Q. Ram & Shyam toss a coin in turn indefinitely untill head occur for the first time. The person getting the head for the first time wins. If Ram start tossing, then find their respective chance of winning.

Sol^n

$$P(\text{Ram wins}) = P(H \text{ or } \bar{H}\bar{H}H \text{ or } \bar{H}\bar{H}\bar{H}\bar{H}H \text{ or} \cdots)$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \cdots$$

$$= \frac{1/2}{1-(1/2)^2} = \frac{1/2}{3/4} = \frac{2}{3}.$$

$$\therefore P(\text{shyam wins}) = 1 - \frac{2}{3} = \frac{1}{3}.$$

Q. A coin is continued tossing until either a head and a tail is obtained for the first time or unless the coin has been tossed for a maximum of five times. If the first two tosses have resulted in both tails, find the chance that the coin will be tossed 5 times.

Sol^n

$\underbrace{T\ T}_{\text{occurred}}\ \underline{T}\ \underline{T}\ \underline{\checkmark}$ H or T

$$\text{Req. prob} = \frac{1}{2}\cdot\frac{1}{2}\cdot(1) = \frac{1}{4}.$$

Q All face cards from a pack of 52 playing cards are removed. From the remaining 40 cards, 4 are drawn.

(i) Find the probability that they are of different suit different denominations.

(ii) Same suit & diff denominations.

(iii) diff suit & same " .

Sol^n

$52 - 12 = 40$ cards

Suits

Club Diamond Heart Spade

(i) $$\frac{{}^{10}C_1 \times {}^{9}C_1 \times {}^{8}C_1 \times {}^{7}C_1}{{}^{40}C_4}$$

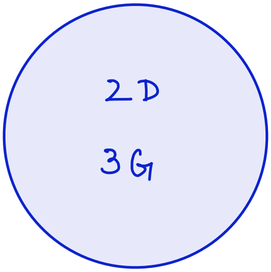
(ii) $$\frac{{}^{4}C_1 \times {}^{10}C_4}{{}^{40}C_4}$$

(iii) $$\frac{{}^{10}C_1 \times 1}{{}^{40}C_4}$$

Q A box contains 5 tubes, 2 of them defective and 3 good one. Tubes are tested by one-by-one till the 2 defective tubes are discovered. What is the probability that the testing procedure comes to an end at the end of

(a) second testing (b) 3rd testing

Sol^n



(i) D D

$$P(D_1 \cap D_2) = P(D_1) \cdot P(D_2/D_1)$$

$$= \left(\frac{2}{5}\right) \cdot \left(\frac{1}{4}\right) = \frac{1}{10} \checkmark$$

(ii) DGD , GDD , GGG

$$P(DGD) + P(GDD) + P(GGG)$$

$$\left(\frac{2}{5} \cdot \frac{3}{4} \cdot \frac{1}{3}\right) + \left(\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3}\right) + \left(\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3}\right) = \frac{3}{10} \checkmark$$

Q Cards are dealt one by one from a well shuffled pack of 52 cards. (i) Find the probability that exactly 4 cards are dealt before the first ace appears. (ii) Find the probability that exactly 4 cards are dealt in all before the second ace.

Sol^n

(i) $\underbrace{\left(\frac{48}{52}\right)\left(\frac{47}{51}\right)\left(\frac{46}{50}\right)\left(\frac{45}{49}\right)} \cdot \underbrace{\left(\frac{4}{48}\right)}$

(ii) $\underbrace{\_\ \_\ \_\ \_}$ $\underline{\text{Ace}}$

1 Ace
3 None ace

$\_\ \_\ \_\ \text{Ace}_1$ $\underline{\text{Ace}_2}$

$\text{Ace}_1\ \_\ \_\ \_$ $\underline{\text{Ace}_2}$

$\_\ \text{Ace}_1\ \_\ \_$ $\underline{\text{Ace}_2}$

$\_\ \_\ \text{Ace}_1\ \_$ $\underline{\text{Ace}_2}$

$$4 \times \left(\left(\frac{48}{52} \cdot \frac{47}{51} \cdot \frac{46}{50} \cdot \frac{4}{49}\right) \cdot \frac{3}{48}\right)$$

Q HW

A pair of dice is rolled until a total of 5 or 7 is obtained. Find the probability that the total of 5 comes before a total of 7.

