THREE EVENTS DEFINED ON AN EXPERIMENTAL PERFORMANCE

For any three events A,B and C

Rem

(i) $\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C) - [P(A \cap B) + P(B \cap C) + P(C \cap A)] + P(A \cap B \cap C)}$

[Proof : Let $B \cup C = D$

L.H.S. $P(A \cup D) = P(A) + P(D) - P(A \cap D) = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap (B \cup C))]$

$= \sum P(A) - P(B \cap C) - [P(A \cap B) \cup (A \cap C)]$

$= \sum P(A) - P(B \cap C) - [P(A \cap B) + (A \cap C) - P(A \cap B \cap C)]$

$\therefore \quad P(A \cup B \cup C) = \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$

(ii) P (exactly one appearing) = P(E – 1)

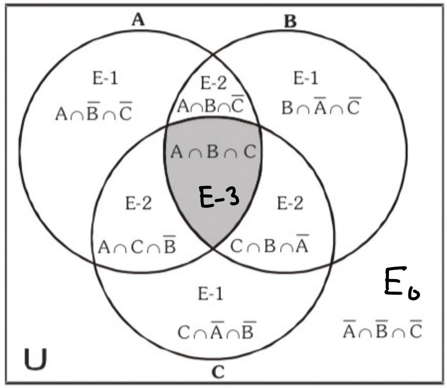
$P(A) + P(B) + P(C) - 2[P(A \cap B) + P(B \cap C) + P(C \cap A)] + 3P(A \cap B \cap C)$

(iii) P(exactly two of occurring) = P(E – 2)

$P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$

(iv) P(at least two occurring) = P(E – 2) + P(E – 3)

$P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$



E-1 $\rightarrow$ $A \cap \bar{B} \cap \bar{C} + \bar{A} \cap B \cap \bar{C} + \bar{A} \cap \bar{B} \cap C$

E-2 $\rightarrow$ $A \cap B \cap \bar{C} + A \cap \bar{B} \cap C + \bar{A} \cap B \cap C$

E-3 $\rightarrow$ $A \cap B \cap C$

E_0 $\rightarrow$ $\bar{A} \cap \bar{B} \cap \bar{C}$ (None of them occurs).

$P(\bar{A} \cap \bar{B} \cap \bar{C}) = 1 - P(A \cup B \cup C)$

Imp

Note :

(i) If A,B,C are pairwise independent $\Rightarrow P(A\cap B) = P(A).P(B), P(B\cap C) = P(B).P(C), P(C\cap A) = P(C).P(A)$

(ii) However, if A, B, C are pair wise independent $\not\Rightarrow$ they are independent. Infact for 3 events A, B and C to be independent :

$$P(A \cap B) = P(A) . P(B); P(B \cap C) = P(B) . P(C) ; P(C \cap A) = P(C) . P(A)$$ i.e Pairwise

and $$P(A \cap B \cap C) = P(A) . P(B) . P(C)$$ i.e mutually.

* Similarly for n independent events, the total number of conditions would be

$$\boxed{{}^nC_2 + {}^nC_3 + \ldots\ldots + {}^nC_n = 2^n - n - 1}$$

Q A fair coin is tossed two times resulting in an equiprobable space {HT, TH, TT, HH}. Three events are defined as

A : Head occuring on 1^{st} toss {HT, HH} $\Rightarrow P(A) = 1/2$

B : Head occuring on 2^{nd} toss {TH, HH} $\Rightarrow P(B) = 1/2$

C : Head occuring exactly on one toss {HT, TH} $\Rightarrow P(C) = 1/2$

$A\cap B$: {HH}

$B\cap C$: {TH}

$C\cap A$: {HT}

(i) Check whether the events are pair wise independent ? YES

$A\cap B\cap C = \phi$

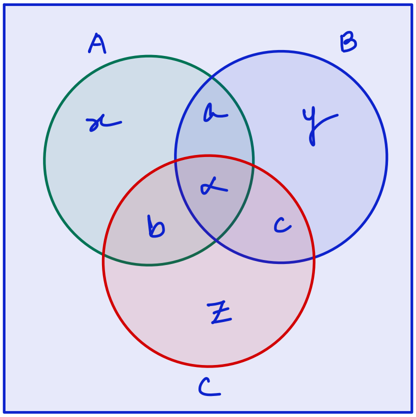
(ii) Check whether the events are independent ? NO

