ALLEN Career Institute (Kota)

Vector

(Solutions for Leader)

Do-yourself (I)

(i) if $\bar{a} = 2\hat{i} + \mu\hat{j} - 7\hat{k}$ &
$\bar{b} = \lambda\hat{i} + \sqrt{3}\hat{j} - 7\hat{k}$ are two equal vector than find $\lambda^2 + \mu^2$

Solⁿ equal vector = every component equal

$$2\hat{i} + \mu\hat{j} - 7\hat{k} = \lambda\hat{i} + \sqrt{3}\hat{j} - 7\hat{k}$$

$$\lambda = 2 \qquad \mu = \sqrt{3}$$

$$\lambda^2 + \mu^2 = 4 + 3 = 7$$

(ii) if $\bar{a}$ & $\bar{b}$ are two vectors, which of the following statement is/are correct.

Soln. (A) $\bar{a} = -\bar{b} \Rightarrow |\vec{a}| = |\vec{b}|$

Correct (−) sign shows change in direction only (anti-parallel).

(B) $|\vec{a}| = |\vec{b}| \Rightarrow \bar{a} = \pm\bar{b}$ ✗

(C) $|\vec{a}| = |\vec{b}| \Rightarrow \bar{a} = \bar{b}$ ✗

(D) $|\vec{a}| = |\vec{b}| \Rightarrow \bar{a} = \pm 2\bar{b}$ ✗

(A)

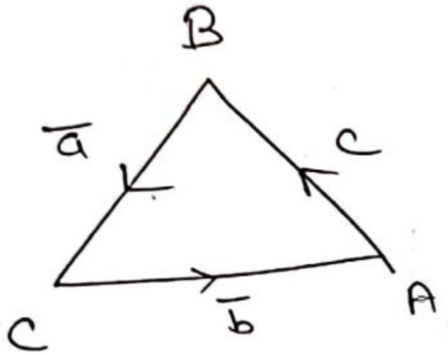
B, C, D all are incorrect, eg. $\hat{i}, \hat{j}, \hat{k}$ vectors have same magnitude.

Do yourself (2)

(i) If $\bar{a}, \bar{b}, \bar{c}$ be the vectors represented by the sides of a triangle taken in order, then prove that

$$\bar{a} + \bar{b} + \bar{c} = \bar{0}$$

<u>Sol^n</u>



By triangle rule

$$\overline{BC} + \overline{CA} = \overline{BA}$$

$$\Rightarrow \quad \bar{a} + \bar{b} = -\bar{c}$$

$$\Rightarrow \quad \boxed{\bar{a} + \bar{b} + \bar{c} = \bar{0}}$$

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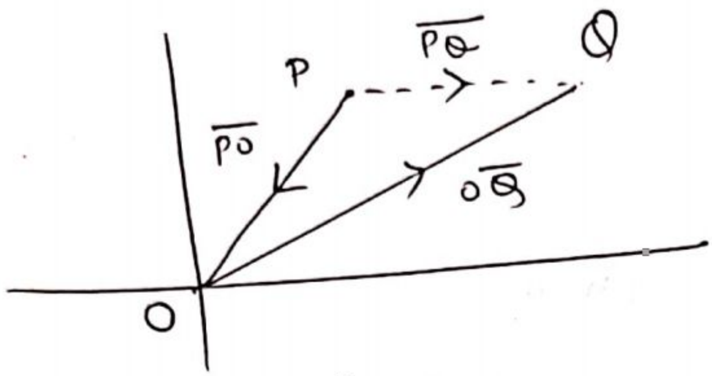
(ii) If $\overline{PO} + \overline{OQ} = \overline{QO} + \overline{OR}$ then prove that the points P, Q, R are colinear

Soln



By triangle law

$$\overline{PO} + \overline{OQ} = \overline{PQ}$$

Similarly

$$\overline{QO} + \overline{OR} = \overline{QR}$$

Given $\overline{PO} + \overline{OQ} = \overline{QO} + \overline{OR}$

So $\overline{PQ} = \overline{QR}$

vetor equal means direction as well as magnitude equal.

So P, Q, R colinear points.

(iii) For any two vectors $\bar{a}$ & $\bar{b}$

Prove that

(a) $|\bar{a}+\bar{b}| \leq |\bar{a}| + |\bar{b}|$

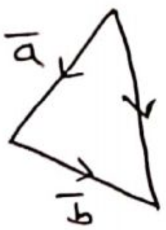
(b) $|\bar{a}-\bar{b}| \leq |\bar{a}| + |\bar{b}|$

(c) $|\bar{a}+\bar{b}| \geq \big||\bar{a}| - |\bar{b}|\big|$

<u>Solⁿ</u>

(a) $|\bar{a}+\bar{b}| \leq |\vec{a}| + |\bar{b}|$

Case (I) $\bar{a}$, $\bar{b}$ are not colinear



By triangle property

(Sum of two sides always greater than third side)

$|\bar{a}| + |\bar{b}| \geq |\bar{a}+\bar{b}|$

Case (II) $\bar{a}$, $\bar{b}$ are colinear

if they are parallel then equality holds.

$|\vec{a}| + |\vec{b}| = |\bar{a}+\bar{b}|$

(b) for second case

$$|\bar{a}-\bar{b}| \leq |\vec{a}| + |\vec{b}|$$

replace $\bar{b}$ by $-\bar{b}$ in first case.

(c) $$|\bar{a}+\bar{b}| \geq ||\vec{a}| - |\vec{b}||$$

Case (I) $\bar{a}$ & $\bar{b}$ are not collinear.

In this case they form a triangle similar to option 'a'.

And for triangle.

$\Rightarrow$ Diffrence of two sides always less than third side

$$|\bar{a}+\bar{b}| > ||\vec{a}| - |\vec{b}||$$

Case (II) $\bar{a}$ & $\bar{b}$ are co-linear.

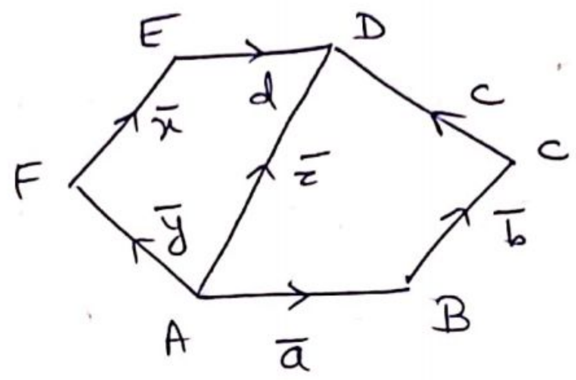
equality holds when $\bar{b} = \bar{0}$ (one possibility)

So combine both case.

$$\boxed{|\bar{a}+\bar{b}| \geq ||\vec{a}| - |\vec{b}||}$$

Do your self (3)

(i) In a given figure of regular hexagon which vectors are (provided vertices A, B, C, D, E, F all are fixed.):



Sol^n (a) Parallel

(i) $\bar{a}, \bar{d}$ (ii) $\bar{b}, \bar{x}, \bar{z}$ (iii) $\bar{c}, \bar{y}$

(b) equal.

(i) $\bar{b}, \bar{x}$ (ii) $\bar{a}, \bar{d}$ (iii) $\bar{c}, \bar{y}$

(C) Co initial

$\bar{a}, \bar{y}, \bar{z}$

(d) Parallel but not equal.

(i) $\bar{b}, \bar{z}$ (ii) $\bar{x}, \bar{z}$

(ii) if $\bar{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\bar{b} = 8\hat{i} - 12\hat{j} + 16\hat{k}$ Such that

$\bar{a} = \lambda \bar{b}$ then λ equal to

Sol^n

$$\bar{b} = 8\hat{i} - 12\hat{j} + 16\hat{k}$$

$$\bar{b} = 4(2\hat{i} - 3\hat{j} + 4\hat{k})$$

$$\bar{b} = 4(\bar{a})$$

$$\bar{b} = 1/\lambda\ (\bar{a})$$

$$(\lambda = 1/4)$$

(iii) if $3\bar{a} + 2\bar{b} = 5\bar{c}$

& $8\bar{a} - 7\bar{b} = 4\bar{c}$

then which Statement is/are true:

Soln

$$8 \times (3\bar{a} + 2\bar{b} = 5\bar{c})$$
$$3 \times (8\bar{a} - 7\bar{b} = 4\bar{c})$$

[eliminating $\bar{a}$]

$$16\bar{b} + 21\bar{b} = 40\bar{c} - 12\bar{c}$$

$$37\bar{b} = 28\bar{c}$$

$$\bar{b} = \frac{28}{37}\bar{c}$$

clearly $|\vec{b}| < |\vec{c}|$

$\Rightarrow$ eliminating $\bar{b}$

$$21\bar{a} + 16\bar{a} = 35\bar{c} + 8\bar{c}$$

$$37\bar{a} = 43\bar{c}$$

$$\frac{37}{43}\bar{a} = \bar{c}$$

clearly $|\vec{c}| < |\vec{a}|$

as we are able to represent $\vec{a} = \lambda_1\vec{b}$, $\vec{b} = \lambda_2\vec{c}$ & so on

so $\bar{a}, \bar{b}, \bar{c}$ are colinear

A, B, C

Do yourself (4)

(i) find the position vectors of the point which divide the join of the points $2\bar{a} - 3\bar{b}$ & $3\bar{a} - 2\bar{b}$ internally & externally in the ratio $2:3$

<u>Soln</u> Internally

$2 : 3$

$(2\bar{a} - 3\bar{b})$ P $(3\bar{a} - 2\bar{b})$

$$P = \frac{3(2\bar{a} - 3\bar{b}) + 2(3\bar{a} - 2\bar{b})}{2+3}$$

$$P = \frac{12\bar{a} - 13\bar{b}}{5}$$

Externally. $\bar{P} = \frac{m(\ ) - n(\ )}{m-n}$

$$\bar{P} = \frac{2(3\bar{a} - 2\bar{b}) - 3(2\bar{a} - 3\bar{b})}{2-3}$$

$$\bar{P} = \frac{5\bar{b}}{-1} = -5\bar{b}$$

(ii) ABCD is a parallelogram and P is the point of Intersection of its diagonal. If O is the origin of reference show that

$$\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD} = 4\,\overline{OP}$$

<u>Soln</u>

In parallelogram digonal bisect each other

So $$\frac{\overline{OA} + \overline{OC}}{2} = \overline{OP}$$

$$\overline{OA} + \overline{OC} = 2\overline{OP} \quad \text{—— (i)}$$

Similarly $\overline{OB} + \overline{OD} = 2\overline{OP}$ —— (ii)

adding eq. (i) & (ii)

$$\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD} = 4\,\overline{OP}$$

(H.P)

(iii) find the unit vector in direction of $3\hat{i} - 6\hat{j} + 2\hat{k}$

<u>Soln</u>

$$\hat{a} = \frac{\bar{a}}{|a|}$$

$$\bar{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$$

$$|a| = \sqrt{3^2 + 6^2 + 2^2}$$

$$\Rightarrow \sqrt{49} = 7$$

$$\hat{a} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{7}$$

$$\hat{a} = \left(\frac{3}{7}\right)\hat{i} - \left(\frac{6}{7}\right)\hat{j} + \left(\frac{2}{7}\right)\hat{k}$$

Do yourself - 5 :

(i) The position vectors of the points P, Q, R are $\hat{i}+2\hat{j}+3\hat{k}$, $-2\hat{i}+3\hat{j}+5\hat{k}$ and $7\hat{i}-\hat{k}$ respectively. Prove that P, Q and R are collinear.

Sol

$P(\hat{i}+2\hat{j}+3\hat{k})$; $Q(-2\hat{i}+3\hat{j}+5\hat{k})$; $R(7\hat{i}-\hat{k})$

$\therefore \overrightarrow{PQ} = -3\hat{i}+\hat{j}+2\hat{k}$; $\overrightarrow{QR} = 9\hat{i}-3\hat{j}-6\hat{k}$

$\because \overrightarrow{QR} = -3(\overrightarrow{PQ})$

$\Rightarrow$ P, Q, R are collinear

Do yourself (5)

M2

(i) The position vectors of the points P, Q, R are $\hat{i}+2\hat{j}+3\hat{k}$, $-2\hat{i}+3\hat{j}+5\hat{k}$ & $7\hat{i}-\hat{k}$ respectively

Prove that P, Q, R collinear.

Sol^n

$$l(P) + m(Q) + n(R) = 0$$

$$\& \quad l + m + n = 0$$

$$l(\hat{i}+2\hat{j}+3\hat{k}) + m(-2\hat{i}+3\hat{j}+5\hat{k}) + n(7\hat{i}-\hat{k}) = 0$$

$$\left.\begin{array}{l} l - 2m + 7n = 0 \\ 2l + 3m = 0 \\ 3l + 5m - n = 0 \end{array}\right] \rightarrow \text{Solve}$$

By solving this

$$l = -3n \qquad m = 2n$$

Clearly $l + m + n = 0$

Hence proved.

Do your self (6)

(i) find the angle between vectors $\bar{a}$ & $\bar{b}$ with magnitude 2 & 1 respectively and Such that $a \cdot b = \sqrt{3}$

Sol^n

$$\bar{a} \cdot \bar{b} = |a| |b| \cos\theta$$

$$\cos\theta = \frac{\bar{a} \cdot \bar{b}}{|a| |b|}$$

$$|a| = 2, \quad |b| = 1$$

$$\cos\theta = \frac{\sqrt{3}}{2 \cdot (1)}$$

$$\cos\theta = \frac{\sqrt{3}}{2} \qquad \theta = 30^\circ = \frac{\pi}{6}$$

(ii) find value of $(\vec{a}+3\bar{b}) \cdot (2\bar{a}-\vec{b})$

if $\bar{a} = \hat{i} + \hat{j} + 2\hat{k}$

$\bar{b} = 3\hat{i} + 2\hat{j} - \hat{k}$

Solⁿ.

$|\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$

$|\vec{b}| = \sqrt{14}$

$\Rightarrow (\bar{a} + 3\bar{b}).(2\bar{a} - b)$

$\Rightarrow 2\,\bar{a}.\bar{a} - \bar{a}.\bar{b} + 6\,\bar{b}.\bar{a} - 3\,\bar{b}.\bar{b}$

$\Rightarrow 2\,|\vec{a}|^2 + 5\,\bar{a}.\bar{b} - 3\,|\vec{b}|^2 \quad (\bar{a}.\bar{b} = \bar{b}.\bar{a})$

$\Rightarrow 2\,(6) + 5\,(3+2-2) - 3\,(14)$

$\Rightarrow -15$

(iii) The Scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ & $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to 1. find 'λ'

<u>Sol^n</u>

$$\bar{a} = \hat{i}+\hat{j}+\hat{k}$$
$$\bar{b} = 2\hat{i}+4\hat{j}-5\hat{k}$$
$$\bar{c} = \lambda\hat{i}+2\hat{j}+3\hat{k}$$

$$\bar{b}+\bar{c} = (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\hat{P} = \frac{(\bar{b}+\bar{c})}{|b+c|} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+36+4}}$$

$$\bar{a}\cdot\hat{P} = 1$$

$$(\hat{i}+\hat{j}+\hat{k})\cdot\left(\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+40}}\right) = 1$$

$$2+\lambda+6-2 = \sqrt{\lambda^2+4\lambda+44}$$

$$(\lambda+6)^2 = \lambda^2+4\lambda+44$$

$$\lambda^2+12\lambda+36 = \lambda^2+4\lambda+44$$

$$8\lambda = 8$$

$$\underline{\underline{\lambda = 1}}$$

(iv) find the projection of the vector $\bar{a} = 4\hat{i} - 2\hat{j} + \hat{k}$ on $\bar{b} = 3\hat{i} + 6\hat{j} + 2\hat{k}$ also find vector component of $\bar{a}$ along $\bar{b}$ & perpendicular to $\bar{b}$

Soln

Projection of $\bar{a}$ on $\bar{b}$ $= \dfrac{\bar{a}\cdot\bar{b}}{|\bar{b}|}$

$$\Rightarrow \frac{(4\hat{i} - 2\hat{j} + \hat{k})\cdot(3\hat{i} + 6\hat{j} + 2\hat{k})}{\sqrt{9 + 36 + 4}} \Rightarrow \frac{2}{7}$$

Vector component of $\bar{a}$ along $\bar{b}$

$$= \left(\frac{\bar{a}\cdot\bar{b}}{|b|^2}\right)\bar{b}$$

$$\Rightarrow \frac{2}{(7)^2}\cdot(3\hat{i} + 6\hat{j} + 2\hat{k})$$

Vector component of $\bar{a}$ perpendicular to $\bar{b}$

$$\Rightarrow \bar{a} - \left(\frac{\bar{a}\cdot\bar{b}}{|b|^2}\right)\bar{b}$$

$$\Rightarrow 4\hat{i} - 2\hat{j} + \hat{k} - \left(\frac{2}{49}\right)(3\hat{i} + 6\hat{j} + 2\hat{k})$$

$$\Rightarrow \frac{190\hat{i} - 110\hat{j} + 45\hat{k}}{49}$$

(v) find the unit vector along the angle bisector between the vector $\bar{a} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\bar{b} = -3\hat{i} + 6\hat{j} + 2\hat{k}$

Sol^n

Vector along angle bisector will be

$$\bar{P} \Rightarrow \lambda(\hat{a} + \hat{b})$$

$$\hat{a} = \frac{\hat{i} + 2\hat{j} - 2\hat{k}}{\sqrt{1+4+4}} = \frac{\hat{i} + 2\hat{j} - 2\hat{k}}{3}$$

$$\hat{b} = \frac{-3\hat{i} + 6\hat{j} + 2\hat{k}}{\sqrt{9+36+4}} = \frac{-3\hat{i} + 6\hat{j} + 2\hat{k}}{7}$$

$$\bar{P} \Rightarrow \lambda\left(\frac{\hat{i} + 2\hat{j} - 2\hat{k})}{3} + \frac{-3\hat{i} + 6\hat{j} + 2\hat{k}}{7}\right)$$

We have to find unit vector

so

$$\lambda^2\left(\left(\frac{1}{3} - \frac{3}{7}\right)^2 + \left(\frac{2}{3} + \frac{6}{7}\right)^2 + \left(\frac{-2}{3} + \frac{2}{7}\right)^2\right) = 1$$

$\lambda = 1/3 \quad (\lambda \in R^+)$

$\lambda = -1/3$ reject.

unit vector along angle bisector will be

$$\Rightarrow \frac{1}{3}\left(-\frac{2}{7}\hat{i} + \frac{32}{7}\hat{j} - \frac{8}{7}\hat{k}\right)$$

$$\Rightarrow -\frac{2}{21}\hat{i} + \frac{32}{21}\hat{j} - \frac{8}{21}\hat{k}$$

Do-yourself (7)

(i) find a vector $\bar{r}$ in the plane of $\bar{p} = -\hat{i} + \hat{j}$ & $\bar{q} = -\hat{j} + \hat{k}$, Such that $\bar{r}$ is perpendicular to $\bar{p}$ and $\bar{r} \cdot \bar{q} = -2$

<u>Soln</u> any vector in plane of $\bar{p}$ & $\bar{q}$

$$\bar{r} = x\bar{p} + y\bar{q} \text{ (linear Combination)}$$

given $\bar{r} \cdot \bar{q} = -2$

$$(x\bar{p} + y\bar{q}) \cdot (\bar{q}) = -2$$

$$x\,\bar{p} \cdot \bar{q} + y \cdot |q|^2 = -2$$

$$x(0 - 1 + 0) + y(2) = -2$$

$$-x + 2y = -2 \quad \text{—— (i)}$$

$\vec{r}$ is perpendicular to $\bar{p}$

$$\bar{r} \cdot \bar{p} = 0$$

$$(x\bar{p} + y\vec{q}) \cdot \bar{p} = 0$$

$$x|p|^2 + y(\vec{p} \cdot \vec{q}) = 0$$

$$x(2) + y(-1) = 0$$

by (i) & (ii) $\quad y = 2x \quad$ —— (ii)

$$x = -2/3 \qquad y = -4/3$$

$$\bar{r} = \frac{2}{3}(\hat{i} + \hat{j} - 2\hat{k})$$

Do-yourself (8)

(i) if $\bar{a}\times\bar{b} = \bar{c}\times\bar{d}$ & $\bar{a}\times\bar{c} = \bar{b}\times\bar{d}$ than show that $\bar{a}-\bar{d}$ parallel to $\bar{b}-\bar{c}$ when $\bar{a}\neq\bar{d}$ & $\bar{b}\neq\bar{c}$

Sol^n

$$\bar{a}\times\bar{b} = \bar{c}\times\bar{d} \quad \text{—(i)}$$

$$\bar{a}\times\bar{c} = \bar{b}\times\bar{d} \quad \text{—(ii)}$$

Sub-tract

$$\bar{a}\times\bar{b} - \bar{a}\times\bar{c} = \bar{c}\times\bar{d} - \bar{b}\times\bar{d}$$

$$\bar{a}\times(\bar{b}-\bar{c}) = (\bar{c}-\bar{b})\times\bar{d}$$

$$\bar{a}\times(\vec{b}-\vec{c}) + (\bar{b}-\bar{c})\times\bar{d} = 0$$

$$\bar{a}\times(\vec{b}-\vec{c}) - \vec{d}\times(\bar{b}-\bar{c}) = 0$$

$$(\bar{a}-\bar{d})\times(\bar{b}-\bar{c}) = 0$$

$\vec{a}=\vec{d}$ reject

$\bar{a}-\bar{d} \parallel \bar{b}-\bar{c}$

$\vec{b}=\vec{c}$ reject

Hence. Proved.

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(ii) Find $\bar{a} \times \bar{b}$ if $\bar{a} = 2\hat{i} + \hat{k}$

$\bar{b} = \hat{i} + \hat{j} + \hat{k}$

Sol^n

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow \quad \hat{i}(0-1) - \hat{j}(2-1) + \hat{k}(2-0)$$

$$\bar{a} \times \bar{b} = -\hat{i} - \hat{j} + 2\hat{k}$$

(iii) (a) $(u \cdot v)^2 + |u \times v|^2 = |u|^2 |v|^2$

Soln

$u \cdot v = |u||v| \cos\theta$

$(u \cdot v)^2 = |u|^2 |v|^2 \cos^2\theta$

$|u \times v| = |u| |v| \sin\theta \qquad \{ |\hat{n}| = 1 \}$

$|u \times v|^2 = |u|^2 |v|^2 \sin^2\theta$

Hence

$\Rightarrow \quad |u|^2 |v|^2 \cos^2\theta + |u|^2 |v|^2 \sin^2\theta$

$\Rightarrow \quad |u|^2 |v|^2 (\cos^2\theta + \sin^2\theta)$

$\Rightarrow \quad |u|^2 |v|^2 \qquad$ H.P.

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(b) $\left(1+|\vec{\mu}|^2\right)\left(1+|\vec{v}|^2\right) = \left(1-\bar{\mu}.\bar{v}\right)^2 + |\bar{\mu}+\bar{v}+(\vec{\mu}\times\vec{v})|^2$

Sol^n: R.H.S

$$|\bar{\mu}+\bar{v}+(\bar{\mu}\times\bar{v})|^2$$

$$= |\mu|^2 + |v|^2 + (\vec{\mu}\times\vec{v})^2 + 2\vec{\mu}.\vec{v} + 2\vec{\mu}.\cancelto{0}{(\vec{\mu}\times\vec{v})} + 2\vec{v}.\cancelto{0}{(\vec{\mu}\times\vec{v})} \quad \text{—(i)}$$

$$(1-\vec{\mu}.\vec{v})^2 = 1+(\vec{\mu}.\vec{v})^2 - 2\vec{\mu}.\vec{v} \quad \text{—(ii)}$$

adding both eq.

$$|\vec{\mu}|^2 + |\vec{v}|^2 + (\vec{\mu}\times\vec{v})^2 + (\vec{\mu}.\vec{v})^2 + 1 + \cancel{2\vec{\mu}.\vec{v}} - \cancel{2\vec{\mu}.\vec{v}}$$

$$\Rightarrow 1+|\vec{\mu}|^2+|\vec{v}|^2 + \left\{(\vec{\mu}.\vec{v})^2 + (\vec{\mu}\times\vec{v})^2\right\}$$

$$\Rightarrow 1+|\vec{\mu}|^2+|\vec{v}|^2 + |\vec{\mu}|^2|\vec{v}|^2$$

$$\Rightarrow \left(1+|\vec{\mu}|^2\right)\left(1+|\vec{v}|^2\right) = \text{L.H.S}$$

Hence prove.

Do yourself - 9

(i) Find the shortest distance b/w the lines :-

$\vec{r_1} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(2\hat{i}+3\hat{j}+4\hat{k})$ & $\vec{r_2} = (2\hat{i}+4\hat{j}+5\hat{k}) + \mu(3\hat{i}+4\hat{j}+5\hat{k})$

Solution:- $d = \left| \dfrac{(\vec{a_2}-\vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}[15-16] - \hat{j}(10-12) + \hat{k}(8-9)$$

$$\vec{b_1} \times \vec{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{1+4+1} = \sqrt{6}$$

$$\vec{a_2} - \vec{a_1} = (2\hat{i}+4\hat{j}+5\hat{k}) - (\hat{i}+2\hat{j}+3\hat{k})$$

$$= \hat{i} + 2\hat{j} + 2\hat{k}$$

$$d = \left| \frac{(\hat{i}+2\hat{j}+2\hat{k}) \cdot (-\hat{i}+2\hat{j}-\hat{k})}{\sqrt{6}} \right|$$

$$d = \left| \frac{-1+4-2}{\sqrt{6}} \right|$$

$$d = \frac{1}{\sqrt{6}}$$

DO-yourself 10

(i) If $\vec{a}\ \vec{b}\ \vec{c}$ are three non coplanar mutually $\perp^r$ unit vectors then find $[\vec{a}\ \vec{b}\ \vec{c}]$

Solⁿ:- $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$

$\because$ $\vec{a}\ \vec{b}\ \vec{c}$ are 3 non coplanar mutually $\perp^r$ unit vec.

so angle b/w $\vec{a}$ & $(\vec{b}\times\vec{c})$ is either 0° or 180°

so -

$$[\vec{a}\ \vec{b}\ \vec{c}] \begin{cases} |\vec{a}|\,|\vec{b}\times\vec{c}|\cos 0^\circ = 1 \\ |\vec{a}|\,|\vec{b}\times\vec{c}|\cos 180^\circ = -1 \end{cases}$$

$[\vec{a}\ \vec{b}\ \vec{c}] \Rightarrow \pm 1$

(ii) If $\vec{r}$ be a vector $\perp^r$ to $\vec{a}+\vec{b}+\vec{c}$ where $[\vec{a}\,\vec{b}\,\vec{c}]=2$ and $\vec{r}=l(\vec{b}\times\vec{c})+m(\vec{c}\times\vec{a})+n(\vec{a}\times\vec{b})$ then find $l+m+n$.

Solⁿ:- $\because \vec{r}$ is $\perp^r$ to $\vec{a}+\vec{b}+\vec{c}$ then

$$\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})=0$$

$$\vec{r}\cdot\vec{a}+\vec{r}\cdot\vec{b}+\vec{r}\cdot\vec{c}=0 \quad \text{--(1)}$$

$\Rightarrow \because \vec{r}=l(\vec{b}\times\vec{c})+m(\vec{c}\times\vec{a})+n(\vec{a}\times\vec{b})$

$$\vec{r}\cdot\vec{a}=l(\vec{b}\times\vec{c})\cdot\vec{a}+m(\vec{c}\times\vec{a})\cdot\vec{a}+n(\vec{a}\times\vec{b})\cdot\vec{a}$$

$$\vec{r}\cdot\vec{a}=l[abc]+0+0$$

$$\vec{r}\cdot\vec{a}=l\cdot 2 \quad \text{--(2)}$$

similarly $\vec{r}\cdot\vec{b}=m\cdot 2$ --(3)

$\vec{r}\cdot\vec{c}=n\cdot 2$ --(4)

So from eqⁿ (1) (2) (3) & (4)

$$\vec{r}\cdot\vec{a}+\vec{r}\cdot\vec{b}+\vec{r}\cdot\vec{c}=0=(l+m+n)\,2$$

So, $\boxed{l+m+n=0}$

(iii) Find the volm of the parallelepiped whose coterminous edges are represented by $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

Soln:- volume of parallelepiped is : $[\vec{a}\,\vec{b}\,\vec{c}]$

$$= \vec{a} \cdot (\vec{b} \times \vec{c})$$

$$\Rightarrow \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$$\vec{b} \times \vec{c} = 3\hat{i} - 5\hat{j} - 7\hat{k}$$

$$[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$$

$$= (2\hat{i} - 3\hat{j} + 4\hat{k}) \cdot (3\hat{i} - 5\hat{j} - 7\hat{k})$$

$$= 6 + 15 - 28$$

$$[\vec{a}\,\vec{b}\,\vec{c}] = -7$$

So volume of parallelepiped is $\underline{\underline{7}}$

(iv) Examine whether the vectors $\vec{a} = 2\hat{i}+3\hat{j}+2\hat{k}$, $\vec{b} = \hat{i}-\hat{j}+2\hat{k}$ & $\vec{c} = 3\hat{i}+2\hat{j}-4\hat{k}$ form a left handed or right handed system.

