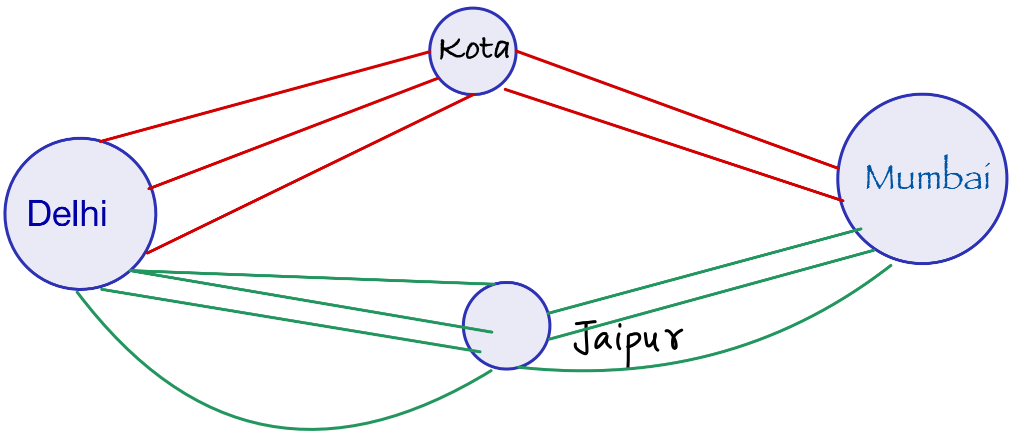
Permutations and Combinations

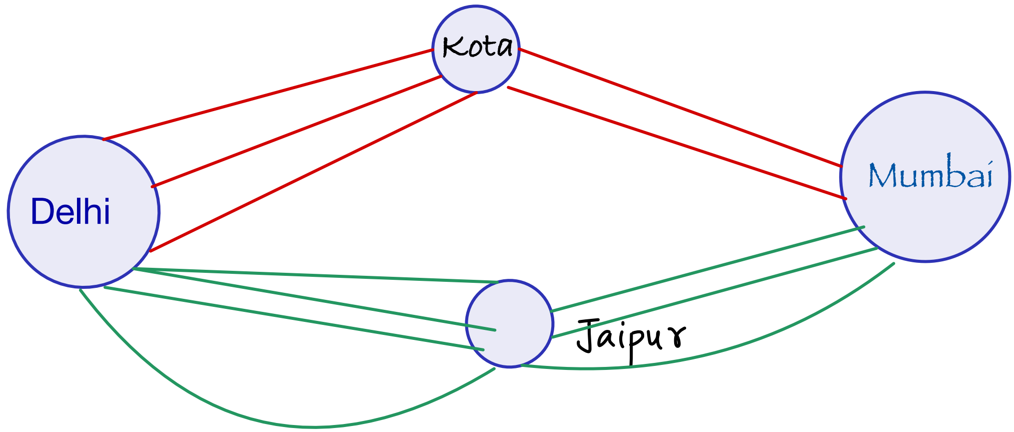


(1) In how many ways a person can travel from Delhi to Mumbai ?

$$3 \times 2 + 4 \times 3 = 6 + 12 = 18 \text{ ways.}$$

(2) In how many ways a person can complete a circular trip from Delhi to Mumbai and again back to delhi via Kota in to and fro journey.

$$3 \times 2 \times 2 \times 3 = 36 \text{ ways.}$$



(How many ways)

(3) In H.M.W a person can complete a circular trip from delhi to Mumbai and again back to Delhi via Kota in to and fro journey without using the same route between every pair of cities?

$$3 \times 2 \times 1 \times 2 = 12 \text{ ways.}$$

Q A question paper consists of 10 questions of MCQ type in which one or more than one option out of 4 options may be CORRECT. In HMW a student can attempt if each question is compulsoly to attempt?

Sol^n 15 ways to attempt one question

$(4+6+4+1)$

$15 \times 15 \times 15 \times \ldots \times 15 = (15)^{10}.$

Q A series of 5 matches is played between INDIA & ENGLAND in which each match results in a WIN, LOSE OR DRAW wrt a team.

(i) How many total forecasting can be made on the matches of series?

$$3 \times 3 \times 3 \times 3 \times 3 = 3^5.$$

(ii) How many such forecasting contains one error?

$$\underset{2}{\overset{*}{\_}}\ \underset{1}{\overset{\checkmark}{\_}}\ \underset{1}{\overset{\checkmark}{\_}}\ \underset{1}{\overset{\checkmark}{\_}}\ \underset{1}{\overset{\checkmark}{\_}}$$

2

$$\overset{\checkmark}{\_}\ \underset{2}{\overset{*}{\_}}\ \overset{\checkmark}{\_}\ \overset{\checkmark}{\_}\ \overset{\checkmark}{\_}$$

$$\overset{\checkmark}{\_}\ \overset{\checkmark}{\_}\ \overset{*}{\_}\ \overset{\checkmark}{\_}\ \overset{\checkmark}{\_}$$

$$\overset{\checkmark}{\_}\ \overset{\checkmark}{\_}\ \overset{\checkmark}{\_}\ \overset{*}{\_}\ \overset{\checkmark}{\_}$$

$$\overset{\checkmark}{\_}\ \overset{\checkmark}{\_}\ \overset{\checkmark}{\_}\ \overset{\checkmark}{\_}\ \overset{*}{\_}$$

$$2 + 2 + 2 + 2 + 2 = 10 \checkmark$$

Q In HMW 10 different toys can be distributed among 3 kids such that any one gets none, one or more than one toys?

Sol^n

$$3 \times 3 \times 3 \times \cdots \times 3 = 3^{10}.$$

~~WRONG ANS GIVEN BY STUDENTS $= 10^3$~~

Q In HMW 3 different toys can be distributed among 10 kids such that :-

(i) anyone gets any no. of toys.

$$10 \times 10 \times 10 = 10^3.$$

(ii) everyone gets none or one toy. (atmost one)

$$10 \times 9 \times 8 = 720.$$

(iii) atleast one kid gets more than one toy.

$$10^3 - 720.$$

Fundamental principle of counting :-

Multiplication Rule :

If an event can occur in m different ways following which another event could occur in n different ways, then total number of ways of simultaneous occurence of both the events in a definite order is m x n (This can be extended to any number of events)

$$A \overset{m}{\frown} B \overset{n}{\frown} C \overset{p}{\frown} D \cdots$$

$m \times n \times p \times \cdots$

Addition rule :

If an event occurs in ‘m’ different ways and another event which is independent of first event occurs in ‘n’ different ways then the no. Of ways in which either of the event is completed is (m + n)

(option)

Q Using letters of the word "EDUCATION"
How many 4 letters words can be formed if:

(i) starts with 'U' and ends with 'I'.

U _ _ I

7×6

42 ways.

(ii) Containing letter 'U'.

_ _ _ _

'U' placed → $4 \times (8 \times 7 \times 6)$

OR

Total − without 'U'

$9 \times 8 \times 7 \times 6 - (8 \times 7 \times 6 \times 5)$

(iii) 'U' comes before 'I'.

_ _ _ _

place U & I first → $\dfrac{(4 \times 3) \times (7 \times 6)}{2}$

Q By using the digits 1, 3, 4, 5, 7 five digit numbers are formed with different digit arranged in ascending order of their values.

(i) Find position of 53147 ?

(ii) Find number at 100^{th} position ?

71453

13457 → 1^{st} position

13475 → 2^{nd} "

fix

1 _ _ _ _ → $4 \times 3 \times 2 \times 1 = 24$

3 _ _ _ _ → $= 24.$

4 _ _ _ _ → $= 24.$

5 1 _ _ _ → $3 \times 2 \times 1 = 6$

5 3 1 4 7 → 1

$\overline{79^{th}}$

(ii)

1 _ _ _ _ → 24

3 _ _ _ _ → 24

4 _ _ _ _ → 24

5 _ _ _ _ → 24

96

7 1 3 4 5 → 97^{th}

7 1 3 5 4 → 98^{th}

7 1 4 3 5 → 99^{th}

Q By using digits 0, 1, 3, 4, 6, how many 5 digit numbers can be formed if :-

(i) repetition of digits is allowed.

$4 \times 5 \times 5 \times 5 \times 5$ $\quad 4(5)^4.$

(ii) repetition of digits is NOT allowed.

$4 \times 4 \times 3 \times 2 \times 1 = 16 \times 6 = 96.$

Q By using digits 0, 1, 3, 4, 6, how many even 5 digit numbers can be formed if :-

(i) repetition of digits is allowed. → 12×125

(ii) repetition of digits is NOT allowed. → 60.

(i)

0/4/6

$4 \times 5 \times 5 \times 5 \times 3$

(ii)

ⓐ _ _ _ _ 0 fix

$4 \times 3 \times 2 \times 1 = 24$

ⓑ _ _ _ _ 4/6

$(3 \times 2 \times 1) \times 3 \times 2 = 36$

Q How many 4 digit numbers can be formed which contains atleast one of the digits as '5'?

Sol^n Total N.O. Numbers — N.O. Numbers without '5'

$9 \times 10 \times 10 \times 10 - 8 \times 9 \times 9 \times 9$

Q How many 4 digit numbers can be formed whose atleast one of the digits is repeated?

Total — (all 4 digits diff)

$9 \times 10 \times 10 \times 10 - (9 \times 9 \times 8 \times 7)$

Q How many 4 digit numbers can be formed greater than 5000 with all of the digits being different. How many of them are ODD?

Sol^n

f — — —

5|6|7|8|9

(i) $5 \times 9 \times 8 \times 7$

$\underbrace{1, 3}_{\text{I}}, \underbrace{5, 7, 9}_{\text{II}}$

(ii) — — — —

a '1' or '3' at units place

$5 \times 8 \times 7 \times 2$

b '5' or '7' or '9' is at units place

— — — —

$4 \times 8 \times 7 \times 3$

Q By Using the digits 0, 1, 2, 3, 4, 6, how many 4 digit numbers can be formed when digits are all different and the number is divisible by 11 ?

$\underline{a}\ \underline{b}\ \underline{c}\ \underline{d}\ \underline{e}$

$(b+d) = Y$

$(a+c+e) = X$

$(X-Y) \Rightarrow$ multiple of '11'

$11\lambda \; ; \; \lambda \in I$

$0; \pm 11; \pm 22; \ldots \ldots$

Factorial :-

$n \in W$

n-factorial $= n! = \lfloor n$

$n! = n \times (n-1) \times (n-2) \times (n-3) \times \cdots\cdots 3 \times 2 \times 1.$

$$\frac{n!}{n} = (n-1)(n-2)(n-3)\cdots\cdots 3.2.1$$

$$\boxed{\frac{n!}{n} = (n-1)!} \Rightarrow$$

$0! = 1$

$1! = 1$

$2! = 2$

$3! = 6$

$4! = 24$

$5! = 120$

$6! = 720$

Rem

$$\boxed{(2n)! = 2^n.n!\,[1.3.5\ldots\ldots.(2n-1)].}$$

$$(2n)! = (2n)(2n-1)(2n-2)(2n-3)\cdots\cdots 4 \times 3 \times 2 \times 1$$

$$= 2^n\Big(\underbrace{n.(n-1)\ldots 2 \times 1}_{}\Big).\Big((2n-1)(2n-3)\ldots 3 \times 1\Big)$$

Rem

Note that sum of all the factorials ≥ 4 is never a perfect square.

Units digit in 'I'	Units digit in 'I^2'
0	0
1	1
2	4
3	9
4	6
5	5
6	6
7	9
8	4
9	1

$1! = 1$

$2! = 2$

$3! = 6$

$4! = 24$

Sum $= 33$

$5! = 120$

$6! = 720$

$\vdots$

0

Q By using the digits 0, 1, 2, 3, 4, 6, how many 4 digit numbers can be formed when digits are all different and the number is divisible by 11?

Sol^n

$$\underset{O_1}{\uparrow}\ \ \underset{E_1}{\uparrow}\ \ \underset{O_2}{\uparrow}\ \ \underset{E_2}{\uparrow}$$

$(O_1 + O_2) - (E_1 + E_2) = 11\lambda;\ \lambda \in I$

$0;\ \pm 11;\ \pm 22;\ \cdots$

$O_1 + O_2 = E_1 + E_2$

(i) 1, 6 & 3, 4.
8 cases

O_1	O_2
1	6
6	1

E_1	E_2
3	4
4	3

E_1	E_2
1	6
6	1

O_1	O_2
3	4
4	3

8

(ii) 1, 4 & 2, 3.
8 cases

(iii) 0, 6 & 4, 2.
6 cases

O_1	O_2
6	0

E_1	E_2
4	2
2	4

E_1	E_2
0	6
6	0

O_1	O_2
4	2
2	4

6

(iv) 0, 4 & 1, 3.
6 cases

(v) 0, 3 & 1, 2.
6 cases

Total = 34

Q If $\sum_{r=1}^{n} r!$ is a perfect square of natural number then find n?

Sol^n

$\sum_{r=1}^{1} r! = 1$ ✓

$\sum_{r=1}^{2} r! = 1! + 2! = 3$ ✗

$\sum_{r=1}^{3} r! = 1! + 2! + 3! = 9$ ✓

$n = 1$ ✓✓

$n = 3$ ✓✓

Q Find the remainder when $\sum_{r=1}^{15} r!$ is divided

(i) by 15 (ii) by 28.

Sol^n (i) $15 = 3 \times 5$

$$\sum_{r=1}^{15} r! = \underbrace{1! + 2! + 3! + 4!}_{33} + \underbrace{5! + 6! + \cdots + 15!}_{15I_1}$$

$$N = 15I_1 + 33 \xrightarrow{15} \boxed{Rem = 3}$$

(ii) $28 = 4 \times 7 = 2^2 \times 7^1$

$$\sum_{r=1}^{15} r! = 1! + 2! + 3! + 4! + 5! + 6! + \underbrace{7! + 8! + \cdots + 15!}_{28I_1}$$

$$1 + 2 + 6 + 24 + 120 + 720.$$

Rem

If p is a prime number, then exponent of 'p' in n! is $\left[\frac{n}{p^1}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\ldots\ldots+$ till on when greatest integral value becomes zero.

$5! = 5 \times 4 \times 3 \times 2 \times 1.$

$5! = 2^K \times (\text{odd integer})$

$K = ?$

$\frac{5}{2} = 2$

$\frac{2}{2} = 1$

$\rightarrow \ 3$

$15! = 2^K \times (\text{odd integer})$

$\frac{15}{2} = 7$

$+$

$\frac{7}{2} = 3$

$+$

$\frac{3}{2} = 1$

11

eg $15! = 10^K \times \text{Integer}$

K largest = ?

$10 = 2 \times 5$

$\frac{15}{5} = 3$

Q Find exponent of 11 in $\lfloor 2020$?

$$\left[\frac{2020}{11}\right] + \left[\frac{2020}{11^2}\right] + \left[\frac{2020}{11^3}\right] + \underbrace{\left[\frac{2020}{11^4}\right]}_{0}$$

Q Find number of zeroes at end of $\lfloor 100$?

Soln

$\lfloor 100 = 10^{K}$ (Int) $K \to$ largest ?

$\frac{100}{5} = 20$

$\frac{20}{5} = \frac{4}{24}$ Ans

Q Find number of zeroes at end of

$$\frac{(2020)!}{(1010)!\,(1010)!} \, ?$$

Ans 1

$$\frac{10^{503} \cdot I_1}{\underbrace{10^{251} \cdot 10^{251}}\; I_2}$$

$\frac{2020}{5} = 404$

$\frac{404}{5} = 80$

$\frac{80}{5} = 16$

$\frac{16}{5} = 3$

$\frac{1010}{5} = 202$

$\frac{202}{5} = 40$

$\frac{40}{5} = 8$

$\frac{8}{5} = 1$

Q Find exponent of 160 in $\lfloor 100$? 19 Ans

Sol^n

$$160 = 2^5 \cdot 5^1 \cdot$$

$$\lfloor 100 = 2^{97} \cdot 3^{\beta} \cdot 5^{24} \cdot 7^{\delta} \cdots\cdots = (2^{95} \cdot 2^2 \cdot 5^{24})\, I.$$

$$= \underbrace{(2^5)^{19} \cdot 5^{19}} \cdot 2^2 \cdot 5^5 \cdot I$$

$\frac{100}{2} = 50$

$\frac{50}{2} = 25$

$\frac{25}{2} = 12$

$\frac{12}{2} = 6$

$\frac{6}{2} = 3$

$\frac{3}{2} = 1$

$\frac{100}{5} = 20$

$\frac{20}{5} = 4$

$$(160)^{19} \cdot (\text{Integer})$$

Permutation / Arrangement :—

Meaning of ${}^nP_r$:—

Permutation of 'n' different things taken 'r' at a time is same as arrangement of those 'n' things at 'r' places, when repition is not allowed. It is denoted by

Rem

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

. ${}^nP_r$ OR $P(n,r)$

$n \geq r$

'r' places.

_ _ _ _ . . . . _

$$\big(n.(n-1)(n-2)\ldots\ldots(n-(r-1))\big)\frac{(n-r)(n-r-1)\ldots 2\times 1}{(n-r)(n-r-1)\ldots 2\times 1}$$

Rem

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

$${}^7P_4 = \frac{7!}{3!}$$

Combination / Selection :-

Meaning of ${}^{n}C_r$:-

No. of combination of 'n different things' taken 'r' at a time or Selection of 'r' things out of 'n' different things, when repetition is not allowed, is denoted by

$$ {}^{n}C_r = C(n,r) = \binom{n}{r} $$

Rem

$$\boxed{ {}^{n}C_r = \frac{n!}{r!\,(n-r)!} }$$

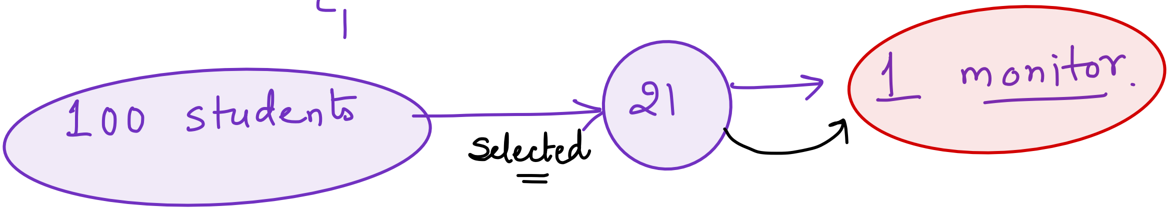
$$ \binom{7}{2} = {}^{7}C_2 = \frac{7!}{2!\;5!} $$

No. of ways of selecting 'r' different things out of 'n' diff things is same as no. of ways of rejecting 'n-r' diff things out of 'n' diff things

$$\boxed{ {}^{n}C_r = {}^{n}C_{n-r} }$$

$$\boxed{{}^{n}C_{r} = \frac{n}{r}\,{}^{n-1}C_{r-1}}$$ Rem

$${}^{n}C_{r} = \frac{{}^{n}C_{1}}{{}^{r}C_{1}} \cdot {}^{n-1}C_{r-1}$$



$${}^{100}C_{21} \cdot {}^{21}C_{1} \equiv {}^{100}C_{1} \cdot {}^{99}C_{20}$$

$$\boxed{{}^{100}C_{21} = \frac{{}^{100}C_{1}}{{}^{21}C_{1}} \cdot {}^{99}C_{20}}$$

$$\boxed{{}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}}$$

$${}^{n}C_{r} = \frac{n}{r}\ {}^{n-1}C_{r-1}$$

$${}^{n-1}C_{r-1} = \frac{n-1}{r-1}\ {}^{n-2}C_{r-2}$$

$${}^{n-2}C_{r-2} = \frac{n-2}{r-2}\cdot\ {}^{n-3}C_{r-3}$$

$$\vdots \qquad\qquad\qquad \vdots$$

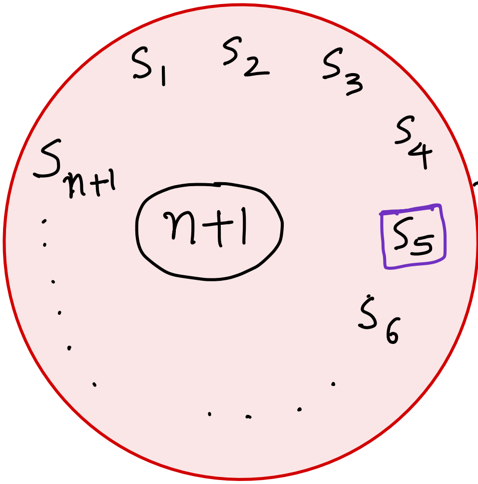
$${}^{n-r+1}C_{1} = \frac{n-r+1}{1}\ {}^{n-r}C_{0}$$

$${}^{n}C_{r} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r(r-1)\ldots 2\times 1}\cdot\frac{(n-r)!}{(n-r)!}$$

$${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}.$$

$$\boxed{{}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}}$$ Rem

$${}^{71}C_{4} + {}^{71}C_{5} = {}^{72}C_{5}$$



Select 'r' out of '$n+1$'

S_5 included	S_5 excluded
${}^{n}C_{r-1}$	${}^{n}C_{r}$

Note :

(i) $^nP_0 = 1;\ ^nP_n = n!$

(ii) $^nC_r = {^nC_{n-r}};\ ^nC_0 = 1;\ ^nC_n = 1$

(iii) $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

(iv) $^nC_x = {^nC_y} \Rightarrow x = y$ or $x + y = n$

(v) $^nP_r = r!\ ^nC_r$

$$^{17}C_3 = \frac{17!}{3!\ 14!} \quad \frac{17 \times 16 \times 15}{6}$$

$$^{22}C_{18} = \frac{22 \times 21 \times 20 \times 19}{4 \times 3 \times 2 \times 1}.$$

Q From a group of persons, the number of ways of selecting 5 persons is equal to that of 8 persons. The number of person in the group is

Sol^n

$$^xC_5 = {^xC_8}$$

$$\therefore x = 13 \text{ Ans}$$

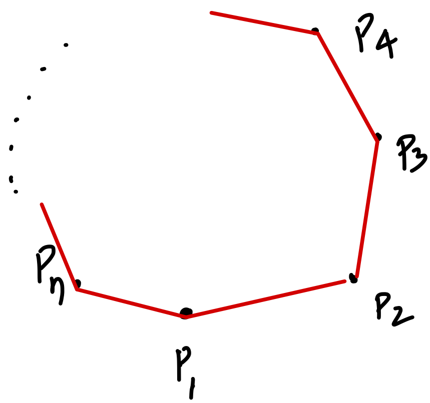
Q There are n points in a xy plane, no 3 of which are collinear. Find

(a) Number of straight lines by joining these points.

