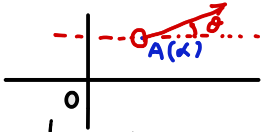
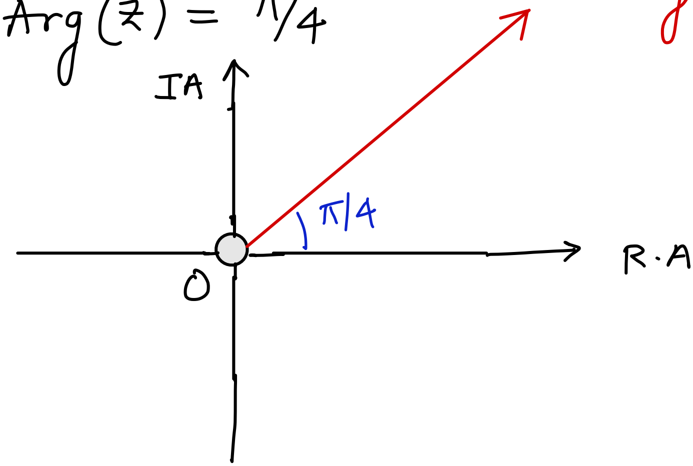
Note:-

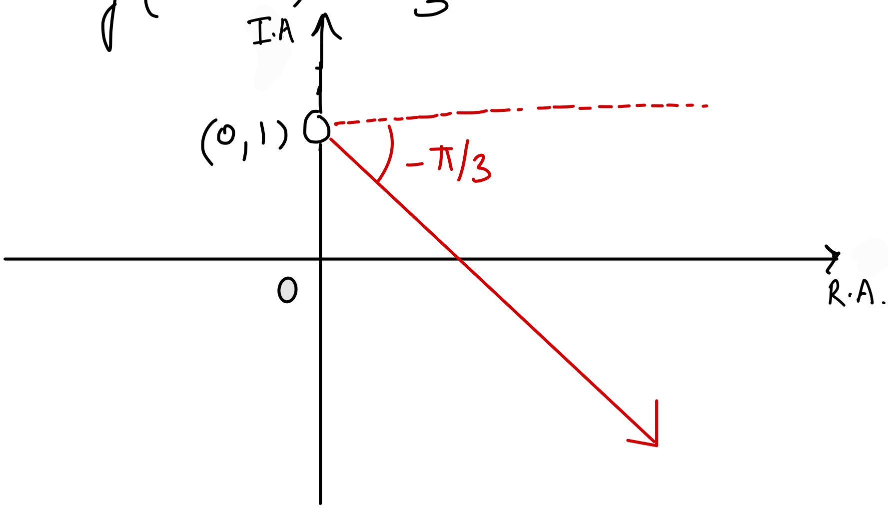


$\mathrm{Arg}(z-\alpha)=\theta$ denotes a ray emanating from the point $A(\alpha)$ moving away from A.

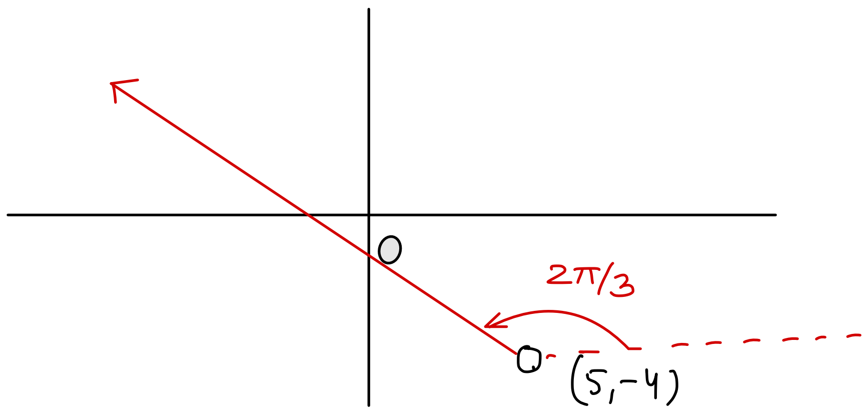
eg: $\mathrm{Arg}(z) = \pi/4$ $\quad$ $\mathrm{Arg}(z-0) = \dfrac{\pi}{4}$.



