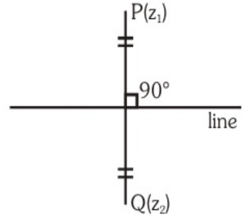
Reflection points for A line :



Q Prove that two points $P(z_1)$ & $Q(z_2)$ will be reflection points of each other in the straight line $\bar{\alpha}z + \alpha\bar{z} + r = 0$, iff $\bar{\alpha}z_1 + \alpha\bar{z}_2 + r = 0$, where r is real and α is non zero complex constant. **[IIT-1997]**

OR $\bar{\alpha}z_2 + \alpha\bar{z}_1 + r = 0$

Proof :

Let $P(z_1)$ is the reflection point of $Q(z_2)$ then the perpendicular bisector of z_1 & z_2 must be the line

$\bar{\alpha}z + \alpha\bar{z} + r = 0$ .... (i)

Now perpendicular bisector of z_1 & z_2 is, $|z - z_1| = |z - z_2|$

or $(z - z_1)(\bar{z} - \bar{z}_1) = (z - z_2)(\bar{z} - \bar{z}_2)$

$-z\bar{z}_1 - z_1\bar{z} + z_1\bar{z}_1 = -z\bar{z}_2 - z_2\bar{z} + z_2\bar{z}_2$ ($z\bar{z}$ cancels on either side)

or $(\bar{z}_2 - \bar{z}_1)z + (z_2 - z_1)\bar{z} + z_1\bar{z}_1 - z_2\bar{z}_2 = 0$ .... (ii)

Comparing (i) & (ii) $\dfrac{\bar{\alpha}}{\bar{z}_2 - \bar{z}_1} = \dfrac{\alpha}{z_2 - z_1} = \dfrac{r}{z_1\bar{z}_1 - z_2\bar{z}} = \lambda$

$\therefore$ $\bar{\alpha} = \lambda(\bar{z}_2 - \bar{z}_1)$ .... (iii) $\alpha = \lambda(z_2 - z_1)$ .... (iv)

and $r = \lambda(z_1\bar{z}_1 - z_2\bar{z}_2)$ .... (v)

Multiply (iii) by z_1 ; (iv) by $\bar{z}_2$ and adding

$\bar{\alpha}z_1 + \alpha\bar{z}_2 + r = \lambda\,[(\cancel{z_1\bar{z}_2} - \cancel{z_1\bar{z}_1}) + \cancel{z_2\bar{z}_2} - \cancel{\bar{z}_2 z_1} + \cancel{z_1\bar{z}_1} - \cancel{z_2\bar{z}_2}] = 0$

Note that we could also multiply (iii) by z_2 & (4) by $\bar{z}_1$ & add to get the same result.

Hence the $\bar{\alpha}z_1 + \alpha\bar{z}_2 + r = 0$.

Again, let $\bar{\alpha}z_1 + \alpha\bar{z}_2 + r = 0$ is true w.r.t. the line $\bar{\alpha}z + \alpha\bar{z} + r = 0$.

Subtracting $\bar{\alpha}(z - z_1) + \alpha(\bar{z} - \bar{z}_2) = 0$

or $|(z - z_1)|\,|\cancel{\bar{\alpha}}| = |\cancel{\alpha}|\,|(\bar{z} - \bar{z}_2)|$

Hence 'z' lies on the perpendicular bisector of joins of z_1 & z_2.

Q Find reflection of $\underbrace{(2-3i)}_{z_1}$ in $(1+2i)z + (1-2i)\bar{z} + 5 = 0$

Sol^n

$\bar{\alpha}z_1 + \alpha\bar{z}_2 + r = 0$

$(1+2i)(2-3i) + (1-2i)\bar{z}_2 + 5 = 0 \rightarrow$ get z_2

Probability

BASIC DEFINITIONS :

(all outcomes are known to us & every out come is by chance)

(i) Random Experiment : An experiment or an action resulting in two or more possible out comes.

e.g. tossing a coin throwing a die, drawing a card

(ii) Sample space : The set of all possible outcomes of an experiment is called sample space of an experiment and each outcome is called a sample point.

