1(-\lambda + 1) + 1(1 + \lambda) = 0$$

$$\Rightarrow \lambda^3 + \lambda + \lambda \cancel{-1} \cancel{+1} + \lambda = 0$$

$$\Rightarrow \lambda^3 + 3\lambda = 0$$

$$\lambda(\lambda^2 + 3) = 0$$

$\boxed{\lambda = 0}$, $\lambda^2 + 3 \neq 0$ (for real λ)

A

EXERCISE (O-1)

EXERCISE (O-1)

1. $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$ equals -

(A) $x^2y^2z^2$ (B) $4x^2y^2z^2$ (C) xyz (D) 4xyz

Sol^n (1) $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$ ~~$R_1 \to R_1 - R_2 - R_3$~~

$R_1 \to R_1 - R_2 - R_3$ $\begin{vmatrix} 0 & -2z & -2y \\ y & x+z & y \\ z & z & x+y \end{vmatrix}$

$R_2 \to R_2 + \frac{1}{2}R_1$
$R_3 \to R_3 + \frac{1}{2}R_1$ $\begin{vmatrix} 0 & -2z & -2y \\ y & x & 0 \\ z & 0 & x \end{vmatrix}$

$0[x^2-0] + 2z[yx-0] - 2y[0-xz]$

$+ 2xyz + 2xyz$

$\boxed{4xyz}$ (D)

2. If $\begin{vmatrix} 1 & 3 & 4 \\ 1 & x-1 & 2x+2 \\ 2 & 5 & 9 \end{vmatrix} = 0$, then x is equal to-

(A) 2 (B) 1 (C) 4 (D) 0

Sol^n 2

$$\begin{vmatrix} 1 & 3 & 4 \\ 1 & x-1 & 2x+2 \\ 2 & 5 & 9 \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 2R_1$

$$\begin{vmatrix} 1 & 3 & 4 \\ 0 & x-4 & 2x-2 \\ 0 & -1 & 1 \end{vmatrix} = 0$$

$$1\left[(x-4)(1) - (-1)(2x-2)\right] = 0$$

$$1\left[x-4+2x-2\right] = 0$$

$$3x-6 = 0$$

$$3x = 6 \Rightarrow \boxed{x = 2}$$

A

3. If a, b, c are in AP, then $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ equals -

(A) a + b + c (B) x + a + b + c (C) 0 (D) none of these

Solution:

$\Rightarrow$ a, b, c in A.P. $\Rightarrow$ $2b = a+c$ $\Rightarrow$ $b-a = c-b$

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$R_3 \to R_3 - R_2$ $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ 1 & 1 & c-b \end{vmatrix}$

$R_2 \Rightarrow R_2 - R_1$ $\begin{vmatrix} x+1 & x+2 & x+a \\ 1 & 1 & b-a \\ 1 & 1 & b-a \end{vmatrix}$ $\Rightarrow \boxed{0}$

(C)

4. If $px^4 + qx^3 + rx^2 + sx + t = \begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & 2-x & x-3 \\ x-3 & x+4 & 3x \end{vmatrix}$ then t is equal to -

(A) 33 (B) 0 (C) 21 (D) none

$$px^4 + qx^3 + rx^2 + sx + t = \begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & 2-x & x-3 \\ x-3 & x+4 & 3x \end{vmatrix}$$

put $x = 0$ we have

$$t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix}$$

$$t = 0[(0)-(-12)] + 1[0-(9)] + 3[4+6]$$

$$t = -9 + 30 \qquad \boxed{t = 21}$$

(C)

5. For positive numbers x, y and z, the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is-

(A) 0 (B) log xyz (C) log(x + y + z) (D) logx logy logz

Solution:

$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

$$1\left[1 - \log_y z \cdot \log_z y\right] - \log_x y\left[\log_y x - \log_y z \cdot \log_z x\right]$$

$$+ \log_x z\left[\log_y x \cdot \log_z y - \log_z x\right]$$

$$0 + 0 + 0 \Rightarrow \boxed{0} \quad A$$

6. Let a determinant is given by $A = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ and suppose det. A = 6. If $B = \begin{vmatrix} p+x & q+y & r+z \\ a+x & b+y & c+z \\ a+p & b+q & c+r \end{vmatrix}$

then

(A) det. B = 6 (B) det. B = – 6 (C) det. B = 12 (D) det. B = – 12

Solution:

$$B = \begin{vmatrix} p+x & q+y & r+z \\ a+x & b+y & c+z \\ a+p & b+q & c+r \end{vmatrix}$$

$R_2 \to R_2 - R_1 + R_3$

$$B = \begin{vmatrix} p+x & q+y & r+z \\ 2a - \cancel{p} & 2b - \cancel{q} & 2c - \cancel{r} \\ a+p & b+q & c+r \end{vmatrix}$$

$R_3 \to R_3 - \frac{R_2}{2}$

$$B = \begin{vmatrix} p+x & q+y & r+z \\ 2a & 2b & 2c \\ p & q & r \end{vmatrix}$$

$R_1 \to R_1 - R_3$

$$B = \begin{vmatrix} x & y & z \\ 2a & 2b & 2c \\ p & q & r \end{vmatrix}$$

$$B = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

$|B| = 2|A|$

$|B| = 2 \times 6$

$|B| = 12$

7. If in the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, A_1, B_1, C_1 etc. be the co-factors of a_1, b_1, c_1 etc., then which of the following relations is incorrect-

(A) $a_1A_1 + b_1B_1 + c_1C_1 = \Delta$

(B) $a_2A_2 + b_2B_2 + c_2C_2 = \Delta$

(C) $a_3A_3 + b_3B_3 + c_3C_3 = \Delta$

(D) $a_1A_2 + b_1B_2 + c_1C_2 = \Delta$

Solution:

The sum of the product of elements of any row (column) with their corresponding cofactors is always equal to the value of the determinant.

The sum of the product of elements of any row (column) with the cofactors of other row (column) is always equal to zero.

8. If $S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$ then $\sum_{r=1}^{n} S_r$ does not depend on-

(A) x (B) y (C) n (D) all of these

Solution:

$$\sum_{r=1}^{n} S_r = \begin{vmatrix} \Sigma(2r) & x & n(n+1) \\ \Sigma(6r^2-1) & y & n^2(2n+3) \\ \Sigma(4r^3-2nr) & z & n^2(n+1) \end{vmatrix}$$

$$\sum_{r=1}^{n} S_r = \begin{vmatrix} n(n+1) & x & n(n+1) \\ n^2(2n+3) & y & n^2(2n+3) \\ n^2(n+1) & z & n^2(n+1) \end{vmatrix}$$

$$\sum_{r=1}^{n} S_r = 0$$

9. The value of k for which the set of equations 3x+ky– 2z=0, x + ky + 3z = 0 and 2x+3y – 4z=0 has a non-trivial solution is-

(A) 15 (B) 16 (C) 31/2 (D) 33/2

Sol^n:

This set of eq^n is Homogenous So for Non trivial Sol^n

$D = 0$

$$D = \begin{vmatrix} 3 & K & -2 \\ 1 & K & 3 \\ 2 & 3 & -4 \end{vmatrix}$$

$= 3[-4K-9] - K[-4-6] -2[3-2K]$

$= -12K - 27 + 10K - 6 + 4K$

$= 2K - 33 = 0$

$K = 33/2$

10. If the system of linear equations [JEE-MAIN Online 2013]

$$x_1 + 2x_2 + 3x_3 = 6$$
$$x_1 + 3x_2 + 5x_3 = 9$$
$$2x_1 + 5x_2 + ax_3 = b$$

is consistent and has infinite number of solutions, then :-

(A) $a \in R - \{8\}$ and $b \in R - \{15\}$
(B) $a = 8$, b can be any real number
(C) $a = 8$, $b = 15$
(D) $b = 15$, a can be any real number

Solution:

- For infinite sol^n $D = D_1 = D_2 = D_3 = 0$

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{vmatrix} \Rightarrow 1[3a-25] - 2[a-10] + 3[5-6] = 0$$
$$\Rightarrow a = 8$$

$$D_3 = \begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 9 \\ 2 & 5 & b \end{vmatrix}$$

$$\Rightarrow 1[3b-45] - 2[b-18] + 6[5-6] = 0$$
$$\Rightarrow b = 15$$

11. Consider the system of equations : x + ay = 0, y + az = 0 and z + ax = 0. Then the set of all real values of 'a' for which the system has a unique solution is : [JEE-MAIN Online 2013]

(A) {1, –1}
(B) R – {–1}
(C) {1, 0, –1}
(D) R – {1}

Solution:

For unique Sol^n $D \neq 0$

$$D = \begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix}$$

$$= 1[1] - a[-a^2]$$

$$D = 1 + a^3 \neq 0$$

$$a \neq -1$$

12. If a, b, c > 0 and x, y, z ∈ R, then the determinant $\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^y+b^{-y})^2 & (b^y-b^{-y})^2 & 1 \\ (c^z+c^{-z})^2 & (c^z-c^{-z})^2 & 1 \end{vmatrix}$ is equal to -

