Q.

For a complex number $z(\alpha,\beta) = \alpha + i\beta$, where α,β are determined by a throw of single dice twice. Events X and Y are defined as

$X \equiv Z(\alpha,\beta)$ lies on $|Z| = |Z - 7(1 + i)|$,

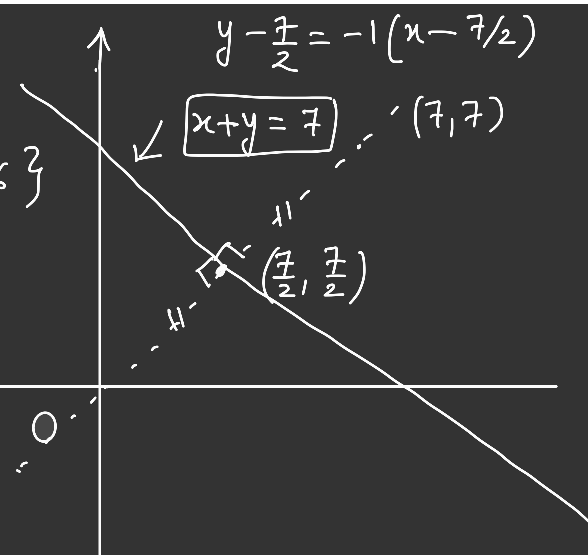
$Y \equiv Z(\alpha,\beta)$ lies on $\left|\arg\left(Z - \frac{7}{2}(1+i)\right)\right| = \frac{\pi}{4}$. Then

(A) $P(X) = \frac{1}{6}$ (B) $P(Y) = \frac{1}{6}$ (C) $P\left(\frac{X}{Y}\right) = \frac{1}{2}$ (D) $P\left(\frac{Y}{X}\right) = \frac{1}{2}$

Sol^n

$|z - 0| = |z - (7 + 7i)|$

$y - \frac{7}{2} = -1(x - 7/2)$



$\alpha + \beta = 7$ $\alpha, \beta \in \{1, 2, 3, 4, 5, 6\}$

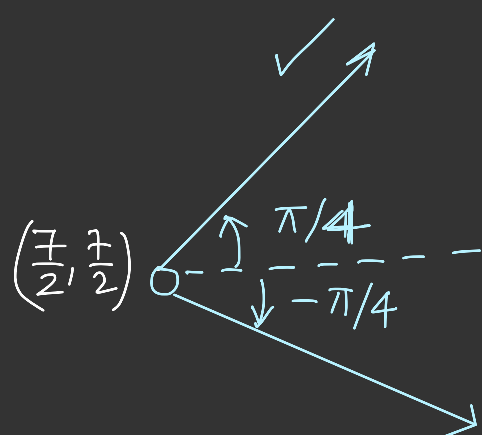
X : { (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) }

$P(X) = \frac{1}{6}$

$y \longrightarrow \arg\left(z - \left(\frac{7}{2} + \frac{7}{2}i\right)\right) = \pm \pi/4$

[ABCD]

Y : { (4,4), (5,5), (6,6), (4,3), (5,2), (6,1) }



$P(Y) = \frac{1}{6}$

$x \cap y$: {(4,3), (5,2), (6,1)}

$P(X/Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$

$P(Y/X) = \frac{P(X \cap Y)}{P(X)} = \frac{1}{2}$

Let A and B be two independent events such that $P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{6}$. Then, which of the following is TRUE ?

(1) $P(A/B)=\frac{2}{3}$ (2) $P(A/(A\cup B))=\frac{1}{4}$ (3) $P(A/B')=\frac{1}{3}$ (4) $P(A'/B')=\frac{1}{3}$

Sol^n $P(A)=\frac{1}{3}$; $P(B)=1/6$ $P(A\cap B)=\frac{1}{3}\cdot\frac{1}{6}=\frac{1}{18}$

$P(A/B)=P(A)=\frac{1}{3}$

$P(A'/B')=P(A')=\frac{2}{3}$

$P(A/B')=P(A)=\frac{1}{3} \longrightarrow [C]$ ✓

$$P(A/A\cup B)=\frac{P(A\cap(A\cup B))}{P(A\cup B)}=\frac{P(A)}{P(A)+P(B)-P(A\cap B)}$$

$$=\frac{1/3}{\frac{1}{3}+\frac{1}{6}-\frac{1}{18}}$$

$$=\frac{3}{4}$$

Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$. Then $P(E_2^C \cap E_3^C / E_1)$ is equal to :

