ALLEN CAREER INSTITUTE
KOTA

Permutations and Combinations
Sheet Solutions
(2020)

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P & C

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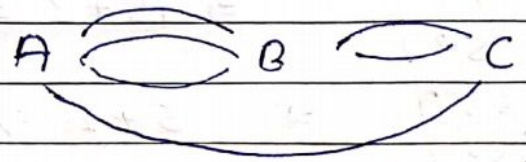
Do yourself (1)

(i) There are 3 ways to go from A to B, 2 ways to go from B to C and 1 way to go from A to C. In How many Ways Can a person travel from A to C?



(A to B) x (B to C)
+ A to C

$3 \times 2 + 1 = \boxed{7}$

(ii) There are 2 Red balls and 3 Green balls, All balls are identical exept colour. in How many Ways Can a Person select two balls?

$R^2 \quad G^3$

∵ all ball's are identical

All different = 1
Same Red = 1
Same green = 1

$\boxed{3 \text{ ways}}$

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Do your self (2)

(i) Find the exponent of 10 in ${}^{75}C_{25}$.

$${}^{75}C_{25} \Rightarrow \frac{\lfloor 75}{\lfloor 25 \lfloor 50} \Rightarrow \frac{5^{18}}{5^{18}} \Rightarrow \boxed{0}$$

$$\left[\frac{75}{5}\right] + \left[\frac{75}{5^2}\right] \Rightarrow 15+3 \Rightarrow 18$$

$$\left[\frac{25}{5}\right] + \left[\frac{25}{5^2}\right] \Rightarrow 5+1 \Rightarrow 6$$

$$\left[\frac{50}{5}\right] + \left[\frac{50}{5^2}\right] \Rightarrow 10+2 \Rightarrow 12$$

(ii) If ${}^{10}P_r = 5040$ then find the value of r.

$$\frac{\lfloor 10}{\lfloor 10-r} = 5040 \Rightarrow \frac{\lfloor 10}{5040} = \frac{\lfloor 10-r}{}$$

$$\frac{\cancel{10}\times\cancel{9}\times\cancel{8}\times\cancel{7}\times\cancel{6}}{\cancel{5040}\ \cancel{56}\ \cancel{7}} = \lfloor 10-r$$

$$\therefore \lfloor 6 = \lfloor 10-r$$

$$\therefore \boxed{r=4}$$

(iii) Find the No. of ways of selecting 4 even Numbers from the set of first 100 Natural Numbers.

$\because$ In first 100 Natural Numbers 50 even & 50 odd Numbers

$\therefore$ 4 even Number's selected from 50 even Numbers

$$\therefore \boxed{{}^{50}C_4}$$

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(iv) If all letters of word "RANK" are arranged in all possible manner as they are in a disctionary, then find the Rank of the Word "RANK".

RANK

Alphabet order A K N R

A - - - = 3!
K - - - = 3!
N - - - = 3!
R A K - = 1!

19

R A N K = 1 → 20th

$\boxed{20}$

(v) How many Words can be formed Using all letters of the Word "LEARN"? In How many of these words Vowels are together?

5 4 3 2 1 ⇒ $\lfloor 5 = \boxed{120}$

& Vowel's are together

∴ (EA) LRN

$\lfloor 4 \times \lfloor 2$ → (arrangement of E & A)

∴ 24 × 2 ⇒ $\boxed{48}$

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Do your self (3)

(i) Find the No. of ways of selecting 5 members from a Committee of 5 men & 2 women such that all women are always included.

Selection of 5 members from 5 men & 2 women is $^{7}C_{5}$

women are always included

$\therefore$ $^{7-2}C_{5-2} \Rightarrow {}^{5}C_{3} \Rightarrow \boxed{10}$

(ii) Out of first 20 Natural Numbers, 3 numbers are selected such that there is exactly one even Number. How many different selection can be made?

in first 20 Natural Number's — even Number's 10, odd Number's 10

Selection of exactly one even number is

$^{10}C_{1} \; {}^{10}C_{2} \Rightarrow 10 \times \frac{\lfloor 10}{\lfloor 2 \, \lfloor 8} \Rightarrow \boxed{450}$

(iii) How many four letter words can be made from the letters of the word "PROBLEM". How many of these start as well as end with a vowel?

$_ \; _ \; _ \; _ \quad {}^{7}C_{4} \times 4! \rightarrow$ [arrengment of four letter's]

$\frac{\lfloor 7}{\lfloor 4 \, \lfloor 3} \times \lfloor 4 \Rightarrow \boxed{840}$

Vowel → O/E _ ↓ 5, ↓ 4, O/E _ ↓ Vowel

$5 \times 4 \times \lfloor 2 \Rightarrow \boxed{40}$

↓ arrengment of vowel's

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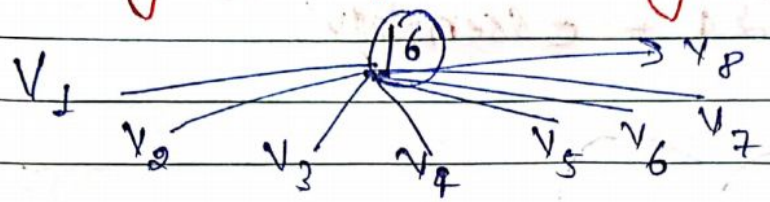
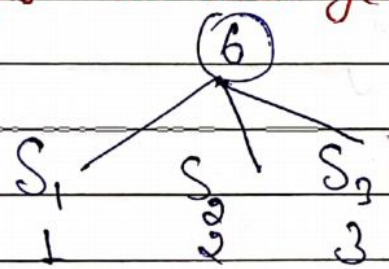
Do yourself (4)

(i) Find the Number of Ways in which 16 Constables can be assigned to patrol 8 village, 2 for each.



$\therefore \dfrac{16!}{(2!)^8 \times 8!} \times 8! \rightarrow$ [arrengment of 8 village]

(ii) In How many Ways can 6 different books be distributed among 3 Students s.t. None gets equal Number of books and each gets atleast one book?



$\dfrac{6!}{1!\,2!\,\cancel{3!}} \times \cancel{3!} \rightarrow$ [arrengement of Students]

$\dfrac{6 \times 5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}} \Rightarrow \boxed{360}$

(iii) n different toys are to be distributed among n children. find the Number of Ways in which these toys can be distributed so that exactly one child gets no toy.

$\because$ exactly one child gets No toy i.e. there are one toy Remaining

$\therefore$

$\Rightarrow \dfrac{n!}{0!\,2!\,(1)^{n-2}\,\underline{|n-2}}$

$\therefore \boxed{{}^nC_2 \times \underline{|n}}$

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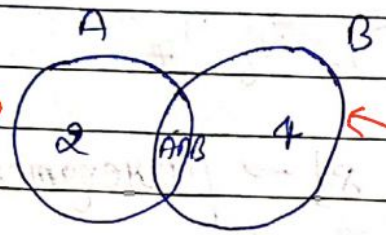
Do your Self (5)

(i) Find the Number of n digit Numbers formed using first 5 Natural Numbers, which contain the digits 2 & 4 essentially.



No 2 (use only 1, 3, 4, 5)

No 4 (use only 1, 2, 3, 5)

A∩B : use only 1, 3, 5

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 4^n + 4^n - 3^n$$

Total − $n(A \cup B)$

$$5^n - [4^n + 4^n - 3^n]$$

$$\boxed{5^n - 4^n - 4^n + 3^n}$$

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Do your self (6)

(i) In How many Ways can the letters of the Word "ALLEN" be arranged? Also find its Rank if all these words are arranged as they are in disctionary?

$$ALLEN = \frac{5!}{2!} = \frac{5\times4\times3\times\cancel{2}}{\cancel{2}} = \boxed{60}$$

A E L L N → Alphabet order

$$A\ E\ _\ _\ _ = \frac{3!}{2!} = 3$$

$$A\ L\ E\ _\ _ = 2! = 2$$

$$A\ L\ L\ E\ N = 1 = 1 \rightarrow 6^{th}$$

5

$$\boxed{6^{th}}$$

(ii) How many Numbers greater than 1000 can be formed from the digits 1, 1, 2, 2, 3?

5 digits: $_\ _\ _\ _\ _ = \frac{5!}{2!\,2!} = 30$

4 digits: $_\ _\ _\ _$

i) when 3 Not include $\frac{4!}{2!\,2!} = 6$

ii) when 2 not include $\frac{4!}{2!} = 12$

iii) when 1 not include $\frac{4!}{2!} = 12$

$$\therefore 30+6+12+12 = \boxed{60}$$

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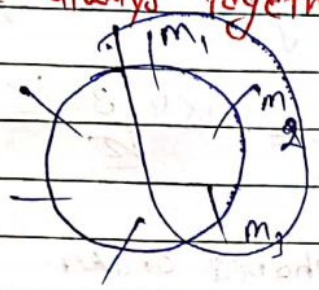
Do your self (7)

(i) In How many ways can 3 men & 3 Women be Seated around a round table Such that all men are always together?



$\lfloor n-1$ arrengement of 4 person

$\therefore \lfloor 4-1 = \lfloor 3$

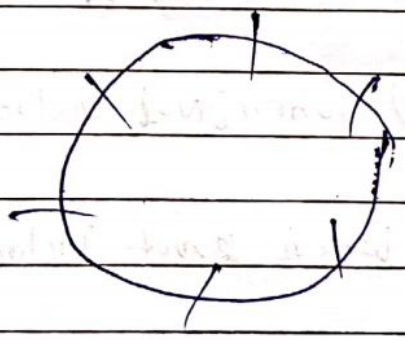
& arrengment of three men is $\lfloor 3$

$\therefore 3! \times 3! \Rightarrow \boxed{36}$

(ii) Find the Number of Ways in Which 10 different diamonds can be arranged to make a neklace.

$$\frac{\lfloor n-1}{2} \Rightarrow \frac{\lfloor 10-1}{2} \Rightarrow \boxed{\frac{\lfloor 9}{2}}$$

(iii) Find the Number of Ways in Which 6 persons out of 5 men & 5 Women Can be Seated at a Round table So to 2 Men are Never together.

5w 1m =  $^5C_5 \; ^5C_1 \times 5! = 600$

4w 2m = $^5C_4 \; ^5C_2 \times 3! \; ^4P_2 = 360$

3w 3m = $^5C_3 \; ^5C_3 [2! \times 3!] = 1200$

$\boxed{5400}$

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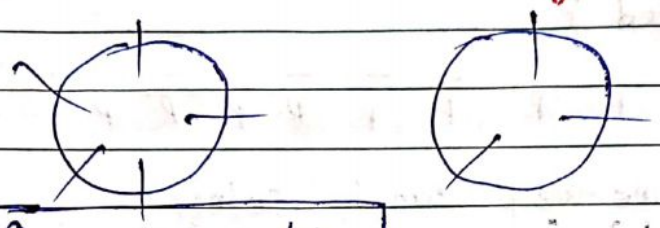
(iv) In How many ways can 8 person's be seated on two round table of capacity 5 & 3.



$$\boxed{{}^{8}C_{5} \times \lfloor 4 \times \lfloor 2}$$

Arrengement of 3 persons

Selection of 5 person

Arrengement of 5 person's

$= 2688$

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Do your self (8)

(i) There are p copies each of n different books. Find the number of ways in which atleast one book can be selected?

$\overline{p}\ \overline{p}\ \overline{p}\ \overline{p}\ \overline{p}\ \overline{p}\ \overline{p}\ \overline{p}\ \overline{p}\ \overline{p}\ \overline{p}\ \overline{p}$ ---- n p copies each

atleast one book can be select

$\therefore (0,1,2 \cdots p)\ (0,1,2 \cdots p)$ ------ $(0,1,2,\cdots p)$

$\therefore p+1,\ p+1,\ p+1,$ ------- $p+1$ n times

$\therefore (p+1)^n$

But selecting atleast one $\therefore \boxed{(p+1)^n - 1}$

(ii) There are 10 questions in an Examination. In How many ways can a Candidate answer the questions, if he attempts atleast one question.

if Given n different objects, the Number of ways of selecting atleast one of them is $2^n - 1$

$\therefore \boxed{2^{10} - 1}$

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Do yourself (9)

(i) Find the Number of Ways in Which the Number 94864 can be resolved as a product of two factors.

$N = 94864 = 2^4 \cdot 7^2 \cdot 11^2$

$N = 94864$ is perfect Square

$\therefore \frac{1}{2}\left[(a+1)(b+1)(c+1)+1\right]$

$\frac{1}{2}\left[(4+1)(2+1)(2+1)+1\right] \Rightarrow \boxed{23}$

(ii) Find the Number of different sets of Solution of $xy = 1440$

2	1440
2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

$2^5 \cdot 3^2 \cdot 5^1$

$(5+1)(2+1)(1+1)$

$6 \times 3 \times 2$

$\boxed{36}$

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Do your self (10)

(i) In How many ways can 12 identical apples be distributed among 4 boys.

(a) If each boy receives any number of apples.

(b) if each boy receives atleast 2 apples.

Sol^n (a) $^{n+r-1}C_{r-1} \Rightarrow {}^{4+12-1}C_{4-1} \Rightarrow \boxed{{}^{15}C_3}$

(b) B_1 B_2 B_3 B_4

2 2 2 2 $\rightarrow$ 1 way

Now Remaining 4 apples distributed among 4 boys

$^{n+r-1}C_{r-1} = \boxed{{}^{7}C_3}$

(ii) Find the Number of Non-Negative integral solutions of the eq^n $x+y+z=10$

$^{n+r-1}C_{r-1} = {}^{10+3-1}C_{3-1} \Rightarrow \boxed{{}^{12}C_2}$

(iii) Find the Number of integral solutions of $x+y+z=20$ if $x \geq -4$, $y \geq 1$, $z \geq 2$

$\underbrace{x+4}_{x_1} \geq 0$, $\underbrace{y-1}_{y_1} \geq 0$, $\underbrace{z-2}_{z_1} \geq 0$

$x_1 + y_1 + z_1 = 21$

$\therefore$ Now $^{n+r-1}C_{r-1} = {}^{21+3-1}C_{3-1}$

$\boxed{{}^{23}C_2}$

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Do your self (11)

(i) There are four balls of different colours and four boxes of colours same as those of the balls. Find the number of ways in which the balls, one in each box, could be placed in such a way that a ball does not go to box of its own colour.

$$D(n) = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + \frac{(-1)^n}{n!}\right]$$

$$D(4) = 4!\left[\cancel{1} - \cancel{\frac{1}{1!}} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right]$$

$$24\left[\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right]$$

$$\cancel{24}\left[\frac{12 - 4 + 1}{\cancel{24}}\right] \Rightarrow \boxed{9}$$

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Exercise 01

(1) Number of natural numbers between 100 and 1000 such that at least one of their digits is 7, is

(A) 225 (B) 243 (C) 252 (D) none

Soln

total nos. b/w 100 and 1000

$= 900 - 1$ (Not including 100 & 1000)

total 3 digits nos. with none of the digits as seven:

$$\underbrace{8}_{\text{first place cannot have zero or seven}} \times \underbrace{9 \times 9}_{\text{Remaining places cannot have seven}} - \underbrace{1}_{\text{Exclude 100}} = 648 - 1$$

$\therefore$ Ans: $900 - 1 - (648 - 1)$

$= 252$ Ans.

