Permutation of alike objects taken some at a time

Ex:- How many 5 lettered words can be formed using letters of word "INDEPENDENCE".

I	N	D	E	P	C
	N	D	E		
	N		E		
			E		

$\boxed{3N,\ 2D,\ 4E,\ I,\ P,\ C}$

5 Alike	0
4 Alike 1 Diffnt	$\{1 \cdot {}^5C_1\} \cdot \frac{5!}{4!} = 25$ EEEE D
3 Alike 2 Diffnt	$\{{}^2C_1\ {}^5C_2\} \cdot \frac{5!}{3!} = 400$ NNN IP
3 Alike → 1st kind 2 Alike → 2nd kind	$\{{}^2C_1\ \ {}^2C_1\} \frac{5!}{3!\,2!} = 40$ NNN DD

3N, 2D, 4E, I P C

2 alike 3 Differnt	$\{{}^3C_1\ {}^5C_3\} \cdot \frac{5!}{2!} = 1800$ D
2 alike → 1st kind 2 alike → 2nd kind 1 D	$\{{}^3C_2\ \ {}^4C_1\}\ \frac{5!}{2!\,2!} = 360$ N N D D P
5 diffnt	$\{{}^6C_5\}\ 5! = 720$

Ex:- M I S S I S S I P P I

How many 5 letter words can be formed.

Soln 4 S, 4 I , P P, M

Category	Selection	Total no. of words
5 Alike	0	0
4 Alike 1 Differnt	${}^2C_1\ {}^3C_1 = 6$	${}^2C_1\ {}^3C_1\ \frac{5!}{4!}$
3 alike 2 Differnt	${}^2C_1\ {}^3C_2 = 6$	${}^2C_1\ {}^3C_2\ \frac{5!}{3!}$

3 alike → Ist kind 2 alike → 2nd	$^2C_1\ ^2C_1 = 4$	$^2C_1\ ^2C_1\ \frac{5!}{3!\,2!}$
2 alike 3 D	$^3C_1\ ^3C_3 = 3$	$^3C_1\ ^3C_3\ \frac{5!}{2!}$
2 alike → Ist kind 2 alike → 2nd 1 D	$^3C_2 \times {}^2C_1 = 6$	$^3C_2\ ^2C_1\ \frac{5!}{2!\,2!}$
5 Diff^{nt}	0	0

selections = 25

TOTAL words = 550

2 alike → Ist

2 alike → 2nd

1 D

wrong

Repeat

$^3C_1\ ^2C_1\ ^2C_1$

SS PP II   PP II

SS II

SS	PP
I	II

PP	SS
I	II

Circular permutation

(I) No. of permutations of n diff^nt thing taken all at a time in a circle $= (n-1)!$

(II) No. of circular permutations of r diff^nt things out of n diff^nt things $= {}^nC_r \cdot (r-1)!$

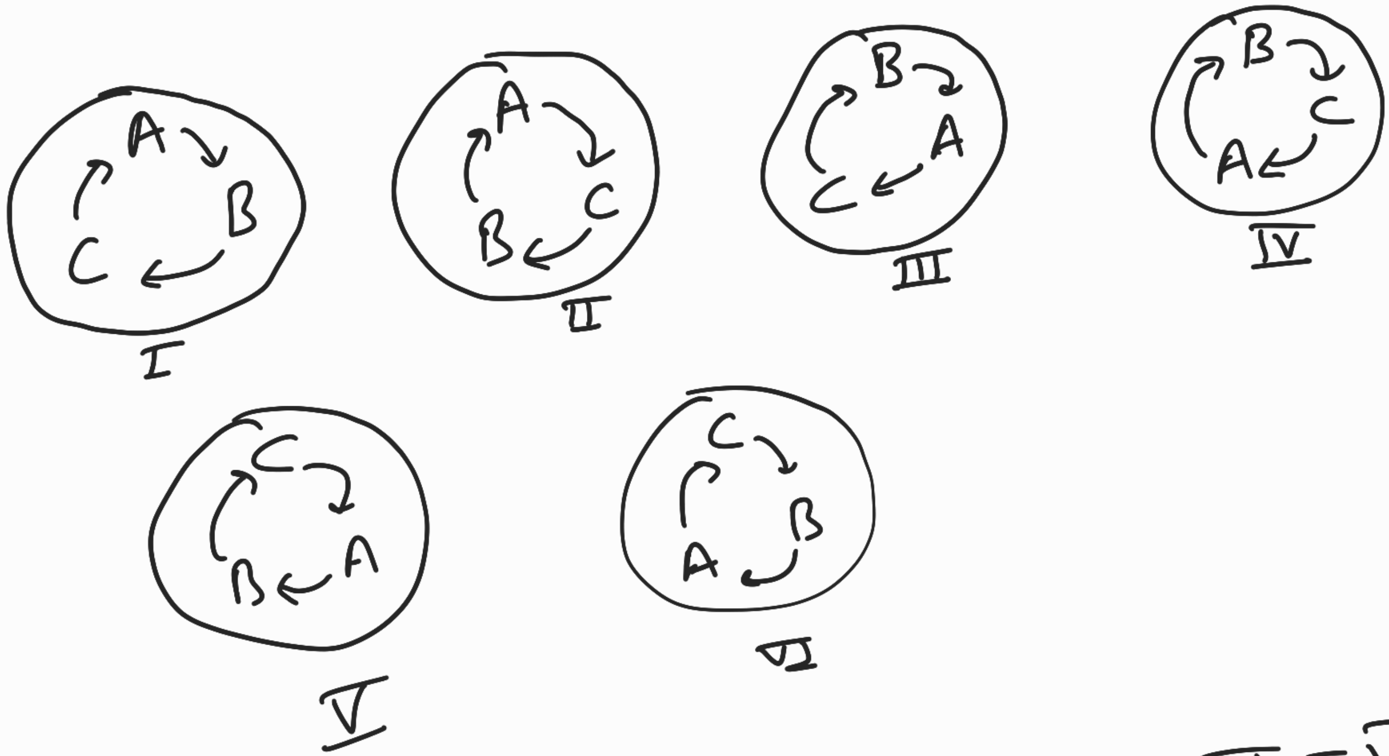
(III) If clockwise and anticlockwise Arrangements are considered as same then

