9. With usual notation, if in a $\Delta$ ABC, $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$; then prove that, $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$.

Given $\Rightarrow \frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$

$\Rightarrow \frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}=\frac{a+b+c}{18}$

$\Rightarrow a=7\lambda,\ b=6\lambda,\ c=5\lambda$

Using cosine formula

$\cos A=\frac{36+25-49}{60}=\frac{1}{5}=\frac{7}{35}$

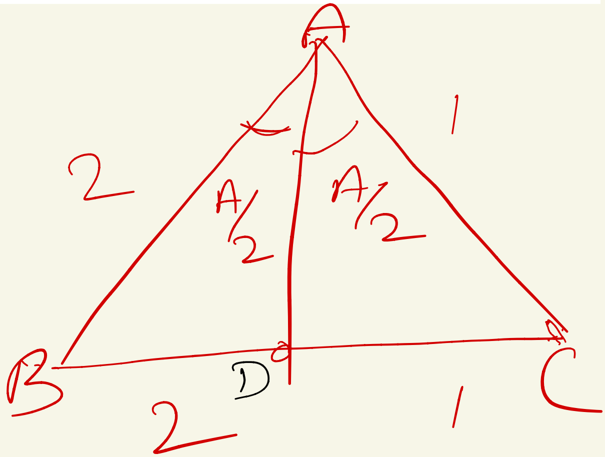
$\cos B=\frac{49+25-36}{70}=\frac{19}{35}=\frac{19}{35}$

$\cos C=\frac{49+36-25}{84}=\frac{5}{7}=\frac{25}{35}$

$\Rightarrow \frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$

# EXERCISE (S-2)

2 Given a triangle ABC with AB = 2 and AC = 1. Internal bisector of $\angle BAC$ intersects BC at D. If AD = BD and $\Delta$ is the area of triangle ABC, then find the value of $12\Delta^2$.




$AD = BD \Rightarrow \angle B = A/2$

$\angle D = \pi - A$

Apply Sine Rule in $\Delta ABD$

$\Rightarrow \dfrac{2}{\sin A} = \dfrac{BD}{\sin A/2} \Rightarrow BD = \dfrac{1}{\cos A/2}$

Apply Sine Rule in $\Delta ABC$

$\Rightarrow \dfrac{BC}{\sin A} = \dfrac{1}{\sin A/2} \Rightarrow BC = 2\cos A/2$

$\dfrac{BD}{DC} = \dfrac{2}{1} \Rightarrow BD = 2x, \quad BC = 3x$

$\Rightarrow (2x)(3x) = 2 \Rightarrow x^2 = \dfrac{1}{3}$

3 For any triangle ABC, if $B = 3C$, show that $\cos C = \sqrt{\dfrac{b+c}{4c}}$ & $\sin\dfrac{A}{2} = \dfrac{b-c}{2c}$.

$A + B + C = \pi$ & $B = 3C$

$\Rightarrow A = \pi - 4C$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin 4C} = \frac{b}{\sin 3C} = \frac{c}{\sin C} \quad \text{---} \text{(I)}$$

