Q Show that $\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} = 1$

is an identity? (Given a, b, c are distinct).

Sol^n put $x = a$.

$$0 + \frac{\cancel{(a-b)}\cancel{(a-c)}}{\cancel{(a-b)}\cancel{(a-c)}} + 0 = 1 \Rightarrow 1 = 1.$$

$||^{ly}$ $x = b$ & $x = c$ also satisfy the given eqn.

$\therefore$ It is satisfied by more than 2 distinct values

Hence it is an identity.

Condition of common roots :-

$$a_1x^2+b_1x+c_1=0 < \begin{matrix}\alpha\\ \beta\end{matrix}$$

$$a_2x^2+b_2x+c_2=0 < \begin{matrix}\alpha\\ \beta\end{matrix}$$

I : Condition for both roots to be common.

$$\boxed{\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}}$$

Rem

II : Atleast one common root :-

$$a_1x^2+b_1x+c_1=0 < \begin{matrix}\alpha\\ \beta\end{matrix}$$

$$a_2x^2+b_2x+c_2=0 < \begin{matrix}\alpha\\ \gamma\end{matrix}$$

Let 'α' be common root.

$$a_1\alpha^2+b_1\alpha+c_1=0$$

$$a_2\alpha^2+b_2\alpha+c_2=0.$$

$$\frac{\alpha^2}{b_1c_2-b_2c_1}=\frac{\alpha}{a_2c_1-a_1c_2}=\frac{1}{a_1b_2-a_2b_1}$$

$$(a_2c_1-a_1c_2)^2=(b_1c_2-b_2c_1)(a_1b_2-a_2b_1). \Leftarrow \alpha=\frac{b_1c_2-b_2c_1}{a_2c_1-a_1c_2}$$

$$\alpha=\frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}$$

Alt:

$$(a_1\alpha^2 + b_1\alpha + c_1 = 0.) \times a_2. \checkmark$$

$$(a_2\alpha^2 + b_2\alpha + c_2 = 0.) \times a_1 \checkmark$$

Subtract $\Rightarrow \alpha = (\qquad) \checkmark$

Q1 If the equation $x^2 - 4x + 5 = 0$ and $x^2 + ax + b = 0$ have common root then find $(a+b)$?

Sol^n

$x^2 - 4x + 5 = 0 \to \alpha + i\beta \to x_1$, $\alpha - i\beta \to x_2$

$D < 0 \to$ Roots are imaginary

$x^2 + ax + b = 0 \to x_1 = \alpha + i\beta$, $x_2 = \alpha - i\beta$.

$\therefore$ Both roots are common.

$$\frac{1}{1} = \frac{-4}{a} = \frac{5}{b}$$

$a = -4$ & $b = 5$.

Q2 If equation $4x^2\sin^2\theta-(4\sin\theta)x+1=0$ and $a^2(b^2-c^2)x^2+b^2(c^2-a^2)x+c^2(a^2-b^2)=0$ have a common root and 2^{nd} equation has equal roots then find possible value(s) of θ in $(0,\pi)$?

Soln

$$a^2(b^2-c^2)x^2+b^2(c^2-a^2)x+c^2(a^2-b^2)=0.$$

Coeff are cyclic $\Rightarrow$ $x=1$

Given equal roots $\therefore$ both roots are '1'.

$\therefore$ $x=1$ will satisfy 1^{st} eqn.

$$4\sin^2\theta-4\sin\theta+1=0 \Rightarrow (2\sin\theta-1)^2=0 \Rightarrow \sin\theta=\frac{1}{2}$$

$$\theta=\pi/6\,;\ 5\pi/6$$ Ans

Q. If the QE $x^2+bx+c=0$ and $x^2+cx+b=0$ $(b\neq c)$ have a common root then prove that their uncommon roots are roots of equation $x^2+x+bc=0$.

Sol^n

$x^2+bx+c=0$ — α, β

$x^2+cx+b=0$ — α, γ.

$\alpha+\beta=-b$; $\alpha\beta=c \rightarrow \beta=c$

$\alpha+\gamma=-c$; $\alpha\gamma=b. \rightarrow \gamma=b$

$\cancel{\alpha^2}+b\alpha+c=0.$

$\cancel{\alpha^2}+c\alpha+b=0$

$(b-c)\alpha+(c-b)=0$

$\alpha=\dfrac{\cancel{b-c}}{\cancel{b-c}}$

$\boxed{\alpha=1}$

$1+b+c=0$

$\boxed{b+c=-1}$

QE whose roots are β & γ.

