ALLEN CAREER INSTITUTE
KOTA

TANGENT & NORMAL
(Solutions)

EXERCISE O-1.

1. The slope of the curve $y = \sin x + \cos^2 x$ is zero at the point, where-

(A) $x = \frac{\pi}{4}$ (B) $x = \frac{\pi}{2}$ (C) $x = \pi$ (D) No where

1. Slope $= \frac{dy}{dx} = \cos x - 2\cos x \cdot \sin x = 0.$

$\Rightarrow (\cos x)(1 - 2\sin x) = 0 \Rightarrow \underbrace{\cos x = 0}_{x = \pi/2}$ or $\underbrace{\sin x = 1/2}_{\text{Not in options.}}$

(Ans. B)

2. The equation of tangent at the point (at^2, at^3) on the curve $ay^2 = x^3$ is-

(A) $3tx - 2y = at^3$ (B) $tx - 3y = at^3$ (C) $3tx + 2y = at^3$ (D) None of these

2. $ay^2 = x^3 \Rightarrow 2ayy' = 3x^2 \Rightarrow y' = \frac{3x^2}{2ay}$

At $x = at^2,\ y = at^3$: $\frac{dy}{dx} = \frac{3(at^2)^2}{2a(at^3)} \Rightarrow \frac{dy}{dx} = \frac{3t}{2}$

$\therefore$ Eq^n of tangent: $y - at^3 = \frac{3t}{2}(x - at^2)$

$\Rightarrow y - at^3 = \frac{3t}{2}x - \frac{3at^3}{2} \Rightarrow 3tx - 2y = at^3.$

(Ans. A)

3. The equation of normal to the curve $y = x^3 - 2x^2 + 4$ at the point $x = 2$ is-

(A) $x + 4y = 0$ (B) $4x - y = 0$ (C) $x + 4y = 18$ (D) $4x - y = 18$

3. $y = x^3 - 2x^2 + 4$

At $x = 2$: $y = 2^3 - 2(2)^2 + 4 \Rightarrow y = 4$.

$\therefore$ Point $(2, 4)$

Also, $\frac{dy}{dx} = 3x^2 - 4x \Big|_{x=2} \Rightarrow \frac{dy}{dx} = 12 - 8 \Rightarrow m_T = 4$

$\therefore m_N = \frac{-1}{m_T} \Rightarrow m_N = -\frac{1}{4}$.

$\therefore$ Eq^n. of Normal: $y - 4 = \frac{-1}{4}(x - 2)$

$\Rightarrow 4y - 16 = -x + 2 \Rightarrow x + 4y = 18$. (Ans (C))

4. The slope of the normal to the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ at point $\theta = \pi/2$ is-

(A) 0 (B) 1 (C) –1 (D) $1/\sqrt{2}$

4. $m_N = -\dfrac{dx}{dy} = -\dfrac{dx/d\theta}{dy/d\theta} = -\dfrac{\cancel{a}(1-\cos\theta)}{\cancel{a}(0+\sin\theta)}\Bigg|_{\theta=\pi/2}$

$\Rightarrow m_N = -\dfrac{1}{1} = -1$ (Ans. C).

S.

Consider the curve represented parametrically by the equation

$x = t^3 - 4t^2 - 3t$ and $y = 2t^2 + 3t - 5$ where $t \in R$.

If H denotes the number of point on the curve where the tangent is horizontal and V the number of point where the tangent is vertical then

(A) H = 2 and V = 1

(B) H = 1 and V = 2

(C) H = 2 and V = 2

(D) H = 1 and V = 1

S. $\frac{dy}{dt} = 4t + 3$; $\frac{dx}{dt} = 3t^2 - 8t - 3$; $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

for horizontal tangent : $\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dt} = 0 \Rightarrow 4t + 3 = 0$

$\Rightarrow t = -3/4$

for Vertical tangent : $\frac{dx}{dy} = 0 \Rightarrow \frac{dx}{dt} = 0$

$\Rightarrow 3t^2 - 8t - 3 = 0 \Rightarrow t = 3 ; t = -1/3$.

$\therefore$ H : 1 value ; V : 2 values

6. The line x/a + y/b = 1 touches the curve y = be$^{-x/a}$ at the point-

(A) (0, a) (B) (0, 0) (C) (0, b) (D) (b, 0)

6. $y = be^{-x/a} \Rightarrow y' = -\frac{b}{a} e^{-x/a}$

for point of contact: $-\frac{b}{a} \cdot e^{-x/a} = -\frac{b}{a}$ (slope of given line)

$\Rightarrow e^{-x/a} = 1 \Rightarrow -x/a = 0 \Rightarrow \boxed{x = 0}$

At $x = 0$: $y = b \cdot e^0 \Rightarrow \boxed{y = b}$ $\therefore (0, b)$ Ans.

7. The curve $x^2 - y^2 = 5$ and $\frac{x^2}{18} + \frac{y^2}{8} = 1$ cut each other at any common point at an angle-

(A) $\pi/4$ (B) $\pi/3$ (C) $\pi/2$ (D) None of these

7. $C_1: x^2 - y^2 = 5$ (Hyperbola) ; $C_2: \frac{x^2}{18} + \frac{y^2}{8} = 1$ (Ellipse).

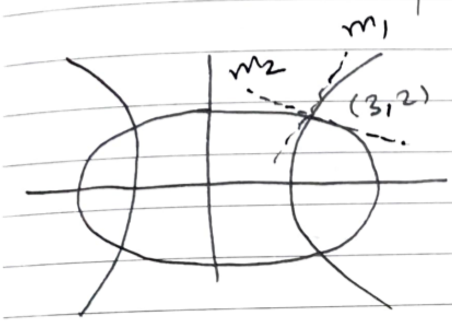
Point of intersection: $x^2 - y^2 = 5 \Rightarrow x^2 = y^2 + 5$.

put in ellipse: $\frac{y^2+5}{18} + \frac{y^2}{8} = 1 \Rightarrow y^2 = 4$

$\Rightarrow y = \pm 2$

$\therefore x^2 = y^2 + 5 \Rightarrow x^2 = 4 + 5 \Rightarrow x = \pm 3$.

$\therefore$ Four points of intersection $(\pm 3, \pm 2)$



By Symmetry, angle of intersection at all points will be same.

$C_1: x^2 - y^2 = 5$.

$\Rightarrow 2x - 2yy' = 0 \Rightarrow y' = \frac{x}{y}$

$\Rightarrow m_1 = \frac{3}{2}$

C_2: $\frac{x^2}{18} + \frac{y^2}{8} = 1 \Rightarrow \frac{2x}{18} + \frac{2yy'}{8} = 0 \Rightarrow y' = -\frac{8x}{18y}$.

$\Rightarrow y' = -\frac{8}{18} \times \frac{3}{2} \Rightarrow m_2 = -2/3$.

$\because m_1 m_2 = -1 \Rightarrow$ Angle of intersection $= \pi/2$.

Curves are orthogonal.

