Probability

BASIC DEFINITIONS :

(i) **Random Experiment :** An experiment or an action resulting in two or more possible out comes.

e.g. tossing a coin throwing a die, drawing a card

(ii) **Sample space :** The set of all possible outcomes of an experiment is called sample space of an experiment and each outcome is called a sample point.

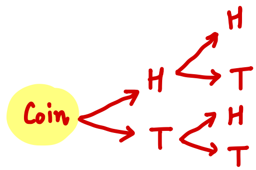
(1) Exp. : Throwing a coin

S {H, T}

E_1 : H occurs ≡ {H}

E_2 : T occurs ≡ {T}

(2) Exp. : Throwing 2 coin



S ≡ {HH, HT, TH, TT}

E_1 ≡ Atleast one head ≡ {HT, HH,TH}

E_2 ≡ Exactly one tail ≡ {HT, TH}

(3) Throwing a die

S ≡ {1,2,3,4,5,6}

E_1 ≡ Getting even number ≡ {2, 4, 6}

(4) Throwing 3 coins

S = {HHH, TTT, HHT, HTH, THH, TTH, THT, HTT}

E_1 ≡ Getting two consecutive head ≡ {HHH, HHT, THH}

(iii) **Event of an experiment :** An event is defined occurrence or situation.

(1) tossing a coin and the coin landing up head.

(2) scoring a six on the throw of a die,

(3) winning the first prize in a raffle.

Note : Event is subset of sample space. If outcome of an experiment is an element of A we say that event A has occurred.

An event consisting of a single point of S is called a simple or elementary event.

If an event has more than one simple point it is called compound event.

ϕ is called impossible event and S (sample space) is called sure event.

3 important terms :

(iv) **Equally likely Event :**

Events are said to be equally likely when no particular event has a preference to occur in relation to the other event.

For Example :

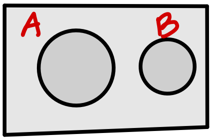
(1) The outcomes as a result of throwing a die are equally likely, as no particular face is more likely to occur as compared to the other faces. That is way we normally write as fair die or unbiased die.

Note that events which are equally likely are also **"equiprobable"**.

Mutually exclusive Event : (ME events).

Events are said to be ME if their simultaneously occurrence is impossible. Two events A and B defined on an experimental performance are said to be ME/disjoint/incompatible if occurrence of one precludes (rules out) the simultaneous occurrence of other.

$A \cap B = \phi$
$n(A \cap B) = 0$.



Exp: Throwing a die
S : $\{1, 2, 3, 4, 5, 6\}$
E_1 : Getting even prime number
E_2 : " odd " "
E_3 : " prime number

E_1 : $\{2\}$
E_2 : $\{3, 5\}$
E_3 : $\{2, 3, 5\}$

E_1 & E_2 are M.E but E_1 & E_3 are not M.E.

Exhaustive event :

Events as a consequence of an experimental performance are said to be exhaustive if nothing beyond than those listed in the set of possible outcomes/sample space can occur. e.g. the possible outcomes of tossing a fair coin in {H, T}.

In general if the events A and B are exhaustive defined on a sample space then $A \cup B$ given the complete sample space and $P(A \cup B) = 1$.

$n(A \cup B) = n(S)$.

Exp: Throwing a die
S : $\{1, 2, 3, 4, 5, 6\}$
E_1 : Getting even no. $E_1 : \{2, 4, 6\}$
E_2 : " odd no. $E_2 : \{1, 3, 5\}$
E_3 : " prime no. $E_3 : \{2, 3, 5\}$

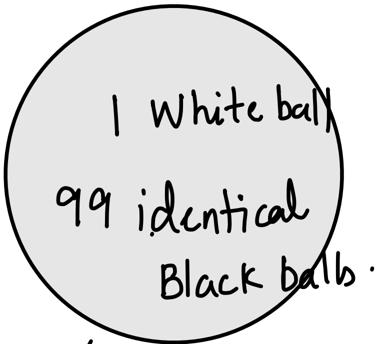
E_2 & E_3 are not exhaustive events.

	Experiment	Events	E/L	M/E	Exhaustive
1.	Throwing of a die	A : throwing an odd face {1, 3, 5} B : throwing a composite face {4,.6}	✗	✓	✗
2.	A ball is drawn from an urn containing 2W, 3R and 4G balls	E_1 : getting a W ball E_2 : getting a R ball E_3 : getting a G ball	✗	✓	✓
3.	Throwing a pair of dice	A : throwing a doublet {11, 22, 33, 44, 55, 66} B : throwing a total of 10 or more {46, 64, 55, 56, 65, 66 }	✓	✗	✗
4.	From a well shuffled pack of cards a card is drawn	E_1 : getting a heart E_2 : getting a spade E_3 : getting a diamond E_4 : getting a club	✓	✓	✓
5.	From a well shuffled pack of cards a card is drawn	A = getting a heart B = getting a face card	✗	✗	✗

(1) S: {1, 2, 3, 4, 5, 6}

Pair of Dice:

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66.



$S : \{W, B\}$

$P(W) = p.$

$P(B) = 99p.$

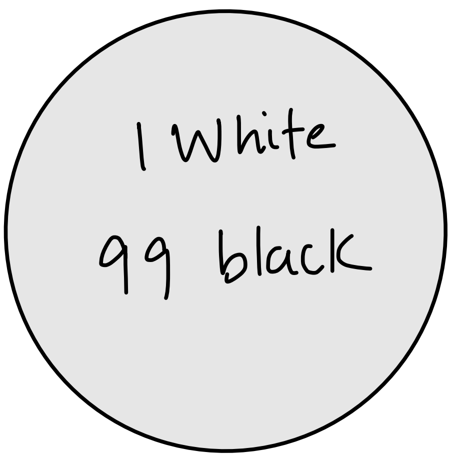
$P(W) + P(B) = 1$

$p + 99p = 1$

$$\boxed{p = \frac{1}{100}}$$

$$\left.\begin{aligned} P(W) &= 1/100 \\ P(B) &= 99/100 \end{aligned}\right\}$$

Alt.



$S : \{W, B_1, B_2, \ldots, B_{99}\}$

The classical definition of probability :
If an experiment gives n possible out comes which are equally likely, mutually exclusive & exhaustive & out of these outcomes if m outcomes favour event A, then $P(A) = \frac{m}{n}$

as $m < n \Rightarrow 0 < P(A) \leq 1$

Complement of an event : The set of all out comes which are in S but not in A is called the Complement Of The Event A denoted by $\bar{A}$, A^c, A' or 'not A'.

$$P(A^c) = P(\bar{A}) = \frac{n-m}{n}$$

$$P(A^c) = 1 - P(A)$$

Note that A and A^C makes an event a sure event and Pr. of a sure event is one.

Odds in favour & Odd against :-

$$\text{Odds in fav} = \frac{\text{no. of favourable outcomes}}{\text{no. of unfavourable outcomes}}$$

$$= \frac{a}{b}$$

S :

$$\text{Odds in fav} = \frac{\left(\frac{a}{a+b}\right)}{\left(\frac{b}{a+b}\right)} = \frac{P(A)}{P(\bar{A})}$$

$$\text{Odds against} = \frac{\text{no. of unfav outcomes}}{\text{no. of fav. outcomes}}$$

$$= \frac{b}{a} = \frac{P(\bar{A})}{P(A)}$$

Q 4 Apples and 3 Oranges are randomly placed in a line. Find the chance that the two extreme fruits are both oranges.

Sol^n

M-1

Apples are alike.
Oranges " " .

A A A A
O O O

$$\underset{\uparrow}{\underline{O}}\ \underbrace{\_\ \_\ \_\ \_\ \_}_{OAAAA}\ \underset{\uparrow}{\underline{O}}$$

$$n(A) = \frac{5!}{4!}$$

$$n(S) = \frac{7!}{4!\ 3!}$$

$$P(A) = \frac{5!/\cancel{4!}}{\dfrac{7!}{\cancel{4!}\ 3!}} = \frac{5! \times 3!}{7!} = \frac{6}{7\times 6} = \frac{1}{7}.$$

M-2

$A_1\ A_2\ A_3\ A_4$

$\underbrace{O_1\ \ O_2\ \ O_3}$

$$\underset{\uparrow}{\underline{O_2}}\ \underbrace{\_\ \_\ \_\ \_\ \_}_{O_1\, A_1 A_2 A_3 A_4}\ \underset{\uparrow}{\underline{O_3}}$$

$$n(S) = \underline{|7}$$

$$n(A) = \left({}^3C_2 \cdot 2!\right) \times 5!$$

$$P(A) = \frac{3.(2!)(5!)}{7!} = \frac{1}{7}.$$

M-3

$$\underset{\uparrow}{\_}\ \overbrace{\_\ \_\ \_\ \_\ \_}\ \underset{\uparrow}{\_}$$

$$\frac{{}^3C_2 \cdot 2!}{{}^7C_2 \cdot 2!}$$

$$\frac{\cancel{3}}{7\times \cancel{6}_{\,2}} = \frac{1}{7}$$

Q A single letter is selected from word PROBABILITY. Find the probability it is vowel?

Soln

$$\frac{4}{11}.$$

Q A card is drawn randomly from a well shuffled pack of 52 cards. The Pr. that the drawn card is

(a) neither a heart nor a face card.

(b) "neither a heart nor a king

(c) red face card

(d) king or a red card .

Sol^n

$$n(S) = {}^{52}C_1 = 52.$$

(a) $n(A) = {}^{30}C_1 = 30.$

$$P(A) = \frac{30}{52}.$$

(b) $n(B) = 36$

$$P(B) = \frac{36}{52}.$$

(C) $n(C) = 6.$

$$P(C) = \frac{6}{52}$$

(d) $n(D) = 28$

$$P(D) = \frac{28}{52}.$$

Q Two natural numbers are randomly selected from the set of first 20 natural numbers. Find the probability that :

(i) their sum is odd

(ii) sum is even

(iii) selected pair is twin prime

Soln $n(S) = {}^{20}C_2 = \frac{20 \times 19}{2} = 190.$

(i) $n(A) = {}^{10}C_1 \cdot {}^{10}C_1 = 100$

$P(A) = \frac{100}{190} = \frac{10}{19}.$

(ii) $n(B) = {}^{10}C_2 + {}^{10}C_2$

$P(B) = \frac{2 \cdot {}^{10}C_2}{190}$

(iii) $\{3,5\}, \{5,7\}, \{11,13\}, \{17,19\}$

$P(C) = \frac{4}{190}.$

Q. Bag $\left\langle \begin{array}{l} 5G \text{ balls} \\ 4W \text{ balls} \end{array} \right.$ $\xrightarrow[\text{drawn}]{\text{3 balls are}}$ find the odds against these being all green.

Sol^n

$$P(GGG) = \frac{{}^5C_3}{{}^9C_3} = \frac{10}{\frac{\cancel{9}^{3} \times \cancel{8}^{4} \times 7}{\cancel{6}}} = \frac{5}{42} = \frac{5}{5+37}$$

Odd's in fav of all being green $= \frac{5}{37}$.

odd's against " " " $= \frac{37}{5}$.

Q. A leap year is selected at random. Find the probability that it has :

(A) 53 Sundays and Mondays (B) 53 Sundays (C) 53 Sundays or 53 Mondays

Sol^n

366 days $= 52 \times 7 + 2$.

Mon, Tue.
Tue, Wed.
Wed, Thur.
Thur, Fri.
Fri, Sat.
Sat, Sun.
Sun, Mon *

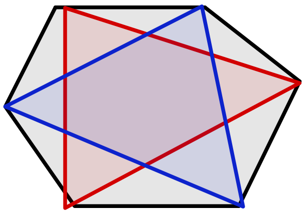
S.S. 5

(A) 1/7.

(B) 2/7.

(C) 3/7.

Q 3 vertices of a regular hexagon are joined randomly. Probability that an equilateral Δ is formed.



$n(S) = {}^6C_3 = 20.$

$n(A) = 2$

$P(A) = \frac{2}{20} = \frac{1}{10}.$

Q An old man while dialing a seven digit telephone number, after having dialed the first five digits, suddenly forgets the last two. But he remembered that the last two digits were different. On this assumption he randomly dials the last two digits. What is the Probability that the correct telephone number is dialed.

Sol^n

$\underbrace{\_\ \_\ \_\ \_\ \_}_{\text{already dialed}}\quad \downarrow\ \downarrow$

$n(S) = 10 \times 9 = 90.$

$n(A) = 1$

$P(A) = \frac{1}{90}.$

Q What is the chance that the fourth power of an integer chosen randomly ends in the digit six.

Units digit in I	U.D in I^2	U.D in I^4
0	0	0
1	1	1
2	4	6
3	9	1
4	6	6
5	5	5
6	6	6
7	9	1
8	4	6
9	1	1

$n(S) = 10.$

$n(A) = 4.$

$P(A) = \frac{4}{10}.$

Q 2 numbers are randomly selected from the set of first six natural numbers. Find the probability that the selected pair is coprime.

Soln

$\{1, 2, 3, 4, 5, 6\}$

$n(S) = {}^6C_2$.

$P(A) = \dfrac{11}{{}^6C_2}$

$\{(1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,3), (2,5)$
$(3,4), (3,5)$.
$(4,5), (5,6)$. $\}$

Q Three tickets are drawn randomly from a pack of 51 tickets numbered from 1 to 51. Find the probability that :

(i) numbers are in AP.

(ii) their sum is odd.

Sol^n (i) $n(S) = {}^{51}C_3$.

$x, y, z \longrightarrow AP \quad \Rightarrow \quad \underset{\text{Even}}{2y} = \underbrace{x+z}_{\text{Even}}$

$n(A) = {}^{26}C_2 + {}^{25}C_2$

$P(A) = \dfrac{n(A)}{n(S)}$

(ii) $x+y+z = \text{odd}$.

$\left.\begin{array}{ll} \text{(i)} & O \quad O \quad O \\ \text{(ii)} & E \quad E \quad O. \end{array}\right\}$

$n(B) = {}^{26}C_3 + {}^{25}C_2 \cdot {}^{26}C_1$

Q There are 10 Keys, 3 Keys are randomly selected to open a lock. Find the chance that you have choosen the correct key.

Sol^n

$$\frac{{}^9C_2}{{}^{10}C_3} = \frac{3}{10}.$$

Q A pack of well shuffled 52 cards are randomly distributed equally among four brothers. Find the probability that all the queens are held by the youngest brother?

