EXERCISE (O-1)

1. Coordinates of the vertices of a triangle ABC are (12,8), (–2,6) and (6,0) then the **correct** statement is-

(A) triangle is right but not isosceles

(B) triangle is isosceles but not right

(C) triangle is obtuse

(D) the product of the abscissa of the centroid, orthocenter and circumcenter is 160.

Ans. ΔABC

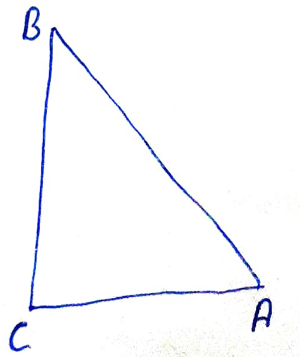
$A(12, 8)$, $B(-2, 6)$, $C(6, 0)$

$AB = \sqrt{(12+2)^2 + (8-6)^2} = \sqrt{196+4} = 10\sqrt{2}$

$BC = \sqrt{(6+2)^2 + (6-0)^2} = 10$

$AC = \sqrt{(12-6)^2 + (8-0)^2} = 10$

$\because AB^2 = BC^2 + AC^2$ Right angle Δ



$\text{Centroid} \equiv \left(\frac{12-2+6}{3}, \frac{8+6+0}{3}\right)$

$\equiv \left(\frac{16}{3}, \frac{14}{3}\right)$

$\text{orthocenter} \equiv C \equiv (6, 0)$

$\text{Circumcenter} \equiv \frac{A+B}{2} = (5, 7)$

So $\frac{16}{3} \times 6 \times 5 = 160$ the product of the abscissa of the centroid, orthocenter & circumcenter

(D)

2. Find the value of x_1 if the distance between the points $(x_1, 2)$ and $(3, 4)$ be 8.

(A) $3 \pm 2\sqrt{15}$ (B) $3 \pm \sqrt{15}$ (C) $2 \pm 3\sqrt{15}$ (D) $2 \pm \sqrt{15}$

Ans

$A(x_1, 2)$

$B(3, 4)$

$AB = 8 = \sqrt{(x_1-3)^2 + (2-4)^2}$

$\Rightarrow 64 = (x_1-3)^2 + 4$

$\Rightarrow 64 = x_1^2 - 6x_1 + 9 + 4$

$\Rightarrow \cancel{x_1^2 - 6x_1 - 51 = 0}$

$\Rightarrow (x_1-3)^2 = 60$

$x_1 = 3 \pm 2\sqrt{15}$ B

3. If P(1,2), Q(4,6), R(5,7) & S(a,b) are the vertices of a parallelogram PQRS, then :

(A) a = 2, b = 4 (B) a = 3, b = 4 (C) a = 2, b = 3 (D) a = 3, b = 5

Ans

P(1, 2) R(5, 7)

Q (4, 6) S(a, b)

In parallelogram we know that

$$\frac{P+R}{2} = \frac{Q+S}{2}$$

$$S = P+R-Q$$

$$a = 1+5-4 = 2$$

$$b = 2+7-6 = 3$$

$$\boxed{a = 2, \quad b = 3}$$

C

4. The length of a line segment AB is 10 units. If the coordinates of one extremity are (2,–3) and the abscissa of the other extremity is 10 then the sum of all possible values of the ordinate of the other extremity is -

(A) 3 (B) –4 (C) 12 (D) –6

A (2, -3)

B (10, y)

AB = 10

$AB^2 = 10^2 = (10-2)^2 + (y+3)^2$

$\Rightarrow (y+3)^2 = 10^2 - 8^2$

$\Rightarrow (y+3)^2 = 6^2$

$\Rightarrow y+3 = \pm 6$

→ + $y_1 = 3$

→ - $y_2 = -9$

$y_1 + y_2 = 3 - 9 = -6$

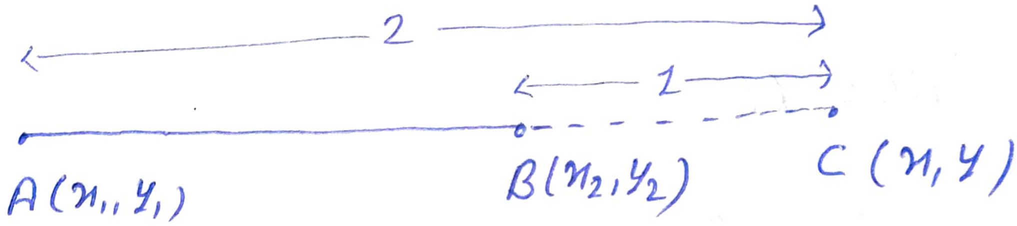
D

5. If A and B are the points (–3,4) and (2,1), then the co-ordinates of the point C on AB produced such that AC = 2BC are :

(A) (2,4) (B) (3,7) (C) (7,–2) (D) $\left(-\frac{1}{2}, \frac{5}{2}\right)$

A (-3, 4)

B (2, 1)



$$x = \frac{mx_2 - nx_1}{m-n} \qquad y = \frac{my_2 - ny_1}{m-n}$$

$$x = \frac{2(2) - 1(-3)}{2-1} \qquad y = \frac{2(1) - 1(4)}{2-1}$$

$$x = \frac{4+3}{1} \qquad y = \frac{2-4}{1}$$

$$x = 7 \qquad y = -2$$

point c (7, -2) ©

6. The orthocenter of the triangle ABC is 'B' and the circumcenter is 'S' (a,b). If A is the origin then the co-ordinates of C are :

(A) (2a,2b) (B) $\left(\frac{a}{2},\frac{b}{2}\right)$ (C) $\left(\sqrt{a^2+b^2},0\right)$ (D) none

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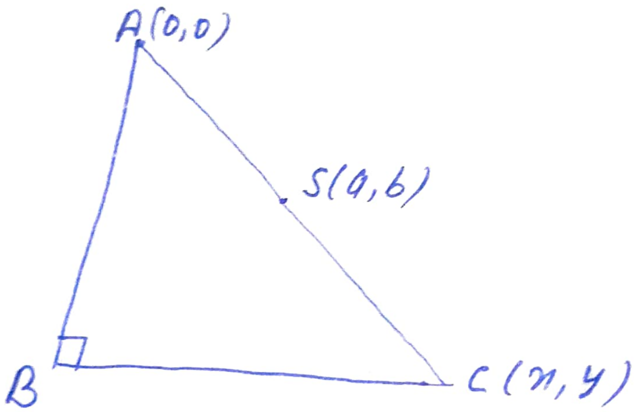
ΔABC

$B \equiv$ orthocenter

$S \equiv$ circumcenter $\equiv (a,b)$

$A \equiv$ origin $\equiv (0,0)$



Right angle Δ

So $S = \dfrac{A+C}{2}$

$C = 2S - A$

$x = 2a - 0$ | $y = 2b - 0$

$x = 2a$ | $y = 2b$

Vertex $C \equiv (2a, 2b)$ A

7. The medians of a triangle meet at (0,–3) and its two vertices are at (–1,4) and (5,2). Then the third vertex is at -

(A) (4,15) (B) (–4,–15) (C) (–4,15) (D) (4,–15)

n/

Centroid $\equiv G\ (0,-3)$

ΔABC

$A(-1,4)$

$B(5,2)$

$C(x,y)$

$\Rightarrow \quad 3G = A + B + C$

$C = 3G - A - B$

$x = 3(0) - (-1) - (5)$

$x = -4$

$y = 3(-3) - 4 - 2$

$y = -15$

Vertex $C \equiv (-4,-15)$ B

8. If the two vertices of a triangle are (7,2) and (1,6) and its centroid is (4,6) then the coordinate of the third vertex are (a,b). The value of (a + b), is-

(A) 13 (B) 14 (C) 15 (D) 16

Ans

ΔABC

$A(7,2)$

$B(1,6)$ $G \equiv \text{Centroid} \equiv (4,6)$

$C(a,b)$

$3G = A + B + C$

$\Rightarrow C = 3G - A - B$

$a = 3(4) - 7 - 1$

$a = 12 - 7 - 1 = 4$

$\Rightarrow b = 3(6) - 2 - 6$

$= 18 - 2 - 6$

$b = 10$

$\boxed{a + b = 14}$ (B)

9. If in triangle ABC, A ≡ (1,10), circumcenter ≡ $\left(-\frac{1}{3}, \frac{2}{3}\right)$ and orthocenter ≡ $\left(\frac{11}{3}, \frac{4}{3}\right)$ then the co-ordinates of mid-point of side opposite to A is-

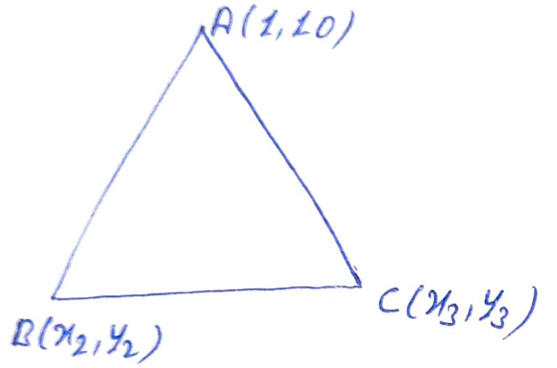
(A) (1,–11/3) (B) (1,5) (C) (1,–3) (D) (1,6)

Ans

Ans ΔABC $A \equiv (1, 10)$

Circumcenter $\equiv C \equiv \left(-\frac{1}{3}, \frac{2}{3}\right)$

Orthocenter $\equiv O \equiv \left(\frac{11}{3}, \frac{4}{3}\right)$



$G \equiv$ centroid

Mid point of side opposite to $A \equiv \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

$O \leftarrow 2 \rightarrow G \rightarrow 1 \rightarrow C$

$$G = \frac{2C+O}{3}$$

$$\Rightarrow 3G = 2C + O$$

$$\Rightarrow 3\left(\frac{x_1+x_2+x_3}{3}\right) = 2\left(-\frac{1}{3}\right) + \frac{11}{3}$$

$$x_2 + x_3 = 2$$

$$\Rightarrow 3\left(\frac{y_1+y_2+y_3}{3}\right) = 2\left(\frac{2}{3}\right) + \frac{4}{3} = 8/3$$

$$y_2 + y_3 = -22/3$$

$$\frac{x_2+x_3}{2} = 1 \qquad \frac{y_2+y_3}{2} = -11/3$$

$(1, -11/3)$ (A)

10. Consider the points P(2,–4); Q(4,–2) and R(7,1). The points P,Q,R -

(A) form an equilateral triangle

(B) form a right angled triangle

(C) form an isosceles triangle which is not equilateral

(D) are collinear.

Ans

$P(2,-4)$

$Q(4,-2)$

$R(7,1)$

$$PQ = \sqrt{(4-2)^2 + (-2+4)^2} = 2\sqrt{2}$$

$$QR = \sqrt{(7-4)^2 + (1+2)^2} = 3\sqrt{2}$$

$$PR = \sqrt{(7-2)^2 + (1+4)^2} = 5\sqrt{2}$$

$$PQ + QR = 2\sqrt{2} + 3\sqrt{2}$$

$$= 5\sqrt{2} = PR$$

Are collinear (D)

11. A triangle has two of its vertices at (0,1) and (2,2) in the cartesian plane. Its third vertex lies on the x-axis. If the area of the triangle is 2 square units then the sum of the possible abscissae of the third vertex, is-

(A) –4 (B) 0 (C) 5 (D) 6

Ans ΔABC

$$A(0,1),\quad B(2,2)$$

$$C(x,0)$$

$$\text{Area}(\Delta ABC) = 2$$

$$\frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 2 & 2 & 1 \\ x & 0 & 1 \end{vmatrix} = 2$$

$$C_2 \to C_2 - C_3$$

$$\begin{vmatrix} 0 & 0 & 1 \\ 2 & 1 & 1 \\ x & -1 & 1 \end{vmatrix} = 4$$

$$|x+2| = 4$$

$$x_1 = -6 \quad \text{or} \quad x_2 = 2$$

$$x_1 + x_2 = -6 + 2$$

$$= -4$$

(A)

12. A point P(x,y) moves so that the sum of the distance from P to the coordinate axes is equal to the distance from P to the point A(1,1). The equation of the locus of P in the first quadrant is -

(A) $(x + 1)(y + 1) = 1$ (B) $(x + 1)(y + 1) = 2$

(C) $(x - 1)(y - 1) = 1$ (D) $(x - 1)(y - 1) = 2$

A.

Ans

P(x, y) $\quad$ I^{st} quadrant

$x > 0,\ y > 0$

$$|x| + |y| = \sqrt{(x-1)^2 + (y-1)^2}$$

$$\Rightarrow x + y = \sqrt{(x-1)^2 + (y-1)^2}$$

$$\Rightarrow x^2 + y^2 + 2xy = x^2 - 2x + 1 + y^2 - 2y + 1$$

$$\Rightarrow xy = -x - y + 1$$

$$\Rightarrow xy + x + y + 1 = 2$$

$$\Rightarrow (x+1)(y+1) = 2$$

B

3. Let A(2,–3) and B(–2,1) be vertices of a ΔABC. If the centroid of ΔABC moves on the line $2x + 3y = 1$, then the locus of the vertex C is-

(A) $2x + 3y = 9$ (B) $2x - 3y = 7$ (C) $3x + 2y = 5$ (D) $3x - 2y = 3$

ΔABC

$A(2,-3)$ $B(-2,1)$ $C(h,k)$

$$\text{Centroid} \equiv \left(\frac{2-2+h}{3}, \frac{-3+1+k}{3}\right)$$

$$\equiv \left(\frac{h}{3}, \frac{k-2}{3}\right)$$

centroid moves on the line $2x+3y+1=0$

$$\Rightarrow 2\left(\frac{h}{3}\right)+3\left(\frac{k-2}{3}\right) - 1 = 0$$

$$\Rightarrow 2h + 3K - 9 = 0$$

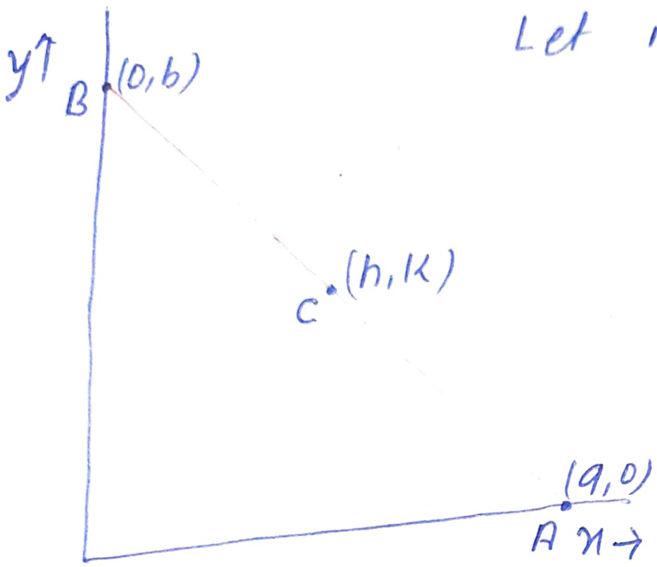
$$\Rightarrow 2x + 3y = 9$$

(A)

14. A stick of length 10 units rests against the floor and a wall of a room. If the stick begins to slide on the floor then the locus of its middle point is :

(A) $x^2 + y^2 = 2.5$ (B) $x^2 + y^2 = 25$ (C) $x^2 + y^2 = 100$ (D) none

Ans



Let middle pt is (h,K)

AB = 10

~~10~~

$$100 = a^2 + b^2 \quad \text{—— (1)}$$

C is mid pt A & B

$$C = \frac{A+B}{2}$$

$$h = \frac{a}{2} \quad , \quad K = \frac{b}{2}$$

$$a = 2h \quad , \quad b = 2K$$

Put a & b in eqn (i)

$$100 = (2h)^2 + (2K)^2$$

$$\Rightarrow \boxed{x^2 + y^2 = 25}$$

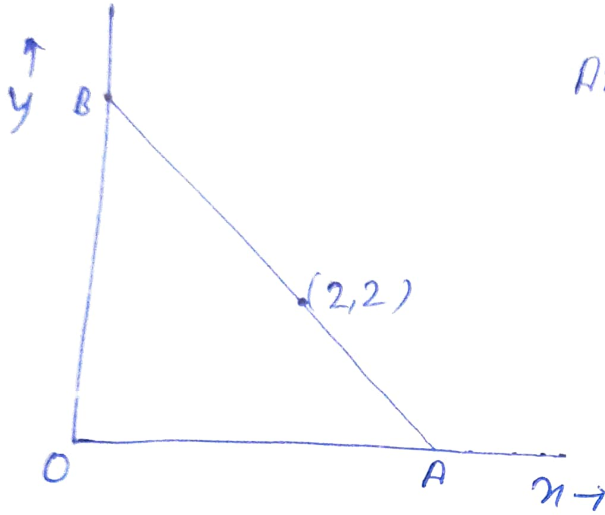
B

15. A line passes through (2,2) and cuts a triangle of area 9 square units from the first quadrant. The sum of all possible values for the slope of such a line, is-

(A) –2.5 (B) –2 (C) –1.5 (D) –1

Ans

line passes through (2,2)



Area(ΔOAB) = 9

line → $y-2 = m(x-2)$

point A $\left(-\frac{2}{m}+2, 0\right)$
point B $(0, 2-2m)$

I^{st} Quadrant

Now →
$$9 = \frac{1}{2}\left|\left(-\frac{2}{m}+2\right)(2-2m)\right|$$

+ive +ive

$$\Rightarrow 18 = \left(-\frac{2}{m}+2\right)(2-2m)$$

$$\Rightarrow -4+4m+4m-4m^2 = 18m$$

$$\Rightarrow 4m^2+10m-4=0$$

Sum of slopes $= -\frac{10}{4}$

$= -2.5$

(A)

16. The diagonals of a parallelogram PQRS are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then PQRS must be a :

(A) rectangle
(B) square
(C) cyclic quadrilateral
(D) rhombus

- Slope of diagonal (1)

$$m_1 = -1/3$$

- Slope of diagonal (2)

$$m_2 = \frac{6}{2} = 3$$

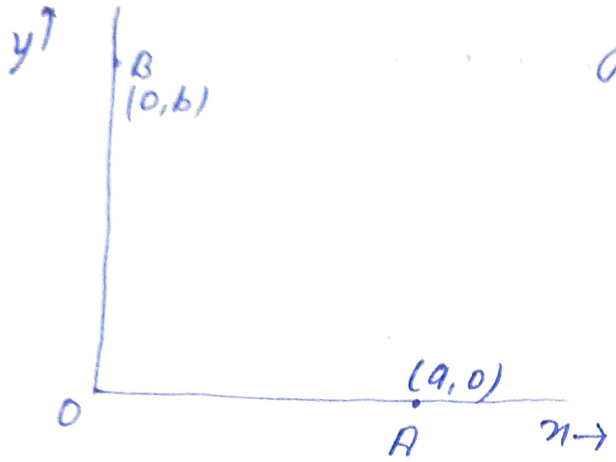
$$m_1 \times m_2 = -\frac{1}{3} \times 3 = -1$$

So diagonal are $\perp$

17. A and B are any two points on the positive x and y axis respectively satisfying 2(OA) + 3(OB) = 10. If P is the middle point of AB then the locus of P is-

(A) $2x + 3y = 5$ (B) $2x + 3y = 10$ (C) $3x + 2y = 5$ (D) $3x + 2y = 10$

Ans



Given

$2(OA) + 3(OB) = 10$

$\Rightarrow 2a + 3b = 10 \quad —(1)$

$P(h, k)$ mid point of AB

$P = \frac{A+B}{2}$

$h = \frac{a}{2}$, $k = \frac{b}{2}$

$a = 2h$, $b = 2k$

using eqn (1)

$2.(2h) + 3(2k) = 10$

$\Rightarrow 4h + 6k = 10$

$\Rightarrow 2h + 3k = 5$

$2x + 3y = 5$

(A)

18. The co-ordinates of the orthocentre of the triangle bounded by the lines, $4x - 7y + 10 = 0$; $x + y = 5$ and $7x + 4y = 15$ is-

(A) (2,1) (B) (–1,2) (C) (1,2) (D) (1,–2)

Orthocenter of Δ bounded by lines

$4x - 7y + 10 = 0$ —— (i)

$x + y = 5$ —— (ii)

$7x + 4y = 15$ —— (iii)

slope of (i) $m_1 = 4/7$

slope of (iii) $m_3 = -7/4$

Now $m_1 \times m_3 = -1$

Δ is Right angle

Orthocenter is intersection point of (i) & (iii)