(1) Exp. : Throwing a coin

S {H, T}

E_1 : H occurs ≡ {H}

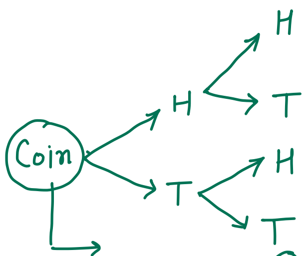
E_2 : T occurs ≡ {T}

(2) Exp. : Throwing 2 coin

S ≡ {HH, HT, TH, TT}

E_1 ≡ Atleast one head ≡ {HT, HH,TH}

E_2 ≡ Exactly one tail ≡ {HT, TH}



$2 \times 2 = 4$

(3) Throwing a die

S ≡ {1,2,3,4,5,6}

E_1 ≡ Getting even number ≡ {2, 4, 6}

(4) Throwing 3 coins $\longrightarrow$ $2 \times 2 \times 2 = 8$

S = {HHH, TTT, HHT, HTH, THH, TTH, THT, HTT}

E_1 ≡ Getting two consecutive head ≡ {HHH, HHT, THH}

(iii) Event of an experiment : An event is defined occurrence or situation.

(1) tossing a coin and the coin landing up head.

(2) scoring a six on the throw of a die,

(3) winning the first prize in a raffle.

Note : Event is subset of sample space. If outcome of an experiment is an element of A we say that event A has occurred.

An event consisting of a single point of S is called a simple or elementary event.

If an event has more than one simple point it is called compound event.

ϕ is called impossible event and S (sample space) is called sure event.

3 important terms :

(iv) Equally likely Event :

Events are said to be equally likely when no particular event has a preference to occur in relation to the other event.

For Example :

(1) The outcomes as a result of throwing a die are equally likely, as no particular face is more likely to occur as compared to the other faces. That is way we normally write as fair die or unbiased die.

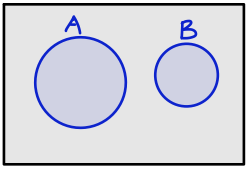
Note that events which are equally likely are also **"equiprobable"**.

Mutually exclusive Event : (ME events).

Events are said to be ME if their simultaneously occurrence is impossible. Two events A and B defined on an experimental performance are said to be ME/disjoint/incompatible if occurrence of one precludes (rules out) the simultaneous occurrence of other.

$A \cap B = \phi$

$n(A \cap B) = 0$



Exp : Throwing a die

$S : \{1, 2, 3, 4, 5, 6\}$

E_1 : Getting even prime number : $\{2\}$

E_2 : " odd " " $\{3, 5\}$

E_3 : " prime number $\{2, 3, 5\}$

E_1 & E_2 are M.E but

E_1 & E_3 are not M.E.

Exhaustive event :

Events as a consequence of an experimental performance are said to be exhaustive if nothing beyond than those listed in the set of possible outcomes/sample space can occur. e.g. the possible outcomes of tossing a fair coin in {H, T}.

In general if the events A and B are exhaustive defined on a sample space then $A \cup B$ given the complete sample space and $P(A \cup B) = 1$. $n(A \cup B) = n(S)$ S = sample space

Exp : Throwing a die

$S : \{1, 2, 3, 4, 5, 6\}$

E_1 : Getting even number $\{2, 4, 6\}$

E_2 : " odd number $\{1, 3, 5\}$

E_3 : " prime number $\{2, 3, 5\}$

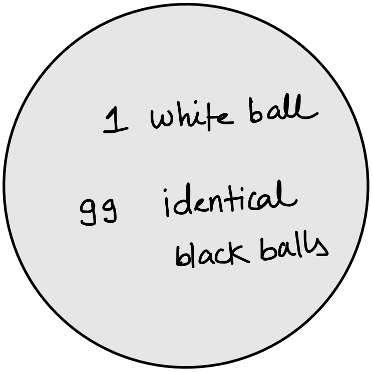
E_1 & E_2 are exhaustive events.

E_1 & E_3 are not exhaustive events.

	Experiment	Events	E/L	M/E	Exhaustive
1.	Throwing of a die	A : throwing an odd face {1, 3, 5} B : throwing a composite face {4,.6}	✗	✓	✗
2.	A ball is drawn from an urn containing 2W, 3R and 4G balls	E_1 : getting a W ball E_2 : getting a R ball E_3 : getting a G ball	✗	✓	✓
3.	Throwing a pair of dice	A : throwing a doublet {11, 22, 33, 44, 55, 66} B : throwing a total of 10 or more {46, 64, 55, 56, 65, 66 }	✓	✗	✗
4.	From a well shuffled pack of cards a card is drawn	E_1 : getting a heart E_2 : getting a spade E_3 : getting a diamond E_4 : getting a club	✓	✓	✓
5.	From a well shuffled pack of cards a card is drawn	A = getting a heart B = getting a face card	✗	✗	✗

Pair of dice :

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66.



