Q Let z_1 lies on $|z| = 1$ and z_2 lies on $|z| = 2$, then which of the following is/are true -

(A) maximum value of $|z_1 + z_2|$ is 3 (B) minimum value of $|2z_1 - z_2|$ is 0

(C) maximum value of $|2z_1 + z_2|$ is 4 (D) minimum value of $|2z_1 - 3z_2|$ is 5

Sol^n (ABC)

$|z| = 1 \leftarrow z_1 \Rightarrow |z_1| = 1$

$|z| = 2 \leftarrow z_2 \Rightarrow |z_2| = 2$

(A) $|z_1 + z_2| \leq |z_1| + |z_2|$

$|z_1| + |z_2| \leq 3.$

(B) $\big||2z_1| - |z_2|\big| \leq |2z_1 + (-z_2)|$

$|2 - 2| \leq |2z_1 - z_2| \Rightarrow |2z_1 - z_2| \geq 0.$

(C) $|2z_1 + z_2| \leq |2z_1| + |z_2|$

$\leq 2 + 2$

≤ 4

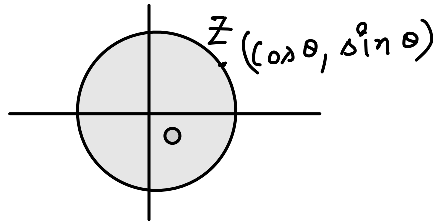
HW

Q Let z is a complex number such that $|z| = 1$, then maximum value of $|z+1| + |z^2 - z + 1|$

Sol^n $|z|^2 = 1 \Rightarrow \boxed{z\bar{z} = 1}$.

$E = |z+1| + |z^2 - z + 1| = |z+1| + |z^2 - z + z\bar{z}|$

$E = |z+1| + \underbrace{|z|}_{1}\, |z - 1 + \bar{z}|$



$E = |\cos\theta + 1 + i\sin\theta| + |2\cos\theta - 1|$

$E = \sqrt{(\cos\theta+1)^2 + \sin^2\theta} + |2\cos\theta - 1|$

$E = \sqrt{4\cos^2\frac{\theta}{2}} + \left|4\cos^2\frac{\theta}{2} - 3\right| = 2\left|\cos\frac{\theta}{2}\right| + \left|4\cos^2\frac{\theta}{2} - 3\right|$

Let $\left|\cos\frac{\theta}{2}\right| = t$; $t \in [0, 1]$

$E = 2t + |4t^2 - 3|$

$-4t^2 + 2t + 3$

$t \in \left[0, \frac{\sqrt{3}}{2}\right]$

max value at $t = \frac{1}{4}$

equal to $\frac{13}{4}$

$4t^2 + 2t - 3$

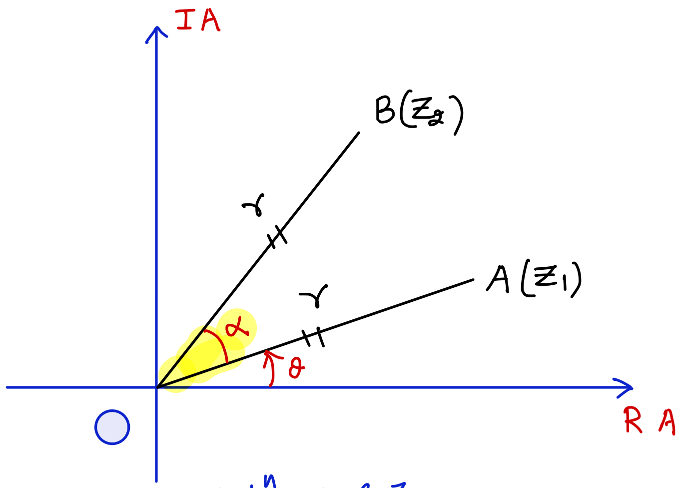
$t \in \left[\frac{\sqrt{3}}{2}, 1\right]$

max value at $t = 1$ equal to

3

Hence, max value $= \frac{13}{4}$ Ans

Rotation of complex number:



$\alpha \rightarrow$ angle betⁿ z_1 & z_2

$z_1 = re^{i\theta}$

$z_2 = r\,e^{i(\theta+\alpha)}$

$\frac{z_2}{z_1} = e^{i\alpha}$ ∠ rotate

$$\boxed{z_2 = z_1 e^{i\alpha}}$$

z_2 → final vector

z_1 → initial vector

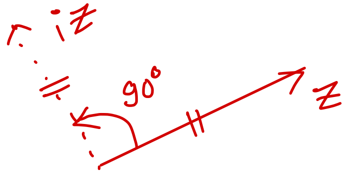
$e^{i\alpha} = \cos\alpha + i\sin\alpha$

$|e^{i\alpha}| = 1.$

Note :- ① $i = e^{i\pi/2}$

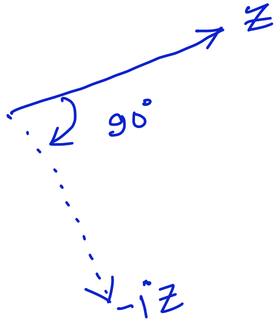
$z' = zi = ze^{i\pi/2}$

$|i| = 1$



② $-i = e^{-i\pi/2}$

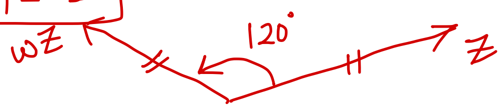
$z' = z(-i) = ze^{-i\pi/2}$



③ $\omega = e^{i2\pi/3}$; $\omega \rightarrow$ cube root of unity

$\boxed{|\omega| = 1}$

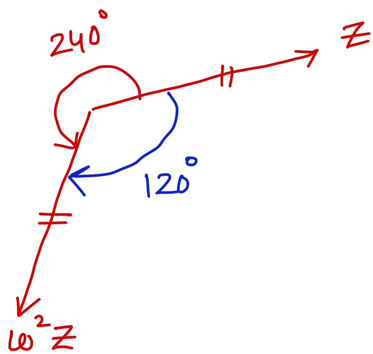
$z' = z\omega = ze^{i2\pi/3}$



(4) $w^2 = e^{i4\pi/3}$

$|w^2| = 1$

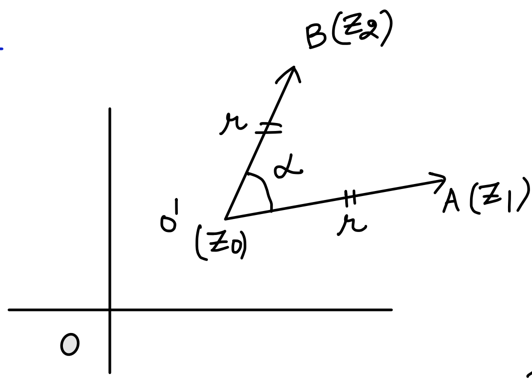
$z' = zw^2 = ze^{i4\pi/3}$



$w^2 \to$ cube root of unity

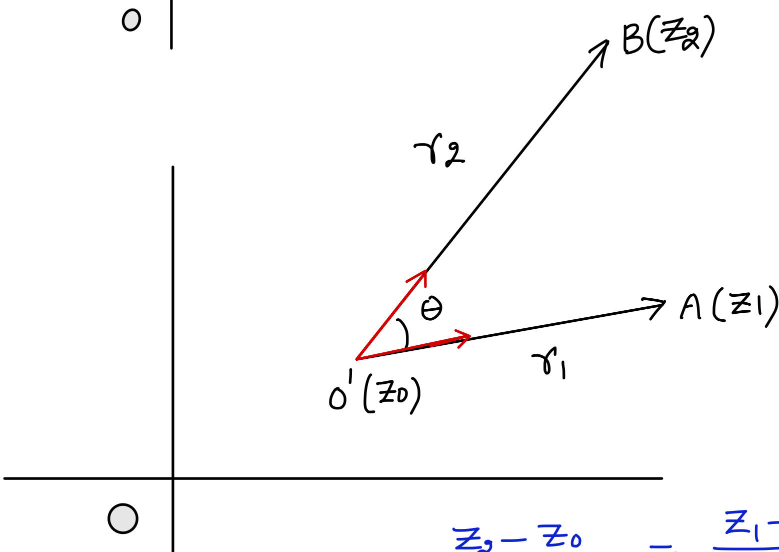
Note :-

①