eg: $\mathrm{Arg}(z-i) = \dfrac{-\pi}{3}$



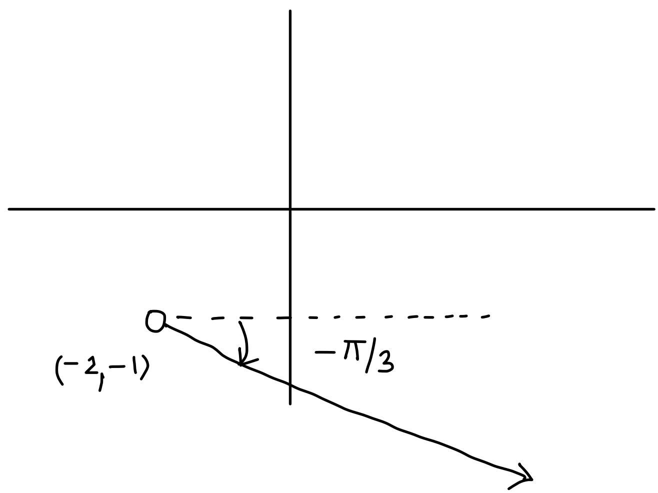
eg: $\operatorname{Arg}(z-5+4i) = \frac{2\pi}{3}$ $\longrightarrow$ $\arg(z-(5-4i)) = \frac{2\pi}{3}$



eg $\operatorname{Arg}(z+2+i) = -\pi/3$

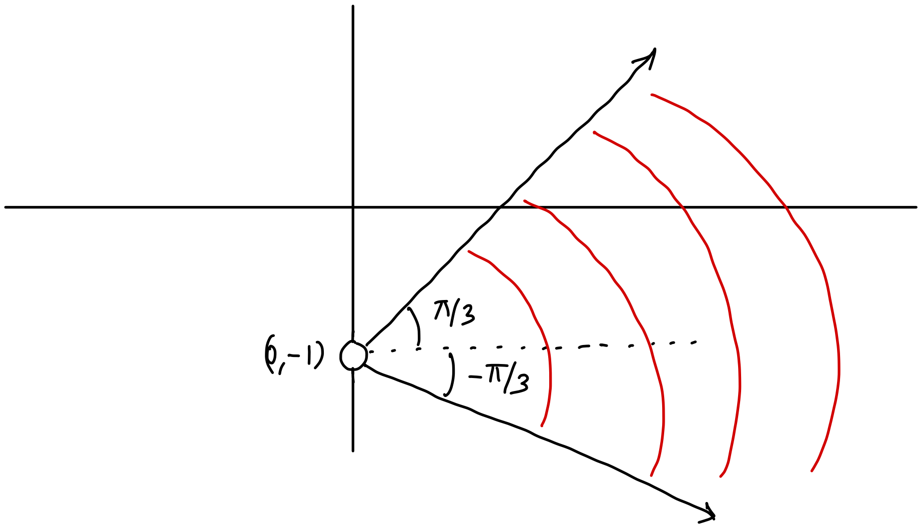
$\downarrow$

$\arg(z-(-2-i)) = -\pi/3$



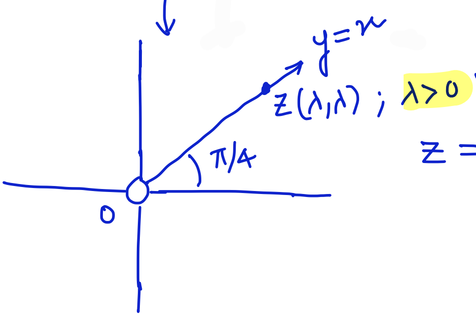
eg

$$\left|Amp(z+i)\right| \le \frac{\pi}{3}. \qquad -\frac{\pi}{3} \le amp(z-(-i)) \le \frac{\pi}{3}$$



Q If $\arg z = \frac{\pi}{4}$ and $|z+3-i| = 4$ then find z.