(A) $a^x b^y c^x$ (B) $a^{-x} b^{-y} c^{-z}$ (C) $a^{2x} b^{2y} c^{2z}$ (D) zero

Solution:

$$\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^y+b^{-y})^2 & (b^y-b^{-y})^2 & 1 \\ (c^z+c^{-z})^2 & (c^z-c^{-z})^2 & 1 \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$\begin{vmatrix} 4 & (a^x-a^{-x})^2 & 1 \\ 4 & (b^y-b^{-y})^2 & 1 \\ 4 & (c^z-c^{-z})^2 & 1 \end{vmatrix} = 4\begin{vmatrix} 1 & (a^x-a^{-x})^2 & 1 \\ 1 & (b^y-b^{-y})^2 & 1 \\ 1 & (c^z-c^{-z})^2 & 1 \end{vmatrix}$$

c_1 & c_3 are identical

so value of det is $\underline{0}$

13. There are two numbers x making the value of the determinant $\begin{vmatrix} 1 & -2 & 5 \\ 2 & x & -1 \\ 0 & 4 & 2x \end{vmatrix}$ equal to 86. The sum of these two numbers, is-

(A) –4 (B) 5 (C) –3 (D) 9

Solution:

$$\begin{vmatrix} 1 & -2 & 5 \\ 2 & x & -1 \\ 0 & 4 & 2x \end{vmatrix} = 86$$

$$1[2x^2+4] + 2[4x+0] + 5[8-0] = 86$$

$$2x^2+4+8x+40 = 86$$

$$x^2+4x-21=0$$

$$(x+7)(x-3)=0$$

$$x = -7, 3$$

$$x_1+x_2 = -7+3 = \boxed{-4}$$

(A)

14. If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and A_1, B_1, C_1 denote the co-factors of a_1, b_1, c_1 respectively, then the value of the determinant $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$ is -

(A) Δ (B) Δ^2 (C) Δ^3 (D) 0

Soln:

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} ; \quad \Delta_c = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} \xrightarrow[\text{transpose}]{\text{Take its}} \Delta_c^T = \begin{vmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{vmatrix}$$

$$\Delta\,\Delta_c^T = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{vmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{vmatrix} = \begin{vmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{vmatrix} = \Delta^3$$

$$\therefore \Delta_c^T = \Delta^2 \Rightarrow \boxed{\Delta_c = \Delta^2}$$

Note:- In general $\boxed{\Delta_c = \Delta^{n-1}}$ where n is order of det.

EXERCISE (O-2)

EXERCISE (O-2)

1. Let $f(x)=\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$, then the maximum value of f(x), is-

(A) 2 (B) 4 (C) 6 (D) 8

(1) $C_1 \to C_1 + C_2 + C_3$

$$(2+4\sin 2x)\begin{vmatrix} 1 & \cos^2 x & 4\sin 2x \\ 1 & 1+\cos^2 x & 4\sin 2x \\ 1 & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$(2+4\sin 2x)\begin{vmatrix} 1 & \cos^2 x & 4\sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$\Rightarrow (2+4\sin 2x)(1)$

$(f(x))_{max} = 2+4 = 6$ (C)

2. The determinant $\begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$ is -

(A) 0

(B) independent of θ

(C) independent of ϕ

(D) independent of θ & ϕ both

$\underline{\text{Sol}^n}$ Apply

$R_1 \to R_1 + (\sin\phi) R_2 + (\cos\phi) R_3$

$$\begin{vmatrix} 0 & 0 & 2\cos^2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$$

$= 2\cos^2\phi\,(\sin^2\phi + \cos^2\phi)$

$= 2\cos^2\phi$ independent of (θ)

(B)

3. Let $D_1 = \begin{vmatrix} a & b & a+b \\ c & d & c+d \\ a & b & a-b \end{vmatrix}$ and $D_2 = \begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$ then the value of $\dfrac{D_1}{D_2}$ where $b \neq 0$ and $ad \neq bc$, is

(A) – 2 (B) 0 (C) – 2b (D) 2b

(3)

Page

$$D_1 = \begin{vmatrix} a & b & a+b \\ c & d & c+d \\ a & b & a-b \end{vmatrix}$$

$$D_1 = \cancelto{0}{\begin{vmatrix} a & b & a \\ c & d & c \\ a & b & a \end{vmatrix}} + \begin{vmatrix} a & b & b \\ c & d & d \\ a & b & -b \end{vmatrix}$$

$R_1 \to R_1 - R_3$

$$\begin{vmatrix} 0 & 0 & 2b \\ c & d & d \\ a & b & -b \end{vmatrix} = 2b\,(bc-ad)$$

$$D_2 = \begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$$

$$\Rightarrow \cancelto{0}{\begin{vmatrix} a & c & a \\ b & d & b \\ a & c & a \end{vmatrix}} + \begin{vmatrix} a & c & c \\ b & d & d \\ a & c & b+c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a & c & 0 \\ b & d & 0 \\ a & c & b \end{vmatrix} + \cancelto{0}{\begin{vmatrix} a & c & c \\ b & d & d \\ a & c & c \end{vmatrix}}$$

$R_1 \to R_1 - R_3$

$$D_2 = -b\,(bc-ad)$$

$$\frac{D_1}{D_2} = -2 \qquad \text{(A)}$$

4. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$ then $f(x)$ is a polynomial of degree-

(A) 0 (B) 1 (C) 2 (D) 3

Solution:

$C_1 \rightarrow C_1 + C_2 + C_3$

$$(1 + x + x - (2)x) \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$a^2+b^2+c^2$

$R_2 \rightarrow R_2 - R_1$, $R_3 - R_3 - R_1$

$$\begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$$

$f(x) \Rightarrow (1-x)^2$ (C)

5. If the system of equation, $a^2x - ay = 1 - a$ & $bx + (3 - 2b)y = 3 + a$ possess a unique solution $x = 1, y = 1$ than :

(A) $a = 1; b = -1$ (B) $a = -1, b = 1$ (C) $a = 0, b = 0$ (D) none

Sol^n.

$a^2x - ay = 1 - a$

$bx + (3-2b)y = 3 + a$

$x = y = 1$ will satisfy both equation

$a^2 - a = 1 - a \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$

I: $a = 1$ | II: $a = -1$

$b + 3 - 2b = 3 + a$

$a + b = 0$

$b = -1$ | $b = 1$

$x - y = 0$

$-x + 5y = 4$ | $x + y = 2$

| $x + y = 2$

Unique sol^n | $\times$ Coincident

infinite sol^n

(A)

6. The number of real values of x satisfying $\begin{vmatrix} x & 3x+2 & 2x-1 \\ 2x-1 & 4x & 3x+1 \\ 7x-2 & 17x+6 & 12x-1 \end{vmatrix} = 0$ is -

(A) 3 (B) 0 (C) 1 (D) infinite

Solⁿ Apply

$R_3 \to R_3 - 3R_1 - 2R_2$

$$\begin{vmatrix} x & 3x+2 & 2x-1 \\ 2x-1 & 4x & 3x+1 \\ 0 & 0 & 0 \end{vmatrix} = 0 \text{ always}$$

irrespective of x so infinite sol^n

(D)

[ONE OR MORE THAN ONE ARE CORRECT]

7. Which of the following determinant(s) vanish(es) ?

(A) $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}$

(B) $\begin{vmatrix} 1 & ab & \frac{1}{a}+\frac{1}{b} \\ 1 & bc & \frac{1}{b}+\frac{1}{c} \\ 1 & ca & \frac{1}{c}+\frac{1}{a} \end{vmatrix}$

(C) $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$

(D) $\begin{vmatrix} \log_x xyz & \log_x y & \log_x z \\ \log_y xyz & 1 & \log_y z \\ \log_z xyz & \log_z y & 1 \end{vmatrix}$

Solution:

(A) $\left.\begin{matrix} R_1 \to aR_1 \\ R_2 \to bR_2 \\ R_3 \to cR_3 \end{matrix}\right\}$ $\therefore \Delta = \frac{1}{abc}\begin{vmatrix} a & abc & abc(b+c) \\ b & abc & abc(c+a) \\ c & abc & abc(a+b) \end{vmatrix}$

Take abc common from C_2 & C_3

$$\Delta = \frac{1}{abc}(abc)^2\begin{vmatrix} a & 1 & (b+c) \\ b & 1 & (c+a) \\ c & 1 & (a+b) \end{vmatrix}$$

Apply $C_1 \to C_1 + C_3$

$$\Delta = abc\begin{vmatrix} a+b+c & 1 & b+c \\ b+c+a & 1 & c+a \\ c+a+b & 1 & a+b \end{vmatrix} = (a+b+c)abc\begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}$$

$\Downarrow$ 0.