$$\frac{b}{c} = \frac{\sin 3C}{\sin C} = 3 - 4\sin^2 C$$

$$\Rightarrow 1 + \frac{b}{c} = 4\cos^2 C$$

$$\Rightarrow \frac{b+c}{4c} = \cos^2 C$$

$$\Rightarrow \cos C = \sqrt{\frac{b+c}{4c}} \quad \text{(part I)}$$

We have $A/2 = \pi/2 - 2C$

$$\Rightarrow \sin A/2 = \cos 2C = 2\cos^2 C - 1$$

$$= \frac{b+c}{2c} - 1 = \frac{b-c}{2c} \quad \text{(part II)}$$

4 In a triangle ABC if $a^2 + b^2 = 101c^2$ then find the value of $\dfrac{\cot C}{\cot A + \cot B}$.

using $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 + b^2 = 101c^2$

$\Rightarrow \sin^2 A + \sin^2 B - \sin^2 C = 100 \sin^2 C$

$\Rightarrow \sin^2 A + \sin(B+C)\sin(B-C) = 100 \sin^2 C$

$\Rightarrow \sin A[\sin A + \sin(B-C)] = 100 \sin^2 C$

$\Rightarrow \sin A[\sin(B+C) + \sin(B-C)] = 100 \sin^2 C$

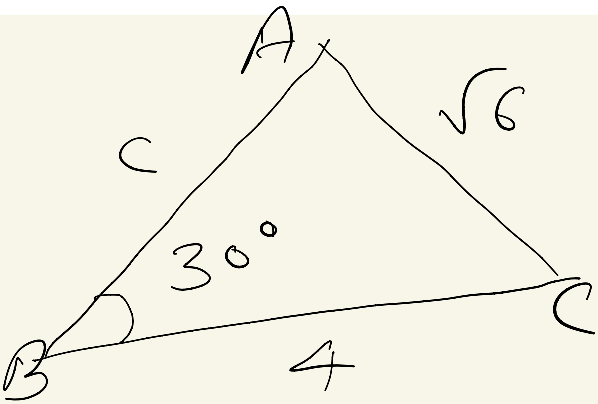
$\Rightarrow \sin A \times 2 \sin B \cos C = 100 \sin^2 C$

$\Rightarrow \dfrac{2 \sin A \cdot \sin B \cos C}{\sin C \cdot \sin C} = 100$

$\Rightarrow \dfrac{\cot C}{\dfrac{\sin A \cdot \cos B + \cos A \sin B}{\sin A \cdot \sin B}} = 50$

$\Rightarrow \dfrac{\cot C}{\cot A + \cot B} = 50$

15 Two sides of a triangle are of lengths $\sqrt{6}$ and 4 and the angle opposite to smaller side is $30^0$. How many such triangles are possible? Find the length of their third side and area.




Apply cosine rule

$$\Rightarrow \cos B = \frac{c^2+a^2-b^2}{2ac}$$

$$\Rightarrow \cos 30° = \frac{c^2+16-6}{8c}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{c^2+10}{8c}$$

$$\Rightarrow 4\sqrt{3}c = c^2+10$$

$$\Rightarrow c^2 - 4\sqrt{3}c + 10 = 0$$

$$\Rightarrow c = 2\sqrt{3}+\sqrt{2},\ 2\sqrt{3}-\sqrt{2}$$

$$\text{Area} = \frac{1}{2} 4 \times c \sin 30° = c$$

$$= 2\sqrt{3}+\sqrt{2},\ 2\sqrt{3}-\sqrt{2}$$

6 The triangle ABC (with side lengths a, b, c as usual) satisfies $\log a^2 = \log b^2 + \log c^2 - \log(2bc\cos A)$.

What can you say about this triangle?

$$\log a^2 = \log b^2 + \log c^2 - \log(2bc\cos A)$$

$$\Rightarrow \log(2bc\cos A) = \log b^2 + \log c^2 - \log a^2$$

$$\Rightarrow 2bc\cos A = \frac{b^2c^2}{a^2}$$

$$\Rightarrow b^2+c^2-a^2 = \frac{b^2c^2}{a^2}$$

$$\Rightarrow c^2-a^2 = \frac{b^2c^2}{a^2} - b^2 = \frac{b^2(c^2-a^2)}{a^2}$$

$\Rightarrow$ Either $c^2-a^2=0$ OR $b^2=a^2$

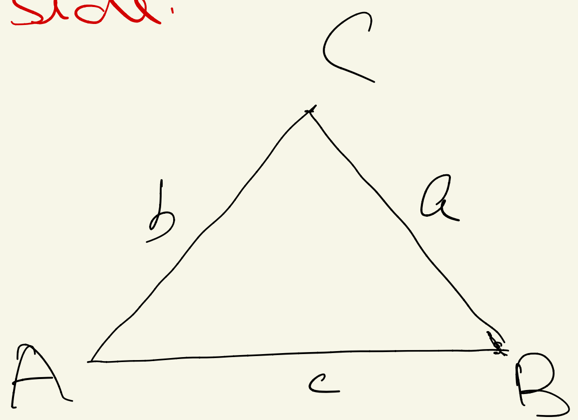
$\Rightarrow$ $a^2=c^2$ OR $b^2=a^2$

$\Rightarrow$ Always isosceles triangle

# EXERCISE (S-2)

7. The sides of a triangle are consecutive integers $n$, $n+1$ and $n+2$ and the largest angle is twice the smallest angle. Find $n$.