$4x - 7y = -10$

$7x + 4y = 15$

$x = 1, \quad y = 2$

orthocenter $(1, 2)$

C

19. If the x intercept of the line y = mx + 2 is greater than 1/2 then the gradient of the line lies in the interval-

(A) (–1,0) (B) (–1/4,0) (C) (–∞,–4) (D) (–4,0)

line $y = mx + 2$

x- intercept $= -\dfrac{2}{m}$

$\because \quad -\dfrac{2}{m} > \dfrac{1}{2}$

$\Rightarrow \quad +\dfrac{2}{m} + \dfrac{1}{2} < 0$

$\Rightarrow \quad \dfrac{4+m}{2m} < 0$

$\Rightarrow \quad \dfrac{m+4}{m} < 0$

$m \in (-4, 0)$

D

20. The greatest slope along the graph represented by the equation $4x^2 - y^2 + 2y - 1 = 0$, is-

(A) -3 (B) -2 (C) 2 (D) 3

Ans

equation

$$4x^2 - y^2 + 2y - 1 = 0$$

$$\Rightarrow \quad 4x^2 = (y-1)^2$$

$$\Rightarrow \quad y - 1 = \pm 2x$$

$y - 1 = 2x$ | $y - 1 = -2x$

$m_1 = 2$ | $m_2 = -2$

greatest slope $= 2$

C

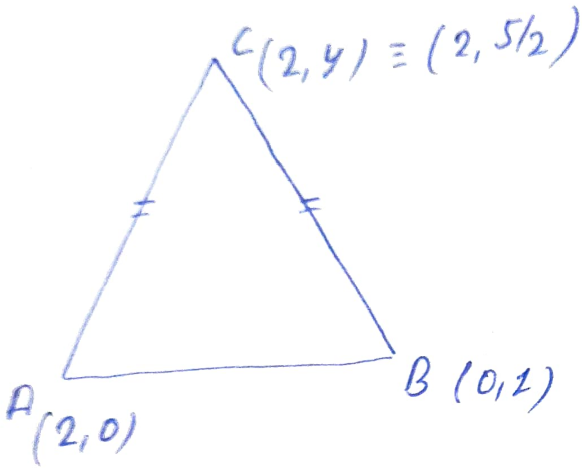
)e

21. The extremities of the base of an isosceles triangle ABC are the points A(2,0) and B(0,1). If the equation of the side AC is x = 2 then the slope of the side BC is -

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\sqrt{3}$

Ans

Isosceles Δ ABC



$AC = BC$

$\Rightarrow \quad y = \sqrt{2^2 + (y-1)^2}$

$\Rightarrow \quad y^2 = 4 + y^2 - 2y + 1$

$2y = 5 \quad \boxed{y = \frac{5}{2}}$

Slope of side BC $= \dfrac{\frac{5}{2} - 1}{2 - 0}$

$= 3/4$ (A)

22. Number of lines that can be drawn through the point(4,–5) so that its distance from (–2,3) will be equal to 12 is equal to-

(A) 0 (B) 1 (C) 2 (D) 3

---

lines passes through the point $(4,-5)$

$$y+5 = m(x-4)$$

$$\Rightarrow \quad y - mx + 5 + 4m = 0$$

distance from $(-2,3)$

$$\left| \frac{3+2m+5+4m}{\sqrt{1+m^2}} \right| = 12 \text{ (Given)}$$

$$\left| \frac{8+6m}{\sqrt{1+m^2}} \right| = 12$$

$$\Rightarrow \quad 16 + 9m^2 + 24m = 36(1+m^2)$$

$$\Rightarrow \quad 27m^2 - 24m + 20 = 0$$

$$D = (24)^2 - 4 \times 27 \times 20$$

$$= 576 - 2160 < 0$$

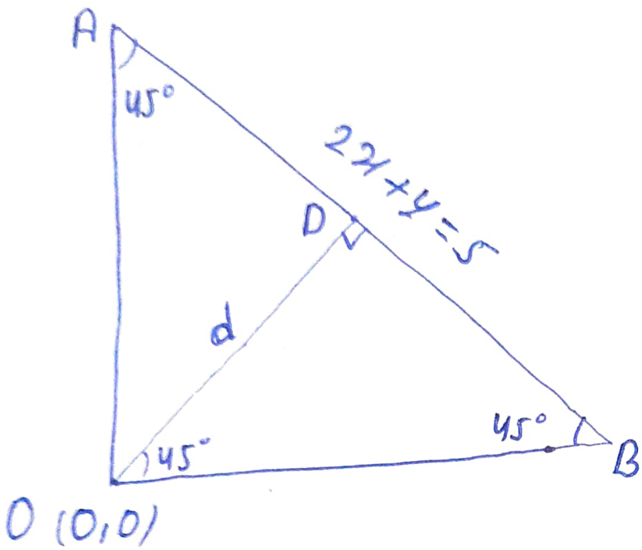
No real values of m

No. of lines $= 0$

A

23. Two mutually perpendicular straight lines through the origin from an isosceles triangle with the line $2x + y = 5$. Then the area of the triangle is :

(A) 5 (B) 3 (C) 5/2 (D) 1



$$d = \left| \frac{2.0 + 0 - 5}{\sqrt{1+4}} \right| = \frac{5}{\sqrt{5}} = \sqrt{5}$$

$$BD = \tan 45^\circ \times d$$

$$= AD = \sqrt{5}$$

$$\text{Area}(\Delta OAB) = \frac{1}{2} \times \text{Base} \times \text{height}$$

$$= \frac{1}{2} \times (2\sqrt{5}) \times \sqrt{5}$$

$$= 5$$

(A)

24. The area of the parallelogram formed by the lines $3x + 4y = 7a$; $3x + 4y = 7b$; $4x + 3y = 7c$ and $4x + 3y = 7d$ is-

(A) $\frac{|(a-b)(c-d)|}{7}$ (B) $|(a-b)(c-d)|$ (C) $\frac{|(a-b)(c-d)|}{49}$ (D) $7|(a-b)(c-d)|$

Sides of Parallelogram (lines)

$$3x + 4y = 7a, \Rightarrow y = \frac{7}{4}a - \frac{3x}{4}.$$

$$3x + 4y = 7b, \Rightarrow y = \frac{7}{4}b - \frac{3x}{4}$$

$$4x + 3y = 7c, \Rightarrow y = \frac{7c}{3} - \frac{4x}{3}$$

$$4x + 3y = 7d. \Rightarrow y = \frac{7d}{3} - \frac{4x}{3}$$

Area of parallelogram $= \left| \frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2} \right|$

$$= \left| \frac{\left(\frac{7}{4}a - \frac{7}{4}b\right)\left(\frac{7c}{3} - \frac{7d}{3}\right)}{-\frac{3}{4} + \frac{4}{3}} \right|$$

$$= 7|(a-b)(c-d)|$$

(D)

25. Consider a parallelogram whose sides are represented by the lines $2x + 3y = 0$; $2x + 3y - 5 = 0$; $3x - 4y = 0$ and $3x - 4y = 3$. The equation of the diagonal not passing through the origin, is-

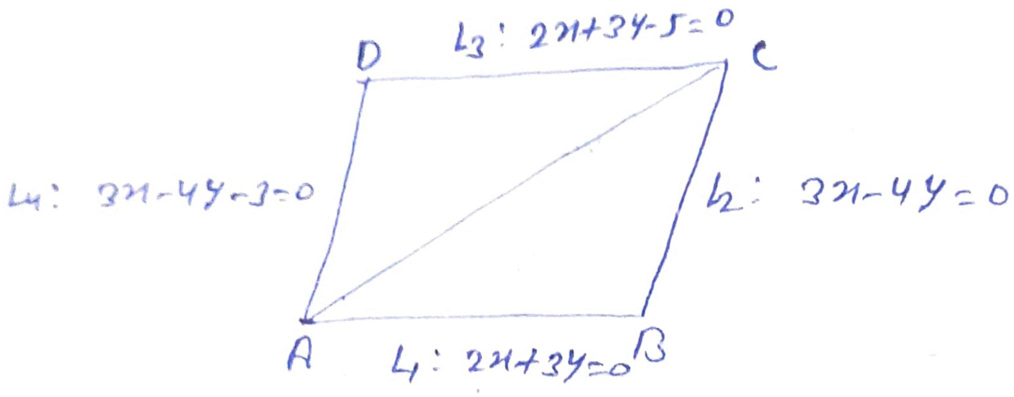
(A) $21x - 11y + 15 = 0$

(B) $9x - 11y + 15 = 0$

(C) $21x - 29y - 15 = 0$

(D) $21x - 11y - 15 = 0$

Ans



eqn of AC: $L_1L_2 = L_3L_4$

$\Rightarrow \quad (2x+3y)(3x-4y) = (2x+3y-5)(3x-4y-3)$

$\Rightarrow \quad 21x - 11y - 15 = 0$ Ans

(D)

26. If the straight lines, $ax + amy + 1 = 0$, $bx + (m+1)by + 1 = 0$ and $cx + (m+2)cy + 1 = 0$, $m \neq 0$ are concurrent then a,b,c are in :

(A) A.P. only for $m = 1$

(B) A.P. for all m

(C) G.P. for all m

(D) H.P. for all m

Straight lines are concurrent –

$$\begin{vmatrix} a & am & 1 \\ b & (m+1)b & 1 \\ c & (m+2)c & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & am & 1 \\ b & bm & 1 \\ c & cm & 1 \end{vmatrix} + \begin{vmatrix} a & 0 & 1 \\ b & b & 1 \\ c & 2c & 1 \end{vmatrix}$$

$$\Rightarrow 0 + ab + 2bc - cb - 2ac = 0$$

$$ab + bc = 2ac$$

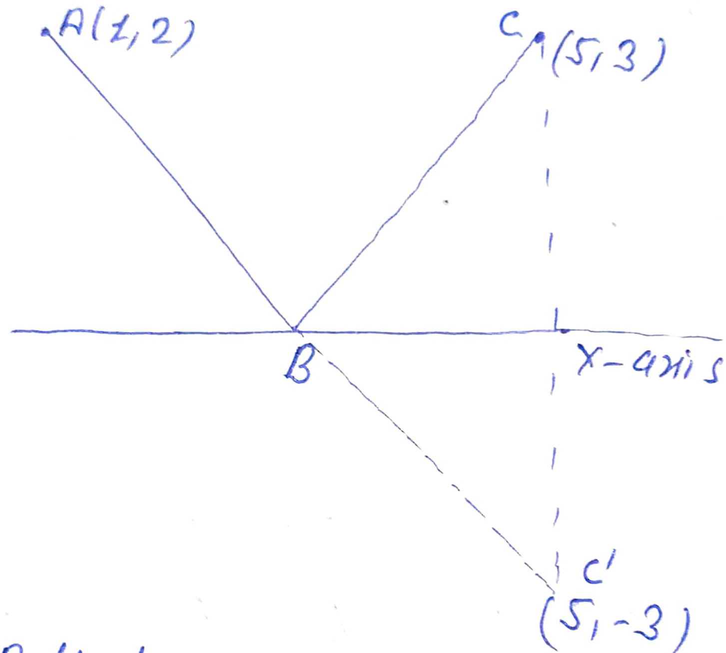
$$\Rightarrow \frac{1}{c} + \frac{1}{a} = \frac{2}{b} \quad \underline{H.P}$$

(D)

27. A ray of light passing through the point A(1,2) is reflected at a point B on the x-axis and then passes through (5,3). Then the equation of AB is :

(A) $5x + 4y = 13$
(B) $5x - 4y = -3$
(C) $4x + 5y = 14$
(D) $4x - 5y = -6$

Ans



Reflection of point C on x-axis is C'

equation of line AB

$$y-2 = \frac{2+3}{1-5}(x-1) \Rightarrow (y-2) = \frac{5}{-4}(x-1)$$

$$\Rightarrow 5x+4y = 13$$

(A)

28. If the line $y = mx$ bisects the angle between the lines $ax^2 + 2h\,xy + by^2 = 0$ then m is a root of the quadratic equation :

(A) $hx^2 + (a - b)x - h = 0$

(B) $x^2 + h(a - b)x - 1 = 0$

(C) $(a - b)x^2 + hx - (a - b) = 0$

(D) $(a - b)x^2 - hx - (a - b) = 0$

Ans

lines $ax^2 + 2hxy + by^2 = 0$

bisectors →

$y = mx$

$$\frac{x^2 - y^2}{a-b} = \frac{xy}{h}$$

$$\Rightarrow \frac{x^2 - m^2x^2}{a-b} = \frac{mx^2}{h}$$

$$\Rightarrow hm^2 + (a-b)m - h = 0$$

So $\Rightarrow hx^2 + (a-b)x - h = 0$

A

29. If the equation $ax^2 - 6xy + y^2 + 2gx + 2fy + c = 0$ represents a pair of lines whose slopes are m and m^2, then sum of all possible values of a is-

(A) 17 (B) –19 (C) 19 (D) –17

Ans

Sum of slopes $= \frac{6}{1} = 6$

$m + m^2 = 6$ —— (i)

product of the slopes $= a$

$m \times m^2 = a$

$m^3 = a$ —— (ii)

(i) $\rightarrow$ $m^2 + m - 6 = 0$

$(m+3)(m-2) = 0$

$m = 2$, $m = -3$

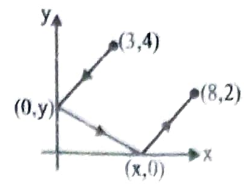
$m = 2 \rightarrow a = 8$

$m = -3 \Rightarrow a = -27$

} sum $= -19$

(B)

30. Suppose that a ray of light leaves the point (3,4), reflects off the y-axis towards the x-axis, reflects off the x-axis, and finally arrives at the point (8,2). The value of x, is-



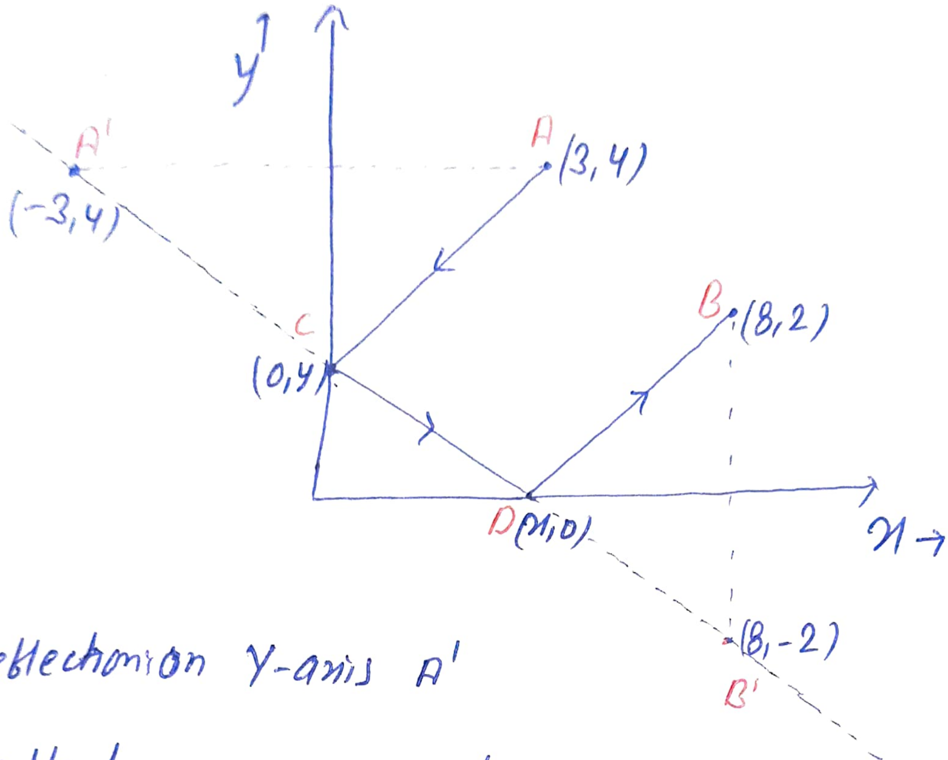
(A) $x = 4\frac{1}{2}$

(B) $x = 4\frac{1}{3}$

(C) $x = 4\frac{2}{3}$

(D) $x = 5\frac{1}{3}$

Ans



A reflection on Y-axis A'

B reflection on x-axis B'

slope A'D = slope DB'

$$\frac{4-0}{-3-x} = \frac{0+2}{x-8}$$

$\Rightarrow 4x - 8\cdot4 = -2x - 6$

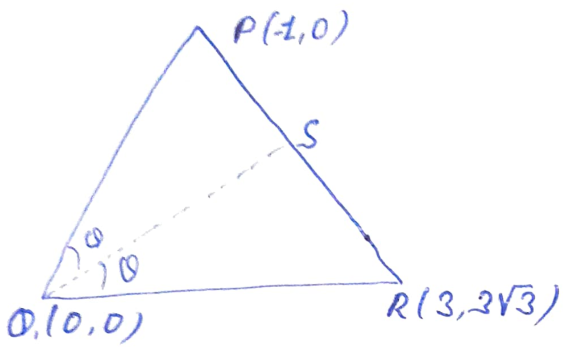
$\Rightarrow 6x = 26$

$x = 4\frac{1}{3}$ (B)

31. Let P(–1,0) Q(0,0) and R$(3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is-

(A) $\sqrt{3}x + y = 0$ (B) $x + \frac{\sqrt{3}}{2}y = 0$ (C) $\frac{\sqrt{3}}{2}x + y = 0$ (D) $x + \sqrt{3}y = 0$

Ans ΔPQR



(QR) Slope of line(Q) $= \frac{3\sqrt{3}-0}{3-0} = \sqrt{3}$

$\alpha = \tan^{-1}(\sqrt{3}) = 60°$

QP line slope $= \frac{0-0}{0+1} = 0$

$\beta = 180°$

angle of line (QS) $= \frac{\alpha+\beta}{2} = \frac{60+180}{2}$

$= 120°$

eqn of line QS $\Rightarrow$ $y - 0 = \tan(120°)(x-0)$

$y = -\cot(30°)(x)$

$y + \sqrt{3}x = 0$

(A)

32. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy=0$, then m is-

(A) $-\frac{1}{2}$ (B) -2 (C) 1 (D) 2

---

lines $my^2+(1-m^2)xy-mx^2=0$

$xy=0 \longrightarrow$ Angle bisectors are $\left.\begin{array}{l} x=y \\ x=-y \end{array}\right\}$

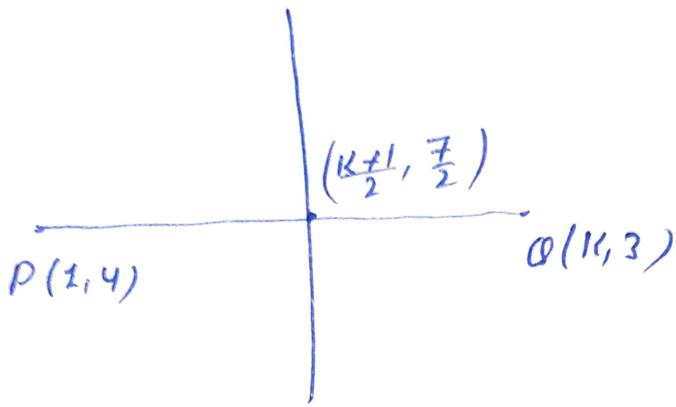
So $m=1$ or $m=-1$

$m=1$ (C)

33. The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y–intercept –4. Then a possible value of k is-

(A) 1 (B) 2 (C) –2 (D) –4

Ans



$y = mx - 4$

m_1 = slope of PQ = $\frac{4-3}{1-k}$

$m_1 \times m = -1 \implies \frac{1}{1-k} m = -1$

$\implies \boxed{m = k-1}$

mid pt of PQ $= \left(\frac{k+1}{2}, \frac{7}{2}\right)$

So $\frac{7}{2} = (k-1)\left(\frac{k+1}{2}\right) - 4$

$\implies k^2 = 16$

$\boxed{k = -4}$

(D)

[MATRIX LIST TYPE]

34. Find the equation to the straight line :

Column-I	Column-II
(P) passing through the point (2, 3) and perpendicular to the straight line $4x - 3y = 10$.	(1) $4y + 11x = 10$
(Q) passing through the point (–6, 10) and perpendicular to the straight line $7x + 8y = 5$.	(2) $4y + 3x = 18$
(R) passing through the point (2, –3) and perpendicular to the straight line joining the points (5, 7) and (–6, 3).	(3) $x + 4y + 16 = 0$
(S) passing through the point (–4, –3) and perpendicular to the straight line joining (1, 3) and (2, 7).	(4) $9y - 8x = 118$

Codes :

	P	Q	R	S
(A)	1	2	3	4
(B)	2	4	1	3
(C)	4	3	2	1
(D)	1	3	4	2

Ans (B)

(P) line $\perp$ to $4x-3y=10$ is $3x+4y=K$

Also passing through (2,3)

$\therefore \quad K = 3(2)+4(3) = 18$

$\therefore \quad 3x+4y=18$

(Q) Line $\perp$ to $7x+8y=5$ is $8x-7y=K$

Also passing through (-6,10)

$\therefore \quad K = 8(-6)-7(10) = -118$

$\therefore \quad 118 = 7y-8x$

(R) $m_1 \times m_2 = -1$

$\left(\frac{7-3}{5+6}\right) \times m_2 = -1 \Rightarrow m_2 = \frac{-11}{4}$

$\Rightarrow \quad y+3 = -\frac{11}{4}(x-2)$

$\Rightarrow \quad 11x+4y=10$

(S) $m_1 \times m_2 = -1$

$\left(\frac{3-7}{1-2}\right) m_2 = -1 \Rightarrow m_2 = -1/4$

$\Rightarrow \quad y+3 = -\frac{1}{4}(x+4)$

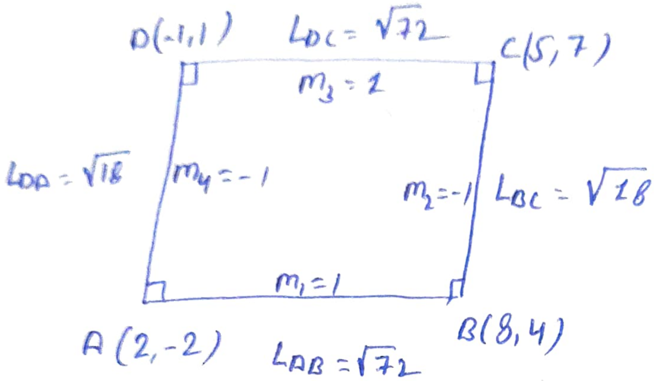
$x+4y+16=0$

[MATRIX MATCH]

35.

