A certain kind of bacteria either die, split into two or split into three bacteria. All splits are exact copies. The chances of dying is $\frac{1}{4}$, the chances of splitting into two is $\frac{1}{2}$ and splitting into three is $\frac{1}{4}$. If the probability that it survives for infinite length of time is $\frac{m-\sqrt{13}}{n}$ (m,n $\in$ N), then the value of (m+n) is

Soln

Let 'P' be prob hat single bateria is going to die.

$$P = \frac{1}{4} + \frac{1}{2} \cdot P \cdot P + \frac{1}{4} \cdot P \cdot P \cdot P$$

$$\cancel{(P-1)}\,(P^2+3P-1) = 0$$

$$P = \frac{\sqrt{13}-3}{2}$$

$$\text{Req prob} = 1 - P = 1 - \left(\frac{\sqrt{13}-3}{2}\right) = \frac{5-\sqrt{13}}{2}$$

$$\boxed{m+n = 7}$$ Ans

Comprehension (3 questions)

Q. There are n urns each containing $n+1$ balls such that the i^{th} urn contains i white balls and $(n+1-i)$ red balls. Let u_i be the event of selecting i^{th} urn, $i = 1, 2, 3, \ldots\ldots, n$ and w denotes the event of getting a white ball.

(a) If $P(u_i) \propto i$ where $i = 1, 2, 3, \ldots\ldots, n$ then $\lim\limits_{n\to\infty} P(w)$ is equal to

(A) 1 (B) 2/3 (C) 3/4 (D) 1/4

(b) If $P(u_i) = c$, where c is a constant then $P(u_n/w)$ is equal to

(A) $\frac{2}{n+1}$ (B) $\frac{1}{n+1}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{2}$

(c) If n is even and E denotes the event of choosing even numbered urn ($P(u_i) = \frac{1}{n}$), then the value of $P(w/E)$, is

(A) $\frac{n+2}{2n+1}$ (B) $\frac{n+2}{2(n+1)}$ (C) $\frac{n}{n+1}$ (B) $\frac{1}{n+1}$

Sol^n

$u_1, u_2, \ldots u_n$ urns

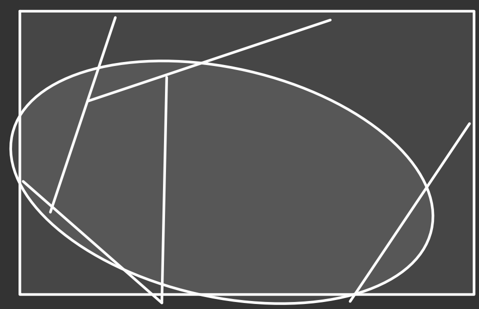
$u_1 = 1W + nR$

$u_2 = 2W + (n-1)R$

$u_3 = 3W + (n-2)R$

$\vdots \qquad \vdots \qquad \vdots$

$u_n = nW + 1R$



$P(u_i) = Ki$

$P(U_1) = K \ , \ P(U_2) = 2K, \ldots\ldots$

$P(u_1) + P(u_2) + \ldots + P(u_n) = 1$

$K + 2K + 3K + \ldots + nK = 1 \Rightarrow K = \frac{2}{n(n+1)}$

$$\boxed{P(u_i) = \frac{2i}{n(n+1)}}$$

$P(W) = P(W \cap U_1) + P(W \cap U_2) + \ldots + P(W \cap U_n)$

$$P(U_1 \cap W) = P(U_1) \cdot P(W/U_1) \qquad P(u_i) = \frac{i(2)}{n(n+1)}$$

$$P(W) = P(U_1)P(W/U_1) + P(U_2)P(W/U_2) + \cdots + P(U_n)P(W/U_n)$$

$$= \sum_{i=1}^{n} \frac{2i}{n(n+1)} \cdot \left(\frac{i}{n+1}\right)$$

$$P(W) = \frac{2}{n(n+1)^2} \sum_{i=1}^{n} i^2 = \frac{\cancel{2}}{\cancel{n}(n+1)^{\cancel{2}}} \frac{\cancel{n}(\cancel{n+1})(2n+1)}{\cancel{6}\ 3}$$

$$P(W) = \frac{2n+1}{3(n+1)}$$

$$\lim_{n \to \infty} \frac{\cancel{n}\left(2 + \left(\frac{1}{n}\right)\right)}{3\cancel{n}\left(1 + \left(\frac{1}{n}\right)\right)} = \frac{2}{3} \quad [B]$$

