Q If for unit vectors $\hat{a}, \hat{b}$ and non-zero $\vec{c}$, $\hat{a} \times \hat{b} + \hat{a} = \vec{c}$ and $\hat{b}.\vec{c} = 0$, then volume of parallelopiped with coterminous edges $\hat{a}, \hat{b}$ and $\vec{c}$ will be (in cu.units)-

(A) 6 (B) 4 (C) 1 (D) $\frac{1}{2}$

Sol^n $V = [\hat{a}\ \hat{b}\ \vec{c}] = (\hat{a} \times \hat{b}).\vec{c}$

$\hat{b}.\vec{c} = 0$ — ① —

$\boxed{\hat{a} \times \hat{b} + \hat{a} = \vec{c}}$ — ② —

dot with $\vec{c}$:

$$\underbrace{(\hat{a} \times \hat{b}).\vec{c}}_{V} + \hat{a}.\vec{c} = c^2$$

$$\boxed{V = c^2 - \hat{a}.\vec{c}} \quad \text{— ③ —}$$

② dot with $\hat{b}$:—

$$\underbrace{(\hat{a} \times \hat{b}).\hat{b}}_{0} + \hat{a}.\hat{b} = \underbrace{\hat{b}.\vec{c}}_{0} \Rightarrow \boxed{\hat{a}.\hat{b} = 0} \Rightarrow \theta(\hat{a} \wedge \hat{b}) = \frac{\pi}{2}$$

② dot with $\hat{a}$:—

$$\underbrace{(\hat{a} \times \hat{b}).\hat{a}}_{0} + \underbrace{\hat{a}.\hat{a}}_{1} = \vec{c}.\hat{a} \Rightarrow \boxed{\vec{c}.\hat{a} = 1}$$

$$(\hat{a} \times \hat{b})^2 = (\vec{c} - \hat{a})^2$$

$$\underbrace{\hat{a}^2 \hat{b}^2}_{1} \underbrace{\sin^2 \frac{\pi}{2}}_{1} = c^2 + \underbrace{\hat{a}^2}_{1} - 2\underbrace{\vec{c}.\hat{a}}_{1}$$

$$\cancel{1} = c^2 + \cancel{1} - 2$$

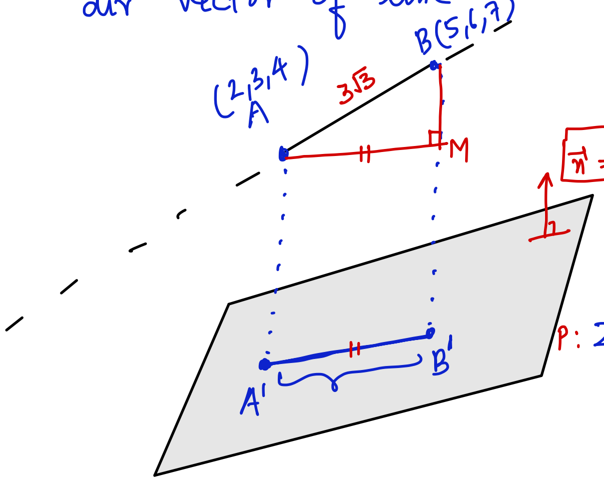
$\boxed{|\vec{c}| = \sqrt{2}}$ put in ③

$$\boxed{V = 2 - 1 = 1}$$ Ans

Q Projection of line segment joining (2,3,4) and (5,6,7) on plane $2x + y + z = 1$ is :-

(A) 2 (B) $\sqrt{3}$ (C) 3 (D) $3\sqrt{3}$

Sol^n dir^n vector of line $3\hat{i}+3\hat{j}+3\hat{k} = \overrightarrow{AB}$



$BM = \left|\text{proj of } \overrightarrow{AB} \text{ on } \vec{n}\right|$

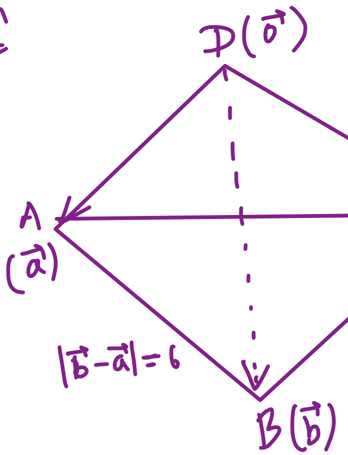
$\boxed{\vec{n} = 2\hat{i}+\hat{j}+\hat{k}}$

