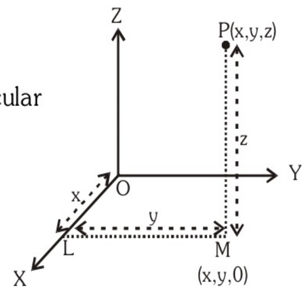
3 - D

COORDINATES OF A POINT IN SPACE :



In two dimensional geometry, magnitude of x & y coordinates are perpendicular distances from y & x axis respectively.

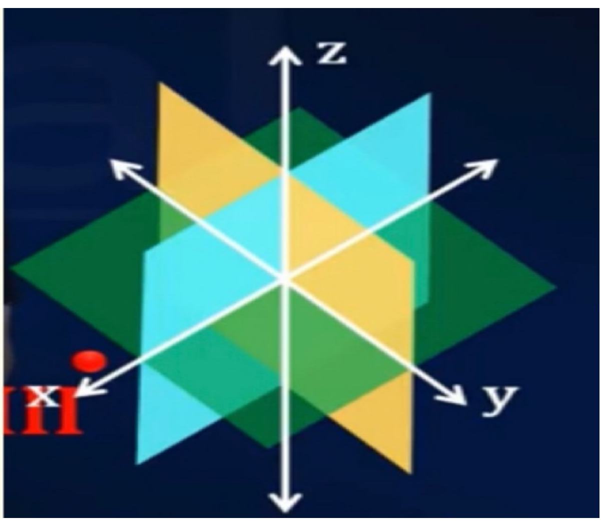
But in case of three dimensional geometry it is understood in different way.

(a) Magnitude of x-coordinate is perpendicular distance from y-z plane, similarly magnitude of y & z coordinates are perpendicular distances from x-z and x-y plane

(b) If a point lies on x-axis then its coordinates are (x, 0, 0) and similarly on y-axis & z-axis the coordinates are (0, y, 0) and (0, 0, z)

Remark : The sign of the coordinates of a point determine the octant in which the point lies. The following table shows the signs of the coordinates in eight octants.

Octants / Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−



DISTANCE FORMULA :

The distance between two points A (x_1, y_1, z_1) and B (x_2, y_2, z_2) is given by

$$AB = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$$

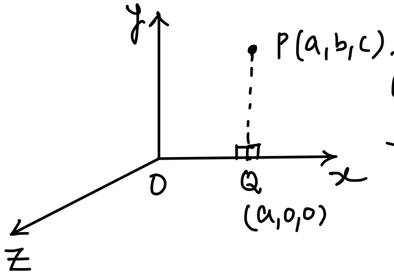
SECTION FORMULA :

Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points and let $R(x, y, z)$ divide PQ in the ratio $m_1 : m_2$. Then co-ordinates of

$$R(x, y, z) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$$

Q Find the distance of P(a,b,c) from x-axis ?

Sol^n



$d = \sqrt{b^2+c^2}$.

Q

Find the locus of the point which move such that its distance from x-axis is $\frac{1}{2}$ of its distance from zy-plane.

Sol^n $P(\alpha, \beta, \gamma)$

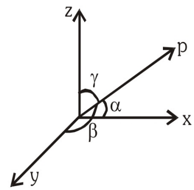
$$\sqrt{\beta^2+\gamma^2} = \frac{1}{2}|\alpha|$$

$$4(\beta^2+\gamma^2) = \alpha^2 \Rightarrow x^2 - 4y^2 - 4z^2 = 0. \text{ Ans}$$

DIRECTION COSINES OF VECTOR :

Let $\vec{a} = a_1 i + a_2 j + a_3 k$ the angles which this vector makes with the +ve directions OX, OY & OZ are called Direction Angles & their cosine are called the direction cosine hence if α, β, γ are the direction angles then the d.c's are

$$\cos\alpha = \frac{a_1}{|\vec{a}|},\ \cos\beta = \frac{a_2}{|\vec{a}|},\ \cos\gamma = \frac{a_3}{|\vec{a}|}$$