As we know that side opposite to largest angle is largest side.




Suppose $a=n$, $b=n+1$, $c=n+2$

$\Leftrightarrow$ $\angle C$ is the largest and angle $\angle A$ is the smallest

From question $C = 2A$

In $\Delta ABC$, $A+B+C=\pi$

$B = \pi - 3A$

Using Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{n}{\sin A} = \frac{n+1}{\sin 3A} = \frac{n+2}{\sin 2A}$$

$$\Rightarrow \frac{n}{1} = \frac{n+1}{3-4\sin^2 A} = \frac{n+2}{2\cos A}$$

$$\Rightarrow \frac{2n+1}{4-4\sin^2 A} = \frac{n+2}{2\cos A}$$

$$\Rightarrow \frac{2n+1}{2\cos A} = \frac{n+2}{1}$$

$$\Rightarrow 2\cos A = \frac{2n+1}{n+2}$$

So, $\frac{2n+1}{n+2} = \frac{n+2}{n}$ (Using above Ratio)

$$\Rightarrow 2n^2+n = n^2+4+4n$$

$$\Rightarrow n^2-3n-4=0$$

$$\Rightarrow n=-1,\ 4$$

n can't be -ve $\Rightarrow$ $n=4$

# EXERCISE (JM)

**1.** If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to : **[JEE(Main)-Jan 2019]**

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$ (3) $\frac{5}{4}$ (4) $\frac{7}{4}$

**Ans.** **(4)**

**Sol.** r = 1 is obviously true.

Let $0 < r < 1$

$\Rightarrow \quad r + r^2 > 1$

$\Rightarrow r^2 + r - 1 > 0$

$$\left(r - \frac{-1-\sqrt{5}}{2}\right)\left(r - \left(\frac{-1+\sqrt{5}}{2}\right)\right)$$

$$\Rightarrow \quad r - \frac{-1-\sqrt{5}}{2} \text{ or } r > \frac{-1+\sqrt{5}}{2}$$

$$r \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$$

$$\frac{\sqrt{5}-1}{2} < r < 1$$

When $r > 1$

$$\Rightarrow \quad \frac{\sqrt{5}+1}{2} > \frac{1}{r} > 1$$

$$\Rightarrow \quad r \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$$

Now check options

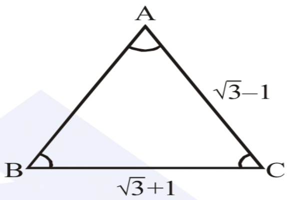
# EXERCISE (JM)

**2.** With the usual notation, in $\Delta ABC$, if $\angle A + \angle B = 120^{\circ}$, $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, then the ratio $\angle A : \angle B$, is : **[JEE(Main)-Jan 2019]**

(1) 7 : 1 (2) 5 : 3 (3) 9 : 7 (4) 3 : 1

**Ans.** **(1)**

**Sol.** $A + B = 120^{\circ}$




$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$= \frac{\sqrt{3}+1-\sqrt{3}+1}{2(\sqrt{3})}\cot(30^{\circ}) = \frac{1}{\sqrt{3}}.\sqrt{3} = 1$$

$$\frac{A-B}{2} = 45^{\circ} \Rightarrow A - B = 90^{\circ}$$

$$A + B = 120^{\circ}$$

$$2A = 210^{\circ}$$

$$A = 105^{\circ}$$

$$B = 15^{\circ}$$

$\therefore$ Option (1)

