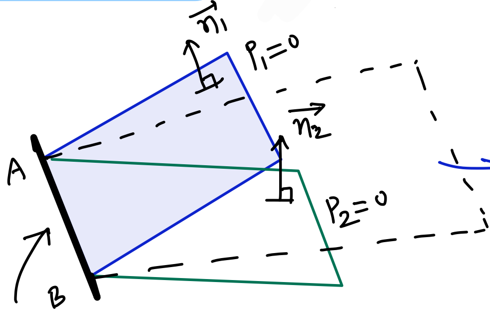
Family of Planes :-



line of intersection of $P_1 = 0$ & $P_2 = 0$

$\vec{n_1} \times \vec{n_2}$

$\boxed{aP_1 + bP_2 = 0}$; $a, b \in \mathbb{R}$

if $a \neq 0$

$P_1 + \frac{b}{a} P_2 = 0$

$\boxed{P_1 + \lambda P_2 = 0}$

family of planes containing line of intersection of 2 plane $P_1 = 0$ & $P_2 = 0$.

Q To find the equation of the plane coaxial with the planes $\vec{r}.\vec{n}_1 = q_1$ and $\vec{r}.\vec{n}_2 = q_2$ .... (i)

and passing through the point with position vector $\vec{a}$.

Sol^n $P_1 + \lambda P_2 = 0$

$$(\vec{r}_1 \cdot \vec{n}_1 - q_1) + \lambda(\vec{r} \cdot \vec{n}_2 - q_2) = 0.$$

P : $\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = q_1 + \lambda q_2$

put $\vec{r} = \vec{a}$ get λ

Q Find the equation of the plane containing the line of intersection of the planes $\vec{r}.\vec{n}_1 = q_1$; $\vec{r}.\vec{n}_2 = q_2$ and is parallel to the line of intersection of the planes $\vec{r}.\vec{n}_3 = q_3$ and $\vec{r}.\vec{n}_4 = q_4$.

Sol^n $P_1 + \lambda P_2 = 0 \Rightarrow (\vec{r} \cdot \vec{n}_1 - q_1) + \lambda(\vec{r} \cdot \vec{n}_2 - q_2) = 0$

$$\vec{r} \cdot (\underbrace{\vec{n}_1 + \lambda \vec{n}_2}_{\downarrow}) = q_1 + \lambda q_2.$$

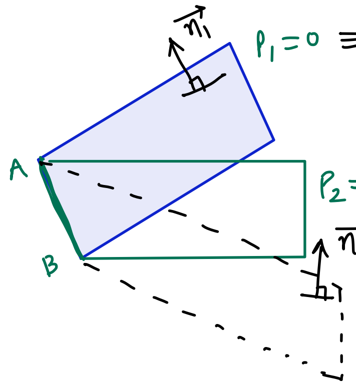
normal vector of req. plane

$$(\vec{n}_1 + \lambda \vec{n}_2) \cdot (\vec{n}_3 \times \vec{n}_4) = 0$$

$$[\vec{n}_1 \ \vec{n}_3 \ \vec{n}_4] + \underline{\underline{\lambda}} [\vec{n}_2 \ \vec{n}_3 \ \vec{n}_4] = 0$$

Q The plane x − y − z = 4 is rotated through 90° about its line of intersection with the plane x + y + 2z = 4. Find its equation in the new position.

Sol^{n}



$P_1 = 0 \Rightarrow x - y - z - 4 = 0$ $\qquad \vec{n} \cdot \vec{n_1} = 0$

$P_2 = 0 \Rightarrow x + y + 2z - 4 = 0$

$P_1 + \lambda P_2 = 0$

$\Downarrow$

$$(x - y - z - 4) + \lambda(x + y + 2z - 4) = 0$$

$$x(1+\lambda) + y(\lambda - 1) + z(2\lambda - 1) - 4 - 4\lambda = 0$$

$$\vec{n} = (1+\lambda)\hat{i} + (\lambda - 1)\hat{j} + (2\lambda - 1)\hat{k}$$

$$\vec{n_1} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{n} \cdot \vec{n_1} = 0 \Rightarrow (1 + \cancel{\lambda}) - (\cancel{\lambda} - 1) - (2\lambda - 1) = 0$$

$$2 - 2\lambda + 1 = 0 \Rightarrow \lambda = \frac{3}{2}$$

STRAIGHT LINES

SYMMETRICAL FORM OF STRAIGHT LINE (CARTESIAN FORM) :

(i) $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=\lambda$ or $\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}=\lambda$ (derived from $\vec{r}=\vec{r}_0+\lambda\vec{v}$)

(ii) $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$ (Two point form)

where a, b, c are the dr's of line or the vector along which the line is travelling.