8. The lines tangent to the curve $y^3 - x^2y + 5y - 2x = 0$ and $x^4 - x^3y^2 + 5x + 2y = 0$ at the origin intersect at an angle θ equal to-

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

8. We have rational algebraic integral curves.

Eq^n. of tangent at origin :

$C_1 : y^3 - x^2y + 5y - 2x = 0 \Rightarrow 5y - 2x = 0 \longrightarrow T_1$

$C_2 : x^4 - x^3y^2 + 5x + 2y = 0 \Rightarrow 5x + 2y = 0 \longrightarrow T_2$.

$\therefore m_1 = 2/5$; $m_2 = -5/2 \Rightarrow m_1 m_2 = -1$.

$\therefore \theta = \pi/2$ Ans.

9. The angle of intersection between the curves $y^2 = 8x$ and $x^2 = 4y - 12$ at (2, 4) is-

(A) 90° (B) 60° (C) 45° (D) 0°

9. $C_1: y^2 = 8x \Rightarrow 2yy' = 8 \Rightarrow y'\big|_{(2,4)} = 1$
$C_2: x^2 = 4y - 12 \Rightarrow 2x = 4y' \Rightarrow y'\big|_{(2,4)} = 1$
$\left.\right\} \theta = 0°$

The curves touch each other at this point.

10. The length of subtangent to the curve $x^2 + xy + y^2 = 7$ at the point $(1, -3)$ is-

(A) 3 (B) 5 (C) 15 (D) 3/5

10. $x^2 + xy + y^2 = 7 \Rightarrow 2x + xy' + y + 2yy' = 0$

At $x = 1$, $y = -3$: $2(1) + (1)y' + (-3) + 2(-3)y' = 0$

$\Rightarrow y' = -1/5 \Rightarrow L_{ST} = \left|\frac{y}{y'}\right| = \left|\frac{-3}{-1/5}\right| = 15$ (Ans)

11. Let S be a square with sides of length x. If we approximate the change in size of the area of S by $h \cdot \left.\frac{dA}{dx}\right|_{x=x_0}$, when the sides are changed from x_0 to $x_0 + h$, then the absolute value of the error in our approximation, is

(A) h^2 (B) $2hx_0$ (C) x_0^2 (D) h

11. Area of Square $A = x^2$. $\Rightarrow \frac{dA}{dx} = 2x$.

Old area $= x_0^2$; New Area $= (x_0+h)^2 = x_0^2+h^2+2hx_0$ (Exact)

Exact change in Area $= x_0^2+h^2+2hx_0 - x_0^2 = \boxed{h^2+2hx_0}$

Approximate change in Area $= h \cdot \left.\frac{dA}{dx}\right|_{x=x_0} = \boxed{h \cdot 2x_0}$

Difference $= h^2$. (Ans A).

12. A Spherical balloon is being inflated at the rate of 35 cc/min. The rate of increase in the surface area (in cm^2/min.) of the balloon when its diameter is 14 cm, is :

(A) $\sqrt{10}$ (B) $10\sqrt{10}$ (C) 100 (D) 10

12. $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow 35 = 4\pi (7)^2 \frac{dr}{dt}$

$\Rightarrow \cancel{35}^{5} = 4 \cdot \frac{22}{\cancel{7}} \cdot \cancel{7} \cdot \cancel{7} \cdot \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{5}{88} \longrightarrow (1)$

$A = 4\pi r^2 \Rightarrow \frac{dA}{dt} = 8\pi r \frac{dr}{dt} \Rightarrow \frac{dA}{dt} = \cancel{8}^{2} \cdot \frac{\cancel{22}}{\cancel{7}} \cdot \cancel{7} \cdot \frac{5}{\cancel{88}_{\cancel{4}}}$ (by (1))

$\Rightarrow \frac{dA}{dt} = 10 \ cm^2/s.$ (Ans D)

13. If the surface area of a sphere of radius r is increasing uniformly at the rate $8cm^2/s$, then the rate of change of its volume is :

(A) proportional to r^2

(B) constant

(C) proportional to r

(D) proportional to $\sqrt{r}$

SA[4]

13. $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt} = 8 \Rightarrow \frac{dr}{dt} = \frac{1}{\pi r}.$

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dV}{dt} = 4\pi r^2 . \frac{1}{\pi r}$

$\Rightarrow \frac{dV}{dt} = 4r.$ (proportimal to r) (Ans. C).

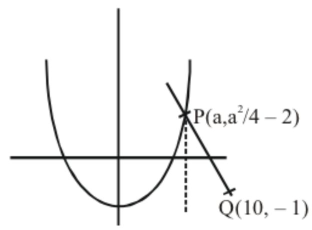
EXERCISE 02.

1. A line L is perpendicular to the curve $y = \frac{x^2}{4} - 2$ at its point P and passes through (10, –1). The coordinates of the point P are

(A) (2, –1) (B) (6, 7) (C) (0, –2) (D) (4, 2)

1. D

$$y = \frac{x^2}{4} - 2$$



$$y' = \frac{2x}{4}$$

$$m_N \text{ at } P = -\frac{2}{a}$$

$$\therefore\ -\frac{2}{a} = m_{PQ} = \frac{\frac{a^2}{4} - 2 + 1}{a - 10}$$

$\Rightarrow a^3 = -4a + 80$

$a^3 + 4a - 80 = 0 \Rightarrow a = 4$

$\therefore$ P(4,2)

2. The x-intercept of the tangent at any arbitrary point of the curve $\frac{a}{x^2}+\frac{b}{y^2}=1$ is proportional to:

(A) square of the abscissa of the point of tangency

(B) square root of the abscissa of the point of tangency

(C) cube of the abscissa of the point of tangency

(D) cube root of the abscissa of the point of tangency.

Sol. Ans. (C)

$$\frac{a}{x^2}+\frac{b}{y^2}=1 \quad \Rightarrow \quad ay^2+bx^2=x^2y^2 \qquad .....(1)$$

$$-\frac{2a}{x^3}-\frac{2b}{y^3}\frac{dy}{dx}=0 \quad \Rightarrow \quad \frac{dy}{dx}=-\frac{ay^3}{bx^3}$$

equation of tangent

$$Y-y=-\frac{ay^3}{bx^3}(X-x)$$

for x-intercept, put $Y=0$

$$\therefore \qquad X=\frac{bx^3}{ay^2}+x$$

$$X=x\left[\frac{bx^2+ay^2}{ay^2}\right]=x\left[\frac{x^2y^2}{ay^2}\right]=\frac{x^3}{a}$$

$\Rightarrow$ x-intercept is proportional to cube of abscissa

3. At any two points of the curve represented parametrically by $x = a(2\cos t - \cos 2t)$; $y = a(2\sin t - \sin 2t)$ the tangents are parallel to the axis of x corresponding to the values of the parameter t differing from each other by :

(A) $2\pi/3$ (B) $3\pi/4$ (C) $\pi/2$ (D) $\pi/3$

3. A

Sol. $\frac{dy}{dx} = \frac{\cos 2t - \cos t}{\sin 2t - \sin t} = 0 \Rightarrow \cos 2t = \cos t \Rightarrow t = \frac{2\pi}{3}, y - \frac{\pi}{3} \Rightarrow \text{diff} = \frac{2\pi}{3} \Rightarrow t = 2\pi/3]$

Sol^n $f(x) = \int_2^x (2t-5)\,dt = t^2 - 5t\Big]_2^x$.

