Q The direction cosines of the projection of the line $\frac{1}{2}(x-1) = -y = z+2$ on the plane $2x + y - 3z = 4$ are-

$\vec{n} = 2\hat{i} + \hat{j} - 3\hat{k}$

(A) $\left(\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

(B) $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

(C) $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$

(D) None of these

Sol^n

$$L: \quad \frac{x-1}{2} = \frac{y}{-1} = \frac{z-(-2)}{1}$$

$$(2\hat{i} - \hat{j} + \hat{k}) \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 0.$$

$\Rightarrow$ Line is $\parallel$ to plane.

$\therefore \boxed{A}$

Q The direction ratios of a normal to the plane passing through (1,0,0), (0,1,0) and making an angle $\frac{\pi}{4}$ with plane x + y = 3 can be-

(A) 0,1,0 (B) $1,1,\sqrt{2}$

(C) 1,0,0 (D) $\sqrt{2},1,1$

Sol^n Let eqn of the plane is

$$P: a(x-1) + b(y-0) + c(z-0) = 0 \longrightarrow \vec{n} = a\hat{i} + b\hat{j} + c\hat{k}.$$

(0,1,0)

$$-a + b + 0 = 0 \Rightarrow \boxed{a = b} \quad -①-$$

$$\vec{n_1} = \hat{i} + \hat{j}$$

$$\vec{n}\cdot\vec{n_1} = |\vec{n_1}||\vec{n}| \cos\frac{\pi}{4}$$

$$a+b = \cancel{\sqrt{2}}\sqrt{a^2+b^2+c^2}\cdot\frac{1}{\cancel{\sqrt{2}}}$$

$$\cancel{a^2} + \cancel{b^2} + 2ab = \cancel{a^2} + \cancel{b^2} + c^2$$

$$2ab = c^2 \Rightarrow 2a^2 = c^2$$

$$\boxed{c = \pm\sqrt{2}a} \quad -②-$$

$$a : b : c = \cancel{a} : \cancel{a} : \pm\sqrt{2}\cancel{a}$$

$$1 : 1 : \pm\sqrt{2}$$

[B]

Q If for unit vectors $\hat{a}, \hat{b}$ and non-zero $\vec{c}$, $\hat{a} \times \hat{b} + \hat{a} = \vec{c}$ and $\hat{b}.\vec{c} = 0$, then volume of parallelopiped with coterminous edges $\hat{a}, \hat{b}$ and $\vec{c}$ will be (in cu.units)-

(A) 6 (B) 4 (C) 1 (D) $\frac{1}{2}$

Sol^n $[C]$

$$V = [\hat{a}\ \hat{b}\ \vec{c}] = (\hat{a} \times \hat{b}).\vec{c}$$

$$\hat{a} \times \hat{b} + \hat{a} = \vec{c} \quad -①- \quad \text{and} \quad \hat{b}.\vec{c} = 0 \quad -②-$$

dot with $\vec{c}$

$$\underbrace{(\hat{a} \times \hat{b}).\vec{c}}_{V} + \hat{a}.\vec{c} = \vec{c}^{\,2}$$

$$\boxed{V = c^2 - \hat{a}.\vec{c}} = 2 - 1 = 1$$

① dot with $\hat{b}$

$$\cancel{(\hat{a} \times \hat{b}).\hat{b}}^{\,0} + \hat{a}.\hat{b} = \cancel{\vec{c}.\hat{b}}^{\,0} \Rightarrow \boxed{\hat{a}.\hat{b} = 0} \quad -③-$$

$$\theta(\hat{a} \wedge \hat{b}) = 90^\circ$$

① doth with $\hat{a}$:

$$\cancel{(\hat{a} \times \hat{b}).\hat{a}}^{\,0} + \hat{a}.\hat{a} = \hat{a}.\vec{c}$$

$$\boxed{1 = \hat{a}.\vec{c}} \quad -④-$$

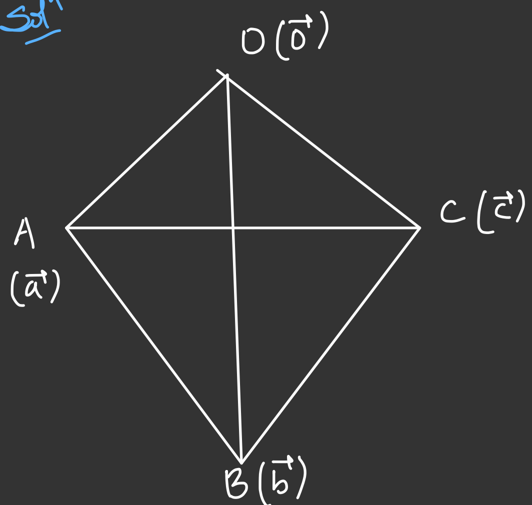
$$(\hat{a} \times \hat{b})^2 = (\vec{c} - \hat{a})^2$$

$$(1)^2 = c^2 + 1 - 2\vec{c}.\hat{a} \Rightarrow \cancel{1} = c^2 + \cancel{1} - 2 \Rightarrow \boxed{c^2 = 2}$$

