Q If $f(x) = x^4 - 4x^3 + 4x^2 + 8x + 44$, find $f(3 + 2i)$. $\rightarrow$ 5

Sol^n

$x = 3+2i$

$(x-3)^2 = (2i)^2 \Rightarrow x^2 + 9 - 6x = -4$

$\boxed{x^2 - 6x + 13 = 0}$ — (1) —

$x^4 - 4x^3 + 4x^2 + 8x + 44 = \underbrace{(x^2 - 6x + 13)}_{0}$ Quotient + Remainder

Quotient

$x^2 - 6x + 13 \,\big)\, x^4 - 4x^3 + 4x^2 + 8x + 44$

Rem = 5

Q If sum of reciprocal of roots of equation $x^2 + ipx = 4x - i$ is $2 - qi$, $p, q \in R$, then find q.

Solⁿ

$$x^2 + (ip - 4)x + i = 0 \quad \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \qquad p, q \in \mathbb{R}$$

$$\alpha + \beta = -(ip - 4)$$
$$\alpha\beta = i$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = 2 - qi$$

$$\frac{\alpha + \beta}{\alpha\beta} = 2 - qi \Rightarrow \frac{-ip + 4}{i} = 2 - qi$$

$$-ip + 4 = 2i + q$$

$$\left.\begin{matrix} p = -2 \\ q = 4 \end{matrix}\right\}$$

Q If $(4-3i)$ is one root of equation $(1+i)x^2 - (7+3i)x + 6+8i = 0$ then find the other root ?

Sol^n

$$(4-3i)\alpha = \frac{6+8i}{(1+i)}$$

$$\alpha = \frac{6+8i}{(1+i)(4-3i)} \times \frac{(1-i)}{(1-i)} \frac{(4+3i)}{(4+3i)}$$

$$\alpha = \frac{(6+8i)(1-i)(4+3i)}{2 \times 25}$$

Q If $\arg(z_1 z_2) = \frac{5\pi}{6}$ and $\arg(z_1^2 \bar{z}_2) = \pi/6$ then $\arg(z_1) = ?$

Sol^n $\arg(z_1 z_2) = \frac{5\pi}{6}$

$K \in$ Int
$\lambda \in$ Int

$K+\lambda \in$ Int

$$\arg z_1 + \arg z_2 + 2K\pi = \frac{5\pi}{6} \quad -①-$$

$$2\arg z_1 - \arg z_2 + 2\lambda\pi = \pi/6 \quad -②-$$

add

$$3\arg(z_1) + 2\underbrace{(K+\lambda)}_{\text{integer}}\pi = \pi.$$

$I = K+\lambda$

$$\underbrace{\arg z_1}_{(-\pi,\ \pi]} = \frac{\pi}{3} - \frac{2\pi}{3}(I)$$

if $I = 1$

$\arg z_1 = -\pi/3$ ✓

if $I = 0$ then

$\arg z_1 = \pi/3$ ✓

if $I = -1$ then

$\arg z_1 = \pi.$ ✓

Q If $|z_1| = |z_2| = |z_3| = 1$ and $z_1 + z_2 + z_3 = 0$ then find value of $(z_1^2 + z_2^2 + z_3^2) = ?$

Sol^n

$$\boxed{z_1 + z_2 + z_3 = 0} \Rightarrow \overline{z_1 + z_2 + z_3} = 0 \qquad z\bar{z} = |z|^2$$

$$\Downarrow$$

$$\boxed{\bar{z}_1 + \bar{z}_2 + \bar{z}_3 = 0}$$

Since $|z| = 1$ therefore $z\bar{z} = 1 \Rightarrow \bar{z} = \frac{1}{z}$

squaring

$$(z_1 + z_2 + z_3)^2 = 0 \Rightarrow \underbrace{(z_1^2 + z_2^2 + z_3^2)}_{E} + 2(z_1 z_2 + z_2 z_3 + z_3 z_1)$$

$$= 0$$

$$E + 2 z_1 z_2 z_3 \left(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right) = 0$$

$$E + 2 z_1 z_2 z_3 (\underbrace{\bar{z}_1 + \bar{z}_2 + \bar{z}_3}_{0}) = 0.$$

$$0$$

$\therefore \boxed{E = 0}$ Ans

Q If $(a + ib)^5 = \alpha + i\beta$ (where $a,b,\alpha,\beta \in R$), then show that $(b + ia)^5 = \beta + i\alpha$

Sol^n

$$(a+ib)^5 = \alpha+i\beta \qquad\qquad i^5 = i$$

$$i^5(a+ib)^5 = \underbrace{i^5}(\alpha+i\beta)$$

$$(ai-b)^5 = \alpha i-\beta$$

$$(b-ai)^5 = \beta-\alpha i$$

Take conjugate both sides :-

$$\overline{(b-ai)^5} = \overline{(\beta-\alpha i)}$$

$$(b+ai)^5 = \beta+\alpha i \qquad \text{(Hence proved)}$$

Q. If $\left|\frac{6z-i}{2+3iz}\right| \le 1$ then find $|z|_{max} = ?$

$|z|^2 = z\bar{z}$

Sol^n

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$\frac{|6z-i|}{|2+3iz|} \le 1 \Rightarrow |6z-i| \le |2+3iz|$