$\boxed{BM = 2\sqrt{6}}$

$AM = \sqrt{AB^2 - BM^2} = \sqrt{3}$

Q Let shortest distance between two opposite edges of a tetrahedron is '4 unit' and the length of these opposite edges are same and equal to 6 unit. If angle between these two opposite edges is 30° and volume of tetrahedron is V, the value of $\frac{V}{6}$ is

Sol^n



$$\boxed{V=\frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]}$$

AB & CD are opp edges

$|\vec{AB}| = |\vec{CD}| = 6.$

S.D = 4

$$S.D = \left|\frac{\vec{a}\cdot((\vec{a}-\vec{b})\times\vec{c})}{|(\vec{a}-\vec{b})\times\vec{c}|}\right|$$

$$\underbrace{S.D}_{4} = \left|\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{\underbrace{|(\vec{a}-\vec{b})\times\vec{c}|}_{18}}\right| \Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]=72$$

$$|(\vec{a}-\vec{b})\times\vec{c}| = |\vec{a}-\vec{b}||\vec{c}|\sin 30^\circ$$

$$= 6.\ 6.\ \frac{1}{2}$$

$$= 18$$

$$V=\frac{1}{6}(72)$$

$$\boxed{V=12}$$

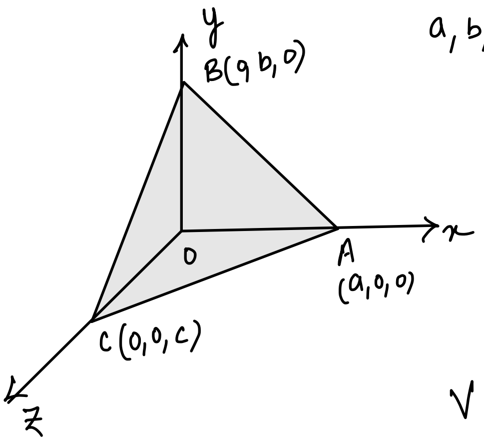
$$\boxed{\frac{V}{6}=2}$$

Ans

Q A plane passing through (1, 1, 1) cut positive direction of co-ordinate axes at A, B and C, then the volume of tetrahedron OABC (as V) satisfies

(1) $V \le \frac{9}{2}$ (2) $V \ge \frac{9}{2}$ (3) $V = \frac{9}{2}$ (4) None

Sol^n



$a, b, c > 0$

$P: \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$(1,1,1)$

$$\boxed{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1} \quad \text{— ① —}$$

$$V = \frac{1}{6}\left[\vec{OA}\ \vec{OB}\ \vec{OC}\right] = \frac{1}{6}(abc)$$

$$GM \geq HM$$

$$(abc)^{1/3} \geq \frac{3}{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)}$$

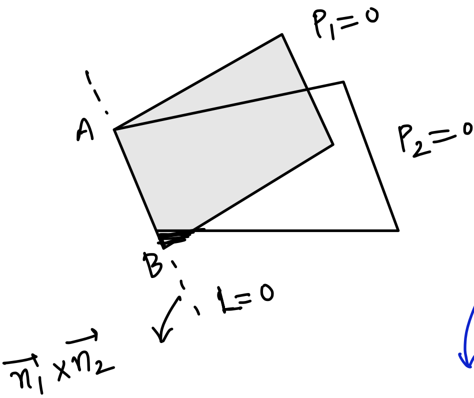
$$(abc)^{1/3} \geq 3 \Rightarrow abc \geq 27$$

$$V \geq \frac{27}{6}$$

$$V \geq \frac{9}{2}$$

Q Let $P_1 = x + y + z + 1 = 0$, $P_2 = x - y + 2z + 1 = 0$, $P_3 = 3x + y + 4z + 7 = 0$ be three planes. Find the distance of line of intersection of planes $P_1 = 0$ and $P_2 = 0$ from the plane $P_3 = 0$.