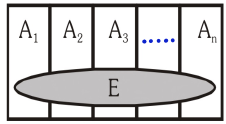
V. Imp

LAW OF TOTAL PROBABILITY :

Suppose S is the sample space of an experiment & $A_1, A_2, A_3 \ldots\ldots\ldots A_n$ are n mutually exclusive & exhaustive events defined on the experiment. Let E be another event of this experiment as shown in figure then probability of occurrence of E i.e. $\boxed{P(E) = P(E \cap A_1) + P(E \cap A_2) + \cdots + P(E \cap A_n)}$

$$P(E) = P(A_1)\,P(E/A_1) + P(A_2)\,P(E/A_2) + \ldots\ldots\ldots + P(A_n)\,P(E/A_n)$$

$$= \sum_{k=1}^{n} P(A_k)P(E/A_k)$$

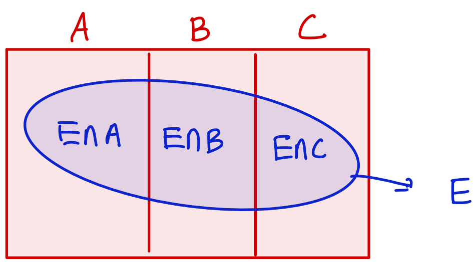


- $A_i \cap A_j = \phi$
- $A_1 \cup A_2 \cup A_3 \cdots \cup A_n = S$

Q. A lady has 3 compartments in her purse. I[st] compartment contains 1 Rupee & 2 Paise coins, II[nd] compartment contains 2 Rupee & 3 Paise coins, & IIIrd compartment contain 3 Rupee & 4 Paise coins. She randomly selects a compartment to draw a coin. What is probability that the drawn coin is a rupee coin.

Sol^n

I	II	III
1 R 2 P	2 R 3 P	3 R 4 P



A : Compartment I is selected $\quad P(A) = \frac{1}{3}$

B : " II " " $\quad P(B) = \frac{1}{3}$

C : " III " " $\quad P(C) = \frac{1}{3}$

E : She draws a rupee coin.

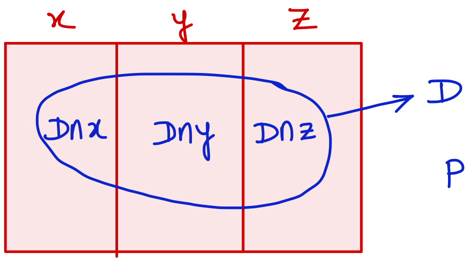
$$P(E) = P(E \cap A) + P(E \cap B) + P(E \cap C) = P(A)\cdot P(E/A) + P(B)\cdot P(E/B) + P(C)\cdot P(E/C)$$

$$= \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{5} + \frac{1}{3} \times \frac{3}{7}.$$

Q. In a company there are 3 machine x,y & z. Machine x produces 50% of total production of which 3% are defective. Machine y produces 30% of total production of which 4% are defective and machine z produces 20% of total production of which 5% is defective. Out of total production lot 1 item is randomly selected. What is the probability that the selected item is defective ?

Sol^n

$D \rightarrow$ Defective



$$P(D) = P(D \cap x) + P(D \cap y) + P(D \cap z)$$

$$= P(x) \cdot P(D/x) + P(y) \cdot D(D/y) + P(z) \cdot P(D/z)$$

$$= \left(\frac{50}{100}\right)\left(\frac{3}{100}\right) + \left(\frac{30}{100}\right)\left(\frac{4}{100}\right) + \left(\frac{20}{100}\right)\left(\frac{5}{100}\right)$$

√ Imp

BAYE'S THEOREM :

Here, with the knowledge of present, we predict the past.

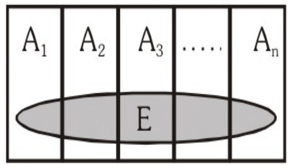
If an event E can occur only with one of the n mutually exclusive and exhaustive events $A_1, A_2, \ldots\ldots A_n$ & if the conditional probabilities of the events $P(E/A_1)$, $P(E/A_2)$ .... $P(E/A_n)$ are known then,

$$\boxed{P(A_1/E) = \frac{P(A_i).P(E/A_i)}{\sum_{i=1}^{n} P(A_i).P(E/A_i)}}$$

Rem

Proof :

The events E occurs with one of the n mutually exclusive & exhaustive events $A_1, A_2, A_3, \ldots\ldots\ldots A_n$



$$E = EA_1 + EA_2 + EA_3 + \ldots\ldots + EA_n$$

$$P(E) = P(EA_1) + P(EA_2) + \ldots\ldots + P(EA_n) = \sum_{i=1}^{n} P(EA_i)$$

Note : $E \equiv$ event what we have

$A_1 \equiv$ event what we want ;

$A_2, A_3, \ldots\ldots A_n$ are alternative event.