Column-I	Column-II
(A) The points (2,–2), (8,4), (5,7) and (–1,1) taken in order constitute the vertices of a	(P) square
(B) The points (0,–1),(2,1),(0,3) and (–2,1) taken in order are the vertices of a	(Q) rectangle
(C) The points (3,–5), (–5,–4),(7,10),(15,9) taken in order are the vertices of a	(R) trapezium
(D) The points (–3,4), (–1,0), (1,0) and (3,4) taken in order are the vertices of a	(S) parallelogram
	(T) cyclic quadrilateral

Ans (A)

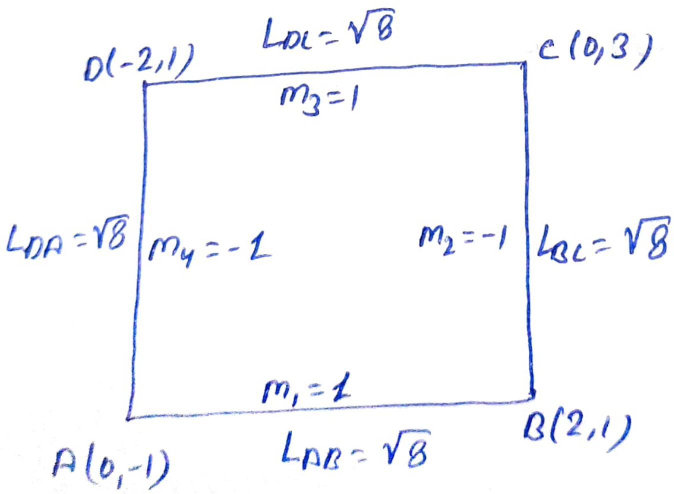


Also $m_{AB} \times m_{BC} = -1$

& $m_{CD} \times m_{DA} = -1$

Ans QST

(B)



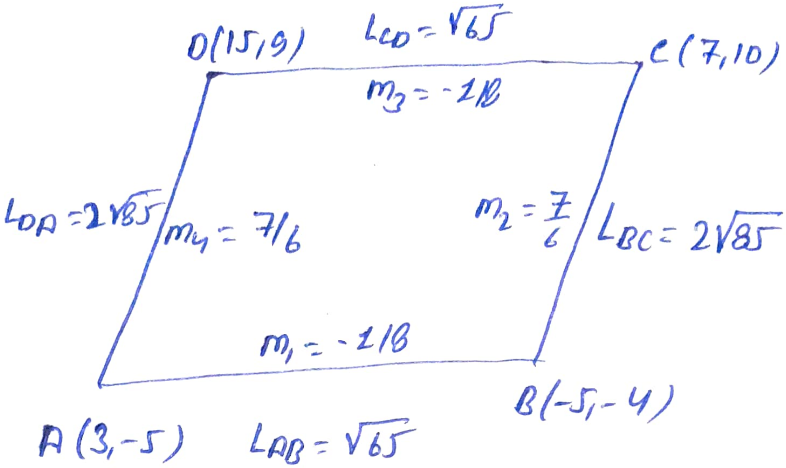
Also

$m_{AB} \times m_{BC} = -1$

& $m_{CD} \times m_{DA} = -1$

Ans PQST

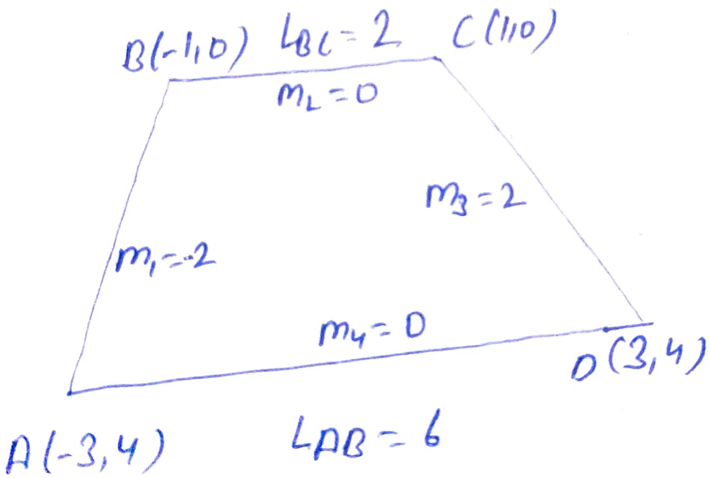
(C)



Also $m_1 = m_3$ & $M_2 = M_4$

Ans S

(D)



Ans RT

36. Equation of Straight Line

	Column-I		Column-II
(A)	Which cuts-off an intercept 4 on the x-axis and passes through the point (2,–3).	(P)	$2x + y + 1 = 0$
(B)	Which cuts-off equal intercepts on the co-ordinate axes and passes through (2,5)	(Q)	$x + y = 7$
(C)	Which makes an angle of 135° with the axis of x and which cuts the axis of y at a distance -8 from the origin and	(R)	$3x - 2y = 12$
(D)	Through the point (4,1) and making with the axes in the first quadrant a triangle whose area is 8.	(S)	$x + 4y = 8$
		(T)	$x + y + 8 = 0$

Ans

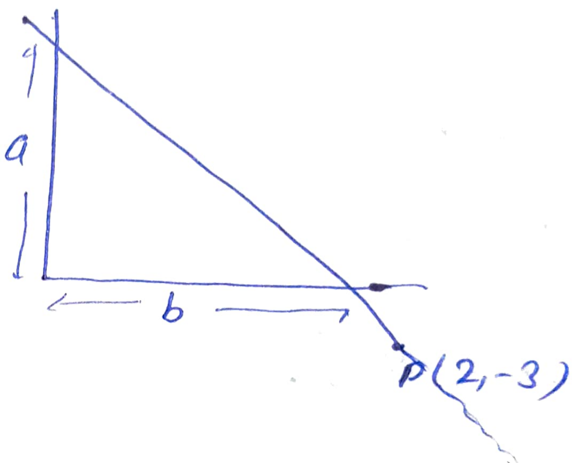
(A)

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{2}{4} - \frac{3}{b} = 1$$

$$b = -6$$

$\Rightarrow$ $3x - 2y = 12$ (R)



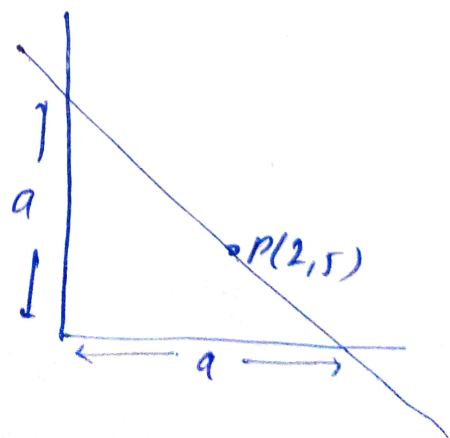
(B)

$$x + y = a$$

$$2 + 5 = a = 7$$

$$x + y = 7$$

(Q)



(C)

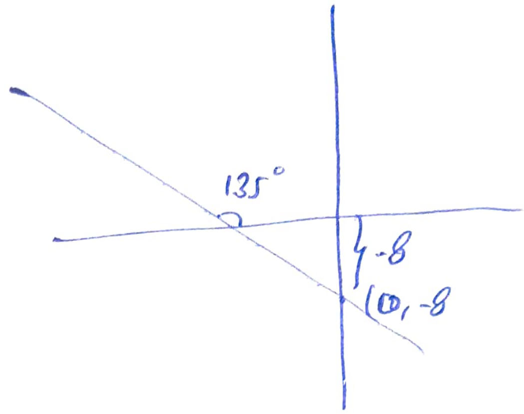
$m = -1$

$c = -8$

$y = -x - 8$

$\Rightarrow \quad y + x + 8 = 0$

T



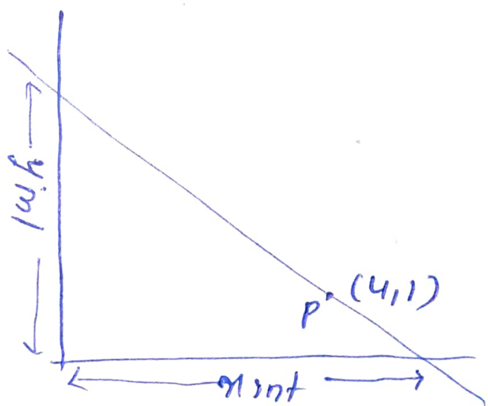
(D)

St line eqn

$y - 1 = m(x - 4)$

$x\,\text{int} = 4 - \frac{1}{m}$

$y\,\text{int} = 1 - 4m$



$\Delta = \frac{1}{2}\left(4 - \frac{1}{m}\right)(1 - 4m)$

$m = -1/4$

$\Rightarrow \quad y - 1 = -\frac{1}{4}(x - 4)$

$\Rightarrow \quad x + 4y = 8$

S

EXERCISE (O-2)

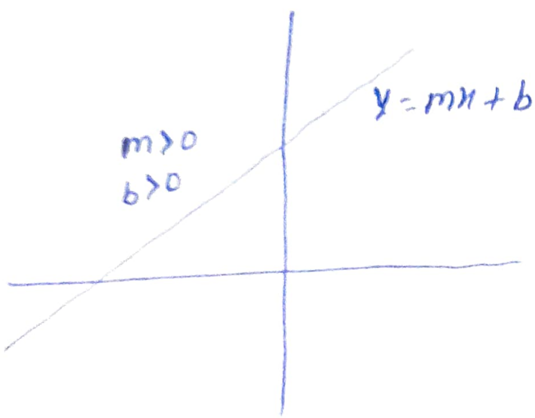
1. If m and b are real numbers and $mb > 0$, then the line whose equation is $y = mx + b$ cannot contain the point-

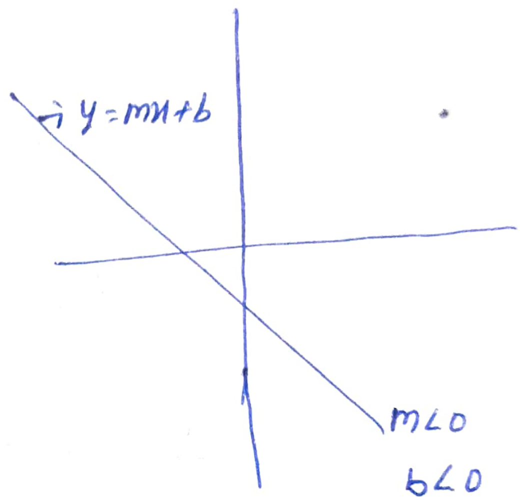
(A) (0,2009) (B) (2009,0) (C) (0,–2009) (D) (20,–100)

$mb > 0$

$m > 0$ & $b > 0$ or $m < 0$, $b < 0$

$y = mx + b$





Clearly see that

+ive x-axis not intersected

by those lines

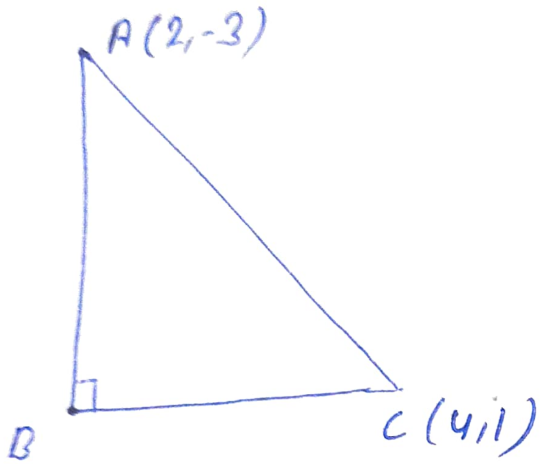
So

(2009, 0)

(B)

2. The vertex of the right angle of a right angled triangle lies on the straight line $2x - y - 10 = 0$ and the two other vertices, at points $(2,-3)$ and $(4,1)$ then the area of triangle in sq. units is-

(A) $\sqrt{10}$ (B) 3 (C) $\frac{33}{5}$ (D) 11



B lies on the line

$2x - y - 10 = 0$

$B(a, 2a-10)$

$\because m_{AB} \cdot m_{BC} = -1$

$\Rightarrow \left(\frac{2a-10+3}{a-2}\right)\left(\frac{2a-10-1}{a-4}\right) = -1$

$a = 5$ or $\frac{19}{5}$ (reject)

$\therefore$ Area of ΔABC

$= \frac{1}{2} AB \cdot AC$

$= \frac{1}{2} \sqrt{18} \cdot \sqrt{2}$

$= 3$ Ans

(B)

3. A triangle ABC is formed by the lines $2x - 3y - 6 = 0$; $3x - y + 3 = 0$ and $3x + 4y - 12 = 0$. If the points $P(\alpha,0)$ and $Q(0,\beta)$ always lie on or inside the ΔABC, then :

(A) $\alpha \in [-1,2]$ and $\beta \in [-2,3]$

(B) $\alpha \in [-1,3]$ and $\beta \in [-2,4]$

(C) $\alpha \in [-2,4]$ and $\beta \in [-3,4]$

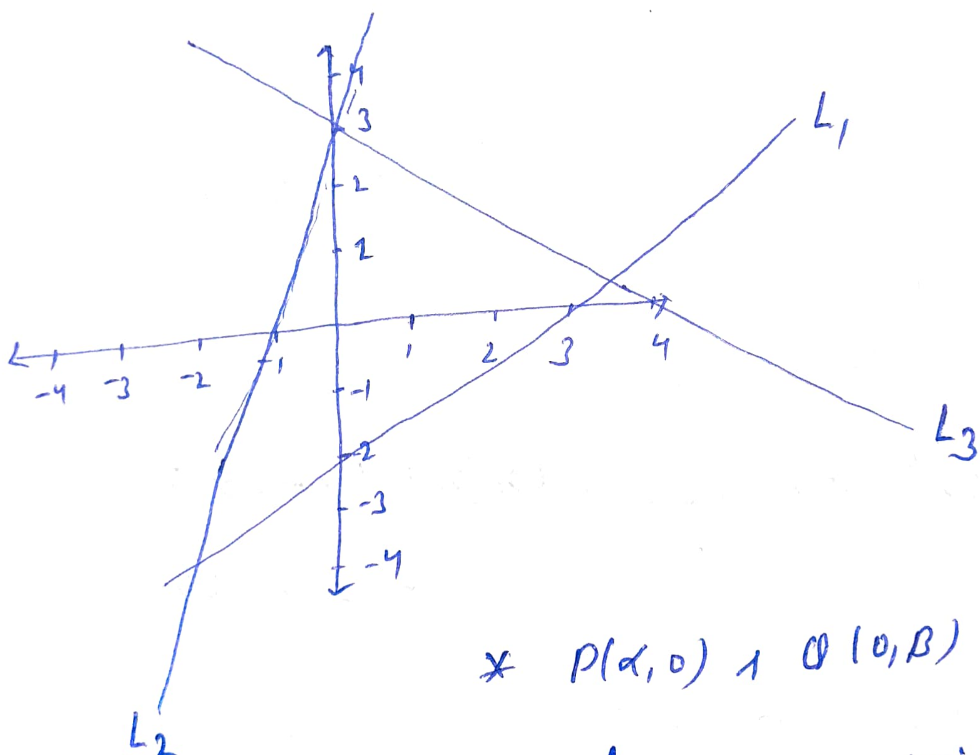
(D) $\alpha \in [-1,3]$ and $\beta \in [-2,3]$

Ans

$L_1 : \frac{x}{3} + \frac{y}{-2} = 1$

$L_2 : \frac{x}{-1} + \frac{y}{3} = 1$

$L_3 : \frac{x}{4} + \frac{y}{3} = 1$



* $P(\alpha, 0)$ & $Q(0,\beta)$ lies on or inside Δ

* P lie on x-axis & Q lie on y-axis

$\therefore \alpha \in [-1, 3]$

& $\beta \in [-2, 3]$ (D)

4. If the straight lines joining the origin and the points of intersection of the curve

$5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$

are equally inclined to the x- axis then the value of k :

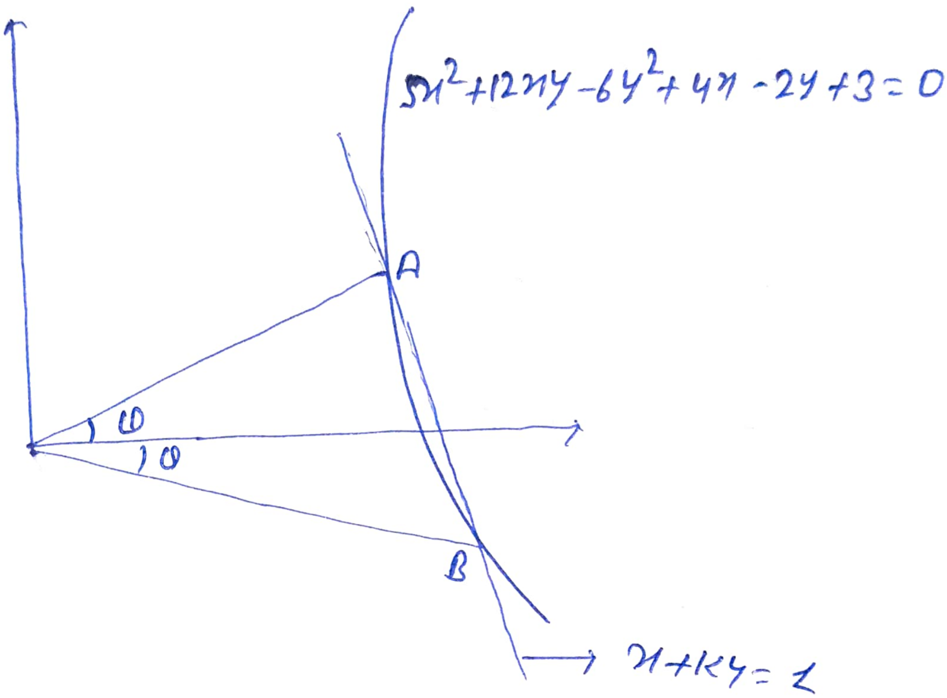
(A) is equal to 1

(B) is equal to –1

(C) is equal to 2

(D) does not exist in the set of real numbers

Ans



Pair eqn of OA & OB

{Homogenization}

$$5x^2 + 12xy - 6y^2 + 4x(x+ky) - 2y(x+ky) + 3(x+ky)^2 = 0$$

$$\Rightarrow \quad 12x^2 + (10k+10)xy + (3k^2 - 2k - 6)y^2 = 0$$

$\because$ lines are equally inclined with x-axis

$\therefore$ Coefficient of $xy = 0$

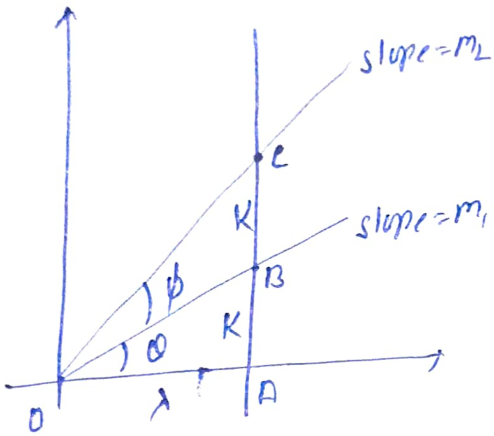
$10k + 10 = 0$ $\quad \boxed{k = -1}$

(B)

5. Through a point A on the x-axis a straight line is drawn parallel to y-axis so as to meet the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ in B and C. If AB = BC then-

(A) $h^2 = 4ab$ (B) $8h^2 = 9ab$ (C) $9h^2 = 8ab$ (D) $4h^2 = ab$

Ans



OB & OC pair of st. line

$ax^2+2hxy+by^2=0$

Let. AB = BC = K

& OA = λ

$m_1 = \tan\theta = \frac{K}{\lambda}$ & $m_2 = \tan(\theta+\phi) = \frac{2K}{h}$

$\Rightarrow$ $m_2 = 2m_1$

$\frac{m_2}{m_1} = 2 \Rightarrow \frac{m_2+m_1}{m_2-m_1} = \frac{2+1}{2-1} = 3$

$\Rightarrow \left(\frac{m_2+m_1}{m_2-m_1}\right)^2 = 9$

$\Rightarrow \frac{(m_1+m_2)^2}{(m_1+m_2)^2 - 4m_1m_2} = 9$

$\Rightarrow \frac{(-2h/b)^2}{\left(\frac{-2h}{b}\right)^2 - 4\left(\frac{a}{b}\right)} = 9$

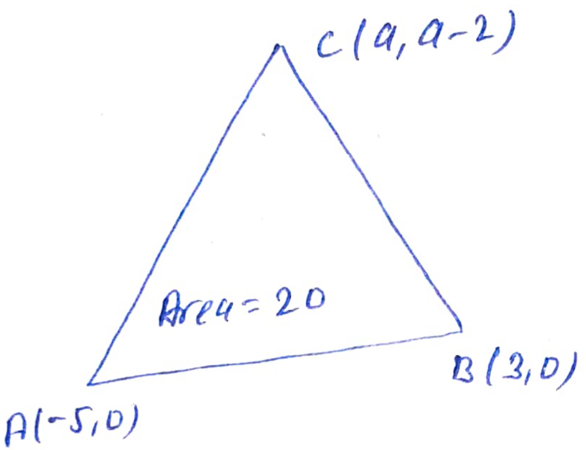
$\Rightarrow 8h^2 = 9ab$ (B)

[MULTIPLE CHOICE]

6. The area of triangle ABC is 20 square units. The co-ordinates of vertex A are (–5,0) and B are (3,0). The vertex C lies on the line, x – y = 2. The co-ordinates of C are -

(A) (5,3) (B) (–3,–5) (C) (–5,–7) (D) (7,5)

Ans.



