LIMIT - (L)

Complete Solutions Of DYS & Exercises

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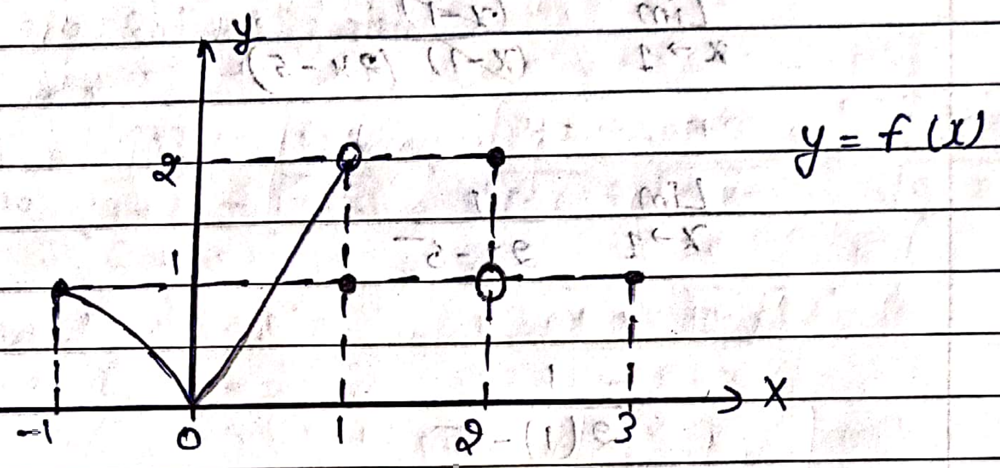
Subject

Do yourself ①

① Which of the following statements about the function $y = f(x)$ graphed here are true and which are false?



Ⓐ $\lim_{x \to -1^+} f(x) = 1$ $\boxed{T}$

Ⓑ $\lim_{x \to 2} f(x) = D.N.E.$ $\boxed{F}$

Ⓒ $\lim_{x \to 2} f(x) = 2$ $\boxed{F}$

Ⓓ $\lim_{x \to 1^-} f(x) = 2$ $\boxed{T}$

Ⓔ $\lim_{x \to 1} f(x)$ D.N.E. $\boxed{T}$

Ⓕ $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$ $\boxed{T}$

Ⓖ $\lim_{x \to c} f(x)$ exist at every $c \in (-1, 1)$ $\boxed{T}$

Ⓗ $\lim_{x \to c} f(x)$ exists at every $c \in (1, 3)$ $\boxed{T}$

Ⓘ $\lim_{x \to 1^-} f(x) = 0$ $\boxed{F}$

Ⓙ $\lim_{x \to 3^+} f(x)$ DNE. $\boxed{T}$

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Do your self (2)

(i) Evaluate $\lim_{x \to 1} \dfrac{x-1}{2x^2-7x+5}$

$$\lim_{x \to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(2x-5)}$$

$$\lim_{x \to 1} \frac{1}{2x-5}$$

$$\frac{1}{2(1)-5}$$

$$\boxed{-\frac{1}{3}}$$

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Do your self (2)

(i) Evaluate :- $\lim_{x \to 0} \dfrac{\sqrt{p+x} - \sqrt{p-x}}{\sqrt{q+x} - \sqrt{q-x}}$

By Rationalization

$$\lim_{x \to 0} \frac{\sqrt{p+x} - \sqrt{p-x}}{\sqrt{q+x} - \sqrt{q-x}} \times \frac{\sqrt{p+x} + \sqrt{p-x}}{\sqrt{p+x} + \sqrt{p-x}} \times \frac{\sqrt{q+x} + \sqrt{q-x}}{\sqrt{q+x} + \sqrt{q-x}}$$

$$\lim_{x \to 0} \frac{\cancel{p}+x-\cancel{p}+x}{\sqrt{p+x} + \sqrt{p-x}} \times \frac{\sqrt{q+x} + \sqrt{q-x}}{\cancel{q}+x-\cancel{q}+x}$$

$$\lim_{x \to 0} \frac{\cancel{2x}}{\cancel{2x}} \frac{\sqrt{q+x} + \sqrt{q-x}}{\sqrt{p+x} + \sqrt{p-x}}$$

$$\frac{\sqrt{q} + \sqrt{q}}{\sqrt{p} + \sqrt{p}}$$

$$\frac{\cancel{2}\sqrt{q}}{\cancel{2}\sqrt{p}}$$

$$\boxed{\sqrt{\frac{q}{p}}}$$

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(ii) Evaluate : $\lim\limits_{x\to a} \dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$, $a \neq 0$

By Rationalization

$$\lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}$$

$$\lim_{x\to a} \frac{a+2x-3x}{\sqrt{a+2x}+\sqrt{3x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{3a+x-4x}$$

$$\lim_{x\to a} \frac{\cancel{(a-x)}\,\sqrt{3a+x}+2\sqrt{x}}{3\,\cancel{(a-x)}\,\sqrt{a+2x}+\sqrt{3x}}$$

$$\frac{1}{3}\ \frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$\frac{1}{3} \times \frac{4}{2\sqrt{3}} \times \frac{\sqrt{a}}{\sqrt{a}}$$

$$\boxed{\frac{2}{3\sqrt{3}}}$$

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(iii) If $G(x) = -\sqrt{25-x^2}$ then find the $\lim_{x\to 1}\left[\frac{G(x)-G(1)}{x-1}\right]$

$$\lim_{x\to 2} \frac{-\sqrt{25-x^2}+\sqrt{24}}{x-1}$$

By Rationalization

$$\lim_{x\to 1} \frac{-\sqrt{25-x^2}+\sqrt{24}}{x-1} \times \frac{x+1}{x+1} \times \frac{\sqrt{24}+\sqrt{25-x^2}}{\sqrt{24}+\sqrt{25-x^2}}$$

$$\lim_{x\to 1} \frac{(x+1)\ (24-25+x^2)}{x^2-1\ (\sqrt{24}+\sqrt{25-x^2})}$$

$$\lim_{x\to 2} \frac{(x+1)\ \cancel{(x^2-1)}}{\cancel{(x^2-1)}\ (\sqrt{24}+\sqrt{25-x^2})}$$

$$\frac{2}{\sqrt{24}+\sqrt{24}}$$

$$\frac{\cancel{2}}{\cancel{2}\sqrt{24}}$$

$$\boxed{\frac{1}{\sqrt{24}}}$$

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Do your self (4)

(i) Evaluate: $\displaystyle \lim_{n\to\infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!}$

$$\lim_{n\to\infty} \frac{(n+2)\,(n+1)! + (n+1)!}{(n+2)\,(n+1)! - (n+1)!}$$

$$\lim_{n\to\infty} \frac{\cancel{(n+1)!}\,[n+2+1]}{\cancel{(n+1)!}\,[n+2-1]}$$

$$\lim_{n\to\infty} \frac{n+3}{n+1}$$

$$\lim_{n\to\infty} \frac{\cancel{n}\left[1+3/n\right]}{\cancel{n}\left[1+1/n\right]}$$

$$\boxed{1}$$

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(ii) Evaluate: $\lim\limits_{n\to\infty} \left[n - \sqrt{n^2+n}\right]$

By Rationalization

$$\lim_{n\to\infty} \left[n - \sqrt{n^2+n}\right] \times \frac{\left[n+\sqrt{n^2+n}\right]}{\left[n+\sqrt{n^2+n}\right]}$$

$$\lim_{n\to\infty} \frac{\cancel{n^2} - \cancel{n^2} - n}{n+\sqrt{n^2+n}}$$

$$\lim_{n\to\infty} \frac{-n}{n + n\sqrt{1+1/n}}$$

$$\lim_{n\to\infty} \frac{-\cancel{n}}{\cancel{n}\left(1+\sqrt{1+\frac{1}{n}}\right)}$$

$$\boxed{\frac{-1}{2}}$$

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Do your self (5)

(1) Evaluate :-

(A) $\lim_{x \to 0} \frac{\sin \alpha x}{\tan \beta x}$

$$\lim_{x \to 0} \frac{\frac{\sin \alpha x}{\alpha x} \times \alpha x}{\frac{\tan \beta x}{\beta x} \times \beta x}$$

$$\lim_{x \to 0} \frac{\alpha \cancel{x}}{\beta \cancel{x}}$$

$$\boxed{\frac{\alpha}{\beta}}$$

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(B) $$\lim_{x \to y} \frac{\sin^2 x - \sin^2 y}{x^2 - y^2}$$

$$\lim_{x \to y} \frac{\sin(x+y)}{(x+y)} \cdot \frac{\sin(x-y)}{(x-y)}$$

$$\lim_{x \to y} \frac{\sin(x+y)}{(x+y)}$$

$$\boxed{\frac{\sin 2y}{2y}}$$

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(c) $\lim_{h\to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$

Which is also cane be written as

$\lim_{h\to 0} \frac{(a^2+h^2+2ah)\sin(a+h) - a^2 \sin a}{h}$

$\lim_{h\to 0} \frac{a^2[\sin(a+h) - \sin a]}{h} + \lim_{h\to 0} \frac{(h^2+2ah)\sin(a+h)}{h}$

$\lim_{h\to 0} \frac{a^2\left[2\cos\left(a+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]}{h} + \lim_{h\to 0} \frac{\cancel{h}[\sin(a+h)(h+2a)]}{\cancel{h}}$

$\lim_{h\to 0} a^2 \cos(a+h/2) \frac{\sin(h/2)}{(h/2)} + 2a\sin a$

$\boxed{a^2\cos a + 2a\sin a}$

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Do your self (6)

(i) Evaluate ; $\lim_{x \to a} \frac{e^x - e^a}{x - a}$

$x \to a + h \quad ; \quad h \to 0$

$$\lim_{h \to 0} \frac{e^{a+h} - e^a}{\cancel{a} + h - \cancel{a}}$$

$$\lim_{h \to 0} \frac{e^{a+h} - e^a}{h}$$

$$e^a \lim_{h \to 0} \frac{e^{\cancel{a}+h-\cancel{a}} - 1}{h}$$

$$e^a \lim_{h \to 0} \frac{e^h - 1}{h} \qquad \because \lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

$$\boxed{e^a}$$

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(ii) Evaluate : $\lim_{x \to 0} \dfrac{2^x - 1}{(1+x)^{1/2} - 1}$

$$\lim_{x \to 0} \frac{2^x - 1}{\cancel{1} + \frac{1}{2} \cdot x + \cdots - \cancel{1}}$$

$$\lim_{x \to 0} \frac{2^x - 1}{\frac{x}{2}}$$

$$\lim_{x \to 0} 2 \cdot \frac{2^x - 1}{x}$$

$\because \lim_{x \to 0} \dfrac{a^x - 1}{x} = \ln a \quad (a > 0)$

$\therefore \boxed{2 \cdot \ln 2}$

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Do your self (7)

(i) Evaluate :- $\lim_{x \to \infty} x\left\{\ln(x+a) - \ln x\right\}$

$x \to \frac{1}{y}$, $y \to 0$

$\lim_{y \to 0} \frac{1}{y}\left\{\ln\left(\frac{1}{y}+a\right) - \ln\left(\frac{1}{y}\right)\right\}$

$\lim_{y \to 0} \frac{1}{y}\left\{\ln\left(\frac{1+ay}{y}\right) - \ln\left(\frac{1}{y}\right)\right\}$

$\lim_{y \to 0} \frac{1}{y}\left\{\ln(1+ay) \cancel{-\ln y} - \cancel{\ln 1} + \cancel{\ln y}\right\}$ 0

$\lim_{y \to 0} \frac{\ln(1+ay)}{y \times a} \times a$

$\because \lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$

$\therefore \boxed{a}$

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(ii) Evaluate : $\lim_{n\to\infty}\left[1+\frac{1}{n}\right]^{pn+q}$

1^{∞} form

$e^{\lim_{n\to\infty}\left[\cancel{1}+\frac{1}{n}-\cancel{1}\right](pn+q)}$

$e^{\lim_{n\to\infty}\left(\frac{pn+q}{n}\right)}$

$e^{\lim_{n\to\infty} p+q/n}$

$\boxed{e^{p}}$

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(iii) Evaluate: $\lim_{x\to 0}\left[1+\tan^2\sqrt{x}\right]^{\frac{1}{2x}}$

1^{∞} form

$$e^{\lim_{x\to 0}\left[\cancel{1}+\tan^2\sqrt{x}\cancel{-1}\right]\cdot\frac{1}{2x}}$$

$$e^{\lim_{x\to 0}\frac{\tan^2\sqrt{x}}{2x}}$$

$$e^{\lim_{x\to 0}\frac{\tan^2\sqrt{x}}{2(\sqrt{x})^2}}$$

$$\boxed{e^{1/2}}$$

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(iv) Evaluate :- $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}$

$\lim_{x\to\infty}\left[\frac{x+1+5}{x+1}\right]^{x+4}$

$\lim_{x\to\infty}\left[1+\frac{5}{x+1}\right]^{x+4}$

1^{∞} form

$e^{\lim_{x\to\infty}\left[\cancel{1}+\frac{5}{x+1}-\cancel{1}\right](x+4)}$

$e^{\lim_{x\to\infty}\frac{5}{x+1}\times(x+4)}$

$e^{\lim_{x\to\infty}\frac{5\left[\cancel{x}\left(1+4/x\right)\right]}{\cancel{x}\left[1+1/x\right]}}$

$e^{\lim_{x\to\infty}\frac{5\left(1+4/x\right)}{\left(1+1/x\right)}}$

$\boxed{e^{5}}$

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Do your self (8)

(i) Evaluate :- $\lim_{x\to\infty} \left[\frac{1+5x^2}{1+3x^2}\right]^{-x^2}$

Here $f(x) = \frac{1+5x^2}{1+3x^2}$ & $g(x) = -x^2$

$$\lim_{x\to\infty}\left[\frac{\cancel{x^2}\left(\frac{1}{x^2}+5\right)}{\cancel{x^2}\left(\frac{1}{x^2}+3\right)}\right]^{-x^2}$$

$$\lim_{x\to\infty}\left(\frac{5}{3}\right)^{-x^2}$$

$\because \frac{5}{3} > 1$

$\therefore \lim_{x\to\infty}\left(f(x)\right)^{g(x)}$ & $g(x)\to -\infty$

$$\left(\frac{5}{3}\right)^{-\infty}$$

$$\boxed{0}$$

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Do your self (9)

(i) Evaluate : $\lim_{x\to 0} \frac{x-\sin x}{\sin(x^3)}$

$\because \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

$\therefore \lim_{x\to 0} \frac{x - \left\{ x - \frac{x^3}{3!} + \dots \right\}}{x^3 + \dots}$

$\lim_{x\to 0} \frac{\cancel{x} - \cancel{x} + \frac{x^3}{3!}}{x^3}$

$\lim_{x\to 0} \frac{\cancel{x^3}}{3!\cdot \cancel{x^3}}$

$\boxed{\frac{1}{6}}$

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(ii) Evaluate :- $\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3}$

$\because \tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$

$$\lim_{x \to 0} \frac{x - \left\{ x - \frac{x^3}{3} + \cdots \right\}}{x^3}$$

$$\lim_{x \to 0} \frac{\cancel{x} - \cancel{x} + \cancel{x^3}}{3 \times \cancel{x^3}}$$

$$\boxed{\frac{1}{3}}$$

EXERCISE (O-1)
[SINGLE CORRECT CHOICE TYPE]

EXERCISE (O-1)

1. $\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$ is equal to

(A) –1 (B) 0 (C) 1 (D) D.N.E.