52 → 13, 13, 13, 13

Sol^n M-1

$$n(S) = \frac{52!}{(13!)^4 \cdot 4!} \times 4!$$

48 → 13, 13, 13, 9

$$n(A) = \frac{48!}{(13!)^3 \cdot 3! \; 9!} \times 3!$$

$$P(A) = \frac{n(A)}{n(S)}.$$

M-2

$$\frac{{}^{48}C_9}{{}^{52}C_{13}}$$

Q. HW Let x and y are 2 randomly selected integers.

Find the probability that :

(i) $(x+y)$ is divisible by '7'.

(ii) (x^2+y^2) is divisible by '3'

(iii) $(x^2 + y^2)$ is divisible by 5.

Q. HW Let x and y are 2 randomly selected integers.

Find the probability that :

(i) $(x+y)$ is divisible by '7'.

$$7I \longrightarrow p = 1/7$$
$$7I+1 \longrightarrow \text{"}$$
$$7I+2 \longrightarrow \text{"}$$
$$7I+3 \longrightarrow \text{"}$$
$$7I+4 \longrightarrow \text{"}$$
$$7I+5 \longrightarrow \text{"}$$
$$7I+6 \longrightarrow \text{"}$$

$$\cancel{7}\left(\frac{1}{\cancel{7}} \times \frac{1}{7}\right) = \frac{1}{7}$$

(ii) (x^2+y^2) is divisible by '3'

$x, y \longrightarrow \begin{cases} 3I \\ 3I+1 \\ 3I+2 \end{cases}$

$3I \longrightarrow (3I)^2 = 9I^2 = 3\lambda$

$3I+1 \longrightarrow (3I+1)^2 = 9I^2+6I+1$

$= \underbrace{3\lambda_1+1}$

$3I+2 \longrightarrow (3I+2)^2 = 9I^2+12I+\underbrace{4}_{3+1}$

$= \underbrace{3\lambda_2+1}.$

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}.$$

(iii) (x^2+y^2) is divisible by 5.

$$x, y \rightarrow \begin{cases} 5I \rightarrow 25I^2 = 5\lambda \\ 5I+1 \rightarrow = 5\lambda_1+1 \\ 5I+2 \rightarrow = 5\lambda_2+4 \\ 5I+3 \rightarrow = 5\lambda_3+4 \\ 5I+4 \rightarrow = 5\lambda_4+1 \end{cases}$$

$\frac{9}{25}$. ✓

<u>Alt</u> :-

$n(s) = 10 \times 10 = 100.$

$n(A) = 36.$

$$\frac{n(A)}{n(s)} = \frac{36}{100}$$

$$= \frac{9}{25} \text{ Ans}$$

✓

Units digit in I	Unit digit in I^2
0	0
1	1
2	4
3	9
4	6
5	5
6	6
7	9
8	4
9	1

Now two important sample spaces are

(a) Throwing a pair of dice given 36 ME/EL and Exhaustive cases which are :

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	11	12	13	14	15	16
⚁	21	22	23	24	25	26
⚂	31	32	33	34	35	36
⚃	41	42	43	44	45	46
⚄	51	52	53	54	55	56
⚅	61	62	63	64	65	66

Note that :

(i) $P(\text{Total } 2) = \frac{1}{36}$; $P(3) = \frac{2}{36}$; $P(4) = \frac{3}{36}$; $P(5) = \frac{4}{36}$ etc.

(ii) Probability of throwing a doublet $= \frac{6}{36}$

(iii) Probability of throwing a total which is prime $= P(2, 3, 5, 7, 11) = \frac{15}{36}$

Note : Let $p : N \to [0, 1)$ where p is the probability function. N → Natural number.

If p(x) = Probability of getting the sum of scores when two dice are thrown.

$$P(x) - \begin{cases} \frac{x-1}{36} & \text{for } 1 \le x \le 7 \\ \frac{13-x}{36} & \text{for } 8 \le x \le 13 \end{cases}$$

P(x) is a many one function ; Domain of p = N, Range of p $= \left\{ \frac{x}{36} \middle| 0 \le x \le 6,\ x \in N \right\}$

& into

$P(s=2) = \frac{1}{36}$

$P(s=3) = \frac{2}{36}$

$P(s=4) = \frac{3}{36}$

$P(s=5) = \frac{4}{36}$

$P(s=6) = \frac{5}{36}$

$P(s=12) = \frac{1}{36}$

$P(s=11) = \frac{2}{36}$.

$P(s=10) = \frac{3}{36}$.

$P(s=9) = \frac{4}{36}$.

$P(s=8) = \frac{5}{36}$

$\boxed{P(s=7)}$

$P(\text{sum} \le 6) = \;?$

$$p + p + \underbrace{P(\text{sum}=7)}_{\downarrow} = 1$$

$$2p + \frac{6}{36} = 1$$

$$p = \frac{1}{2}\left(1 - \frac{6}{36}\right) = \frac{15}{36}.$$

Q A pair of dice is thrown and shows different faces. Find the probability :

(i) sum of two faces is prime.

(ii) one die shows '1'.

Sol^n (i) $P(\underbrace{2}_{\substack{\times \\ (1,1)}}, \underbrace{3}_{\substack{2 \text{ cases} \\ (1,2) \\ (2,1)}}, \underbrace{5}_{4 \text{ cases}}, \underbrace{7}_{6 \text{ cases}}, \underbrace{11}_{2 \text{ cases}}) = \dfrac{14}{30}$.

(ii) $\dfrac{10}{30}$.

Q Let P_i $(i = 1, 2, \dots, 6)$ denotes the probability of getting the face 'i', when a biased die is thrown where $P_i \propto i$.

Find the probability of getting :-

(i) Composite faces.

Sol^n

$$P_i = Ki$$

$$P_1 = \frac{1}{21} \quad \dots \quad P_6 = \frac{6}{21}$$

$$P_2 = \frac{2}{21}$$

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 1$$

$$K + 2K + 3K + 4K + 5K + 6K = 1 \Rightarrow \boxed{K = \frac{1}{21}}$$

(i)

$$P_4 + P_6 = \frac{10}{21}.$$

(2) When 2 such dice are thrown then find the prob. of getting a doublet.

$$\left(\frac{1}{21}\right)^2 + \left(\frac{2}{21}\right)^2 + \dots + \left(\frac{6}{21}\right)^2$$

Q When 3 normal dice are thrown, find the probability that

(a) sum on the faces is less than 11.

(b) " " " " is 6.

(c) " " " " is 9.

$n(S) = \underline{\underline{216}}$

(a)

$P(s=3)$	$P(s=18)$
$P(s=4)$	$P(s=17)$
$\vdots$	
$P(s=10)$	$P(s=11)$

$$\underbrace{P(\text{sum} \leq 10)}_{x} + \underbrace{P(\text{sum} \geq 11)}_{x} = 1$$

$$x + x = 1 \Rightarrow \boxed{x = 1/2}$$

(b)

$$x + y + z = 6.$$

$1 \leq x \leq 6$

$1 \leq y \leq 6$

$1 \leq z \leq 6.$

$$X + Y + Z = 3. \rightarrow {}^5C_2 = 10. \text{ cases.}$$

Req. prob $= \dfrac{10}{216}$.

(c) $x + y + z = 9$

$1 \le x \le 6$

$1 \le y \le 6$

$1 \le z \le 6$

$x + y + z = \boxed{6}$

6, 0, 0.

0, 6, 0.

0, 0, 6.

${}^{8}C_{2} - 3 = 25$ cases.

Req. prob $= \dfrac{25}{216}$ Ans

Q Find the prob that in a random throw of 12 dice, each face occurs twice ?

Sol^n

1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6

$$\frac{\frac{12!}{(2!)^6}}{6^{12}}$$

Throwing of a coin say 4 times :

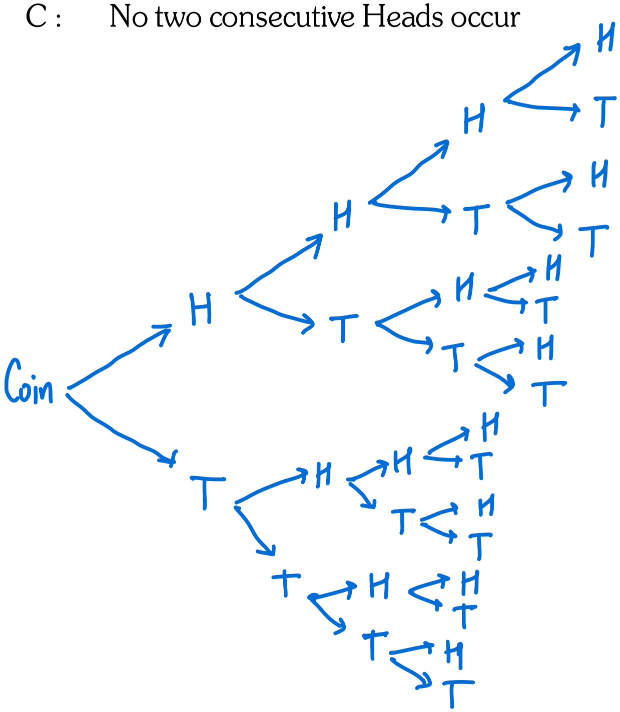
S = {HTTT, HTTH, THTT, THTH, HHTT, HHTH, TTTT, TTTH, HTHT, HTHH, THHT, THHH, HHHT, HHHH, TTHT, TTHH}

Find the probability of the following events

A : H and T come alternately

B : number of H occurring is more than the number of T.

C : No two consecutive Heads occur



$P(A) = \frac{2}{16}$.

$P(B) = \frac{5}{16}$.

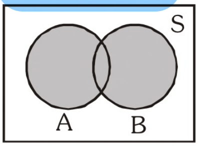
$P(C) = \frac{8}{16}$.

VENN DIAGRAMS :

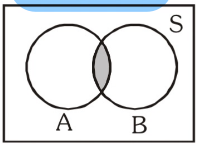
A diagram used to illustrate relationships between sets. Commonly, a rectangle represents the universal set and a circle within it represents a given set (all members of the given set are represented by points within the circle). A subset is represented by a circle within a circle and union and intersection are indicated by overlapping circles.

Let S is the sample space of an experiment and A, B, C are three events corresponding to it :

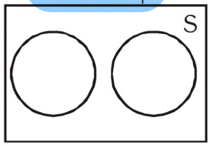
$A \cup B$ or $A+B$



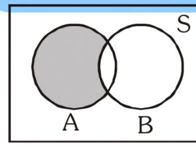
$A \cap B$ or AB



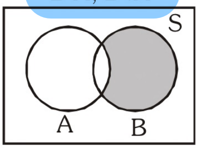
$A \cap B = \phi$



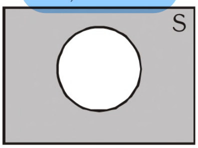
$A - B$ or $A \setminus B$ or $A \cap \bar{B}$ or only A.



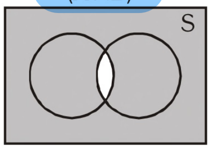
$B - A$, $B \setminus A$



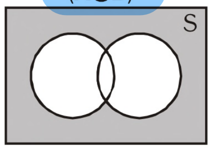
A', A^C or $\bar{A}$



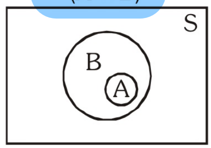
$(A \cap B)'$



$(A \cup B)'$



$(A \subset B)$

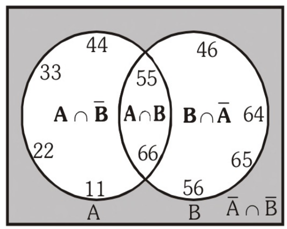


Example : Let us now conduct an experiment of tossing a pair of dice.

Two events defined on the experiment are

A : getting a doublet $\{11, 22, 33, 44, 55, 66\}$

B : getting total score of 10 or more $\{64, 46, 55, 56, 65, 66\}$



Note :

(i) If A & B are two subsets of a universal set U, then

(a) $(A \cup B)^c = A^c \cap B^c$ **(b)** $(A \cap B)^c = A^c \cup B^c$ **(DE MORGAN'S LAW)**

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

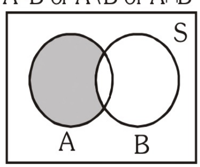
(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iv) $(A \setminus B) \rightarrow$ read as A minus B

$(A \setminus B) = (A \cap B^c)$

$P(A \setminus B) = P(A) - P(A \cap B)$

$A - B$ or $A \setminus B$ or $A \cap \bar{B}$

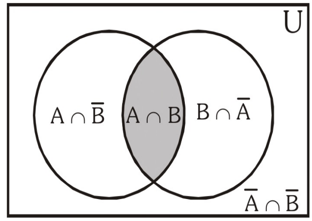


ADDITION THEOREM :

$A \cup B = A + B$ = A or B denotes occurrence of at least A or B.

For 2 events A & B :

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$



Note :

(i) $P(A \cup B)$

$P(A + B)$

$P(A \text{ or } B)$

P(occurence of atleast one A or B)

P(either A or B)

= $P(A) + P(B) - P(A \cap B)$ (This is known as generallised addition theorem)

$P(A) + P(B \cap \bar{A})$

$P(B) + P(A \cap \bar{B})$

$P(A \cap \bar{B}) + P(A \cap B) + P(B \cap \bar{A})$

$1 - P(A^C \cap B^C)$

$1 - P(A \cup B)^C$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$P(A \cup B \cup C) = \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$$

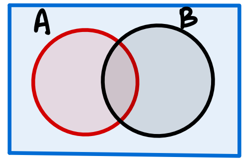
Probability of neither A nor B $= P(\bar{A} \cap \bar{B})$

$$P(\overline{A \cup B}) = 1 - P(A \cup B)$$

Probability of neither A nor B = 1 − P (either A or B).

* **If A & B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.**

* For any two events A & B, P(exactly one of A, B occurs)



$$= P(A \cap \bar{B}) + P(B \cap \bar{A}) = P(A) + P(B) - 2P(A \cap B)$$

$$= P(A \cup B) - P(A \cap B) = P(A^c \cup B^c) - P(A^c \cap B^c)$$

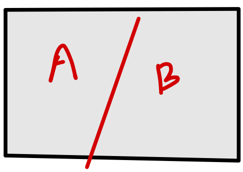
** **If E_1, E_2 .... E_n are mutually exclusive & exhaustive, then $P(E_1 \cup E_2 \text{ .... } \cup E_n) = 1$**

$\Rightarrow \quad$ **$P(E_1) + P(E_2)$ .... $+ P(E_n) = 1$**

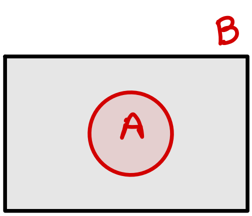
$P(A \cap B) \le P(A)$ or $P(B) \le P(A \cup B) \le P(A) + P(B)$

If A and B are M.E and exhaustive, then

$$\boxed{P(A \cup B) = P(A) + P(B) = 1.}$$



* A implies B ($A \Rightarrow B$) i.e occurance of A ensures occurance of B.



