Q Two natural numbers are randomly selected from the set of first 20 natural numbers. Find the probability that :

(i) their sum is odd

(ii) sum is even

(iii) selected pair is twin prime

Soln $\{1, 2, 3, 4, \ldots, 20\}$; $n(S) = {}^{20}C_2 = \frac{20 \times 19}{2} = 190$

(i) $n(A) = {}^{10}C_1 \times {}^{10}C_1 = 100$

$P(A) = \frac{n(A)}{n(S)} = \frac{100}{190} = \frac{10}{19}$

(ii) $P(B) = 1 - \frac{10}{19} = \frac{9}{19}$

alt :-

$n(B) = {}^{10}C_2 + {}^{10}C_2$

$P(B) = \frac{n(B)}{n(S)} = \frac{9}{19}$.

(iii) $\{3, 5\}$; $\{5, 7\}$; $\{11, 13\}$; $\{17, 19\}$

$P(C) = \frac{4}{190}$

Q Bag $\left\langle \begin{matrix} 5G \text{ balls} \\ 4W \text{ balls} \end{matrix} \right.$ $\xrightarrow[\text{drawn}]{\text{3 balls are}}$ find the odds against these being all green.

Sol^n

$$P(GGG) = \frac{{}^5C_3}{{}^9C_3} = \frac{\cancel{10}\ 5}{\left(\frac{\cancel{9}^3 \times \cancel{8}^4 \times 7}{6\cancel{2}}\right)} = \frac{5}{42} = \frac{5}{5+37}.$$

Odd. against all 3 being green $= \frac{37}{5}$.

Q A leap year is selected at random. Find the probability that it has :

(A) 53 Sundays and Mondays (B) 53 Sundays (C) 53 Sundays or 53 Mondays

Sol^n

$$366 \text{ days} = 52 \times 7 + \underbrace{2}_{2 \text{ days}}$$

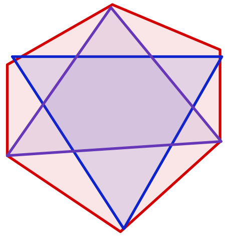
$\left.\begin{matrix} \text{Mon, Tue} \\ \text{Tue, Wed} \\ \text{Wed, Thur} \\ \text{Thur, Fri} \\ \text{Fri, Sat} \\ \text{Sat, Sun} \\ \text{Sun, Mon} \end{matrix}\right\}$ Sample space

(A) $\frac{1}{7}$.

(B) $\frac{2}{7}$.

(C) $\frac{3}{7}$.

Q 3 vertices of a regular hexagon are joined randomly. Probability that an equilateral Δ is formed.



$n(S) = {}^6C_3 = 20$

$n(A) = 2$

$P(A) = \frac{2}{20} = \frac{1}{10}$.

Q An old man while dialing a seven digit telephone number, after having dialed the first five digits, suddenly forgets the last two. But he remembered that the last two digits were different. On this assumption he randomly dials the last two digits. What is the Probability that the correct telephone number is dialed.

Sol^n

$\underbrace{\_\ \_\ \_\ \_\ \_}_{\text{already dialed}}\ \_\ \_$

$n(S) = 10 \times 9 = 90$

$n(A) = 1$

$P(A) = \frac{1}{90}$.

Q What is the chance that the fourth power of an integer chosen randomly ends in the digit six.

Unit digit in I	Unit digit in I^2	Unit digit in I^4
0	0	0
1	1	1
2	4	6
3	9	1
4	6	6
5	5	5
6	6	6
7	9	1
8	4	6
9	1	1

$n(A) = 4$

$n(S) = 10$

$P(A) = \frac{4}{10} = 0.40$

Q 2 numbers are randomly selected from the set of first six natural numbers. Find the probability that the selected pair is coprime.

Soln

$\{1, 2, 3, 4, 5, 6\}$

$n(S) = {}^6C_2 = \frac{6\times5}{2} = 15$; $n(A) = 11$

$\{(1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,3), (2,5), (3,4), (3,5), (4,5), (5,6)\}$

$P(A) = \frac{11}{15}$

Q Three tickets are drawn randomly from a pack of 51 tickets numbered from 1 to 51. Find the probability that :

(i) numbers are in AP.

(ii) their sum is odd.

$1, 2, 3, \ldots, 51$

odd = 26

even = 25

Solⁿ

$n(S) = {}^{51}C_3$

(i) $x, y, z \longrightarrow$ A.P.

$\underbrace{2y}_{\text{even}} = \underbrace{x+z}_{\text{even}}$

$n(A) = {}^{26}C_2 + {}^{25}C_2$

$P(A) = \dfrac{n(A)}{n(S)}$

(ii) $x+y+z = \text{odd}$

$\left.\begin{array}{lccc} \text{(i)} & O & O & O \\ \text{(ii)} & E & E & O \end{array}\right\}$

$n(B) = {}^{26}C_3 + {}^{26}C_1 \cdot {}^{25}C_2$

$P(B) = \dfrac{n(B)}{n(S)}$.