# EXERCISE (JM)

3. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is : **[JEE(Main)-Jan 2019]**

(1) $\frac{y}{\sqrt{3}}$ (2) $\frac{c}{\sqrt{3}}$ (3) $\frac{c}{3}$ (4) $\frac{3}{2}y$

**Ans. (2)**

**Sol.** Given $a + b = x$ and $ab = y$

If $x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$

$\Rightarrow a^2 + b^2 - c^2 = -ab$

$$\Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = -\frac{1}{2}$$

$$\Rightarrow \cos C = -\frac{1}{2}$$

$$\Rightarrow \angle C = \frac{2\pi}{3}$$

$$R = \frac{c}{2\sin C} = \frac{c}{\sqrt{3}}$$

# EXERCISE (JM)

**4.** Given $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$ for a $\Delta ABC$ with usual notation. If $\frac{\cos A}{\alpha}=\frac{\cos B}{\beta}=\frac{\cos C}{\gamma}$, then the ordered triad $(\alpha, \beta, \gamma)$ has a value :- **[JEE(Main)-Jan 2019]**

(1) (3, 4, 5) (2) (19, 7, 25) (3) (7, 19, 25) (4) (5, 12, 13)

**Ans. (3)**

**Sol.** $b + c = 11\lambda,\ c + a = 12\lambda,\ a + b = 13\lambda$

$\Rightarrow a = 7\lambda,\ b = 6\lambda,\ c = 5\lambda$

(using cosine formula)

$$\cos A=\frac{1}{5},\cos B=\frac{19}{35},\cos C=\frac{5}{7}$$

$\alpha : \beta : \gamma \Rightarrow 7 : 19 : 25$

# EXERCISE (JM)

**5.** If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is : **[JEE(Main)-Apr 2019]**

(1) 5 : 9 : 13 (2) 5 : 6 : 7 (3) 4 : 5 : 6 (4) 3 : 4 : 5

**Sol.** $a < b < c$ are in A.P.

$\angle C = 2\angle A$ (Given)

$$\Rightarrow \sin C = \sin 2A$$

$$\Rightarrow \sin C = 2\sin A.\cos A$$

$$\Rightarrow \frac{\sin C}{\sin A} = 2\cos A$$

$$\Rightarrow \frac{c}{a} = 2\frac{b^2 + c^2 - a^2}{2bc}$$

put $a = b - \lambda$, $c = b + \lambda$, $\lambda > 0$

$$\Rightarrow \lambda = \frac{b}{5}$$

$$\Rightarrow a = b - \frac{b}{5} = \frac{4}{5}b, c = b + \frac{b}{5} = \frac{6b}{5}$$

$\Rightarrow$ required ratio $= 4 : 5 : 6$

# EXERCISE (JM)

**6.** The angles A, B and C of a triangle ABC are in A.P. and a : b = 1 : $\sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is : **[JEE(Main)-Apr 2019]**

(1) $4\sqrt{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $2\sqrt{3}$ (4) $\frac{4}{\sqrt{3}}$

**Sol.** $\angle B = \frac{\pi}{3}$, by sine Rule

$$\sin A = \frac{1}{2}$$

$$\Rightarrow A = 30°, \quad a = 2, \ b = 2\sqrt{3}, \ c = 4$$

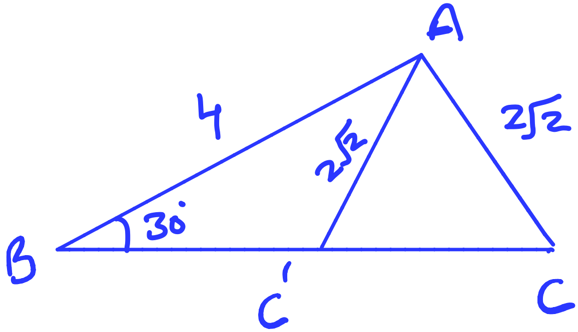
$$\Delta = \frac{1}{2} \times 2\sqrt{3} \times 2 = 2\sqrt{3} \text{ sq. cm}$$

