Q HW A pair of dice is rolled until a total of 5 or 7 is obtained. Find the probability that the total of 5 comes before a total of 7.

Ans → $\frac{2}{5}$.

Sol^n

E: Sum on pair of dice = 5 ; $P(E) = 4/36$.

F: " " " " " = 7 ; $P(F) = 6/36$.

$$P\left(E \text{ or } (\bar{E} \cap \bar{F}) E \text{ or } (\bar{E} \cap \bar{F})^2 E \text{ or } \cdots\cdots\right) = \text{Req. prob.}$$

$$P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$$

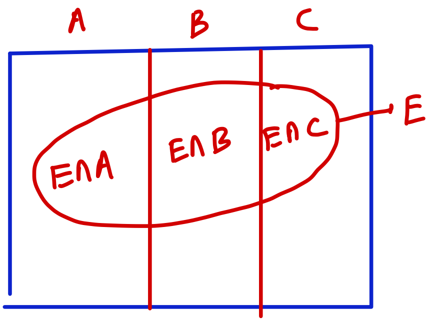
$$= 1 - \left(P(E) + P(F) - \cancel{P(E \cap F)}^{\,0}\right)$$

$$= 1 - \frac{10}{36} = \frac{26}{36}.$$

$$P(E) + P(\bar{E} \cap \bar{F}) P(E) + \left(P(\bar{E} \cap \bar{F})\right)^2 P(E) + \cdots\cdots = \frac{2}{5} \text{ Ans}$$

Q In a test an examine either guesses or copies or knows the answer to a multiple choice question with 4 choices (one or more than one correct). The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it, is 1/8. Find the probability that he knows the answer to the question given that he correctly answered it.

Sol^n



A : He knows $\quad P(A) = 1/2$

B : He Copies $\quad P(B) = 1/6$

C : He guesses. $\quad P(C) = 1/3$

E : He answered correctly

$$P(A/E) = \frac{P(E\cap A)}{P(E\cap A) + P(E\cap B) + P(E\cap C)} = \frac{P(A)\cdot P(E/A)}{P(A)P(E/A) + P(B)P(E/B) + P(C)\cdot P(E/C)}$$

$$= \frac{\frac{1}{2}\times 1}{\frac{1}{2}\times 1 + \frac{1}{6}\times\frac{1}{8} + \frac{1}{3}\times\frac{1}{15}}$$

$$\underbrace{{}^4C_1} + \underbrace{{}^4C_2} + \underbrace{{}^4C_3} + \underbrace{{}^4C_4} = 15 \checkmark$$

Ⓐ

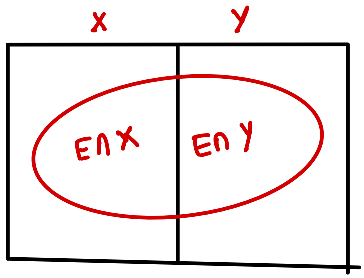
Ⓑ

Ⓒ

Ⓓ

Q 'A' writes a letter to his friend and gives it to his son to post it in a letter box, the reliability of his son being 3/4. The probability that a letter posted will get delivered is 8/9. At a later date 'A' hears from 'B' that the letter has not reached him. Find the probability that the son did not post the letter at all.

Sol^n



X : Son post the letter

Y : Son did not post the letter

$P(X) = \frac{3}{4}$

$P(Y) = \frac{1}{4}$

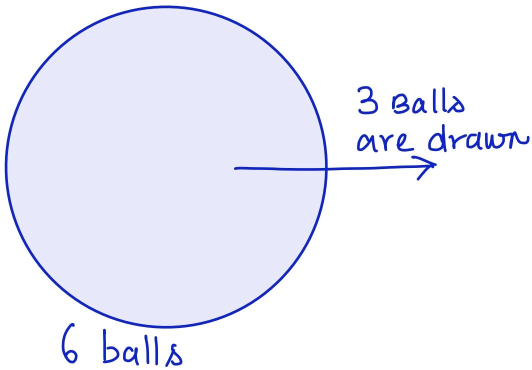
E : Letter did not reach destination

$$P(Y/E) = \frac{P(E \cap Y)}{P(E \cap X) + P(E \cap Y)} = \frac{P(Y) \cdot P(E/Y)}{P(X)\, P(E/X) + P(Y) \cdot P(E/Y)}$$

$$= \frac{\frac{1}{\cancel{4}} \times 1}{\frac{3}{\cancel{4}} \times \frac{1}{9} + \frac{1}{\cancel{4}} \times 1}$$

$$= \frac{1}{\frac{1}{3} + 1} = \frac{3}{4} \text{ Ans}$$

Q A bag contains 6 balls and it is not known of what colours they are. 3 balls are drawn from the bag and found to be all black. Find the probability that no black balls are left in the bag now. (Assume all numbers of black balls in the bag to be equally likely)