Sol^n:

$$= \lim_{x\to 1}\left(\frac{1+x+x^2-3}{1-x^3}\right) = \lim_{x\to 1} \frac{\overset{-1}{\cancel{(x-1)}}(x+2)}{\cancel{(1-x)}(1+x+x^2)} = -1$$

2. $\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$ is equal to

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Sol^n:

$$= \lim_{x\to0} \frac{(1+x)-(1-x)}{2x\left[\sqrt{1+x}+\sqrt{1-x}\right]}$$

$$= \lim_{x\to0} \frac{1}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \frac{1}{2}$$

3. $\lim_{x \to 2} \frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$ is equal to

(A) $\frac{1}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) $\frac{1}{4\sqrt{3}}$ (D) $\frac{1}{8\sqrt{3}}$

Sol^n:

$$= \lim_{x \to 2} \frac{1+\sqrt{x+2}-3}{(x-2)\left[\sqrt{1+\sqrt{x+2}}+\sqrt{3}\right]}$$

$$= \frac{1}{2\sqrt{3}} \lim_{x \to 2} \left(\frac{\sqrt{x+2}-2}{x-2}\right) \times \left(\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}\right)$$

$$= \frac{1}{2\sqrt{3}} \times \frac{1}{4} \lim_{x \to 2} \frac{(x+2)-4}{x-2}$$

$$= \frac{1}{8\sqrt{3}}$$

4. $\lim_{x\to 1}\frac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}$ (m and n integers) is equal to

(A) 0 (B) 1 (C) $\frac{m}{n}$ (D) $\frac{n}{m}$

Sol^n:

$$= \lim_{x\to 1}\frac{\left(\frac{x^{1/n}-1^{1/n}}{x-1}\right)}{\left(\frac{x^{1/m}-1}{x-1}\right)} = \frac{\frac{1}{n}(1)^{\frac{1}{n}-1}}{1/m\,(1)^{\frac{1}{m}-1}}$$

$$= m/n$$ **Ans**

5. If $\lim\limits_{x\to a}\dfrac{2x-\sqrt{x^2+3a^2}}{\sqrt{x+a}-\sqrt{2a}}=\sqrt{2}$ (where $a\in\mathbb{R}^+$), then a is equal to -

(A) $\dfrac{1}{3}$ (B) $\dfrac{1}{2\sqrt{2}}$ (C) $\dfrac{1}{3\sqrt{2}}$ (D) $\dfrac{1}{9}$

Sol^n:

$$\Rightarrow \lim_{x\to a}\frac{4x^2-(x^2+3a^2)}{x+a-2a}\times\left(\frac{\sqrt{x+a}+\sqrt{2a}}{2x+\sqrt{x^2+3a^2}}\right)=\sqrt{2}$$

$$\Rightarrow \frac{2\sqrt{2a}}{4a}\lim_{x\to a}\frac{3(x^2-a^2)}{x-a}=\sqrt{2}$$

$$\Rightarrow \frac{1}{\sqrt{2a}}\lim_{x\to a}\frac{3\cancel{(x-a)}(x+a)}{\cancel{(x-a)}}=\sqrt{2}$$

$$\Rightarrow \frac{1}{\sqrt{2a}}\cdot 3(2a)=\sqrt{2}$$

$$\Rightarrow 3\sqrt{2}\sqrt{a}=\sqrt{2} \Rightarrow \sqrt{a}=1/3$$

$$\Rightarrow a=1/9$$

Ans

6. $\lim_{x\to 0} \frac{\ell n(\sin 3x)}{\ell n(\sin x)}$ is equal to

(A) 0 (B) 1 (C) 2 (D) Non existent

6)

Sol:

$$= \lim_{x\to 0} \frac{\ln[3\sin x - 4\sin^3 x]}{\ln(\sin x)} = \lim_{x\to 0} \frac{\ln(\sin x(3-4\sin^2 x))}{\ln \sin x}$$

$$= \lim_{x\to 0} \frac{\ln(\sin x) + \ln(3-4\sin^2 x)}{\ln(\sin x)}$$

$$= \lim_{x\to 0}\left[1 + \frac{\ln(3-4\sin^2 x)}{\ln(\sin x)}\right] = 1 + \frac{\ln 3}{-\infty}$$

$$= 1 + 0 \quad (\text{as } \ln_{x\to 0}(\sin x) \to -\infty)$$

$$= 1$$ Ans

7. $\lim_{n\to\infty}\frac{(n+1)^4-(n-1)^4}{(n+1)^4+(n-1)^4}$ is equal to

(A) -1 (B) 0 (C) 1 (D) D.N.E.

Sol^n:

$$= \lim_{n\to\infty}\left(\frac{\text{Polynomial of degree } 3}{\text{Polynomial of degree } 4}\right)$$

$$= 0$$

8. $\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+....+(x+100)^{10}}{x^{10}+10^{10}}$ is equal to

(A) 1 (B) 100 (C) 200 (D) 10

Sol:

$$= \lim_{x\to\infty} \frac{x^{10}\left(1+\frac{1}{x}\right)^{10} + x^{10}\left(1+\frac{2}{x}\right)^{10} + \dots + x^{10}\left(1+\frac{100}{x}\right)^{10}}{x^{10}\left(1+\frac{10^{10}}{x^{10}}\right)}$$

$$= \frac{(1+0)+(1+0)+\dots+(1+0)}{(1+0)}$$

$$= \underline{\underline{100}}$$

9. $\lim_{x \to -\infty}\left(\sqrt{x^2-2x-1}-\sqrt{x^2-7x+3}\right)$ is equal to

(A) $-\frac{5}{2}$ (B) $\frac{5}{2}$ (C) 0 (D) D.N.E

Sol^n:

put $x = \frac{1}{t}$

$$= \lim_{t \to 0^-} \frac{\sqrt{1-2t-t^2} - \sqrt{1-7t+3t^2}}{|t|}$$

$$= \lim_{t \to 0^-} \frac{(\cancel{1}-2t-t^2) - (\cancel{1}-7t+3t^2)}{|t|\left[\sqrt{1-2t-t^2}+\sqrt{1-7t+3t^2}\right]}$$

$$= \lim_{t \to 0^-} \frac{5t-4t^2}{-t\left[\sqrt{1-2t-t^2}+\sqrt{1-7t+3t^2}\right]}$$

$$= -\frac{5}{2}$$

10. $\lim_{x \to -1} \frac{\cos 2 - \cos 2x}{x^2 - |x|}$ is equal to

(A) 0 (B) cos2 (C) 2sin2 (D) sin1

sol^n:

$$= \lim_{x \to -1} \frac{2\sin(1+x)\cdot\sin(x-1)}{x^2+x}$$

$$= \lim_{x \to -1} \frac{2\sin(1+x)\sin(x-1)}{(1+x)x} = 1$$

$$= \frac{2\sin(-1-1)}{-1} = 2\sin 2$$

11. $\lim_{x\to 0}\left(\left[\frac{-5\sin x}{x}\right]+\left[\frac{6\sin x}{x}\right]\right)$ (where [.] denotes greatest integer function) is equal to -

(A) 0 (B) –12 (C) 1 (D) 2

Sol^n:

$= [-5(1^-)] + [6(1^-)]$

$= -5 + 5 = 0$

12. Let $f(x) = \left[\frac{\sin x}{x}\right] + \left[\frac{2\sin 2x}{x}\right] + ... + \left[\frac{10\sin 10x}{x}\right]$ (where [y] is the largest integer ≤ y). The value of

$\lim_{x\to 0} f(x)$ equals

(A) 55 (B) 164 (C) 165 (D) 375

sol^n:

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\left[\frac{\sin x}{x}\right] + \left[\frac{2\sin 2x}{x}\right] + \cdots + \left[\frac{10\sin 10x}{x}\right]\right)$$

$$= [1^-] + [(2^2)^-] + [(3^2)^-] + \cdots + [(10^2)^-]$$

$$= (1^2-1) + (2^2-1) + (3^2-1) + \cdots + (10^2-1)$$

$$= (1^2+2^2+3^2+\cdots+10^2) - 10$$

$$= \frac{10(10+1)(2(10)+1)}{6} - 10 = 375$$

Ans

$$\because \left\{1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}\right\}$$

13. Let $f(x)=\dfrac{\sin\{x\}}{x^2+ax+b}$. If $f(5^+)$ & $f(3^+)$ exists finitely and are not zero, then the value of (a + b) is (where {.} represents fractional part function) -

(A) 7 (B) 10 (C) 11 (D) 20

Sol:

$\because$ at $x \to 5^+$ and $x \to 3^+ \Rightarrow \sin\{x\} \to 0$

$\therefore$ $x^2+ax+b \to 0$ at $x \to 5^+$ and $x \to 3^+$

$\Rightarrow$ $x^2+ax+b = (x-5)(x-3)$

$= x^2-8x+15$

$\Rightarrow$ $a=-8,\ b=15$

$\therefore$ $a+b = -8+15 = 7$

14. $\lim_{x\to 0}\frac{|\cos(\sin(3x))|-1}{x^2}$ equals

(A) $\frac{-9}{2}$ (B) $\frac{-3}{2}$ (C) $\frac{3}{2}$ (D) $\frac{9}{2}$

Sol^n:

$$= \lim_{x\to 0} \frac{\cos(\sin 3x)-1}{x^2} \qquad \left(\because |\cos(\sin 3x)| = \cos(\sin 3x) \text{ as } x\to 0\right)$$

$$= \lim_{x\to 0} (-)\left(\frac{1-\cos(\sin 3x)}{\sin^2 3x}\right)\cdot\left(\frac{\sin^2 3x}{9x^2}\right)\cdot(9)$$

$$= (-1)\left(\frac{1}{2}\right)(1)(9) = -9/2$$

Ans

15. $\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$ is equal to

(A) 1 (B) e (C) $\frac{1}{e^2}$ (D) e^2

Sol^n:

$= \lim_{x\to\infty} e^{x\left[\frac{x^2-2x+1}{x^2-4x+2}-1\right]}$ $(\because 1^\infty)$

$= e^{\lim_{x\to\infty} \frac{x[2x-1]}{x^2-4x+2}}$

$= e^2$

16. $\lim_{x\to 0}(1+\sin x)^{\cos x}$ is equal to

(A) 0 (B) e (C) 1 (D) $\frac{1}{e}$

Sol:

$$= e^{\lim_{x\to 0} \cos x[1+\sin x - 1]} \quad (\text{as } 1^{\infty})$$

$$= e^{0} = 1$$

17. $\lim_{x\to 0}(\cos x + a\sin bx)^{\frac{1}{x}}$ is equal to

(A) e^{a} (B) e^{ab} (C) e^{b} (D) $e^{a/b}$

Sol:

$$= e^{\lim_{x\to 0} \frac{1}{x}[\cos x + a\sin bx - 1]} \quad (\text{as } 1^{\infty})$$

$$= e^{\lim_{x\to 0}\left[\frac{\cos x - 1}{x} + \frac{a\sin bx}{x}\right]}$$

$$= e^{0 + ab} = e^{ab}$$

18. $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to

(A) e^{-2} (B) $\frac{1}{e}$ (C) e (D) e^{2}

Sol:

$$= \lim_{x\to 0} e^{\frac{1}{x}\left[\tan\left(\frac{\pi}{4}+x\right)-1\right]} \quad (\text{as } 1^{\infty})$$

$$= \lim_{x\to 0} e^{\left[\frac{1+\tan x}{1-\tan x}-1\right]\cdot\frac{1}{x}}$$

$$= \lim_{x\to 0} e^{\frac{2\tan x}{x(1-\tan x)}}$$

$$= e^{2}$$

19. $\lim_{n\to\infty}(4^n+5^n)^{1/n}$ is equal to

(A) 5 (B) 4 (C) 0 (D) D.N.E.