(B) $C_2 \Rightarrow \frac{C_2}{abc}$

$$\therefore \Delta = abc\begin{vmatrix} 1 & 1/c & 1/a + 1/b \\ 1 & 1/a & 1/b + 1/c \\ 1 & 1/b & 1/c + 1/a \end{vmatrix}$$

$C_3 \to C_3 + C_2$

$$\to \begin{vmatrix} 1 & 1/a + 1/b + 1/c & 1/a + 1/b \\ 1 & 1/a + 1/b + 1/c & 1/b + 1/c \\ 1 & 1/a + 1/b + 1/c & 1/c + \frac{1}{a} \end{vmatrix} = 0$$

7 Continues...

(C) $C_1 \to C_1 - C_2$
$C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} b-a & c-b & a-c \\ b-a & c-b & b-c \\ b-a & c-b & 0 \end{vmatrix}.$$

$$\Delta = (b-a)(c-b) \begin{vmatrix} 1 & 1 & a-c \\ 1 & 1 & b-c \\ 1 & 1 & 0 \end{vmatrix} = 0.$$

(D)
$$\Delta = \begin{vmatrix} \frac{\log xyz}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log xyz}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log xyz}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix}$$

Take $\log(xyz)$ common from C_1
$\log(y)$ —— " —— C_2
$\log(z)$ —— " —— C_3

$$\Delta = \log(xyz) \cdot \log y \cdot \log z \begin{vmatrix} \frac{1}{\log x} & \frac{1}{\log x} & \frac{1}{\log x} \\ \frac{1}{\log y} & \frac{1}{\log y} & \frac{1}{\log y} \\ \frac{1}{\log z} & \frac{1}{\log z} & \frac{1}{\log z} \end{vmatrix}$$

$= 0$

identical

8. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$ is equal to zero, if -

(A) a, b, c are in AP

(B) a, b, c are in GP

(C) α is a root of the equation $ax^2+bx+c=0$

(D) $(x-\alpha)$ is a factor of $ax^2+2bx+c$

Sol^n

$R_3 \to R_3 - (\alpha R_1 + R_2)$

$$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha+b & b\alpha+c & -(\alpha(a\alpha+b) + b\alpha+c) \end{vmatrix} = 0$$

Expand by R_3 :

$$-\underbrace{(a\alpha^2+2b\alpha+c)}_{\Downarrow \atop 0} \text{ OR } \underbrace{(b^2-ac)}_{\Downarrow \atop 0} = 0$$

If $a\alpha^2 + 2b\alpha + C = 0 \Rightarrow x=\alpha$ is root of QE $ax^2+2bx+C=0$

If $b^2 - ac = 0 \Rightarrow a, b, c$ are in G.P.

(B & D)

9. System of linear equations in x,y,z

$$2x + y + z = 1$$
$$x - 2y + z = 2$$
$$3x - y + 2z = 3$$ have infinite solutions which

(A) can be written as $(-3\lambda - 1, \lambda, 5\lambda + 3)\ \forall\ \lambda \in R$

(B) can be written as $(3\lambda - 1, -\lambda, -5\lambda + 3)\ \forall\ \lambda \in R$

(C) are such that every solution satisfy $x - 3y + 1 = 0$

(D) are such that none of them satisfy $5x + 3z = 1$

Solution: Let $y = \lambda \in R$

$2x + z = 1 - \lambda$ —— (1)

$x + z = 2 + 2\lambda$ —— (2)

(1) − (2)

$\Rightarrow \boxed{x = -1 - 3\lambda}$ Put in (1), we get $\boxed{z = 3 + 5\lambda}$

opt. (A)

for opt (B). Replace λ by $(-\lambda)$.

—— (D) Put value of x, y, z in given eqn

[MATRIX MATCH TYPE]

10 Consider a system of linear equations $a_ix + b_iy + c_iz = d_i$(where $a_i, b_i, c_i \neq 0$ and $i = 1,2,3$) & (α,β,γ) is its unique solution, then match the following conditions.

	Column-I		**Column-II**
(A)	If $a_i = k$, $d_i = k^2$, $(k \neq 0)$ and $\alpha + \beta + \gamma = 2$, then k is	(P)	1
(B)	If $a_i = d_i = k \neq 0$, then $\alpha + \beta + \gamma$ is	(Q)	2
(C)	If $a_i = k > 0$, $d_i = k + 1$, then $\alpha + \beta + \gamma$ can be	(R)	0
(D)	If $a_i = k < 0$, $d_i = k + 1$, then $\alpha + \beta + \gamma$ can be	(S)	3
		(T)	–1

Solution:

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad \Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \qquad \Delta_z = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(A)

$$\Delta = \begin{vmatrix} k & b_1 & c_1 \\ k & b_2 & c_2 \\ k & b_3 & c_3 \end{vmatrix} = k\begin{vmatrix} 1 & b_1 & c_1 \\ 1 & b_2 & c_2 \\ 1 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_x = \begin{vmatrix} k^2 & b_1 & c_1 \\ k^2 & b_2 & c_2 \\ k^2 & b_3 & c_3 \end{vmatrix} = k^2 \cdot \begin{vmatrix} - " - \end{vmatrix}$$

$$\Delta_y = \begin{vmatrix} k & k^2 & c_1 \\ k & k^2 & c_2 \\ k & k^2 & c_3 \end{vmatrix} = 0 \qquad \text{Similarly } \Delta_z = 0.$$

$\therefore\ x = \frac{\Delta_x}{\Delta}$ | $y = \frac{\Delta_y}{\Delta}$ | $z = \frac{\Delta_z}{\Delta}$

$\alpha = k$ | $\beta = 0$ | $\gamma = 0$

Given $\alpha+\beta+\gamma = 2$

$k+0+0 = 2$

$k = 2$ // (Q)

(B)

$$\Delta_x = \begin{vmatrix} k & b_1 & c_1 \\ k & b_2 & c_2 \\ k & b_3 & c_3 \end{vmatrix} = \Delta \qquad \therefore x = 1$$

$\boxed{\alpha = 1}$

$$\Delta_y = \begin{vmatrix} k & k & c_1 \\ k & k & c_2 \\ k & k & c_3 \end{vmatrix} = 0 \Rightarrow \boxed{\beta = 0}$$

Similarly $\Delta_z = 0 \Rightarrow \boxed{\gamma = 0}$

$\alpha+\beta+\gamma = 1$ (P)

$\boxed{10}$ Continues....

(C),(D) $\Delta_x = \begin{vmatrix} k+1 & b_1 & c_1 \\ k+1 & b_2 & c_2 \\ k+1 & b_3 & c_3 \end{vmatrix} \Rightarrow x = \dfrac{\Delta_x}{\Delta} = \dfrac{k+1}{k} = \alpha$

$\Delta_y = \begin{vmatrix} k & k+1 & c_1 \\ k & k+1 & c_2 \\ k & k+1 & c_3 \end{vmatrix} = 0 \Rightarrow y = 0 = \beta$

Similarly $\Delta_z = 0 \Longrightarrow z = 0 = \gamma$

$$\boxed{\alpha+\beta+\gamma = \frac{k+1}{k} = 1+\frac{1}{k}}$$

If $k>0$, then $\alpha+\beta+\gamma > 1 \longrightarrow$ Opt. Q, S

If $k<0$ —— " —— $<1 \longrightarrow$ Opt R, T

EXERCISE (S-1)

EXERCISE (S-1)

1. **(a)** On which one of the parameter out of a, p, d or x the value of the determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix} \text{ does not depend.}$$

(b) If $\begin{vmatrix} x^3+1 & x^2 & x \\ y^3+1 & y^2 & y \\ z^3+1 & z^2 & z \end{vmatrix} = 0$ and x, y, z are all different then, prove that $xyz = -1$.

(a) Expand through R_1

$= \sin(p+d-p)x - a.\sin(p+d-p+d)x + a^2 \sin(p-p+d)x$

$= \sin(d)x - a\sin(2d)x + a^2 \sin(d)x$

So this determinant is independent of p.

