

Radial wave function:-

for S-orbital $\Rightarrow \begin{cases} R(r) = +ve & \text{at } r=0 \\ R(r) = 0 & \text{at } r=\infty \end{cases}$

for other orbitals $\Rightarrow \begin{cases} R(r) = 0 & \text{at } r=0 \\ R(r) = 0 & \text{at } r=\infty \end{cases}$

General form :

$$R_{n\ell}(r) = K \cdot e^{-\sigma/2} \cdot \sigma^\ell \quad (\text{Polynomial of order } n - \ell - 1)$$

where $\sigma = \frac{2Zr}{na_0}$ $a_0 = 1^{\text{st}} \text{ Bohr's radius} = 0.529 \text{ \AA}$

#

1s $(n = 1, \ell = 0) : R_{1s}(r) = 2 \cdot \left(\frac{Z}{a_0} \right)^{3/2} \cdot e^{-\sigma/2}$

$$\sigma^l = 1 \Rightarrow l = 0$$

$$n-l-1 = 0$$

$$n = 1$$

1(s)

where $\sigma = \frac{2Zr}{na_0}$ $a_0 = 1^{\text{st}} \text{ Bohr's radius} = 0.529 \text{ \AA}$

2s $(n = 2, \ell = 0) : R_{2s}(r) = \frac{1}{2\sqrt{2}} \cdot \left(\frac{Z}{a_0} \right)^{3/2} \cdot (2-\sigma)e^{-\sigma/2}$

$$\sigma^l = 1 \Rightarrow l = 0$$

$$n-l-1 = 1$$

$$n = 2 \Rightarrow 2(s)$$

2p $(n = 2, \ell = 1) : R_{2p}(r) = \frac{1}{2\sqrt{6}} \cdot \left(\frac{Z}{a_0} \right)^{3/2} \cdot \sigma \cdot e^{-\sigma/2}$

$$\sigma^l = \sigma \Rightarrow l = 1$$

$$n-l-1 = 0 \Rightarrow n = 2 \Rightarrow (2p)$$

3s $(n = 3, \ell = 0) : R_{3s}(r) = \frac{1}{9\sqrt{3}} \cdot \left(\frac{Z}{a_0} \right)^{3/2} \cdot (6-6\sigma+\sigma^2)e^{-\sigma/2}$

$$\sigma^l = 1 \Rightarrow l = 0$$

$$n-l-1 = 2$$

$$n = 3 \Rightarrow 3(s)$$

3p $(n = 3, \ell = 1) : R_{3p}(r) = \frac{1}{9\sqrt{6}} \cdot \left(\frac{Z}{a_0} \right)^{3/2} \cdot \sigma(4-\sigma)e^{-\sigma/2}$

$$\sigma^l = \sigma \Rightarrow l = 1$$

$$n-l-1 = 1 \Rightarrow n = 3$$

(3p)

3d $(n = 3, \ell = 2) : R_{3d}(r) = \frac{1}{9\sqrt{30}} \cdot \left(\frac{Z}{a_0} \right)^{3/2} \cdot \sigma^2 \cdot e^{-\sigma/2}$

$$\sigma^2 = \sigma^1 \Rightarrow l = 2$$

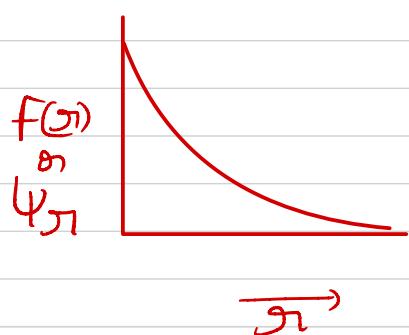
$$n-l-1 = 0 \Rightarrow n = 3$$

(3d)

Graph R(r) vs r:-

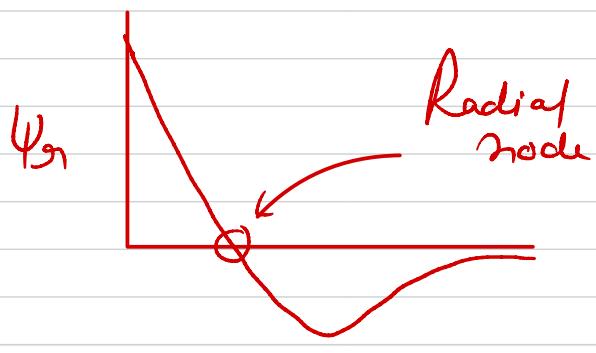
① 1(S)

$$n-l-1 = 0$$



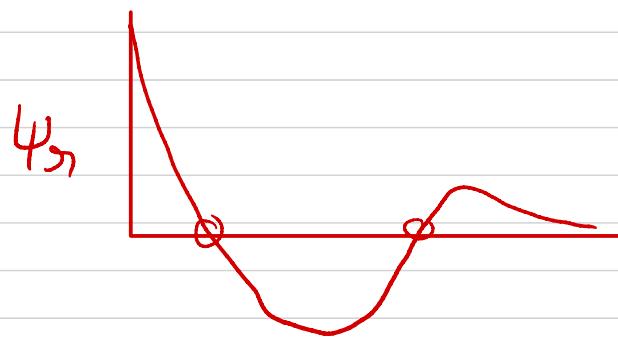
② 2(S)

$$n-l-1 = 1$$



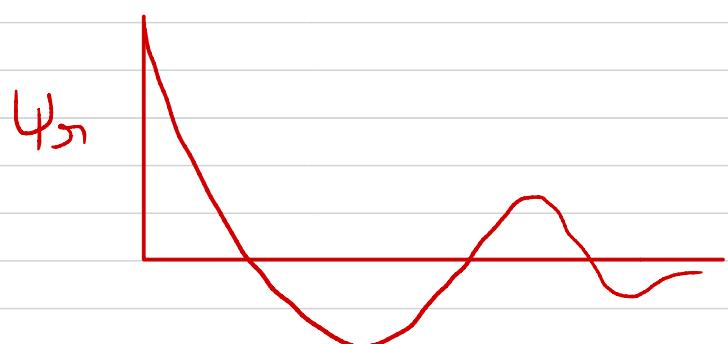
③ 3(S)