(i)
$$\left.\begin{aligned} P(AB) &= P(A\cap B) = \frac{1}{4} = P(A).P(B) \\ P(BC) &= P(B\cap C) = \frac{1}{4} = P(B).P(C) \\ P(CA) &= P(C\cap A) = \frac{1}{4} = P(A).P(B) \end{aligned}\right\}$$

$$P(A\cap B\cap C) = 0 \neq P(A).P(B).P(C)$$

Q There are 3 clubs A, B and C in a town with 40, 50 and 60 members respectively. 10 people are members of all the three clubs, 70 are members in only one club. A member is randomly selected. Find the probability that he had membership of exactly two clubs.

(A) $\frac{7}{15}$ (B) $\frac{1}{6}$ (C) $\frac{3}{21}$ (D) $\frac{5}{21}$



$n(A) = x + a + \alpha + b = 40.$

$n(B) = y + a + \alpha + c = 50.$

$n(C) = z + b + \alpha + c = 60$

$n(A \cap B \cap C) = \boxed{\alpha = 10}$

$x + y + z = 70.$

$$\left.\begin{aligned} x + a + b &= 30. \\ y + a + c &= 40. \\ z + b + c &= 50. \end{aligned}\right\} \text{add}$$

$$\boxed{\text{Req. prob} = \frac{a+b+c}{\text{Total}}}$$

$\underbrace{x+y+z}_{0} + 2(a+b+c) = 120$

$2(a+b+c) = 50$

$a+b+c = 25.$

$\text{Total} = (x+y+z) + (a+b+c) + \alpha$

$= 70 + 25 + 10 = 105.$

$\text{Prob} = \frac{25}{105} = \frac{5}{21}$

[D] ✓

Q For 3 events A, B and C

P(exactly one of the events A or B occurs)
P(exactly one of the events B or C occurs)
P(exactly one of the events C or A occurs)

$\left.\right\} = p$

P(all the 3 events occurs simultaneously) = p^2 → $P(A \cap B \cap C) = p^2$

If A, B, C are exhaustive then find the value(s) of p.

↳ $P(A \cup B \cup C) = 1$

$$\left.\begin{aligned} P(A) + P(B) - 2P(A \cap B) &= p. \\ P(B) + P(C) - 2P(B \cap C) &= p \\ P(C) + P(A) - 2P(C \cap A) &= p \end{aligned}\right\}$$

add: $2\sum P(A) - 2\sum P(A \cap B) = 3p$

$\sum P(A) - \sum P(A \cap B) = \frac{3p}{2}$

$$P(A \cup B \cup C) = \underbrace{P(A) + P(B) + P(C) - \left(P(A \cap B) + P(B \cap C) + P(C \cap A)\right)} + P(A \cap B \cap C)$$

$$1 = \frac{3p}{2} + p^2$$

$$2p^2 + 3p - 2 = 0 \Rightarrow p = \cancel{-2} \text{ or } \boxed{p = \frac{1}{2}} ✓$$

Q If A,B,C are independent events such that P(A) = p, P(B) = q, P(C) = r where $p,q,r \in (0,1)$. Let

E_1: only A occurs

E_2: none of A,B,C occurs

E_3: only event C occurs.

If probabilities of events E_1, E_2, E_3 are in GP. Then find value of p+r ?