# EXERCISE (JA)

**1.** Let ABC and ABC′ be two non-congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30°. The absolute value of the difference between the areas of these triangles is **[JEE 2009, 5]**

Solⁿ

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$




$$\cos 30^\circ = \frac{a^2+16-8}{2\cdot a\cdot(4)}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{a^2+8}{8a}$$

$$\Rightarrow a^2 - 4\sqrt{3}a + 8 = 0 \quad \begin{cases} a_1 \\ a_2 \end{cases}$$

$$a_1 + a_2 = 4\sqrt{3}, \quad a_1 a_2 = 8$$

$$\Rightarrow |a_1 - a_2| = \sqrt{(a_1+a_2)^2 - 4a_1a_2}$$

$$= 4$$

Now

$$|\Delta_1 - \Delta_2| = \left|\frac{1}{2}a_1\cdot 4\sin 30^\circ - \frac{1}{2}a_2\cdot 4\sin 30^\circ\right|$$

$$= |a_1 - a_2| = 4 \quad \underline{\text{Ans}}$$

# EXERCISE (JA)

**2.** (a) If the angle A,B and C of a triangle are in an arithmetic progression and if a,b and c denote the length of the sides opposite to A,B and C respectively, then the value of the expression

$\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$, is -

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$

Sol$^n$ Given $2B = A + C$ — ①

wkt $A + B + C = \pi$ — ②

$\therefore B = 60^\circ$

Now using Sine Law

$$\frac{a}{c} = \frac{\sin A}{\sin C}$$

therefore

$$\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$$

$$\Rightarrow \frac{\sin A}{\cancel{\sin C}} \times 2\cancel{\sin C}\cos C + \frac{\sin C}{\cancel{\sin A}} \times 2\cancel{\sin A}\cos A$$

$$\Rightarrow 2\sin(A+C)$$

$$\Rightarrow 2\sin B = 2\sin 60^\circ$$

$$= \sqrt{3} \text{ Ans}$$

# EXERCISE (JA)

2

(b) Consider a triangle ABC and let a,b and c denote the length of the sides opposite to vertices A,B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then $r^2$ is equal to

Solt$^n$:- $\text{Area} = \frac{1}{2}\left|ab\sin C\right|$

$\Rightarrow \quad 15\sqrt{3} = \left|\frac{1}{2}\times 6\times 10\times \sin C\right|$

$\Rightarrow \quad \sin C = \pm\frac{\sqrt{3}}{2}$

Given $\angle C$ is obtuse, so $\angle C = 120^\circ$

Now

$\cos C = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \quad -\frac{1}{\cancel{2}} = \frac{36+100-c^2}{\cancel{2}\times 6\times 10} \Rightarrow c = 14$

Now $r = \frac{\Delta}{s}$

$\Rightarrow \quad r = \frac{15\sqrt{3}}{\left(\frac{6+10+14}{2}\right)} \Rightarrow r = \sqrt{3}$

$\therefore \quad r^2 = 3$ Ans

2

# EXERCISE (JA)

(c) Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a,b and c denote the lengths of the sides opposite to A,B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is/are **[JEE 2010, 3+3+3]**

(A) $-\left(2+\sqrt{3}\right)$ (B) $1+\sqrt{3}$ (C) $2+\sqrt{3}$ (D) $4\sqrt{3}$

Soltⁿ :- $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \cos A = \frac{(x^2-1)^2+(2x+1)^2-(x^2+x+1)^2}{2\times(x^2-1)\times(2x+1)}$

$\Rightarrow \cos A = \frac{\cancel{x^4}-\cancel{2x^2}+1+\cancel{4x^2}+4x+\cancel{1}-\cancel{x^4}-x^2-\cancel{1}-2x^3-\cancel{2x^2}-2x}{2(2x^3+x^2-2x-1)}$

$\Rightarrow \cos A = \frac{-(2x^3+x^2-2x-1)}{2(2x^3+x^2-2x-1)} \Rightarrow -\frac{1}{2}$

ie $\angle A = \frac{2\pi}{3}$, given $\angle C = \frac{\pi}{6}$

$\therefore \angle B = \pi/6$ ie $\Delta$ is isosceles $\Delta$.