Black	non-black	
0	6	$\rightarrow$ 1/7
1	5	$\rightarrow$ 1/7
2	4	$\rightarrow$ 1/7
3	3	$\rightarrow$ "
4	2	$\rightarrow$ "
5	1	$\rightarrow$ "
6	0	$\rightarrow$ "

$$\text{Req. prob} = \frac{\frac{1}{7} \times \left(\frac{{}^3C_3}{{}^6C_3}\right)}{\frac{1}{7} \times 0 + \frac{1}{7} \times 0 + \frac{1}{7} \times 0 + \frac{1}{7}\left(\frac{{}^3C_3}{{}^6C_3}\right) + \frac{1}{7}\left(\frac{{}^4C_3}{{}^6C_3}\right) + \frac{1}{7}\left(\frac{{}^5C_3}{{}^6C_3}\right) + \frac{1}{7} \times \left(\frac{{}^6C_3}{{}^6C_3}\right)}$$

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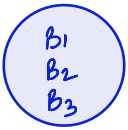
EXTENDED BAYES :

Here, with the knowledge of present, first we predict the past and then the future.

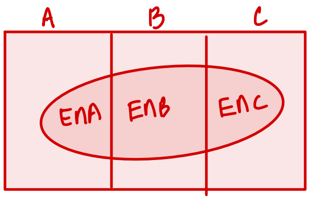
Q A bag contains 3 biased coins B_1, B_2 and B_3 whose probabilities of falling head wise are 1/3, 2/3 and 3/4 respectively. A coin is drawn randomly and tossed, fell head wise. Find the probabilities that the same coin when tossed again will fall head wise.



A : Coin B_1 is selected $\rightarrow P(A) = \frac{1}{3}$

B : " B_2 " " $P(B) = \frac{1}{3}$

C : " B_3 " " $P(C) = \frac{1}{3}$



$$P(A/E) = \frac{P(E \cap A)}{\Sigma P(E \cap A)} = \frac{P(A) \cdot P(E/A)}{\Sigma P(A) \cdot P(E/A)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{3}}{\underbrace{\frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{3}{4}}_{K}}$$

$$P(B/E) = \frac{\frac{1}{3} \times \frac{2}{3}}{K}$$

$$P(C/E) = \frac{\frac{1}{3} \times \frac{3}{4}}{K}$$

$$\text{Req. prob} = P(A/E) \times \frac{1}{3} + P(B/E) \times \frac{2}{3} + P(C/E) \times \frac{3}{4}$$

$$= \frac{23}{36} \quad \text{Ans}$$

Q HW A bag contains 5 balls of unknown colours. A ball is drawn twice with replacement from the bag found to be red on both the occasions. The contents of the bag were replenished. If now two ball are drawn simultaneously from the bag, find the probabilities that they will be both red. Assume all number of red balls in the bag to be equally likely.

BINOMIAL PROBABILITY DISTRIBUTION : (BPD)

Suppose that we have an experiment such as tossing a coin or die repeatedly or choosing a marble from an urn repeatedly. Each toss or selection is called a trial. In any single trial there will be a probability associated with a particular event such as head on the coin, 4 on the die, or selection of a red marble. In some cases this probability will not change from one trial to the next (as in tossing a coin or die). Such trials are then said to be independent and are often called Bernoulli trials after James Bernoulli who investigated them at the end of the seventeenth century.

Definition : Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

(i) There should be a finite number of trials.

(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes : success or failure.

(iv) The probability of success remains the same in each trial.

For Independent Trials

Experiment

↓

n independent trials (Bernoulli's trials)

↓

If each of these trails have two possible outcomes

Success	$p + q = 1$	Failure
$P(S) = p$		$P(F) = q$ $q = 1-p$

Rem

$\boxed{\text{Then, } P(r \text{ successes}) = {}^nC_r\, p^r q^{n-r}}$

Proof : Consider the compound event where 'r' successes are in succession and (n – r) failures are in succession.

$$P\left(\underbrace{SSS....S}_{r}\underbrace{FFF.....F}_{(n-r)}\right) = \underbrace{P(S).P(S)....P(S)}_{r \text{ times}}\underbrace{P(F).P(F).....P(F)}_{(n-r) \text{ times}} = p^r.q^{n-r}$$

But these r successes and (n – r) failures can be arranged in $\dfrac{n!}{r!(n-r)!} = {}^nC_r$ ways and in each arrangement the probability will be $p^r \,.\, q^{n-r}$

Hence total pr. $= P(r) = {}^nC_r p^r q^{n-r}$ .... (i)

$\boxed{\text{Recurrence relation}}$

$$P(r+1) = {}^nC_{r+1}p^{r+1} \,.\, q^{n-r-1}$$

$$\therefore \quad \frac{P(r+1)}{P(r)} = \frac{{}^nC_{r+1}}{{}^nC_r}\frac{p}{q} = \frac{n-r}{r+1}\frac{p}{1-p}$$

$$\therefore \quad \boxed{P(r+1) = \frac{n-r}{r+1}.\frac{p}{1-p}P(r)} \quad ...... \text{(ii)}$$

equation (ii) is used for completely the probabilities of P(1) ; P(2) ; P(3); .... etc. once P(0) is determined.

