Q HW If $l^2+m^2+n^2=125$, $a^2+b^2+c^2=5$ and $al+bm+cn=25$, where $a,b,c,l,m,n \in R$, then value of $\frac{lmn}{abc}$ is μ, where sum of digits of μ is

$$\vec{v_1}\cdot\vec{v_2}=25$$

Sol^n

$$\vec{v_1} = l\hat{i}+m\hat{j}+n\hat{k}$$

$$\vec{v_2} = a\hat{i}+b\hat{j}+c\hat{k}$$

$$|\vec{v_1}| = \sqrt{l^2+m^2+n^2} = \sqrt{125}$$

$$|\vec{v_2}| = \sqrt{a^2+b^2+c^2} = \sqrt{5}$$

$$\vec{v_1}\cdot\vec{v_2} = |\vec{v_1}||\vec{v_2}|\cos\theta$$

$$25 = \sqrt{125}\sqrt{5}\cos\theta \Rightarrow \cos\theta = 1 \Rightarrow \boxed{\theta = 0^\circ}$$

$\therefore \vec{v_1}$ is collinear with $\vec{v_2}$ i.e $\vec{v_1} \parallel \vec{v_2}$

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \lambda \text{ (say)} ; \lambda > 0.$$

$$l = a\lambda ; m = b\lambda ; n = c\lambda .$$

$$l^2+m^2+n^2 = (a^2+b^2+c^2)\lambda^2$$

$$125 = 5\lambda^2 \Rightarrow \boxed{\lambda = 5}$$

$$\frac{l}{a} = \lambda = 5$$

$$\frac{lmn}{abc} = \left(\frac{l}{a}\right)\left(\frac{m}{b}\right)\left(\frac{n}{c}\right)$$

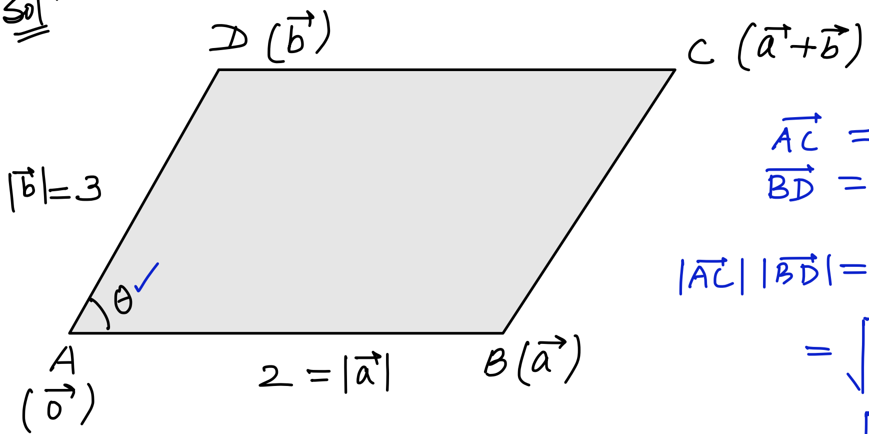
$$= \lambda\cdot\lambda\cdot\lambda = \lambda^3$$

$$\mu = 125$$

sum of digits of $\mu = 8$ Ans

Q Given in a parallelogram ABCD, AB = 2, AD = 3 and M, m denotes the maximum and minimum integral value of product $|\vec{AC}||\vec{BD}|$ then $(M - m)$ is ______

Sol^n



$\vec{AC} = \vec{a} + \vec{b}$

$\vec{BD} = \vec{b} - \vec{a}$

$|\vec{AC}||\vec{BD}| = |\vec{a}+\vec{b}||\vec{b}-\vec{a}|$

$= \sqrt{a^2 + b^2 + 2\vec{a}\cdot\vec{b}}\sqrt{a^2 + b^2 - 2\vec{a}\cdot\vec{b}}$

$E = |\vec{AC}||\vec{BD}| = \sqrt{\underbrace{4+9}\,+ 12\cos\theta}\sqrt{\underbrace{4+9} - 12\cos\theta}$ ★