(1) $P(E_3^C) - P(E_2)$ (2) $P(E_2^C) + P(E_3)$ (3) $P(E_3^C) - P(E_2^C)$ (4) $P(E_3) - P(E_2^C)$

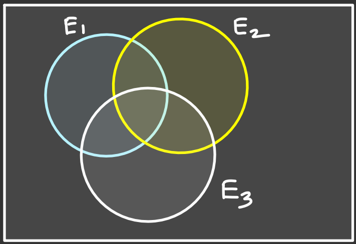
Sol^n

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$

$$P(E_2 \cap E_3) = P(E_2) \cdot P(E_3)$$

$$P(E_3 \cap E_1) = P(E_3) \cdot P(E_1)$$

$$P(E_1 \cap E_2 \cap E_3) = 0$$



$$P(\bar{E}_2 \cap \bar{E}_3 / E_1) = \frac{P((\bar{E}_2 \cap \bar{E}_3) \cap E_1)}{P(E_1)}$$

$$= \frac{P(E_1) - P(E_1 \cap E_2) - P(E_1 \cap E_3) + \cancel{P(E_1 \cap E_2 \cap E_3)}^{0}}{P(E_1)}$$

$$= \frac{\cancel{P(E_1)}(1 - P(E_2) - P(E_3))}{\cancel{P(E_1)}}$$

$$= P(\bar{E}_2) - P(E_3)$$

OR $= P(\bar{E}_3) - P(E_2) \longrightarrow [1]$

In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six The game stops as soon as either of the players wins. The probability of A winning the game is :

(1) $\frac{31}{61}$ (2) $\frac{5}{6}$ (3) $\frac{5}{31}$ (4) $\frac{30}{61}$

Sol^n $P(6) = \frac{5}{36}$; $P(7) = \frac{6}{36}$

(Do yourself)

[4]

The probabilities of three events A, B and C are given by P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \le \alpha \le 0.95$, then β lies in the interval:

(1) [0.36, 0.40] (2) [0.35, 0.36] (3) [0.25, 0.35] (4) [0.20, 0.25]

Soln

$$P(A \cup B \cup C) = \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$$

$$\alpha = 1.5 - (\cancel{0.2} + 0.3 + \beta) + \cancel{0.2}$$

$$\boxed{\alpha = 1.2 - \beta}$$

$\alpha \in [0.85, 0.95]$

$0.85 \le 1.2 - \beta \le 0.95$

$\boxed{0.25 \le \beta \le 0.35} \rightarrow [3]$

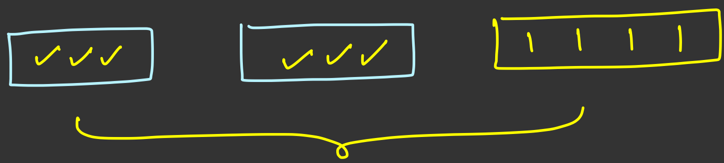
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.8 = 0.6 + 0.4 - P(A \cap B) \Rightarrow P(A \cap B) = 0.2$$

Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?

(1) $2!3!4!$ (2) $(3!)^3.(4!)$ (3) $(3!)^2.(4!)$ (4) $3!(4!)^3$

Sol^n

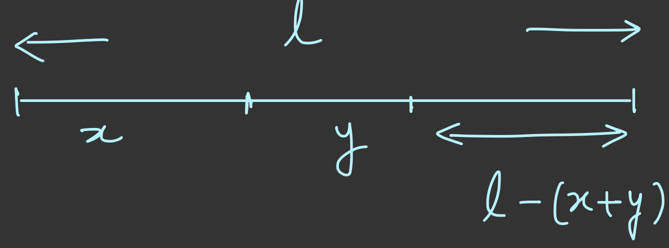


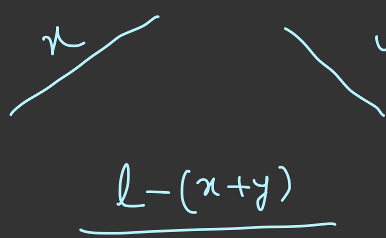
$$3! \times (3! \times 3! \times 4!) = (3!)^3 (4!)^1$$

[2]

Q (HW) A line segment is divided into 3 parts, what is the prob. that they form sides of Δ.