E(1) A card is drawn from a pack of 52 cards

$A \to$ red card drawn; $\quad B \to$ face card drawn

find the following :

1. $P(A) =$

2. $P(B) =$

3. $P(A \cap B) =$

4. $P(A \cup B) = P(A) + P(B) - P(A \cap B) =$

5. $P(A^c) = 1 - P(A) =$

6. $P(A \setminus B) = P(A) - P(A \cap B) =$

7. $P(B \cap A^c) = P(B \setminus A) = P(B) - P(A \cap B) =$

8. $P(A^c \cup B^c) = P(A \cap B)^c = 1 - P(A \cap B) =$

9. $P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B) =$

10. $P(A^c \setminus B^c) = P(A^c) - P(A^c \cap B^c) =$

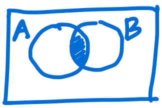
Sol^n

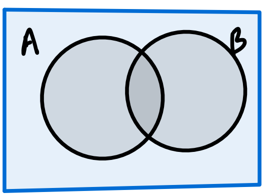
$P(A) = \frac{26}{52} = \frac{1}{2}$; $P(B) = \frac{12}{52} = \frac{3}{13}$

$P(A \cap B) = \frac{6}{52} = \frac{3}{26}$; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A^c) = 1 - P(A) = \frac{1}{2}$; $P(A \setminus B) = P(A - B)$

$= P(A) - P(A \cap B)$





$P(B \cap \bar{A}) = P(\text{only } B)$

$= P(B) - P(A \cap B).$

$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$P(\bar{A} \setminus \bar{B}) = P(\bar{A} - \bar{B}) = P(\bar{A}) - P(\bar{A} \cap \bar{B})$

E(2) For any two events A and B.

(a) Let $P(A \cup B) = 7/8$; $P(A \cap B) = 1/4$ and $P(A^c) = 5/8$.

Find P(A), P(B) and $P(A\backslash B)$

HW

(b) If A and B are any two events with $P(A) = 3/8$; $P(B) = 1/2$ and $P(A \cap B) = 1/4$. Find

(i) $P(A \cup B)$ (ii) $P(A^c)$ and $P(B^c)$ (iii) $P(A^c \cap B^c)$ (iv) $P(A^c \cup B^c)$

(v) $P(A \cap B^c)$ (vi) $P(B \cap A^c)$

[**Ans.** (i) 5/8; (ii) 5/8 & 1/2; (iii) 3/8 ; (iv) 3/4 ; (v) 1/8 ; (vi) 1/4]

(a) $P(\bar{A}) = \frac{5}{8}$; $P(A) = \frac{3}{8}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{7}{8} = \frac{3}{8} + P(B) - \frac{1}{4}.$$

$$P(A \backslash B) = P(A - B) = P(A) - P(A \cap B).$$

Q. A divisor of N = 75600 is randomly chosen. Find the probability that

(i) divisor N is divisible by 12 but not by 30.

(ii) divisor N is neither divisible by 12 nor 30.

Sol^n

$$N = 75600 = 2^4 \cdot 3^3 \cdot 5^2 \cdot 7^1.$$

$$\text{Total no. of divisors} = 5 \times 4 \times 3 \times 2 = 120.$$

A : divisors which are divisible by $12 = 2^2 \cdot 3^1$

B : " " " " " $30. = 2^1 \cdot 3^1 \cdot 5^1$

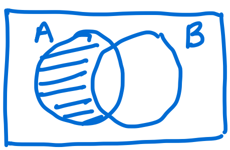
$$n(A) = 3 \times 3 \times 3 \times 2 = 54. \qquad P(A) = \frac{54}{120}.$$

$$n(B) = 4 \times 3 \times 2 \times 2 = 48 \qquad P(B) = \frac{48}{120}.$$

$= 2^2 \cdot 3^1 5^1$

LCM of $(12, 30) = 60.$

(i)



$$P(A) - P(A \cap B)$$

$$\frac{54}{120} - \frac{36}{120}.$$

$$n(A \cap B) = 3 \times 3 \times 2 \times 2 = 36$$

$$P(A \cap B) = \frac{36}{120}$$

(ii) $$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$

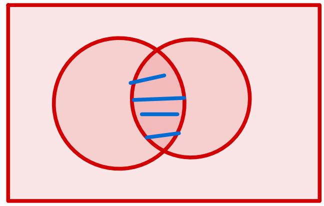
$$= 1 - \left(P(A) + P(B) - P(A \cap B) \right)$$

Q. Consider a set S = { 1,2,3,.......10 } . Three numbers are randomly chosen. Find the probability that their minimum is 4 and max. is 8.

Sol^n $n(S) = {}^{10}C_3$

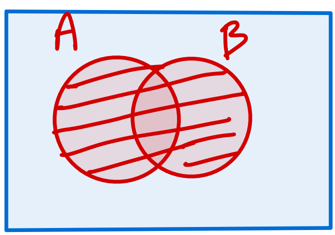
$n(A) = {}^{3}C_1$

Req. prob $= \dfrac{{}^{3}C_1}{{}^{10}C_3}$



Q. Consider a set S = { 1,2,3,.......10 } . Three numbers are randomly chosen. Find the probability that their minimum is 4 or max. is 8.

Req. prob $= \dfrac{{}^{6}C_2 + {}^{7}C_2 - {}^{3}C_1}{{}^{10}C_3}$

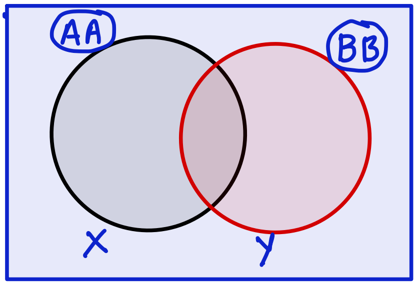


Q. By using all the letters of the word ' GABBAR ' , words are formed randomly. Find the probability in a randomly chosen word no two alike letters are together.

GABBAR

Total no. of words $= \frac{6!}{2!2!} = 180.$

G, B, B, R, AA



$n(X) = \frac{5!}{2!} = 60$

$n(Y) = \frac{5!}{2!} = 60$

G, A, A, R, BB

AA BB G R

$n(X \cap Y) = 4! = 24$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= \frac{60}{180} + \frac{60}{180} - \frac{24}{180}$

$= \frac{96}{180}$

Req. prob $= P(\overline{X \cup Y}) = 1 - P(X \cup Y)$

$= 1 - \frac{96}{180}.$

CONDITIONAL PROBABILITY : / Restricted sample space (Reduced)

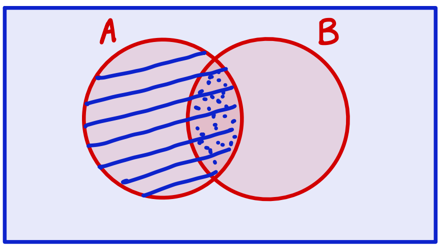
Let A and B be two events such that P (A) >0. Then P(B/A) denote the conditional probability of B given that A has occurred. Since A is known to have occurred, it becomes the new sample space replacing the original S. From this we led to the definition.

Rem

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)},}$$ which is called conditional probability of B given A

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\boxed{P(B/A) = P(B|A) = P\left(\frac{B}{A}\right)}$$



$$P(B/A) = \frac{n(A \cap B)}{n(A)}$$

$$= \frac{\left(\frac{n(A \cap B)}{n(S)}\right)}{\left(\frac{n(A)}{n(S)}\right)}$$

$$= \frac{P(A \cap B)}{P(A)}.$$

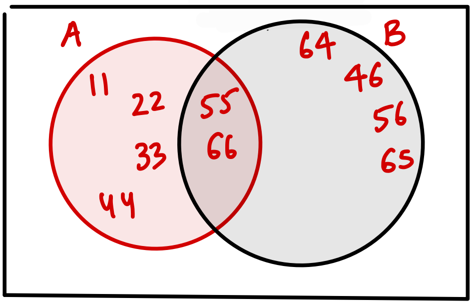
eg:-

Exp. : Throwing a pair of dice

Event : A : getting a doublet

B : getting a sum of 10 or more

P(A/B) = conditional prob. of occurrence of A given B has occurred



$$P(A/B) = \frac{2}{6} = 1/3$$

$$P(A \cap B) = 2/36 = 1/18$$

$$P(B) = 6/36 = 1/6.$$

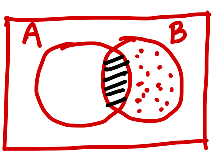
$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/18}{1/6} = 1/3.$$

Note:- (i) $P(A/B) + P(\bar{A}/B) = 1.$

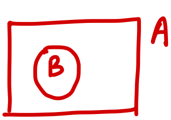
(ii) If B is subset of A then

$$P(B/A) = \frac{P(B)}{P(A)} \quad \text{and} \quad P(A/B) = 1.$$

(i) $P(A/B) + P(\bar{A}/B) = \frac{P(A \cap B)}{P(B)} + \frac{P(\bar{A} \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1.$



(ii)



$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(B)}{P(A)}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\cancel{P(B)}}{\cancel{P(B)}} = 1$$

Q. Given P(A) = 1/2 ; P(B) = 1/3 and P(A ∩ B) = 1/4. Find

(i) P(A/B) (ii) P(B/A) (iii) P(A ∪ B) (iv) P(A^C/B^C) (v) P(B^C/A^C)

(i) $P(A/B) = \dfrac{P(A\cap B)}{P(B)}$

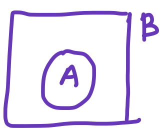
(ii) $P(B/A) = \dfrac{P(B\cap A)}{P(A)}$

(iii) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

(iv) $P(\overline{A}/\overline{B}) = \dfrac{P(\overline{A}\cap\overline{B})}{P(\overline{B})} = \dfrac{P(\overline{A\cup B})}{P(\overline{B})} = \dfrac{1-P(A\cup B)}{1-P(B)}$

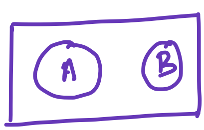
Q. Find P(B/A) if (i) A is subset of B (ii) A and B are disjoint

(i)



$P(B/A) = \dfrac{P(A\cap B)}{P(A)} = 1$

(ii)



$A\cap B = \phi$

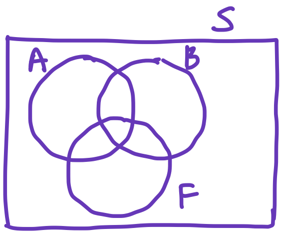
$P(A\cap B) = 0$

$P(B/A) = 0.$

Q. If A and B are any two events of a sample space S and F is an event of S such that $P(F) \neq 0$. Prove that $P((A \cup B)/F) = P(A/F) + P(B/F) - (P(A \cap B)/F)$

Sol^n

$$P((A \cup B)/F) = \frac{P((A \cup B) \cap F)}{P(F)} = \frac{P(A \cap F) + P(B \cap F) - P((A \cap B) \cap F)}{P(F)}$$



$$= P(A/F) + P(B/F) - P(A \cap B/F)$$

(H.P)

Imp

MULTIPLICATION THEOREM :

Rem

$$\boxed{\begin{array}{l} P(A \cap B) = P(A)\,P(B/A),\ P(A) \neq 0 \\ \text{or}\quad P(A \cap B) = P(B)\,P(A/B),\ P(B) \neq 0 \end{array}}$$

$P(A/B) = \dfrac{P(A\cap B)}{P(B)} \Rightarrow P(A\cap B) = P(B)\cdot P(A/B)$

$= P(A)\cdot P(B/A)$

(simultaneous occur)

which is called compound probability or multiplication theorem. It says the probability that both A and B occur is equal to the probability that A occur times the probability that B occurs given that A has occurred.

For three events A,B & C:

$$P(ABC) = P(A\cap B\cap C) = P(A).P(B/A).\ P(C/A\cap B)$$

$D = A\cap B$

$A\cap B = AB$

$A\cup B = A+B$

$A\cap B\cap C = ABC.$

$P(D\cap C) = P(D)\cdot P(C/D)$

$= P(A\cap B)\cdot P(C/A\cap B)$

$= P(A)\cdot P(B/A)\cdot P(C/A\cap B)$

Q An urn contains 3 W, 4 G, 5 R balls. Three balls are drawn one by one at random with replacement. Find the prob. that

(i) first two balls are white & third is red.

(ii) first white, second green & third red.

(iii) all are of different colours.

(iv) two of them are white & other is red.

3W
4G
5R

(i) WWR

$$\left(\frac{3}{12}\right)\left(\frac{3}{12}\right)\left(\frac{5}{12}\right)$$

(ii) WGR

$$\left(\frac{3}{12}\right)\left(\frac{4}{12}\right)\left(\frac{5}{12}\right)$$

(iii) one is red, one is white & one is green.

$$6 \text{ cases} \begin{cases} R\,W\,G \longrightarrow \left(\frac{5}{12}\right)\left(\frac{3}{12}\right)\left(\frac{4}{12}\right) \\ R\,G\,W \longrightarrow \left(\frac{5}{12}\right)\left(\frac{4}{12}\right)\left(\frac{3}{12}\right) \\ W\,R\,G \qquad\qquad '' \\ W\,G\,R \qquad\qquad '' \\ G\,W\,R \qquad\qquad '' \\ G\,R\,W \qquad\qquad '' \end{cases}$$

$$6 \times \left(\frac{5}{12} \times \frac{3}{12} \times \frac{4}{12} \right)$$

(iv) two are white & one is red.

$$\begin{cases} W\,W\,R \longrightarrow \left(\frac{3}{12}\right)\left(\frac{3}{12}\right)\left(\frac{5}{12}\right) \\ W\,R\,W \longrightarrow \quad '' \\ R\,W\,W \longrightarrow \quad '' \end{cases}$$

$$3\left(\frac{3}{12} \times \frac{3}{12} \times \frac{5}{12} \right).$$

Q An urn contains 3 W, 4 G, 5 R balls. Three balls are drawn one by one at random without replacement. Find the prob. that

(i) first two balls are white & third is red.

(ii) first white, second green & third red.

(iii) all are of different colours.

(iv) two of them are white & other is red.

3W
4G
5R

(i) WWR

$$\left(\frac{3}{12}\right)\left(\frac{2}{11}\right)\left(\frac{5}{10}\right).$$

(ii) WGR

$$\left(\frac{3}{12}\right)\left(\frac{4}{11}\right)\left(\frac{5}{10}\right)$$

(iii) all are of diff colours.