Note: Equation of x-axis $\frac{x}{1}=\frac{y}{0}=\frac{z}{0}$; y-axis $\frac{x}{0}=\frac{y}{1}=\frac{z}{0}$; z-axis $\frac{x}{0}=\frac{y}{0}=\frac{z}{1}$, 0 in denominator shows that the line is perpendicular to the axis.

eg:- $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z-2}{0}$ $\Rightarrow$ OR $\frac{x-2}{3}=\frac{y+1}{-2}$ and $z=2$

represent a line parallel to xy plane at a distance 2 units.

gener l point : $\left(\lambda+1, \frac{3\lambda-3}{2}, 2\lambda-5\right)$ $\lambda \in$ scalar

eg:- $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ $=\lambda$ passes through $1, -\frac{3}{2}$ & -5 with dr's $1, \frac{3}{2}$ & 2.

$\frac{x-1}{1}=\frac{2\left(y+\frac{3}{2}\right)}{3}=\frac{z-(-5)}{2}$

$\frac{x-1}{1}=\frac{y-(-3/2)}{3/2}=\frac{z-(-5)}{2}$

OR

dr's $2:3:4$

OR

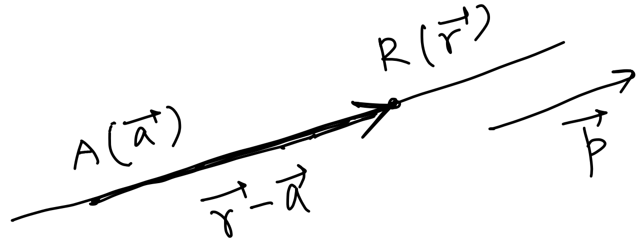
dr's $\sqrt{2}, \frac{3}{\sqrt{2}}, 2\sqrt{2}$

and so on.

Vector form :

$\vec{r}=\left(\hat{i}-\frac{3}{2}\hat{j}-5\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$

Vector equ of line :



$$\vec{r} = \vec{a} + \lambda \vec{p}$$

$$\vec{r} - \vec{a} = \lambda \vec{p}$$

Let, $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{p} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$(x-a_1)\hat{i} + (y-a_2)\hat{j} + (z-a_3)\hat{k} = \lambda(\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$$

$$\boxed{\frac{x-a_1}{\alpha} = \frac{y-a_2}{\beta} = \frac{z-a_3}{\gamma} = \lambda} \text{ (say)}$$

Cartesian form or symmetrical form.

general pt : $(a_1+\lambda\alpha,\ a_2+\lambda\beta,\ a_3+\lambda\gamma)$

Q Convert the equation $3x + 1 = 6y - 2 = 1 - z$ in vector form and find its direction ratios.

Sol^n

$$3\left(x+\tfrac{1}{3}\right) = 6\left(y-\tfrac{2}{6}\right) = -(z-1)$$

$$\frac{x-\left(-\frac{1}{3}\right)}{\frac{1}{3}} = \frac{y-\frac{1}{3}}{\frac{1}{6}} = \frac{z-1}{-1}$$

dr's : $\left(\frac{1}{3}, \frac{1}{6}, -1\right)$ or

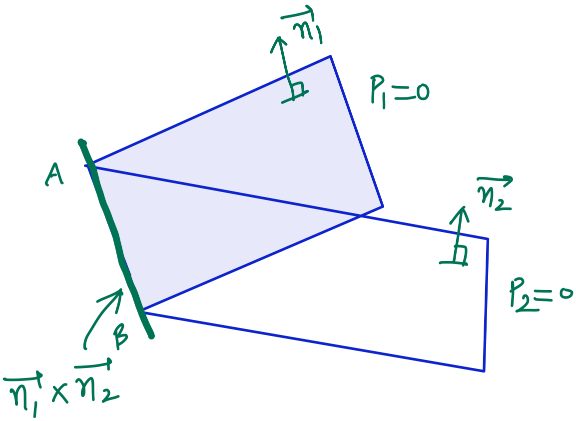
$(2, 1, -6)$ or .....