Sol^n



$z = \lambda(1+i)$

$|\lambda+\lambda i+3-i| = 4$

$\sqrt{(\lambda+3)^2+(\lambda-1)^2} = 4$

$2\lambda^2+10+4\lambda = 16.$

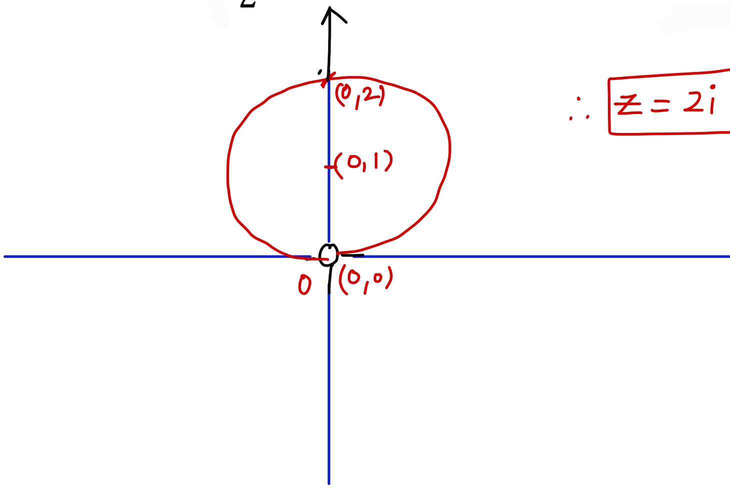
$\lambda^2+2\lambda-3=0$

$\boxed{\lambda=1}$ $\lambda=-3$ ✗✗

$\therefore \boxed{z = 1+i}$ Ans

Q If $|z-i| = 1$ and $\text{Arg } z = \frac{\pi}{2}$, find the number of complex number.

Sol^n

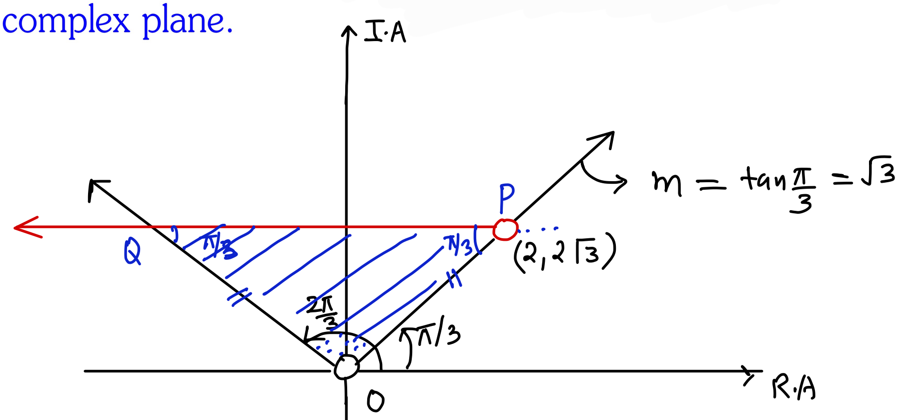


$\therefore \boxed{z = 2i}$ Ans

Q Find the area bounded by the curves $\operatorname{Arg} z = \frac{\pi}{3}$,

$\operatorname{Arg} z = \frac{2\pi}{3}$ & $\operatorname{Arg}(z - 2 - 2\sqrt{3}i) = \pi$ on the complex plane.

Sol^n

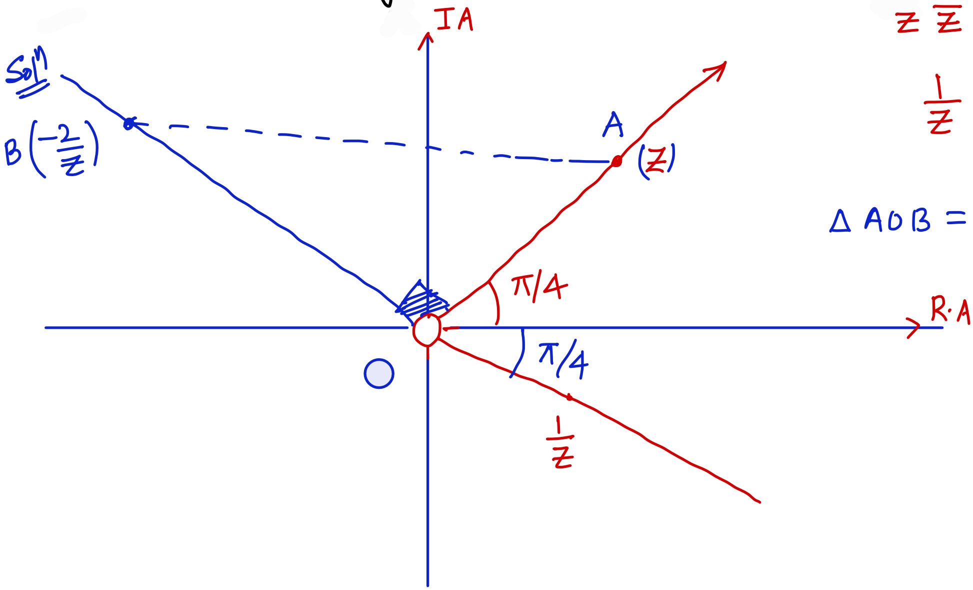


$OP = \sqrt{2^2 + (2\sqrt{3})^2} = 4.$

$\Delta OPQ = \frac{\sqrt{3}}{4} \cdot (4)^2 = 4\sqrt{3}$ sq. units

Q If $amp(z) = \frac{\pi}{4}$, then find the area of Δ formed by z, $\frac{-2}{z}$ and origin?