(b) Number of triangles

(c) Number of diagonals is a polygon

Sol^n (a) ${}^nC_2$. (b) ${}^nC_3$



${}^nC_2 - n$

Q A Grandfather with 7 grandchildren takes them to zoological garden without taking the same three children together. How frequently each child goes and how frequently Grand father goes.

Sol^n $\underbrace{A, B, C, D, E, F, G}_{\text{grand children}} \rightarrow {}^7C_3$

Grand father goes to zoo ${}^7C_3$ times.

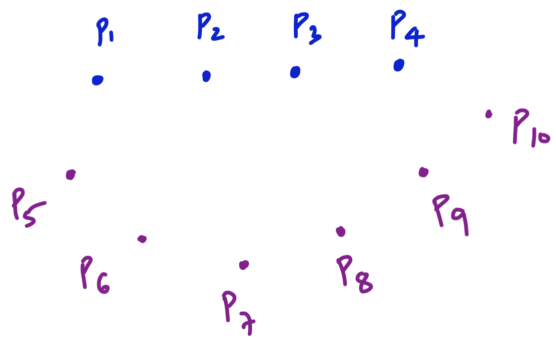
Frequency of each child going to zoo $= {}^6C_2$ times

Q. 10 points in a plane no 3 being collinear except 4 which are in the same line. Find diff

(i) number of line ✓

(ii) number of triangles

Sol^n



(ii) $^{10}C_3 - {}^4C_3$

OR

$^6C_3 + {}^6C_1 \cdot {}^4C_2 + {}^6C_2 \cdot {}^4C_1$

(i) $^{10}C_2 - {}^4C_2 + 1.$

$45 - 6 + 1 = 40$

OR $^6C_2 + {}^6C_1 \cdot {}^4C_1 + 1$

$15 + 24 + 1 = 40.$

HW

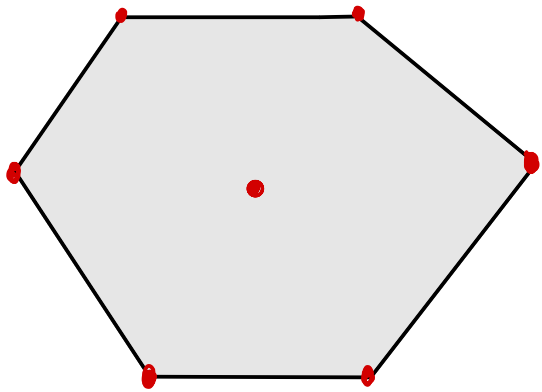
Q. Consider 6 vertices of a regular Hexagon and its centre. How many

(i) Straight lines ; (ii) Triangles can be formed with these 7 points.

Q. Consider 6 vertices of a regular Hexagon and its centre. How many

(i) Straight lines ; (ii) Triangles can be formed with these 7 points.

(i)

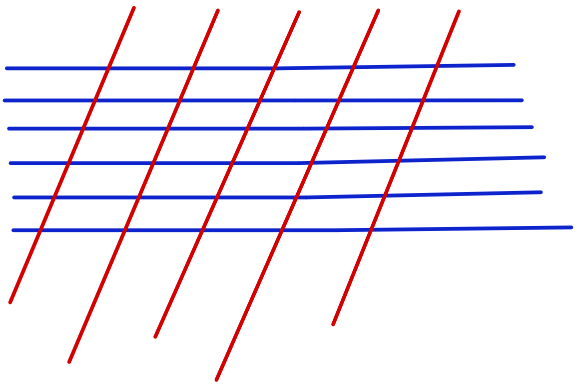


M-1 $\quad {}^6C_2 = 15.$

M-2 $\quad {}^7C_2 - 6 = 15.$

(ii) $\quad {}^7C_3 - 3 = 32.$

Q 6 parallel lines which are cut by another set of 5 parallel lines. Number of parallelograms.



$^6C_2 \cdot {}^5C_2$

Q Number of words with 10 different consonants & 4 different vowels if each word has 3C and 2V and begins with a consonant.

a, i, e, o, u → vowels (5)

b, c, ... , z → Consonants (21)

Sol^n

M-1

$$\left({}^{10}C_3 \cdot {}^4C_2\right) \cdot {}^3C_1 \cdot 4!$$

$C_1 \; C_2 \; C_3$

$V_1 \; V_2$

M-2

$$\left({}^{10}C_1\right) \cdot \left({}^9C_2 \cdot {}^4C_2\right) \cdot 4!$$

$\underset{\text{Consonant}}{\underline{\downarrow}} \; \underbrace{\_\_ \; \_\_ \; \_\_ \; \_\_}$

Q A cricket team comprising of A, B, C,..... J, K is to be sent for batting. If 'A' wants to bat before 'J' and 'J' wants to bat after 'G'. Then the number of batting orders if other players could go in any order.

Solⁿ

_ _ _ _ _ _ _ _ _ _ _

$$\underbrace{\begin{matrix} A \\ G \\ J \end{matrix}} \qquad \underbrace{\begin{matrix} G \\ A \\ J \end{matrix}} \qquad {}^{11}C_3 \cdot 2 \cdot 8!$$

OR

$$\left({}^{11}C_8 \cdot 8!\right) \cdot (2)$$

Q Out of 10 men & 5 women, how many arrangements of 3 men & 3 women are possible if each arrangement starts with man & ends with woman?

$M_1 \; M_2 \; M_3$

$W_1 \; W_2 \; W_3$

Solⁿ

M-1 $\left({}^{10}C_3 \cdot {}^{5}C_3\right)\left({}^{3}C_1 \cdot {}^{3}C_1 \cdot 4!\right)$

$$* \; \underbrace{\_ \; \_ \; \_ \; \_} \; *$$

M-2 $\left({}^{10}C_1 \cdot {}^{5}C_1\right)\left({}^{9}C_2 \cdot {}^{4}C_2 \cdot 4!\right)$

Q How many 10 digit numbers can be formed using the digits 1, 4, 7, 9 such that digit '9' appears exactly 5 times ?

Sol^n

_ _ _ _ _ _ _ _ _ _

$\left({}^{10}C_5 \cdot 1\right) \cdot 3^5$.

Q How many 4 digit numbers can be formed whose digits are : (i) In ascending order.
(ii) In descending order.

(i) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$\underbrace{{}^{9}C_4} \cdot \underbrace{1} = x$

(ii) ${}^{10}C_4 \cdot \underbrace{1} = y$

*Q Let $a, b, c, d \in \{1, 2, 3, \ldots, 9\}$ and $N = abcd$ (a four digit number).
Find number of N such that $a < b \leq c < d$?

Sol^n **M-1**

C-I : $a < b < c < d$

$$^9C_4 \cdot 1 = {^9C_4}$$

C-II $a < b = c < d$

$$^9C_3 \cdot 1 = {^9C_3}$$

Total no. of $N = {^9C_4} + {^9C_3} = {^{10}C_4}$.

M-2

$$a < b \leq c < d$$

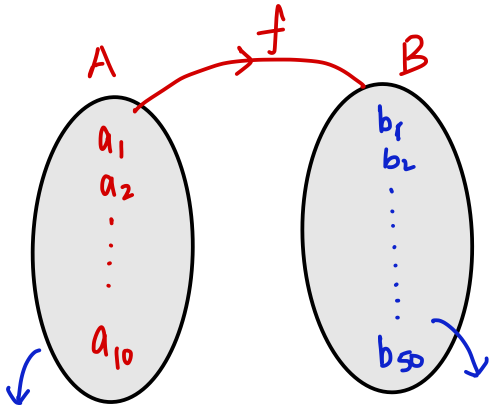
$$a, b, c, d \in \{1, 2, 3, \ldots, 9\} \; \text{`}\omega\text{'}$$

dummy

10

$$^{10}C_4 \checkmark$$

Q*



$a_1 < a_2 < \dots < a_{10}$

$b_1 < b_2 < \dots < b_{50}$

$f : A \to B$, find number of :

① strictly increasing fns from A to B.

$$^{50}C_{10}$$

② increasing fns from A to B

$$\underbrace{f(a_1) \le f(a_2) \le f(a_3) \le \dots \le f(a_{10})}$$

$$= \, ^{59}C_{10} \quad \text{Ans}$$

Q If first 20 natural numbers are there, the number of ways in which

(i) '3' distinct selected nos. are in A.P.

(ii) '5' distinct selected nos. " " ".

N: {1, 2, 3, ..., 20}

Sol^n (i) $x, y, z \longrightarrow AP$

$\underbrace{2y}_{\text{Even}} = \underbrace{x+z}_{\text{Even}}$.

I: Even + Even = Even

II: odd + odd = Even

$\underbrace{{}^{10}C_2}_{\text{I}} + \underbrace{{}^{10}C_2}_{\text{II}} = 2 \cdot {}^{10}C_2 = 90.$

** (ii)

$\underbrace{a_1, a_2, a_3, a_4, a_5}_{4d} \longrightarrow$ A.P.

d = Common difference

$4d = 20 - 1 \Rightarrow 4d = 19$

$d = 4, 3, 2, 1$

C-I $d=4$

1, 5, 9, 13, 17
2, 6, 10, 14, 18
3, 7, 11, 15, 19
4, 8, 12, 16, 20

4

C-II $d=3$

1, 4, 7, 10, 13
2, 5, 8, 11, 14
3, 6, 9, 12, 15
4, 16
5, 17
6, 18
7, 19
8, 20

8

C-III $d=2$

12

C-IV $d=1$

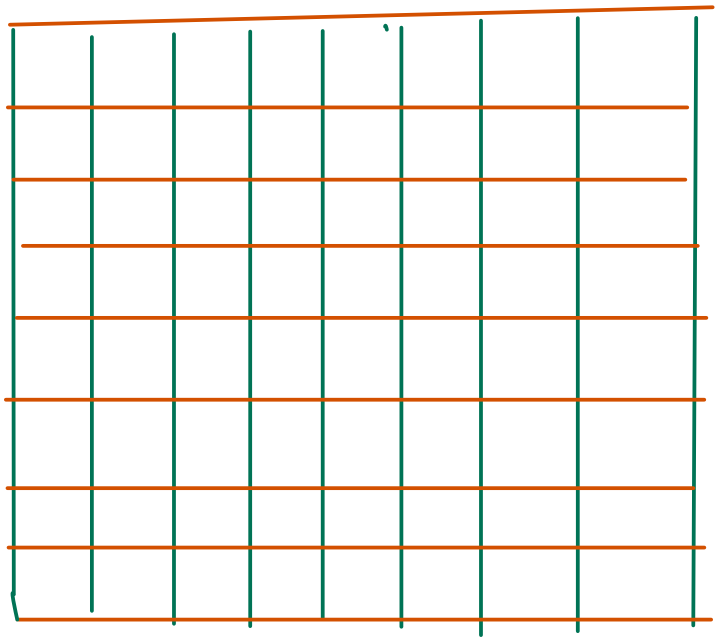
16

Total = 40 Ans

Q In a 8 X 8 CHESSBOARD, how many :

(i) rectangles can be counted.

(ii) squares can be counted.



(i) ${}^9C_2 \cdot {}^9C_2$

(ii) $8^2 + 7^2 + 6^2 + \cdots \cdots + 1^2$

$$\frac{\cancel{8}^{4} \cdot \cancel{9}^{3} \cdot 17}{\cancel{6}\,\cancel{2}} = 68 \times 3$$

$$= 204$$

Q[HW] In a grid formed by 17 horizontal and 10 vertical lines.

(i) How many rectangles can be counted?

(ii) " " squares " " " ?

Note :-

1. No. of squares in a grid of $n \times n$ is

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

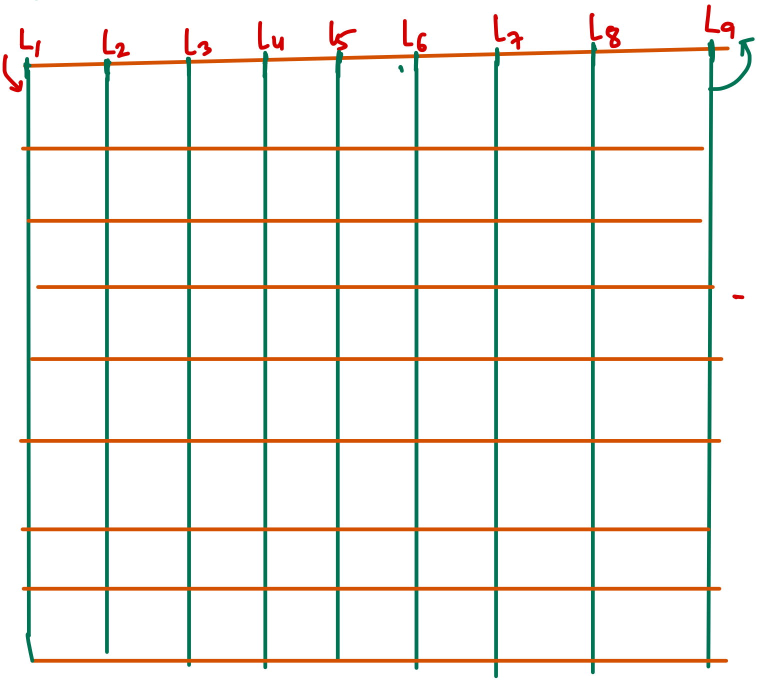
2. No. of rectangles in a grid of $n \times n$ is

$$^{n+1}C_2 \cdot {}^{n+1}C_2 = \left(\frac{n(n+1)}{2}\right)^2 = 1^3 + 2^3 + \cdots + n^3.$$

3. No. of diagonals in n-sided polygon is

$$^{n}C_2 - n = \frac{n(n-3)}{2}.$$

*Q Find number of rectangles in a grid of 8×8 (1) whose both dimensions are <u>ODD</u>.



E − E = E.
O − O = E.
O − E = O.
E − O = O.

(1) $\left({}^5C_1 \cdot {}^4C_1\right)\left({}^5C_1 \cdot {}^4C_1\right)$

(2) whose both dimensions are <u>EVEN</u>

$$\left({}^5C_2 + {}^4C_2\right) \cdot \left({}^5C_2 + {}^4C_2\right)$$

Q By using all the letters of the word "MIRACLE"

I, A, E → 3 Vowels

(i) How many words can be formed.

$\lfloor 7$

(ii) Vowels occupy odd places.

1st — 3rd — 5th — 7th

$({}^4C_3 \cdot 3!) \cdot 4!$

* (iii) Vowels are alternate.

V _ V _ V _ _

_ V _ V _ V _

_ _ V _ V _ V

$(3!\ 4!) \times 3$

(iv) Vowels & consonants are alternate / vowels & Consonants are seperated.

$\underbrace{C\ V\ C\ V\ C\ V\ C}$

$3! \times 4!$

** (V) Vowels are all together.

packet.

I, A, E

M, R, C, L

$5!\ (3!)$

(vi) Vowels are in alphabetical order.

$({}^7C_3 \cdot 1) \cdot \lfloor 4$

_ _ _ _ _ _ _

(vii) 'L' comes before 'M' and 'A' comes before 'E'.

_ _ _ _ _ _ _

$({}^7C_2 \cdot 1) \cdot ({}^5C_2 \cdot 1) \cdot (3!)$

M-1 $\frac{7!}{\cancel{5!}\,2!} \times \frac{\cancel{5!}}{\cancel{3!}\,2!} \times \cancel{3!} = \frac{7!}{4}$.

M-2 $\left(\frac{7!}{2}\right)/2 = \frac{7!}{4}$

HW

Q* 10 different lines are in a plane. Then find number of new lines formed by point of intersection of given lines?

Q In how many ways can the seven different colours of a rainbow be arranged so that the blue and green never come together.

Packet / Tie → Method.

VIBGYOR

M-1

T.N.O.W − (N.O.W when B & G are together)

$7! - 6! \cdot 2!$

V, I, $\boxed{BG}$, Y, O, R

M-2

Gap- Method.