2. How many of the 900 three digit numbers have at least one even digit?

(A) 775 (B) 875 (C) 450 (D) 750

Soln

Ans: Total 900 nos. — When none of the digits is even

_ _ _

$\Rightarrow$ Ans: $900 - \underbrace{(5 \times 5 \times 5)}$

Each place has 5 options: 1/3/5/7/9

$\Rightarrow$ Ans: $900 - 125 = 775$

EXERCISE (O-1)

ONLY ONE CORRECT :

3

The number of different seven digit numbers that can be written using only three digits 1, 2 & 3 under the condition that the digit 2 occurs exactly twice in each number is

(A) 672 (B) 640 (C) 512 (D) none

Soln

_ _ _ _ _ _ _

$$\underbrace{{}^7C_2 \times 1}_{\text{Choose 2 places out of 7 for both 2}} \times \underbrace{2\times2\times2\times2\times2}_{\text{2 options 1/3 for each of remaining 5 places}} = 672$$

(4) Out of seven consonants and four vowels, the number of words of six letters, formed by taking four consonants and two vowels is (Assume that each ordered group of letter is a word):

(A) 210 (B) 462 (C) 151200 (D) 332640

Soln

7C 4V

4C 2V

Total no. of words

$= {}^7C_4 \cdot {}^4C_2 \cdot 6!$

Choose 4 consonants

Choose 2 vowels

Arrange 6 letters in a line

$= \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} \cdot \frac{4 \cdot 3}{2 \cdot 1} \cdot 720$

$= 151200$ Ans.

(5) Number of 5 digit numbers which are divisible by 5 and each number containing the digit 5, digits being all different is equal to $k(4!)$, the value of k is,

(A) 84 (B) 168 (C) 188 (D) 208

Sol. unit place must contain 5 or 0

(i)

				5
$8\times$	$8\times$	$7\times$	$6\times$	1

No. of ways $= 8\times8\times7\times6\times1$

(ii)

				0
				1

$^4C_1 \times 1 \times 8\times7\times6$

sel. of one place out of 4 place for 5

filling of 5 by one way

No. of ways $= 4\times8\times7\times6$

$\therefore$ Ans $\Rightarrow 8\times8\times7\times6 + 4\times8\times7\times6$

$= 8\times7\times6\times12$

$= 168\times\underline{|4}$

$\therefore \boxed{k=168}$

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6

The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 & 7 so that digits do not repeat and the terminal digits are even is :

(A) 144 (B) 72 (C) 288 ~~(D)~~ 720

Soln

_ _ _ _ _ _ _

Even Even

$\underbrace{1, 3, 5, 7}_{\text{odd digits}}$; $\underbrace{2, 4, 6}_{\text{Even digits}}$

$\underbrace{3 \times 2}_{\text{Ways to fill terminal digits by Even digits}} \times \underbrace{5!}_{\text{ways to fill remaining 5 places by remaining digits}} = 720$ Ans.

P&C 0-1 (Q.16 - Q.20)

(7) If the letters of the word VARUN are written in all possible ways and then are arranged as in a dictionary, then the rank of the word VARUN is

(A) 98 (B) 99 ✓(C) 100 (D) 101

Sol. A, N, R, U, V

No. of words starting with A = $\lfloor 4$ = 24

A				

No. of words starting with N = $\lfloor 4$ = 24

N				

No. of words starting with R = $\lfloor 4$ = 24

R				

No. of words starting with U = $\lfloor 4$ = 24

U				

No. of words starting with VAN = $\lfloor 2$ = 2

V	A	N	R	U

V	A	N	U	R

And then

V	A	R	N	U

→ 1

V	A	R	U	N

→ 1

∴ Rank is 24 + 24 + 24 + 24 + 2 + 2 + 1 + 1

= 100 th

Hence option (c)

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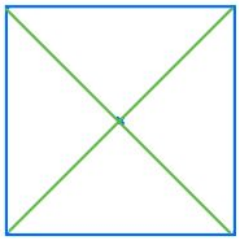
8

Consider the five points comprising of the vertices of a square and the intersection point of its diagonals. How many triangles can be formed using these points?

(A) 4 (B) 6 ~~(C)~~ 8 (D) 10

Soln



$$\underbrace{{}^5C_3}_{} - \underbrace{{}^3C_3 \times 2}_{} = 10 - 2 = 8$$

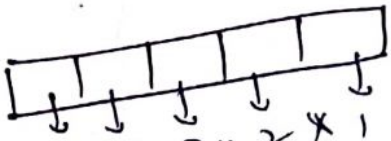
Choose any 3 pts. out of given 5 for a triangle

If we choose 3 pts. out of 3 pts along a diagonal we do not get a triangle. This is done for 2 diagonals

(9) A 5 digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 & 5 without repetition. The total no of ways this can be done is

(A) 3125 (B) 600 (C) 240 (D) 216

Sol. Case-1 we can use 1, 2, 3, 4, 5



$5 \times 4 \times 3 \times 2 \times 1$

No. of ways $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Case-2 we can use 0, 1, 2, 4, 5

$4 \times 4 \times 3 \times 2 \times 1$

No. of ways $= 4 \times 4 \times 3 \times 2 \times 1$
$= 96$

Ans. $= 120 + 96$
$= 216$

Hence option (D)

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(10) Number of permutations of 1, 2, 3, 4, 5, 6, 7, 8 and 9 taken all at a time, such that the digit 1 appearing somewhere to the left of 2 3 appearing to the left of 4 and 5 somewhere to the left of 6, is (eg. 815723946 would be one such permutation)

✓(A) $9.\lfloor 7$ (B) $\lfloor 8$ (C) $\lfloor 5 . \lfloor 4$ (D) $\lfloor 8 . \lfloor 4$

Sol.

$$\frac{\lfloor 9}{\lfloor 2 \lfloor 2 \lfloor 2}$$

$= 9.\lfloor 7$

Hence option (A)

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(11) 5 Indian & 5 American couples meet at a party & shake hands . If no wife shakes hands with her own husband & no Indian wife shakes hands with a male, then the number of hand shakes that takes place in the party is :

(A) 95 (B) 110 (C) 135 (D) 150

Sol^n

AH_1	AW_1	IH_1	IW_1
AH_2	AW_2	IH_2	IW_2
AH_3	AW_3	IH_3	IW_3
AH_4	AW_4	IH_4	IW_4
AH_5	AW_5	IH_5	IW_5

3 types of handshakes:

M = Male
F = Female

M-M + F-F + M-F

$$\underbrace{{}^{10}C_2}_{\text{Select 2 males out of 10}} + \underbrace{{}^{10}C_2}_{\text{Select 2 females out of 10}} + \underbrace{{}^{5}C_1}_{\text{Choose 1 female out of 5 American females}} \cdot \underbrace{{}^{9}C_1}_{\text{Choose 1 male out of 9 available (excluding husband)}}$$

$= 135$ Ans.

(12) A student has to answer 10 out of 13 questions in an examination . The number of ways in which he can answer if he must answer atleast 3 of the first five questions is :

(A) 276 (B) 267 (C) 80 (D) 1200

Soln

$\underbrace{1\ 2\ 3\ 4\ 5}\quad \underbrace{6\ 7\ 8\ 9\ 10\ 11\ 12\ 13}$

selected no. of questions		
	3	7
	4	6
	5	5

total ways =

$${}^5C_3 \cdot {}^8C_7 + {}^5C_4 \cdot {}^8C_6 + {}^5C_5 \cdot {}^8C_5$$

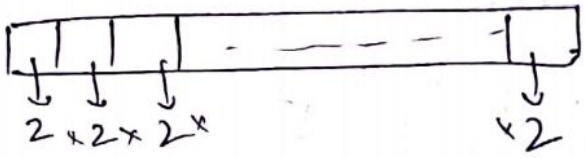
$$= 10 \cdot 8 + 5 \cdot 28 + 1 \cdot 56$$

$$= 276 \text{ Ans.}$$

(13) The number of 'n' digit numbers which consists of the digits 1 & 2 only if each digit is to be used atleast once, is equal to 510 then n is equal to

(A) 7 (B) 8 (C) 9 (D) 10

Sol.



Total no. of 'n' digit numbers using 1 & 2 only $= \underbrace{2 \times 2 \times 2 \times \cdots \times 2}_{n \text{ times}}$

$= 2^n$

These consist one number having only 1's and one number having only 2's

$\therefore \quad 2^n - 2 = 510$

$\Rightarrow \quad 2^n = 512 = 2^9$

$\Rightarrow \quad n = 9$

hence option (C)

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(14) A question paper on mathematics consists of twelve questions divided into three parts A, B and C, each containing four questions. In how many ways can an examinee answer five questions, selecting atleast one from each part.

(A) 624 (B) 208 (C) 1248 (D) 2304

Sol^n

$$\underbrace{(A_1A_2A_3A_4)}\underbrace{(B_1B_2B_3B_4)}\underbrace{(C_1C_2C_3C_4)}$$

$$\begin{cases} 1 \\ 1 \\ 3 \end{cases} \quad \begin{matrix} 1 \\ 3 \\ 1 \end{matrix} \quad \begin{cases} 3 \\ 1 \\ 1 \end{cases}$$

$$\begin{cases} 2 \\ 2 \\ 1 \end{cases} \quad \begin{matrix} 2 \\ 1 \\ 2 \end{matrix} \quad \begin{cases} 1 \\ 2 \\ 2 \end{cases}$$

Total no. of ways

$$= ({}^4C_1 \cdot {}^4C_1 \cdot {}^4C_3)3 + ({}^4C_2 \cdot {}^4C_2 \cdot {}^4C_1)3$$

$$= 64 \cdot 3 + 36 \cdot 4 \cdot 3 = 624$$

(15) If m denotes the number of 5 digit if each successive digits are in their descending order of magnitude and n is the corresponding figure, when the digits are their in ascending order of magnitude then (m−n) has the value.

(A) $^{10}C_4$ (B) $^{9}C_5$ (C) $^{10}C_3$ (D) $^{9}C_3$

Sol.

$$m = {}^{10}C_5 \times 1$$

sel. of 5 digits out of 10 digits

arrengement in desecending order

$$n = {}^{9}C_5 \times 1$$

sel. of 5 digits out of 9 digits (we can not select 0)

arrengement in ascending order

$$m - n = {}^{10}C_5 - {}^{9}C_5 = {}^{9}C_4 = {}^{9}C_5$$

Hence option (B)

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(16) A rack has 5 different pair of shoes. The number of ways in which 4 shoes can be chosen from it, so that there will be no complete pair is

(A) 1920 (B) 200 (C) 110 (D) 80

Sol.

$$\underbrace{{}^5C_4}_{\text{sel. of 4 pair out of 5 pair}} \times \underbrace{2 \times 2 \times 2 \times 2}_{\text{sel. of 1 shoe out 2 shoes of any pair}}$$

Ans. $= {}^5C_4 \times 2 \times 2 \times 2 \times 2$

$= 80$

Hence option (D)

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(17) Number of ways in which 8 people can be arranged in a line if A and B must be next to each other and C must be somewhere behind D, is equal to

(A) 10080 (B) 5040 (C) 5050 (D) 10100

Sol. (AB) C D E F G H

$$\frac{\lfloor 7 \times \lfloor 2}{\lfloor 2} = 5040$$

A & B are tied with string.

so there are total 7 units.

which can be arranged by $\lfloor 7 \times \lfloor 2$ ways

(A & B can be interchanged)

Now C is somewhere behind D

∴ we devide it by $\lfloor 2$

hence option (B)

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(18) The number of ways in which 8 distinguishable apples can be distributed among 3 boys such that every boy should get atleast 1 apple & atmost 4 apples is $K \cdot {}^7P_3$ where K has the value equal to

(A) 14 (B) 66 (C) 44 (D) 22

45 no of ways =

$$8 \begin{cases} 3 \\ 3 \\ 2 \end{cases} + 8 \begin{cases} 4 \\ 3 \\ 1 \end{cases} + 8 \begin{cases} 4 \\ 2 \\ 2 \end{cases}$$

$$\frac{\lfloor 8}{\lfloor 3 \cancel{\lfloor 3} \lfloor 2 \lfloor 2} \times \cancel{\lfloor 3} + \frac{\lfloor 8}{\lfloor 4 \cancel{\lfloor 3} \lfloor 1} \times \cancel{\lfloor 3} + \frac{\lfloor 8}{\underset{4}{\cancel{\lfloor 4}} \lfloor 2 \lfloor 2 \lfloor 2} \times \cancel{\lfloor 3}$$

$$\left(\frac{\lfloor 8}{\lfloor 4} + \frac{\lfloor 8}{\lfloor 4} + \frac{\lfloor 8}{8 \times 4} \times \frac{\lfloor 3}{\lfloor 3} \right)$$

$$\frac{\lfloor 7}{\lfloor 4} (8 + 8 + 6)$$

$$22 \cdot {}^7P_3$$

so $\boxed{K = 22}$

(D)