$\Rightarrow b = c$

$\Rightarrow x^2-1 = 2x+1$

$\Rightarrow x^2-2x-2=0 \Rightarrow x = 1+\sqrt{3}$ Ans

# EXERCISE (JA)

**3.** Let PQR be a triangle of area $\Delta$ with $a = 2$, $b = \frac{7}{2}$ and $c = \frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals **[JEE 2012, 3M, –1M]**

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

Solt^n

$$\Rightarrow \frac{2\sin P - \sin 2P}{2\sin P + \sin 2P} \Rightarrow \frac{\cancel{2\sin P}(1-\cos P)}{\cancel{2\sin P}(1+\cos P)}$$

$$\Rightarrow \frac{2\sin^2(P/2)}{2\cos^2(P/2)}$$

$$\Rightarrow \tan^2(P/2) = \left(\frac{(s-b)(s-c)}{\Delta}\right)^2$$

$\Rightarrow$ where $s = \dfrac{2 + 7/2 + 5/2}{2} = 4$

$$\therefore \left(\frac{(s-b)(s-c)}{\Delta}\right)^2 = \left(\frac{\frac{1}{2}\times\frac{3}{2}}{\Delta}\right)^2$$

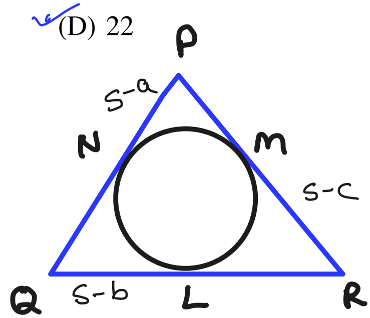
$$= \left(\frac{3}{4\Delta}\right)^2 \text{ Ans}$$

# EXERCISE (JA)

**4.** In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are)

**[JEE(Advanced) 2013, 3, (–1)]**

(A) 16 (B) 18 (C) 24 (D) 22




Sol$^n$ :- $2s = a+b+c$

$\therefore$ $a = l(QR) = 2s-b-c$

$\therefore$ $QL = s-b$, $LR = s-c$

Now, Let

$PN = s-a = 2k-2$

$QL = s-b = 2k$

$MR = s-c = 2k+2$, where $k \in I$ & $k > 1$

adding we get $s = 6k$

$\therefore$ $a = 4k+2$, $b = 4k$, $c = 4k-2$

Now given $\cos P = \frac{1}{3}$

$\therefore$ $\frac{b^2+c^2-a^2}{2bc} = \frac{1}{3}$

$\Rightarrow$ $3\left((4k)^2+(4k-2)^2-(4k+2)^2\right) = 2 \times 4k \times 4k-2$

$\Rightarrow$ on solving we get $k = 5$

$\therefore$ $a = 22$, $b = 20$, $c = 18$

# EXERCISE (JA)

**5.** In a triangle the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is a third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is - **[JEE(Advanced)-2014, 3(–1)]**

(A) $\frac{3y}{2x(x+c)}$ ✓(B) $\frac{3y}{2c(x+c)}$ (C) $\frac{3y}{4x(x+c)}$ (D) $\frac{3y}{4c(x+c)}$

Sol$^n$:- given

$$a+b = x \text{ — (1)} \quad \& \quad ab = y \text{ — (2)}$$

Also

$$x^2 - c^2 = y$$

$$\Rightarrow (a+b)^2 - c^2 = ab$$

$$\Rightarrow a^2 + b^2 - c^2 = -ab$$

$\Rightarrow$ $\div$ the eq$^n$ by $2ab$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = -\frac{ab}{2ab} \Rightarrow \cos C = -\frac{1}{2} \Rightarrow \angle C = \frac{2\pi}{3}$$