$E = \sqrt{169 - 144\cos^2\theta}$; $\boxed{0 \le \cos^2\theta < 1}$

Range of $E = (5, 13]$

$M = 13$

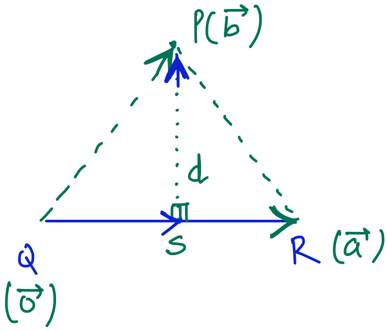
$m = 6$

$\therefore M - m = 13 - 6 = 7$ Ans

Q Let P be a point not on the line L that passes through the points Q and R where $\overrightarrow{QR} = \vec{a}$ & $\overrightarrow{QP} = \vec{b}$. The distance d from the point P to the line L is equal to-

(A) $\dfrac{|\vec{a}\times\vec{b}|}{|\vec{b}|}$ (B) $\dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|}$ (C) $\left|\vec{b}-\left(\dfrac{\vec{a}.\vec{b}}{\vec{a}^2}\right)\vec{a}\right|$ (D) $\sqrt{|\vec{b}|^2-\left(\dfrac{\vec{b}.\vec{a}}{|\vec{a}|}\right)^2}$

Sol^n



$$\Delta PQR = \cancel{\tfrac{1}{2}}\,|\vec{a}\times\vec{b}| = \cancel{\tfrac{1}{2}}\,\underbrace{|\overrightarrow{QR}|}_{|\vec{a}|}\cdot d$$

$$d = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}|} \rightarrow [B]$$

$$\overrightarrow{QS} + \overrightarrow{SP} = \overrightarrow{QP}$$

$$|\overrightarrow{QS}| = |\text{proj of } \overrightarrow{QP} \text{ on } \vec{a}|$$

$$|\overrightarrow{QS}| = |\vec{b}\cdot\hat{a}|$$

$$\boxed{\overrightarrow{QS} = |\vec{b}\cdot\hat{a}|\,\hat{a}.}$$

$$\overrightarrow{SP} = \overrightarrow{QP} - \overrightarrow{QS}$$

$$\overrightarrow{SP} = \vec{b} - |\vec{b}\cdot\hat{a}|\,\hat{a}$$

$$|\overrightarrow{SP}| = \left|\vec{b} - \frac{|\vec{b}\cdot\vec{a}|\,\vec{a}}{|\vec{a}|^2}\right|$$

$\rightarrow$ [C]

$$PS^2 = QP^2 - QS^2$$

$$= b^2 - ((\vec{b}\cdot\hat{a})\hat{a})^2$$

$\rightarrow$ [D]

[B, C, D]

Q The angle θ between two non-zero vectors $\vec{a}$ & $\vec{b}$ satisfies the relation

$$\cos\theta = (\vec{a}\times\hat{i}).(\vec{b}\times\hat{i}) + (\vec{a}\times\hat{j}).(\vec{b}\times\hat{j}) + (\vec{a}\times\hat{k}).(\vec{b}\times\hat{k}),$$

then the least value of $|\vec{a}|+|\vec{b}|$ is equal to (where $\theta \neq 90°$)

(A) $\frac{1}{2}$ (B) 2 (C) $\sqrt{2}$ (D) 4

Sol^n

$$\cos\theta = (\vec{a}\cdot\vec{b}) - (\vec{a}\cdot\hat{i})(\vec{b}\cdot\hat{i}) + (\vec{a}\cdot\vec{b}) - (\vec{a}\cdot\hat{j})(\vec{b}\cdot\hat{j}) + (\vec{a}\cdot\vec{b}) - (\vec{a}\cdot\hat{k})(\vec{b}\cdot\hat{k})$$

$$\cos\theta = 3(\vec{a}\cdot\vec{b}) - \vec{a}\cdot\vec{b} \Rightarrow \cancel{\cos\theta} = 2\vec{a}\cdot\vec{b} = 2|\vec{a}||\vec{b}|\cancel{\cos\theta}$$

$$|\vec{a}||\vec{b}| = \frac{1}{2}$$

AM $\geq$ GM

$$\frac{|\vec{a}|+|\vec{b}|}{2} \geq \sqrt{|\vec{a}||\vec{b}|}$$

$$|\vec{a}|+|\vec{b}| \geq \cancel{2}\cdot\frac{1}{\cancel{\sqrt{2}}}$$

$\sqrt{2}$ [C]

Q If $x^2+y^2+z^2=1$ where $x, y, z \in \mathbb{R}$ and maximum value of $(2x-y)^2+(3y-2z)^2+(z-3x)^2$ is λ then $\lambda =$?