$S.O.R=\beta+\gamma=b+c$; $P.O.R=\beta\gamma=bc.$

$\boxed{x^2-(b+c)x+bc=0.}$

$x^2-(-1)x+bc=0$

(H.P)

Q If $Q_1(x) = x^2 + (k-29)x - k$ and $Q_2(x) = 2x^2 + (2k-43)x + k$ both are factors of a cubic poly $P(x)$ then find largest value of k?

Soln

$Q_1(x) = 0$: α, β

$Q_2(x) = 0$: α, γ

'α' shd be common root.

$$(\alpha^2 + (k-29)\alpha - k = 0) \times 2$$

$$\cancel{2\alpha^2} + (2k-43)\alpha + k = 0$$

$$\cancel{2\alpha^2} + (2k-58)\alpha - 2k = 0$$

$$\quad - \qquad - \qquad\qquad + $$

$$15\alpha + 3k = 0 \Rightarrow \alpha = -\frac{k}{5}$$

$$\alpha^2 + (k-29)\alpha - k = 0$$

$k = 0$ OR $k = 30$ ✓ Ans

Q If the quadratic equation $x^2+ax+12=0$ and $x^2+bx+15=0$ and $x^2+(a+b)x+36=0$ have a positive common root then find 'a' & 'b'?

Sol^n Let 'α' be common root.

$$\left.\begin{array}{l}\alpha^2+a\alpha+12=0\\ \alpha^2+b\alpha+15=0.\end{array}\right] \longrightarrow \text{Add}$$

$$\alpha^2+(a+b)\alpha+36=0.$$

$$2\alpha^2+\cancel{(a+b)\alpha}+27=0.$$

$$\underline{\alpha^2+\cancel{(a+b)\alpha}+36=0.}$$

$$\alpha^2=9. \Rightarrow \boxed{\alpha=3} \text{ or } \alpha=-3.$$

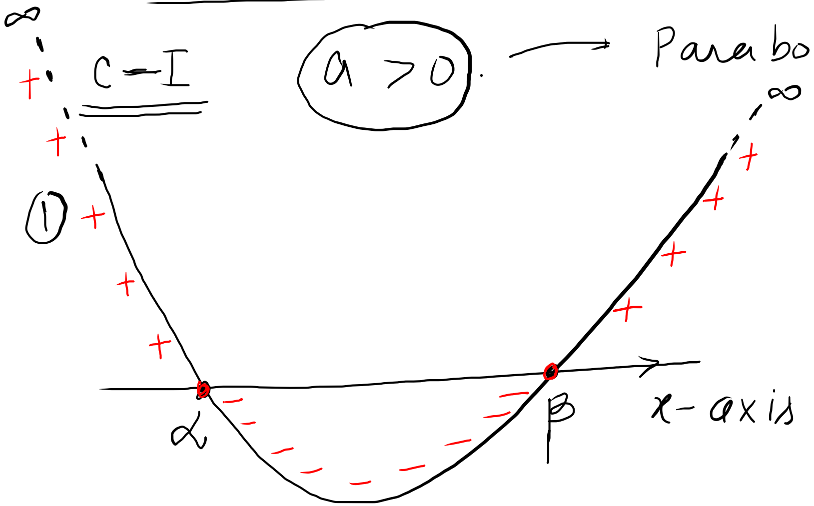
$$\left.\begin{array}{l}9+3a+12=0 \Rightarrow a=-7\\ 9+3b+15=0 \Rightarrow b=-8\end{array}\right\}$$

Graphs of Quadratic expression ie $y = ax^2 + bx + c$; $a \neq 0$.

C-I

$a > 0$ → Parabola mouth opening upwards.