C lies on → x − y = 2

id,

$$\frac{1}{2}\begin{vmatrix} a & a-2 & 1 \\ -5 & 0 & 1 \\ 3 & 0 & 1 \end{vmatrix} = 20$$

$\Rightarrow \quad a = 7 \quad$ or $\quad a = -3$

C [7, 5] or [−3, −5]

B, D

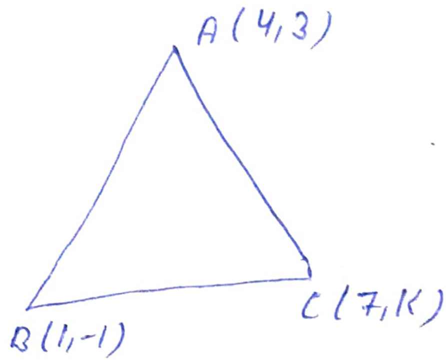
7. Three vertices of a triangle are A(4,3); B(1,–1) and C(7,k). Value(s) of k for which centroid, orthocentre, incentre and circumcentre of the ΔABC lie on the same straight line is/are-

(A) 7 (B) –1 (C) –19/8 (D) none

Any

$\because$ Only in Isosceles Δ the points are collinear

✗ $AB = BC$



$\sqrt{25} = \sqrt{36+(K+1)^2}$

No value of K

✗ $BC = CA$

$\sqrt{36+(K+1)^2} = \sqrt{9+(K-3)^2}$

$K = -19/8$

✗ $CA = AB$

$\sqrt{9+(K-3)^2} = \sqrt{25}$

$K = 7,$ or -1

But, for $K = 7$ points A, B, C(7,7) are collinear

($K = 7$ reject)

$$\boxed{\begin{array}{l} K = -1 \\ \text{or } K = -\dfrac{19}{8} \end{array}}$$

B, C

8. A line passes through the origin and makes an angle of $\pi/4$ with the line $x - y + 1 = 0$. Then :

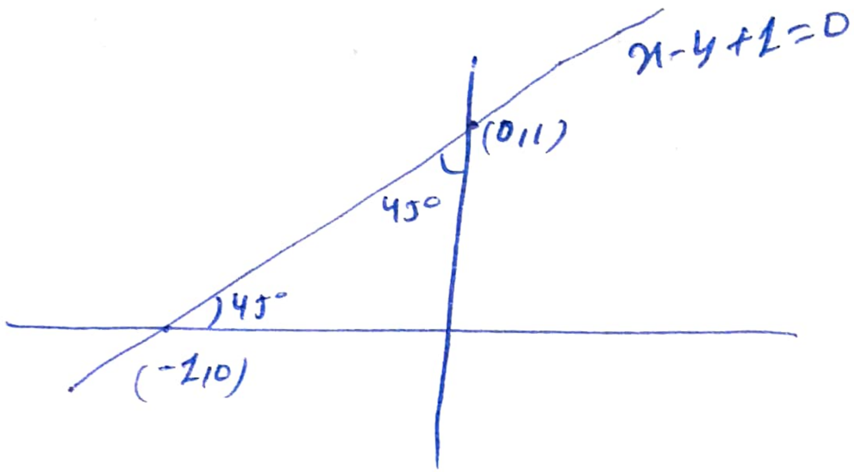
(A) equation of the line is $x = 0$

(B) the equation of the line is $y = 0$

(C) the point of intersection of the line with the given line is $(-1,0)$

(D) the point of intersection of the line with the given line is $(0,1)$

Ans



Req^d line can be x-axis

So or y-axis

$x = 0$

$y = 0$

9. Equation of a straight line passing through the point (2,3) and inclined at an angle of arc tan $\frac{1}{2}$ with the line $y + 2x = 5$ is-

(A) $y = 3$ (B) $x = 2$ (C) $3x + 4y - 18 = 0$ (D) $4x + 3y - 17 = 0$

Ans

$y + 2x = 5$ $m_1 = -2$

Let $m_2 = m$

So $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\frac{1}{2} = \left|\frac{m+2}{1-2m}\right|$

$m = -3/4$ | $2m + 4 = -1 + 2m$

⊥ to x-axis

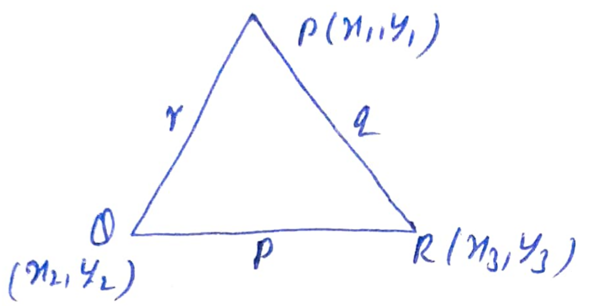
$y - 3 = -\frac{3}{4}(x - 2)$

$3x + 4y = 18$ or $x = 2$

10. If the vertices P,Q,R of a triangle PQR are rational points, which of the following points of the triangle PQR is/are always rational point(s) ?

(A) centroid (B) incentre (C) circumcentre (D) orthocentre

Ans



Given (x_1, y_1), (x_2, y_2), (x_3, y_3) Rational.

Ⓐ Centroid $G\left[\frac{\Sigma x}{3}, \frac{\Sigma y}{3}\right]$ Rational

(B) Incenter $I\left[\frac{px_1+qx_2+rx_3}{p+q+r}, \frac{py_1+qy_2+ry_3}{p+q+r}\right]$

$p, q, r \rightarrow$ may or may not rational

So $p+q+r$ is not always rational

Ⓒ Circumcenter (O) → Given that, vertices are rational. So slope of sides must be rational. Then coefficient of $\perp^r$ Bisectors of sides also must be rational. By solving eqn of $\perp^r$ bisectors, we get co-ordinate of circumcenter which must be rational.

Ⓓ Orthocenter (H) →

slope of sides must be rational. Then coefficient of altitudes must be rational. Co-ordinate of orthocenter → rational.

A C D

11. The lines L_1 and L_2 denoted by $3x^2 + 10xy + 8y^2 + 14x + 22y + 15 = 0$ intersect at the point P and have gradients m_1 and m_2 respectively. The acute angles between them is θ. Which of the following relations hold good ?

(A) $m_1 + m_2 = 5/4$

(B) $m_1 m_2 = 3/8$

(C) acute angle between L_1 and L_2 is $\sin^{-1}\left(\frac{2}{5\sqrt{5}}\right)$.

(D) sum of the abscissa and ordinate of the point P is -1.

Ans

$8y^2 + 10xy + 3x^2 = 0$

$m_1 + m_2 = \frac{-10}{8} = -5/4$

& $m_1 \cdot m_2 = 3/8$

angle b/w line is

$\tan\theta = \frac{\sqrt{\frac{25}{16} - \frac{24}{16}}}{1 + 3/8} = 2/11$

$\sin\theta = \frac{2}{\sqrt{125}} = \frac{2}{5\sqrt{5}}$

$\Rightarrow$ $6x + 10y + 14 = 0$

$10x + 16y + 22 = 0$

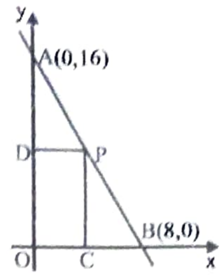
on solving $x = 1$

$y = -2$

Sum $= -2 + 1 = -1$

Paragraph for Question Nos. 12 to 14

In the diagram, a line is drawn through the points A(0,16) and B(8,0). Point P is chosen in the first quadrant on the line through A and B. Points C and D are chosen on the x and y axis respectively, so that PDOC is a rectangle.



12. Perpendicular distance of the line AB from the point (2, 2) is -

(A) $\sqrt{4}$ (B) $\sqrt{10}$

(C) $\sqrt{20}$ (D) $\sqrt{50}$

13. Sum of the coordinates of the point P if PDOC is a square is -

(A) $\frac{32}{3}$ (B) $\frac{16}{3}$ (C) 16 (D) 11

14. Number of possible ordered pair(s) of all positions of the point P on AB so that the area of the rectangle PDOC is 30 sq. units, is-

(A) three (B) two (C) one (D) zero

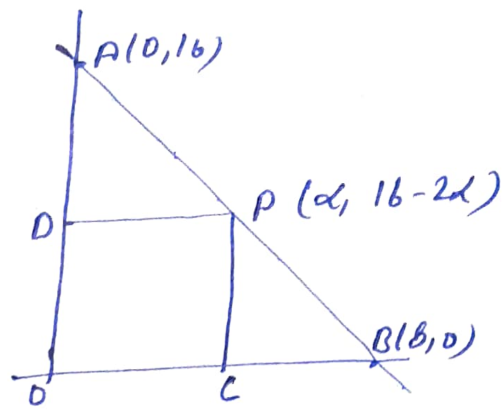
---

(12) line AB

$\Rightarrow \frac{x}{8} + \frac{y}{16} = 1$

$\Rightarrow 2x + y = 16$

$P(\alpha, 16-2\alpha)$



length of $\perp^r$ from (2,2)

$\left| \frac{4+2-16}{\sqrt{4+1}} \right| = 2\sqrt{5}$ (C)

$\boxed{\begin{matrix} \alpha > 0 \\ 0 < 16-2\alpha < 8 \end{matrix}}$

(13) $\alpha = 16 - 2\alpha \Rightarrow \alpha = \frac{16}{3}$

At $P\left(\frac{16}{3}, \frac{16}{3}\right)$ sum $= \frac{32}{3}$ Ans

(A)

(14) $\alpha \cdot (16 - 2\alpha) = 30$

$\Rightarrow 16\alpha - 2\alpha^2 = 30 \Rightarrow \alpha^2 - 8\alpha + 15 = 0$

$D < 0$

No value of α

(D)

Paragraph for question nos. 15 and 16

An equilateral triangle ABC has its centroid at the origin and the base BC lies along the line $x + y = 1$.

15. Area of the equilateral ΔABC is -

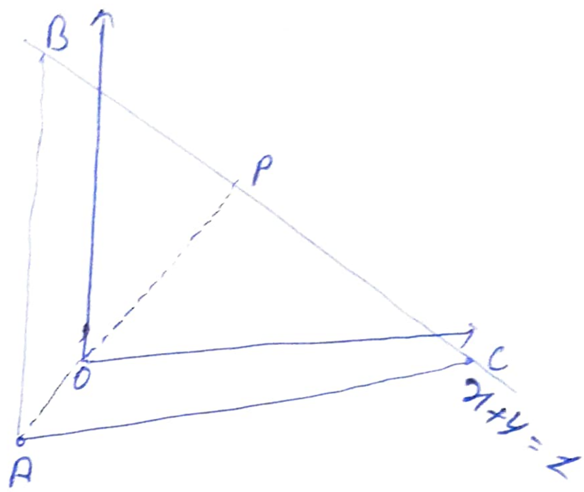
(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{3\sqrt{3}}{4}$ (C) $\frac{3\sqrt{2}}{2}$ (D) $\frac{2\sqrt{3}}{4}$

16. Gradient of the other two lines are -

(A) $\sqrt{3}, \sqrt{2}$ (B) $\sqrt{3}, \frac{1}{\sqrt{3}}$ (C) $\sqrt{2}+1, \sqrt{2}-1$ (D) $2+\sqrt{3}, 2-\sqrt{3}$

Ans (a)

(15)



$OP = \frac{1}{\sqrt{2}}$

$\therefore AP = \frac{3}{\sqrt{2}}$

In ΔAPC

$\tan 60^\circ = \frac{AP}{PC}$

$\Rightarrow PC = \frac{3}{\sqrt{2}} \times \frac{1}{\sqrt{3}} = \sqrt{\frac{3}{2}}$

$\therefore$ Side $AB = BC = CA = \sqrt{6}$

Area of $\Delta ABC = \frac{\sqrt{3}}{4} \times 6 = \frac{3\sqrt{3}}{2}$

(16) let slope of other side line is m

$\therefore \tan 60^\circ = \left| \frac{m+1}{1-m} \right| \Rightarrow \frac{m+1}{1-m} = \pm\sqrt{3}$

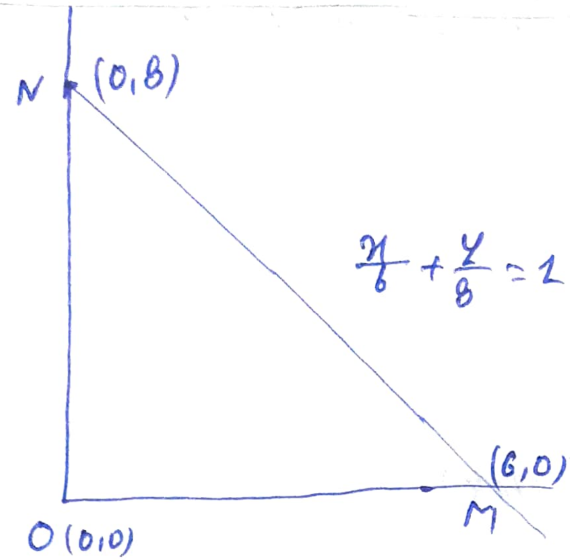
$\Rightarrow \frac{m+1}{1-m} = \sqrt{3}$ or $\frac{m+1}{1-m} = -\sqrt{3}$

$m = 2-\sqrt{3}$, $m = 2-\sqrt{3}$ Ans

EXERCISE (S-1)

1. Line $\frac{x}{6}+\frac{y}{8}=1$ intersects the x and y axes at M and N respectively. If the coordinates of the point P lying inside the triangle OMN (where 'O' is origin) are (a, b) such that the areas of the triangle POM, PON and PMN are equal. Find the coordinates of the point P.

Ans 1



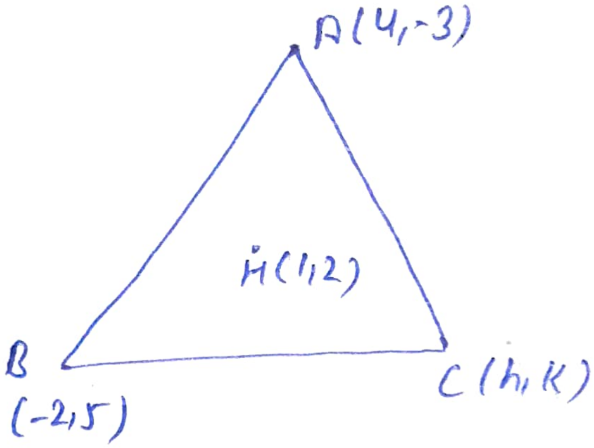
Point 'P' is centroid of Δ OMN.

$$\therefore\ P(a,b) \equiv \left(\frac{0+6+0}{3},\ \frac{0+0+8}{3}\right)$$

$$\equiv \left(2,\ 8/3\right)$$

2. Two vertices of a triangle are $(4, -3)$ & $(-2, 5)$. If the orthocentre of the triangle is at $(1, 2)$, find the coordinates of the third vertex.

Let co-ordinates of third vertex be (h, k)



$\Rightarrow \quad m_{AH} \cdot m_{BC} = -1$

$\Rightarrow \left(-\frac{5}{3}\right)\left(\frac{K-5}{h+2}\right) = -1$

$\Rightarrow \quad 3h - 5K + 31 = 0 \quad \text{—— (1)}$

Also $\quad m_{CH} \cdot m_{AB} = -1$

$\Rightarrow \left(\frac{K-2}{h-1}\right)\left(\frac{8}{-6}\right) = -1$

$\Rightarrow \quad 3h - 4K + 5 = 0 \quad \text{—— (2)}$

from (1) & (2)

$h = 33$
$K = 26$ } Ans

Ans. 3.

3. The point A divides the join of P(–5, 1) & Q(3, 5) in the ratio K : 1. Find the two values of K for which the area of triangle ABC, where B is (1, 5) & C is (7, –2), is equal to 2 units in magnitude.

Cordinates of A $\left(\frac{3K-5}{K+1} , \frac{5K+1}{K+1} \right)$

$$\text{Area of } \Delta ABC = \frac{1}{2} \begin{vmatrix} 1 & 5 \\ 7 & -2 \\ \frac{3K-5}{K+1} & \frac{5K+1}{K+1} \\ 1 & 5 \end{vmatrix} = 2$$

$$\Rightarrow -2 - 35 + \frac{35K+7}{K+1} \cancel{+} \frac{6K-10}{K+1} + \frac{15K-25}{K+1} - \frac{5K+1}{K+1} = \pm 4$$

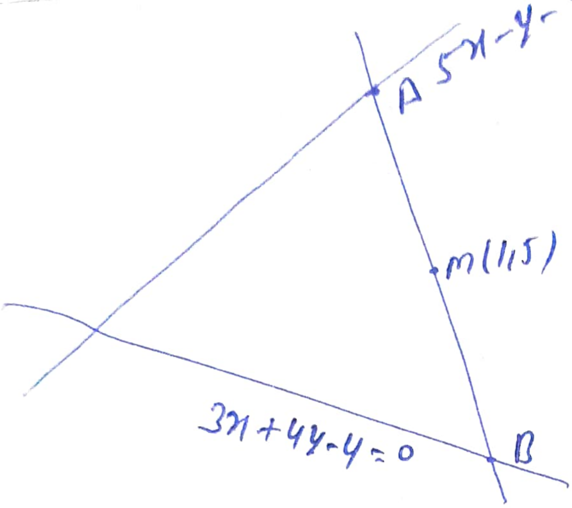
$K = 7$

or

$K = 31/9$

4. A line is such that its segment between the straight lines $5x - y - 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain the equation.

Ans. 4



$A(x_1, 5x_1-4)$

$B\left(x_2, \frac{4-3x_2}{4}\right)$

$\frac{x_1+x_2}{2} = 1$ $\frac{5x_1 - 4 + \frac{4-3x_2}{4}}{2} = 5$

$x_1 = \frac{58}{23}$

pt $A\left(\frac{58}{23}, \frac{198}{23}\right)$

eqn of line AM is

$$y - 5 = \frac{\frac{198}{23} - 5}{\frac{58}{23} - 1}(x-1)$$

$\Rightarrow$ $83x - 35y + 92 = 0$ Ans

|

5. The area of a triangle is 5. Two of its vertices are (2, 1) & (3, –2). The third vertex lies on y = x + 3. Find the third vertex.

Ans. 5

third vertex be (x_1, y_1), then $y_1 = x_1 + 3$

$$\frac{1}{2}\left|\begin{matrix} 2 & 1 \\ 3 & -2 \\ x_1 & x_1+3 \\ 2 & 1 \end{matrix}\right| = 5$$

$$\Rightarrow \quad -4 - 3 + 3x_1 + 9 + 2x_1 + x_1 - 2x_1 - 6 = \pm 10$$

$$x_1 = \frac{7}{2} \text{ or } -\frac{3}{2}$$

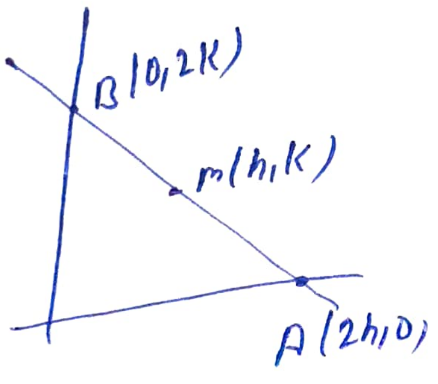
$$(x_1, y_1) = \left(\frac{7}{2}, \frac{13}{2}\right) \text{ or } \left(-\frac{3}{2}, \frac{3}{2}\right)$$

6. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ & $\frac{x}{b}+\frac{y}{a}=1$, meets the coordinate axes in A & B. Find the locus of the mid point of AB.

A. 6. Point of intersection of given lines is

$$\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$$

Now let the line passing through this point cuts cordinate Axis at A & B Now let mid pt of AB is (h,k)



Thus eqn of line AB is

$$\frac{x}{2h}+\frac{y}{2k}=1$$

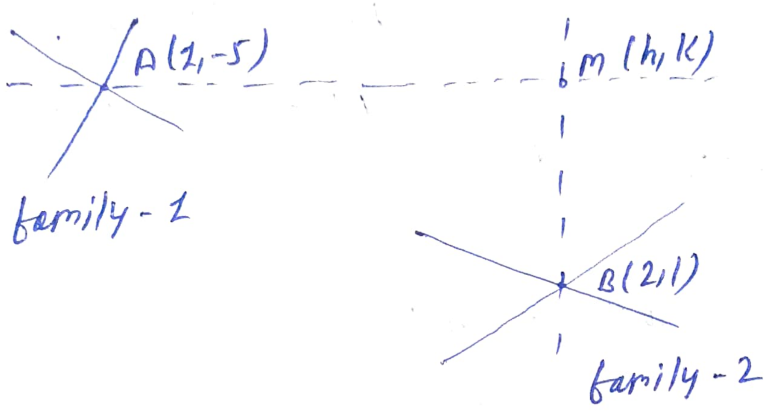
passes $\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$

so $$\frac{ab}{2h(a+b)}+\frac{ab}{2k(a+b)}=1$$

$$\Rightarrow \frac{1}{x}+\frac{1}{y}=\frac{2(a+b)}{ab} \quad \text{Ar}$$

Ans 7

7. Consider the family of lines $(x - y - 6) + \lambda(2x + y + 3) = 0$ and $(x + 2y - 4) + \lambda(3x - 2y - 4) = 0$. If the lines of these 2 families are at right angle to each other then find the locus of their point of intersection.