Sol^n:

$$= \lim_{n\to\infty}\left[5^n\left(\left(\frac{4}{5}\right)^n+1\right)\right]^{1/n}$$

$$= \lim_{n\to\infty} 5\left[(4/5)^n+1\right]^{1/n}$$

$$= 5\,[0+1]^0 = \underline{\underline{5}}$$

20. $\lim_{x\to\infty}\left(\frac{1^{1/x}+2^{1/x}+3^{1/x}+.....+n^{1/x}}{n}\right)^{nx}$, $n \in \mathbb{N}$ is equal to

(A) $n!$ (B) 1 (C) $\frac{1}{n!}$ (D) 0

Sol:

$$= e^{\lim_{x\to\infty} \cancel{n}x\left[\frac{1^{\frac{1}{x}}+2^{\frac{1}{x}}+\cdots+n^{\frac{1}{x}}}{\cancel{n}}-1\right]} \quad (\text{as } 1^{\infty})$$

$$= e^{\lim_{x\to\infty} \frac{\cancel{n}x\left[1^{\frac{1}{x}}+2^{\frac{1}{x}}+\cdots+n^{\frac{1}{x}}-n\right]}{\cancel{n}}}$$

$$= e^{\lim_{x\to\infty} \frac{\left(1^{\frac{1}{x}}+2^{\frac{1}{x}}+\cdots-\{1+1+\cdots+1\}\right]}{\frac{1}{x}}}$$

$$= e^{\lim_{x\to\infty}\left(\frac{1^{\frac{1}{x}}-1}{\frac{1}{x}}+\frac{2^{\frac{1}{x}}-1}{\frac{1}{x}}+\cdots+\frac{n^{\frac{1}{x}}-1}{\frac{1}{x}}\right]}$$

$$= e^{(\ln 1+\ln 2+\cdots \ln n)}$$

$$= e^{\ln(1.2.\cdots\cdots n)} = n!$$

21. If $\lim_{x \to 0}(1 + ax + bx^2)^{2/x} = e^3$, then

(A) $a = \frac{3}{2}$ and $b \in \mathbb{R}$

(B) $a = \frac{3}{2}$ and $b \in \mathbb{R}^+$

(C) a = 0 and b = 1

(D) a = 1 and b = 0

Sol^n:

$$\Rightarrow \lim_{x \to 0} e^{\frac{2}{x}\left[1+ax+bx^2-1\right]} = e^3 \quad (\text{as } 1^\infty)$$

$$\Rightarrow \lim_{x \to 0} \left(\frac{ax+bx^2}{x}\right)2 = 3$$

$$\Rightarrow 2a = 3 \Rightarrow a = \frac{3}{2}, \quad b \in R$$

22. If f(x) is a polynomial of least degree, such that $\lim_{x\to 0}\left(1+\frac{f(x)+x^2}{x^2}\right)^{1/x} = e^2$, then f(2) is -

(A) 2 (B) 8 (C) 10 (D) 12

Solⁿ: $\frac{f(x)+x^2}{x^2} \to 0$ as $x \to 0$ ($\because f(x)$ is of degree 3)

$\therefore \lim_{x\to 0}\left(1+\frac{f(x)+x^2}{x^2}\right)^{1/x} = e^2$ (as 1^∞ form)

$$\Rightarrow e^{\lim_{x\to 0}\frac{1}{x}\left[1+\frac{f(x)+x^2}{x^2}-1\right]} = e^2$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)+x^2}{x^3} = 2$$

Now, let $f(x) = ax^3+bx^2+cx+d$

$$\therefore \lim_{x\to 0}\frac{f(x)+x^2}{x^3} = 2$$

$$\Rightarrow \lim_{x\to 0}\frac{ax^3+(b+1)x^2+cx+d}{x^3} = 2$$

$\Rightarrow a=2$ | $b+1=0 \Rightarrow b=-1$ | $c=0$ | $d=0$

$\therefore f(x) = 2x^3 - x^2$

$f(2) = 2(2)^3 - 2^2 = 12$ //

Ans

23. $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3} = 1$ then

(A) $a = -5/2$ (B) $a = -3/2, b = -1/2$ (C) $a = -3/2, b = -5/2$ (D) $a = -5/2, b = -3/2$

Soln.

$$\lim_{x\to 0}\frac{x + ax\left\{1-\frac{x^2}{2!}+\frac{x^4}{4!}\cdots\right\} - b\left\{x-\frac{x^3}{3!}+\cdots\right\}}{x^3} = 1$$

$$= \lim_{x\to 0}\frac{(1+a-b)x + \left(-\frac{a}{2}+\frac{b}{6}\right)x^3 \cdots\cdots}{x^3} = 1$$

$\Rightarrow$ $1+a-b=0$ —— (1)

$-\frac{a}{2}+\frac{b}{6}=1$ —— (2)

from (1) & (2)

$\Rightarrow$ $\boxed{a=-\frac{5}{2},\quad b=-\frac{3}{2}}$

[MULTIPLE CORRECT CHOICE TYPE]

24. Consider following statements and identify correct options

(i) $\lim\limits_{x\to4}\left(\frac{2x}{x-4}-\frac{8}{x-4}\right)=\lim\limits_{x\to4}\frac{2x}{x-4}-\lim\limits_{x\to4}\frac{8}{x-4}$

(ii) $\lim\limits_{x\to1}\frac{x^2+6x-7}{x^2+5x-6}=\frac{\lim\limits_{x\to1}(x^2+6x-7)}{\lim\limits_{x\to1}(x^2+5x-6)}$

(iii) $\lim\limits_{x\to1}\frac{x-3}{x^2+2x-4}=\frac{\lim\limits_{x\to1}(x-3)}{\lim\limits_{x\to1}(x^2+2x-4)}$

(iv) If $\lim\limits_{x\to5}f(x)=2$ and $\lim\limits_{x\to5}g(x)=0$, then $\lim\limits_{x\to5}\frac{f(x)}{g(x)}$ does not exist.

(v) If $\lim\limits_{x\to5}f(x)=0$ and $\lim\limits_{x\to5}g(x)=2$, then $\lim\limits_{x\to5}\frac{f(x)}{g(x)}$ does not exist.

(A) Only one is true. (B) Only two are true.
(C) Only three are false. (D) Only two are false.

Sol:

36. (i) $\lim\limits_{x\to4}\left[\frac{2x}{x-4}-\frac{8}{x-4}\right] = \lim\limits_{x\to4}\frac{2x-8}{x-4} = 2$

but $\lim\limits_{x\to4}\frac{2x}{x-4} - \lim\limits_{x\to4}\frac{8}{x-4}$

Not defined - Not defined

False

(ii) $\lim\limits_{x\to1}\frac{x^2+6x-7}{x^2+5x-6} = \frac{\lim\limits_{x\to1}(x^2+6x-7)}{\lim\limits_{x\to1}(x^2+5x-6)}$

R.H.S. : $\frac{\lim\limits_{x\to1}(x^2+6x-7)}{\lim\limits_{x\to1}(x^2+5x-6)} = \frac{0}{0}$ = Not defined.

L.H.S. : $\lim\limits_{x\to1}\frac{x^2+6x-7}{x^2+5x-6} = \lim\limits_{x\to1}\frac{(x+7)\cancel{(x-1)}}{(x+6)\cancel{(x-1)}} = \frac{8}{7}$

False

(iii) $\lim\limits_{x\to1}\frac{x-3}{x^2+2x-4} = \frac{\lim\limits_{x\to1}(x-3)}{\lim\limits_{x\to1}(x^2+2x-4)}$

L.H.S $\Rightarrow \frac{-2}{-1}=2$

R.H.S. $\Rightarrow \frac{-2}{-1}=2$

True

(iv) $\lim\limits_{x\to5}f(x)=2$ & $\lim\limits_{x\to5}g(x)=0$

so $\lim\limits_{x\to5}\frac{f(x)}{g(x)} = \frac{2}{0}$ = does not exist

True

(v) $\lim\limits_{x\to5}f(x)=0$ & $\lim\limits_{x\to5}g(x)=2$

so $\lim\limits_{x\to5}\frac{f(x)}{g(x)} = \frac{0}{2} = 0$

False

Ans: B, C

25 Let $f(x) = \begin{cases} \sin x \,;\text{where } x = \text{integer} \\ 0 \;;\; \text{otherwise} \end{cases}$: $g(x) = \begin{cases} x^2+1 & ; & x \neq 0, 2 \\ 4 & ; & x = 0 \\ 5 & ; & x = 2 \end{cases}$, then

(A) $\lim_{x\to 0} g(f(x)) = 4$ (B) $\lim_{x\to 0} f(g(x)) = 0$ (C) $\lim_{x\to 1} f(g(x)) = 0$ (D) $\lim_{x\to 1} g(f(x)) = 5$

Sol:

(A) $\lim_{x\to 0} g(f(x)) = ?$

$\Rightarrow \lim_{x\to 0} g(f(x)) \begin{cases} \lim_{x\to 0^+} g(f(x)) \\ \lim_{x\to 0^-} g(f(x)) \end{cases}$

$\Rightarrow \because \lim_{x\to 0^+} f(x) = 0 \;\&\; \lim_{x\to 0^-} f(x) = 0$

So $\lim_{x\to 0} g(f(x)) \begin{cases} \lim_{x\to 0^+} g(f(x)) = g(0) = 4 \\ \lim_{x\to 0^-} g(f(x)) = g(0) = 4 \end{cases}$] Ans $= \underline{\underline{4}}$

(B) $\lim_{x\to 0} f(g(x)) \begin{cases} \lim_{x\to 0^+} f(g(x)) \\ \lim_{x\to 0^-} f(g(x)) \end{cases}$

$\Rightarrow \because \lim_{x\to 0^+} g(x) = \lim_{h\to 0} (0+h)^2+1 = 1^+$

$\lim_{x\to 0^-} g(x) = \lim_{h\to 0} (0-h)^2+1 = 1^+$

So. $\lim_{x\to 0} f(g(x)) \begin{cases} \lim_{x\to 0^+} f(g(x)) = f(1^+) = 0 \\ \lim_{x\to 0^-} f(g(x)) = f(1^+) = 0 \end{cases}$] Ans $= 0$

(C) $\lim_{x\to 1} f(g(x)) = 0$

$\lim_{x\to 1} f(g(x)) \begin{cases} \lim_{x\to 1^+} f(g(x)) \\ \lim_{x\to 1^-} f(g(x)) \end{cases}$

$\because \lim_{x\to 1^+} g(x) = (1+h)^2+1 = 2+2h+h^2 = 2^+$

$\lim_{x\to 1^-} g(x) = (1-h)^2+1 = 2-2h+h^2 = 2^-$

$\lim_{x\to 1} f(g(x)) \begin{cases} \lim_{x\to 1^+} f(g(x)) = f(2^+) = 0 \\ \lim_{x\to 1^-} f(g(x)) = f(2^-) = 0 \end{cases}$] Ans $= 0$

(D) $\lim_{x \to 1} g(f(x)) = 5$

$$\Rightarrow \lim_{x \to 1} g(f(x)) \begin{cases} \lim_{x \to 1^+} g(f(x)) \\ \lim_{x \to 1^-} g(f(x)) \end{cases}$$

$\Rightarrow \because \lim_{x \to 1^+} f(x) = 0$

$\lim_{x \to 1^-} f(x) = 0$

$$\Rightarrow \therefore \lim_{x \to 1} g(f(x)) \begin{cases} \lim_{x \to 1^+} g(f(x)) = g(0) = 4 \\ \lim_{x \to 1^-} g(f(x)) = g(0) = 4 \end{cases} \text{Ans} = 4$$

26. If $\dfrac{\sin x + ae^{x} + be^{-x} + c\ln(1+x)}{x^{3}}$ has a finite limit L as $x \to 0$, then

(A) $a = -\dfrac{1}{2}$ (B) $b = \dfrac{1}{2}$ (C) $c = 0$ (D) $L = -\dfrac{1}{3}$

Sol

41. $\lim\limits_{x\to 0} \dfrac{\sin x + ae^{x} + be^{-x} + c\ln(1+x)}{x^3}$

$$\lim_{x\to 0} \frac{\left(x - \frac{x^3}{3!} + \ldots\right) + a\left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\right] + b\left[1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots\right] + c\left[x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\right]}{x^3}$$

$$\lim_{x\to 0} \frac{(a+b) + x\left[1 + a - b + c\right] + x^2\left[\frac{a}{2} + \frac{b}{2} - \frac{c}{2}\right] + x^3\left[-\frac{1}{6} + \frac{a}{6} - \frac{b}{6} + \frac{c}{3}\right]}{x^3}$$

$\because$ for limit to be exist coff. of below x^3 must be zero

$\therefore$ $a + b = 0$ —(1)

$1 + a - b + c = 0$ —(2)

$\frac{a}{2} + \frac{b}{2} - \frac{c}{2} = 0$ —(3)

then $L = -\frac{1}{6} + \frac{a}{6} - \frac{b}{6} + \frac{c}{3}$

by eqn (1) & (3) $c = 0$

by eqn (1) & (2) $a + b = 0$

$1 + a - b = 0$

$a = -1/2$, $b = 1/2$

$$\Rightarrow \lim_{x\to 0} \frac{\cancel{x^3}\left[-\frac{1}{6} - \frac{1}{12} - \frac{1}{12} + 0\right]}{\cancel{x^3}}$$

$L = -1/3$

27. Let $\ell = \lim\limits_{x\to\infty} \frac{a^x - a^{-x}}{a^x + a^{-x}} (a > 0)$, then

(A) $\ell = 1\ \forall\ a > 0$ (B) $\ell = -1\ \forall\ a \in (0, 1)$ (C) $\ell = 0$, if $a = 1$ (D) $\ell = 1\ \forall\ a > 1$

sol^n: 42.

$$\ell = \lim_{x\to\infty} \frac{a^x - a^{-x}}{a^x + a^{-x}} \quad (a>0)$$

Case-1: $a \in (0,1)$

$$\lim_{x\to\infty} \left(\frac{a^{2x}-1}{a^{2x}+1}\right) = \frac{0-1}{0+1} = -1$$

Case-2 $a = 1$

$$\lim_{x\to\infty} \frac{1-1}{1+1} = 0$$

Case-3: $a > 1$

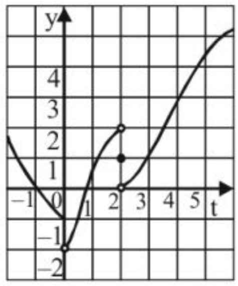
$$\lim_{x\to\infty} \frac{a^{2x}-1}{a^{2x}+1} = \lim_{x\to\infty} \frac{1-a^{-2x}}{1+a^{-2x}}$$

$$\Rightarrow \frac{1-0}{1+0} = 1$$

[MATCH THE COLUMN TYPE]

28. For the function g(t) whose graph is given, match the entries of column-I to column-II

Column-I	**Column-II**
(A) $\lim\limits_{t\to 0^+} g(t)+\lim\limits_{t\to 2^-} g(t)$	(P) $\lim\limits_{t\to 2^+} g(t)$
(B) $\lim\limits_{t\to 0^-} g(t)+g(2)$	(Q) does not exist
(C) $\lim\limits_{t\to 0} g(t)$	(R) 0
(D) $\lim\limits_{t\to 2} g(t)$	(S) $\lim\limits_{t\to 4} g(t)$



Sol

43. (A) $\lim\limits_{t\to 0^+} g(t) + \lim\limits_{t\to 2^-} g(t)$

$\Rightarrow \quad -2 \quad +2 \quad = 0$

(B) $\lim\limits_{t\to 0^-} g(t) + g(2)$

$-1 + 1 = 0$

(C) $\lim\limits_{t\to 0} g(t) = ?$

$\Rightarrow \because \lim\limits_{t\to 0^+} g(t) = -2$

$\Rightarrow \lim\limits_{t\to 0^-} g(t) = -1$

So $\lim\limits_{t\to 0} g(t)$ does not exist

(D) $\lim\limits_{t\to 2} g(t)$

L.H.L. $\lim\limits_{t\to 2^-} g(t) = 2$

R.H.L $\lim\limits_{t\to 2^+} g(t) = 0$

$\because$ L.H.L $\neq$ R.H.L. So does not exist

(P) $\lim\limits_{t\to 2^+} g(t) = 0$

(S) $\lim\limits_{t\to 4} g(t)$

L.H.L: $\lim\limits_{t\to 4^-} g(t) = 3$

R.H.L - $\lim\limits_{t\to 4^+} g(t) = 3$

$\left.\begin{matrix} \\ \\ \end{matrix}\right] = 3$

Ans:

43. (A)→(P,R); (B)→(P,R); (C)→(Q); (D)→(Q)

29. **Column-I** | **Column-II**

(A) $\lim_{n\to\infty} n\sin\left(\frac{\pi}{4n}\right)\cos\left(\frac{\pi}{4n}\right)$ is equal to — (P) 0

(B) $\lim_{x\to 0}\frac{\sin x^\circ}{x}$ is equal to — (Q) $\frac{1}{2}$

(C) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)$ is equal to — (R) $\frac{\pi}{4}$

(D) $\lim_{x\to \pi/2}\frac{1+\cos 2x}{(\pi-2x)^2}$ is equal to — (S) $\frac{\pi}{180}$

sol) 44.