$$\boxed{z_2 - z_0 = (z_1 - z_0)\, e^{i\alpha}}$$

②

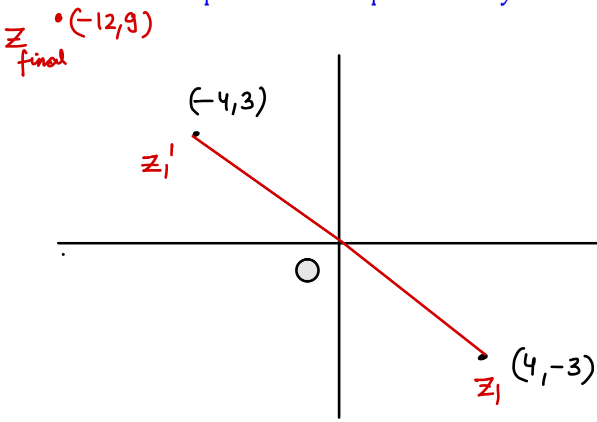


$$\frac{z_2 - z_0}{|z_2 - z_0|} = \frac{z_1 - z_0}{|z_1 - z_0|}\, e^{i\theta}$$

Rem

$$\boxed{\frac{z_2 - z_0}{z_1 - z_0} = \left(\frac{r_2}{r_1}\right) e^{i\theta}}$$

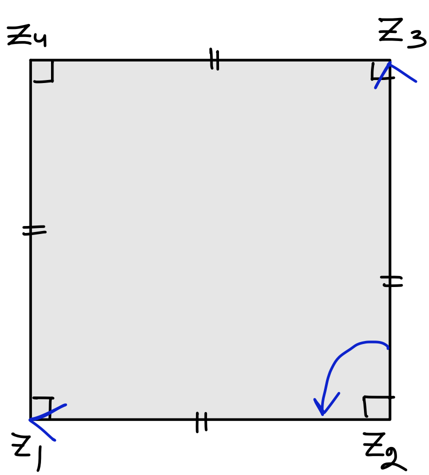
Q In the Argand plane, the vector $z = 4 - 3i$ is turned in the clockwise sense through 180° and stretched three times. The complex number represented by the new vector is



$z_{final} = -12 + 9i$

Q If z_1, z_2, z_3, z_4 are the vertices of square taken in order, then prove that $2z_2 = (1+i)z_1 + (1-i)z_3$.

Sol^n



$$(z_1 - z_2) = (z_3 - z_2)\, i^{\circ}$$

$$z_1 - i z_3 = z_2(1 - i)$$

$$z_2 = \frac{z_1}{1-i} - \frac{i z_3}{1-i}$$

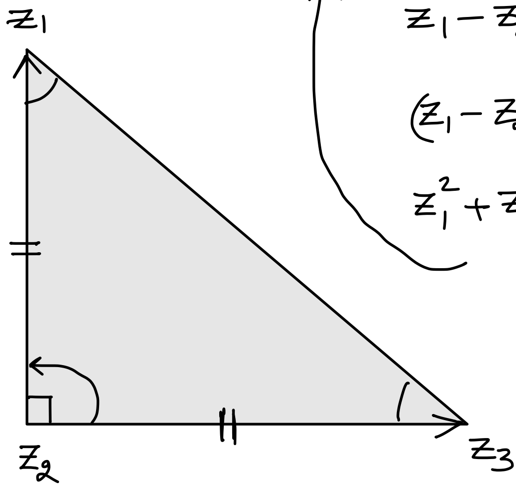
$$z_2 = \frac{z_1(1+i)}{2} - \frac{i(1+i)z_3}{2}$$

$$2z_2 = (1+i)z_1 + (1-i)z_3$$

(H.P.)