Solⁿ:- $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$

$$\Rightarrow \vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 2 & -4 \end{vmatrix}$$

$$= \hat{i}[4-4] - \hat{j}[-4-6] + \hat{k}(2+3)$$

$$= +10\hat{j} + 5\hat{k}$$

$$[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$$

$$= (2\hat{i}+3\hat{j}+2\hat{k})\cdot(10\hat{j}+5\hat{k})$$

$$= 0 + 30 + 10$$

$$[\vec{a}\ \vec{b}\ \vec{c}] = 40 > 0$$

So it is a Right handed System

Do yourself - 11

(i) If $\vec{a} = 2\hat{i} - 4\hat{j} + 7\hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{j} - 9\hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ then find $[\vec{a}\,\vec{b}\,\vec{c}]$ and also $\vec{a} \times (\vec{b} \times \vec{c})$

<u>Solⁿ:-</u> $[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & -9 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}[5+9] - \hat{j}[3+9] + \hat{k}(3-5)$$

$$\vec{b} \times \vec{c} = 14\hat{i} - 12\hat{j} - 2\hat{k}$$

$$[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$$
$$= (2\hat{i} - 4\hat{j} + 7\hat{k}) \cdot (14\hat{i} - 12\hat{j} - 2\hat{k})$$
$$= 28 + 48 - 14$$
$$[\vec{a}\,\vec{b}\,\vec{c}] = 62$$

$\Rightarrow$ $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$\because$ $\vec{a} \cdot \vec{c} = (2\hat{i} - 4\hat{j} + 7\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 2 - 4 + 7 = 5$

$\vec{a} \cdot \vec{b} = (2\hat{i} - 4\hat{j} + 7\hat{k}) \cdot (3\hat{i} + 5\hat{j} - 9\hat{k}) = 6 - 20 - 63 = -77$

$$\vec{a} \times (\vec{b} \times \vec{c}) = 5(3\hat{i} + 5\hat{j} - 9\hat{k}) - (-77)(\hat{i} + \hat{j} + \hat{k})$$
$$= 15\hat{i} + 25\hat{j} - 45\hat{k} + 77\hat{i} + 77\hat{j} + 77\hat{k}$$
$$= 92\hat{i} + 102\hat{j} + 32\hat{k}$$

Do-yourself:-12

(i) If $\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{b}\,\vec{c}\,\vec{a}]}$, $\vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{c}\,\vec{a}\,\vec{b}]}$, $\vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

then find the value of $(\vec{a}+\vec{b})\cdot\vec{p} + (\vec{b}+\vec{c})\cdot\vec{q} + (\vec{c}+\vec{a})\cdot\vec{r}$

Solⁿ: $\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{b}\,\vec{c}\,\vec{a}]}$

$\because [\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}]$

$\Rightarrow \vec{p}\cdot\vec{a} = \frac{\vec{a}\cdot[\vec{b}\times\vec{c}]}{[\vec{a}\,\vec{b}\,\vec{c}]} = 1$

$\vec{p}\cdot\vec{b} = \frac{\vec{b}\cdot(\vec{b}\times\vec{c})}{[\vec{a}\,\vec{b}\,\vec{c}]} = 0$

$\Rightarrow \vec{q}\cdot\vec{b} = \frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{[\vec{c}\,\vec{a}\,\vec{b}]}$

$= \frac{[\vec{a}\,\vec{b}\,\vec{c}]}{(\vec{a}\,\vec{b}\,\vec{c}]} = 1$

$\vec{q}\cdot\vec{c} = 0$

$\Rightarrow$ similarly $\vec{r}\cdot\vec{c} = 1$

$\vec{r}\cdot\vec{a} = 0$

So. $\Rightarrow (\vec{a}+\vec{b})\cdot\vec{p} + (\vec{b}+\vec{c})\vec{q} + (\vec{c}+\vec{a})\vec{r}$

$\Rightarrow (\vec{a}\cdot\vec{p}) + (\vec{b}\cdot\vec{p}) + \vec{b}\cdot\vec{q} + \vec{c}\cdot\vec{q} + \vec{c}\cdot\vec{r} + \vec{a}\cdot\vec{r}$

$\Rightarrow 1 + 0 + 1 + 0 + 1 + 0$

$\Rightarrow 3$

(ii) If $\vec{a}, \vec{b}$ & $\vec{c}$ are non zero, non coplanar vectors determine whether the vectors $\vec{r_1} = 2\vec{a} - 3\vec{b} + \vec{c}$, $\vec{r_2} = 3\vec{a} - 5\vec{b} + 2\vec{c}$ & $\vec{r_3} = 4\vec{a} - 5\vec{b} + \vec{c}$ are linearly independent or dependent.

<u>Soln</u>:- $k_1\vec{r_1} + k_2\vec{r_2} + k_3\vec{r_3} = 0$

$\Rightarrow \quad k_1(2\vec{a} - 3\vec{b} + \vec{c}) + k_2(3\vec{a} - 5\vec{b} + 2\vec{c}) + k_3(4\vec{a} - 5\vec{b} + \vec{c}) = 0$

$\Rightarrow \quad \vec{a}(2k_1 + 3k_2 + 4k_3) + \vec{b}(-3k_1 - 5k_2 - 5k_3) + \vec{c}(k_1 + 2k_2 + k_3) = 0$

$\because \vec{a}\ \vec{b}\ \vec{c}$ are non coplanar so

$2k_1 + 3k_2 + 4k_3 = 0$

$3k_1 + 5k_2 + 5k_3 = 0$

$k_1 + 2k_2 + k_3 = 0$

$$D = \begin{vmatrix} 2 & 3 & 4 \\ 3 & 5 & 5 \\ 1 & 2 & 1 \end{vmatrix}$$

$= 2[5-10] - 3[3-5] + 4[6-5]$

$= -10 + 6 + 4 = 0$

So It has Non trivial Soln i.e. atleast one of the $k_1\ k_2\ k_3$ is non zero.

So $\vec{r_1}, \vec{r_2}$ & $\vec{r_3}$ are linearly dependendent vectors.

EXERCISE 01

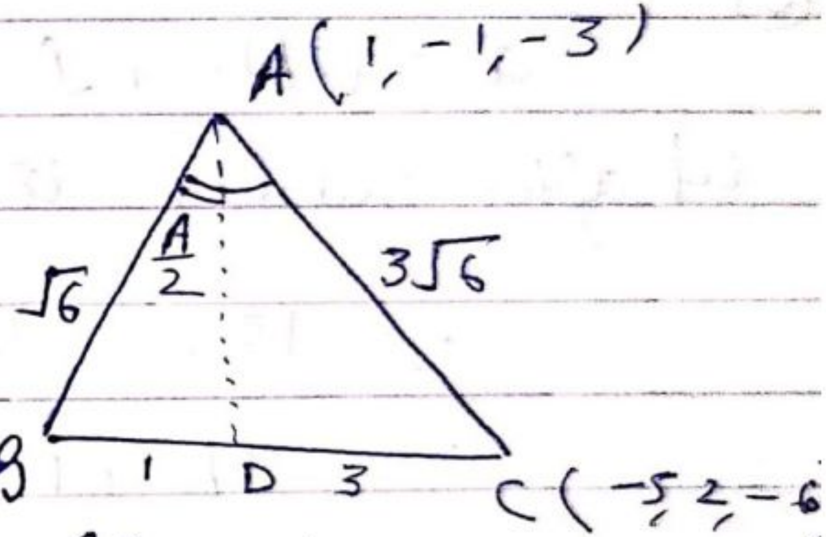
1. A (1, –1, –3), B (2, 1, –2) & C (–5, 2, –6) are the position vectors of the vertices of a triangle ABC. The length of the bisector of its internal angle at A is :

(A) $\sqrt{10}/4$ (B) $3\sqrt{10}/4$ (C) $\sqrt{10}$ (D) none

(I)

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$= \frac{1}{3}$$



$\therefore$ D will divide BC in the ratio 1 : 3.

$\therefore$ co-ordinates of D are $\left(\frac{1}{4}, \frac{5}{4}, -3\right)$

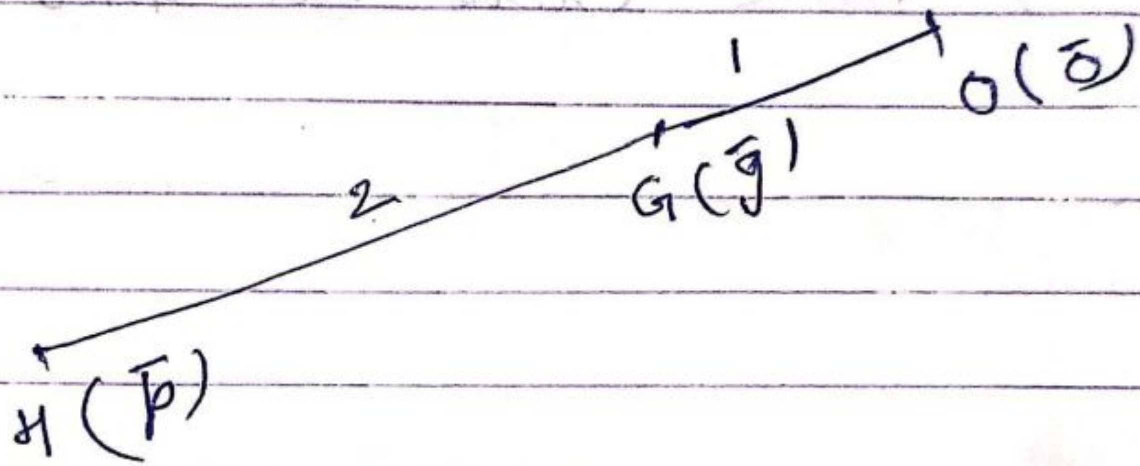
$\Rightarrow$ length of angle bisector $AD = \sqrt{\frac{9}{16} + \frac{81}{16} + 0}$

$$= \frac{3\sqrt{10}}{4}$$

2. Let $\vec{p}$ is the p.v. of the orthocentre & $\vec{g}$ is the p.v. of the centroid of the triangle ABC where circumcentre is the origin. If $\vec{p} = K\vec{g}$, then K =

(A) 3 (B) 2 (C) 1/3 (D) 2/3

(2)



$$\Rightarrow \frac{2\cdot\bar{o} + 1\cdot\bar{p}}{3} = \bar{g}$$

$$\Rightarrow \bar{p} = 3\bar{g} \Rightarrow k = 3$$

3

Four points A(+1, –1, 1) ; B(1, 3, 1) ; C(4, 3, 1) and D(4, – 1, 1) taken in order are the vertices of

(A) a parallelogram which is neither a rectangle nor a rhombus

(B) rhombus

(C) an isosceles trapezium

(D) a cyclic quadrilateral.

$\vec{AB} = 4\hat{j}$, $\vec{BC} = 3\hat{i}$, $\vec{CD} = -4\hat{j}$, $\vec{DA} = -3\hat{i}$

$\Rightarrow$ $\vec{AB} \parallel^{al} \vec{CD}$ and $\vec{BC} \parallel^{al} \vec{DA}$, $\vec{AB} \perp^{ar} \vec{BC}$

$\Rightarrow$ $|\vec{AB}| \neq |\vec{BC}|$

$\Rightarrow$ ABCD is rectangle which is cyclic.

④

Let α, β & γ be distinct real numbers. The points whose position vector's are $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$; $\beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}$ and $\gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$

(A) are collinear

~~(B)~~ form an equilateral triangle

(C) form a scalene triangle

(D) form a right angled triangle

Let $A(\alpha, \beta, \gamma)$, $B(\beta, \gamma, \alpha)$, $C(\gamma, \alpha, \beta)$

$\Rightarrow AB = BC = CA$

$\Rightarrow$ equilateral triangle.

5 If the vectors $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}=-\hat{i}+3\hat{j}+4\hat{k}$ & $\vec{c}=4\hat{i}-2\hat{j}-6\hat{k}$ constitute the sides of a Δ ABC, then the length of the median bisecting the vector $\vec{c}$ is

(A) $\sqrt{2}$ (B) $\sqrt{14}$ (C) $\sqrt{74}$ (D) $\sqrt{6}$

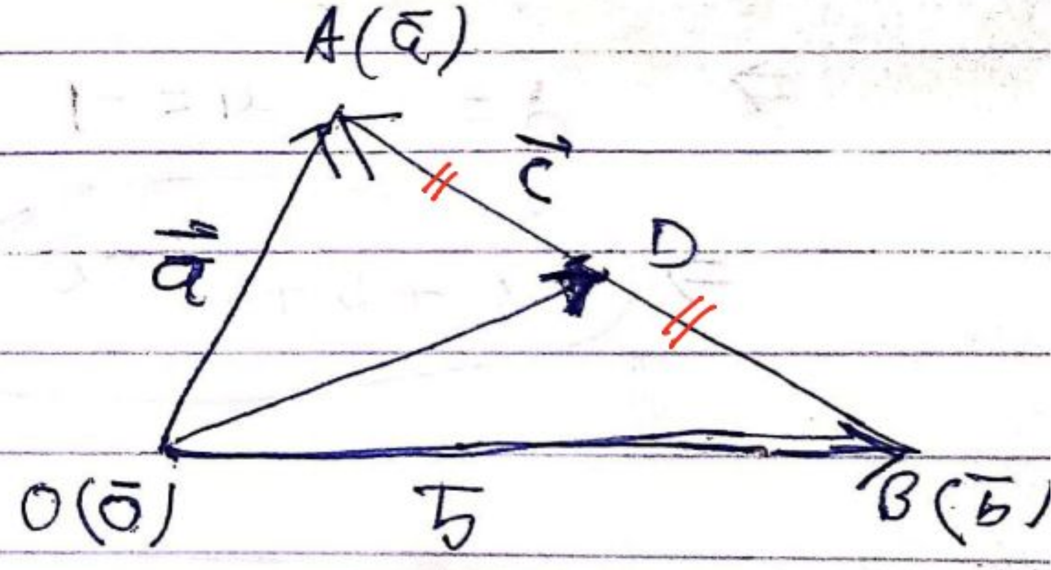
$\vec{b}+\vec{c}=\vec{a} \Rightarrow$

$\Rightarrow$ p.v. of D is $\dfrac{\bar{a}+\bar{b}}{2}$,

$\Rightarrow$ length of Median $=|\overline{OD}|$

$=\left|\dfrac{\bar{a}+\bar{b}}{2}\right|$

$=\sqrt{6}$



6 $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, no two of which are collinear and the vector $\vec{a}+\vec{b}$ is collinear with $\vec{c}$, $\vec{b}+\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+\vec{b}+\vec{c}$ is equal to -

(A) $\vec{a}$ (B) $\vec{b}$ (C) $\vec{c}$ ~~(D)~~ none of these

$$\bar{a} + \cancel{\bar{b}} = \lambda \bar{c}$$

$$\cancel{\bar{b}} \pm \bar{c} = \mu \bar{a}$$

$$\bar{a} - \bar{c} = \lambda \bar{c} - \mu \bar{a}$$

$$\Rightarrow \quad \lambda = -1, \ \mu = -1$$

$$\Rightarrow \bar{a} + \bar{b} + \bar{c} = 0$$

7. Consider the following 3 lines in space

$L_1 : \vec{r} = 3\hat{i} - \hat{j} + 2\hat{k} + \lambda(2\hat{i} + 4\hat{j} - \hat{k})$

$L_2 : \vec{r} = \hat{i} + \hat{j} - 3\hat{k} + \mu(4\hat{i} + 2\hat{j} + 4\hat{k})$

$L_3 : \vec{r} = 3\hat{i} + 2\hat{j} - 2\hat{k} + t(2\hat{i} + \hat{j} + 2\hat{k})$

Then which one of the following pair(s) are in the same plane.

(A) only L_1L_2 (B) only L_2L_3 (C) only L_3L_1 (D) L_1L_2 and L_2L_3

for L_1 and L_2

$$S.T.P = \begin{vmatrix} 2 & 4 & -1 \\ 4 & 2 & 4 \\ (3-1) & (-1-1) & (2+3) \end{vmatrix} = (20+32+8) - (-4-16+80)$$

$$= 0$$

$\Rightarrow$ Coplanar lines

for L_2 and L_3

$$S.T.P. = \begin{vmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ (3-1) & (2-1) & (-2+3) \end{vmatrix} = 0$$

$\Rightarrow$ Coplanar lines

7 Continues...

M2

Consider the following 3 lines in space

$L_1 : \vec{r} = 3\hat{i} - \hat{j} + 2\hat{k} + \lambda(2\hat{i} + 4\hat{j} - \hat{k})$

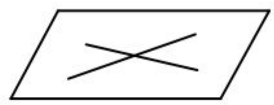
$L_2 : \vec{r} = \hat{i} + \hat{j} - 3\hat{k} + \mu(4\hat{i} + 2\hat{j} + 4\hat{k})$

$L_3 : \vec{r} = 3\hat{i} + 2\hat{j} - 2\hat{k} + t(2\hat{i} + \hat{j} + 2\hat{k})$

Then which one of the following pair(s) are in the same plane.

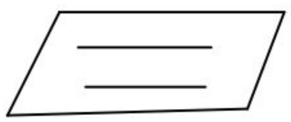
(A) only L_1L_2 (B) only L_2L_3 (C) only L_3L_1 (D) L_1L_2 and L_2L_3

Sol^n Non parallel intersecting lines lie in same plane



L_1 and L_2 are non parallel and intersect at $(5, 3, 1)$

Parallel distinct lines are also coplaner



L_2 and L_3 are parallel and distinct lines

$\therefore$ (Ans D)

8

The vectors $3\hat{i} - 2\hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} + 5\hat{k}$ & $2\hat{i} + \hat{j} - 4\hat{k}$ form the sides of a triangle. Then triangle is

(A) an acute angled triangle

(B) an obtuse angled triangle

(C) an equilateral triangle

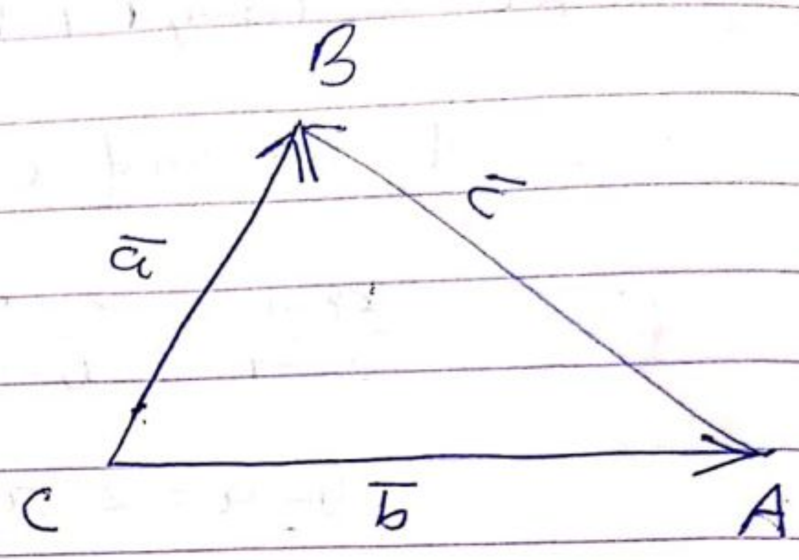
(D) a right angled triangle

let $\bar{a} = 3\hat{i} - 2\hat{j} + \hat{k}$

$\bar{b} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\bar{c} = 2\hat{i} + \hat{j} - 4k$

$\Rightarrow \quad \bar{b} + \bar{c} = \bar{a}$



$$\cos B = \frac{\bar{a} \cdot \bar{c}}{|\bar{a}||\bar{c}|} = 0$$

$\Rightarrow$ Δ^{le} is right angled.

Scanned with CamScanner

9 If the vectors $3\bar{p}+\bar{q}\,;5\bar{p}-3\bar{q}$ and $2\bar{p}+\bar{q}\,;4\bar{p}-2\bar{q}$ are pairs of mutually perpendicular vectors then $\sin\left(\bar{p}\,\hat{}\,\bar{q}\right)$ is

(A) $\sqrt{55}/4$ ~~(B)~~ $\sqrt{55}/8$ (C) $3/16$ (D) $\sqrt{247}/16$

14 $(3\bar{p}+\bar{q}).(5\bar{p}-3\bar{q})=0$

$(2\bar{p}+\bar{q}).(4\bar{p}-2\bar{q})=0$

$\Rightarrow \quad 15|\bar{p}|^2-4\,\bar{p}.\bar{q}-3|\bar{q}|^2=0$

and $8|\bar{p}|^2-2|\bar{q}|^2=0$

$\Rightarrow \quad |\bar{q}|=2|\bar{p}|$ and $\bar{p}.\bar{q}=\frac{3}{4}|\bar{p}|^2$

Now $\cos(\bar{p}\,\hat{}\,\bar{q})=\frac{\bar{p}.\bar{q}}{|\bar{p}||\bar{q}|}$

$=\frac{\frac{3}{4}|\bar{p}|^2}{|\bar{p}|.2|\bar{p}|}=\frac{3}{8}$

$\Rightarrow \sin(\bar{p}\,\hat{}\,\bar{q})=\frac{\sqrt{55}}{8}$

10) The set of values of c for which the angle between the vectors $cx\hat{i}-6\hat{j}+3\hat{k}$ & $x\hat{i}-2\hat{j}+2cx\hat{k}$ is acute for every $x \in R$ is

(A) (0, 4/3) (B) [0, 4/3] (C) (11/9, 4/3) (D) [0, 4/3)

---

10)

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

If θ is acute, $\cos\theta > 0$

so $\vec{a}\cdot\vec{b} > 0$

$$(cx\hat{i}-6\hat{j}+3\hat{k})\cdot(x\hat{i}-2\hat{j}+2cx\hat{k}) > 0$$

$$cx^2+12+6cx > 0$$

$$cx^2+6cx+12 > 0 \quad \forall\ x \in R$$

so $a > 0$, $D < 0$

$c > 0$ and $36c^2 - 4 \times c \times 12 < 0$

$$12c(3c-4) < 0$$

$$\underbrace{c > 0 \text{ and } c \in (0, \tfrac{4}{3})}$$

so $c \in (0, \frac{4}{3})$

at $c = 0 \Rightarrow 0 + 0 + 12 > 0 \ \forall\ x \in R$

$c = 0$ satisfies the eqn.

so final answer is $\boxed{c \in [0, \frac{4}{3})}$

11 Let $\vec{u}=\hat{i}+\hat{j}, \vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\cdot\hat{n}=0$ and $\vec{v}\cdot\hat{n}=0$, then $|\vec{w}\cdot\hat{n}|$ is equal to

(A) 1 (B) 2 (C) 3 (D) 0

---

11 $\hat{n}$ is Perpendicular to both $\vec{u}$ and $\vec{v}$

so $\hat{n} = \lambda(\vec{u} \times \vec{v})$

$$= \lambda(\hat{i}+\hat{j}) \times (\hat{i}-\hat{j})$$

$$= \lambda(\underbrace{\hat{i}\times\hat{i}}_{0} - \underbrace{\hat{i}\times\hat{j}}_{\hat{k}} + \underbrace{\hat{j}\times\hat{i}}_{-\hat{k}} - \underbrace{\hat{j}\times\hat{j}}_{0})$$

$$\hat{n} = -2\lambda\hat{k}$$

$\hat{n}$ is a unit vector so

$$|\hat{n}| = 2|\lambda| = 1$$

$$\lambda = \pm 1/2$$

$$\hat{n} = \hat{k} \text{ or } -\hat{k}$$

$$\boxed{|\vec{\omega}\cdot\hat{n}| = |(\hat{i}+2\hat{j}+3\hat{k})\cdot\hat{n}| = 3}$$

(11) Continues...

OR

Alitter

Let $\hat{n} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{u} \cdot \hat{n} = 0 \Rightarrow a + b = 0$ —— (1)

$\vec{v} \cdot \hat{n} = 0 \Rightarrow a - b = 0$ —— (2)

by eq^n (1) & (2) we get

$$\boxed{a = b = 0}$$

$$\hat{n} = c\hat{k}$$

$\hat{n}$ is a unit vector so

$$c = \pm 1$$

$$\hat{n} = \hat{k} \text{ or } -\hat{k}$$

$$|\vec{\omega} \cdot \hat{n}| = 3$$

(12) If the vector $6\hat{i} - 3\hat{j} - 6\hat{k}$ is decomposed into vectors parallel and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ then the vectors are :

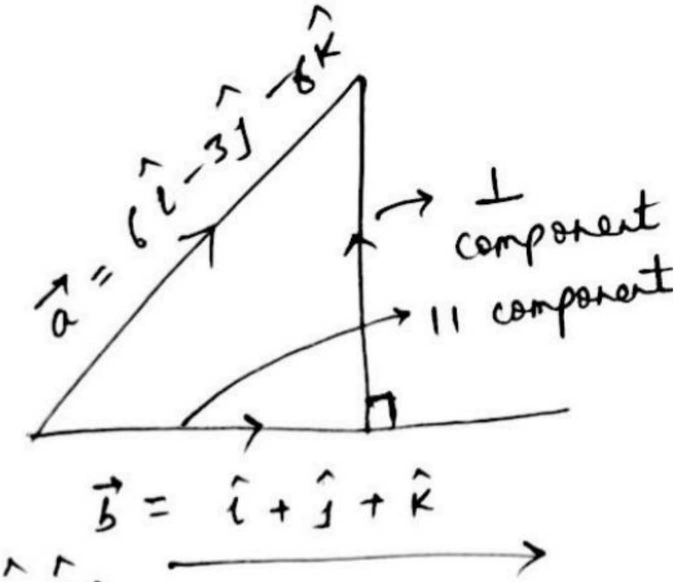
(A) $-(\hat{i} + \hat{j} + \hat{k})$ & $7\hat{i} - 2\hat{j} - 5\hat{k}$ ✓

(B) $-2(\hat{i} + \hat{j} + \hat{k})$ & $8\hat{i} - \hat{j} - 4\hat{k}$

(C) $+2(\hat{i} + \hat{j} + \hat{k})$ & $4\hat{i} - 5\hat{j} - 8\hat{k}$

(D) none

The vector component $\vec{a}$ along $\vec{b} = (\vec{a} \cdot \hat{b})\hat{b}$



Parallel component

$$= \left[(6\hat{i} - 3\hat{j} - 6\hat{k}) \cdot \left(\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}\right)\right]\left(\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}\right)$$

$$= \left(\frac{6 - 3 - 6}{3}\right)(\hat{i} + \hat{j} + \hat{k})$$

$$= \boxed{-(\hat{i} + \hat{j} + \hat{k})}$$

Perpendicular component $= \vec{a} - (\text{Parallel component})$

(By Triangle Rule)

Perpendicular component $= (6\hat{i} - 3\hat{j} - 6\hat{k}) + \hat{i} + \hat{j} + \hat{k}$

$$= \boxed{7\hat{i} - 2\hat{j} - 5\hat{k}}$$

13 Let $\vec{r}=\vec{a}+\lambda\vec{l}$ and $\vec{r}=\vec{b}+\mu\vec{m}$ be two lines in space where $\vec{a}=5\hat{i}+\hat{j}+2\hat{k}$, $\vec{b}=-\hat{i}+7\hat{j}+8\hat{k}$, $\vec{l}=-4\hat{i}+\hat{j}-\hat{k}$ and $\vec{m}=2\hat{i}-5\hat{j}-7\hat{k}$ then the p.v. of a point which lies on both of these lines, is

(A) $\hat{i}+2\hat{j}+\hat{k}$ (B) $2\hat{i}+\hat{j}+\hat{k}$

(C) $\hat{i}+\hat{j}+2\hat{k}$ (D) non existent as the lines are skew

---

for the Point of Intersection

$$\vec{a}+\lambda\vec{l}=\vec{b}+\mu\vec{m}$$

$$(5\hat{i}+\hat{j}+2\hat{k})+\lambda(-4\hat{i}+\hat{j}-\hat{k})=-\hat{i}+7\hat{j}+8\hat{k}+\mu(2\hat{i}-5\hat{j}-7\hat{k})$$

$$(5-4\lambda)\hat{i}+(1+\lambda)\hat{j}+(2-\lambda)\hat{k}=(-1+2\mu)\hat{i}+(7-5\mu)\hat{j}+(8-7\mu)\hat{k}$$

$$(6-4\lambda-2\mu)\hat{i}+(\lambda-6+5\mu)\hat{j}+(7\mu-\lambda-6)\hat{k}=\vec{0}$$

$$6-4\lambda-2\mu=0$$

$$\mu+2\lambda=3 \quad \text{---} \quad (1)$$

$$\lambda+5\mu=6 \quad \text{---} \quad (2)$$

by eq^n (1) & (2) $\boxed{\lambda=\mu=1}$,

for which $7\mu-\lambda-6$ is also vanishes.

so the Point of Intersection is

$$\vec{r}=\vec{a}+\vec{l} \quad (\text{for } \lambda=1)$$

$$=\boxed{\hat{i}+2\hat{j}+\hat{k}}$$

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(14) Let A(1, 2, 3), B(0, 0, 1), C(–1, 1, 1) are the vertices of a ΔABC.