①



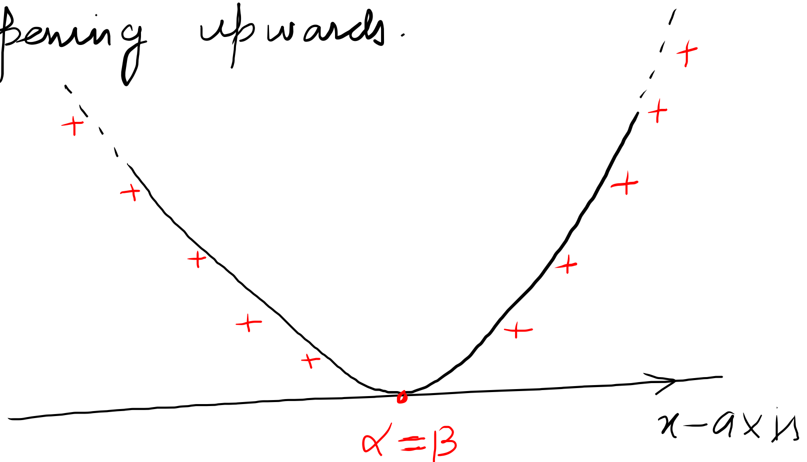
$\left.\begin{array}{l} a > 0 \\ D > 0 \end{array}\right\}$

$y > 0$ ✓✓

$y = 0$ ✓✓

$y < 0$ ✓✓

②



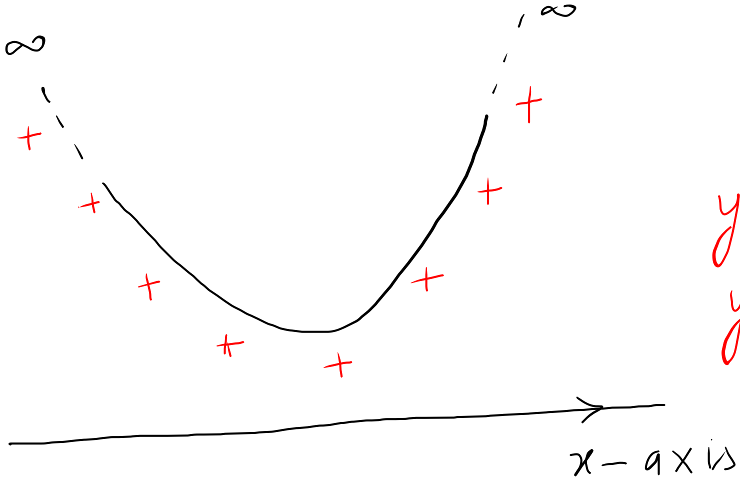
$\boxed{\begin{array}{l} a > 0 \\ D = 0 \end{array}}$

$y > 0$ ✓

$y = 0$ ✓

$y < 0$ ××

③



$y > 0$ ✓

$y = 0$ ×

$y < 0$ ×

$\boxed{\begin{matrix} a > 0 \\ D < 0 \end{matrix}}$

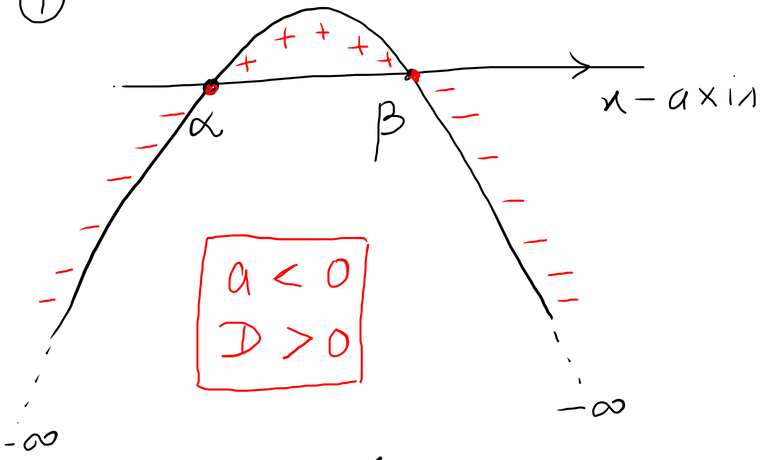
Note :

① Condition for quad expression $y = ax^2 + bx + c > 0 \ \forall x \in R$ is $a > 0$ & $D < 0$.

② Condition for quad exp $y = ax^2 + bx + c \geq 0 \ \forall x \in R$ is $a > 0$ & $D \leq 0$.

C-II $a < 0 \Rightarrow$ parabola mouth opening downwards.