(b) Since each element of C_1 is the sum of two elements so we can write it as a sum of two determinants

$$= \begin{vmatrix} x^3 & x^2 & x \\ y^3 & y^2 & y \\ z^3 & z^2 & z \end{vmatrix} + \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix}$$

$$= xyz \begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix} + \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (xyz+1)$$

$$= -(x-y)(y-z)(z-x)(xyz+1)$$

Since determinant is zero and x, y, z are distinct

So $xyz + 1 = 0 \Rightarrow xyz = -1$

2. Prove that :

(a) $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

(b) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = [(x-y)(y-z)(z-x)(x+y+z)]$

Solution:

a) $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$C_1 \to C_1 - 2C_2 + 3C_3$; $C_2 \to C_2 - 3C_3$.

$$\begin{vmatrix} a^2-2a+1 & 2(a-1) & 1 \\ 0 & a-1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} (a-1)^2 & 2(a-1) & 1 \\ 0 & (a-1) & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (a-1)^3$$

(b) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}$

$C_1 \to C_1 - C_2$; $C_2 \to C_2 - C_3$.

$$\begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^3-y^3 & y^3-z^3 & z^3 \end{vmatrix}$$

expanding about R_1.

$$(x-y)(y^3-z^3) - (y-z)(x^3-y^3)$$

$$= (x-y)(y-z)\left[y^2+yz+z^2-x^2-xy-y^2\right]$$

$$= (x-y)(y-z)(z-x)(x+y+z).$$

3. (a) Let $f(x) = \begin{vmatrix} x & 1 & \frac{-3}{2} \\ 2 & 2 & 1 \\ \frac{1}{x-1} & 0 & \frac{1}{2} \end{vmatrix}$. Find the minimum value of f(x) (given x > 1).

(b) If $a^2 + b^2 + c^2 + ab + bc + ca \le 0 \ \forall \ a, b, c \in R$, then find the value of the determinant

$$\begin{vmatrix} (a+b+2)^2 & a^2+b^2 & 1 \\ 1 & (b+c+2)^2 & b^2+c^2 \\ c^2+a^2 & 1 & (c+a+2)^2 \end{vmatrix}.$$

Solution:

(a)

$$\begin{vmatrix} x & 1 & -3/2 \\ 2 & 2 & 1 \\ \frac{1}{x-1} & 0 & 1/2 \end{vmatrix} = (x-1) + \frac{4}{(x-1)}$$

as $(x-1) > 0$ then Apply AM ≥ GM

$$\frac{(x-1) + \frac{4}{(x-1)}}{2} \geq \sqrt{4}$$

$$\geq 4 \quad \text{Ans.}$$

(b)

Sol. $a^2+b^2+c^2+ab+bc+ca \le 0$

or $(a-b)^2+(b-c)^2+(c-a)^2 \le 0$

or $a=b=c=0$

$$\begin{vmatrix} 4 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 65$$

4. If $D = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ and $D' = \begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix}$, then prove that $D' = 2D$.

Solution:

$$D = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$D' = \begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$D' = 2\begin{vmatrix} a+b+c & c+a & a+b \\ a+b+c & b+c & c+a \\ a+b+c & a+b & b+c \end{vmatrix}$$

$C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$$D' = 2\begin{vmatrix} a+b+c & -b & -c \\ a+b+c & -a & -b \\ a+b+c & -c & -a \end{vmatrix} = 2\begin{vmatrix} a+b+c & b & c \\ a+b+c & a & b \\ a+b+c & c & a \end{vmatrix}$$

$C_1 \to C_1 - C_2 - C_3$

$$D' = 2\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = 2D$$

5. If $a+b+c=0$, solve for x: $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0.$

Solution:

$R_1 \to R_1 + R_2 + R_3$

$$\underbrace{(a+b+c-x)}\begin{vmatrix} 1 & 1 & 1 \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$$

$\Downarrow$

0

$\boxed{\therefore x = 0}$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ x+c-b & b-x-a & a \\ b-c & a-c+x & c-x \end{vmatrix} = 0$$

Now expand and solve.

to get $x = \pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$ Ans

6. Let a, b, c are the solutions of the cubic $x^3 - 5x^2 + 3x - 1 = 0$, then find the value of the determinant

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}.$$

Solution:

$$a+b+c = 5$$
$$ab+bc+ca = 3$$
$$abc = 1$$

$C_1 \to C_1 + C_2 + C_3$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$$

$R_3 \to R_3 - 2R_1$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$$

Now expanding about R_1

$$= (a+b+c)\left[(b-c)(a+b-2c) - (c-a)(c+a-2b)\right]$$
$$= (a+b+c)\left[a^2+b^2+c^2-ab-bc-ca\right]$$
$$= (a+b+c)\left[(a+b+c)^2 - 3(ab+bc+ca)\right]$$
$$= 5 \times (5^2 - 3 \times 3) = 80$$

7. Show that, $\begin{vmatrix} a^2+\lambda & ab & ac \\ ab & b^2+\lambda & bc \\ ac & bc & c^2+\lambda \end{vmatrix}$ is divisible by λ^2 and find the other factor.

multiply R_1 by 'a' and take 'a' common by C_1 and similarly multiply R_2 by 'b' and take out 'b' common by C_2 multiply R_3 by 'c' and take out 'c' common by C_3.

$$\frac{1}{abc}\begin{vmatrix} a(a^2+\lambda) & a^2b & a^2c \\ ab^2 & b(b^2+\lambda) & b^2c \\ ac^2 & bc^2 & c(c^2+\lambda) \end{vmatrix}$$

$$\frac{\cancel{abc}}{\cancel{abc}}\begin{vmatrix} a^2+\lambda & a^2 & a^2 \\ b^2 & b^2+\lambda & b^2 \\ c^2 & c^2 & c^2+\lambda \end{vmatrix}$$

⑦ Continues...

Apply $R_1 \to R_1 + R_2 + R_3$

and take $(a^2+b^2+c^2+\lambda)$ common from R_1.

$$(a^2+b^2+c^2+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+\lambda & b^2 \\ c^2 & c^2 & c^2+\lambda \end{vmatrix}$$

Apply $C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ -\lambda & \lambda & b^2 \\ 0 & -\lambda & c^2+\lambda \end{vmatrix}$$

$\therefore \ (a^2+b^2+c^2+\lambda)\lambda^2$

Hence other factor is $(a^2+b^2+c^2+\lambda)$.

8. If $\Delta(x) = \begin{vmatrix} 0 & 2x-2 & 2x+8 \\ x-1 & 4 & x^2+7 \\ 0 & 0 & x+4 \end{vmatrix}$ and $f(x) = \sum_{j=1}^{3}\sum_{i=1}^{3} a_{ij}c_{ij}$, where a_{ij} is the element of i^{th} and j^{th} column in $\Delta(x)$ and c_{ij} is the cofactor a_{ij} $\forall$ i and j, then find the greatest value of f(x), where $x \in [-3, 18]$

Solution:

$$\Delta(x) = -(x-1)(2x-2)(x+4)$$
$$= -2(x^3 + 2x^2 - 7x + 4)$$

$$f(x) = \sum_{j=1}^{3}\sum_{i=1}^{3} a_{ij} C_{ij}$$

$$f(x) = \sum_{j=1}^{3} a_{1j} C_{1j} + \sum_{j=1}^{3} a_{2j} C_{2j} + \sum_{j=1} a_{3j} C_{3j}$$

$$f(x) = (a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}) + (a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23}) + (a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33})$$

$$f(x) = 3\,\Delta(x) = -6(x^3 + 2x^2 - 7x + 4)$$

See, $f(x) = -6\,(x-1)^2(x+4)$

In interval, $x \in [-3, 18]$

$(x-1)^2 \geq 0$ & $x+4 \geq 0$

$\therefore$ $f(x) \leq 0$ $\left[-6(x-1)^2(x+4) \leq 0\right]$

& $f(x) = 0$ when $x = 1$ in $[-3, 18]$ $\{x \neq -4\}$

So, maximum value of $f(x)$ in $[-3, 18] = 0$

9. If $S_r = \alpha^r + \beta^r + \gamma^r$ then show that $\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix} = (\alpha-\beta)^2(\beta-\gamma)^2(\gamma-\alpha)^2$.

Solution:

$$\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix} = \begin{vmatrix} 1+1+1 & \alpha+\beta+\gamma & \alpha^2+\beta^2+\gamma^2 \\ \alpha+\beta+\gamma & \alpha^2+\beta^2+\gamma^2 & \alpha^3+\beta^3+\gamma^3 \\ \alpha^2+\beta^2+\gamma^2 & \alpha^3+\beta^3+\gamma^3 & \alpha^4+\beta^4+\gamma^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix} \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix}$$

using product of 2 determin-ants

$$= \left(\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix} \right)^2 = (\alpha-\beta)^2(\beta-\gamma)^2(\gamma-\alpha)^2$$

10. Solve the following sets of equations using Cramer's rule and remark about their consistency.

(a) $\begin{aligned} x + y + z - 6 &= 0 \\ 2x + y - z - 1 &= 0 \\ x + y - 2z + 3 &= 0 \end{aligned}$ (b) $\begin{aligned} x + 2y + z &= 1 \\ 3x + y + z &= 6 \\ x + 2y &= 0 \end{aligned}$ (c) $\begin{aligned} 7x - 7y + 5z &= 3 \\ 3x + y + 5z &= 7 \\ 2x + 3y + 5z &= 5 \end{aligned}$

10

(a)

$$x + y + z = 6$$
$$2x + y - z = 1$$
$$x + y - 2z = -3$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{vmatrix} = 3, \quad D_x = \begin{vmatrix} 6 & 1 & 1 \\ 1 & 1 & -1 \\ -3 & 1 & -2 \end{vmatrix} = 3$$

$$D_y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 1 & -1 \\ 1 & -3 & -2 \end{vmatrix} = 6, \quad D_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 9$$

$$x = \frac{D_x}{D} = 1, \quad y = \frac{D_y}{D} = 2, \quad z = \frac{D_z}{D} = 3$$

(b)

$$x + 2y + z = 1$$
$$3x + y + z = 6$$
$$x + 2y + 0z = 0$$

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 5, \quad D_1 = \begin{vmatrix} 1 & 2 & 1 \\ 6 & 1 & 1 \\ 0 & 2 & 0 \end{vmatrix} = 10$$

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 6 & 1 \\ 1 & 0 & 0 \end{vmatrix} = -5, \quad D_3 = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 6 \\ 1 & 2 & 0 \end{vmatrix} = 5$$

(10) Continues...