(b) $P(U_1) = P(U_2) = \cdots = P(U_n) = c = \frac{1}{n}$.

$$\boxed{P(W) = P(U_1 \cap W) + P(U_2 \cap W) + \cdots + P(U_n \cap W)}$$

$$P(U_n/W) = \frac{P(U_n \cap W)}{P(W)}$$

$$= \frac{P(U_n) \cdot P(W/U_n)}{\sum_{i=1}^{n} P(U_i) P(W/U_i)} = \frac{\left(\frac{\cancel{1}}{\cancel{n}}\right)\left(\frac{n}{\cancel{n+1}}\right)}{\left(\frac{\cancel{1}}{\cancel{n}}\right)\sum_{i=1}^{n}\left(\frac{i}{\cancel{n+1}}\right)} = \frac{\cancel{n}}{\frac{\cancel{n}(n+1)}{2}} = \frac{2}{n+1}.$$

A

Q The angle θ between two non-zero vectors $\vec{a}$ & $\vec{b}$ satisfies the relation

$$\cos\theta = (\vec{a}\times\hat{i}).(\vec{b}\times\hat{i}) + (\vec{a}\times\hat{j}).(\vec{b}\times\hat{j}) + (\vec{a}\times\hat{k}).(\vec{b}\times\hat{k}),$$

then the least value of $|\vec{a}|+|\vec{b}|$ is equal to (where $\theta \neq 90°$)

Sol^n

$$\cos\theta = \vec{a}.\vec{b} - (\vec{a}.\hat{i})(\vec{b}.\hat{i}) + (\vec{a}.\vec{b}) - (\vec{a}.\hat{j})(\vec{b}.\hat{j}) + \vec{a}.\vec{b} - (\vec{a}.\hat{k})(\vec{b}.\hat{k})$$

$$\cos\theta = 3\vec{a}.\vec{b} - \vec{a}.\vec{b} = 2\vec{a}.\vec{b}$$

$$\cancel{\cos\theta} = 2|\vec{a}||\vec{b}|\cancel{\cos\theta} \Rightarrow |\vec{a}||\vec{b}| = \frac{1}{2}.$$

$$AM \geq GM$$

$$\frac{|\vec{a}|+|\vec{b}|}{2} \geq \sqrt{|\vec{a}||\vec{b}|}$$

$$|\vec{a}|+|\vec{b}| \geq 2\sqrt{\frac{1}{2}}$$

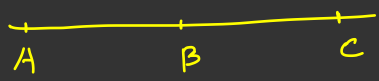
$$\geq \sqrt{2}$$

Q If three points $(2\vec{p}-\vec{q}+3\vec{r})$, $(\vec{p}-2\vec{q}+\alpha\vec{r})$ and $(\beta\vec{p}-5\vec{q})$ (Where $\vec{p}, \vec{q}, \vec{r}$ are non-coplanar vectors) are collinear, then $\frac{1}{(\alpha+\beta)}$ is

$A(2\vec{p}-\vec{q}+3\vec{r})$

$B(\vec{p}-2\vec{q}+\alpha\vec{r})$

$C(\beta\vec{p}-5\vec{q})$



$$\overrightarrow{BA} = \lambda\,\overrightarrow{CB}$$

$$\vec{p}+\vec{q}+(3-\alpha)\vec{r} = \lambda\left((1-\beta)\vec{p}+3\vec{q}+\alpha\vec{r}\right)$$

$$\vec{p}(1-\lambda+\lambda\beta)+\vec{q}(1-3\lambda)+\vec{r}(3-\alpha-\lambda\alpha) = \vec{0}$$

$$\left.\begin{aligned} 1-\lambda+\lambda\beta &= 0 \\ 1-3\lambda &= 0 \\ 3-\alpha-\lambda\alpha &= 0 \end{aligned}\right\}$$

