Exercise 01

Part-I

1. 6 married couples are standing in a room. If 4 people are choosen at random, then the chance that exactly one married couple is among the 4 is -

Soln:- ⇒ No. of way of selecting one married couple out of 6 is -

$^6C_1$

⇒ No. of ways of selecting remaining 2 people out of 5 couples

$^5C_2 \times {^2C_1} \times {^2C_1}$

⇒ So no. of ways of selecting 4 people such that exactly one married couple is among them. is ⇒ $^6C_1 \times {^5C_2} \times {^2C_1} \times {^2C_1}$

⇒ ~~Pr~~ Required probability $= \dfrac{^6C_1 \times {^5C_2} \times {^2C_1} \times {^2C_1}}{^{12}C_4}$

$= \dfrac{16}{33}$

3. The probability that a positive two digit number selected at random has its tens digit atleast three more than its unit digit is.

Soln.

	Tens digit	Unit digit	no. of ways
(1)	3	0	1
(2)	4	0, 1	2
(3)	5	0, 1, 2	3
(4)	6	0, 1, 2, 3	4
(5)	7	0, 1, 2, 3, 4	5
(6)	8	0, 1, 2, 3, 4, 5	6
(7)	9	0, 1, 2, 3, 4, 5, 6	7

Total No. of ways
$= 1+2+3+4+5+6+7$
$= 28$

Required probability $= \frac{28}{90} = \frac{14}{45}$

3. A 5 digit number is formed by using the digits 0, 1, 2, 3, 4, 5 without repetition. The probability that the number is divisible by 6 is -

Solⁿ:- For divisible by 6 the ~~ntta~~ last digit of the no. should be even and sum of the digit should be divisible by 3.

Sum of the given digits $= 0+1+2+3+4+5 = 15$

Case-1 :- When sum is 15 (1, 2, 3, 4, 5)

4	3	2	1	2

↳ Should be even (2, 4)

$= 4 \times 3 \times 2 \times 1 \times 2 \Rightarrow 48$

Case-2 :- When sum is 12 (0, 1, 2, 4, 5)

3	3	2	1	2

↳ should be even (2, 4)

+

4	3	2	1	1

↳ (0)

$= 3 \times 3 \times 2 \times 1 \times 2 + 4 \times 3 \times 2 \times 1 = 36 + 24 = 60$

Case-3 :- When Sum is 9 = Not possible

So Total no. of ways of forming a 5 digit no. divisible by 6 is -

$48 + 60 = 108$

Total no. of ways of forming a 5 digit no =

$5 \times 5 \times 4 \times 3 \times 2 = 600$

Required probability $= \frac{108}{600} = \frac{9}{50} = 18\%$

4 A card is drawn at random from a well shuffled deck of cards. find the probability that the card is a -

Soln.- (i) King or a red card

No. of ways of selecting King or Red card = 26 + 2

Red card (includ 2 Kings) — 2 other King

Required probability = $\frac{28}{52} = \frac{7}{13}$

(ii) Club or a diamond

No. of ways of selecting club or diamond = 13 + 13 = 26

Required Probability = $\frac{26}{52} = \frac{1}{2}$

(iii) King or a queen.

No. of ways of selecting King or queen = 4 + 4 = 8

Required probability = $\frac{8}{52} = \frac{2}{13}$

(iv) King or an Ace

No. of ways of selecting King or an Ace = 4 + 4 = 8

Required probability = $\frac{8}{52} = \frac{2}{13}$

(V) Spade or a Club

No. of ways of selecting spade or a club $= 13+13 = 26$

Required probability $= \frac{26}{52} = \frac{1}{2}$

(VI) Neither a heart nor a king

No. of ways of selecting a heart or a king $= 13+3 = 16$

Required probabil. $= 1 - \frac{16}{52}$

$= 1 - \frac{4}{13}$

$= \frac{9}{13}$

5 A Bag contain 5 white, 7 Black & 4 red balls. Find the chance that three balls drawn at random are all white.

Soln:-

No. of ways of selecting white balls = $5c_3$

No. of ways of selecting 3 balls out of 16 balls is

$= 16c_3$

Required probability $= \frac{5c_3}{16c_3}$

$= \frac{1}{56}$

6. If four coins are tossed. Two events A and B are defined as -

A :- No two consecutive heads occur

B :- At least two consecutive heads occur

Find P(A) and P(B). State whether the ~~e~~ events are equally likely, mutually exclusive and exhaustive.

Soln. A :- No two consecutive heads occur.

Case-1 :- When all are Tails (T T T T)

No. of ways = 1

Case-2 :- When 3 Tails & 1 Head

No. of ways $= \frac{4!}{3!} = 4$

Case-3 :- When 2 T, & 2 H

No. of ways $= 1 \times {}^3C_2 \times 1 = 3$

Total no. of ways $= 1+4+3 = 8$

Required Probability $P(A) = \frac{8}{2^4} = \frac{1}{2}$

B :- P(B) = 1 - No two consecutive heads occurs.

$= 1 - P(A)$

$P(B) = 1 - \frac{1}{2} = \frac{1}{2}$

⇒ ∵ $P(A) = P(B)$ So equally likely

⇒ $P(A \cap B) = 0$ So mutually ~~Ex~~ Exclusive

$P(A) + P(B) = 1$ So Exhaustive

7. Thirteen persons take their places at a round table. Find the odds against two particular persons sitting together.

Sol^n. No. of ways of sitting 2 particular person together is —

$11! \times 2!$

~~No of~~ So Total no. of favourable cases $= 11!\,2!$

Total no. of unfavourable cases $= 12! - 11!\,2!$

Odds in Against $= \dfrac{\text{Unfavourable}}{\text{favourable}}$

$= \dfrac{12! - 11!\,2!}{11!\,2!}$

$= \dfrac{5}{1}$

8. Mr. A forgot to write down a very important phone number. All he remembers is that it started with 713 and that the next set of 4 digit involved are 1, 7 and 9 with one of these numbers appearing twice. He gusses a phone number and dials randomly. The odds in favour of dialing the correct telephone number is -

Soln:- No. of ways of arranging last 4 digit of phone no. is $\Rightarrow \left({}^{3}C_{1} \times \frac{4!}{2!}\right) = {}^{3}C_{1} \times 12 = 36$

For selecting the Repeated no.

for arranging them

odd in favour = $\frac{\text{favaourable}}{\text{Un favourable}}$

$= \frac{1}{36-1}$

$= \frac{1}{35}$

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Q.(a) A fair die is tossed. If the number is odd, find the probability that it is prime.

Solⁿ:- Total no. of ways that the no. is odd = 3 (1, 3, 5)

∴ no. of ways that the no is prime & odd = 2 (3, 5)

Required probability = $\frac{2}{3}$

9. (b) Three fair coins are ~~A fair die is~~ tossed. If both heads and tails appear, determine the probability that exactly one head appears.

Soln. Total no. of cases when both head and tails appear

⇒ Total − when only Head or tail appear

$= 2^3 - 2$

$= 6$

Total no. of cases when exactly one head appear

$= {}^3C_1 = 3$

Required probability $= \frac{3}{6} = \frac{1}{2}$

10 Mr. A lives at origin on the cartesian plane and has his office at (4,5). His friend lives at (2,3) on the same plane. Mr A can go to his office travelling one block at a time either in the +y or +x direction. If all possible paths are equally likely then the probability the Mr. A passed is friend house is -

Solⁿ:-

Mr A ⟶ Friend's House ⟶ Office

(0,0) (2,3) (4,5)

No. of ways of reaching office by passing friends house

$$= \frac{5!}{2!3!} \times \frac{4!}{2!2!}$$

Total no. of ways of reaching office

$$= \frac{9!}{4!5!}$$

Required probability $= \dfrac{\frac{5!}{2!3!} \times \frac{4!}{2!2!}}{\frac{9!}{4!5!}}$

$$= \frac{10 \times 6}{\frac{9 \times 8 \times 7 \times 6}{24}}$$

$$= \frac{10}{21}$$

PART # 2

1. In throwing 3 dice, the probability that atleast 2 of the three numbers obtained are same is -

(A) 1/2 (B) 1/3 (C) 4/9 (D) none

At least 2 of the 3 numbers are same

$\equiv$ Exactly 2 numbers are same

+ Exactly 3 numbers are same.

$A \equiv$ Event that shows 2 numbers are same

$$P(A) = \frac{{}^3C_2 \times 6 \times 5}{6^3} = \frac{15}{36}$$

$B \equiv$ Event that shows 3 numbers are same

$$P(B) = \frac{6}{6^3} = \frac{1}{36}$$

$$P(A \cup B) = P(A) + P(B) = \frac{4}{9}$$

= Required probability

2. From a pack of 52 playing cards, face cards and tens are removed and kept aside then a card is drawn at random from the remaining cards. If

A: The event that the card drawn is an ace

H: The event that the card drawn is a heart

S : The event that the card drawn is a spade

then which of the following holds ?