B, G

✓ ✓ ✓ ✓ ✓ ✓

| V | I | Y | O, | R |

$5! \times \left({}^{6}C_{2} \cdot 2!\right)$

Q **4 boys and 4 girls** are to be seated in a line, find :

(a) Number of ways they can be seated so that 'No two girls are together. (or girls are separated)

$| B_1 | B_2 | B_3 | B_4 |$

$$\underline{|4} \cdot \left({}^5C_4 \cdot \underline{|4} \right)$$

(b) not all the girls are together or at least one girl is separated from the rest of the girls

T.N.O.W − N.O.W when all girls are together

$\underbrace{B_1 B_2 B_3 B_4 \boxed{G_1 G_2 G_3 G_4}}$

$$8! - 5! \times 4!$$

(c) 'boys and girls are alternate' or boys as well as girls are separated or each boy follows the girl and each girl follows the boy

$G_1 \quad G_2 \quad G_3 \quad G_4$

$B_1 \quad B_2 \quad B_3 \quad B_4$

$G_1 \quad G_2 \quad G_3 \quad G_4$

$$\underline{|4} \cdot \underline{|4} \cdot \underset{\downarrow}{2} = y$$

(d) Boys and girls are alternate and a particular boy & girl are never adjacent to each other in any arrangement

Sol^n

B_1 & G_1

$B_2 \quad B_3 \quad B_4$

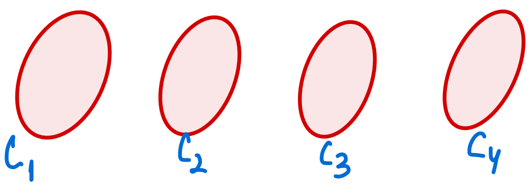
$$\underbrace{\left(|\underline{3} \cdot |\underline{3} \cdot 2\right)} \cdot {}^7C_1 \cdot 1 = x$$

N.O.W in which Boys & girls are alternate and particular Boy & girls are always together

eg: $B_2\, G_2\, B_3\, G_3\, B_4\, G_4$

$$\text{Req. ans} = y - x$$

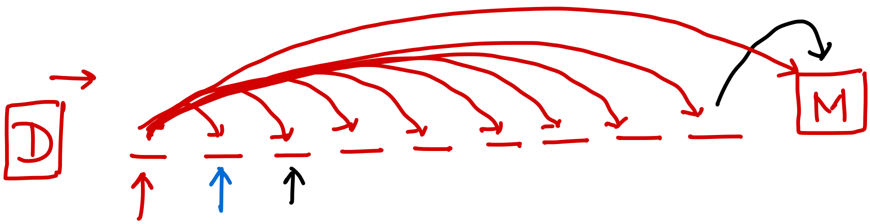
(e) If they are 4 married couples then the number of ways if each couple is together.



$$|\underline{4} \cdot \left(|\underline{2}\; |\underline{2}\; |\underline{2}\; |\underline{2}\right)$$

Q Rajdhani express bound from Delhi to Mumbai stops at 9 intermediate stations.
which has started
6 passengers enter the train during the journey holding 6 different tickets of the same class . How many different sets of tickets they may have had.

Sol^n



$$9 + 8 + 7 + \cdots\cdots + 1 = 45$$

$\therefore$ No. of diff tickets of same class $= 45.$

Req. ans $= {}^{45}C_6.$

*** Station Problem:

Q. A train having 12 stations enroute has to be stopped at 4 stations. Number of ways it can be stopped if no two of the stopping stations are consecutive.

$\to {}^{9}C_4$

S S S S S S S S

Let us solve for 4 stations instead of 12 → stop at 2 stations such that no '2' are consecutive.

$S_1 \quad S_2 \quad S_3 \quad S_4$

$S_1 \quad \cancel{S_2} \quad S_3 \quad \cancel{S_4}$

$S_1 \quad \cancel{S_2} \quad \cancel{S_3} \quad S_4$

$\cancel{S_1} \quad S_2 \quad \cancel{S_3} \quad S_4$

S S S S → remove '2' station

| S | S |

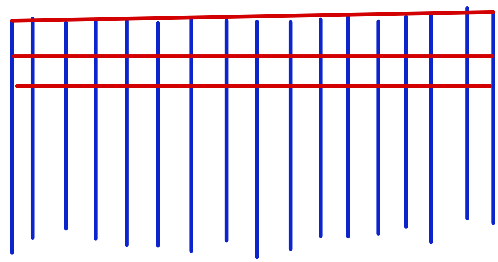
$\to {}^{3}C_2 = 3$ Ans

Q In a grid formed by 17 horizontal and 10 vertical lines.

(i) How many rectangles can be counted?

(ii) " " squares " " " ?

Soln 16×9 grid.



(i) $^{17}C_2 \times {}^{10}C_2$.

(ii) $1 \times 1 \rightarrow 16 \times 9$.

$2 \times 2 \rightarrow 15 \times 8$.

$3 \times 3 \rightarrow 14 \times 7$.

$\vdots \qquad \vdots$

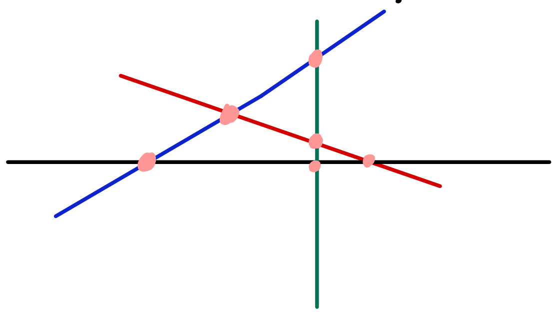
$\rightarrow 8 \times 1$

* In general for $m \times n$ grid where $m \geq n$

$$\boxed{\sum_{r=0}^{n-1} (m-r)(n-r) = \text{No. of squares.}}$$

Q** 10 different lines are in a plane. Then find number of new lines formed by point of intersection of given lines ?

Soln Let us think for 4 lines.



$^4C_2 = 6$. points

$^6C_2 - {}^3C_2 \times 4 = 15 - 12$

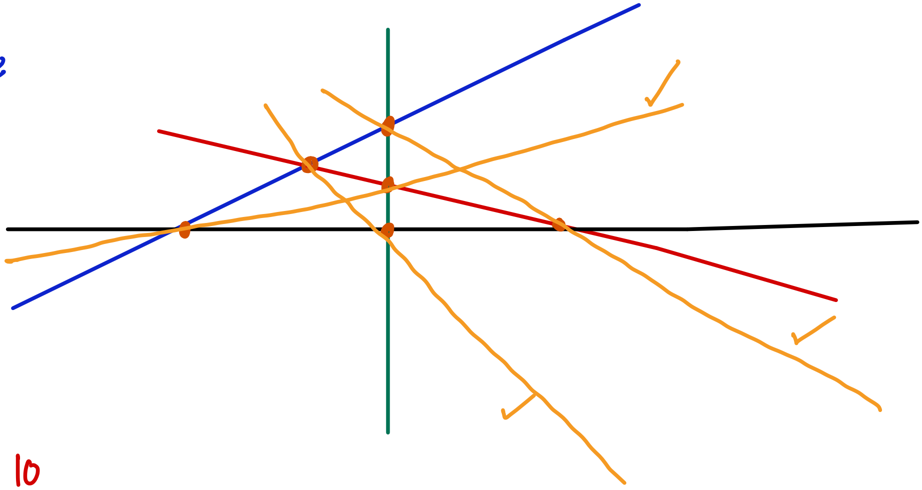
$= 3$. new lines.

10 different lines:

$^{10}C_2 = 45$ points.

$^{45}C_2 - \left({}^9C_2\right) \times 10$

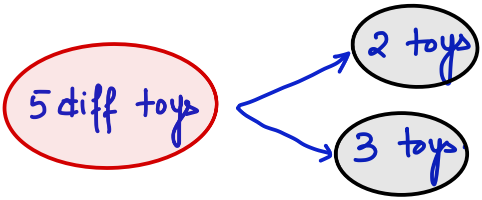
M-2



$^{10}C_4 \times 3.$

Formation of groups

Grouping



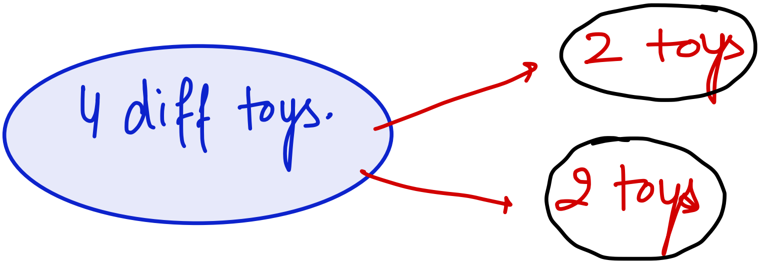
$^5C_3$ OR $^5C_2$

$$\downarrow$$

$$\frac{5!}{3!\,2!}$$

$T_1T_2T_3$	T_4T_5
$T_1T_2T_4$	T_3T_5
$T_1T_2T_5$	T_3T_4
$T_2T_3T_4$	T_1T_5
$T_2T_3T_5$	T_1T_4
$T_3T_4T_5$	T_1T_2
$T_2T_4T_5$	T_1T_3
$T_1T_4T_5$	T_2T_3
$T_1T_3T_5$	T_2T_4
$T_1T_3T_4$	T_2T_5

T_1, T_2, T_3, T_4, T_5

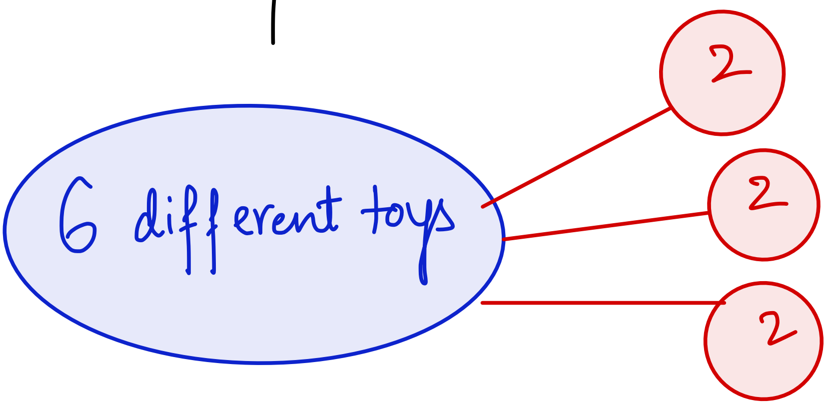


T_1, T_2, T_3, T_4

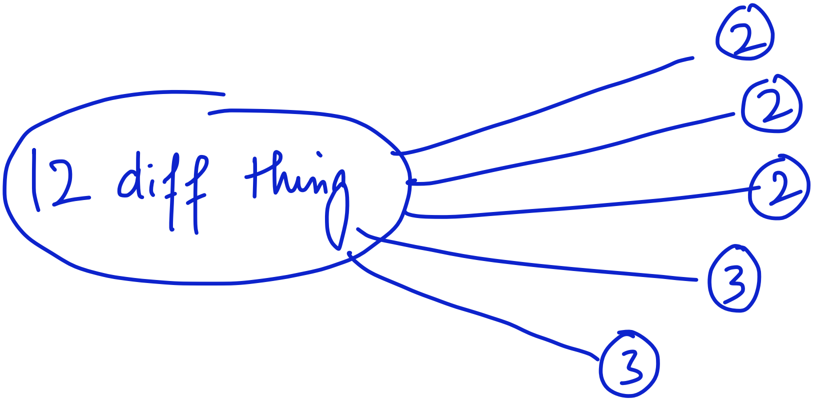
$^4C_2$

	$T_1 T_2$	$T_3 T_4$
	$T_1 T_3$	$T_2 T_4$
2	$T_1 T_4$	$T_2 T_3$
	$T_2 T_3$	$T_1 T_4$
	$T_2 T_4$	$T_1 T_3$
	$T_3 T_4$	$T_1 T_2$

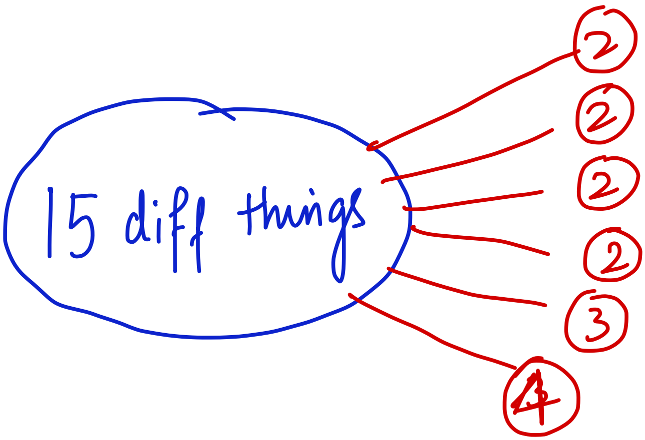
$$\frac{4!}{(2!\,2!)\,2!}$$



$$\frac{6!}{(2!\,2!\,2!)\,3!}$$



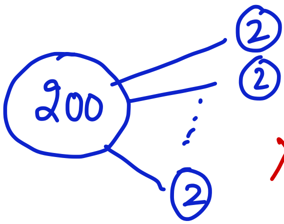
$$\frac{12!}{(2!\,2!\,2!)\,3!\;(3!\,3!)\,2!}$$



$$\frac{15!}{(2!)^4\,4!\;(3!\,4!)}$$

Q number of ways in which 200 people can be divided into 100 couples is ____

Sol^n



$$X = \frac{200!}{(2!)^{100}\ 100!} \quad **$$

$$(2n)! = 2^n \cdot n!\ (1.3.5.\ldots.2n-1)$$

$$(200)! = 2^{100} \cdot 100!\ (1.3.5.\ \ldots\ 199)$$

$$X = \frac{\cancel{2^{100}} \cdot \cancel{100!}\ (1.3\ 5.\ ..\ 199)}{\cancel{(2!)^{100}}\ \cancel{100!}}$$

$$\boxed{X = 1.3.5\ldots.\ 199} \quad **$$

$$X = \frac{200 \times 199 \times 198 \times \ldots \times 101 \times \cancel{100!}}{(2)^{100} \cdot \cancel{100!}}$$

$$X = \frac{101.\ 102.\ \ldots\ 200}{2^{100}} = \left(\frac{101}{2}\right)\left(\frac{102}{2}\right)\cdots\left(\frac{200}{2}\right)$$

Q 5 different books are to distributed among R|S|G if each child gets atleast one book?

Sol^n

$$1 \quad 1 \quad 3 \longrightarrow \frac{5!}{(1!\,1!)\,2!\;3!} \times \underbrace{(3!)}_{} = a$$

$$2 \quad 2 \quad 1 \longrightarrow \frac{5!}{(2!\,2!)\,2!\;1!} \times \underbrace{(3!)}_{} = b$$

Total N.O.W = a+b

Alt :-

$B_1 \quad B_2 \quad B_3 \quad B_4 \quad B_5$

$$\cancel{\left(\left({}^5C_3\right)\cdot 3!\right) \times 3^2}$$

R	S	G
B_1	B_2	B_3
B_4		
B_5		

R	S	G
B_4	B_2	B_3
B_1		
B_5		

Q 8 computers of different make to be distributed in 5 school if each school gets at least one computer.

Sol^n

4 1 1 1 1 $\longrightarrow \dfrac{8!}{4!\,4!} \times 5! = a$

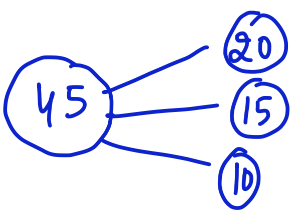
3 2 1 1 1 $\longrightarrow \dfrac{8!}{3!\,2!\,3!} \times 5! = b.$

2 2 2 1 1 $\longrightarrow \dfrac{8!}{(2!)^3 \cdot 3!\,2!} \times 5! = c.$

T.N.O.W = a+b+c.

Q During election 3 districts are to be convassed by 20, 15 and 10 people respectively. If 45 volunteers then the number of ways in which they can be sent.

Sol^n



$$\frac{45!}{20!\,15!\,10!}$$

PKW

Number of ways in which 8 persons can be seated in 3 different taxies each having 3 seats for passangers and duly numbered if

(I) internal arrangement of persons inside the taxi is immaterial.

(II) internal arrangement also matters.

Q In HMW 13 cards to each of the four players be distributed from a pack of 52 cards so that each may have ace, king, queen & jack of same suit?

♦ diamond (13)

♥ heart (13)

♠ spade (13)

♣ club. (13)

(ace, king, queen, jack of 4 suits)

$$52 - 16 = 36$$

$$\left(\frac{36!}{(9!\,9!\,9!\,9!)\,4!} \times 4! \right) \times 4!$$

distributed in 4 players.

Distribution of alike objects :-

$a_1 \ a_2 \ a_3 \longrightarrow 3!$

$a \ a \ a \longrightarrow \frac{3!}{3!} = 1$

$a \ a \ b \ c \longrightarrow \frac{4!}{2!}$

a, a, a, b, b, c, d

taken all at a time

$\frac{7!}{3! \, 2!}$

How many 4 letter words can be formed by them

taken some at a time

$a, a, a, b \longrightarrow 4$

a, a, b, b

$\vdots$

$a, b, c, d \longrightarrow 4!$

Number of permutation of n things
- p of one kind
- q of another kind and
- r are of yet another kind

taken all at a time $= \frac{n!}{p! \, q! \, r!}$

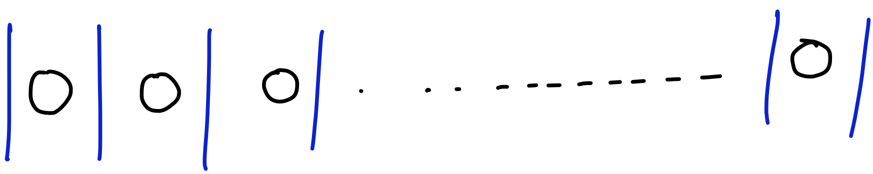
Q number of words which can be formed using all letters of the word "MAHABHARAT"

M → 1
A → 4
H → 2
R → 1
T → 1
B → 1

$$\frac{10!}{4!\ 2!}$$

Q 21 W and 19 B balls are arranged in a line (balls of the same colour alike). Find the number of arrangement if Black balls are separated.

N.O.W → 21 identical white balls arrangement in line = 1



$$^{22}C_{19} \times 1$$

Q In how many ways the letters of word PERMUTATIONS be arranged if word starts with P end with S.

$$\underset{\text{fix}}{P}\ \underbrace{-\,-\,-\,-\,-\,-\,-\,-\,-\,-}\ \underset{\text{fix.}}{S}$$

$$\frac{10!}{2!}$$

P → 1
E → 1
R → 1
M → 1
U → 1
T → 2
A → 1
I → 1
O → 1
N → 1
S → 1

I, P, C → 1 , E → 4, D → 2, N → 3 ; Vowels → I, E
 1 4

Consider the word INDEPENDENCE, then find the number of words taken all the letters if

(a) no two vowels are consecutive.

$$\left(\frac{7!}{2!\,3!}\right) \times {}^{8}C_{5} \times \left(\frac{5!}{4!}\right)$$

(b) E's are separated/no two 'E' occur together.

$$\left(\frac{8!}{3!\,2!}\right) \times {}^{9}C_{4} \times 1$$

(c) without changing the order of vowels

$$\left({}^{12}C_{5} \times 1\right) \times \frac{7!}{3!\,2!}$$

(d) at least one 'E' is separated from the rest of the E's (Total - all four together)

$$\left(\frac{12!}{4!\,2!\,3!} - \frac{9!}{2!\,3!}\right)$$

I, P, C, $\boxed{EEEE}$, D, D, N, N, N

(e) keeping the positions of vowels fixed (each V must remain at its own place)

$$1 \times \frac{7!}{2!\,3!}$$

(f) without changing the relative order of vowels and consonants.

V C C V C V C C V C C V

$$\left(\frac{5!}{4!}\right) \times \left(\frac{7!}{3!\,2!}\right)$$

(g) word $\boxed{\text{DIPEN}}$ forms

$\boxed{\text{DIPEN}}$ 7 letters

D
EEE
NN
C

$$\frac{8!}{3!\,2!}$$

HW

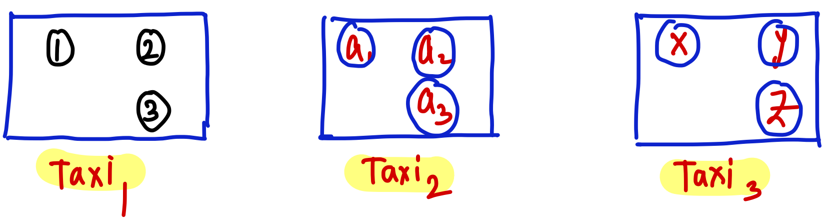
Q Find rank of the word "LEADER"?

Q Number of ways in which 8 persons can be seated in 3 different taxies each having 3 seats for passangers and duly numbered if

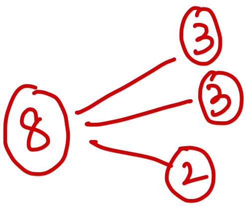
(I) internal arrangement of persons inside the taxi is immaterial.