Solⁿ

$$\vec{v_1} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v_2} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{v_1} \times \vec{v_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 2 & 3 \end{vmatrix} = (3y-2z)\hat{i} + (2x-y)\hat{k} - (3x-z)\hat{j}$$

$$|\vec{v}| = |\vec{v_1} \times \vec{v_2}| = \sqrt{(3y-2z)^2+(2x-y)^2+(3x-z)^2}.$$

$v^2_{max} = ?$

$$|\vec{v}|_{max} = |\vec{v_1} \times \vec{v_2}|_{max} = |v_1|\,|v_2|\,\underbrace{\sin\theta}_{1}$$

$$|\vec{v}|_{max} = \sqrt{x^2+y^2+z^2}\,\sqrt{1^2+2^2+3^2}\,.\,1$$

$$|\vec{v}|_{max} = 1.\sqrt{14}$$

$$|\vec{v}|^2_{max} = 14$$ Ans

Q If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors having magnitude 1, 2, 3 respectively then $[\vec{a}+\vec{b}+\vec{c} \quad \vec{b}-\vec{a} \quad \vec{c}] = ?$

Sol^n $\quad R_1 \to R_1 + R_2 - R_3$

$$[2\vec{b} \quad \vec{b}-\vec{a} \quad \vec{c}]$$

$$\underbrace{\cancel{[2\vec{b} \quad \vec{b} \quad \vec{c}]}}_{0} - \underbrace{[2\vec{b} \quad \vec{a} \quad \vec{c}]}_{0}$$

$$2[\vec{a} \quad \vec{b} \quad \vec{c}] = 2|\vec{a}||\vec{b}||\vec{c}|$$

$$= 2(1)(2)(3)$$

$$= 12 \text{ Ans}$$

Q Let $|\vec{a}|=\sqrt{3}, |\vec{b}|=5, \vec{b}.\vec{c}=10$ angle between $\vec{b}$ & $\vec{c}$ equal to $\frac{\pi}{3}$. If $\underbrace{\vec{a} \text{ is perpendicular to } \vec{b}\times\vec{c}}$ then find the value of $|\vec{a}\times(\vec{b}\times\vec{c})|$.

$\phi(\vec{a} \wedge (\vec{b}\times\vec{c})) = \pi/2$

Sol^n

$$\vec{b}.\vec{c} = |\vec{b}||\vec{c}|\cos\theta = 10$$

$$5\,|\vec{c}|\cdot\frac{1}{2} = 10 \Rightarrow |\vec{c}| = 4$$

$$|\vec{a}\times(\vec{b}\times\vec{c})| = |\vec{a}|\,|\vec{b}\times\vec{c}|\sin 90^\circ$$

$$= \sqrt{3}\,(5)(4)\cdot\sin 60^\circ$$

$$= \sqrt{3}\,(\cancel{20})^{10}\cdot\frac{\sqrt{3}}{\cancel{2}} = 30 \text{ Ans}$$

Paragraph

Consider a plane Π whose equation is x – y = 0 and two points A(3,2,4) and B(7,0,–10)

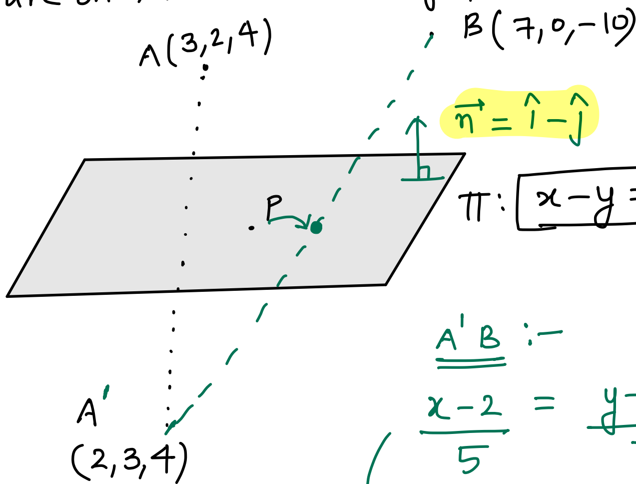
1. Coordinates of point P, which lies on plane Π such that |PA + PB| is minimum, are-