(A) $9P(A) = 4P(H)$ (B) $P(S) = 4P(A \cap H)$

(C) $3P(H) = 4P(A \cup S)$ (D) $P(H) = 12P(A \cap S)$

No. of cards removed = 16

No. of available cards = 36

No. of elements in Sample-Space = 36

$$P(A) = \frac{4}{36} = \frac{1}{9}$$

$$P(H) = \frac{9}{36} = \frac{1}{4}$$

$$\Rightarrow \boxed{4P(H) = 1 = 9P(A)}$$

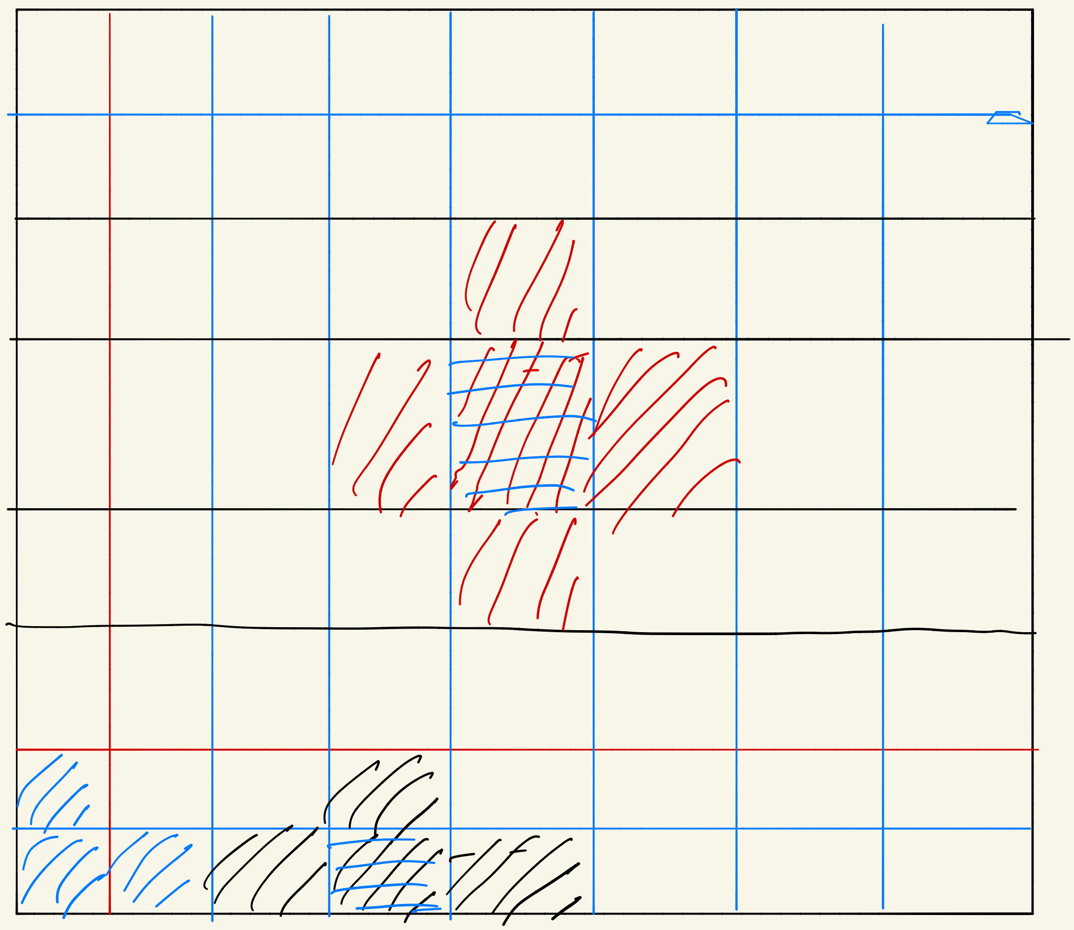
$$P(S) = \frac{9}{36} = \frac{1}{4}$$

$$P(A \cap H) = \frac{1}{36}$$

$$P(A \cap S) = \frac{1}{36}$$

3. If two of the 64 squares are chosen at random on a chess board, the probability that they have a side in common is -

(A) 1/9 (B) 1/18 (C) 2/7 (D) none



No. of elements in sample space

$$= {}^{64}C_2$$

No. of ways of choosing 2 squares having a common side

$$= \frac{2\times4 + 6\times3\times4 + 36\times4}{2} = \frac{224}{2} = 112$$

Required probability $= \dfrac{112\times2}{64\times63} = \dfrac{1}{18}$

4 Two red counters, three green counters and 4 blue counters are placed in a row in random order. The probability that no two blue counters are adjacent is -

(A) $\frac{7}{99}$ (B) $\frac{7}{198}$ (C) $\frac{5}{42}$ (D) none

No. of ways of placing 9 counters

$= 9!$

No. of ways of placing these 9 counters in such a way no 2 blue counters are adjacent

$= 5! \times {}^6C_4 \times 4!$

Required probability

$$= \frac{\cancel{5!} \times \frac{6\times 5}{\cancel{2}} \times \cancel{4!}}{\cancel{9!}} \quad \frac{}{\frac{\cancel{9}\times 8\times 7\times 6}{3}} = \frac{5}{42}$$

5 South African cricket captain lost the toss of a coin 13 times out of 14. The chance of this happening was

(A) $\frac{7}{2^{13}}$ (B) $\frac{1}{2^{13}}$ (C) $\frac{13}{2^{14}}$ (D) $\frac{13}{2^{13}}$

Total no. of trials = 14

probability of winning toss in a single trial = $\frac{1}{2}$

Trials are independent

probability of getting exactly one win

$= {}^{14}C_{13} \times \left(\frac{1}{2}\right)^{13} \times \left(\frac{1}{2}\right)$

$= 14 \times \frac{1}{2^{13}} \times \frac{1}{2} = \frac{7}{2^{13}}$

6 A coin is tossed and a die is thrown. Find the probability that the outcome will be a head or a number greater than 4.

A = Event that outcome is head

B = Event that number is greater than 4

$P(A) = \frac{1}{2}$, $P(B) = \frac{2}{6}$

Here Events A and B are independent

$\Rightarrow P(A \cap B) = P(A)\,P(B) = \frac{1}{6}$

Required probability

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{3+2-1}{6}$

$= \frac{4}{6} = \frac{2}{3}$

~~7~~. A coin is biased so that heads is three times as likely to appear as tails. Find P(H) and P(T). If such a coin is tossed twice find the probability that head occurs at least once.

$P(H) = 3P(T)$

Also we have $P(H) + P(T) = 1$

$\Rightarrow 4P(T) = 1 \Rightarrow P(T) = \frac{1}{4}$

$\Rightarrow P(H) = \frac{3}{4}, \quad P(T) = \frac{1}{4}$

Required probability

$= 1 - P(\text{no heads}) = 1 - \frac{1}{4} \times \frac{1}{4}$

$= \frac{15}{16}$

8. Given two independent events A, B such that P(A) = 0.3, P(B) = 0.6. Determine

(i) P(A and B) (ii) P(A and not B) (iii) P(not A and B) (iv) P(neither A nor B)

(v) P(A or B)

For independent events A & B

$$P(A \cap B) = P(A)\,P(B)$$
$$= .3 \times .6 = .18$$

$$P(A \cap B') = P(A)\,P(B')$$
$$= P(A)(1 - P(B))$$
$$= .3 \times .4 = .12$$

$$P(A' \cap B) = P(A')\,P(B) = (1 - P(A))\,P(B)$$
$$= .7 \times .6 = .42$$

$$P(A' \cap B') = P(A')\,P(B') = (1 - P(A))(1 - P(B))$$
$$= .7 \times .4 = .28$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= .3 + .6 - .18 = \begin{array}{r} .90 \\ -\,.18 \\ \hline .72 \end{array}$$

9. In a single throw of three dice, determine the probability of getting

(i) a total of 5 (ii) a total of atmost 5 (iii) a total of at least 5.

Total of 5 $\equiv$ (1, 1, 3) OR (1, 2, 2)

No. of ways of getting a total of 5 = 3 + 3 = 6

Required probability $= \frac{6}{6 \times 6 \times 6}$

$= \frac{1}{36}$

Total of at most 5 $\equiv$ (1, 1, 3) OR (1, 2, 2) OR (1, 1, 2) OR (1, 1, 1)

No. of ways = 3 + 3 + 3 + 1 = 10

Required probability $= \frac{10}{6 \times 6 \times 6}$

$= \frac{5}{108}$

No. of ways of getting a total of at least 5 = 216 − 1 − 3 = 212

Required probability $= \frac{212}{216} = \frac{53}{54}$

10 3 students A, B and C are in a swimming race. A and B have the same probability of winning and each is twice as likely to win as C. Find the probability that B or C wins. Assume no two reach the winning point simultaneously.

$$P(A) = P(B) = 2P(C)$$

$$P(A) + P(B) + P(C) = 1$$

$$\Rightarrow \quad 5P(C) = 1 \Rightarrow P(C) = \frac{1}{5}$$

$$P(A) = \frac{2}{5}, \; P(B) = \frac{2}{5}$$

Here A, B & C are mutually exclusive and exhaustive events.

$$P(B \cup C) = P(B) + P(C) = \frac{3}{5}$$

11. 5 different marbles are placed in 5 different boxes randomly. Find the probability that exactly two boxes remain empty. Given each box can hold any number of marbles.

No. of ways in which marbles can be placed $= 5^5$

When exactly 2 boxes are empty No. of ways

$$= {}^5C_2 \times \left(3^5 - {}^3C_1 \times 2^5 + {}^3C_2\, 1^5\right)$$

$$= 10 \times (243 - 96 + 3)$$

$$= 10 \times 150 = 1500$$

Required probability

$$= \frac{\cancel{1500}\ 12}{\cancel{5\times5\times5}\times5\times5} = \frac{12}{25}$$

12. Let A and B be events such that $P(\bar{A}) = 4/5$, $P(B) = 1/3$, $P(A/B) = 1/6$, then :

(a) $P(A \cap B)$

(b) $P(A \cup B)$

(c) $P(B/A)$

(d) Are A and B independent?

$$P(A) = 1 - P(\bar{A}) = 1 - \frac{4}{5} = \frac{1}{5}$$

$$P(A \cap B) = P(B)\, P\left(\frac{A}{B}\right)$$

$$= \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{5} + \frac{1}{3} - \frac{1}{18} = \frac{18 + 30 - 5}{90}$$

$$= \frac{43}{90}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{18}}{\frac{1}{5}} = \frac{5}{18}$$

$$P(A) \times P(B) = \frac{1}{5} \times \frac{1}{3} \neq \frac{1}{18} = P(A \cap B)$$

$\Rightarrow$ A & B are not independent

PART # 3

1. Let A & B be two events. Suppose $P(A) = 0.4$, $P(B) = p$ & $P(A \cup B) = 0.7$. The value of p for which A & B are independent is :

(A) 1/3 (B) 1/4 (C) 1/2 (D) 1/5

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow 0.7 = 0.4 + p - P(A \cap B)$$

For Independent events A & B

$$P(A \cap B) = P(A)\,P(B)$$

$$\Rightarrow .7 = .4 + p - .4 \times p$$

$$\Rightarrow .3 = .6p \Rightarrow p = \frac{1}{2}$$

2. A pair of numbers is picked up randomly (without replacement) from the set $\{1, 2, 3, 5, 7, 11, 12, 13, 17, 19\}$. The probability that the number 11 was picked given that the sum of the numbers was even, is nearly :

(A) 0.1 (B) 0.125 (C) 0.24 (D) 0.18

Given that sum of the chosen numbers are even

No. of ways of choosing a pair of even numbers

$= {}^{8}C_{2} + 1 = 29$

If one number is 11, then other must be any one of $\{1, 3, 5, 7, 13, 17, 19\}$

No. of ways $= 7$

Required probability $= \frac{7}{29}$

$\simeq .24$

3. For a biased die the probabilities for the different faces to turn up are given below :

Faces :	1	2	3	4	5	6
Probabilities :	0.10	0.32	0.21	0.15	0.05	0.17

The die is tossed & you are told that either face one or face two has turned up. Then the probability that it is face one is :