$x = \frac{D_1}{D} = 2, \quad y = \frac{D_2}{D} = -1, \quad z = \frac{D_3}{D} = 1$

(c)

$7x - 7y + 5z = 3$

$3x + y + 5z = 7$

$2x + 3y + 5z = 5$

$$D = \begin{vmatrix} 7 & -7 & 5 \\ 3 & 1 & 5 \\ 2 & 3 & 5 \end{vmatrix} = 0$$

$D_1 = 120.$

Since $D = 0$ & $D_1 \neq 0$

$\therefore$ symstem has no solution.

(11) For what value of K do the following system of equations $x + Ky + 3z = 0$, $3x + Ky - 2z = 0$, $2x + 3y - 4z = 0$ possess a non trivial (i.e. not all zero) solution over the set of rationals Q. For that value of K, find all the solutions of the system.

The given system of equations is homogeneous

$$D = \begin{vmatrix} 1 & K & 3 \\ 3 & K & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow K = \frac{33}{2}$

Let $z = \lambda$

$x + \frac{33}{2}y = -3\lambda$

$3x + \frac{33}{2}y = 2\lambda.$

$-2x = -5\lambda$

$x = \frac{5\lambda}{2}$

$\frac{33}{2}y = -3\lambda - x = -3\lambda - \frac{5\lambda}{2}$

$y = -\lambda/3$

$$x : y : z = \frac{5\lambda}{2} : -\frac{\lambda}{3} : \lambda$$

$$= \frac{5}{2} : -\frac{1}{3} : 1$$

$$= \frac{15}{2} : -1 : 3$$

$$= -\frac{15}{2} : 1 : -3 \quad \text{Ans}$$

(12) If the equations a(y + z) = x, b(z + x) = y, c(x + y) = z (where a,b,c ≠ –1)have nontrivial solutions, then find the value of $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}$.

$$\begin{vmatrix} 1 & -a & -a \\ b & -1 & b \\ c & c & -1 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\begin{vmatrix} 1+a & 0 & -a \\ 1+b & -(1+b) & +b \\ 0 & 1+c & -1 \end{vmatrix} = 0$$

$$(1+a)(1+b)(1+c) \begin{vmatrix} 1 & 0 & \frac{-a}{1+a} \\ 1 & -1 & \frac{+b}{1+b} \\ 0 & 1 & \frac{-1}{1+c} \end{vmatrix}$$

Now expand.

$$\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} = 2$$

13. Solve the following sets of equations using Cramer's rule and remark about their consistency.

(a) $x + y + z - 6 = 0$, $2x + y - z - 1 = 0$, $x + y - 2z + 3 = 0$

(b) $x + 2y + z = 1$, $3x + y + z = 6$, $x + 2y = 0$

(c) $7x - 7y + 5z = 3$, $3x + y + 5z = 7$, $2x + 3y + 5z = 5$

13 (a)

$$x + y + z = 6$$
$$2x + y - z = 1$$
$$x + y - 2z = -3$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{vmatrix} = 3, \quad D_x = \begin{vmatrix} 6 & 1 & 1 \\ 1 & 1 & -1 \\ -3 & 1 & -2 \end{vmatrix} = 3$$

$$D_y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 1 & -1 \\ 1 & -3 & -2 \end{vmatrix} = 6, \quad D_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 9$$

$$x = \frac{D_x}{D} = 1, \quad y = \frac{D_y}{D} = 2, \quad z = \frac{D_z}{D} = 3$$

(b)

$$x + 2y + z = 1$$
$$3x + y + z = 6$$
$$x + 2y + 0z = 0$$

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 5, \quad D_1 = \begin{vmatrix} 1 & 2 & 1 \\ 6 & 1 & 1 \\ 0 & 2 & 0 \end{vmatrix} = 10$$

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 6 & 1 \\ 1 & 0 & 0 \end{vmatrix} = -5, \quad D_3 = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 6 \\ 1 & 2 & 0 \end{vmatrix} = 5$$

13 Show that the system of equations $3x - y + 4z = 3$, $x + 2y - 3z = -2$ and $6x + 5y + \lambda z = -3$ has alteast one solution for any real number λ. Find the set of solutions of $\lambda = -5$.

$D = 7(\lambda+5)$, $D_1 = 4(\lambda+5)$

$D_2 = -9(\lambda+5)$ $D_3 = 0$

for unique sol. $D \neq 0$, $\Rightarrow \lambda \neq -5$

then $x = \frac{4}{7}$, $y = -9/7$, $z = 0$ is the sol.

If $\lambda = -5$ $\Rightarrow$ $D_1 = D_2 = D_3 = D = 0$

then $x = \frac{4-5k}{7}$, $y = \frac{13k-9}{7}$, $z = k$ is the sol.

when $k \in R$

EXERCISE (S-2)

EXERCISE (S-2)

1. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$.

Solution:

Step - I

$C_1 - bC_2 \to C_1$

$C_2 + aC_3 \to C_2$

$$\begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}$$

3

Take $(1+a^2+b^2)$ common from C_1 & C_2

$$(1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$

On expanding from R_1 :-

$$= (1+a^2+b^2)^3$$

S-2

(2) Given $x = cy + bz$; $y = az + cx$; $z = bx + ay$, where x, y, z are not all zero, then prove that $a^2 + b^2 + c^2 + 2abc = 1$.

Solution:

$$\left.\begin{array}{l} x - cy - bz = 0 \\ cx - y + az = 0 \\ bx + ay - z = 0 \end{array}\right\}$$

Homogeneous System has two types of solutions (1) trivial (2) Non-trivial

x, y, z are not all zero $\Rightarrow$ Non-trivial

$$D = 0 \Rightarrow \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0 \Rightarrow a^2 + b^2 + c^2 + 2abc = 1$$

③ Investigate for what values of λ, μ the simultaneous equations $x+y+z=6$; $x+2y+3z=10$ & $x+2y+\lambda z=\mu$ have :

(a) A unique solution. (b) An infinite number of solutions.

(c) No solution.

Solution:

$$x+y+z=6 \quad --- \quad P_1$$

$$x+2y+3z=10 \quad --- \quad P_2$$

$$x+2y+\lambda z=\mu \quad --- \quad P_3$$

Solution:- $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = (\lambda-3)$

$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix}$: for $\lambda=3$

$D_x = \mu - 10$

$D_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & \lambda \end{vmatrix}$: for $\lambda=3$

$D_y = 20 - 2\mu$

$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix}$: for $\lambda=3$

$D_z = \mu - 10$

(a) Unique solution: $D \neq 0 \Rightarrow \lambda \neq 3$

(b) Infinite solution:- $D = D_x = D_y = D_z = 0$

& No two are distinct parallel

$\lambda = 3$, $\mu = 10$. P_1 & P_2 distinct, P_2, P_3 Same

(c) No Solution:- $\lambda = 3$, $\mu \neq 10$

4 For what values of p, the equations : $x+y+z=1$; $x+2y+4z=p$ & $x+4y+10z=p^2$ have a solution ? Solve them completely in each case.

Solution:

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ p & 2 & 4 \\ p^2 & 4 & 10 \end{vmatrix} = (20+4p+4p^2) - (2p^2+10p+16)$$

$$= 2(p-1)(p-2)$$

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & p & 4 \\ 1 & p^2 & 10 \end{vmatrix} = (10p+4+p^2) - (p+4p^2+10)$$

$$= -3(p-1)(p-2)$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & p \\ 1 & 4 & p^2 \end{vmatrix} = (2p^2+p+4) - (2+4p+p^2)$$

$$= (p-1)(p-2)$$

for $p=1$, $D = D_1 = D_2 = D_3 = 0 \Rightarrow$ infinite solutions

$\therefore$ system is $x+y+z=1$
$x+2y+4z=1$
$x+4y+10z=1$

put $z=k$ and solve to get $x=1+2k$, $y=-3k$

Similarly for $p=2$, $D=D_1=D_2=D_3=0 \Rightarrow$ infinite solutions

$\therefore$ system is $x+y+z=1$, $x+2y+4z=2$, $x+4y+10z=4$

put $z=k$ to get $x=2k$, $y=1-3k$.