Q If z_1, z_2, z_3 are the vertices of an isosceles triangle right angled at z_2,

then prove that $z_1^2 + 2z_2^2 + z_3^2 = 2z_2(z_1 + z_3)$

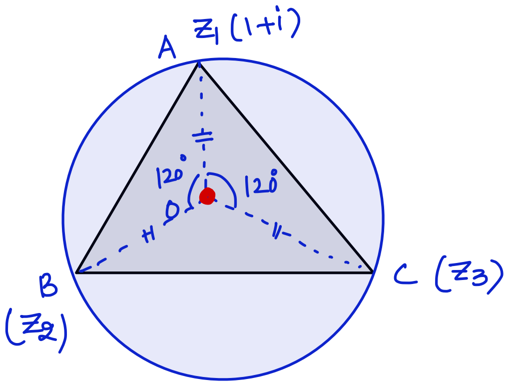


$$z_1 - z_2 = (z_3 - z_2)i$$

$$(z_1 - z_2)^2 = (z_3 - z_2)^2 i^2$$

$$z_1^2 + z_2^2 - 2z_1z_3 = -(z_3^2 + z_2^2 - 2z_2z_3)$$

Q If z_1, z_2, z_3 are the vertices of a equilateral triangle with circumcentre at the origin. If $z_1 = (1 + i)$ then find z_2 and z_3.



$$z_3 = z_1 e^{-i(120°)}$$

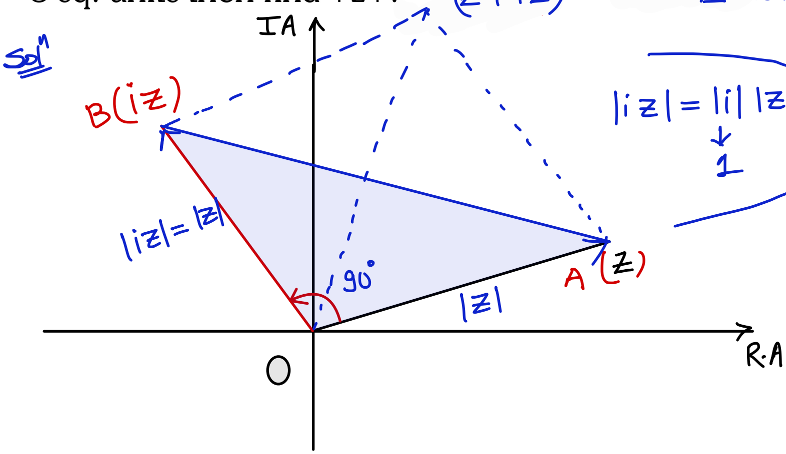
$$z_3 = (1+i)\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$$

$$z_3 = (1+i)\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$\frac{z_1 + z_2 + z_3}{3} = 0.$$

Q If the area of the triangle formed by z, iz and z + iz is 8 sq. units then find | z |.

Sol^n



$|iz| = |i|\,|z|$, where $|i| = 1$

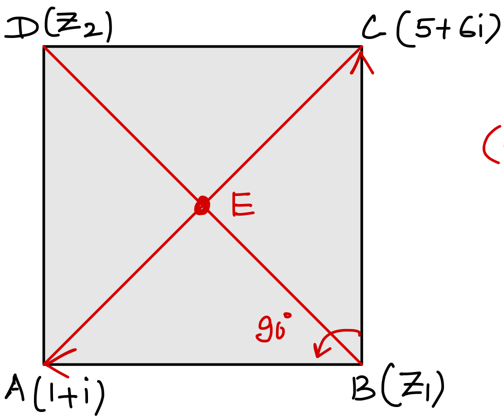
$\Delta AOB = \frac{1}{2}|z|\,|iz|$

$8 = \frac{1}{2}|z|^2$

$|z|^2 = 16$

$\therefore |z| = 4.$

Q If 1 + i and 5 + 6i are the extremities of diagonals of square, then find the remaining vertices.