Family 1 pass the pt $A(1,-5)$ &

family 2 pass pt $B(2,1)$

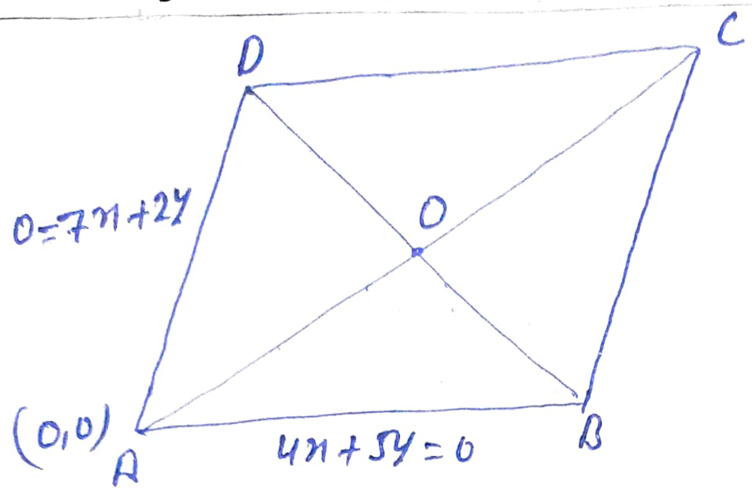
Now slope of $AM \times$ slope of $BM = -1$

$$\Rightarrow \quad \frac{k+5}{h-1} \times \frac{k-1}{h-2} = -1$$

$$\Rightarrow \quad h^2 + k^2 - 3h + 4k - 3 = 0$$

Locus $\rightarrow$ $x^2 + y^2 - 3x - 4y - 3 = 0$

8. Two consecutive sides of a parallelogram are $4x + 5y = 0$ & $7x + 2y = 0$. If the equation to one diagonal is $11x + 7y = 9$, find the equation to the other diagonal.



$BD \rightarrow 11x + 7y = 9$ — ①

$AB \rightarrow 4x + 5y = 0$ — ②

$AD \rightarrow 7x + 2y = 0$ — ③

Solve ① & ②

$B\left(\frac{5}{3}, -\frac{4}{3}\right)$

Solve ① & ③

$D\left(-\frac{2}{3}, \frac{7}{3}\right)$

pt O is mid point of B & D

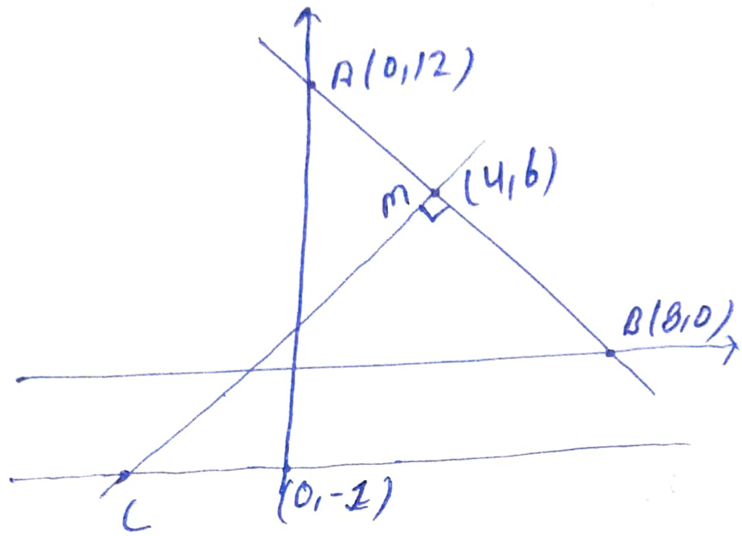
$O\left(\frac{1}{2}, \frac{1}{2}\right)$

Line AO or line AC

$\boxed{y = x}$

9. The line $3x + 2y = 24$ meets the y-axis at A & the x-axis at B. The perpendicular bisector of AB meets the line through $(0, -1)$ parallel to x-axis at C. Find the area of the triangle ABC.

Ans 9



eq^n of $\perp^r$ bisector of AB

$$y - 6 = \frac{2}{3}(x - 4)$$

$$\Rightarrow \quad 2x - 3y + 10 = 0 \quad \text{——} \; ①$$

eq^n of line passing through $(0,-1)$ & slope $= 0$

$$y = -1 \quad \text{—} \; ②$$

intersection ① & ② point

$$C \equiv \left(-\frac{13}{2}, -1\right)$$

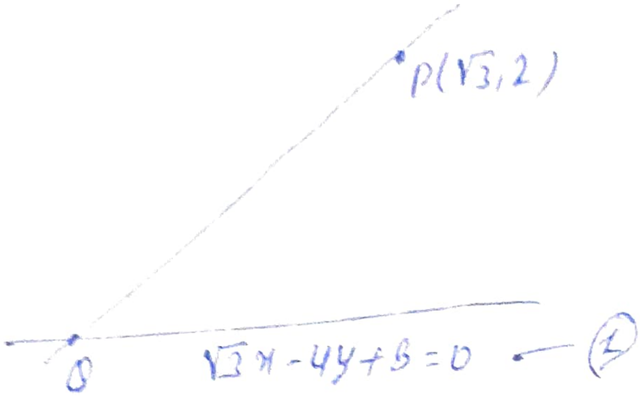
Now

$$\frac{1}{2}\left|\begin{matrix} 0 & 12 \\ 8 & 0 \\ -\frac{13}{2} & -1 \\ 0 & 12 \end{matrix}\right|$$

$= 91$ Ans

10. If the straight line drawn through the point $P(\sqrt{3}, 2)$ & inclined at an angle $\frac{\pi}{6}$ with the x-axis, meets the line $\sqrt{3}x - 4y + 8 = 0$ at Q. Find the length PQ.

Ans (10)



eqⁿ of line PQ is

$$y - 2 = \tan\left(\frac{\pi}{6}\right)(x - \sqrt{3})$$

$$\Rightarrow y - 2 = \frac{1}{\sqrt{3}}(x - \sqrt{3})$$

$$\Rightarrow x - \sqrt{3}y + \sqrt{3} = 0 \quad \text{— (2)}$$

Solve (1) & (2)

$$Q\,(4\sqrt{3}, 5)$$

$$\therefore PQ = \sqrt{(4\sqrt{3} - \sqrt{3})^2 + (5-2)^2}$$

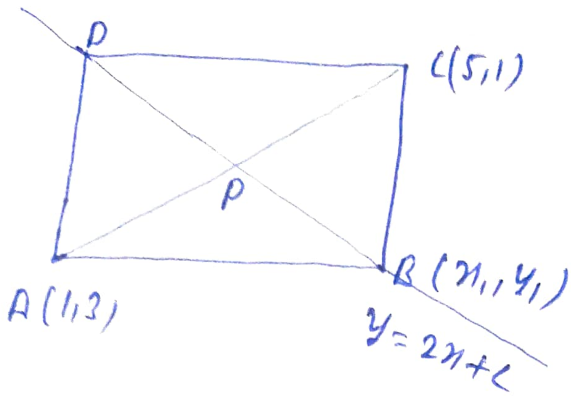
$$= \sqrt{36}$$

$$= 6 \text{ units}$$

11. The points (1, 3) & (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the line y = 2x + c. Find c & the remaining vertices.

Ans. (11)

Ans



$$P = \frac{A+C}{2} = (3,2)$$

P lies on $y = 2x + c$

$$\Rightarrow 2 = 2(3) + c \quad [c = -4]$$

$$\boxed{c = -4}$$

point B lies on the given line $\Rightarrow$

$$2x_1 - y_1 - 4 = 0 \quad \text{— (1)}$$

In ΔABC

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (5-1)^2 + (1-3)^2 = (x_1-1)^2 + (y_1-3)^2 + (x_1-5)^2 + (y_1-1)^2$$

$$\Rightarrow 2x_1^2 + 2y_1^2 - 12x_1 - 8y_1 + 16 = 0$$

$$\Rightarrow x_1^2 + y_1^2 - 6x_1 - 4y_1 + 8 = 0 \quad \text{— (2)}$$

Solve (1) & (2)

B & D $\quad (2,0) \quad \&\quad (4,4)$ Ans

Ans 12

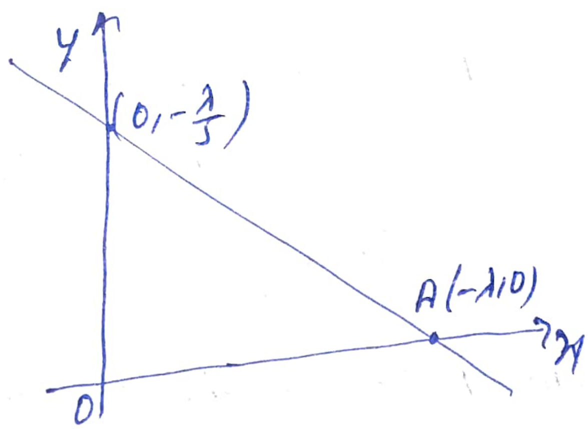
12. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L & the coordinate axes is 5. Find the equation of the line.

Given line $\rightarrow$ $5x - y = 1$ — (1)

Let L is $\perp^r$ to given line

then $x + 5y + \lambda = 0$ — (2)

Put $\begin{cases} x = 0 \Rightarrow y = \frac{-\lambda}{5} \\ y = 0 \Rightarrow x = -\lambda \end{cases}$



$\therefore$ Area of ΔOAB

$= \frac{1}{2}(OA)(OB)$

$\Rightarrow \frac{1}{2} |-\lambda| \left|\frac{-\lambda}{5}\right| = 5$

$\Rightarrow \lambda^2 = 5 \times 5 \times 2$

$\lambda = \pm 5\sqrt{2}$

Put in (2)

$x + 5y \pm 5\sqrt{2} = 0$ Ans

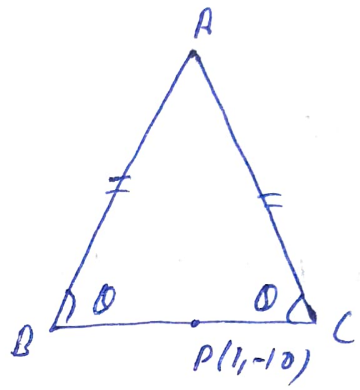
Ans. 13

13. Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ & its third side passes through the point $(1, -10)$. Determine the equation of the third side.

Eqⁿ of AB : $7x - y + 3 = 0$ — (1)

Eqⁿ of AC : $x + y - 3 = 0$ — (2)

$\because$ AB = AC



m_1 = slope of AB = 7

m_2 = slope of AC = −1

Let m is the slope of BC

$$\therefore \tan\theta = \left|\frac{m_1 - m}{1 + m_1 m}\right| = \left|\frac{m_2 - m}{1 + m_2 m}\right|$$

$$\Rightarrow \frac{7 - m}{1 + 7m} = \pm\left(\frac{-1 - m}{1 - m}\right)$$

(+) $\Rightarrow$ $m \Rightarrow$ imaginary

(−) $\Rightarrow$ $m = -3, \frac{1}{3}$

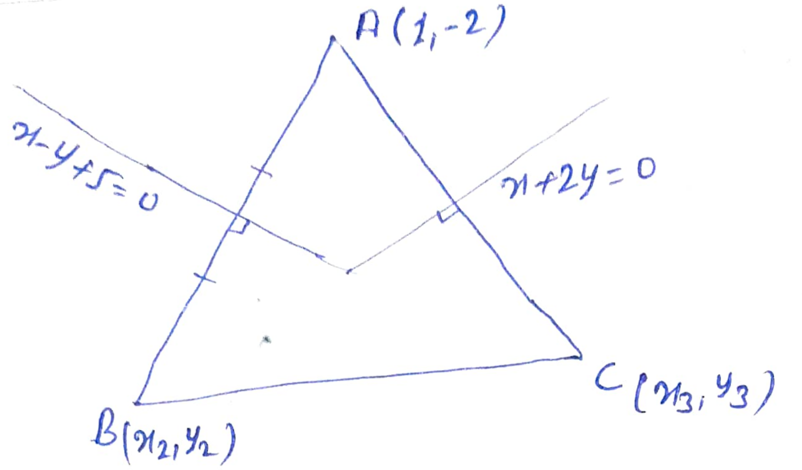
eqⁿ of side BC

$\Rightarrow y + 10 = m(x - 1)$

$$\left[\begin{array}{c} 3x + y + 7 = 0 \\ x - 3y = 31 \end{array}\right]$$ Ans

Q.14

14. The equations of the perpendicular bisectors of the sides AB & AC of a triangle ABC are $x - y + 5 = 0$ & $x + 2y = 0$, respectively. If the point A is $(1, -2)$ find the equation of the line BC.



Point B is the reflection of pt A w.r.t line $x-y+5=0$

$$\frac{x_2-1}{1} = \frac{y_2+2}{-1} = \frac{-2(1+2+5)}{1+1}$$

$x_2 = -7$, $y_2 = 6$ $B(-7, 6)$

Similarly C is the reflection of pt A wrt line $x+2y=0$

$$\frac{x_3-1}{1} = \frac{y_3+2}{2} = \frac{-2[1+2(-2)]}{1^2+2^2}$$

$x_3 = \frac{11}{5}$, $y_3 = 2/5$

$C\left(\frac{11}{5}, \frac{2}{5}\right)$

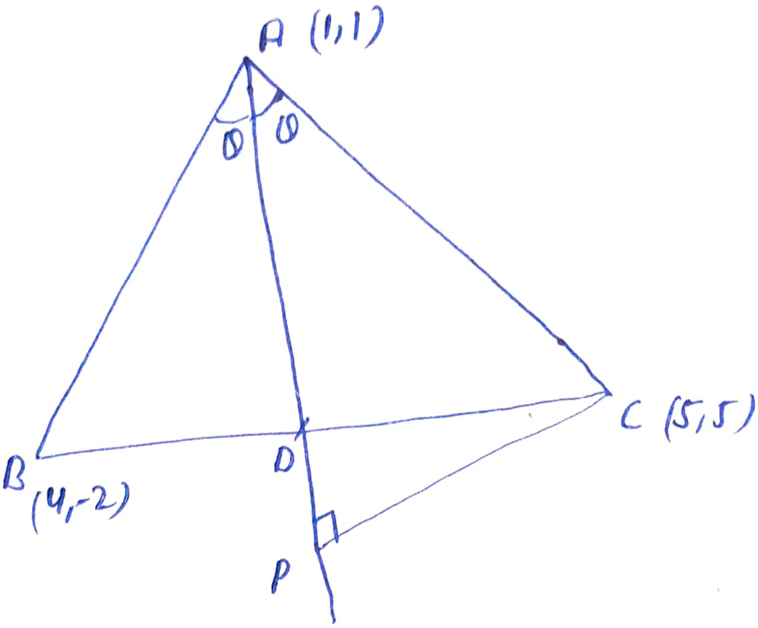
Eq^n of line BC →

$$y-6 = \frac{\frac{2}{5}-6}{\frac{11}{5}+7}(x+7)$$

$14x + 23y = 40$ Ans

15. Given vertices A(1, 1), B(4, −2) & C(5, 5) of a triangle, find the equation of the perpendicular dropped from C to the interior bisector of the angle A.

Ans. 15



slope of line AC ⇒ $m_1 = 1$

slope of line AB ⇒ $m_2 = -1$

∴ slope of line AP ⇒ $m_3 \to \infty$

So slope of line CP ⇒ $m_4 = 0$

line CP $x = k$

$x = k$ satisfy pt C(5,5)

$\boxed{x = 5}$ Ans.

16. Consider a triangle ABC with sides AB and AC having the equations $L_1 = 0$ and $L_2 = 0$. Let the centroid, orthocentre and circumcentre of the ΔABC are G, H and S respectively. $L = 0$ denotes the equation of sides BC.

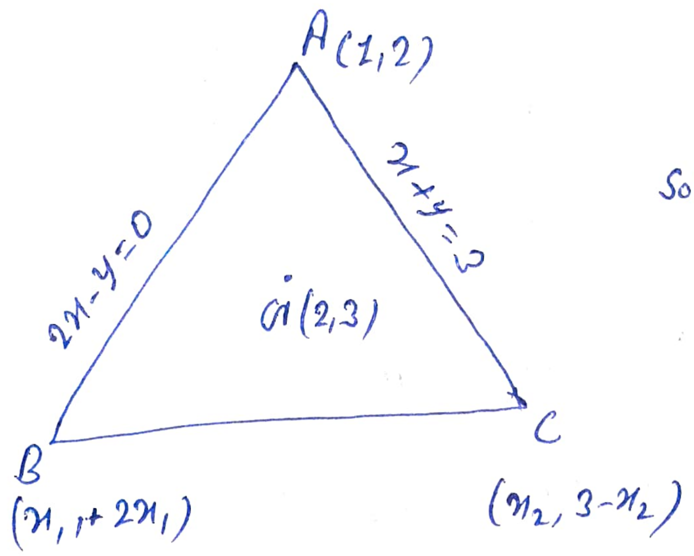
(a) If $L_1 : 2x - y = 0$ and $L_2 : x + y = 3$ and G(2, 3) then find the slope of the line $L = 0$.

(b) If $L_1 : 2x + y = 0$ and $L_2 : x - y + 2 = 0$ and H(2, 3) then find the y-intercept of $L = 0$.

(c) If $L_1 : x + y - 1 = 0$ and $L_2 : 2x - y + 4 = 0$ and S(2, 1) then find the x-intercept of the line L = 0.

Ans 16

(a)



Solve:

$2x - y = 0$

$x + y = 3$

$\downarrow$

$A(1, 2)$

$$G \equiv \left(\frac{x_1 + x_2 + 1}{3}, \frac{2x_1 + 2 + 3 - x_2}{3} \right) \equiv (2, 3)$$

$x_1 + x_2 = 5$

$2x_1 - x_2 = 4$

$x_1 = 3$

$x_2 = 2$

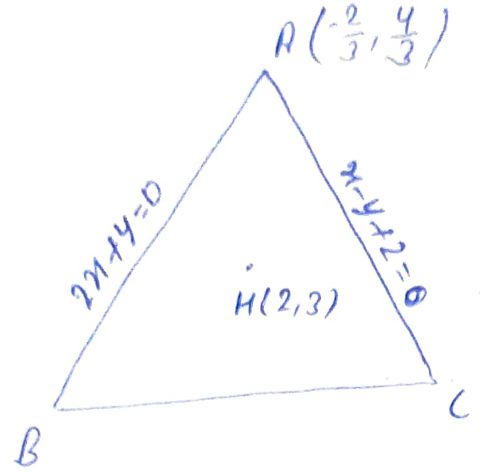
$B(3, 6)$

$C(2, 1)$

slope of line BC $= \dfrac{6-1}{3-2} = 5$ Ans

Ans 16 (b)

Ans



Solve →

$2x+y=0$

$x-y+2=0$

slope of line AH $= \dfrac{3-4/3}{2+2/3} = \dfrac{5}{8}$

so slope of line → $m(BC) = -\dfrac{8}{5}$

point $B(x_1, -2x_1)$

$m_{BH} \times m_{AC} = -1$

$\Rightarrow \left(\dfrac{2-x_1}{3+2x_1}\right)(1) = -1$

$\Rightarrow 2-x_1 = -3-2x_1 \Rightarrow x_1 = -5$

pt $B(-5, 10)$

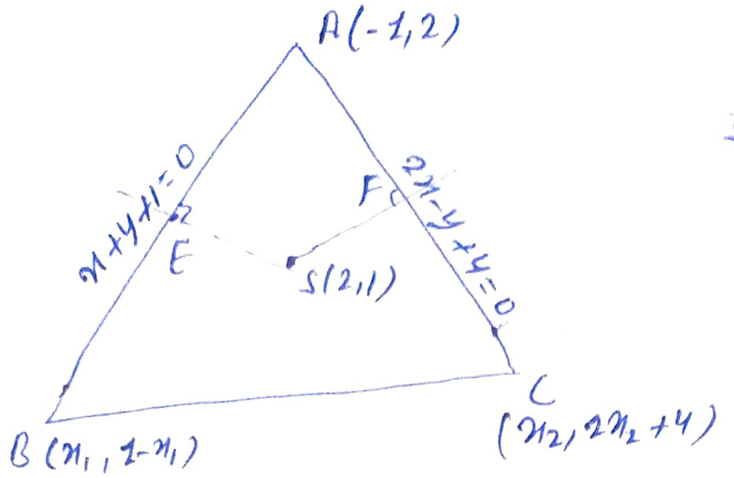
line BC → $y-10 = \dfrac{-8}{5}(x+5)$

put $x=0$

$\boxed{y_{inter} = 2}$

(16)

(c)



Solve

$x+y+1=0$

$2x-y+4=0$

$\downarrow$

$A(-1,2)$

- Eq^n of line SE $\rightarrow$ $x-y+\lambda_1=0$

~~passes~~ $\cancel{(-1,2)}$ $\Rightarrow$ ~~$-1-2+\lambda_1=0$~~

Passes $(2,1)$ $\cancel{(2,1)}$

$2-1+\lambda_1=0 \Rightarrow \lambda_1=-1$

SE $\rightarrow$ $\boxed{x-y-1=0}$

- Eq^n of SF $\rightarrow$ $x+2y+\lambda_2=0$ passes $(2,1)$

$\lambda_2=-4$

$\boxed{x+2y-4=0}$

$E\left(\frac{x_1-1}{2}, \frac{3-x_1}{2}\right)$ $\rightarrow$ lies on $x-y-1=0$

$\frac{x_1-1}{2}-\left(\frac{3-x_1}{2}\right)-1=0$

$x_1=3$

pt $B(3,-2)$

$F\left(\frac{x_2-1}{2}, x_2+3\right)$ lies on $x+2y-4=0$

$x_2=-3/5$

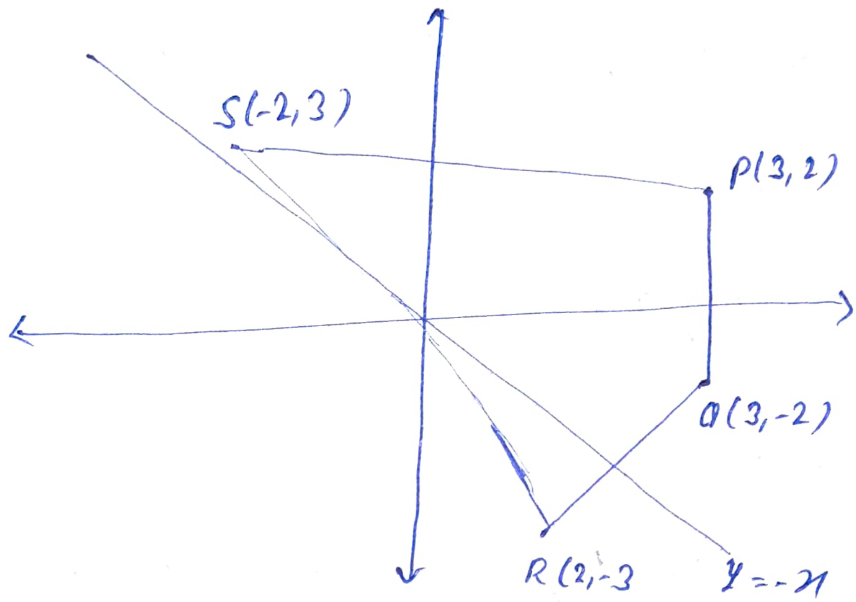
$C\left(-\frac{3}{5}, \frac{11}{4}\right)$

line BC $\rightarrow$ $4x+3y=6$

Put $y=0$ $\Rightarrow$ $\boxed{x_{int}=3/2}$

17. Let P be the point (3, 2). Let Q be the reflection of P about the x-axis. Let R be the reflection of Q about the line y = –x and let S be the reflection of R through the origin. PQRS is a convex quadrilateral. Find the area of PQRS.