$$n-l-1 = 2$$



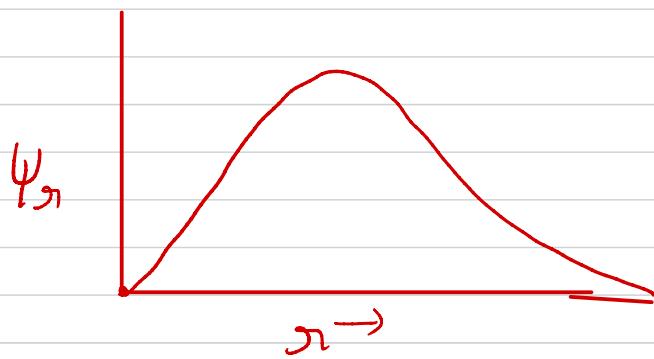
④ 4(S)

$$n-l-1 = 3$$



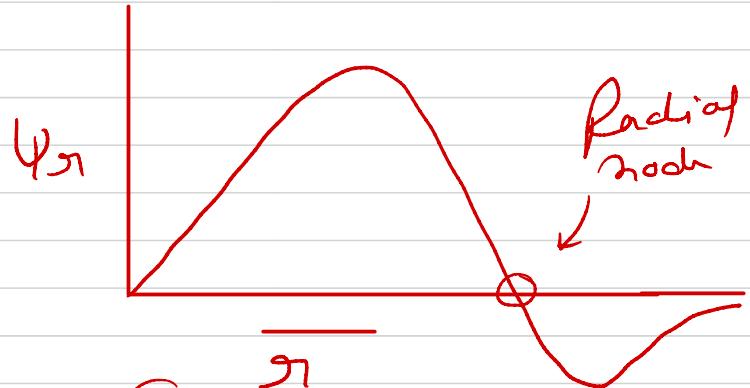
⑤ 2(P)

$$n-l-1 = 0$$

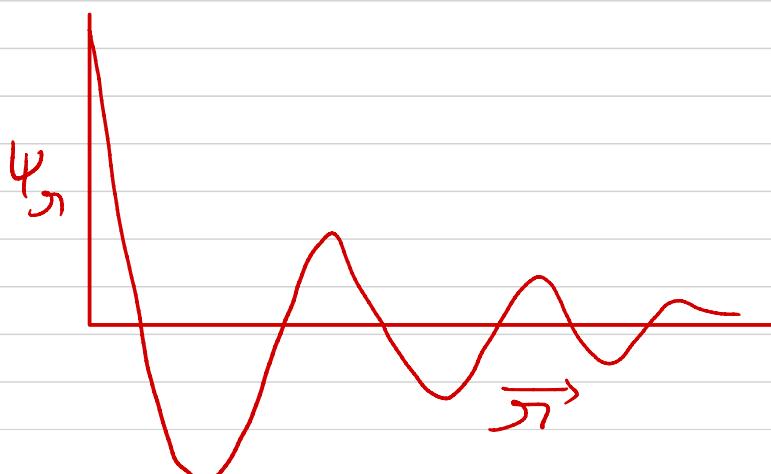


⑥ 3(P)