Soln



$z\bar{z} = |z|^2$

$\frac{1}{z} = \frac{\bar{z}}{|z|^2}$

$\Delta AOB = \frac{1}{2} OA \cdot OB$

$= \frac{1}{2}|z| \cdot \left|-\frac{2}{z}\right|$

$= \frac{1}{2}\cancel{|z|} \cdot \frac{2}{\cancel{|z|}}$

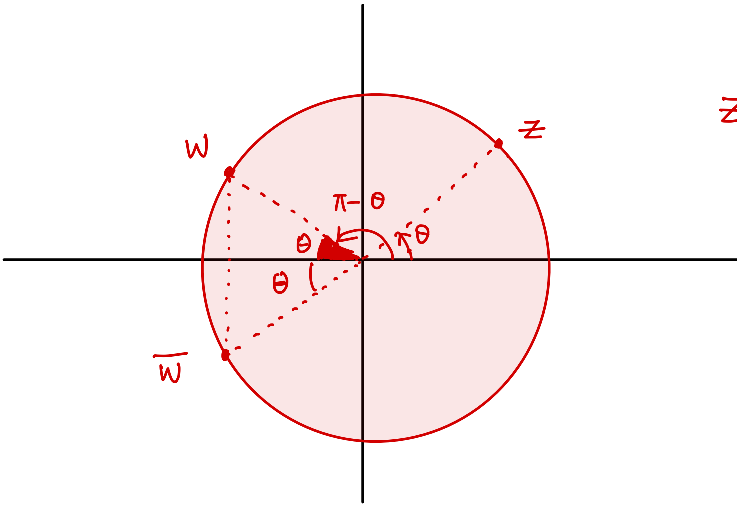
$= 1$ sq. units.

Q If z and w are two non zero complex number such that $|z| = |w|$ and $\operatorname{amp} z + \operatorname{amp} w = \pi$, then prove that $z = -\overline{w}$.

$\theta + \operatorname{amp} w = \pi$

$\operatorname{amp} w = \pi - \theta$

Sol^n



$z = -\overline{w}$

Q If $|\bar{z}| - 2z = 2i$ then find $|z|$ & $\text{amp}(z)$

Sol^n

$$z = x + iy \; ; \quad x, y \in \mathbb{R}$$

$$\bar{z} = x - iy$$

$$\sqrt{x^2+y^2} - 2(x+iy) = 2i$$

$$\sqrt{x^2+y^2} - 2x = 0 \quad —(1)— \; ; \qquad -2y = 2 \Rightarrow \boxed{y = -1}$$

$$\boxed{\sqrt{x^2+1} = 2x}$$

$$x^2 + 1 = 4x^2$$

$$x^2 = \frac{1}{3} \Rightarrow \boxed{x = \frac{1}{\sqrt{3}}} \text{ or } x = -\frac{1}{\sqrt{3}} \;\times\times$$

$$\therefore \; z = \frac{1}{\sqrt{3}} + (-i)$$

Ans

Q Prove that $|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2\,\mathrm{Re}(z_1\bar{z}_2)$

Sol^n

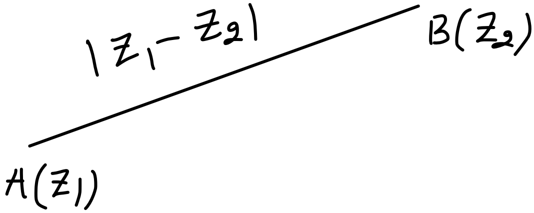
LHS: $(z_1+z_2)(\overline{z_1+z_2}) = (z_1+z_2)(\bar{z}_1+\bar{z}_2)$

$= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2$

$= |z_1|^2 + |z_2|^2 + (\underbrace{z_1\bar{z}_2}_{z} + \underbrace{z_2\bar{z}_1}_{\bar{z}})$