(A) (21,21,9) (B) $\left(\frac{21}{4},\frac{21}{4},\frac{9}{4}\right)$ (C) $\left(\frac{21}{8},\frac{21}{8},\frac{9}{4}\right)$ (D) $\left(\frac{21}{16},\frac{21}{16},\frac{9}{16}\right)$

2. Angle between $\overrightarrow{AB}$ and vector perpendicular to plane Π is-

(A) $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$ (B) $\frac{\pi}{4}$ (C) $\cos^{-1}\left(\frac{1}{6}\right)$ (D) $\cos^{-1}\left(\frac{1}{12}\right)$

Sol^n A & B are on the same side of plane $x-y=0$



$\Pi : \boxed{x - y = 0}$

A'B :–

$$\frac{x-2}{5} = \frac{y-3}{-3} = \frac{z-4}{-14} = \lambda$$

general pt

$x = 5\lambda + 2$; $z = 4 - 14\lambda$

$y = -3\lambda + 3$

put in eqn of given plane :

$$(5\lambda+2) - (-3\lambda+3) = 0$$

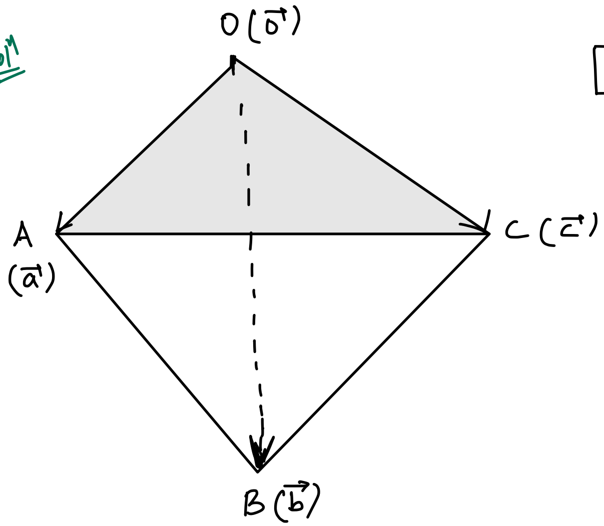
$$8\lambda - 1 = 0 \Rightarrow \boxed{\lambda = \frac{1}{8}}$$

$\therefore$ P(   ) → [C]

Q In a tetrahedron OABC, the measures of the $\angle BOC$, $\angle COA$ & $\angle AOB$ are α, β & γ respectively, then $(\cos^2\alpha + \cos^2\beta + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma)$ can attain-

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\pi}{4}$ (C) 1 (D) 2

Sol^n



$$[\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix}$$

$$\begin{vmatrix} a^2 & ab\cos\gamma & ac\cos\beta \\ ab\cos\gamma & b^2 & bc\cos\alpha \\ ac\cos\beta & bc\cos\alpha & c^2 \end{vmatrix}$$

$$[\vec{a}\ \vec{b}\ \vec{c}]^2 = a^2b^2c^2 \begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$$

$$\underbrace{[\vec{a}\ \vec{b}\ \vec{c}]^2}_{>0} = \underbrace{a^2b^2c^2}_{>0} \left(\underbrace{1 - \Sigma\cos^2\alpha + 2\cos\alpha\cos\beta\cos\gamma}_{>0} \right)$$

$$\Sigma\cos^2\alpha - 2\cos\alpha\cos\beta\cos\gamma < 1$$

$[A, B]$

Q Let $\vec{a} = 3\hat{i} - 5\hat{k}$, $\vec{b} = 2\hat{i} + 7\hat{j}$ and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$. Consider $\vec{r}$ such that $\vec{r}.\vec{a} = -1$, $\vec{r}.\vec{b} = 6$ and $\vec{r}.\vec{c} = 5$. Then the vector component of $2\hat{i} + 3\hat{j} + 4\hat{k}$ along $\vec{r}$ is $n\left(\frac{\ell\hat{i} + m\hat{k}}{\ell^2 + m^2}\right)$, where ℓ & m are coprimes, then $\left(\ell^2 + m^2 + n^2\right)$ is equal to