(A) $\lim_{n\to\infty} n\sin\left(\frac{\pi}{4n}\right)\cos\left(\frac{\pi}{4n}\right)$

$= \lim_{n\to\infty} \frac{\pi}{2}\underbrace{\frac{\sin\frac{\pi}{2n}}{\frac{\pi}{2n}}}_{\to 1} \times \frac{\pi}{2\pi}$

$= \pi/4$

(B) $\lim_{x\to 0}\frac{\sin x^\circ}{x}$

$\lim_{x\to 0}\underbrace{\frac{\sin\frac{\pi}{180}\times x}{x\times\frac{\pi}{180}}}_{\to 1}\times\frac{\pi}{180}$

$= \frac{\pi}{180}$

(C) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)$

$\lim_{x\to 0}\frac{1-\cos x}{\underbrace{\left(\frac{\sin x}{x}\right)}_{\to 1}\times x}$

$\lim_{x\to 0}\left(\frac{1-\cos x}{x\times x}\right)\times x$

$= \frac{1}{2}\times x$

$= 0$

(D) $\lim_{x\to \pi/2}\frac{1+\cos 2x}{(\pi-2x)^2} = \lim_{h\to 0}\frac{1+\cos 2(\frac{\pi}{2}+h)}{(\pi-2(\frac{\pi}{2}+h))^2}$

$\Rightarrow \lim_{h\to 0}\frac{1-\cos 2h}{4h^2} = 1/2$

Ans:

44. (A)→(R);(B)→(S);(C)→(P);(D)→(Q)

30.

	Column-I		Column-II
(A)	$\lim\limits_{x\to\infty}\dfrac{a^x}{a^x+1}$ $(a>0)$ can be equal to	(P)	$\lim\limits_{x\to\infty} x(e^{1/x}-1)$
(B)	$\lim\limits_{x\to 2}\dfrac{\sin(e^{x-2}-1)}{\log(x-1)}$ is equal to	(Q)	$\lim\limits_{x\to 0}\dfrac{a^x+b^x+c^x-3}{x}$ $(a,b,c>0\ \&\ abc=1)$
(C)	$\lim\limits_{x\to e}\dfrac{(\ell n x-1)e}{x-e}$ is equal to	(R)	$\lim\limits_{x\to 0}\dfrac{e^{4x}-e^{3x}}{x}$
(D)	$\lim\limits_{x\to 0}\dfrac{x(5^x-1)}{(1-\cos x)4\ell n5}$ is equal to	(S)	$\dfrac{1}{2}$
		(T)	0

Sol^n: 45. (A) $\lim\limits_{x\to\infty}\dfrac{a^x}{a^x+1}$ $(a>0)$

Case-1 :- $a\in(0,1)$

$\Rightarrow \dfrac{0}{0+1} = 0$

Case-2 :- $a=1$

$\Rightarrow \dfrac{1}{1+1} = \dfrac{1}{2}$

Case-3 :- $a>1$

$\lim\limits_{x\to\infty}\dfrac{\cancel{a^x}}{\cancel{a^x}}\left[\dfrac{1}{1+a^{-x}}\right]$

$= 1$

(B) $\lim\limits_{x\to 2}\dfrac{\sin(e^{x-2}-1)}{\log(x-1)}$

$= \lim\limits_{h\to 0}\dfrac{\sin(e^h-1)}{\underbrace{\dfrac{\log(1+h)}{h}}_{\to 1}\times h}$

$= \lim\limits_{h\to 0}\dfrac{\sin\left(\dfrac{e^h-1}{h}\times h\right)}{h}$

$= \lim\limits_{h\to 0}\dfrac{\sin h}{h} = 1$

C) $= \lim\limits_{x\to e}\dfrac{(\ln x-\ln e)e}{x-e}$

$= \lim\limits_{x\to e}\dfrac{\ln(x/e)}{\frac{x}{e}-1} =$

$= \lim\limits_{x\to e}\dfrac{\ln\left(\left(\frac{x}{e}-1\right)+1\right)}{\left(\frac{x}{e}-1\right)}$

$= 1$

(D) $\lim\limits_{x\to 0}\dfrac{x(5^x-1)}{(1-\cos x)\,4\ln 5}$

$= \lim\limits_{x\to 0}\dfrac{x\left[\dfrac{5^x-1}{x}\right]\times x}{\dfrac{1-\cos x}{x^2}\times x^2\times 4\ln 5}$

$= \dfrac{x^2\times\ln 5}{\frac{1}{2}\times x^2\times 4\ln 5}$

$= 1/2$

(P) $\lim_{x\to\infty} x\left(e^{1/x}-1\right)$

$\Rightarrow \lim_{x\to\infty} \frac{(e^{1/x}-1)}{1/x} = \underline{1}$

($\underline{Q.}$) $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x-3}{x}\right)$; $(a,b,c>0$ & $abc=1)$

$= \lim_{x\to 0}\left[\frac{(a^x-1)}{x} + \frac{(b^x-1)}{x} + \left(\frac{c^x-1}{x}\right)\right] = \ln a + \ln b + \ln c$

$= \ln(abc) = \ln(1)$

$= 0$ //

($\underline{R.}$) $\lim_{x\to 0} \frac{e^{4x}-e^{3x}}{x}$

$\lim_{x\to 0} \frac{1+4x-1-3x}{x}$

$= \underline{1}$

$\underline{\text{Ans}}$:

45. (A)→(P,Q,R,S,T);(B)→(P,R);(C)→(P,R); (D)→(S)

EXERCISE (O-2)

[SINGLE CORRECT CHOICE TYPE]

1. $\lim\limits_{h\to 0}\dfrac{\tan(a+2h)-2\tan(a+h)+\tan a}{h^2}$ is equal to

(A) tana (B) $\tan^2 a$ (C) seca (D) $2(\sec^2 a)(\tan a)$

Sol^n $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B}$

$\Rightarrow \tan A - \tan B = (\tan(A-B))(1+\tan A \tan B)$

$\therefore L = \lim\limits_{h\to 0}\dfrac{(\tan(a+2h)-\tan(a+h)) - (\tan(a+h)-\tan a)}{h^2}$

$\Rightarrow L = \lim\limits_{h\to 0}\dfrac{(\tan h)\left[(1+\tan(a+2h)\tan(a+h)) - (1+\tan(a+h)\tan a)\right]}{h^2}.$

$\Rightarrow L = \lim\limits_{h\to 0}\left(\dfrac{\tan h}{h}\right)\left(\dfrac{\tan(a+h)(\tan(a+2h)-\tan a)}{h}\right).$

$\Rightarrow L = 1 \cdot \tan a \cdot \lim\limits_{h\to 0}\dfrac{\tan(2h)(1+(\tan(a+2h))\tan a)}{h}.$

$\Rightarrow L = (\tan a)(2(\sec^2 a.))$

2. $\lim_{x \to 0}\left(2^{x-1}+\frac{1}{2}\right)^{1/x}$ equals

(A) $\sqrt{2}$ (B) $\frac{1}{2}\ell n2$ (C) $\ell n2$ (D) 2

Solⁿ 1^{∞} form.

$\Rightarrow$ ~~Ans~~ $= e^{L}$, where $L = \lim_{x \to 0} \frac{1}{x}\left(2^{x-1}+\frac{1}{2}-1\right)$

$\Rightarrow L = \lim_{x \to 0} \frac{1}{x}\left(\frac{2^{x}}{2}-\frac{1}{2}\right) \Rightarrow L = \lim_{x \to 0} \frac{1}{x}\left(\frac{2^{x}-1}{2}\right)$

$\Rightarrow L = \frac{1}{2}\ln 2 \Rightarrow L = \ln\sqrt{2}$

Ans: $e^{\ln\sqrt{2}} \Rightarrow \sqrt{2}$

3. If $\lim\limits_{x \to 0} \left(\cos x + a^3 \sin\left(b^6 x\right)\right)^{\frac{1}{x}} = e^{512}$, then the value of ab^2 is equal to

(A) –512 (B) 512 (C) 8 (D) $8\sqrt{8}$

Soln 1^{∞} Form.

$\Rightarrow e^{512} = e^{L}$, where $L = \lim\limits_{x \to 0} \left(\frac{1}{x}\right)\left(\cos x + a^3 \sin b^6 x - 1\right)$

$\Rightarrow 512 = \lim\limits_{x \to 0} \frac{\cos x - 1}{x} + \frac{a^3 \sin b^6 x}{x}$

$\Rightarrow 512 = a^3 b^6 \Rightarrow ab^2 = 8.$

4. The value of $\lim_{x \to 0} \frac{\sin\left(\sqrt[3]{x}\right)\ell n(1+3x)}{\left(\tan^{-1}\sqrt{x}\right)^2\left(e^{5\left(\sqrt[3]{x}\right)}-1\right)}$ is equal to

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\frac{4}{5}$

sol:

$$= \lim_{x \to 0} \frac{\underbrace{\left(\frac{\sin(x^{1/3})}{x^{1/3}}\right)}_{=1} \cancel{x^{1/3}} \underbrace{\left(\frac{\ln(1+3x)}{3x}\right)}_{=1} \cdot 3\cancel{x}}{(\cancel{\sqrt{x}})^2 \underbrace{\left(\frac{\tan^{-1}(\sqrt{x})}{\sqrt{x}}\right)^2}_{=1} \underbrace{\left(\frac{e^{5x^{1/3}}-1}{5x^{1/3}}\right)}_{=1} \cdot 5\cancel{x^{1/3}}}$$

$= 3/5$ Ans

5. The value of $\lim\limits_{x \to 2} \dfrac{\sec^x \theta - \tan^x \theta - 1}{x - 2}$ is equal to

(A) $\sec^2\theta.\ \ell n \sec\theta + \tan^2\theta.\ \ell n \tan\theta$

(B) $\sec^2\theta.\ \ell n \tan\theta + \tan^2\theta.\ \ell n \sec\theta$

(C) $\sec^2\theta.\ \ell n \tan\theta - \tan^2\theta.\ \ell n \sec\theta$

(D) $\sec^2\theta.\ \ell n \sec\theta - \tan^2\theta.\ \ell n \tan\theta$

Solⁿ $x = 2 + h$:

$$\lim_{h \to 0} \frac{(\sec\theta)^{2+h} - (\tan\theta)^{2+h} - (\sec^2\theta - \tan^2\theta)}{h}$$

$$= \lim_{h \to 0} \frac{\sec^2\theta\left((\sec\theta)^h - 1\right) - (\tan^2\theta)\left((\tan\theta)^h - 1\right)}{h}$$

$$= \lim_{h \to 0} \sec^2\theta \ln \sec\theta - \tan^2\theta \ln \tan\theta$$

6. Consider the function $f(x) = \begin{bmatrix} 1-x, & 0 \le x \le 1 \\ x+2, & 1 < x < 2 \\ 4-x, & 2 \le x \le 4 \end{bmatrix}$. Let $\lim\limits_{x\to 1} f(f(x)) = \ell$ and $\lim\limits_{x\to 2} f(f(x)) = m$ then which one of the following hold good ?

(A) ℓ exists but m does not.
(B) m exists but ℓ does not.
(C) Both ℓ and m exist
(D) Neither ℓ nor m exist.

Solⁿ

$$\ell = \begin{cases} \lim\limits_{x\to 1^+} f(f(x)) = f(f(1^+)) = f(3^+) = 1. \\ \lim\limits_{x\to 1^-} f(f(x)) = f(f(1^-)) = f(0^+) = 1. \end{cases}$$

$f(1^+ + 2) =$

$= f(1 - 1^-)$

$\therefore \ \ell = 1$ (exists).

$$m = \begin{cases} \lim\limits_{x\to 2^+} f(f(x)) = f(f(2^+)) = f(2^-) = 4. \\ \lim\limits_{x\to 2^-} f(f(x)) = f(f(2^-)) = f(4^-) = 0. \end{cases}$$

$f(4 - 2^+) =$

$f(2^- + 2) =$

$\therefore$ m DNE.

7. Let $f(x)$ be a quadratic function such that $f(0) = f(1) = 0$ & $f(2) = 1$, then $\lim_{x\to 0} \frac{\cos\left(\frac{\pi}{2}\cos^2 x\right)}{f^2(x)}$ is equal to

(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) 4π

Solⁿ. Let $f(x) = k(x-0)(x-1)$ $(\because 0, 1 \text{ are roots})$

$\because f(2) = 1 \Rightarrow k = 1/2$

$\therefore f(x) = \frac{x^2 - x}{2} = \frac{x(x-1)}{2}$

$\therefore L = \lim_{x\to 0} \frac{\cos\left(\frac{\pi}{2}(1-\sin^2 x)\right)}{f^2(x)}$

$\Rightarrow L = \lim_{x\to 0} \frac{\sin\left(\frac{\pi}{2}\sin^2 x\right)}{\frac{x^2(x-1)^2}{4}}$ $\Rightarrow L = \lim_{x\to 0} \frac{\frac{\pi}{2}\cdot\sin^2 x}{\frac{x^2\cdot(x-1)^2}{4}}$

$\Rightarrow L = 2\pi$

[MULTIPLE CORRECT CHOICE TYPE]

8. If $\ell = \lim_{x \to a} \dfrac{\sqrt{3x^2 + a^2} - \sqrt{x^2 + 3a^2}}{(x-a)}$ then -

(A) $\ell = 1\ \forall\ a \in \mathbb{R}$ (B) $\ell = 1\ \forall\ a > 0$ (C) $\ell = -1\ \forall\ a < 0$ (D) $\ell =$ D.N.E. if $a = 0$

Sol^n $\ell = \lim_{x \to a} \dfrac{3x^2 + a^2 - x^2 - 3a^2}{(x-a)\left(\sqrt{3x^2+a^2} + \sqrt{x^2+3a^2}\right)}$

$\Rightarrow \ell = \lim_{x \to a} \dfrac{2\cancel{(x-a)}(x+a)}{\cancel{(x-a)}\cdot(2|a| + 2|a|)} = \dfrac{\cancel{2}\cdot \cancel{2}a}{\cancel{4}|a|} = \dfrac{a}{|a|}$.