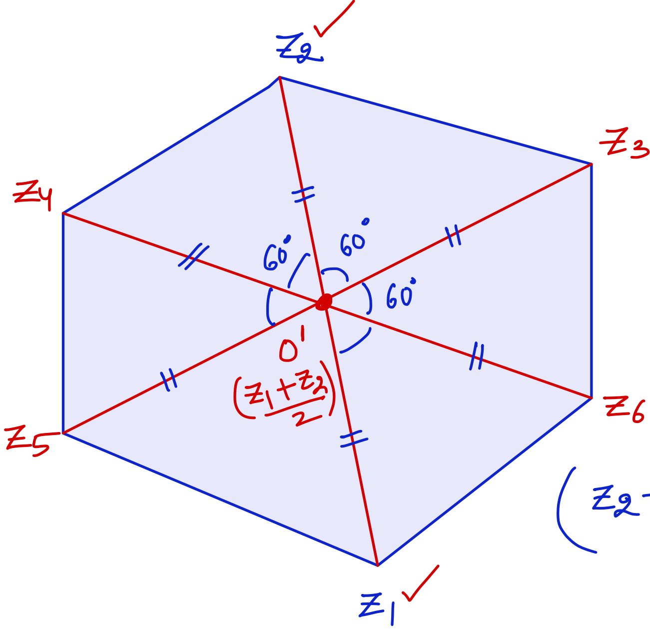


$(5+6i - z_1)i = (1+i-z_1)$

$\|^{ly}$ get z_2

$$\frac{z_1+z_2}{2} = \frac{(1+i)+(6i+5)}{2}$$

eg:



$$(O'z_2)e^{i\pi/3} = (O'z_4)$$

$$\downarrow$$

$$\left(z_2 - \left(\frac{z_1+z_2}{2}\right)\right)e^{i\pi/3}$$

$$= z_4 - \left(\frac{z_1+z_2}{2}\right)$$

||ly get remaining vertices.

Rem :-

Q If z_1, z_2, z_3 are vertices of an equilateral Δ & 'z_0' is circumcentre of Δ then :-

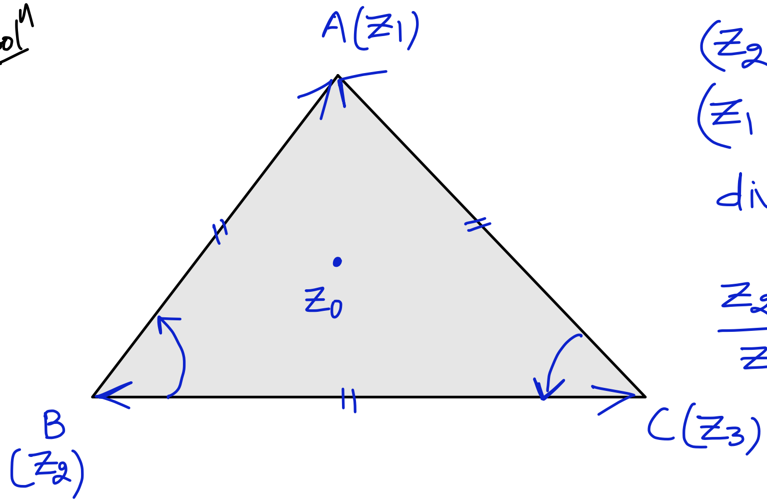
(i) $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$

i.e $\Sigma z_1^2 = \Sigma z_1 z_2$.

(ii) $z_1^2 + z_2^2 + z_3^2 = 3 z_0^2$.

(iii) $\dfrac{1}{z_1 - z_2} + \dfrac{1}{z_2 - z_3} + \dfrac{1}{z_3 - z_1} = 0$.

Solⁿ



$(z_2 - z_3) = (z_1 - z_3) e^{i\pi/3}$

$(z_1 - z_2) = (z_3 - z_2) e^{i\pi/3}$

divide :-

$$\frac{z_2 - z_3}{z_1 - z_2} = \frac{z_1 - z_3}{z_3 - z_2}.$$

$$\boxed{z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1}$$

(H.P.)