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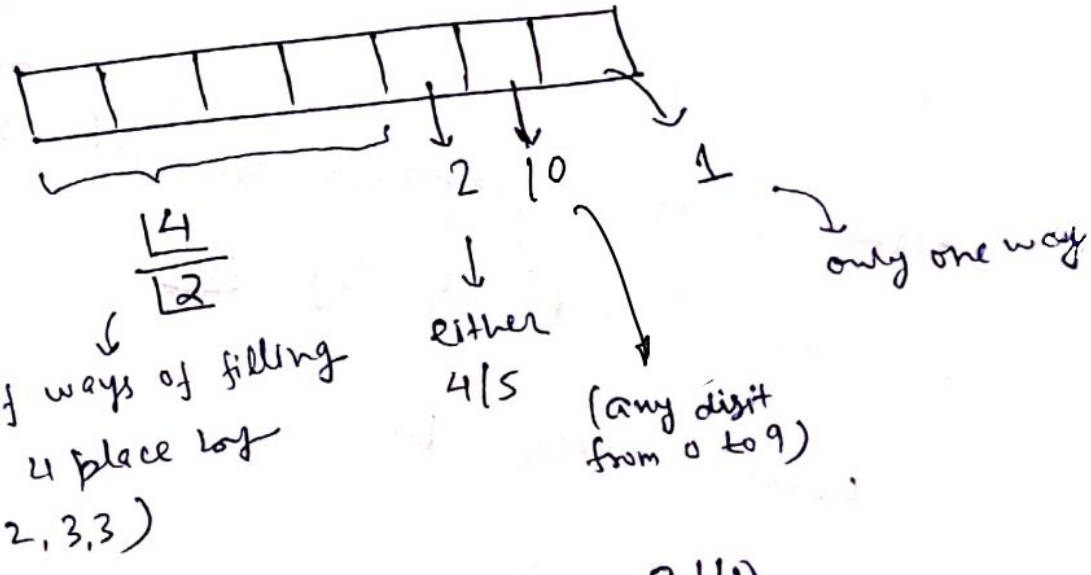
(19) An old man while dialing a 7 digit telephone number remembers that the first four digits consists of one 1's, one 2's and two 3's. He also remembers that the fifth digit is either a 4 or 5 while has no memorising of the sixth digit, he remembers that the seventh digit is 9 minus the sixth digit. Maximum no of distinct trials he has to try to make sure that he dials the correct telephone number is

(A) 360 (B) 240 (C) 216 (D) None

Sol.



$$\text{Ans.} = \frac{\lfloor 4}{\lfloor 2} \times 2 \times 10 \times 1 = 240$$

Hence option (B)

Scanned with CamScanner

(20) Number of ways in which 9 different toys be distributed among 4 children belonging to different age groups in such a way that distribution among the 3 elder children is even and the youngest one is to receive one toy more is

(A) $\frac{(\lfloor 5)^2}{8}$ (B) $\frac{\lfloor 9}{2}$ (C) $\frac{\lfloor 9}{\lfloor 3 (\lfloor 2)^3}$ (D) None

Sol.

$$\underset{\substack{\downarrow \\ 3}}{a} \quad \underbrace{B \quad C \quad D}_{(2+2+2)}$$

$$\underbrace{{}^9C_3}_{\text{3 to youngest out of 9}} \times \underbrace{\frac{\lfloor 6}{\lfloor 2 \lfloor 2 \lfloor 2 \lfloor 3}}_{\text{grouping of remaining 6 coins}} \times \underbrace{\lfloor 3}_{\text{Distributed in 3 children}}$$

$$= \frac{\lfloor 9}{\cancel{\lfloor 6} \lfloor 3} \times \frac{\cancel{\lfloor 6}}{\lfloor 2 \lfloor 2 \lfloor 2 \cancel{\lfloor 3}} \times \cancel{\lfloor 3}$$

$$= \frac{\lfloor 9}{\lfloor 3 (\lfloor 2)^3}$$

Hence Option (C)

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(21) Let P_n denotes the number of ways in which three people can be selected out of 'n' people sitting in a row, if no two of them are consecutive. If $P_{n+1} - P_n = 15$ then then value of 'n' is

(A) 7 (B) 8 (C) 9 (D) 10

Sol.

$$P_n = {}^{n-3+1}C_3 = {}^{n-2}C_3$$

$$P_{n+1} = {}^{n+1-3+1}C_3 = {}^{n-1}C_3$$

$$P_{n+1} - P_n = 15$$

$$\Rightarrow {}^{n-1}C_3 - {}^{n-2}C_3 = 15$$

$$\Rightarrow \frac{(n-1)(n-2)(n-3)}{3\times 2} - \frac{(n-2)(n-3)(n-4)}{3\times 2} = 15$$

$$\Rightarrow (n-2)(n-3)[n-1-n+4] = 90$$

$$\Rightarrow (n-2)(n-3) = 30$$

$$\Rightarrow n^2 - 5n - 24 = 0$$

$$\Rightarrow (n+3)(n-8) = 0$$

$$\therefore n = 8$$

Hence option (B)

Scanned with CamScanner

(22) Number of ways in which 7 green bottles and 8 blue bottles can be arranged in a row if exactly 1 pair of green bottles is side by side, is (Assume all bottles to be alike except for the colour).

(A) 84 (B) 360 (C) 504 (D) none

(39) one Pair of green bottles is side by side

$\underbrace{G\ G\ G\ G\ G\ G\ G}_{\text{now treat them as 6 bottles}}$

no. of ways $= 1 \times {}^{9}C_{6} \times \frac{\lfloor 6}{\lfloor 5} = 504$ $\boxed{\text{(option-C)}}$

Arrangements of eight alike blue bottles

selection of 6 gaps out of 9

Arrangements of green bottles

Scanned with CamScanner

(23) A team of 8 students goes on an excursion, in two cars, of which one can seat 5 and the other only 4. If internal arrangement inside the car does not matter then the number of ways in which they can travel, is

(A) 91 (B) 182 (C) 126 (D) 3920

(36) Team can be divided in two ways

(1) $8 \begin{cases} 5 \\ 3 \end{cases}$ no. of ways $= \dfrac{\lfloor 8}{\lfloor 5 \lfloor 3} = 56$

(2) $8 \begin{cases} 4 \\ 4 \end{cases}$ no of ways $= \dfrac{\lfloor 8}{\lfloor 4 \lfloor 4 \lfloor 2} \times \lfloor 2 = 70$

Total number of ways $= 56 + 70 = 126$

$\boxed{\text{option - C}}$

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(24) Number of cyphers at the end of ${}^{2002}C_{1001}$ is

(A) 0 (B) 1 (C) 2 (D) 200

Soln

$$N = {}^{2002}C_{1001} = \frac{2002!}{(1001!)^2}$$

We find exponent of 10 in (2002)! and 1001!. It will be same as exponent of 5 in these numbers.

$\therefore$ Exponent of 5 in (2002)!

$$= \left[\frac{2002}{5}\right] + \left[\frac{2002}{5^2}\right] + \left[\frac{2002}{5^3}\right] + \left[\frac{2002}{5^4}\right] \cdots$$

$$= 400 + 80 + 16 + 3 = 499$$

Exponent of 5 in (1001)!

$$= \left[\frac{1001}{5}\right] + \left[\frac{1001}{5^2}\right] + \left[\frac{1001}{5^3}\right] + \left[\frac{1001}{5^4}\right] \cdots$$

$$= 200 + 40 + 8 + 1 = 249$$

Overall Exponent of 10: $\frac{10^{499}}{(10^{249})^2} = 10^1$

$\therefore$ No. of zeroes is 1

25. Number of 5 digit numbers divisible by 25 that can be formed using only the digits 1, 2, 3, 4, 5 & 0 taken 5 at a time is

(A) 2 (B) 32 ✓(C) 42 (D) 52

Sol. (i)

			2	5

$3 \times 3 \times 2 = 18$

(ii)

			5	0

$4 \quad 3 \quad 2 = 24$

No. of ways $= 18 + 24 = 42$

Hence option (C)

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(26) Number of 7 digit numbers the sum of whose digits is 61 is.

(A) 12 (B) 24 (C) 28 ✓ (D) None

Sol. If all digits are 9 then sum = 63

If sum is 61 then digits can be used

9, 9, 9, 9, 9, 9, 7

or 9, 9, 9, 9, 9, 8, 8

Case-1 9, 9, 9, 9, 9, 9, 7

No. of Numbers $= \frac{\lfloor 7}{\lfloor 6} = 7$

Case-2 9, 9, 9, 9, 9, 8, 8

No. of Numbers $= \frac{\lfloor 7}{\lfloor 5 \times \lfloor 2}$

$= 21$

Ans ⇒ $7 + 21 = 28$

Hence option (C)

Scanned with CamScanner

(27) Number of ways in which four different toys and five indistinguishable marbles can be distributed between Amar, Akbar and Anthony, if each child receives atleast one toy and one marble, is

(A) 42 (B) 100 (C) 150 (D) 216

(43) no. of distribution of 4 different toys b/w Amar, Akbar and anthony..

$4 \leftarrow \begin{matrix} 1 \\ 1 \\ 2 \end{matrix} \Rightarrow \frac{\lfloor 4}{\lfloor 1 \lfloor 1 \lfloor 2 \lfloor 2} \times \lfloor 3 = 36$

or $3^4 - {}^3C_1(2^4 - 2) - {}^3C_2(1)^5 = 36$

Total distributions

(distribution in which one get nothing)

(distribution in which two gets nothing)

no. of ways of distributing 5 alike marbles such that each receive at least one marble

$x + y + z = 5$

$x, y, z \in N,$

$x + y + z = 5 - 3 = 2$

$x, y, z \in W$

no. of ways $= {}^{2+3-1}C_{3-1} = {}^4C_2 = 6$

so no. of distribution of toys and marbles, are $= 36 \times 6 = 216$

(D)

Scanned with CamScanner

28. In a unique hockey series between India and Pakistan, they decide to play on till a team wins 5 matches. The no. of ways inwhich the series can be won by India, if no match ends in a draw is

(A) 126 (B) 252 (C) 225 (D) None

Sol.

$$^4C_4 \times {}^1C_1 + {}^5C_4 \times {}^1C_1 + {}^6C_4 \times {}^1C_1 + {}^7C_4 \times {}^1C_1 + {}^8C_4 \times {}^1C_1$$

$$= 1 + 5 + 15 + 35 + 70$$

$$= 126$$

Hence option (A)

Scanned with CamScanner

29. There are 100 different books in a shelf. Number of ways in which 3 books can be selected so that no two of which are neighbours is

(A) $^{100}C_3 - 98$ (B) $^{97}C_3$ (C) $^{96}C_3$ (D) $^{98}C_3$

Sol. $^{100-3+1}C_3$ (Train - Station problem)

$= {}^{98}C_3$

Hence option (D)

Scanned with CamScanner

(30) Number of positive integral solutions satisfying the equation $(x_1 + x_2 + x_3)(y_1 + y_2) = 77$, is

(A) 150 (B) 270 (C) 420 (D) 1024

(47) $(x_1 + x_2 + x_3)(y_1 + y_2) = 77$

$$\left.\begin{aligned} x_1 + x_2 + x_3 &= 11 \\ y_1 + y_2 &= 7 \end{aligned}\right\}$$

x_i & $y_i \in N$

$x_1 + x_2 + x_3 = 11 - 3 = 8$

$y_1 + y_2 = 7 - 2 = 5$

$x_i, y_i \in W$

$$\left.\begin{aligned} x_1 + x_2 + x_3 &= 7 \\ y_1 + y_2 &= 11 \end{aligned}\right\}$$

$x_i, y_i \in N$

$x_1 + x_2 + x_3 = 7 - 3 = 4$

$y_1 + y_2 = 11 - 2 = 9$

$x_i, y_i \in W$

solutions (Total) $= {}^{8+3-1}C_{3-1} \times {}^{5+2-1}C_{2-1} + {}^{4+3-1}C_{3-1} \times {}^{9+2-1}C_{2-1}$

$= {}^{10}C_2 \times {}^{6}C_1 + {}^{6}C_2 \times {}^{10}C_1$

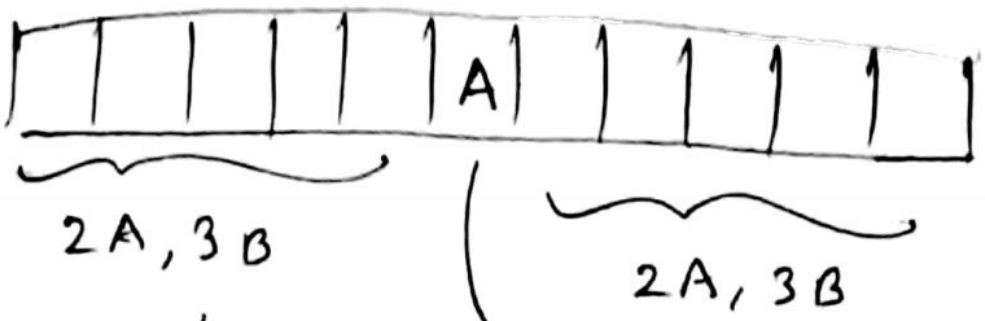
$= 270 + 150 = 420$

Scanned with CamScanner

(31) Number of ways in which 5 A's and 6 B's can be arranged in a row which reads the same backwards and forwards, is

(A) 6 (B) 8 (C) 10 (D) 12

(42)



no. of ways $= \frac{\lfloor 5}{\lfloor 2 \lfloor 3} \times 1 \times 1$

(one Possibility according as left hand side of 'A')

no of ways = 10

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(32) A person writes letters to his 5 friends and addresses the corresponding envelopes. Number of ways in which the letters can be placed in the envelope, so that atleast two of them are in the wrong envelopes,is,

(A) 1 (B) 2 (C) 118 (D) 119

(Pii)

Total ways $= \lfloor 5 = \underbrace{5R}_{1} + \underbrace{3R2W + 2R3W + 1R4W + 5W}_{\text{At least two are wrong}}$

required ways $= \lfloor 5 - 1$

$= 120 - 1$

$= 119$ — (D)

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(33)