$= |z_1|^2 + |z_2|^2 + 2\,\mathrm{Re}(z_1\bar{z}_2)$

Distance :-



$z_1(1-3i)$

$z_2(-2+i)$

$z_1 = x_1 + iy_1$ & $z_2 = x_2 + iy_2$

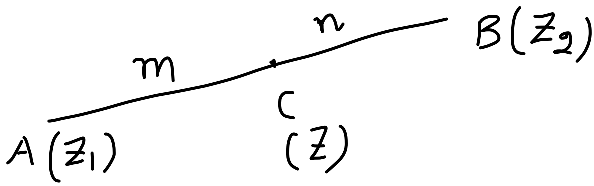
$$AB = |z_1 - z_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}.$$

$$|z_1 - z_2|$$

$$= \sqrt{3^2 + 4^2}$$

$$= 5.$$

Section Formula :-



$$\boxed{z = \frac{m z_2 + n z_1}{m+n}}$$

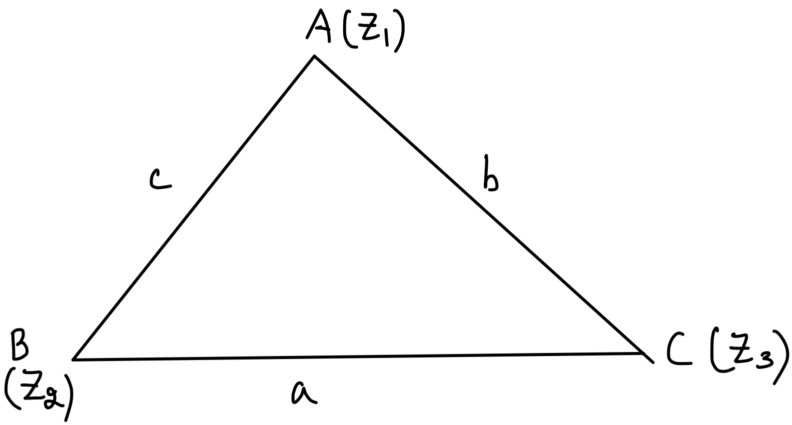
CENTROID, INCENTRE, ORTHOCENTRE & CIRCUMCENTRE OF A TRIANGLE ON A COMPLEX PLANE :

(i) Centroid 'G' = $\dfrac{z_1 + z_2 + z_3}{3} = Z_G$

(ii) Incentre 'I' = $\dfrac{az_1 + bz_2 + cz_3}{a+b+c}$

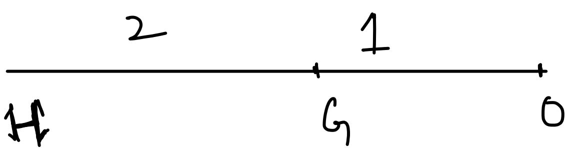
(iii) Orthocentre : $Z_H = \dfrac{z_1 \tan A + z_2 \tan B + z_3 \tan C}{\Sigma \tan A \text{ OR } (\prod \tan A)}$

(iv) Circumcentre : $Z_O = \dfrac{z_1 \sin 2A + z_2 \sin 2B + z_3 \sin 2C}{\Sigma \sin 2A \text{ OR } 4(\prod \sin A)}$



$a = |z_2 - z_3|$ $\quad$ $c = |z_1 - z_2|$

$b = |z_1 - z_3|$



$$\boxed{\frac{HG}{GO} = \frac{2}{1}}$$

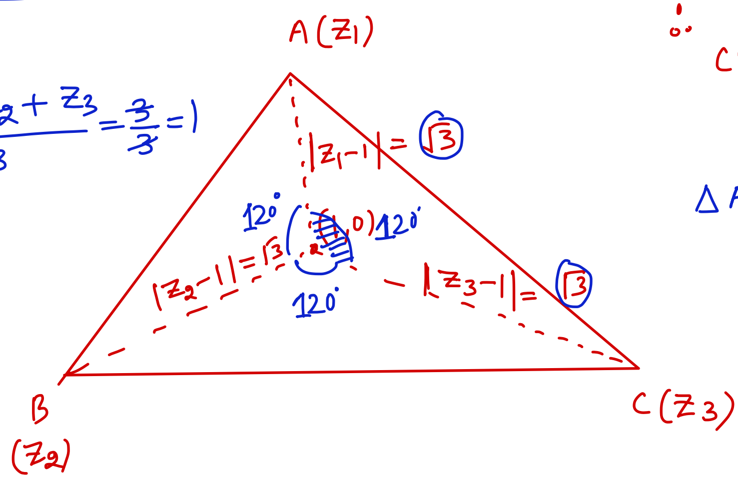
Q If z_1, z_2, z_3 are vertices of Δ such that $|z_1 - 1| = |z_2 - 1| = |z_3 - 1| = \sqrt{3}$ and $\underbrace{z_1 + z_2 + z_3 = 3}$, find area of Δ ?

$$z_G = \frac{z_1 + z_2 + z_3}{3} = \frac{\cancel{3}}{\cancel{3}} = 1$$

$\therefore \boxed{z_G = 1}$

$\Downarrow$

$\therefore \Delta ABC$ is equilateral



$\therefore$ $\underline{\underline{z = 1}}$ is circum centre of ΔABC.