④

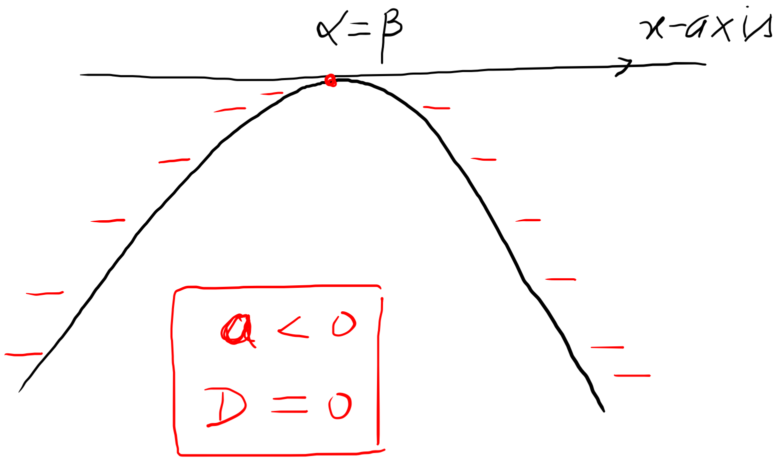


$y > 0$ ✓

$y = 0$ ✓

$y < 0$ ✓

⑤

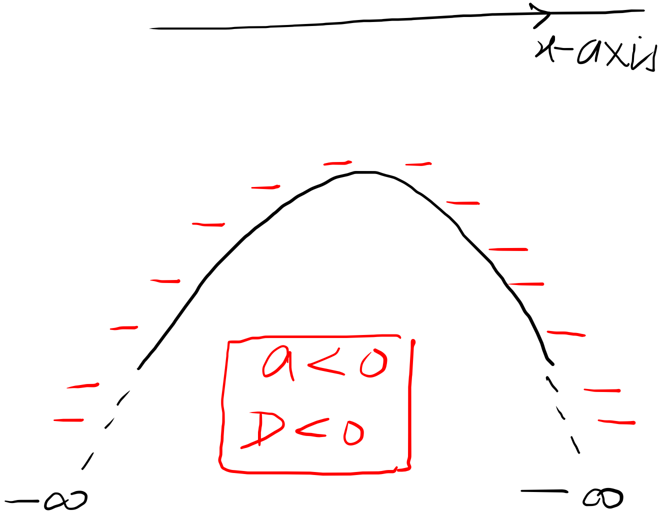


$y > 0$ ✗

$y = 0$ ✓

$y < 0$ ✓

⑥



$y > 0$ ✗

$y = 0$ ✗

$y < 0$ ✓

Note: (1) For quadratic expression $y = ax^2+bx+c < 0 \quad \forall x \in R$

Condition $\rightarrow$ $a<0$ and $D<0$.

(2) For quad. expression $y = ax^2+bx+c \le 0 \quad \forall x \in R$.

Condition $\rightarrow$ $a<0$ and $D \le 0$.

★★ The quadratic expression ax^2+bx+c is perfect square then

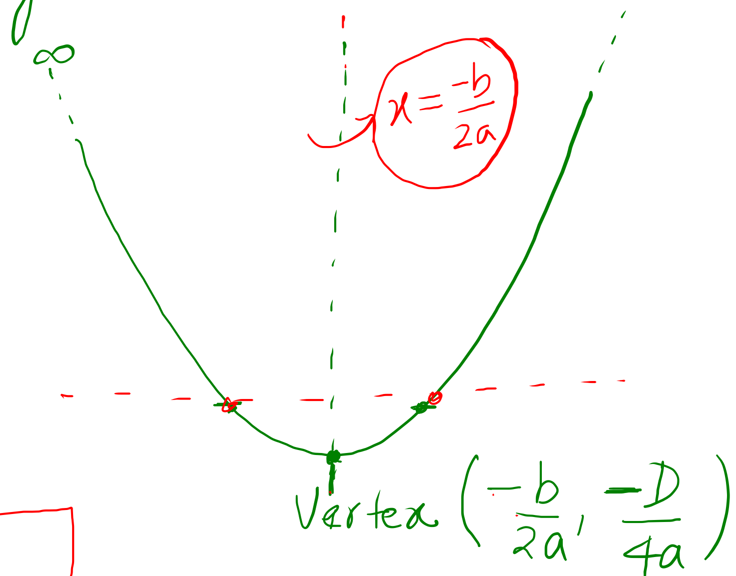
$$\boxed{a>0 \text{ and } D=0.}$$

★★

$$x^2-2x+1 = (x-1)^2 \checkmark\checkmark$$

$$-(x^2-2x+1) = -(x-1)^2 \times\times$$

$\underline{a>0}$ $\qquad y = ax^2+bx+c$



Rem

$$y = a\left(x^2+\frac{b}{a}x\right)+c.$$

$$y = a\left(\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}\right)+c$$

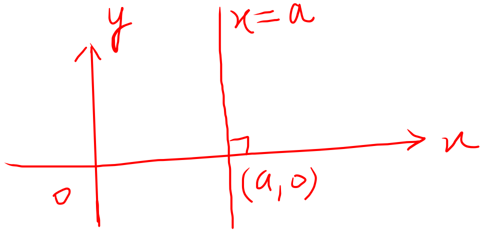
$$y = a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c.$$