(II) internal arrangement also matters.

Soln



(I)



$$\frac{8!}{(3!\,3!)\,2!\;2!} \times \underbrace{3!}$$

(II)

$$\left(\frac{8!}{(3!\,3!)\,2!\;2!} \times 3!\right) \times 3! \times 3! \times {}^3C_2 \times 2!$$

Alt:

$$\underbrace{{}^9C_8} \times 8!$$

Q Find rank of the word "LEADER" ?

Soln

fix → A _ _ _ _ _ $\frac{5!}{2!} = 60$

fix → D _ _ _ _ _ $\frac{5!}{2!} = 60$

fix * → E $\underbrace{\_ \_ \_ \_ \_}$ $5! = 120$

L A _ _ _ _ $\frac{4!}{2!} = 12$

fix

L D _ _ _ _ $\frac{4!}{2!} = 12$

LEADER ⇒ 1

$$\underline{265}$$

Q Number of numbers greater than a million using the digits 2, 3, 0, 3, 4, 2, 5 how many of these are div. by 5.

Sol^n

1 million = 10 lacs = $10 \times 10^5 = 10^6$. 1000000

M-1

T.N.O. Numbers formed by 2, 3, 0, 3, 4, 2, 5

0 _ _ _ _ _ _

$$\frac{7!}{2!\,2!} - \frac{6!}{2!\,2!} = \frac{6!\,(7-1)}{2!\,2!} = \frac{6 \cdot 6!}{2!\,2!}$$

M-2

$$\underbrace{\_}\;\overbrace{\_\;\_\;\_\;\_\;\_\;\_}^{0}$$

$$\underbrace{({}^{6}C_1 \cdot 1)}_{0} \times \frac{6!}{2!\,2!}$$

(ii) C-I

$$\underbrace{\_\;\_\;\_\;\_\;\_\;\_}_{0}\;\;\underline{0}\text{ fix}$$

$$\frac{6!}{2!\,2!} = a$$

C-II

$$\_\;\underbrace{\_\;\_\;\_\;\_\;\_}_{0}\;\;\underline{5}\text{ fix}$$

$$ {}^{5}C_1 \times \frac{5!}{2!\,2!} = b$$

a + b = Ans

Q Find the number of 7 digit numbers if the sum of their digits is 59. Digits to be used can be 5, 6, 7, 8, 9.

Sol^n

$$\left.\begin{aligned}5+5+5+5+5+5+5 &= 35\\ 9+9+9+9+9+9+9 &= 63\end{aligned}\right\}$$

(I) 9, 9, 9, 9, 9, 9, 5 $\longrightarrow \dfrac{7!}{6!}$

(II) 9, 9, 9, 9, 9, 8, 6 $\longrightarrow \dfrac{7!}{5!}$

(III) 9, 9, 9, 9, 9, 7, 7 $\longrightarrow \dfrac{7!}{5!\,2!}$

(IV) 9, 9, 9, 9, 8, 8, 7 $\longrightarrow \dfrac{7!}{4!\,2!}$ Total = $\overline{\underline{210}}$

(V) 9, 9, 9, 8, 8, 8, 8 $\longrightarrow \dfrac{7!}{4!\,3!}$

HW

Q By using all the letters of the word "SUCCESS", how many words can be formed such that no two 'C' are together as well as no two 'S' are together?

Q
HW

Four faces of a tetrahedral dice are marked with 2, 3, 4, 5. The lowest face being considered as the outcome. In how many ways a total of 30 can occur in 7 throws.

$\left\{ \begin{matrix} C, V, A, L, S, T, N, O \\ I \to 4 \end{matrix} \right\}$

****

CAUTION : Note the language which can be used in the problems

Number of other words are asked OR number of ways in which the given word is rearranged OR if as many more words as possible be formed.

For e.g. Use the letters of the word "CIVILISATION" without changing the relative order of vowels and consonants.

in above all the cases the correct answer is $\frac{6!}{4!} \times 6! - 1 = 21599$

$I, P, C \to 1$

$D \to 2$, $E \to 4$, $N \to 3$

Case-II : Examples on some at the time :

E(1) Find the number of words formed by taking 5 letters of the word INDEPENDENCE at a time.

Category	Selection	Arrangement
(1) 4 alike + 1 diff	${}^1C_1 \times {}^5C_1 = 5$ eg: E, E, E, E, D	$5 \times \frac{5!}{4!}$
(2) 3 alike + 2 diff	${}^2C_1 \cdot {}^5C_2 = 20.$ eg: E E E P N	$20 \times \frac{5!}{3!}$
(3) 3 alike + 2 others alike	${}^2C_1 \cdot {}^2C_1 \cdot = 4$ eg: N N N D D	$4 \times \frac{5!}{3!\,2!}$
** (4) 2 alike + 2 others alike + 1 diff	${}^3C_2 \times {}^4C_1 = 12$ eg: N, N, D, D, P	$12 \times \frac{5!}{2!\,2!}$
(5) 2 alike + 3 diff	${}^3C_1 \cdot {}^5C_3 = 30$ eg: D, D, E, N, P	$30 \times \frac{5!}{2!}$
(6) All 5 diff	${}^6C_5 = 6$ E, N, D, P, C	$6 \times 5!$

${}^3C_1 \cdot {}^2C_1 \cdot {}^4C_1$

eg: E, E, $\overbrace{N, N}$, C

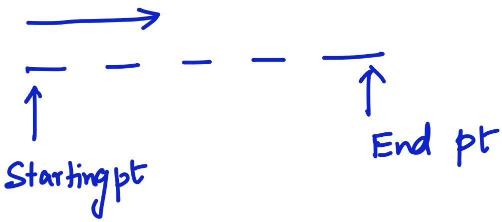
eg: N, N, $\overbrace{E, E}$, C

Q HW From word MISSISSIPPI selection of 5 letters can be done in how many ways.

HW

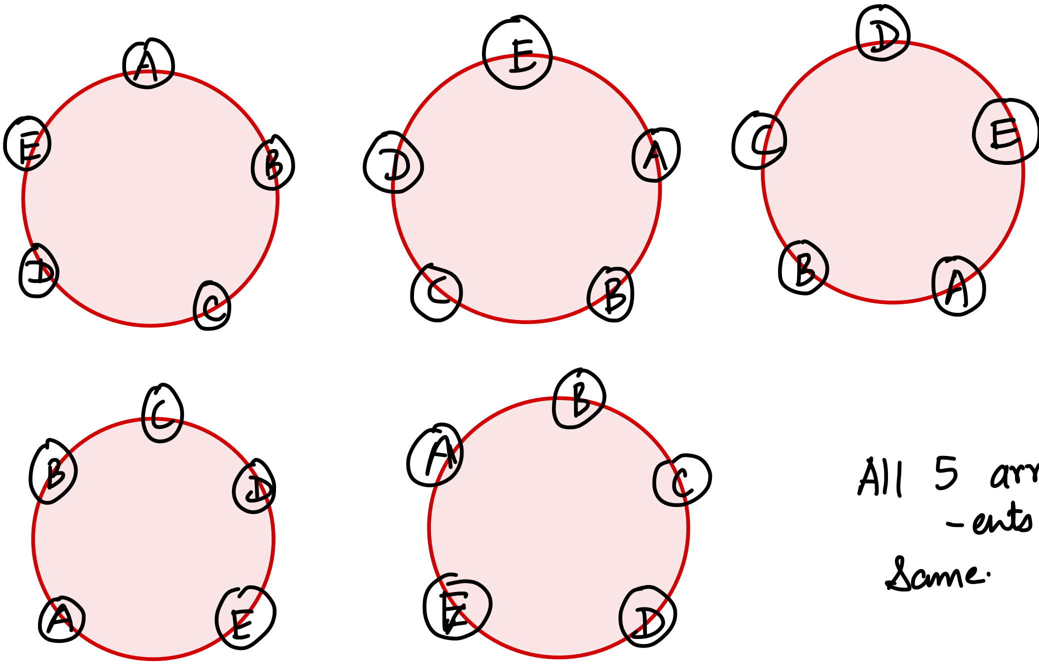
Q Number of numbers greater than 1000 from the digits 1,1,2,3,4,0 taken 4 at a time.

Circular permutation



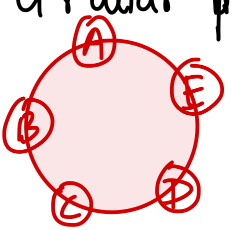
A , B , C, D, E $\longrightarrow$ Linear arrangement

5!



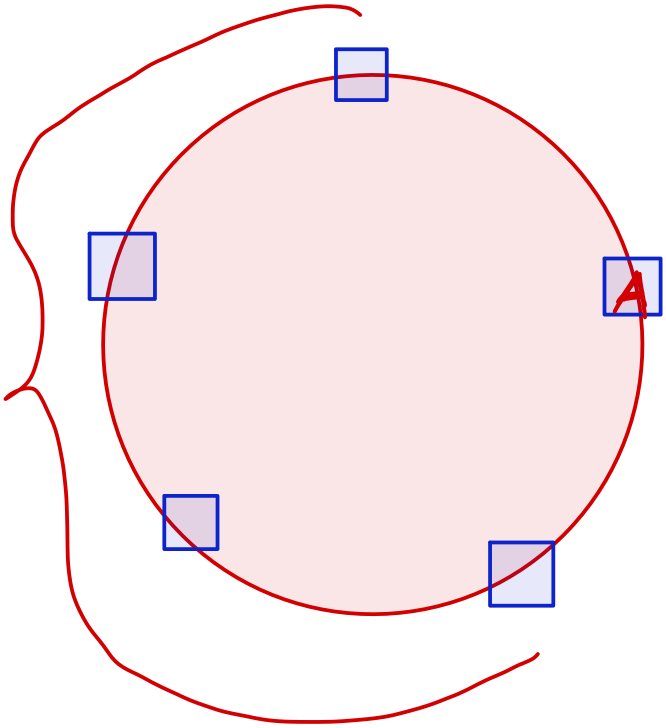
All 5 arrangem-ents are same.

No. of Circular permutation of 5 diff things



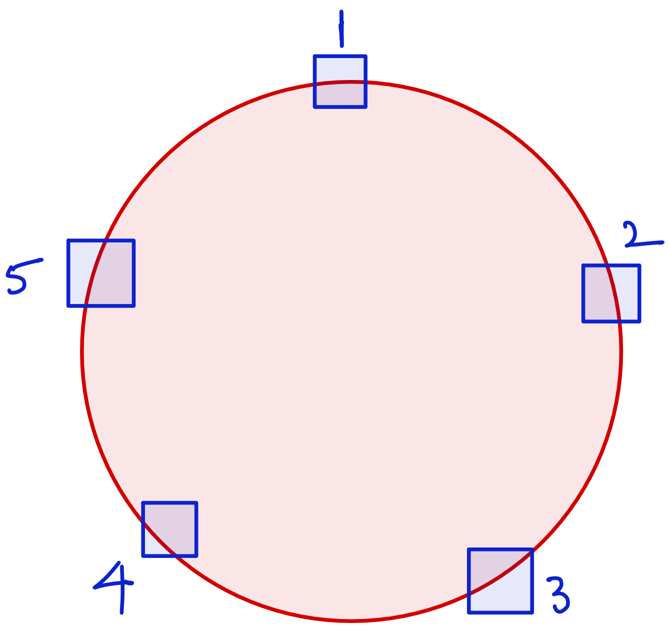
$$= \frac{5!}{5} = 4!$$

A, B, C, D, E



4!

A, B, C, D, E



5!

No. of Circular permutations :

(a) (i) of n different things taken all at a time = $(n-1)!$

(ii) taken r at a time = ${}^nC_r(r-1)!$

This distinguishes/discriminates/considers to be different the clockwise and anticlockwise arrangement.

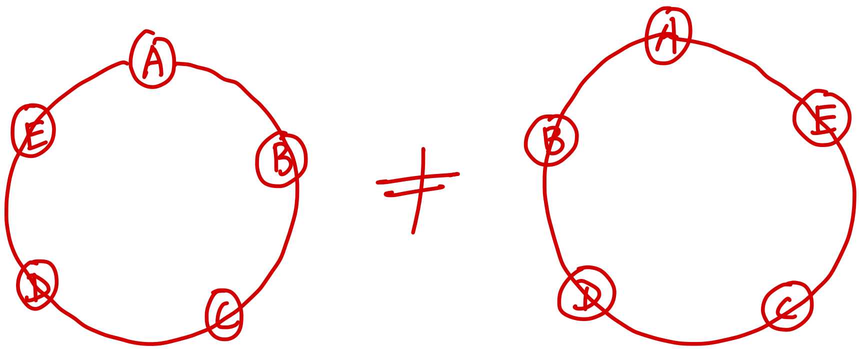
(b) If clockwise and anticlockwise arrangements are considered as same then the number of circular permutations of n different things = $\dfrac{(n-1)!}{2}$

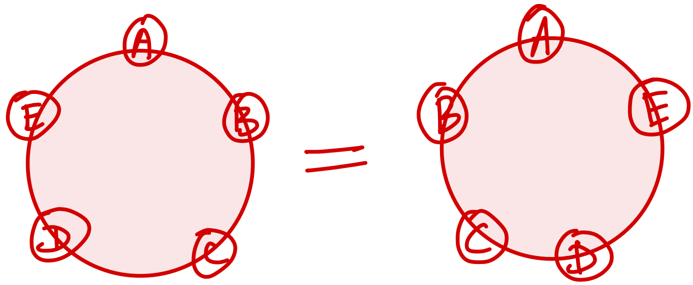
Note : In case of garland anticlockwise & clockwise sense are same or necklace made from beads

Q Number of ways in which 8 persons can be seated on a round table so that "all shall not have the same neighbours in any two arrangement".

$\dfrac{7!}{2}$

Soln Let us see for 5 persons





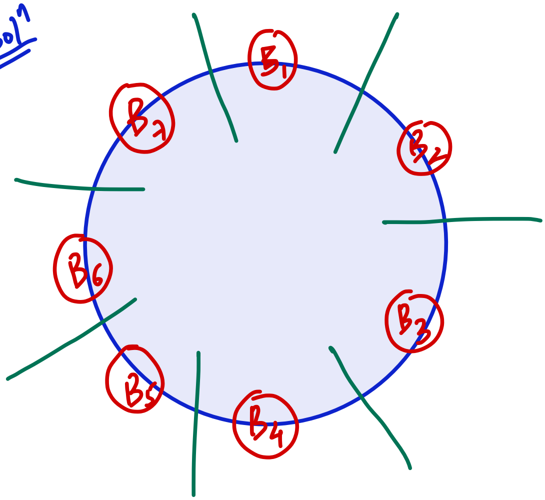
Q Out of 10 flowers of different colours, how many different garlands can be made if each garland consists of 6 flower of different colour.

Soln

$$^{10}C_6 \times \frac{5!}{2!}$$

Q Number of ways in which 7 Americans and 7 British people can be seated on a round table so that no two Americans are consecutive.

Sol^n

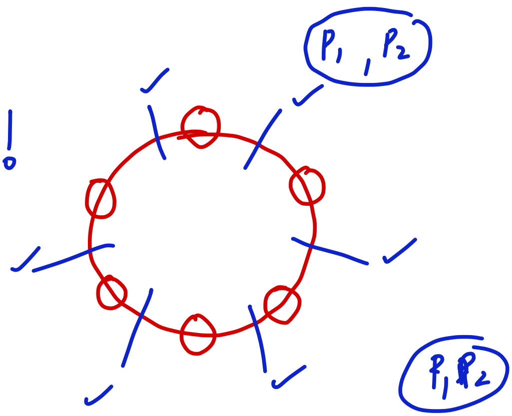


$$6! \times \underbrace{{}^{7}C_{7} \times 7!}$$

Q Number of ways in which 8 persons can be seated on a round table so that two particular person are never together.

$$8 \rightarrow 6, \quad 8 \rightarrow 2 \ ✓$$

$$5! \times {}^{6}C_{2} \cdot 2!$$



Alt:

T.N.O.W − When those 2 particular persons are together

$$7! - 6! \times 2!$$

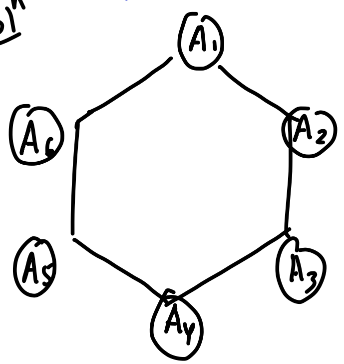
HW Q Number of ways in which 5B and 5G can be seated on a circle alternately if a particular B and G are never adjecent to each other in any arrangement.

HW

Q How many hexagons can be constructed by joining the vertices of a quindecagon (15 sides) if none of the sides of the hexagon is also the side of the 15-gon.

(ii) How many triangles can be formed using vertices of a hexagon if none of the side of hexagon is side of Δ ?

Sol^n



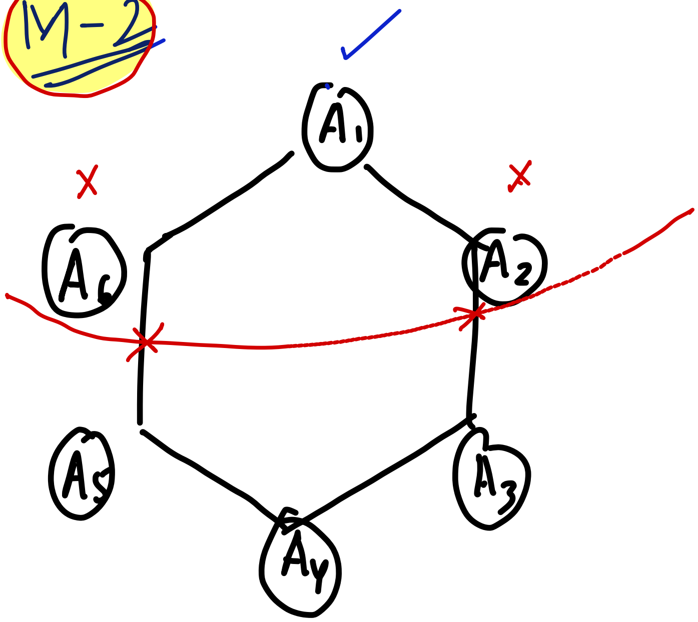
M-1

$$^6C_3 - \left(\overbrace{\text{When all 3 vertices Choosen are consecutive}} + \text{When exactly 2 vertices are consecutive}\right)$$

$$^6C_3 - \left(\overbrace{6} + \overbrace{6 \times {}^2C_1}\right) = 20 - 18$$

$$= 2.$$

M-2



$$\frac{{}^{6}C_{1}\;{}^{2}C_{2}}{3} = 2$$

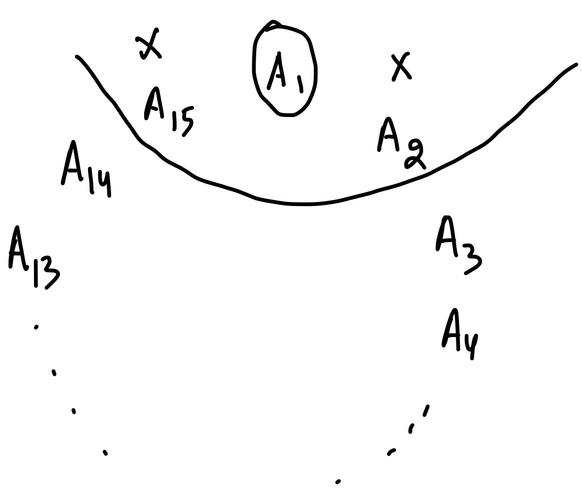
A_1 A_3 A_5

A_3 A_1 A_5

A_5 A_1 A_3

Q How many hexagons can be constructed by joining the vertices of a quindecagon (15 sides) if none of the sides of the hexagon is also the side of the 15-gon.