UNSYMMETRICAL FORM OF STRAIGHT LINE :

The equations $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ together represents a line in unsymmetrical form.

$a_1x+b_1y+c_1z+d_1 = a_2x+b_2y+c_2z+d_2=0.$

Note : Vector along the line of intersection of planes $\vec{r}.\vec{n}_1 = q_1$ and $\vec{r}.\vec{n}_2 = q_2$ is $\vec{n}_1 \times \vec{n}_2$.



Q1 Find eqn of line in symmetrical form

$$x+y+z-6=0=2x+y-z-1$$

Sol^n

$$\vec{n_1}\times\vec{n_2} = -2\hat{i}+3\hat{j}-\hat{k}$$

put $\boxed{z=0}$

$$\left.\begin{aligned} x+y &= 6 \\ 2x+y &= 1 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x &= -5 \\ y &= 11 \end{aligned}\right\}$$

$$L: \frac{x+5}{-2} = \frac{y-11}{3} = \frac{z-0}{-1}$$

thr can be infinite ways.

A $(-5, 11, 0)$

DEFINITION :

A straight line in space is characterised by the intersection of two planes which are not parallel and, therefore, the equation of a straight line is present as a solution of the system constituted by the equations of the two planes : $a_1 x + b_1 y + c_1 z + d_1 = 0$; $a_2 x + b_2 y + c_2 z + d_2 = 0$

This form is also known as **unsymmetrical form**.

Some particular straight lines :

	Straight lines	Equation
(i)	Through the origin	$y = mx,\ z = nx$
(ii)	x-axis	$y = 0,\ z = 0$ or $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$
(iii)	y-axis	$x = 0,\ z = 0$ or $\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$
(iv)	z-axis	$x = 0,\ y = 0$ or $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$
(v)	parallel to x-axis	$y = p,\ z = q$
(vi)	parallel to y-axis	$x = h,\ z = q$
(vii)	parallel to z-axis	$x = h,\ y = p$

EQUATION OF A STRAIGHT LINE IN SYMMETRICAL FORM :

(a) **One point form :** Let $A(x_1, y_1, z_1)$ be a given point on the straight line and ℓ, m, n be the d.c's of the line, then its equation is

$$\boxed{\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = r \quad \text{(say)}}$$

It should be noted that $P(x_1 + \ell r, y_1 + mr, z_1 + nr)$ is a general point on this line at a distance r from the point $A(x_1, y_1, z_1)$ i.e. AP = r. One should note that for AP = r; ℓ, m, n must be d.c.'s not d.r.'s. If a, b, c are direction ratios of the line, then equation of the line is

$$\boxed{\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = r} \quad \text{but here } AP \neq r$$

(b) Equation of the line through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

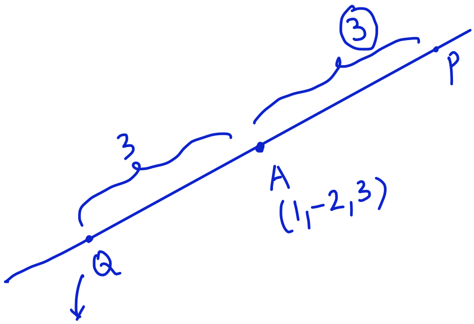
$$\boxed{\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}}$$

Q. Find the co-ordinates of those points on the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{6}$ which is at a distance of 3 units from point (1,–2, 3).