$p \rightarrow$ prob of success

$q \rightarrow$ " " failure

$p+q = 1$

$$(\underbrace{p+q}_{1})^n = {}^nC_0\, p^n \cdot q^0 + {}^nC_1\, p^{n-1} q^1 + {}^nC_2\, p^{n-2} q^2 + \cdots + {}^nC_r\, p^{n-r} q^r + \cdots + {}^nC_n\, p^0 q^n$$

n- success
0- failure

$P(x=n)$

$n-1$ success
1 failure

$P(x=n-1)$

$n-r$ success
r failure

0 success
n failures

$P(x=0)$

Q A pair of dice is thrown 6 times, getting a doublet is considered a success. Compute the probability of
(i) no success (ii) exactly one success (iii) at least one success (iv) at most one success

Sol^n

$n = 6$

Success : $P(S) = \frac{6}{36} = \frac{1}{6}$

failure $P(F) = \frac{5}{6}$

(i) ${}^6C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^6 = \left(\frac{5}{6}\right)^6$ (ii) ${}^6C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^5$

(iii) $P(S=1) + P(S=2) + \cdots + P(S=6) = 1 - P(S=0)$

$= 1 - \left(\frac{5}{6}\right)^6$

(iv) $P(S=0) + P(S=1) \Rightarrow$ (i) + (ii)

Q A coin is twice as likely to land heads as tails. In a sequence of independent tosses, find the probability that the third head occurs on the fifth toss.

Sol^n

$P(H) = 2x$ $x + 2x = 1 \Rightarrow x = \frac{1}{3}$

$P(T) = x$

$P(H) = \frac{2}{3}$; $P(T) = \frac{1}{3}$

$n = 5$

$\underbrace{\_\ \_\ \_\ \_}_{2H + 2T}$ $\boxed{\text{Head}}$ 3^{rd} time

$\left(\underbrace{{}^4C_2 \left(\frac{2}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^2}_{H,\, H,\, T,\, T} \right) \times \underbrace{\frac{2}{3}}_{H}$

Q A drunkard takes a step forward or backward. The probability that he takes a step forward is 0.4. Find the probability that at the end of 11 steps he is one step away from the starting point.

Solⁿ

Success = step fwd = 0.4 $n = 11$

Failure = " backward = 0.6

(*) — Starting point

6 step fwd
5 " backward

OR

— (*) Starting point

5 Step Fwd
6 step backward

$$^{11}C_6 (0.4)^6 (0.6)^5 + {}^{11}C_5 (0.4)^5 (0.6)^6$$

Comprehension : [Q.1 & Q.2]

Research has shown that studying improves a student's chances to 80% of selecting the correct answer to a multiple choice question. A multiple choice test has 15 questions. Each question has 4 choices and exactly one is correct.

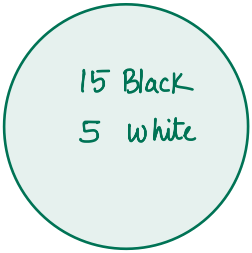
(1) Probability that the student does exactly 7 or 8 correct answer when he studies and attempts all the questions.

$$ {}^{15}C_7 \left(\frac{4}{5}\right)^7 \left(\frac{1}{5}\right)^8 + {}^{15}C_8 \left(\frac{4}{5}\right)^8 \left(\frac{1}{5}\right)^7 . $$

(2) Suppose that a student does not study for the test but randomly guesses the answers. The probability that the student will answer 7 or 8 questions correctly when he attempts all, is

$$ {}^{15}C_7 \left(\frac{1}{4}\right)^7 \left(\frac{3}{4}\right)^8 + {}^{15}C_8 \left(\frac{1}{4}\right)^8 \left(\frac{3}{4}\right)^7 . $$

Q. An urn contains 15 black balls and 5 white balls. 10 balls are drawn one by one with replacement. Then find the probability of getting exactly 5 black balls.