(i) The equation of internal angle bisector through A to side BC is

(A) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} + 2\hat{j} + 3\hat{k})$ (B) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} + 4\hat{j} + 3\hat{k})$

(C) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} + 3\hat{j} + 2\hat{k})$ (D) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} + 3\hat{j} + 4\hat{k})$

(ii) The equation of median through C to side AB is

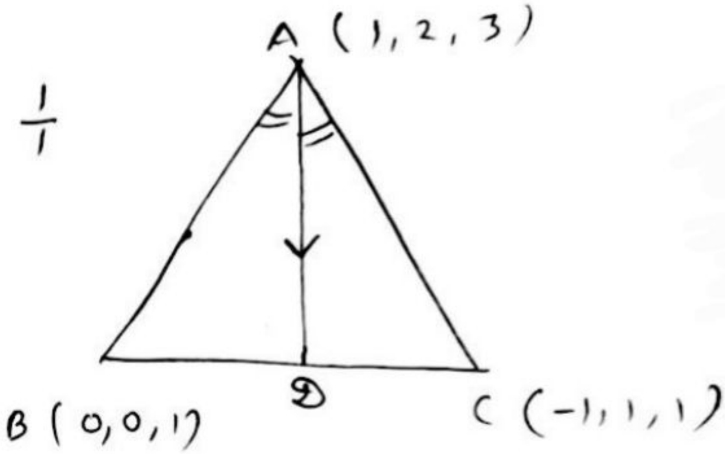
(A) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p(3\hat{i} - 2\hat{k})$ (B) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p(3\hat{i} + 2\hat{k})$

(C) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p(-3\hat{i} + 2\hat{k})$ (D) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p(3\hat{i} + 2\hat{j})$

(iii) The area (ΔABC) is equal to

(A) $\frac{9}{2}$ (B) $\frac{\sqrt{17}}{2}$ (C) $\frac{17}{2}$ (D) $\frac{7}{2}$

(i) $\frac{BD}{DC} = \frac{AB}{AC} = \frac{3}{3} = \frac{1}{1}$



so △ABC is Isosceles △

and D is mid point of BC

so $D \equiv \left(-\frac{1}{2}, \frac{1}{2}, 1\right)$

vector $\overrightarrow{AD} = \left(-\frac{1}{2} - 1\right)\hat{i} + \left(\frac{1}{2} - 2\right)\hat{j} + (1 - 3)\hat{k}$

$\overrightarrow{AD} = -\frac{3}{2}\hat{i} - \frac{3}{2}\hat{j} - 2\hat{k} = -\frac{(3\hat{i} + 3\hat{j} + 4\hat{k})}{2}$

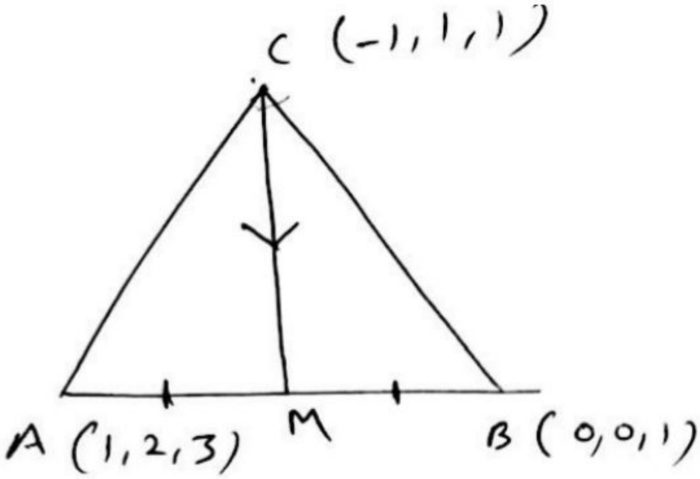
eqⁿ of line is given by $\vec{r} = \vec{a} + \lambda\vec{b}$, $\lambda \in R$

where $\vec{a}$ is a position vector on the line and $\vec{b}$ is parallel vector to the line

so eqⁿ of line AD =)

$$\boxed{\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} + 3\hat{j} + 4\hat{k})}$$

(14) Continues...



ii) $M \equiv \left(\frac{1}{2}, 1, 2\right)$

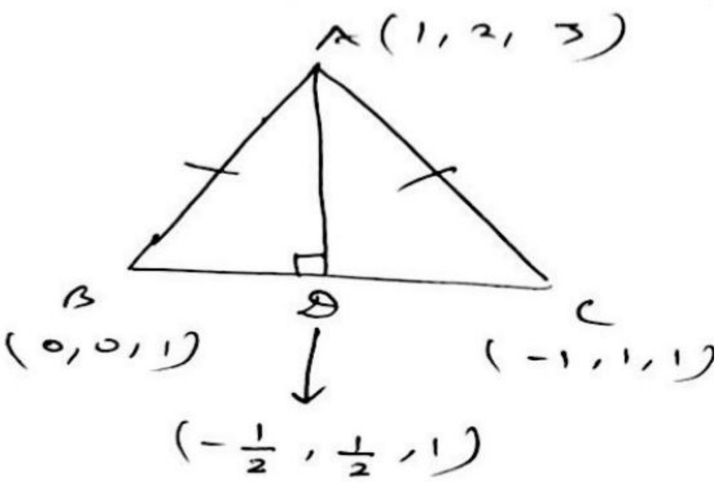
$\vec{CM} = \left(\frac{1}{2}+1\right)\hat{i} + (1-1)\hat{j} + (2-1)\hat{k}$

$\vec{CM} = \frac{3}{2}\hat{i} + \hat{k}$

$\vec{CM} = \frac{3\hat{i} + 2\hat{k}}{2}$

eqn of median CM $\Rightarrow$

$$\boxed{\vec{r} = -\hat{i} + \hat{j} + \hat{k} + \rho\,(3\hat{i} + 2\hat{k})}$$



iii) $BC = \sqrt{2}$

$AD = \sqrt{\left(1+\frac{1}{2}\right)^2 + \left(2-\frac{1}{2}\right)^2 + (3-1)^2}$

$AD = \sqrt{\frac{9}{4} + \frac{9}{4} + 4}$

$AD = \sqrt{\frac{17}{2}}$

Area of $\Delta ABC = \frac{1}{2} \times \sqrt{\frac{17}{2}} \times \sqrt{2}$

$= \frac{\sqrt{17}}{2}$

OR

$\text{Area} = \frac{1}{2}\left|\vec{AB} \times \vec{AC}\right| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & -2 \\ -2 & -1 & -2 \end{vmatrix}$

$= \frac{1}{2}\left|(2\hat{i} + 2\hat{j} - 3\hat{k})\right|$

$\text{Area} = \frac{\sqrt{17}}{2}$

15 If $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=7$, then the angle between $\vec{a}$ & $\vec{b}$ is :

(A) $\pi/6$ (B) $2\pi/3$ (C) $5\pi/3$ (D) $\pi/3$

$$\vec{a}+\vec{b}+\vec{c}=0$$

$$\vec{a}+\vec{b}=-\vec{c}$$

$$|\vec{a}+\vec{b}|^2=|-\vec{c}|^2$$

$$|\vec{a}|^2+|\vec{b}|^2+2\,\vec{a}\cdot\vec{b}=|\vec{c}|^2$$

$$9+25+2(3\cdot5\cos\theta)=49$$

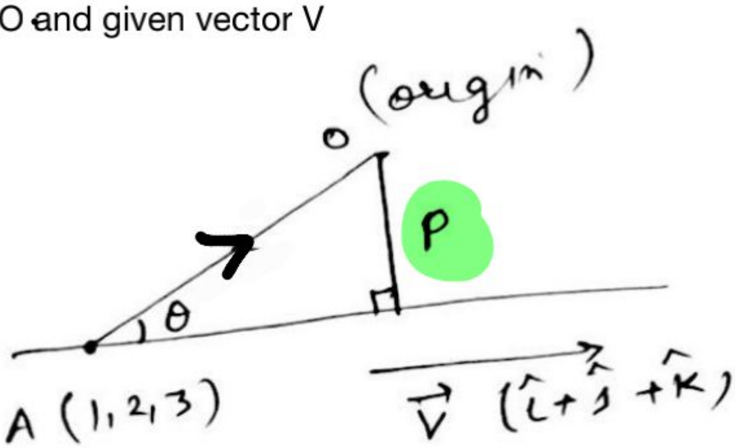
$$\cos\theta=\frac{1}{2}$$

so angle b/w $\vec{a}$ & $\vec{b}$ = $\boxed{\theta=\frac{\pi}{3}}$

16 A line passes through the point $A(\hat{i}+2\hat{j}+3\hat{k})$ and is parallel to the vector $\vec{V}(\hat{i}+\hat{j}+\hat{k})$. The shortest distance from the origin, of the line is -

(A) $\sqrt{2}$ (B) $\sqrt{4}$ (C) $\sqrt{5}$ (D) $\sqrt{6}$

θ is the angle between vector AO and given vector V



$$\cos\theta = \frac{-(1+2+3)}{\sqrt{14}\cdot\sqrt{3}}$$

$$\cos\theta = -\frac{\sqrt{6}}{\sqrt{7}}$$

$$\sin\theta = \frac{1}{\sqrt{7}}$$

$$P = |OA| \sin\theta$$

$$P = \sqrt{14}\cdot\frac{1}{\sqrt{7}} = \sqrt{2}$$

OR

$$\text{shortest distant} = \left|\frac{(a_2-a_1)\times\vec{b}}{|b|}\right|$$

$$= \left|\frac{(\hat{i}+2\hat{j}+3\hat{k})\times(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}\right|$$

$$= \frac{|-\hat{i}+2\hat{j}-\hat{k}|}{\sqrt{3}}$$

$$\boxed{\text{shortest distance} = \sqrt{2}}$$

17 Let $\vec{a}, \vec{b}, \vec{c}$ be vectors of length 3, 4, 5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ to $\vec{c}+\vec{a}$ & $\vec{c}$ to $\vec{a}+\vec{b}$. Then $|\vec{a}+\vec{b}+\vec{c}|$ is :

(A) $2\sqrt{5}$ (B) $2\sqrt{2}$ (C) $10\sqrt{5}$ (D) $5\sqrt{2}$

$$\vec{a}\cdot(\vec{b}+\vec{c}) = 0$$

$$\vec{b}\cdot(\vec{c}+\vec{a}) = 0$$

$$\vec{c}\cdot(\vec{a}+\vec{b}) = 0$$

By adding all above eq^n,

we get $2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

Now $|\vec{a}+\vec{b}+\vec{c}|^2 = |a|^2 + |b|^2 + |c|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$$= 9 + 16 + 25 + 0$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 50$$

$$\boxed{|\vec{a}+\vec{b}+\vec{c}| = 5\sqrt{2}}$$

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18 If a vector $\vec{a}$ of magnitude 50 is collinear with vector $\vec{b} = 6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k}$ and makes an acute angle with positive z-axis then :

(A) $\vec{a} = 4\vec{b}$ (B) $\vec{a} = -4\vec{b}$ (C) $\vec{b} = 4\vec{a}$ (D) none

---

$|\vec{a}| = 50$, Let $\vec{a} = \lambda\vec{b}$

$\vec{a} = \lambda\left(6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k}\right)$

$|\vec{a}| = |\lambda|\sqrt{36+64+\frac{225}{4}} = |\lambda| \cdot \frac{25}{2}$

$50 = \frac{25}{2}|\lambda|$

$|\lambda| = 4$

$\lambda = \pm 4$

and $\vec{a} \cdot \hat{k} > 0$

$-\frac{15\lambda}{2} > 0$

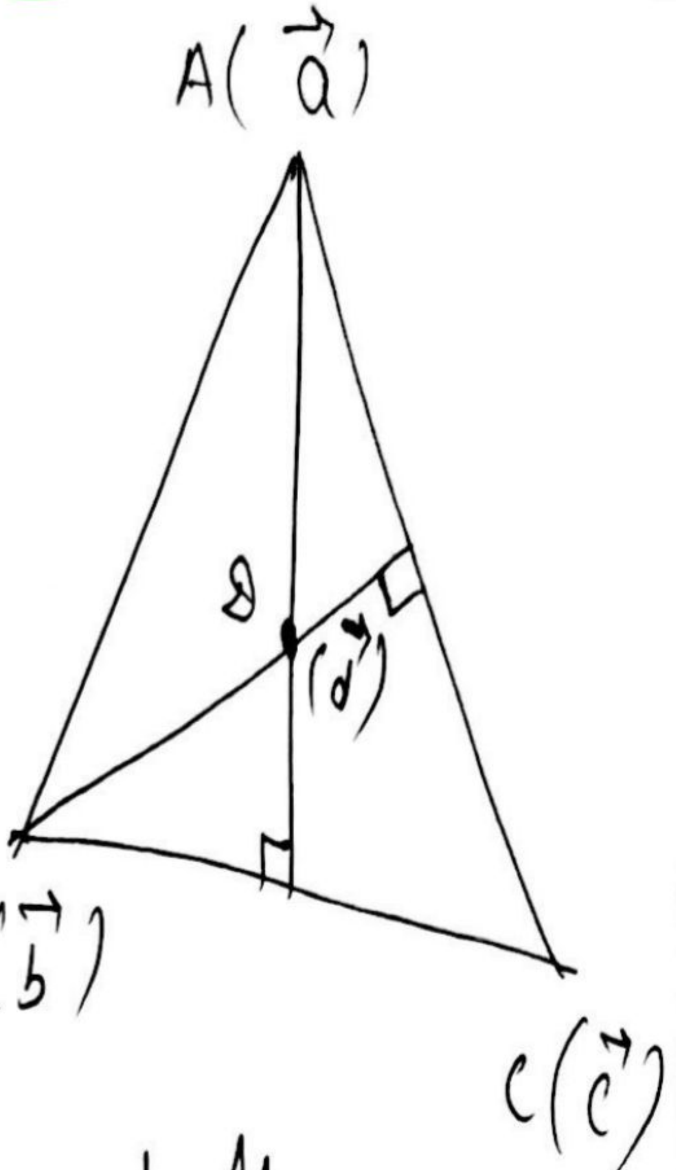
$\boxed{\lambda < 0}$

so $\boxed{\lambda = -4}$

$\boxed{\vec{a} = -4\vec{b}}$

19 A, B, C & D are four points in a plane with pv's $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ respectively such that $(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c}) = (\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a}) = 0$. Then for the triangle ABC, D is its

(A) incentre (B) circumcentre (C) orthocentre (D) centroid



$(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c}) = 0$

so $\vec{AD}$ and $\vec{BC}$ are mutually perpendicular

and $(\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a}) = 0$

so $\vec{BD}$ and $\vec{AC}$ is also mutually perpendicular

so D is orthocentre of $\triangle ABC$

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⑳

$\vec{a}$ and $\vec{b}$ are unit vectors inclined to each other at an angle α, $\alpha \in (0, \pi)$ and $\left|\vec{a}+\vec{b}\right| < 1$. Then $\alpha \in$

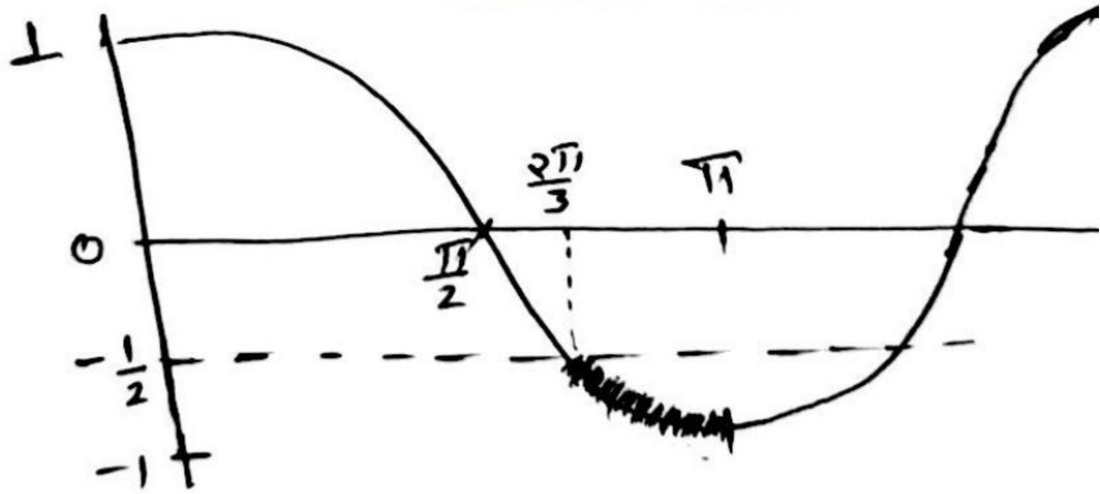
(A) $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$ (B) $\left(\frac{2\pi}{3}, \pi\right)$ (C) $\left(0, \frac{\pi}{3}\right)$ (D) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$$|\vec{a}+\vec{b}|^2 < 1$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}\cdot\vec{b} < 1$$

$$1 + 1 + 2(1\cdot 1\cdot \cos\alpha) < 1$$

$$\cos\alpha < -\frac{1}{2}$$



so $\boxed{\alpha \in \left(\frac{2\pi}{3}, \pi\right)}$

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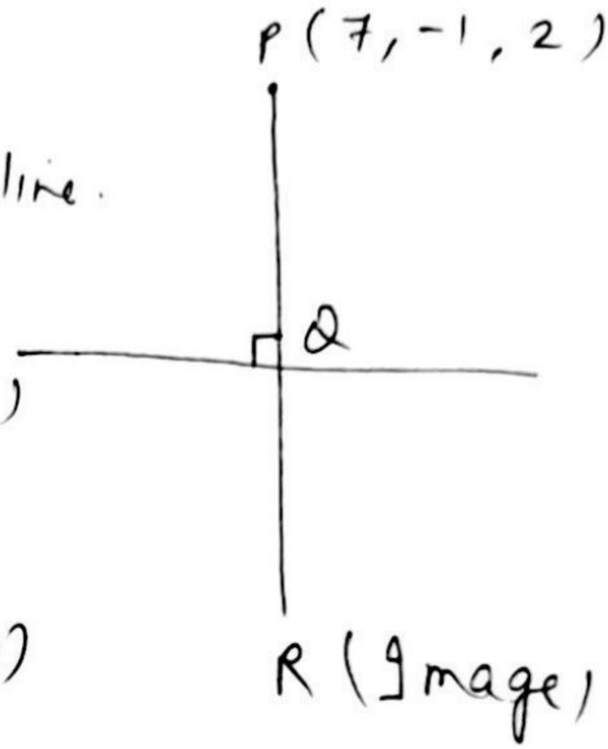
21 Image of the point P with position vector $7\hat{i} - \hat{j} + 2\hat{k}$ in the line whose vector equation is $\vec{r} = 9\hat{i}+5\hat{j}+5\hat{k}+\lambda(\hat{i}+3\hat{j}+5\hat{k})$ has the position vector

(A) $(-9, 5, 2)$ (B) $(9, 5, -2)$ (C) $(9, -5, -2)$ (D) none

Let Q is foot of Perpendicular of P on the line.



$\vec{r} = 9\hat{i}+5\hat{j}+5\hat{k}+\lambda(\hat{i}+3\hat{j}+5\hat{k})$

Let P.V of Q

is $\equiv (9+\lambda, 5+3\lambda, +5+5\lambda)$

$\vec{PQ}$ and line is mutually Perpendicular

so $\left((9+\lambda-7)\hat{i}+(5+3\lambda+1)\hat{j}+(5+5\lambda-2)\hat{k}\right)\cdot(\hat{i}+3\hat{j}+5\hat{k}) = 0$

$(2+\lambda)+3(6+3\lambda)+5(3+5\lambda)=0$

$\boxed{\lambda=-1}$

so $Q \equiv (8, 2, 0)$

Now $\boxed{R \equiv (9, 5, -2)}$ By mid-Point theorem

22

Let $\hat{a}$, $\hat{b}$, $\hat{c}$ are three unit vectors such that $\hat{a} + \hat{b} + \hat{c}$ is also a unit vector. If pairwise angles between $\hat{a}$, $\hat{b}$, $\hat{c}$ are θ_1, θ_2 and θ_3 respectively then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3$ equals

(A) 3 (B) – 3 (C) 1 (D) – 1

$$|\hat{a} + \hat{b} + \hat{c}|^2 = 1$$

$$|\hat{a}|^2 + |\hat{b}|^2 + |\hat{c}|^2 + 2(\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot\hat{a}) = 1$$

$$1 + 1 + 1 + 2(\cos\theta_1 + \cos\theta_2 + \cos\theta_3) = 1$$

$$\boxed{\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = -1}$$

23

Cosine of an angle between the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ if $|\vec{a}|=2$, $|\vec{b}|=1$ and $\vec{a}\wedge\vec{b}=60°$ is

(A) $\sqrt{3/7}$ (B) $9/\sqrt{21}$ (C) $3/\sqrt{7}$ (D) none

Let angle is θ

$$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}+\vec{b}|\,|\vec{a}-\vec{b}|\cos\theta$$

$$|\vec{a}|^2-|\vec{b}|^2 = |\vec{a}+\vec{b}|\,|\vec{a}-\vec{b}|\cos\theta$$

$$|\vec{a}+\vec{b}| = \sqrt{|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}}$$

given $\vec{a}\wedge\vec{b}=60°$, $|\vec{a}|=2$, $|\vec{b}|=1$

$$= \sqrt{4+1+2(2\cdot1\cdot\tfrac{1}{2})}$$

$$|\vec{a}+\vec{b}| = \sqrt{7}$$

$$|\vec{a}-\vec{b}| = \sqrt{|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}}$$

$$= \sqrt{4+1-2(2\cdot1\cdot\tfrac{1}{2})}$$

$$|\vec{a}-\vec{b}| = \sqrt{3}$$

$$\cos\theta = \frac{|a|^2-|b|^2}{|a+b|\,|a-b|}$$

$$= \frac{4-1}{\sqrt{7}\cdot\sqrt{3}}$$

$$\boxed{\cos\theta = \sqrt{\frac{3}{7}}}$$

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(24) Given three vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$ each two of which are non collinear. Further if $(\vec{a}+\vec{b})$ is collinear with $\vec{c}$, $(\vec{b}+\vec{c})$ is collinear with $\vec{a}$ & $|\vec{a}| = |\vec{b}| = |\vec{c}| = \sqrt{2}$. Then the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$:

(A) is 3 (B) is -3 (C) is 0 (D) cannot be evaluated

Let $\vec{a}+\vec{b} = \lambda \vec{c}$

and $\vec{b}+\vec{c} = \mu \vec{a}$

After substracting the eq^n

$$\vec{a}-\vec{c} = \lambda\vec{c} - \mu\vec{a}$$

$$(1+\mu)\vec{a} = (1+\lambda)\vec{c}$$

$\vec{a}$ and $\vec{c}$ are non-collinear so

$$1+\mu = 0 \quad \text{and} \quad 1+\lambda = 0$$

$$\mu = \lambda = -1$$

$$\vec{a}+\vec{b} = -\vec{c}$$

$$\vec{a}+\vec{b}+\vec{c} = \vec{0}$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 0$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

$$2 + 2 + 2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

$$\boxed{\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -3}$$

25

The vector equations of two lines L_1 and L_2 are respectively

$\vec{r} = 17\hat{i} - 9\hat{j} + 9\hat{k} + \lambda(3\hat{i} + \hat{j} + 5\hat{k})$ and $\vec{r} = 15\hat{i} - 8\hat{j} - \hat{k} + \mu(4\hat{i} + 3\hat{j})$

I L_1 and L_2 are skew lines

II $(11, -11, -1)$ is the point of intersection of L_1 and L_2

III $(-11, 11, 1)$ is the point of intersection of L_1 and L_2

IV $\cos^{-1}(3/\sqrt{35})$ is the acute angle between L_1 and L_2

then, which of the following is true?

(A) II and IV (B) I and IV (C) IV only (D) III and IV

$\Rightarrow$ for point of intersection

$$17\hat{i} - 9\hat{j} + 9\hat{k} + \lambda(3\hat{i} + \hat{j} + 5\hat{k}) = 15\hat{i} - 8\hat{j} - \hat{k} + \mu(4\hat{i} + 3\hat{j})$$

$$(2 + 3\lambda - 4\mu)\hat{i} + (-1 + \lambda - 3\mu)\hat{j} + (10 + 5\lambda)\hat{k} = \vec{0}$$

$$10 + 5\lambda = 0$$

$$\boxed{\lambda = -2}$$

$$2 + 3\lambda - 4\mu = 0$$

$$\boxed{\mu = -1}$$

$-1 + \lambda - 3\mu = 0$ for $\lambda = -2$ and $\mu = -1$

$$\vec{r} = 15\hat{i} - 8\hat{j} - \hat{k} + \mu(4\hat{i} + 3\hat{j})$$

for $\mu = -1$

Point of Intersection $\equiv (11, -11, -1)$

clearly lines are not skew because they are intersecting.

Angle between two intersecting lines is angle between their parallel vectors.

angle b/w line θ (let)

$$(3\hat{i} + \hat{j} + 5\hat{k}) \cdot (4\hat{i} + 3\hat{j}) = \sqrt{35} \cdot 5 \cos\theta$$

$$12 + 3 = 5\sqrt{35}\cos\theta$$

$$\cos\theta = \frac{3}{\sqrt{35}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{3}{\sqrt{35}}\right)}$$

26 Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1$, $|\vec{v}| = 2$, $|\vec{w}| = 3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v}$, $\vec{w}$ are perpendicular to each other then $|\vec{u} - \vec{v} + \vec{w}|$ equals

(A) 2 (B) $\sqrt{7}$ (C) $\sqrt{14}$ (D) 14

Soln Projection of $\vec{v}$ along $\vec{u}$ $= \dfrac{\vec{v}.\vec{u}}{|\vec{u}|}$ —— (1)

Projection of $\vec{w}$ along $\vec{u}$ $= \dfrac{\vec{w}.\vec{u}}{|\vec{u}|}$ —— (2)

$\because \vec{v} \perp \vec{w} \Rightarrow \vec{v}.\vec{w} = 0$ —— (3)

(1) = (2) (given in question)

$\Rightarrow \dfrac{\vec{v}.\vec{u}}{|\vec{u}|} = \dfrac{\vec{w}.\vec{u}}{|\vec{u}|} \Rightarrow \vec{v}.\vec{u} = \vec{w}.\vec{u}$ — (4)

Now $|\vec{u} - \vec{v} + \vec{w}|^2 = \vec{u}^2 + \vec{v}^2 + \vec{w}^2 \cancel{- 2\vec{u}.\vec{v}} - 2\vec{v}.\vec{w} \cancel{+ 2\vec{u}.\vec{w}}$ [from Eqn (3) & (4)]

$= 1 + 4 + 9 - 0$

$= 14$

$$\boxed{|\vec{u} - \vec{v} + \vec{w}| = \sqrt{14}}$$

27

If $\vec{a}$ and $\vec{b}$ are non zero, non collinear, and the linear combination

$(2x-y)\vec{a}+4\vec{b}=5\vec{a}+(x-2y)\vec{b}$ holds for real x and y then x + y has the value equal to

(A) – 3 (B) 1 (C) 17 (D) 3

Soln $(2x-y)\vec{a}+4\vec{b}=5\vec{a}+(x-2y)\vec{b}$

$(2x-y-5)\vec{a}=(x-2y-4)\vec{b}$ —— (1)

$\because$ $\vec{a}$ and $\vec{b}$ are non zero, non collinear

$\Rightarrow$ eqn (1) is true only when

$2x-y-5=0$ & $x-2y-4=0$

by solving $x=2$, $y=-1$

$\Rightarrow$ $\boxed{x+y=1}$

(28)

If $\vec{p}$ & $\vec{s}$ are not perpendicular to each other and $\vec{r} \times \vec{p} = \vec{q} \times \vec{p}$ & $\vec{r} \cdot \vec{s} = 0$, then $\vec{r}$ =

(A) $\vec{p} \cdot \vec{s}$ (B) $\vec{q} + \left(\frac{\vec{q} \cdot \vec{p}}{\vec{p} \cdot \vec{s}}\right)\vec{p}$ (C) $\vec{q} - \left(\frac{\vec{q} \cdot \vec{s}}{\vec{p} \cdot \vec{s}}\right)\vec{p}$ (D) $\vec{q} + \mu\vec{p}$ for all scalars μ

Solⁿ

$\vec{r} \times \vec{p} = \vec{q} \times \vec{p} \Rightarrow (\vec{r} - \vec{q}) \times \vec{p} = 0$

$\Rightarrow$ Vector $\vec{r} - \vec{q}$ and $\vec{p}$ are collinear

$\Rightarrow \vec{r} - \vec{q} = \lambda \vec{p}$

$\Rightarrow \vec{r} = \vec{q} + \lambda \vec{p}$ —— ①

Now taking dot product with vector $\vec{s}$

$\Rightarrow \vec{r} \cdot \vec{s} = \vec{q} \cdot \vec{s} + \lambda \vec{p} \cdot \vec{s}$

$0 = \vec{q} \cdot \vec{s} + \lambda \vec{p} \cdot \vec{s}$ $[\because \vec{r} \cdot \vec{s} = 0$ given$]$

$\Rightarrow \lambda = \dfrac{-(\vec{q} \cdot \vec{s})}{(\vec{p} \cdot \vec{s})}$ Put in Eqⁿ ①

$$\boxed{\vec{r} = \vec{q} - \left(\frac{\vec{q} \cdot \vec{s}}{\vec{p} \cdot \vec{s}}\right)\vec{p}}$$

29

If $\vec{u}$ and $\vec{v}$ are two vectors such that $|\vec{u}|=3$; $|\vec{v}|=2$ and $|\vec{u}\times\vec{v}|=6$ then the correct statement is

(A) $\vec{u} \wedge \vec{v} \in (0, 90°)$ (B) $\vec{u} \wedge \vec{v} \in (90°, 180°)$ (C) $\vec{u} \wedge \vec{v} = 90°$ (D) $(\vec{u}\times\vec{v})\times\vec{u} = 6\vec{v}$

Solⁿ $\vec{u}\times\vec{v} = |\vec{u}|\,|\vec{v}|\,(\sin\theta)\,\hat{n}$ where $\theta = \vec{u} \wedge \vec{v}$

$\Rightarrow |\vec{u}\times\vec{v}| = 3 \cdot 2 \sin\theta$

$6 = 6\sin\theta \Rightarrow \sin\theta = 1$

$\Rightarrow \theta = 90° \Rightarrow \boxed{\vec{u} \wedge \vec{v} = 90°}$

30

Given a parallelogram OACB. The lengths of the vectors $\vec{OA}$, $\vec{OB}$ & $\vec{AB}$ are a, b & c respectively. The scalar product of the vectors $\vec{OC}$ & $\vec{OB}$ is :

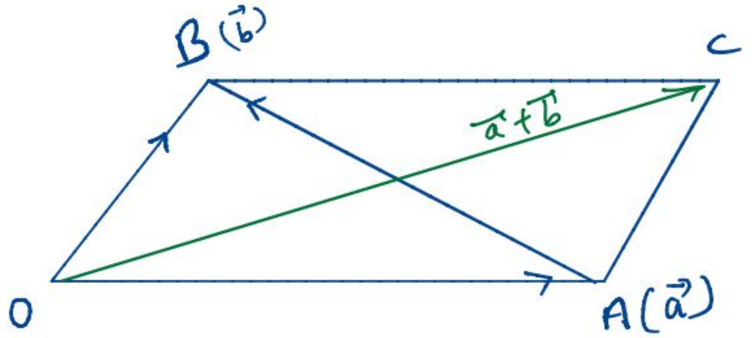
(A) $\frac{a^2 - 3b^2 + c^2}{2}$ (B) $\frac{3a^2 + b^2 - c^2}{2}$ (C) $\frac{3a^2 - b^2 + c^2}{2}$ (D) $\frac{a^2 + 3b^2 - c^2}{2}$

Soln

$\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$

$\vec{OC} = \vec{OA} + \vec{AC}$

$\vec{OC} = \vec{a} + \vec{b}$ — (1)



$\Rightarrow \vec{OC} \cdot \vec{OB} = (\vec{a} + \vec{b}) \cdot \vec{b} = \vec{a} \cdot \vec{b} + b^2$ — (2)

in Δ OAB

$\vec{OA} + \vec{AB} = \vec{OB}$

$\vec{a} + \vec{AB} = \vec{b}$ $[\because |\vec{AB}| = c]$

$\Rightarrow \vec{AB} = \vec{b} - \vec{a}$

$\Rightarrow (\vec{AB})^2 = b^2 + a^2 - 2\vec{a}\cdot\vec{b}$

$c^2 = b^2 + a^2 - 2\vec{a}\cdot\vec{b}$

$\Rightarrow \vec{a}\cdot\vec{b} = \frac{b^2 + a^2 - c^2}{2}$ Put in Eqn (2)

$$\Rightarrow \boxed{\vec{OC} \cdot \vec{OB} = \left(\frac{b^2 + a^2 - c^2}{2}\right) + b^2 = \frac{a^2 + 3b^2 - c^2}{2}}$$

31

Vectors $\vec{a}$ & $\vec{b}$ make an angle $\theta = \frac{2\pi}{3}$. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ then $\left\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\right\}^2 =$

(A) 225 (B) 250 (C) 275 ($\cancel{D}$) 300

Sol^n $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|(\sin\theta)\,\hat{n} = 1.2.\sin\left(\frac{2\pi}{3}\right)\hat{n}$

$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{3}$ ——— (1)

Now $\left\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\right\}^2 = \left\{3\vec{a} \times \vec{a} - \vec{a} \times \vec{b} + 9\,\vec{b} \times \vec{a} - 3\,\vec{b} \times \vec{b}\right\}^2$

$= \left\{0 - \vec{a} \times \vec{b} - 9\,\vec{a} \times \vec{b} - 0\right\}^2$

$= 100\,(\vec{a} \times \vec{b})^2 = 100\,(\sqrt{3})^2$

$= \underline{\underline{300}}$

32

Given the vectors

$$\vec{u} = 2\hat{i} - \hat{j} - \hat{k}$$
$$\vec{v} = \hat{i} - \hat{j} + 2\hat{k}$$
$$\vec{w} = \hat{i} - \hat{k}$$

If the volume of the parallelopiped having $-c\vec{u}$, $\vec{v}$ and $c\vec{w}$ as concurrent edges, is 8 then 'c' can be equal to

(A) ± 2 (B) 4 (C) 8 (D) can not be determined

Soln

$$\text{Volume} = [-c\vec{u}\ \vec{v}\ c\vec{w}] = 8$$

$$= -c^2[\vec{u}\ \vec{v}\ \vec{w}] = 8$$

$$\Rightarrow -c^2\begin{vmatrix} 2 & -1 & -1 \\ 1 & -1 & 2 \\ 1 & 0 & -1 \end{vmatrix} = 8$$

$$(-c^2)\left[2(1-0) + 1(-1-2) - 1(0+1)\right] = 8$$

$$2c^2 = 8 \Rightarrow \boxed{c = \pm 2}$$

33

Given $\bar{a} = x\hat{i} + y\hat{j} + 2\hat{k}$, $\bar{b} = \hat{i} - \hat{j} + \hat{k}$, $\bar{c} = \hat{i} + 2\hat{j}$; $(\bar{a} \,\hat{}\, \bar{b}) = \pi/2$, $\bar{a} \cdot \bar{c} = 4$ then

(A) $[\bar{a}\ \bar{b}\ \bar{c}]^2 = |\bar{a}|$ (B) $[\bar{a}\ \bar{b}\ \bar{c}] = |\bar{a}|$ (C) $[\bar{a}\ \bar{b}\ \bar{c}] = 0$ (D) $[\bar{a}\ \bar{b}\ \bar{c}] = |\bar{a}|^2$

Soln $\vec{a} \wedge \vec{b} = \pi/2 \Rightarrow \vec{a} \cdot \vec{b} = 0$

$\Rightarrow (x\hat{i} + y\hat{j} + 2\hat{k}) \cdot (\hat{i} - \hat{j} + \hat{k}) = 0$

$\Rightarrow x - y + 2 = 0$ —— ①

Now $\vec{a} \cdot \vec{c} = 4$

$\Rightarrow (x\hat{i} + y\hat{j} + 2\hat{k}) \cdot (\hat{i} + 2\hat{j}) = 4$

$x + 2y = 4$ —— ②

Solving ① & ②

$\boxed{x = 0 \ \& \ y = 2}$

$\Rightarrow \vec{a} = 2\hat{j} + 2\hat{k}$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 0 & 2 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 8 \text{ —— ③}$$

$|\vec{a}| = 2\sqrt{2} \Rightarrow |\bar{a}|^2 = 8$ —— ④

$\Rightarrow \boxed{[\vec{a}\ \vec{b}\ \vec{c}] = |\bar{a}|^2}$ from ③ + ④

34

For three vectors $\vec{u}$, $\vec{v}$, $\vec{w}$ which of the following expressions is not equal to any of the remaining three?