6 cases $\begin{cases} WGR \longrightarrow \left(\frac{3}{12}\right)\left(\frac{4}{11}\right)\left(\frac{5}{10}\right) \\ \text{---} \quad = \\ \text{---} \quad = \\ \text{---} \quad = \\ \text{---} \quad = \\ \text{---} \quad = \end{cases}$

$$6 \times \left(\frac{3}{12} \times \frac{4}{11} \times \frac{5}{10}\right)$$

(iv) Two are white and one is red.

3 cases $\begin{cases} WWR \\ WRW \\ RWW \end{cases}$

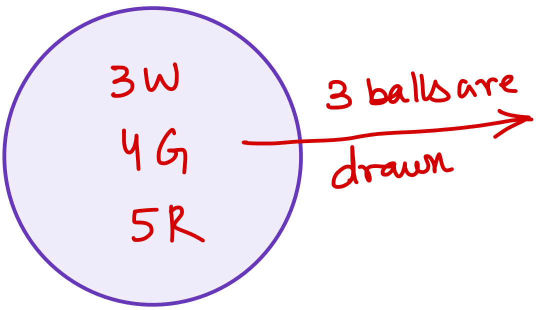
$$3\left(\frac{3}{12} \cdot \frac{2}{11} \cdot \frac{5}{10}\right)$$

Q An urn contains 3 W, 4 G, 5 R balls. Three balls are drawn simultaneously at random.

Find the prob. that :

(i) All of them are of different colours.

(ii) two of them are white & other is red.



(i)

$$\frac{{}^3C_1 \cdot {}^4C_1 \cdot {}^5C_1}{{}^{12}C_3} = \left(\frac{3.4.5}{12 \times 11 \times 10}\right) \times 6$$

$$= 6\left(\frac{3}{12} \cdot \frac{4}{11} \cdot \frac{5}{10}\right)$$

(ii)

$$\frac{{}^3C_2 \cdot {}^5C_1}{{}^{12}C_3} = \left(\frac{3.5}{12 \times 11 \times 10}\right)6$$

Q A lot contains 12 items of which 4 are defective. 3 items are drawn one after the other without replacement find the probability that all 3 are non defective.

Solⁿ

8 Good
4 defective

$\xrightarrow{\text{3 items are drawn}}$

$$\frac{{}^{8}C_3}{{}^{12}C_3}$$

Alt:

$$\frac{8}{12} \cdot \frac{7}{11} \cdot \frac{6}{10}$$

Imp

INDEPENDENT EVENTS :

If A & B are two events such that P(A/B) = P(A) and P(B/A) = P(B) then they are called independent events.

$$P(A/B) = P(A) \quad ; \quad P(A/\bar{B}) = P(A)$$

$$\frac{P(A \cap B)}{P(B)} = P(A) \Rightarrow \boxed{P(A \cap B) = P(A) \cdot P(B)} \;**.$$

2 Events A and B are said to be independent if occurrence or non-occurrence of one does not affect the probability of occurrence or non-occurrence of the other.

Exp. Drawing a card

$E_A \rightarrow$ The card is spade ; $E_B \rightarrow$ card is an ace

$P(A) = 1/4$; $P(B) = 1/13$; $P(A \cap B) = 1/52$

$P(B/A) = 1/13$ Here A & B are independent events.

Rem

Note :

(i) $P(A \cap B) = P(A) . P(B)$ is generally used as a defining equation for independent events.

(ii) If A and B are independent then $P(A \cup B) = P(A) + P(B) - P(A) . P(B)$

(iii) If A and B are independent then $\bar{A}$ and $\bar{B}$; A and $\bar{B}$; $\bar{A}$ and B are also independent.

** [**Proof :** (iii) Given A and B are independent $\Rightarrow$ $P(A \cap B) = P(A) . P(B)$

T.P.T. $\bar{A}$ and $\bar{B}$ are also independent i.e $P(\bar{A} \cap \bar{B}) = P(\bar{A}) . P(\bar{B})$

R.H.S. $[1 - P(A)][1 - P(B)] = 1 - P(A) - P(B) + P(A) . P(B)$

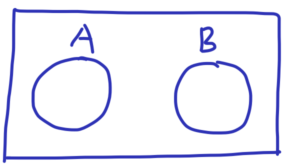
$= 1 - [P(A) + P(B) - P(A) . P(B)]$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A \cup B) = P(\overline{A \cup B})$

$= P(\bar{A} \cap \bar{B}) = $ L.H.S.]

* If A and B are M.E as well as independent events it would mean $P(A) = 0$ or $P(B) = 0$ or both zero.



$A \cap B = \phi$

$P(A \cap B) = 0$

$\Rightarrow P(A) \cdot P(B) = 0$

Hence atleast one of A or B must be impossible event.

Q If A and B are independent events such that $P(A) = p$, $P(B) = q$ where $p, q \in [0, 1]$ then

(i) $P(\bar{A}/\bar{B})$

(ii) $P(\bar{A} \setminus \bar{B})$

(iii) $P(\bar{A}/A \cup B)$

① $$P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(\bar{A}) \cdot P(\bar{B})}{P(\bar{B})}$$

$$= \frac{(1-p)\cancel{(1-q)}}{\cancel{(1-q)}} = (1-p)$$

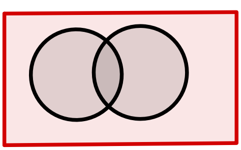
② $$P(\bar{A} \setminus \bar{B}) = P(\bar{A}) - P(\bar{A} \cap \bar{B})$$

$$= (1-p) - P(\bar{A}) \cdot P(\bar{B})$$

$$= (1-p) - (1-p)(1-q)$$

$$= (1-p)(\cancel{1} - \cancel{1} + q) = q(1-p).$$

ⓒ $$P(\bar{A}/A \cup B) = \frac{P(\overbrace{\bar{A} \cap (A \cup B)})}{P(A \cup B)}$$



$$= \frac{P(\bar{A} \cap B)}{P(A \cup B)} = \frac{P(\bar{A}) \cdot P(B)}{P(A) + P(B) - P(A)P(B)}.$$

Imp

If $p_1, p_2, \ldots\ldots p_n$ are probabilities of n independent events $E_1, E_2, \ldots\ldots E_n$ respectively, then

P(occurance of atleast one) $= P(E_1 \cup E_2 \cup \ldots.. E_n)$

$$= 1 - P\left(\overline{E_1 \cup E_2 \cup \ldots.. E_n}\right)$$

$$= 1 - P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \ldots.. \bar{E}_n)$$

$$= 1 - P(\bar{E}_1) P(\bar{E}_2) \ldots\ldots P(\bar{E}_n)$$

$$= 1 - (1 - p_1)(1 - p_2) \ldots\ldots (1 - p_n)$$

Q The probability that an antiaircraft gun can hit an enemy plane at the first, second and third shot are 0.6,0.7 and 0.1 respectively. Find the probability that the gun hits the plane.

Soln

$$1 - P(\bar{1} \cap \bar{2} \cap \bar{3})$$

$$1 - P(\bar{1}) \cdot P(\bar{2}) \cdot P(\bar{3})$$

$$1 - (0.4)(0.3)(0.9)$$

Note :-

If cards are dealt one by one from a well shuffled pack of 52 cards without replacement then the probability that a particular card is drawn at the nth draw ($1 \leq n \leq 52$) is independent of n and is equal to 1/52.

52 playing cards. $\longrightarrow$ Ace of spade.

Ace of spade in 1^{st} draw $= \frac{1}{52}$.

" " " " 5^{th} draw $= \frac{\cancel{51}}{52} \cdot \frac{\cancel{50}}{\cancel{51}} \cdot \frac{\cancel{49}}{\cancel{50}} \cdot \frac{\cancel{48}}{\cancel{49}} \cdot \frac{1}{\cancel{48}} = \frac{1}{52}$.

" " " " 41^{st} draw $= \frac{1}{52}$.

THREE EVENTS DEFINED ON AN EXPERIMENTAL PERFORMANCE

For any three events A,B and C

(i) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - [P(A \cap B) + P(B \cap C) + P(C \cap A)] + P(A \cap B \cap C)$ or P(A or B or C)

[Proof : Let $B \cup C = D$

L.H.S. $P(A \cup D) = P(A) + P(D) - P(A \cap D) = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap (B \cup C))]$

$= \sum P(A) - P(B \cap C) - [P(A \cap B) \cup (A \cap C)]$

$= \sum P(A) - P(B \cap C) - [P(A \cap B) + (A \cap C) - P(A \cap B \cap C)]$

$\therefore \quad P(A \cup B \cup C) = \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$

(ii) P (exactly one appearing) = P(E – 1)

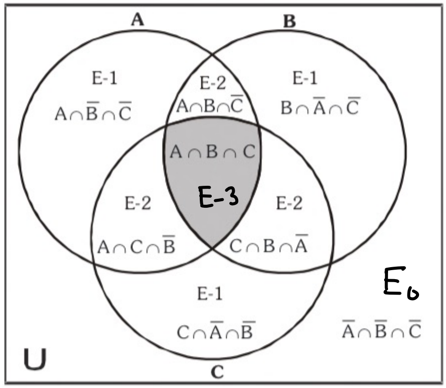
$P(A) + P(B) + P(C) - 2[P(A \cap B) + P(B \cap C) + P(C \cap A) + 3P(A \cap B \cap C)$

(iii) P(exactly two of occurring) = P(E – 2)

$P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$

(iv) P(at least two occurring) = P(E – 2) + P(E – 3)

$P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$



E-1 $\rightarrow A \cap \bar{B} \cap \bar{C} + \bar{A} \cap B \cap \bar{C} + \bar{A} \cap \bar{B} \cap C$

E-2 $\rightarrow A \cap B \cap \bar{C} + A \cap \bar{B} \cap C + \bar{A} \cap B \cap C$

E-3 $\rightarrow A \cap B \cap C$

$E_0 \rightarrow \bar{A} \cap \bar{B} \cap \bar{C}$ (None of them occurs).

$P(\bar{A} \cap \bar{B} \cap \bar{C}) = 1 - P(A \cup B \cup C)$

Imp

Note :

(i) If A,B,C are pairwise independent $\Rightarrow P(A \cap B) = P(A).P(B)$, $P(B \cap C) = P(B).P(C)$, $P(C \cap A) = P(C).P(A)$

(ii) However, if A, B, C are pair wise independent $\not\Rightarrow$ they are independent. Infact for 3 events A, B and C to be independent :

$$P(A \cap B) = P(A) . P(B); P(B \cap C) = P(B) . P(C) ; P(C \cap A) = P(C) . P(A)$$ i.e Pairwise

and $$P(A \cap B \cap C) = P(A) . P(B) . P(C)$$ i.e mutually.

* Similarly for n independent events, the total number of conditions would be

$$\boxed{{}^nC_2 + {}^nC_3 + \ldots\ldots + {}^nC_n = 2^n - n - 1}$$

Q A fair coin is tossed two times resulting in an equiprobable space {HT, TH, TT, HH}. Three events are defined as

A : Head occuring on 1st toss {HT, HH} $\Rightarrow P(A) = 1/2$

B : Head occuring on 2nd toss {TH, HH} $\Rightarrow P(B) = 1/2$

C : Head occuring exactly on one toss {HT, TH} $\Rightarrow P(C) = 1/2$

(i) Check whether the events are pair wise independent ? Yes

(ii) Check whether the events are independent ? No

(i) $P(AB) = P(A \cap B) = \frac{1}{4} = P(A).P(B)$ ✓

$P(BC) = P(B \cap C) = \frac{1}{4} = P(B).P(C)$ ✓

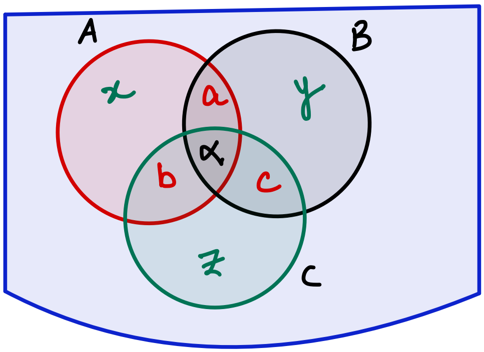
$P(CA) = P(C \cap A) = \frac{1}{4} = P(C).P(A)$ ✓

$P(A \cap B \cap C) = 0 \neq P(A).P(B).P(C)$

Q There are 3 clubs A, B and C in a town with 40, 50 and 60 members respectively. 10 people are members of all the three clubs, 70 are members in only one club. A member is randomly selected. Find the probability that he had membership of exactly two clubs.

(A) $\frac{7}{15}$ (B) $\frac{1}{6}$ (C) $\frac{3}{21}$ (D) $\frac{5}{21}$

Sol^n



$n(A) = 40 = x + a + \alpha + b.$

$n(B) = 50 = a + \alpha + c + y.$

$n(C) = 60. = b + \alpha + c + z.$

$n(A \cap B \cap C) = 10. = \alpha.$

$x + y + z = 70.$

$$\left.\begin{aligned} x + a + b &= 30. \\ a + c + y &= 40. \\ b + c + z &= 50. \end{aligned}\right\}$$

$2(a+b+c) + \underbrace{x+y+z}_{70} = 120.$

$\boxed{a+b+c = 25.}$

$$\text{Req. prob} = \frac{a+b+c}{\text{Total.}}$$

$$= \frac{25}{105} = \frac{5}{21.}$$

$\text{Total} = (x+y+z) + (a+b+c) + \alpha$

$= 70 + 25 + 10 = 105.$

(D)

Q For 3 events A, B and C

P(exactly one of the events A or B occurs)
P(exactly one of the events B or C occurs)
P(exactly one of the events C or A occurs) $\Bigg\} = p$

P(all the 3 events occurs simultaneously) $= p^2$

If A, B, C are exhaustive then find the value(s) of p.

Solⁿ

$$\left.\begin{aligned} P(A) + P(B) - 2P(A\cap B) &= p. \\ P(B) + P(C) - 2P(B\cap C) &= p. \\ P(C) + P(A) - 2P(C\cap A) &= p. \end{aligned}\right\} \quad \text{Add: } 2\Sigma P(A) - 2\Sigma P(A\cap B) = 3p$$

$$P(A\cap B\cap C) = p^2.$$

A, B, C → exhaustive. $\Rightarrow$ $P(A\cup B\cup C) = 1.$

$$\underbrace{\Sigma P(A) - \Sigma P(A\cap B)}_{} + P(A\cap B\cap C) = 1.$$

$$\frac{3p}{2} + p^2 = 1.$$

$$2p^2 + 3p - 2 = 0$$

$$2p^2 + 4p - p - 2 = 0$$

$$(2p-1)(p+2) = 0$$

$$\boxed{p = \frac{1}{2}} \text{ or } p = -2 \quad \times\times$$

Q If A,B,C are independent events such that P(A) = p, P(B) = q, P(C) = r where p,q,r (0,1). Let

E_1: only A occurs

E_2: none of A,B,C occurs

E_3: only event C occurs.