(5) Solve the equations : $Kx+2y-2z=1$, $4x+2Ky-z=2$, $6x+6y+Kz=3$ considering specially the case when $K=2$.

Solution:

$kx+2y-2z=1$

$4x+2ky-z=2$

$6x+6y+kz=3$

$$D=\begin{vmatrix} k & 2 & -2 \\ 4 & 2k & -1 \\ 6 & 6 & k \end{vmatrix} = (2k^3-12-\cancel{4}8)-(-24k-6k+8k)$$

$$=2k^3+22k-60=2(k^2+2k+15)(k-2)$$

$D_1=2(k-2)(k+6)$, $D_2=(k-2)(2k+3)$, $D_3=6(k-2)^2$

$\Rightarrow x=\frac{D_1}{D} \Rightarrow \frac{x}{D_1}=\frac{1}{D}=\frac{y}{D_2}=\frac{z}{D_2}$ for $k\neq 2$

for $\cancel{k\neq}$ $k=2$, $D=D_1=D_2=D_3=0 \Rightarrow$ infinite solutions

put $x=\lambda$, we get $y=\frac{1-2\lambda}{2}$, $z=0$.

6. Find the sum of all positive integral values of a for which every solution to the system of equation $x + ay = 3$ and $ax + 4y = 6$ satisfy the inequalities $x > 1$, $y > 0$.

Solⁿ $a(x + ay = 3)$ where $x > 1$, $y > 0$

$-\ ax + 4y = 6$

$(a^2 - 4)y = 3a - 6 \Rightarrow \boxed{y = \frac{3}{a+2}}$, $a \neq 2$

$\Rightarrow \boxed{x = 3 - \frac{a \cdot 3}{a+2}}$

Now, $\because x > 1$, $y > 0$

$\therefore \begin{cases} 3 - \frac{3a}{a+2} > 1 \\ \frac{3}{a+2} > 0 \end{cases}$

$\Rightarrow \begin{cases} a + 2 > 0 \\ \frac{3a}{a+2} < 2 \end{cases}$

$\Rightarrow \begin{cases} a > -2 \\ 3a < 2a + 4 \end{cases}$

$\Rightarrow -2 < a < 4$, $a \neq 2$

Positive integral values of a are $1, 3$.

7. Given $a = \frac{x}{y-z}$; $b = \frac{y}{z-x}$; $c = \frac{z}{x-y}$, where x, y, z are not all zero, prove that : $1 + ab + bc + ca = 0$.

Solution:

$a = \frac{x}{y-z}$, $b = \frac{y}{z-x}$, $c = \frac{z}{x-y}$

$x - ay + az = 0$

$-bx - y + bz = 0$ where x, y, z NOT all ZERO

$cx - cy - z = 0$

$\Rightarrow$ Non trivial solutions

$\Rightarrow D = 0 \Rightarrow \begin{vmatrix} 1 & -a & a \\ -b & -1 & b \\ c & -c & -1 \end{vmatrix} = 0.$

$\Rightarrow \boxed{1 + ab + bc + ca = 0}$

8 Solve the system of equations : $\left.\begin{array}{l} z + a y + a^2 x + a^3 = 0 \\ z + b y + b^2 x + b^3 = 0 \\ z + c y + c^2 x + c^3 = 0 \end{array}\right]$ where $a \neq b \neq c$.

Solution:

$a^2 x + a y + z = -a^3$

$b^2 x + b y + z = -b^3 \qquad a \neq b \neq c$

$c^2 x + c y + z = -c^3$

$$D = \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} = -(a-b)(b-c)(c-a) \neq 0$$

$\Rightarrow$ unique solution

$$D_1 = \begin{vmatrix} -a^3 & a & 1 \\ -b^3 & b & 1 \\ -c^3 & c & 1 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

$$D_2 = \begin{vmatrix} a^2 & -a^3 & 1 \\ b^2 & -b^3 & 1 \\ c^2 & -c^3 & 1 \end{vmatrix} = -(a-b)(b-c)(c-a)(ab+bc+ca)$$

$$D_3 = \begin{vmatrix} a^2 & a & -a^3 \\ b^2 & b & -b^3 \\ c^2 & c & -c^3 \end{vmatrix} = -abc \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix} = abc(a-b)(b-c)(c-a)$$

$\Rightarrow x = -(a+b+c), \quad y = ab+bc+ca, \quad z = -abc.$

EXERCISE (JM)

1. The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$, $2x + 2y + z = 0$ possess a non-zero solution is : **[AIEEE - 2011]**

(1) 1 (2) zero (3) 3 ~~(4)~~ 2

2. If the trivial solution is the only solution of the system of equations $x - ky + z = 0$, $kx + 3y - kz = 0$, $3x + y - z = 0$ Then the set of all values of k is: **[AIEEE - 2011]**

(1) $\{2, -3\}$ ~~(2)~~ $R - \{2, -3\}$ (3) $R - \{2\}$ (4) $R - \{-3\}$

1.
$$4x + ky + 2z = 0$$
$$kx + 4y + z = 0$$
$$2x + 2y + z = 0$$

has Non-trivial sol^n

$D = 0$

Expand through R_1

$$\begin{vmatrix} 4 & K & 2 \\ K & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$$4(2) - K(K-2) + 2(2K-8) = 0$$

$$3K^2 + 2K - 8 = 0 \Rightarrow (3K-4)(K+2) = 0$$

$$K = \frac{4}{3}, -2$$

2.
$$x - Ky + z = 0$$
$$Kx + 3y - Kz = 0 \quad , \quad D \neq 0$$
$$3x + y - z = 0$$

Expand through R_1

$$\begin{vmatrix} 1 & -K & 1 \\ K & 3 & -K \\ 3 & 1 & -1 \end{vmatrix} \neq 0$$

$$(K-3) + 2K^2 + (K-9) \neq 0 \Rightarrow K \neq -3, 2$$

3. The number of values of k, for which the system of equations : [JEE(Main)-2013]

$(k+1)x + 8y = 4k$, $kx + (k+3)y = 3k - 1$ has no solution, is -

(1) infinite ~~(2)~~ 1 (3) 2 (4) 3

4. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to : [JEE(Main)-2014]

(1) $\alpha\beta$ (2) $\frac{1}{\alpha\beta}$ ~~(3)~~ 1 (4) -1

---

3. $(K+1)x + 8y = 4K$ has No soln

$Kx + (K+3)y = 3K-1 \rightarrow (2)$

$$\frac{K+1}{K} = \frac{8}{K+3} \neq \frac{4K}{3K-1}$$

$K^2 - 4K + 3 \Rightarrow K = 1, 3$

But $K=1$ does not satisfy the (2) equation

$K = 3$

---

4. $f(n) = \alpha^n + \beta^n$

$$\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

$$\Rightarrow ((\alpha-\beta)(\beta-1)(1-\alpha))^2 = (\alpha-\beta)^2(\beta-1)^2(\alpha-1)^2$$

$$\Rightarrow \boxed{K=1}$$

5 The set of all values of λ for which the system of linear equations :

$2x_1 - 2x_2 + x_3 = \lambda x_1$, $2x_1 - 3x_2 + 2x_3 = \lambda x_2$, $-x_1 + 2x_2 = \lambda x_3$ has a non-trivial solution

[JEE(Main)-2015]

(1) contains two elements
(2) contains more than two elements
(3) is an empty set
(4) is a singleton

Sol^n:

$$(2-\lambda)x_1 - 2x_2 + x_3 = 0$$

$$2x_1 - (3+\lambda)x_2 + 2x_3 = 0$$

$$-x_1 + 2x_2 - \lambda x_3 = 0$$

has non-trivial sol^n

$$\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -\lambda-3 & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$C_1 \to C_1 - C_3$

$C_2 \to C_2 + 2C_3$

$$\begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & -\lambda+1 & 2 \\ \lambda-1 & 2(1-\lambda) & -\lambda \end{vmatrix} = 0$$

$$(\lambda-1)^2 \begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -2 & -\lambda \end{vmatrix} = 0$$

$$(\lambda-1)^2(\lambda+3) = 0 \Rightarrow \lambda = -3, 1$$

6 The system of linear equations $x + \lambda y - z = 0$, $\lambda x - y - z = 0$, $x + y - \lambda z = 0$ has a non-trivial solution for : **[JEE(Main)-2016]**

(1) exactly three values of λ.
(2) infinitely many values of λ.
(3) exactly one value of λ.
(4) exactly two values of λ.