(ii) $\dfrac{z_1 + z_2 + z_3}{3} = z_0$

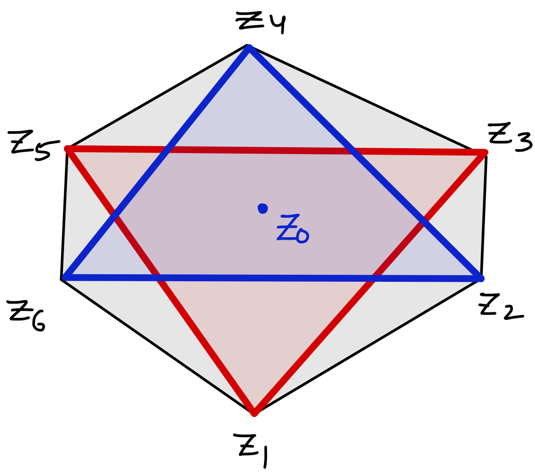
$(z_1 + z_2 + z_3)^2 = (3 z_0)^2$

$\Sigma z_1^2 + 2 \underbrace{\Sigma z_1 z_2} = 9 z_0^2 \Rightarrow \cancel{3} \Sigma z_1^2 = \overset{3}{\cancel{9}} z_0^2 \Rightarrow \Sigma z_1^2 = 3 z_0^2$

(H.P.)

Q If $z_r (r = 1, 2, \ldots 6)$ are the vertices of a regular hexagon then $\sum_{r=1}^{6} z_r^2 = 6z_0^2$, where z_0 is the circumcentre.

Sol^n



$z_1^2 + z_3^2 + z_5^2 = 3z_0^2$

$z_2^2 + z_4^2 + z_6^2 = 3z_0^2$

add

$\Sigma z_1^2 = 6z_0^2$

Q HW

Let z_1 and z_2 be roots of the equation $z^2 + pz + q = 0$, where the coefficients p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane. If $\angle AOB = \alpha \neq 0$ and $OA = OB$, where O is the origin, prove that $p^2 = 4q \cos^2\alpha/2$.

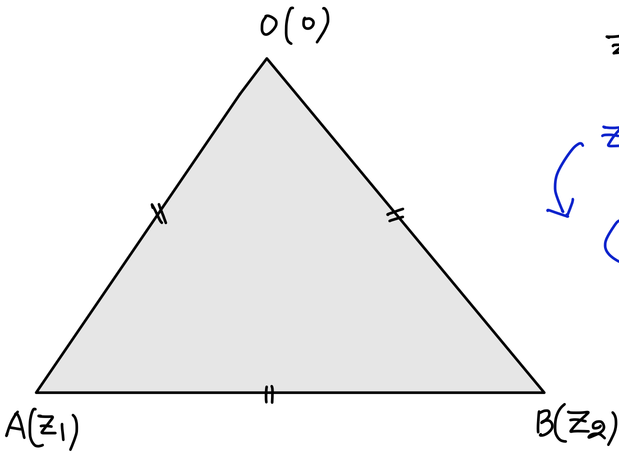
HW

Q A, B, C are the points representing the complex numbers z_1, z_2 z_3 respectively and the circumcentre of the triangle ABC lies at the origin. If the altitudes of the triangle through the opposite vertices meets the circumcircle at D, E, F respectively. Find the complex numbers corresponding to the D, E, F in terms of

z_1, z_2, z_3.

Q z_1 and z_2 are roots of $3z^2+3z+b=0$.
If O (origin), $A(z_1)$, $B(z_2)$ form an equilateral Δ then find 'b'?

Sol^n



$3z^2+3z+b=0 \begin{matrix} \nearrow z_1 \\ \searrow z_2 \end{matrix}$

$z_1+z_2=-1$; $z_1z_2=b/3$

$z_1^2+z_2^2+0^2 = z_1z_2+0+0$

$z_1^2+z_2^2 = z_1z_2$

$(z_1+z_2)^2 = 3z_1z_2$

$(-1)^2 = \cancel{3}\left(\frac{b}{\cancel{3}}\right)$

$\therefore \boxed{b=1}$

HW :-

O-1 Q 21 to 32.

O-2 Q 1, 2, 11, 12, 14, 16 to 19.

JA Q 1, 3, 5, 7, 8.