Sol^n

dr's : $(2, 3, 6)$

dc's : $\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$



$P\left(1+3\cdot\left(\frac{2}{7}\right), -2+3\left(\frac{3}{7}\right), 3+3\left(\frac{6}{7}\right)\right)$

$Q \downarrow$

$\left(1+(-3)\left(\frac{2}{7}\right), -2+(-3)\left(\frac{3}{7}\right), 3+(-3)\cdot\frac{6}{7}\right)$

Q. Find the points in which the line x = 1 + 2t ; y = –1 – t and z = 3t meets the coordinate planes i.e. xy, yz, zx plane P, Q, R, Find the equation of the plane containing the point P, Q, R.

Sol^n

(i) xy plane $\Rightarrow$ $z=0$

$\Downarrow$

$3t=0 \Rightarrow t=0$

$\therefore x = 1+2t \Rightarrow x=1$

$y = -1-t \Rightarrow y=-1$

(ii) yz plane $\Rightarrow$ $x=0$

$1+2t=0 \Rightarrow t=-1/2$

$y=-1-t \Rightarrow y=-1+\frac{1}{2}=-\frac{1}{2}$

$z=3t \Rightarrow z=-\frac{3}{2}$

Q Show that the straight lines

$$L_1 : 3x + 2y + z - 5 = 0 = x + y - 2z - 3$$

and $L_2 : 8x - 4y - 4z = 0 = 7x + 10y - 8z$ are at right angle.

Sol^n

$$\vec{a} = \vec{n_1} \times \vec{n_2}$$

$$\vec{b} = \vec{n_3} \times \vec{n_4}$$

$$(\vec{n_1} \times \vec{n_2}) \cdot (\vec{n_3} \times \vec{n_4}) = 0.$$

(do yourself)

Q Find the equation of the line through (1, 4, –2) and parallel to the planes 6x + 2y + 2z + 3 = 0 &

x + 2y – 6z + 4 = 0.

Sol^n

$$L : \frac{x-1}{a} = \frac{y-4}{b} = \frac{z+2}{c}$$

$$\left.\begin{aligned} \vec{n_1} &= 6\hat{i} + 2\hat{j} + 2\hat{k} \\ \vec{n_2} &= \hat{i} + 2\hat{j} - 6\hat{k} \end{aligned}\right\}$$

$$\vec{n_1} \times \vec{n_2} = a\hat{i} + b\hat{j} + c\hat{k}$$

Q Find the equation of the straight line which passes through the point (2, –1, –1) ; is parallel to the plane $4x + y + z + 2 = 0$ and is perpendicular to the line of intersection of the planes $2x + y = 0 = x - y + z$.

Sol^n

$$L: \quad \frac{x-2}{a} = \frac{y+1}{b} = \frac{z+1}{c}$$

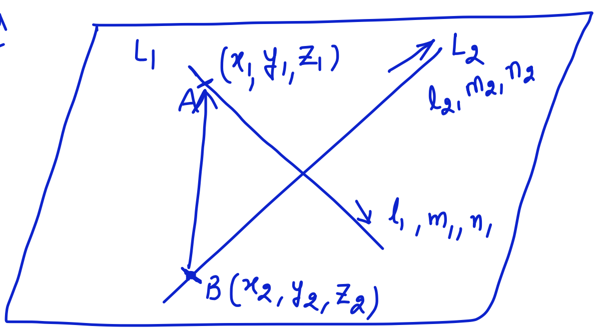
$$a\hat{i} + b\hat{j} + c\hat{k} = (4\hat{i} + \hat{j} + \hat{k}) \times \left((2\hat{i} + \hat{j}) \times (\hat{i} - \hat{j} + \hat{k}) \right)$$

(VTP)

Q If the lines $\dfrac{x-x_1}{l_1} = \dfrac{y-y_1}{m_1} = \dfrac{z-z_1}{n_1}$ and $\dfrac{x-x_2}{l_2} = \dfrac{y-y_2}{m_2} = \dfrac{z-z_2}{n_2}$ are coplanar/intersect then

Value of $\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \,?\ 0$ Ans

Sol^n



[$\vec{BA}$ $\quad l_1\hat{i} + m_1\hat{j} + n_1\hat{k} \quad l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$]
$= 0$