(A) 1/6 (B) 1/10 (C) 5/49 (D) 5/21

$E_1 \equiv$ Event that 1 turned up

$E_2 \equiv$ Event that 2 turned up

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

Here E_1 & E_2 are mutually exclusive events

$$\Rightarrow P(E_1 \cap E_2) = 0$$

$$P(E_1 \cup E_2) = .10 + .32 = .42$$

$$P\left(\frac{E_1}{E_1 \cup E_2}\right) = \frac{.10}{.42} = \frac{5}{21}$$

4. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the determinant chosen has the value non negative is :

(A) 3/16 (B) 6/16 (C) 10/16 (D) 13/16

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc < 0$$

$\Rightarrow ad < bc$

$\Rightarrow a = 0, d = 0$ | $b = 1, c = 1$

$d = 1$

$a = 1, d = 0$

No. of determinants whose determinant value is $-ve$ is 3

probability that the determinant chosen has the value non-negative

$$= 1 - \frac{3}{16} = \frac{13}{16}$$

Total no. of determinants $= 2^4 = 16$

5 A license plate is 3 capital letters (of English alphabets) followed by 3 digits. If all possible license plates are equally likely, the probability that a plate has either a letter palindrome or a digit palindrome (or both), is

(A) $\frac{7}{52}$ (B) $\frac{9}{65}$ (C) $\frac{8}{65}$ (D) none

$$\boxed{L_1 L_2 L_3 D_1 D_2 D_3}$$

$L_1, L_2, L_3 \in \{A, B, \ldots Z\}$

$D_1, D_2, D_3 \in \{0, 1, 2, -- 9\}$

$n(S) = 26 \times 26 \times 26 \times 10 \times 10 \times 10$

A = Event that plate has a letter palindrome

B = Event that plate has a digit palindrome

Required probability = $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

Now $P(A) = \dfrac{26 \times 10^3 + {}^{26}C_2 \times 2 \times 10^3}{n(S)}$

$= \dfrac{26 \times 10^3 (1 + 25)}{n(S)} = \dfrac{1}{26}$

$$P(B) = \frac{10\times 26^3 + {}^{10}C_2 \times 2 \times 26^3}{n(S)}$$

$$= \frac{(26)^3 \times 10(1+9)}{n(S)}$$

$$= \frac{1}{10}$$

$$P(A\cap B) = \frac{(26 + {}^{26}C_2\times 2)(10 + {}^{10}C_2 \times 2)}{n(S)}$$

$$= \frac{(26+26\times 25)(10+10\times 9)}{n(S)}$$

$$= \frac{(26)^2 \times (10)^2}{(26)^3 \times (10)^3} = \frac{1}{260}$$

Required probability

$$= \frac{1}{26} + \frac{1}{10} - \frac{1}{260} = \frac{10+26-1}{260}$$

$$= \frac{\cancel{35}\,7}{\cancel{260}\,52} = \frac{7}{52}$$

6. A committee of three persons is to be randomly selected from a group of three men and two women and the chair person will be randomly selected from the committee. The probability that the committee will have exactly two women and one man, and that the chair person will be a woman, is/are

(A) 1/5 (B) 8/15 (C) 2/3 (D) 3/10

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$M_1 M_2 M_3 + W_1 W_2$

P(Exactly 2 Women + One man)

$$= \frac{{}^2C_2 \times {}^3C_1}{{}^5C_3} = \frac{3}{10}$$

Required probability

$$= \frac{3}{10} \times \frac{2}{3} = \frac{1}{5}$$

7. The probability that an automobile will be stolen and found within one week is 0.0006. The probability that an automobile will be stolen is 0.0015. The probability that a stolen automobile will be found in one week is

(A) 0.3 (B) 0.4 (C) 0.5 (D) 0.6

$E_1 \equiv$ Automobile will be Stolen

$E_2 \equiv$ Stolen automobile will be found.

$P(E_1 \cap E_2) = 0.0006$

$P(E_1) = 0.0015$

$$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_2 \cap E_1)}{P(E_1)}$$

$$= \frac{0.0006}{0.0015} = \frac{6}{15}$$

$$= 0.40$$

8. A box contains 100 tickets numbered 1, 2, 3,.... ,100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5, with probability

(A) $\frac{1}{9}$ (B) $\frac{2}{11}$ (C) $\frac{3}{19}$ (D) none

As per the given condition 2 tickets are chosen from $\{1, 2, 3, \ldots 10\}$

This set will be effective sample space

Required probability

$$= \frac{{}^5C_1}{{}^{10}C_2}$$

$$= \frac{5 \times 2}{10 \times 9} = \frac{1}{9}$$

9. Two boys A and B find the jumble of n ropes lying on the floor. Each takes hold of one loose end randomly. If the probability that they are both holding the same rope is $\frac{1}{101}$ then the number of ropes is equal to

(A) 101 (B) 100 (C) 51 (D) 50

There are $(2n)$ free ends lying on the floor.

probability of holding the same rope

$$= 1 \times \frac{1}{2n-1}$$

$$\Rightarrow \frac{1}{101} = \frac{1}{2n-1}$$

$$\Rightarrow 2n-1 = 101$$

$$\Rightarrow 2n = 102$$

$$\Rightarrow n = 51$$

For C. probability of getting Even score in a single trial

$$= \frac{3^5 + {}^5C_1 \times 3^5 + {}^5C_3 \times 3^5}{6 \times 6 \times 6 \times 6 \times 6}$$

$$= \frac{3^5 (1 + 5 + 10)}{6^5} = \frac{16}{2^5} = \frac{1}{2}$$

For D probability of getting Even score in a single trial

$$= \frac{3^7 + {}^7C_1 \times 3^7 + {}^7C_3 \times 3^7 + {}^7C_5 \times 3^7}{6^7}$$

$$= \frac{3^7 (1 + 7 + 35 + 21)}{6^7} = \frac{64}{2^7} = \frac{1}{2}$$

All the 4 children have the same probability if they roll dice simultaneously node06\B0AH-AI\Kota\JEE(Advanced)\Enthusiast\Advance Batch\Maths\Sheet\Det.\Eng.p65

10. In one day test match between India and Australia the umpire continues tossing a fair coin until the two consecutive throws either H T or T T are obtained for the first time. If it is H T, India wins and if it is T T, Australia wins.

Statement-1: Both India and Australia have equal probability of winning the toss.

Statement-2: If a coin is tossed twice then the events HT or TT are equiprobable.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

If coin is tossed 2 times only then sample-space —

$\{HH, HT, TH, TT\}$

$P(\text{India wins}) = \frac{1}{4}$

$P(\text{Australia wins}) = \frac{1}{4}$

$\Rightarrow$ Both teams are equiprobable

$\Rightarrow$ Statement 2 is TRUE.

After getting first H, Austrailia can't win.

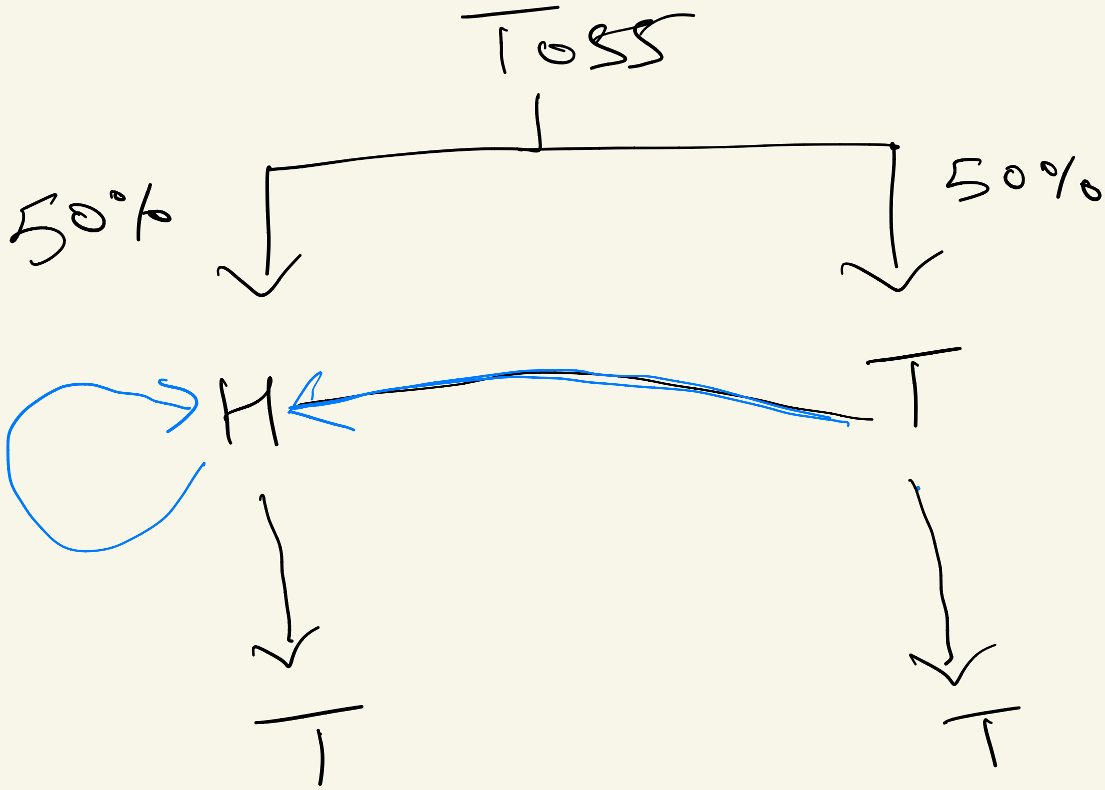
That means
probability of winning India
is greater than probability
of winning Australia.
$\Rightarrow$ Statement 1 is FALSE.

Explanation:

H T
OR H H $\longrightarrow$ H H T
OR H H H $\longrightarrow$ H H H T
OR H H H H $\longrightarrow$

H H H H T
OR H H H H H $\longrightarrow$ H H H H H T
OR H H H H H H $\longrightarrow$

to be continued till
India wins.



probability of Austrailia winning

$= P(TT) = \frac{1}{2} \times \frac{1}{2}$

Probability of India winning

$= P(HT) + P(HHT) + P(HHHT) + \ldots \text{ to } \infty$

$+ P(THT) + P(THHT) + \ldots \text{ to } \infty$

$= \frac{1}{4} + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4} + \ldots \text{ to } \infty$

to ∞

$\frac{1}{8} + \frac{1}{8} \times 2 + \ldots$

$= \frac{3}{4}$

4. Events A and C are independent. If the probabilities relating A, B and C are P (A) = 1/5; P (B) = 1/6;
P(A ∩ C) = 1/20; P(B ∪ C) = 3/8 then
(A) events B and C are independent
(B) events B and C are mutually exclusive
(C) events B and C are neither independent nor mutually exclusive
(D) events A and C are equiprobable

Sol^n $P(A\cap C) = P(A)\cdot P(C) = \frac{1}{20}$ (given)

and $P(A) = \frac{1}{5} \Rightarrow \boxed{P(C) = \frac{1}{4}}$

$P(B\cup C) = P(B) + P(C) - P(B\cap C)$

$\frac{3}{8} = \frac{1}{6} + \frac{1}{4} - P(B\cap C)$

$\Rightarrow P(B\cap C) = \frac{1}{6} + \frac{1}{4} - \frac{3}{8} = \frac{4+6-9}{24}$

$= \frac{1}{24}$

Now $P(B)\cdot P(C) = \left(\frac{1}{6}\right)\left(\frac{1}{4}\right) = \frac{1}{24}$

$\Rightarrow P(B\cap C) = P(B)\cdot P(C)$

$\Rightarrow$ B and C are independent events.