Sol^n

$$P(E_1) = P(A \cap \bar{B} \cap \bar{C}) = P(A).P(\bar{B}).P(\bar{C})$$
$$= p(1-q)(1-r)$$

$$P(E_2) = P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}).\ P(\bar{B})\ P(\bar{C}) = (1-p)(1-q)(1-r)$$

$$P(E_3) = P(\bar{A} \cap \bar{B} \cap C) = (1-p)(1-q)\, r.$$

$$(P(E_2))^2 = P(E_1) \cdot P(E_3) \Rightarrow (1-p)^2 \cancel{(1-q)^2} (1-r)^2 = p\cancel{(1-q)}\cancel{(1-r)} \times \cancel{(1-p)}\cancel{(1-q)} \cdot r$$

$$(1-p)(1-r) = pr$$

$$1 - p - r + \cancel{pr} = \cancel{pr}$$

$$p + r = 1 \quad \text{Ans}$$

Q A purse contains 10 tickets, five printed with I and five printed with T. 3 tickets are drawn without replacement and arranged in the same order in which they are drawn on the table. Find the probability that IIT is formed.

Solⁿ $P(IIT) = P(I) \cdot P(I/I) \cdot P(T/I \cap I)$

$= \left(\frac{5}{10}\right)\left(\frac{4}{9}\right)\left(\frac{5}{8}\right)$

Q A problem in mathematics is given to 2 children who solve it independently. If probability of A solving it is 1/2 and probability of B solving it is 2/3. Find the probability that the problem is solved.

Solⁿ

$P(\text{prob is solved}) = 1 - P(\bar{A} \cap \bar{B})$

$= 1 - P(\bar{A}) \cdot P(\bar{B})$

$= 1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{5}{6}.$

Q Probability that a teacher takes a surprise test is 1/3. If a student remains absent for two days then find the probability that he misses

(i) exactly one test (ii) at least one test (iii) atmost one test

$S : \{ TT, T\bar{T}, \bar{T}T, \bar{T}\bar{T} \}$

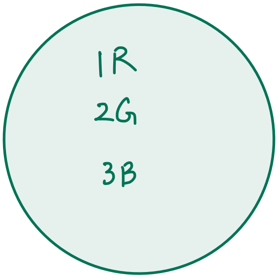
(i) $P(T\bar{T}) + P(\bar{T}T) = \frac{1}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{1}{3} = \frac{4}{9}$

(ii) $1 - P(\bar{T}\bar{T}) = 1 - \frac{2}{3} \cdot \frac{2}{3} = \frac{5}{9}$

(iii) $1 - P(TT) = 1 - \frac{1}{3} \cdot \frac{1}{3} = \frac{8}{9}$.

Q An urn contains 1R, 2G and 3B balls. Three people A, B & C in order draw one ball from the urn and put it back after noting its colour. They continue doing it indefinitely unless one who draws a red ball first wins the game. Compute their respective chances of winning the game.

$$P(R) = \frac{1}{6} \quad ; \quad P(\bar{R}) = \frac{5}{6}$$



$$P(A \text{ wins}) = P(R \text{ or } \bar{R}\bar{R}\bar{R}R \text{ or } \bar{R}\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}R \text{ or } \ldots\ldots)$$

$$= P(R) + P(\bar{R}\bar{R}\bar{R}R) + P(\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}R) + \ldots\ldots$$

$$= P(R) + P(\bar{R})P(\bar{R})P(\bar{R})P(R) + (P(\bar{R}))^6 P(R) + \ldots\ldots$$

$$= \frac{P(R)}{1-(P(\bar{R}))^3} = \frac{1/6}{1-\left(\frac{5}{6}\right)^3} = \frac{1/6}{\left(\frac{216-125}{6^3}\right)}$$

$$= \frac{36}{91} \checkmark$$

$$P(B \text{ win}) = P(\bar{R}R \text{ or } \bar{R}\bar{R}\bar{R}\bar{R}R \text{ or } \ldots\ldots)$$

$$= P(\bar{R})P(R) + (P(\bar{R}))^4 P(R) + \ldots\ldots$$

$$= \frac{P(\bar{R})P(R)}{1-(P(\bar{R}))^3} = \frac{\frac{1}{6}\cdot\frac{5}{6}}{1-\left(\frac{5}{6}\right)^3} = \frac{5\times 6}{91}$$

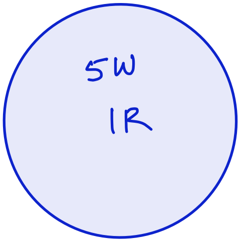
$$= \frac{30}{91} \checkmark$$

$$P(C \text{ wins}) = 1 - \left(\frac{36}{91} + \frac{30}{91}\right) = \frac{25}{91} \text{ Ans}$$

Q. Two persons A and B one by one in order drawn one ball each from a purse containing 5W and 1R balls and retain it. The person who gets a red ball wins the game.