$$y = a\left(x+\frac{b}{2a}\right)^2-\left(\frac{b^2-4ac}{4a}\right)$$

$$y = a\underbrace{\left(x+\frac{b}{2a}\right)^2}-\frac{D}{4a}.$$

For $a>0$

$$\boxed{y = ax^2+bx+c \in \left[\frac{-D}{4a}, \infty\right).}$$

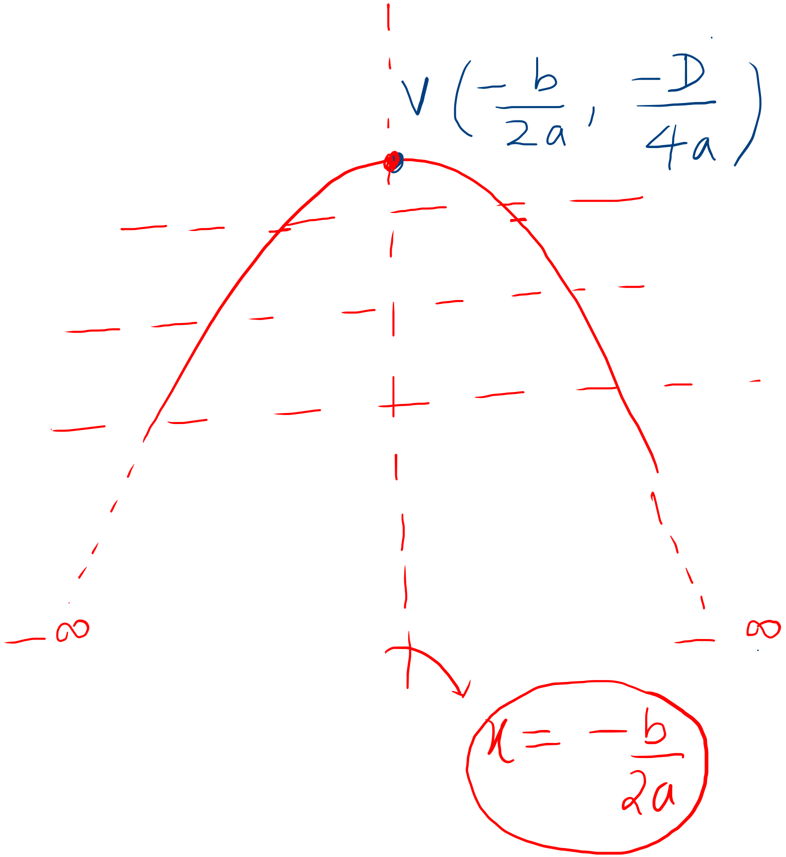


Note: For $y=f(x)=ax^2+bx+c$ is symmetric about line $x=\frac{-b}{2a}$. i.e

$$f\left(\frac{-b}{2a}-x\right)=f\left(\frac{-b}{2a}+x\right)$$
$$\forall\, x\in\mathbb{R}.$$