Column-I	Column-II
(A) No of increasing permutations of 'm' symbols are there from the 'n' set numbers $\{a_1, a_2, \ldots, a_n\}$ where the order among the nos is given by $a_1 < a_2 < a_3 < a_4 \ldots < a_n$	(P) n^m
(B) There are m men and n monkeys. No. of ways inwhich every monkey has a master, if a man can have any no. of monkeys.	(Q) ${}^mC_n$
(C) No. of ways inwhich 'n' red balls and (m-1) green balls can be arranged in a line, so that no two red balls are together is (balls of same colour are alike)	(R) ${}^nC_m$
(D) No. of ways inwhich m different toys can be distributed in n children if every child may receive any no. of toys is	(S) m^n

Sol. (A) ${}^nC_m \Rightarrow$ (R)

(B) $m^n \Rightarrow$ (S)

(C) ${}^mC_n \Rightarrow$ (Q)

(D) $n^m \Rightarrow$ (P)

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Exercise 02

1. In a certain strange language, words are written with letters from the following six-letter alphabet : A, G, K, N, R, U. Each word consists of six letters and none of the letters repeat. Each combination of these six letters is a word in this language. The word "KANGUR" remains in the dictionary at,

(A) 248th (B) 247th (C) 246th (D) 253rd

Soln

~~A G K A~~ R U

words start with A → 5! = 120

G → 5! = 120

[K][A] G → 3! = 6

[KA][N][G] R → 1! = 1

[KANG][U][R] → 0! = 1

$\boxed{248}$

(2) All possible three digits even numbers which can be formed with the condition that if 5 is one of the digit, then 7 is the next digit is :

(A) 5 (B) 325 (C) 345 (D) 365

Soln

5	7	0, 2, 4, 6, 8
1	1	5

$1 \times 1 \times 5$

+

		0, 2, 4, 6, 8
8	9	5

$8 \times 9 \times 5$

Number of ways $= 5 + 360 = \underline{\underline{365}}$

3 Number of 3 digit numbers in which the digit at hundredth's place is greater than the other two digit is

(A) 285 (B) 281 (C) 240 (D) 204

Soln

xyz + xyy | $x > y$, $x > z$

xzy

Required no of ways $= \left({}^{10}C_3 \times 2!\right) + {}^{10}C_2$

$= 240 + 45 = \underline{\underline{285}}$

(4) The number of three digit numbers having only two consecutive digits identical is :

(A) 153 (B) 162 (C) 180 (D) 161

Solⁿ (M1) $XOO + XXO + \begin{cases} XXY \\ YXX \end{cases} + \begin{matrix} YYX \\ XYY \end{matrix}$ $x \neq 0$, $y \neq 0$

Required ways $= {}^9C_1 + {}^9C_1 + ({}^9C_2)2 + ({}^9C_2)2$

$= 9 + 9 + 72 + 72 = \underline{\underline{162}}$

(M2)

x	x	

$9 \times 9 = 81$

or

	x	x

8×9

or

x	0	0

9×1

Required ways $= 81 + 72 + 9 = \underline{\underline{162}}$

EXERCISE (O-2)

ONLY ONE CORRECT :

5 A committee of 5 is to be chosen from a group of 9 people. Number of ways in which it can be formed if two particular persons either serve together or not at all and two other particular persons refuse to serve with each other, is

(A) 41 (B) 36 (C) 47 (D) 76

1

9 — AB (together)
— C/D (separated)
— 5 others

① When AB is included $= {}^7C_3 - {}^5C_1 = 30$

↓ when C & D both taken

② When AB is excluded $= {}^7C_5 - {}^5C_3 = 11$

↓ when C & D both taken

Total ways $= 30 + 11 = 41$ (A)

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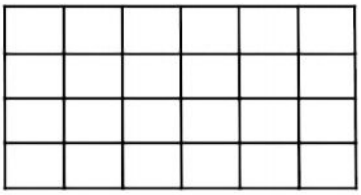
6 Number of rectangles in the grid shown which are not squares is



(A) 160 (B) 162 (C) 170 (D) 185

4

Total rectangles $= {}^5C_2 \times {}^7C_2 = 210$

no of squares of size $1\times1 = 4\times6$

" " " $2\times2 = 3\times5$

" " " $3\times3 = 2\times4$

" " " $4\times4 = 1\times3$

Total squares $= 24+15+8+3 = 50$

number of rectangles which are not squares $= 210 - 50 = 160$

A

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7 Six people are going to sit in a row on a bench. A and B are adjacent, C does not want to sit adjacent to D. E and F can sit anywhere. Number of ways in which these six people can be seated, is

(A) 200 (B) 144 (C) 120 (D) 56

Soln Required number of ways =

A+B are adjacent, C, D, E & F sit anywhere

− A&B are adjacent, C&D adjacent, E&F sit anywhere

$= 5! \times 2! - 4! \times 2! \times 2!$

$= \underline{\underline{144}}$

(8) Given 11 points, of which 5 lie on one circle, other than these 5, no 4 lie on one circle. Then the maximum number of circles that can be drawn so that each contains atleast three of the given points is :

(A) 216 (B) 156 (C) 172 (D) none

(5)

maximum number of circles $= {}^{11}C_3 - {}^{5}C_3 + 1$

repeating circles

(repeating circles forming a circle)

$= 156$

(B)

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Subjective :

(9) Six married couple are sitting in a room. Find the number of ways in which 4 people can be selected so that :

(a) they do not form a couple

(b) they form exactly one couple

(c) they form at least one couple

(d) they form atmost one couple

(A) they do not form a couple $= {}^6C_4 \cdot {}^2C_1 \cdot {}^2C_1 \cdot {}^2C_1 \cdot {}^2C_1$

$= \boxed{240}$

(B) they form exactly one couple $= {}^6C_1 \times {}^5C_2 \times {}^2C_1 \cdot {}^2C_1$

$= \boxed{240}$

(C) they form at least one couple = They form Exactly one couple or Exctly two couples

$= {}^6C_1 \times {}^5C_2 \cdot {}^2C_1 \, {}^2C_1 + {}^6C_2$

$= 240 + 15 = \boxed{255}$

(D) they form atmost one couple =

No couple or Exactly one couple

$= 240 + 240 = \boxed{480}$

EXERCISE (O-2)

(10) The number of ways of choosing a committee of 2 women & 3 men from 5 women & 6 men, if Mr. A refuses to serve on the committee if Mr. B is a member & Mr. B can only serve, if Miss C is the member of the committee, is

(A) 60 (B) 84 (C) 124 (D) none

Sol^n

Miss C ✓ B ✓ A ✗ $\longrightarrow {}^4C_1 \times {}^4C_2 = 24$

C ✓ B ✗ A ✓ — ${}^4C_1 \cdot {}^4C_2 = 24$

C ✓ B ✗ A ✗ — ${}^4C_1 \cdot {}^4C_3 = 16$

C ✗ B ✗ $\longrightarrow {}^4C_2 \times {}^5C_3 = 60$

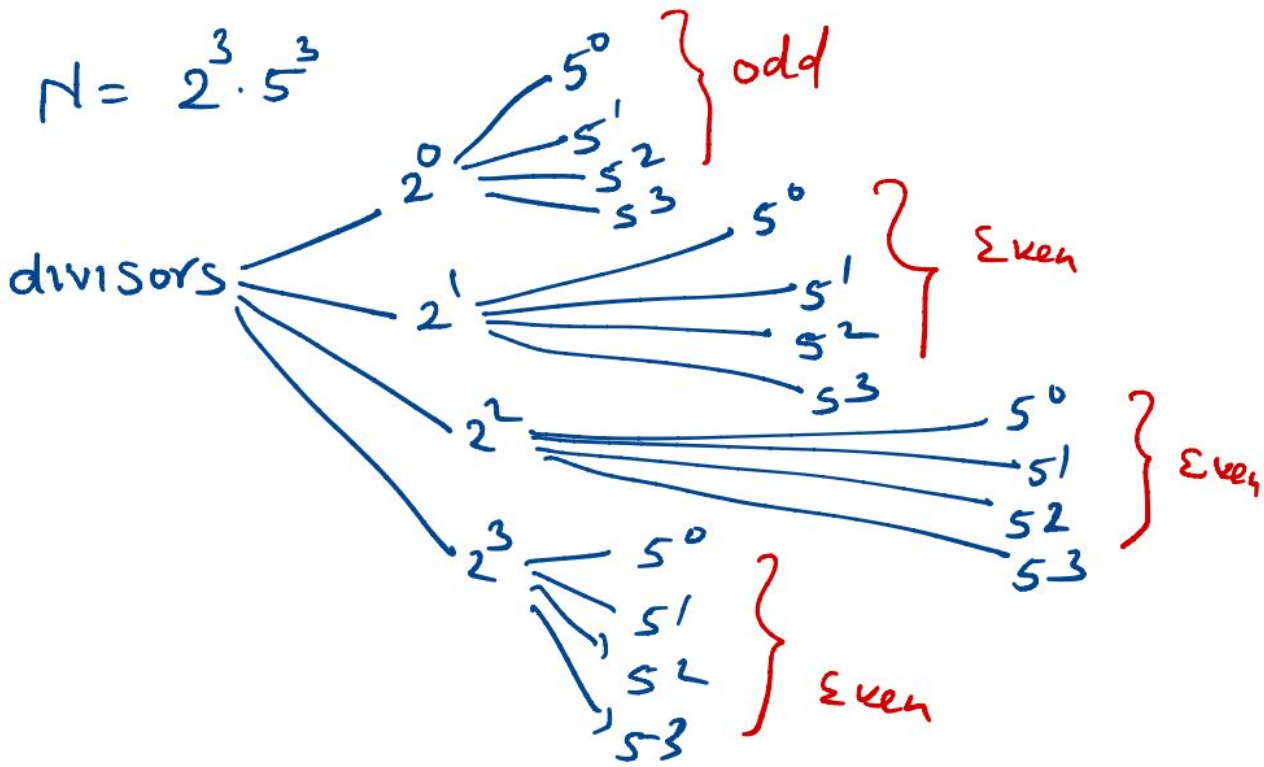
$\Rightarrow$ Number of ways choosing committee $= 24+24+16+60$

$= \underline{\underline{124}}$

(11) Product of all the even divisors of $N = 1000$, is

(A) $32 \cdot 10^2$ (B) $64 \cdot 2^{14}$ ✓(C) $64 \cdot 10^{18}$ (D) $128 \cdot 10^6$

Soln $N = 2^3 \cdot 5^3$



Product of Even divisors

$$= \{(2^1 \cdot 5^0)(2^1 \cdot 5^1)(2^1 5^2)(2^1 \cdot 5^3)\} \cdot \{(2^2 \cdot 5^0) \cdot (2^2 5^1)(2^2 5^2)(2^2 \cdot 5^3)\} \{(2^3 \cdot 5^0) \cdot (2^3 \cdot 5^1)(2^3 5^2)(2^3 \cdot 5^3)\}$$

$$= (2^4 \cdot 5^6)(2^8 \cdot 5^6)(2^{12} \cdot 5^6) = 2^{24} \cdot 5^{18}$$

$$= 2^6 \cdot 10^{18} = \underline{64 \cdot 10^{18}}$$

(12) A guardian with 6 wards wishes everyone of them to study either Law or Medicine or Engineering. Number of ways in which he can make up his mind with regard to the education of his wards if every one of them be fit for any of the branches to study, and atleast one child is to be sent in each discipline is :

(A) 120 (B) 216 (C) 729 (D) 540

Sol:- 6 Wards

Law, Medicine, Engineering

Atleast one child in each discipline

Possible Cases

Divide and distribute

1) 4, 1, 1 - $\frac{6!}{4!1!1!2!} \times 3! = 90$

2) 3, 2, 1 - $\frac{6!}{3!2!1!} \times 3! = 360$

3) 2, 2, 2 - $\frac{6!}{2!2!2!\cancel{3!}} \times \cancel{3!} = 90$

540

(13) There are 10 questions, each question is either True or False. Number of different sequences of incorrect answers is also equal to

(A) Number of ways in which a normal coin tossed 10 times would fall in a definite order if both Heads and Tails are present.

(B) Number of ways in which a multiple choice question containing 10 alternatives with one or more than one correct alternatives, can be answered.

(C) Number of ways in which it is possible to draw a sum of money with 10 coins of different denominations taken some or all at a time.

(D) Number of different selections of 10 indistinguishable things taken some or all at a time.

Soln

There are 10 questions, each question is either True or False. Number of different sequences of incorrect answers is equal to $= 2^{10} - 1$

(A) $2^{10} - 2$

(B) $2^{10} - 1$

(C) $2^{10} - 1$

(D) 10 things are identical

$\Rightarrow$ number of different selection taken some or all at a time = selection of (one thing or two things or - - - all things)

$= 1 + 1 + \cdots + 1 = 10$

(14) Number of ways in which 3 numbers in A.P. can be selected from 1, 2, 3, ... n is :

(A) $\left(\frac{n-1}{2}\right)^2$ if n is even

(B) $\frac{n(n-2)}{4}$ if n is odd

(C) $\frac{(n-1)^2}{4}$ if n is odd

(D) $\frac{n(n-2)}{4}$ if n is even

Soln

Case I if n is even

Let $n = 2m$: $m \in N$

$\Rightarrow \{1, 3, 5 \text{ ----- } (2m-1)\}$ $\{2, 4, 6, \text{ ---- } 2m\}$

Required number of ways $= {}^{m}C_2 + {}^{m}C_2$

$= 2({}^{m}C_2) = 2\,\frac{m(m-1)}{2}$

$= m(m-1) = \frac{n}{2}\left(\frac{n}{2}-1\right)$

$= \boxed{\frac{n(n-2)}{4} \text{ if } n \text{ is even}}$

Case II if n is odd Let $n = 2m+1 \Rightarrow m = \frac{n-1}{2}$

$\{1, 3, 5 \text{ ---- } (2m+1)\}$ $\{2, 4, \text{ ---- } 2m\}$

Required number of ways $= {}^{m+1}C_2 + {}^{m}C_2 = m^2$

$= \boxed{\left(\frac{n-1}{2}\right)^2 \text{ if } n \text{ is odd}}$

(15) The combinatorial coefficient ${}^{n-1}C_p$ denotes

(A) the number of ways in which n things of which p are alike and rest different can be arranged in a circle.

(B) the number of ways in which p different things can be selected out of n different thing if a particular thing is always excluded.

(C) number of ways in which n alike balls can be distributed in p different boxes so that no box remains empty and each box can hold any number of balls.

