RACE :-3

1. $\int \frac{dx}{x^2-7x+10}$

Soln. $\int \frac{dx}{x^2-2x\frac{7}{2}x+\frac{49}{4}+10-\frac{49}{4}}$

$\Rightarrow \int \frac{dx}{(x-7/2)^2-(\frac{3}{2})^2}$ $\quad \because \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

$\Rightarrow \frac{1}{2\times\frac{3}{2}} \ln\left|\frac{(x-7/2)-3/2}{(x-7/2)+3/2}\right| + C$

$\Rightarrow \frac{1}{3}\ln\left|\frac{x-5}{x-2}\right| + C$

(2) $\int \frac{dx}{4x^2-9}$

$\Rightarrow \frac{1}{4}\int \frac{dx}{x^2-(3/2)^2}$

$\Rightarrow \frac{1}{4} \times \frac{1}{2\times 3/2} \ln\left|\frac{x-3/2}{x+3/2}\right| + c$

$\Rightarrow \frac{1}{12} \ln\left|\frac{x-3/2}{x+3/2}\right| + c$

(3) $\int \frac{dx}{(x-1)^2+4}$

$\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1} x/a + c$

$\Rightarrow \frac{1}{2}\tan^{-1}\frac{x-1}{2} + c$

(4) $\int \frac{dx}{x^2+2x+3}$

$\Rightarrow \int \frac{dx}{(x+1)^2+2}$

$\Rightarrow \because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1} x/a + C$