(a) no. of circular permutations of n diff^nt things $= \frac{(n-1)!}{2}$

(b) no. of circular permutations of r diff^nt things out of n diff^nt things $= {}^{n}C_{r} \dfrac{(r-1)!}{2}$

NOTE :- when number of people have to be seated around a table so as not to have same neighbours then clockwise and anticlockwise sense are same.

ABC — linear arrangement $= 3!$

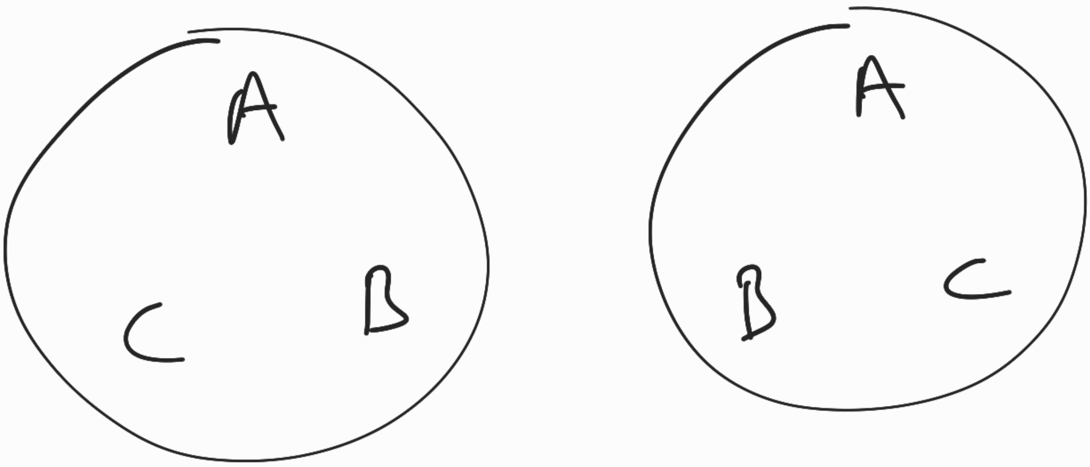


$\underbrace{\mathrm{I} = \mathrm{IV} = \mathrm{V}}$ or $\underbrace{\mathrm{II} = \mathrm{III} = \mathrm{VI}}$

circular $= \dfrac{3!}{3} = 2!$

NOTE :- In case of garland or necklace considered clockwise and anticlock wise Arrangements as same.

Ex :-



Circle $= (3-1)! = 2!$

not same neighbours $= \frac{2!}{2} = 1$

Ex) Number of ways in 10 children can be sit on a merry go round ??

$(10-1)! = 9!$

Ex :- number of ways in which 8 person can be seated on round table so that all shall not have same neighbour in any arrangement ??