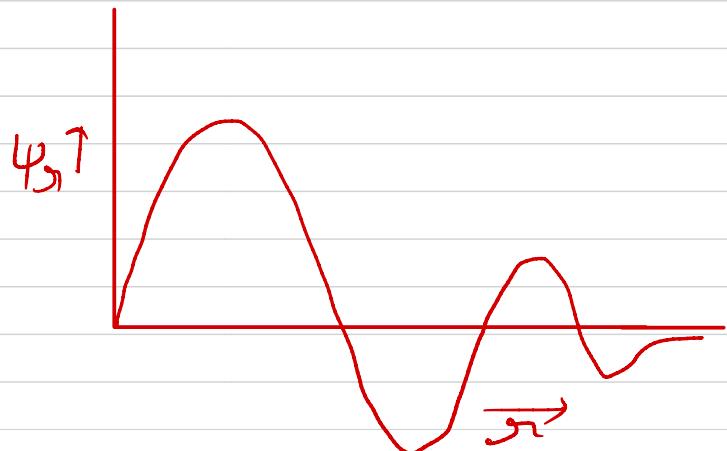
$$n-l-1 = 1$$



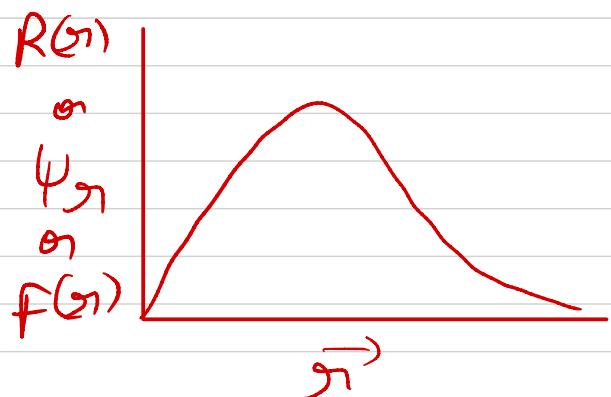
⑦ 7(S) $n-l-1 = 6$



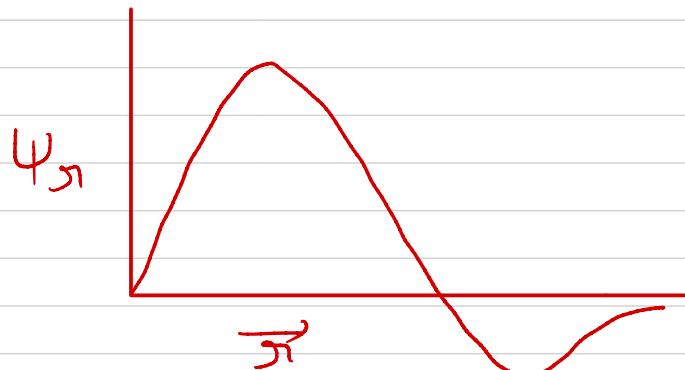
⑧ 5(P) $n-l-1 = 3$



$$\textcircled{9} \quad \underline{3(d)} \quad n-l-1 = 0$$



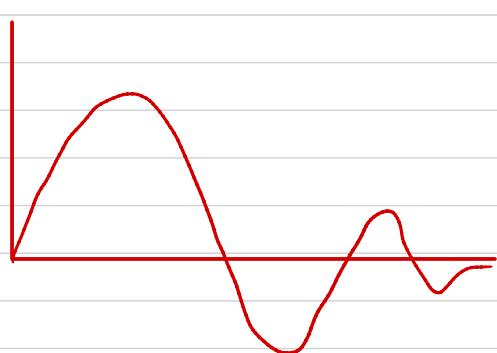
$$\textcircled{10} \quad \underline{4(d)} \quad n-l-1 = 1$$



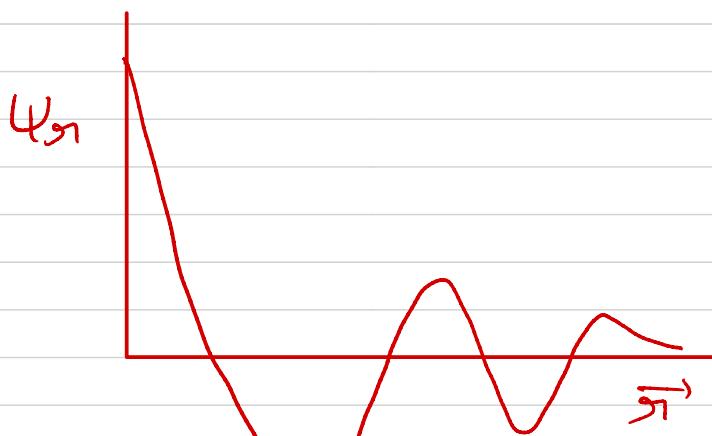
$$2P \equiv 3d \equiv 4f$$

$$3P \equiv 4d \equiv 5f$$

eg: $\textcircled{1} \underline{5P} \quad n-l-1 = 3$

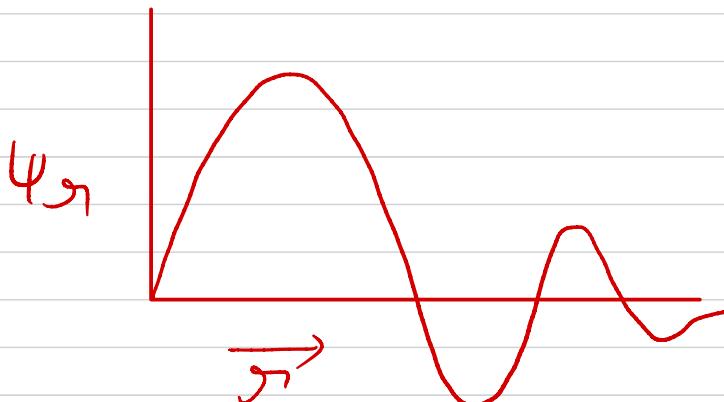


$$\textcircled{2} \quad 5(g) \quad n-l-1 = 4$$

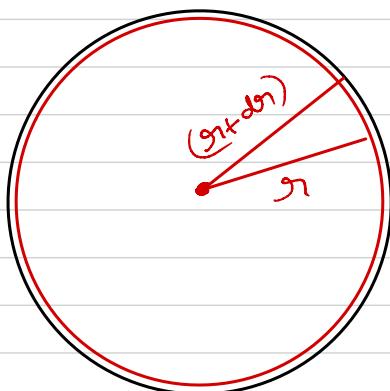


$$\textcircled{3} \quad 6(d)$$

$$n-l-1 = 3$$



Radial probability distribution function:-



$$\# dV = 4\pi r^2 \cdot dr$$

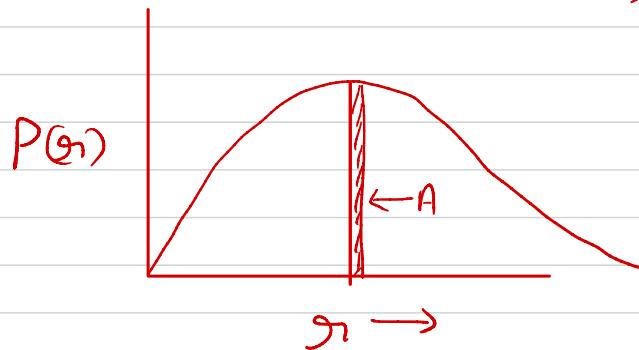
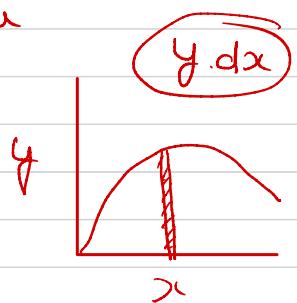
$[R(r)]^2$ = Radial probability density

= Probability of finding e⁻ at a distance r from the nucleus.