Sol^n



$$\frac{{}^{15}C_{1} \times {}^{8}C_{5}}{6}$$

A_1	A_3	A_6	A_{10}	A_{12}	A_{14}
A_3	A_1	A_6	A_{10}	A_{12}	A_{14}
⋮					
A_{14}	—	—	—	—	—

HW

Q n people A_1 ,A_2, ...,A_n sitting on a circle. Number of ways in which 3 people can be selected if no two of them are consecutive. [Ans. ${}^nC_3 - (n^2 - 3n)$]

2 Methods

TOTAL NUMBER OF COMBINATIONS : Means selecting at least one out of n things.

Case-I : When things are all different

The number of ways of selecting atleast one thing out of n distinct things = ${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots.. + {}^nC_n = 2^n - 1$

Case-II : When some alike, some other alike and remaining all the different

The number of ways of selecting atleast one thing = $\left((p+1)(q+1)2^{n-p-q}\right) - 1$

where p alike are of one kind, q alike are of other kind and rest are different

4 alike oranges

5 alike mangoes

6 diff bananas

$\xrightarrow[\text{atleast one fruit}]{\text{select}}$ $(5 \times 6 \times 2^6) - 1$

Q By using all the letters of the word "SUCCESS", how many words can be formed such that no two 'C' are together as well as no two 'S' are together?

U, C, C, E

Sol^n

$S \to 3$
$C \to 2$
$U, E \to 1$

Total No. of words when no two 's' are together −

T.N.O words when no two 's' are together and two 'c' are always together.

$$\frac{4!}{2!} \times {}^5C_3 - 3! \times {}^4C_3 \times 1 = 120 - 24 = 96$$

Ans

Q Four faces of a tetrahedral dice are marked with 2, 3, 4, 5. The lowest face being considered as the outcome. In how many ways a total of 30 can occur in 7 throws.

$5\ 5\ 5\ 5\ 5\ 2\ 3 \longrightarrow \frac{7!}{5!}$

$5\ 5\ 5\ 5\ 4\ 4\ 2 \longrightarrow \frac{7!}{4!\,2!}$

$5\ 5\ 5\ 5\ 3\ 3\ 4. \longrightarrow \frac{7!}{4!\,2!}$

$5\ 5\ 5\ 4\ 4\ 4\ 3. \longrightarrow \frac{7!}{3!\,3!}$

$5\ 5\ 4\ 4\ 4\ 4\ 4 \longrightarrow \frac{7!}{2!\,5!}$

Total = 413

Q From word MISSISSIPPI selection of 5 letters can be done in how many ways.

M → 1
S → 4
I → 4
P → 2

Category	Selection
① 4A + 1 diff	${}^2C_1 \cdot {}^3C_1 = 6$ eg: IIIIS
② 3A + 2A'	${}^2C_1 \cdot {}^2C_1 = 4$ eg: SSSII
③ 3A + 2 diff	${}^2C_1 \cdot {}^3C_2 = 6$ eg: SSSPM
④ 2A + 2A' + 1 diff	${}^3C_2 \cdot {}^2C_1 = 6$ eg: SSPPM
⑤ 2A + 3 diff	${}^3C_1 \cdot {}^3C_3 = 3$ eg: PPSIM

Total = 25

Q Number of numbers greater than 1000 from the digits 1,1,2,3,4,0 taken 4 at a time.

1, 1, 2, 3, 4, 0

Category	Selection	Arrangement
① 2A + 2 Diff	(i) '0' excluded $^1C_1 \cdot {}^3C_2 = 3$ eg: 1, 1, 2, 4	$3 \times \frac{4!}{2!} = 36.$
	(ii) '0' included. $^1C_1 \cdot {}^3C_1 = 3$ eg: 1, 1, 0, 2	$3 \times {}^3C_1 \times \frac{3!}{2!} = 27$
② All 4 Diff	(i) '0' excluded $^4C_4 = 1$ eg: 1, 2, 3, 4	$1 \times 4! = 24$
	(ii) '0' included. $^4C_3 = 4$ eg: 2, 3, 4, 0.	$4 \times {}^3C_1 \times 3! = 72$

Total = 159

Ans..

Q n people $A_1, A_2, \ldots\ldots, A_n$ sitting on a circle. Number of ways in which 3 people can be selected if no two of them are consecutive. [Ans. ${}^{n}C_3 - (n^2 - 3n)$]

M-1

${}^{n}C_3 - \left(\text{when all 3 Consecutive} + \text{when exactly '2' consecutive}\right)$

$${}^{n}C_3 - \left(n + n \times {}^{n-4}C_1\right)$$

$${}^{n}C_3 - \left(n + n.(n-4)\right)$$

$${}^{n}C_3 - \left(n^2 - 3n\right).$$

M-2

$$\frac{{}^{n}C_1 \times {}^{n-4}C_2}{3} = \frac{n.(n-4)!}{3 \times 2!(n-6)!}$$

$$= \frac{n.(n-4)(n-5)}{6}$$

TOTAL NUMBER OF COMBINATIONS : Means selecting at least one out of n things.

Case-I : When things are all different

The number of ways of selecting atleast one thing out of n distinct things = ${}^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 + \ldots.. + {}^{n}C_n = 2^n - 1$

Case-II : When some alike, some other alike and remaining all the different

The number of ways of selecting atleast one thing = $\left((p+1)(q+1)2^{n-p-q}\right) - 1$

where p alike are of one kind, q alike are of other kind and rest are different

4 alike oranges

5 alike mangoes

6 diff bananas

select atleast one fruit

$(5 \times 6 \times 2^{6}) - 1$

Q Out of 3 books of maths, 4 books of physics and 5 books of chemistry, how many different selections/combinations can be made so that each selection consists of :-

(i) One book on each subject.

(ii) atleast one book on each subject

(iii) atleast one book.

(iv) atleast one book of maths and atmost two books of chemistry.

Solve the question for two cases :-

C-I :- When books are all different.

C-II :- When books are alike except for the subject.

C-I :

1. $^3C_1 \cdot {}^4C_1 \cdot {}^5C_1$.

2. $({}^3C_1+{}^3C_2+{}^3C_3)({}^4C_1+{}^4C_2+{}^4C_3+{}^4C_4)$

   $({}^5C_1+{}^5C_2+\cdots+{}^5C_5) = (2^3-1)(2^4-1)(2^5-1)$

3. $^{12}C_1+{}^{12}C_2+\cdots+{}^{12}C_{12} = 2^{12}-1$.

4. $({}^3C_1+{}^3C_2+{}^3C_3)\cdot 2^4 \cdot ({}^5C_0+{}^5C_1+{}^5C_2)$.

C–II

MMM PPPP CCCCC

① $1 \times 1 \times 1 = 1$

② $3 \times 4 \times 5$

③ $4 \times 5 \times 6 - 1$

④ $3 \times 5 \times 3.$

Q. A shopkeeper places before you n different books each having p copies. Find the number of different selections that can be made.

$$(p+1)(p+1)\ldots(p+1) - 1 = (p+1)^n - 1.$$

Q Let $A = \{1, 2, 3, 4, 5\}$

Find no. of subsets of A ?

$\phi \equiv \{\ \} \equiv$ Null set

Sol^n 2^5.

Q Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ then find no. of subsets of $A \times A$ containing atleast 1 and not more than 47 ordered pairs.

Sol^n $A \times A = 7 \times 7 = 49$

$${}^{49}C_1 + {}^{49}C_2 + {}^{49}C_3 + \cdots + {}^{49}C_{47} = \text{N.O.W} = P$$

$$\underbrace{{}^{49}C_0} + {}^{49}C_1 + \cdots + {}^{49}C_{47} + \underbrace{{}^{49}C_{48} + {}^{49}C_{49}} = 2^{49}$$

$$\therefore P = 2^{49} - 1 - {}^{49}C_{48} - 1 = 2^{49} - 2 - 49$$

$$= 2^{49} - 51 \text{ Ans}$$

Q Let $S = \{1,2\}$ then find no. of unordered pairs of disjoint subsets of S?

Sol^n

$S = \{1,2\}$

ϕ

$\{1\}$

$\{2\}$

$\{1,2\}$

ϕ, ϕ

$\phi, \{1\}$

$\phi, \{2\}$

$\phi, \{1,2\}$ same

same

$\{1\}, \{2\}$

same

$\{2\}, \{1\}$

same

$\{1\}, \phi$

same

$\{2\}, \phi$

$\{1,2\}, \phi$

$\therefore$ No. of unordered pairs of disjoint subsets of 'S' $= 5$

$$\frac{3 \times 3 - 1}{2} + 1 = 5$$

② $S = \{1,2,3,4\} \longrightarrow \left(\frac{3^4 - 1}{2} + 1\right)$

No. of +ve divisors of 12 ?

$\underbrace{1, 2, 3, 4, 6, 12}$ $\Rightarrow$ Total '6' divisors

$12 = 2^2 \cdot 3^1$

$2^1, 2^2, 2^0$

$3^1, 3^0$

$3 \times 2 = 6$ divisors

Sum of all +ve divisors $= 1 + 2 + 3 + 4 + 6 + 12$

$= 28.$

$$(2^0 + 2^1 + 2^2)(3^0 + 3^1)$$

$$= 2^0 3^0 + 2^0 \cdot 3^1 + 2^1 \cdot 3^0 + 2^1 \cdot 3^1 + 2^2 \cdot 3^0 + 2^2 \cdot 3^1$$

Q

Consider the number $N = 75600\ (2^4.3^3.5^2.7^1)$

(i) Number of positive divisors $\rightarrow 5 \times 4 \times 3 \times 2 = 120$

(ii) Number of positive proper divisors $= 120 - 1 = 119$ (note that N is not the proper divisor)

(iii) Number of positive odd divisors = no 2 should be taken = $1 \times 4 \times 3 \times 2.$

(iv) Number of positive even divisors $\cancel{(2\lambda+1)};\ \lambda \in W$ = atleast one 2 should be taken = $4 \times 4 \times 3 \times 2 = 96.$

(v) Number of positive divisors divisible by 5 = atleast one 5 must be taken = $5 \times 4 \times 2 \times 2 = 80$

(vi) Number of positive divisors divisible by 10 = atleast one 5 and atleast one 2 must be taken = $4 \times 4 \times 2 \times 2$

(vii) Sum of all the positive divisors =

(viii) No. of +ve divisors of form $(4n+2)$ where $n \in W$. $= 1 \times 4 \times 3 \times 2$

$2(2n+1)$

(vii) $(2^0 + 2^1 + 2^2 + \dots + 2^4)(3^0 + 3^1 + 3^2 + 3^3)(5^0 + 5^1 + 5^2)(7^0 + 7^1)$

$$\left(\frac{2^5 - 1}{2-1}\right) \cdot \left(\frac{3^4 - 1}{3-1}\right) \cdot (31)(8)$$

(ix) sum of all odd positive divisors:

$$(2^0)(3^0 + 3^1 + 3^2 + 3^3)(5^0 + 5^1 + 5^2)(7^0 + 7^1)$$

Note:- If given number is a perfect square, then total number +ve divisors will be odd

(h) Number of ways in which N can be resolved as a product of two divisors

$$(N = p^a \cdot q^b \,\ldots\ldots;\ p, q, \ldots \text{ are prime numbers.}) \quad = \begin{cases} \dfrac{n}{2} & \text{when } n \text{ is even} \\ \dfrac{n+1}{2} & \text{when } n \text{ is odd} \end{cases}, \text{ where } n \text{ is number of divisiors of } N.$$

(i) Number of ways in which N can be resolved as a product of 2 divisors which are relatively prime $= 2^{n-1}$ where n is the number of primes involved in the prime factorisation of N.

$N = a^{p_1} \cdot b^{p_2} \cdot c^{p_3} \cdot d^{p_4} \ldots\ldots$; where $a, b, c, d, \ldots\ldots$ are prime numbers

Relatively prime = Co-prime = HCF is 1

$N = 2^4 \cdot 3^3 \cdot 5^2 \cdot 7^1$

$\Rightarrow 2^{4-1} = 2^3 = 8$ ways.

Q In how many different ways the number $N = 2^4.3^2.5^3.7^2.11^4.13^2$ can be resolved as product of two divisors which are relatively prime.

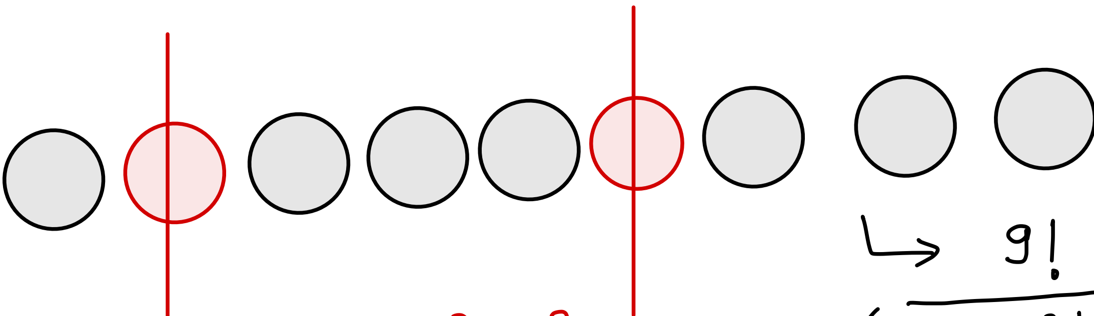
$$2^{6-1} = 2^5 = 32.$$

V.V. Imp

Distribution of alike objects :

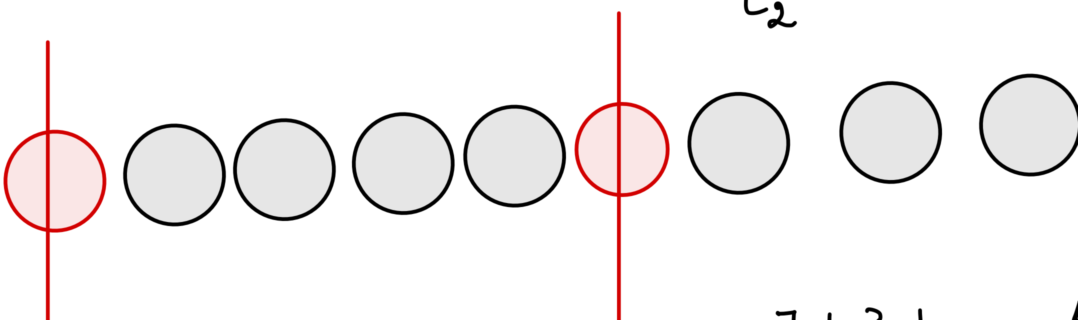
(also known as Beggar Method)

Let we are having 7 identical coins which needs to be distributed among 3 beggers such that each begger gets none, one or more no. of coins.



$B_1 = 1$; $B_2 = 3$; $B_3 = 3$

$$\frac{9!}{7!\,2!} = {}^9C_2$$



$B_1 = 0$; $B_2 = 4$; $B_3 = 3$

$$^{7+3-1}C_{3-1} = {}^9C_2$$

Number of ways in which n identical coins can be distributed between p persons, if each person receiving none, one or more coins $= \dfrac{(n+p-1)!}{n!(p-1)!} = {}^{n+p-1}C_{p-1}$

$= {}^{n+p-1}C_{n}$

Q A shelf contains 6 separate compartments. Number of ways in which 12 indistinguishable (like) marbles can be placed in the compartments so that no compartment is empty.

Sol^n

$$c_1 + c_2 + c_3 + c_4 + c_5 + c_6 = 12$$

$c_1 > 0$
$c_2 > 0$
$c_3 > 0$
$c_4 > 0$
$c_5 > 0$
$c_6 > 0$

$$\underbrace{c_1 + c_2 + c_3 + \cdots + c_6}_{} = 6.$$

$$\text{T.N.O.W} = {}^{6+5}C_5$$

Q There are unlimited number of Red, White, Black & Green balls. (balls are all alike except for the colour)

(i) Number of ways in which a selection of 10 balls can be made.

(ii) Number of ways in which selection of 10 balls contain balls of all four different colours.

(i) $R + W + B + G = 10.$; $R \geq 0$, $W \geq 0$, $B \geq 0$, $G \geq 0$

$^{10+3}C_3$ ✓

(ii) $R + W + B + G = 10$; $R \geq 1$, $W \geq 1$, $B \geq 1$, $G \geq 1$

$R + W + B + G = 6$

$^{6+3}C_3$ ✓

Q Number of different dissimilar terms in $(x_1 + x_2 + \ldots\ldots + x_n)^m$ is $^{m+n-1}C_{n-1}$

Sol^n

eg: $(a+b)^2 = a^2 + 2ab + b^2$

ab ab

$(x_1 + x_2 + \cdots + x_n)^m$ $^{m+(n-1)}C_{(n-1)}$

$x_1, x_2, \ldots, x_n \rightarrow$ beggars

$m \rightarrow$ identical coins.

Q Number of natural solution of the equation x+y+z = 102

$$x+y+z=102\ ; \quad x\geq 1,\ y\geq 1,\ z\geq 1$$

$$x_1+y_1+z_1=99 \quad x_1\geq 0,\ y_1\geq 0,\ z_1\geq 0.$$

$${}^{99+2}C_2$$

Q Number of non negative integral solution of the inequality $x+y+z+t \leq 30$

$$x+y+z+t+w=30\ ; \quad w\geq 0$$

$x, y, z, t \in$ Whole number

$${}^{30+4}C_4$$

Q Number of non-negative integral solution of inequality $x+y+z+t<30$, such that $x\geq 1,\ y>1,\ z\geq 2,\ t>0$

Sol^n

$$x+y+z+t+w=30\ ; \quad x\geq 1,\ y\geq 2,\ z\geq 2,\ t\geq 1,\ w\geq 1$$

$$x_1+y_1+z_1+t_1+w_1=23$$

$${}^{23+4}C_4$$

HW

Q. Number of non negative integral solution of the inequality $10 \le x+y+z+t \le 30$

Q Find number of odd natural solution(s) of the equation ① $x+y+z = 18$.

② $x+y+z = 19$.

Sol^n

$$\left.\begin{aligned} x &= 2n_1+1 & n_1 \geq 0 \\ y &= 2n_2+1 & n_2 \geq 0 \\ z &= 2n_3+1 & n_3 \geq 0 \end{aligned}\right\} n_i \in N$$

① No sol^n

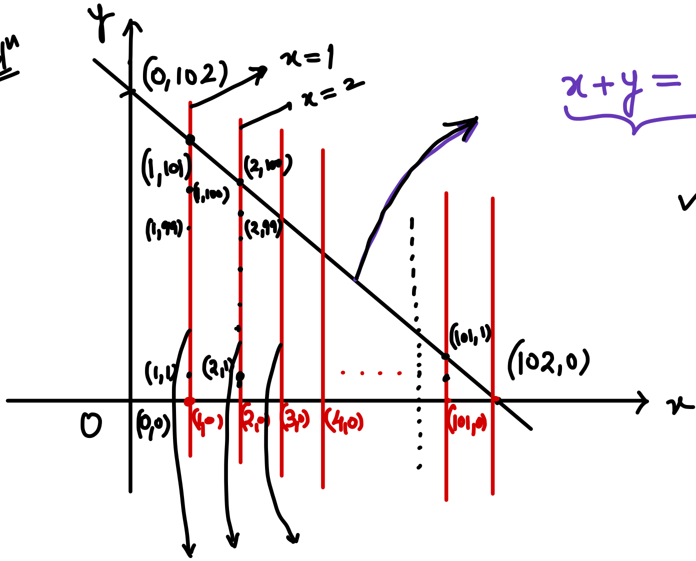
② $(2n_1+1)+(2n_2+1)+(2n_3+1) = 19$

$\cancel{2}(n_1+n_2+n_3) = \cancel{16}\ 8$

$n_1+n_2+n_3 = 8. \Longrightarrow {}^{8+2}C_2 = {}^{10}C_2$

Q Number of points with integral co-ordinates that lie inside a triangle whose vertices are (0, 0), (0, 102) and (102, 0) is.