If probabilities of events E_1, E_2, E_3 are in GP. Then find value of p+r ?

Sol^n

$$P(E_1) = P(A \cap \bar{B} \cap \bar{C}) = P(A) \cdot P(\bar{B}) \cdot P(\bar{C})$$
$$= p(1-q)(1-r).$$

$$P(E_2) = P(\bar{A} \cap \bar{B} \cap \bar{C}) = (1-p)(1-q)(1-r).$$
$$P(E_3) = P(\bar{A} \cap \bar{B} \cap C) = (1-p)(1-q) \cdot r.$$

$$(P(E_2))^2 = P(E_1) \cdot P(E_3)$$

$$(1-p)^{\cancel{2}} \cancel{(1-q)}^{\cancel{2}} (1-r)^{\cancel{2}} = p \cdot \cancel{(1-q)} \cancel{(1-r)} \cdot \cancel{(1-p)} \cancel{(1-q)} \cdot r$$

$$(1-p)(1-r) = pr$$
$$1 - r - p + \cancel{pr} = \cancel{pr} \Rightarrow p + r = 1 \quad \text{Ans}$$

Q A purse contains 10 tickets, five printed with I and five printed with T. 3 tickets are drawn without replacement and arranged in the same order in which they are drawn on the table. Find the probability that IIT is formed.

Sol^n

$$P(IIT) = P(I) \cdot P(I/I) \cdot P(T/I \cap I)$$

$$= \left(\frac{5}{10}\right) \cdot \left(\frac{4}{9}\right)\left(\frac{5}{8}\right)$$

Q A problem in mathematics is given to 2 children who solve it independently. If probability of A solving it is 1/2 and probability of B solving it is 2/3. Find the probability that the problem is solved.

Sol^n

$$P(\text{prob is solved}) = 1 - P(\bar{A} \cap \bar{B})$$

$$= 1 - P(\bar{A}) \cdot P(\bar{B})$$

$$= 1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$$

$$= 1 - \frac{1}{6} = \frac{5}{6} \text{ Ans}$$

Q Probability that a teacher takes a surprise test is 1/3. If a student remains absent for two days then find the probability that he misses

(i) exactly one test (ii) at least one test (iii) atmost one test

Sol^n

$$S: \{TT,\ T\bar{T},\ \bar{T}T,\ \bar{T}\bar{T}\}$$

(i) $P(T\bar{T}) + P(\bar{T}T) = \frac{1}{3}\cdot\frac{2}{3} + \frac{2}{3}\cdot\frac{1}{3} = \frac{4}{9}.$

(ii) $P(T\bar{T}) + P(\bar{T}T) + P(TT)$

$$\frac{1}{3}\cdot\frac{2}{3} + \frac{2}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{3} = \frac{5}{9}.$$

OR

$$1 - P(\bar{T}\bar{T}) = 1 - \frac{2}{3}\cdot\frac{2}{3} = \frac{5}{9}.$$

(iii) $P(T\bar{T}) + P(\bar{T}T) + P(\bar{T}\bar{T})$

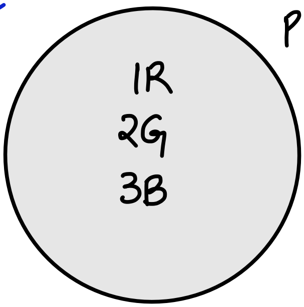
$$\frac{1}{3}\cdot\frac{2}{3} + \frac{2}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{2}{3} = \frac{8}{9}$$

OR

$$1 - P(TT) = 1 - \frac{1}{3}\cdot\frac{1}{3} = \frac{8}{9}.$$

Q An urn contains 1R, 2G and 3B balls. Three people A, B & C in order draw one ball from the urn and put it back after noting its colour. They continue doing it indefinitely unless one who draws a red ball first wins the game. Compute their respective chances of winning the game.

Sol^n



$$P(\underline{A}\text{ wins}) = P(R \text{ or } \bar{R}\bar{R}\bar{R}R \text{ or } \bar{R}\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}R \text{ or } \ldots)$$

$$= P(R) + P(\bar{R}\bar{R}\bar{R}R) + P(\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}R) + \ldots$$

$$= \frac{1}{6} + \left(\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\right)\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^6\cdot\frac{1}{6} + \ldots$$

$$= \frac{1}{6}\left(\frac{1}{1-\frac{125}{216}}\right) = \frac{36}{91}$$

$$P(B\text{ wins}) = P(\bar{R}R \text{ or } \bar{R}\bar{R}\bar{R}\bar{R}R \text{ or} \ldots)$$

$$= P(\bar{R})P(R) + (P(\bar{R}))^4 P(R) + \ldots$$

$$= \frac{5}{6}\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^4\cdot\frac{1}{6} + \ldots$$

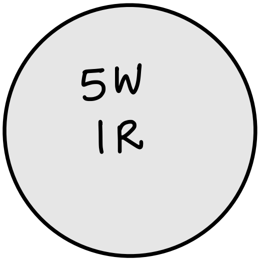
$$= \frac{5}{6}\cdot\frac{1}{6}\left(\frac{1}{1-\frac{125}{216}}\right) = \frac{30}{91}.$$

$$P(C\text{ wins}) = 1 - \left(\frac{66}{91}\right) = \frac{25}{91}.$$

Q. Two persons A and B one by one in order drawn one ball each from a purse containing 5W and 1R balls and retain it. The person who gets a red ball wins the game.

E : event that 'A' wins and F : event that 'B' wins . Compute E and F.

Solⁿ



$$P(E) = P(R \text{ or } \bar{R}\bar{R}R \text{ or } \bar{R}\bar{R}\bar{R}\bar{R}R)$$

$$= P(R) + P(\bar{R}\bar{R}R) + P(\bar{R}\bar{R}\bar{R}\bar{R}R)$$

$$= \frac{1}{6} + \left(\frac{\cancel{5}}{6} \cdot \frac{\cancel{4}}{\cancel{5}} \cdot \frac{1}{\cancel{4}}\right) + \left(\frac{\cancel{5}}{6} \cdot \frac{\cancel{4}}{\cancel{5}} \cdot \frac{\cancel{3}}{\cancel{4}} \cdot \frac{\cancel{2}}{\cancel{3}} \cdot \frac{1}{\cancel{2}}\right)$$

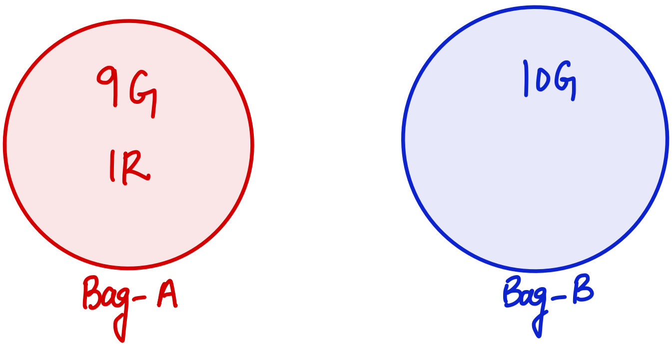
$$P(E) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

$$P(F) = \frac{1}{2}.$$

HW

Q. Ram & Shyam toss a coin in turn indefinitely untill head occur for the first time. The person getting the head for the first time wins. If Ram start tossing, then find their respective chance of winning.

Q Bag A has 9G and 1R balls & Bag B has 10G balls and no red ball. 9 balls are transferred from Bag A to Bag B & then 9 balls are transfered from Bag B to Bag A. After these pair of operation are performed find the probability that red ball is still in Bag A.



$$\left(\frac{{}^{9}C_{9}}{{}^{10}C_{9}}\right)\cdot\left(\frac{{}^{19}C_{9}}{{}^{19}C_{9}}\right)$$

$$+\left(\frac{{}^{9}C_{8}\,{}^{1}C_{1}}{{}^{10}C_{9}}\right)\left(\frac{{}^{18}C_{8}\cdot{}^{1}C_{1}}{{}^{19}C_{9}}\right)$$

Q A fair coin whose faces are marked with one & two is thrown for four times. Find the probability of throwing a total of : (a) 6 (b) atleast 5 (c) atmost 7

Sol^n

(a) $\underbrace{1, 1, 2, 2}_{}$ $\rightarrow \left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) = \frac{1}{16}$

6 cases

$\frac{4!}{2!\,2!} = 6$

Prob of total '6' $= 6\times\left(\frac{1}{16}\right)$.

(b) 4, 5, 6, 7, 8

1, 1, 1, 1

$P(\text{atleast } 5) = 1 - P(\text{getting sum} = 4)$

$= 1 - \left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) = 1 - \frac{1}{16} = \frac{15}{16}$.

(c) $\rightarrow 1 - P(\text{sum} = 8)$

$= \frac{15}{16}$.

Q HW = A coin is continued tossing until either a head and a tail is obtained for the first time or unless the coin has been tossed for a maximum of five times. If the first two tosses have resulted in both tails, find the chance that the coin will be tossed 5 times.

Q Ram & Shyam toss a coin in turn indefinitely untill head occur for the first time. The person getting the head for the first time wins. If Ram start tossing, then find their respective chance of winning.

Sol^n

$$P(\text{Ram wins}) = P(H \text{ or } \bar{H}\bar{H}H \text{ or } \bar{H}\bar{H}\bar{H}\bar{H}H \text{ or} \cdots)$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \cdots\cdots$$

$$= \frac{1/2}{1-(1/2)^2} = \frac{1/2}{3/4} = \frac{2}{3}.$$

$$\therefore P(\text{shyam wins}) = 1 - \frac{2}{3} = \frac{1}{3}.$$

Q. A coin is continued tossing until either a head and a tail is obtained for the first time or unless the coin has been tossed for a maximum of five times. If the first two tosses have resulted in both tails, find the chance that the coin will be tossed 5 times.

Sol^n

$$\underbrace{T\ T}_{\text{occurred}}\ \ \underline{T}\ \ \underline{T}\ \ \underline{\checkmark}\ \text{H or T}$$

$$\text{Req. prob} = \frac{1}{2}\cdot\frac{1}{2}\cdot(1) = \frac{1}{4}.$$

Q All face cards from a pack of 52 playing cards are removed. From the remaining 40 cards, 4 are drawn.

(i) Find the probability that they are of different suit different denominations.

(ii) Same suit & diff denominations.

(iii) diff suit & same "

Solⁿ

$52 - 12 = 40$ Cards.

Suit

Spade Club Diamond Heart

(i) $$\frac{{}^{10}C_1 \cdot {}^{9}C_1 \cdot {}^{8}C_1 \cdot {}^{7}C_1}{{}^{40}C_4}$$

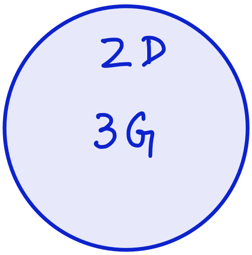
(ii) $$\frac{{}^{4}C_1 \cdot {}^{10}C_4}{{}^{40}C_4}$$

(iii) $$\frac{{}^{10}C_1 \cdot 1}{{}^{40}C_4}.$$

Q A box contains 5 tubes, 2 of them defective and 3 good one. Tubes are tested by one-by-one till the 2 defective tubes are discovered. What is the probability that the testing procedure comes to an end at the end of

(a) second testing (b) 3rd testing

Sol^n



(a) $\underline{D}\ \underline{D}$

$$P(D_1 \cap D_2) = P(D_1) \cdot P(D_2/D_1)$$

$$= \frac{{}^2C_1}{{}^5C_1} \cdot \frac{{}^1C_1}{{}^4C_1} = \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10}.$$

(b) DGD, GDD, GGG

$$P(DGD) + P(GDD) + P(GGG)$$

$$= \left(\frac{2}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3}\right)$$

$$= \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}.$$

Q Cards are dealt one by one from a well shuffled pack of 52 cards. (i) Find the probability that exactly 4 cards are dealt before the first ace appears. (ii) Find the probability that exactly 4 cards are dealt in all before the second ace.

Sol^n

(i) $$\left(\frac{48}{52}\right)\left(\frac{47}{51}\right)\left(\frac{46}{50}\right)\left(\frac{45}{49}\right)\left(\frac{4}{48}\right)$$

(ii) $$\underbrace{\_\ \_\ \_\ \_}_{\substack{1\text{ Ace}\\ 3\text{ None ace}}}\qquad \underline{\text{Ace}}$$

$$\underbrace{\_\ \_\ \_}\quad \underline{\text{Ace}_1}\qquad \text{Ace}_2$$

$$4.\left(\frac{48}{52}\cdot\frac{47}{51}\cdot\frac{46}{50}\cdot\frac{4}{49}\right)\cdot\frac{3}{48}$$

Q HW A pair of dice is rolled until a total of 5 or 7 is obtained. Find the probability that the total of 5 comes before a total of 7.

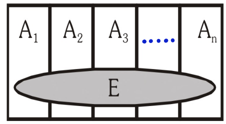
V. Imp

LAW OF TOTAL PROBABILITY :

Suppose S is the sample space of an experiment & $A_1, A_2, A_3 \ldots\ldots\ldots A_n$ are n mutually exclusive & exhaustive events defined on the experiment. Let E be another event of this experiment as shown in figure then probability of occurrence of E i.e. $\boxed{P(E) = P(E \cap A_1) + P(E \cap A_2) + \cdots + P(E \cap A_n)}$.

$$P(E) = P(A_1)\,P(E/A_1) + P(A_2)\,P(E/A_2) + \ldots\ldots\ldots + P(A_n)\,P(E/A_n)$$

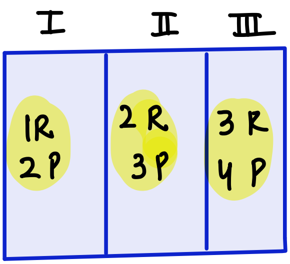
$$= \sum_{k=1}^{n} P(A_k)P(E/A_k)$$

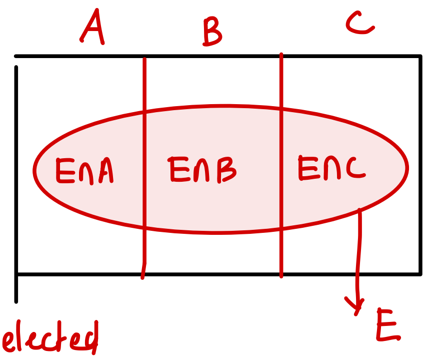


- $A_i \cap A_j = \phi$
- $A_1 \cup A_2 \cup A_3 \ldots \cup A_n = S$

Q. A lady has 3 compartments in her purse. I[st] compartment contains 1Rupee & 2 Paise coins, II[nd] compartment contains 2 Rupee & 3 Paise coins, & IIIrd compartment contain 3 Rupee & 4 Paise coins. She randomly selects a compartment to draw a coin. What is probability that the drawn coin is a rupee coin.