$\therefore$ B, C holds.

Also, at $a = 0$: $\ell = \lim_{x \to 0} \dfrac{\sqrt{3x^2} - \sqrt{x^2}}{x}$.

$\Rightarrow \ell = \lim_{x \to 0} \dfrac{(\sqrt{3}-1)|x|}{x} = \underline{DNE}$.

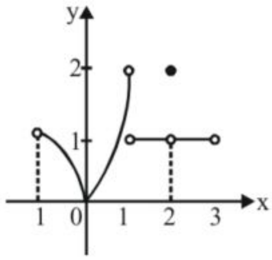
9. Which of the following limits vanish ?

(A) $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}$ (B) $\lim\limits_{x\to\infty}\dfrac{\arctan x}{x}$ (C) $\lim\limits_{x\to\infty}\dfrac{x+\sin x}{x+\cos x}$ (D) $\lim\limits_{x\to 1}\dfrac{\arcsin x}{\tan\dfrac{\pi x}{2}}$

Solⁿ. (A) $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}=0$; (B) $\lim\limits_{x\to\infty}\dfrac{\tan^{-1}x}{x}=\dfrac{\pi/2}{\infty}=0$

(C) $\lim\limits_{x\to\infty}\dfrac{x+\sin x}{x+\cos x}=1$ (D) $\lim\limits_{x\to 1}\dfrac{\sin^{-1}x}{\tan\frac{\pi x}{2}}=0$.

10. Which of the following statement are true for the function f defined for $-1 \le x \le 3$ in the figure shown.



(A) $\lim_{x \to -1^+} f(x) = 1$

(B) $\lim_{x \to 2} f(x)$ does not exist

(C) $\lim_{x \to 1^-} f(x) = 2$

(D) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

Solⁿ $\lim_{x \to -1^+} f(x) = 1.$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = 2$. (Exists)

$\lim_{x \to 1^-} f(x) = 2$

$\lim_{x \to 0^+} = \lim_{x \to 0^-} = 0.$

11. Let $f(x) = x + \sqrt{x^2+2x}$ and $g(x) = \sqrt{x^2+2x} - x$, then

(A) $\lim_{x\to\infty} g(x) = 1$ (B) $\lim_{x\to\infty} f(x) = 1$ (C) $\lim_{x\to-\infty} f(x) = -1$ (D) $\lim_{x\to-\infty} g(x) = -1$

Solⁿ (A) $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \sqrt{x^2+2x} - x = \lim_{x\to\infty} \dfrac{\cancel{x^2}+2x-\cancel{x^2}}{\sqrt{x^2+2x}+x}$

$\Rightarrow \lim_{x\to\infty} g(x) = \lim_{x\to\infty} \dfrac{2x}{x\sqrt{1+2/x}+x} = 1.$

(D) $\lim_{x\to-\infty} g(x) = \lim_{x\to-\infty}\left(\sqrt{x^2+2x} - x\right) = \infty + \infty$ DNE.

(B) $\lim_{x\to\infty}\left(x + \sqrt{x^2+2x}\right) = \infty + \infty = \text{DNE}$

(C) $\lim_{x\to-\infty}\left(x + \sqrt{x^2+2x}\right)$

Put $x = -t$: $L = \lim_{t\to\infty} \sqrt{t^2-2t} - t$

$\Rightarrow L = \lim_{t\to\infty} \dfrac{\cancel{t^2}-2t-\cancel{t^2}}{\sqrt{t^2-2t}+t} = -1.$ Ans (A, C)

12. The value(s) of 'n' for which $\lim_{x \to 1} \frac{e^{x-1} - x}{(x-1)^n}$ exists is/are -

(A) 1 (B) 2 (C) 3 (D) 4

Sol^n Put $x = 1 + h$: $L = \lim_{h \to 0} \frac{e^h - h - 1}{h^n}$

$$L = \frac{\left(\cancel{1} + \cancel{h} + \frac{h^2}{2} + \cdots\right) - \cancel{h} - \cancel{1}}{h^n}$$

for L to be exists ; $n \leq 2$ //

EXERCISE (S-1)

1. $$\lim_{x \to 1} \frac{x^2 - x.\ln x + \ln x - 1}{x - 1}$$

Solⁿ:

$$= \lim_{x \to 1} \frac{(x^2-1) - \ln x\,(x-1)}{x-1}$$

$$= \lim_{x \to 1} \frac{(x+1)(x-1) - \ln x\,(x-1)}{x-1}$$

$$= \lim_{x \to 1} x+1-\ln x$$

$$= 1+1-\ln 1 = 2$$

2. $$\lim_{x\to 1} \frac{\left[\sum_{K=1}^{100} x^k\right] - 100}{x-1}$$

Sol^n:

$$= \lim_{x \to 1} \frac{(x^1-1)+(x^2-1)+(x^3-1)+(x^4-1)+\cdots+(x^{100}-1)}{x-1}$$

$$= \lim_{x\to 1}\frac{(x^1-1)}{x-1} + \lim_{x\to 1}\frac{(x^2-1)}{(x-1)} + \lim_{x\to 1}\frac{(x^3-1)}{x-1} + \cdots + \lim_{x\to 1}\frac{(x^{100}-1)}{x-1}$$

$$= 1+2+3+4+\cdots+100$$

$$\left(\because \frac{n(n+1)}{2}\right)$$

$$\therefore = \frac{100 \times 101}{2} \Rightarrow \boxed{5050}$$

3. $$\lim_{x\to\frac{\pi}{4}} \frac{1-\tan x}{1-\sqrt{2}\sin x}$$

Sol^n Replace $x \to \frac{\pi}{4} + h \Rightarrow h \to 0$

$$\therefore \lim_{h\to 0} \frac{1-\tan(\pi/4+h)}{1-\sqrt{2}\sin\left(\frac{\pi}{4}+h\right)}$$

$$\lim_{h\to 0} \frac{1-\frac{(1+\tan h)}{(1-\tan h)}}{1-\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos h+\frac{1}{\sqrt{2}}\sin h\right]}$$

$$\lim_{h\to 0} \frac{\cancel{1}-\tan h-\cancel{1}-\tan h}{(1-\tan h)\left[1-\cos h-\sin h\right]}$$

$$\lim_{h\to 0} \frac{(-2\,\underbrace{\tan h)\times h}_{\to =1}}{(1-\tan h)\times h\times\left[1-\cos h-\sin h\right]}$$

$$\lim_{h\to 0} \frac{-2h}{1-\frac{\sin h}{\cos h}\left[2\sin^2\frac{h}{2}-2\sin\frac{h}{2}\cos\frac{h}{2}\right]}$$

$$\lim_{h\to 0} \frac{-2\times h/2\times\cancel{2}\times\cos h}{\left[\cos h-\sin h\right]\cancel{2}\underbrace{\sin\frac{h}{2}}_{=1}\left[\sin\frac{h}{2}-\cos\frac{h}{2}\right]}$$

$$\therefore \frac{-2\times 1}{(1-0)(0-1)} = \boxed{2}$$

Ans

4. $\lim_{x\to 0}\frac{8}{x^8}\left[1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right]$

sol^n:

$$= \lim_{x\to 0}\frac{8}{x^8}\left(1-\cos\frac{x^2}{2}\right)\left(1-\cos\frac{x^2}{4}\right)$$

$$\because \lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$$

$$\therefore \lim_{x\to 0}\left(\frac{8}{x^8}\right)\left(\frac{1-\cos\frac{x^2}{2}}{\frac{x^4}{4}}\right)\left(\frac{x^4}{4}\right)\left(\frac{1-\cos\frac{x^2}{4}}{\frac{x^4}{16}}\right)\times\frac{x^4}{16}$$

$$= 8\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{16}\right) = \frac{1}{32}$$

5. $\lim\limits_{x\to\infty} x^2\left(\sqrt{\frac{x+2}{x}} - \sqrt[3]{\frac{x+3}{x}}\right)$

Solⁿ $\lim\limits_{x\to\infty} x^2\left[\left(1+\frac{2}{x}\right)^{\frac{1}{2}} - \left(1+\frac{3}{x}\right)^{\frac{1}{3}}\right]$

~~Lim~~ Replace $x\to\infty$ to $t\to 0$

$$\lim_{t\to 0} \frac{(1+2t)^{\frac{1}{2}} - (1+3t)^{\frac{1}{3}}}{t^2}$$

$$\lim_{t\to 0} \frac{\cancel{1}+\cancel{2t\times\left(\frac{1}{2}\right)} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(2t)^2 \cancel{-1} - \cancel{\frac{1}{3}\times 3t} - \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2}(3t)^2}{t^2}$$

$$\lim_{t\to 0} \frac{\left(-\frac{1}{8}\times 4 + \frac{1}{1}\right)\cancel{t^2}}{\cancel{t^2}} \Rightarrow \boxed{\frac{1}{2}}$$

6. $\lim\limits_{x\to-\infty}\dfrac{(3x^4+2x^2)\sin\frac{1}{x}+|x|^3+5}{|x|^3+|x|^2+|x|+1}$

Sol^n $= \lim\limits_{x\to-\infty}\dfrac{(3x^4+2x^2)\sin\frac{1}{x}+|x|^3+5}{(|x|+1)(|x|^2+1)}$

Replace $x \to \frac{1}{t}$ $\quad t\to 0^-$

$= \lim\limits_{t\to 0^-}\dfrac{\left(\frac{3}{t^4}+\frac{2}{t^2}\right)\sin t+\left|\frac{1}{t^3}\right|+5}{\left(\left|\frac{1}{t}\right|+1\right)\left(\left|\frac{1}{t}\right|^2+1\right)}$

$= \lim\limits_{t\to 0^-}\dfrac{\left(\frac{3+2t^2}{t^4}\right)\sin t-\frac{1}{t^3}+5}{\left(1-\frac{1}{t}\right)\left(1+\frac{1}{t^2}\right)}$

$= \lim\limits_{t\to 0^-}\dfrac{(3+2t^2)-1+5t^3}{(t-1)(t^2+1)}$

$= \lim\limits_{t\to 0^-}\dfrac{5t^3+2t^2+2}{(t-1)(t^2+1)} = \dfrac{0+0+2}{(0-1)(0+1)} = -2$ //

7. If $\ell = \lim_{n\to\infty} \sum_{r=2}^{n} \left((r+1)\sin\frac{\pi}{r+1} - r\sin\frac{\pi}{r} \right)$ then find $\{\ell\}$. (where {} denotes the fractional part function)

sol:

$$l = \lim_{n\to\infty} \left[\left(\cancel{3\cdot \sin\frac{\pi}{3}} - 2\cdot \sin\frac{\pi}{2}\right) + \left(\cancel{4\sin\frac{\pi}{4}} - \cancel{3\sin\frac{\pi}{3}}\right) + \left(\cancel{5\sin\frac{\pi}{5}} - \cancel{4\sin\frac{\pi}{4}}\right) + \cdots \left((n+1)\sin\frac{\pi}{(n+1)} - \cancel{n\sin\frac{\pi}{n}}\right) \right]$$

$$l = \lim_{n\to\infty} \left[(n+1)\sin\frac{\pi}{n+1} - 2\times 1 \right]$$

$$l = \lim_{n\to\infty} \cancel{(n+1)} \frac{\sin\left(\frac{\pi}{n+1}\right)}{\left(\frac{\pi}{n+1}\right)} \times \left(\frac{\pi}{\cancel{n+1}}\right) - 2$$

$$l = \pi - 2$$

$$\therefore \{l\} = \{\pi - 2\} = (\pi-2) - [\pi-2] = \pi - 2 - 1 = \boxed{\pi - 3}$$

8. Find a & b if : (i) $\lim_{x\to\infty}\left[\frac{x^2+1}{x+1}-ax-b\right]=0$

Solⁿ (i) $x \to \frac{1}{t} \Rightarrow t \to 0$

$\therefore \lim_{t\to 0}\left[\frac{1+t^2}{t(1+t)} - \frac{a}{t} - b\right] = 0$

$\Rightarrow \lim_{t\to 0} \frac{1+t^2-a-at-bt-bt^2}{t(1+t)} = 0$

$\Rightarrow \lim_{t\to 0} \frac{(1-b)t^2-(a+b)t+(1-a)}{t(t+1)} = 0$

$\Rightarrow 1-a=0 \quad \therefore \Rightarrow \boxed{a=1}$

$\lim_{t\to 0} \frac{(1-b)t-(a+b)}{t+1} = 0$

$\frac{-(a+b)}{1} = 0 \quad \Rightarrow \quad a+b=0 \Rightarrow \boxed{b=-1}$

(ii) $\lim_{x\to-\infty}\left[\sqrt{x^2-x+1}-ax-b\right]=0$

(ii) $\lim_{x\to-\infty} \quad x \to \frac{1}{t} \Rightarrow t \to 0^-$

$\lim_{t\to 0^-} \sqrt{\frac{1}{t^2}-\frac{1}{t}+1} - \left(\frac{a}{t}+b\right) = 0$

$\lim_{t\to 0^-} \frac{\sqrt{1-t+t^2}}{-t} - \left(\frac{a+bt}{t}\right) = 0$

$\lim_{t\to 0^-} \frac{\sqrt{t^2-t+1}+(a+bt)}{-t} = 0$

$\lim_{t\to 0^-} \frac{(1+t^2-t)^{\frac{1}{2}}+a+bt}{-t} = 0$

$\lim_{t\to 0^-} \frac{1+\frac{1}{2}(t^2-t)+a+bt}{-t} = 0 \qquad a+1=0 \Rightarrow \boxed{a=-1}$

$\lim_{t\to 0^-} \frac{\frac{t^{\cancel{2}}}{2}+\left(b-\frac{1}{2}\right)\cancel{t}}{-\cancel{t}} = 0 \quad \Rightarrow \boxed{b=\frac{1}{2}}$

9. $\lim\limits_{x\to 0} [\ell n\ (1 + \sin^2 x).\ \cot\ (\ell n^2\ (1 + x))]$

Sol:

$$= \lim_{x\to 0} \frac{\ln(1+\sin^2 x)}{\tan(\ln^2(1+x))} \times \frac{\sin^2 x}{\sin^2 x} \times \frac{\ln^2(1+x)}{\ln^2(1+x)}$$

$$= \lim_{x\to 0} \frac{\ln(1+\sin^2 x)}{\sin^2 x} \; \frac{\ln^2(1+x)}{\tan(\ln^2(1+x))} \; \frac{\sin^2 x}{\ln^2(1+x)}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{\ln^2(1+x)} \times \frac{x^2}{x^2} \Rightarrow \boxed{1}$$