Ans ~~18~~ (17)



Area of Quadrilateral PQRS

$$\frac{1}{2}\begin{vmatrix} x_1-x_3 & x_2-x_4 \\ y_1-y_3 & y_2-y_4 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3-2 & 3+2 \\ 2+3 & -2-3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 5 \\ 5 & -5 \end{vmatrix}$$

$$= 15 \text{ Ans}$$

18. A straight line is drawn from the point (1, 0) to the curve $x^2 + y^2 + 6x - 10y + 1 = 0$, such that the intercept made on it by the curve subtends a right angle at the origin. Find the equations of the line.

Ans 18

Curve : $x^2+y^2+6x-10y+1=0$ —— ①

eqn of line passing through A(1,0)

$y = m(x-1)$

$\Rightarrow \frac{mx-y}{m} = 1$ —— ②

Homogenization

$x^2+y^2+6x\left(\frac{mx-y}{m}\right)-10y\left(\frac{mx-y}{m}\right)+\left(\frac{mx-y}{m}\right)^2=0$

if at Right angle

coeff of x^2 + coeff. of $y^2=0$

$(1+6+1)+\left(1+\frac{10}{m}+\frac{1}{m^2}\right)=0$

$\Rightarrow 9m^2+10m+1=0$

$m=-1, -1/9$

line by ②

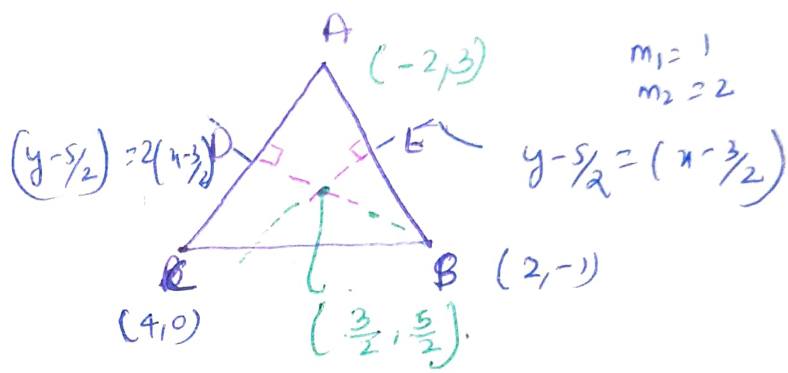
$y=-1(x-1)$ | $y=-1/9(x-1)$

$x+y-1=0$ | $x+9y-1=0$

S-2

1. The equations of perpendiculars of the sides AB & AC of triangle ABC are $x - y - 4 = 0$ and $2x - y - 5 = 0$ respectively. If the vertex A is $(-2, 3)$ and point of intersection of perpendiculars bisectors is $\left(\frac{3}{2}, \frac{5}{2}\right)$, find the equation of medians to the sides AB & AC respectively.

Soln:-



$m_1 = 1$

$m_2 = 2$

$2y - 5 = 4(x - 3/2)$

$2y - 4x = 5 - 6 = -1$

$2y - 4x + 1 = 0$

$2y - 5 = 2(x - 3/2)$

$2y - 5 = 2x - 3$

$y - x = 1$

$\frac{h+2}{-4} = \frac{k-3}{2} = -2\left(\frac{15}{20}\right)$

$h \Rightarrow \frac{3}{2} \times 4 - 2 = 4$

$k = -3 + 3 = 0$

$\frac{h+2}{-1} = \frac{k-3}{1} = \frac{-2(4)}{2}$

$h = 2$

$k = -1$

$G = \left(\frac{4}{3}, \frac{2}{3}\right)$

$GC = \quad y = (x-4)\left(\frac{2/3}{-8/3}\right)$

$-4y = x - 4$

$\boxed{x + 4y = 4}$

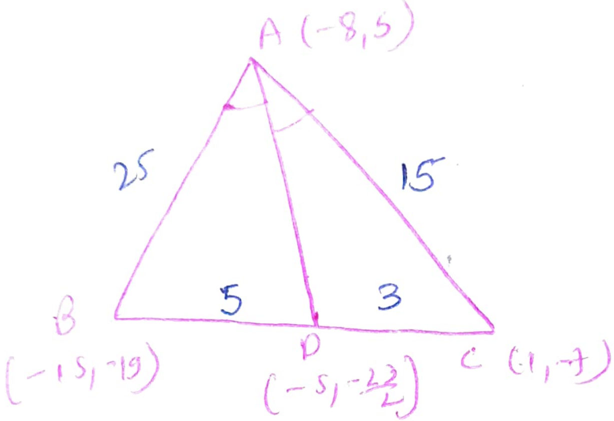
$BG = \quad y + 1 = (x-2)\left(\frac{5/3}{-4/3}\right)$

$-2y - 2 = 5x - 10$

$\boxed{6x + 2y = 8}$

2. The interior angle bisector of angle A for the triangle ABC whose coordinates of the vertices are A(–8, 5); B(–15, –19) and C(1, – 7) has the equation ax + 2y + c = 0. Find 'a' and 'c'.

Soln.



$\frac{AB}{AC} = \frac{BD}{DC} = \frac{5}{3}$. , use section formula to find D.

$x = \frac{5(1)+3(-15)}{8}$, $y = \frac{5(-7)+3(-19)}{8}$ $\left(-5, \frac{-23}{2}\right)$

$y-5 = (x+8)\left(\frac{\frac{-23}{2}-5}{-5+8}\right)$

$y-5 = \frac{-33}{3\times 2}(x+8)$

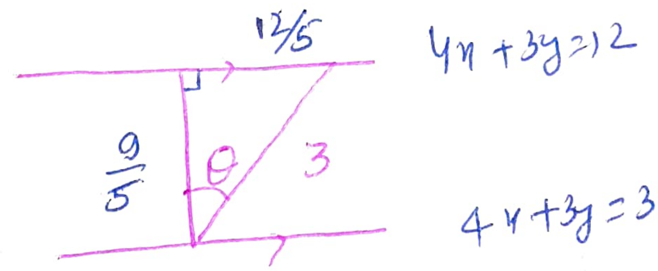
$2y-10 = -11x-88$

$11x+2y+78=0$

3. Find the equation of the straight lines passing through (–2, –7) & having an intercept of length 3 between the straight lines $4x + 3y = 12$, $4x + 3y = 3$.

Sol^n:-

$$\left|\frac{C_1 - C_2}{\sqrt{25}}\right| = \frac{9}{5}$$



$$\tan\theta = \frac{12/5}{9/5} = 4/3.$$

$$4/3 = \left|\frac{m - 3/4}{1 + \frac{3}{4}m}\right|$$

$$\pm 4/3 = \frac{4m-3}{4+3m}$$

$$m = -7/24.$$

$$y + 7 = -\frac{7}{24}(x+2), \quad 24y + 168 = -7x - 14$$

$$\boxed{7x + 24y + 182 = 0}$$

$$\boxed{x = -2}$$

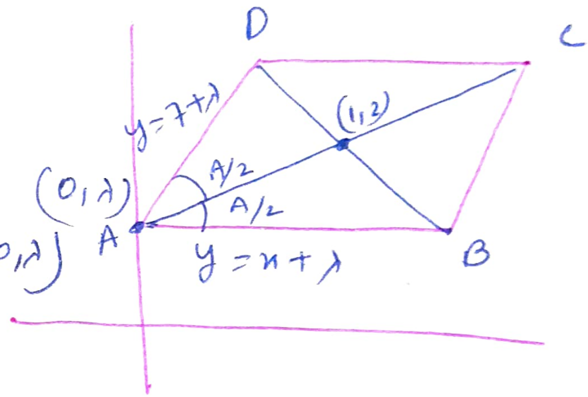
4. Two sides of a rhombus ABCD are parallel to the lines $y = x + 2$ & $y = 7x + 3$. If the diagonals of the rhombus intersect at the point (1, 2) & the vertex A is on the y-axis, find the possible coordinates of A.

Soln :-

Let lines ∥ to ~~AB~~ lines

$y = x+2$ & $y = 7x+3$

intersect on y-axis at $(0,\lambda)$



⊥r distance from P(1,2) to AD =

⊥r distance P to AB

$$\left|\frac{7-2+\lambda}{\sqrt{50}}\right| = \left|\frac{1-2+\lambda}{\sqrt{2}}\right|$$

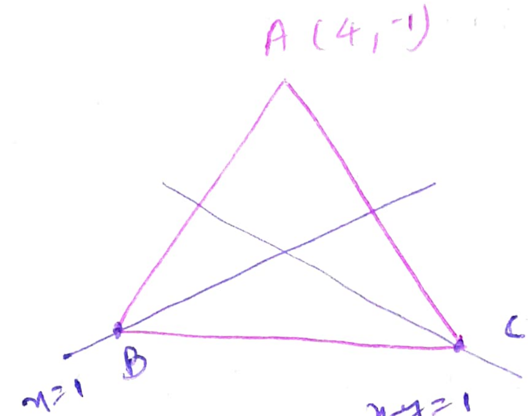
$$(5+\lambda) = \pm 5(\lambda-1)$$

$$\boxed{\lambda = 5/2, 0}$$

$$A = (0, 5/2),\ (0,0).$$

5. Find the equations of the sides of a triangle having (4, –1) as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles.

Soln.:-



Reflection of A in Angular bisector lies in base BC

$$\frac{h-4}{1} = \frac{k+1}{-1} = \frac{-2(4)}{2}$$ $\boxed{(0,3)}$

eqn.

$y-3 = \frac{4}{2}(x)$

$\boxed{2x-y+3=0}$

$$\frac{h-4}{1} = \frac{k+1}{0} = \frac{-2(3)}{1}$$ $\boxed{(-2,-1)}$

$h = -2, \quad k = -1$

B(1,5) → C(-4,-5),

$y+1 = \frac{6}{-3}(x-4)$

$-y-1 = 2x-8$

$2x\ 4y-7=0$

$y+1 = \frac{-4}{-8}(x-4)$

$2y+2 = x-4$

$x-2y-6=0$

$y-3 = 2+2y$

$\boxed{y=-5}$

$|\ \sqrt{1^2+4^2}\ |$ $\sqrt{17}$ "

EXERCISE (JM)

1. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is : **[AIEEE-2010]**

(1) $\frac{23}{\sqrt{15}}$ (2) $\sqrt{17}$ (3) $\frac{17}{\sqrt{15}}$ (4) $\frac{23}{\sqrt{17}}$

Soln :- Given line L

$\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32)

then the point satisfy the given eqn,

$\frac{13}{5}+\frac{32}{b}=1$,

$\frac{32}{b}=1-\frac{13}{5}$, $\frac{32}{b}=-\frac{8}{5}$

$b=-\frac{(5\times 32)}{8}=-20$, $\boxed{b=-20}$

The line K has eqn :- $\frac{x}{c}+\frac{y}{3}=1$

The line K is parallel to given line L.

then slope of both line are same.

$m_1=m_2$

$3x+yc=3c$

$yc=-3x+3c$

$y=-\frac{3}{c}x+3$ —(i)

first eqn is $\frac{x}{5}-\frac{y}{20}=1$

$\frac{4x-y}{20}=1$

$4x-y=20$, $y=4x-20$ —(ii)

from (i) & (ii) we get

$4=-\frac{3}{c}$, $\boxed{c=-3/4}$

The distance betn L and K is,

$y-4x+20=0$

$y-4x-3=0$

$\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|=\left|\frac{23}{\sqrt{1^2+4^2}}\right|=\frac{23}{\sqrt{17}}$ //.

2. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[AIEEE 2011]**

Statement - 1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement - 2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

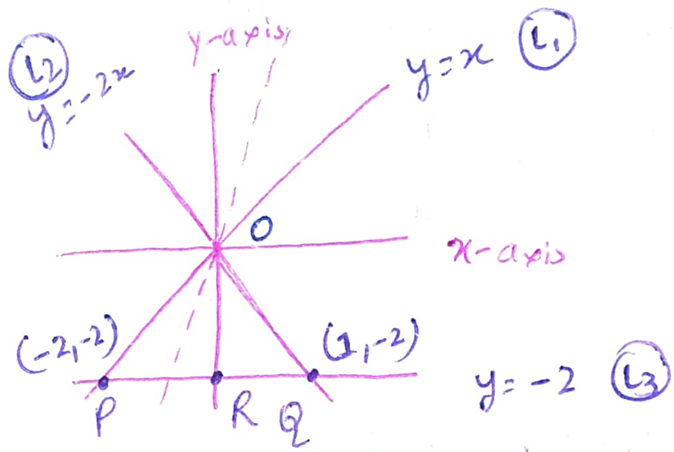
(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

Sol^n:- $L_1 : y - x = 0$ The line L_1 and line L_2

$L_2 : 2x + y = 0$ intersect the line L_3 at P and Q.

$L_3 : y + 2 = 0$,



The bisector of the acute angle between L_1 & L_2 interect at (L_3) at R.

St-1:- $\frac{PR}{RQ}$ equals $2\sqrt{2} : \sqrt{5}$

use angle bisector that is

$$\frac{OP}{OQ} = \frac{PR}{RQ}.$$

O is origin (0,0)
P (-2,-2), Q (1,-2)

$OP = \sqrt{8} = 2\sqrt{2}$, $OQ = \sqrt{5}$

$$\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}} \quad - (i)$$

and, In any Δ, bisector of the angle divide the Δ into two similar Δ's.

So, Statement - 1 is true, statement - 2 is false.

3. The lines x + y = |a| and ax – y = 1 intersect each other in the first quadrant. Then the set of all possible values of a is the interval : **[AIEEE 2011]**

(1) (–1, 1] (2) (0, ∞) (3) [1, ∞) (4) (–1, ∞)

Soln:- $x+y=|a|$ and $ax-y=1$

both lines intersect each other in the first quadrant.

$$x+y=|a|$$
$$ax-y=1$$
$$x(1+a)=|a|+1$$
$$\boxed{x=\frac{|a|+1}{1+a}}, \qquad y=|a|-\left(\frac{|a|+1}{1+a}\right)$$

so, both intersect in the first quadrant, so,

$\begin{cases} x>0 \\ y>0 \end{cases}$ $\qquad \frac{|a|+1}{1+a}>0$

let $a>0$. case (i)

$$x=\frac{a+1}{1+a}>0$$

$$y=a-\frac{(a+1)}{1+a}>0$$

$$\frac{a+a^2-a-1}{1+a}>0$$

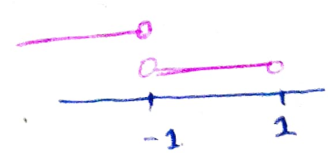
$$\frac{(a-1)(a+1)}{1+a}>0$$

$$\boxed{a>1}$$

case (ii) $a<0$

$$\frac{1-a}{1+a}>0 \qquad \frac{-a-(-a+1)}{1+a}>0$$

$$\frac{a-1}{a+1}<0 \qquad \frac{1}{1+a}<0$$



$a \in \phi$

so, $a \in [1,\infty)$

4. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is : **[AIEEE 2012]**

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) –4 (4) –2

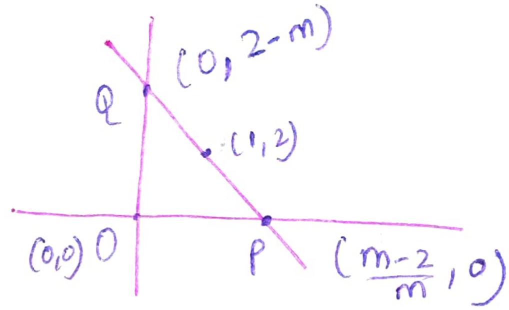
Solnː- Line pt (1, 2) meet coordinate axes at P and Q, form Δ OPQ, O- origin.

If area of Δ OPQ is least, then slope of the line PQ is :

$(y-2) = m(x-1)$

$y-2 = mx - m$

$y = mx - m + 2$



at x-coordinate, (P)

$y=0, \quad \frac{m-2}{m} = x,$

at Q, x-coordinate = 0,

$y = -m+2,$

$P\left(\frac{m-2}{m}, 0\right)$

$Q(0, -m+2)$.

If in Δ OPQ area is least then,

$$\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} \frac{m-2}{m} & 0 & 1 \\ 0 & 2-m & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$\left(\frac{m-2}{m}\right)(2-m) \rightarrow$ least,

$\left| -\frac{(m-2)^2}{m} \right| = \Delta$ for least differentiate its,

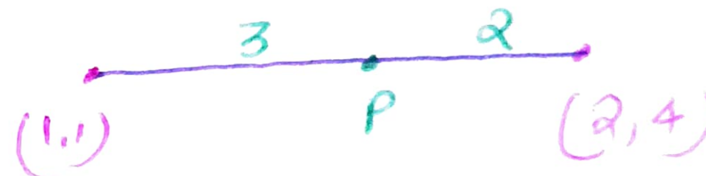
$\frac{m(2)(m-2) - (m-2)^2}{m^2} = 0,$

$\boxed{m = -2}$

5. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals : **[AIEEE 2012]**

(1) $\frac{11}{5}$ (2) $\frac{29}{5}$ (3) 5 (4) 6

Soln:- line $2x+y=k$ passes through pt. divide the points (1,1) and (2,4) in ratio 3:2 then k.



$$P \equiv \left(\frac{3\times2+2\times1}{5}, \frac{12+2}{5} \right)$$

$P \equiv \left(\frac{8}{5}, \frac{14}{5} \right)$ then point P satisfy the line,

$$2\left(\frac{8}{5}\right) + \frac{14}{5} = K$$

$$K = \frac{16+14}{5} = \frac{30}{5} = 6, \quad K = 6$$

6. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is : **[JEE-MAIN 2013]**

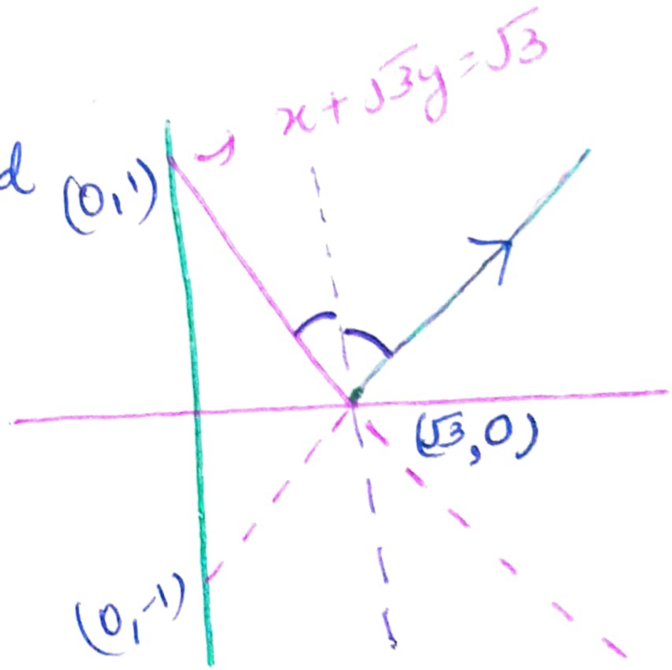
(1) $y=x+\sqrt{3}$ (2) $\sqrt{3}y=x-\sqrt{3}$ (3) $y=\sqrt{3}x-\sqrt{3}$ (4) $\sqrt{3}y=x-1$

Soln:-

ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis,

have the same slope the reflected ray,

$$y=-\frac{x}{\sqrt{3}}+1.$$



$$\frac{y-0}{x-\sqrt{3}}=+\frac{1}{\sqrt{3}}$$

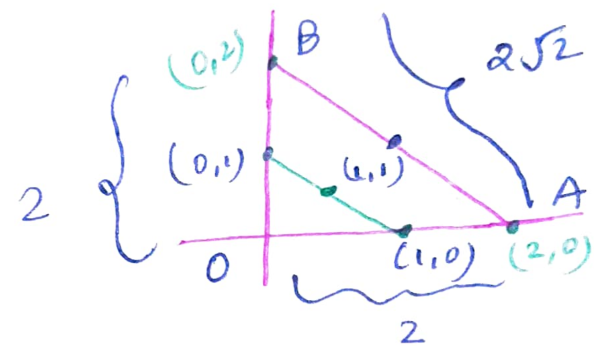
$$\sqrt{3}y=+x+\sqrt{3}$$

$$\sqrt{3}y=x-\sqrt{3}$$

7. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1)(1, 1) and (1, 0) is : **[JEE-MAIN 2013]**

(1) $2+\sqrt{2}$ (2) $2-\sqrt{2}$ (3) $1+\sqrt{2}$ (4) $1-\sqrt{2}$

Soln:- x - coordinate of the incentre of the triangle, that has the coordinates of mid pt of its sides as (0,1) (1,1) and (1,0) is ?



