Q In a Workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that atmost two machines will be out of service on the same day is

$\left(\frac{3}{4}\right)^3 K$ then $K = ?$

Sol^n

'0' out of services $\longrightarrow {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^5$

'1' " " " $\longrightarrow {}^5C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^4$

'2' " " " $\longrightarrow {}^5C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3$

Add

$\left(\frac{3}{4}\right)^3 \cdot \left(\frac{17}{8}\right)$

$K = \frac{17}{8}$

Q. Mean and variance of a binomial probability distribution are 4 and $\frac{4}{3}$ respectively then the probability of atleast two success is equal to

(A) $\frac{552}{729}$ (B) $\frac{201}{243}$ (C) $\frac{298}{343}$ (D) $\frac{716}{729}$

Sol^n

$$\left.\begin{array}{l} np = 4 \\ npq = \frac{4}{3} \end{array}\right\} \Rightarrow \begin{array}{l} n = 6 \\ p = \frac{2}{3} \\ q = 1/3 \end{array}$$

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$

$$= 1 - \left({}^6C_0 \left(\frac{1}{3}\right)^6 + {}^6C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^5 \right) = \frac{716}{729}$$

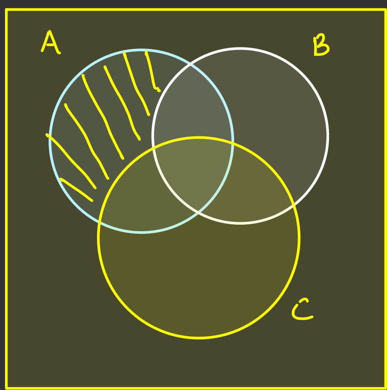
[D].

Q If the probability that a number selected from the set {1, 2, 3, ..., 1000} is divisible by 3 but neither divisible by 5 nor by 7, is $\frac{m}{n}$, then $5\left[\frac{5m}{n}\right]$ is (where [.] represents greatest integer function less than or equal to x)?

Sol^n

A → div by 3
B → " " 5
C → " " 7

$n(A) = 333$
$n(B) = 200$
$n(C) = 142$



$n(A \cap \bar{B} \cap \bar{C}) = \underbrace{n(A)}_{333} - \left(\underbrace{n(A \cap B)}_{66} + \underbrace{n(A \cap C)}_{47}\right) + \underbrace{n(A \cap B \cap C)}_{9}$

$n(A \cap \bar{B} \cap \bar{C}) = 229$

$\text{Req. prob} = \frac{229}{1000} = \frac{m}{n}$

$5\left[\frac{5m}{n}\right] = 5$ Ans

A die is weighted such that the probability of rolling the face numbered n is proportional to n^2 (n = 1, 2, 3, 4, 5, 6). The die is rolled twice, yielding the numbers a and b. If the probability that a < b can be expressed in lowest rational as $\frac{p}{q}$ find (p + q).

Sol^n

$$P(n) = kn^2$$

$$P(1) = k; \quad P(2) = 2^2 k; \quad P(3) = 3^2 k; \quad \cdots \quad P(6) = 6^2 k$$

$$P(1) + P(2) + \cdots + P(6) = 1$$

$$k + 2^2 k + \cdots + 6^2 k = 1 \Rightarrow \boxed{k = \frac{1}{91}}$$

$$\boxed{P(1) = \frac{1}{91}}; \quad P(2) = \frac{4}{91} \quad \cdots\cdots$$

→ A : a > b ✓ $\quad P(A) = P(C)$

B : a = b ←

→ C : a < b ✓

$$P(A) + P(B) + P(C) = 1 \Rightarrow \boxed{2P(A) + P(B) = 1} \quad \text{—(i)—}$$

$$P(B) = \left(\frac{1}{91}\right)^2 + \left(\frac{4}{91}\right)^2 + \left(\frac{9}{91}\right)^2 + \left(\frac{16}{91}\right)^2 + \left(\frac{25}{91}\right)^2 + \left(\frac{36}{91}\right)^2$$

$$P(B) = \frac{2275}{(91)^2} = \frac{25}{91}$$

$$P(A) = \frac{1 - P(B)}{2} = \frac{33}{91} = \frac{p}{q}$$

$$\therefore (p + q) = 124 \text{ Ans}$$

3 coins are thrown at a time and we remove those coins which show tails. The trial is done repeatedly until all of coins are removed. Then the probability that trial will end in the 2^{nd} round :-

(A) $\frac{19}{64}$ (B) $\frac{13}{64}$

(C) $\frac{17}{64}$ (D) $\frac{21}{64}$

Sol^n

C–I

$$\underset{1^{st}}{HHH} \longrightarrow \underset{2^{nd}}{TTT}$$

$$\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}.$$

C–II

$$\underset{1^{st}}{H\,\underbrace{TT}} \longrightarrow \underset{2^{nd}}{T}$$

$$3 \times \left(\frac{1}{8}\right) \times \left(\frac{1}{2}\right) = \frac{3}{16}$$

C–III

$$HHT \longrightarrow TT$$

$$\left(3 \times \frac{1}{8}\right) \times \left(\frac{1}{4}\right) = \frac{3}{32}$$

$$\frac{1}{64} + \frac{3}{16} + \frac{3}{32} = \frac{19}{64}$$

[A]

A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is :-

(A) $\frac{8}{243}$ (B) $\frac{1}{729}$ (C) $\frac{8}{9}$ (D) $\frac{8}{729}$

Sol^n

$P(9) = \frac{4}{36} = \frac{1}{9}$

success

Failure $P(F) = \frac{8}{9}$

${}^3C_2 \left(\frac{1}{9}\right)^2 \cdot \left(\frac{8}{9}\right)^1$

$\frac{8}{243}$ [A]

Let A,B & C are three independent events such that $P(B \cup C) = \frac{1}{2}$, $P\left(A \cap \bar{B} \cap \bar{C}\right) = \frac{1}{8}$.

If $P\left(A \cup B \cup C\right) = \frac{p}{q}$ (p & q are coprime), then q – p is

Sol^n

$$\underbrace{P(B \cup C) + P(A \cap \bar{B} \cap \bar{C})}_{P(A \cup B \cup C)} = \frac{1}{2} + \frac{1}{8} = \frac{5}{8} = \frac{p}{q}$$