Now

$$\frac{r}{R} = \frac{\Delta/s}{\frac{abc}{4\Delta}} \Rightarrow \frac{4\Delta^2}{s \times (abc)}$$

$$\Rightarrow \frac{r}{R} = \frac{4 \times \left(\frac{1}{2} ab \sin C\right)^2}{\left(\frac{a+b+c}{2}\right)(ab)\,c}$$

$$= \frac{\left(y \cdot \frac{\sqrt{3}}{2}\right)^2}{\left(\frac{x+c}{2}\right) y \cdot c} \Rightarrow \frac{3y}{2c(x+c)}$$

# EXERCISE (JA)

**6.** In a triangle XYZ, let x,y,z be the lengths of sides opposite to the angles X,Y,Z, respectively and $2s = x + y + z$. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then-

✓(A) area of the triangle XYZ is $6\sqrt{6}$

(B) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

✓(C) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2} = \frac{4}{35}$

✓(D) $\sin^2\left(\frac{X+Y}{2}\right) = \frac{3}{5}$

**[JEE(Advanced)-2016, 4(–2)]**



Soln:

$\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2} = k$

$\Rightarrow x = s-4k$

$y = s-3k$

$z = s-2k$

adding $\{\because 2s = x+y+z\}$

$2s = 3s - 9k$

$\Rightarrow s = 9k$

$\therefore x = 5k,\ y = 6k,\ z = 7k$

Now given $\cancel{\pi} r^2 = \frac{8\cancel{\pi}}{3}$

$\Rightarrow \left(\frac{\Delta}{s}\right)^2 = \frac{8}{3}$

$\Rightarrow \frac{s(s-x)(s-y)(s-z)}{s^2} = \frac{8}{3}$

$\Rightarrow \frac{\cancel{4}k \times 3k \times \cancel{2}k}{\left(\frac{5k+6k+7k}{2}\right)} = \frac{\cancel{8}}{3}$

$\Rightarrow k = 1$

$\therefore x = 5,\ y = 6,\ z = 7$

Now using $r = \frac{\Delta}{s} \Rightarrow \Delta = rs$

$\Rightarrow \Delta = \sqrt{\frac{8}{3}} \times 9 = 6\sqrt{6}$

(option A)

opt$^n$-B) $R = \frac{xyz}{4\Delta}$

$\Rightarrow R = \frac{5\times6\times7}{4\times6\sqrt{6}} = \frac{35}{4\sqrt{6}}$ or $\frac{35\sqrt{6}}{24}$

Option C) $\frac{r}{R} = 4\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2}$

$\Rightarrow \sin\frac{X}{2}\times\sin\frac{Y}{2}\times\sin\frac{Z}{2} = \frac{r}{4R}$

$\Rightarrow \frac{\sqrt{8/3}}{4\times\frac{35}{4\sqrt{6}}} = \frac{4}{35}$

option-D) $\sin^2\left(\frac{X+Y}{2}\right) = \cos^2\left(\frac{Z}{2}\right)$

$\Rightarrow \frac{1+\cos Z}{2}$

$\Rightarrow \frac{1 + \frac{5^2+6^2-7^2}{2\times5\times6}}{2}$

$\Rightarrow \frac{3}{5}$

# EXERCISE (JA)

**7.** In a triangle PQR, let $\angle PQR = 30°$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE ? **[JEE(Advanced)-2018, 4(–2)]**

(A) $\angle QPR = 45°$

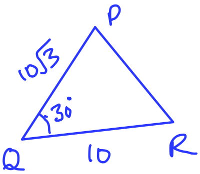
✓(B) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120°$

✓(C) The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$

✓(D) The area of the circumcircle of the triangle PQR is $100\pi$.