E : event that 'A' wins and F : event that 'B' wins . Compute E and F.

Sol^n



$$P(A \text{ wins}) = P(R \text{ or } \bar{R}\bar{R}R \text{ or } \bar{R}\bar{R}\bar{R}\bar{R}R)$$

$$= P(R) + P(\bar{R}\bar{R}R) + P(\bar{R}\bar{R}\bar{R}\bar{R}R)$$

$$= \frac{1}{6} + \left(\frac{\cancel{5}}{6}\cdot\frac{\cancel{4}}{\cancel{5}}\cdot\frac{1}{\cancel{4}}\right) + \left(\frac{\cancel{5}}{6}\cdot\frac{\cancel{4}}{\cancel{5}}\cdot\frac{\cancel{3}}{\cancel{4}}\cdot\frac{\cancel{2}}{\cancel{3}}\cdot\frac{1}{\cancel{2}}\right)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

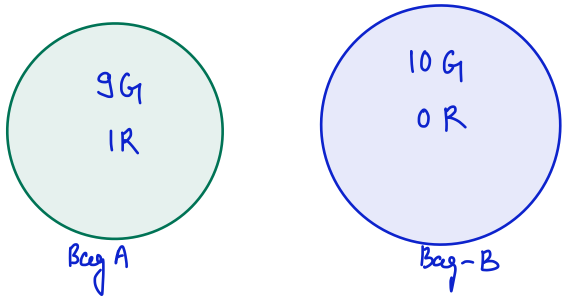
$$P(A \text{ wins}) = \frac{1}{2} = P(B \text{ wins}).$$

HW

Q. Ram & Shyam toss a coin in turn indefinitely untill head occur for the first time. The person getting the head for the first time wins. If Ram start tossing, then find their respective chance of winning.

Q Bag A has 9G and 1R balls & Bag B has 10G balls and no red ball. 9 balls are transferred from Bag A to Bag B & then 9 balls are transfered from Bag B to Bag A. After these pair of operation are performed find the probability that red ball is still in Bag A.

Sol^n



C-I : 9G

$$\left(\frac{{}^{9}C_9}{{}^{10}C_9}\right)\left(\frac{{}^{19}C_9}{{}^{19}C_9}\right) = a$$

C-II $8G + 1R$

$$\left(\frac{{}^{9}C_8 \cdot {}^{1}C_1}{{}^{10}C_9}\right)\left(\frac{{}^{18}C_8 \cdot {}^{1}C_1}{{}^{19}C_9}\right) = b$$

$(a+b)$ Ans

$8 \to 2, 2, 2, 2$

Q A fair coin whose faces are marked with one & two is thrown for four times. Find the probability of throwing a total of : (a) 6 (b) atleast 5 (c) atmost 7

Sol^n (a) $\underbrace{1, 1, 2, 2}_{\text{6 cases}} \to \left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) = \frac{1}{16}$

$\frac{4!}{2!\,2!} = 6$

$\text{Prob} = 6 \times \frac{1}{16} = \frac{3}{8}$

(c) $1 - P(\text{sum} = 8)$

$1 - \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$

$= \frac{15}{16}$

1 1 1 1

(b) $P(\text{atleast } 5) = 1 - P(\text{sum} = 4)$

$= 1 - \left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) = \frac{15}{16}$

Q HW
A coin is continued tossing until either a head and a tail is obtained for the first time or unless the coin has been tossed for a maximum of five times. If the first two tosses have resulted in both tails, find the chance that the coin will be tossed 5 times.

HW

Q All face cards from a pack of 52 playing cards are removed. From the remaining 40 cards, 4 are drawn.

(i)

Find the probability that they are of different suit different denominations.

(ii) Same suit & diff denominations.

(iii) diff suit & same " .

Ex O-1 Part - 3 & 4.