Radial

Probability of e⁻ at a distance r from the nucleus = $[R(r)]^2 \times dV$

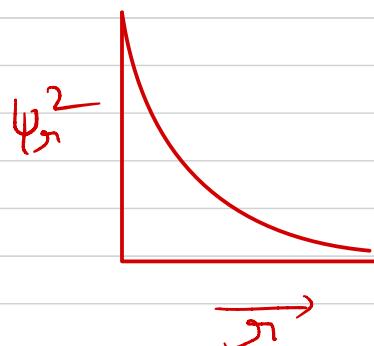
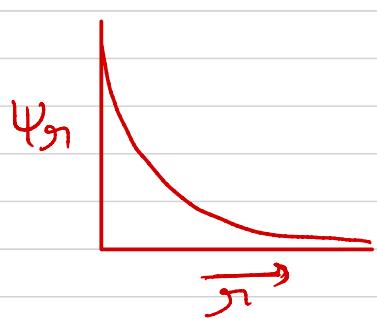
$$= \frac{4\pi r^2 [R(r)]^2 \cdot dr}{P(r)}$$



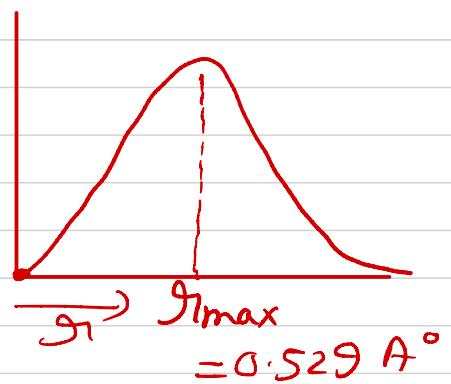
$$P(r) \cdot dr = A$$

$$\underline{P(r) \propto r^{-3}}$$