10. $$\lim_{x\to 0}\frac{27^x-9^x-3^x+1}{\sqrt{2}-\sqrt{1+\cos x}}$$

sol:

$$= \lim_{x\to 0}\frac{(9^x-1)(3^x-1)}{(\sqrt{2}-\sqrt{1+\cos x})(\sqrt{2}+\sqrt{1+\cos x})}\times\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

$$= \lim_{x\to 0}\frac{(9^x-1)(3^x-1)}{x\cdot x}\times\frac{x^2\times\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{(2-1-\cos x)}$$

$$= \frac{\ln 9\cdot\ln 3\cdot\left(\sqrt{2}+\sqrt{2}\right)}{\frac{1}{2}}$$

$$= 4\sqrt{2}\,\ln 9\cdot\ln 3$$

$$= \boxed{8\sqrt{2}\,(\ln 3)^2}$$

11. (a) $\lim\limits_{x \to 0} \tan^{-1} \dfrac{a}{x^2}$, where $a \in \mathbb{R}$; (b) Plot the graph of the function $f(x) = \lim\limits_{t \to 0} \left(\dfrac{2x}{\pi} \tan^{-1} \dfrac{x}{t^2} \right)$

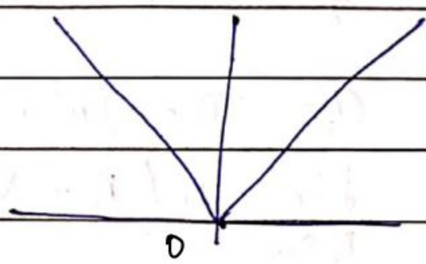
Solⁿ A

$$= \begin{cases} \dfrac{\pi}{2} & ; \ a > 0 \\ 0 & ; \ a = 0 \\ -\pi/2 & ; \ a < 0 \end{cases}$$

B

$$= \begin{cases} \dfrac{2x}{\pi} \cdot \dfrac{\pi}{2} = x & ; \ x > 0 \\ 0 & ; \ x = 0 \\ -x & ; \ x < 0 \end{cases}$$

$f(x) = |x|$



12. Let $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = x^2 + x - 2$.

If $\lim_{x\to 1}\frac{f(x)}{g(x)} = 1$ and $\lim_{x\to -2}\frac{f(x)}{g(x)} = 4$, then find the value of $\frac{c^2+d^2}{a^2+b^2}$.

Solⁿ $g(x) = x^2 + x - 2$

$= (x-1)(x+2)$

$\therefore \lim_{x\to 1} \frac{ax^3+bx^2+cx+d}{(x-1)(x+2)} = 1$ $\quad (x-1)$ is a factor

& $\lim_{x\to -2} \frac{ax^3+bx^2+cx+d}{(x-1)(x+2)} = 4$ $\quad (x+2)$ is a factor

$\therefore \lim_{x\to 1} a(x+\lambda) = 1$

$a(1+\lambda) = 1$ —— (1)

& $\lim_{x\to -2} a(x+\lambda) = 4$

$a(-2+\lambda) = 4$ —— (2)

from eqⁿ (1) & (2) we get

$a = -1$ & $\lambda = -2$

$\therefore (-1)(x-1)(x+2)(x-2)$

$= (1-x)(x^2-4)$

$= x^2 - x^3 - 4 + 4x$

$a = -1, \quad b = 1, \quad c = 4, \quad d = -4$

$\therefore \frac{c^2+d^2}{a^2+b^2} = \frac{16+16}{1+1} \Rightarrow \boxed{16}$

13. $\lim\limits_{x\to\infty}\left[\dfrac{2x^2+3}{2x^2+5}\right]^{8x^2+3}$

Sol^n $\lim\limits_{x\to\infty}\left[\dfrac{2x^2+5-2}{2x^2+5}\right]^{8x^2+3}$

$= \lim\limits_{x\to\infty}\left[1-\dfrac{2}{2x^2+5}\right]^{8x^2+3}$

$\because 1^\infty$ form

$\therefore$

$= e^{\lim\limits_{x\to\infty}\left[\cancel{1}-\frac{2}{2x^2+5}-\cancel{1}\right](8x^2+3)}$

$= e^{\lim\limits_{x\to\infty}\frac{-2(8x^2+3)}{2x^2+5}}$

$= e^{\frac{-16}{2}} \Rightarrow \boxed{e^{-8}}$

14. $\lim_{x\to\infty}\left(\frac{x+c}{x-c}\right)^x = 4$ then find c

Sol^n $\lim_{x\to\infty}\left[\frac{x-c+2c}{x-c}\right]^x = 4$

$\lim_{x\to\infty}\left[1+\frac{2c}{x-c}\right]^x = 4$

$\because$ 1^∞ form

$\therefore$ $e^{\lim_{x\to\infty}\left[\cancel{1}+\frac{2c}{x-c}-\cancel{1}\right]x} = 4$

$\Rightarrow e^{\lim_{x\to\infty}\frac{2cx}{x-c}} = 4$

$\Rightarrow e^{\lim_{x\to\infty}\frac{2c}{1-c/x}} = 4$

$\Rightarrow e^{2c} = 4$

$\Rightarrow 2c = \ln 4$

$\boxed{c = \ln 2}$

15. $\lim_{x\to 1}\left(\tan\frac{\pi x}{4}\right)^{\tan\frac{\pi x}{2}}$

Sol: 1^∞ form

$$e^{\lim_{x\to 1}\left[\tan\frac{\pi x}{4}-1\right]\tan\frac{\pi x}{2}}$$

$$e^{\lim_{x\to 1}\left[\tan\frac{\pi x}{4}-1\right]\frac{2\tan\frac{\pi x}{4}}{1-\tan^2\frac{\pi x}{4}}}$$

$$e^{\lim_{x\to 1}\frac{-\left(+2\tan\frac{\pi x}{4}\right)\cancel{\left(1-\tan\frac{\pi x}{4}\right)}}{\left(1+\tan\frac{\pi x}{4}\right)\cancel{\left(1-\tan\frac{\pi x}{4}\right)}}}$$

$$e^{\lim_{x\to 1}\frac{-2\tan\frac{\pi x}{4}}{1+\tan\frac{\pi x}{4}}}$$

$$e^{\frac{-2}{1+1}} \Rightarrow \boxed{e^{-1}}$$

16. $\lim_{x\to 0}\left(\frac{x-1+\cos x}{x}\right)^{\frac{1}{x}}$

Sol^n $= \lim_{x\to 0}\left[1+\frac{\cos x-1}{x}\right]^{\frac{1}{x}}$

$= e^{\lim_{x\to 0}\left[\cancel{1}+\frac{\cos x-1}{x}-\cancel{1}\right]\frac{1}{x}}$

$= e^{\lim_{x\to 0}\left(\frac{\cos x-1}{x^2}\right)}$

$= \boxed{e^{-\frac{1}{2}}}$

EXERCISE (S-2)

EXERCISE (S-2)

1. $$\lim_{n\to\infty}\left(\frac{\sqrt{n^2+n-1}}{n}\right)^{2\sqrt{n^2+n}-1}$$

Sol:- It is 1^∞ indeterminate form

$$= e^{\underset{n\to\infty}{Lt}\left(2\sqrt{n^2+n}-1\right)\left(\frac{\sqrt{n^2+n}-1}{n}-1\right)}$$

$$= e^{\underset{n\to\infty}{Lt}\left(2\sqrt{n^2+n}-1\right)\left(\frac{\sqrt{n^2+n}-n-1}{n}\right)}$$

Put $n=\frac{1}{y}$

$$= e^{\underset{y\to 0}{Lt}\left(2\sqrt{\frac{1}{y^2}+\frac{1}{y}}-1\right)\left(\frac{\sqrt{\frac{1}{y^2}+\frac{1}{y}}-\frac{1}{y}-1}{\frac{1}{y}}\right)}$$

$$= e^{\underset{y\to 0}{Lt}\left(\frac{2\sqrt{1+y}-y}{y}\right)\left(\frac{\sqrt{1+y}-1-y}{\cancel{y}\cdot\frac{1}{\cancel{y}}}\right)}$$

$$= e^{\underset{y\to 0}{Lt}\left(\frac{2\sqrt{1+y}-y}{y}\right)\frac{(1+y)-(1+y)^2}{\sqrt{1+y}+(1+y)}}\ \text{[Rationalisation]}$$

$$= e^{\underset{y\to 0}{Lt}\left[\frac{2\sqrt{1+y}-y}{y}\right]\frac{\cancel{1}+y-\cancel{1}-y^2-2y}{\sqrt{1+y}+(1+y)}}$$

$$= e^{\underset{y\to 0}{Lt}\frac{2\sqrt{1+y}-y}{\sqrt{1+y}+(1+y)}\ \underset{y\to 0}{Lt}\frac{-y^2-y}{y}}$$

$$= e^{\underset{y\to 0}{Lt}\frac{2\sqrt{1+y}-y}{\sqrt{1+y}+(1+y)}\ \underset{y\to 0}{Lt}(-1-y)}$$

$$= e^{\frac{2}{2}(-1)}$$

$$= e^{-1}$$

2. $\lim_{x\to\infty}\left(\dfrac{a_1^{\frac{1}{x}}+a_2^{\frac{1}{x}}+a_3^{\frac{1}{x}}+\dots\dots+a_n^{\frac{1}{x}}}{n}\right)^{nx}$, $n \in \mathbb{N}$, where $a_1, a_2, a_3, \dots . a_n > 0$

Sol:- Form is 1^∞

$$= e^{\underset{x\to\infty}{Lt}\; nx\left(\frac{a_1^{1/x}+a_2^{1/x}+\dots+a_n^{1/x}}{n}-1\right)}$$

$$= e^{\underset{x\to\infty}{Lt}\; \cancel{n}x\left(\frac{a_1^{1/x}+a_2^{1/x}+\dots+a_n^{1/x}-n}{\cancel{n}}\right)}$$

$$= e^{\underset{x\to\infty}{Lt}\; x\left[(a_1^{1/x}-1)+(a_2^{1/x}-1)+\dots+(a_n^{1/x}-1)\right]}$$

Put $x = \frac{1}{y}$

$$= e^{\underset{y\to 0}{Lt}\; \frac{(a_1^{1/x}-1)+(a_2^{1/x}-1)+\dots+(a_n^{1/x}-1)}{1/x}}$$

$$= e^{\underset{y\to 0}{Lt}\frac{a_1^y-1}{y}+\underset{y\to 0}{Lt}\frac{a_2^y-1}{y}+\dots+\underset{y\to 0}{Lt}\frac{a_n^y-1}{y}}$$

$$= e^{\ln a_1+\ln a_2+\dots+\ln a_n}$$

$$= e^{\ln(a_1 a_2 a_2 \dots a_n)}$$

$$= a_1 a_2 a_3 \dots a_n$$

3. $\displaystyle \lim_{x\to 0}\left[\frac{(1+x)^{1/x}}{e}\right]^{1/x}$

<u>Sol</u>:- $= e^{\displaystyle \operatorname*{Lt}_{x\to 0} \frac{1}{x}\left[\frac{(1+x)^{1/x}}{e}-1\right]}$ $\left[\because \operatorname*{Lt}_{x\to 0}(1+x)^{1/x}=e\right.$ This is 1^{∞} form$\left.\right]$

$$= e^{\displaystyle \operatorname*{Lt}_{x\to 0} \frac{1}{x}\left[\frac{e^{\ln(1+x)^{1/x}}-e}{e}\right]}$$

$$= e^{\displaystyle \operatorname*{Lt}_{x\to 0}\left[\frac{e^{\frac{1}{x}\ln(1+x)-1}-1}{x}\right]}$$

Now, as $x\to 0$, $\frac{1}{x}\ln(1+x)-1 \to \frac{1}{x}\left(x-\frac{x^2}{2}+\dots\right)-1 \to 0$

$$= e^{\displaystyle \operatorname*{Lt}_{x\to 0} \frac{e^{\frac{1}{x}\ln(1+x)-1}-1}{\frac{1}{x}\ln(1+x)-1}\cdot\frac{\frac{1}{x}\ln(1+x)-1}{x}}$$

$$= e^{\displaystyle \operatorname*{Lt}_{x\to 0} \frac{e^{\frac{1}{x}\ln(1+x)-1}-1}{\frac{1}{x}\ln(1+x)-1}\cdot\operatorname*{Lt}_{x\to 0}\frac{\frac{1}{x}\ln(1+x)-1}{x}}$$

Put $\frac{1}{x}\ln(1+x)-1=y$; As $x\to 0$, $y\to 0$

$$= e^{\displaystyle \operatorname*{Lt}_{y\to 0}\frac{e^y-1}{y}\cdot\operatorname*{Lt}_{x\to 0}\frac{\frac{1}{x}\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\dots\right)-1}{x}}$$

$$= e^{\displaystyle (1)\operatorname*{Lt}_{x\to 0}\frac{\left(1-\frac{x}{2}+\frac{x^2}{3}-\dots\right)-1}{x}}$$

$$= e^{\displaystyle \operatorname*{Lt}_{x\to 0}\left(-\frac{1}{2}+\frac{x}{3}-\dots\right)}$$

$$= e^{-1/2}$$

4. If $\lim\limits_{x\to\infty}\dfrac{a(2x^3-x^2)+b(x^3+5x^2-1)-c(3x^3+x^2)}{a(5x^4-x)-bx^4+c(4x^4+1)+2x^2+5x}=1$, then the value of $(a+b+c)$ can be expressed in the lowest form as $\dfrac{p}{q}$. Find the value of $(p+q)$.