Ans → option (A)

2. An unbiased cubic die marked with 1, 2, 2, 3, 3, 3 is rolled 3 times. The probability of getting a total score of 4 or 6 is

(A) $\frac{16}{216}$ (B) $\frac{50}{216}$ (C) $\frac{60}{216}$ (D) none

Sol^n

$$1+1+2=4 \} \rightarrow \left(\frac{1}{6}\right)\cdot\left(\frac{1}{6}\right)\left(\frac{2}{6}\right)\times 3=\frac{1}{36}$$

$$1+2+3=6 \} \rightarrow \left(\frac{1}{6}\right)\cdot\left(\frac{2}{6}\right)\cdot\left(\frac{3}{6}\right)\times \underline{|3}=\frac{1}{6}$$

$$2+2+2=6 \} \rightarrow \left(\frac{2}{6}\right)\cdot\left(\frac{2}{6}\right)\cdot\left(\frac{2}{6}\right)=\frac{1}{27}$$

$$P=\frac{1}{36}+\frac{1}{6}+\frac{1}{27}=\frac{3+18+4}{108}=\frac{25}{108}=\frac{50}{216}$$

Ans $\rightarrow$ option (B)

3. A biased coin with probability P, $0 < P < 1$, of heads, is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then the value of P is

(A) 1/4 (B) 1/6 (C) 1/3 (D) 1/2

Sol^n

$$P(TH) + P(TTTH) + P(TTTTTH) + \cdots = \frac{2}{5}$$

$$\Rightarrow (1-p)p + (1-p)^3 p + (1-p)^5 p + \cdots = \frac{2}{5}$$

$$\Rightarrow \frac{(1-p)p}{1-(1-p)^2} = \frac{2}{5}$$

$$\Rightarrow 5p - 5p^2 = -2p^2 + 4p \Rightarrow 3p^2 - p = 0$$

$$\Rightarrow \boxed{p = \frac{1}{3}}$$

Ans → option (C)

4. Two numbers a and b are selected from the set of natural number then the probability that $a^2 + b^2$ is divisible by 5 is

(A) $\frac{9}{25}$ (B) $\frac{7}{18}$ (C) $\frac{11}{36}$ (D) $\frac{17}{81}$

Sol^n a^2+b^2 is divisible by 5.

So here we are just concerned about the unit place digit in 'a^2+b^2' which should be '0' and '5'.

i.e. we are ultimately concerned about unit place digits in a^2 & b^2 both.

$$\left.\begin{array}{l} 1^2 \to 1 \\ 2^2 \to 4 \\ 3^2 \to 9 \\ 4^2 \to 6 \\ 5^2 \to 5 \\ 6^2 \to 6 \\ 7^2 \to 9 \\ 8^2 \to 4 \\ 9^2 \to 1 \\ 10^2 \to 0 \end{array}\right\} \text{unit places}$$

For unit place digit of a^2+b^2 to be '0' or '5'

So possible combinations are :-

$1^2 + 2^2 = \sim 5$ (2^2+1^2)

$1^2 + 3^2 = \sim 0$ (3^2+1^2)

$1^2 + 7^2 = \sim 0$

$2^2 + 4^2 = \sim 0$

$2^2 + 6^2 = \sim 0$

$3^2 + 9^2 = \sim 0$

$1^2 + 8^2 = \sim 5$

$2^2 + 9^2 = \sim 5$
$3^2 + 4^2 = \sim 5$
$3^2 + 6^2 = \sim 5$
$4^2 + 7^2 = \sim 5$
$4^2 + 8^2 = \sim 0$
$5^2 + 5^2 = \sim 0$ ✓
$5^2 + 10^2 = \sim 5$
$6^2 + 8^2 = \sim 0$
$7^2 + 9^2 = \sim 0$
$8^2 + 9^2 = \sim 5$
$10^2 + 10^2 = \sim 0$ ✓

favourable cases $= 2 \times 18 = 36$

Total no. of combinations of 2 no's out of these 10 no's $= {}^{10}C_1 \cdot {}^{10}C_1 = 100$

So required Prob $= \dfrac{36}{100} = \dfrac{9}{25}$

option (A) Ans.

5 An urn contains 10 balls coloured either black or red. When selecting two balls from the urn at random, the probability that a ball of each colour is selected is $8/15$. Assuming that the urn contains more black balls than red balls, the probability that at least one black ball is selected, when selecting two balls, is

(A) $\frac{18}{45}$ (B) $\frac{30}{45}$ (C) $\frac{39}{45}$ (D) $\frac{41}{45}$

Sol^n Let urn contains 'x' no. of Black balls and '$10-x$' red balls.

$\boxed{x > 5}$

Black → x
Red → $10-x$

Given $\frac{{}^{x}C_1 \cdot {}^{10-x}C_1}{{}^{10}C_2} = \frac{8}{15}$

$\Rightarrow x(10-x) = \frac{8}{15} \times 45 = 24$

$\Rightarrow x^2 - 10x + 24 = 0 \Rightarrow x = 4$ or 6
(Not possible)

So $\boxed{x = 6} \Rightarrow$ Black balls '6'
Red balls '4'

Now required probability

$= 1 - P(RR) = 1 - \left(\frac{{}^{4}C_2}{{}^{10}C_2}\right) = 1 - \frac{6}{45}$

$= \frac{39}{45}$ Ans

option (C)

6 An unbiased die with numbers 1, 2, 3, 4, 6 and 8 on its six faces is rolled. After this roll if an odd number appears on the top face, all odd numbers on the die are doubled. If an even number appears on the top face, all the even numbers are halved. If the given die changes in this way then the probability that the face 2 will appear on the second roll is -

(A) 2/18 (B) 3/18 (C) 2/9 (D) 5/18

sol^n If odd no. appears in first roll
Resulting die → 2, 2, 6, 4, 6, 8

In this case req. probability
$= P(\text{odd in 1st Roll}) \times P(2 \text{ in 2nd Roll})$

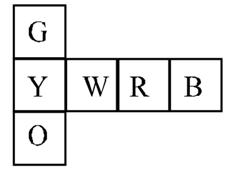
$= \left(\frac{2}{6}\right)\left(\frac{2}{6}\right) = \frac{1}{9}$

If even no. appears in first Roll
Resulting die → 1, 1, 3, 2, 3, 4

In this case req. prob =
$P(\text{even in 1st Roll}) \times P(2 \text{ in 2nd Roll})$
$= \left(\frac{4}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{9}$

so, required Prob $= \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$ option (C)

7. A butterfly randomly lands on one of the six squares of the T-shaped figure shown and then randomly moves to an adjacent square. The probability that the butterfly ends up on the R square is



(A) 1/4 (B) 1/3

(C) 2/3 (D) 1/6

Sol^n To end up on R, it shall land on 'B' or 'W'

So required Prob. $= P(BR) + P(WR)$

$$= \left(\frac{1}{6}\right)(1) + \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) = \frac{3}{2} \times \frac{1}{6} = \frac{1}{4}$$

Because, from W it has two choices 'Y' and 'R'.

Ans ⇒ option (A)

12. The number 'a' is randomly selected from the set {0, 1, 2, 3, ... 98, 99}. The number 'b' is selected from the same set. Probability that the number $3^a + 7^b$ has a digit equal to 8 at the units place, is

(A) $\frac{1}{16}$ (B) $\frac{2}{16}$ (C) $\frac{4}{16}$ (D) $\frac{3}{16}$

Soln

$3^0 \rightarrow 1$

$3^1 \rightarrow 3$

$3^2 \rightarrow 9$

$3^3 \rightarrow 7$

unit places of 3^a

$3^4 \rightarrow 1$

$3^5 \rightarrow 3$

$3^6 \rightarrow 9$

$3^7 \rightarrow 1$

NOW

$7^0 \rightarrow 1$

$7^1 \rightarrow 7$

$7^2 \rightarrow 9$

$7^3 \rightarrow 6$

unit places of 7^b

$7^4 \rightarrow 1$

$7^5 \rightarrow 7$

$7^6 \rightarrow 9$

$7^7 \rightarrow 6$

so possible combinations

$1+7 = 8$

$7+1 = 8$

$9+9 = 1\underline{8}$

3 Combinations

Total Combinations in one segment $= 16$

PART # 5

1. An examination consists of 8 questions in each of which one of the 5 alternatives is the correct one. On the assumption that a candidate who has done no preparatory work chooses for each question any one of the five alternatives with equal probability, the probability that he gets more than one correct answer is equal to

(A) $(0.8)^8$ (B) $3(0.8)^8$ (C) $1-(0.8)^8$ (D) $1-3(0.8)^8$

Solⁿ

P(more than one correct answer)

$$= 1 - \underbrace{P(\text{atmost one correct answer})}$$

exactly one correct answer or No correct answer

$$= 1 - \left(P(X=1) + P(X=0)\right)$$

$$= 1 - \left({}^8C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^7 + {}^8C_0\left(\frac{4}{5}\right)^8\right)$$

$$= 1 - \frac{(4)^7}{(5)^8}(8+4) = 1 - \frac{(4)^8}{(5)^8} \times 3$$

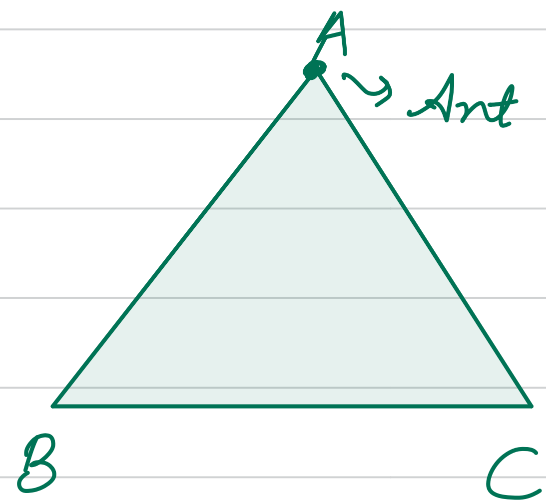
$$= 1 - 3(0.8)^8$$

Ans → option (D)

$\boxed{\text{Binomial Distribution}}$

2. An ant is situated at the vertex A of the triangle ABC. Every movement of the ant consists of moving to one of other two adjacent vertices from the vertex where it is situated. The probability of going to any of the other two adjacent vertices of the triangle is equal. The probability that at the end of the fourth movement the ant will be back to the vertex A, is :

(A) 4/16 (B) 6/16 (C) 7/16 (D) 8/16



sol^n

$P(ABABA)+P(ACACA)$

$+P(ABACA)+P(ACABA)$

$+P(ABCBA)+P(ACBCA)$

$=\left(\frac{1}{2}\right)^4+\left(\frac{1}{2}\right)^4+ \text{----} \text{ (6 times)}$

$=\frac{1}{16}\times 6=\frac{6}{16}$

Ans → option (B)

3. A key to room number C_3 is dropped into a jar with five other keys, and the jar is throughly mixed. If keys are randomly drawn from the jar without replacement until the key to room C_3 is chosen, then what are the odds in favour that the key to room C_3 will be obtained on the 2nd try?