Sol^n



$$\underbrace{x+y=102}$$

✓ **M-1**

$$\left.\begin{matrix} x>0 \\ y>0 \end{matrix}\right\}$$

$$x+y<102$$

$$x+y+w=102$$

$$\left.\begin{matrix} x>0 \\ y>0 \\ w>0 \end{matrix}\right\}$$

$$x+y+w=99$$

$$^{99+2}C_2 = {}^{101}C_2$$

$$= 5050.$$

M-2

$$100+99+98+\cdots+1 = 5050.$$

Q In HMW 15 identical coins can be distributed among R|S|G such that 'R' gets atleast one, 'S' gets atleast two and 'G' gets exactly three coins?

Sol^n

$$\underbrace{R \geq 1}\,;\ \underbrace{S \geq 2}\,;\ \underbrace{G = 3}.$$

$$R + S + G = 15$$

$$R_1 + S_1 = 9\ ;\quad R_1 \geq 0,\ S_1 \geq 0$$

$$\downarrow$$

$${}^{10}C_1$$

Q In HMW 25 identical books can be distributed among R|S|G such that 'R' gets atleast one, 'S' gets atleast two & 'G' gets at most 10 books.

Sol^n $R \geq 1$; $S \geq 2$; $G \leq 10$.

① $R+S+G = 25$; $R \geq 1$; $S \geq 2$; $G \geq 0$

② $R+S+G = 25$; $R \geq 1$; $S \geq 2$; $G \geq 11$

Req. ans $= {}^{22+2}C_2 - {}^{13}C_2$

Q. HW Number of ways in which 3 apples, 4 oranges and 5 mangoes can be distributed between R|S|G if each receives :

(i) None, one or more no. of fruits.

(ii) atleast one fruit of each species.

Solve the problem for 2 cases :

C–I Fruits of same species are alike.

C–II Apples are identical, oranges are identical and mangoes are all different.

Q Find no. of positive integral solⁿ of equation $xyz = 24$?

Solⁿ

$$xyz = 2^3 \cdot 3^1$$

$$x = 2^{a_1} \cdot 3^{b_1}$$

$$y = 2^{a_2} \cdot 3^{b_2}$$

$$z = 2^{a_3} \cdot 3^{b_3}$$

$\left.\begin{array}{l} a_i \geq 0 \\ b_i \geq 0 \end{array}\right\}$

$$2^{a_1+a_2+a_3} \cdot 3^{b_1+b_2+b_3} = 2^3 \cdot 3^1$$

$a_1+a_2+a_3 = 3$ and $b_1+b_2+b_3 = 1$.

$$^5C_2 \times {}^{1+2}C_2 = 10 \times 3 = \underline{\underline{30}}$$

Q **Statement - 1 :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $^9C_3$. **[AIEEE-2011]**

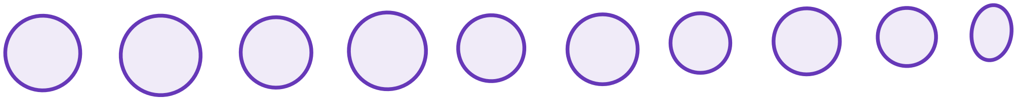
Statement - 2 : The number of ways of choosing any 3 places from 9 different places is $^9C_3$.

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.



HW

Q In how many ways 30 can be partioned into 3 unequal natural parts ?

Q HW

If 9 balls are to be distributed in 3 boxes, then find the number of ways of distribution if

(i) Balls & boxes are distinct.

(ii) Balls are identical & boxes are distinct.

(iii) Both are identical

GENERAL/MISCELLANEOUS :

$n \in$ Constant

Maximise ^nC_r : The value of r for which $^nC_r > {^nC_{r+1}}$ and $^nC_r > {^nC_{r-1}}$

$$\Rightarrow \quad {^nC_r} \text{ is maximum if } \begin{cases} r = \dfrac{n}{2} & \text{if n is even} \\ r = \dfrac{n-1}{2} \text{ or } \dfrac{n+1}{2} & \text{if n is odd} \end{cases}$$

E(1) Find the difference between the greatest value of $^{15}C_r$ and $^{12}C_r$.

Sol^n

$({^{15}C_r})_{max}$ when $r = \dfrac{15-1}{2}$ or $\dfrac{15+1}{2}$

$r = 7$ or 8

$|{^{15}C_7} - {^{12}C_6}| =$ Ans

Q A man wants to give as many number of parties as he can out of his 20 friends by calling the same number of people but not the same group. How many should he call so that the number of parties is maximum.

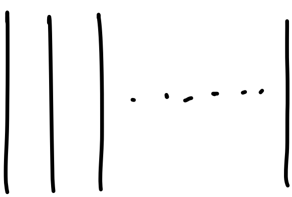
Sol^n Let he is inviting 'r' friends.

$$^{20}C_r \longrightarrow \text{maximise.}$$

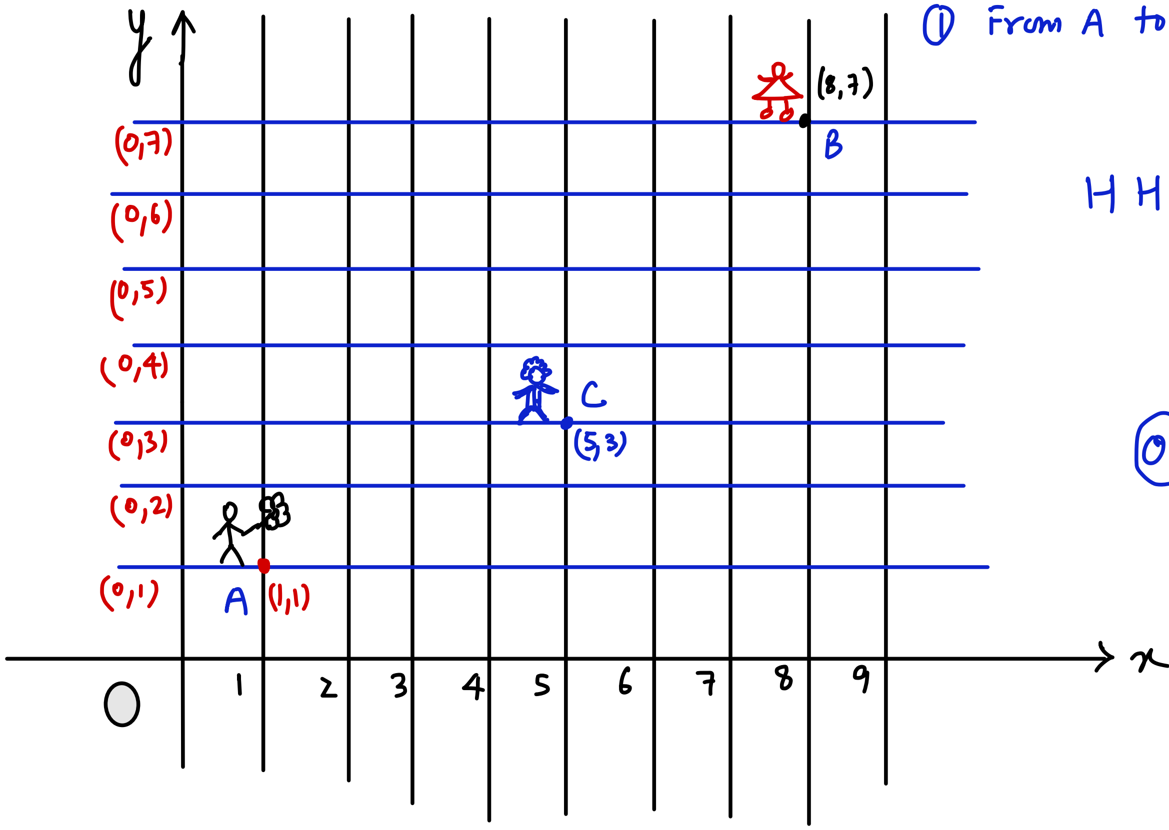
$$^{20}C_{10} \checkmark$$

GRID PROBLEM:

m horizontal lines $\longrightarrow$ $(m-1)$ vertical path



n vertical lines $\longrightarrow$ $(n-1)$ horizontal paths.



① From A to B in minimum steps

13 steps

H H H H H H H V V V V V V

$$^{13}C_7 = K$$

OR

$$\frac{13!}{7!\,6!}$$

② A to B without using C in minimum steps

$$K - (\underbrace{A \text{ to } C}\ \&\ C \text{ to } B)$$

$$^{13}C_7 - \left({}^{6}C_4 \cdot {}^{7}C_3 \right)$$

eg

1, 5, 8 $\longrightarrow$ 3 digit Nos (no repition)

$3! = 6.$

$$\begin{array}{ccc} 3 & 2 & \\ 1 & 5 & 8 \\ 1 & 8 & 5 \\ 5 & 1 & 8 \\ 5 & 8 & 1 \\ 8 & 1 & 5 \\ 8 & 5 & 1 \\ \hline 31 & 0 & 8 \\ \hline \end{array}$$

Unit place $\rightarrow 2(1+8+5)$

Tens " $\rightarrow 2\times10(1+8+5)$

Hundreds " $\rightarrow 2\times10^2(1+8+5)$

Add

$2.(1+8+5)\underbrace{(1+10+10^2)}$

$2\times(14)\times(111)$

28×111

1, 2, 4, 9 $\longrightarrow$ 4 digit

$6(1+2+4+9)(1111)$

(10) Summation of numbers – (3 different ways)

(a) Sum of all the numbers greater then 10000 formed by the digits 1,3,5,7,9 no digit being repeated.

Method - 1 All possible numbers = 5! = 120

If one occupies the units place then total numbers = 24.

Hence 1 enjoys units place 24 times

				1

|||ly 1 enjoys each place 24 times

Sum due to 1 = $1 \times 24\,(1 + 10 + 10^2 + 10^3 + 10^4)$

|||ly Sum due to the

digit 3 = $3 \times 24\,(1 + 10 + 10^2 + 10^3 + 10^4)$

⋮ ⋮ ⋮ ⋮ ⋮ ⋮ ⋮

Required total sum = $24\,(1 + 10 + 10^2 + 10^3 + 10^4)\,(1 + 3 + 5 + 7 + 9)$

Method –2 In 1st column there are twenty four 1's , Twenty four 3's & so on and their sum is

$= 24 \times 25 = 600$

Hence add. in vertical column normally we get = 6666600

	5th			2nd	1st
120 Number	X	X	X	X	X
	X	X	X	X	X
	⋮	⋮	⋮	⋮	⋮
	⋮	⋮	⋮	⋮	⋮
	X	X	X	X	X
	666	6	6	0	0

= 6666600]

Method–3 Applicable only if the digits used are such that they have the same common difference.

Writing all the numbers in ascending order of magnitude

S = (13579 + 13597 + ... + 97513 + 97531)

S = (13579 + 97531) + (13597 + 97513) + ...

= (111110) 60 time = 6666600 Ans

$$S = \frac{n}{2}(l + L)$$ where n = number of numbers, l = smallest, L = Largest

Q Sum of all the numbers greater then 10000 formed by the digits 0, 2, 4, 6, 8 no digit being repeated.

Sol^n $0, 2, 4, 6, 8 \longrightarrow 5! = 120$

$\underline{0}\ \underbrace{-\ -\ -\ -}$

$$\text{Req. sum} = \frac{120}{2}(02468 + 86420) - \frac{24}{2}(2468 + 8642)$$

Q Sum of all the numbers that can be formed using all the digits 2, 3, 3, 4, 4, 4 is :

T.N.O Numbers $= \dfrac{6!}{3!\,2!} = 60$

$$
\begin{array}{cccccc}
22 & 22 & 22 & 22 & 20 & \downarrow \\
X & X & X & X & X & X \\
X & X & X & X & X & X \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
X & X & X & X & X & X \\
\hline
\end{array}
$$

22222200

$2 \to 10$

$3 \to 20$

$4 \to 30.$

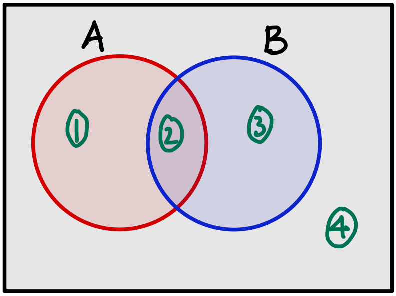
$2\times 10 + 3\times 20$

$+ 4\times 30 =$

$20 + 60 + 120 = 200$

Imp

Principle of inclusion and exclusion



$A \longrightarrow 1 + 2$

$B \longrightarrow 2 + 3$

$A \cap B \longrightarrow 2$ A and B both

$A \cap \bar{B}$ OR $A \cap B^c = 1$

$\bar{A} \cap B = 3$

$\bar{A} \cap \bar{B} = 4$

$n(A) =$ no. of elements in set A.

$A \cup B$ = A or B $= 1 + 2 + 3$

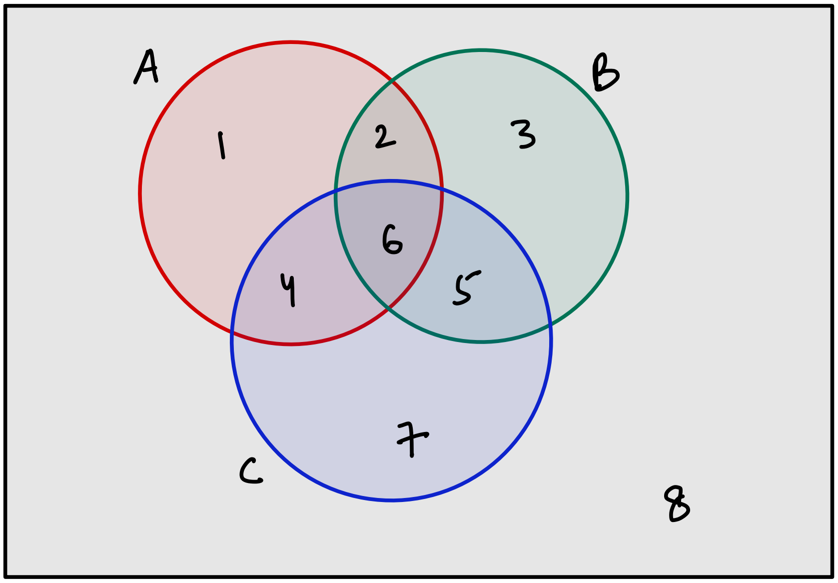
(atleast one of A or B)

Rem

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

$$n(A \cap \bar{B}) = n(A) - n(A \cap B)$$

$$n(\text{exactly 1 of A or B}) = n(A) + n(B) - 2n(A \cap B)$$



$A \to 1+2+6+4$

$A\cap B \to 2+6.$

$A\cap B\cap \bar{C} \to 2$

$\bar{A}\cap B\cap \bar{C} \to 3.$

$\bar{A}\cap \bar{B}\cap \bar{C} \to 8.$

$A\cap B\cap C \to 6.$

$A\cup B\cup C \to 1+2+3+4+5+6+7$

$$n(A\cup B\cup C) = n(\text{atleast one of } A \text{ or } B \text{ or } C)$$

$$= \underbrace{n(A)+n(B)+n(C)} - \big(n(A\cap B)+n(B\cap C)+n(C\cap A)\big) + n(A\cap B\cap C)$$

$$\boxed{n(A\cup B\cup C) = \Sigma\, \underbrace{n(A)}_{{}^3C_1} - \Sigma\, \underbrace{n(A\cap B)}_{{}^3C_2} + \underbrace{n(A\cap B\cap C)}_{{}^3C_3}}$$

$$n(A\cup B\cup C\cup D) = \underbrace{\Sigma\, n(A)}_{{}^4C_1} - \overbrace{\Sigma\, n(A\cap B)}^{{}^4C_2} + \overbrace{\Sigma\, n(A\cap B\cap C)}^{{}^4C_3} - \underbrace{n(A\cap B\cap C\cap D)}_{{}^4C_4}$$

$$n(A_1 \cup A_2 \cup A_3 \cup \dots A_n) = \overbrace{\sum n(A_1)}^{{}^nC_1} - \overbrace{\sum n(A_1 \cap A_2)}^{{}^nC_2} + \overbrace{\sum n(A_1 \cap A_2 \cap A_3)}^{{}^nC_3}$$

$$- \sum n(A_1 \cap A_2 \cap A_3 \cap A_4) + \dots$$

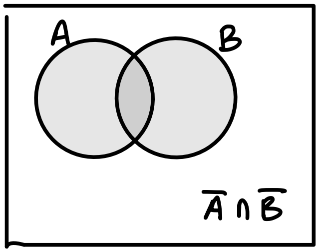
$$\dots + (-1)^{n-1} \underbrace{n(A_1 \cap A_2 \cap A_3 \dots \cap A_n)}_{{}^nC_n}$$

Q Find no. of permutation of letters a, b, c, d, e, f, g taken all together if neither 'cad' nor 'beg' pattern appear?

A → words having 'cad' pattern.

B → " " 'beg' " .

Sol^n



$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\text{Total} - n(A \cup B) = n(\bar{A} \cap \bar{B})$$

↓ Req. ans.

$\underbrace{\boxed{\text{Cad}}}$ b, e, f, g $\longrightarrow$ $5! = n(A)$

$\boxed{\text{beg}}$ c, a, d, g $\longrightarrow$ $5! = n(B)$

$\boxed{\text{cad}}$ $\boxed{\text{beg}}$ f $\longrightarrow$ $3! = n(A \cap B)$

Total $= 7!$

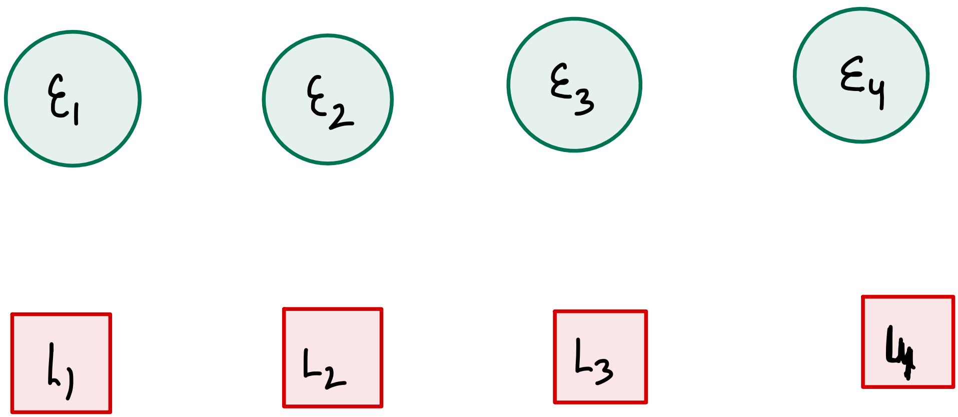
HW

Q How many different words can be formed using all the letters of the word HONOLULU if no two alike letters are together

Dearrangement / Mismatching :-

If n letters are to be kept in n directed envelopes, then number of ways in which they can be placed when none of the letter goes into its own envelope is $= n!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots\ldots + (-1)^n \frac{1}{n!}\right] = D(n)$

Remember : D(2) = 1, D(3) = 2, D(4) = 9, D(5) = 44, D(6) = 265



$$T.N.O.W = \overbrace{\text{All 4 wrong}}^{D(4)} + \text{Exactly 3 wrong \& 1 correct}$$

$$+ \text{Exactly 2 wrong \& 2 correct}$$

$$+ \text{All 4 correct}$$

$$4! = x + {}^4C_1 \times 2 + {}^4C_2 \cdot 1 + 1$$

Dearrangement of 'n' distinct elements → $D(n)$

= Total no. of permutation without any restriction

− No. of permutation in which atleast one element appears in its original position.

$$= \text{Total} - n(A_1 \cup A_2 \cup A_3 \cup \dots A_n)$$

$$= n! - \Big(\Sigma n(A_1) - \Sigma n(A_1 \cap A_2) + \Sigma n(A_1 \cap A_2 \cap A_3) - \Sigma n(A_1 \cap A_2 \cap A_3 \cap A_4) + \dots \dots \Big)$$

$$= n! - \Big({}^nC_1 \cdot (n-1)! - {}^nC_2 (n-2)! + {}^nC_3 (n-3)! - {}^nC_4 (n-4)! + \dots + (-1)^n \, {}^nC_n (1) \Big)$$

A_i → permutation in which i^{th} element appears in its original position

Q In HMW 5 different balls can be placed in their 5 boxes if atleast one ball goes in wrong box ?