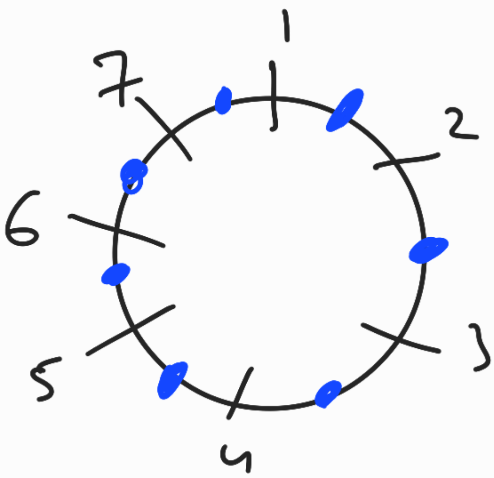
$$\frac{(8-1)!}{2} = \frac{7!}{2}$$

Ex:- find no. of ways in which 7 Americans and 7 Indians can be seated on a round table so that no two Americans are consecutive.

$$= 6! \quad {}^{7}C_{7} \quad 7!$$

7 Indians (circular)

7 Americans



Ex:- 10 diff^nt flowers are given How many garland can be formed using 6 flowers ?

$$= {}^{10}C_{6} \cdot \frac{(6-1)!}{2}$$

Ex:- 5 Boys, 5 Girls are to be seated

on round table. In how many ways a particular boy and a particular girl never sit together.

$\Rightarrow$ $B_1\ B_2\ B_3\ B_4\ G_1\ G_2\ G_3\ G_4\ \underbrace{B_5\ G_5}_{\text{particular}}$

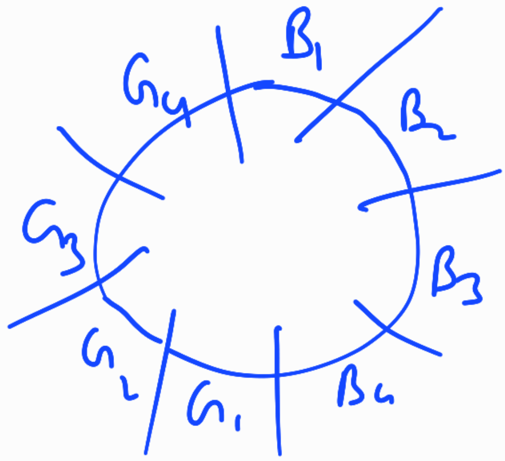
M#1

$$\underbrace{(10-1)!}_{\text{TOTAL}} - \underbrace{(9-1)!\ 2!}_{\text{Both are together}}$$

M#2

gap method

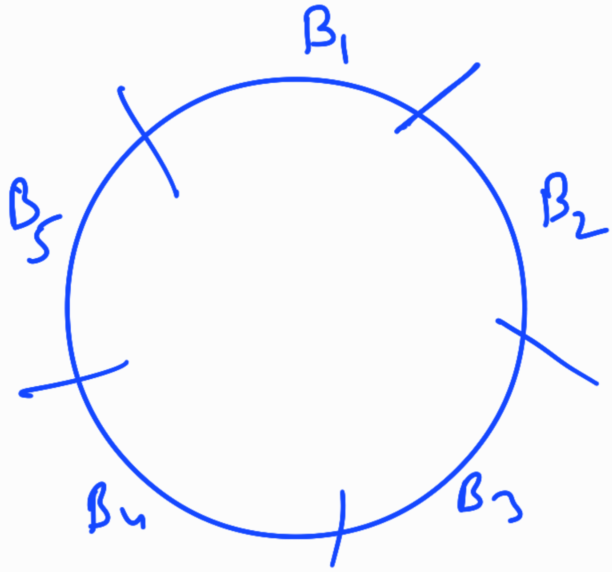
$$(8-1)!\ {}^8C_2\ 2!$$



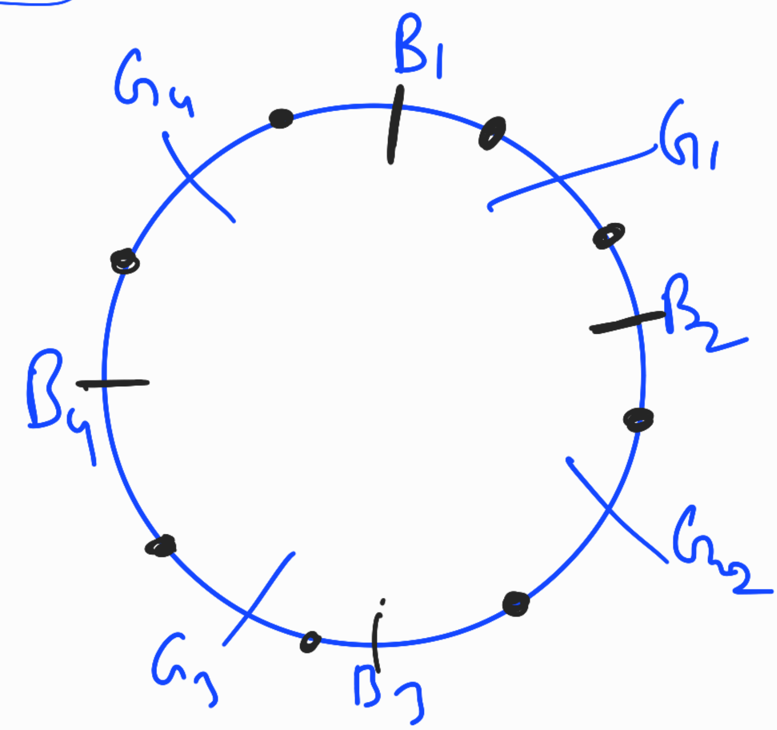
Ex:- 5B, 5G, Alternate on round table. B_5 & G_5 are never sit together.