$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\frac{x+1}{\sqrt{2}} + C$

(5) $\int \frac{dx}{4x^2+4x+5}$

$\Rightarrow \int \frac{dx}{4x^2+4x+4+1}$

$\Rightarrow \int \frac{dx}{(2x+1)^2+(2)^2}$

Let $2x+1 = t$

$2dx = dt$

$\Rightarrow \frac{1}{2} \int \frac{dt}{t^2+(2)^2}$

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \tan^{-1} \frac{t}{2} + c$

(6) $\int \frac{dx}{\sqrt{1-(2x+3)^2}}$

let $2x+3 = t$

$2\,dx = dt$

$\Rightarrow \frac{1}{2}\int \frac{dt}{\sqrt{(1)^2-t^2}}$

$\because \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}x/a + C$

$\Rightarrow \frac{1}{2}\sin^{-1}t + C$

$\Rightarrow \frac{1}{2}\sin^{-1}2x+3 + C$

(7) $\int \frac{dx}{\sqrt{4x-3-x^2}}$

$\Rightarrow \int \frac{dx}{\sqrt{1-(x^2-4x+4)}}$

$\Rightarrow \int \frac{dx}{\sqrt{(1)^2-(x-2)^2}}$

$\because \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}x/a + c$

$\Rightarrow \because \quad \sin^{-1}\frac{x-2}{1} + c$

(8) $\int \cos^2 x \, dx$

$\because \cos 2x = 2\cos^2 x - 1$

$\therefore \cos^2 x = \frac{1+\cos 2x}{2}$

$\Rightarrow \int \frac{1+\cos 2x}{2} \, dx$

$\Rightarrow \int \frac{1}{2} dx + \frac{1}{2} \int \cos 2x \, dx$

$\Rightarrow \frac{1}{2}x + \frac{1}{2} \times \frac{1}{2} \sin 2x + C$

$\Rightarrow \frac{x}{2} + \frac{\sin 2x}{4} + C$

(9) $\int \sin^2 x \, dx$

$\because \cos 2x = 1 - 2\sin^2 x$

$\therefore \sin^2 x = \frac{1-\cos 2x}{2}$

$\Rightarrow \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx$

$\Rightarrow \frac{1}{2}x - \frac{1}{2} \times \frac{1}{2}(\sin 2x) + C$

$\Rightarrow \frac{x}{2} - \frac{\sin 2x}{4} + C$

(10) $\int \frac{dx}{1-\cos x}$

$\because 1-\cos x = 2\sin^2 x/2$

$\Rightarrow \int \frac{dx}{2\sin^2 x/2}$

$\Rightarrow \frac{1}{2}\int \operatorname{cosec}^2 x/2 \, dx$

$\Rightarrow \frac{1}{2} \times 2 \left(-\cot x/2\right) + C$

$\Rightarrow -\cot x/2 + C$

(11) $\int \frac{dx}{1+\sin x}$

$\Rightarrow \int \frac{dx}{\sin^2 x/2 + \cos^2 x/2 + 2\sin x/2 \cos x/2}$

$\Rightarrow \int \frac{dx}{(\sin x/2 + \cos x/2)^2}$

$\Rightarrow \int \frac{dx}{\cos^2 x/2 (1+\tan x/2)^2}$

$\Rightarrow \int \frac{\sec^2 x/2 \, dx}{(1+\tan x/2)^2}$

Let $1+\tan x/2 = t$

$\frac{1}{2}\sec^2 x/2 \, dx = dt$

$\Rightarrow \int \frac{2dt}{t^2}$

$\Rightarrow 2\left[\frac{t^{-1}}{-1}\right] + C$

$\Rightarrow -\frac{2}{t} + C$

$\Rightarrow -\frac{2}{1+\tan x/2} + C$

$\Rightarrow 1 - \frac{2}{1+\tan x/2} - 1 + C$

$\Rightarrow \frac{1+\tan x/2 - 2}{1+\tan x/2} - 1 + C$

$\Rightarrow \frac{\tan x/2 - \tan \pi/4}{1 + \tan \pi/4 \cdot \tan x/2} + C$

$\Rightarrow \tan\left[\frac{x}{2} - \frac{\pi}{4}\right] + C$

(12) $\int (\tan^2 x + \tan^4 x)\, dx$

$\Rightarrow \int \tan^2 x (1 + \tan^2 x)\, dx$

$\Rightarrow \int \tan^2 x \sec^2 x\, dx$

let $\tan x = t$

$\sec^2 x\, dx = dt$

$\Rightarrow \int t^2\, dt$

$\Rightarrow \frac{t^3}{3} + c$

$\Rightarrow \frac{\tan^3 x}{3} + c$

(13) $\int \frac{\cos 2x}{1+\sin x \cos x}$

$\Rightarrow \int \frac{\cos 2x}{1+\frac{\sin 2x}{2}} dx$

Let $1+\frac{\sin 2x}{2} = t$

$\frac{2\cos 2x}{2} dx = dt$

$\Rightarrow \int \frac{1}{t} dt$

$\Rightarrow \ln t + c$

$\Rightarrow \ln\left(1+\frac{\sin 2x}{2}\right) + c$

(14) $\int \cos x \sin 3x \, dx$

$\Rightarrow \frac{1}{2}\int 2\cos x \sin 3x \, dx$

$\Rightarrow \frac{1}{2}\int \sin 4x + \sin 2x$

$\Rightarrow \frac{1}{2}\left[\int \sin 4x \, dx + \int \sin 2x \, dx\right]$

$\Rightarrow \frac{1}{2}\left[-\frac{1}{4}\cos 4x - \frac{1}{2}\cos 2x\right] + C$

$\Rightarrow C - \frac{1}{4}\left[\frac{\cos 4x}{2} + \cos 2x\right]$

(15) $\int \frac{dx}{\cos x}$

$\Rightarrow \int \sec x\,dx \int \frac{dx}{\frac{1-\tan^2 x/2}{1+\tan^2 x/2}}$

$\Rightarrow \int \frac{\sec^2 x/2}{1-\tan^2 x/2}$

let $\tan x/2 = t$

$\frac{1}{2}\sec^2 x/2\,dx = dt$

$\Rightarrow \int \frac{2\,dt}{1-t^2}$

$\Rightarrow \int \left(\frac{1}{1-t} + \frac{1}{1+t}\right) dt$

$\Rightarrow \ln(1+t) - \ln(1-t) + C$

$\Rightarrow \ln \frac{1+t}{1-t} + C$

$\Rightarrow \ln \frac{1+\tan x/2}{1-\tan x/2} + C \Rightarrow \ln \tan(\pi/4 + x/2) + C$

(16) $\int \frac{1-\sin x}{\cos x}\,dx$

$\Rightarrow \int \frac{\sin^2 x/2 + \cos^2 x/2 - 2\sin x/2 \cos x/2}{\cos^2 x/2 - \sin^2 x/2}\,dx$