$= x^2 - 5x + 6$

$f(x) = 0 \Rightarrow x = 2$ or 3.

$f'(x) = 2x - 5$.

$f'(2) = -1$

$f'(3) = 1$.

$f'(2) \cdot f'(3) = -1$.

$\therefore$ Angle between tangent lines $= \pi/2$.

5. The coordinates of the points on the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, where tangent is inclined an angle $\pi/4$ to the x-axis are-

(A) (a, a) (B) $\left(a\left(\frac{\pi}{2}-1\right), a\right)$ (C) $\left(a\left(\frac{\pi}{2}+1\right), a\right)$ (D) $\left(a, a\left(\frac{\pi}{2}+1\right)\right)$

5.

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a(0-(-\sin\theta))}{a(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta}.$$

$$\frac{dy}{dx} = \tan\left(\frac{\pi}{4}\right) = \frac{\sin\theta}{1+\cos\theta} \Rightarrow \tan(\pi/4) = \frac{2\sin\theta/2\cdot\cos\theta/2}{2\cos^2\theta/2}$$

$$\Rightarrow \tan\theta/2 = \tan\pi/4 \Rightarrow \boxed{\theta = \pi/2}$$

Ans (C)

$$\therefore x = a(\pi/2+1) \quad ; \quad y = a(1-0) = a.$$

6. If the tangent to the curve $2y^3 = ax^2 + x^3$ at a point (a, a) cuts off intercepts p and q on the coordinates axes, where $p^2 + q^2 = 61$, then a equals-

(A) 30 (B) –30 (C) 0 (D) ±30

6. $2y^3 = ax^2 + x^3 \Rightarrow 6y^2 y' = 2ax + 3x^2$

At $x = a$, $y = a$: $y' = \frac{5a^2}{6a^2} \Rightarrow y' = 5/6$.

$\therefore$ Eqn of tangent : $y - a = \frac{5}{6}(x - a)$

At $x = 0$: $y = a/6$; At $y = 0$: $x = -a/5$.

One value is p, other is q

$\therefore p^2 + q^2 = 61 \Rightarrow \frac{a^2}{36} + \frac{a^2}{25} = 61$ (Ans)

$\Rightarrow \frac{61 a^2}{25 \cdot 36} = 61 \Rightarrow a^2 = (5 \times 6)^2 \Rightarrow \boxed{a = \pm 30}$

7. The sum of the intercepts made by a tangent to the curve $\sqrt{x}+\sqrt{y}=4$ at point (4, 4) on coordinate axes is-

(A) $4\sqrt{2}$ (B) $6\sqrt{3}$ (C) $8\sqrt{2}$ (D) $\sqrt{256}$

7.

$\frac{x^n}{a^n}+\frac{y^n}{b^n}=1.$

Eq^n of T at (x_1, y_1) is

$\frac{x x_1^{n-1}}{a^n}+\frac{y y_1^{n-1}}{b^n}=1$

$x^{1/2}+y^{1/2}=4$

Tangent at (4, 4):

$x\cdot(4)^{1/2-1}+y(4)^{1/2-1}=4$

$\Rightarrow x/2+y/2=4 \Rightarrow x+y=8.$

$\therefore x_{int}=8;\ y_{int}=8 \Rightarrow$ Sum $=16=\sqrt{256}$

8. If the tangent at a point P, with parameter t, on the curve $x = 4t^2 + 3$, $y = 8t^3 - 1$, $t \in R$, meets the curve again at a point Q, then the coordinates of Q are :

(A) $(t^2 + 3, t^3 - 1)$

(B) $(t^2 + 3, -t^3 - 1)$

(C) $(16t^2 + 3, -64t^3 - 1)$

(D) $(4t^2 + 3, -8t^3 - 1)$

8. $(4t^2+3, 8t^3-1)$ P

$Q(4t_1^3+3, 8t_1^3-1)$ (let).

$m_{PQ} = \left.\frac{dy}{dx}\right|_P$

$\Rightarrow \frac{8t_1^3 - 8t^3}{4t_1^2 - 4t^2} = \frac{\cancel{24}^{\,3}\, t^{\cancel{2}}}{\cancel{8t}}.$

$\Rightarrow \frac{2\cancel{(t_1 - t)}(t_1^2 + t^2 + tt_1)}{\cancel{(t_1 - t)}(t_1 + t)} = 3t.$ $(\because t_1 \neq t)$

$\Rightarrow 2t_1^2 + 2t^2 + 2tt_1 = 3tt_1 + 3t^2.$

$\Rightarrow t^2 + tt_1 - 2t_1^2 = 0 \Rightarrow (t - t_1)(t + 2t_1) = 0$

$\Rightarrow t = t_1$ (reject) or $t_1 = -t/2$

Put $t_1 = -t/2$, we get: $Q\left(\left(+4 \cdot \frac{t^2}{4} + 3\right), \left(8\left(-\frac{t^3}{8}\right) - 1\right)\right)$

$\Rightarrow Q\left((+t^2 + 3), (-t^3 - 1)\right)$ (Ans B).

9. For a curve $\frac{(\text{length of normal})^2}{(\text{length of tangent})^2}$ is equal to -

(A) (subnormal)/(subtangent)

(B) (subtangent)/(subnormal)

(C) (subtangent × subnormal)

(D) constant

9. $\left(\frac{L_N}{L_T}\right)^2 = \left(\frac{y\sqrt{1+(y')^2}}{\frac{y\sqrt{1+(y')^2}}{y'}}\right)^2 = (y')^2.$

Now check options

(A) $\frac{L_{SN}}{L_{ST}} = \frac{|y y'|}{|y/y'|} = (y')^2$ (Ans.)

10. At any point of a curve (subtangent) × (subnormal) is equal to the square of the-

(A) slope of the tangent at that point

(B) slope of the normal at that point

(C) abscissa of that point

(D) ordinate of that point

10.

$$\frac{L_{ST}}{L_{SN}} = \frac{1}{(y')^2} = \left(-\frac{1}{y'}\right)^2 = (\text{Slope of normal})^2$$

(Ans B)

EXERCISE S1.

1. Find the equation of the normal to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$.

1. $y = (1+x)^y + \sin^{-1}(\sin^2 x)$

At $x = 0$, $y = 1 + 0 \Rightarrow y = 1$.

$y' = y(1+x)^{y-1} \cdot 1 + (1+x)^y \cdot \ln(1+x) \cdot y' + \dfrac{2\sin x \cos x}{\sqrt{1 - \sin^4 x}}$

(using shortcut of exponential differentiation)

Put $x = 0$, $y = 1$:

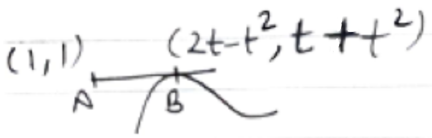
$y' = 1 + 0 + 0 \Rightarrow y' = 1 = m_T \Rightarrow m_N = -1$

$\therefore$ Eq^n. of normal: $y - 1 = -1(x - 0)$

$\Rightarrow x + y - 1 = 0$. (Ans).

2. Find all the lines that pass through the point (1, 1) and are tangent to the curve represented parametrically as $x = 2t - t^2$ and $y = t + t^2$.