$$\Delta AOC = \frac{1}{2}(\sqrt{3})(\sqrt{3}) \sin 120^\circ$$

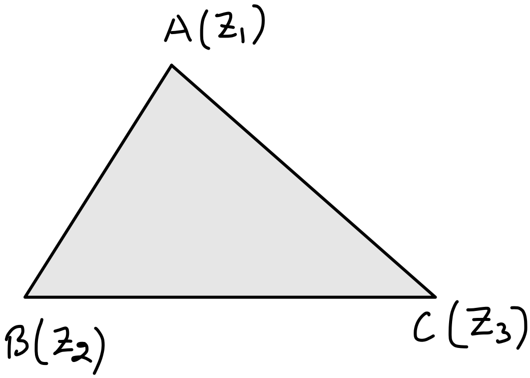
$$= \frac{1}{2} \cdot 3 \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{4}$$

$$\Delta ABC = 3 \times \Delta AOC$$

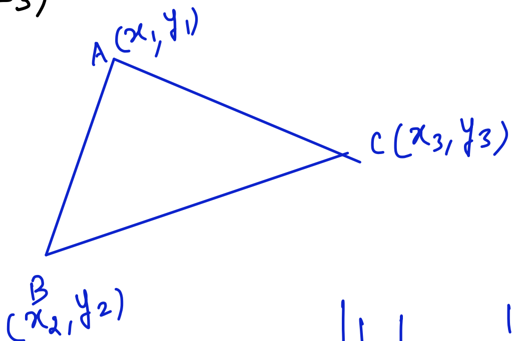
$$= 3 \times \frac{3\sqrt{3}}{4} = \frac{9\sqrt{3}}{4} \text{ sq. units}$$

Area of Δ :-



Rem

$$\Delta = \left| \frac{i}{4} \begin{vmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ \bar{z}_1 & \bar{z}_2 & \bar{z}_3 \end{vmatrix} \right|$$



Let $z_1 = (x_1, y_1)$

$z_2 = (x_2, y_2)$

$z_3 = (x_3, y_3)$

$$\Delta = \frac{1}{2} \left\| \begin{matrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{matrix} \right\|$$

$$\left. \begin{aligned} x_1 &= \frac{z_1 + \bar{z}_1}{2} \\ y_1 &= \frac{z_1 - \bar{z}_1}{2i} \end{aligned} \right\}$$