(D) the number of ways in which (n – 2) white balls and p black balls can be arranged in a line if black balls are separated, balls are all alike except for the colour.

Soln

(A) $\boxed{\dfrac{(n-1)!}{p!}}$

(B) $\boxed{{}^{n-1}C_p}$

(C) $B_1 + B_2 + \cdots + B_p = n$

Put 1 ball in each box.

$\Rightarrow B_1' + B_2' + \text{——} + B_p' = n-p$

using beggars method

Number of ways $= {}^{n-p+(p-1)}C_{p-1} = \boxed{{}^{n-1}C_{p-1}}$

(D) $\because$ balls of same colour are identical

$\Rightarrow$ Arrange white balls $\rightarrow$ No of ways $= 1$

gaps in white balls $= (n-1)$

Now p black balls put in gaps

$\Rightarrow$ No of ways (D) $= \boxed{{}^{(n-1)}C_p}$

Exercise S1

EXERCISE (S-1)

1. Four visitors A, B, C & D arrive at a town which has 5 hotels. In how many ways can they disperse themselves among 5 hotels, if 4 hotels are used to accommodate them.

Sol:- 4 out of 5 hotels are to be used.

So select 4 out of 5 hotels in $5C_4$ ways.

Now A, B, C, D should be accomodated to 4 hotels and each hotel should be used.

So 4 persons can be arranged in 4 hotels [1 in each hotel] in 4! ways.

So total ways to accomodate them $= 5C_4 \cdot 4! = 5 \cdot 4! = 5!$

$= 120.$

2. There are 6 roads between A & B and 4 roads between B & C.

(i) In how many ways can one drive from A to C by way of B ?

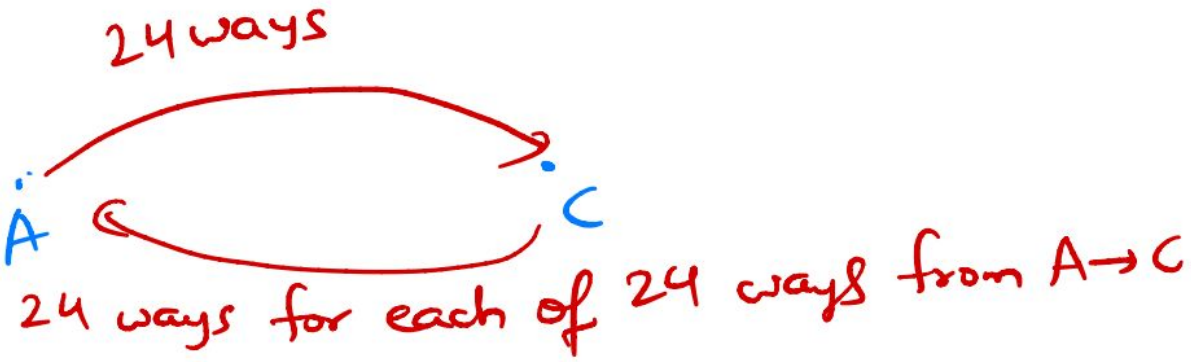
(ii) In how many ways can one drive from A to C and back to A, passing through B on both trips ?

(iii) In how many ways can one drive the circular trip described in (ii) without using the same road more than once.

(i) $A \longrightarrow B$ in 6 ways, for each of these 6 ways there are 4 ways from $B \longrightarrow C$. So by fundamental principle of multiplication, total number of ways from $A \longrightarrow C$ $= 6 \times 4 = 24$

(ii) No of ways from $A \rightarrow C = 24\ (6 \times 4)$

|||ly No of ways from $C \rightarrow A = 24\ (4 \times 6)$



So total number of ways $= 24 \times 24 = 576$.

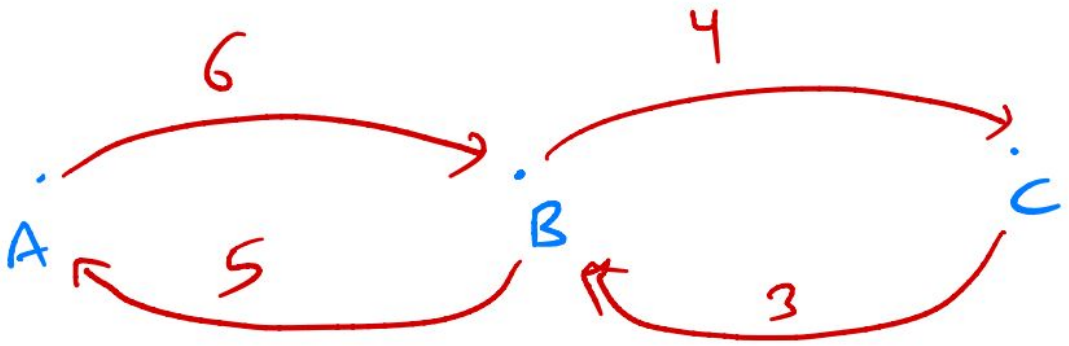
(iii) No of ways from

$A \to B = 6$

$B \to C = 4$

$C \to B = 3$ (Since road used while travelling from $B \to C$ can't be used now)

$B \to A = 5$ (Since road used while travelling from $A \to B$ can't be used now)



So total ways $= 6 \times 4 \times 3 \times 5$

$= 360.$

3. (i) Find the number of four letter word that can be formed from the letters of the word HISTORY. (each letter to be used atmost once)

Sol:-

H I S T O R Y

→ Four letter word $\underset{1^{st}}{_}\ \underset{2^{nd}}{_}\ \underset{3^{rd}}{_}\ \underset{4^{th}}{_}$

Can be filled by any of the 7 letters.

→ Now for each way of filling the first place, second place can be filled in 6 ways [∵ The letter used in first place can't be used again]

→ For each way of filling $1^{st}, 2^{nd}$ places, 3rd place can be filled in 5 ways. [∵ The letter used in $1^{st}, 2^{nd}$ places can't be used again]

→ For each way of filling the first 3 places, 4th place can be filled in 4 ways [∵ The letters used in the first 3 places can't be used again]

So total number of filling all the four places by the Fundamental principle of Multiplication is $7 \times 6 \times 5 \times 4 = 840$.

Aliter:- Number of permutations of 7 letters taken 4 at a time

$$= {}^7P_4$$

$$= 7 \times 6 \times 5 \times 4$$

$$= 840$$

(ii) How many of them contain only consonants?

Sol: HISTORY

Consonants are H, S T, R, Y

So total 5 consonants.

Four letter word _ _ _ _

No of permutations of 5 letters taken 4 at a time $= {}^5P_4$

$= 5 \cdot 4 \cdot 3 \cdot 2$

$= 120$.

(iii) How many of them begin & end in a consonant?

Sol:- HISTORY (5 consonants, 2 vowels)

Four letter word

$\underline{5}$ $_\ {}^5P_2\ _$ $\underline{4}$

Step 1

Any of the 5 consonants

Step 2

After 1st place, any of the 4 consonants left

Step 3

Any of the 5 letters left after filling 1st, 4th place

So total ways $= 5 \times {}^5P_2 \times 4$

$= 5 \times 5 \times 4 \times 4$

$= 400$

(iv) How many of them begin with a vowel?

Sol:- HISTORY (5 consonants, 2 vowels)

Four letter word

$\underline{2}$ $\quad$ $\underline{\quad}\ \underline{\quad}\ \underline{\quad}$ $\quad$ $^6P_3$

Step 1

Any of the two vowels

Step 2

Use the 6 letters left after filling 1st place to fill 3 places in $^6P_3$ ways.

So total ways $= 2 \times {}^6P_3$

$= 2 \times 6 \times 5 \times 4$

$= 240.$

(v) How many contain the letters Y?

Sol:- HISTORY

Four letter word

_ _ _ _

Step 1: Choose Y in 1 way

Step 2: Choose 3 letters from remaining 6 letters in $6c_3$ ways.

Step 3: Arrange the chosen 4 letters in 4! ways.

Step 4: Calculate the total no of ways $= 1 \times 6c_3 \times 4!$

$= 20 \times 24$

$= 480.$

Step 5: ENJOY. ☺

(vi) How many begin with T & end in a vowel?

Sol:- HISTORY (5 consonants, 2 vowels)

Four letter word

1 _ _ $^5P_2$ _ _ 2

Step 1

Select T and put it in the first place in 1 way.

Step 3

Use 5 letters left after step 1, step 2 to fill the two places in $^5P_2$ ways.

Step 2

Select any of the two vowels and put in the last place in $^2C_1$ ways.

So total no of ways $= 1 \times {}^5P_2 \times 2$

$= 5 \times 4 \times 2$

$= 40.$

(vii) How many begin with T & also contain S?

Sol:- HISTORY

Four letter word

1 _ _ _

Step 1: Take T and put it in the first place in 1 way.

Step 2: Select S in 1 way

Step 3: Select 2 more letters required from the remaining 5 letters in $5c_2$ ways.

Step 4: Arrange S, 2 letters chosen in step 2, step 3 in 3! ways

Step 5: Calculate the total no of ways $= 1 \times 1 \times 5c_2 \times 3!$

$= 10 \times 6$

$= 60$

Step 6: ENJOY ☺.

(viii) How many contain both vowels?

Sol:- HISTORY (5 consonants, 2 vowels)

Four letter word

__ __ __ __

Step 1: Select both vowels in $^2C_2$ ways

Step 2: Select more two letters required from the remaining 5 in $^5C_2$ ways.

Step 3: Arrange the 4 letters chosen in Step 1 and Step 2 in 4! ways.

Step 4: Calculate the total number of ways $= {}^2C_2 \times {}^5C_2 \times 4!$

$= 1 \times 10 \times 24$

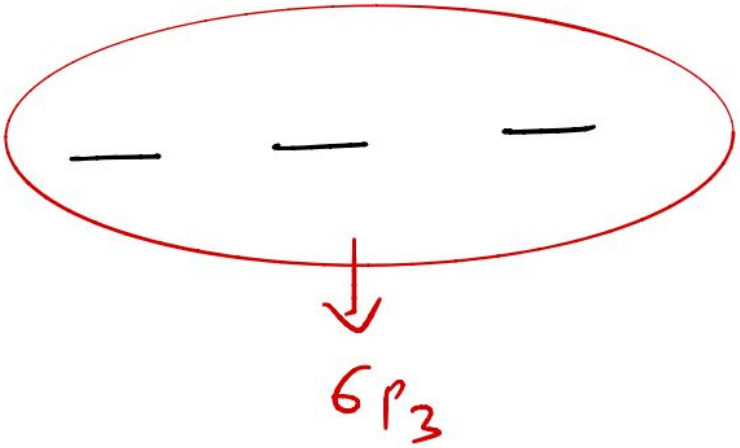
$= 240.$

Step 5: Enough of Enjoying. Now focus on the next question.

4. If repetitions are not permitted

(i) How many 3 digit numbers can be formed from the six digits 2, 3, 5, 6, 7 & 9 ?

Sol :- Given 6 digits 2, 3, 5, 6, 7, 9



Number of 3 digit numbers

= Number of permutations of 6 digits taken 3 at a time

$= {}^6P_3$

$= 6 \times 5 \times 4$

$= 120.$

Aliter :- Select 3 from given 6 in $^6C_3$ ways and each selection can be arranged in 3! ways.

So total ways $= {}^6C_3 \cdot 3!$

$= 20 \cdot 6$

$= 120.$

(ii) How many of these are less than 400 ?

Sol :- Given digits 2, 3, 5, 6, 7, 9

$\underline{2}$ $\underline{\quad}\ {}^5C_2 \cdot 2!\ \underline{\quad}$

Notice that for this number to be less than 400, leading digit can be 2 or 3 only.

Step 1 : Fill the first place by 2 or 3. This can be done in 2 ways.

Step 2 : Select the two digits still required from the remaining 5 digits in ${}^5C_2$ ways.

Step 3 : Arrange the 2 digits chosen in step 2 in 2! ways

Step 4 : Calculate the total number of ways as $2 \times {}^5C_2 \times 2!$

$= 2 \times 10 \times 2$

$= 40.$

(iii) How many are even ?

Sol :-

2, 3, 5, 6, 7, 9

$^5C_2 \times 2!$

__ __

Step 2:

Select 2 digits from 5 digits in $^5C_2$ ways. Arrange them in 2! ways.

2

__

Step 1 :-

Fill units place by either 2 or 6 for the number to be even. This can be done in 2 ways.

Total number of ways

$= 2 \times {}^5C_2 \times 2!$

$= 2 \times 10 \times 2$

$= 40$

(iv) How many are odd ?

Sol:- 2, 3, 5, 6, 7, 9

$5c_2 \cdot 2!$

__ __

Step 2

Select 2 more from remaining 5 in $5c_2$ ways. Arrange these 2 in 2! ways.

4

__

Step 1

Fill units place by 3, 5, 7 or 9. This can be done in 4 ways

Total no of ways

$= 4 \times 5c_2 \times 2!$

$= 4 \times 10 \times 2$

$= 80.$

(v) How many are multiples of 5 ?

Sol: Given digits 2, 3, 5, 6, 7, 9

$5c_2 \cdot 2!$

____ ____

Step 2

Select 2 more from remaining 5 in $5c_2$ ways. Arrange these 2 in 2! ways.

1

____

Step 1

Fill the units place by 5 in one way

Total number of ways

$= 1 \times 5c_2 \times 2!$

$= 1 \times 10 \times 2$

$= 20.$

5. How many two digit numbers are there in which the tens digit and the units digit are different and odd ?

Sol:- 2 digit number with odd and different digits.

5 _ 4 _

Step 1

Fill this place by one of 1, 3, 5, 7, 9 in 5 ways

Step 2

Fill this place from the remaining 4 odd digits left in 4 ways.

Total number of ways

$= 5 \times 4$

$= 20$

ALITER:- Fill two places using 5 odd digits in $5P_2$ ways

$= 5 \times 4$

$= 20$

PNC

Q.6 Every telephone no. consists of 7 digits. How many telephone numbers are there which donot include any other digits but 2, 3, 5 & 7?

Solⁿ:- There are 7 digits in the telephone no. So there are 7 places. and each place has 4 choices

4	4	4	4	4	4	4

So total no. of telephone no. are $\Rightarrow 4^7$

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7. (a) In how many ways can four passanger be accommodated in three railway carriages if each carriage can accommodate any no. of passangers.