$\Rightarrow \int \frac{(\sin x/2 - \cos x/2)^2}{(\cos^2 x/2 - \sin^2 x/2)}\,dx$

$\Rightarrow \int \frac{\cos x/2 - \sin x/2}{\cos x/2 + \sin x/2}\,dx$

$\Rightarrow \int \left(\frac{1-\tan x/2}{1+\tan x/2}\right)dx$

$\Rightarrow \int \tan(\pi/4 - x/2)\,dx$

$\because \int \tan x\,dx = \ln \sec x + C$

$\Rightarrow \because$ ~~$-2\ln(\pi/4$~~ $-2\ln\sec(\pi/4 - x/2) + C$

$\Rightarrow \ln \frac{1}{\sec^2(\pi/4 - x/2)} + C$

$\Rightarrow \ln \frac{1}{1+\tan^2(\pi/4 - x/2)} + C$

$\Rightarrow \ln \frac{1}{\frac{1+(1-\tan x/2)^2}{(1+\tan x/2)^2}} + C$

$\Rightarrow \ln \frac{(1+\tan x/2)^2}{2+2\tan^2 x/2} + C$

$\Rightarrow \ln \left[\frac{1}{2}\left(1 + \frac{2\tan x/2}{1+\tan^2 x/2}\right)\right] + C$

$\Rightarrow \ln\left(\frac{1}{2}(1+\sin x)\right) + C$

$\Rightarrow \ln(1+\sin x) + (\ln 1/2 + C) \rightarrow$ Const.