$$\textcircled{1} \quad l(s) \quad n-l-1=0$$

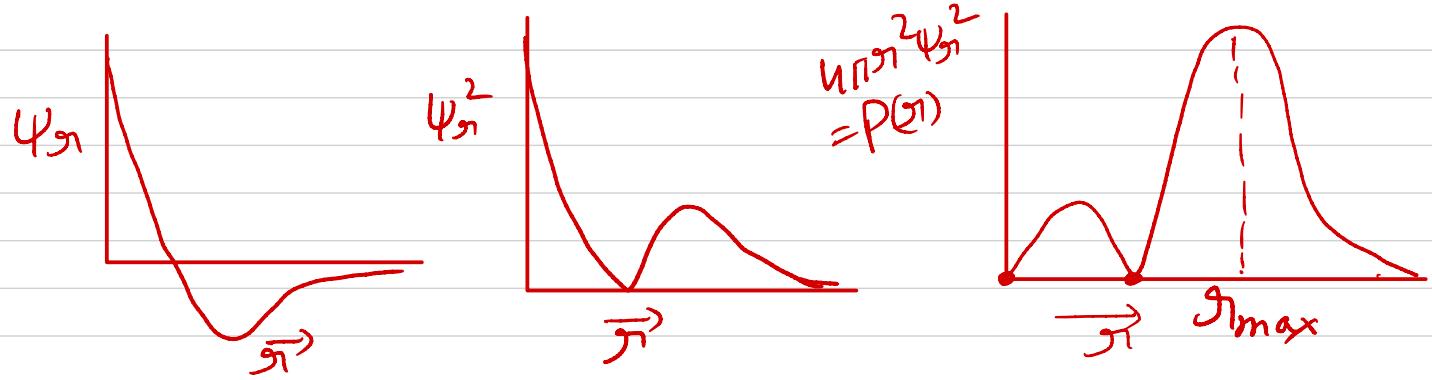


$$\frac{4\pi r^2 \Psi_r^2}{= P(r)}$$

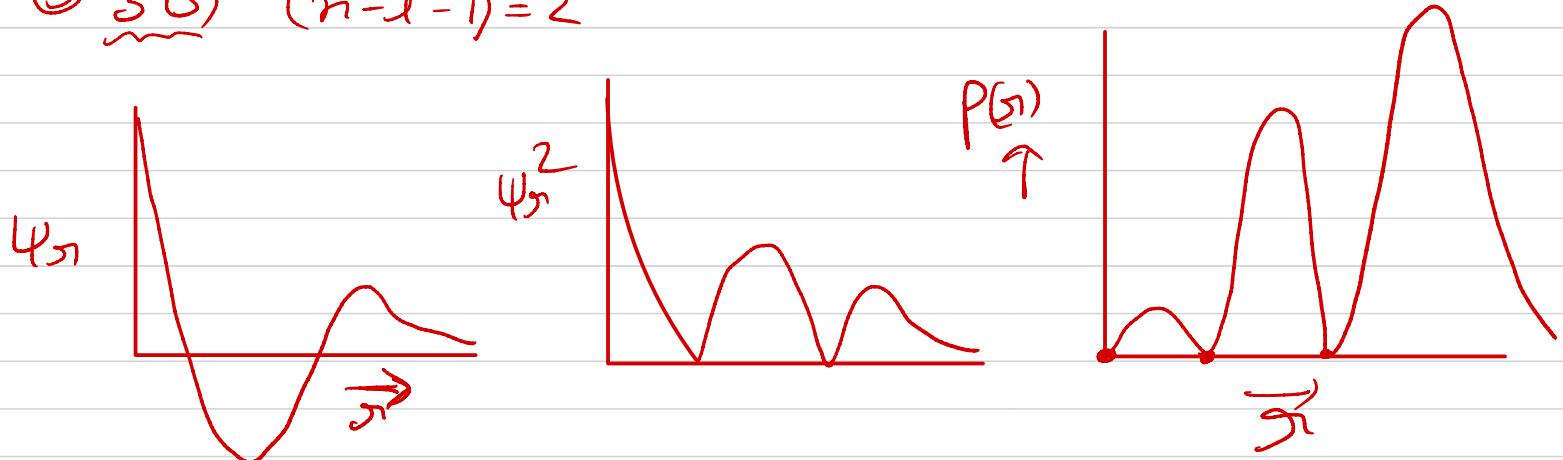


For 1s in H atom

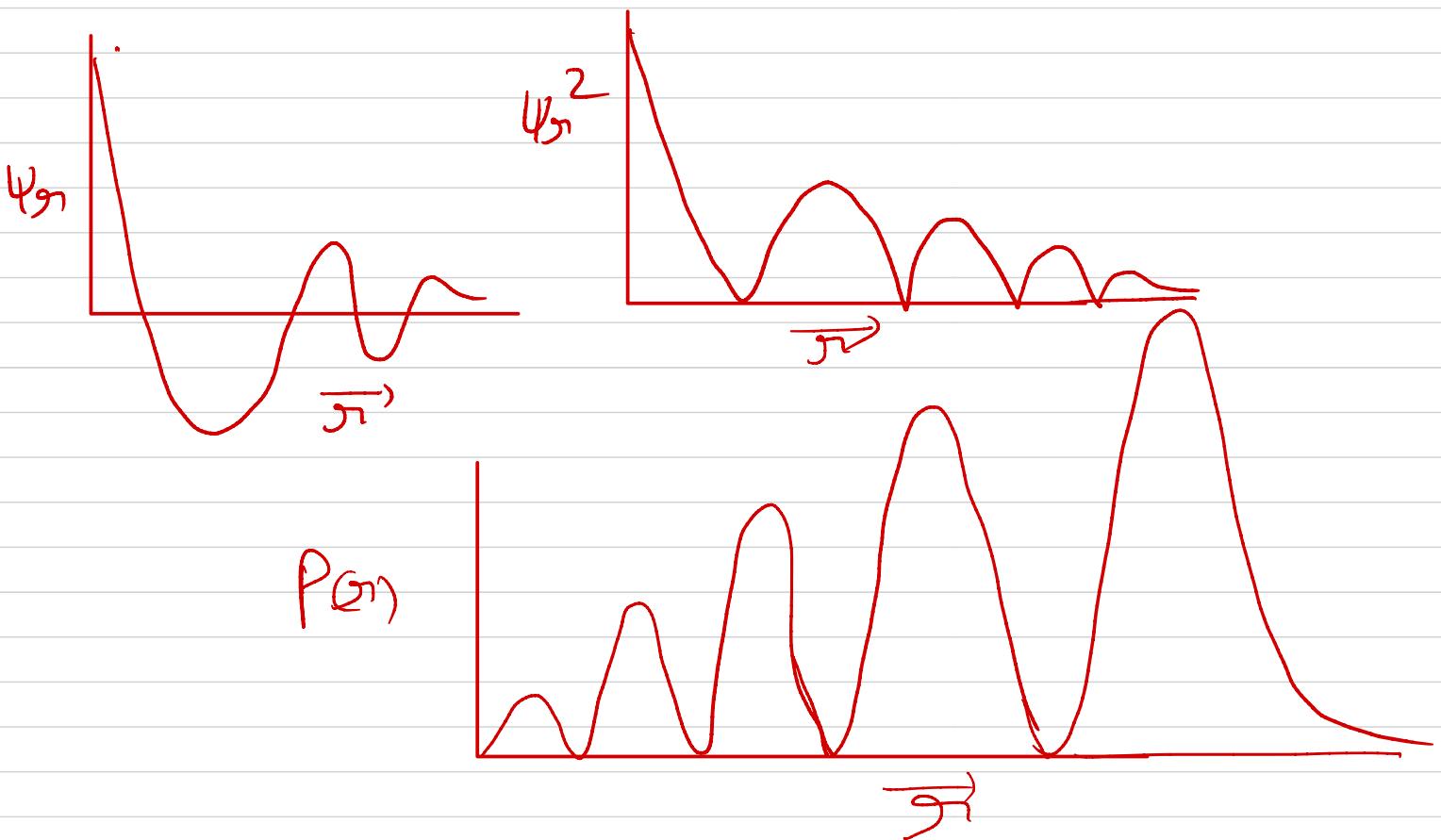
② 2(S) $(n-l-1)=1$



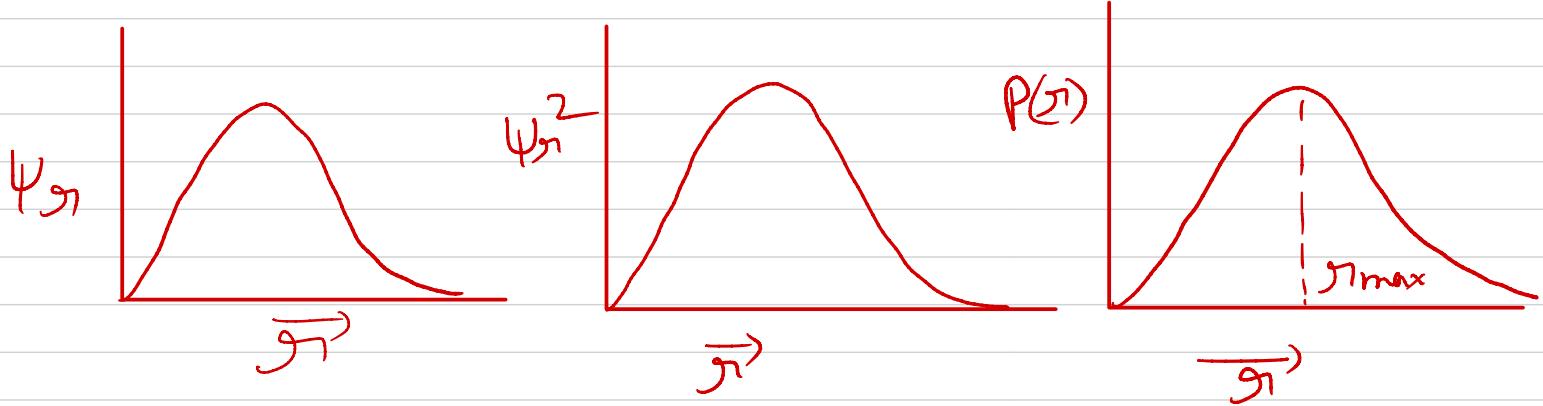