$\Downarrow$

$\begin{vmatrix} & & \\ & & \\ & & \end{vmatrix} = 0$

Q Find the equation of the line through the point with p.v. $\vec{c}$ and || to the plane $\vec{r}.\vec{n} = 1$ and perpendicular to the line $\vec{r} = \vec{a} + t\vec{b}$ $\Rightarrow$ $\vec{r} = \vec{c} + \lambda(\vec{n} \times \vec{b})$; $\lambda \in$ scalar

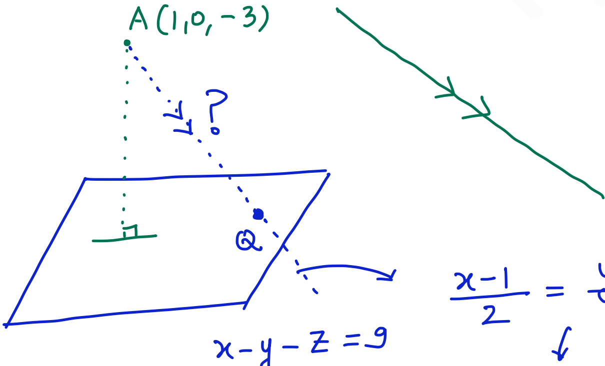
Q Find the equation of the lines passing through the point with p.v. $\vec{a}$ and parallel to the line of intersection of the planes $\vec{r}.\vec{n}_1 = 1$ and $\vec{r}.\vec{n}_2 = 1$.

$\vec{r} = \vec{a} + \lambda(\vec{n}_1 \times \vec{n}_2)$; $\lambda \in$ scalar

E(a) Find the distance of the point A(1, 0, –3) from the plane P : x – y – z = 9 measured parallel to the line L : $\frac{x-2}{2} = \frac{y+2}{3} = \frac{z-6}{-6}$.

(b) Find also the distance of A from L parallel to the plane P.

(a)



$$\frac{x-2}{2} = \frac{y+2}{3} = \frac{z-6}{-6}$$

$$\frac{x-1}{2} = \frac{y-0}{3} = \frac{z+3}{-6} = \lambda$$

general pt

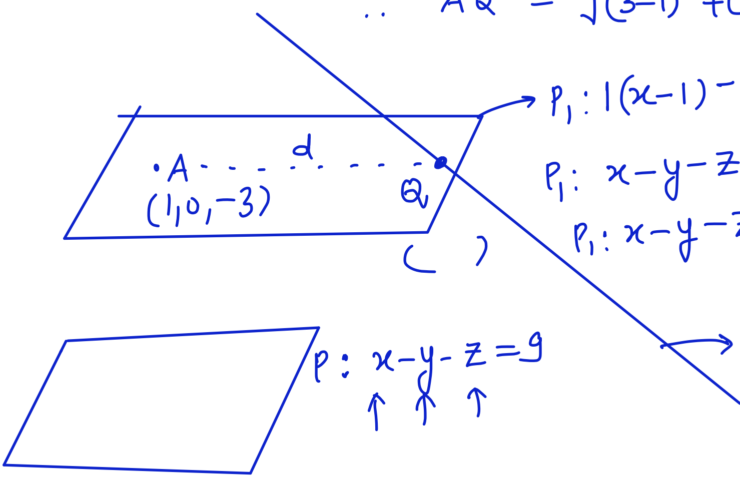
$(2\lambda+1,\ 3\lambda,\ -6\lambda-3)$

$(2\lambda+1)-(3\lambda)-(-6\lambda-3) = 9$

$-\lambda+6\lambda+4 = 9 \Rightarrow 5\lambda = 5 \Rightarrow \boxed{\lambda=1} \Rightarrow Q(3,3,-9)$

$\therefore\ AQ = \sqrt{(3-1)^2+(3-0)^2+(-9+3)^2} = 7$

(b)