Bag

$S : \{W, B\}$

$P(W) = p$

$P(B) = 99p.$

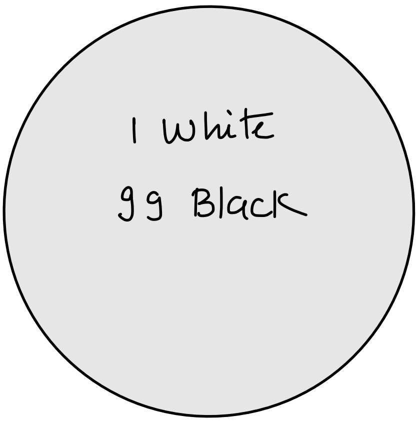
$P(W) + P(B) = 1$

$p = \frac{1}{100}$

$P(W) = \frac{1}{100}$

$P(B) = \frac{99}{100}.$

Alt :-



$S : \{W, B_1, B_2, B_3, \ldots, B_{99}\}$

The classical definition of probability :
If an experiment gives n possible out comes which are equally likely, mutually exclusive & exhaustive & out of these outcomes if m outcomes favour event A, then $\boxed{P(A) = \frac{m}{n}}$

as $0 \le m < n \Rightarrow 0 < P(A) \le 1$

Complement of an event : The set of all out comes which are in S but not in A is called the Complement Of The Event A denoted by $\bar{A}$, A^c, A' or 'not A'.

$$P(A^c) = P(\bar{A}) = \frac{n-m}{n}$$

$$\boxed{P(A^c) = 1 - P(A)}$$

Note that A and A^C makes an event a sure event and Pr. of a sure event is one.

Odds in favour & Odd against :-

$$\text{odd's in favour} = \frac{\text{no. of favourable outcomes}}{\text{no. of unfavourable outcomes}} = \frac{a}{b}$$

$n(S) = a+b$

$$\text{odd's in favour} = \frac{a}{b} = \frac{\left(\frac{a}{a+b}\right)}{\left(\frac{b}{a+b}\right)} = \frac{P(A)}{P(\bar{A})}$$

$$\text{odd's against} = \frac{\text{no. of unfavourable out comes}}{\text{no. of favourable out comes}} = \frac{b}{a}$$

$$= \frac{P(\bar{A})}{P(A)}$$

Q 4 Apples and 3 Oranges are randomly placed in a line. Find the chance that the two extreme fruits are both oranges.

Sol^n

M-1

Apples are alike A A A A

oranges are alike O O O

$$O \underbrace{\_\ \_\ \_\ \_\ \_}_{O,\ A,\ A,\ A,\ A} O$$

$$n(A) = \frac{5!}{4!}$$

$$n(S) = \frac{7!}{4!\,3!}$$

$$P(A) = \frac{\frac{5!}{\cancel{4!}}}{\frac{7!}{\cancel{4!}\,3!}} = \frac{5!\,3!}{7!} = \frac{\cancel{6}}{7 \times \cancel{6}} = \frac{1}{7}.$$

M-2

$A_1\ A_2\ A_3\ A_4$

$O_1\ O_2\ O_3$

$$n(S) = 7!$$

eg

$$O_1 \underbrace{\_\ \_\ \_\ \_\ \_}_{A_1 A_2 A_3 A_4 O_3} O_2$$

$$n(A) = \left({}^3C_2 \cdot 2!\right) 5!$$

$$P(A) = \frac{{}^3C_2 \cdot 2!\ 5!}{7!} = \frac{1}{7}.$$

M-3

$$\checkmark\ \underbrace{\_\ \_\ \_\ \_\ \_}_{e}\ \checkmark$$

$$n(S) = {}^7C_2 \cdot 2!$$

$$n(A) = {}^3C_2 \cdot 2!$$

$$P(A) = \frac{{}^3C_2 \cdot 2!}{{}^7C_2 \cdot 2!} = \frac{1}{7}.$$

Q A single letter is selected from word PROBABILITY. Find the probability it is vowel?

Solⁿ

$$\frac{4}{11}$$

Q A card is drawn randomly from a well shuffled pack of 52 cards. The Pr. that the drawn card is

(a) neither a heart nor a face card.

(b) "neither a heart nor a king

(c) red face card

(d) king or a red card

Sol^n $n(S) = {}^{52}C_1 = 52.$

(a) $n(A) = 30.$

$P(A) = \frac{30}{52}.$

(b) $n(B) = 36$

$P(B) = \frac{36}{52}$

(c) $n(C) = 6$

$P(C) = \frac{6}{52}$

(d) $n(d) = 28$

$P(D) = \frac{28}{52}.$

HW:

Remaining sheet of Complex Number