(A) $\vec{u} . (\vec{v} \times \vec{w})$ (B) $(\vec{v} \times \vec{w}) . \vec{u}$ (C) $\vec{v} . (\vec{u} \times \vec{w})$ (D) $(\vec{u} \times \vec{v}) . \vec{w}$

Sol^n

(A) $\vec{u}.(\vec{v}\times\vec{w}) = [\vec{u}\ \vec{v}\ \vec{w}]$

(B) $(\vec{v}\times\vec{w}).\vec{u} = \vec{u}.(\vec{v}\times\vec{w}) = [\vec{u}\ \vec{v}\ \vec{w}]$

(C) $\vec{v}.(\vec{u}\times\vec{w}) = -\vec{v}.(\vec{w}\times\vec{u}) = -[\vec{v}\ \vec{w}\ \vec{u}]$

$= -[\vec{u}\ \vec{v}\ \vec{w}]$

(D) $(\vec{u}\times\vec{v}).\vec{w} = [\vec{u}\ \vec{v}\ \vec{w}]$

clearly (C) Ans

35

Let $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$ & $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. If the vectors , $\hat{i} - 2\hat{j} + \hat{k}$, $3\hat{i} + 2\hat{j} - \hat{k}$ & $\vec{c}$ are coplanar then $\frac{\alpha}{\beta}$ is

(A) 1 (B) 2 (C) 3 ~~(D)~~ – 3

Soln $\vec{c} = \alpha\vec{a} + \beta\vec{b} = \alpha(\hat{i}+\hat{j}) + \beta(\hat{j}+\hat{k})$

$\vec{c} = \alpha\hat{i} + (\alpha+\beta)\hat{j} + \beta\hat{k}$

Let $\vec{u} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{v} = 3\hat{i} + 2\hat{j} - \hat{k}$

$\because$ $\vec{c}$, $\vec{u}$ & $\vec{v}$ are coplanar

$\Rightarrow [\vec{c}\ \vec{u}\ \vec{v}] = 0$

$$\Rightarrow \begin{vmatrix} \alpha & \alpha+\beta & \beta \\ 1 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = 0$$

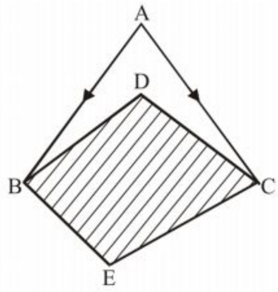
$\alpha(2-2) - (\alpha+\beta)(-1-3) + \beta(2+6) = 0$

$4\alpha + 12\beta = 0$

$\Rightarrow \boxed{\frac{\alpha}{\beta} = -3}$

36

Let $\overrightarrow{AB} = 3\hat{i} - \hat{j}$, $\overrightarrow{AC} = 2\hat{i} + 3\hat{j}$ and $\overrightarrow{DE} = 4\hat{i} - 2\hat{j}$. The area of the shaded region in the adjacent figure, is-



(A) 5

(B) 6

(C) 7

(D) 8

Soln

in ΔABC

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = (2\hat{i} + 3\hat{j}) - (3\hat{i} - \hat{j})$$

$$\overrightarrow{BC} = -\hat{i} + 4\hat{j}$$

Now $$\overrightarrow{BC} \times \overrightarrow{DE} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & 0 \\ 4 & -2 & 0 \end{vmatrix} = 14\hat{k}$$

$$\Rightarrow \text{Shaded Area} = \frac{1}{2} |\overrightarrow{BC} \times \overrightarrow{DE}| = 7$$

37

If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-2\hat{j}+\hat{k}$ then the vector $\vec{c}$ such that $\vec{a}\cdot\vec{c} = 2$ & $\vec{a}\times\vec{c} = \vec{b}$ is :-

let $\vec{c} = p\hat{i}+q\hat{j}+r\hat{k}$ & $\because$ $\vec{a}\times\vec{c} = \vec{b}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ p & q & r \end{vmatrix} = \vec{b} \Rightarrow \hat{i}(r-q) - \hat{j}(r-p) + \hat{k}(q-p) = \vec{b}$$

$\therefore$ $r-q = 1$ $\quad r-p = 2$

$q-p = 1$ $\quad r = 2+p$

$q = 1+p$

$\because$ $\vec{a}\cdot\vec{c} = 2$

$(\hat{i}+\hat{j}+\hat{k})\cdot(p\hat{i}+q\hat{j}+r\hat{k}) = 2$

$p+q+r = 2$

$p+1+p+2+p = 2 \Rightarrow p = -1/3$

$q = 2/3$

$r = 5/3$

$\therefore$ $\vec{c} = -\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{5}{3}\hat{k}$

$$\boxed{\vec{c} = \frac{1}{3}\left[-\hat{i}+2\hat{j}+5\hat{k}\right]}$$

(Ans B)

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38

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors & $|\vec{c}| = \sqrt{3}$ then find α & β :-

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

$$(3\beta + 4\alpha + 4) - (3 + 4\beta + 4\alpha) = 0$$

$$3\beta \cancel{+4\alpha} - 4 - 3 - 4\beta \cancel{-4\alpha} = 0 \quad \Rightarrow \boxed{\beta = 1}$$

$$\vec{c} = \sqrt{3}$$

$$\sqrt{1 + \alpha^2 + \beta^2} = \sqrt{3}$$

$$1 + \alpha^2 + 1 = 3$$

$$\alpha^2 = 1 \quad \Rightarrow \boxed{\alpha = \pm 1}$$

(AmD)

39

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Subject

MATRIX MATCH TYPE

if $A(0,1,0)$, $B(0,0,0)$, $C(1,0,1)$ are the vertices of a ΔABC. Match the entries of Column - I with Column - II

Column - I	Column - II
(A) Orthocentre of ΔABC	(P) $\sqrt{2}/2$
(B) Circumcentre of ΔABC	(Q) $\sqrt{3}/2$
(C) Area of ΔABC	(R) $\sqrt{3}/3$
(D) Distance B/W orthocentre & centroid	(S) $\sqrt{3}/6$
(E) Distance B/W orthocentre & circumcentre	(T) $(0,0,0)$
(F) Distance B/W circumcentre & centroid	(U) $(1/2, 1/2, 1/2)$
(G) Incentre of ΔABC	(V) $(1/3, 1/3, 1/3)$
(H) Centroid of ΔABC	(W) $\left[\frac{1}{\sqrt{1}+\sqrt{2}+\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{1}+\sqrt{2}+\sqrt{3}}, \frac{1}{\sqrt{1}+\sqrt{2}+\sqrt{3}}\right]$

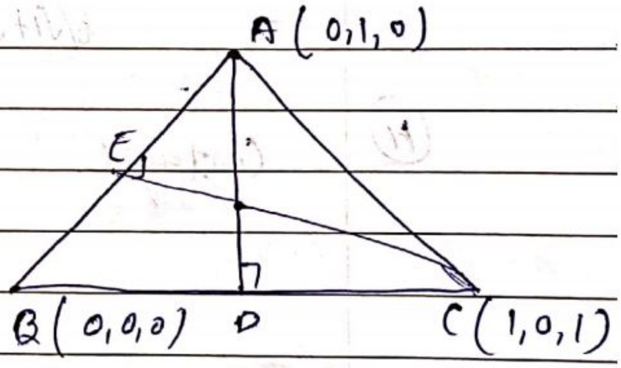
Solⁿ

$\overrightarrow{BA} = \hat{i}$

$\overrightarrow{BC} = \hat{i} + \hat{k}$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = 0$

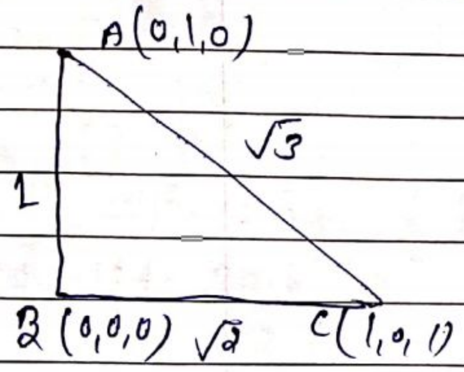
$\overrightarrow{BA} \perp \overrightarrow{BC}$



(A) Orthocentre $(0,0,0)$

(B) Circumcentre $\left[\frac{0+1}{2}, \frac{1+0}{2}, \frac{0+1}{2}\right]$

$\therefore \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right]$



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(39) Continues...

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(C) Area $= \frac{1}{2} \times \overrightarrow{AB} \times BC$ $\quad |\overrightarrow{AB}| = 1$

$|\overrightarrow{BC}| = \sqrt{2}$

$\frac{1}{2} \times 1 \times \sqrt{2}$

$\boxed{\frac{\sqrt{2}}{2}}$

(D) centroid $= \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$\therefore$ Distance B/w Centroid & orthocentre $= \boxed{\frac{\sqrt{3}}{3}}$

(E) $\boxed{\frac{\sqrt{3}}{2}}$

(F) $\left(\frac{1}{2} - \frac{1}{3}\right)\sqrt{3} \Rightarrow \boxed{\frac{\sqrt{3}}{6}}$

(G) INcentre $= \left[\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}, \frac{az_1+bz_2+cz_3}{a+b+c}\right]$

$= \left[\frac{1}{\sqrt{1}+\sqrt{2}+\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{1}+\sqrt{2}+\sqrt{3}}, \frac{1}{\sqrt{1}+\sqrt{2}+\sqrt{3}}\right]$

(H) Centroid $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

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EXERCISE 02

Exercise # O-2

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Subject ______

1. Position Vector of the four angular pints of a tetrahedron ABCD are $A(3,-2,1)$, $B(3,1,5)$, $C(4,0,3)$, $D(1,0,0)$. Acute angle b/w the plane faces ADC & ABC is :-

$\because$

Angle b/w plance faces = Angle b/w their Normal.

$\therefore$ Normal of ADC can be found out by $\vec{AD} \times \vec{CD}$

Similarly for ABC $\vec{AB} \times \vec{BC}$

$\vec{AD} = -2\hat{i} + 2\hat{j} - \hat{k}$ $\qquad$ $\vec{AB} = 3\hat{j} + 4\hat{k}$

$\vec{CD} = -3\hat{i} - 3\hat{k}$. $\qquad$ $\vec{BC} = \hat{i} - \hat{j} - 2\hat{k}$

$$\vec{AD} \times \vec{CD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -2 & -1 \\ -3 & 0 & -3 \end{vmatrix} \qquad \vec{AB} \times \vec{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 4 \\ 1 & -1 & -2 \end{vmatrix}$$

$\hat{i}(-6) - \hat{j}(3) + \hat{k}(6)$ $\qquad$ $\hat{i}(-2) - \hat{j}(-4) + \hat{k}(-3)$

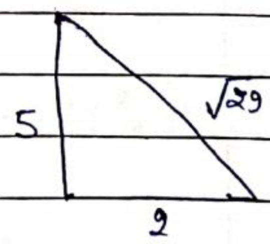
$-6\hat{i} - 3\hat{j} + 6\hat{k}$ $\qquad$ $-2\hat{i} + 4\hat{j} - 3\hat{k}$

$3(-2\hat{i} - \hat{j} + 2\hat{k})$

$$\cos\theta = \frac{(-2\hat{i} - \hat{j} + 2\hat{k}) \cdot (-2\hat{i} + 4\hat{j} - 3\hat{k})}{3 \cdot \sqrt{29}}$$

$$\frac{|4 - 4 - 6|}{3 \times \sqrt{29}}$$

$$\cos\theta = \frac{2}{\sqrt{29}}$$



$$\boxed{\therefore \theta = \tan^{-1} 5/2}$$

(Ans A)

②

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Subject

The Volume of the tetrahedron formed by the coterminus edge $\vec{a}, \vec{b}, \vec{c}$ is 3. Then the volume of the parallelepiped formed by the coterminus edges $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ is:-

Volume of tetrahedron $= \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}] = 3$

$\therefore [\vec{a}\ \vec{b}\ \vec{c}] = 18$

Volume of parallelepiped $= [\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]$

$= 2[\vec{a}\ \vec{b}\ \vec{c}]$

$= 2 \times 18 = \boxed{36}$

(Ans C)

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③ Subject

$\vec{a}, \vec{b}, \vec{c}$ be three vectors having magnitudes 1, 1, 2 Respectively. If $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = 0$, then the acute angle b/w $\vec{a}$ & $\vec{c}$ is :-

$$\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = 0$$

$$\vec{a} \times (\vec{a} \times \vec{c}) = -\vec{b}$$

$$(\vec{a} \times \vec{c}) \times \vec{a} = \vec{b}$$

$$|(\vec{a} \times \vec{c}) \times \vec{a}| = |\vec{b}|$$

$\vec{a} \perp (\vec{a} \times \vec{c})$ $\quad$ $|\vec{a} \times \vec{c}|\,|\vec{a}| \sin(\pi/2) = |\vec{b}|$

$$|\vec{a} \times \vec{c}| \quad 1 \times 1 = 1$$

$$|\vec{a}|\,|\vec{c}| \sin\theta = 1$$

$$1 \times 2 \sin\theta = 1 \quad \Rightarrow \boxed{\theta = \pi/6}$$

(AmA)

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④

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4) A vector of magnitude $5\sqrt{5}$ coplanar with vectors $\hat{i}+2\hat{j}$ & $\hat{j}+2\hat{k}$ & the perpendicular vector $2\hat{i}+\hat{j}+2\hat{k}$ is :-

$$\vec{r} = \lambda(\hat{i}+2\hat{j}) + \mu(\hat{j}+2\hat{k})$$

$$= \lambda\hat{i} + (2\lambda+\mu)\hat{j} + 2\mu\hat{k}$$

$$\vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k}) = 0$$

$$2\lambda + 2\lambda + \mu + 4\mu = 0$$

$$4\lambda + 5\mu = 0$$

$$\because |\vec{r}| = 5\sqrt{5} \qquad |\vec{r}|^2 = 125$$

$$\lambda^2 + (2\lambda+\mu)^2 + (2\mu)^2 = 125$$

$$\left(\frac{-5\mu}{4}\right)^2 + \left[2\left(\frac{-5\mu}{4}\right)+\mu\right]^2 + 4\mu^2 = 125$$

$$\frac{25\mu^2}{16} + \frac{36\mu^2}{16} + 4\mu^2 = 125$$

$$\frac{61}{16}\mu^2 + \frac{64\mu^2}{16} = 125$$

$$\frac{125\mu^2}{16} = 125 \Rightarrow \mu = \pm 4$$

$$\therefore \lambda = \frac{-5}{4}(\pm 4) \Rightarrow \lambda = \mp 5$$

$$\therefore \mu = 4, \ \lambda = -5$$

$$\vec{r} = -5\hat{i} - 6\hat{j} + 8\hat{k}$$

$$\boxed{\vec{r} = \pm[5\hat{i} + 6\hat{j} - 8\hat{k}]}$$ (Ans D)

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④ Continues. . . .

M2 Vector coplaner with $\vec{a}, \vec{b}$ and perpendicular to $\vec{c}$ is given by

$$\vec{r} = \lambda((\vec{a} \times \vec{b}) \times \vec{c})$$

$$\Rightarrow \vec{r} = \lambda((\hat{i}+2\hat{j}) \times (\hat{j}+2\hat{k})) \times (2\hat{i}+\hat{j}+2\hat{k})$$

$$= \lambda(4\hat{i}-2\hat{j}+\hat{k}) \times (2\hat{i}+\hat{j}+2\hat{k})$$

$$= \lambda(-5\hat{i}-6\hat{j}+8\hat{k})$$

$$\because |\vec{r}| = 5\sqrt{5}$$

$$\Rightarrow |\lambda| \cdot \sqrt{125} = 5\sqrt{5}$$

$$\Rightarrow |\lambda| = 1 \Rightarrow \lambda = \pm 1$$

$$\therefore \vec{r} = \pm(5\hat{i}+6\hat{j}-8\hat{k})$$

⑤

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5 Let $\vec{\alpha} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{\beta} = \hat{i} + \hat{j}$. if $\vec{r}$ is a unit vector, then the maximum value of $[\vec{\alpha}\times\vec{\beta} \;\; \vec{\beta}\times\vec{r} \;\; \vec{r}\times\vec{\alpha}]$ is :-

Solⁿ $\because [\vec{\alpha}\times\vec{\beta} \;\; \vec{\beta}\times\vec{r} \;\; \vec{r}\times\vec{\alpha}] = [\vec{\alpha} \;\; \vec{\beta} \;\; \vec{r}]^2$

$((\vec{\alpha}\times\vec{\beta})\cdot\vec{r})^2$

$$\vec{\alpha}\times\vec{\beta} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$\hat{i} - \hat{j} - \hat{k}$$

$(\vec{\alpha}\times\vec{\beta})\cdot\vec{r}$ will be maximum when $\vec{r}$ is parallel to $\vec{\alpha}\times\vec{\beta}$

$\vec{r} = \lambda(\hat{i} - \hat{j} - \hat{k}) \Rightarrow |\vec{r}| = 1$

$\sqrt{\lambda^2 + \lambda^2 + \lambda^2} = 1 \Rightarrow \lambda = \frac{1}{\sqrt{3}}$

$$(\vec{\alpha}\times\vec{\beta})\cdot\vec{r} = (\hat{i} - \hat{j} - \hat{k})\lambda(\hat{i} - \hat{j} - \hat{k})$$
$$= \lambda(1+1+1)$$
$$= 3\lambda$$

$$((\vec{\alpha}\times\vec{\beta})\cdot\vec{r})^2 = 9\lambda^2$$
$$\Rightarrow 9\times\frac{1}{3}$$
$$\Rightarrow \boxed{3}$$

(Ans)

6 **[MULTIPLE OBJECTIVE TYPE]**

Subject

(AC) If $\vec{a}, \vec{b}, \vec{c}$ be three non zero vectors satisfying the condition $\vec{a}\times\vec{b} = \vec{c}$ & $\vec{b}\times\vec{c} = \vec{a}$ then which of the following always hold(s) good?

Sol:

$\vec{a}\times\vec{b} = \vec{c}$

$\vec{a}\cdot(\vec{a}\times\vec{b}) = \vec{a}\cdot\vec{c}$

$0 = \vec{a}\cdot\vec{c}$

$\vec{b}\cdot(\vec{a}\times\vec{b}) = \vec{b}\cdot\vec{c}$

$0 = \vec{b}\cdot\vec{c}$

$\vec{b}\times\vec{c} = \vec{a}$

$\vec{b}\cdot(\vec{b}\times\vec{c}) = \vec{b}\cdot\vec{a}$

$0 = \vec{b}\cdot\vec{a}$

$$\boxed{\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0}$$

$\vec{a}\times\vec{b} = \vec{c}$

$\vec{c}\cdot(\vec{a}\times\vec{b}) = \vec{c}\cdot\vec{c}$

$[\vec{c}\ \vec{a}\ \vec{b}] = |\vec{c}|^2$

$$\boxed{[\vec{a}\ \vec{b}\ \vec{c}] = |\vec{c}|^2}$$ ✓

$\vec{a}\times\vec{b} = \vec{c}$

$\vec{b}\times\vec{c} = \vec{a} \Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = |\vec{a}|^2$

$\vec{b}\times(\vec{a}\times\vec{b}) = \vec{a}$

$(\vec{b}\cdot\vec{b})\,\vec{a} - \underbrace{(\vec{b}\cdot\vec{a})}_{0}\cdot\vec{b} = \vec{a}$

$|\vec{b}|^2\,\vec{a} = \vec{a} \Rightarrow |\vec{b}|^2 = 1 \Rightarrow |\vec{b}| = 1$

$\therefore (\vec{b}\times\vec{c})\times\vec{b} = \vec{c}$

$(\vec{b}\cdot\vec{b})\,\vec{c} - \cancel{(\vec{b}\cdot\vec{c})}^{0}\,\vec{b} = \vec{c}$

$|\vec{a}|^2 = |\vec{c}|^2 \Rightarrow |\vec{a}| = |\vec{c}|$.

$\therefore \vec{a}\times\vec{b} = \vec{c}$

$|\vec{a}\times\vec{b}| = |\vec{c}|$

$|\vec{a}\times\vec{b}| = |\vec{c}|$

$|\vec{a}|\,|\vec{b}|\sin\theta = |\vec{c}| \Rightarrow |\vec{a}| = |\vec{c}|$

$\therefore \vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs ✓

7

Given the following information about the non zero vectors $\vec{A}, \vec{B}$ and $\vec{C}$

(i) $(\vec{A}\times\vec{B})\times\vec{A}=\vec{0}$ (ii) $\vec{B}\cdot\vec{B}=4$ (iii) $\vec{A}\cdot\vec{B}=-6$ (iv) $\vec{B}\cdot\vec{C}=6$

Which one of the following holds good?

~~(A)~~ $\vec{A}\times\vec{B}=\vec{0}$ ~~(B)~~ $\vec{A}\cdot(\vec{B}\times\vec{C})=0$ (C) $\vec{A}\cdot\vec{A}=8$ ~~(D)~~ $\vec{A}\cdot\vec{C}=-9$

Soln

Given $(\vec{A}\times\vec{B})\times\vec{A}=0$

$(\vec{A}\cdot\vec{A})\vec{B}-(\vec{B}\cdot\vec{A})\vec{A}=0$

$|\vec{A}|^2\vec{B}+6\vec{A}=0$ —— ①

$|\vec{A}|^2\vec{B}\cdot\vec{B}+6\vec{A}\cdot\vec{B}=0$ (Dot with vector $\vec{B}$ both sides)

$|\vec{A}|^2|\vec{B}|^2+6(-6)=0$ ($\vec{B}\cdot\vec{B}=4$ or $|\vec{B}|^2=4$

$4|\vec{A}|^2=36$

$\therefore |\vec{A}|=3$ [option C is wrong)

From ① $9\vec{B}+6\vec{A}=0$ $\therefore$ $\vec{A}=-\frac{3}{2}\vec{B}$ OR $\vec{A}=\lambda\vec{B}$

For option B $\vec{A}\cdot(\vec{B}\times\vec{C})=0$ {because $\vec{A}=\lambda\vec{B}$}

For option D

$9\vec{B}+6\vec{A}=0$

$9\vec{B}\cdot\vec{C}+6\vec{A}\cdot\vec{C}=0$ {Dot with $\vec{A}$}

$9(6)+6\vec{A}\cdot\vec{C}=0$ { $\vec{B}\cdot\vec{C}=6$ }

$\vec{A}\cdot\vec{C}=-9$ [option D is correct]

For option A: $9\vec{B}+6\vec{A}=0$

$9\vec{A}\times\vec{B}+6\vec{A}\times\vec{A}=0$

$\therefore \vec{A}\times\vec{B}=0$

8

If $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are four non zero vectors in the same plane no two of which are collinear then which of the following hold(s) good?

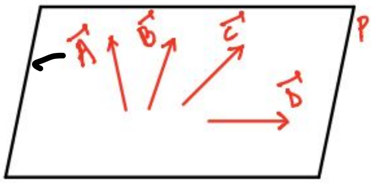
(A) $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D}) = 0$

(B) $(\vec{A}\times\vec{C})\cdot(\vec{B}\times\vec{D}) \neq 0$

(C) $(\vec{A}\times\vec{B})\times(\vec{C}\times\vec{D}) = \vec{0}$

(D) $(\vec{A}\times\vec{C})\times(\vec{B}\times\vec{D}) \neq \vec{0}$

Soln



: Let P be a plane containing four vectors $\vec{A}, \vec{B}, \vec{C}, \vec{D}$

$\vec{A}\times\vec{B}$ = is a vector perpendicular to plane P

$\vec{C}\times\vec{D}$ = is again a vector perpendicular to plane $\vec{P}$

similarly $\vec{A}\times\vec{C}$ and $\vec{B}\times\vec{D}$

Thus $\vec{A}\times\vec{B} = \lambda\,\vec{C}\times\vec{D}$ $\therefore$ $(\vec{A}\times\vec{B})\times(\vec{C}\times\vec{D}) = 0$

Hence option C is correct.

Also $(\vec{A}\times\vec{C})\times(\vec{B}\times\vec{D}) = 0$

But $(\vec{A}\times\vec{C})\cdot(\vec{B}\times\vec{D}) \neq 0$ {$\vec{A}, \vec{B}, \vec{C}, \vec{D}$ are Non-zero Non-collinear}

Hence B is correct.

9

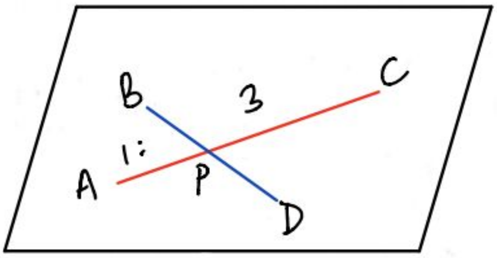
If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are the pv's of the points A, B, C & D respectively in three dimensional space & satisfy the relation $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$, then :

(A) A, B, C & D are coplanar

(B) the line joining the points B & D divides the line joining the point A & C in the ratio 2 : 1.

(C) the line joining the points A & C divides the line joining the points B & D in the ratio 1 : 1

(D) the four vectors $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are linearly dependent.



Sol^n

$$3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$$

$$3\vec{a} + \vec{c} = 2\vec{b} + 2\vec{d}$$

$$\frac{3\vec{a} + 1\cdot\vec{c}}{3+1} = \frac{2\vec{b} + 2\vec{d}}{4}$$

$$\frac{3\vec{a} + 1\cdot\vec{c}}{3+1} = \frac{\vec{b} + \vec{d}}{2}$$

Thus we can say P is point of intersection of line AC and BD such that AC divides BD in ratio 1:1 and BD divides AC in ratio 1:3

Hence C is correct.

Also $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = 0$ where $x+y+z+w=0$ then $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are representing four points A, B, C, D which are coplanar and hence linearly dependent.

Thus: A, C, D are correct.

(10)

If a, b, c are different real numbers and $a\hat{i}+b\hat{j}+c\hat{k}$; $b\hat{i}+c\hat{j}+a\hat{k}$ & $c\hat{i}+a\hat{j}+b\hat{k}$ are position vectors of three non-collinear points A, B & C then :

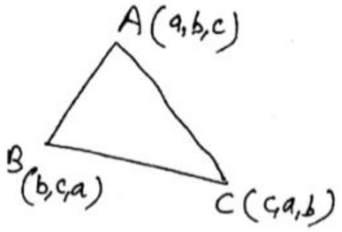
(A) centroid of triangle ABC is $\frac{a+b+c}{3}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(B) $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the three vectors

(C) perpendicular from the origin to the plane of triangle ABC meet at centroid

(D) triangle ABC is an equilateral triangle.

Sol^n



$\vec{AB} = (b-a)\hat{i} + (c-b)\hat{j} + (a-c)\hat{k}$

$|\vec{AB}| = \sqrt{(b-a)^2+(c-b)^2+(a-c)^2}$

$\vec{BC} = (c-b)\hat{i} + (a-c)\hat{j} + (b-a)\hat{k}$

$|\vec{BC}| = \sqrt{(c-b)^2+(a-c)^2+(b-a)^2}$

$\vec{CA} = (a-c)\hat{i} + (b-a)\hat{j} + (c-b)\hat{k}$

$|\vec{CA}| = \sqrt{(a-c)^2+(b-a)^2+(c-b)^2}$

Thus $|\vec{AB}| = |\vec{BC}| = |\vec{CA}|$ Hence $\triangle ABC$ is Equilateral.