$AB = 2\sqrt{2}$, $OA = 2$, $OB = 2$,

x - coordinate of incentre,

$\Rightarrow \dfrac{ax_1 + bx_3 + cx_3}{a+b+c}$

$$\frac{0 \times 2\sqrt{2} + 2 \times 2 + 0 \times 2}{2+2+2\sqrt{2}} = \frac{4}{4+2\sqrt{2}} = \frac{2}{(2+\sqrt{2})} \frac{\times (2-\sqrt{2})}{\times (2-\sqrt{2})}$$

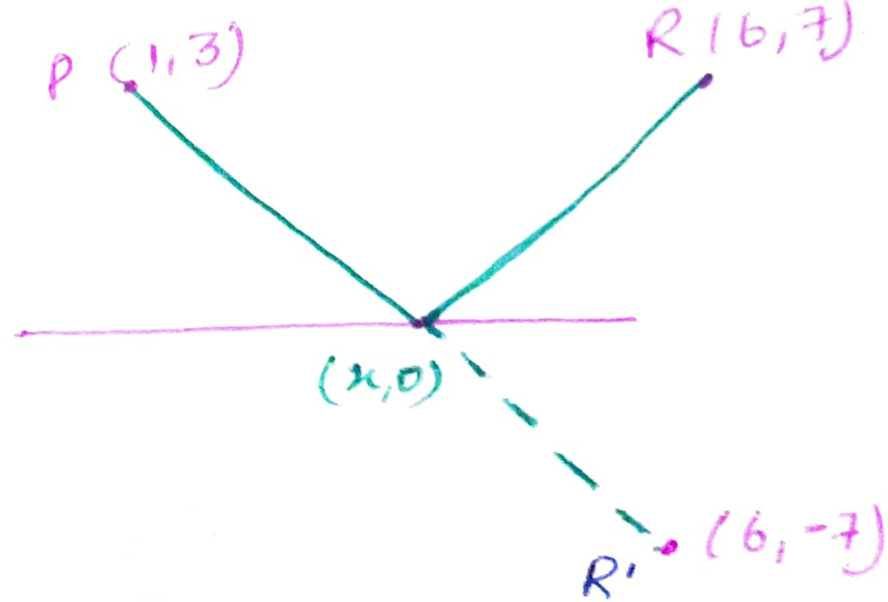
$$= \frac{2(2-\sqrt{2})}{(4-2)} = 2-\underline{\sqrt{2}}$$

8. A light ray emerging from the point source placed at P(1,3) is reflected at a point Q in the axis of x. If the reflected ray passes through the point R(6,7), then the abscissa of Q is : **[JEE-MAIN Online 2013]**

(1) 3 (2) $\frac{7}{2}$ (3) 1 (4) $\frac{5}{2}$

Solⁿ:- P (1,3) is reflected at a point Q in the axis of x.

reflected ray passes through the pt R(6,7).



Slope of PR' = $\frac{-7-3}{6-1} = \frac{-10}{5} = -2$

eqⁿ passing through P is -

$\frac{y-3}{x-1} = -2$, $y-3 = -2x+2$

$y + 2x = 5$, passes through (x,0).

$0 + 2(x) = 5$, $\boxed{x = \frac{5}{2}}$

9. If the three lines $x-3y = p$, $ax + 2y = q$ and $ax + y = r$ from a right – angled triangle then:

[JEE-MAIN Online 2013]

(1) $a^2 - 6a - 12 = 0$ (2) $a^2 - 9a + 12 = 0$ (3) $a^2 - 9a + 18 = 0$ (4) $a^2 - 6a - 18 = 0$

Soln:-

three lines $x-3y=p$, $ax+2y=q$ and $ax+y=r$ form right angle Δ.

$m_1 = +\frac{1}{3}$, $m_2 = -\frac{a}{2}$, $m_3 = -a$.

$m_1 m_2 = -1$

$-\frac{a}{6} = -1$

$\boxed{a=6}$

$m_1 m_3 = -1$

$-\frac{a}{3} = -1$

$\boxed{a=3}$

a^2 - (sum of roots)a + product of roots

$a^2 - 9a + 18 = 0$

10. If the x-intercept of some line L is double as that of the line, $3x + 4y = 12$ and the y-intercept of L is half as that of the same line, then the slope of L is :- **[JEE-MAIN Online 2013]**

(1) -3 (2) $-3/2$ (3) $-3/8$ (4) $-3/16$

Soln:- x-intercept of some line L is double of line $3x + 4y = 12$

$$\frac{x}{4} + \frac{y}{3} = 1,$$

x-intercept $= 2 \times 4 = 8$

y-intercept half $= 3/2 = 3/2$.

$$\frac{x}{8} + \frac{y}{3/2} = 1$$

$$\frac{x}{8} + \frac{2y}{3} = 1, \quad \frac{3x + 16y}{24} = 1.$$

$$3x + 16y = 24$$

$$16y = -3x + 24, \quad y = \frac{-3}{16}x + \frac{24}{16}$$

$$\boxed{\text{Slope} = -\frac{3}{16}}$$

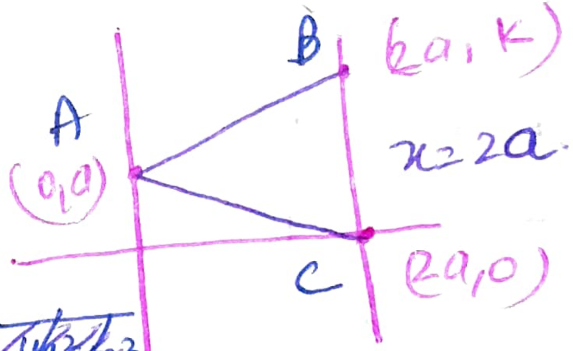
11. If the extremities of the base of an isoscelestriangle are the points (2a, 0) and (0, a) and the equation of one of the sides isx = 2a, then the area of the triangle, in square units, is :

[JEE-MAIN Online 2013]

(1) $\frac{5}{2}a^2$ (2) $\frac{5}{4}a^2$ (3) $\frac{25a^2}{4}$ (4) $5a^2$

Soln:-

Area = ?



AB = BC

$(k-a)^2 + 4a^2 = k^2$

$k = \frac{5a}{2}$

Area $= \frac{1}{2}\begin{vmatrix} 0 & 2a & 2a & 0 \\ a & 0 & \frac{5a}{2} & a \end{vmatrix} = \boxed{\frac{5a^3}{2}}$ //.

12. Let θ_1 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$, and θ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$, where c_1, c_2, c_3 are any real numbers :

Statement–1 : If c_2 and c_3 are proportional, then $\theta_1 = \theta_2$. **[JEE-MAIN Online 2013]**

Statement–2 : $\theta_1 = \theta_2$ for all c_2 and c_3.

(1) Statement-1 is true and Statement - 2 is true, Statement-2 is not a correct explanation for Statement-1.

(2) Statement-1 is false and Statement-2 is true.

(3) Statement-1 is true and Statement-2 is false.

(4) Statement-1 is true and Statement - 2 is true, Statement-2 is a correct explanation for Statement-1.

Soln:- two lines $2x+3y+c_1=0$ and $-x+5y+c_2=0$ — angle θ_1

$2x+3y+c_1=0$ and $-x+5y+c_3=0$ — angle θ_2

$m_1 = -2/3$
$m_2 = \frac{1}{5}$
$m_3 = \frac{1}{5}$

$$\tan\theta_1 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta_2 = \left|\frac{m_1 - m_3}{1 + m_1 m_3}\right|$$

$$\tan\theta_1 = \left|\frac{-\frac{2}{3} - \frac{1}{5}}{1 + \frac{2}{15}}\right|, \quad \tan\theta_2 = \left|\frac{-\frac{2}{3} - \frac{1}{5}}{1 + \frac{2}{15}}\right|.$$

from above $\boxed{\theta_1 = \theta_2}$

So, above both statement is true.

13. Let A (-3, 2) and B (-2,1) be the vertices of a triangle ABC. If the centroid of this triangle lies on the line $3x + 4y + 2 = 0$, then the vertex C lies on the line : **[JEE-MAIN Online 2013]**

(1) $4x + 3y + 5 = 0$ (2) $3x + 4y + 5 = 0$ (3) $3x + 4y + 3 = 0$ (4) $4x + 3y + 3 = 0$

Soln:- A(-3,2), B(-2,1) be vertices of ΔABC, centroid lies on line $3x+4y+2=0$ then vertex C lies on line-

$G(A_1, B_1)$ $\quad A_1 = \frac{-3+-2+h}{3}$, $\quad C(h,k)$.

$B_1 = \frac{2+1+k}{3}$

$A_1 = \frac{-5+h}{3}$.

$B_1 = \frac{3+k}{3}$.

(A_1, B_1) lies on $3x+4y+2=0$

$3\left(\frac{-5+h}{3}\right) + 4\left(\frac{3+k}{3}\right) + 2 = 0$

$(-5+h) + 12 + 4k = -2 \times 3$ $\quad 3h + 4k = -6 + 3$.

$3h + 4k + 3 = 0$

$3x + 4y + 3 = 0$ //

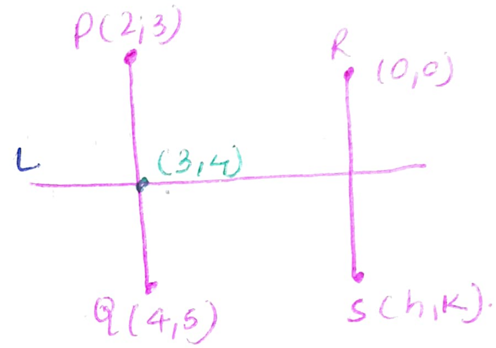
14. If the image of point P(2, 3) in a line L is Q (4, 5) then, the image of point R (0, 0) in the same line is : **[JEE-MAIN Online 2013]**

(1) (4, 5) (2) (2, 2) (3) (3, 4) (4) (7, 7)

Soln: - image of a point P (2,3) in a line L is Q (4,5) the image of point R (0,0) in the same line is -.

slope of line PQ

$$\frac{5-3}{4-2} = 1$$



Let slope of line l is given by

$$m \times m_1 = -1$$

$$1 \times m_1 = -1$$

$$\boxed{m_1 = -1}$$

eqn of line l passing through (3,4) is

$$\frac{y-4}{x-3} = -1$$

$$y-4 = -x+3$$

$$y+x-7=0$$

image of S passing through line L, R(0,0)

$$\frac{h-0}{1} = \frac{k-0}{1} = \frac{-2(0+0-7)}{1^2+1^2}$$

$$\frac{h}{1} = \frac{k}{1} = +7$$

(7,7) pt

15. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then :

[JEE(Main)-2014]

(1) $2bc - 3ad = 0$ (2) $2bc + 3ad = 0$ (3) $3bc - 2ad = 0$ (4) $3bc + 2ad = 0$

Soln:-

$a, b, c, d \to$ non-zero numbers.

pt of intersection of the lines

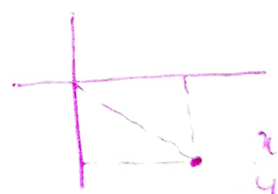
$$\left.\begin{aligned} 4ax + 2ay + c = 0 \\ 5bx + 2by + d = 0 \end{aligned}\right]$$

lies in fourth quadrant & is equidistant from axes. (equal)

$$4abx + 2aby + cb = 0$$
$$5abx + 2aby + ad = 0$$
$$- \quad - \quad -$$
$$-abx + cb - ad = 0,$$

$$\boxed{x = \frac{cb - ad}{ab}}$$

$$2ay = -c - 4ax$$
$$y = \frac{1}{2a}\left[-c - 4a \times \left[\frac{cb - ad}{ab}\right]\right]$$
$$= \frac{1}{2ab}\left[-cb - 4cb + 4ad\right].$$
$$= \frac{4ad - 5bc}{2ab}$$



$x > 0$

$y < 0$

equal distance

$$\sqrt{\left(\frac{cb - ad}{ab}\right)^2} = \sqrt{\left(\frac{4ad - 5bc}{2ab}\right)^2}$$

$$\left|\frac{cb - ad}{ab}\right| = \left|\frac{4ad - 5bc}{2ab}\right|. \quad y < 0,$$

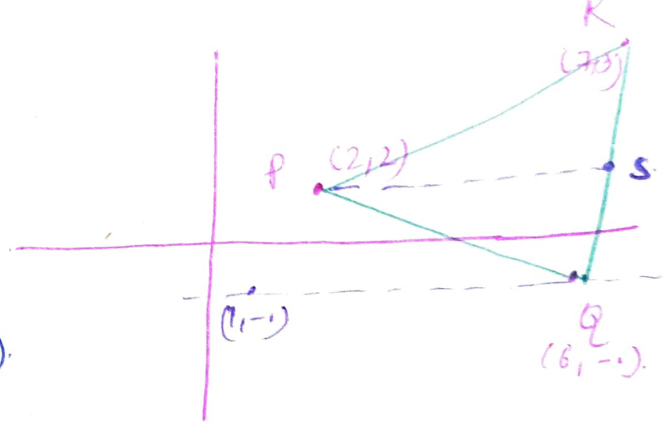
$$2cb - 2ad = -(4ad - 5bc)$$
$$-3bc = 2ad$$
$$\boxed{2ad + 3bc = 0}$$

16. Let PS be the median of the triangle with vertices P (2, 2), Q (6, –1) and R (7, 3). The equation of the line passing through (1, –1) and parallel to PS is : **[JEE(Main)-2014]**

(1) $4x - 7y - 11 = 0$ (2) $2x + 9y + 7 = 0$ (3) $4x + 7y + 3 = 0$ (4) $2x - 9y - 11 = 0$

Soln :- PS median,

P (2,2), Q (6,−1) and R (7,3), eqn of line passing through (1,−1) & parallel to PS is –



S mid pt of RQ

$$S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right)$$

$$= \left(\frac{13}{2}, 1\right), \; P(2,2).$$

$$\text{Slope of } PS = \frac{1-2}{\frac{13}{2}-2} = \frac{-2}{+9}$$

So, eqn of line passing through (1,−1) & have slope $-2/9$

$$\frac{y+1}{x-1} = \frac{-2}{9}$$

$$9y + 9 = -2x + 2$$

$$9y + 2x + 7 = 0$$

17. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a **[JEE(Main)-2015]**

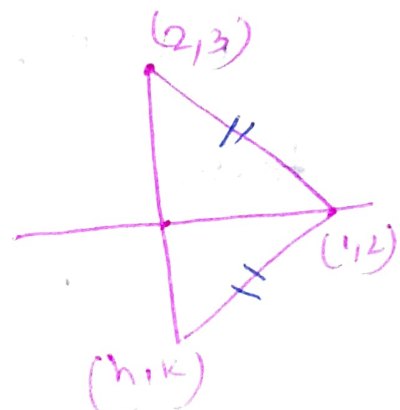
(1) circle of radius $\sqrt{2}$ (2) circle of radius $\sqrt{3}$

(3) straight line parallel to x-axis (4) straight line parallel to y-axis

Soln:- locus of the image of the pt (2,3) in the line $(2x-3y+4)+k(x-2y+3)=0$ $k \in R$, is a,

$$2x-3y+4=0, \quad x-2y+3=0$$

$$x=1, \; y=2.$$



$$\sqrt{(2-3)^2+(1-2)^2} = \sqrt{(k-2)^2+(h-1)^2}$$

$$\sqrt{1+1} = \sqrt{(k-2)^2+(h-1)^2}.$$

$$(k-2)^2+(h-1)^2 = (R)^2.$$

circle of radius $\sqrt{2}$.

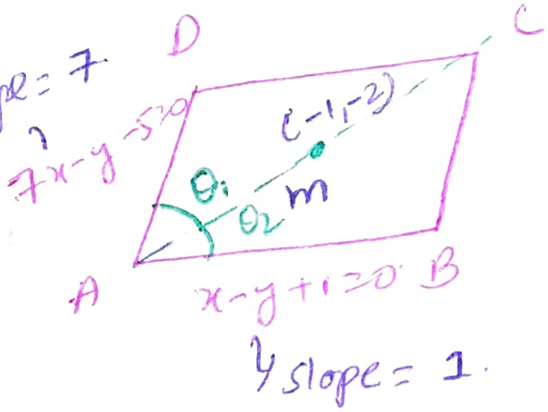
18. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus ? **[JEE(Main)-2016]**

(1) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (2) $(-3, -9)$ (3) $(-3, -8)$ (4) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

Soln.- Two sides of rhombus are along the lines

$x - y + 1 = 0$

$7x - y - 5 = 0$



$\tan\theta_1 = \left|\frac{7-m}{1+7m}\right|$

$\tan\theta_2 = \left|\frac{m-1}{1+m}\right|$

$\frac{7-m}{1+7m} = \frac{m-1}{1+m}$

$(7-m)(1+m) = (m-1)(1+7m)$

$7 + 7m - m - m^2 = m + 7m^2 - 1 - 7m$

$8m^2 - 12m - 8 = 0,$

$(m-2)(2m+1) = 0.$

$m = 2$, $m = -1/2.$

$\frac{y+2}{x+1} = 2.$

$y = 2x$

$\frac{y+2}{x+1} = -1/2$

$2y + 4 = -x - 1,$ $2y + x + 5 = 0,$

Here pt $(1/3, -8/3)$

satisfy (D).

19. Let k be an integer such that triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[JEE(Main)-2017]**

(1) $\left(2, \frac{1}{2}\right)$ (2) $\left(2, -\frac{1}{2}\right)$ (3) $\left(1, \frac{3}{4}\right)$ (4) $\left(1, -\frac{3}{4}\right)$

Soln:- $K \in I$, vertices, $(K, -3K)$, $(5, K)$, $(-K, 2)$ has area 28 sq unit.

$$\frac{1}{2}\begin{vmatrix} K & -3K & 1 \\ 5 & K & 1 \\ -K & 2 & 1 \end{vmatrix} = 28$$

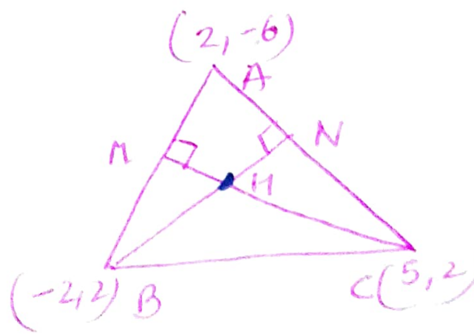
$K(K-2) + 3K(5+K) + 1(10+K^2) = 56$

$K^2 - 2K + 15K + 3K^2 + 10 + K^2 = 56$

$5K^2 + 13K - 46 = 0$

$\boxed{K=2}$ ✓.

vertices $(2, -6)$, $(5, 2)$, $(-2, 2)$



slope of AB $= \frac{8}{-4} = -2$

slope of MC $= m \times -2 = -1$

$\boxed{m = 1/2}$

eqn of MC $= \frac{y-2}{x-5} = 1/2$, $2y - 4 = x - 5$

$\boxed{x - 2y - 1 = 0}$ — (i)

slope of AC $= \frac{2+6}{5-2} = 8/3$

slope of BN $= m \times + 8/3 = -1$, $\boxed{m = -3/8}$

eqn of BN $= \frac{y+2}{x+2} = \frac{-3}{8}$, $8y - 16 = -3x - 6$

$\boxed{8y + 3x - 10 = 0}$ — (ii)

from (i) & (ii),

$3x - 6y - 3 = 0$

$3x + 8y - 10 = 0$

$-14y + 7 = 0$

$y = 1/2$, $x = 2$

orthocentre is $(2, 1/2)$

20. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:

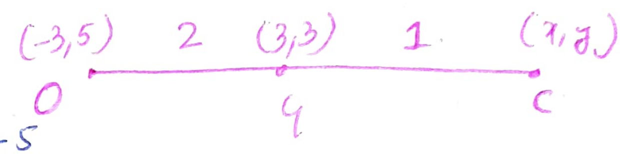
[JEE(Main)-2018]

(1) $2\sqrt{10}$ (2) $3\sqrt{\frac{5}{2}}$ (3) $\frac{3\sqrt{5}}{2}$ (4) $\sqrt{10}$

Sol^n:- $A \simeq O(-3,5)$, $\overset{B \simeq}{G}(3,3)$, O - orthocentre, G - centroid

$C \to$ circumcentre,

$r = ?$, AC - diameter,



$$3 = \frac{2x + -3}{3}, \quad 3 = \frac{2y+5}{3}.$$

$$9+3 = 2x, \quad 9-5 = 2y.$$

$$x = 6, \quad y = 2, \quad C \simeq \underline{(6,2)}.$$

$$OC = \sqrt{(2-5)^2 + (6+3)^2}$$

$$= \sqrt{9+81} = \sqrt{90}.$$

radius $= \frac{1}{2} \times$ diameter

$$= \frac{1}{2} \times 3\sqrt{10}, \quad \frac{3 \times \sqrt{5} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

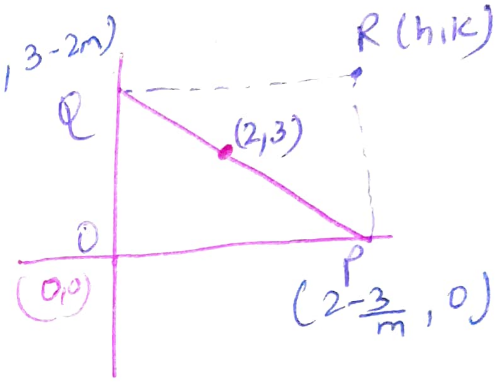
$$= 3\sqrt{\frac{5}{2}}$$

21. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is : **[JEE(Main)-2018]**

(1) $2x + 3y = xy$ (2) $3x + 2y = xy$ (3) $3x + 2y = 6xy$ (4) $3x + 2y = 6$

Soln:- St line passes through fixed pt (2,3) intersect at axes at distinct pt P & Q.