$\Rightarrow \boxed{\ln(1+\sin x) + C}$

(17) $\int \frac{\sin^3 x}{\cos x}\, dx$

$\Rightarrow \int \frac{\sin^2 x}{\cos x} \sin x\, dx$

$\Rightarrow \int \frac{1-\cos^2 x}{\cos x} \sin x\, dx$

Let $\cos x = t$

$-\sin x = dt$

$\Rightarrow -\int \frac{1-t^2}{t}\, dt$

$\Rightarrow \int t - \frac{1}{t}\, dt$

$\Rightarrow \frac{t^2}{2} - \ln t + C$

$\Rightarrow \frac{\cos^2 x}{2} - \ln|\cos x| + C$

(18) $\int \frac{\cos^3 x \, dx}{\sin^4 x}$

$\Rightarrow \int \frac{\cos^2 x}{\sin^4 x} \cos x \, dx$

$\Rightarrow \int \frac{1-\sin^2 x}{\sin^4 x} \cos x \, dx$

$\Rightarrow$ Let $\sin x = t$

$\cos x = dt$

$\Rightarrow \int \frac{1-t^2}{t^4} \, dt$

$\Rightarrow \int \frac{1}{t^4} - \frac{t^2}{t^4} \, dt$

$\Rightarrow \frac{t^{-3}}{-3} - \frac{t^{-1}}{-1} + C$

$\Rightarrow \frac{1}{t} - \frac{1}{3t^3} + C$

$\Rightarrow \frac{1}{\sin x} - \frac{1}{3\sin^3 x} + C$

(19) $\int \frac{\sin^3\alpha}{\sqrt{\cos\alpha}}\,d\alpha$

$\Rightarrow \int \frac{\sin^2\alpha}{\sqrt{\cos\alpha}} \sin\alpha\, d\alpha$

$\Rightarrow \int \frac{1-\cos^2\alpha}{\sqrt{\cos\alpha}} \sin\alpha\, d\alpha$

$\because$ let $\cos\alpha = t$

$-\sin\alpha\, d\alpha = dt$

$\Rightarrow -\int \frac{1-t^2}{\sqrt{t}}\,dt$

$-\int \frac{1}{\sqrt{t}} - t^{3/2}\,dt$

$\Rightarrow -\left[\frac{t^{+1/2}}{+1/2} - \frac{t^{5/2}}{5/2}\right] + C$

$\Rightarrow 2\left[-t^{1/2} + \frac{t^{+5/2}}{5}\right] + C$

$\Rightarrow 2\left[\sqrt{\cos\alpha} + \frac{1}{\sqrt{\cos\alpha}}\right] + C$

$\Rightarrow 2\sqrt{t}\left[\frac{t^2}{5} - 1\right] + C$

$\Rightarrow 2\sqrt{\cos\alpha}\left[\frac{\cos^2\alpha}{5} - 1\right] + C$

(20) $\int \frac{dx}{\cos^4 x}$

$\Rightarrow \int \sec^4 x \, dx$

$\Rightarrow \int \sec^2 x \cdot \sec^2 x \, dx$

$\Rightarrow \int (1+\tan^2 x) \sec^2 x \, dx$

Let $\tan x = t$

$\sec^2 x \, dx = dt$

$\Rightarrow \int (1+t^2) \, dt$

$\Rightarrow \left[t + \frac{t^3}{3} \right] + C$

$\tan x + \frac{\tan^3 x}{3} + C$

(21) $\int \tan^4 x \, dx$

$\Rightarrow \int \tan^2 x \cdot \tan^2 x \, dx$

$\Rightarrow \int (\sec^2 x - 1) \tan^2 x \, dx$

$\Rightarrow \int (\tan^2 x \sec^2 x - \tan^2 x) \, dx$

$\Rightarrow \int \tan^2 x \sec^2 x - \int \sec^2 x \, dx + \int 1 \, dx$

$\Rightarrow \frac{\tan^3 x}{3} - \tan x + x + C$

(22) $\int \sin^5 x \, dx$

$\Rightarrow \int \sin^4 x \cdot \sin x \, dx$

$\Rightarrow \int (1-\cos^2 x)^2 \sin x \, dx$

Let $\cos x = t$

$-\sin x \, dx = dt$

$\Rightarrow -\int (1-t^2)^2 \, dt$

$\Rightarrow -\int (t^4 - 2t^2 + 1) \, dt$

$\Rightarrow -\frac{t^5}{5} + \frac{2t^3}{3} - t + C$

$\Rightarrow -\frac{\cos^5 x}{5} + 2\frac{\cos^3 x}{3} - \cos x + C$

(23) $\int \sin^4 x \, dx$

$\Rightarrow \int \frac{\sin^4 x}{\cos^4 x} \int \sin^2 x (1 - \cos^2 x) \, dx$

$\Rightarrow \int \sin^2 x \, dx - \int \sin^2 x \cos^2 x \, dx$

$\Rightarrow \int \frac{1 - \cos 2x}{2} dx - \frac{1}{4} \int \sin^2 2x \, dx$

$\Rightarrow \int \frac{1 - \cos 2x}{2} dx - \frac{1}{4} \int \frac{1 - \cos 4x}{2} dx$

$\Rightarrow \frac{x}{2} - \frac{1}{4} \sin 2x - \frac{x}{8} + \frac{\sin 4x}{32} + C$

$\Rightarrow \frac{3}{8} x - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$

(24) $\int \tan^3 x \, dx$

$\Rightarrow \int \tan x (\tan^2 x) \, dx$

$\Rightarrow \int \tan x (\sec^2 x - 1) \, dx$

$\Rightarrow \int \tan x \sec^2 x \, dx - \int \tan x \, dx$ $\quad \because \int \tan^2 x = \ln|\sec x| + c$

$\Rightarrow \frac{\tan^2 x}{2} - \ln|\sec x| + c$

$\Rightarrow \frac{\tan^2 x}{2} + \ln \frac{1}{\sec x} + c$

$\Rightarrow \frac{\tan^2 x}{2} + \ln|\cos x| + c$

(25) $\int \frac{dx}{\sin^6 x}$

$\Rightarrow \int \operatorname{cosec}^6 x \, dx$

$\Rightarrow \int (1+\cot^2 x)^2 \operatorname{cosec}^2 x \, dx$

Let $\cot x = t$

$-\operatorname{cosec}^2 x \, dx = dt$

$\Rightarrow -\int (1+t^2)^2 \, dt$

$\Rightarrow -\int (t^4 + 2t^2 + 1) \, dt$

$\Rightarrow -\left[\frac{t^5}{5} + 2\frac{t^3}{3} + t\right] + C$

$\Rightarrow C - \left[\frac{\cot^5 x}{5} + \frac{2\cot^3 x}{3} + \cot x\right]$

(26) $\int (x+1)\sqrt{x^2+2x}\,dx$

$\Rightarrow \int (x+1)\sqrt{(x+1)^2-1}$

Let $(x+1)^2-1 = t^2$

$2(x+1)\,dx = 2t\,dt$

$\Rightarrow \int t \cdot t\,dt$

$\Rightarrow \int t^2\,dt$

$\Rightarrow \frac{t^3}{3} + C$

$\Rightarrow \frac{((x+1)^2-1)^{3/2}}{3} + C$

$\Rightarrow \frac{1}{3}\sqrt{(x^2+2x)^3} + C$

(27) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

let $\sqrt{x} = t$

$\frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \int 2e^t dt$

$\Rightarrow 2e^t + c$

$\Rightarrow 2e^{\sqrt{x}} + c$

(28) $\int \frac{\sqrt{x}\,dx}{1+x^{3/2}}$

let $1+x^{3/2} = t$

$\left(\frac{3}{2}x^{1/2}\right)dx = dt$

$\Rightarrow \frac{2}{3}\int \frac{dt}{t}$

$\Rightarrow 2/3 \ln t + C$

$\Rightarrow 2/3 \ln(1+x^{3/2}) + C$

(29) $I = \int \frac{2x+3}{\sqrt{1+x^2}}\,dx$

$\Rightarrow I = \underbrace{\int \frac{2x}{\sqrt{1+x^2}}\,dx}_{I} + \underbrace{\int \frac{3}{\sqrt{1+x^2}}}_{II}$