Sol^n:

$$x + \lambda y - z = 0$$
$$\lambda x - y - z = 0 \quad \text{has Non-trivial sol}^n$$
$$x + y - \lambda z = 0$$

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$C_1 \to C_1 + C_3$

$C_2 \to C_2 - C_3$

$$(\lambda-1)(\lambda+1)\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & \lambda \end{vmatrix} = 0$$

$$\lambda(\lambda-1)(\lambda+1) = 0 \Rightarrow \lambda = 0, 1, -1$$

7 If S is the set of distinct values of 'b' for which the following system of linear equations

$$x + y + z = 1$$
$$x + ay + z = 1$$
$$ax + by + z = 0$$

has no solution, then S is : **[JEE(Main)-2017]**

(1) a singleton (2) an empty set

(3) an infinite set (4) a finite set containing two or more elements

Soln:

for no solution $\Delta = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$$

$$(a-b) - (1-a) + b - a^2 = 0$$

$$a - \cancel{b} - 1 + a + \cancel{b} - a^2 = 0$$

$$a^2 - 2a + 1 = 0 \Rightarrow a = 1$$

Put $a = 1$

$$x + y + z = 1$$
$$x + y + z = 1$$
$$x + by + z = 0$$

when plane become parallel then no soln. possible when $\boxed{b = 1}$

8 If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to :

[JEE(Main)-2018]

(1) (–4, 3) (2) (–4, 5) (3) (4, 5) (4) (–4, –5)

Solution: Use $R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 5x-4 & 5x-4 & 5x-4 \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

$$(5x-4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

Use $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$$(5x-4)\begin{vmatrix} 1 & 0 & 0 \\ 2x & -x-4 & 0 \\ 2x & x+4 & -x-4 \end{vmatrix}$$

$$(5x-4)(x+4)^2 = (A+Bx)(x-A)^2$$

$$\therefore \quad A = -4 \ ; \ B = 5$$

J-M

9 If the system of linear equations

$x + ky + 3z = 0$ —— 1

$3x + ky - 2z = 0$ —— 2

$2x + 4y - 3z = 0$ —— 3

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) 30 (2) –10

(3) 10 (4) –30

Sol. (3)

For non–zero solution D = 0

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$\Rightarrow$ $1(-3k + 8) - k(-9 + 4) + 3(12 - 2k) = 0$

$\Rightarrow$ $-3k + 8 + 5k + 36 - 6k = 0$

$\Rightarrow$ $-4k + 44 = 0$

$k = 11$

hence equations are $x + 11y + 3z = 0$

$3x + 11y - 2z = 0$

and $2x + 4y - 3z = 0$

let $z = t$

then we get, $x = \frac{5}{2}t$ and $y = -\frac{t}{2}$

thus, $\frac{xz}{y^2} = \frac{\left(\frac{5}{2}t\right)(t)}{\frac{t^2}{4}} = 10$

Aliter: 1 – 2 $\Rightarrow -2x + 5z = 0$

$\Rightarrow 2x = 5z$

Put in 3

$5z + 4y - 3z = 0$

$\Rightarrow z = -2y$

$\therefore \frac{xz}{y^2} = \frac{\frac{5z}{2}\cdot z}{\frac{z^2}{4}} = 10$

11 If the system of linear equations

$x - 4y + 7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then :

(1) $g + h + k = 0$

(2) $2g + h + k = 0$

(3) $g + h + 2k = 0$

(4) $g + 2h + k = 0$

Ans. (2)

Sol. $D = \begin{vmatrix} 1 & -4 & 7 \\ 0 & 3 & -5 \\ -2 & 5 & -9 \end{vmatrix} = 0$

so if system is consistent than infinite soln $\Rightarrow D_1 = D_2 = D_3 = 0$

$D_1 = \begin{vmatrix} g & -4 & 7 \\ h & 3 & -5 \\ k & 5 & -9 \end{vmatrix} = 0 \Rightarrow \boxed{2g + h + k = 0}$ Ans

10. The system of linear equations

$$x + y + z = 2$$
$$2x + 3y + 2z = 5$$
$$2x + 3y + (a^2 - 1)z = a + 1$$

[JEE(Main)-2019]

(1) has infinitely many solutions for $a = 4$

(2) is inconsistent when $|a| = \sqrt{3}$

(3) is inconsistent when $a = 4$

(4) has a unique solution for $|a| = \sqrt{3}$

Solution:

$x + y + z = 2 \quad —(1)$

$2x + 3y + 2z = 5 \quad —(2)$

$2x + 3y + (a^2 - 1)z = a + 1 \quad —(3)$

For $|a| = \sqrt{3}$, planes (2) & (3) are parallel

so inconsistent system.

For $a = 4$,

(2) − (3) $\Rightarrow 2z - 15z = 0 \Rightarrow z = 0$

$$\Rightarrow \left.\begin{array}{l} x + y = 2 \\ \& \ 2x + 3y = 5 \end{array}\right\} \quad y = 1;\ x = 1$$

unique solution.

$\therefore$ option (2) is correct.

11. If the system of linear equations

$x-4y+7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then :

(1) $g + h + k = 0$

(2) $2g + h + k = 0$

(3) $g + h + 2k = 0$

(4) $g + 2h + k = 0$

Ans. (2)

Sol. $D = \begin{vmatrix} 1 & -4 & 7 \\ 0 & 3 & -5 \\ -2 & 5 & -9 \end{vmatrix} = 0$ so if system is consistent than infinite solⁿ $\Rightarrow D_1 = D_2 = D_3 = 0$

$D_1 = \begin{vmatrix} g & -4 & 7 \\ h & 3 & -5 \\ k & 5 & -9 \end{vmatrix} = 0 \Rightarrow \boxed{2g+h+k=0}$ Ans

12. If the system of equations

$$x+y+z = 5$$
$$x+2y+3z = 9$$
$$x+3y+\alpha z = \beta$$

has infinitely many solutions, then $\beta-\alpha$ equals: **[JEE(Main)-2019]**

(1) 5 (2) 18 (3) 21 (4) 8

Solution: for infinitely many Solutions $D = D_x = D_y = D_z = 0$.

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \alpha-1 \end{vmatrix} = 0 \Rightarrow \boxed{\alpha = 5}$$

$$D_x = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & \alpha \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 5 & 1 & 1 \\ -1 & 0 & 1 \\ \beta & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 6 & 1 & 1 \\ 0 & 0 & 1 \\ \beta+5 & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow -1\,(18 - \beta - 5) = 0 \Rightarrow \boxed{\beta = 13}$$

$$\therefore \boxed{\beta - \alpha = 8}$$

13. Let $d \in R$, and $A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix}$, $\theta \in [0,2\pi]$. If the minimum value of det(A) is 8, then a value of d is : **[JEE(Main) 2019]**

(1) –7 (2) $2(\sqrt{2}+2)$ (3) –5 (4) $2(\sqrt{2}+1)$

Sol^n

$$|A| = \begin{vmatrix} -2 & 4+d & (\sin\theta-2) \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 4+d & (\sin\theta-2) \\ 1 & (\sin\theta) & d \\ 1 & 0 & 0 \end{vmatrix} \qquad (\text{New } R_3 = R_3 - 2R_2 + R_1)$$

$$= (4+d)d - \sin^2\theta + 4 = (d+2)^2 - \sin^2\theta$$

Because minimum value of $|A| = 8 \Rightarrow (d+2)^2 = 9 \Rightarrow d = 1$ or –5

14. Let $a_1, a_2, a_3, \ldots, a_{10}$ be in G.P. with $a_i > 0$ for $i = 1,2,\ldots,10$ and S be the set of pairs (r,k), r, k $\in$ N (the set of natural numbers) for which $\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$. Then the number of elements in S, is : **[JEE(Main) 2019]**

(1) Infinitely many (2) 4 (3) 10 (4) 2

Sol. Apply

$C_3 \to C_3 - C_2$

$C_2 \to C_2 - C_1$

We get D = 0

Option (1)

Let common ratio be 'R'.