$\underline{\underline{a < 0}}$

$y = ax^2 + bx + c$



For $a < 0$

$$y = ax^2 + bx + c \in \left(-\infty, \frac{-D}{4a}\right]$$

Q. The quadratic equation $ax^2+bx+c=0$ has no real roots then comment upon sign of $c(a+b+c)$?

Sol^n

$ax^2+bx+c=0$

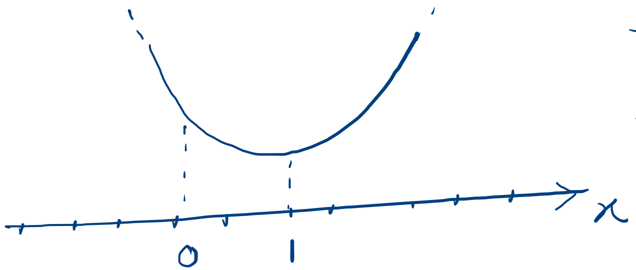
$\boxed{D<0}$

$f(x) = ax^2+bx+c$

$f(0) = c$

$f(1) = a+b+c$

C–I $a>0$ & $D<0$

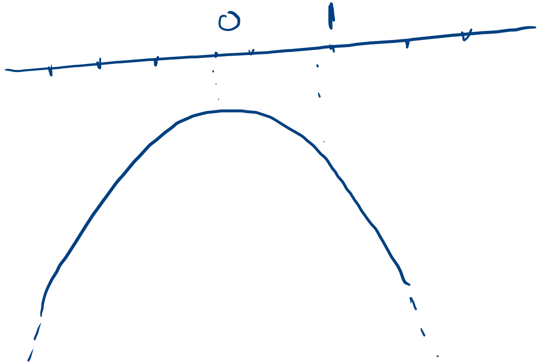


$f(0)>0$

$f(1)>0$

$\boxed{c(a+b+c)>0}$

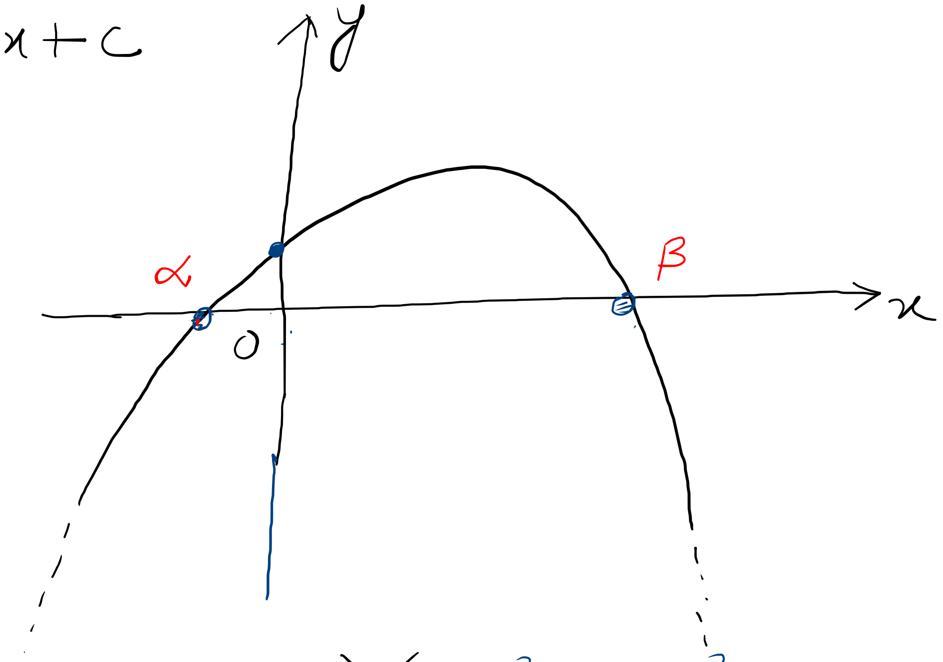
C–II $a<0$ & $D<0$



$\left.\begin{matrix} f(0)<0 \\ f(1)<0 \end{matrix}\right] \rightarrow f(0)\,f(1)>0$

$\boxed{c(a+b+c)>0}$

Q2 Given graph of quadratic $y = ax^2 + bx + c$



Sol^n

$\boxed{a < 0}$ ☆

$f(x) = ax^2 + bx + c$

$f(0) > 0 \Rightarrow \boxed{c > 0}$.

$\boxed{D > 0}$

$\left.\begin{array}{l}\alpha < 0\\ \beta > 0\end{array}\right] \rightarrow P.O.R < 0$