(A) 1 : 4 (B) 1 : 5 (C) 1 : 6 (D) 5 : 6

Sol^n Let Event A → Key of C_3 is obtained in 2nd trial.

$$\Rightarrow P(A) = \left(\frac{5}{6}\right)\left(\frac{1}{5}\right) = \frac{1}{6}$$

$$\text{Odds in favour of } A = \frac{P(A)}{P(\bar{A})} = \frac{1/6}{5/6} = \frac{1}{5}$$

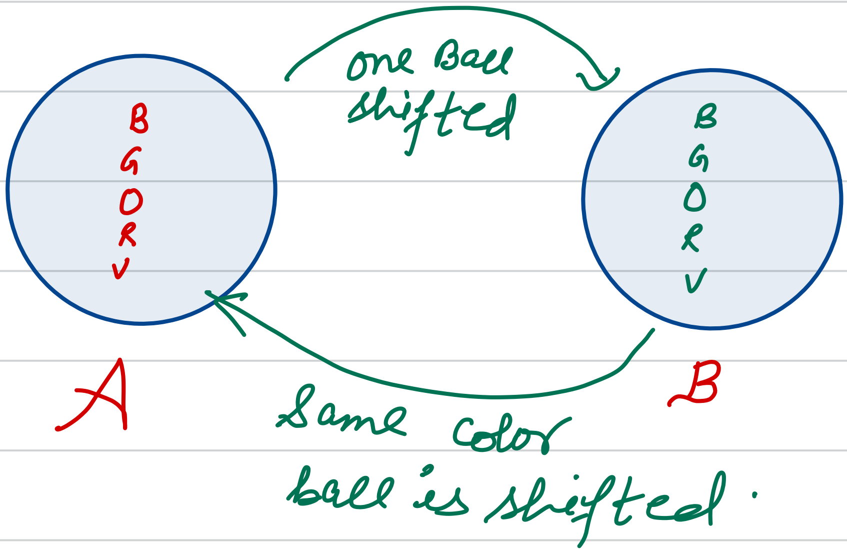
$$= 1:5$$

Ans → option (B)

4. Mr. A and Mr. B each have a bag that contains one ball of each of the colours blue, green, orange, red and violet. 'A' randomly selects one ball from his bag and puts it into B's bag. 'B' then randomly selects one ball from his bag and puts it into A's bag. The probability that after this process the contents of the two bags are the same, is

(A) 1/6 (B) 1/5 (C) 1/3 (D) 1/2

Sol^n



required Prob $= \frac{2}{6} = \frac{1}{3}$

Ans → option (C)

5 An instrument consists of two units. Each unit must function for the instrument to operate. The reliability of the first unit is 0.9 & that of the second unit is 0.8. The instrument is tested & fails. The probability that "only the first unit failed & the second unit is sound" is :

(A) 1/7 (B) 2/7 (C) 3/7 (D) 4/7

Solⁿ P(Instrument failed) = 1 − P(Instrument working)

$$= 1-(0.9)(0.8) = 1-\frac{72}{100} = \frac{28}{100}$$

$$P\left(\frac{1^{st}\text{ unit failed, }2^{nd}\text{ unit sound}}{\text{Instrument failed}}\right)$$

$$= \frac{(0.1)(0.8)}{\frac{28}{100}} = \frac{8}{28} = \frac{2}{7}$$

Ans → option (B)

6 A box contains 10 tickets numbered from 1 to 10. Two tickets are drawn one by one without replacement. The probability that the "absolute value of difference between the first drawn ticket number and the second is not less than 4" is

(A) $\frac{7}{30}$ (B) $\frac{7}{15}$ (C) $\frac{11}{30}$ (D) $\frac{10}{30}$

sol^n $(1,5), (5,1), (1,6), (6,1)$ --- $(1,10), (10,1)$

$(2,6), (6,2)$ - - - - - - - $, (2,10), (10,2)$

$(3,7), (7,3),$ - - - - - - - - - $(3,10), (10,3)$

⋮

$(6,10), (10,6)$

favourable Cases $= 12+10+8+6+4+2$

$= 42$

total Cases $= 2 \cdot {}^{10}C_2 = 90$

Req. Probability $= \frac{42}{90} = \frac{7}{15}$

Ans → option (B)

7. A purse contains 2 six sided dice. One is a normal fair die, while the other has 2 ones, 2 threes, and 2 fives. A die is picked up and rolled. Because of some secret magnetic attraction of the unfair die, there is 75% chance of picking the unfair die and a 25% chance of picking a fair die. The die is rolled and shows up the face 3. The probability that a fair die was picked up, is

(A) 1/7 (B) 1/4 (C) 1/6 (D) 1/24

Solⁿ fair Dice unfair Dice

1 2

3 4

5 6

1, 1, 3, 3, 5, 5

$P(F) = .25\% = \frac{1}{4}$ (given)

$P(U) = 75\% = \frac{3}{4}$ (given)

Now by Baye's Theorem

$$P\left(\frac{\text{Fair dice was picked up}}{\text{Dice has shown face '3'}}\right)$$

$$= \frac{\left(\frac{1}{4}\right)\left(\frac{1}{6}\right)}{\left(\frac{1}{4}\right)\left(\frac{1}{6}\right) + \left(\frac{3}{4}\right)\left(\frac{2}{6}\right)} = \frac{1}{1+6} = \frac{1}{7}$$

Ans → option (A)

8 On a Saturday night 20% of all drivers is U.S.A. are under the influence of alcohol. The probability that a driver under the influence of alcohol will have an accident is 0.001. The probability that a sober driver will have an accident is 0.0001. If a car on a Saturday night smashed into a tree, the probability that the driver was under the influence of alcohol, is -

(A) 3/7 (B) 4/7 (C) 5/7 (D) 6/7

Sol^n Let Event A → Car had an accident

B → Driver is Drunk.

By using Baye's theorem

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{(0.20)(0.001)}{(0.20)(0.001) + (0.80)(0.0001)}$$

$$= \frac{20}{20+8} = \frac{20}{28} = \frac{5}{7}$$

Ans → Option (C)

Paragraph for question nos. 9 to 11

A JEE aspirant estimates that she will be successful with an 80 percent chance if she studies 10 hours per day, with a 60 percent chance if she studies 7 hours per day and with a 40 percent chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively

9. The chance she will be successful, is

(A) 0.28 (B) 0.38 (C) 0.48 (D) 0.58

10. Given that she is successful, the chance she studied for 4 hours, is

(A) $\frac{6}{12}$ (B) $\frac{7}{12}$ (C) $\frac{8}{12}$ (D) $\frac{9}{12}$

11. Given that she does not achieve success, the chance she studied for 4 hour, is

(A) $\frac{18}{26}$ (B) $\frac{19}{26}$ (C) $\frac{20}{26}$ (D) $\frac{21}{26}$

Sol^n

12) $P(\text{Success}) = (0.1)(0.80) + (0.2)(0.60) + (0.7)(0.40)$

$= 0.08 + 0.12 + 0.28 = 0.48$

option (C)

13) $P\left(\frac{\text{Studied for 4 hr}}{\text{Success}}\right) = \frac{(0.7)(0.40)}{0.48} = \frac{28}{48} = \frac{7}{12}$

option (B)

14) $P\left(\frac{\text{Studied for 4 hr}}{\text{not Successful}}\right) = \frac{(0.7)(0.60)}{1-0.48} = \frac{42}{52}$

$= \frac{21}{26}$

option (D)

Part-6

1. A bowl has 6 red marbles and 3 green marbles. The probability that a blind folded person will draw a red marble on the second drawn from the bowl without replacing the marble from the first draw is -

Solⁿ:- Case-I :- When first draw is Red

$$\Rightarrow \frac{6}{9} \times \frac{5}{8} = \frac{5}{12}$$

Case-II :- When first draw is green

$$\Rightarrow \frac{3}{9} \times \frac{6}{8} = \frac{1}{4}$$

Required Probability $= \frac{2}{3}\ \frac{5}{12} + \frac{1}{4}$

$$= \frac{8}{12} = \frac{2}{3}$$

2. In a certain factory, machines A, B and C produce bolts. of their production machines A, B and C produce 2%, 1% and 3% defective bolts respectively. machine A produces 35% of the total output of the bolts, machine B produces 25% and machine C produces 40%. A bolts is choosen at random from the factory's production and is found to be defective. The probability it was produced on machine C is -

Solⁿ:- Let us suppose total production = 100

Bolts produced by A = 35

" B = 25

" C = 40

No. of defective bolts by A = $\frac{2}{100} \times 35$

by B = $\frac{1}{100} \times 25$

by C = $\frac{3}{100} \times 40$

So probability that the bolt is produced by C given that its is defective is

$$= \frac{\frac{3}{100} \times 40}{\frac{2}{100} \times 35 + \frac{1}{100} \times 25 + \frac{3}{100} \times 40}$$

$$= \frac{120}{70 + 25 + 120}$$

$$= \frac{120}{215}$$

$$= \frac{24}{23}$$

3. Three numbers are choosen at random without replacement $\{1, 2, 3, \ldots, 10\}$. The probability that the min^m of the choosen numbers is 3 or their max^m is 7 is -

Sol^n. P(A) = Probability that min^m of chosen no. is 3

$$= \frac{{}^7C_2}{{}^{10}C_3} = \frac{\frac{7 \times \cancel{6}3}{\cancel{2}}}{\frac{10 \times \cancel{9} \times \cancel{8}4}{3 \times 2}} = \frac{\cancel{21}}{} \frac{7}{40}$$