2. $\left.\begin{aligned} x &= 2t - t^2 \\ y &= t + t^2 \end{aligned}\right\}$ (1,1) does not lie on this curve.



$$m_{AB} = \left.\frac{dy}{dx}\right|_B$$

$$\Rightarrow \frac{t^2+t-1}{2t-t^2-1} = \frac{dy/dt}{dx/dt} \qquad \text{for } (t \neq 1)$$

$$\Rightarrow \frac{t^2+t-1}{-(t-1)^2} = \frac{1+2t}{2-2t} \Rightarrow \frac{t^2+t-1}{-(t-1)^{\cancel{2}}} = \frac{2t+1}{-2\cancel{(t-1)}}$$

$$\Rightarrow 2t^2+2t-2 = (2t+1)(t-1)$$

$$\Rightarrow 2t^2+2t-2 = 2t^2 - 2t + t - 1 \Rightarrow t = 1/3.$$

At $t = 1/3$: $B(5/9, 4/9) \Rightarrow AB : 5x - 4y = 1$.

At $t = 1$: $B(1, 2)$; $A(1, 1)$

$\therefore$ Eqⁿ. of AB : $x = 1$ (vertical line).

3. The tangent to $y = ax^2 + bx + \frac{7}{2}$ at (1, 2) is parallel to the normal at the point (–2, 2) on the curve $y = x^2 + 6x + 10$. Find the value of a and b.

3. (1,2) lies on $y = ax^2 + bx + 7/2 \Rightarrow \boxed{a+b+3/2=0}$

$y = ax^2 + bx + 7/2 \Rightarrow y' = 2ax + b \big|_{(1,2)} \Rightarrow y' = 2a + b.$

$y = x^2 + 6x + 10 \Rightarrow y' = 2x + 6 \big|_{(-2,2)} \Rightarrow y' = -4 + 6 = 2$

$\Rightarrow m_N = -1/2$

$\therefore \boxed{2a + b = -1/2}$ Solving, we get $a = 1, b = -5/2$

4. Find all the tangents to the curve $y = \cos(x + y)$, $-2\pi \le x \le 2\pi$, that are parallel to the line $x + 2y = 0$.

4. $y = \cos(x+y)$, $x \in [-2\pi, 2\pi]$ $\{$Also $y \in [-1,1]\}$

$y' = -\sin(x+y)\cdot(1+y') \Rightarrow y' = -\dfrac{\sin(x+y)}{1+\sin(x+y)} = \dfrac{-1}{2}$

$\Rightarrow 2\sin(x+y) = 1 + \sin(x+y) \Rightarrow \sin(x+y) = 1$

$\Rightarrow x+y = \pi/2,\ x+y = -\dfrac{3\pi}{2}$ only.

At $x+y = \pi/2$ or $-3\pi/2$; $y = \cos(x+y) = 0$ only

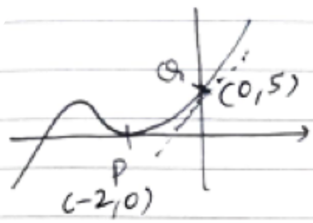
$\therefore$ Points of contact can be $\left(\dfrac{\pi}{2}, 0\right)$ or $\left(\dfrac{-3\pi}{2}, 0\right)$

For $(\pi/2, 0)$: $T \equiv y - 0 = -1/2(x - \pi/2) \Rightarrow x + 2y = \pi/2$ Ans.

For $(-3\pi/2, 0)$: $T \equiv y - 0 = -1/2(x + \frac{3\pi}{2}) \Rightarrow x + 2y = -3\pi/2$ Ans

5. The curve $y = ax^3 + bx^2 + cx + 5$, touches the x - axis at P $(-2, 0)$ & cuts the y-axis at a point Q where its gradient is 3. Find a, b, c.

5.



We have 3 variables a, b, c and we can write 3 conditions. P lies on curve; $\frac{dy}{dx}\Big|_P = 0$ and $\frac{dy}{dx}\Big|_Q = 3$.

$y = ax^3 + bx^2 + cx + 5$.

(i) $P(-2,0)$ lies on curve $\Rightarrow 0 = a(-8) + b(4) + c(-2) + 5$.

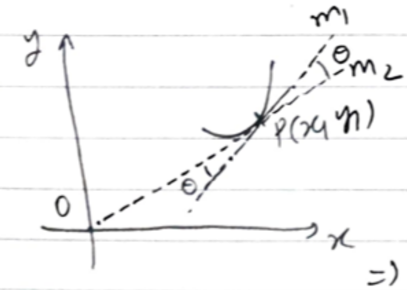
(ii) $(dy/dx)_P = 0 \Rightarrow 3ax^2 + 2bx + c\big|_{x=-2} = 0 \Rightarrow 12a - 4b + c = 0$.

(iii) $(dy/dx)_Q = 3 \Rightarrow 3ax^2 + 2bx + c\big|_{x=0} = 3 \Rightarrow c = 3$.

by solving (i), (ii) and (iii) : $a = -1/2$; $b = -3/4$; $c = 3$

6. Show that the angle between the tangent at any point 'A' of the curve $\ln(x^2+y^2) = C\tan^{-1}\frac{y}{x}$ and the line joining A to the origin is independent of the position of A on the curve.

6.



$m_2 = y_1/x_1$

$m_1 = \left.\frac{dy}{dx}\right|_{(x_1, y_1)}$

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

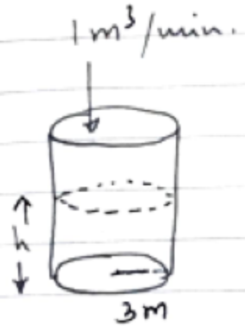
$\ln(x^2+y^2) = C\tan^{-1}(y/x)$

$\Rightarrow \frac{2x + 2yy'}{x^2+y^2} = \left(\frac{C}{1+(y/x)^2}\right)\left(\frac{x \cdot y' - y}{x^2}\right)$

$\Rightarrow \frac{2(x + yy')}{\cancel{x^2+y^2}} = \left(\frac{C}{\cancel{x^2+y^2}}\right)(xy' - y) \Rightarrow y' = \frac{2x_1 + Cy_1}{Cx_1 - 2y_1}$

7. Water is being poured on to a cylindrical vessel at the rate of 1 m^3/min. If the vessel has a circular base of radius 3 m, find the rate at which the level of water is rising in the vessel.

7.



$$V_t = \pi (3)^2 h.$$

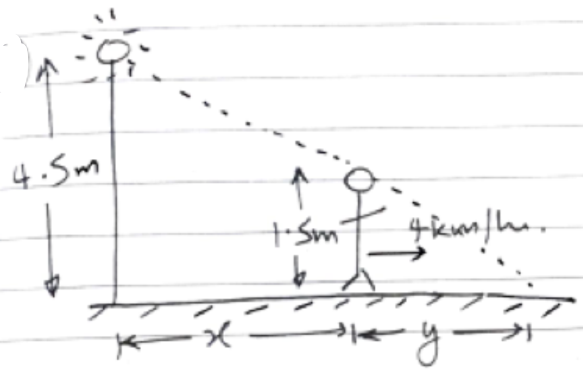
$$\frac{dV}{dt} = 9\pi \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{9\pi} \text{ m/min}$$

(Ans.)