Sol[n]





A : Compartment I is selected

B : " II

C : " III

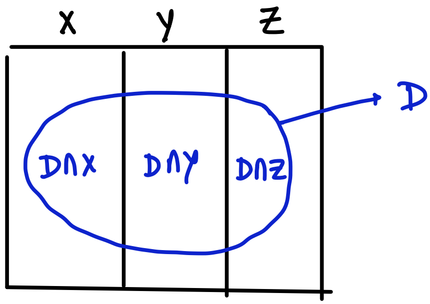
E : She draws a rupee coin.

$P(E) = P(E \cap A) + P(E \cap B) + P(E \cap C) = P(A)\cdot P(E/A) + P(B)\cdot P(E/B) + P(C)\cdot P(E/C)$

$= \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{5} + \frac{1}{3} \times \frac{3}{7}$.

Q In a company there are 3 machine x,y & z. Machine x produces 50% of total production of which 3% are defective. Machine y produces 30% of total production of which 4% are defective and machine z produces 20% of total production of which 5% is defective. Out of total production lot 1 item is randomly selected. What is the probability that the selected item is defective ?

Sol^n



$$P(D) = P(D \cap X) + P(D \cap Y) + P(D \cap Z)$$

$$P(\text{Defective}) = P(X) \cdot P(D/X) + P(Y) \cdot P(D/Y) + P(Z) \cdot P(D/Z)$$

$$= \left(\frac{50}{100}\right) \cdot \left(\frac{3}{100}\right) + \left(\frac{30}{100}\right)\left(\frac{4}{100}\right) + \left(\frac{20}{100}\right)\left(\frac{5}{100}\right)$$

V Imp

BAYE'S THEOREM :

Here, with the knowledge of present, we predict the past.

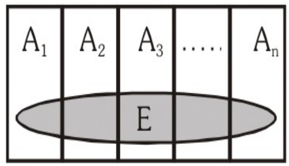
If an event E can occur only with one of the n mutually exclusive and exhaustive events $A_1, A_2, \ldots. A_n$ & if the conditional probabilities of the events $P(E/A_1)$, $P(E/A_2)$ .... $P(E/A_n)$ are known then,

$$\boxed{P(A_1/E) = \frac{P(A_i).P(E/A_i)}{\sum_{i=1}^{n} P(A_i).P(E/A_i)}}$$

Rem

Proof :

The events E occurs with one of the n mutually exclusive & exhaustive events $A_1, A_2, A_3, \ldots\ldots.A_n$



$E = EA_1 + EA_2 + EA_3 + \ldots\ldots. + EA_n$

$$P(E) = P(EA_1) + P(EA_2) + \ldots\ldots.+ P(EA_n) = \sum_{i=1}^{n} P(EA_i)$$

Note : $E \equiv$ event what we have

$A_1 \equiv$ event what we want ;

$A_2, A_3, \ldots.A_n$ are alternative event.

Now,

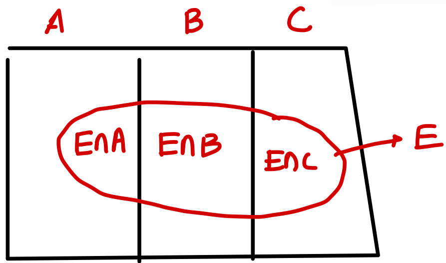
$P(EA_i) = P(E) . P(A_i/E) = P(A_i) . P(E/A_i)$

$$P(A_i/E) = \frac{P(A_i).P(E/A_i)}{P(E)} = \frac{P(A_i).P(E/A_i)}{\sum_{i=1}^{n} P(EA_i)}; \quad P(A_i/E) = \frac{P(A_i).P(E/A_i)}{\sum P(A_i).P(E/A_i)}$$

Q A lady has 10 coins in her purse, 8 of them are normal coins, one of them is a DH and one is a DT coin. She randomly draws a coin and tosses it for 5 times. The coin is found to fall headwise on all the 5 occasions. Find the probability that the drawn coin was a DH coin.

Soln

$$10 \begin{cases} 8\ N \\ 1\ DH \\ 1\ DT \end{cases}$$



A : Normal coin selected ; $P(A) = 8/10$

B : DH " " ; $P(B) = 1/10$

C : DT " " ; $P(C) = 1/10$

E : Coin tossed 5 times fell headwise

$$\text{Req. prob} = \frac{P(E \cap B)}{P(E)}$$

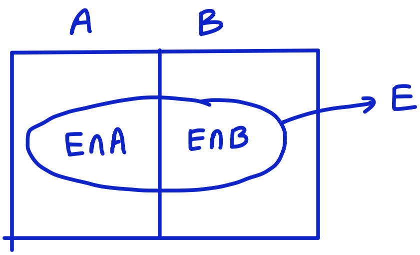
$$= \frac{P(B) \cdot P(E/B)}{P(A) \cdot P(E/A) + P(B) \cdot P(E/B) + P(C) \cdot P(E/C)}$$

$$P(B/E) = \frac{\frac{1}{10} \times 1}{\frac{8}{10} \times \left(\frac{1}{2}\right)^5 + \frac{1}{10} \times 1 + \frac{1}{10} \times (0)}$$

Q The contents of the urn I and urn II are as follows
One urn is chosen at random and a ball is drawn and its colour noted and replaced back in the same urn. Again a ball is drawn from the same urn, colour noted and replaced. The process is repeated 4 times and as a result one ball of white colour and 3 of black colour are noted. What is the probability that the chosen urn was I.

Urn	W	B
I	4	5
II	3	6

Solⁿ



A : Urn I is selected
B : " II " "

E : 1 W & 3 B balls were observed.

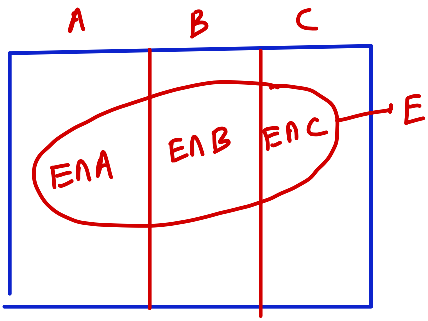
W B B B

$$\text{Req. prob} = P(A/E) = \frac{P(A)\cdot P(E/A)}{P(A)\, P(E/A) + P(B)\cdot P(E/B)}$$

$$= \frac{\cancel{\frac{1}{2}} \times \left(\frac{4}{9}\cdot\frac{5}{9}\cdot\frac{5}{9}\cdot\frac{5}{9}\right) \times \cancel{4}}{\cancel{\frac{1}{2}}\left(\frac{4}{9}\cdot\frac{5}{9}\cdot\frac{5}{9}\cdot\frac{5}{9}\right)\times\cancel{4} + \cancel{\frac{1}{2}}\left(\frac{3}{9}\cdot\frac{6}{9}\cdot\frac{6}{9}\cdot\frac{6}{9}\right)\times\cancel{4}}$$

Q In a test an examine either guesses or copies or knows the answer to a multiple choice question with 4 choices (one or more than one correct). The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it, is 1/8. Find the probability that he knows the answer to the question given that he correctly answered it.

Sol^n



A : He knows $\qquad P(A) = 1/2$

B : He copies $\qquad P(B) = 1/6$

C : He guesses. $\qquad P(C) = 1/3$

E : He answered correctly

$$P(A/E) = \frac{P(E\cap A)}{P(E\cap A) + P(E\cap B) + P(E\cap C)} = \frac{P(A)\cdot P(E/A)}{P(A)\,P(E/A) + P(B)\,P(E/B) + P(C)\cdot P(E/C)}$$

$$= \frac{\frac{1}{2}\times 1}{\frac{1}{2}\times 1 + \frac{1}{6}\times\frac{1}{8} + \frac{1}{3}\times\frac{1}{15}}.$$

$$\underbrace{{}^4C_1} + \underbrace{{}^4C_2} + \underbrace{{}^4C_3} + \underbrace{{}^4C_4} = 15 \checkmark$$

(A)

(B)

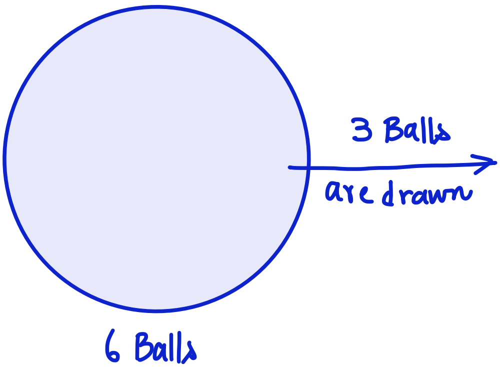
(C)

(D)

HW

Q. 'A' writes a latter to his friend and given it to his son to post it in a letter box, the reliability of his son being 3/4. The probability that a letter posted will get delivered is 8/9. At a later date 'A' hears from 'B' that the latter has not reached him. Find the probability that the son did not post the latter at all.

Q A bag contains 6 balls and it is not known of what colours they are. 3 balls are drawn from the bag and found to be all black. Find the probability that no black balls are left in the bag now. (Assume all numbers of black balls in the bag to be equally likely)



B	Non-Black	
0	6 →	$1/7$
1	5 →	"
2	4 →	"
3	3 →	"
4	2 →	"
5	1 →	"
6	0 →	"

$$\text{Req. prob} = \frac{\frac{1}{7} \times \frac{{}^3C_3}{{}^6C_3}}{\frac{1}{7}\times 0 + \frac{1}{7}\times 0 + \frac{1}{7}\times 0 + \frac{1}{7}\left(\frac{{}^3C_3}{{}^6C_3}\right) + \frac{1}{7}\left(\frac{{}^4C_3}{{}^6C_3}\right) + \frac{1}{7}\left(\frac{{}^5C_3}{{}^6C_3}\right) + \frac{1}{7}\left(\frac{{}^6C_3}{{}^6C_3}\right)}$$

★★★

EXTENDED BAYES :

Here, with the knowledge of present, first we predict the past and then the future.

HW

Q A bag contains 3 biased coins B_1, B_2 and B_3 whose probabilities of falling head wise are 1/3, 2/3 and 3/4 respectively. A coin is drawn randomly and tossed, fell head wise. Find the probabilities that the same coin when tossed again will fall head wise.

Q HW A bag contains 5 balls of unknown colours. A ball is drawn twice with replacement from the bag found to be red on both the occasions. The contents of the bag were replenished. If now two ball are drawn simultaneously from the bag, find the probabilities that they will be both red. Assume all number of red balls in the bag to be equally likely.

BINOMIAL PROBABILITY DISTRIBUTION : (BPD)

Suppose that we have an experiment such as tossing a coin or die repeatedly or choosing a marble from an urn repeatedly. Each toss or selection is called a trial. In any single trial there will be a probability associated with a particular event such as head on the coin, 4 on the die, or selection of a red marble. In some cases this probability will not change from one trial to the next (as in tossing a coin or die). Such trials are then said to be independent and are often called Bernoulli trials after James Bernoulli who investigated them at the end of the seventeenth century.

Definition : Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

(i) There should be a finite number of trials.

(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes : success or failure.

(iv) The probability of success remains the same in each trial.

For Independent Trials

Experiment

↓

n independent trials (Bernoulli's trials)

↓

If each of these trails have two possible outcomes

Success		Failure
$P(S) = p$	$p + q = 1$	$P(F) = q$

$$\boxed{\text{Then, } P(r \text{ successes}) = {}^{n}C_r\, p^r q^{n-r}}$$

Proof : Consider the compound event where 'r' successes are in succession and (n – r) failures are in succession.

$$P\left(\underbrace{SSS....S}_{r}\underbrace{FFF.....F}_{(n-r)}\right) = \underbrace{P(S).P(S)....P(S)}_{r\text{ times}}\underbrace{P(F).P(F).....P(F)}_{(n-r)\text{ times}} = p^r.q^{n-r}$$

But these r successes and (n – r) failures can be arranged in $\frac{n!}{r!(n-r)!} = {}^{n}C_r$ ways and in each arrangement the probability will be $p^r \cdot q^{n-r}$

Hence total pr. $= P(r) = {}^{n}C_r p^r q^{n-r}$ .... (i)

Recurrence relation

$$P(r+1) = {}^{n}C_{r+1} p^{r+1} \cdot q^{n-r-1}$$

$$\therefore \quad \frac{P(r+1)}{P(r)} = \frac{{}^{n}C_{r+1}}{{}^{n}C_r}\frac{p}{q} = \frac{n-r}{r+1}\frac{p}{1-p}$$

$$\therefore \quad \boxed{P(r+1) = \frac{n-r}{r+1}\cdot\frac{p}{1-p}P(r)} \quad ...... \text{(ii)}$$

equation (ii) is used for completely the probabilities of P(1) ; P(2) ; P(3); .... etc. once P(0) is determined.

Q A pair of dice is thrown 6 times, getting a doublet is considered a success. Compute the probability of
(i) no success (ii) exactly one success (iii) at least one success (iv) at most one success

Sol^n

$$P(S) = 1/6$$
$$P(F) = 5/6.$$

(i) $${}^6C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^6$$

(ii) $${}^6C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^5$$

(iii) $$P(S=1) + P(S=2) + \cdots + P(S=6) = 1 - P(S=0)$$
$$= 1 - \left(\frac{5}{6}\right)^6$$

(iv) $$P(\text{one success}) + P(\text{0 success})$$
$${}^6C_1 \cdot \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^5 + \left(\frac{5}{6}\right)^6.$$

HW

Q A coin is twice as likely to land heads as tails. In a sequence of independent tosses, find the probability that the third head occurs on the fifth toss.

Q HW A pair of dice is rolled until a total of 5 or 7 is obtained. Find the probability that the total of 5 comes before a total of 7.

Ans $\rightarrow \frac{2}{5}$.

Sol^n

E: Sum on pair of dice $= 5$; $P(E) = 4/36$.

F: " " " " " $= 7$; $P(F) = 6/36$.