Sol^n

$$120 - 1 = 119$$

Column I	Column II
A	1
B	2
C	3
D	4
E	5
F	6

(Match the Column)

Q From a given number of 6 different question each question has only one correct answer. For each correct entry a student will be awarded 2 marks & for each wrong entry 1 mark will be deducted. Total number of ways in which a student can secure non-negative markes.

Sol^n

All 6 Wrong	5W & 1C	4W+2C
✗	$-5+2=-3$	$-4+2\times2=0$
	✗	✓

3W + 3C	2W + 4C		All 6 C.

$$4W+2C \rightarrow {}^6C_2 \cdot \times D(4) = {}^6C_2 \times 9$$

$$3W+3C \rightarrow {}^6C_3 \times D(3) = {}^6C_3 \times 2.$$

HEIGHT AND DISTANCE

1. INTRODUCTION :

One of the important application of trigonometry is in finding the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

2. ANGLES OF ELEVATION AND DEPRESSION :

Let OP be a horizontal line in the vertical plane in which an object R is given and let OR be joined.

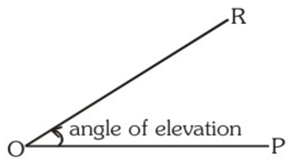


Fig. (a)

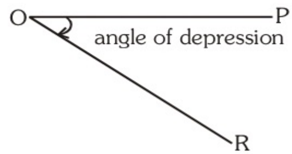


Fig. (b)

In Fig. (a), where the object R is above the horizontal line OP, the angle POR is called the angle of elevation of the object R as seen from the point O. In Fig. (b) where the object R is below the horizontal line OP, the angle POR is called the angle of depression of the object R as seen from the point O.

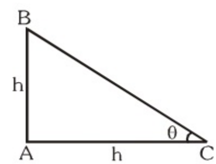
Remark :

Unless stated to the contrary, it is assumed that the height of the observer is neglected, and that the angles of elevation are measured from the ground.

Ex.1 Find the angle of elevation of the sum when the length of shadow of a vertical pole is equal to its height.

Sol. Let height of the pole AB = h and

length of the shadow of the Pole (AC) = h



In ΔABC $\tan\theta = \dfrac{AB}{AC} = \dfrac{h}{h} = 1 \Rightarrow \tan\theta = 1$

$$\Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°$$

Ex.2 The shadow of the tower standing on a level ground is found to be 60 metres longer when the sun's altitude is 30° than when it is 45°. The height of the tower is-

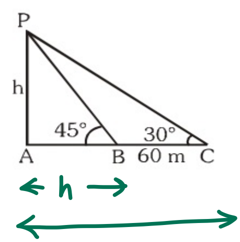
(1) 60 m (2) $30(\sqrt{3}-1)$m (3) $60\sqrt{3}$ m (4) $30(\sqrt{3}+1)$ m.

Sol. $AC = h\cot 30° = \sqrt{3}\,h$

$AB = h\cot 45° = h$

$\therefore\ BC = AC - AB = h(\sqrt{3}-1) \Rightarrow 60 = h(\sqrt{3}-1)$

$\therefore\ h = \dfrac{60}{\sqrt{3}-1} = \dfrac{60(\sqrt{3}+1)}{3-1} = 30(\sqrt{3}+1)$



Q AB is a vertical tower. The point A is on the ground and C is the mid point of AB. The part CB subtend an angle α at a point P on the ground. If $AP = n\,AB$ then the correct relation is

(A) $n = (n^2+1)\tan\alpha$.

(B) $n = (2n^2-1)\tan\alpha$.

(C) $n^2 = (2n^2+1)\tan\alpha$.

(D) $n = (2n^2+1)\tan\alpha$.

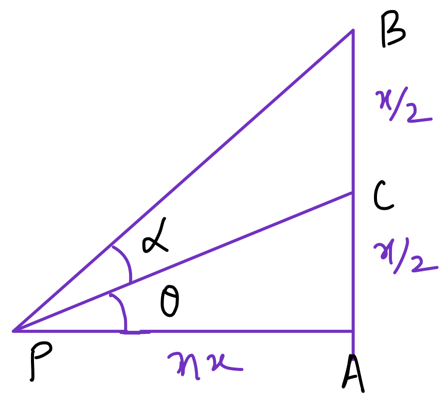
Sol^n

$$\tan\theta = \frac{1}{2n}$$

$$\tan(\theta+\alpha) = \frac{1}{n}.$$

$$\frac{\tan\theta + \tan\alpha}{1-\tan\theta\tan\alpha} = \frac{1}{n}$$

$$\frac{\frac{1}{2n} + \tan\alpha}{1 - \frac{\tan\alpha}{2n}} = \frac{1}{n}$$



$$n(1+2n\tan\alpha) = 2n - \tan\alpha.$$

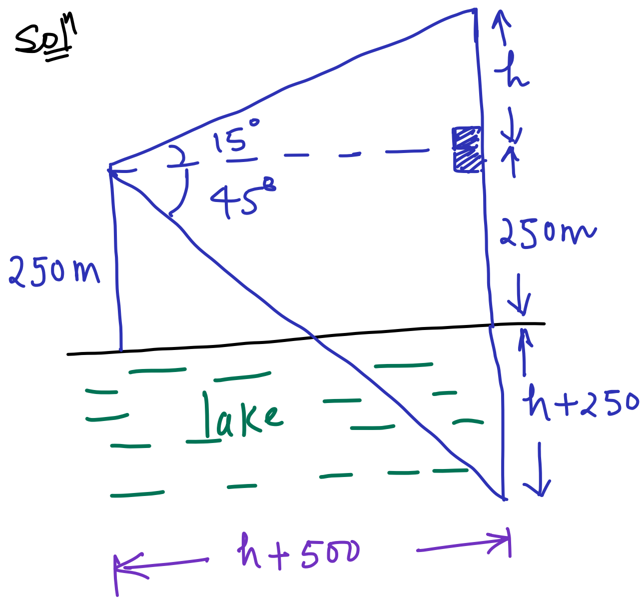
$$2n^2\tan\alpha + \tan\alpha = n$$

[D]

Q The angle of elevation of a cloud from a point 250 m above a lake is 15° and angle of depression of its reflection in the lake is 45°. The height of cloud is

(A) $250\sqrt{3}$ m (B) 250m (C) $\frac{250}{\sqrt{3}}$ m (D) None.

Solⁿ



Height of Cloud $= h+250$

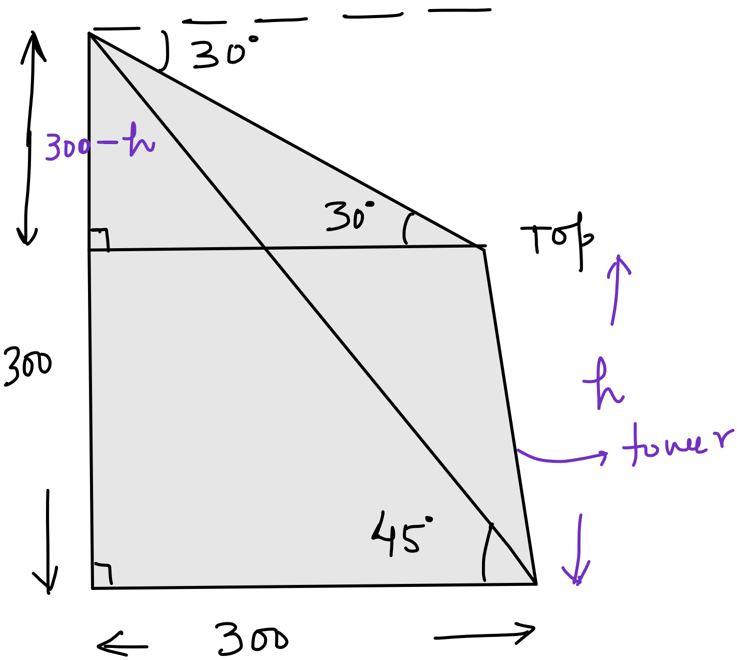
$$\frac{h}{h+500} = 2-\sqrt{3}$$

tan15°

$$h = 250(\sqrt{3}-1)$$

$$h+250 = 250\sqrt{3}\text{ m.}$$

Q From the top of cliff 300 m high, the top of a tower was observed at an angle of depression 30° and from the foot of the tower the top of the cliff was observed at an angle of elevation of 45°. The height of tower is ______



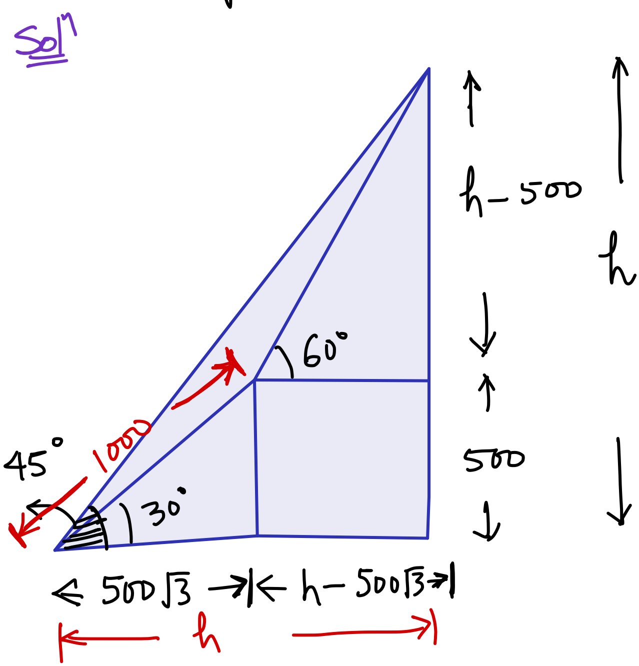
$$\tan 30^\circ = \frac{300-h}{300}$$

$$h = 100(3-\sqrt{3})$$

Ans

Q At the foot of a mountain the elevation of its summit is 45° after ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. The height of mountain is ________

Soln

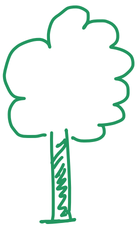


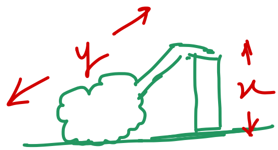
$$\tan 60° = \frac{h-500}{h-500\sqrt{3}}$$

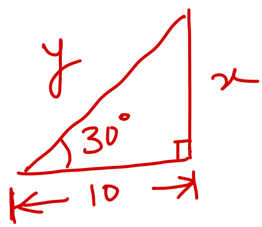
$$h = 1000\left(\frac{\sqrt{3}+1}{2}\right)\text{m.}$$

Q The upper part of a tree broken by wind makes an angle of 30° with the ground and the distance from the root (bottom) to the point where the top of the tree touches the ground is 10 m. The height of the tree is ______

Sol^n







$x = \frac{10}{\sqrt{3}}$; $y = \frac{20}{\sqrt{3}}$

Height of tree $= \frac{30}{\sqrt{3}} = 10\sqrt{3}$ m

Q Number of non negative integral solution of the inequality $10 \leq x+y+z+t \leq 30$

Sol^n

$$x+y+z+t \leq 30$$

$$x+y+z+t+w = 30; \quad x,y,z,t,w \geq 0$$

$$^{34}C_4$$

$$x+y+z+t \leq 9$$

$$x+y+z+t+w' = 9; \quad x,y,z,t,w' \geq 0$$

$$^{13}C_4$$

$\therefore$ No. of non-negative integral solution

$$= {}^{34}C_4 - {}^{13}C_4 \quad \text{Ans}$$

Q Number of ways in which 3 apples, 4 oranges and 5 mangoes can be distributed between R|S|G if each receives :

(i) None, one or more no. of fruits.

(ii) atleast one fruit of each species.

Solve the problem for 2 cases :

C–I Fruits of same species are alike.

C–II Apples are identical, oranges are identical and mangoes are all different.

C–I

(i) $R+S+G=3$; $R+S+G=4$; $R+S+G=5$

$R, S, G \geq 0$.

$^5C_2 \times {^6C_2} \times {^7C_2}$.

(ii) $R+S+G=3$; $R+S+G=4$; $R+S+G=5$

$R, S, G \geq 1$

$1 \times {^3C_2} \times {^4C_2}$

C – II

(i) $^{5}C_{2} \times {}^{6}C_{2} \times 3^{5}$

(ii) $1 \times {}^{3}C_{2} \times 150.$

<u>Mangoes</u>

1 1 3 $\longrightarrow \dfrac{5!}{2!\,3!} \times 3! = 60$

1 2 2 $\longrightarrow \dfrac{5!}{2!\,2!\,2!} \times 3! = 90$

$\overline{150}$

Q In how many ways 30 can be partioned into 3 unequal natural parts?

Solⁿ

$$x_1 + x_2 + x_3 = 30$$

$x_1 \neq x_2$
$x_2 \neq x_3$
$x_3 \neq x_1$
$x_i \geq 1$

$$X_1 + X_2 + X_3 = 27$$

Total ways − (Atleast two equal parts)

$$^{29}C_2 - \left((X_1 = X_2 = X_3) + (X_1 = X_2 \neq X_3) + (X_1 \neq X_2 = X_3) + (X_1 = X_3 \neq X_2) \right)$$

$$^{29}C_2 - (1 + 13 \times 3)$$

$$\Rightarrow 406 - 40 = 366.$$

$$\frac{366}{6} = 61$$

Ans

0	0	27
1	1	25
2	2	23
3	3	21
⋮	⋮	⋮
9	9	9
10	10	7
⋮		
13	13	1

Q

If 9 balls are to be distributed in 3 boxes, then find the number of ways of distribution if

(i) Balls & boxes are distinct.

(ii) Balls are identical & boxes are distinct.

* (iii) Both are identical

(i) 3^9

B → Boxes

(ii) $B_1+B_2+B_3 = 9$; $B_i \geq 0$.

$$^{9+2}C_2 = {}^{11}C_2 = \frac{11 \times 10}{2} = 55 \text{. Ans}$$

(iii)

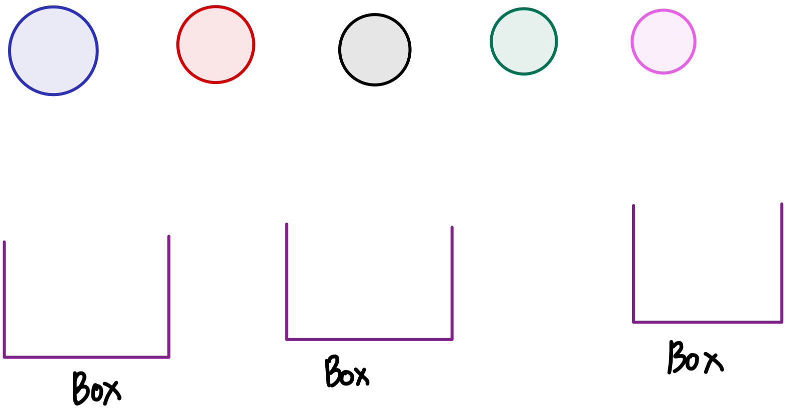
B_1	B_2	B_3		
5	3	1	→ 6 ways.	7
* 3	3	3	→ 1 way.	1
0	0	9	→ 3 ways.	4
1	1	7	→ 3 ways	
2	2	5	→ 3 ways	
3	3	3	→ 1 way	
4	4	1	→ 3 ways	

$$55 - (1 + 4 \times 3) = 42 \Rightarrow \frac{42}{6} = 7$$

∴ T.N.O.W = 12. Ans

Q In how many ways 5 distinct balls are to be distributed in 3 identical boxes such that every box contains any number of balls ?

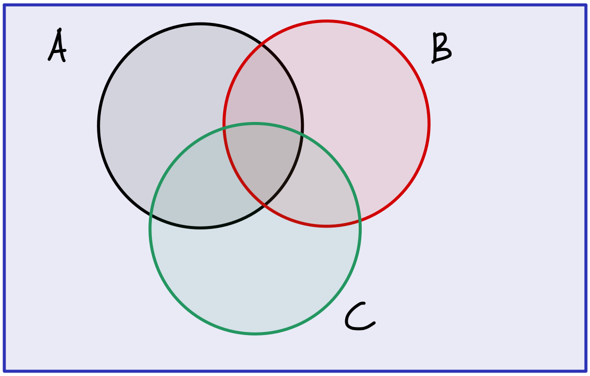
Sol^n



5	0	0
4	1	0
3	2	0
3	1	1
2	2	1

$$1 + \frac{5!}{4!\,1!\,0!} + \frac{5!}{3!\,2!\,0!} + \frac{5!}{(3!\,1!\,1!)\,2!} + \frac{5!}{(2!\,2!)\,2!}$$

Q. How many different words can be formed using all the letters of the word HONOLULU if no two alike letters are together



$A \rightarrow \boxed{OO}$

$B \rightarrow \boxed{LL}$

$C \rightarrow \boxed{UU}$

$A \rightarrow \boxed{OO}$ HNLULU

$B \rightarrow \boxed{LL}$ HONOUU

Total no. of words formed using all the letters of word $= \dfrac{8!}{2!\,2!\,2!} = K$

$$n(A \cup B \cup C) = \sum n(A) - \sum n(A \cap B) + n(A \cap B \cap C)$$

$$= 3\left(\frac{7!}{2!\,2!}\right) - 3\left(\frac{6!}{2!}\right) + 5!$$

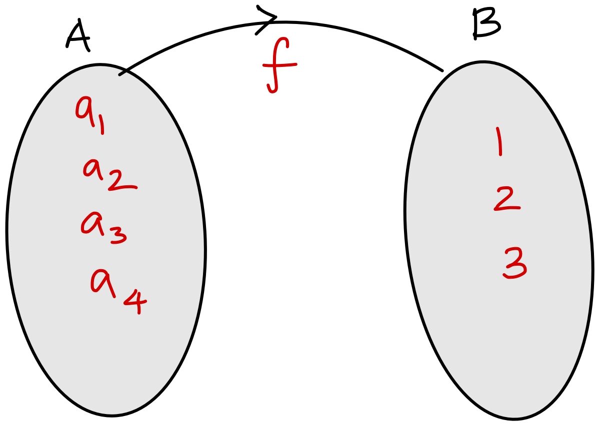
No. of words $= K - n(A \cup B \cup C)$

$= 2220$. Ans

$A \cap B \rightarrow \boxed{OO}\ \boxed{LL}$ HNUU

$A \cap B \cap C \rightarrow \boxed{OO}\ \boxed{LL}\ \boxed{UU}$ H N

Q



Find number of :-

(i) functions from A to B.

(ii) one-one fns from A to B.

(iii) many-one " " " " .

(iv) onto fns " " " .

(v) into " " " .