③ 3(S) $(n-l-1)=2$



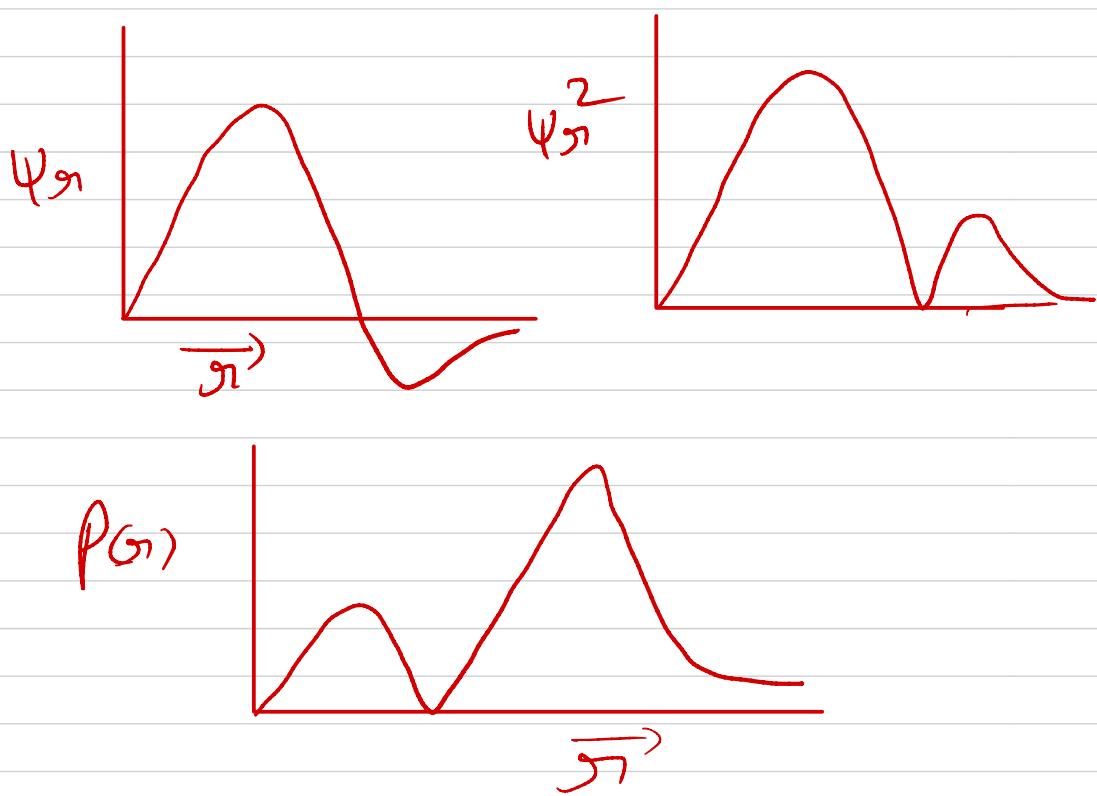
④ 5(S) $(n-l-1)=4$



$$\textcircled{5} \quad 2(P) \quad n-l-1 = 0$$



$$\textcircled{6} \quad 3(P) \quad (n-l-1)=1$$



Angular function:-

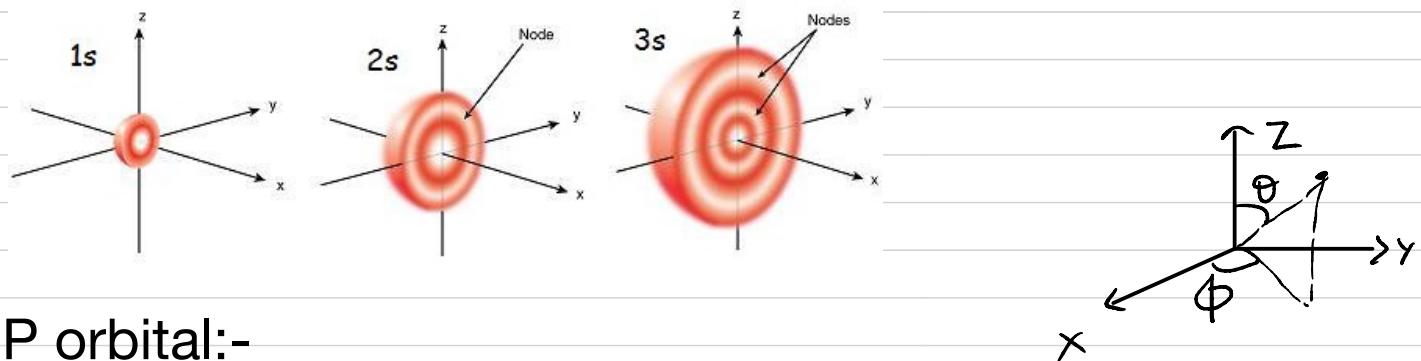
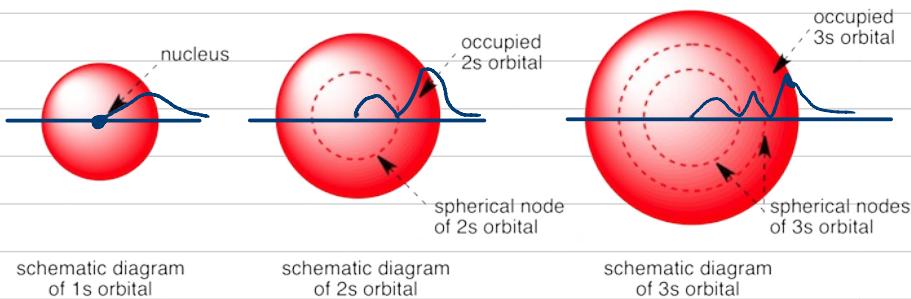
$$Y(\theta, \phi)$$