Solⁿ :-

$$\cos 30° = \frac{10^2 + (10\sqrt{3})^2 - (PR)^2}{2 \times 10 \times 10\sqrt{3}}$$




$\Rightarrow \quad PR = 10$

ie $\Delta$ is isosceles triangle

$\therefore \quad \angle Q = \angle P = \frac{\pi}{6}$ & $\angle R = \frac{2\pi}{3}$

option B)

Area of $\Delta PQR = \frac{1}{2} \times 10 \times 10\sqrt{3} \times \sin 30°$

$= 25\sqrt{3}$

option c)

$$r = \frac{\Delta}{s}$$

$$= \frac{25\sqrt{3}}{\frac{10+10+10\sqrt{3}}{2}}$$

$$= \frac{25\sqrt{3}}{10+5\sqrt{3}} \times \frac{10-5\sqrt{3}}{10-5\sqrt{3}}$$

$$= 10\sqrt{3} - 15$$

option D :-

$$R = \frac{a}{2\sin A} = \frac{10}{2 \times \sin 30°}$$

$\Rightarrow \quad R = 10$

$\therefore$ Area of circumcircle

$= \pi R^2$

$= 100\pi$

# EXERCISE (JA)

**8.** In a non-right-angled triangle $\Delta PQR$, let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}$, $q = 1$, and the radius of the circumcircle of the $\Delta PQR$ equals 1, then which of the following options is/are correct ?

**[JEE(Advanced)-2019, 4(–1)]**

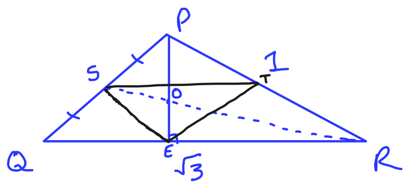
(1) Area of $\Delta SOE = \frac{\sqrt{3}}{12}$

✓(2) Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2}(2-\sqrt{3})$

✓(3) Length of $RS = \frac{\sqrt{7}}{2}$

✓(4) Length of $OE = \frac{1}{6}$

Solⁿ :-




given circumradius = 1, So by using Sine rule

$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R} = 2\times 1$$

$$\Rightarrow \frac{\sqrt{3}}{\sin P} = \frac{1}{\sin Q} = 2$$

$$\Rightarrow \sin P = \frac{\sqrt{3}}{2} \ \& \ \sin Q = \frac{1}{2}$$

$$\frac{1/2}{\sin\theta} = \frac{\sqrt{7}/6}{\sqrt{3}/2}$$

$$\frac{\sqrt{3}}{4}\times\frac{6}{\sqrt{7}} = \sin\theta$$

$$\frac{3\sqrt{3}}{2\sqrt{7}} = \sin\theta$$

as it is given that $\Delta$ is not right angled, so

$P = 120°$, $Q = 30°$ & $\therefore R = 30°$

So $\Delta PQR$ is isosceles $\Delta$, and RS and PE are medians of $\Delta PQR$, so 'O' is centroid of $\Delta$ & $\ell(PQ) = 1$

Optⁿ-D) length of median (PE)

$$= \frac{1}{2}\sqrt{2r^2 + 2q^2 - p^2}$$

$$\Rightarrow \frac{1}{2}\sqrt{2+2-3} = \frac{1}{2}$$

Now $\ell(OE) = \frac{1}{3}P(E)$ [$\because$ O divides PE in ratio 2:1]

$$= \frac{1}{3}\times\frac{1}{2} = \frac{1}{6}$$

Optⁿ-C) length of median (RS)

$$= \frac{1}{2}\sqrt{2(\sqrt{3})^2 + 2(1)^2 - 1^2}$$

$$= \sqrt{7}/2$$

option-B) $r = \frac{\Delta}{S} = \frac{\frac{1}{2}pq\sin R}{\left(\frac{p+q+r}{2}\right)}$ (incird.)

$$\Rightarrow r = \frac{\sqrt{3}}{2}(2-\sqrt{3})$$

option-A) let T be the midpt of PR then area of $\Delta EST = \frac{\text{Area of } \Delta PQR}{4}$

ie Ar $\Delta EST = \frac{\sqrt{3}/4}{4} = \frac{\sqrt{3}}{16}$

Now since 'O' is centroid of $\Delta EST$

So

$$\text{Area of } \Delta SOE = \frac{\text{Area of } \Delta EST}{3}$$

$$= \frac{\sqrt{3}}{48}$$