Sol^n





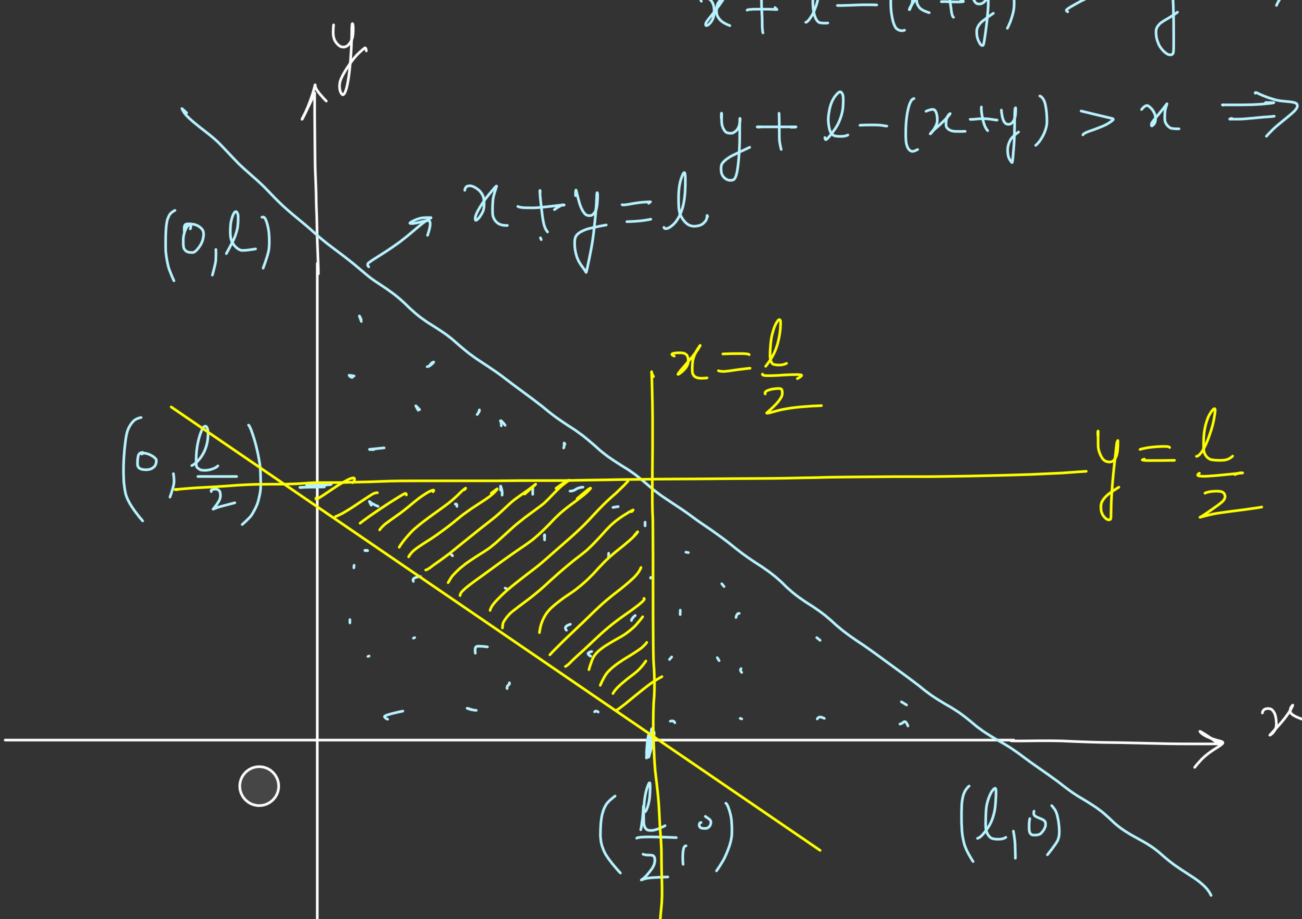
$$\left.\begin{array}{l} x>0 \\ y>0 \\ l-(x+y)>0 \end{array}\right\}$$

Sum of 2 sides of Δ > third side

$x+y > l-(x+y) \Rightarrow \boxed{x+y > \frac{l}{2}}$

$x + l-(x+y) > y \Rightarrow \boxed{y < \frac{l}{2}}$

$y + l-(x+y) > x \Rightarrow \boxed{x < \frac{l}{2}}$



$$\frac{\frac{1}{2}\cdot\frac{l}{2}\cdot\frac{l}{2}}{\frac{1}{2}\cdot l\cdot l}$$

$\frac{1}{4}$

HW

JA Q1 to 14

S2 Q1 to 5