Solⁿ:- There are 4 passanger and each passange can go in any three of the carriages. so each passanger has 3 choices -

3	3	3	3

So total No. of ways $= 3^4 = 81$

(b) In how many ways four persons can be accomodated in three ~~railway carriages~~ different chairs if each person can occupy only one chair

Solⁿ:- There are four person and each person can occupy only one chair. So 1st chair has 4 choices 2nd chair has 3 choices and so on -.

4	3	2

So, total no. of ways $= 4 \times 3 \times 2 = 24$.

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Q. 8. How many odd no. of five distinct digit no. can be formed with the digits 0, 1, 2, 3, 4?

Sol^n. There are five digits in a no. so there are 5 places. For to be odd last place is filled with only 2 choices (1, 3).

3	3	2	1	2

So. Total no. of ways are :- $3 \times 3 \times 2 \times 1 \times 2 \Rightarrow 36$

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Q.9. No. of ways in which 7 different colours in a rainbow can be arranged if green is always in the middle.

Sol^n:- There are 7 colours in a rainbow. so there are 7 places and middle place (4th place) has only one choice which is green so-

6	5	4	1	3	2	1

4th

So. total no. of ways is :- $6 \times 5 \times 4 \times 1 \times 3 \times 2 \times 1$

$= 720$

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(10) A letter lock consists of three rings each marked with 10 different letters. Find the number of ways in which it is possible to make an unsuccesful attempts to open the lock:

Sol^n:- There are 3 rings so 3 places and each places has 10 choices so -

10	10	10

total no. of combination :- $10^3 = 1000$

Total no. of unsuccesful attempts $= 1000 - 1$

$= 999$

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(11) Find the no. of ways in which letters of the word 'VALEDICTORY' be arranged so that the vowels may never be separated.

Soln. No. of Vowels => 4 (A E I O)

No of Consonant = 7 (V L D C T R Y)

$\boxed{A\ E\ I\ O}$ VLDCTRY

$8! \times 4! = 967680$

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12. (i) $^{n}P_{r} = {}^{n-1}P_{r} + r \cdot {}^{n-1}P_{r-1}$

Sol^n:- taking R.H.S

$\Rightarrow {}^{n-1}P_{r} + r \cdot {}^{n-1}P_{r-1}$

$\Rightarrow \frac{(n-1)!}{(n-r-1)!} + r \cdot \frac{(n-1)!}{(n-r)!}$

$\Rightarrow \frac{(n-1)!}{(n-r-1)!}\left[1 + \frac{r}{n-r}\right]$

$\Rightarrow \frac{(n-1)!}{(n-r-1)!} \times \frac{n}{n-r} = \frac{n!}{n-r!} = {}^{n}P_{r}$ Hence proved

(ii) If $^{20}C_{r+2} = {}^{20}C_{2r-3}$ find $^{12}C_{r}$

Case -1

$r+2 = 2r-3$

$r = 5$

Case-2

$r+2 = 20-(2r-3)$

$3r = 21$

$r = 7$

$\therefore {}^{12}C_{5} = {}^{12}C_{7} = \frac{12!}{5!\,7!} = 792$

(iii) Prove that :- ${}^{n-1}C_{3} + {}^{n-1}C_{4} > {}^{n}C_{3}$ if $n > 7$

$\because {}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

$\therefore {}^{n-1}C_{3} + {}^{n-1}C_{4} = {}^{n}C_{4}$

${}^{n}C_{4} > {}^{n}C_{3}$

$\because$ If ~~$n \leq 7$~~ $n > 7$

then in middle term r is always greater than 4

so ${}^{n}C_{4} > {}^{n}C_{3}$

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(iv) Find r if ${}^{15}C_{3r} = {}^{15}C_{r+3}$

Case-1:

$3r = r+3$

$2r = 3$

$r \neq 3/2$

Case-2

$3r = 15-(r+3)$

$4r = 12$

$\boxed{r = 3}$

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(13) There are 2 women participating in a chess tournament. Every participant played 2 games with the other participants. The no. of games that the men played between themselves exceeded by 66 as compared to the no. of games that the men played ~~b/w~~ with the women. Find the no. of participants & the total no. of games played in the tournament.

Soln Let there be n men

$\Rightarrow$ No. of games played b/w men $= 2 \times {}^nC_2$

$\Rightarrow$ No of games played b/w men & women $= 2(n \times 2)$

So acc. to question,

$$2 \times {}^nC_2 = 66 + 4n$$

$$n(n-1) = 66 + 4n$$

$$n^2 - 5n - 66 = 0$$

$$n = 11$$

Total No. of of Participants $= 11 + 2 = 13$

total no. of games $= {}^{13}C_2 \times 2$

$= 156$

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(14) An examination paper consist of 12 question divided into parts A & B. Part A contains 7 question & Part B contains 5 question. A candidate is required to attempt 8 question selecting atleast 3 from each part. In how many maximum ways can the candidate select the questions.

Solⁿ:- Case-1:- From part A 3 question & from part B 5 questi-

No. of ways = $^7C_3 \times {}^5C_5$

Case-2:- From part A 4 question & from part B 4 questin

No. of ways = $^7C_4 \times {}^5C_4$

Case-3:- From part A 5 question & from part B 3 questi.

No. of ways = $^7C_5 \times {}^5C_3$

So Total no. of ways = $^7C_3 \times {}^5C_5 + {}^7C_4 \times {}^5C_4 + {}^7C_5 \times {}^5C_3$

$= 420$

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(15) In How many ways can a team of 6 horses be selected out of a stud of 16, so that there shall always be 3 out of $ABCA'B'C'$, but never AA', BB' or CC' together.

Sol^n: ⇒ we have to select 3 horses from $ABCA'B'C$ such that AA', BB' or CC' not come together so we select one horse each from the pair AA', BB' or CC'. So No. of ways of selecting these 3 horses is =

$$^2C_1 \times {}^2C_1 \times {}^2C_1$$

⇒ No of ways of selecting remaining 3 horse from remaining 10 horses is = $^{10}C_3$

⇒ total No of ways = $^2C_1 \times {}^2C_1 \times {}^2C_1 \times {}^{10}C_3$

$= 8 \times 120$

$= 960$

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(16) In how many ways can you divide a pack of 52 cards equally among 4 players. In how many ways the cards can be divided in 4 sets, 3 of them having 17 cards each & the 4th with 1 card.

Sol^n (31)

(I) (52) cards in 4 equal groups and then distribute to 4 diff. persons

$$\underbrace{\frac{(52)!}{(13)!^4 \cdot (4!)}}_{\text{grouping}} \times \underbrace{4!}_{\text{distribution}} = \frac{(52)!}{(13!)^4}$$

(II) 52 in 17, 17, 17, 1

$$\frac{(52)!}{(17!)^3 \cdot (1!) \times (3!)} \Rightarrow \frac{(52)!}{(17!)^3 (3!)}$$

$\Rightarrow$ Question says $\rightarrow$ divide (only grouping)

(17) In a certain algebrical exercise book there are 4 examples on A.P., 5 examples on 'PNC' and 6 examples on B.T., No. of ways a teacher can select for his pupils atleast one but not more than 2 examples from each of these sets is -

Sol^n ⇒ No. of ways of selecting examples for A.P. $= {}^4C_1 + {}^4C_2$

⇒ No. of ways of selecting examples for PNC $= {}^5C_1 + {}^5C_2$

⇒ No. of ways of selecting examples for B.T. $= {}^6C_1 + {}^6C_2$

So total no. of ways $= ({}^4C_1 + {}^4C_2)({}^5C_1 + {}^5C_2)({}^6C_1 + {}^6C_2)$

$= 10 \times 15 \times (5+6)$

$= 3150$

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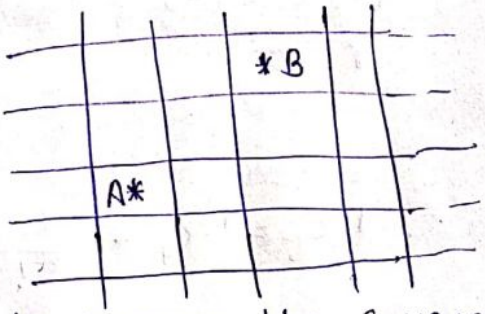
(18) Find the no. of ways in which two squares can be selected from an 8 by 8 chess board of size 1×1 so that they are not in the same row & in the same column.

Sol^n. It is a 8 by 8 chess board
So No of 1×1 Squares is = 64



⇒ No. of selecting one square is = $^{64}C_1$

⇒ By selecting 1 square we have to remove the square of that row & colowmn so No. of square left ⇒ 64 − 15 = 49

⇒ So No. of selecting one square from 49 is :- $^{49}C_1$

So total no. of ways = $\frac{^{64}C_1 \times {}^{49}C_1}{2}$ = 1568

There are two cases < AB, BA
These two cases are same
So we have to divide it by 2.

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(19) Find the number of permutations of the word "AUROBIND" in which vowels appear in an alphabetical order.

Sol^n (26)

A U O I → vowels

In alphabetical order ⇒ A I O U

(only one way of arrangement)

_ _ _ _ _ _ _

$^8C_4 \times (1) \times (4!)$

Selecting 4 places for vowels

only one way of arranging vowels

arranging consonent

$\Rightarrow {}^8C_4 \; 4!$

(20) In how many different ways a grandfather along with two of his grandsons and four grand daughters can be seated in a line for a photograph so that he is always in the middle and the two grandsons are never adjacent to each other.

Sol^n (28)

_ _ _ Grand father _ _ _

Case I Both grand son on same side

$$\underbrace{(2)}_{\text{either left of G.F or Right}} \times \underbrace{4!}_{\text{arrangement of grand daughter}} \times \underbrace{2!}_{\text{arrangement of grand son}}$$

Case II Grand sons on each side

$$\underbrace{{}^3C_1}_{\text{Picking one place on left side}} \times \underbrace{{}^3C_1}_{\text{Picking one place on Right side}} \times \underbrace{4!}_{\text{arrangent of Grand daughters}} \times \underbrace{2!}_{\text{arrangent of grand sons}}$$

Total = Case I + Case II

$\Rightarrow$ 528

(21) In how many other ways can the letters of the word **MULTIPLE** be arranged;

(i) without changing the order of the vowels

Sol^n (34)

Consonant → M T P L L

Vowel → U I E

(i) $\left({}^8C_3\right) \times (1) \times \left(\frac{5!}{2!}\right) - 1$

Selecting three position for vowel

arranging vowel

arranging Consonant

other than "multiple"

$\Rightarrow 3359$

(ii) keeping the position of each vowel fixed &

(ii)

_ U _ _ I _ _ E

Vowels fixed

$$\left(\frac{5!}{2!}\right) - (1) = 59$$

other than "multiple"

arranging consonant

(iii) without changing the relative order/position of vowels & consonants.

(iii) Relative Position

$$(3!) \times \left(\frac{5!}{2!}\right) - 1 \Rightarrow 359$$

arranging vowel

arranging consonant

other than "multiple"

(22) How many 6 digits odd numbers greater than 60,0000 can be formed from the digits 5, 6, 7, 8, 9, 0 if

(i) repetitions are not allowed

Sol^n (33)

Number greater than 60000 Start → 6, 7, 8, 9

odd number end → 5, 7, 9

(i) repetition not allowed.

(5)* → last digit

↑ ↑ ↑ ↑ ↑

4 choice 4c 3c 2c 1c

(6, 7, 8, 9)

$\Rightarrow 4 \times 4 \times 3 \times 2 \times 1 = 96$

(7/9)* last digit

↑ ↑ ↑ ↑ ↑

3 choice 4c 3c 2c 1c

(6, 8, (9/7))

$\Rightarrow 3 \times 4 \times 3 \times 2 \times 1 \times 2$

$\Rightarrow 144$

C stands for choice

Total = 144 + 96 = 240

(ii) repetitions are allowed.

(ii) repetition allowed

↑	—	—	—	—	↑
(6, 7, 8, 9)	6C	6C	6C	6C	(5, 7, 9)

$\Rightarrow 6^4 \times 4 \times 3 \Rightarrow 15552$

(23) If as many more words as possible be formed out of the letters of the word "DOGMATIC" then find the number of words in which the relative order of vowels and consonants remain unchanged.

Sol^n (29)

Consonant → C
vowel → V

D	O*	G	M	A*	T	I*	C
C	V	C	C	V	C	V	C

2nd, 5th, 7th positions are for Vowel
Rest positions for Consonant.

So

$$(3!) \times (5!) - (1)$$

arrangement of vowel

arrangement of consonant

other than "DOGMATIC"

$$\Rightarrow 720 - 1 = \underline{\underline{719}}$$

(24) Find the no. of ways in which 3 distinct no. can be selected from the set $\{3^1, 3^2, 3^3, \ldots, 3^{100}, 3^{101}\}$ so that they form a G.P.

Sol^n:- For G.P. Power on the 3 should be in A.P. So we have to find 3 distinct no. from 1, 2, 3, ---- 101 which can form A.P.

Let suppose these no. are a, b, c and for

A.P $\Rightarrow$ $2b = a + c$

$b = \frac{a+c}{2}$ → summation of a & c must be an even no. for this there are two cases.

Case-1

when both a & c are even

$\Rightarrow$ Total no. of even number from 1, 2, --101 is 50

$\Rightarrow$ So no. of ways of selecting 2 even no. ~~of~~ from 50 even no. is $\Rightarrow {}^{50}C_2$

Case-2

when both a & c are odd

$\Rightarrow$ Total no. of odd number from 1, 2, ----101 is 51

$\Rightarrow$ so no. of ways of selecting 2 odd no. from 51 odd no. is $\Rightarrow {}^{51}C_2$

So total no. of ways $= {}^{50}C_2 + {}^{51}C_2$

$= 2500$

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25

Consider the number N = 2910600.