P(B) = Probability that max^m of chosen no. is 7

$$= \frac{{}^6C_2}{{}^{10}C_3} = \frac{\frac{\cancel{3}\cancel{6} \times 5}{\cancel{2}}}{\frac{10 \times \cancel{9} \times \cancel{8}4}{\cancel{3} \times 2}} = \frac{5}{40}$$

P(A∩B) = Probability that min^m is 3 & max^m is 7

$$= \frac{{}^3C_1}{{}^{10}C_3} = \frac{\cancel{3} \times \cancel{6}}{10 \times \cancel{9} \times \cancel{8}4} = \frac{1}{40}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{7}{40} + \frac{5}{40} - \frac{1}{40}$$

$$P(A \cup B) = \frac{11}{40}$$

4. Two buses A & B are scheduled to arrive at a town Central bus station at noon. The probability that bus A will be late is 1/5. The probability the bus B will be late is 7/25. The probability that the bus B is late given that bus A is late is 9/10. Then the probabilities

(i) neither bus will be late on a particular day and

$P(A)$ = Probability that Bus A is late

$= 1/5$

$P(B)$ = probability that Bus B is late

$= 7/25$

$P(B/A)$ = Probability that Bus B is late given that bus A is late is

$= \frac{9}{10}$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\frac{9}{10} = \frac{P(A \cap B)}{1/5}$$

$$P(A \cap B) = \frac{9}{50}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{5} + \frac{7}{25} - \frac{9}{50}$$

$$P(A \cup B) = \frac{3}{10}$$

Probability that neither Bus will be late

$$= 1 - P(A \cup B)$$

$$= 1 - \frac{3}{10} = \frac{7}{10}$$

(ii) Bus A is late given that Bus B is late

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{9}{50}}{7/25}$$

$$= \frac{9}{14} = \frac{18}{28}$$

5. If at least one child in a family with 3 children is a boy then the probability that exactly 2 of the children are boys is -

Soln. Case-I   1 Boy & 2 Girl

BGG
GBG   = 3
GGB

Case-II   2 Boy & 1 Girl

BBG
BGB   = 3
GBB

Case-III   3 Boy

BBB   = 1

Required probability = $\frac{3}{3+3+1} = \frac{3}{7}$

6. From an urn containing six balls, 3 white & 3 black ones, a person selects at random an even number of balls (all the different ways of drawing an even number of balls are considered equally probable irrespective of their number). Then the probability that there will be the same number of black and white balls among them.

Sol^n:- There are three ways of drawing even ball.
(2 balls, 4 balls, 6 balls)

So each have Probability $= \frac{1}{3}$

Case-1 :- When 2 balls are drawn.

P_1 = Probability of drawing two balls when same no. of white & Black balls are among them.

$$P_1 = \left(\frac{1}{3}\right) \times \left[\frac{{}^3C_1 \times {}^3C_1}{{}^6C_2}\right] = \frac{3}{15}$$

Probability of drawing 2 balls

Case-2 :- When 4 balls are drawn

P_2 = Probability of drawing 4 balls when same no. of white & Black balls are among them

$$P_2 = \left(\frac{1}{3}\right) \times \frac{{}^3C_2 \times {}^3C_2}{{}^6C_4} = \frac{3}{15}$$

Probability of drawing 4 balls

Case-3 :. ~~Probability~~ When 6 balls are drawn

P_3 = Probability that 6 balls are drawn when equal no. of Black & white balls are among them

$$P_3 = \left(\frac{1}{3}\right) \times \left[\frac{{}^3C_3 \times {}^3C_3}{{}^6C_6}\right]$$

Probability of drawing 6 balls

$$P = P_1 + P_2 + P_3$$

$$= \frac{3}{15} + \frac{3}{15} + \frac{1}{3}$$

$$\boxed{P = \frac{11}{15}}$$

7. There are three main political parties namely 1, 2, 3. If in the adjoining table P_{ij} $(i, j = 1, 2, 3)$ denotes the probability that party j wins the general election contested when party i is in the power. What is the probability that the party 2 will be in power after the next two election given that the party 1 is in the power.

$P_{11} = 0.7$	$P_{12} = 0.2$	$P_{13} = 0.1$
$P_{21} = 0.5$	$P_{22} = 0.3$	$P_{23} = 0.2$
$P_{31} = 0.3$	$P_{32} = 0.4$	$P_{33} = 0.3$

Solⁿ:-

1st election   Second Election

$P_1 \xrightarrow{P_{11}=0.7} P_1 \xrightarrow{P_{12}=0.2} P_2 = 0.7 \times 0.2 = 0.14$

$P_1 \xrightarrow{P_{12}=0.2} P_2 \xrightarrow{P_{22}=0.3} P_2 = 0.2 \times 0.3 = 0.06$

$P_1 \xrightarrow{P_{13}=0.1} P_3 \xrightarrow{P_{32}=0.4} P_2 = 0.1 \times 0.4 = 0.04$

Required Probability $= 0.14 + 0.06 + 0.04$

$= 0.24$

8. Shalu bought two cages of birds : Cage-I contains 5 parrots and 1 owl and Cage-II contains 6 parrots as shown. One day shalu forgot to lock both cages and two birds flew from Cage-I to Cage-II Then two birds flew back from Cage-II to Cage-I Assume that all birds have equal chance of flying the probability that the owl is still in Cage-I is-

Sol^n:-

Cage-I	Cage-II
5P, 1O	6P

Case-I: When two parrots flew from Cage I to II & after that 2 parrots from Cage II → I

$$P_1 = \underbrace{\frac{{}^5C_2}{{}^6C_2}}_{\text{For Cage-I}} \times \underbrace{1}_{\text{For Cage-II}}$$

$$P_1 = \frac{2}{3}$$

Case-2: When 1 parrots & 1 owl flew from Cage I to II and after that 1 P & 1 owl flew back from Cage II to I

$$P_2 = \underbrace{\frac{{}^5C_1 \times {}^1C_1}{{}^6C_2}}_{\text{From Cage I to II}} \times \underbrace{\frac{{}^7C_1 \times {}^1C_1}{{}^8C_2}}_{\text{From Cage II - I}}$$

$$P_2 = \frac{1}{12}$$

$$P = P_1 + P_2 = 2/3 + 1/12 = \frac{3}{4}$$

9. Miss C has either Tea or Coffee at morning break. If she has tea one morning, the probability she has tea the next morning is 0.4. If she has Coffee one morning, the probability she has coffee next morning is 0.3. Suppose she has Coffee on a monday morning. The probability that she has tea on the following wednesday morning is –

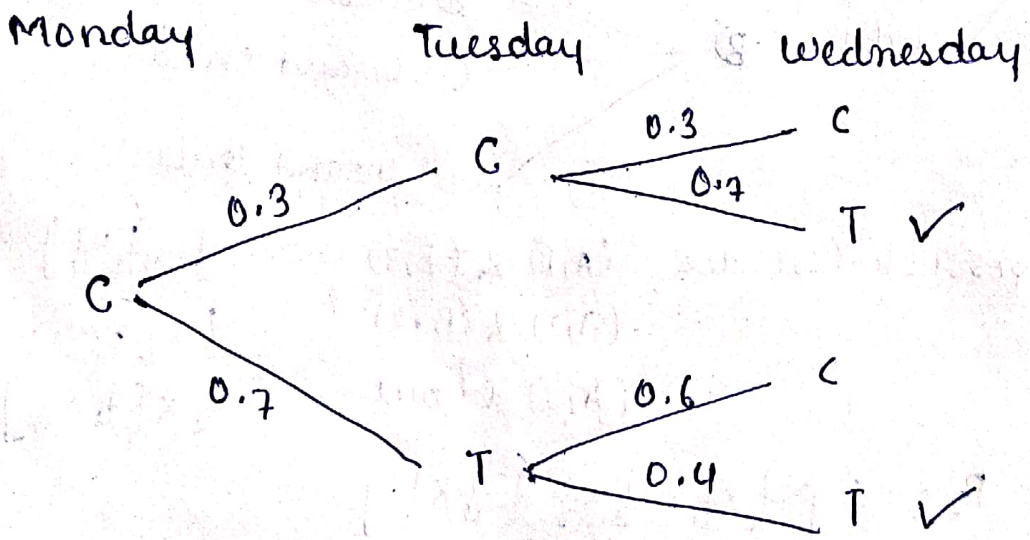
Soln:- Let $P(C)$ = Probability that Miss has Coffee one morning

$P(T)$ = Probability that Miss C has Tea one morning.

So $P(C/C) = 0.3$ then $P(T/C) = 0.7$

$P(T/T) = 0.4$ $P(\bar{T}/T) = 0.6$

When $P(C/C)$: she has Coffee in one morning & she has Coffee in the next morning



$P = 0.3 \times 0.7 + 0.7 \times 0.4$

$p = 0.49$

Exercise 02

EXERCISE (O-2)

[STRAIGHT OBJECTIVE TYPE]

1. n different books ($n \geq 3$) are put at random in a shelf. Among these books there is a particular book 'A' and a particular book B. The probability that there are exactly 'r' books between A and B is -

(A) $\frac{2}{n(n-1)}$ (B) $\frac{2(n-r-1)}{n(n-1)}$ (C) $\frac{2(n-r-2)}{n(n-1)}$ (D) $\frac{(n-r)}{n(n-1)}$

Solution:

$n(S) = \lfloor n$

For $n(E)$, we make a block of $(r+2)$ books having books A & B at the ends of this block.

So, $n(E) = \underbrace{{}^{(n-2)}C_r \times \lfloor r}_{\text{Selection of r books and their arrangement between A and B}} \times \underbrace{\lfloor 2}_{\text{A and B can be interchanged}} \times \underbrace{\lfloor n-r-1}_{\text{Arrangement of (n-r-2)+1 objects}}$

Hence, $n(E) = \frac{\lfloor n-2}{\lfloor r \lfloor n-r-2} \times \lfloor r \times \lfloor 2 \times \lfloor n-r-1$

$\Rightarrow n(E) = \lfloor n-2 \lfloor 2 \, (n-r-1)$

Thus $P(E) = \frac{\lfloor n-2 \lfloor 2 \, (n-r-1)}{\lfloor n}$

$= \frac{2(n-r-1)}{n(n-1)}$

2. A fair die is thrown 3 times. The chance that sum of three numbers appearing on the die is less than 11, is equal to -

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{6}$ (D) $\frac{5}{8}$

Solution: $n(S) = 6^3$

Let x, y and z be face values of first, second and third throw respectively.