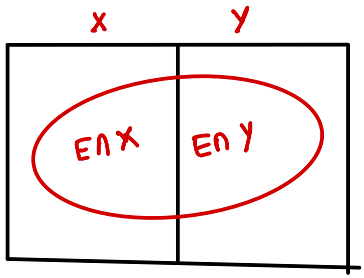
$$P\left(E \text{ or } (\bar{E} \cap \bar{F}) E \text{ or } (\bar{E} \cap \bar{F})^2 E \text{ or } \cdots\cdots\right) = \text{Req. prob.}$$

$$P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$$
$$= 1 - \left(P(E) + P(F) - \cancel{P(E \cap F)}^{\,0}\right)$$
$$= 1 - \frac{10}{36} = \frac{26}{36}.$$

$$P(E) + P(\bar{E} \cap \bar{F})\,P(E) + \left(P(\bar{E} \cap \bar{F})\right)^2 P(E) + \cdots = \frac{2}{5} \text{ Ans}$$

Q 'A' writes a letter to his friend and gives it to his son to post it in a letter box, the reliability of his son being 3/4. The probability that a letter posted will get delivered is 8/9. At a later date 'A' hears from 'B' that the letter has not reached him. Find the probability that the son did not post the letter at all.

Sol^n



X : Son post the letter

Y : Son did not post the letter

$P(X) = \frac{3}{4}$

$P(Y) = \frac{1}{4}$

E : Letter did not reach destination

$$P(Y/E) = \frac{P(E\cap Y)}{P(E\cap X) + P(E\cap Y)} = \frac{P(Y)\cdot P(E/Y)}{P(X)\,P(E/X) + P(Y)\cdot P(E/Y)}$$

$$= \frac{\frac{1}{\cancel{4}}\times 1}{\frac{3}{\cancel{4}}\times\frac{1}{9} + \frac{1}{\cancel{4}}\times 1}$$

$$= \frac{1}{\frac{1}{3}+1} = \frac{3}{4} \text{ Ans}$$

EXTENDED BAYES :

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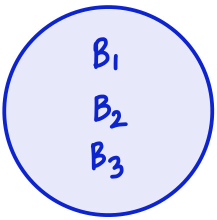
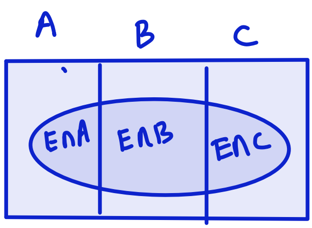
Q A bag contains 3 biased coins B_1, B_2 and B_3 whose probabilities of falling head wise are 1/3, 2/3 and 3/4 respectively. A coin is drawn randomly and tossed, fell head wise. Find the probabilities that the same coin when tossed again will fall head wise.

Sol^n





$$P(A/E) = \frac{\frac{1}{3} \times \frac{1}{3}}{\frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{3}{4}}$$

$$P(B/E) = \frac{1/3 \times 2/3}{1/3 \times 1/3 + 1/3 \times 2/3 + 1/3 \times 3/4}$$

$$P(C/E) = \frac{\frac{1}{3} \times \frac{3}{4}}{\frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{3}{4}}$$

A : Coin B_1 selected
B : " B_2 "
C : " B_3 "
E : Coin tossed fell headwise

$$\text{Req. prob} = P(A/E) \times \frac{1}{3} + P(B/E) \times \frac{2}{3} + P(C/E) \times \frac{3}{4}$$

$$= \frac{23}{36} \text{ Ans}$$

Q A bag contains 5 balls of unknown colours. A ball is drawn twice with replacement from the bag found to be red on both the occasions. The contents of the bag were replenished. If now two ball are drawn simultaneously from the bag, find the probabilities that they will be both red. Assume all number of red balls in the bag to be equally likely.

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Experiment

↓

n independent trials (Bernoulli's trials)

↓

If each of these trails have two possible outcomes

Success	$p + q = 1$	Failure
$P(S) = p$		$P(F) = q$

Then, $P(r \text{ successes}) = {}^nC_r\, p^r q^{n-r}$

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But these r successes and (n – r) failures can be arranged in $\dfrac{n!}{r!(n-r)!} = {}^nC_r$ ways and in each arrangement the probability will be $p^r \,.\, q^{n-r}$

Hence total pr. $= P(r) = {}^nC_r p^r q^{n-r}$ .... (i)

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$$P(r + 1) = {}^nC_{r+1}p^{r+1} \,.\, q^{n-r-1}$$

$$\therefore \quad \frac{P(r+1)}{P(r)} = \frac{{}^nC_{r+1}}{{}^nC_r}\frac{p}{q} = \frac{n-r}{r+1}\frac{p}{1-p}$$

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equation (ii) is used for completely the probabilities of P(1) ; P(2) ; P(3); .... etc. once P(0) is determined.

$$\underbrace{(p+q)}_{1}{}^{n} = \underbrace{{}^{n}C_0\, p^{n} q^{0}}_{\substack{n \text{ - success} \\ 0 \text{ - Failure} \\ P(x=n)}} + \overbrace{{}^{n}C_1 \cdot p^{n-1} q^{1}}^{\substack{P(x=n-1) \\ n-1 \to S \\ 1 \to F}} + {}^{n}C_2\, p^{n-2} q^{2} + \cdots$$

$$\cdots + \underbrace{{}^{n}C_r\, p^{n-r} q^{r}}_{\substack{n-r \to \text{success} \\ r \to \text{Failure}}} + \cdots + \underbrace{{}^{n}C_n\, p^{0} q^{n}}_{\substack{0 \to \text{success} \\ n \to \text{Failure} \\ P(x=0)}}.$$

Q A coin is twice as likely to land heads as tails. In a sequence of independent tosses, find the probability that the third head occurs on the fifth toss.

Sol^n

$P(H) = 2x$

$P(T) = x$

$P(H) + P(T) = 1$

$x = 1/3$

$P(H) = 2/3$

$P(T) = 1/3$.

$\underbrace{\_\ \_\ \_\ \_}_{\substack{\text{2 Head.} \\ \text{2 Tail.}}}\ \ \boxed{\text{Head}} \rightarrow 3^{rd}$ time

$$\left({}^4C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2 \right) \times \left(\frac{2}{3}\right)$$

Q A drunkard takes a step forward or backward. The probability that he takes a step forward is 0.4. Find the probability that at the end of 11 steps he is one step away from the starting point.

Solⁿ

Step Fwd = 0.4 = success
Step Backward = 0.6 = Failure

⊛ — OR — ⊛

Starting point

6 step Fwd
5 " Bck

6 step Backward
5 " Fwd

$${}^{11}C_6 (0.4)^6 (0.6)^5 + {}^{11}C_5 (0.4)^5 (0.6)^6$$

Comprehension : [Q.1 & Q.2]

Research has shown that studying improves a student's chances to 80% of selecting the correct answer to a multiple choice question. A multiple choice test has 15 questions. Each question has 4 choices and exactly one is correct.

(1) Probability that the student does exactly 7 or 8 correct answer when he studies and attempts all the questions.

$${}^{15}C_7 \left(\frac{4}{5}\right)^7 \left(\frac{1}{5}\right)^8 + {}^{15}C_8 \left(\frac{4}{5}\right)^8 \left(\frac{1}{5}\right)^7$$

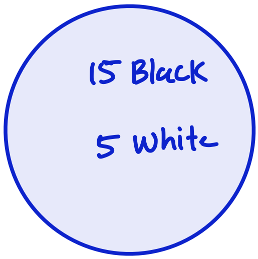
(2) Suppose that a student does not study for the test but randomly guesses the answers. The probability that the student will answer 7 or 8 questions correctly when he attempts all, is

$${}^{15}C_7 \left(\frac{1}{4}\right)^7 \left(\frac{3}{4}\right)^8 + {}^{15}C_8 \left(\frac{1}{4}\right)^8 \left(\frac{3}{4}\right)^7$$

Q. An urn contains 15 black balls and 5 white balls. 10 balls are drawn one by one with replacement. Then find the probability of getting exactly 5 black balls.

Success : White Ball $\longrightarrow P(S) = \frac{5}{20} = \frac{1}{4}$

Failure : Black Ball $\longrightarrow P(F) = 3/4$



$$^{10}C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^5 .$$

* fair

Q. Find the minimums number of times a fair coin is tossed such that probability of getting at least 2 heads is greater than 0.5 ?

n times tossed.

$$P(2H) + P(3H) + P(4H) + \cdots + P(n\,H) > 0.5$$

$$1 - P(0\,H) - P(1\,H) > 0.5$$

$$1 - \left({}^nC_0 \left(\tfrac{1}{2}\right)^0 \cdot \left(\tfrac{1}{2}\right)^n + {}^nC_1 \cdot \left(\tfrac{1}{2}\right)^1 \left(\tfrac{1}{2}\right)^{n-1} \right) > \frac{1}{2}$$

$$\frac{1}{2} > \frac{1}{2^n} + \frac{n}{2^n} \Rightarrow \frac{n+1}{2^n} < \frac{1}{2}$$

$n=1$ $\quad \frac{2}{2} < \frac{1}{2}$ ✗

$n=2$ $\quad \frac{3}{4} < \frac{1}{2}$ ✗

$n=3$ $\quad \frac{4}{8} < \frac{1}{2}$ ✗

$\boxed{n=4}$ $\quad \frac{5}{16} < \frac{1}{2}$ ✓

$\therefore$ minimum = 4.

PROBABILITY THROUGH STATISTICAL (STOCHASTIC) TREE DIAGRAM :

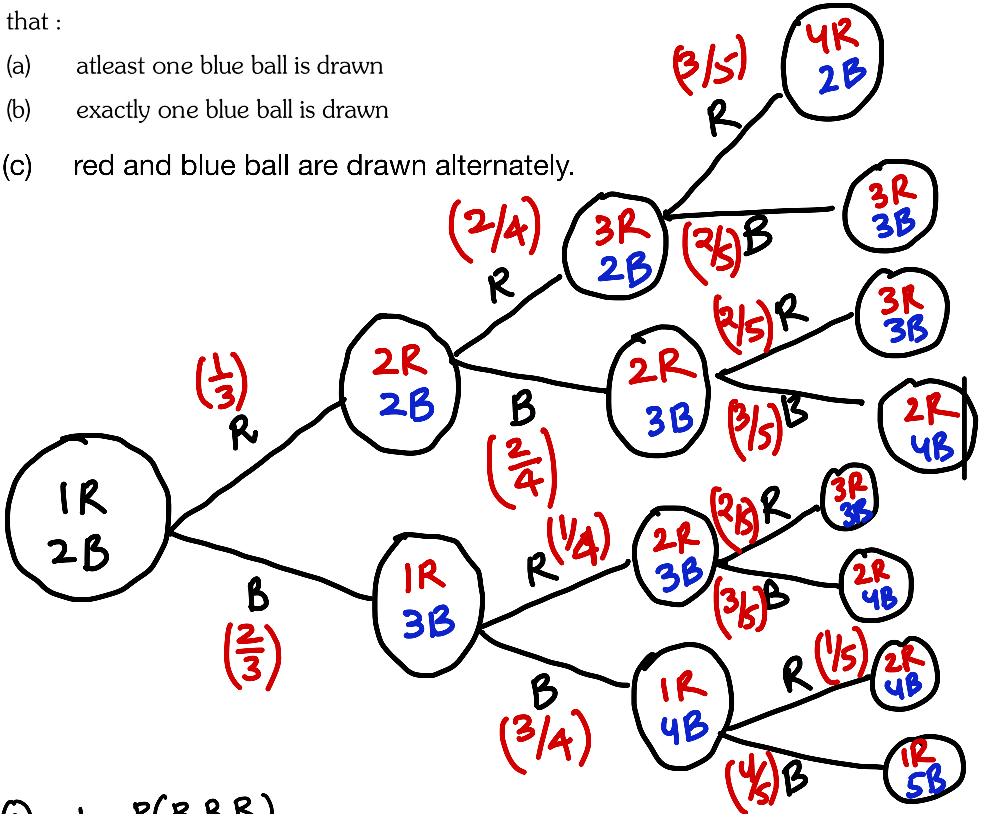
These tree diagrams are generally drawn by economist and give a simple approach to solve a problem.

E(1) A bag initially contains 1 red ball and 2 blue balls. A trial consists of selecting a ball at random, noting its colour and replacing it together with an additional ball of the same colour. Given that three trials are made, draw a tree diagram illustrating the various probabilities. Hence, or otherwise, find the probability that :

(a) atleast one blue ball is drawn

(b) exactly one blue ball is drawn

(c) red and blue ball are drawn alternately.



(a) $1 - P(RRR)$

$1 - \left(\frac{1}{3}\right)\left(\frac{2}{4}\right)\left(\frac{3}{5}\right)$

(b) $P(RRB) + P(RBR) + P(BRR)$

$\frac{1}{3}\cdot\frac{2}{4}\cdot\frac{2}{5} + \frac{1}{3}\cdot\frac{2}{4}\cdot\frac{2}{5} + \frac{2}{3}\cdot\frac{1}{4}\cdot\frac{2}{5}$

(c) $P(BRB) + P(RBR)$

$\frac{2}{3}\cdot\frac{1}{4}\cdot\frac{3}{5} + \frac{1}{3}\cdot\frac{2}{4}\cdot\frac{2}{5}$

E(2) Box A contains nine cards numbered 1 through 9 and box B contains five cards numbered I through 5. A box is chosen at random and a card drawn; if the card shows an even number, another card is drawn from the same box. If the card shows an odd number, a card is drawn from the other box ;

HW

(a) What is the probability that both cards show even numbers ?

(b) What is the probability that both cards show odd numbers ?

Probability Distribution :- (FOR JEE MAINS ONLY)

Rem **(a)** Mean of any probability distribution of a random variable (x_i) is given by :

$$\boxed{\mu = \sum p_i \, x_i} \quad (\text{Since } \Sigma p_i = 1)$$

Rem **(b)** Variance of a random variable is given by, $\sigma^2 = \sum (x_i - \mu)^2 \cdot p_i$

$$\boxed{\sigma^2 = \sum p_i x_i^2 - \mu^2} \qquad (\text{Note that } SD = +\sqrt{\sigma^2})$$

Rem **(c)** Standard deviation $= \sigma = \sqrt{\text{variance}}$

$$\sigma^2 = \Sigma (x_i - \mu)^2 p_i$$

$$= \Sigma (x_i^2 p_i + \mu^2 p_i - 2 x_i \mu p_i)$$

$$= \Sigma p_i x_i^2 + \mu^2 \Sigma p_i - 2\mu \Sigma x_i p_i$$

$$= \Sigma p_i x_i^2 + \mu^2 - 2\mu(\mu).$$

$$\boxed{\sigma^2 = \Sigma p_i x_i^2 - \mu^2.}$$

H.P

Binomial probability distribution :

If there are n Bernoulli trials & Probability of success = p

Probability of failure = q, then

(i) Rem Mean of BPD = np ; (ii) Rem variance of BPD = npq ; (iii) Rem $SD = +\sqrt{npq}$

Q A pair of fair dice is thrown. Let X be the random variable which denotes the minimum of the two numbers which appear. Find the probability distribution, mean and variance of X.