||ly

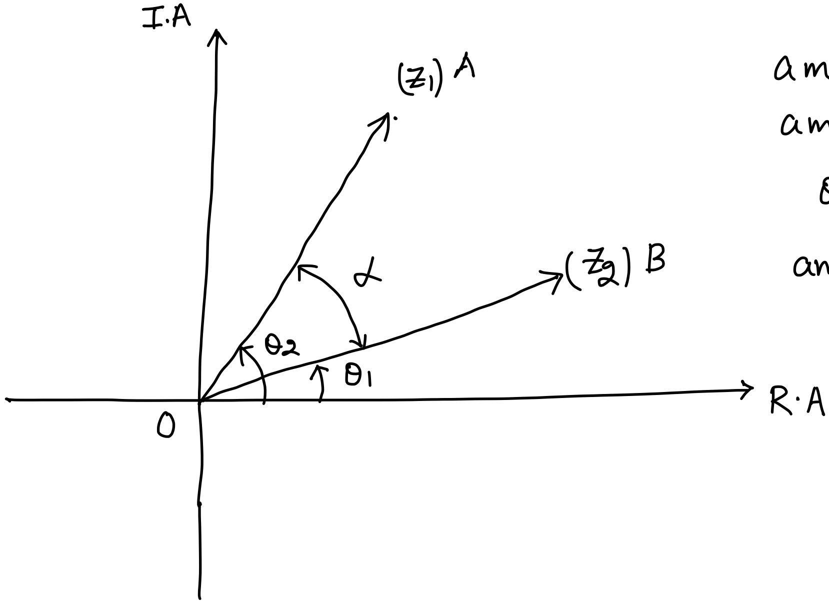
x_2

y_2

x_3

y_3

Angle between 2 complex nos. :-



$amp(z_1) = \theta_2$

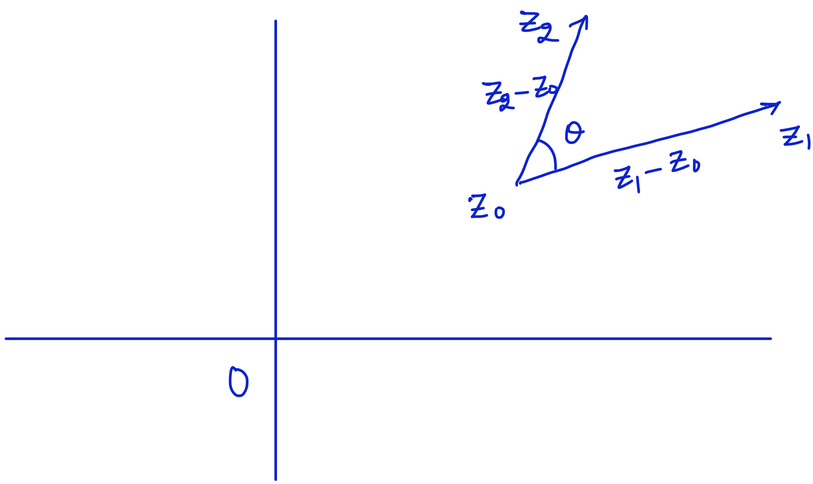
$amp(z_2) = \theta_1$

$\theta_2 - \theta_1 = \alpha$

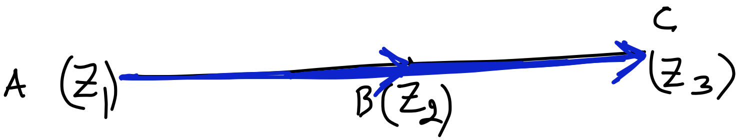
$amp\, z_1 - amp\, z_2 = \alpha$

$amp\left(\frac{z_1}{z_2}\right) = \alpha.$

$amp\left(\frac{z_2 - z_0}{z_1 - z_0}\right) = \theta.$



Condition of Collinearity :-



M-1 $\quad AB + BC = AC$

$$|z_1 - z_2| + |z_2 - z_3| = |z_1 - z_3|.$$

M-2

$$\begin{vmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ \bar{z}_1 & \bar{z}_2 & \bar{z}_3 \end{vmatrix} = 0.$$

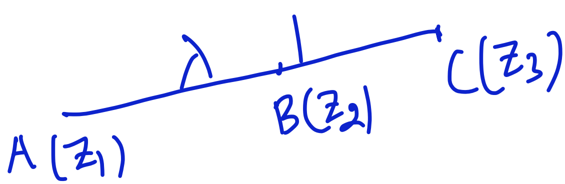
M-3

$$\operatorname{Arg}\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = 0.$$

$\Rightarrow \quad \dfrac{z_3 - z_1}{z_2 - z_1}$ is P.R.

$$\Rightarrow \quad \frac{z_3 - z_1}{z_2 - z_1} = \overline{\left(\frac{z_3 - z_1}{z_2 - z_1}\right)}$$