On the basis of above information, answer the following questions :

Total number of divisors of N, which are divisible by 15 but not by 36 are-

(A) 92 (B) 94 (C) 96 (D) 98

(Sol^n) (35)

$N = 2910600$

$N = 2^3 . 3^3 . 5^2 . 7^2 . 11$

Divisible by (15)

$N = (5.(3)) \, (2^3 . 3^2 . 5 . 7^2 . 11)$

$\downarrow$

(15)

$\frac{N}{15} = (2^3)(3^2)(5)(7^2)(11)$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$(3+1) \quad (2+1) \quad (1+1) \quad (2+1) \quad (1+1)$

$\Rightarrow 144$

Divisible by 180 $\longrightarrow$ (L.c.m of 15 & 36)

$N = (2^2 . 3^2 . 5) \, ((2)(3)(5).(7^2)(11))$

$\frac{N}{180} = (2)(3)(5)(7^2)(11)$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$(1+1) \quad (1+1) \quad (1+1) \quad (2+1) \quad (1+1)$

$\Rightarrow 48$

Divisible by 15 not by 36

$\Rightarrow \quad 144 - 48 = 96$

(26) Determine the number of paths from the origin to the point (9, 9) in the cartesian plane which never pass through (5, 5) in paths consisting only of steps going 1 unit North and 1 unit East.

(Sol^n) (41)

$$\left(\frac{18!}{9!\,9!}\right) - \left(\frac{10!}{5!\,5!}\right) \times \left(\frac{8!}{4!\,4!}\right)$$

Total ways in 9×9 Grid

Ways in which it passes through (5,5)

Desireable ways ⇒ Total − Ways (5,5)

= 30980

(27) There are 20 books on Algebra & Calculus in our library. Prove that the greatest number of selections each of which consists of 5 books on each topic is possible only when there are 10 books on each topic in the library.

(Sol^n)(4-2)

let Algebra books $= x$

then Calculus book $= 20-x$

Selecting 5 Book each

$\Rightarrow {}^{x}C_5 \cdot {}^{20-x}C_5$

Maximum Iff $x = 10$

$\boxed{{}^{2n}C_x \Rightarrow \text{maximum} \circledast \ (\text{If } x = n)}$

(28) There are 10 different books in a shelf. Find the number of ways in which 3 books can be selected so that exactly two of them are consecutive.

Solⁿ (30)

10 Books in Shelf

1	2	3	4	5	6	7	8	9	10

exactly one pair → 1, 2
2, 3
⋮ ⋮
9, 10 } total 9 pair

→ (1,2) & (9,10) → for these two pair - for the remaining book we have 7 choice except the neighbour of 2/9.

→ for other pairs we have only 6 choice
let for (2,3) pair we can't select 1 & 4.
So By this

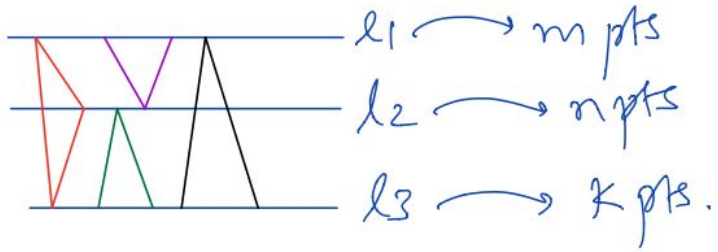
$$\underset{\substack{\uparrow \\ (1,2),\ (9,10) \\ \text{pair}}}{(2)} \times \underset{\substack{\uparrow \\ \text{choice}}}{(7)} + \underset{\substack{\uparrow \\ \text{rest} \\ \text{pairs}}}{(7)} \times \underset{\substack{\uparrow \\ \text{choice}}}{(6)} \Rightarrow \underline{\underline{56}}$$

Exercise S2

EXERCISE (S-2)

1. The straight lines l_1, l_2 & l_3 are parallel & lie in the same plane. A total of m points are taken on the line l_1, n points on l_2 & k points on l_3. How many maximum number of triangles are there whose vertices are at these points ?

Soln



No triangle will be formed if all three pts. are taken on same line

$\therefore$ Total no. of triangles :

$$= {}^{m+n+k}C_3 - ({}^{m}C_3 + {}^{n}C_3 + {}^{k}C_3) \text{ (Ans)}$$

Selecting 3 pts out of total (m+n+k) pts

All 3 pts taken on l_1

All 3 pts taken on l_2

All 3 pts taken on l_3

2. (a) How many five digits numbers divisible by 3 can be formed using the digits 0, 1, 2, 3, 4, 7 and 8 if each digit is to be used atmost once.

Soln Given 7 digits : 0, 1, 2, 3, 4, 7, 8

Sum of these digits = 25

for 5 digit no to be divisible by 3, sum of digits must be divisible by 3

∴ Out of given 7 digits, two digits should be removed so that sum of remaining digits is divisible by 3

	Removed digits	Used digits	No. of Nos.	
CI	0, 1	2,3,4,7,8	5!	120
CII	0,4	1,2,3,7,8	5!	120
CIII	0,7	1,2,3,4,8	5!	120
CIV	1,3	0,2,4,7,8	5! − 4!	96
CV	4,3	0,1,2,7,8	5! − 4!	96
CVI	2,8	0,1,3,4,7	5! − 4!	96
CVII	7,3	0,1,2,4,8	5! − 4!	96

Total no. of nos. = 120×3 + 96×4

= 744 Ans.

(b) Find the number of 4 digit positive integers if the product of their digits is divisible by 3.

Soln

Product of digits is divisible by 3 if atleast one of the digits is 0, 3, 6 or 9

$\therefore$ Reqd. Ans = (total no. of 4 digit nos.) − (No. of 4 digit nos. with no 0, 3, 6 or 9)

$\Rightarrow$ Reqd. Ans. $= 9 \times 10 \times 10 \times 10 - (6 \times 6 \times 6 \times 6)$

$= 9000 - 1296$

$= 7704.$

(3) Find the number of three elements sets of positive integers {a, b, c} such that a × b × c = 2310.

Soln

$$2310 = 2 \times 3 \times 5 \times 7 \times 11$$

$$= (a \times b \times c)$$

Each of the numbers 2, 3, 5, 7 or 11 can be assigned to either a, b or c

Assign each to a, b or c

when all nos are assigned to a, b or c

$$\frac{3^5 - 3}{3!}$$

order of elements in final set {a, b, c} is not important.

$= 40$ Ans.

(4) How many 4 digit numbers are there which contains not more than 2 different digits?

Soln (here X, Y are distinct non zero digits)

Case I : All 4 digits same : XXXX : $\boxed{9}$

Case II : 3 digits same, 1 different :

(a) XXX0 $\Rightarrow$ ${}^9C_1 \times \frac{3!}{2!} = \boxed{27}$

(b) X000 $\Rightarrow$ ${}^9C_1 \times 1 = \boxed{9}$

(c) XXXY $\Rightarrow$ ${}^9C_2 \times 2 \times \frac{4!}{3!} = \boxed{288}$

Case III : 2 digits same, 2 others same

(a) XX00 $\Rightarrow$ ${}^9C_1 \times \frac{3!}{2!} = \boxed{27}$

(b) XXYY $\Rightarrow$ ${}^9C_2 \times \left(\frac{4!}{2!2!}\right) = \boxed{216}$

$\therefore$ Total no. of nos. $= \boxed{576}$ Ans.

5. A shop sells 6 different flavours of ice-cream. In how many ways can a customer choose 4 ice-cream cones if

(i) they are all of different flavours

(ii) they are non necessarily of different flavours

(iii) they contain only 3 different flavours

(iv) they contain only 2 or 3 different flavours?

Soln (i) ${}^6C_4$ (Select any 4 out of given 6 flavours)

(ii) $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 4$

No. of non negative integral solns

$= {}^{4+6-1}C_{6-1} = {}^9C_5 = 126$

(iii) $\underbrace{{}^6C_3}_{\text{Select 3 flavours out of given 6}} \cdot \underbrace{{}^3C_1}_{\text{Select 1 flavour to be repeated}} = 20 \cdot 3 = 60$

(iv) $\overbrace{\underbrace{{}^6C_2}_{\text{Select 2 flavours out of given 6}} \cdot \underbrace{(3)}_{\text{Let A \& B are 2 selected flavours}}}^{\text{choose only 2 flavours}} + \overbrace{\underbrace{(60)}_{\text{part (iii) above}}}^{\text{choose 3 flavours}} = 105$

Possible quantity of A & B (3)

A	B
1	3
2	2
3	1

6

How many different ways can 15 Candy bars be distributed between Ram, Shyam, Ghanshyam and Balram, if Ram can not have more than 5 candy bars and Shyam must have at least two. Assume all Candy bars to be alike.

Sol^n

$$r + \cancel{s} + g + b = 15$$

$$\downarrow 2$$

$$S = \cancel{s} + 2$$

$$r + S + g + b = 13$$

$r=0$	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$
$S+b+g=13$	$S+b+g=12$	$S+b+g=11$			
${}^{13+3-1}C_{3-1}$	${}^{12+3-1}C_{3-1}$	${}^{11+3-1}C_{3-1}$	${}^{10+3-1}C_{3-1}$	${}^{9+3-1}C_{3-1}$	${}^{8+3-1}C_{3-1}$

$\therefore$ total: ${}^{15}C_2 + {}^{14}C_2 + {}^{13}C_2 + {}^{12}C_2 + {}^{11}C_2 + {}^{10}C_2$

$= 105 + 91 + 78 + 66 + 55 + 45$

$= 440$ Ans.

7 Find the sum of all numbers greater than 10000 formed by using the digits 0 , 1 , 2 , 4 , 5 no digit being repeated in any number.

Soln

$$\text{Reqd Ans} = \underbrace{\left(\begin{array}{c}\text{Sum of nos.}\\ \text{using}\\ 0,1,2,4,5\end{array}\right)}_{a} - \underbrace{\left(\begin{array}{c}\text{Sum of 4}\\ \text{digit nos.}\\ \text{using } 1,2,4,5\end{array}\right)}_{b}$$

$$\therefore a = (5-1)!\,(0+1+2+4+5)(11111)$$

$$b = (4-1)!\,(1+2+4+5)\,(1111)$$

$$\therefore a-b = 24\cdot 12\cdot(11111) - 6\cdot 12\,(1111)$$

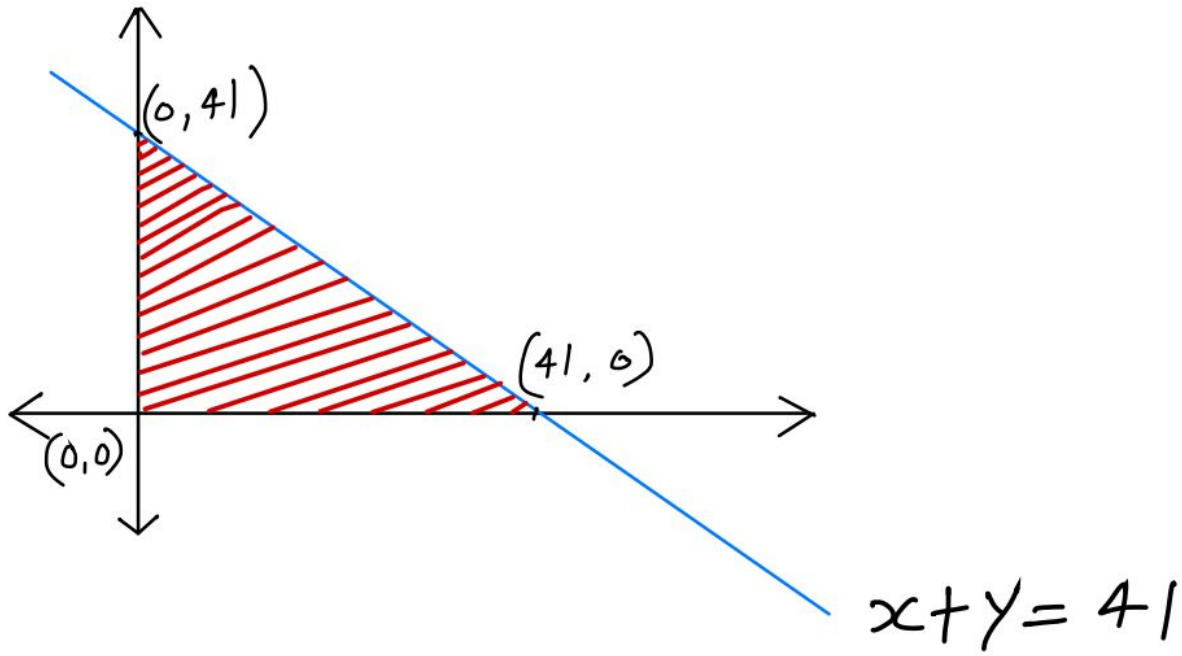
$$\Rightarrow a-b = 3{,}119{,}976 \quad \underline{\text{Ans}}$$

Exercise JM

(1) The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is : **[JEE (Main)-2015]**

(1) 820 (2) 780 (3) 901 (4) 861

Sol:-



Points (x, y) lie in Red shaded Region satisfying $\Rightarrow$ $\boxed{x+y-41<0}$ $\boxed{x>0}$ $\boxed{y>0}$

$\therefore$ $x+y+k=41$, where $k \in I^+$

Now 41 alike '1' distributed b/w 3 variable x, y & k such that each one get atleast one '1'.

So, No. of +ve integral sol^n of (x, y) $= {}^{(41-3)+2}C_2 = \boxed{780}$

Ans.

(2) Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is : **[JEE (Main)-2015]**

(1) 275 (2) 510 (3) 219 (4) 256

Sol:-

$n(A) = 4, n(B) = 2$

$n(A \times B) = 8$

number of subsets having atleast 3 elements

$$= 2^8 - \left(1 + {}^8C_1 + {}^8C_2\right) = 219$$

(3) The number of integers greater than 6000 that can be formed, using the digits 3,5,6,7 and 8 without repetition, is : **[JEE (Main)-2015]**

(1) 120 (2) 72 (3) 216 ~~(4)~~ 192

Sol:-

6/7/8			
3	4	3	2

4 - digit No.

$= 3 \cdot 4 \cdot 3 \cdot 2 = \boxed{72}$

5	4	3	2	1

5-digit No.

$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{120}$

$\therefore$ Total No. of integers $= 120 + 72$

$= \boxed{192}$ Ans.