(i) 3^4

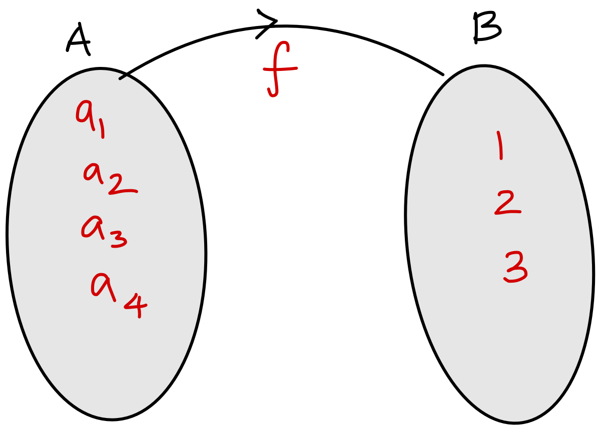
(ii) 0

(iii) $3^4 - 0 = 3^4$

(iv) 1 1 2 $\longrightarrow \dfrac{4!}{2!\,2!} \times 3! = 36.$

(v) $3^4 - 36 = 45.$

Q



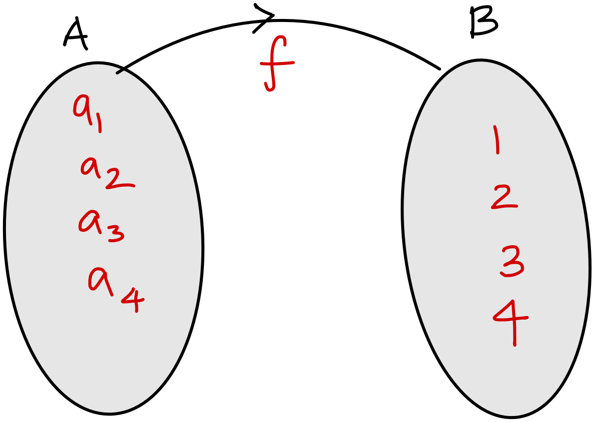
Find number of onto fns from A to B.

M-2

Total no. of functions − No. of into fns

$$3^4 - \left({}^3C_1 \cdot 2^4 - {}^3C_2 \cdot 1^4 \right)$$

$$81 - (3 \times 16 - 3) = 81 - 45 = 36.$$



Find number of :-

(i) functions from A to B. $\rightarrow 4^4$

(ii) one-one fns from A to B. $\rightarrow {}^4C_4 \cdot 4!$

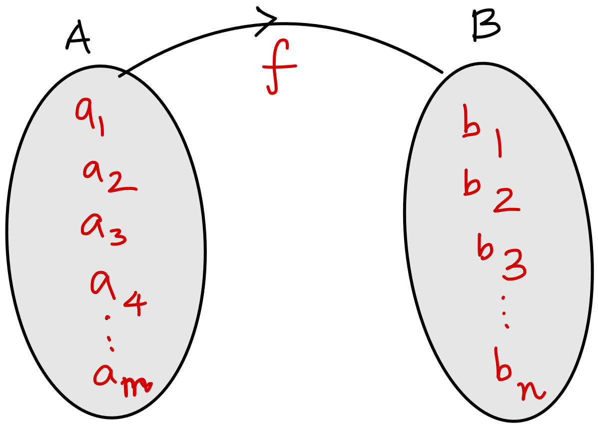
(iii) many-one " " " . $\rightarrow 4^4 - ({}^4C_4 \cdot 4!)$

(iv) onto fns " " .

$| \quad | \quad | \quad | \longrightarrow \frac{4!}{4!} \times 4! = 4!$

(v) into " " " . $\rightarrow 4^4 - 4!$

Q



where $n > m$

Find number of :-

(i) functions from A to B. $\longrightarrow n^m$

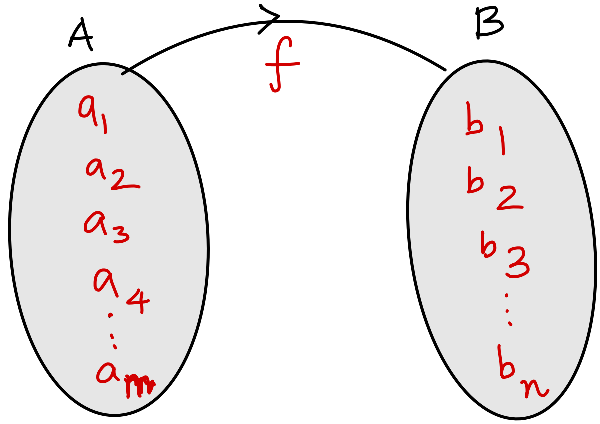
(ii) one-one fns from A to B. $\longrightarrow {}^nC_m \times m!$

(iii) many-one " " ". $\longrightarrow n^m - {}^nC_m(m!)$

(iv) onto fns " ". $\longrightarrow 0$

(v) into " " ". $\longrightarrow n^m$.

Q



where $n < m$

Find number of :-

(i) functions from A to B. $\rightarrow n^m$

(ii) one-one fns from A to B. $\rightarrow 0$

(iii) many-one " " " . $\rightarrow n^m$

(iv) onto fns " " $\rightarrow$

$$n^m - \left({}^nC_1 . (n-1)^m - {}^nC_2 (n-2)^m + {}^nC_3 (n-3)^m - \cdots (-1)^{n-1}\, {}^nC_n (n-n)^m \right)$$

(v) into " " " .

Mathematical Reasoning

STATEMENT : OR Mathematical statement.

A sentence which is either true or false but cannot be both are called a statement. A sentence which is an exclamatory or a wish or an imperative or an interrogative can not be a statement.

If a statement is true then its truth value is T and if it is false then its truth value is F

For ex.

(i) "New Delhi is the capital of India", a true statement

(ii) "$3 + 2 = 6$", a false statement

(iii) "Where are you going ?" not a statement beasuse it connot be defined as true or false

Note : A statement cannot be both true and false at a time

SIMPLE STATEMENT :

Any statement whose truth value does not depend on other statement are called simple statement

For ex. (i) "$\sqrt{2}$ is an irrational number" (ii) "The set of real number is an infinite set"

COMPOUND STATEMENT :

A statement which is a combination of two or more simple statements are called compound statement
Here the simple statements which form a compound statement are known as its sub statements

For ex.

(i) "If x is divisible by 2 then x is even number"

(ii) "ΔABC is equilatral if and only if its three sides are equal"

LOGICAL CONNECTIVES :

The words or phrases which combined simple statements to form a compound statement are called logical connectives.

In the following table some possible connectives, their symbols and the nature of the compound statement formed by them

S.N.	Connectives	symbol	use	operation
1.	and	$\wedge$	$p \wedge q$	conjunction
2.	or	$\vee$	$p \vee q$	disjunction
3.	not	$\sim$ or $'$	$\sim p$ or p'	negation
4.	If .... then	$\Rightarrow$ or $\rightarrow$	$p \Rightarrow q$ or $p \rightarrow q$	Implication or conditional
5.	If and only if (iff)	$\Leftrightarrow$ or $\leftrightarrow$	$p \Leftrightarrow q$ or $p \leftrightarrow q$	Equivalence or Bi-conditional

Explanation :

(i) $p \wedge q \equiv$ statement p and q

($p \wedge q$ is true only when p and q both are true otherwise it is false)

(ii) $p \vee q \equiv$ statement p or q

($p \vee q$ is true if at least one from p and q is true i.e. $p \vee q$ is false only when p and q both are false)

(iii) $\sim p \equiv$ not statement p

($\sim p$ is true when p is false and $\sim p$ is false when p is true)

(iv) $p \Rightarrow q \equiv$ statement p then statement q

($p \Rightarrow q$ is false only when p is true and q is false otherwise it is true for all other cases)

(v) $p \Leftrightarrow q \equiv$ statement p if and only if statement q

($p \Leftrightarrow q$ is true only when p and q both are true or false otherwise it is false)

TRUTH TABLE :

A table which shows the relationship between the truth value of compound statement S(p, q, r ....) and the truth values of its sub statements p, q, r, ... is said to be truth table of compound statement S

If p and q are two simple statements then truth table for basic logical connectives are given below

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

p	$(\sim p)$
T	F
F	T

Conditional

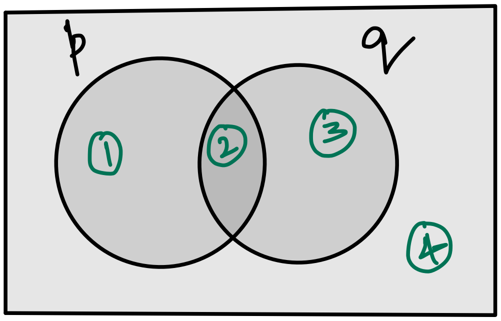
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$ or $p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Note : If the compound statement contain n sub statements then its truth table will contain 2^n rows.

Use of Set theory :-



$p \vee q \rightarrow$ 1, 2, 3

$p \wedge q \rightarrow$ 2.

$p \rightarrow q \rightarrow$ 2, 3, 4

$(p \rightarrow q) \wedge (q \rightarrow p)$

Region	p	q	$p \vee q$	$p \wedge q$	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
1	T	F	T	F	F	T	F
2	T	T	T	T	T	T	T
3	F	T	T	F	T	F	F
4	F	F	F	F	T	T	T

De-morgan's law :-

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

Rem

$$\boxed{p \rightarrow q \equiv \sim p \vee q}$$

$$p \rightarrow q \equiv \sim 1 \equiv \sim (p \wedge \sim q)$$

$$\equiv \sim p \vee q$$

LOGICAL EQUIVALENCE :

Two compound statements $S_1(p, q, r...)$ and $S_2(p, q, r ....)$ are said to be logically equivalent or simply equivalent if they have same truth values for all logically possibilities

Two statements S_1 and S_2 are equivalent if they have identical truth table i.e. the entries in the last column of their truth table are same. If statements S_1 and S_2 are equivalent then we write $S_1 \equiv S_2$

For ex. The truth table for $(p \to q)$ and $(\sim p \vee q)$ given as below

p	q	$(\sim p)$	$p \to q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

We observe that last two columns of the above truth table are identical hence compound statements $(p \to q)$ and $(\sim p \vee q)$ are equivalent

i.e. $\boxed{p \to q \equiv \sim p \vee q}$

TAUTOLOGY AND CONTRADICTION :

(i) Tautology : A statement is said to be a tautology if it is true for all logical possibilities

i.e. its truth value always T. it is denoted by t.

For ex. the statement $p \vee \sim (p \wedge q)$ is a tautology

p	q	$p \wedge q$	$\sim (p \wedge q)$	$p \vee \sim (p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Clearly, The truth value of $p \vee \sim (p \wedge q)$ is T for all values of p and q. so $p \wedge \sim (p \wedge q)$ is a tautology

(ii) Contradiction : A statement is a contradiction if it is false for all logical possibilities.

i.e. its truth value always F. It is denoted by c.

Contradiction = Fallacy.

For ex. The statement $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction

p	q	$\sim p$	$\sim q$	$p \vee q$	$(\sim p \wedge \sim q)$	$(p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

Clearly, then truth value of $(p \vee q) \wedge (\sim p \wedge \sim q)$ is F for all value of p and q. So $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

Note : The negation of a tautology is a contradiction and negation of a contradiction is a tautology

DUALITY :

Two compound statements S_1 and S_2 are said to be duals of each other if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$

If a compound statement contains the special variable t (tautology) and c (contradiction) then obtain its dual we replaced t by c and c by t in addition to replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

Note :

(i) the connectives $\wedge$ and $\vee$ are also called dual of each other.

(ii) If $S^*(p, q)$ is the dual of the compound statement $S(p, q)$ then

(a) $S^*(\sim p, \sim q) \equiv \sim S(p, q)$ (ii) $\sim S^*(p, q) \equiv S(\sim p, \sim q)$

For ex. The duals of the following statements

(i) $(p \wedge q) \vee (r \vee s)$ (ii) $(p \vee t) \wedge (p \vee c)$

(iii) $\sim(p \wedge q) \vee [p \wedge \sim(q \vee \sim s)]$

are as given below

(i) $(p \vee q) \wedge (r \wedge s)$

(ii) $(p \wedge c) \vee (p \wedge t)$

(iii) $\sim(p \vee q) \wedge [p \vee \sim(q \wedge \sim s)]$

CONVERSE, INVERSE AND CONTRAPOSITIVE OF THE CONDITIONAL STATEMENT $(p \to q)$:

(i) **Converse** : The converse of the conditional statement $p \to q$ is defined as $q \to p$

same

(ii) **Inverse** : The inverse of the conditional statement $p \to q$ is defined as $\sim p \to \sim q$

(iii) **Contrapositive** : The contrapositive of conditional statement $p \to q$ is defined as $\sim q \to \sim p$

NEGATION OF COMPOUND STATEMENTS :

same

If p and q are two statements then

(i) **Negation of conjunction :** $\boxed{\sim(p \wedge q) \equiv \sim p \vee \sim q}$

p	q	~ p	~ q	$(p \wedge q)$	$\sim(p \wedge q)$	$(\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

(ii) **Negation of disjunction :** $\boxed{\sim(p \vee q) \equiv \sim p \wedge \sim q}$

p	q	~ p	~ q	$(p \vee q)$	$(\sim p \vee q)$	$(\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

(iii) **Negation of conditional :** $\boxed{\sim(p \to q) \equiv p \wedge \sim q}$

p	q	~ q	$(p \to q)$	$\sim(p \to q)$	$(p \wedge \sim q)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

(iv) **Negation of biconditional :** $\boxed{\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)}$

we know that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$$\therefore \sim(p \leftrightarrow q) \equiv \sim[(p \to q) \wedge (q \to p)]$$

$$\equiv \sim(p \to q) \vee \sim(q \to p)$$

$$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$$

Note : The above result also can be proved by preparing truth table for $\sim(p \leftrightarrow q)$ and $(p \wedge \sim q) \vee (q \wedge \sim p)$

ALGEBRA OF STATEMENTS :

If p, q, r are any three statements then the some low of algebra of statements are as follow

(i) Idempotent Laws :

(a) $p \wedge p \equiv p$ (b) $p \vee p \equiv p$

i.e. $p \wedge p \equiv p \equiv p \vee p$

p	$(p \wedge p)$	$(p \vee p)$
T	T	T
F	F	F

(ii) Comutative laws :

(a) $p \wedge q \equiv q \wedge p$ (b) $p \vee q \equiv q \vee p$

p	q	$(p \wedge q)$	$(q \wedge p)$	$(p \vee q)$	$(q \vee p)$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	F	F	T	T
F	F	F	F	F	F

(iii) Associative laws :

(a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

p	q	r	$(p \wedge q)$	$(q \wedge r)$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Similarly we can proved result (b)

(iv) Distributive laws :

(a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ (c) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge (p \wedge r)$

(b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (d) $p \vee (q \vee r) \equiv (p \vee q) \vee (p \vee r)$

p	q	r	$(q \vee r)$	$(p \wedge q)$	$(p \wedge r)$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Similarly we can prove result (b), (c), (d)

(v) De Morgan Laws : (a) $\sim (p \wedge q) \equiv \sim p \vee \sim q$

(b) $\sim (p \vee q) \equiv \sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$(p \wedge q)$	$\sim (p \wedge q)$	$(\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Similarly we can proved resulty (b)

(vi) Involution laws (or Double negation laws) : $\sim(\sim p) \equiv p$

p	$\sim p$	$\sim (\sim p)$
T	F	T
F	T	F

(vii) Identity Laws : If p is a statement and t and c are tautology and contradiction respectively then

(a) $p \wedge t \equiv p$ (b) $p \vee t \equiv t$ (c) $p \wedge c \equiv c$ (d) $p \vee c \equiv p$

p	t	c	$(p \wedge t)$	$(p \vee t)$	$(p \wedge c)$	$(p \vee c)$
T	T	F	T	T	F	T
F	T	F	F	T	F	F

(viii) Complement Laws :

(a) $p \wedge (\sim p) \equiv c$ (b) $p \vee (\sim p) \equiv t$ (c) $(\sim t) \equiv c$ (d) $(\sim c) \equiv t$

p	$\sim p$	$(p \wedge \sim p)$	$(p \vee \sim p)$
T	F	F	T
F	T	F	T

(ix) Contrapositive laws : $\boxed{p \to q \equiv \sim q \to \sim p}$

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

QUANTIFIED STATEMENTS AND QUANTIFIERS :

The words or phrases "All", "Some", "None", "There exists a" are examples of quantifiers.

A statement containing one or more of these words (or phrases) is a quantified statement.

E.g. (1) All dogs are poodles

(2) Some books have hard covers

(3) There exists an odd number which is prime.

Note : Phrases "There exists a" and "Atleast one" and the word "some" have the same meaning.

NEGATION OF QUANTIFIED STATEMENTS :

(1) '**None**' is the negation of '**at least one**' or '**some**' or '**few**'

Statement : Some dogs are poodles.

Negation : No dogs are poodles.

Similarly negation of '**some**' is '**none**'

(2) The negation of "**some A are B**" or "**There exist A which is B**" is "**No A are (is) B**" or "**There does not exist any A which is B".**

Statement-1 : Some boys in the class are smart

Statement-2 : There exists a boy in the class who is smart

Statement-3 :Alteast one boy in the class is smart

All the three above statements have same meaning as they all indicate "**existence**" of smart boy in the class.

Negation of these statements is

No boy in the class is smart.

or

There does not exist any boy in the class who is smart.

(3) Negation of "**All A are B**" is "**Some A are not B**".

Statement : All boys in the class are smart.

Negation : Some boys in the class are not smart.

or

There exists a boy in the class who is not smart.

Q Contrapositive of the statement
"If two numbers are not equal, then their squares are not equal." is :-

(1) If the squares of two numbers are equal, then the numbers are equal. ✓

(2) If the squares of two numbers are equal, then the numbers are not equal.

(3) If the squares of two numbers are not equal, then the numbers are equal.

(4) If the squares of two numbers are not equal, then the numbers are not equal.

If ___ then ___

Contrapositive of $p \to q$ is $\sim q \to \sim p$.

Q The expression ~(~p→q) is logically equvalent to :

(1) ~ p ^ ~ q (2) p ^ q

(3) ~ p ^ q (4) p ^ ~ q

$$p \to q \equiv \sim p \vee q$$

$$\sim p \to q \equiv \sim(\sim p) \vee q$$
$$\equiv p \vee q$$

$$\sim(\sim p \to q) \equiv \sim(p \vee q)$$
$$\equiv \sim p \wedge \sim q$$

Q The negation of the boolean expression

$\sim s \vee (\sim r \wedge s)$ is equivalent to :

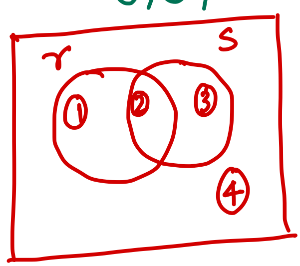
(1) r (2) $s \wedge r$

(3) $s \vee r$ (4) $\sim s \wedge \sim r$

$\sim r \wedge s \to$ 3

$\sim s \to$ 1, 4

1, 4, 3



$$\sim s \vee (\sim r \wedge s)$$

Q The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to :

(1) $(\sim p) \Rightarrow q$ (2) $p \vee q$

(3) $q \Rightarrow \sim p$ (4) $p \wedge q$

$\sim(p \to \sim q)$

$\sim(\sim p \vee \sim q)$

$p \wedge q$

Q If the truth value of the statement $P \to (\sim p \vee r)$ is false(F), then the truth values of the statements p, q, r are respectively :

(1) F, T, T (2) T, F, F

(3) T, T, F (4) T, F, T

Q The boolean expression

$((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \vee q)$ is equivalent to

(A) $p \leftrightarrow q$ (B) $p \rightarrow q$

(C) $\sim p \vee q$ (D) $p \wedge \sim q$

Q The statement $(p \rightarrow q) \vee (q \rightarrow p)$ is

(A) Tautology

(B) Contradiction

(C) Neither tautology nor contradiction

(D) None of these

Q The negation of the statement

"If I become a teacher, then I will open a school", is :

(1) I will not become a teacher or I will open a school.

(2) I will become a teacher and I will not open a school.

(3) Either I will not become a teacher or I will not open a school.

(4) Neither I will become a teacher nor I will open a school.

$p \to$ I become teacher

$q \to$ I will open school

$\sim(p \to q)$

$\sim(\sim p \vee q)$

$p \wedge \sim q$