Also centroid of $\triangle ABC$ is $\left(\frac{a+b+c}{3}, \frac{a+b+c}{3}, \frac{a+b+c}{3}\right)$

$\Rightarrow \frac{a+b+c}{3}(\hat{i}+\hat{j}+\hat{k})$

Let $\hat{i}+\hat{j}+\hat{k}$ makes angle α, β, γ with three vectors

$\cos\alpha = \frac{a+b+c}{\sqrt{3}\cdot\sqrt{a^2+b^2+c^2}}$; $\cos\beta = \frac{a+b+c}{\sqrt{3}\cdot\sqrt{a^2+b^2+c^2}}$; $\cos\gamma = \frac{a+b+c}{\sqrt{3}\cdot\sqrt{a^2+b^2+c^2}}$

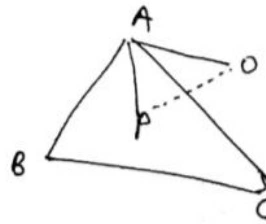
Let P be the foot of perpendicular from origin to plane containing A, B, C

Let O (0, 0, 0)

we observe that

$\vec{AP} + \vec{PO} = \vec{AO}$

$\vec{PO} = \vec{AO} - \vec{AP}$



$\vec{PO} = -\left(\frac{a+b+c}{3}\right)(\hat{i}+\hat{j}+\hat{k})$

This is obtained by

$\vec{BP} + \vec{PO} = \vec{BO}$ and $\vec{CP} + \vec{PO} = \vec{CO}$

which implies that perpendicular from origin meets plane at centroid of $\triangle ABC$

11

A vector of magnitude 10 along the normal to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at its point P(1, 0) can be

(A) $6\hat{i}+8\hat{j}$ (B) $-6\hat{i}+8\hat{j}$ (C) $6\hat{i}-8\hat{j}$ (D) $-6\hat{i}-8\hat{j}$

Soln. curve is $3x^2+8xy+2y^2-3=0$

Differentiating

$$6x+8\left[x\frac{dy}{dx}+y\right]+4y\frac{dy}{dx}=0$$

$$(8x+4y)\frac{dy}{dx}+6x+8y=0$$

$$\frac{dy}{dx}=\frac{-(6x+8y)}{8x+4y}$$

$$\left(\frac{dy}{dx}\right)_{(1,0)}=\frac{-(6+0)}{8+0}=-\frac{3}{4}$$

Slope of Normal is $\frac{4}{3}$

Equation of Normal is $y-0=\frac{4}{3}(x-1)$

$$4x-3y-4=0$$

Since vector along a line $ax+by+c=0$ is $\lambda(b\hat{i}-a\hat{j})$

$\therefore$ vector along Normal is $\lambda(3\hat{i}+4\hat{j})$

$\sqrt{9\lambda^2+16\lambda^2}=10 \quad \therefore \lambda=\pm 2$

vector along Normal is $6\hat{i}+8\hat{j}$ OR $-6\hat{i}-8\hat{j}$

Hence A, D

(12)

Let OAB be a regular triangle with side length unity (O being the origin). Also M,N are the points of trisection of AB,M being closer to A and N closer to B. Position vectors of A,B,M and N are $\vec{a}, \vec{b}, \vec{m}$ and $\vec{n}$ respectively. Which of the following hold(s) good ?

(A) $\vec{m} = x\vec{a} + y\vec{b} \Rightarrow \frac{2}{3}$ and $y = \frac{1}{3}$ (B) $\vec{m} = x\vec{a} + y\vec{b} \Rightarrow x = \frac{5}{6}$ and $y = \frac{1}{6}$

(C) $\vec{m}.\vec{n}$ equals $\frac{13}{18}$ (D) $\vec{m}.\vec{n}$ equals $\frac{15}{18}$

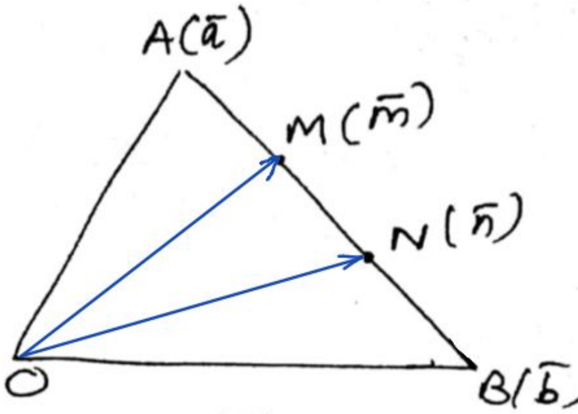
Soln

M and N are point of trisection of AB

AM = MN = NB

Thus AM : MB = 1 : 2

$\therefore \bar{m} = \frac{2\bar{a} + \bar{b}}{3}$.



$|\bar{a}| = |\bar{b}| = 1$

triange is Equilateral.

Similarly $\bar{n} = \frac{\bar{a} + 2\bar{b}}{3}$

Now $\bar{m} = x\bar{a} + y\bar{b}$ $\therefore x = \frac{2}{3}$; $y = \frac{1}{3}$

$$\bar{m}\cdot\bar{n} = \left(\frac{2\bar{a} + \bar{b}}{3}\right)\cdot\left(\frac{\bar{a} + 2\bar{b}}{3}\right) = \frac{2|\bar{a}|^2 + 5\bar{a}\cdot\bar{b} + 2|\bar{b}|^2}{9}$$

$$\bar{m}\cdot\bar{n} = \frac{2(1) + 5|\bar{a}||\bar{b}|\cos 60^\circ + 2}{9} = \frac{4 + \frac{5}{2}}{9} = \frac{13}{18}$$

13

Given three vectors $\vec{U}=2\hat{i}+3\hat{j}-6\hat{k}$; $\vec{V}=6\hat{i}+2\hat{j}+3\hat{k}$; $\vec{W}=3\hat{i}-6\hat{j}-2\hat{k}$

Which of the following hold good for the vectors $\vec{U}$, $\vec{V}$ and $\vec{W}$?

(A) $\vec{U}$, $\vec{V}$ and $\vec{W}$ are linearly dependent

(B) $(\vec{U}\times\vec{V})\times\vec{W}=\vec{0}$

(C) $\vec{U}$, $\vec{V}$ and $\vec{W}$ form a triplet of mutually perpendicular vectors

(D) $\vec{U}\times(\vec{V}\times\vec{W})=\vec{0}$

soln

$$\vec{u} = 2i+3j-6\hat{k}$$
$$\vec{v} = 6i+2j+3\hat{k}$$
$$\vec{w} = 3i-6j-2\hat{k}$$

$$[\bar{u}\ \bar{v}\ \bar{w}] = \begin{vmatrix} 2 & 3 & -6 \\ 6 & 2 & 3 \\ 3 & -6 & -2 \end{vmatrix} = 343$$

Thus $\bar{u}, \vec{v}, \bar{w}$ are non-coplannar therefore $\bar{u}, \bar{v}\ \bar{w}$ are linearly Independent

$$(\bar{u}\times\bar{v})\times\vec{w} = (\bar{u}\cdot\bar{w})\vec{v} - (\bar{v}\cdot\bar{w})\vec{u}$$
$$= 0 - 0 = 0$$

$\bar{u}\cdot\bar{w} = 6-18+12=0$

$\bar{v}\cdot\bar{w} = 18-12-6=0$

$\bar{u}\cdot\bar{v} = 12+6-18=0$

Also $\bar{u}, \bar{v}, \vec{w}$ forms triplet of mutually perpendicular vectors because $\bar{u}\cdot\bar{v} = \bar{v}\cdot\bar{w} = \bar{w}\cdot\vec{u} = 0$

similarly $\bar{u}\times(\bar{v}\times\bar{w}) = 0$

Thus B, C, D are correct.

(14)

Let $\vec{a}$ and $\vec{b}$ be two non-zero and non-collinear vectors then which of the following is/are always correct?

(A) $\vec{a}\times\vec{b}=[\vec{a}\ \vec{b}\ \hat{i}]\hat{i}+[\vec{a}\ \vec{b}\ \hat{j}]\hat{j}+[\vec{a}\ \vec{b}\ \hat{k}]\hat{k}$

(B) $\vec{a}.\vec{b}=(\vec{a}.\hat{i})(\vec{b}.\hat{i})+(\vec{a}.\hat{j}).(\vec{b}.\hat{j})+(\vec{a}.\hat{k})(\vec{b}.\hat{k})$

(C) if $\vec{u}=\hat{a}-(\hat{a}.\hat{b})\hat{b}$ and $\vec{v}=\hat{a}\times\hat{b}$ then $|\vec{u}|=|\vec{v}|$

(D) if $\vec{c}=\vec{a}\times(\vec{a}\times\vec{b})$ and $\vec{d}=\vec{b}\times(\vec{a}\times\vec{b})$ then $\vec{c}+\vec{d}=\vec{0}$

Soln. $\bar{a}$ and $\bar{b}$ are two Non-zero vectors and non-collinear

Since we know that

$$\vec{a}=(\bar{a}\cdot\hat{i})\hat{i}+(\bar{a}\cdot j)j+(\bar{a}\cdot\hat{k})\hat{k} \quad (1)$$

Replace $\vec{a}$ by $\bar{a}\times\bar{b}$

$$\bar{a}\times\bar{b}=((\bar{a}\times\bar{b})\cdot\hat{i})\hat{i}+((\bar{a}\times b)\cdot j)j+((\bar{a}\times\bar{b})\cdot\hat{k})\hat{k}$$

$$\bar{a}\times\bar{b}=[\vec{a}\,\bar{b}\,\hat{i}]\hat{i}+[\bar{a}\,\bar{b}\,j]j+[\bar{a}\,\bar{b}\,\hat{k}]\hat{k}$$ [option A]

From (1)

$$\bar{a}\cdot\vec{b}=(\bar{a}\cdot i)(\bar{b}\cdot\hat{i})+(\bar{a}\cdot j)(\bar{b}\cdot j)+(\bar{a}\cdot\hat{k})(\bar{b}\cdot\hat{k})$$

(Hence option B)

(C)

$$\vec{u}=\hat{a}-(\hat{a}\cdot\hat{b})\hat{b}$$

$$|\bar{u}|^2=(\hat{a}-(\hat{a}\cdot\hat{b})\hat{b})^2$$

$$=|\hat{a}|^2+|(\hat{a}\cdot\hat{b})\hat{b}|^2-2(\hat{a}\cdot\hat{b})^2$$

$$|\vec{u}|^2=1+(\hat{a}\cdot\hat{b})^2-2(\hat{a}\cdot\hat{b})^2=1-(\hat{a}\cdot\hat{b})^2$$

Also $|\bar{v}|^2=|\bar{a}\times\bar{b}|^2=|\hat{a}|^2|\hat{b}|^2\sin^2\theta$

$$=|\hat{a}|^2|\hat{b}|^2(1-\cos^2\theta)$$

$$=|\hat{a}|^2|\hat{b}|^2-(\hat{a}\cdot\hat{b})^2$$

$$|\bar{v}|^2=1-(\hat{a}\cdot\hat{b})^2$$

Thus $|\bar{u}|^2=|\bar{v}|^2 \Rightarrow |\vec{u}|=|\bar{v}|$

(D)

$$\bar{c}=\bar{a}\times(\bar{a}\times\bar{b})=(\bar{a}\cdot\bar{b})\vec{a}-(\bar{a}\cdot\bar{a})\vec{b}$$

$$\bar{d}=\bar{b}\times(\bar{a}\times\bar{b})=(\bar{b}\cdot\bar{b})\vec{a}-(\bar{b}\cdot\bar{a})\vec{b}$$

$$\bar{c}+\vec{d}\neq 0$$

[COMPREHENSION TYPE]

Paragraph for questions nos. 15 and 16

15

Consider three vectors $\vec{p}=\hat{i}+\hat{j}+\hat{k}$, $\vec{q}=2\hat{i}+4\hat{j}-\hat{k}$ and $\vec{r}=\hat{i}+\hat{j}+3\hat{k}$ and let $\vec{s}$ be a unit vector, then

$\vec{p}$, $\vec{q}$ and $\vec{r}$ are

(A) linearly dependent

(B) can form the sides of a possible triangle

(C) such that the vectors $(\vec{q}-\vec{r})$ is orthogonal to $\vec{p}$

(D) such that each one of these can be expressed as a linear combination of the other two

Soln

$\because \vec{p} = i + j + \hat{k}$

$\vec{q} = 2i + 4j - \hat{k}$

$\vec{r} = i + j + 3\hat{k}$ and $\vec{s}$ be a unit vector

$$[\vec{p}\,\vec{q}\,\vec{r}] = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 1 & 1 & 3 \end{vmatrix} = 4$$

Thus $\bar{p}, \vec{q}, \bar{r}$ are Non-coplannar Thus option A and D are incorrect.

Now $\bar{q} - \bar{r} = \hat{i} + 3j - 4\hat{k}$

$(\bar{q} - \bar{r})\cdot\bar{p} = (\hat{i} + 3j - 4\hat{k})\cdot(i + j + \hat{k})$

$\Rightarrow 1 + 3 - 4 = 0$

Thus option C is correct

$|\vec{p}| = \sqrt{3}$ $|\bar{q}| = \sqrt{21}$ $|\bar{r}| = \sqrt{11}$

Also $\bar{p} + \bar{q} + \bar{r} = 4i + 6j + 3\hat{k}$ Thus $\bar{p} + \bar{q} + \bar{r} \neq 0$

Does not forms the triangle

Also $\bar{p}, \bar{q}, \bar{r}$ are Non-coplannar.

Thus B is also wrong

Consider three vectors $\vec{p}=\hat{i}+\hat{j}+\hat{k}$, $\vec{q}=2\hat{i}+4\hat{j}-\hat{k}$ and $\vec{r}=\hat{i}+\hat{j}+3\hat{k}$ and let $\vec{s}$ be a unit vector, then

16 If $(\vec{p}\times\vec{q})\times\vec{r}=u\vec{p}+v\vec{q}+w\vec{r}$, then (u + v + w) equals to

(A) 8 (B) 2 (C) – 2 (D) 4

Sol^n

$$(\vec{p}\times\vec{q})\times\vec{r}=u\vec{p}+v\vec{q}+w\vec{r}$$

$$(\vec{p}\cdot\vec{r})\vec{q}-(\vec{q}\cdot\vec{r})\vec{p}=u\vec{p}+v\vec{q}+w\vec{r}$$

$$\vec{p}\cdot\vec{r}=(i+j+k)\cdot(i+j+3k)=5$$

$$\vec{q}\cdot\vec{r}=(2i+4j-k)\cdot(i+j+3k)=2+4-3=3$$

Thus $5\vec{q}-3\vec{p}=u\vec{p}+v\vec{q}+w\vec{r}$

$v=5,\quad u=-3,\quad w=0$

$u+v+w=2$

(Hence B)

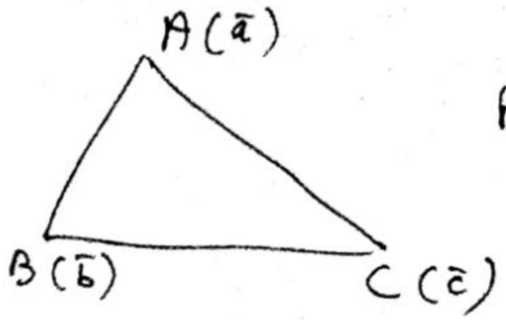
17

[MATRIX MATCH TYPE]

	Column-I		Column-II
(A)	P is point in the plane of the triangle ABC. pv's of A,B and C are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively with respect to P as the origin. If $(\vec{b}+\vec{c}).(\vec{b}-\vec{c})=0$ and $(\vec{c}+\vec{a}).(\vec{c}-\vec{a})=0$, then w.r.t. the triangle ABC,P is its	(P)	centroid
		(Q)	orthocentre
(B)	If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the three non collinear points A,B and C respectively such that the vector $\vec{V}=\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ is a null vector then w.r.t. the ΔABC, P is its	(R)	Incentre
(C)	If P is a point inside the ΔABC such that the vector $\vec{R}=(BC)\overrightarrow{PA}+(CA)(\overrightarrow{PB})+(AB)(\overrightarrow{PC})$ is a null vector then w.r.t. the ΔABC, P is its	(S)	circumcentre
(D)	If P is a point in the plane of the triangle ABC such that the scalar product $\overrightarrow{PA}.\overrightarrow{CB}$ and $\overrightarrow{PB}.\overrightarrow{AC}$ vanishes, then w.r.t. the ΔABC, P is its		

Soln :

(A)



P is in plane of ΔABC as origin

Now $(\bar{b}+\bar{c})\cdot(\bar{b}-\bar{c})=0$

$\bar{b}^2-\bar{c}^2=0 \quad \therefore |\bar{b}|=|\bar{c}|$

Also $(\bar{c}+\bar{a})\cdot(\bar{c}-\bar{a})=0$

$\bar{c}^2-\bar{a}^2=0 \quad \therefore |\bar{c}|=|\bar{a}|$

$\Rightarrow |\bar{a}|=|\bar{b}|=|\bar{c}|$

$|\overline{OA}|=|\overline{OB}|=|\overline{OC}|=$ constant

Hence P is circumcentre [S]

(17) Continues ...

(B) $\vec{V} = \overline{PA} + \overline{PB} + \overline{PC} = 0 \therefore \bar{a} - \bar{p} + \bar{b} - \bar{p} + \bar{c} - \bar{p} = 0$

$\bar{a} + \bar{b} + \bar{c} = 0 + 3\bar{p}$ OR

$$\bar{p} = \frac{\bar{a} + \bar{b} + \bar{c}}{3}$$

then P is centroid. [P]

(C)

$BC(\overline{PA}) + CA(\overline{PB}) + AB(\overline{PC}) = 0$

$BC(\bar{a} - \bar{p}) + CA(\bar{b} - \bar{p}) + AB(\vec{c} - \bar{p}) = 0$

$$\bar{p} = \frac{BC\,\bar{a} + CA\,\bar{b} + AB\,\bar{c}}{BC + CA + AB}$$

Thus P is Incentre [R]

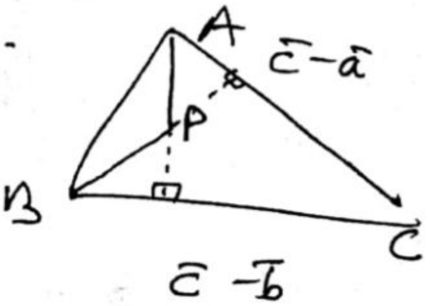
(D) $\overline{PA} \cdot \overline{CB}$ ~~and~~ $\overline{PB} \cdot \overline{AC}$ vanishes

$\overline{PA} \cdot \overline{CB} = (\bar{a} - \bar{p}) \cdot (\bar{b} - \bar{c}) = 0 \Rightarrow PA \perp CB$

$\overline{PB} \cdot \overline{AC} = (\bar{b} - \bar{p}) \cdot (\bar{c} - \bar{a}) = 0 \Rightarrow PB \perp AC$

Thus P is orthocentre, i.e point of intersection of altitudes.

[Q]



EXERCISE S1

1. Given the vector $\vec{PQ} = -6\hat{i} - 4\hat{j}$ and Q is the point $(3,3)$. Find the point P.

Soln:

Let $\vec{P} = x\hat{i} + y\hat{j}$

$\vec{Q} = 3\hat{i} + 3\hat{j}$

$\vec{PQ} = \vec{Q} - \vec{P}$

$= 3\hat{i} + 3\hat{j} - x\hat{i} - y\hat{j}$

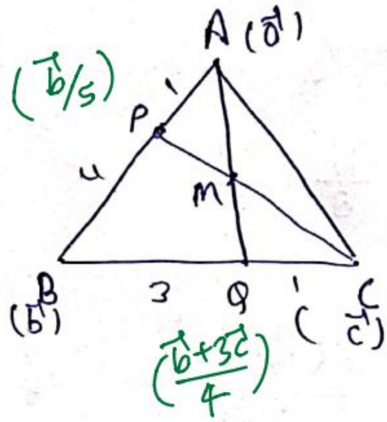
$-6\hat{i} - 4\hat{j} = (3-x)\hat{i} + (3-y)\hat{j}$.

$3 - x = -6$ $\quad$ $3 - y = -4$

$x = 9$ $\quad$ $y = 7$

So P is $(9, 7)$

In ΔABC, a point P is chosen on side $\overrightarrow{AB}$ So that
(2) $AP:PB = 1:4$ and a point Q is chosen on the side $\overrightarrow{BC}$ so that $CQ:QB = 1:3$ segment $\overrightarrow{CP}$ & $\overrightarrow{AQ}$ intersect at M. If the ratio $\frac{MC}{PC}$ is expressed as a rational no. in the lowest term as a/b, find $(a+b)$.



Sol^n: Let $A = \vec{0}$
$B = \vec{b}$
$C = \vec{c}$

So $\vec{p} = \dfrac{\vec{0}\times 4 + 1\times\vec{b}}{5} = \dfrac{\vec{b}}{5}$

$\vec{q} = \dfrac{1\times\vec{b} + 3\times\vec{c}}{4} = \dfrac{\vec{b}+3\vec{c}}{4}$

$\Rightarrow$ eq^n of PC $=$

$\vec{r} = \vec{c} + \vec{d}(\vec{p}-\vec{c})$

$\vec{r} = \vec{c} + d(\vec{b}/5 - \vec{c})$

$\Rightarrow$ eq^n of $\overrightarrow{AQ}$

$\vec{r} = \vec{0} + \mu\left(\dfrac{\vec{b}+3\vec{c}}{4} - 0\right)$

$\Rightarrow$ Intersection point M is

$\vec{r} = \vec{r}$

$\vec{c} + d\left(\dfrac{\vec{b}}{5} - \vec{c}\right) = \mu\left(\dfrac{\vec{b}+3\vec{c}}{4} - 0\right)$

$(1-d)\vec{c} + \dfrac{d\vec{b}}{5} = \dfrac{\mu\vec{b}}{4} + \dfrac{3\mu\vec{c}}{4}$

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$\boxed{\text{② Continues...}}$

$\Rightarrow \quad 1-\lambda = \frac{3\mu}{4}$, $\lambda/5 = \frac{\mu}{4}$

$1-\lambda = \frac{3\lambda}{5}$

$5-5\lambda = 3\lambda$

$\lambda = 5/8$

$\mu = 1/2$

point M $\vec{r} = \frac{\vec{b}}{8} + \frac{3\vec{c}}{8}$

Let $MC:MP = K:1$

So. $\frac{K(\vec{b}/5)+\vec{c}}{K+1} = \frac{\vec{b}}{8} + \frac{3\vec{c}}{8}$

$\Rightarrow \quad \frac{K}{5(K+1)} = \frac{1}{8}$

$8K = 5K+5$

$3K = 5$

$K = 5/3$

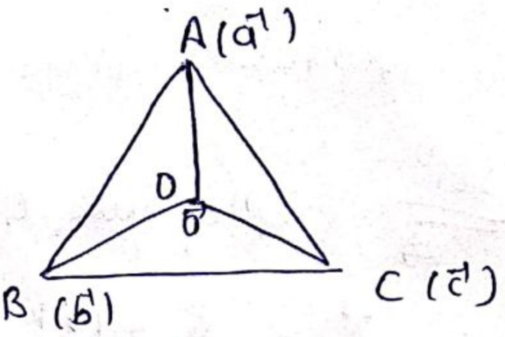
$\frac{MC}{MP} = \frac{5}{3} \Rightarrow \frac{MC}{PC} = \frac{5}{8} = \frac{a}{b}$

$\frac{a}{b} = \frac{5}{8}$

$\boxed{a+b = 5+8 = 13}$

3. Let O be an interior point of ΔABC such that $2\overrightarrow{OA}+5\overrightarrow{OB}+10\overrightarrow{OC}=\vec{0}$. If the ratio of the area of ΔABC to the area of ΔAOC is t, where 'O' is the origin. Find [t].

(where [ ] denotes greatest integer function)



Sol^n:- let O is the origin

$\Rightarrow 2\vec{OA} + 5\vec{OB} + 10\vec{OC} = 0$

$\Rightarrow 2\vec{a} + 5\vec{b} + 10\vec{c} = 0$

$\because \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta AOC} = \dfrac{\left|\frac{1}{2}(\vec{a}-\vec{b})\times(\vec{c}-\vec{b})\right|}{\frac{1}{2}|\vec{a}\times\vec{c}|}$

$$= \frac{|\vec{a}\times\vec{c} - \vec{b}\times\vec{c} - \vec{a}\times\vec{b} + \vec{b}\times\vec{b}|}{|\vec{a}\times\vec{c}|}$$

$$= \frac{|\vec{a}\times\vec{c} + (\vec{c}-\vec{a})\times\vec{b} + 0|}{|\vec{a}\times\vec{c}|}$$

$$= \frac{\left|\vec{a}\times\vec{c} + (\vec{c}-\vec{a})\times\left(\frac{-2\vec{a}-10\vec{c}}{5}\right)\right|}{|\vec{a}\times\vec{c}|}$$

$$= \frac{|\vec{a}\times\vec{c} - 2/5\,\vec{c}\times\vec{a} + 0 + 0 + 2\vec{a}\times\vec{c}|}{|\vec{a}\times\vec{c}|}$$

$$= 17/5\,\frac{|\vec{a}\times\vec{c}|}{|\vec{a}\times\vec{c}|} = 17/5 = t$$

$[t] = [17/5] = \underline{\underline{3}}$

④ If the distance from the point $P(1,1,1)$ to the line passing through the points $Q(0,6,8)$ and $R(-1,4,7)$ is expressed in the form $\sqrt{p/q}$ where p and q are coprime, then the value of $\frac{(p+q)(p+q-1)}{2}$

Soln:-

$\Rightarrow$ eqn of line QR

$\vec{r} = \vec{Q} + d(\vec{R} - \vec{Q})$

$\vec{r} = 6\hat{j} + 8\hat{k} + d(-\hat{i} + 4\hat{j} + 7\hat{k} - 6\hat{j} - 8\hat{k})$

$\vec{r} = 6\hat{j} + 8\hat{k} + d(-\hat{i} - 2\hat{j} - \hat{k})$

$\Rightarrow$ let A on the line QR such that $\underline{PA \perp^r QR}$

$A = (-d, 6-2d, 8-d)$

$\Rightarrow \vec{PA} \cdot (-\hat{i} - 2\hat{j} - \hat{k}) = 0$

$((-d-1)\hat{i} + (5-2d)\hat{j} + (7-d)\hat{k}) \cdot (-\hat{i} - 2\hat{j} - \hat{k}) = 0$

$d + 1 + 4d - 10 - 7 + d = 0$

$6d - 16 = 0$

$d = 8/3$

$\vec{PA} = -11/3\,\hat{i} - 1/3\,\hat{j} + 13/3\,\hat{k}$

$|\vec{PA}| = \frac{1}{3}\sqrt{121 + 1 + 169}$

$= \sqrt{\frac{291}{9}} = \sqrt{\frac{97}{3}} = \sqrt{p/q}$

$p = 97,\ q = 3$

$\frac{(p+q)(p+q-1)}{2} = \frac{100 * 99}{2} = 4950$

5

If $\vec{a}$ & $\vec{b}$ are non collinear vectors such that $\vec{p} = (x+4y)\vec{a} + (2x+y+1)\vec{b}$ & $\vec{q} = (y-2x+2)\vec{a} + (2x-3y-1)\vec{b}$, find x & y such that $3\vec{p} = 2\vec{q}$.

Solⁿ

$$3\vec{p} = 2\vec{q}$$

$$\Rightarrow (3x+12y)\vec{a} + (6x+3y+3)\vec{b} = (2y-4x+4)\vec{a} + (4x-6y-2)\vec{b}$$

On comparing:

$$3x+12y = 2y-4x+4 \Rightarrow 7x+10y = 4 \longrightarrow (I)$$

$$6x+3y+3 = 4x-6y-2 \Rightarrow 2x+9y = -5 \longrightarrow (II)$$

by (I) and (II) : $x = 2,\ y = -1$

6

(a) Show that the points $\vec{a}-2\vec{b}+3\vec{c}; 2\vec{a}+3\vec{b}-4\vec{c}$ & $-7\vec{b}+10\vec{c}$ are collinear.

(b) Prove that the points A (1, 2, 3), B(3, 4, 7), C(–3, –2, –5) are collinear & find the ratio in which B divides AC.

Sol

(a) $A(\vec{a}-2\vec{b}+3\vec{c})$; $B(2\vec{a}+3\vec{b}-4\vec{c})$; $C(-7\vec{b}+10\vec{c})$

$$\overrightarrow{AB} = \vec{a}+5\vec{b}-7\vec{c}$$
$$\overrightarrow{BC} = -2\vec{a}-10\vec{b}+14\vec{c}$$

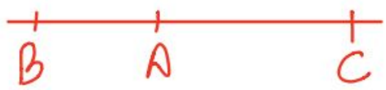
$\because$ $\overrightarrow{BC} = -2\overrightarrow{AB} \Rightarrow$ A, B and C are collinear

(b) $\overrightarrow{AB} = 2\hat{i}+2\hat{j}+4\hat{k}$
$\overrightarrow{BC} = -6\hat{i}-6\hat{j}-12\hat{k}$

$\because$ $-3\overrightarrow{AB} = \overrightarrow{BC}$

$\Rightarrow 3\overrightarrow{AB} = \overrightarrow{CB} \Rightarrow$ A, B and C are collinear

Also,
$3|\overrightarrow{AB}| = |\overrightarrow{BC}|$



$\therefore$ B divides AC in 1 : 3 externally

7

Find out whether the following pairs of lines are parallel, non-parallel & intersecting, or non-parallel non-intersecting.

(a) $\vec{r}_1 = \hat{i} + \hat{j} + 2\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$
$\vec{r}_2 = 2\hat{i} + \hat{j} + 3\hat{k} + \mu(-6\hat{i} + 4\hat{j} - 8\hat{k})$

(b) $\vec{r}_1 = \hat{i} - \hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$
$\vec{r}_2 = 2\hat{i} + 4\hat{j} + 6\hat{k} + \mu(2\hat{i} + \hat{j} + 3\hat{k})$

(c) $\vec{r}_1 = \hat{i} + \hat{k} + \lambda(\hat{i} + 3\hat{j} + 4\hat{k})$
$\vec{r}_2 = 2\hat{i} + 3\hat{j} + \mu(4\hat{i} - \hat{j} + \hat{k})$

Solⁿ (a) $L_1: \vec{r}_1 = \hat{i} + \hat{j} + 2\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$
$L_2: \vec{r}_2 = 2\hat{i} + \hat{j} + 3\hat{k} + \alpha(3\hat{i} - 2\hat{j} + 4\hat{k})$

$\therefore L_1 \parallel L_2$ (parallel to same vector)

Fixed pt. of L_1: $(1, 1, 2)$
Any pt. of L_2: $(2+3\alpha, 1-2\alpha, 3+4\alpha)$

Check: $2 + 3\alpha = 1 \Rightarrow \alpha = -1/3$
$1 - 2\alpha = 1 \Rightarrow \alpha = 0$
$3 + 4\alpha = 2 \Rightarrow \alpha = -1/4$
$\therefore \alpha \in \emptyset$

$\therefore$ Parallel non intersecting lines

(b) lines are non parallel
Any pt. on L_1: $(1+\lambda, -1-\lambda, 3+\lambda)$
Any pt. on L_2: $(2+2\mu, 4+\mu, 6+3\mu)$

$\therefore 1 + \lambda = 2 + 2\mu \Rightarrow \lambda - 2\mu = 1$
$-1 - \lambda = 4 + \mu \Rightarrow \lambda + \mu = -5$
$\lambda = -3$, $\mu = -2$

$3 + \lambda = 6 + 3\mu \longrightarrow$ Satisfied by $\lambda = -3, \mu = -2$

$\therefore$ lines are intersecting and pt. of intersection is $P(-2, 2, 0)$

7 Continues...

(c) lines are non parallel

Any pt. on L_1 : $(1+\lambda, 3\lambda, 4\lambda+1)$
Any pt. on L_2 : $(2+4\mu, 3-\mu, \mu)$

$$\therefore \quad \left.\begin{array}{l} 1+\lambda = 2+4\mu \Rightarrow \lambda - 4\mu = 1 \\ 3\lambda = 3-\mu \Rightarrow 3\lambda + \mu = 3 \end{array}\right\} \begin{array}{l} \lambda = 1 \\ \mu = 0 \end{array}$$

$4\lambda + 1 = \mu \longrightarrow$ Not satisfied by $\lambda = 1, \mu = 0$

$\therefore$ lines are neither parallel nor intersecting (skew lines)

⑧

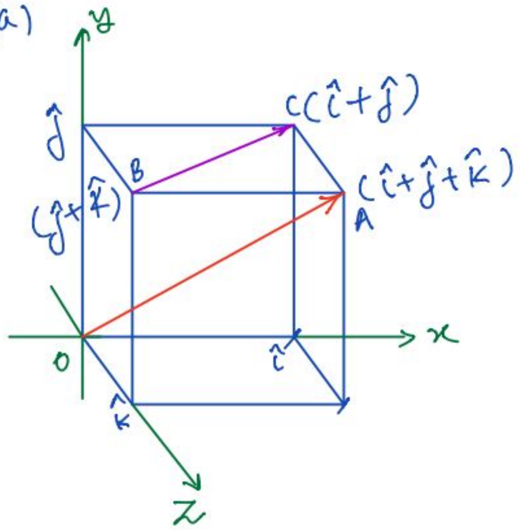
In a unit cube. Find

(a) The angle between the diagonal of the cube and a diagonal of a face skew to it.