8. A man 1.5 m tall walks away from a lamp post 4.5 m high at the rate of 4 km/hr.

(i) how fast is the farther end of the shadow moving on the pavement ?

(ii) how fast is his shadow lengthening ?

8. 

$\frac{dx}{dt} = 4 km/h.$ (Speed of man)

$y =$ length of shadow

$\frac{dy}{dt} =$ rate at which shadow is lengthening.

$\frac{4.5}{1.5} = \frac{x+y}{y} \Rightarrow 3y = x+y \Rightarrow 2y = x.$

(by $\sim \Delta s$.)

$\Rightarrow 2\frac{dy}{dt} = \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = 2$

(i) for farther end of shadow's speed:

$\frac{d}{dt}(x+y) = \frac{dx}{dt} + \frac{dy}{dt} = 6$ Km/h (Ans)

(ii) Rate of change of length of shadow $= \frac{dy}{dt} = 2$ (Ans)

EXERCISE S2.

1. Find the point of intersection of the tangents drawn to the curve $x^2y = 1 - y$ at the points where it is intersected by the curve $xy = 1 - y$.

1. (0, 1)

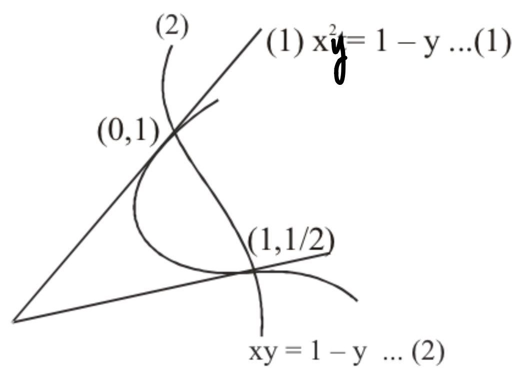
Sol. $xy = x^2y$ ...(1)

$xy(x - 1) = 0$

$x = 0, y = 0, x = 1$

$xy = 1 - y$ ...(2)

when $x = 0$ then $y = 1 \Rightarrow (x, y) = (0, 1)$



when $x = 1$ then $y = \frac{1}{2} \Rightarrow (x, y) = \left(1, \frac{1}{2}\right)$

when $y = 0$ then $0 = 1$ (which not possible so reject it)

using (1) & (2)

$y(x^2 + 1) = 1$

$y = \frac{1}{x^2+1}$

$\frac{dy}{dx} = \frac{-2x}{(1+x^2)^2}$

$\left.\frac{dy}{dx}\right|_{(0,1)} = 0$

Equation of tangent

$y - 1 = 0$

$\boxed{y = 1}$ ...(3)

$\left.\frac{dy}{dx}\right|_{(1,1/2)} = \frac{-2}{4} = \frac{-1}{2}$

Equation of tangent $\Rightarrow$

$y - \frac{1}{2} = -\frac{1}{2}(x - 1)$

$2y - 1 = -x + 1$

$2y + x = 2$ ...(4)

so by using (3) & (4) point of intersection of tangent is (0, 1) **Ans**

2. A function is defined parametrically by the equations

$$f(t) = x = \left[\begin{array}{ll} 2t + t^2 \sin\frac{1}{t} & \text{if } t \neq 0 \\ 0 & \text{if } t = 0 \end{array}\right] \quad \text{and } g(t) = y = \left[\begin{array}{ll} \frac{1}{t}\sin t^2 & \text{if } t \neq 0 \\ o & \text{if } t = 0 \end{array}\right.$$

2. Find the equation of the tangent and normal at the point for $t = 0$ if exist.

$T : x - 2y = 0 \ ; \ N : 2x + y = 0$

Sol. $$\frac{dy}{dx} = \frac{g'(t)}{f'(t)} \qquad \text{...(1)}$$

$$g'(t) = \lim_{t\to 0}\left(\frac{g(t) - g(0)}{t - 0}\right)$$

$$= \lim_{t\to 0}\left(\frac{\frac{1}{t}\sin t^2 - 0}{t}\right) = 1$$

$$f'(t) = \lim_{t\to 0}\left(\frac{f(t) - f(0)}{t - 0}\right)$$

$$= \lim_{t\to 0}\frac{2t + t^2 \sin\frac{1}{t}}{t} = 2 + 0 = 2$$

then $\frac{dy}{dx} = \frac{1}{2}$

so the equation of tangent

$$y - 0 = \frac{1}{2}(x - 0)$$

$\boxed{x - 2y = 0}$ **Ans.**

equation of normal

3. Show that the condition that the curves $x^{2/3} + y^{2/3} = c^{2/3}$ & $(x^2/a^2) + (y^2/b^2) = 1$ may touch if $c = a + b$.

3. When the two curve meet at a point then their $\frac{dy}{dx}$ are equal.

$x^{2/3} + y^{2/3} = c^{2/3}$ ...(1)

(α, β) lies on (1) so

$\alpha^{2/3} + \beta^{2/3} = c^{2/3}$ ...(3)

similarly lies on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(2) then

$\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1$ ...(4)

$\frac{dy}{dx}$ of equation(1)

$$\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$$

$$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)} = -\frac{\beta^{1/3}}{\alpha^{1/3}}$$

$\frac{dy}{dx}$ of equation (2)

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)} = -\frac{b^2\alpha}{\alpha^2\beta}$$

curve touches when

$$-\frac{\beta^{1/3}}{\alpha^{1/3}} = -\frac{b^2\alpha}{a^2\beta}$$

$$a^2\beta^{4/3} = b^2\,\alpha^{4/3}$$

$$\frac{\alpha^{4/3}}{a^2} = \frac{\beta^{4/3}}{b^2} = \lambda \text{ let}$$

$\alpha^{4/3} = \lambda a^2 \Rightarrow \alpha = (\lambda a^2)^{3/4}$

$\Rightarrow \alpha = \lambda^{3/4}.\ a^{3/2}$

$\beta^{4/3} = \lambda b^2 \Rightarrow \beta = (\lambda b^2)^{3/4}$

$\Rightarrow \beta = \lambda^{3/4}.\ b^{3/2}$

put these value in equation (4)

$$\frac{\lambda^{3/2}.a^3}{a^2} + \frac{\lambda^{3/2}b^3}{b^2} = 1$$

$$\lambda^{3/2}\,(a + b) = 1$$

$$\lambda^{3/2} = \frac{1}{a+b}$$

$$\lambda = \left(\frac{1}{a+b}\right)^{2/3}$$

again put value of (α, β) in (3)

$$(\lambda^{3/4}.\ a^{3/2})^{2/3} + (\lambda^{3/4}.\ b^{3/2})^{2/3} = c^{2/3}$$

$$\sqrt{\lambda}\,(a + b) = c^{2/3}$$

$$\lambda\,(a + b)^2 = c^{4/3}$$

$$\frac{1}{(a+b)^{2/3}}.(a+b)^2 = c^{4/3}$$

$$(a + b)^{4/3} = c^{4/3}$$

$a + b = c$ **Ans**

4. Prove that the segment of the normal to the curve $x = 2a \sin t + a \sin t \cos^2 t$; $y = -a \cos^3 t$ contained between the co-ordinate axes is equal to 2a.