Sol^n

$$\text{TOTAL} = \underbrace{(5-1)!}_{\text{circle}}\ 5! = 4!\ 5!$$



when $\boxed{B_5 \; G_5}$ are together



$3! \; 4! \; {}^8C_1$

4 Boys circle

4 girls

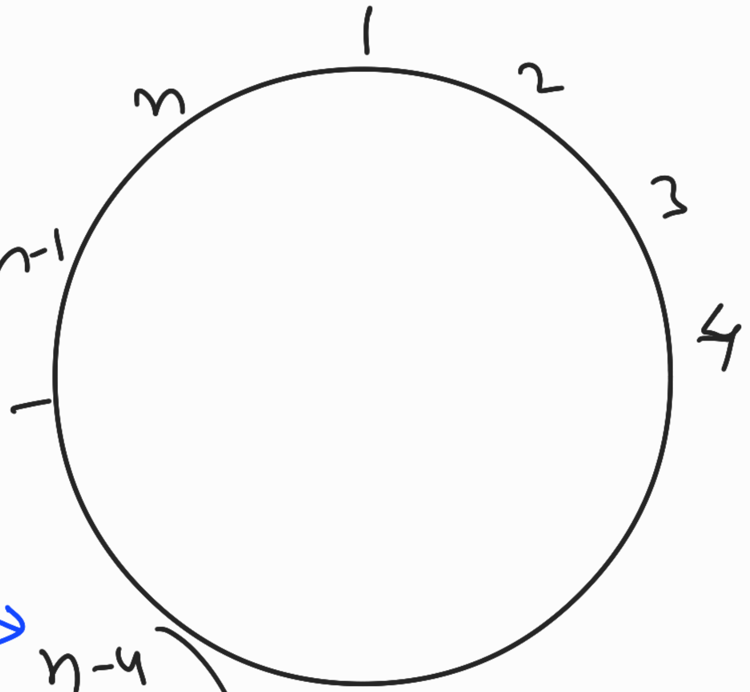
1 gap from 8 gaps betn 4B4G Alternate

$$Ans = 4! \, 5! - 3! \, 4! \, 8$$

Important Concept

n persons are sitting in a circle.

Select 3 out of n if no two of them are consecutive. find Total of ways of selection



Ans.

$^{n-3+1}C_3$ = linear

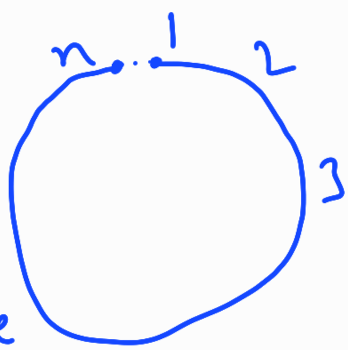
circle = $\left({}^{n-2}C_3 - {}^{n-4}C_1 \right)$ Ans.

-2 -2

1 2 3 - - - (n-2) (n-1) n

⇓

$^{n-2}C_3$ = linear



in line 1, n → non consecutive

but in circle 1, n → consecutive

TOTAL linear $= {}^{n-2}C_3$

Subtract when we select

$\underbrace{1,\ n,\ \boxed{3^{rd}}}$ = non cosecutive in line

$$1\quad \underset{\times}{2}\quad \underbrace{3\quad 4\quad -\ -\ (n-2)}_{3^{rd}\text{ person} = {}^{n-4}C_1}\quad \underset{\times}{(n-1)}\quad n$$

There are n persons in a circle

No. of ways in which r persons are selected so that no of them are consecutive $= {}^{n-r+1}C_r - {}^{n-r-1}C_{r-2}$

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linear $= {}^{n-r+1}C_r$

circle $= {}^{n-r+1}C_r - {}^{n-r-1}C_{r-2}$

-2

-2