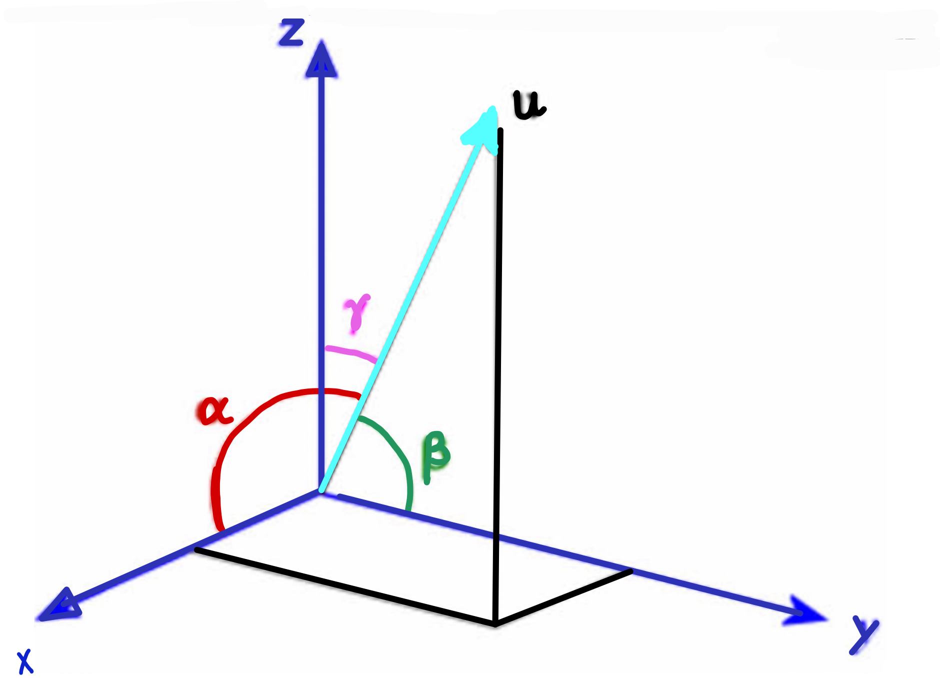


$\cos\alpha$, $\cos\beta$, $\cos\gamma$ are popularly denoted by ℓ, m and n.

Note :

(i) $$\boldsymbol{\ell^2 + m^2 + n^2 = 1} \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

(ii) Components of the unit vector denotes the dc's of the vector : $\hat{\mathbf{a}} = \ell\hat{\mathbf{i}} + \mathbf{m}\hat{\mathbf{j}} + \mathbf{n}\hat{\mathbf{k}}$



eg D.C's of the vector $\underbrace{2\hat{j}-2\hat{j}+\hat{k}}_{\vec{a}}$ are $\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}$

$|\vec{a}| = 3$

eg There exists a vector with direction angles $\alpha = 30°$ and $\beta = 30°$ True or False ?

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

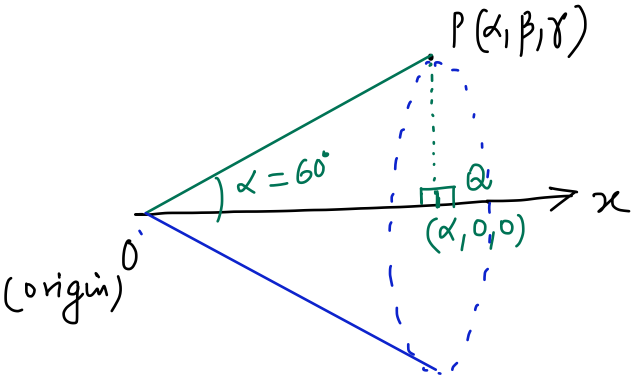
$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \cos^2\gamma = 1$

$\cos^2\gamma = -ve \Rightarrow$ This is not possible

(False)

Q (O is Origin)

Find the locus of all points the P for which $\overrightarrow{OP}$ represents a vector whose direction cosine $\cos\alpha = \frac{1}{2}$



$\tan 60° = \frac{|PQ|}{|OQ|}$

$\sqrt{3} = \frac{\sqrt{\beta^2+\gamma^2}}{|\alpha|}$

$(\sqrt{3}|\alpha|)^2 = \beta^2 + \gamma^2$

$3x^2 - y^2 - z^2 = 0.$

Locus of all such points 'P' will be a Cone Concentric with x-axis.