*Q Let $a, b, c, d \in \{1, 2, 3, \dots, 9\}$ and $N = abcd$ (a four digit number).
Find number of N such that $a < b \leq c < d$?

Sol^n C-I $\quad a < b < c < d$

$${}^9C_4 \times 1 = {}^9C_4$$

C-II $\quad a < b = c < d$

$${}^9C_3 \times 1 = {}^9C_3$$

$${}^9C_4 + {}^9C_3 = {}^{10}C_4 = 210.$$

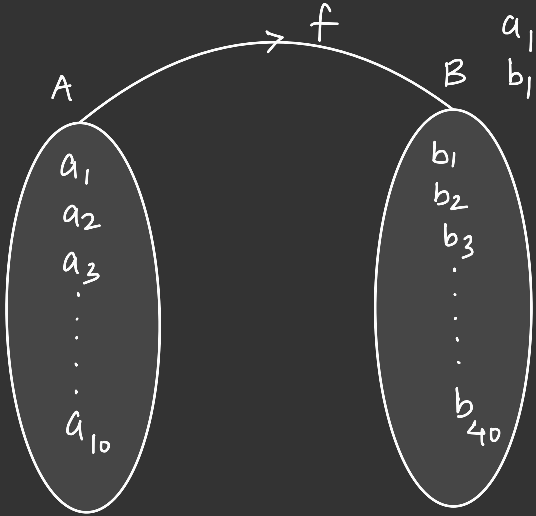
M-2 $\quad a < b \leq c < d$

$a, b, c, d \in \{1, 2, 3, \dots, 9\}$ dummy variable 'ω'

10

${}^{10}C_4$ ✓

Q



$\left.\begin{array}{l} a_1 < a_2 < a_3 < \dots < a_{10} \\ b_1 < b_2 < b_3 < \dots < b_{40} \end{array}\right\}$

$f : A \to B$

Find number of

(i) Strictly ↑ fns from A to B.

(ii) increasing fns from A to B.

(i) $\quad {}^{40}C_{10} \times 1 = {}^{40}C_{10}$

(ii) $\quad f(a_1) \le f(a_2) \le f(a_3) \le \dots \le f(a_{10})$

$\quad\quad\quad\; \uparrow \quad\quad\; \uparrow \quad\quad\; \uparrow \quad\quad\quad\quad \uparrow$

${}^{49}C_{10}$ ✓

Q If first 20 natural numbers are there, the number of ways in which

(i) '3' distinct selected nos. are in A.P.

(ii) '5' distinct selected nos. " " "

N : {1, 2, 3, ... 20}

$x, y, z \longrightarrow$ A.P.

$$\underbrace{2y}_{\text{Even}} = x + z$$

odd	odd	$\longrightarrow {}^{10}C_2$
Even	Even	$\longrightarrow {}^{10}C_2$

$$\text{Total} = 2 \cdot {}^{10}C_2 = 90.$$

(ii) $a_1, a_2, a_3, a_4, a_5 \longrightarrow$ A.P. $\hookrightarrow$ c.d = d

$a_5 - a_1 = 4d$

$$(4d)_{max} = a_5 - a_1 = 20 - 1 = 19$$

possible values of 'd' = 4, 3, 2, 1

C-I If d = 4

1, 5, 9, 13, 17.
2, 6, 10, 14, 18.
3, 7, 11, 15, 19.
4, 8, 12, 16, 20.

4 AP's

C-II   If $d=3$

$$\left.\begin{array}{l} 1, 4, 7, 10, 13 \\ 2, 5, 8, 11, 14 \\ \vdots \qquad\qquad\qquad\qquad 15 \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ 8, 11, 14, 17, 20 \end{array}\right\}$$ ✓ 8 AP's

C-III   If $d=2$ $\rightarrow$ 12 AP's

C-IV   If $d=1$ $\rightarrow$ 16 AP's

Total AP's $= 16+12+8+4 = 40$

Q Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ then find no. of subsets of $A \times A$ containing atleast 1 and not more than 47 ordered pairs.

Sol^n $A \times A = 7 \times 7 = 49$

$${}^{49}C_1 + {}^{49}C_2 + \cdots\cdots + {}^{49}C_{47} = \text{No. of subsets required.}$$

$$\underbrace{{}^{49}C_0} + \underbrace{{}^{49}C_1 + \cdots + {}^{49}C_{47}} + \underbrace{{}^{49}C_{48} + {}^{49}C_{49}} = 2^{49}$$

$${}^{49}C_1 + {}^{49}C_2 + \cdots\cdots + {}^{49}C_{47} = 2^{49} - 2 - 49$$

$$= 2^{49} - 51 \quad \text{Ans}$$

Q Let $S = \{1, 2\}$ then find no. of unordered pairs of disjoint subsets of S?

Sol^n $S = \{1, 2\}$

$$\left\{ \begin{array}{l} \phi \\ \{1\} \\ \{2\} \\ \{1,2\} \end{array} \right.$$

Same

ϕ	ϕ
ϕ	$\{1\}$
ϕ	$\{2\}$
ϕ	$\{1,2\}$
$\{1\}$	$\{2\}$
$\{2\}$	$\{1\}$
$\{1\}$	ϕ
$\{2\}$	ϕ
$\{1,2\}$	ϕ

Same

No. of unordered pairs of disjoint subsets of 'S' $= 5$

M-2

$$\frac{3 \times 3 - 1}{2} + 1 = 5.$$

Let N be the number of ordered pairs of non-empty sets A and B. If A and B satisfy

(i) $A \cup B = \{1,2,3,....12\}$

(ii) $A \cap B = \phi$

(iii) (number of elements of A) $\notin$ A

(iv) (number of elements of B) $\notin$ B

then choose correct options

(A) N is a 3-digits number

(B) Sum of the digits of N is 11

(C) When N is divided by 10, remainder is 2

(D) N is an odd number

C-I

$n(A) = 1$ $\qquad$ $n(B) = 11$

$A = \{11\}$ $\qquad$ $B : \{1, \dots\}$ $\quad$ $\underline{\underline{10}}$

${}^{10}C_0$

C-I* $\quad n(A) = 6$; $\cancel{n(B) = 6}$

C-II $\quad n(A) = 2$ $\qquad$ $n(B) = 10$

$A = \{10, \_\_\}$ $\quad {}^{10}C_1$ $\qquad$ $B = \{2, \dots\}$

C-III $\quad n(A) = 3$ $\qquad$ $n(B) = 9$

$A = \{9, \_, \_\}$ $\quad {}^{10}C_2$ $\qquad$ $B = \{3, \_\_\}$

${}^{10}C_5$

$$\Big(\underbrace{{}^{10}C_0 + {}^{10}C_1 + \dots + {}^{10}C_{10}}_{1024}\Big) - {}^{10}C_5 = 772$$

$$1024 - {}^{10}C_5$$