So, we require $x + y + z \le 10$; $x, y, z \in \{1, 2, \ldots, 6\}$

$\Rightarrow x + y + z + w = 10$; $w \in \{0, 1, 2, \ldots, 7\}$

The number of solutions of this equation is same as the coefficient of x^{10} in the expansion of

$$(x + x^2 + \ldots + x^6)^3 \quad (1 + x + \ldots + x^7)$$

consequently, we require coefficient of x^7 in

$$(1 + x + x^2 + \ldots + x^5)^3 \quad (1 + x + \ldots + x^7)$$

$$= (1 - x^6)^3 (1 - x^8)(1 - x)^{-4}$$

$$= (1 - 3x^6 + \ldots)(1 - x^8)(1 + {}^4C_1 x + {}^5C_2 x^2 + \ldots)$$

Thus, $n(E) = {}^{10}C_7 - 3 \times {}^4C_1 = 108$

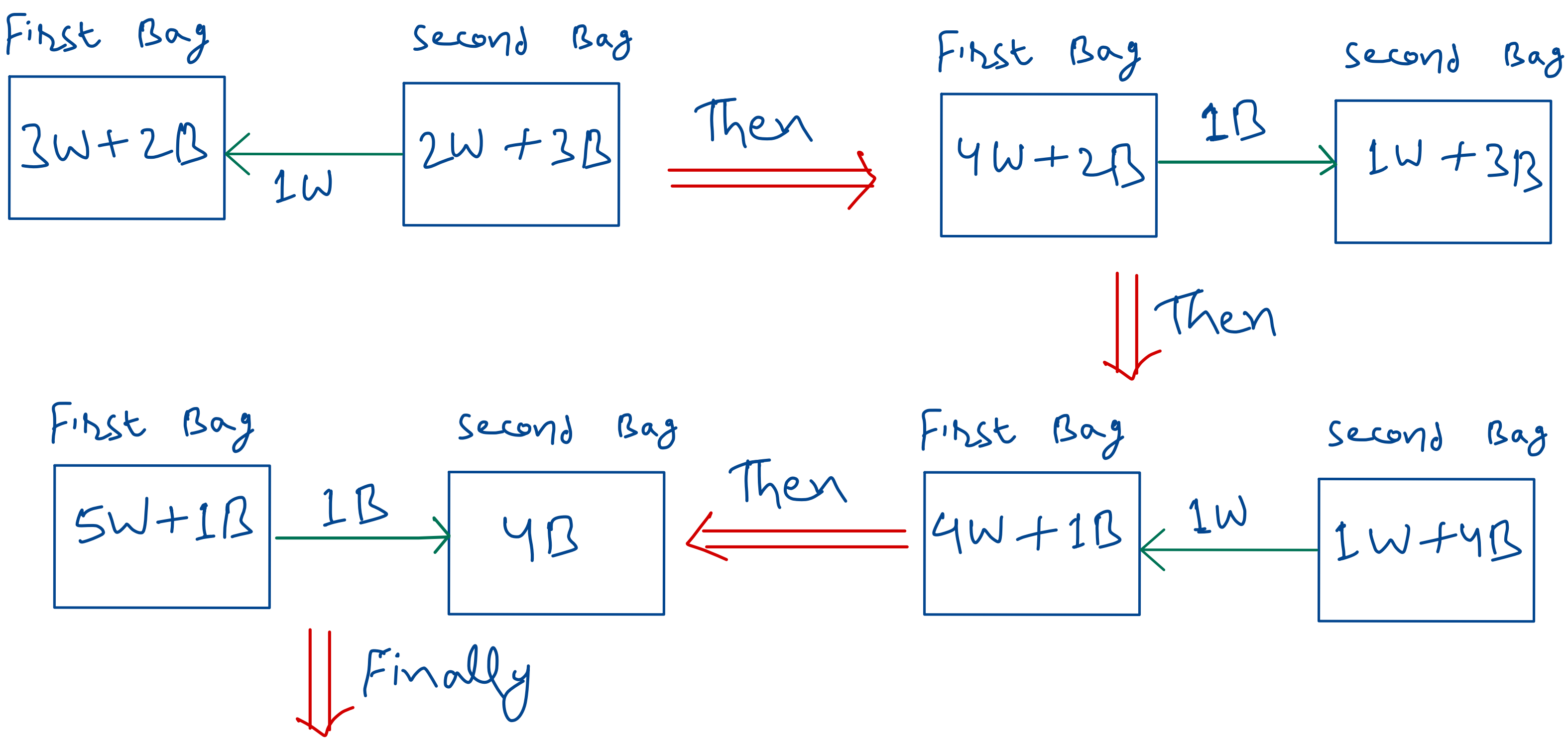
Hence, $P(E) = \frac{36 \times 3}{36 \times 6} = \frac{1}{2}$

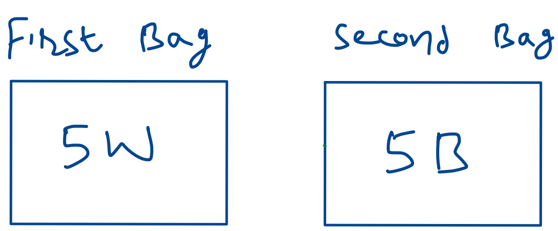
3 One bag contains 3 white & 2 black balls, and another contains 2 white & 3 black balls. A ball is drawn from the second bag & placed in the first, then a ball is drawn from the first bag & placed in the second. When the pair of the operations is repeated, the probability that the first bag will contain 5 white balls is:

(A) 1/25 (B) 1/125 (C) 1/225 (D) 2/15

Solution:

Initially there are five balls in first bag. Now two balls goes into this bag and two balls goes out of this bag. So total number of balls remains five in the first bag after the pair of operations is repeated. Since we require five white balls in the first bag at the end. So two white balls must goes in and two black balls must goes out from the first bag as it contains only three white balls initially. So, series of required operations is:





Hence, required probality is

$$\frac{2}{5} \times \frac{2}{6} \times \frac{1}{5} \times \frac{1}{6} = \frac{1}{225}$$

Note:- We have used $P(E_1 \cap E_2 \cap E_3 \cap E_4)$

$$= P(E_1) \times P\left(\frac{E_2}{E_1}\right) \times P\left(\frac{E_3}{E_1 \cap E_2}\right) \times P\left(\frac{E_4}{E_1 \cap E_2 \cap E_3}\right)$$

4. If a, b and c are three numbers (not necessarily different) chosen randomly and with replacement from the set {1, 2, 3, 4, 5}, the probability that (ab + c) is even, is

(A) $\frac{35}{125}$ (B) $\frac{59}{125}$ (C) $\frac{64}{125}$ (D) $\frac{75}{125}$

Solution: $n(S) = 5^3$

For $n(E)$; we consider two cases:—

Case I: c is even

Now, ab must be even i.e. both a & b cannot be odd simultaneously.

So, $n(E_1) = \underbrace{(5\times 5 - 3\times 3)}_{\text{both a \& b cannot be odd}} \times \underbrace{2}_{\text{c can be 2 or 4}}$

Case II: c is odd

Now, ab must be odd i.e. both a & b must be odd.

So, $n(E_2) = 3\times 3\times 3$

Hence, $n(E) = n(E_1) + n(E_2) = 59$

Thus, $P(E) = \frac{59}{125}$

5 A purse contains 100 coins of unknown value, a coin drawn at random is found to be a rupee. The chance that it is the only rupee in the purse, is (Assume all numbers of rupee coins in the purse to be equally likely.)

(A) $\frac{1}{5050}$ (B) $\frac{2}{5151}$ (C) $\frac{1}{4950}$ (D) $\frac{2}{4950}$

Solution: Let's define the following events:

E_r :The purse contains exactly r rupee coins; r= 1,2, ..., 100

So, $P(E_r) = \frac{1}{100}$; as they are equally likely.

Also, event F : a coin is drawn from the purse and found to be a rupee coin. Note we need to find $P(E_1/F)$. So we take the recourse of Baye's Theorem.

Hence, $$P\left(\frac{E_1}{F}\right) = \frac{P\left(\frac{F}{E_1}\right) P(E_1)}{\sum_{r=1}^{100} P\left(\frac{F}{E_r}\right) P(E_r)}$$

$$= \frac{\frac{1}{100} \times \frac{1}{100}}{\sum_{r=1}^{100} \frac{r}{100} \times \frac{1}{100}} = \frac{1}{\sum_{r=1}^{100} r}$$

$$= \frac{2}{100 \times 101} = \frac{1}{5050}$$

6 Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French, and will mistake it for a Californian wine with probability 0.1. When given a Californian wine, he will identify it with probability 0.8 correctly as Californian, and will mistake it for a French wine with probability 0.2. Suppose that Mr. Dupont is given ten unlabelled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine, and solemnly says : "French". The probability that the wine he tasted was Californian, is nearly equal to

(A) 0.14 (B) 0.24 (C) 0.34 ✓ (D) 0.44

Solution:

Let's define following events:

F: Mr. Dupont is given French wine.
C: Mr. Dupont is given Californian wine.
F1: He identifies the given wine as French.
C1: He identifies the given wine as Californian.

So, we have been given :—

$P\left(\frac{F_1}{F}\right) = 0.9$, $P\left(\frac{C_1}{F}\right) = 0.1$, $P\left(\frac{C_1}{C}\right) = 0.8$, $P\left(\frac{F_1}{C}\right) = 0.2$

Now, to find $P\left(\frac{C}{F_1}\right)$ we apply Baye's theorem.

$$P\left(\frac{C}{F_1}\right) = \frac{P\left(\frac{F_1}{C}\right) P(C)}{P\left(\frac{F_1}{C}\right) P(C) + P\left(\frac{F_1}{F}\right) \times P(F)}$$

$$= \frac{0.2 \times \frac{7}{10}}{0.2 \times \frac{7}{10} + 0.9 \times \frac{3}{10}} = \frac{14}{41}$$

7. Identify the correct statement :

(A) If the probability that a computer will fail during the first hour of operation is 0.01, then if we turn on 100 computers, exactly one will fail in the first hour of operation.

(B) A man has ten keys only one of which fits the lock. He tries them in a door one by one discarding the one he has tried. The probability that fifth key fits the lock is 1/10.

(C) Given the events A and B in a sample space. If P(A) = 1, then A and B are independent.

(D) When a fair six sided die is tossed on a table top, the bottom face can not be seen. The probability that the product of the numbers on the five faces that can be seen is divisible by 6 is one.

Solution:

A) since, we know only the probability that a computer will fail in first hour of operation. So, we cannot comment that how many computers would fail during that interval.