4 If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is :

[JEE (Main)-2016]

(1) 58th (2) 46th (3) 59th (4) 52nd

Sol:-

A LL MS

A (LL MS) $\rightarrow \frac{4!}{2!} = \frac{24}{2} = 12$

L (AL MS) $\rightarrow 4! = 24$

M (ALLS) $\rightarrow \frac{4!}{2!} = \frac{24}{2} = 12$

SA (MLL) $\rightarrow \frac{3!}{2!} = 3$

SL (ALM) $\rightarrow 3! = 6$

Total words = 12 + 24 + 12 + 3 + 6 = 57

SMALL 58th

$\therefore$ the position of the word SMALL is $58^{th.}$.

5 A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is : **[JEE (Main)-2017]**

(1) 484 (2) 485 (3) 468 (4) 469

Sol.

X → 4 Ladies
X → 3 Men

y → 3 Ladies
y → 4 Men

Total number of ways

$$^4C_0 \cdot {}^3C_3 \cdot {}^3C_3 \cdot {}^4C_0 + {}^4C_1 \cdot {}^3C_2 \cdot {}^3C_2 \cdot {}^4C_1 + {}^4C_2 \cdot {}^3C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^4C_3 \cdot {}^3C_0 \cdot {}^3C_0 \cdot {}^4C_3$$

$$= 485$$

6 From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is- **[JEE(Main)-2018]**

(1) less than 500

(2) at least 500 but less than 750

(3) at least 750 but less than 1000

(4) at least 1000

Sol. Number of ways $= \binom{6}{4}\binom{3}{1}4!$

$$= 15 \times 3 \times 24$$

$$= 1080$$

7 The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repitition of digits allowed) is equal to :

(1) 250 (2) 374 (3) 372 (4) 375

(2)

a_1	a_2	a_3

Number of numbers $= 5^3 - 1$

a_4	a_1	a_2	a_3

2 ways for a_4

Number of numbers $= 2 \times 5^3$

Required number $= 5^3 + 2 \times 5^3 - 1$

$= 374$

8 The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is :

(1) 1365 (2) 1256 (3) 1465 (4) 1356

(4)

$$\sum_{r=2}^{13}(7r+2) = 7.\frac{2+13}{2}\times 6+2\times 12$$

$= 7 \times 90 + 24 = 654$

$$\sum_{r=1}^{13}(7r+5) = 7\left(\frac{1+13}{2}\right)\times 13+5\times 13 = 702$$

Total = 654 + 702 = 1356

(9) Let S = {1,2,3, ..., 100}. The number of non-empty subsets A of S such that the product of elements in A is even is :- **[JEE(Main)-2019]**

(1) $2^{50}(2^{50}-1)$ (2) $2^{100}-1$ (3) $2^{50}-1$ (4) $2^{50}+1$

Sol. S = {1,2,3------100}

= Total non empty subsets-subsets with product of element is odd $= \underbrace{(2^{100}-1)}_{\text{Total subset}} - \underbrace{(2^{50}-1)}_{\text{Total subset with odd Numbers only.}}$

$= 2^{50}(2^{50}-1)$ Ans

(16) There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is :

(1) 9 (2) 11 (3) 12 (4) 7

(3)

Let m-men, 2-women

$^{m}C_2 \times 2 = {}^{m}C_1 \, {}^{2}C_1 \, . \, 2 + 84$

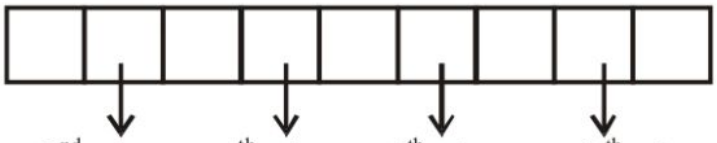
$m^2 - 5m - 84 = 0 \Rightarrow (m - 12)(m + 7) = 0$

$m = 12$

(11) All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is : **[JEE(Main)-2019]**

(1) 175 (2) 162 (3) 160 (4) 180

Sol.



2nd place 4th place 6th place 8th place (even places)

Number of such numbers

$$= {}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{2!4!} = 180$$

sel. of 3 places for odd Numbers

arr. of 3 Numbers {1, 1, 3}

arr. of {2, 2, 2, 2, 4, 4} at remaining places

12. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are addded to the total number of balls used in forming the equilaterial triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :- **[JEE(Main)-2019]**

(1) 190 (2) 262 (3) 225 (4) 157

Sol. $$\frac{n(n+1)}{2} + 99 = (n-2)^2$$

$$n^2 + n + 198 = 2(n^2 + 4 - 4n)$$

$$n^2 - 9n - 190 = 0$$

$$n^2 - 19n + 10 - 190 = 0$$

$$n(n - 19) + 10(n - 19) = 0$$

$$n = 19$$

No. of balls $= \frac{19 \times 20}{2} = 190.$

13 A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to :

(1) 25 (2) 28 (3) 27 (4) 24

$^5C_1 \cdot {}^nC_2 + {}^5C_2 \cdot {}^nC_1 = 1750$

$n^2 + 3n = 700$

$\therefore\ n = 25$

Exercise JA

1. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

(A) 55 (B) 66 (C) 77 (D) 88 **[JEE 2009, 3]**

Sol. **[C]**

Possible cases are

$$3\,2\,1\,1\,1\,1\,1 \rightarrow \frac{7!}{5!} = 42$$

or $$2\,2\,2\,1\,1\,1\,1 \rightarrow \frac{7!}{4!\cdot 3!} = 35$$

$\therefore$ number of 7 digits number is $42 + 35 = 77$ **Ans.** $\Rightarrow$ **(C)**

(2) Let S = {1,2,3,4}. The total number of unordered pairs of disjoint subsets of S is equal to -

[JEE 10, 5M, –2M]

(A) 25 (B) 34 (C) 42 (D) 41

Sol:- $S = \{1, 2, 3, 4\}$

Every element have 3 choices either go for A, B or Not.

sub sets ⇒ $A = \{\ \}$, $B = \{\ \}$ ⇒ Disjoints means $A \cap B = \phi$

$\therefore \frac{3^4 - 1}{2} + 1$

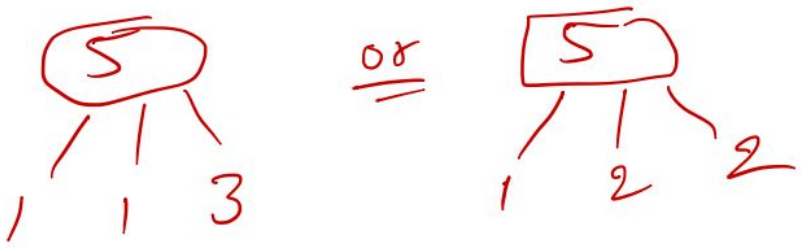
$= 41$

(3) The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is -

[JEE 2012, 3M, –1M]

(A) 75 (B) 150 (C) 210 (D) 243

Sol:-



$$\left[\frac{5!}{3!\,(1!)^2 \cdot 2!} + \frac{5!}{1!\,(2!)^2 \cdot 2!}\right] \cdot 3!$$

Paragraph for Question 4 and 5 :

Let a_n denotes the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

4 The value of b_6 is **[JEE 2012, 3M, –1M]**

(A) 7 (B) 8 (C) 9 (D) 11

5 Which of the following is correct ? **[JEE 2012, 3M, –1M]**

(A) $a_{17} = a_{16} + a_{15}$ (B) $c_{17} \neq c_{16} + c_{15}$ (C) $b_{17} \neq b_{16} + c_{16}$ (D) $a_{17} = c_{17} + b_{16}$

Sol:-

for b_n

$\underbrace{\boxed{}\boxed{}\boxed{}\boxed{}\cdots\boxed{}}_{(n-1)\text{ places}}\boxed{1}$

fill acc. to 'a_n'

$\therefore \boxed{b_n = a_{n-1}}$

for c_n $\underbrace{\boxed{}\boxed{}\boxed{}\boxed{}\cdots\boxed{}}_{(n-2)\text{ places}}\boxed{1}\boxed{0}$

$\therefore \boxed{c_n = a_{n-2}}$

$\because a_n = b_n + c_n$

$\therefore \boxed{a_n = a_{n-1} + a_{n-2}}$

$\Downarrow$

$a_{17} = a_{16} + a_{15}$

$\because a_1 = 1$ $\boxed{1}$

$a_2 = 2$ $\boxed{11}, \boxed{10}$

$a_3 = 2+1 = 3$

$a_4 = 3+2 = 5$

$a_5 = 5+3 = 8$

so, $b_6 = a_5 = 8$

$\therefore \boxed{b_6 = 8}$ 1.

(6) Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. The number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is **[JEE(Advanced)-2014, 3]**

Sol:- $n_1 + n_2 + n_3 + n_4 + n_5 = 20$

	n_1	n_2	n_3	n_4	n_5
1.)	1	2	3	4	10
2.)	1	2	3	5	9
3.)	1	2	3	6	8
4.)	1	2	4	5	8
5.)	1	2	4	6	7
6.)	1	3	4	5	7
7.)	2	3	4	5	6

Total 7 ways (Ans.)

(7) Let $n \geq 2$ b an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

[JEE(Advanced)-2014, 3]

Sol:-

$$^nC_2 = \underbrace{n}_{\text{side}} + \underbrace{n}_{\text{diagonal}}$$

$$\Rightarrow \frac{n(n-1)}{2} = 2n$$

$$\Rightarrow \boxed{n = 5}$$

(8) Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 in always placed in envelope numbered 2. Then the number of ways it can be done is - **[JEE(Advanced)-2014, 3(–1)]**

(A) 264 (B) 265 (C) 53 (D) 67

Sol:-

$$\underbrace{\text{Total}}_{D(6)} = \underbrace{x}_{\text{letter 1 go in to Envelop 2}} + \underbrace{x}_{\text{letter 1 go in to Envelop 3}} + \underbrace{x}_{\text{letter 1 go in to Envelop 4}} + \underbrace{x}_{\text{letter 1 go in to Envelop 5}} + \underbrace{x}_{\text{letter 1 go in to Envelop 6}}$$

$$\Rightarrow 265 = 5x$$

$$\Rightarrow x = \boxed{53}$$ Ans.

(9) Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is **[JEE (Advanced) 2015, 4M, –0M]**

Sol:-

$* \; B_1 \; B_2 \; B_3 \; B_4 \; B_5 \; (G_1 G_2 G_3 G_4 G_5)$

$$\boxed{n = 6!\,5!}$$

$* \; | B_1 | B_2 | B_3 | B_4 | B_5 |$ $\quad (G_1 G_2 G_3 G_4 G_5)$ → 4, 1

$$m = \underbrace{5!}_{\text{arrangment of 5 Boys}} \cdot \underbrace{{}^6C_2}_{\text{selection of 2 place for 4 \& 1 girls}} \cdot \underbrace{\left(\frac{5!}{4!\,1!}\right)}_{\text{grouping of 5 Girls, group of 4 \& 1}} \cdot \underbrace{2!}_{\text{arr. of both group at two places}} \cdot \underbrace{4!}_{\text{Internal arr. of 4 Girls in a group.}}$$

(10) A debate club consists of 6 girls and 4 body. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 member) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

[JEE(Advanced)-2016, 3(–1)]

(A) 380 (B) 320 (C) 260 (D) 95

Sol:-

$$\underbrace{\left[{}^6C_4 + {}^6C_3 \cdot {}^4C_1\right]}_{\substack{\Downarrow \\ \text{selection of 4 members}}} \cdot \underbrace{{}^4C_1}_{\substack{\Downarrow \\ \text{sel. of Captin from 4-selected member.}}}$$

(11) Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then $\frac{y}{9x} =$

[JEE(Advanced)-2017, 3]

Sol. $x = 10!$

$$y = {}^{10}C_1 \, {}^9C_8 \, \frac{10!}{2!}$$

$$\frac{y}{9x} = \frac{5.9.10!}{9.10!} = 5$$

(12) Let S = {1, 2, 3,.....,9}. For k = 1,2, ..., 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

[JEE(Advanced)-2017, 3(–1)]

(A) 125 (B) 252 (C) 210 (D) 126

Sol. $N_1 + N_2 + N_3 + N_4 + N_5$ = Total ways – {when no odd}

Total ways = ${}^9C_5$

Number of ways when no odd, is zero ($\because$ only available even are 2, 4, 6, 8)

$\therefore$ **Ans :** ${}^9C_5$ – zero = 126

(13) The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is _____ **[JEE(Advanced)-2018, 3(0)]**

Sol. Option for last two digits are (12), (24), (32), (44) are (52).

$\therefore$ Total No. of digits

$= 5 \times 5 \times 5 \times 5 = 625$

(14) In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boy and 2 girls.

(ii) Let α_2 be the total number of ways in which the committe can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committe can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the commitee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

LIST-I	**LIST-II**
P. The value of α_1 is	**1.** 136
Q. The value of α_2 is	**2.** 189
R. The value of α_3 is	**3.** 192
S. The value of α_4 is	**4.** 200
	5. 381
	6. 461

The correct option is :-

(A) **P → 4; Q → 6, R → 2; S → 1**

(B) **P → 1; Q → 4; R → 2; S → 3**

(C) **P → 4; Q → 6, R → 5; S → 2**

(D) **P → 4; Q → 2; R → 3; S → 1**

[JEE(Advanced)-2018, 3(–1)]

Sol. (1) $\alpha_1 = \binom{6}{3}\binom{5}{2} = 200$

So $P \to 4$

(2) $\alpha_2 = \binom{6}{1}\binom{5}{1} + \binom{6}{2}\binom{5}{2} + \binom{6}{3}\binom{5}{3} + \binom{6}{4}\binom{5}{4} + \binom{6}{5}\binom{5}{5}$

$= \binom{11}{5} - 1$

$= 46!$

So $Q \to 6$

(3) $\alpha_3 = \binom{5}{2}\binom{6}{3} + \binom{5}{3}\binom{6}{2} + \binom{5}{4}\binom{6}{1} + \binom{5}{5}\binom{6}{0}$

$= \binom{11}{5} - \binom{5}{0}\binom{6}{5} - \binom{5}{1}\binom{6}{4}$

$= 381$

So $R \to 5$

(4) $\alpha_2 = \binom{5}{2}\binom{6}{2} - \binom{4}{1}\binom{5}{1} + \binom{5}{3}\binom{6}{1} - \binom{4}{2}\binom{1}{1} + \binom{5}{4} = 189$

So $S \to 2$