(A) $\frac{2}{\sqrt{26}}$ (B) $\frac{4}{\sqrt{26}}$ ✓ (C) $\sqrt{\frac{1}{26}}$ (D) $\frac{7}{\sqrt{26}}$

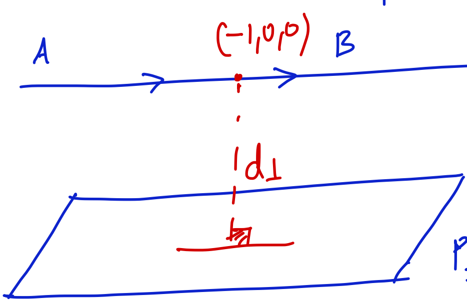


✓ $z = 0$ in both planes

$$\left.\begin{aligned} x+y &= -1 \\ x-y &= -1 \end{aligned}\right\} \quad \checkmark\, x = -1,\ \checkmark\, y = 0$$

$$L:\ \frac{x+1}{3} = \frac{y-0}{-1} = \frac{z-0}{-2}.$$

We can see that Line L is parallel to $P_3 = 0$.



$$d_\perp = \frac{|-3+0+0+7|}{\sqrt{3^2+1^2+4^2}}$$

$$= \frac{4}{\sqrt{26}}$$

Paragraph

A plane p contains the line $L_1: \frac{y}{b}+\frac{z}{c}=1, x=0$ and is parallel to the line $L_2: \frac{x}{a}-\frac{z}{c}=1, y=0$

1. If the shortest distance between L_1 and L_2 is $\frac{1}{4}$ then the value of $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$ equals :

(A) 16 ✓(B) 64 (C) 128 (D) 192

2. Distance of image of A(a, 0, 0) in the plane p from $M\left(-\frac{5}{3}, \frac{8}{3}, \frac{11}{3}\right)$. where a = b = c = 1 is equal to :

(A) 1 (B) 2 ✓(C) 3 (D) 4

Solⁿ

$$L_1: \frac{y}{b}-1 = -\frac{z}{c}, \; x=0.$$

$$L_1: \frac{x-0}{0} = \frac{y-b}{-b} = \frac{z}{c}.$$

$$L_2: \frac{x-a}{a} = \frac{y}{0} = \frac{z}{c}.$$

Equation of plane P : $\begin{vmatrix} x & y-b & z \\ 0 & -b & c \\ a & 0 & c \end{vmatrix} = 0$

$$P: \frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0.$$

(where $a=b=c=1$)

Image of A $(a,0,0)$ in plane $P=0$ is $A'\left(-\frac{1}{3}, \frac{4}{3}, \frac{4}{3}\right)$

Hence distance between A' & M is 3.

L_1: $\vec{r} = b\hat{j} + \lambda(-b\hat{j} + c\hat{k})$

L_2: $\vec{r} = a\hat{j} + \mu(a\hat{i} + c\hat{k})$

$$S.D = \frac{1}{4} = \frac{\begin{vmatrix} -a & b & 0 \\ 0 & -b & c \\ a & 0 & c \end{vmatrix}}{\sqrt{(ab)^2 + (bc)^2 + (ca)^2}}.$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 64.$$

Q If three points $(2\vec{p}-\vec{q}+3\vec{r})$, $(\vec{p}-2\vec{q}+\alpha\vec{r})$ and $(\beta\vec{p}-5\vec{q})$ (Where $\vec{p}, \vec{q}, \vec{r}$ are non-coplanar vectors) are collinear, then $\frac{1}{(\alpha+\beta)}$ is

(A) 4 ✓ (B) 5 (C) 6 (D) 7

Sol^n

$$\left.\begin{array}{l} A(2\vec{p}-\vec{q}+3\vec{r}) \\ B(\vec{p}-2\vec{q}+\alpha\vec{r}) \\ C(\beta\vec{p}-5\vec{q}) \end{array}\right\} \text{Collinear points}$$

$$\overrightarrow{BA} = \vec{p}+\vec{q}+(3-\alpha)\vec{r}$$

$$\overrightarrow{CB} = (\beta-1)\vec{p}-3\vec{q}-\alpha\vec{r}$$

$$\overrightarrow{BA} = \lambda\,\overrightarrow{CB}$$

$$\vec{p}(1-\lambda(\beta-1))+\vec{q}(1+3\lambda)+\vec{r}(3-\alpha+\alpha\lambda)=\vec{0}$$

$$1-\lambda(\beta-1)=0\,;\quad 1+3\lambda=0\,;\quad 3-\alpha+\alpha\lambda=0.$$

$$\lambda=-\frac{1}{3}\,;\quad \beta=-2\,;\quad \alpha=\frac{9}{4}.$$

HW:

JM Q8 to 30.

JA Complete.