$\Rightarrow$ Part-I $\int \frac{2x}{\sqrt{1+x^2}}\,dx$

Let $1+x^2 = t^2$

$2x\,dx = 2t\,dt$

$\Rightarrow \int \frac{2t\,dt}{t}$

$\Rightarrow 2t + c$

$\Rightarrow 2\sqrt{1+x^2} + C_1$

Part II $\int \frac{3}{\sqrt{1+x^2}}$

$\because \int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left[x+\sqrt{x^2+a^2}\right]+C$

$\Rightarrow 3\ln\left(x+\sqrt{x^2+1}\right) + C_2$

$\Rightarrow I = 2\sqrt{1+x^2} + 3\ln\left(x+\sqrt{x^2+1}\right) + C$

(30) $\int \frac{dx}{x\sqrt{3-\ln^2 x}}$

let $\ln x = t$

$\frac{1}{x}dx = dt$

$\Rightarrow \int \frac{dt}{\sqrt{3-t^2}}$

$\Rightarrow \sin^{-1}\frac{t}{\sqrt{3}} + c$

$\Rightarrow \sin^{-1}\frac{\ln x}{\sqrt{3}} + c$

(31) $\int \frac{(\tan^{-1}x)^n}{1+x^2}\,dx$

let $\tan^{-1}x = t$

$\frac{1}{1+x^2}dx = dt$

$\Rightarrow \int t^n\,dt$

$\Rightarrow \frac{t^{n+1}}{n+1} + c$

$\Rightarrow \frac{(\tan^{-1}x)^{n+1}}{n+1} + c$

(32) $\int \frac{d\phi}{\sin^2\phi \cos^2\phi}$

$\Rightarrow \int \frac{d\phi}{\tan^2\phi \cos^4\phi}$

$\Rightarrow \int \frac{\sec^4\phi \, d\phi}{\tan^2\phi}$

$\Rightarrow \int \frac{\sec^2\phi \cdot \sec^2\phi \, d\phi}{\tan^2\phi}$

$\Rightarrow \int \left(\frac{1+\tan^2\phi}{\tan^2\phi}\right) \sec^2\phi \, d\phi$

Let ~~$\sec^2\phi$~~ $\tan\phi = t$

$\sec^2\phi \, d\phi = dt$

$\Rightarrow \int \left(\frac{1}{t^2}+1\right) dt$

$\Rightarrow \left[\frac{t^{-1}}{-1} + t\right] + C$

$\Rightarrow t - \frac{1}{t} + C$

(33) $\int \frac{x^3}{x+1} dx$

$\Rightarrow \int \frac{(x^3+1)-1}{x+1} dx$

$\Rightarrow \int \frac{(x+1)(x^2-x+1)}{x+1} dx - \int \frac{1}{x+1} dx$

$\Rightarrow \int (x^2+1-x)\, dx - \int \frac{1}{x+1} dx$

$\Rightarrow \frac{x^3}{3} - \frac{x^2}{2} + x - \ln(x+1) + C$

(34) $\int \frac{x\,dx}{(x+1)^3}$

$\Rightarrow \int \frac{(x+1)-1}{(x+1)^3}\,dx$

$\Rightarrow \int \frac{x+1}{(x+1)^3}\,dx - \frac{1}{(x+1)^3}\,dx$

$\Rightarrow \int \frac{1}{(x+1)^2}\,dx - \int \frac{1}{(x+1)^3}\,dx$

$\Rightarrow \frac{(x+1)^{-1}}{-1} - \frac{(x+1)^{-2}}{-2} + C$

$\Rightarrow \frac{1}{2(x+1)^2} - \frac{1}{x+1} + C$

$\Rightarrow \cancel{\frac{2x+1}{(x+1)^2} + C}$

(35) $\int \frac{x+2}{x^2+2x+2}\,dx$

$\Rightarrow \int \frac{(x+1)+1}{(x+1)^2+1}\,dx$

$\Rightarrow \underbrace{\int \frac{x+1}{(x+1)^2+1}\,dx}_{I} + \underbrace{\int \frac{1}{(x+1)^2+1}\,dx}_{II}$