Q In a tetrahedron OABC, the measures of the ∠BOC, ∠COA & ∠AOB are α, β & γ respectively, then $(\cos^2\alpha + \cos^2\beta + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma)$ can attain-

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\pi}{4}$ (C) 1 (D) 2

Sol^n



$$[\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & ab\cos\gamma & ac\cos\beta \\ ab\cos\gamma & b^2 & bc\cos\alpha \\ ac\cos\beta & bc\cos\alpha & c^2 \end{vmatrix}$$

$$\underbrace{[\vec{a}\ \vec{b}\ \vec{c}]^2}_{>0} = a^2b^2c^2 \underbrace{\begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}}_{>0}$$

$$1 - \Sigma\cos^2\alpha + \cos\alpha\cos\beta\cos\gamma > 0$$

$$\Sigma\cos^2\alpha - \cos\alpha\cos\beta\cos\gamma < 1$$

[A, B]

Q Let $\vec{a} = 3\hat{i} - 5\hat{k}$, $\vec{b} = 2\hat{i} + 7\hat{j}$ and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$. Consider $\vec{r}$ such that $\vec{r}.\vec{a} = -1$, $\vec{r}.\vec{b} = 6$ and $\vec{r}.\vec{c} = 5$. Then the vector component of $2\hat{i} + 3\hat{j} + 4\hat{k}$ along $\vec{r}$ is $n\left(\dfrac{\ell\hat{i} + m\hat{k}}{\ell^2 + m^2}\right)$, where ℓ & m are coprimes, then $\ell^2 + m^2 + n^2$ is equal to

Sol

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$

$$\left.\begin{aligned} \vec{r}.\vec{a} &= -1 \Rightarrow & 3x - 5z &= -1. \\ \vec{r}.\vec{b} &= 6 \Rightarrow & 2x + 7y &= 6. \\ \vec{r}.\vec{c} &= 5 \Rightarrow & x + y + z &= 5. \end{aligned}\right\}$$

$$\vec{r} =$$

Vector component of $(2\hat{i} + 3\hat{j} + 4\hat{k})$ along $\vec{r}$

$$\left((2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot \hat{r}\right) \hat{r}$$

$$14\left(\frac{3\hat{i} + 2\hat{k}}{13}\right)$$

$n = 14$

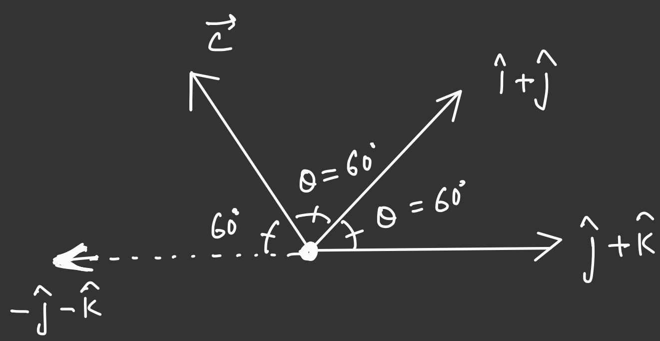
$l = 3$

$m = 2$

$$l^2 + m^2 + n^2 = 209$$

Ans

Q. If $\hat{i}+\hat{j}$ bisects the angle between $\vec{c}$ & $\hat{j}+\hat{k}$, then $\vec{c}.\hat{j}$ is equal to



$(\hat{i}+\hat{j}).(\hat{j}+\hat{k}) = \sqrt{2}.\sqrt{2}\cos\theta$

$1 = 2\cos\theta$

$\cos\theta = \frac{1}{2}$

$\boxed{\theta = 60^\circ}$

$$\vec{c} = \lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} + \left(\frac{-\hat{j}-\hat{k}}{\sqrt{2}}\right)\right)$$

$$\vec{c} = \frac{\lambda}{\sqrt{2}}(\hat{i}-\hat{k})$$

$\vec{c}.\hat{j} = 0$ Ans