Now,

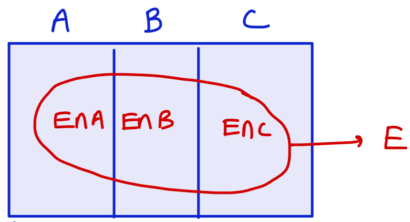
$$P(EA_i) = P(E) \,.\, P(A_i/E) = P(A_i) \,.\, P(E/A_i)$$

$$P(A_i/E) = \frac{P(A_i) \,.\, P(E/A_i)}{P(E)} = \frac{P(A_i) \,.\, P(E/A_i)}{\sum_{i=1}^{n} P(EA_i)} ; \quad P(A_i/E) = \frac{P(A_i) \,.\, P(E/A_i)}{\sum P(A_i) \,.\, P(E/A_i)}$$

Q A lady has 10 coins in her purse, 8 of them are normal coins, one of them is a DH and one is a DT coin. She randomly draws a coin and tosses it for 5 times. The coin is found to fall headwise on all the 5 occasions. Find the probability that the drawn coin was a DH coin.

Sol^n

10 Coins → 8 Normal
10 Coins → 1 DH
10 Coins → 1 DT



A: Normal coin is selected ; $P(A) = \frac{8}{10}$.

B: DH coin " " ; $P(B) = \frac{1}{10}$.

C: DT " " " ; $P(C) = \frac{1}{10}$.

E: Coin tossed 5 times fell headwise

$$\text{Req. prob} = \frac{P(E\cap B)}{P(E\cap A) + P(E\cap B) + P(E\cap C)} = \frac{P(B)\cdot P(E/B)}{P(A)P(E/A) + P(B)\cdot P(E/B) + P(C)\cdot P(E/C)}$$

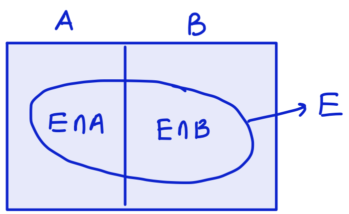
$$= \frac{\frac{1}{10}\times 1}{\frac{8}{10}\times\left(\frac{1}{2}\right)^5 + \frac{1}{10}\times 1 + \frac{1}{10}\times 0}$$

Q. The contents of the urn I and urn II are as follows

One urn is chosen at random and a ball is drawn and its colour noted and replaced back in the same urn. Again a ball is drawn from the same urn, colour noted and replaced. The process is repeated 4 times and as a result one ball of white colour and 3 of black colour are noted. What is the probability that the chosen urn was I.

Urn	W	B
I	4	5
II	3	6

Solⁿ



A : Urn I is selected ; $P(A) = \frac{1}{2}$

B : Urn II " " ; $P(B) = \frac{1}{2}$

E : 1W & 3B balls were observed

W B B B

$$\text{Req. prob} = \frac{P(E\cap A)}{P(E\cap A) + P(E\cap B)} = \frac{P(A)\cdot P(E/A)}{P(A)\cdot P(E/A) + P(B)\cdot P(E/B)}$$

$$= \frac{\cancel{\frac{1}{2}}\times\left(\frac{4}{\cancel{9}}\cdot\frac{5}{\cancel{9}}\cdot\frac{5}{\cancel{9}}\cdot\frac{5}{\cancel{9}}\right)\times\cancel{4}}{\cancel{\frac{1}{2}}\left(\frac{4}{\cancel{9}}\cdot\frac{5}{\cancel{9}}\cdot\frac{5}{\cancel{9}}\cdot\frac{5}{\cancel{9}}\right)\times\cancel{4} + \cancel{\frac{1}{2}}\left(\frac{3}{\cancel{9}}\cdot\frac{6}{\cancel{9}}\cdot\frac{6}{\cancel{9}}\cdot\frac{6}{\cancel{9}}\right)\times\cancel{4}}$$

Q HW

In a test an examine either guesses or copies or knows the answer to a multiple choice question with 4 choices (one or more than one correct). The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it, is 1/8. Find the probability that he knows the answer to the question given that he correctly answered it.

HW

Q 'A' writes a letter to his friend and given it to his son to post it in a letter box, the reliability of his son being 3/4. The probability that a letter posted will get delivered is 8/9. At a later date 'A' hears from 'B' that the letter has not reached him. Find the probability that the son did not post the letter at all.

HW

O-1 Part # 5.

S-1 Q4 to 16.