M-4

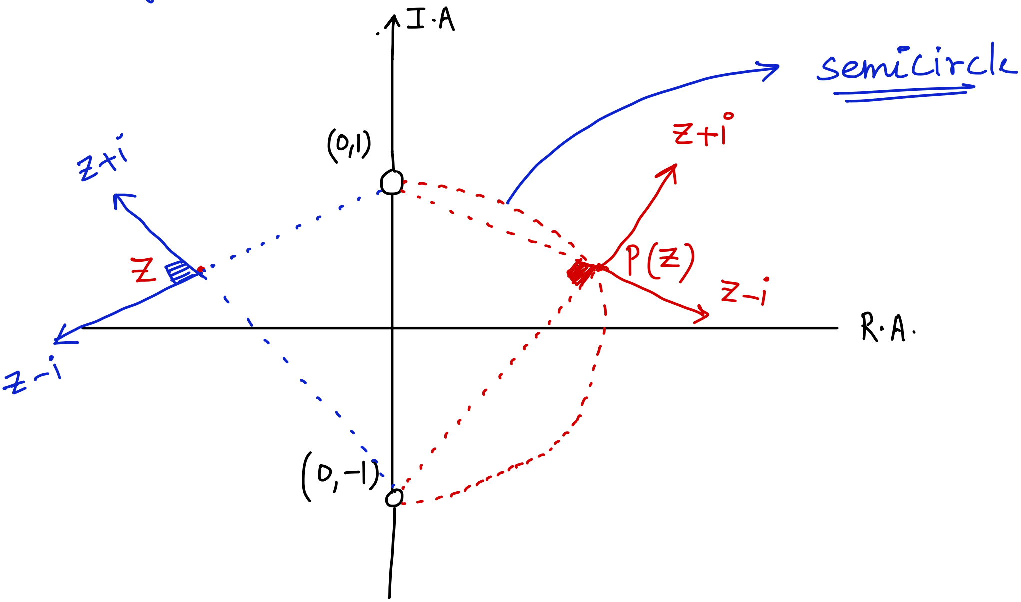


$$\boxed{z_2 = \frac{\lambda z_3 + z_1}{\lambda + 1}}$$

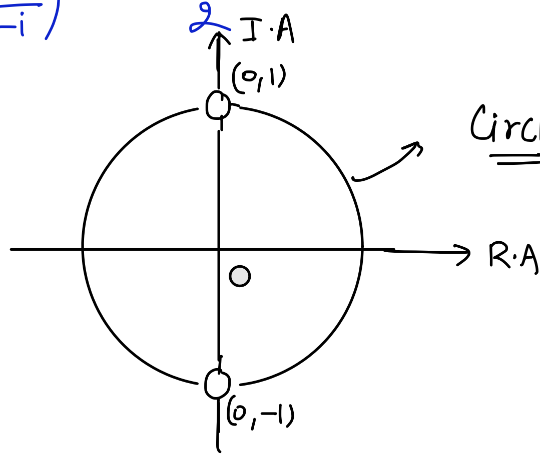
Q $\text{Arg}\left(\frac{z+i}{z-i}\right) = \frac{\pi}{2}$ Find Locus of z ?

Sol^n

$\arg(z+i) - \arg(z-i) = \frac{\pi}{2}$.



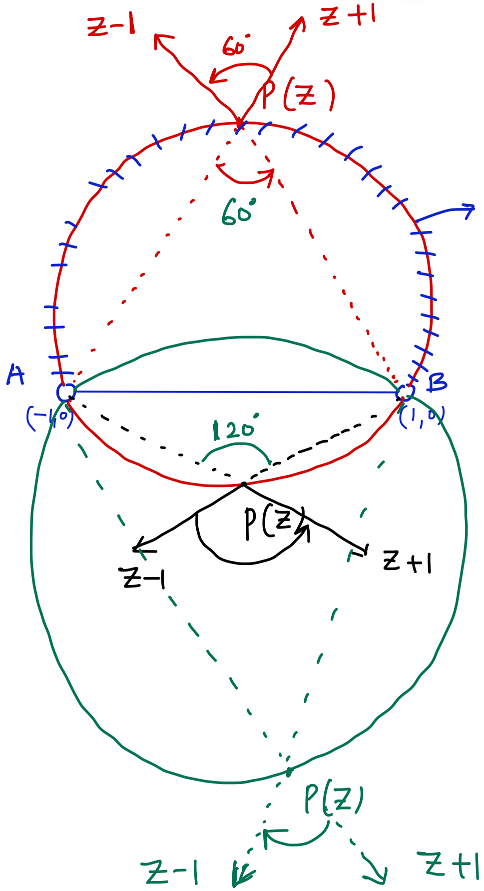
$\text{Arg}\left(\frac{z+i}{z-i}\right) = \pm\frac{\pi}{2}$



Q $\operatorname{Arg}\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$, find locus of z ?

$\arg(z-1)-\arg(z+1)=\frac{\pi}{3}$

Sol^n



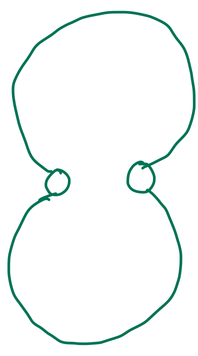
This major arc

$\arg(z-1)-\arg(z+1)=\pi/3$

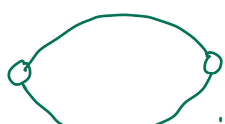
$\arg\left(\frac{z-1}{z+1}\right)=\pi/3$

$\arg\left(\frac{z+1}{z-1}\right)=-\frac{\pi}{3}$

$$\boxed{\arg\left(\frac{z-1}{z+1}\right)=\pm\frac{\pi}{3}}$$



$\arg\left(\frac{z-1}{z+1}\right)=\pm\frac{2\pi}{3}$



Note :-

If $\arg\left(\dfrac{z-z_1}{z-z_2}\right)=\theta$ then locus of 'z' (where z_1 & z_2 are fixed).

is a circular arc.

(i) If θ is acute then its major arc.

(ii) If θ is obtuse " " minor arc.

(iii) If θ is $\dfrac{\pi}{2}$ " " semi-circle.

HW :-

O-1 Q1 to 20.

S-1 Q 1 to 5.