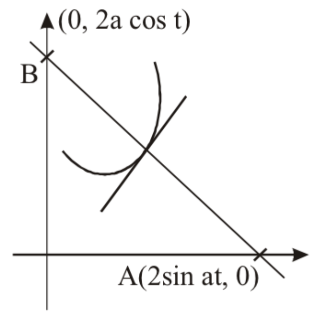
Sol. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$$= \frac{3a\cos^2 t \sin t}{2a\cos t + a\cos^3 t - 2a\sin^2 t \cos t}$$

$$= \frac{3a\cos^2 t \sin t}{2a\cos t(1-\sin^2 t) + a\cos^3 t}$$

$$= \frac{3a\cos^2 t \sin t}{2a\cos^3 t + a\cos^3 t}$$

$$= \frac{3a\cos^2 t \sin t}{3a\cos^3 t}$$



$\frac{dy}{dx} = \tan t$

equation of normal

$$y + a\cos^3 t = -\frac{\cos t}{\sin t}(x - 2a \sin t - a \sin t \cos^2 t)$$

$y \sin t + a \cos^3 t \sin t = -x \cos t + 2a \sin t \cos t + a \sin t \cos^3 t$

$x \cos t + y \sin t = 2a \sin t \cos t$ ...(1)

at $A(2a \sin t, 0)$

5. (a) Show that the curves $\frac{x^2}{a^2+K_1}+\frac{y^2}{b^2+K_1}=1$ & $\frac{x^2}{a^2+K_2}+\frac{y^2}{b^2+K_2}=1$ $(K_1 \neq K_2)$ intersect orthogonally.

(b) If the two curves $C_1 : x = y^2$ and $C_2 : xy = k$ cut at right angles find the value of k.

5. (a) Let $P(x_1\, y_1)$ be point of intersection.

$C_1: \frac{x^2}{a^2+k_1}+\frac{y^2}{b^2+k_1}=1$; $C_2 = \frac{x^2}{a^2+k_2}+\frac{y^2}{b^2+k_2}=1.$

for C_1: $\frac{\cancel{2}x}{a^2+k_1}+\frac{\cancel{2}yy'}{b^2+k_1}=0 \Rightarrow y'\Big|_{(x_1 y_1)} = -\left(\frac{b^2+k_1}{a^2+k_1}\right)\cdot\frac{x_1}{y_1} = m_1$

similarly for C_2: $y'\Big|_{(x_1 y_1)} = -\left(\frac{b^2+k_2}{a^2+k_2}\right)\cdot\frac{x_1}{y_1} = m_2$

for orthogonal : We need to prove that

$$\frac{x_1^2}{y_1^2}\frac{(b^2+k_1)(b^2+k_2)}{(a^2+k_1)(a^2+k_2)} = -1.$$

LHS : $\because$ $P(x_1\, y_1)$ lies on C_1 & C_2

$\Rightarrow \frac{x_1^2}{a^2+k_1}+\frac{y_1^2}{b^2+k_1}=1$ and $\frac{x_1^2}{a^2+k_2}+\frac{y_1^2}{b^2+k_2}=1.$

$\Rightarrow \frac{x_1^2}{a^2+k_1}+\frac{y_1^2}{b^2+k_1} = \frac{x_1^2}{a^2+k_2}+\frac{y_1^2}{b^2+k_2}$

$\Rightarrow x_1^2\left[\frac{k_2-k_1}{(a^2+k_1)(a^2+k_2)}\right] = y_1^2\left[\frac{k_1-k_2}{(b^2+k_2)(b^2+k_1)}\right]$

$\Rightarrow \frac{x_1^2}{y_1^2} = -\left(\frac{(a^2+k_1)(a^2+k_2)}{(b^2+k_1)(b^2+k_2)}\right)$

$\Rightarrow \left(\frac{x_1^2}{y_1^2}\right)\left(\frac{(b^2+k_1)(b^2+k_2)}{(a^2+k_1)(a^2+k_2)}\right) = -1.$ H.P.

5.

(a) Show that the curves $\frac{x^2}{a^2+K_1}+\frac{y^2}{b^2+K_1}=1$ & $\frac{x^2}{a^2+K_2}+\frac{y^2}{b^2+K_2}=1$ $(K_1 \neq K_2)$ intersect orthogonally.

(b) If the two curves $C_1 : x = y^2$ and $C_2 : xy = k$ cut at right angles find the value of k.

5. Let Pt. of intersection $P(x_1\, y_1) \Rightarrow y_1^2 = x_1$ & $x_1 y_1 = k$.

Also : $1 = 2yy' \Rightarrow y' = 1/2y_1$; $xy' + y = 0 \Rightarrow y' = -y_1/x_1$

$m_1 m_2 = -1 \Rightarrow \frac{-y_1}{2x_1 y_1} = -1 \Rightarrow x_1 = 1/2 \Rightarrow \therefore y_1 = \pm \frac{1}{\sqrt{2}}$

$\therefore x_1 y_1 = k \Rightarrow k = \pm \frac{1}{2\sqrt{2}}$

6. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y coordinate is changing 8 times as fast as the x coordinate.

6. $6y = x^3 + 2 \Rightarrow 6\frac{dy}{dt} = 3x^2 \frac{dx}{dt}$.

We want $\frac{dy}{dt} = 8\frac{dx}{dt} \Rightarrow 6 \cdot 8 \cdot \frac{dx}{dt} = 3x^2 \frac{dx}{dt}$

$\Rightarrow x^2 = 16 \Rightarrow x = 4$ and $x = -4$.

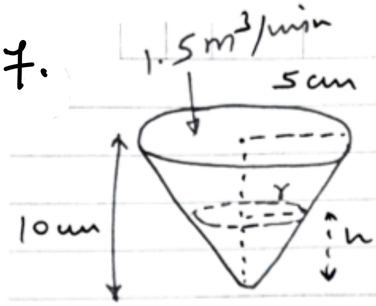
At $x = 4$: $y = \frac{4^3+2}{6} \Rightarrow y = 11 \Rightarrow (4, 11)$ (Ans.)

At $x = -4$: $y = \frac{(-4)^3+2}{6} \Rightarrow y = -\frac{31}{3} \Rightarrow \left(-4, -\frac{31}{3}\right)$ (Ans)

7.

An inverted cone has a depth of 10 cm & a base of radius 5 cm. Water is poured into it at the rate of 1.5 cm³/min. Find the rate at which level of water in the cone is rising, when the depth of water is 4 cm.

7.



$\frac{r}{h} = \frac{5}{10}. \Rightarrow r = \frac{h}{2}.$

$V = \frac{1}{3}\pi r^2 h.$

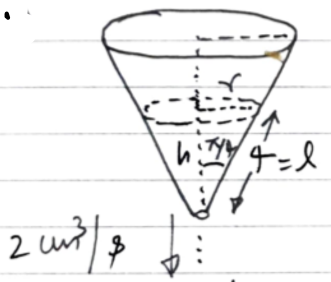
$\Rightarrow V = \frac{1}{3}\pi \frac{h^2}{4}.h \Rightarrow V = \frac{\pi}{12}.h^3.$

$\Rightarrow \frac{dV}{dt} = \frac{\pi}{4}.h^2\frac{dh}{dt} \Rightarrow 1.5 = \frac{\pi}{4}(4)^2\frac{dh}{dt}.$

$\Rightarrow \frac{dh}{dt} = \frac{3}{8\pi}$ cm/min (Ans)

8. Water is dripping out from a conical funnel of semi vertical angle $\pi/4$, at the uniform rate of $2\ cm^3/sec$ through a tiny hole at the vertex at the bottom. When the slant height of the water is 4 cm, find the rate of decrease of the slant height of the water.