B) we can arrange these ten keys in any random order. So, n(S)= 10!
But, we require the correct key at fifth place and other nine keys can be put in any random order. So, n(E)= 9!
Thus, required probability is 1/10.

C.) Now, A is the sample space.

So, $B \subseteq A \Rightarrow B \cap A = B$

$\Rightarrow P(A \cap B) = P(B) = P(B)\,P(A)$ $[\text{As } P(A) = 1]$

$\Rightarrow$ A & B are independent events.

D) There are two possibilities, either bottom face is six or not six. If the bottom face is six, then visible faces are 1,2,3,4 and 5, whose product is divisible by six. Further if the bottom face is not six, then we must have a visible face 6. So, again product of five visible faces is divisible by six. Thus, in all cases product of five visible faces is divisible by six. Hence, it is a sure event.

8 For $P(A)=\frac{3}{8}$; $P(B)=\frac{1}{2}$; $P(A\cup B)=\frac{5}{8}$ which of the following do/does hold good?

(A) $P(A^c/B)=2P(A/B^c)$

(B) $P(B)=P(A/B)$

(C) $15P(A^c/B^c)=8P(B/A^c)$

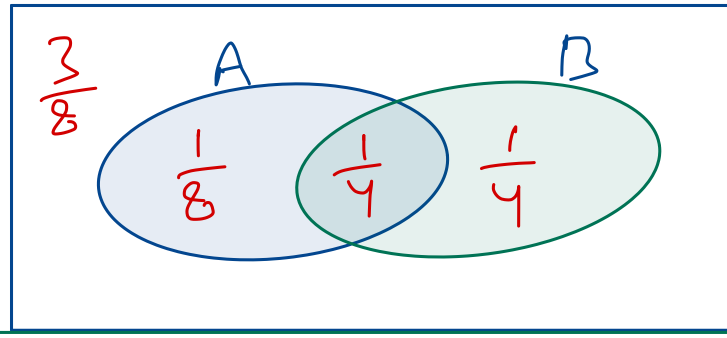
(D) $P(A/B^c)=P(A\cap B)$

Solution:

$$\because P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\Rightarrow \frac{5}{8}=\frac{3}{8}+\frac{1}{2}-P(A\cap B)$$

$$\Rightarrow P(A\cap B)=\frac{1}{4}$$



A) or (D) $\quad P(\bar{A}/B)=\dfrac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)}=\dfrac{1}{2}$

And, $\quad P(A/\bar{B})=\dfrac{\left(\frac{1}{8}\right)}{\left(\frac{1}{8}+\frac{3}{8}\right)}=\dfrac{1}{4}=P(A\cap B)$

B) $\quad P(A/B)=\dfrac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)}=\dfrac{1}{2}=P(B)$

C) $\quad P(\bar{A}/\bar{B})=\dfrac{\frac{3}{8}}{\left(\frac{1}{8}+\frac{3}{8}\right)}=\dfrac{3}{4}$

And, $\quad P(B/\bar{A})=\dfrac{1/4}{\frac{1}{4}+\frac{3}{8}}=\dfrac{2}{5}$

9. If E_1 and E_2 are two events such that $P(E_1) = 1/4$, $P(E_2/E_1) = 1/2$ and $P(E_1/E_2) = 1/4$

(A) then E_1 and E_2 are independent

(B) E_1 and E_2 are exhaustive

(C) E_2 is twice as likely to occur as E_1

(D) Probabilities of the events $E_1 \cap E_2$, E_1 and E_2 are in G.P.

Solution:

$$P(E_1 \cap E_2) = P(E_2/E_1) \times P(E_1) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

Also, $$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

$$\Rightarrow \frac{1}{4} = \frac{\left(\frac{1}{8}\right)}{P(E_2)} \Rightarrow P(E_2) = \frac{1}{2} = 2P(E_1)$$

Also, $P(E_1 \cap E_2) = \frac{1}{8} = P(E_1)\,P(E_2)$

$\Rightarrow E_1$ & E_2 are independent

But, $P(E_1 \cup E_2) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} \neq 1$

So, E_1 & E_2 are not exhaustive.

Note that $P(E_1 \cap E_2)$, $P(E_1)$ & $P(E_2)$ are in G.P. with common ratio 2.

10 Two events A and B are such that the probability that at least one of them occurs is 5/6 and both of them occurring simultaneously is 1/3. If the probability of not occurrence of B is 1/2 then

(A) A and B are equally likely

(B) A and B are independent

(C) P(A/B) = 2/3

(D) 3 P(A) = 4 P(B)

Solution:

Given $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ & $P(\overline{B}) = \frac{1}{2}$

So, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{6} = P(A) + \frac{1}{2} - \frac{1}{3} \Rightarrow P(A) = \frac{2}{3} \neq P(B)$

Now, $P(A \cap B) = \frac{1}{3} = \frac{2}{3} \times \frac{1}{2} = P(A)P(B)$

$\Rightarrow$ A & B are independent.

Also, $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\left(\frac{1}{3}\right)}{\left(\frac{1}{2}\right)} = \frac{2}{3}$

Again, $3P(A) = 2 = 4P(B)$

[redacted]. The probabilities of events, $A \cap B$, A, B & $A \cup B$ are respectively in A.P. with probability of second term equal to the common difference. Therefore the events A and B are

(A) mutually exclusive
(B) independent
(C) such that one of them must occur
(D) such that one is twice as likely as the other

Solution: Let $P(A\cap B)$, $P(A)$, $P(B)$ & $P(A\cup B)$ be respectively α, $\alpha+\beta$, $\alpha+2\beta$ & $\alpha+3\beta$

Also, $\alpha+\beta=\beta \Rightarrow \alpha = P(A\cap B)=0$

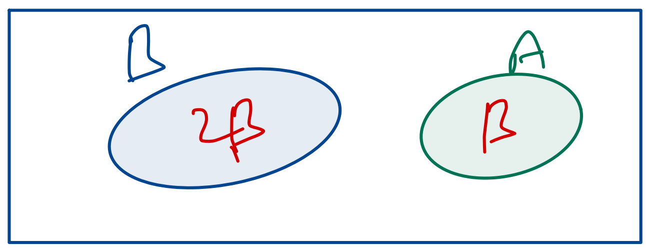
Since, $P(A\cup B) = P(A)+P(B)-P(A\cap B)$

$\Rightarrow \alpha+3\beta = (\alpha+\beta)+(\alpha+2\beta)-\alpha$

$\Rightarrow 3\beta = 3\beta$ (Trivial result)

Since $A\cap B=\phi$

$\Rightarrow$ A & B are exclusive.



Also, $P(A\cap B)=0 \neq P(A)\,P(B)$

(equality holds when $\beta=0$, which is not necessary)

Further, $P(A\cup B)=3\beta$ can be zero as well when $\beta=0$. So, option (C) is wrong.

12 A box contains 11 tickets numbered from 1 to 11. Six tickets are drawn simultaneously at random. Let E_1 denotes the event that the sum of the numbers on the tickets drawn is even and E_2 denotes the event that the sum of the numbers on the tickets drawn is odd. Which of the following hold good?

(A) E_1 and E_2 are equally likely
(B) E_1 and E_2 are exhaustive
(C) $P(E_2) > P(E_1)$
(D) $P(E_1/E_2) = P(E_2 / E_1)$

Solution:

Obviously sum of the numbers on the six drawn tickets will be either odd or even. So the events E1and E2 are mutually exclusive as well as exhaustive.

Hence, $P(E_1) + P(E_2) = 1$

Now, $n(S) = {}^{11}C_6 = 462$

From 1 to 11 there are five even numbers and six odd numbers. So for event E1, we need to choose either (4 even + 2 odd) or (2 even + 4 odd) or (6 odd).

So, $n(E_1) = {}^5C_4 \times {}^6C_2 + {}^5C_2 \times {}^6C_4 + {}^6C_6 = 226$

Now, $P(E_1) = \frac{226}{462} < \frac{231}{462} = \frac{1}{2}$

$\Rightarrow P(E_2) = 1 - P(E_1) > \frac{1}{2} > P(E_1)$

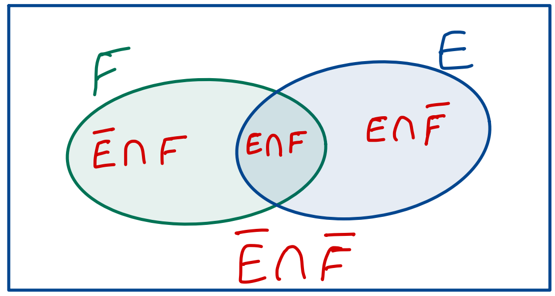
Also, $P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} \neq \frac{P(E_1 \cap E_2)}{P(E_1)} = P(E_2/E_1)$

13. If $\overline{E}$ & $\overline{F}$ are the complementary events of events E & F respectively & if $0 < P(F) < 1$, then :

(A) $P(E|F) + P(\overline{E}|F) = 1$ (B) $P(E|F) + P(E|\overline{F}) = 1$

(C) $P(\overline{E}|F) + P(E|\overline{F}) = 1$ (D) $P(E|\overline{F}) + P(\overline{E}|\overline{F}) = 1$

Solution:

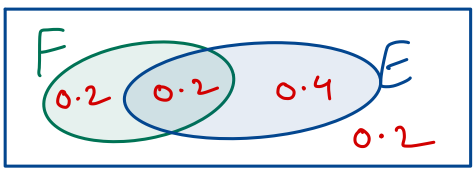


A) $P(E/F) + P(\bar{E}/F)$

$$= \frac{P(E\cap F) + P(\bar{E}\cap F)}{P(F)}$$

$$= \frac{P(F)}{P(F)} = 1$$

B) $P(E/F) + P(E/\bar{F}) = \dfrac{P(E\cap F)}{P(F)} + \dfrac{P(E\cap\bar{F})}{P(\bar{F})}$



may not be equal to 1. For instance, as per our example the value of given expression is $\dfrac{0.2}{0.4} + \dfrac{0.4}{0.6} = \dfrac{7}{6}$

c) To disprove this statement, again we take counterexample of option B.

$$P(\bar{E}/F) + P(E/\bar{F}) = \frac{P(\bar{E}\cap F)}{P(F)} + \frac{P(E\cap\bar{F})}{P(\bar{F})} = \frac{0.2}{0.4} + \frac{0.4}{0.6} = \frac{7}{6}$$

D.) $P(E/\bar{F}) + P(\bar{E}|\bar{F}) = \dfrac{P(E\cap\bar{F}) + P(\bar{E}\cap\bar{F})}{P(\bar{F})}$

$$= \frac{P(\bar{F})}{P(\bar{F})} = 1$$