209 Ans

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Sol^n

$\vec{r} \cdot (\vec{b} + \vec{a} - \vec{c}) = 0$

$\hat{r} = \frac{3\hat{i} + 2\hat{k}}{\sqrt{13}}$

$$\left(\underbrace{(2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot \hat{r}}\right)\hat{r}$$

$$\left(\frac{14}{\sqrt{13}}\right)\frac{(3\hat{i} + 2\hat{k})}{\sqrt{13}} = 14\left(\frac{3\hat{i} + 2\hat{k}}{13}\right)$$

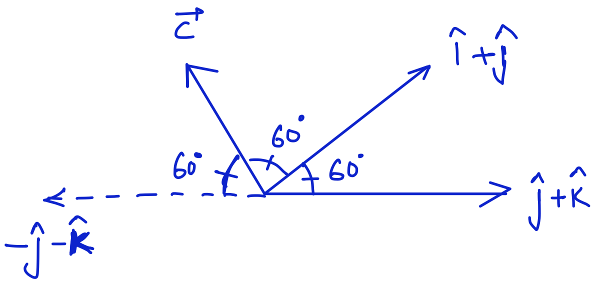
$$\left.\begin{aligned} n &= 14 \\ \ell &= 3 \\ m &= 2 \end{aligned}\right\}$$

Q If $\hat{i}+\hat{j}$ bisects the angle between $\vec{c}$ & $\hat{j}+\hat{k}$, then $\vec{c}.\hat{j}$ is equal to

(A) 0 (B) $\frac{1}{\sqrt{2}}$ (C) $-\frac{1}{\sqrt{2}}$ (D) 1

$$(\hat{i}+\hat{j})\cdot(\hat{j}+\hat{k}) = \sqrt{2}\,\sqrt{2}\cos\theta$$

$$1 = 2\cos\theta \Rightarrow \theta = 60^\circ$$



$$\vec{c} = \lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} - \frac{(\hat{j}+\hat{k})}{\sqrt{2}}\right) \Rightarrow \vec{c} = \frac{\lambda}{\sqrt{2}}(\hat{i}-\hat{k})$$

$\vec{c}\cdot\hat{j} = 0.$ [A]

Q The direction cosines of the projection of the line $\frac{1}{2}(x-1) = -y = z+2$ on the plane $2x + y - 3z = 4$ are-

(A) $\left(\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

(B) $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

(C) $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$

(D) None of these

$\vec{n} = 2\hat{i} + \hat{j} - 3\hat{k}$

$L: \quad \frac{x-1}{2} = \frac{y}{-1} = \frac{z-(-2)}{1}$

$\vec{a}$ = dir^n vector : $2\hat{i} - \hat{j} + \hat{k}$ → dc's : $\pm\left(\frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

$\vec{a} \cdot \vec{n} = 4 - 1 - 3 = 0$

$\Rightarrow$ line is $\parallel$ to plane

[A]

Q The direction ratios of a normal to the plane passing through (1,0,0), (0,1,0) and making an angle $\frac{\pi}{4}$ with plane x + y = 3 can be-

(A) 0,1,0 (B) $1,1,\sqrt{2}$

(C) 1,0,0 (D) $\sqrt{2},1,1$

Sol^n Let eqn of plane be

$$P: \boxed{a(x-1)+b(y-0)+c(z-0)=0} \quad -①-$$

(0,1,0)

$$a(-1)+b+0=0 \Rightarrow \boxed{b=a} \quad -②-$$

$$\vec{n} = a\hat{i}+b\hat{j}+c\hat{k}$$

$$\vec{n_1} = \hat{i}+\hat{j}$$

$$\vec{n}\cdot\vec{n_1} = |\vec{n}||\vec{n_1}|\cos 45^\circ$$

$$a+b = \sqrt{a^2+b^2+c^2}\,\cancel{\sqrt{2}}\cdot\frac{1}{\cancel{\sqrt{2}}}$$

$$\cancel{a^2}+\cancel{b^2}+2ab = \cancel{a^2}+\cancel{b^2}+c^2$$

$$2a^2 = c^2 \Rightarrow \boxed{c=\sqrt{2}a} \text{ or } \boxed{c=-\sqrt{2}a}$$

$$a:b:c \equiv 1:1:\sqrt{2}$$

or

$$1:1:-\sqrt{2}$$

[B]

HW:

3-D Sheet

O-1 Q12 to 20.

S-1 Complete

J-M Q1 to 7.