(b) The angle between the diagonals of two faces of the cube through the same vertex.

(c) The angle between a diagonal of a cube and a diagonal of a face intersecting it.

Sol^n (a)



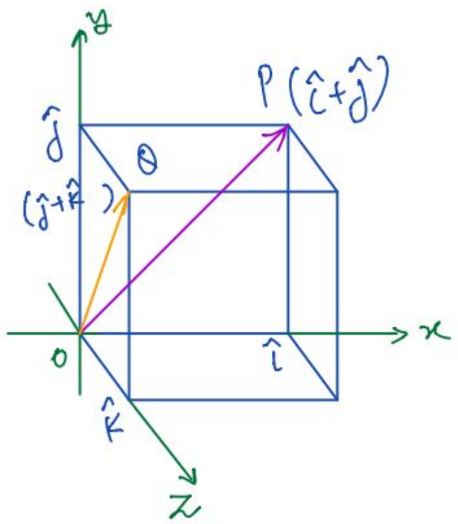
We need to find angle b/w $\vec{OA}$ and $\vec{BC}$

$\therefore (\hat{i}+\hat{j}+\hat{k}) \wedge (\hat{i}-\hat{k}) = ?$

$\Rightarrow \cos\theta = \dfrac{1-1}{\sqrt{3}\cdot\sqrt{2}}$

$\Rightarrow \theta = \pi/2$

(b)



Angle between $\vec{OP}$ and $\vec{OQ}$

$\therefore (\hat{i}+\hat{j}) \wedge (\hat{j}+\hat{k})$

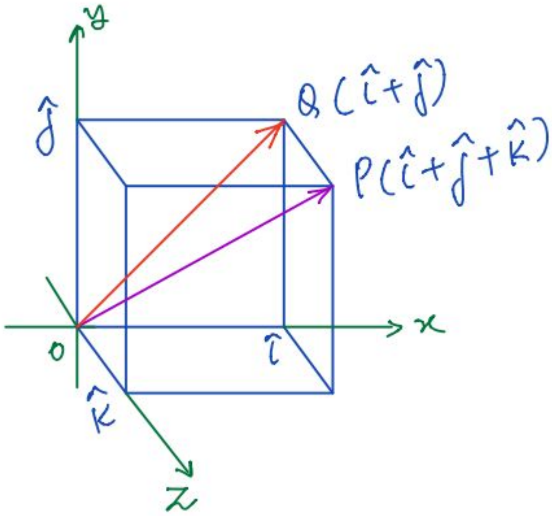
$\Rightarrow \cos\theta = \dfrac{0+1+0}{\sqrt{2}\,\sqrt{2}}$

$\Rightarrow \cos\theta = 1/2$

$\Rightarrow \theta = \pi/3$

⑧ Continues....

(c)



Angle b/w $\vec{OP}$ and $\vec{OQ}$

$\therefore (\hat{i}+\hat{j}+\hat{k})^\wedge(\hat{i}+\hat{j})$

$\therefore \cos\theta = \dfrac{1+1}{\sqrt{3}\cdot\sqrt{2}}$

$\Rightarrow \cos\theta = \dfrac{\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \theta = \cos^{-1}\sqrt{\dfrac{2}{3}}$

9

(a) Find the minimum area of the triangle whose vertices are A(–1,1,2); B(1,2,3) and C(t,1,1) where t is a real number.

Sol.

$$\vec{AB} = 2\hat{i} + \hat{j} + \hat{k}$$
$$\vec{AC} = (t+1)\hat{i} + 0\hat{j} - \hat{k}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ t+1 & 0 & -1 \end{vmatrix}$$

$$\vec{AB} \times \vec{AC} = -\hat{i} + (t+3)\hat{j} - (t+1)\hat{k}$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{1^2 + (t+3)^2 + (t+1)^2}$$
$$= \sqrt{2t^2 + 8t + 11}$$

$$Ar(\Delta ABC) = \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{\sqrt{2t^2+8t+11}}{2}$$
$$= \frac{1}{2}\sqrt{2(t+2)^2+3}$$

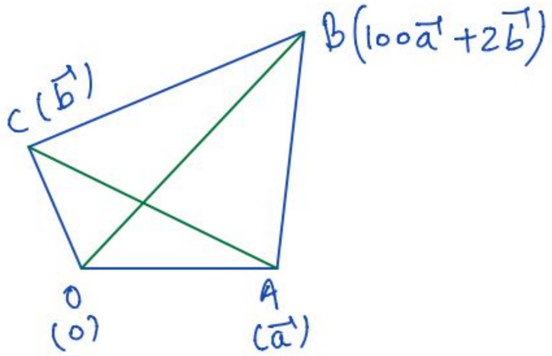
A_{min} at $t = -2$

$A_{min} = \frac{\sqrt{3}}{2}$ sq. units

⑨

(b) Let $\overrightarrow{OA}=\vec{a}; \overrightarrow{OB}=100\vec{a}+2\vec{b}$ and $\overrightarrow{OC}=\vec{b}$ where O,A and C are non collinear points. Let P denotes the area of the parallelogram with $\overrightarrow{OA}$ and $\overrightarrow{OC}$ as adjacent sides and Q denotes the area of the quadrilateral OABC. If $Q=\lambda P$. Find the value of λ.

Solⁿ



$P = |\overrightarrow{OA}\times\overrightarrow{OC}| \Rightarrow |\vec{a}\times\vec{b}| = P \longrightarrow (1)$

$Q = \frac{1}{2}|\overrightarrow{OB}\times\overrightarrow{AC}| = \frac{1}{2}\left|(100\vec{a}+2\vec{b})\times(\vec{b}-\vec{a})\right|$

$\Rightarrow Q = \frac{1}{2}\left|100\vec{a}\times\vec{b} - \vec{0} + \vec{0} - 2\vec{b}\times\vec{a}\right|$

$\Rightarrow Q = \frac{1}{2}\left|100\vec{a}\times\vec{b} + 2\vec{a}\times\vec{b}\right| \Rightarrow Q = 51|\vec{a}\times\vec{b}|$

$\therefore\ Q = 51\cdot(P)$ (by (1))

$\Rightarrow \lambda = 51$ (Ans)

10

Given that $\vec{a}$ and $\vec{b}$ are two unit vectors such that angle between $\vec{a}$ and $\vec{b}$ is $\cos^{-1}\left(\frac{1}{4}\right)$. If $\vec{c}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$, such that $|\vec{c}| = 4, \vec{c}\times\vec{b} = 2\vec{a}\times\vec{b}$ and $\vec{c} = \lambda\vec{a} + \mu\vec{b}$ then, find

(a) the value of λ, **(b)** the sum of values of μ and **(c)** the product of all possible values of μ.

Solⁿ $a = b = 1$; $\vec{a} \wedge \vec{b} = \cos^{-1}(1/4)$

$\vec{c} \times \vec{b} = 2\vec{a} \times \vec{b} \Rightarrow \vec{c} \times \vec{b} - 2\vec{a} \times \vec{b} = 0$

$\Rightarrow (\vec{c} - 2\vec{a}) \times \vec{b} = \vec{0}$

$\Rightarrow \vec{c} - 2\vec{a} = k\vec{b} \Rightarrow \vec{c} = 2\vec{a} + k\vec{b}$

$\therefore \lambda = 2$; $\mu = k$

$|\vec{c}|^2 = 4a^2 + k^2 b^2 + 4k\,\vec{a}\cdot\vec{b}$

$\Rightarrow 16 = 4 + k^2 + 4 \times k \times 1 \times \cos(\cos^{-1}(1/4))$

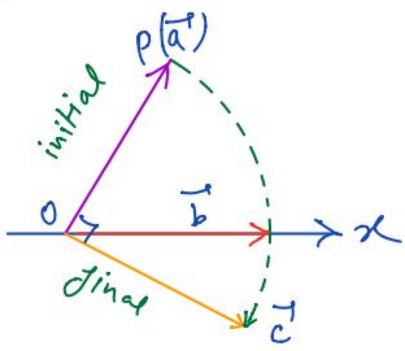
$\Rightarrow 16 = 4 + k^2 + 4k \cdot \frac{1}{4} \Rightarrow k^2 + k - 12 = 0$

$\Rightarrow \mu^2 + \mu - 12 = 0$ < Sum $= -1$, Product $= -12$

(11)

The vector $\overrightarrow{OP} = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle, passing through the positive x-axis on the way. Find the vector in its new position.

Sol^n



$|\vec{a}| = |\vec{b}| = |\vec{c}| = 3$

$\therefore \vec{b} = 3\hat{i}$

Given: $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{c}$ is a vector in the plane of $\vec{a}$ and $\vec{b}$ and $\perp$ to $\vec{a}$

Also, $\vec{c}$ makes acute angle with $\hat{i}$

$$\therefore \vec{c} = \lambda\left((\vec{a} \times \vec{b}) \times \vec{a}\right)$$
$$= \lambda\left(a^2\vec{b} - (\vec{a}\cdot\vec{b})\vec{a}\right)$$
$$= \lambda\left(9(3\hat{i}) - (3)(\hat{i} + 2\hat{j} + 2\hat{k})\right)$$
$$= \lambda(24\hat{i} - 6\hat{j} - 6\hat{k})$$
$$\vec{c} = \mu(4\hat{i} - \hat{j} - \hat{k})$$

$|\vec{c}| = \sqrt{18}\,|\mu| = 3 \Rightarrow \mu = \pm\frac{1}{\sqrt{2}}$

$\therefore \vec{c} = \frac{4}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$ (reject $\mu = -1/\sqrt{2}$)

12

If $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying $[(a-2)\alpha^2 + (b-3)\alpha + c]\,\vec{x} + [(a-2)\beta^2 + (b-3)\beta + c]\,\vec{y} + [(a-2)\gamma^2 + (b-3)\gamma + c]\,(\vec{x} \times \vec{y}) = 0$ where α, β, γ are three distinct real numbers, then find the value of $(a^2 + b^2 + c^2)$.

Solⁿ

$$\left.\begin{aligned} (a-2)\alpha^2 + (b-3)\alpha + c &= 0 \\ (a-2)\beta^2 + (b-3)\beta + c &= 0 \\ (a-2)\gamma^2 + (b-3)\gamma + c &= 0 \end{aligned}\right\}$$

$$\therefore \ (a-2)x^2 + (b-3)x + c = 0 \ \begin{array}{l} \alpha \\ \beta \\ \gamma \end{array}$$

$\Rightarrow$ Quadratic Equation is an identity

$$\Rightarrow \left.\begin{aligned} a &= 2 \\ b &= 3 \\ c &= 0 \end{aligned}\right\} \quad a^2 + b^2 + c^2 = 13$$

EXERCISE S2

1

A vector $\vec{V} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ satisfies the following conditions :

(i) magnitude of $\vec{V}$ is $7\sqrt{2}$

(ii) $\vec{V}$ is parallel to the plane x – 2y + z = 6

(iii) $\vec{V}$ is orthogonal to the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ and (iv) $\vec{V}.\hat{i} > 0$

Find the value of $(v_1 + v_2 + v_3)$.

Sol :- $\bar{V} \parallel$ to $x - 2y + z = 6$

$\Rightarrow \bar{V} \perp ar \quad \hat{i} - 2\hat{j} + \hat{k}$

$\Rightarrow \bar{V} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$

$\Rightarrow v_1 - 2v_2 + v_3 = 0$ —— (1)

$\bar{V} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k}) = 0$

$\Rightarrow 2v_1 - 3v_2 + 6v_3 = 0$ —— (2)

$2 \times (1) - (2) \Rightarrow -v_2 - 4v_3 = 0$

$\Rightarrow v_2 = -4v_3$

$\Rightarrow v_1 = -9v_3$

$|\bar{v}| = 7\sqrt{2}$

$\Rightarrow v_1^2 + v_2^2 + v_3^2 = 98$

$\Rightarrow v_3^2(81 + 16 + 1) = 98$

$\Rightarrow v_3 = \pm 1$

But since $\bar{v} \cdot \hat{i} > 0 \Rightarrow v_1 > 0$

$\Rightarrow -9v_3 > 0$

$\Rightarrow v_3 < 0$

$\Rightarrow v_3 = -1$

$\Rightarrow v_1 = 9, v_2 = 4$

$\Rightarrow \bar{v} = 9\hat{i} + 4\hat{j} - \hat{k}$

Now $v_1 + v_2 + v_3 = 9 + 4 - 1 = 12$

1 Continues...

2 Let $(\vec{p}\times\vec{q})\times\vec{r}+(\vec{q}.\vec{r})\vec{q}=(x^2+y^2)\vec{q}+(14-4x-6y)\vec{p}$ and $(\vec{r}.\vec{r})\vec{p}=\vec{r}$ where $\vec{p}$ and $\vec{q}$ are two non-zero non-collinear vectors and x and y are scalars. Find the value of (x + y).

Sol:- $(\bar{p}\times\bar{q})\times\bar{r}+(\bar{q}\cdot\bar{r})\bar{q}=(x^2+y^2)\bar{q}+(14-4x-6y)\bar{p}$

$\Rightarrow (\bar{p}\cdot\bar{r})\bar{q}-(\bar{q}\cdot\bar{r})\bar{p}+(\bar{q}\cdot\bar{r})\bar{q}=(x^2+y^2)\bar{q}+(14-4x-6y)\bar{p}$

$\Rightarrow \bar{q}\left[\bar{p}\cdot\bar{r}+\bar{q}\cdot\bar{r}-(x^2+y^2)\right]=\bar{p}\left[\bar{q}\cdot\bar{r}+14-4x-6y\right]$

Since $\bar{p},\bar{q}$ are non collinear vectors,

$\bar{p}\cdot\bar{r}+\bar{q}\cdot\bar{r}-(x^2+y^2)=0$ —— (1)

$\bar{q}\cdot\bar{r}+14-4x-6y=0$ —— (2)

Given $(\bar{r}\cdot\bar{r})\bar{p}=\bar{r}$

$\Rightarrow |r|^2\bar{p}=\bar{r}$

$\Rightarrow |r|^2\cdot\bar{p}\cdot\bar{r}=\bar{r}\cdot\bar{r}=|r|^2$

$\Rightarrow \bar{p}\cdot\bar{r}=1$

From (1) $\bar{q}\cdot\bar{r}=x^2+y^2-1$

From (2) $\bar{q}\cdot\bar{r}=4x+6y-14$

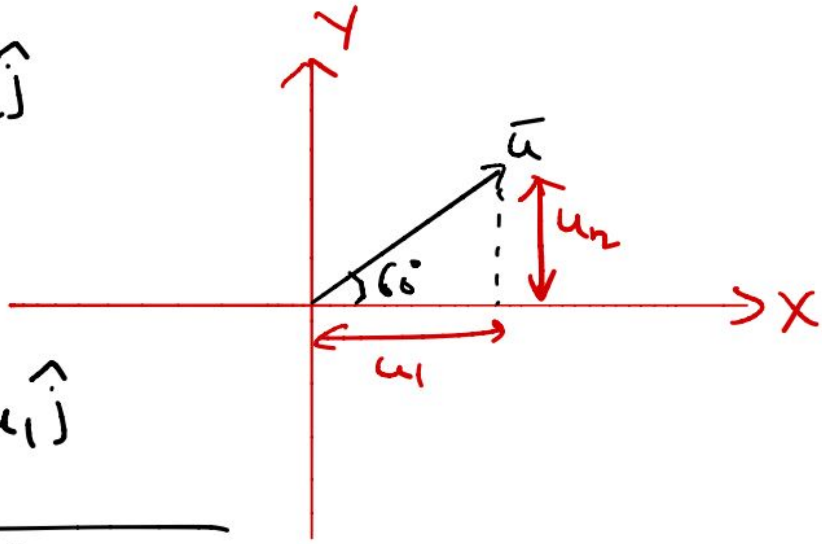
$\Rightarrow x^2+y^2-1=4x+6y-14$

$\Rightarrow x^2+y^2-4x-6y+13=0$

$\Rightarrow (x-2)^2+(y-3)^2=0$

$\Rightarrow x=2,\ y=3 \Rightarrow x+y=5$

③ Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle 60°. Suppose that $|\vec{u}-\hat{i}|$ is geometric mean of $|\vec{u}|$ and $|\vec{u}-2\hat{i}|$ where $\hat{i}$ is the unit vector along x-axis then find the value of $|\vec{u}|$.



Sol:- Let $\bar{u} = u_1\hat{i} + u_2\hat{j}$

$$\tan 60^\circ = \frac{u_2}{u_1} = \sqrt{3}$$

$$\Rightarrow \bar{u} = u_1\hat{i} + \sqrt{3}u_1\hat{j}$$

$$|\bar{u}-\hat{i}| = \sqrt{(u_1-1)^2 + 3u_1^2}$$

$$|\bar{u}| = \sqrt{u_1^2 + 3u_1^2} = \sqrt{4u_1^2}$$

$$|\bar{u}-2\hat{i}| = \sqrt{(u_1-2)^2 + 3u_1^2}$$

$$\left[(u_1-1)^2 + 3u_1^2\right]^2 = 4u_1^2\left[(u_1-2)^2 + 3u_1^2\right]$$

$$\Rightarrow (4u_1^2 - 2u_1 + 1)^2 = 4u_1^2(4u_1^2 - 4u_1 + 4)$$

$$\Rightarrow \cancel{16u_1^4} + 4u_1^2 + 1 - \cancel{16u_1^3} - 4u_1 + 8u_1^2$$

$$= \cancel{16u_1^4} - \cancel{16u_1^3} + 16u_1^2$$

$$\Rightarrow 4u_1^2 + 4u_1 - 1 = 0$$

$$\Rightarrow u_1 = \frac{-4 + \sqrt{16+16}}{8}$$

$$\Rightarrow u_1 = \frac{-1+\sqrt{2}}{2}$$

3 Continues...

Now $|\bar{u}| = \sqrt{4u_1^2} = 2|u_1|$

$|\bar{u}| = \sqrt{2}-1$

4 The position vectors of the points A, B, C are respectively (1, 1, 1) ; (1, –1, 2) ; (0, 2, –1). Find a unit vector parallel to the plane determined by ABC & perpendicular to the vector (1, 0, 1).

Sol :

Vector determined by plane ABC is

$\bar{AB} \times \bar{AC}$

Required vector is $\parallel$ to plane ABC

$\Rightarrow$ It is $\perp$ar to $\bar{AB} \times \bar{AC}$

i.e $\perp$ar to $(0,-2,1) \times (-1,1,-2)$

Also it is $\perp$ar to $(1,0,1)$

$\Rightarrow$ It is $\parallel$ to cross product of

$\bar{AB} \times AC$ and $(1,0,1)$

$\Rightarrow \parallel$ to $\left[(0,-2,1) \times (-1,1,-2)\right] \times (1,0,1)$

$\parallel$ to $\{(0,-2,1).(1,0,1)\}(-1,1,-2)$

$-\{(-1,1,-2).(1,0,1)\}(0,-2,1)$

$\parallel$ to $(-1,1,-2) - (-3)(0,-2,1)$

$\parallel$ to $(-1,1,-2) + (0,-6,3)$

$\parallel$ to $(-1,-5,1)$

Since it is a unit vector, it is

equal to $\pm\left(\frac{-1}{\sqrt{27}}, \frac{-5}{\sqrt{27}}, \frac{1}{\sqrt{27}}\right)$

5 (a) Find a unit vector $\hat{a}$ which makes an angle $(\pi/4)$ with axis of z & is such that $\hat{a}+\hat{i}+\hat{j}$ is a unit vector.

Sol:- Let $\hat{a} = x\hat{i}+y\hat{j}+z\hat{k}$

$(\hat{a}, \hat{k}) = \pi/4$

$\Rightarrow \quad \hat{a}\cdot\hat{k} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad z = \frac{1}{\sqrt{2}}$

also $\quad x^2+y^2+z^2 = 1 \Rightarrow x^2+y^2 = \frac{1}{2}$ — (1)

$\hat{a}+\hat{i}+\hat{j}$ is unit vector

$\Rightarrow \quad (x+1)\hat{i}+(y+1)\hat{j}+\frac{1}{\sqrt{2}}\hat{k}$ is unit

$\Rightarrow \quad (x+1)^2+(y+1)^2+\frac{1}{2} = 1$

$\Rightarrow \quad x^2+y^2+2x+2y+2 = \frac{1}{2}$

$\Rightarrow \quad \cancel{\frac{1}{2}}+2x+2y+2 = \cancel{\frac{1}{2}}$ [From (1)]

$\Rightarrow \quad x+y = -1$ — (2)

From (1), (2) $\quad x = \frac{-1}{2}, \; y = \frac{-1}{2}$

$\Rightarrow \quad \hat{a} = \frac{-1}{2}\hat{i} - \frac{1}{2}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$

5 b

If $\vec{a}$ and $\vec{b}$ are any two unit vectors, then find the range of $\frac{3|\vec{a}+\vec{b}|}{2}+2|\vec{a}-\vec{b}|$.

Sol:-

Let $(\bar{a}, \bar{b}) = \theta$

$$|\bar{a}+\bar{b}| = \sqrt{1+1+2(1)(1)\cos\theta}$$

$$= \sqrt{2(1+\cos\theta)}$$

$$|\bar{a}+\bar{b}| = 2\left|\cos\frac{\theta}{2}\right|$$

|||ly $$|\bar{a}-\bar{b}| = \sqrt{1+1-2(1)(1)\cos\theta}$$

$$= \sqrt{2(1-\cos\theta)}$$

$$|\bar{a}-\bar{b}| = 2\left|\sin\frac{\theta}{2}\right|$$

$$\frac{3}{2}|\bar{a}+\bar{b}| + 2|\bar{a}-\bar{b}| = 3\left|\cos\frac{\theta}{2}\right| + 4\left|\sin\frac{\theta}{2}\right|$$

$$\text{Max} = \sqrt{3^2+4^2} = 5$$

Min if $\theta = 0$ and min value $= 3$

$\therefore$ Range $= [3, 5]$

6 Given three points on the xy plane on O(0, 0), A(1, 0) and B(–1, 0). Point P is moving on the plane satisfying the condition $\left(\overline{PA}.\overline{PB}\right)+3\left(\overline{OA}.\overline{OB}\right)=0$. If the maximum and minimum values of $\left|\overline{PA}\right|\left|\overline{PB}\right|$ are M and m respectively then find the values of $M^2 + m^2$.

Sol :- $O(0,0)$ $A(1,0)$ $B(-1,0)$

Let $P=(x,y)$

$\overline{PA}\cdot\overline{PB}+3(\overline{OA}\cdot\overline{OB})=0$

$\Rightarrow (1-x,-y)\cdot(-1-x,-y)+3(1,0)\cdot(-1,0)=0$

$\Rightarrow (1-x)(1+x)(-)+y^2+3(-1)=0$

$\Rightarrow x^2+y^2=4$ —— (1)

$|\overline{PA}|\,|\overline{PB}| = \sqrt{(1-x)^2+y^2}\sqrt{(1+x)^2+y^2}$

$= \sqrt{5-2x}\,\sqrt{5+2x}$ [From (1)]

$|\overline{PA}||\overline{PB}| = \sqrt{25-4x^2}$

From (1), $-2\le x\le 2$

$\Rightarrow 0\le x^2\le 4$

$\therefore M=\sqrt{25}$

$m=\sqrt{25-4(4)}=\sqrt{9}$

$M^2+m^2=25+9=34$

7

Let $\vec{a},\vec{b},\vec{c}$ are unit vectors where $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}+\vec{a}|^2=3$, then $|\vec{a}+2\vec{b}+3\vec{c}|^2$ is equal to

Sol:-

$$|\bar{a}-\bar{b}|^2+|\bar{b}-\bar{c}|^2+|\bar{c}+\bar{a}|^2=3$$

$$\Rightarrow 1+1-2\bar{a}\cdot\bar{b}+1+1-2\bar{b}\cdot\bar{c}+1+1+2\bar{a}\cdot\bar{c}=3$$

$$\Rightarrow 3-2\bar{a}\cdot\bar{b}-2\bar{b}\cdot\bar{c}+2\bar{a}\cdot\bar{c}=0 \quad \text{——} (1)$$

$$\Rightarrow |\bar{a}-\bar{b}+\bar{c}|^2=0$$

$$\Rightarrow \bar{a}-\bar{b}+\bar{c}=0 \Rightarrow \bar{b}=\bar{a}+\bar{c}$$

Put $\bar{b}=\bar{a}+\bar{c}$ in (1)

$$3-2\bar{a}\cdot(\bar{a}+\bar{c})-2(\bar{a}+\bar{c})\cdot\bar{c}+2\bar{a}\cdot\bar{c}=0$$

$$\Rightarrow 3-2-2\bar{a}\cdot\bar{c}-\cancel{2\bar{a}\cdot\bar{c}}-2+\cancel{2\bar{a}\cdot\bar{c}}=0$$

$$\Rightarrow \bar{a}\cdot\bar{c}=\frac{-1}{2}$$

$$|\bar{a}+2\bar{b}+3\bar{c}|^2=|\bar{a}+2(\bar{a}+\bar{c})+3\bar{c}|^2$$

$$=|3\bar{a}+5\bar{c}|^2$$

$$=9|a|^2+25|c|^2+30\bar{a}\cdot\bar{c}$$

$$=9+25+30\left(-\frac{1}{2}\right)$$

$$=9+25-15$$

$$=19$$

8

The pv's of the four angular points of a tetrahedron are $A(\hat{j}+2\hat{k})$; $B(3\hat{i}+\hat{k})$; $C(4\hat{i}+3\hat{j}+6\hat{k})$ & $D(2\hat{i}+3\hat{j}+2\hat{k})$. Find :

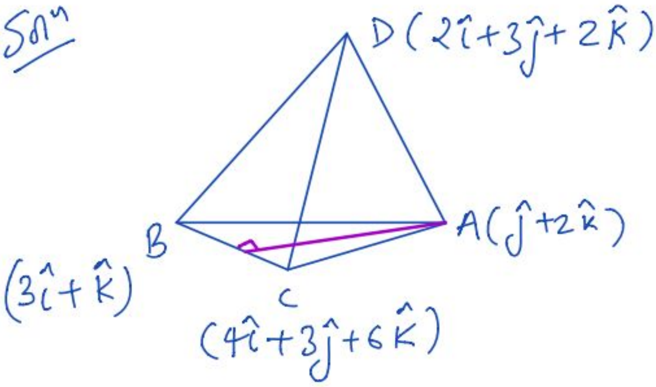
(i) the perpendicular distance from A to the line BC.

(ii) the volume of the tetrahedron ABCD.

(iii) the perpendicular distance from D to the plane ABC.

(iv) the shortest distance between the lines AB & CD.

Solⁿ



(i) Eqⁿ of line BC: $\vec{r} = 3\hat{i}+\hat{k}+\lambda(\hat{i}+3\hat{j}+5\hat{k})$

Let $P(3+\lambda, 3\lambda, 5\lambda+1)$ be a pt. on line BC

$\vec{AP} = (3+\lambda)\hat{i} + (3\lambda-1)\hat{j} + (5\lambda-1)\hat{k}$

$\vec{AP}\cdot\vec{BC} = 0 \Rightarrow 3+\lambda+3(3\lambda-1)+5(5\lambda-1) = 0$

$\Rightarrow 3+\lambda+9\lambda-3+25\lambda-5 = 0.$

$\Rightarrow 35\lambda = 5 \Rightarrow \lambda = 1/7$

$\therefore P\left(\frac{22}{7}, \frac{3}{7}, \frac{12}{7}\right) \Rightarrow AP = \sqrt{\frac{504}{49}} = \frac{6\sqrt{14}}{7}$

(ii) $\vec{AB} = 3\hat{i}-\hat{j}-\hat{k}$

$\vec{AC} = 4\hat{i}+2\hat{j}+4\hat{k}$

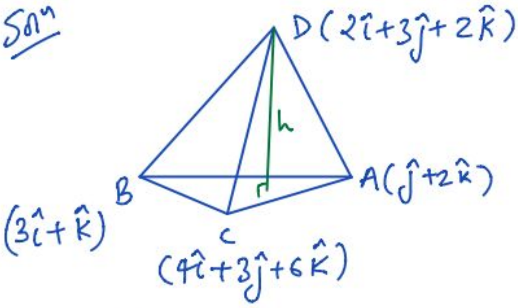
$\vec{AD} = 2\hat{i}+2\hat{j}$

$V = \left|\frac{1}{6}[\vec{AB}\ \vec{AC}\ \vec{AD}]\right|$

$\therefore V = \left|\frac{1}{6}\begin{vmatrix} 3 & -1 & -1 \\ 4 & 2 & 4 \\ 2 & 2 & 0 \end{vmatrix}\right| = 6$ cu. units

8 Continues...

(iii) Solⁿ



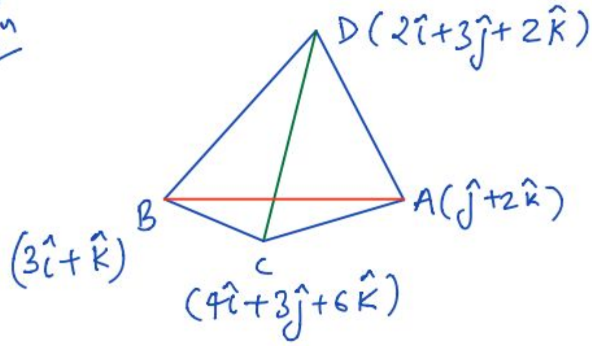
$\vec{AB} = 3\hat{i}-\hat{j}-\hat{k}$; $\vec{AC} = 4\hat{i}+2\hat{j}+4\hat{k}$

$\text{Ar.}(\Delta ABC) = \frac{1}{2}|\vec{AB}\times\vec{AC}| = 3\sqrt{10}\ \text{cm}^2.$

$\text{Vol. of tetrahedron} = \frac{1}{3}(\text{Ar.}(\Delta ABC))\,h$

$\Rightarrow \quad 6 = \frac{1}{3}\cdot 3\sqrt{10}\cdot h \Rightarrow h = \frac{6}{\sqrt{10}}$

(iv)
Solⁿ



Eqⁿ. of line AB: $\vec{r} = 3\hat{i}+\hat{k}+\lambda(3\hat{i}-\hat{j}-\hat{k})$

Eqⁿ. of line CD: $\vec{r} = 2\hat{i}+3\hat{j}+2\hat{k}+\mu(2\hat{i}+4\hat{k})$

Shortest Distance (using formula)
$= \sqrt{6}$ units

9

Let a 3 dimensional vector $\vec{V}$ satisfies the condition $2\vec{V}+\vec{V}\times(\hat{i}+2\hat{j})=2\hat{i}+\hat{k}$.