8.



$$\frac{r}{h} = \tan\frac{\pi}{4} \Rightarrow r = h.$$

$$V = \frac{1}{3}\pi r^2 h.$$

$$V = \frac{1}{3}\pi r^3.$$

At $l = 4$:

$h = l\cos\frac{\pi}{4} = \frac{l}{\sqrt{2}}$

$r = l\sin\frac{\pi}{4} = \frac{l}{\sqrt{2}}$

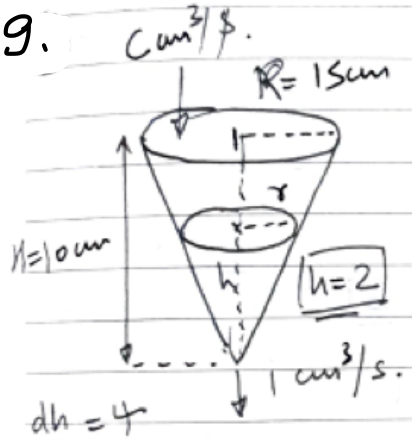
$$\Rightarrow \frac{dV}{dt} = \pi \cdot r^2 \cdot \frac{dr}{dt}$$

$$\Rightarrow \frac{dV}{dt} = \pi \cdot \left(\frac{4}{\sqrt{2}}\right)^2 \cdot \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi}$$

Also, $r = \frac{l}{\sqrt{2}} \Rightarrow \frac{dr}{dt} = \frac{1}{\sqrt{2}}\frac{dl}{dt} \Rightarrow \frac{dl}{dt} = \frac{\sqrt{2}}{4\pi}$ cm/s. (Ans)

9. A water tank has the shape of a right circular cone with its vertex down. Its altitude is 10 cm and the radius of the base is 15 cm. Water leaks out of the bottom at a constant rate of 1 cu. cm/sec. Water is poured into the tank at a constant rate of C cu. cm/sec. Compute C so that the water level will be rising at the rate of 4 cm/sec at the instant when the water is 2 cm deep.

9.



$\frac{dh}{dt} = 4$

$$V = \frac{\pi r^2 h}{3} \qquad \frac{r}{h} = \frac{15}{10} \Rightarrow r = \frac{3h}{2}$$

$$\Rightarrow V = \frac{\pi h}{3}\left(\frac{3h}{2}\right)^2 \Rightarrow V = \frac{3\pi h^3}{4}$$

$$\frac{dV}{dt} = \frac{3\pi}{4}\cdot(3h^2)\,\frac{dh}{dt}$$

$$\Rightarrow c - 1 = \left(\frac{3\pi h^2}{\cancel{4}}\right)(3\cdot\cancel{4})$$

$$\Rightarrow C = 1 + 9\pi(2)^2 \Rightarrow C = 1 + 36\pi.$$

Note: For water level to rise, inlet rate ($c\,cm^3/s$) must be more than outlet rate ($1\,cm^3/s$)

10. Sand is pouring from a pipe at the rate of 12 cc/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always 1/6th of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm.

10.

$12\ cm^3/s$

$h = 4$

$h = r/6 \quad ; \quad V = \frac{1}{3}\pi r^2 h.$

$\Rightarrow r = 6h$

$\therefore V = \frac{1}{3} \cdot \pi \cdot (36h^2) \cdot h.$

$\Rightarrow V = 12\pi h^3 \Rightarrow \frac{dV}{dt} = 36\pi h^2 \frac{dh}{dt}$

$\Rightarrow 12 = 36 \cdot \pi \cdot 16 \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{48\pi}$ cm/s (Ans)

11. A circular ink blot grows at the rate of 2 cm^2 per second. Find the rate at which the radius is increasing after $2\frac{6}{11}$ seconds. Use $\pi = \frac{22}{7}$.

11. At $t=0$, $r=0$ (blot starts).

At $t = 28/11$ secs, $A = 2 \cdot \frac{28}{11} = \frac{56}{11}\text{ cm}^2$.

$A = \frac{56}{11} = \frac{22}{7} \cdot r^2 \Rightarrow r = 14/11$. ; $\frac{dA}{dt} = 2$ (given)

$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \Rightarrow 2 = 2 \cdot \frac{22}{7} \cdot \frac{14}{11} \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = 1/4$ (Ans).

EXERCISE (JM)

1. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is :-

[AIEEE-2010]

(1) $y = 0$ (2) $y = 1$ (3) $y = 2$ (4) $y = 3$

① For tangent to be parallel to x axis: $\frac{dy}{dx} = 0$.

$\frac{dy}{dx} = 1 - \frac{8}{x^3} = 0 \Rightarrow x^3 = 8 \Rightarrow x = 2$.

At $x = 2$: $y = 2 + \frac{4}{4} \Rightarrow y = 3$. $\therefore P(2, 3)$

Eq^n. of line of zero slope thru $(2,3)$ is $y = 3$ (Ans)

$$f(x) = \begin{cases} \dfrac{x^2}{2}. & x \geq 0. \\ -\dfrac{x^2}{2}. & x < 0. \end{cases}$$

$$f'(x) = \begin{cases} x & x \geq 0. \\ -x & x < 0. \end{cases}$$

$\because$ tangent is parallel to $y = 2x$.

$\Rightarrow \quad x = 2 \quad$ if $\; x > 0$.

or $\; -x = 2 \quad$ if $\; x < 0$.

$\Rightarrow \quad x = \pm 2.$

Tangent:

$\underline{x \geq 0}. \quad y - 2 = 2(x - 2).$

$\underline{x < 0}. \quad y + 2 = 2(x + 2).$

$\therefore$ x intercepts $= \pm 1.$

3. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1) : **[JEE-MAIN 2015]**

(1) meets the curve again in the third quadrant

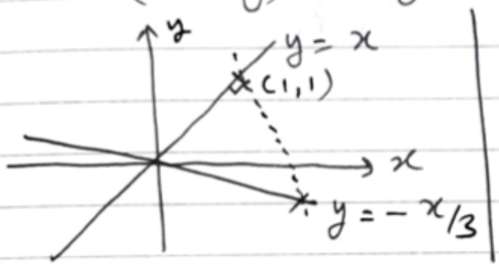
(2) meets the curve again in the fourth quadrant

(3) does not meet the curve again

(4) meets the curve again in the second quadrant

3. $x^2 + 2xy - 3y^2 = 0 \Rightarrow x^2 + 3xy - xy - 3y^2 = 0$

$\Rightarrow (x+3y)(x-y) = 0$ (Pair of St. lines.)



(1,1) lies on $y = x$ line

N at (1, 1) to line $y = x$:

$y - 1 = -1(x-1)$

$\Rightarrow x + y = 2$

Its pt. of intn. with $x + 3y = 0$ is $(3, -1)$ (IV Quad.)