$P_1 : 1(x-1)-1(y-0)-1(z+3) = 0$

$P_1 : x-y-z-1-3=0$

$P_1 : x-y-z=4$

$L : \frac{x-2}{2} = \frac{y+2}{3} = \frac{z-6}{-6}$

general pt on Line L

$(2\lambda+2,\ 3\lambda-2,\ -6\lambda+6)$

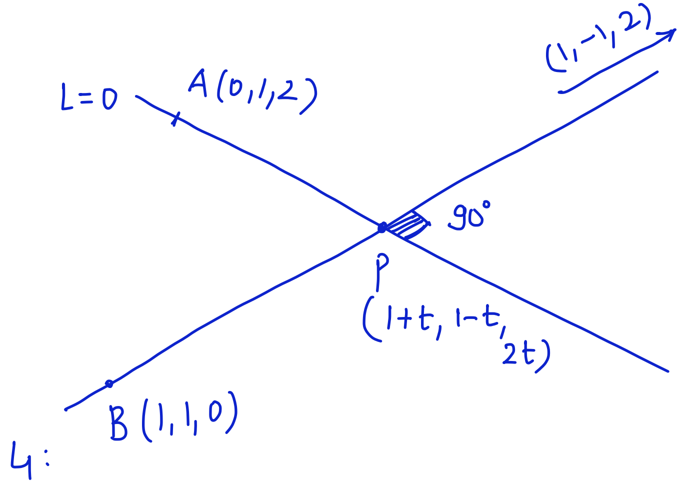
put in P_1 plane

$(2\lambda+2)-(3\lambda-2)-(-6\lambda+6) = 4$

$\lambda =$

Q Find the parametric equation for the line which passes through the point (0, 1, 2) and is perpendicular to the line x = 1 + t, y = 1 – t and z = 2t and also intersects this line.

Sol^n



$L_1: x = 1+t \; ; \; y = 1-t$

$z = 2t$

$$\frac{x-1}{1} = \frac{y-1}{-1} = \frac{z-0}{2}$$

$$\overrightarrow{AP} = (1+t)\hat{i} + (\cancel{1}-t-\cancel{1})\hat{j} + (2t-2)\hat{k}$$

$$\overrightarrow{AP} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 0.$$

$$(1+t) + t + 2(2t-2) = 0$$

$$6t - 4 + 1 = 0$$

$$t = \frac{1}{2}$$

HW Find the equation of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle of $\frac{\pi}{3}$. [Ans: $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ or $\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$]

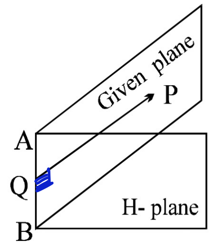
Do yourself

Line of Greatest slope in a plane

It is a line in the plane and perpendicular to the line of intersection of the given plane with the horizontal plane. PQ is the line of greatest slope . Its direction cosines can be determined by the facts that

(i) It lies in G-plane

(ii) It is perpendicular to AB, the line of intersection of G and H plane.



Example

Assuming the plane $4x - 3y + 7z = 0$ to be horizontal find the equation of the line of greatest slope through the point (2, 1, 1) in the plane $\underbrace{2x + y - 5z = 0}_{\text{given plane}}$.

$$L_1: \quad \frac{x-2}{a}=\frac{y-1}{b}=\frac{z-1}{c}$$

$$a\hat{i}+b\hat{j}+c\hat{k} = (2\hat{i}+\hat{j}-5\hat{k}) \times \left((4\hat{i}-3\hat{j}+7\hat{k}) \times (2\hat{i}+\hat{j}-5\hat{k}) \right)$$

(NTP)

Q. Determine whether each statement is true or false

(a) Two lines parallel to a third line are parallel. TRUE

(b) Two lines perpendicular to a third line are parallel. False.

(c) Two planes parallel to a third plane are parallel. TRUE.

(d) Two planes perpendicular to a third plane are parallel. false

(e) Two lines parallel to a plane are parallel. false

(f) Two lines perpendicular to a plane are parallel. TRUE

(g) Two planes parallel to a line are parallel. false.

(h) Two planes perpendicular to a line are parallel. TRUE.

(i) Two planes either intersect or are parallel. True.

(j) Two lines either intersect or are parallel. False

(k) A plane and a line either intersect or are parallel. True

Q If $l^2+m^2+n^2=125$, $a^2+b^2+c^2=5$ and $al+bm+cn=25$, where $a,b,c,l,m,n \in R$, then value of $\frac{lmn}{abc}$ is μ, where sum of digits of μ is

HW

HW

Vector → S-2

3-D → O-1 Q 1 to 11.