If $3|\vec{V}|=\sqrt{m}$, where $m \in N$, then find m.

Soln Let $\vec{v}=x\hat{i}+y\hat{j}+z\hat{k}$.

$$\vec{v}\times(\hat{i}+2\hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 2 & 0 \end{vmatrix}$$

$$= \hat{i}(-2z) - \hat{j}(-z) + \hat{k}(2x-y)$$

$$\therefore 2\vec{v} + \vec{v}\times(\hat{i}+2\hat{j}) = 2\hat{i}+\hat{k}$$

$$\Rightarrow 2(x\hat{i}+y\hat{j}+z\hat{k}) + (-2z)\hat{i}+z\hat{j}+(2x-y)\hat{k} = 2\hat{i}+\hat{k}$$

$$\Rightarrow (2x-2z)\hat{i}+(2y+z)\hat{j}+(2z+2x-y)\hat{k} = 2\hat{i}+\hat{k}$$

$$\Rightarrow \left.\begin{aligned} 2x-2z&=2 \\ 2y+z&=0 \\ 2z+2x-y&=1 \end{aligned}\right\} \quad \begin{aligned} x&=7/9 \\ y&=1/9 \\ z&=-2/9 \end{aligned}$$

$$\therefore \vec{v} = \frac{7}{9}\hat{i}+\frac{1}{9}\hat{j}-\frac{2}{9}\hat{k}$$

$$|\vec{v}| = \sqrt{\frac{49}{81}+\frac{1}{81}+\frac{4}{81}} = \sqrt{\frac{54}{81}} = \frac{\sqrt{6}}{3}$$

$$\therefore 3|\vec{v}| = \sqrt{6} \quad \Rightarrow m=6 \text{ Ans}$$

(10)

Vector $\vec{V}$ is perpendicular to the plane of vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ and satisfies the condition $\vec{V}.(\hat{i} + 2\hat{j} - 7\hat{k}) = 10$. Find $|\vec{V}|^2$.

Solⁿ $\vec{V} = \lambda(\vec{a} \times \vec{b})$

$$\Rightarrow \vec{V} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & -2 & 3 \end{vmatrix} \Rightarrow \vec{V} = \lambda(-7\hat{i} - 5\hat{j} - \hat{k})$$

$$\vec{V} \cdot (\hat{i} + 2\hat{j} - 7\hat{k}) = 10$$

$$\Rightarrow \lambda(-7 - 10 + 7) = 10 \Rightarrow \lambda = -1$$

$$\therefore \vec{V} = 7\hat{i} + 5\hat{j} + \hat{k}$$

$$|\vec{V}| = \sqrt{49 + 25 + 1} = \sqrt{75}$$

$$\therefore |\vec{V}|^2 = 75 \quad \text{Ans}$$

11

Let two non-collinear vectors $\vec{a}$ and $\vec{b}$ inclined at an angle $\frac{2\pi}{3}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = 4$. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given as $\overrightarrow{OP} = (e^t + e^{-t})\vec{a} + (e^t - e^{-t})\vec{b}$. If the least distance of P from origin is $\sqrt{2}\sqrt{\sqrt{p} - q}$ where $p, q \in N$ then find the value of (p + q).

Soln $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\frac{2\pi}{3} \Rightarrow \boxed{\vec{a} \cdot \vec{b} = -6}$

$$|\overrightarrow{OP}| = \sqrt{(e^t+e^{-t})^2 a^2 + (e^t - e^{-t})^2 b^2 + 2\vec{a}\cdot\vec{b}\,(e^{2t} - e^{-2t})}$$

$$|\overrightarrow{OP}| = \sqrt{(e^{2t} + e^{-2t} + 2)9 + (e^{2t} + e^{-2t} - 2)16 + 2(-6)(e^{2t} - e^{-2t})}$$

$$\Rightarrow |\overrightarrow{OP}| = \sqrt{13e^{2t} + 37e^{-2t} - 14}$$

Now, $\frac{13e^{2t} + 37e^{-2t}}{2} \geq \sqrt{13 \cdot 37}$ (AM $\geq$ GM)

$$\Rightarrow 13e^{2t} + 37e^{-2t} \geq 2\sqrt{13 \cdot 37}$$

$$\Rightarrow |\overrightarrow{OP}|_{min} = \sqrt{2\sqrt{13 \cdot 37} - 14}$$

$$= \sqrt{2}\sqrt{\sqrt{13 \cdot 37} - 7}$$

$\therefore p = 13 \cdot 37 = 481$; $q = 7$

$\therefore p + q = 488$ Ans.

EXERCISE JM

EXERCISE (JM)

1. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality **[AIEEE-2009]**
$[3\vec{u}\ p\vec{v}\ p\vec{w}] - [p\vec{v}\ \vec{w}\ q\vec{u}] - [2\vec{w}\ q\vec{v}\ q\vec{u}] = 0$ holds for :-
(1) More than two but not all values of (p,q) (2) All values of (p, q)
(3) Exactly one value of (p, q) (4) Exactly two values of (p, q)

Sol:

$$\left(3p^2 - pq + 2q^2\right)\left[\vec{u}\ \ \vec{v}\ \ \vec{w}\right] = 0$$

But $\left[\vec{u}\ \ \vec{v}\ \ \vec{w}\right] \neq 0$

$$3p^2 - pq + 2q^2 = 0$$

$$2p^2 + p^2 - pq + \left(\frac{q}{2}\right)^2 + \frac{7q^2}{4} = 0 \Rightarrow 2p^2 + \left(p - \frac{q}{2}\right)^2 + \frac{7}{4}q^2 = 0$$

$$\Rightarrow p = 0,\ q = 0,\ p = \frac{q}{2}$$

This possible only when p = 0, q = 0 exactly one value of (p, q)

2. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is :

[AIEEE-2010]

(1) $-\hat{i} + \hat{j} - 2\hat{k}$ (2) $2\hat{i} - \hat{j} + 2\hat{k}$ (3) $\hat{i} - \hat{j} - 2\hat{k}$ (4) $\hat{i} + \hat{j} - 2\hat{k}$

Sol.: $$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 0 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix} = -2\hat{i} - \hat{j} - \hat{k}$$

Given $\vec{a} \times \vec{b} + \vec{c} = 0$

Taking cross with $\vec{a}$ on both sides

$$\vec{a} \times (\vec{a} \times \vec{b}) + \vec{a} \times \vec{c} = \vec{0}$$

$$(\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b} + \vec{a} \times \vec{c} = 0$$

$$3\vec{a} - 2\vec{b} + (-2\hat{i} - \hat{j} + \hat{k}) = \vec{0}$$

$$2\vec{b} = 3(\hat{j} - \hat{k}) + (-2\hat{i} - \hat{j} - \hat{k})$$

$$\vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$$

(Ans 4)

3. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}.\vec{d}=0$. Then the vector $\vec{d}$ is equal to :- **[AIEEE-2011]**

(1) $\vec{b}+\left(\dfrac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (2) $\vec{c}-\left(\dfrac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (3) $\vec{b}-\left(\dfrac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (4) $\vec{c}+\left(\dfrac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

Solⁿ

$$\vec{b}\times\vec{c}-\vec{b}\times\vec{d}=\vec{0}$$

$$\Rightarrow \vec{b}\times(\vec{c}-\vec{d})=\vec{0}$$

$$\Rightarrow \vec{c}-\vec{d}=\lambda\vec{b}$$, where λ is scaler

$$\Rightarrow \vec{d}=\vec{c}-\lambda\vec{b}$$

Dot with $\vec{a}$:

$$\underbrace{\vec{a}\cdot\vec{d}}_{0}=\vec{c}\cdot\vec{a}-\lambda\,\vec{b}\cdot\vec{a}$$

$$\Rightarrow \lambda=\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}$$

$$\therefore \vec{d}=\vec{c}-\left(\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}\right)\vec{b}$$ (Ans. 2)

4. If $\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ and $\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$, then the value of $(2\vec{a} - \vec{b}).[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$ is :-

[AIEEE-2011]

(1) 5 (2) 3 (3) – 5 (4) – 3

Sol. $(2\vec{a} - \vec{b}) . [(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$

$= -(2\vec{a} - \vec{b}) . [(\vec{a} + 2\vec{b}) \times (\vec{a} \times \vec{b})]$

$= -(2\vec{a} - \vec{b}) . [((\vec{a} + 2\vec{b}) . \vec{b})\vec{a} - ((\vec{a} + 2\vec{b}) . \vec{a})\vec{b}]$

$= -(2\vec{a} - \vec{b}) . [(\vec{a} . \vec{b}) + 2\vec{b} . \vec{b})\vec{a} - (\vec{a} . \vec{a} + 2\vec{b} . \vec{a})\vec{b})]$

$= -(2\vec{a} - \vec{b}) . [0 + 2\vec{a} - (0 + \vec{b})]$

$= -(2\vec{a} - \vec{b}) . (2\vec{a} - \vec{b})$

$= -(2\vec{a} - \vec{b})^2 = -4\vec{a}^2 + 4\vec{a} . \vec{b} - \vec{b}^2$

$= -4 + 0 - 1 = -5$

5. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a}+3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+2\vec{c}$ is colliner with $\vec{a}$, then $\vec{a}+3\vec{b}+6\vec{c}$ is : **[AIEEE-2011]**

(1) $\vec{a}+\vec{c}$ (2) $\vec{a}$ (3) $\vec{c}$ (4) $\vec{0}$

Sol:-

* $\vec{a}+3\vec{b}$ is collinear with $\vec{c}$

$\therefore \quad \vec{a}+3\vec{b} = \lambda\vec{c}$ —— ①

* $\vec{b}+2\vec{c}$ is colliner with $\vec{a}$

$\therefore \quad \vec{b}+2\vec{c} = \mu\vec{a}$

$\Rightarrow 3\vec{b}+6\vec{c} = 3\mu\vec{a}$ —— ②

① − ② $\Rightarrow \quad \vec{a}-6\vec{c} = \lambda\vec{c} - 3\mu\vec{a}$

$\Rightarrow (1+3\mu)\vec{a} = (6+\lambda)\vec{c}$

$\because$ Given $\vec{a}$ & $\vec{c}$ are Non-Collinear

so, $\quad 1+3\mu = 6+\lambda = 0$

$\Rightarrow \mu = -\frac{1}{3}$ & $\lambda = -6$ Put in ①

$\therefore \quad \vec{a}+3\vec{b} = -6\vec{c}$

$\Rightarrow \boxed{\vec{a}+3\vec{b}+6\vec{c} = \vec{0}}$

Ans.

6. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is : **[AIEEE-2012]**

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

Sol. $\vec{c} = \hat{a} + 2\hat{b}$

$\vec{d} = 5\hat{a} - 4\hat{b}$

$\vec{c}.\vec{d} = 0$

$\Rightarrow \quad (\hat{a} + 2\hat{b}).(5\hat{a} - 4\hat{b}) = 0 \quad \Rightarrow \quad 5 + 6\hat{a}.\hat{b} - 8 = 0$

$\Rightarrow \quad \hat{a}.\hat{b} = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}$

7. Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : **[AIEEE-2012]**

(1) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

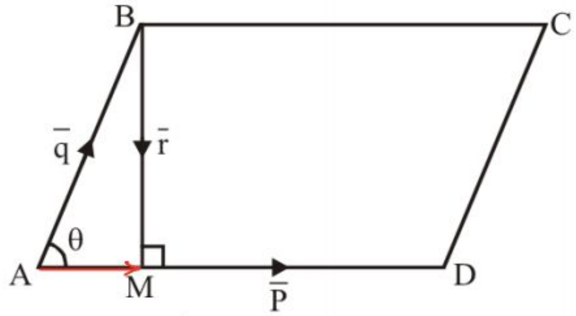
(2) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

(3) $\vec{r} = -\vec{q} + \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

(4) $\vec{r} = \vec{q} - \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

Ans. (3)

Sol.



$$\overline{q} + \overline{r} = \overrightarrow{AM}$$

$$\Rightarrow \overline{r} = -\overline{q} + \overrightarrow{AM}$$

$$\Rightarrow \overline{r} = -\overline{q} + \frac{\overline{p}.\overline{q}}{|\overline{p}|^2}\overline{p}$$ ($\because \overrightarrow{AM}$ is component of $\vec{q}$ along $\vec{p}$)

$$\Rightarrow \overline{r} = -\overline{q} + \left(\frac{\overline{p}.\overline{q}}{\overline{p}.\overline{p}}\right)\overline{p}$$

8. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is : **[JEE-MAINS 2013]**

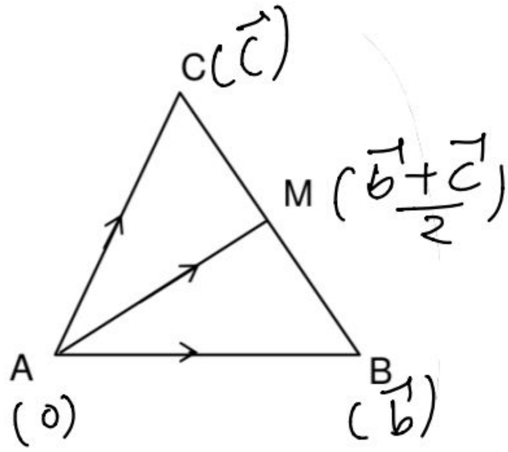
(1) $\sqrt{18}$ (2) $\sqrt{72}$ (3) $\sqrt{33}$ (4) $\sqrt{45}$

Sol. Let $\overrightarrow{AB} = \vec{b}$; $\overrightarrow{AC} = \vec{c}$

$$\overrightarrow{AM} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

$$\overrightarrow{AM} = 4\hat{i} - \hat{j} + 4\hat{k}$$

$$\left|\overrightarrow{AM}\right| = \sqrt{16+16+1} = \sqrt{33}$$



9. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vectors of the type $\vec{b} + \lambda\vec{c}$ for some scalar λ, whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$, is : **[JEE-MAINS Online 2013]**

(1) $2\hat{i} + 3\hat{j} - 3\hat{k}$ (2) $2\hat{i} + \hat{j} + 5\hat{k}$ (3) $2\hat{i} - \hat{j} + 5\hat{k}$ (4) $2\hat{i} + 3\hat{j} + 3\hat{k}$

Sol:- $\because \vec{a} = 2\hat{i} - \hat{j} + \hat{k}$; $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$; $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$

$$\therefore \boxed{\vec{b} + \lambda\vec{c} = (1+\lambda)\hat{i} + (2+\lambda)\hat{j} - (1+2\lambda)\hat{k}}$$

Given $\left[\text{Magnitude of projection of Vector } (\vec{b}+\lambda\vec{c}) \text{ on the vector } \vec{a}\right] = \sqrt{\frac{2}{3}}$

$$\therefore \frac{\vec{a} \cdot (\vec{b} + \lambda\vec{c})}{|\vec{a}|} = \sqrt{\frac{2}{3}}$$

$$\Rightarrow \left|\frac{2(1+\lambda) - (2+\lambda) - (1+2\lambda)}{\sqrt{4+1+1}}\right| = \sqrt{\frac{2}{3}}$$

$$\Rightarrow \lambda = 1 \text{ or } -3$$

$$\therefore b + \lambda\vec{c} \to \boxed{2\hat{i} + 3\hat{j} - 3\hat{k}} \quad \textbf{Ans.}$$

$$\to -2\hat{i} - \hat{j} + 5\hat{k}$$

10. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \bullet \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30º, then $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right|$ equals : **[JEE-MAINS Online 2013]**

(1) $\frac{3}{2}$ (2) 3 (3) $\frac{1}{2}$ (4) $\frac{3\sqrt{3}}{2}$

Sol:- * $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{4+4+1} \Rightarrow \boxed{|\vec{a} \times \vec{b}| = 3}$

* $\because |\vec{c} - \vec{a}| = 2\sqrt{2}$ squaring both side

$\Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2\,\vec{c}\cdot\vec{a} = 8$ $\left\{\because \text{Given } \vec{c}\cdot\vec{a} = |\vec{c}|\right\}$

$\Rightarrow |\vec{c}|^2 + \left(\sqrt{4+1+4}\right)^2 - 2|\vec{c}| = 8$

$\Rightarrow |\vec{c}|^2 - 2|\vec{c}| + 1 = 0$

$\Rightarrow (|\vec{c}| - 1)^2 = 0 \Rightarrow \boxed{|\vec{c}| = 1}$

Find $\Rightarrow |(\vec{a} \times \vec{b}) \times \vec{c}|$

$= |\vec{a} \times \vec{b}| \cdot |\vec{c}| \sin 30^\circ$

$= 3 \cdot 1 \cdot \frac{1}{2} = \boxed{\frac{3}{2}}$ Ans.

11. If $\left[\vec{a}\times\vec{b}\;\vec{b}\times\vec{c}\;\vec{c}\times\vec{a}\right]=\lambda\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$ then λ is equal to : **[JEE(Main)-2014]**

(1) 2 (2) 3 (3) 0 (4) 1

Sol

$$\left[\bar{a}\times\bar{b}\;\;\bar{b}\times\bar{c}\;\;\bar{c}\times\bar{a}\right] \qquad \text{Let } \bar{u}=\bar{b}\times\bar{c}$$

$$=(\vec{a}\times\bar{b})\cdot\left\{\left(\vec{b}\times\bar{c}\right)\times\left(\vec{c}\times\vec{a}\right)\right\}$$

$$=\left(\vec{a}\times\bar{b}\right)\cdot\{\vec{u}\times(\bar{c}\times\bar{a})\}$$

$$=(\vec{a}\times b)\cdot\{(\bar{u}\cdot\vec{a})\,\bar{c}-(\bar{u}\cdot\bar{c})\,\bar{a}\}$$

$$=(\vec{a}\times\bar{b})\cdot\left\{\left[\bar{b}\,\bar{c}\,\bar{a}\right]\vec{c}-\left[\bar{b}\,\bar{c}\,\bar{c}\right]\vec{a}\right\}$$

$$=\left[\bar{b}\,\bar{c}\,\vec{a}\right]\left[\vec{a}\,\bar{b}\,\bar{c}\right]$$

$$=\left[\bar{a}\,\bar{b}\,\bar{c}\right]^2$$

$$\Rightarrow \quad \lambda = 1$$

12. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of sin θ is :

[JEE(Main)-2015]

(1) $\frac{2}{3}$ (2) $\frac{-2\sqrt{3}}{3}$ (3) $\frac{2\sqrt{2}}{3}$ (4) $\frac{-\sqrt{2}}{3}$

Sol :-

$$\underbrace{(\vec{a}\cdot\vec{c})\vec{b}}_{\to 0}-(\vec{b}\cdot\vec{c})\vec{a}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$ (on comparision)

$$\Rightarrow -\,\vec{b}\cdot\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|$$

$$\Rightarrow -|\vec{b}||\vec{c}|\cos\theta=\frac{1}{3}|\vec{b}||\vec{c}|$$

$$\Rightarrow \cos\theta=-\frac{1}{3}$$

$$\Rightarrow \sin\theta=\frac{2\sqrt{2}}{3}.$$

13. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\sqrt{3}}{2}\left(\vec{b} + \vec{c}\right)$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is :- **[JEE(Main)-2016]**

(1) $\frac{5\pi}{6}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{2}$ (4) $\frac{2\pi}{3}$

Sol:-

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\sqrt{3}}{2}\left(\vec{b} + \vec{c}\right)$$

$$\left(\bar{a} \cdot \bar{c}\right)\bar{b} - \left(\bar{a} \cdot \bar{b}\right)\bar{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$$

Equate $\vec{a} \cdot \vec{c} = \frac{\sqrt{3}}{2}$ & $-\left(\bar{a} \cdot \bar{b}\right) = \frac{\sqrt{3}}{2}$

$$|\bar{a}|\ |\bar{b}| \cos\theta = -\frac{\sqrt{3}}{2}$$

$\cos\theta = -\frac{\sqrt{3}}{2}$ as $\bar{a}$ & $\bar{b}$ unit vectors

$$\theta = \frac{5\pi}{6}$$

14. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $\left|(\vec{a} \times \vec{b}) \times \vec{c}\right| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30º. Then $\vec{a} \cdot \vec{c}$ is equal to : **[JEE(Main)-2017]**

(1) $\frac{1}{8}$ (2) $\frac{25}{8}$ (3) 2 (4) 5

Sol. $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$ and $|\vec{a}| = 3$

$\therefore \quad \vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = 3$

Now : $(\vec{a} \times \vec{b}) \times \vec{c} = |\vec{a} \times \vec{b}|\,|\vec{c}| \sin 30\ \hat{n}$

$|(\vec{a} \times \vec{b}) \times \vec{c}| = 3 \cdot |\vec{c}| \cdot \frac{1}{2}$

$3 = 3|\vec{c}| \cdot \frac{1}{2}$

$\therefore \quad |\vec{c}| = 2$

Now : $|\vec{c} - \vec{a}| = 3$

$c^2 + a^2 - 2\,\vec{c} \cdot \vec{a} = 9$

$4 + 9 - 2\vec{a} \cdot \vec{c} = 9$

$\vec{a} \cdot \vec{c} = 2$

15. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b}=24$, then $|\vec{u}|^2$ is equal to - **[JEE(Main)-2018]**

(1) 315 (2) 256 (3) 84 (4) 336

Sol.

$$\vec{u}=\lambda(\vec{a}\times\vec{b})\times\vec{a}$$

$$=\lambda\left\{\vec{a}^2.\vec{b}-(\vec{a}.\vec{b})\vec{a}\right\}$$

$$=\lambda\left\{-4\hat{i}+8\hat{j}+16\hat{k}\right\}$$

$$\vec{u}=\lambda'\left\{-\hat{i}+2\hat{j}+4\hat{k}\right\}$$

$$\vec{u}.\vec{b}=24$$

$$\Rightarrow \lambda'=4$$

$$\vec{u}=-4\hat{i}+8\hat{j}+16\hat{k}$$

$$\vec{u}^2=336$$

M-II

$$\vec{u}=\lambda\vec{a}+\mu\vec{b}$$

$$\vec{u}=2\lambda\hat{i}+(3\lambda+\mu)\hat{j}+(\mu-\lambda)\hat{k}$$

$$\vec{u}\cdot\vec{a}=0$$

$$\Downarrow$$

$$\Rightarrow 4\lambda+3(3\lambda+\mu)+\lambda-\mu=0$$

$$\Rightarrow 14\lambda+2\mu=0$$

$$\Rightarrow \boxed{\mu=-7\lambda} \text{ — (1)}$$

$$* \quad \vec{u}\cdot\vec{b}=24$$

$$\Rightarrow (3\lambda+\mu)+(\mu-\lambda)=24$$

$$\Rightarrow 2\lambda+2\mu=24$$

$$\Rightarrow \lambda+\mu=12$$

$$\Rightarrow \lambda-7\lambda=12$$

$$\Rightarrow \boxed{\lambda=-2}$$ Put in (1)

then $\boxed{\mu=14}$

$$\therefore \quad \vec{u}=-4\hat{i}+8\hat{j}+16\hat{k}$$

$$|\vec{u}|^2=16+64+256$$

$$=\boxed{336}$$

Ans.

16. Let $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}+\vec{b}=\vec{0}$ and $\vec{a}.\vec{c}=4$, then $|\vec{c}|^2$ is equal to :-

[JEE(Main)-Jan 19]

(1) $\frac{19}{2}$ (2) 8 (3) $\frac{17}{2}$ (4) 9

Sol. $\vec{a}\times\vec{c}=-\vec{b}$

$$(\vec{a}\times\vec{c})\times\vec{a}=-\vec{b}\times\vec{a}$$

$$\Rightarrow (\vec{a}\times\vec{c})\times\vec{a}=\vec{a}\times\vec{b}$$

$$\Rightarrow (\vec{a}.\vec{a})\vec{c}-(\vec{c}.\vec{a})\vec{a}=\vec{a}\times\vec{b}$$

$$\Rightarrow 2\vec{c}-4\vec{a}=\vec{a}\times\vec{b}$$

Now $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 0\\ 1 & 1 & 1\end{vmatrix}=-\hat{i}-\hat{j}+2\hat{k}$

So, $2\vec{c}=4\hat{i}-4\hat{j}-\hat{i}-\hat{j}+2\hat{k}$

$$=3\hat{i}-5\hat{j}+2\hat{k}$$

$$\Rightarrow \vec{c}=\frac{3}{2}\hat{i}-\frac{5}{2}\hat{j}+\hat{k}$$

$$|\vec{c}|=\sqrt{\frac{9}{4}+\frac{25}{4}+1}=\sqrt{\frac{38}{4}}=\sqrt{\frac{19}{2}}$$

$$|\vec{c}|^2=\frac{19}{2}$$

17. Let $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is :- **[JEE(Main)-Jan 19]**

(1) 2 (2) 1 (3) 3 (4) 4

Sol. Angle bisector is $x - y = 0$

$$\Rightarrow \frac{|\beta-(1-\beta)|}{\sqrt{2}}=\frac{3}{\sqrt{2}}$$

$$\Rightarrow |2\beta-1|=3$$

$$\Rightarrow \beta = 2 \text{ or } -1$$

18. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to **[JEE(Main)-Apr 19]**

(1) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (2) $3\hat{i} - 9\hat{j} - 5\hat{k}$ (3) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$ (4) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$

Sol. * $\vec{\alpha} = 3\hat{i} + \hat{j}$

given

* $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$

* $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$

$\boxed{\vec{\beta}_1 = \lambda(3\hat{i} + \hat{j})}, \boxed{\vec{\beta}_2 = \lambda(3\hat{i} + \hat{j}) - 2\hat{i} + j - 3\hat{k}}$

$\vec{\beta}_2 . \vec{\alpha} = 0$

$(3\lambda - 2).3 + (\lambda + 1) = 0$

$9\lambda - 6 + \lambda + 1 = 0$

$\lambda = \frac{1}{2}$

$\Rightarrow \vec{\beta}_1 = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$

$\Rightarrow \vec{\beta}_2 = -\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

Now $\vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{3}{2} & -3 \end{vmatrix}$

$= \hat{i}\left(-\frac{3}{2} - 0\right) - \hat{j}\left(-\frac{9}{2} - 0\right) + \hat{k}\left(\frac{9}{4} + \frac{1}{4}\right)$

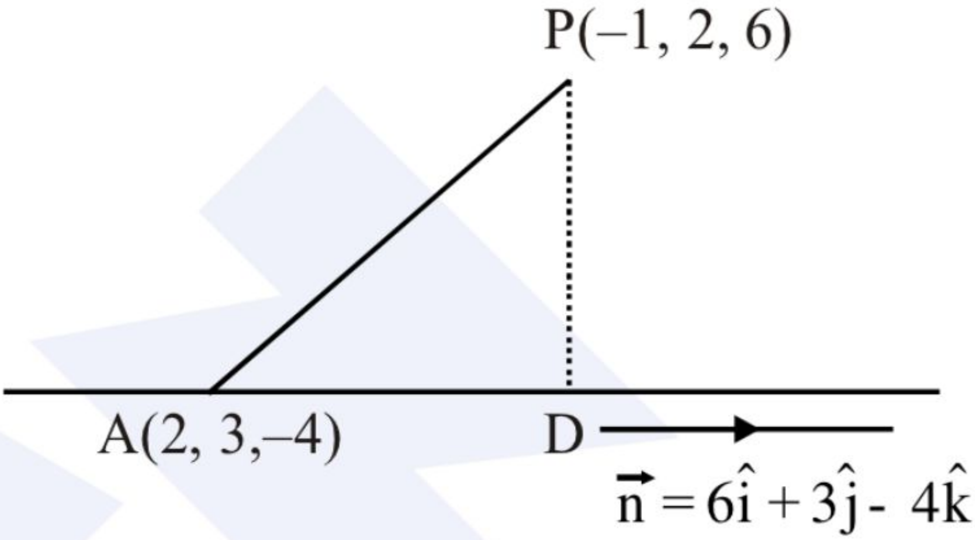
$= -\frac{3}{2}\hat{i} + \frac{9}{2}\hat{j} + \frac{5}{2}\hat{k}$

$= \frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$

19. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is : **[JEE(Main)-Apr 19]**

(1) 7 (2) $4\sqrt{3}$ (3) $2\sqrt{13}$ (4) 6

Sol.



$$AD = \left|\frac{\overrightarrow{AP}.\vec{n}}{|\vec{n}|}\right| = \sqrt{61}$$

$$\Rightarrow \quad PD = \sqrt{AP^2 - AD^2} = \sqrt{110-61} = 7$$

EXERCISE JA

EXERCISE (JA)

1. **(a)** Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by -

(A) $\frac{8}{9}$ (B) $\frac{\sqrt{17}}{9}$ (C) $\frac{1}{9}$ (D) $\frac{4\sqrt{5}}{9}$

Sol. ***(B)***

$$\overrightarrow{AD}=\overrightarrow{AB}\times(\overrightarrow{AB}\times\overrightarrow{AD})=5(61\hat{i}-10\hat{j}-21\hat{k}) \Rightarrow \cos\alpha=\frac{|\overrightarrow{AD'}\cdot\overrightarrow{AD}|}{|\overrightarrow{AD'}||\overrightarrow{AD}|}=\frac{\sqrt{17}}{9}.$$

(b) If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a}=\frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b}=\frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a}+\vec{b}).\left[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})\right]$ is **[JEE 2010, 5+3]**

Sol. **(5)**

$$E=(2\vec{a}+\vec{b})\cdot\left[2|\vec{b}|^2\vec{a}-2(\vec{a}\cdot\vec{b})\vec{b}-(\vec{a}\cdot\vec{b})\vec{a}+|\vec{a}|^2\vec{b}\right]$$

$$\vec{a}\cdot\vec{b}=\frac{2-2}{\sqrt{70}}=0$$

$$|\vec{a}|=1$$

$$|b|=1$$

$$\vec{a}\cdot\vec{b}=0$$

$$E=(2\vec{a}+\vec{b})\cdot\left[2|\vec{b}|^2\vec{a}+|\vec{a}|^2\vec{b}\right]$$

$$=4|\vec{a}|^2|\vec{b}|^2+|\vec{a}|^2(\vec{a}\cdot\vec{b})+2|\vec{b}|^2(\vec{b}\cdot\vec{a})+|\vec{a}|^2|\vec{b}|^2$$

$$=5|\vec{a}|^2|\vec{b}|^2=5$$

2. **(a)** Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(A) $\hat{i}-3\hat{j}+3\hat{k}$ (B) $-3\hat{i}-3\hat{j}-\hat{k}$ (C) $3\hat{i}-\hat{j}+3\hat{k}$ (D) $\hat{i}+3\hat{j}-3\hat{k}$

Sol. **(C)**

$\vec{v}=\lambda\bar{a}+\mu\bar{b}$

$=\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$

Projection of $\bar{v}$ on $\bar{c}$

$$\frac{\bar{v}\cdot\bar{c}}{|\bar{c}|}=\frac{1}{\sqrt{3}} \Rightarrow \frac{\left[(\lambda+\mu)\hat{i}+(\lambda-\mu)\hat{j}+(\lambda+\mu)\hat{k}\right]\cdot\left(\hat{i}-\hat{j}-\hat{k}\right)}{\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$\Rightarrow \lambda+\mu-\lambda+\mu-\lambda-\mu=1 \Rightarrow \mu-\lambda=1 \Rightarrow \lambda=\mu-1$

$\bar{v}=(\mu-1)\left(\hat{i}+\hat{j}+\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right) = \mu\left(2\hat{i}+2\hat{k}\right)-\hat{i}-\hat{j}-\hat{k}$

$\bar{v}=(2\mu-1)\hat{i}-\hat{j}+(2\mu-1)\hat{k}$

At $\mu=2,\ \bar{v}=3\hat{i}-\hat{j}+3\hat{k}$.