$|6z-i|^2 \le |2+3iz|^2$

$(6z-i)\overline{(6z-i)} \le (2+3iz)\overline{(2+3iz)}$

$(6z-i)(6\bar{z}+i) \le (2+3iz)(2-3i\bar{z})$

$36|z|^2 + \cancel{6zi} - \cancel{6i\bar{z}} + 1 \le 4 - \cancel{6i\bar{z}} + \cancel{6iz} + 9|z|^2$

$27|z|^2 \le 3$

$|z|^2 \le \frac{1}{9} \Rightarrow |z|_{max} = \frac{1}{3}$ Ans

* Q Find the set of points on the complex plane for which z^2+z+1 is real and positive.

Sol^n

z^2+z+1 is real and positive.

Let $z = x+iy$; $x, y \in \mathbb{R}$

$z^2 = x^2-y^2+2xyi$

$$\underbrace{z^2+z+1}_{\text{real \& positive}} = (x^2-y^2+2xyi)+(x+iy)+1$$

$$= (x^2-y^2+x+1)+i\underbrace{(2xy+y)}_{0}$$

$2xy+y=0$ and $x^2-y^2+x+1>0$.

$y(2x+1)=0$ and $x^2-y^2+x+1>0$

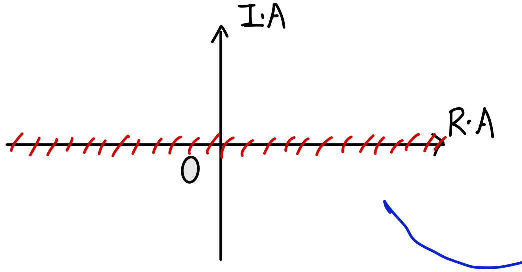
C–I $y=0$

$x^2-y^2+x+1>0$

$x^2+x+1>0$

$x \in \mathbb{R}$

$\therefore$ $\boxed{\text{Real axis}}$



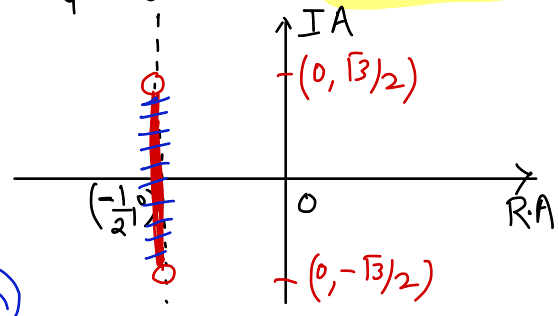
C–II $2x+1=0$

$x=-\frac{1}{2}$

$x^2-y^2+x+1>0$

$\frac{1}{4}-\frac{1}{2}+1-y^2>0$

$\frac{3}{4}-y^2>0 \Rightarrow y \in \left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$



Union

Q Convert into exponential form :—

$$z = \sin\frac{\pi}{7} - i\cos\frac{\pi}{7}.$$

$$z = r e^{i\theta}; \quad \theta \in (-\pi, \pi]$$

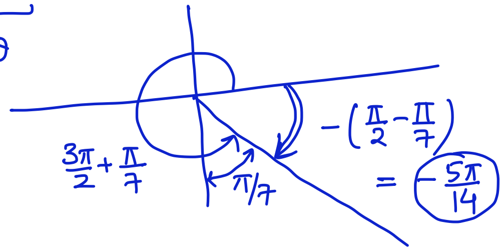
$$r = |z| \geq 0.$$

$$z = r(\cos\theta + i\sin\theta)$$
$$= r\,\underbrace{\text{cis}}\,\theta.$$

Sol^n

$$|z| = \sqrt{\sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7}} = 1$$

$$z = \cos\underbrace{\left(\frac{3\pi}{2} + \frac{\pi}{7}\right)}_{\theta} + i\sin\underbrace{\left(\frac{3\pi}{2} + \frac{\pi}{7}\right)}_{\theta}$$



$$z = 1 \cdot e^{i\left(\frac{-5\pi}{14}\right)}$$

Note:-

Rem

$$\boxed{\begin{aligned}\sqrt{i} &= \pm\frac{1}{\sqrt{2}}(1+i)\\ \sqrt{-i} &= \pm\frac{1}{\sqrt{2}}(1-i)\end{aligned}}$$

$$\left.\begin{aligned} i &= e^{i\pi/2} \\ -i &= e^{-i\pi/2}\end{aligned}\right\}$$

$\sqrt{i} = a+ib \quad ; \quad \underline{\underline{a, b \in \mathbb{R}}}$

$i = a^2-b^2+2ab\,i$

$a^2-b^2=0 \quad ; \quad 2ab=1$

$(a^2+b^2)^2 = (a^2-b^2)^2+4a^2b^2$

$= 0+1$

$$\left.\begin{aligned} a^2+b^2 &= 1\\ a^2-b^2 &= 0\end{aligned}\right\} \Rightarrow \quad \begin{aligned} a &= \\ b &= \end{aligned}$$