$\lambda = \frac{1}{3}$

$(\alpha+\beta) = \frac{1}{4}$

$\frac{1}{\alpha+\beta} = 4.$

Q If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors having magnitude 1, 2, 3 respectively then $[\vec{a}+\vec{b}+\vec{c} \quad \vec{b}-\vec{a} \quad \vec{c}] = ?$

Sol^n

$R_1 \to R_1 + R_2 + R_3$

$[2\vec{b} \quad \vec{b}-\vec{a} \quad \vec{c}]$

$\underbrace{[2\vec{b} \quad \vec{b} \quad \vec{c}]}_{0} + [2\vec{b} \quad -\vec{a} \quad \vec{c}]$

$2[\vec{b} \quad -\vec{a} \quad \vec{c}]$

$2[\vec{a} \quad \vec{b} \quad \vec{c}] =$

$2(|\vec{a}||\vec{b}||\vec{c}|)$

$2(1)(2)(3) = \underline{\underline{12}}$

Q Let $|\vec{a}| = \sqrt{3}, |\vec{b}| = 5, \vec{b}.\vec{c} = 10$ angle between $\vec{b}$ & $\vec{c}$ equal to $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to $\vec{b} \times \vec{c}$ then find the value of $\left|\vec{a} \times \left(\vec{b} \times \vec{c}\right)\right|$.

Solⁿ

$$\vec{b}.\vec{c} = 10 \Rightarrow |\vec{b}||\vec{c}| \cos \frac{\pi}{3} = 10$$

$$|\vec{c}| = 4$$

$$\theta(\vec{a} \wedge \vec{b} \times \vec{c}) = \pi/2$$

$$|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}| \; |\vec{b} \times \vec{c}| \; \sin \pi/2$$

$$= \sqrt{3} \; |\vec{b}||\vec{c}| \sin \pi/3$$

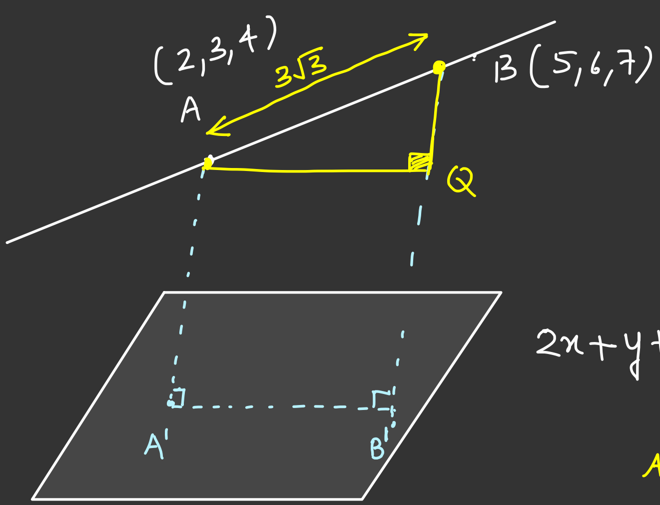
$$= \sqrt{3} \cdot (5)(4) \cdot \frac{\sqrt{3}}{2}$$

$$= 30$$

Ans

Q Projection of line segment joining (2,3,4) and (5,6,7) on plane $2x + y + z = 1$ is :-

Sol^n



$2x+y+z=1$

$AQ = A'B'$

ΔABQ :

$AQ = \sqrt{(AB)^2 - (BQ)^2}$

$BQ = |\text{proj of } \vec{AB} \text{ on } \vec{n}|$

$BQ = 2\sqrt{6}$

$AQ = \sqrt{3}$ Ans

Q Let shortest distance between two opposite edges of a tetrahedron is '4 unit' and the length of these opposite edges are same and equal to 6 unit. If angle between these two opposite edges is $30°$ and volume of tetrahedron is V, the value of $\frac{V}{6}$ is

Done in Notes already.