No of angular nodes = ℓ

at angular node $Y(\theta, \phi) = 0$

① S orbital:-

$$\ell = 0, m = 0, Y(\theta, \phi) = \Theta(\theta) \cdot \Phi(\phi) = \sqrt{\frac{1}{4\pi}} = \text{constant}$$



② P orbital:-

p_x -orbital: $\ell = 1, m = +1$

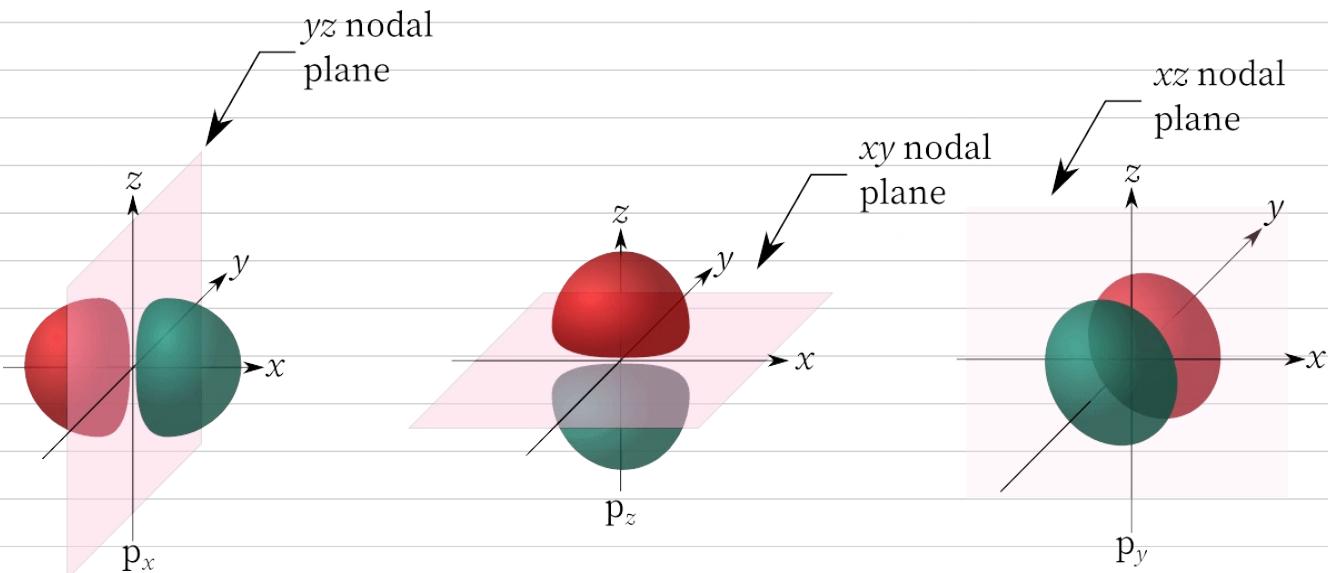
$$\Theta(\theta) \cdot \Phi(\phi) = \sqrt{\frac{3}{4\pi}} \cdot \sin \theta \cos \phi \Rightarrow \theta = 0, \phi = \frac{\pi}{2} \Rightarrow YZ$$

p_y -orbital: $\ell = 1, m = +1$

$$\Theta(\theta) \cdot \Phi(\phi) = \sqrt{\frac{3}{4\pi}} \cdot \sin \theta \cos \phi \quad \boxed{\text{Simplifying}} \Rightarrow \theta = 0, \phi = 0 \Rightarrow XZ$$

p_z -orbital: $\ell = 1, m = 0$

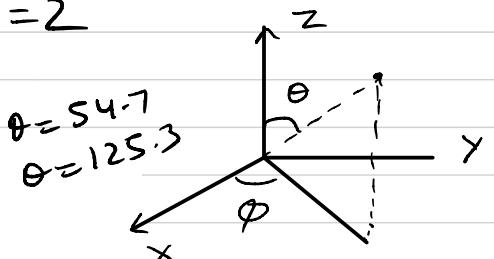
$$\Theta(\theta) \cdot \Phi(\phi) = \sqrt{\frac{3}{4\pi}} \cdot \cos \theta \Rightarrow \theta = -\frac{\pi}{2} \Rightarrow XY$$



③ d orbital:- No of angular node = $\ell = 2$

d_{z^2} -orbital: $\ell = 2, m = 0$

$$\Theta(\theta) \cdot \Phi(\phi) = \sqrt{\frac{5}{16\pi}} (3\cos^2 \theta - 1)$$



$d_{x^2-y^2}$ -orbital: $\ell = 2, m = -2$

$$\Theta(\theta) \cdot \Phi(\phi) = \sqrt{\frac{15}{4\pi}} \cdot \sin^2 \theta \cos 2\phi \Rightarrow (\theta = 0, \phi = \frac{\pi}{4}) \quad (\theta = 0, \phi = \frac{3\pi}{4})$$