Q There are 10 Keys, 3 Keys are randomly selected to open a lock. Find the chance that you have choosen the correct key.

Solⁿ

$$\text{Req. prob} = \frac{{}^{9}C_{2}}{{}^{10}C_{3}}$$

Q A pack of well shuffled 52 cards are randomly distributed equally among four brothers. Find the probability that all the queens are held by the youngest brother?

Solⁿ

$\boxed{52}$ → 13, 13, 13, 13

$$n(S) = \frac{52!}{(13!)^4 \cdot \cancel{4!}} \times \cancel{4!}$$

$\boxed{48}$ → 13, 13, 13, 9

youngest

$$n(A) = \frac{48!}{\left((13!)^3\, 9!\right) \cancel{3!}} \times \cancel{3!}$$

$$\text{Req. prob} = \frac{n(A)}{n(S)}.$$

HW

Q. Let x and y are 2 randomly selected integers.

Find the probability that :

(i) $(x+y)$ is divisible by '7'.

(ii) (x^2+y^2) is divisible by '3'

(iii) $(x^2 + y^2)$ is divisible by 5.

Now two important sample spaces are

(a) **Throwing a pair of dice given 36 ME/EL and Exhaustive cases which are :**

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	11	12	13	14	15	16
⚁	21	22	23	24	25	26
⚂	31	32	33	34	35	36
⚃	41	42	43	44	45	46
⚄	51	52	53	54	55	56
⚅	61	62	63	64	65	66

Note that :

(i) $P(\text{Total } 2) = \frac{1}{36}$; $P(3) = \frac{2}{36}$; $P(4) = \frac{3}{36}$; $P(5) = \frac{4}{36}$ etc.

(ii) Probability of throwing a doublet $= \frac{6}{36}$

(iii) Probability of throwing a total which is prime $= P(2, 3, 5, 7, 11) = \frac{15}{36}$

Note : Let $p : N \to [0, 1)$ where p is the probability function. N → Natural number.

If p(x) = Probability of getting the sum of scores when two dice are thrown.

$$P(x) \;-\; \begin{cases} \dfrac{x-1}{36} & \text{for} \quad 1 \le x \le 7 \\[2ex] \dfrac{13-x}{36} & \text{for} \quad 8 \le x \le 13 \end{cases}$$

P(x) is a many one function ; Domain of p = N, Range of $p = \left\{ \frac{x}{36} \middle| 0 \le x \le 6,\ x \in N \right\}$

& into

$P(\text{sum} = 2) = \frac{1}{36}$

$P(\text{sum} = 3) = \frac{2}{36}$

$P(S = 4) = \frac{3}{36}$

$P(S = 5) = \frac{4}{36}$

$P(S = 6) = \frac{5}{36}$

$P(S = 12) = \frac{1}{36}$

$P(S = 11) = \frac{2}{36}$

$P(S = 10) = \frac{3}{36}$

$P(S = 9) = \frac{4}{36}$

$P(S = 8) = \frac{5}{36}$

$$\boxed{P(S = 7) = \frac{6}{36}.}$$

p

$P(\text{sum} \le 6) = ?$

$$p + p + \frac{6}{36} = 1 \Rightarrow p = \frac{15}{36}$$

Q A pair of dice is thrown and shows different faces. Find the probability :

(i) sum of two faces is prime.

(ii) one die shows '1'.

Solⁿ

(i) P(2, 3, 5, 7, 11)

2 → (1,1) XX

3 → 2 cases (1,2),(2,1)

5 → 4 cases

7 → 6 cases

11 → 2 cases

$$\text{Req. prob} = \frac{14}{30}$$

(ii)

$$\text{Req. prob} = \frac{10}{30}$$

Q Let P_i $(i = 1, 2, \ldots, 6)$ denotes the probability of getting the face 'i', when a biased die is thrown where $P_i \propto i$.

Find the probability of getting :-

(i) Composite faces.

Sol^n $P_i \propto i \Rightarrow \boxed{P_i = Ki}$

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 1$$

$$K + 2K + 3K + 4K + 5K + 6K = 1$$

$\therefore K = \frac{1}{21}$ $\quad P(1) = K = \frac{1}{21}$; $P(2) = \frac{2}{21}$ and so on

(i) $P_4 + P_6 = \frac{4}{21} + \frac{6}{21} = \frac{10}{21}$.

(2) When 2 such dice are thrown then find the prob. of getting a doublet.

$$\left(\frac{1}{21}\right)^2 + \left(\frac{2}{21}\right)^2 + \cdots + \left(\frac{6}{21}\right)^2.$$