(b) The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are

(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

Sol. **(A, D)**

Any vector in the plane of $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ is $\vec{r}=(\lambda+\mu)\hat{i}+(\lambda+2\mu)\hat{j}+(2\lambda+\mu)\hat{k}$

Also, $\vec{r}\cdot\vec{c}=0 \Rightarrow \lambda+\mu=0$

$\Rightarrow \left[\vec{r}\ \vec{a}\ \vec{b}\right]=0$.

(c) Let $\vec{a}=-\hat{i}-\hat{k}$, $\vec{b}=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$, then the value of $\vec{r}.\vec{b}$ is **[JEE 2011, 3+4+4]**

Sol. **(9)**

$\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$

taking cross with a

$\vec{a}\times(\vec{r}\times\vec{b})=\vec{a}\times(\vec{c}\times\vec{b})$

$(\vec{a}\cdot\vec{b})\vec{r}-(\vec{a}\cdot\vec{r})\vec{b}=\vec{a}\times(\vec{c}\times\vec{b})$

$\Rightarrow \vec{r}=-3\hat{i}+6\hat{j}+3\hat{k}$

$\vec{r}\cdot\vec{b}=3+6=9$.

3. **(a)** If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

Sol. **(3)**

As, $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2=3\left(\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2\right)-\left|\vec{a}+\vec{b}+\vec{c}\right|^2$

$\Rightarrow 3\times 3-\left|\vec{a}+\vec{b}+\vec{c}\right|^2=9$

$\Rightarrow \left|\vec{a}+\vec{b}+\vec{c}\right|=0 \Rightarrow \vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow \left|2\vec{a}+5\left(\vec{b}+\vec{c}\right)\right|=\left|-3\vec{a}\right|=3\left|\vec{a}\right|=3$.

(b) If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}|=\sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})=(2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$ is

(A) 0 (B) 3 (C) 4 (D) 8

[JEE 2012, 4+3]

Sol. **(C)**

$\vec{a}\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\times\vec{b}$

$\left(\vec{a}+\vec{b}\right)\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=\vec{0}$

$\Rightarrow \vec{a}+\vec{b}=\pm\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ (as $\left|\vec{a}+\vec{b}\right|=\sqrt{29}$)

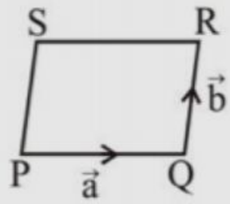
$\Rightarrow \left(\vec{a}+\vec{b}\right)\cdot\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$

$=\pm(-14+6+12)=\pm 4$.

4. Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is **[JEE-Advanced 2013, 2M]**

(A) 5 (B) 20 (C) 10 (D) 30

Sol. $\vec{a} + \vec{b} = \overline{PR}$ & $\vec{a} - \vec{b} = \overline{QS}$



$\vec{a} = \frac{\overline{PR} + \overline{QS}}{2}$ & $\vec{b} = \frac{\overline{PR} - \overline{QS}}{2}$

$\vec{a} = 2\hat{i} - \hat{j} - 3\hat{k}$ & $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

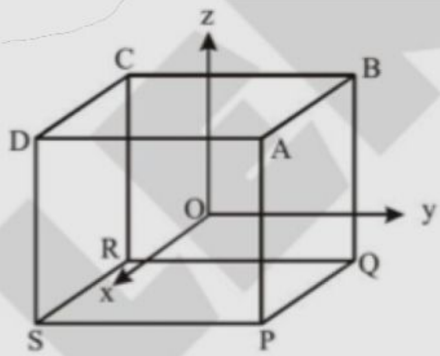
$$\text{Volume} = \begin{vmatrix} 2 & -1 & -3 \\ 1 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$2(4) + (3-1) - 3(2-2)$

$8 + 2 = 10$

5. Consider the set of eight vectors $V = \left\{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\right\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is **[JEE-Advanced 2013, 4, (–1)]**

Sol The 8 vectors will represent $\overline{OA}, \overline{OB} ...... \overline{OD}, \overline{OP}, ... \overline{OS}$ any three out of these 8 will be coplanar when two of them are collinear. There are 4 pairs of collinear vectors



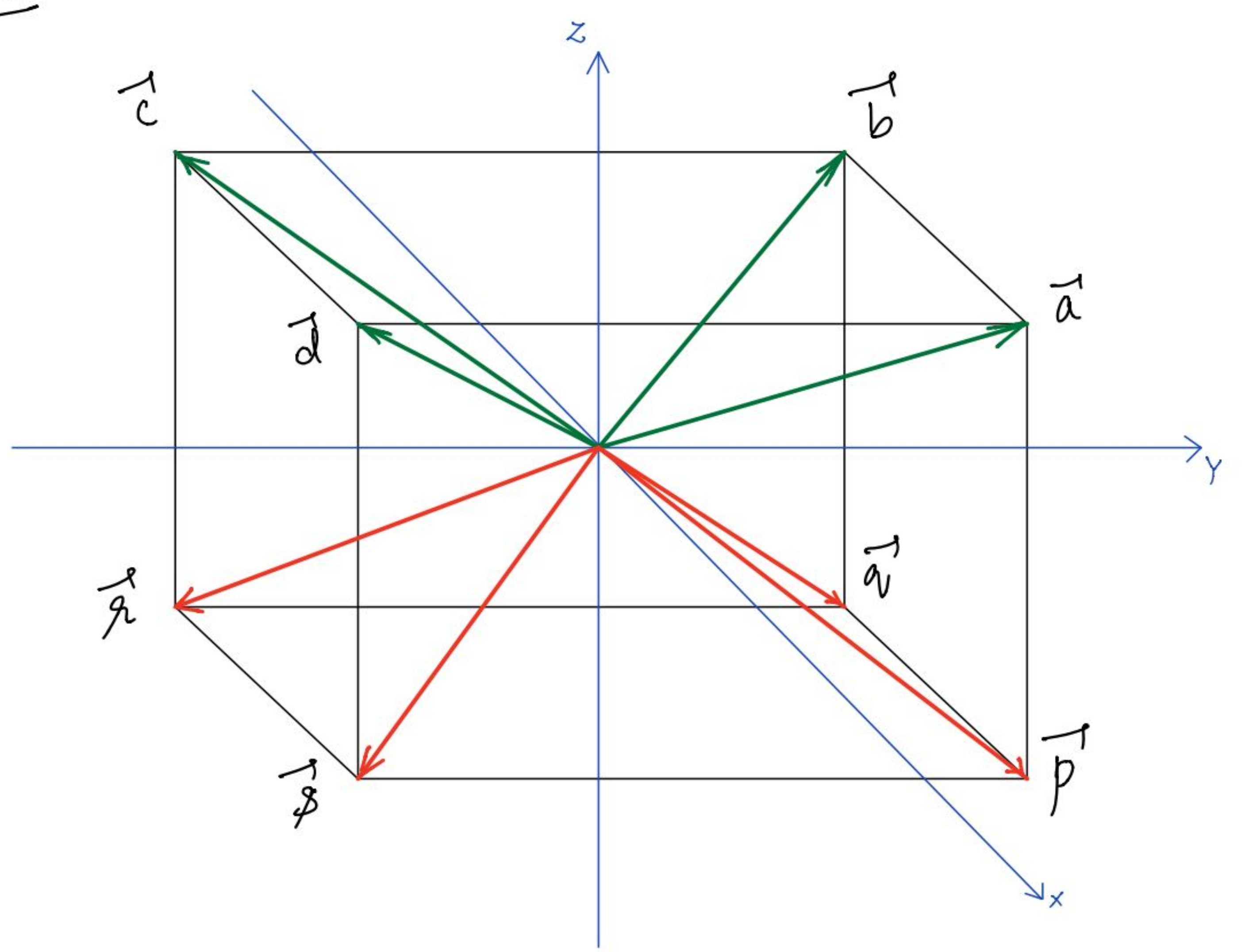
O is at the centre of cube ABCDPQRS

$\overline{OA}$ & $\overline{OR}$, $\overline{OB}$ & $\overline{OS}$, $\overline{OC}$ & $\overline{OP}$, $\overline{OD}$ & $\overline{OQ}$ (it will generate $4 \times 6 = 24$ set of coplanar vectors) rest of the combination of 3 vectors will form three edges of a tetrahedron so they will be not coplanar.

So number of non-coplanar vectors

${}^{8}C_3 - 4.6 = 32$

M2



We divide these 8 vectors in 2 sets of 4 as shown (Above and below x-y plane)

$$\text{Ans}: \underbrace{\text{total ways to select 3 vectors}}_{{}^8C_3} - \underbrace{\text{total ways to select 3 coplaner vectors}}_{K}$$

For K: If we select 2 vectors out of 4 green, then corresponding to these 2 selected green, we have to select one red out of 2 available red to make all three selected vectors coplaner.

Similarly, we can choose 2 vectors from red vectors and one from green vectors

ex: (i) If we select $\vec{a}$ & $\vec{b}$ from green, then we can select one from $\vec{r}$ or $\vec{s}$ and these three selected vectors are coplaner

ex: (ii) If we select $\vec{d}$ & $\vec{b}$ from green, then we can select one from $\vec{q}$ or $\vec{s}$ and these three selected vectors are coplaner

$$\therefore K = \underbrace{{}^4C_2}_{\text{Selecting 2 from 4 vectors}} \times \underbrace{{}^2C_1}_{\text{selecting 1 from corresponding 2 vectors}} \times \underbrace{2}_{\text{Reversing the order of selection}}$$

$$\therefore \text{Ans}: {}^8C_3 - {}^4C_2 \times {}^2C_1 \times 2 = 56 - 24 = \boxed{32}$$

6. Match List-I with List-II and select the correct answer using the code given below the lists.

	List-I		List-II
P.	Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a}\times\vec{b}), 3(\vec{b}\times\vec{c})$ and $(\vec{c}\times\vec{a})$ is	1.	100
Q.	Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a}+\vec{b}), (\vec{b}+\vec{c})$ and $2(\vec{c}+\vec{a})$ is	2.	30
R.	Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a}+3\vec{b})$ and $(\vec{a}-\vec{b})$ is	3.	24
S.	Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a}+\vec{b})$ and $\vec{a}$ is	4.	60

Codes :

	P	Q	R	S
(A)	4	2	3	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	4	3	2

[JEE-Advanced 2013, 3, (–1)]

Sol. (P) Given $[\vec{a}\ \vec{b}\ \vec{c}] = 2$

$$[2(\vec{a}\times\vec{b})\ 3(\vec{b}\times\vec{c})\ (\vec{c}\times\vec{a})] = 6[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$$
$$= 6[\vec{a}\ \vec{b}\ \vec{c}]^2 = 24$$

(Q) Given $[\vec{a}\ \vec{b}\ \vec{c}] = 5$

$$[3(\vec{a}+\vec{b})\ (\vec{b}+\vec{c})\ 2(\vec{c}+\vec{a})] = 12[\vec{a}\ \vec{b}\ \vec{c}] = 60$$

(R) Given $\frac{1}{2}|\vec{a}\times\vec{b}| = 20 \quad \Rightarrow \quad |\vec{a}\times\vec{b}| = 40$

$$\left|\frac{1}{2}(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})\right| = \frac{1}{2}\left|0+3\vec{b}\times\vec{a}-2\vec{a}\times\vec{b}\right|$$
$$= \frac{1}{2}|-5\vec{a}\times\vec{b}| = \frac{5}{2}|\vec{a}\times\vec{b}| = \frac{5}{2}.40 = 100$$

(S) Given $|\vec{a}\times\vec{b}| = 30$

$$|(\vec{a}+\vec{b})\times\vec{a}| = |0+\vec{b}\times\vec{a}| = 30$$

7. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is nonzero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then **[JEE(Advanced)-2014, 3]**

(A) $\vec{b}=(\vec{b}.\vec{z})(\vec{z}-\vec{x})$ (B) $\vec{a}=(\vec{a}.\vec{y})(\vec{y}-\vec{z})$ (C) $\vec{a}.\vec{b}=-(\vec{a}.\vec{y})(\vec{b}.\vec{z})$ (D) $\vec{a}=(\vec{a}.\vec{y})(\vec{z}-\vec{y})$

Sol. Ans. (A,B,C)

Given that $|\vec{x}|=|\vec{y}|=|\vec{z}|=\sqrt{2}$

and angle between each pair is $\frac{\pi}{3}$

$\therefore \quad \vec{x}.\vec{y}=\vec{y}.\vec{z}=\vec{z}.\vec{x}=\sqrt{2}.\sqrt{2}.\frac{1}{2}=1$

Now $\vec{a}$ is $\perp$ to $\vec{x}$ & $(\vec{y}\times\vec{z})$

Let $\vec{a}=\lambda(\vec{x}\times(\vec{y}\times\vec{z}))$

$=\lambda((\vec{x}.\vec{z})\vec{y}-(\vec{x}.\vec{y})\vec{z})=\lambda(\vec{y}-\vec{z})$

$\vec{a}.\vec{y}=\lambda(\vec{y}.\vec{y}-\vec{y}.\vec{z})=\lambda(2-1)=\lambda$

$\Rightarrow \quad \vec{a}=(\vec{a}.\vec{y})(\vec{y}-\vec{z})$

Now let $\vec{b}=\mu\left(\vec{y}\times(\vec{z}\times\vec{x})\right)=\mu(\vec{z}-\vec{x})$

$\vec{b}.\vec{z}=\mu(2-1)=\mu$

$\Rightarrow \quad \vec{b}=(\vec{b}.\vec{z})(\vec{z}-\vec{x})$

Now $\vec{a}.\vec{b}=(\vec{a}.\vec{y})(\vec{y}-\vec{z}).(\vec{b}.\vec{z})(\vec{z}-\vec{x})$

$=(\vec{a}.\vec{y})(\vec{b}.\vec{z})(\vec{y}.\vec{z}-\vec{y}.\vec{x}-\vec{z}.\vec{z}+\vec{z}.\vec{x})$

$=(\vec{a}.\vec{y})(\vec{b}.\vec{z})(1-1-2+1)$

$=-(\vec{a}.\vec{y})(\vec{b}.\vec{z})$

8. Let $\vec{a}, \vec{b}$, and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$, where p,q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is

[JEE(Advanced)-2014, 3]

Sol. Ans. 4

We know $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c}\\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c}\end{vmatrix}$

$$=\begin{vmatrix}1 & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 1 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & 1\end{vmatrix}=\frac{5}{4}-\frac{3}{4}=\frac{1}{2}$$

$\therefore \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]=\frac{1}{\sqrt{2}}$ ...(1)

as given $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$

take dot product with $\vec{a}$

$\Rightarrow \vec{a}.\left(\vec{a}\times\vec{b}\right)+\vec{a}.\left(\vec{b}\times\vec{c}\right)=p\vec{a}^2+q\vec{b}.\vec{a}+r\vec{c}.\vec{a} \Rightarrow 0+\frac{1}{\sqrt{2}}=p+\frac{q}{2}+\frac{r}{2}$ ...(2)

Now, take dot product with $\vec{b}$ & $\vec{c}$

$0=\frac{p}{2}+q+\frac{r}{2}$ ...(3)

& $\frac{1}{\sqrt{2}}=\frac{p}{2}+\frac{q}{2}+r$ ...(4)

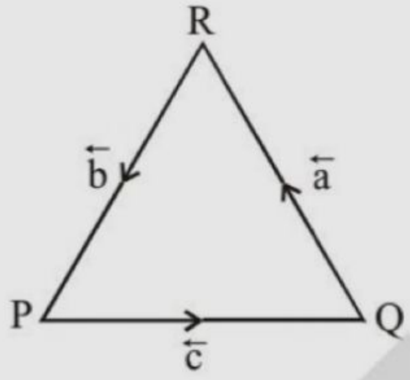
equation (2) – equation (4) $\Rightarrow \frac{p}{2}-\frac{r}{2}=0\Rightarrow p=r\Rightarrow p+q=0$ by equation (3)

$\therefore \frac{p^2+2q^2+r^2}{q^2}=\frac{p^2+2p^2+p^2}{p^2}=4$

9. Let ΔPQR be a triangle. Let $\vec{a}=\overrightarrow{QR}, \vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=12, |\vec{b}|=4\sqrt{3}$ and $\vec{b}.\vec{c}=24$, then which of the following is (are) true ? **[JEE 2015, 4M, –2M]**

(A) $\dfrac{|\vec{c}|^2}{2}-|\vec{a}|=12$ (B) $\dfrac{|\vec{c}|^2}{2}+|\vec{a}|=30$ (C) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$ (D) $\vec{a}.\vec{b}=-72$

Sol. $|\vec{a}|=12, |\vec{b}|=4\sqrt{3}$



$\vec{a}+\vec{b}+\vec{c}=0$ ....(1)

$a^2 = b^2 + c^2 + 2\vec{b}.\vec{c}$

$144 = 48 + c^2 + 48$

$c^2 = 48 \Rightarrow c = 4\sqrt{3}$

Also $c^2 = a^2 + b^2 + 2\vec{a}.\vec{b}$

$48 = 144 + 48 + 2\vec{a}.\vec{b}$

$\Rightarrow \vec{a}.\vec{b} = -72$

Also by (1) $\left\{ \vec{b} = -(\vec{a}+\vec{c}) \Rightarrow \vec{a}\times\vec{b} = -(\underbrace{\vec{a}\times\vec{a}}_{0} + \vec{a}\times\vec{c}) \Rightarrow \vec{a}\times\vec{b} = \vec{c}\times\vec{a} \right.$

$\vec{a}\times\vec{b}=\vec{c}\times\vec{a}$

option C $\Rightarrow |\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=2|\vec{a}\times\vec{b}|$

$=2\sqrt{a^2b^2-(\vec{a}.\vec{b})^2}$

$=2\sqrt{12^2.48-(72)^2}$

$=2.12\sqrt{48-36}=48\sqrt{3}$

$\therefore$ A,C,D are correct & B incorrect

10. Suppose that $\vec{p}, \vec{q}$ and $\vec{r}$ are three non-coplanar vectors in $\mathbb{R}^3$. Let the components of a vector $\vec{s}$ along $\vec{p}, \vec{q}$ and $\vec{r}$ be 4,3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p}+\vec{q}+\vec{r})$, $(\vec{p}-\vec{q}+\vec{r})$ and $(-\vec{p}-\vec{q}+\vec{r})$ are x,y and z, respectively, then the value of 2x + y + z is

[JEE 2015, 4M, –0M]

Ans. $\boxed{\text{Bonus}}$

Sol. Although the language of the question is not appropriate (in complete information) and it must be declare as bonus but as per the theme of problem it must be as follows

$\vec{s} = 4\vec{p} + 3\vec{q} + 5\vec{r}$

$\vec{s} = x(-\vec{p}+\vec{q}+\vec{r}) + y(\vec{p}-\vec{q}+\vec{r}) + z(-\vec{p}-\vec{q}+\vec{r})$

$\vec{s} = \vec{p}(-x+y-z) + \vec{q}(x-y-z) + \vec{r}(x+y+z)$

$-x + y - z = 4$

$x - y - z = 3$

$x + y + z = 5$

$\Rightarrow x = 4, y = \frac{9}{2}, z = -\frac{7}{2}$

$\Rightarrow 2x + y + z = 8 - \frac{7}{2} + \frac{9}{2} = 9$

11. Let $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in $\mathbb{R}^3$ and $\hat{w} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$. Given that there exists a vector $\vec{v}$ in $\mathbb{R}^3$ such that $|\hat{u} \times \vec{v}| = 1$ and $\hat{w}.(\hat{u} \times \vec{v}) = 1$. Which of the following statement(s) is(are) correct?

(A) There is exactly one choice for such $\vec{v}$

(B) There are infinitely many choice for such $\vec{v}$

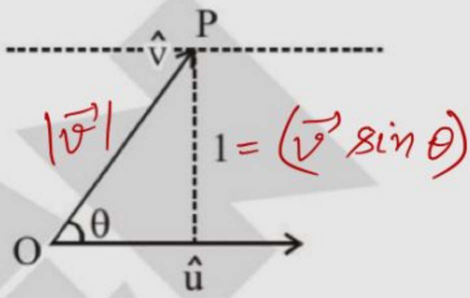
(C) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$

(D) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$ **[JEE(Advanced)-2016, 4(–2)]**

Sol. $|\hat{w}||\hat{u} \times \hat{v}|\cos\phi = 1 \Rightarrow \phi = 0$

$\Rightarrow$ $\underline{\hat{u} \times \vec{v} = \hat{w}}$ also $|\vec{v}|\sin\theta = 1$

$\Rightarrow$ there may be infinite vectors $\vec{v} = \overline{OP}$ such that P is always 1 unit dist. from $\hat{u}$



$$\left\{ \begin{array}{l} \because |\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta \\ 1 = 1.|\vec{v}|\sin\theta \\ \underline{|\vec{v}|\sin\theta = 1} \end{array} \right\}$$

* For option (C) : $\hat{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & 0 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$$\hat{w} = (u_2v_3)\hat{i} - (u_1v_3)\hat{j} + (u_1v_2 - u_2v_1)\hat{k}$$

$$u_2v_3 = \frac{1}{\sqrt{6}}, -u_1v_3 = \frac{1}{\sqrt{6}} \quad \Rightarrow \quad |u_1| = |u_2|$$

* for option (D) : $\hat{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & 0 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$$\hat{w} = (-v_2u_3)\hat{i} - (u_1v_3 - u_3v_1)\hat{j} + (u_1v_2)\hat{k}$$

$$-v_2u_3 = \frac{1}{\sqrt{6}}, u_1v_2 = \frac{2}{\sqrt{6}}$$

$\Rightarrow$ $2|u_3| = |u_1|$ So (D) is wrong

12. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that $\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$. Then the triangle PQR has S as its

[JEE(Advanced)-2017]

(A) incentre (B) orthocenter (C) circumcentre (D) centroid

Sol. Let position vector of $P(\vec{p})$, $Q(\vec{q})$, $R(\vec{r})$ & $S(\vec{s})$ with respect to $O(\vec{o})$

Now, $\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS}$

$\Rightarrow \quad \vec{p}.\vec{q}+\vec{r}.\vec{s}=\vec{r}.\vec{p}+\vec{q}.\vec{s}$

$\Rightarrow \quad (\vec{p}-\vec{s}).(\vec{q}-\vec{r})=0$ .....(1)

Also, $\overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$

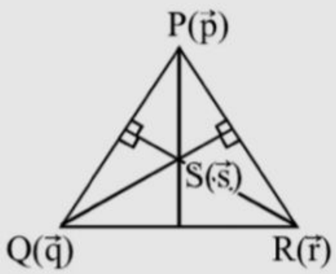
$\Rightarrow \quad \vec{r}.\vec{p}+\vec{q}.\vec{s}=\vec{q}.\vec{r}+\vec{p}.\vec{s}$

$\Rightarrow \quad (\vec{r}-\vec{s}).(\vec{p}-\vec{q})=0$ ......(2)

Also, $\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$

$\Rightarrow \quad \vec{p}.\vec{q}+\vec{r}.\vec{s}=\vec{q}.\vec{r}+\vec{p}.\vec{s}$

$\Rightarrow \quad (\vec{q}-\vec{s}).(\vec{p}-\vec{r})=0$ ......(3)



$\Rightarrow$ Triangle PQR has S as its orthocentre

$\therefore$ option (B) is correct.

PARAGRAPH :

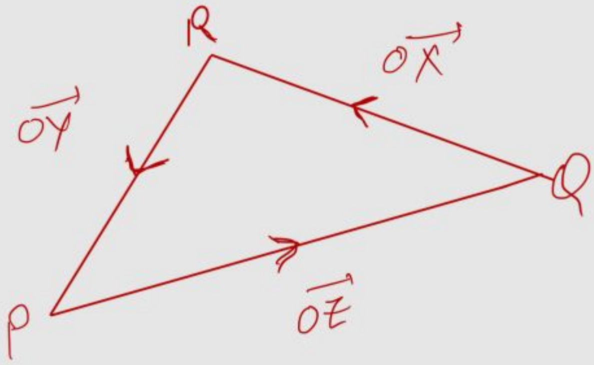
Let O be the origin, and $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}, \overrightarrow{RP}, \overrightarrow{PQ}$, respectively, of a triangle PQR. **[JEE(Advanced)-2017]**

13. $\left|\overrightarrow{OX} \times \overrightarrow{OY}\right| =$

(A) $\sin(Q + R)$ (B) $\sin(P + R)$ (C) $\sin 2R$ (D) $\sin(P + Q)$

Ans. (D)

Sol. $\overrightarrow{OX} = \dfrac{\overrightarrow{QR}}{QR}$



$\overrightarrow{OY} = \dfrac{\overrightarrow{RP}}{RP}$

$\left|\overrightarrow{OX} \times \overrightarrow{OY}\right| = \sin R = \sin\left(P + Q\right)$

14. If the triangle PQR varies, then the minimum value of $\cos(P + Q) + \cos(Q + R) + \cos(R + P)$ is

(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $\frac{5}{3}$ (D) $-\frac{5}{3}$

Ans. **(B)**

$\because C_P + C_Q + C_R \leq 3/2$

Sol. $-(\cos P + \cos Q + \cos R) \geq -\frac{3}{2}$ as we know $\cos P + \cos Q + \cos R$ will take its maximum value when $P = Q = R = \frac{\pi}{3}$

15. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}.\vec{b} = 0$. For some x, y $\in \mathbb{R}$, let $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is ______ **[JEE(Advanced)-2018, 3(0)]**

Sol. $\vec{c} = x\vec{a} + y\vec{b} + \vec{a} \times \vec{b}$

$\vec{c}.\vec{a} = x$ and $x = 2\cos\alpha$

$\vec{c}.\vec{b} = y$ and $y = 2\cos\alpha$

Also, $|\vec{a} \times \vec{b}| = 1$

$\therefore \quad \vec{c} = 2\overset{\cos\alpha.}{\cancel{\cos}}(\vec{a} + \vec{b}) + \vec{a} \times \vec{b}$

$$\vec{c}^{\,2} = 4\cos^2\alpha(\vec{a} + \vec{b})^2 + (\vec{a} \times \vec{b})^2 + 2\cos\alpha\,(\vec{a} + \vec{b}).(\vec{a} \times \vec{b})$$

$$4 = 8\cos^2\alpha + 1$$

$$8\cos^2\alpha = 3$$

16. Three lines $L_1 : \vec{r} = \lambda\hat{i},\ \lambda \in \mathbb{R}$,

$L_2 : \vec{r} = \hat{k} + \mu\hat{j},\ \mu \in \mathbb{R}$ and

$L_3 : \vec{r} = \hat{i} + \hat{j} + \nu\hat{k},\ \nu \in \mathbb{R}$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear ? **[JEE(Advanced)-2019, 4(–1)]**

(1) $\hat{k} + \hat{j}$ (2) $\hat{k}$ (3) $\hat{k} + \frac{1}{2}\hat{j}$ (4) $\hat{k} - \frac{1}{2}\hat{j}$

Sol. Let $P(\lambda, 0, 0)$, $Q(0, \mu, 1)$, $R(1, 1, \nu)$ be points. L_1, L_2 and L_3 respectively

Since P, Q, R are collinear, $\overrightarrow{PQ}$ iscollinear with $\overrightarrow{QR}$

Hence $\frac{-\lambda}{1} = \frac{\mu}{1-\mu} = \frac{1}{\nu-1}$

For every $\mu \in R - \{0, 1\}$ there exist unique $\lambda, \nu \in R$

Hence Q cannot have coordinates (0, 1, 1) and (0, 0, 1).

17. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}$, $\alpha, \beta \in \mathbb{R}$. If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b})).\vec{c}$ equals

[JEE(Advanced)-2019, 3(0)]

Sol. $\vec{c} = (2\alpha + \beta)\hat{i} + \hat{j}(\alpha + 2\beta) + \hat{k}(\beta - \alpha)$

$$\frac{\vec{c}.(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = 3\sqrt{2}$$

$\Rightarrow \quad \alpha + \beta = 2 \quad$ .... (1)

$$(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c} = \vec{c} \cdot \vec{c} - \underbrace{(\vec{a} \times \vec{b}) \cdot \vec{c}}_{0} = c^2$$

$\{\because \vec{c} = \alpha\vec{a} + \beta\vec{b} \Rightarrow \vec{c}$ is coplaner with $\vec{a}$ and $\vec{b}$ and hence $\perp$ to $\vec{a} \times \vec{b}\}$

$\vec{c} = \alpha\vec{a} + \beta\vec{b} \Rightarrow \quad |\vec{c}|^2 = \alpha^2 |\vec{a}|^2 + \beta^2 |b|^2 + 2\alpha\beta(\vec{a}.\vec{b})$

$= 6(\alpha^2 + \beta^2 + \alpha\beta)$

$|\vec{c}|^2 = 6(\alpha^2 + (2 - \alpha)^2 + \alpha(2 - \alpha)) \quad (\because \beta = 2 - \alpha)$

$= 6\,((\alpha - 1)^2 + 3)$

$\Rightarrow \quad$ Min. value $= 18$