Sol: $= \lim\limits_{x\to\infty} \dfrac{x^3(2a+b-3c)+x^2(-a+5b-c)-b}{x^4(5a-b+4c)+x^2(2)+x(-a+5)+c}$

$\lim\limits_{x\to\infty} \dfrac{\text{Polynomial}}{\text{Polynomial}}$ form

For this to exist and nonzero,

x^4 coefficient in denominator $=0$

x^3 coefficient in numerator $=0$

x^2 coefficient in numerator and denominator to be same for the limit value to be $=1$

$\Rightarrow$ $5a-b+4c=0$ —— (1)

$2a+b-3c=0$ —— (2)

$-a+5b-c=2$ —— (3)

Solving these we get, $a=\dfrac{-2}{109}$, $b=\dfrac{46}{109}$, $c=\dfrac{19}{109}$

$\Rightarrow$ $a+b+c=\dfrac{58}{109}=\dfrac{p}{q}$

$p+q=58+109=167$

5. $$\lim_{x\to 0}\left[\frac{\ell n(1+x)^{1+x}}{x^2}-\frac{1}{x}\right]$$

Sol

$$= \underset{x\to 0}{\mathrm{Lt}} \frac{\ln(1+x)^{1+x} - x}{x^2}$$

$$= \underset{x\to 0}{\mathrm{Lt}} \frac{(1+x)\ln(1+x) - x}{x^2}$$

$$= \underset{x\to 0}{\mathrm{Lt}} \frac{(1+x)\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots\right) - x}{x^2}$$

$$= \underset{x\to 0}{\mathrm{Lt}} \frac{\cancel{x}-\frac{x^2}{2}+x^2-\cancel{x}}{x^2}$$ (Neglecting x^3 and higher terms)

$$= \frac{1}{2}$$

6. f(x) is the function such that $\lim_{x\to 0}\frac{f(x)}{x} = 1$. If $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{(f(x))^3} = 1$, then find the value of a and b.

Sol:-

$$\frac{\lim_{x\to 0}\dfrac{x(1+a\cos x)-b\sin x}{x^3}}{\lim_{x\to 0}\dfrac{(f(x))^3}{x^3}} = 1$$

$$\Rightarrow \lim_{x\to 0}\frac{\dfrac{x+ax\cos x-b\sin x}{x^3}}{1} = 1$$

$$\Rightarrow \lim_{x\to 0}\frac{x+ax\left(1-\frac{x^2}{2!}+\cdots\right)-b\left(x-\frac{x^3}{3!}+\cdots\right)}{x^3} = 1$$

[Neglecting x^4 and higher terms] Since maximum in denominator is x^3

$$\Rightarrow \lim_{x\to 0}\frac{x(1+a-b)+x^2(0)+x^3\left(-\frac{a}{2}+\frac{b}{6}\right)}{x^3} = 1$$

$\Rightarrow 1+a-b=0$ and $-\frac{a}{2}+\frac{b}{6} = 1$

Solving these equations,

we get $a = -\frac{5}{2}$, $b = -\frac{3}{2}$

EXERCISE (JM)

1. Let $p = \lim_{x \to 0+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to - **[JEE(Main)-2016]**

(1) $\frac{1}{4}$ (2) 2 (3) 1 (4) $\frac{1}{2}$

Sol^n clearly 1^∞ form

$$\lim_{x \to 0^+} \exp\left(1 + \tan^2\sqrt{x} - 1\right) \cdot \left(\frac{1}{2x}\right)$$

$$\lim_{x \to 0^+} \exp\left(\frac{\tan^2\sqrt{x}}{2x}\right)$$

$$p = \lim_{x \to 0^+} \exp\left(\frac{1}{2}(1)\right) \longrightarrow \text{vicinity of } 0^+$$

x can be written as $(\sqrt{x})^2$

$$\log p = \log e^{(1/2)}$$

$$= \left(\frac{1}{2}\right)$$

D

2. $\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals :- **[JEE(Main)-2017]**

(1) $\frac{1}{4}$ (2) $\frac{1}{24}$ (3) $\frac{1}{16}$ (4) $\frac{1}{8}$

Solⁿ

$$\lim_{h \to 0} \frac{\cot\left(\frac{\pi}{2}+h\right) - \cos\left(\frac{\pi}{2}+h\right)}{\left(\pi - 2\left(\frac{\pi}{2}+h\right)\right)^3}$$

$x = \frac{\pi}{2}+h$

$$\lim_{h \to 0} \frac{-\tan h - (-\sin h)}{-8h^3}$$

$$\lim_{h \to 0} \underbrace{\frac{\sin h - \tan h}{8h^3}}_{\text{sub-standard limit } \left(\frac{1}{2}\right)}$$

$$= \frac{1}{8}\left(\frac{1}{2}\right) = \frac{1}{16}$$

C

3. For each t ∈ R, let [t] be the greatest integer less than or equal to t. Then

$$\lim_{x\to 0+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\dots+\left[\frac{15}{x}\right]\right)$$ **[JEE(Main)-2018]**

(1) is equal to 15. (2) is equal to 120.
(3) does not exist (in R). (4) is equal to 0.

Sol^n

$[x] = x - \{x\}$ → fraction Part

$$\lim_{x\to 0^+} x\left(\frac{1}{x} - \left\{\frac{1}{x}\right\} + \frac{2}{x} - \left\{\frac{2}{x}\right\} - - -\right)$$

$$\lim_{x\to 0^+} x\left(\frac{1}{x}\underbrace{(1+2+---15)}_{\frac{n(n+1)}{2}} - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} - - - - \left\{\frac{15}{x}\right\}\right)\right)$$

$$\lim_{x\to 0^+} \frac{15(16)}{2} - x\left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} - - -\right)$$

$$\lim_{x\to 0^+} 120 - \underbrace{x}_{\text{tending zero}}\{\text{finite value}\}$$

$\{x\}$ lies in $[0,1)$

$120 - 0^{\uparrow} \Rightarrow 120$ B

4. $\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$ **[JEE(Main)-2019]**

(1) exists and equals $\frac{1}{4\sqrt{2}}$

(2) does not exist

(3) exists and equals $\frac{1}{2\sqrt{2}}$

(4) exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

<u>Sol^n</u> $\sqrt{x}-\sqrt{y}=\frac{x-y}{\sqrt{x}+\sqrt{y}}$ (Rationalize)

$$\lim_{y\to 0}\frac{\sqrt{1+y^4}-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$ → Rationalize

→ Partial limit

$$\lim_{y\to 0}\frac{\cancel{y^4}}{\cancel{y^4}\,(2\sqrt{2})\left(\sqrt{1+y^4}+1\right)}$$

$$\Rightarrow \lim_{y\to 0}\frac{1}{2\sqrt{2}\left(\sqrt{1+y^4}+1\right)}$$

$$\Rightarrow \frac{1}{4\sqrt{2}}$$

(A)

5. For each $x \in R$, let [x] be the greatest integer less than or equal to x.

Then $\lim\limits_{x \to 0^-} \dfrac{x([x]+|x|)\sin[x]}{|x|}$ is equal to **[JEE(Main)-2019]**

(1) $-\sin 1$ (2) 0 (3) 1 (4) $\sin 1$

<u>Sol^n</u> in vicinity of $(0^-) \rightarrow$ values of x

$$[x] = -1$$

$$\lim_{h \to 0^+} \frac{-\cancel{h}(-1+h)\sin(-1)}{\cancel{h}}$$

$\underline{\underline{x = -h}}$

$$\lim_{h \to 0^+} \sin(1)(h-1)$$

$$\Rightarrow -\sin(1)$$

6. For each t ∈ R, let [t] be the greatest integer less than or equal to t.

Then, $\lim\limits_{x\to 1+}\dfrac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$ **[JEE(Main)-2019]**

(1) equals –1 (2) equals 1 (3) does not exist (4) equals 0

$\underline{\underline{\text{Sol}^n}}$ in vicinity of 1^+ (values of x)

$$[1-x] = -1$$

$$\therefore L = \lim_{h\to 0} \frac{(1-|1+h|+\sin|-h|)\sin\left(-\frac{\pi}{2}\right)}{|-h|(-1)}$$

$$\lim_{h\to 0}\left(\frac{\cancel{1}-\cancel{1}-h+\sin h}{h}\right)\cdot\frac{(-1)}{(-1)}$$

$$\lim_{h\to 0}\frac{\sin h - h}{h} = \lim_{h\to 0}\frac{-h^3}{3!(h)} = 0$$

(4)

7. Let [x] denote the greatest integer less than or equal to x.

Then $\lim_{x \to 0} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$ **[JEE(Main)-2019]**

(1) equals π (2) equals 0 (3) equals $\pi + 1$ (4) does not exist

Sol^n

$[x] = 0$ (at 0^+)

R.H.L

$$\lim_{x \to 0^+} \frac{\tan(\pi \sin^2 x)}{x^2} + \frac{(x - 0)^2}{x^2}$$

$$\lim_{x \to 0^+} \frac{\tan(\pi \sin^2 x)}{\pi \sin^2 x} \cdot \frac{(\pi \sin^2 x)}{x^2} + \lim_{x \to 0} \frac{x^2}{x^2}$$

$$\Rightarrow \pi + 1$$

L.H.L

$[x] = -1$ (at 0^-)

$$\lim_{x \to 0^-} \frac{\tan(\pi \sin^2 x)}{x^2} + \lim_{x \to 0^-} \frac{(\sin x - x)^2}{x^2}$$

$$\Rightarrow \pi + \lim_{x \to 0^-} \left(\frac{\sin x - x}{x}\right)^2$$

$$\Rightarrow \pi + 0$$

D.N.E

(D)

8. $\lim_{x \to \pi/4} \frac{\cot^3 x - \tan x}{\cos\left(x + \pi/4\right)}$ is : **[JEE(Main)-2019]**

(1) 4 (2) $8\sqrt{2}$ (3) 8 (4) $4\sqrt{2}$

Sol^n

$$\lim_{x \to \frac{\pi}{4}} \frac{(1-\tan^2 x)(1+\tan^2 x)}{\tan x \cos\left(x+\frac{\pi}{4}\right)}$$

$$\lim_{x \to \pi/4} \frac{(1-\tan x)(1+\tan x)(1+\tan^2 x)}{\tan x \cos\left(x+\pi/4\right)}$$

apply partial limit

$$\lim_{x \to \pi/4} \frac{(1-\tan x)(2)(2)}{(1)\cos\left(x+\pi/4\right)}$$

$$\lim_{x \to \pi/4} 4\sqrt{2}\, \frac{1}{\sqrt{2}} \frac{\left(\frac{\cos x - \sin x}{\underset{\left(\frac{1}{\sqrt{2}}\right)}{\cos x}}\right)}{\cos\left(x+\pi/4\right)}$$

$$\lim_{x \to \pi/4} \frac{8\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}{\cos\left(x+\pi/4\right)}$$

$$\lim_{x \to \pi/4} \frac{8\cos\left(x+\pi/4\right)}{\cos\left(x+\pi/4\right)} \Rightarrow 8$$

C

9. $\lim_{x\to 1^-}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ equal to : **[JEE(Main)-2019]**

(1) $\frac{1}{\sqrt{2\pi}}$ (2) $\sqrt{\frac{\pi}{2}}$ (3) $\sqrt{\frac{2}{\pi}}$ (4) $\sqrt{\pi}$

Solⁿ $x = \cos\theta$

So $x \to 1^-$ $\theta \to 0$ (or $0^+, 0^-$)

$$\lim_{\theta\to 0^+} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1}(\cos\theta)}}{\sqrt{1-\cos\theta}}$$

$$\lim_{\theta\to 0^+} \frac{\pi - 2\sin^{-1}(\cos\theta)}{\sqrt{2}\sin(\theta/2)} \frac{1}{\left(\sqrt{\pi}+\sqrt{2\sin^{-1}(\cos\theta)}\right)}$$

$$\lim_{\theta\to 0^+} \frac{2\left(\pi/2 - \sin^{-1}(\cos\theta)\right)}{\sqrt{2}\sin(\theta/2)} \frac{1}{\left(\sqrt{\pi}+\sqrt{2\sin^{-1}(\cos\theta)}\right)}$$

$$\lim_{\theta\to 0^+} \sqrt{2}\,\frac{(\cos^{-1}(\cos\theta))}{(\sin\theta/2)} \cdot \frac{\theta/2}{\theta/2} \cdot \frac{1}{(2\sqrt{\pi})} \to (1)$$

$$\lim_{\theta\to 0^+} \sqrt{2}\,\frac{\cancel{\theta}}{\cancel{\theta}/2}\frac{1}{(2\sqrt{\pi})} \Rightarrow \frac{2\sqrt{2}}{2\sqrt{\pi}} = \sqrt{\frac{2}{\pi}}$$

10. $\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals : **[JEE(Main)-2019]**

(1) $2\sqrt{2}$ (2) $4\sqrt{2}$ (3) $\sqrt{2}$ (4) 4

Sol^n

$$\lim_{x\to 0}\frac{(1-\cos^2 x)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{1-\cos x}$$

$$\lim_{x\to 0}\frac{(1+\cos x)\left(\sqrt{2}+\sqrt{1+\cos x}\right)\cancel{(1-\cos x)}}{\cancel{(1-\cos x)}}$$

$$\Rightarrow (1+1)\left(\sqrt{2}+\sqrt{1+1}\right)$$

$$\Rightarrow 4\sqrt{2}$$

11. Let $f : R \to R$ be a differentiable function satisfying $f'(3) + f'(2) = 0$.

Then $\lim\limits_{x\to 0}\left(\dfrac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to **[JEE(Main)-2019]**

(1) e^2 (2) e (3) e^{-1} (4) 1

Sol^n clearly 1^∞ form

$$\lim_{x\to 0} \exp\left[\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}-1\right)\cdot\frac{1}{x}\right]$$

$$\lim_{x\to 0} \exp\left(\frac{(f(3+x)-f(3)) + f(2)-f(2-x)}{x\,(1+f(2-x)-f(2))}\right)$$

Partial limit (1)

$$\lim_{x\to 0} \exp\left(\frac{f(3+x)-f(3)}{x}+\frac{f(2)-f(2-x)}{x}\right)$$

$$\lim_{x\to 0} \exp\left(f'(3)+f'(2)\right) \Rightarrow e^0 = 1$$

(D)

Alter

Apply L'Hospital rule

12. If $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then k is : **[JEE(Main)-2019]**

(1) $\frac{3}{8}$ (2) $\frac{3}{2}$ (3) $\frac{4}{3}$ (4) $\frac{8}{3}$

Sol^n

$$\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to 1}\frac{\cancel{(x-1)}(x+1)(x^2+1)}{\cancel{(x-1)}}$$

$$\lim_{x\to 1}(x+1)(x^2+1)\Rightarrow 4$$

$$\longrightarrow \lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}=4$$

$$\lim_{x\to k}\frac{\cancel{(x-k)}(x^2+k^2+x(k))}{\cancel{(x-k)}(x+k)}=4$$

$$\frac{k^2+k^2+k^2}{2k}=4$$

$$3k^2=8k$$

$k=0$ (reject) | $k=8/3$ ✓

(D)