d_{xy} -orbital: $\ell = 2, m = +2$

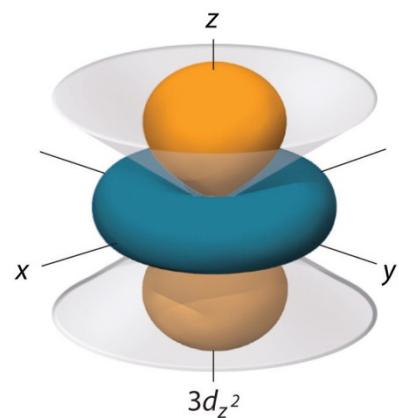
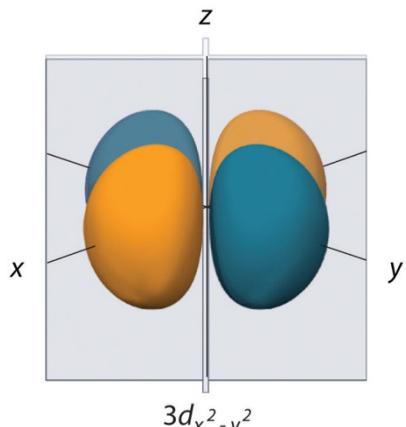
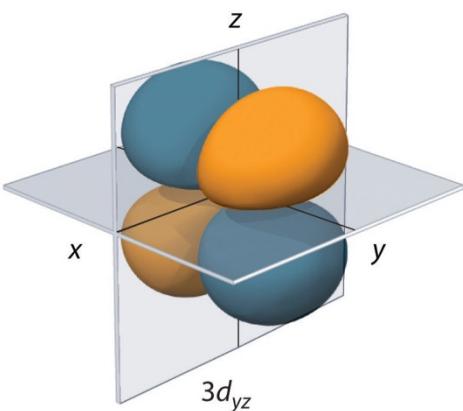
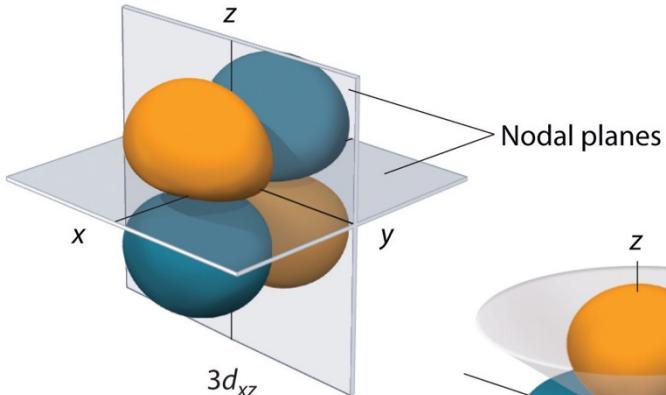
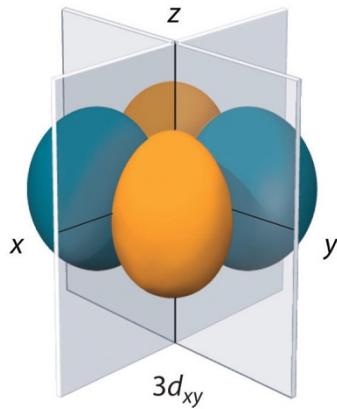
$$\Theta(\theta) \cdot \Phi(\phi) = \sqrt{\frac{15}{4\pi}} \cdot \sin^2 \theta \sin 2\phi \Rightarrow (\theta = 0, \phi = 0), (\theta = 0, \phi = \frac{\pi}{2})$$

d_{xz} -orbital: $\ell = 2, m = +1$

$$\Theta(\theta) \cdot \Phi(\phi) = \sqrt{\frac{15}{4\pi}} \cdot \sin \theta \cos \theta \sin 2\phi$$

d_{yz} -orbital: $\ell = 2, m = +1$

$$\Theta(\theta) \cdot \Phi(\phi) = \sqrt{\frac{15}{4\pi}} \cdot \sin \theta \cos \theta \cdot \sin \phi \quad (\underbrace{\theta = 0, \phi = 0}_{xz}), (\underbrace{\frac{\pi}{2}, 0}_{xy})$$



$$n-l=4$$

$$l=3$$

$$n=7$$

The radial probability distribution curve of an orbital of H has '4' local maxima. If orbital has 3 angular node then orbital will be :

- (a) $7f$ (b) $8f$
(c) $7d$ (d) $8d$

The Schrodinger wave equation for hydrogen atom of 4s-orbital is given by :

$$\psi(r) = \frac{1}{16\sqrt{4}} \left(\frac{1}{a_0}\right)^{3/2} [(\sigma - 1)(\sigma^2 - 8\sigma + 12)] e^{-\sigma/2}$$

where a_0 = 1st Bohr radius and $\sigma = \frac{2r}{a_0}$. The distance

from the nucleus where there will be no radial node will be :

- (a) $r = \frac{a_0}{2}$ (b) $r = 3a_0$
(c) $r = a_0$ (d) $r = 2a_0$

$$\psi(r) = 0$$

$$\sigma = 1 \quad \sigma = 2, 6 \quad = \frac{2r}{a_0}$$

$$r = \frac{a_0}{2}, \quad a_0, \quad 3a_0$$

For an orbital, having no planar angular nodes following the equation :

$$\psi(r) = Ke^{-r/K'} \cdot r^2(K'' - r)$$

Identify the orbital :

- (a) $4d_{z^2}$ (b) $2s$
(c) $3d_{z^2}$ (d) $4d_{xy}$

$$\psi_r = 0$$

$$r = 2a_0 = r_0$$

The Schrodinger wave equation for hydrogen atom is $\psi_{2s} = \frac{1}{4\sqrt{2\pi}} \left(\frac{1}{a_0}\right)^{3/2} \left(2 - \frac{r}{a_0}\right) e^{-r/a_0}$, where a_0 is

Bohr's radius. If the radial node in 2s be at r_0 , then r_0 would be equal to :

- (a) $\frac{a_0}{2}$ (b) $2a_0$
(c) $\sqrt{2}a_0$ (d) $\frac{a_0}{\sqrt{2}}$

Choose the correct statement(s).

- (a) The orbital wave function or ψ for an electron has no physical meaning
- (b) Square of wave function (ψ^2) at a point gives the probability density of the electron at that point.
- (c) Boundary surface diagrams of constant probability density for different orbitals give a fairly good representation of the shapes of the orbitals.
- (d) All of the above

Match the columns.

| Column I (Orbital) | Column II (R vs. r Graph) |
|-----------------------|-----------------------------------|
| (A) $3s$ | (P) |
| (B) $4s$ | (Q) |
| (C) $2p$ | (R) |
| (D) $3p$ | (S) |