$\Rightarrow$ Part-I

$\Rightarrow \int \frac{x+1}{(x+1)^2+1}\,dx$

Let $(x+1)^2+1 = t$

$2(x+1)\,dx = dt$

$\Rightarrow \frac{1}{2}\int \frac{dt}{t}$

$\Rightarrow \frac{1}{2}\ln t + C_1$

$\Rightarrow \frac{1}{2}\ln|(x+1)^2+1| + C_1$

Part-II

$\Rightarrow \int \frac{1}{(x+1)^2+1}\,dx$

$\Rightarrow \tan^{-1}x+1 + C_2$

$I =$ Part I + Part II

$I = \frac{1}{2}\ln|(x+1)^2+1| + \tan^{-1}(x+1) + C$

(36) $\int \frac{(x-3)\,dx}{\sqrt{3-2x-x^2}}$

$\Rightarrow \int \frac{(x-3)\,dx}{\sqrt{4-(x+1)^2}}$

$\Rightarrow \int \frac{(x+1)-4}{\sqrt{4-(x-1)^2}}\,dx$

$\Rightarrow \int \frac{(x+1)}{\sqrt{4-(x+1)^2}}\,dx - \int \frac{4}{\sqrt{4-(x+1)^2}}\,dx$

I_1 I_1

$\Rightarrow I_1 = \int \frac{x+1}{\sqrt{4-(x+1)^2}}$ $\Rightarrow I_2 = \int \frac{4}{\sqrt{4-(x+1)^2}}$

let $4-(x+1)^2 = t^2$

$+2(x+1)\,dx = -2t\,dt$ $I_2 = 4\sin^{-1}\frac{(x+1)}{2} + C_2$

$I_1 = +\int \frac{t\,dt}{t}$

$I_1 = +t + c$

$I_1 = +\sqrt{4-(x+1)^2} + C_1$

$I = I_1 + I_2$

$I = +\sqrt{4-(x+1)^2} + 4\sin^{-1}\frac{(x+1)}{2} + C$

(37) $\int \frac{e^x - 1}{e^x + 1} dx$

$\Rightarrow \int \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}} dx$

let $e^{x/2} + e^{-x/2} = t$

$\frac{1}{2}(e^{x/2} - e^{-x/2})\, dx = dt$

$\Rightarrow \int \frac{2\,dt}{t} dx$

$\Rightarrow 2 \ln t + C$

$\Rightarrow 2 \ln(e^{x/2} + e^{-x/2}) + C$

(38) $\int e^{2x^2+\ln x}\, dx$

$\Rightarrow \int e^{2x^2} \cdot e^{\ln x}\, dx$

$\Rightarrow \int x \cdot e^{2x^2}\, dx$

let $2x^2 = t$

$4x\,dx = dt$

$\Rightarrow \frac{1}{4}\int e^t \cdot dt$

$\Rightarrow \frac{1}{4}(e^t + c)$

$\Rightarrow \frac{1}{4} e^{2x^2} + c$

(39) $\int \frac{\sin 2x\, dx}{4 - \cos^2 2x}$

Let $\cos 2x = t$

$-2\sin 2x = dt$

$\Rightarrow -\frac{1}{2}\int \frac{dt}{4 - t^2}$

$\Rightarrow -\frac{1}{2} \times \frac{1}{2\times 2} \ln\left|\frac{2+t}{2-t}\right| + C$

$\Rightarrow -\frac{1}{8} \ln\left|\frac{2+\cos 2x}{2-\cos 2x}\right| + C$

(40) $I = \int \frac{d\phi}{\sqrt{3}\cos\phi + \sin\phi}$

$I = \int \frac{d\phi}{2\left(\frac{\sqrt{3}}{2}\cos\phi + \frac{1}{2}\sin\phi\right)}$

$I = \int \frac{d\phi}{2\cos(\phi - \pi/6)}$

$I = \frac{1}{2}\int \sec(\phi - \pi/6)$

$I = \frac{1}{2}\ln\left|\tan\left(\frac{\phi}{2} + \pi/6\right)\right| + c$