Sol

x_i	$P(x_i)$	$x_i P(x_i)$	x_i^2	$P(x_i) x_i^2$
1	11/36	11/36	1	11/36
2	9/36	18/36	4	36/36
3	7/36	21/36	9	63/36
4	5/36	20/36	16	80/36
5	3/36	15/36	25	75/36
6	1/36	6/36.	36	36/36
		$\mu = 91/36$		$\Sigma P(x_i) x_i^2 =$ [illegible]

$$\sigma^2 = \Sigma p(x_i) x_i^2 - \mu^2$$

Q A lot contain 8 items of which 5 are good and 3 defective. Getting a defective item is considered as success. If 3 are randomly drawn. Find the probability distribution, mean and S.D. of defective item.

$X_i \rightarrow$ defective item.

X_i	P_i	$X_i P_i$	
0	$\frac{^5C_3}{^8C_3}$		
1	$\frac{^3C_1 \, ^5C_2}{^8C_3}$		
2	$\frac{^3C_2 \cdot ^5C_1}{^8C_3}$		
3	$\frac{^3C_3}{^8C_3}$		
		$\mu =$	

Q If the mean and SD of a binomial variate X are 9 and 3/2 respectively. Find the probability that X takes a value greater than one.

Solⁿ

$$np = 9$$

$$p + q = 1$$

$$\sqrt{npq} = \frac{3}{2}$$

$$npq = \frac{9}{4}$$

$$n = \frac{9}{p} = \frac{9 \times 4}{3} = 12$$

$$q = \frac{1}{4}$$

$$p = \frac{3}{4}$$

$$= P(X=2) + P(X=3) + \cdots + P(X=12)$$

$$= 1 - P(X=0) - P(X=1)$$

$$= 1 - \left({}^{12}C_0 \cdot \left(\frac{1}{4}\right)^{12} + {}^{12}C_1 \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{11} \right).$$

Q. Find the mean (expected) number of dots when a dice is thrown once.

X = No. of dots

x_i	p_i	$x_i p_i$
1	1/6	1/6
2	1/6	2/6
3	1/6	3/6
4	1/6	4/6
5	1/6	5/6
6	1/6	6/6

$\sum p_i x_i = \mu$

Mathematical expectation :-

It is worthwhile indicating that if p represents a person's chance of success in any venture and M the sum of money which he will receive in case of success, then the sum of money denoted by pM is called his expectation.

Q. Four players P_i, i = 1, 2, 3, 4 take part in a tournament, where there is only one winner & chance of P_i being the winner is proportinal to i. If the prize money is Rs. 10,000, then find the amount which the players can expect.

Solⁿ $P_i \propto i \Rightarrow P_i = Ki$

$$\boxed{\begin{matrix} P_1 = K & P_3 = 3K \\ P_2 = 2K & P_4 = 4K \end{matrix}}$$

$P_1 + P_2 + P_3 + P_4 = 1 \Rightarrow 10K = 1 \Rightarrow \boxed{K = \frac{1}{10}}$

Money expected by P_1 = $K \times 10000$ = 1000 Rs

" " " P_2 = $2K \times 10000$ = 2000 Rs.

⋮

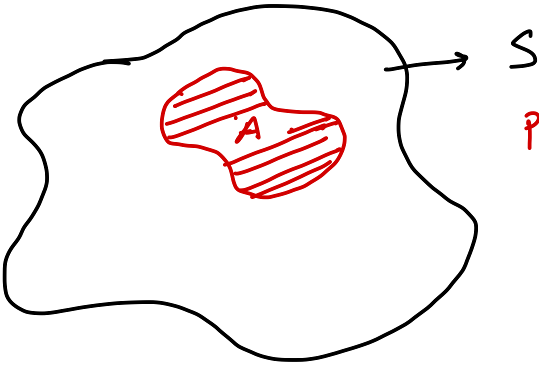
**Q. In a game , a man wins Rs 100 if he gets 5 or 6 on a throw of a fair die and loses Rs 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws , then his expected gain/ lose (in rupees) is

GEOMETRICAL PROBABILITY :

(i) If a point is randomly taken on area S & area A is included in area S then the probability that the point lies in area A is $\frac{A}{S}$

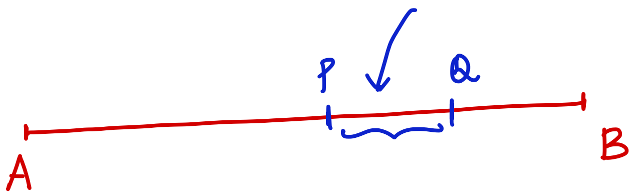
(ii) A point taken randomly on line segment AB lies on line segment PQ contained on it has probability $\frac{\ell(PQ)}{\ell(AB)}$

(i)



$\text{Prob} = \frac{A}{S}$

(ii)



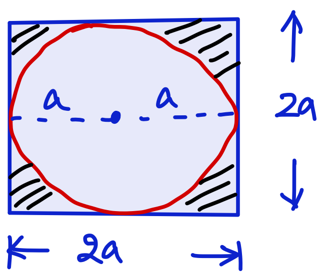
$\text{Prob} = \frac{l_{PQ}}{l_{AB}}$

E(1) A circle of radius 'a' is inscribed in a square of side 2a. Find the probability that a point chosen at random is inside the square but outside the circle.

E(2) A point is selected at random inside a circle. Find the probability that the point is closer to the circumference of the circle than to its centre.

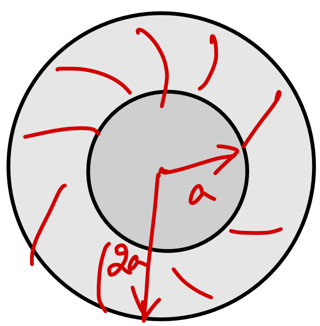
E(3) A point is selected at random inside the equilateral triangle of side 3. What is probability that a randomly selected point on it lies at a distance greater than 1 from any of the 3 corners.

1



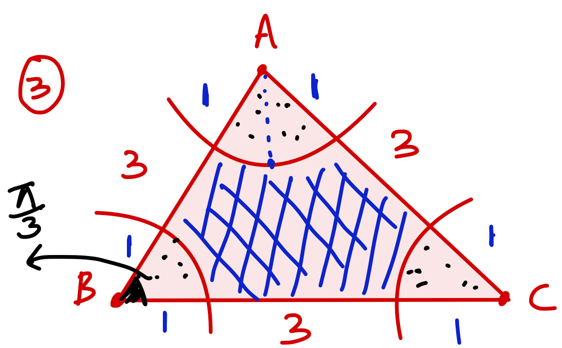
$$P = \frac{(2a)^2 - \pi a^2}{(2a)^2}$$

2



$$P = \frac{\pi (2a)^2 - \pi a^2}{\pi (2a)^2} = \frac{3}{4}.$$

3



$$P = \frac{\frac{\sqrt{3}}{4}(3)^2 - 3\left(\frac{1}{2}\cdot 1^2 \cdot \frac{\pi}{3}\right)}{\frac{\sqrt{3}}{4}\cdot(3)^2}$$

HW

Q A line segment is divided into three parts, what is the chance that they from the sides of a possible triangle.

Q A bag contains 5 balls of unknown colours. A ball is drawn twice with replacement from the bag found to be red on both the occasions. The contents of the bag were replenished. If now two ball are drawn simultaneously from the bag, find the probabilities that they will be both red. Assume all number of red balls in the bag to be equally likely.

$$\frac{\frac{1}{6}\left(\frac{5}{5}\right)^2 \frac{{}^5C_2}{{}^5C_2} + \frac{1}{6}\left(\frac{4}{5}\right)^2 \frac{{}^4C_2}{{}^5C_2} + \frac{1}{6}\left(\frac{3}{5}\right)^2 \cdot \frac{{}^3C_2}{{}^5C_2} + \left(\frac{1}{6}\right)\left(\frac{2}{5}\right)^2 \frac{{}^2C_2}{{}^5C_2} + 0 + 0}{\frac{1}{6}\left(\frac{5}{5}\right)^2 + \frac{1}{6}\left(\frac{4}{5}\right)^2 + \frac{1}{6}\left(\frac{3}{5}\right)^2 + \frac{1}{6}\left(\frac{2}{5}\right)^2 + \frac{1}{6}\left(\frac{1}{5}\right)^2 + 0.}$$

E(2) Box A contains nine cards numbered 1 through 9 and box B contains five cards numbered I through 5. A box is chosen at random and a card drawn; if the card shows an even number, another card is drawn from the same box. If the card shows an odd number, a card is drawn from the other box ;

(a) What is the probability that both cards show even numbers ?

(b) What is the probability that both cards show odd numbers ?

(a) $\frac{1}{2}\left(\frac{4}{9}\right)\left(\frac{3}{8}\right)+\frac{1}{2}\left(\frac{2}{5}\right)\left(\frac{1}{4}\right)$

(b) $\frac{1}{2}\left(\frac{5}{9}\right)\left(\frac{3}{5}\right)+\frac{1}{2}\left(\frac{3}{5}\right)\left(\frac{5}{9}\right)$

Q. In a game , a man wins Rs 100 if he gets 5 or 6 on a throw of a fair die and loses Rs 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws , then his expected gain/ lose (in rupees) is

Sol^n X: expected gain/loss in rupees.

$$P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3} \quad ; \quad P(\overline{5 \text{ or } 6}) = \frac{4}{6} = \frac{2}{3}$$

One throw : $(5 \text{ or } 6) \longrightarrow$ Rs 100

Two throws : $(\overline{5 \text{ or } 6})(5 \text{ or } 6) \longrightarrow -50 + 100 =$ Rs 50

Three throws : $(\overline{5 \text{ or } 6})(\overline{5 \text{ or } 6})(5 \text{ or } 6) \longrightarrow -50 - 50 + 100 =$ Rs 0

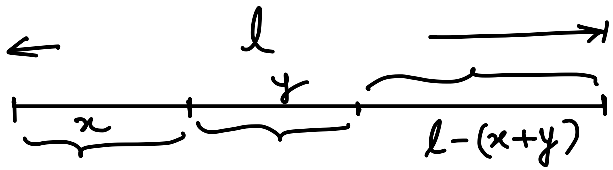
" " : $(\overline{5 \text{ or } 6})(\overline{5 \text{ or } 6})(\overline{5 \text{ or } 6}) \longrightarrow -50 - 50 - 50 = -150$ Rs.

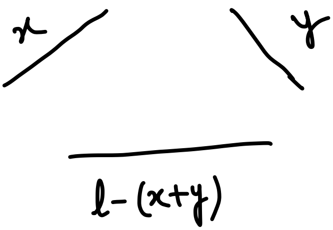
x_i	p_i	$x_i p_i$
100	$1/3$	$100/3$
50	$\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$	$100/9$
0	$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}$	0
-150	$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$	$-150 \times \frac{8}{27}$.
		$\mu = \Sigma x_i p_i = 0$

Ans

Q A line segment is divided into three parts, what is the chance that they from the sides of a possible triangle.

Sol^n





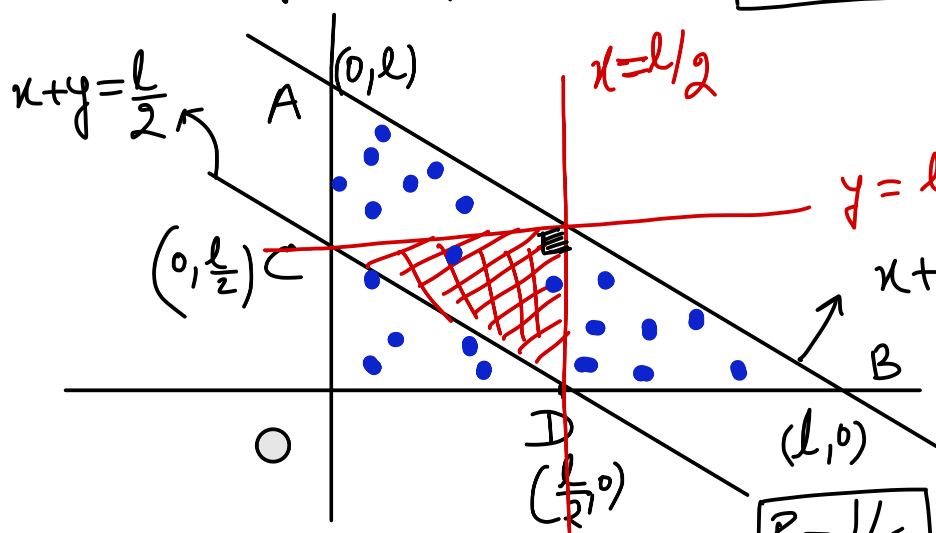
$$\left.\begin{array}{l} x>0 \\ y>0 \\ l-(x+y)>0 \end{array}\right\} \quad \boxed{\begin{array}{l} x, y>0 \\ x+y<l \end{array}}$$

Sum of 2 sides of Δ is greater than 3rd side.

$$x+y > l-(x+y) \Rightarrow x+y > l/2$$

$$\cancel{x} + l - (\cancel{x}+y) > y \Rightarrow y < l/2$$

$$\cancel{y} + l - (x+\cancel{y}) > x \Rightarrow x < l/2$$



$$P = \frac{\frac{1}{2}\cdot\frac{l}{2}\cdot\frac{l}{2}}{\frac{1}{2}\cdot l\cdot l}$$

$$\boxed{P = 1/4} \text{ Ans}$$

An Important Logic :-

n whole numbers taken at random are multiplied together, then the chance that the digit at the unit place of their product is

(a) 1, 3, 7 or 9 is $\left(\frac{2}{5}\right)^n$ (b) 2, 4, 6 or 8 is $\frac{4^n - 2^n}{5^n}$

(c) 5 is $\frac{5^n - 4^n}{10^n}$ (d) 0 is $\frac{10^n - 10^n - 5^n + 4^n}{10^n}$

eg: $31 \times 127 \times 69 = \_\_\_ 3$

$333 \times 127 \times 69 = \_\_\_ 9$

$N_1 \times N_2 \times N_3 = N$

$\left(\frac{4}{10}\right)\left(\frac{4}{10}\right)\left(\frac{4}{10}\right) = \left(\frac{4}{10}\right)^3$

$= \left(\frac{2}{5}\right)^3$

Units : 0, 1, 2, 3, ..., 9

fav : 1, 3, 7 or 9

* Units digit is 1, 3, 5, 7, 9

$N_1 \times N_2 \times N_3 \times N_4 = N$

$\left(\frac{5}{10}\right)\left(\frac{5}{10}\right)\left(\frac{5}{10}\right)\left(\frac{5}{10}\right) = \left(\frac{5}{10}\right)^4$

* Units digit → 1, 2, 3, 4, 6, 7, 8, 9

$N_1 \times N_2 \times N_3 = N$