Success: getting white ball $P(S) = \frac{5}{20} = \frac{1}{4}$

Failure " Black ball $P(F) = \frac{3}{4}$

$${}^{10}C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^5. \checkmark$$

★

Q. Find the minimums number of times a fair coin is tossed such that probability of getting at least 2 heads is greater than 0.5 ?

Sol^n n - times tossed

$$P(2H) + P(3H) + P(4H) + \cdots + P(nH) > 0.5$$

$$1 - P(0H) - P(1H) > 0.5$$

$$1 - \left({}^nC_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n + {}^nC_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{n-1} \right) > 0.5$$

$$\frac{1}{2} > \frac{1}{2^n} + \frac{n}{2^n} \Rightarrow \boxed{\frac{n+1}{2^n} < \frac{1}{2}}$$

$n=1$ $\quad \frac{1+1}{2^1} < \frac{1}{2}$ ✗

$n=2$ $\quad \frac{3}{2^2} < \frac{1}{2}$ ✗

$n=3$ $\quad \frac{4}{8} < \frac{1}{2}$ ✗

$n=4$ $\quad \frac{5}{16} < \frac{1}{2}$ ✓

$\therefore$ minimum $= 4$ ✓

PROBABILITY THROUGH STATISTICAL (STOCHASTIC) TREE DIAGRAM :

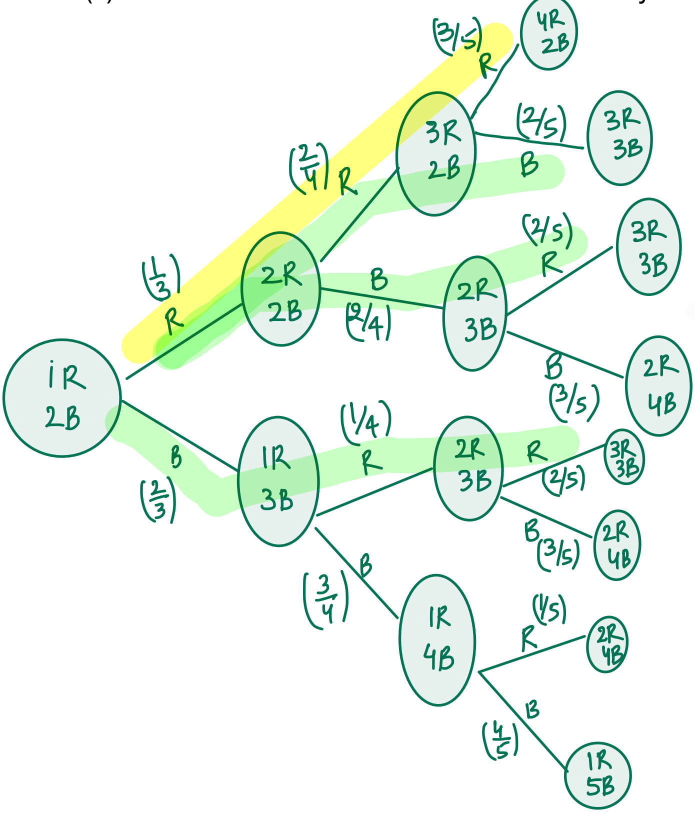
These tree diagrams are generally drawn by economist and give a simple approach to solve a problem.

E(1) A bag initially contains 1 red ball and 2 blue balls. A trial consists of selecting a ball at random, noting its colour and replacing it together with an additional ball of the same colour. Given that three trials are made, draw a tree diagram illustrating the various probabilities. Hence, or otherwise, find the probability that :

(a) atleast one blue ball is drawn

(b) exactly one blue ball is drawn

(c) red and blue ball are drawn alternately.



(a) $1 - P(RRR)$

$1 - \left(\frac{1}{3}\right)\left(\frac{2}{4}\right)\left(\frac{3}{5}\right)$

(b) $P(RRB) + P(RBR) + P(BRR)$

$\left(\frac{1}{3}\cdot\frac{2}{4}\cdot\frac{2}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{2}{4}\right)\left(\frac{2}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{4}\right)\left(\frac{2}{5}\right)$

(c) $P(RBR) + P(BRB)$

$\left(\frac{1}{3}\times\frac{2}{4}\times\frac{2}{5}\right) + \left(\frac{2}{3}\times\frac{1}{4}\times\frac{3}{5}\right)$

E(2) Box A contains nine cards numbered 1 through 9 and box B contains five cards numbered I through 5. A box is chosen at random and a card drawn; if the card shows an even number, another card is drawn from the same box. If the card shows an odd number, a card is drawn from the other box ;

HW

(a) What is the probability that both cards show even numbers ?

(b) What is the probability that both cards show odd numbers ?

HW

O-1 Part #6.

S-1 Q17 to 20.

JM Q1 to 5.