<u>Note</u> : Ans. can also be concluded from figure.

4. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$, $x \in \left(0, \frac{\pi}{2}\right)$. A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point : **[JEE-MAIN 2016]**

(1) $\left(\frac{\pi}{4}, 0\right)$ (2) $(0, 0)$ (3) $\left(0, \frac{2\pi}{3}\right)$ (4) $\left(\frac{\pi}{6}, 0\right)$

4. Consider $\varepsilon = \frac{1+\sin x}{1-\sin x} \Rightarrow \varepsilon = \frac{(\cos x/2 + \sin x/2)^2}{(\cos x/2 - \sin x/2)^2}$.

$\Rightarrow \varepsilon = \left(\frac{\cos x/2 + \sin x/2}{\cos x/2 - \sin x/2}\right)^2 \Rightarrow \varepsilon = \left(\frac{1+\tan x/2}{1-\tan x/2}\right)^2$.

$\Rightarrow \varepsilon = \left(\frac{\tan \pi/4 + \tan x/2}{1 - \tan \pi/4 \tan x/2}\right)^2 \Rightarrow \varepsilon = \left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)^2$.

$\therefore f(x) = \tan^{-1}(\sqrt{\varepsilon}) \Rightarrow f(x) = \tan^{-1}\left|\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)\right|$

$\because x \in \left(0, \frac{\pi}{2}\right) \Rightarrow \frac{x}{2} \in \left(0, \frac{\pi}{4}\right) \Rightarrow \frac{\pi}{4}+\frac{x}{2} \in \left(0, \frac{\pi}{2}\right)$.

$\therefore$ Mod opens positive and $\tan^{-1}\tan M = M$

$\Rightarrow f(x) = \frac{\pi}{4} + \frac{x}{2} = y$ (line) ; Normal at $x = \pi/6$ is
$y - \pi/3 = -2(x - \pi/6) \Rightarrow$ (Ans. 3)

5. The normal to the curve $y(x-2)(x-3) = x+6$ at the point where the curve intersects the y-axis passes through the point : **[JEE-MAIN 2017]**

(1) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (2) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (3) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (4) $\left(\frac{1}{2}, -\frac{1}{3}\right)$

5. $y(x-2)(x-3) = x+6$.

for Pt. of intersection with y axis :

At $x=0$: $y = 1$. $\Rightarrow P(0,1)$

$y = \dfrac{x+6}{(x-2)(x-3)} \Rightarrow y = \dfrac{x+6}{x^2-5x+6} \Rightarrow y' = \dfrac{(x^2-5x+6)(1) - (x+6)(2x-5)}{(x^2-5x+6)^2}$.

At $x=0$: $y' = \dfrac{6-(-30)}{6^2} \Rightarrow y' = 1 = m_T$

$\therefore m_N = -1 \Rightarrow N : y-1 = -1(x-0)$

$\Rightarrow x+y = 1$ $\therefore$ (Ans. 3).

6. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :

(1) $\frac{7}{2}$ (2) 4 (3) $\frac{9}{2}$ (4) 6 **[JEE-MAIN 2018]**

6. $C_1 : y^2 = 6x$; $C_2 : 9x^2 + by^2 = 16$

Let pt. of intersection be $P(x_1, y_1)$

P lies on $C_1 \Rightarrow y_1^2 = 6x_1$ —— (1)

$C_1 : y^2 = 6x \Rightarrow 2yy' = 6\cancel{x}(1) \Rightarrow y' = 3/y_1 = m_1$

$C_2 : 9x^2 + by^2 = 16$

$\Rightarrow 18x + 2byy' = 0 \Rightarrow y' = -\frac{9x_1}{by_1} = m_2$

$\because m_1 \cdot m_2 = -1 \Rightarrow -\frac{27x_1}{by_1^2} = -1.$

by Eq. (1)

$\Rightarrow -\frac{27x_1}{b(6x_1)} = -1 \Rightarrow b = \frac{27}{6} \Rightarrow b = \frac{9}{2}$ Ans

7. The tangent to the curve, $y = xe^{x^2}$ passing through the point (1,e) also passes through the point :

[JEE-MAIN 2019]

(1) $\left(\frac{4}{3}, 2e\right)$ (2) (2,3e) (3) $\left(\frac{5}{3}, 2e\right)$ (4) (3,6e)

7. $y = xe^{x^2} \Rightarrow y' = e^{x^2} + x \cdot e^{x^2}(2x)$

At $x = 1$: $y' = e + 1 \cdot e \cdot (2) \Rightarrow y' = 3e$.

Eqn. of tangent: $y - e = 3e(x - 1)$

$\therefore$ (Ans. 1).

Ex- JA

1. If the normal to the curve, $y = f(x)$ at the point $(3, 4)$ makes an angle $\frac{3\pi}{4}$ with the positive x–axis. Then $f'(3) =$

(A) -1 (B) $-\frac{3}{4}$ (C) $\frac{4}{3}$ (D) 1

[JEE 2000 (Scr.) 1 out of 35]

1.

$m_N = \tan\left(\frac{3\pi}{4}\right) \Rightarrow m_N = -1 \Rightarrow m_T = 1 = f'(3)$.

2. The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is(are)

(A) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (B) $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$ (C) (0, 0) (D) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

[JEE 2002 (Scr.), 3]

2. $y^3 + 3x^2 = 12y \Rightarrow 3y^2y' + 6x = 12y'$

$\Rightarrow y' = \dfrac{6x}{12-3y^2} \Rightarrow y' = \dfrac{2x}{4-y^2}$ = DNE at $y = \pm 2$

$\therefore$ At $y = 2$: $8 + 3x^2 = 24 \Rightarrow x = \pm 4/\sqrt{3}$

At $y = -2$: $-8 + 3x^2 = -24 \Rightarrow 3x^2 = -16 \times\times\times$

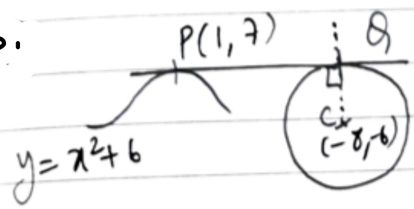
$\therefore (\pm 4/\sqrt{3}, 2)$ (Ans D).

3. Tangent to the curve $y = x^2 + 6$ at a point P (1, 7) touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q. Then the coordinates of Q are

(A) (– 6, –11) (B) (–9, –13) (C) (– 10, – 15) (D) (–6, –7)

[JEE 2005 (Scr.), 3]

3.



Q is point of intersection tangent at P and line $\perp$ to this tangent and passing thru C(−8, −6)

$y = x^2 + 6 \Rightarrow y' = 2x|_P \Rightarrow y' = 2 = m_{T\,at\,P}$.

$\therefore$ Eqn. of tangent: $y - 7 = 2(x-1) \Rightarrow 2x - y + 5 = 0$. ⓘ

Also $m_{line\ CQ} = -1/2 \Rightarrow$ Eqn. of CQ: $y + 6 = (-1/2)(x+8)$

$\Rightarrow x + 2y + 20 = 0. \longrightarrow$ (II)

Solving (I) and (II), we get Q(−6, −7).