Now,

$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots = R.$

$$\begin{vmatrix} \log_e(a_1^r a_2^k) & \log_e\left(\frac{a_2^r a_3^k}{a_1^r a_2^k}\right) & \log_e\left(\frac{a_3^r a_4^k}{a_2^r a_3^k}\right) \\ \log_e(a_4^r a_5^k) & \log_e\left(\frac{a_5^r a_6^k}{a_4^r a_5^k}\right) & \log_e\left(\frac{a_6^r a_7^k}{a_5^r a_6^k}\right) \\ \log_e(a_7^r a_8^k) & \log_e\left(\frac{a_8^r a_9^k}{a_7^r a_8^k}\right) & \log_e\left(\frac{a_9^r a_{10}^k}{a_8^r a_9^k}\right) \end{vmatrix}$$

C_2 & C_3 are Identical

$$= \begin{vmatrix} \log_e(a_1^r a_2^k) & \log_e(R^{r+k}) & \log_e(R^{r+k}) \\ \log_e(a_4^r a_5^k) & \log_e(R^{r+k}) & \log_e(R^{r+k}) \\ \log_e(a_7^r a_8^k) & \log_e(R^{r+k}) & \log_e(R^{r+k}) \end{vmatrix} = 0$$

15. The number of values of $\theta \in (0,\pi)$ for which the system of linear equations

$$x + 3y + 7z = 0$$
$$-x + 4y + 7z = 0$$
$$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$$

has a non-trivial solution, is : **[JEE(Main)-2019]**

(1) One (2) Three (3) Four (4) Two

Solution: $D = 0$

$$\Rightarrow \begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$$

$$\Rightarrow \sin 3\theta\,(21-28) - \cos 2\theta\,(7+7) + 2(4+3) = 0$$

$$\Rightarrow -7\sin 3\theta - 14\cos 2\theta + 14 = 0$$

$$\Rightarrow \sin 3\theta + 2\cos 2\theta = 2$$

$$\Rightarrow 3\sin\theta - 4\sin^3\theta = 2 - 2\cos 2\theta$$

$$\Rightarrow 3\sin\theta - 4\sin^3\theta = 2(2\sin^2\theta)$$

$$\Rightarrow 3\sin\theta - 4\sin^2\theta - 4\sin^3\theta = 0$$

$$\Rightarrow \sin\theta\,(3 - 4\sin\theta - 4\sin^2\theta) = 0$$

$$\Rightarrow \sin\theta\,(4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3) = 0$$

$$\Rightarrow \sin\theta\,(2\sin\theta - 1)(2\sin\theta + 3) = 0$$

$$\Rightarrow \sin\theta = 0 \quad \text{or} \quad \sin\theta = \frac{1}{2} \quad \text{or} \quad \sin\theta = \frac{-3}{2}$$

$\times$ ✓ $\times$

$\boxed{\text{Two}}$ values in $(0,\pi)$.

16. If the system of linear equations

$$2x + 2y + 3z = a$$
$$3x - y + 5z = b$$
$$x - 3y + 2z = c$$

where a, b, c are non-zero real numbers, has more then one solution, then : **[JEE(Main)-2019]**

(1) $b - c - a = 0$ (2) $a + b + c = 0$ (3) $b + c - a = 0$ (4) $b - c + a = 0$

Solution: If there are more than one solutions then there will be infinitely many solutions.

$\therefore D = D_x = D_y = D_z = 0$

$$\therefore \quad D = \begin{vmatrix} 2 & 2 & 3 \\ 3 & -1 & 5 \\ 1 & -3 & 2 \end{vmatrix} = 2(-2+15) - 2(6-5) + 3(-9+1) = 0$$

$$D_x = \begin{vmatrix} a & 2 & 3 \\ b & -1 & 5 \\ c & -3 & 2 \end{vmatrix} = 0 \Rightarrow a(-2+15) - b(4+9) + c(10+3) = 0$$

$$\Rightarrow \quad 13a - 13b + 13c = 0 \Rightarrow \boxed{a - b + c = 0}$$

(A) Option

17. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)(x+a+b+c)^2$, $x \neq 0$ and $a+b+c \neq 0$, then x is equal to :- **[JEE(Main)-2019]**

(1) $-(a+b+c)$ (2) $2(a+b+c)$ (3) abc (4) $-2(a+b+c)$

Solution:

$R_1 \to R_1+R_2+R_3$

$$D = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & then $C_2 \to C_2 - C_3$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b+c+a & -b-c-a & 2b \\ 0 & c+b+a & c-a-b \end{vmatrix}$$

$$= (a+b+c)^3 = (a+b+c)\,(a+b+c)^2$$

$$\therefore (x+a+b+c)^2 = (a+b+c)^2$$

$$\Rightarrow x+a+b+c = \pm(a+b+c)$$

as $x \neq 0 \Rightarrow$ $\boxed{x = -2a-2b-2c}$

18. An ordered pair(α,β) for which the system of linear equations

$(1+\alpha)x + \beta y+z = 2$

$\alpha x+(1+\beta)y+z = 3$

$\alpha x+\beta y+2z = 2$ has a unique solution is **[JEE(Main)-2019]**

(1) (1,–3) (2) (–3,1) (3) (2, 4) (4) (–4, 2)

Solution: For unique Solution, $D \neq 0$

$$\therefore D = \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ \alpha & \alpha+\beta & 2 \end{vmatrix} = -(\alpha+\beta+2)$$

$\therefore \boxed{\alpha+\beta \neq -2}$ (Now check suitable option(s))

19. The set of all values of λ for which the system of linear equations.

$$x - 2y - 2z = \lambda x$$
$$x + 2y + z = \lambda y$$
$$-x - y = \lambda z$$

has a non-trivial solution. **[JEE(Main)-2019]**

(1) contains more than two elements
(2) is a singleton
(3) is an empty set
(4) contains exactly two elements

Solution:

$$(1-\lambda)x - 2y - 2z = 0$$
$$x + (2-\lambda)y + z = 0$$
$$-x - y - \lambda z = 0$$

have a non-trivial solution,

$$\therefore\ D = 0 \Rightarrow \begin{vmatrix} 1-\lambda & -2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix} = 0$$

$$C_2 \to C_2 - C_3 \Rightarrow \begin{vmatrix} 1-\lambda & 0 & -2 \\ 1 & 1-\lambda & 1 \\ -1 & \lambda-1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\begin{vmatrix} 1-\lambda & 0 & -2 \\ 1 & 1 & 1 \\ -1 & -1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\begin{vmatrix} 1-\lambda & 0 & -2 \\ 0 & 1 & 1 \\ 0 & -1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)^3 = 0 \Rightarrow \boxed{\lambda = 1}$$

20. If $A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$; then for all $\theta\in\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$, det(A) lies in the interval :

[JEE(Main)-2019]

(1) $\left[\frac{5}{2},4\right)$ (2) $\left(\frac{3}{2},3\right]$ (3) $\left(0,\frac{3}{2}\right]$ (4) $\left(1,\frac{5}{2}\right]$

Solution:

$$\det(A)=\begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}$$

$$=\begin{vmatrix}0 & 0 & 2\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix} \qquad R_1\to R_1+R_3$$

$$\det(A)=2(\sin^2\theta+1)$$

as $\theta\in\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$

$\Rightarrow \frac{-1}{\sqrt2}<\sin\theta<\frac{1}{\sqrt2} \Rightarrow 0\le\sin^2\theta<\frac{1}{2}$

$\Rightarrow 1\le\sin^2\theta+1<\frac{3}{2}$

$\Rightarrow 2\le2(\sin^2\theta+1)<3 \Rightarrow 2\le\det(A)<3$

So, only $\boxed{\text{option (2)}}$ is $\boxed{\text{subset}}$ of $\boxed{[2,3)}$

EXERCISE (JA)

EXERCISE (JA)

1 Which of the following values of α satisfy the equation $\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$?

[JEE(Advanced)-2015, 4M, –2M]

(A) –4 (B) 9 (C) –9 (D) 4

Ans. (B,C)

Using multiplication of 2 determinants.

Sol. $\begin{vmatrix} 1 & \alpha & \alpha^2 \\ 4 & 2\alpha & \alpha^2 \\ 9 & 3\alpha & \alpha^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 6 \\ 1 & 4 & 9 \end{vmatrix} = -648\alpha$

$$\alpha^3 \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 6 \\ 1 & 4 & 9 \end{vmatrix} = -648\alpha$$

$-8\alpha^3 = -648\alpha$

$\Rightarrow \alpha^3 = 81\alpha$

$\therefore \alpha = 0, 9, -9$

2. The total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

[JEE(Advanced)-2016, 3(0)]

Ans. 2

Sol. $x^3 \begin{vmatrix} 1 & 1 & 1+x^3 \\ 2 & 4 & 1+8x^3 \\ 3 & 9 & 1+27x^3 \end{vmatrix} = 10$

$\Rightarrow \quad x^3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} + x^3.x^3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} = 0$

$\Rightarrow \quad x^3(25 - 23) + 6x^6.2 = 10$

$\Rightarrow \quad 6x^6 + x^3 - 5 = 0$

$\Rightarrow \quad x^3 = \frac{5}{6}, -1$

two real solutions

3

Let a,λ,m $\in \mathbb{R}$. Consider the system of linear equations

$$ax + 2y = \lambda$$
$$3x - 2y = \mu$$

Which of the following statement(s) is(are) correct ?

(A) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ

(B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ

(C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$

(D) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$ **[JEE(Advanced)-2016, 4(–2)]**

Sol. $ax + 2y = \lambda$

$3x - 2y = \mu$

for $a = -3$ above lies will be parallel or coincident

parallel for $\lambda + \mu \neq 0$ and coincident if $\lambda + \mu = 0$

and if $a \neq -3$ lies are intersecting $\Rightarrow$ unique solution.