O – origin, rectangle OPRQ



$y-3 = m(x-2)$

x-intercept → $2 - \frac{3}{m}$

y-intercept → $-2m+3$.

OPRQ is rectangle then,

$$h = 2 - \frac{3}{m}.$$

$$K = 3 - 2m.$$

$$3h = 6 - \frac{9}{m}.$$

$$2K = 6 - 4m.$$

$$3h + 2K = 12 - \frac{9}{m} - 4m = \frac{12m - 9 - 4m^2}{m}$$

$$hK = \left(2 - \frac{3}{m}\right)(3 - 2m)$$

$$= \frac{12m - 9 - 4m^2}{m}$$

$3x + 2y = xy$ //.

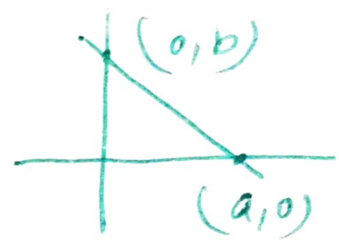
22. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50sq. units, then the number of elements in the set S is : [JEE(Main)-2019]

(1) 9 (2) 18 (3) 32 (4) 36

Soln:- S be the set of all triangles in the x-y plane, one vertex at origin, and other two vertices lie on coordinate axes.

$\left|\frac{1}{2}ab\right| = 50$

$|ab| = 100 = 2^2 5^2.$



No. of divisors $= (2+1)(2+1) = 9.$

No. of ordered pairs $(a,b) = 9.$

for four Quadrants $= 9 \times 4 = 36.$

So, No. of Integers $= 36.$

23. If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is [JEE(Main)-2019]

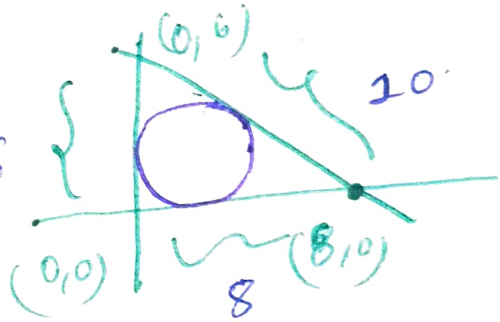
(1) (3, 4) (2) (2, 2) (3) (4, 4) (4) (4, 3)

Soln.:- line $3x + 4y - 24 = 0$

intersect x-axis at point A

and y-axis at point B.

then incentre of triangle OAB,



$$x\text{-coordinate} = \frac{ax_1 + bx_2 + cx_3}{a+b+c}$$

$$= \frac{0\times 10 + 6\times 8 + 8\times 0}{6+8+10}$$

$$= \frac{48}{24} = 2$$

$$y\text{-coordinate} = \frac{ay_1 + by_2 + cy_3}{a+b+c}$$

$$= \frac{0\times 10 + 0\times 6 + 6\times 8}{6+8+10}$$

$$= \frac{48}{24} = 2$$

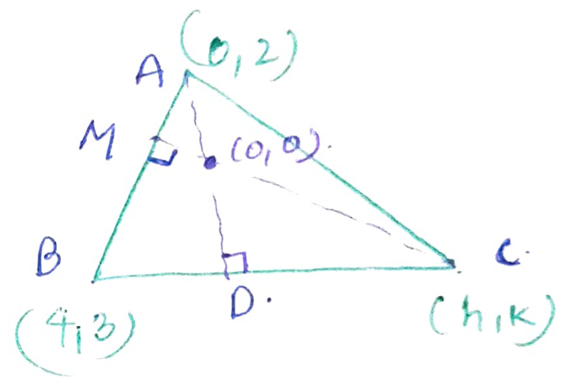
Incentre is (2, 2).

24. Two vertices of a triangle are (0,2) and (4,3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?
[JEE(Main)-2019]

(1) Fourth (2) Second (3) Third (4) First

Soln:- Two vertices of Δ are (0,2) and (4,3).
Orthocentre is at origin,
third vertex lies in which quadrant.

O (0,0),



Slope of CM $\dfrac{k-0}{h-0}$
$= \dfrac{k}{h}$

Slope of AB = $\dfrac{3-2}{4-0} = \dfrac{1}{4}$

Slope of CM × Slope of AB = −1.

$\dfrac{k}{h} \times \dfrac{1}{4} = -1.$

$k = -4h$, $\boxed{k+4h=0}$ — (i)

Slope of AD × Slope of BC = −1.

$\left(\dfrac{0-2}{0-0}\right) \times \left(\dfrac{k-3}{h-4}\right) = -1.$

$\boxed{k=3}$ — put in eqn (i),

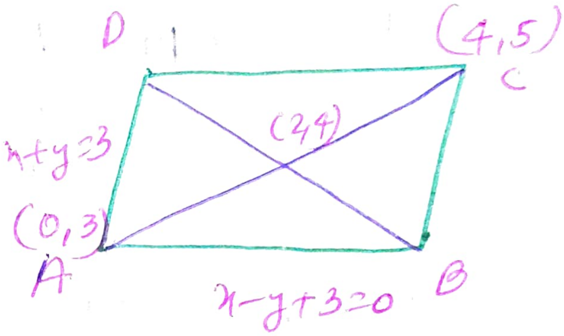
$\boxed{h=-3/4}$

lies in 2nd Quadrant.

25. Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at (2,4), then one of its vertex is : **[JEE(Main)-2019]**

(1) (2,6) (2) (2,1) (3) (3,5) (4) (3,6)

Soln:- Two sides of parallelogram are along the lines $x+y=3$ and $x-y+3=0$, If diagonals intersect at (2,4), then one vertex is -



$$x+y=3$$
$$x-y=-3$$
$$2x=0,\ x=0$$
$$y=3$$

$$C\left(\frac{0+x_1}{2}=2,\ \frac{3+y_1}{2}=4\right).$$
$$(x_1=4,\ y_1=5) \qquad C(4,5)$$

eq^n of BC = $\dfrac{y-5}{x-4}=-1.$

$$y-5=-x+4.$$
$$\boxed{y+x=9}$$

$$x-y=-3$$
$$x+y=9$$

$x=3$, $y=6$, B(3,6)

$$D\left(\frac{a_1+3}{2}=2,\ \frac{b_1+6}{2}=4\right)$$
$$D(1,2).$$

26. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is : [JEE(Main)-2019]

(1) $(x^2 + y^2)^2 = 4Rx^2y^2$

(2) $(x^2 + y^2)(x + y) = R^2xy$

(3) $(x^2 + y^2)^3 = 4R^2x^2y^2$

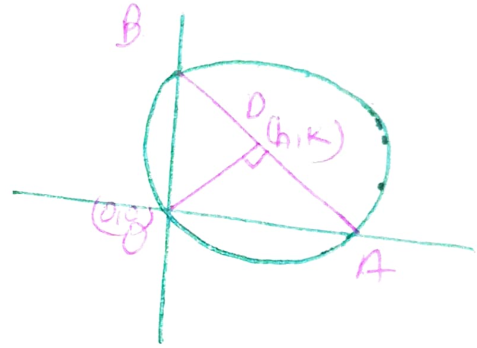
(4) $(x^2 + y^2)^2 = 4R^2x^2y^2$

Soln:- If a circle of radius R passes through origin O

Slope of OD $= \frac{K}{h}$

Slope of AB $= \frac{K}{h} \times m = -1$

$m = -\frac{h}{K}$



eqn of AB $= \frac{y-K}{x-h} = \frac{-h}{K}$

$hx + Ky = h^2 + K^2$

$A \equiv \left(\frac{h^2+K^2}{h}, 0\right)$ and $B = \left(0, \frac{h^2+K^2}{K}\right)$

also, AB = 2R,

$$\frac{(h^2+K^2)^2}{h^2} + \frac{(h^2+K^2)^2}{K^2} = 4R^2$$

$$(x^2+y^2)^3 = 4R^2x^2y^2$$

27. Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is **[JEE(Main)-2019]**

(1) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ (2) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ (3) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$ (4) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$

Soln:- Slope of a line passing through (2,3) intersecting the line, $x+y=7$ at a distance 4 unit from P.

Let slope of line is $\tan\theta$.

Parametric coordinate of point on line which is 4 unit distance from (2,3).

$(4\cos\theta+2,\ 4\sin\theta+3)$

which lie on $x+y=7$.

$4\cos\theta+2+4\sin\theta+3=7$.

$\cos\theta+\sin\theta=\frac{1}{2}$

$\sin 2\theta=-\frac{3}{4}=\frac{2\tan\theta}{1+\tan^2\theta}$

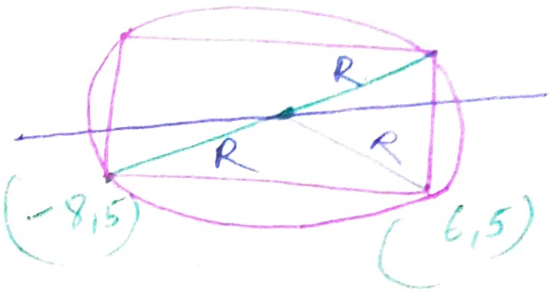
$3\tan^2\theta+8\tan\theta+3=0,$

$\tan\theta=\frac{-8\pm2\sqrt{7}}{6}=\frac{-1+\sqrt{7}}{1+\sqrt{7}}$ or $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

28. A rectangle is inscribed in a circle with a diameter lying along the line $3y = x + 7$. If the two adjacent vertices of the rectangle are $(-8, 5)$ and $(6, 5)$, then the area of the rectangle (in sq. units) is :- **[JEE(Main)-2019]**

(1) 72 (2) 84 (3) 98 (4) 56

Soln :-



let centre be, $\left(\alpha, \frac{\alpha+7}{3}\right)$

$$(\alpha+8)^2 + \left(\frac{\alpha+7}{3} - 5\right)^2 = (\alpha-6)^2 + \left(\frac{\alpha+7}{3} - 5\right)^2$$

$$\cancel{\alpha^2} + 64 + 16\alpha + \frac{1}{9}[\alpha^2 + \cancel{64} - 16\alpha] = \cancel{\alpha^2} + 36 - 12\alpha + \frac{1}{9}[\alpha^2 + \cancel{64} - 16\alpha]$$

$$28\alpha = 36 - 64$$

$$28\alpha = -28, \quad \boxed{\alpha = -1}$$

centre $(-1, 2)$.

vertex opposite side, $\left(\frac{-8+x_1}{2} = -1, \frac{5+y_1}{2} = 2\right)$

$x_1 = 6, \quad y_1 = -1.$

$(6, -1)$ vertex.

area $= l \times b.$

$$= \sqrt{(6+8)^2 + 0^2} \times \sqrt{0^2 + 6^2}$$

$$= 14 \times 6 = 84$$

29. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line L is : **[JEE(Main)-2019]**

(1) $(\sqrt{3}+1)x+(\sqrt{3}-1)y=8\sqrt{2}$

(2) $(\sqrt{3}-1)x+(\sqrt{3}+1)y=8\sqrt{2}$

(3) $\sqrt{3}x+y=8$

(4) $x+\sqrt{3}y=8$

Soln:-

St line L – 4 units from origin,

let slope of line is m.

$x+y=0$

$\boxed{y=-x}$

$\boxed{m_1=-1}$

$$\left|\frac{m-(-1)}{1+m(-1)}\right| = \sqrt{3}$$

$m = 2+\sqrt{3}$, or $2-\sqrt{3}$

$\theta = 75°$, or $15°$

eqn of line using normal form.

$x\cos 15° + y\sin 15° = 4$ or, $x\cos 75° + y\sin 75° = 4$.

$x(\sqrt{3}+1) + y(\sqrt{3}-1) = 8\sqrt{2}$ or, $x(\sqrt{3}-1)+y(\sqrt{3}+1)=8\sqrt{2}$

EXERCISE (JA)

1. Consider the lines given by

$L_1 = x + 3y - 5 = 0$

$L_2 = 3x - ky - 1 = 0$

$L_3 = 5x + 2y - 12 = 0$

Match the statements / Expression in **Column-I** with the statements / Expressions in **Column-II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in OMR.

Column-I		Column-II	
(A)	L_1, L_2, L_3 are concurrent, if	(P)	$k = -9$
(B)	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(Q)	$k = -\frac{6}{5}$
(C)	L_1, L_2, L_3 form a triangle, if	(R)	$k = \frac{5}{6}$
(D)	L_1, L_2, L_3 do not form a triangle, if	(S)	$k = 5$

[JEE 2008, 6]

$L_1 = x + 3y - 5 = 0$

$L_2 = 3x - ky - 1 = 0$

$L_3 = 5x + 2y - 12 = 0$

(A) L_1, L_2, L_3 are concurrent if,

$$\begin{vmatrix} 1 & 3 & -5 \\ 3 & -k & -1 \\ 5 & 2 & -12 \end{vmatrix} = 0$$

$(12K + 2) - 3(-36 + 5) - 5(6 + 5K) = 0$

$12K + 2 + 108 - 15 - 30 - 25K = 0$

$-13K + 65 = 0$, $\boxed{K = 5}$ (A) - (S)

(B) one of L_1, L_2, L_3 is parallel to at least one of the other two, if

$\frac{1}{3} = \frac{3}{-K} \neq \frac{-5}{-1}$ (B) - (P, Q)

$\boxed{K = -9}$.

$\frac{3}{5} = \frac{-K}{2} \neq \frac{-1}{-12}$

$\boxed{K = -\frac{6}{5}}$

(C) L_1, L_2, L_3 form a triangle if,

if L_1, L_2, L_3 are not concurrent & not parallel,

$K \neq 5, -9, -\frac{6}{5}$.

(C) $\rightarrow$ (R)

(D) L_1, L_2, L_3 do not form a triangle if.

$K = 5, -9, -\frac{6}{5}$.

(D) $\rightarrow$ (P), (Q), S.

2. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}$, $4\hat{i}$, $3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a

(A) parallelogram, which is neither a rhombus nor a rectangle

(B) square

(C) rectangle, but not a square

(D) rhombus, but not a square

[JEE 2010, 3]

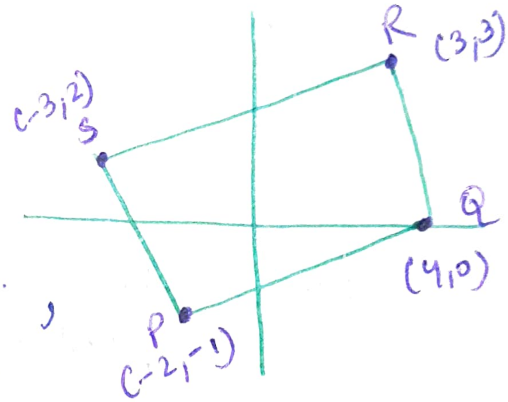
Soln:-

$P \to -2\hat{i}-\hat{j}$

$Q \to 4\hat{i}$

$R \to 3\hat{i}+3\hat{j}$

$S \to -3\hat{i}+2\hat{j}$



$\vec{PQ} = 6\hat{i}+\hat{j}$, $|PQ| = \sqrt{37}$, $m_1 = 1/6$

$RS = \sqrt{37}$, $m_2 = 1/6$

$RQ = \sqrt{10}$, $m_3 = -3$

$PS = \sqrt{10}$, $m_4 = -3$

$$PR = \sqrt{25+16} = \sqrt{41}$$

$$QS = \sqrt{49+4} = \sqrt{53}$$

$$PR \neq QS$$

$$PQ = RS, \quad RQ = PS$$

So, it is a parallelogram which is neither a rhombus, nor a rectangle.

3. A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersect the x-axis, then the equation of L is **[JEE 2011, 3 (–1)]**

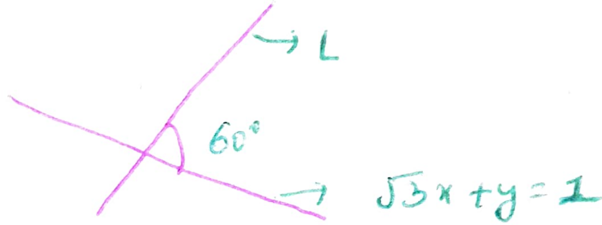
(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$

(B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$

(D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

Soln:-



$\underline{m = -\sqrt{3}}$

$$\tan 60^\circ = \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}.$$

$$\frac{m+\sqrt{3}}{1-\sqrt{3}m} = \sqrt{3}, \qquad \frac{m+\sqrt{3}}{1-\sqrt{3}m} = -\sqrt{3}$$

$$m+\sqrt{3} = \sqrt{3} - 3m \qquad m+\sqrt{3} = -\sqrt{3} + 3m.$$

$$\boxed{m=0}, \qquad 2m = 2\sqrt{3}$$

$$\boxed{m=\sqrt{3}}$$

X.

because line intersect x-axis//

point (3, -2)

$$\frac{y+2}{x-3} = \sqrt{3}, \qquad y+2 = \sqrt{3}x - 3\sqrt{3}.$$

$$y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$$

(B)//

4. For $a > b > c > 0$, the distance between $(1, 1)$ and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then **[JEE-Advanced 2013, 2]**

(A) $a + b - c > 0$ (B) $a - b + c < 0$

(C) $a - b + c > 0$ (D) $a + b - c < 0$

$a > b > c > 0$, $(1,1)$

POI. $ax + by + c = 0 \quad \times b$

$bx + ay + c = 0 \quad \times a$

$abx + b^2y + cb = 0$

$abx + a^2y + ac = 0$

$(b^2 - a^2)y + c(b - a) = 0$

$(b-a)[(b+a)y + c] = 0,$

$$\boxed{y = -\frac{c}{b+a}}$$

$ax - \frac{bc}{b+a} + c = 0$

$ax = \frac{bc}{b+a} - c$, $\quad ax = \frac{bc - cb - ac}{b+a}$

$$\boxed{x = -\frac{c}{b+a}}$$

distance

$(1,1) \quad \left(\frac{-c}{b+a}, \frac{-c}{b+a}\right) < 2\sqrt{2}$

$\sqrt{\left(1 + \frac{c}{a+b}\right)^2 + \left(1 + \frac{c}{a+b}\right)^2} < 2\sqrt{2}$

$\cancel{\sqrt{2}}\left(1 + \frac{c}{a+b}\right) < 2\cancel{\sqrt{2}}$

$a + b - c > 0$ (A)

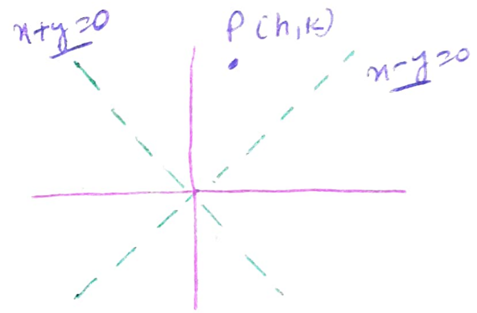
$a - b + c > 0,$

5. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is

[JEE(Advanced)-2014, 3]

Soln:-

$d_1(P)$ and $d_2(P)$ be distance of the point P from the line $x-y=0$ and $x+y=0$.



$$2 \le d_1(P) + d_2(P) \le 4$$

$$2 \le \left|\frac{h-k}{\sqrt{2}}\right| + \left|\frac{h+k}{\sqrt{2}}\right| \le 4$$

$$2\sqrt{2} \le |h-k| + |h+k| \le 4\sqrt{2}$$

Case (i) $h > k$,

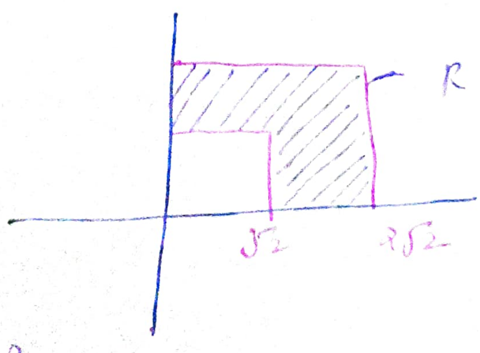
$$2\sqrt{2} \le 2h \le 4\sqrt{2}$$

$$\boxed{\sqrt{2} \le h \le 2\sqrt{2}} \quad -(i)$$

Case (ii) $h < k$,

$$\sqrt{2}\times 2 \le k-h + h+k \le 4\sqrt{2}.$$

$$\boxed{\sqrt{2} \le k \le 2\sqrt{2}} \quad -(ii)$$



$$R = (2\sqrt{2})^2 - (\sqrt{2})^2$$

$$= 8 - 2 = 6.$$