$k=0$ not satisfy given expression

13. If $\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$, then a + b is equal to :- **[JEE(Main)-2019]**

(1) –7 (2) – 4 (3) 5 (4) 1

Sol^n limit is non zero finite

it means

$$x^2 - ax + b = (x-1)(x-\alpha)$$

So

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x-\alpha)}{\cancel{(x-1)}} = 5$$

$$1 - \alpha = 5$$

$$\alpha = -4$$

then

$$x^2 - ax + b = (x-1)(x+4)$$

$$x^2 - ax + b = x^2 + 3x - 4$$

$$a = -3, \quad b = -4$$

$$a + b = -7$$

A

EXERCISE (JA)

1. Let $L = \lim\limits_{x\to 0} \dfrac{a - \sqrt{a^2 - x^2} - \dfrac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then - **[JEE 2009, 4]**

(A) $a = 2$ (B) $a = 1$ (C) $L = \dfrac{1}{64}$ (D) $L = \dfrac{1}{32}$

Sol^n

$$= \lim_{x\to 0} \frac{a - a\left(1 - \frac{x^2}{a^2}\right)^{1/2} - \frac{x^2}{4}}{x^4}$$

$$= \lim_{x\to 0} \frac{a - a\left[1 - \frac{x^2}{2a^2} + \frac{x^4}{a^4} \times \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}\right] - \frac{x^2}{4}}{x^4}$$

$$= \lim_{x\to 0} \frac{\cancel{a} - \cancel{a} + \frac{x^2}{2a} + \frac{1}{8}\frac{x^4}{a^3} - \frac{x^2}{4}}{x^4}$$

$$= \lim_{x\to 0} \frac{x^2\left[\frac{1}{2a} - \frac{1}{4}\right] + \frac{1}{8}\frac{x^4}{a^3}}{x^4}$$

∵ for limit to be exist coff. of $x^2 = 0$

So $\frac{1}{2a} - \frac{1}{4} = 0$

$a = 2$

$$L = \lim_{x\to 0} \frac{\frac{\cancel{x^4}}{64}}{\cancel{x^4}}$$

∴ $\Rightarrow L = \frac{1}{64}$

2. If $\lim_{x\to 0}\left[1+x\ell n(1+b^2)\right]^{\frac{1}{x}} = 2b\sin^2\theta, b>0$ and $\theta \in (-\pi,\pi]$, then the value of θ is-

[JEE 2011, 3M, –1M]

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$ (C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

Sol:

$\Rightarrow e^{\lim_{x\to 0}\frac{1}{\cancel{x}}\times \cancel{x}\ln(1+b^2)} = 2b\sin^2\theta$

$\Rightarrow 1+b^2 = 2b\sin^2\theta$

$\Rightarrow \sin^2\theta = \frac{1}{2}\left(b+\frac{1}{b}\right)$

$\because b>0$

so $b+\frac{1}{b} \geq 2$

$\therefore \sin^2\theta = \frac{1}{2}[2,\infty)$

$\Rightarrow \sin^2\theta = [1,\infty)$

$\Rightarrow \sin\theta \in (-\infty,-1] \cup [1,\infty)$

$\Rightarrow$ so $\sin\theta = \pm 1$

$\theta = \pm \pi/2$ $\quad\because \theta \in (-\pi,\pi]$

3. If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then - **[JEE 2012, 3M, –1M]**

(A) a = 1, b = 4 (B) a = 1, b = –4

(C) a = 2, b = –3 (D) a = 2, b = 3

Sol :

$$\Rightarrow \lim_{x\to\infty}\left[\frac{x^2+x+1-ax^2-ax-bx-b}{x+1}\right]=4$$

$$\Rightarrow \lim_{x\to\infty}\frac{(1-a)x^2+(1-a-b)x+1-b}{x+1}=4$$

For limit to be exist coff. of $x^2=0$

So $1-a=0$

$$\boxed{a=1}$$

$$\Rightarrow \lim_{x\to\infty}\frac{[1-1-b]x+1-b}{x+1}=4$$

$$\Rightarrow \lim_{x\to\infty}\frac{\cancel{x}}{\cancel{x}}\left[\frac{-b+\frac{1-b}{x}}{1+\frac{1}{x}}\right]=4$$

$$\Rightarrow -b=4$$

$$\Rightarrow b=-4$$

4. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where $a>-1$. Then $\lim_{a\to0^+}\alpha(a)$ and $\lim_{a\to0^+}\beta(a)$ are **[JEE 2012, 3M, –1M]**

(A) $-\frac{5}{2}$ and 1 (B) $-\frac{1}{2}$ and -1 (C) $-\frac{7}{2}$ and 2 (D) $-\frac{9}{2}$ and 3

Sol:

$$\lim_{a\to0^+}\left[\left((1+a)^{1/3}-1\right)x^2+\left[(1+a)^{1/2}-1\right]x+\left[(1+a)^{1/6}-1\right]\right]$$

$$\Rightarrow \left[\cancel{1}+\frac{a}{3}-\cancel{1}\right]x^2+\left[\cancel{1}+\frac{a}{2}-\cancel{1}\right]x+\left[\cancel{1}+\frac{a}{6}-\cancel{1}\right]=0$$

$$\Rightarrow 2x^2+3x+1=0$$

$$\Rightarrow 2x^2+2x+x+1=0$$

$$(x+1)(2x+1)=0$$

$$x=-1,\ -1/2$$

5. The largest value of the non-negative integer a for which $\lim_{x\to1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$ is

[JEE(Advanced)-2014, 3]

Solⁿ:

$$\Rightarrow \lim_{x\to1}\left\{\frac{\cancel{-ax}\ a(1-x)+\sin(x-1)}{(x-1)+\sin(x-1)}\right\}^{\frac{(1-\sqrt{x})(1+\sqrt{x})}{(1-\sqrt{x})}} = 1/4$$

$$\Rightarrow \lim_{h\to0}\left\{\frac{ah+\sin h}{h+\sin h}\right\}^{1+\sqrt{1+h}} = 1/4$$

$$\Rightarrow \lim_{h\to0}\left\{\frac{ah+h+\frac{h^2}{2}}{h+h+\frac{h^2}{2}}\right\}^{1+\sqrt{1+h}} = 1/4$$

$$\Rightarrow \lim_{h\to0}\left\{\frac{a+1}{2}\right\}^{1+\sqrt{1+h}} = 1/4$$

$$\Rightarrow \left\{\frac{a+1}{2}\right\}^{2^+} = \frac{1}{4}$$

$\Rightarrow$ ∵ Power of $\frac{a+1}{2}$ is not an integer then $\frac{a+1}{2}$ must be +ve

$$\therefore \frac{a+1}{2} = \frac{1}{2}$$

$$\Rightarrow a = 0$$

6. Let m and n be two positive integers greater than 1. If $\lim_{\alpha \to 0}\left(\frac{e^{\cos(\alpha^n)} - e}{\alpha^m}\right) = -\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is **[JEE 2015, 4M, –0M]**

Sol:

$$\Rightarrow \lim_{\alpha \to 0} \frac{e^{\cos(\alpha^n)} - e}{\alpha^m} = -e/2$$

$$\Rightarrow \lim_{\alpha \to 0} \frac{e^{\left(1 - \frac{(\alpha^n)^2}{2!}\right)} - e}{\alpha^m} = -e/2$$

$$\Rightarrow \lim_{\alpha \to 0} \frac{e \cdot e^{-\frac{\alpha^{2n}}{2}} - e}{\alpha^m} = -e/2$$

$$\Rightarrow \lim_{\alpha \to 0} \frac{e\left[e^{-\frac{\alpha^{2n}}{2}} - 1\right]}{\alpha^m} = -e/2$$

$$\Rightarrow \lim_{\alpha \to 0} \frac{e\left[\cancel{1} - \frac{\alpha^{2n}}{2} - \cancel{1}\right]}{\alpha^m} = -e/2$$

$$\Rightarrow \lim_{\alpha \to 0} \frac{-e}{2} \alpha^{2n-m} = -e/2$$

So $\alpha^{2n-m} = 1$

$2n - m = 0$

$\frac{m}{n} = 2$

7. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim\limits_{x\to 0}\dfrac{x^2\sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals

[JEE(Advanced)-2016, 3(0)]

Solⁿ:

$$\Rightarrow \lim_{x\to 0} \frac{x^2 \times \frac{\sin \beta x}{\beta x} \times \beta x}{\alpha x - \sin x} = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{\beta x^3}{\alpha x - \left[x - \frac{x^3}{3!}\right]} = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{\beta x^3}{x(\alpha - 1) + \frac{x^3}{6}} = 1$$

$\because$ For limit to be exist & Nonzero equal to 1 Coff. of x must be zero

$$\alpha - 1 = 0$$

$$\boxed{\alpha = 1}$$

$$\Rightarrow \lim_{x\to 0} \frac{\beta \cancel{x^3}}{\frac{\cancel{x^3}}{6}} = 1$$

$$\beta = 1/6$$

So $6(\alpha + \beta)$

$$6(1 + 1/6)$$

$$= 7$$

8. Let $f(x) = \dfrac{1 - x(1 + |1 - x|)}{|1 - x|} \cos\left(\dfrac{1}{1 - x}\right)$ for $x \neq 1$. Then **[JEE(Advanced)-2017, 4]**

(A) $\lim_{x \to 1^+} f(x)$ does not exist (B) $\lim_{x \to 1^-} f(x)$ does not exist

(C) $\lim_{x \to 1^-} f(x) = 0$ (D) $\lim_{x \to 1^+} f(x) = 0$

$$\lim_{x \to 1^+} f(x) = ?$$

$$= \lim_{x \to 1^+} \frac{1 - x(1 + |1-x|)}{|1-x|} \cos\left(\frac{1}{1-x}\right)$$

$$= \lim_{h \to 0} \frac{1 - (1+h)(1+h)}{h} \cos\left(\frac{1}{-h}\right)$$

$$= \lim_{h \to 0} \frac{\cancel{1} - (\cancel{1} + 2h + h^2)}{h} \cos\left(\frac{1}{h}\right)$$

$$= \lim_{h \to 0} (-2 + h) \cos\left(\frac{1}{h}\right)$$

$$\nexists \qquad -2 \times (-1, 1) \quad = \text{ so does not exist}$$

$$\lim_{x \to 1^-} f(x) = ?$$

$$= \lim_{x \to 1^-} \frac{1 - x(1 + |1-x|)}{|1-x|} \cos\left(\frac{1}{1-x}\right)$$

$$= \lim_{h \to 0^-} \frac{1 - (1-h)(1+h)}{h} \cos\left(\frac{1}{h}\right)$$

$$= \lim_{h \to 0} \frac{\cancel{1} - (\cancel{1} - h^2)}{h} \cos\frac{1}{h}$$

$$= \lim_{h \to 0} h \times \cos\frac{1}{h}$$

$$0 \times (-1, 1)$$

$$= 0$$

9. For any positive integer n, define $f_n : (0, \infty) \to \mathbb{R}$ as

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right) \text{ for all } x \in (0, \infty).$$

(Here, the inverse trigonometric function $\tan^{-1}x$ assume values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.)

Then, which of the following statement(s) is (are) TRUE ? **[JEE(Advanced)-2018]**

(A) $\sum_{j=1}^{5} \tan^2(f_j(0)) = 55$

(B) $\sum_{j=1}^{10} \left(1+f_j'(0)\right)\sec^2(f_j(0)) = 10$

(C) For any fixed positive integer n, $\lim_{x\to\infty} \tan(f_n(x)) = \frac{1}{n}$

Sol: 9.

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right)$$

$$= \sum_{j=1}^{n} \tan^{-1}\left(\frac{x+j-(x+j-1)}{1+(x+j)(x+j-1)}\right)$$

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}(x+j) - \tan^{-1}(x+j-1)$$

$$= \begin{cases} \tan^{-1}(\cancel{x+1}) - \tan^{-1}x \\ + \tan^{-1}(\cancel{x+2}) - \tan^{-1}\cancel{(x+1)} \\ + \tan^{-1}(\cancel{x+3}) - \tan^{-1}\cancel{(x+2)} \\ \vdots \\ + \tan^{-1}(x+n) - \tan^{-1}\cancel{(x+n-1)} \end{cases}$$

$$f_n(x) = \tan^{-1}(x+n) - \tan^{-1}x$$

(A) $\because$ $f(0)$ is not defined because 0 is not in domain

(B) $\because$ Same Reason

(C) $\because$ $f_n(x) = \tan^{-1}(x+n) - \tan^{-1}x$

$$f_n(x) = \tan^{-1}\frac{n}{1+nx+x^2}$$

So $\lim_{x\to\infty} \tan(f_n(x)) = \lim_{x\to\infty} \tan\left(\tan^{-1}\frac{n}{1+nx+x^2}\right)$

$$= \lim_{x\to\infty} \frac{n}{x^2+nx+1} = 0$$

(D) $\lim_{x\to\infty} \sec^2(f_n(x)) = 1$

$\Rightarrow \lim_{x\to\infty} (1+\tan^2(f_n(x))) =$

$\Rightarrow 1 + 0 = 1$

10. Let F : $\mathbb{R} \to \mathbb{R}$ be a function. We say that f has

PROPERTY 1 if $\lim_{h \to 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim_{h \to 0} \frac{f(h) - f(0)}{h^2}$ exists and is finite.

Then which of the following options is/are correct ? **[JEE(Advanced)-2019, 4(–1)]**

(1) $f(x) = x|x|$ has PROPERTY 2 (2) $f(x) = x^{2/3}$ has PROPERTY 1

(3) $f(x) = \sin x$ has PROPERTY 2 (4) $f(x) = |x|$ has PROPERTY 1

Sol: 10. (A) $f(x) = x|x|$

Property 2 :- $\lim_{h \to 0} \frac{f(h) - f(0)}{h^2}$ exists & finite

So $\lim_{h \to 0^+} \frac{h|h| - 0}{h^2} = 1$

$\lim_{h \to 0^-} \frac{-h|-h| - 0}{h^2} = -1$

L.H.L $\neq$ R.H.L.

So limit not exist

(B) $f(x) = x^{2/3}$

Property 1:-

$\lim_{h \to 0^+} \frac{h^{2/3} - 0}{\sqrt{|h|}}$ $\lim_{h \to 0^-} \frac{(-h)^{2/3} - 0}{h^{1/2}}$

$\lim_{h \to 0^+} h^{1/6} = 0$ $= 0$

R.H.L = L.H.L

So limit exist

(C) $f(x) = \sin x$

Property :- 2

$\lim_{h \to 0^+} \frac{\sin h - 0}{h^2}$

= Not defined

So limit does not exist

(D) $f(x) = |x|$

Property : 1

$\lim_{h \to 0^+} \frac{h - 0}{\sqrt{h}} = 0$ $\lim_{h \to 0^-} = \frac{h - 0}{\sqrt{h}} = 0$

L.H.L = R.H.L = 0

So limit exist and equal to 0