$\dfrac{c}{a} < 0 \Rightarrow \boxed{c > 0}$ ☆

$|\alpha| < |\beta|$

$\alpha + \beta > 0 \Rightarrow -\dfrac{b}{a} > 0 \Rightarrow b > 0$

(A) $abc < 0$ ✓

~~(B)~~ $a^3 - b^3 - c^3 > 0$

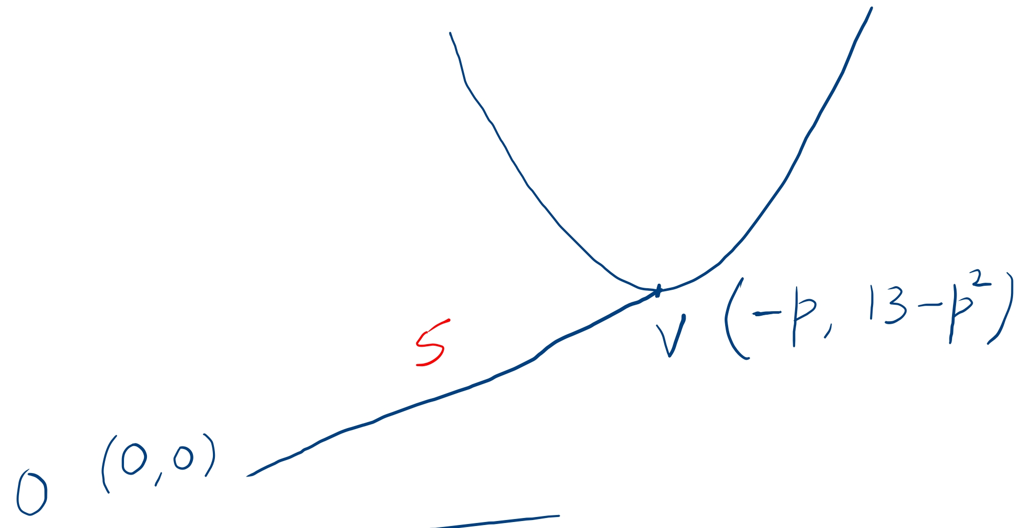
(C) $\underbrace{ab} + \underbrace{ac} - \underbrace{bc} < 0$ ✓

(D) $\alpha - \beta < 0$. ✓

[A C D]

Q1 For what value of 'p' the vertex of parabola $y = x^2 + 2px + 13$ lies at a distance of 5 units from origin?

Soln



$$\left\{\begin{array}{l} x = \dfrac{-b}{2a} = -\dfrac{2p}{2} = -p \\ y = \dfrac{-D}{4a} = \dfrac{-(4p^2-52)}{4} = 13-p^2. \end{array}\right.$$

$$\sqrt{p^2 + (13-p^2)^2} = 5$$

$$p^2 + p^4 - 26p^2 + 169 = 25$$

$$p^4 - 25p^2 + 144 = 0 \Rightarrow (p^2-16)(p^2-9) = 0$$

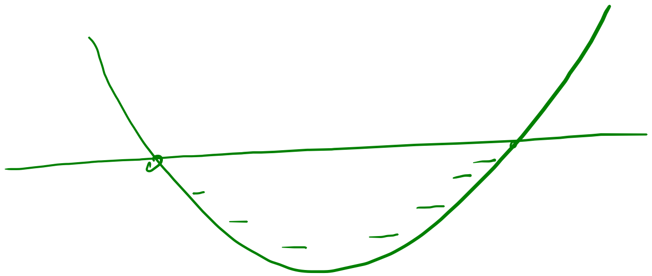
$$\therefore p = \pm 4 ; \pm 3.$$

Q. Find the set of value(s) of 'p' for which $\underbrace{x^2-2px+3p+4}_{y} < 0$ for atleast one real value of x?

Soln

$$y = x^2-2px+3p+4$$

mouth opening parabola

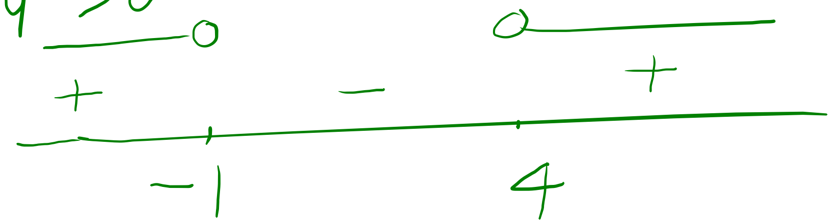


$$\boxed{D>0}$$

$$\cancel{4}p^2 - \cancel{4}(3p+4) > 0$$

$$p^2-3p-4>0 \Rightarrow p^2-4p+p-4>0$$

$$(p-4)(p+1)>0$$



$$p \in (-\infty,-1) \cup (4,\infty)$$

Q. If $y = x^2 - 3x - 4$ then find range of y when ① $x \in R$

② $x \in [0, 3]$

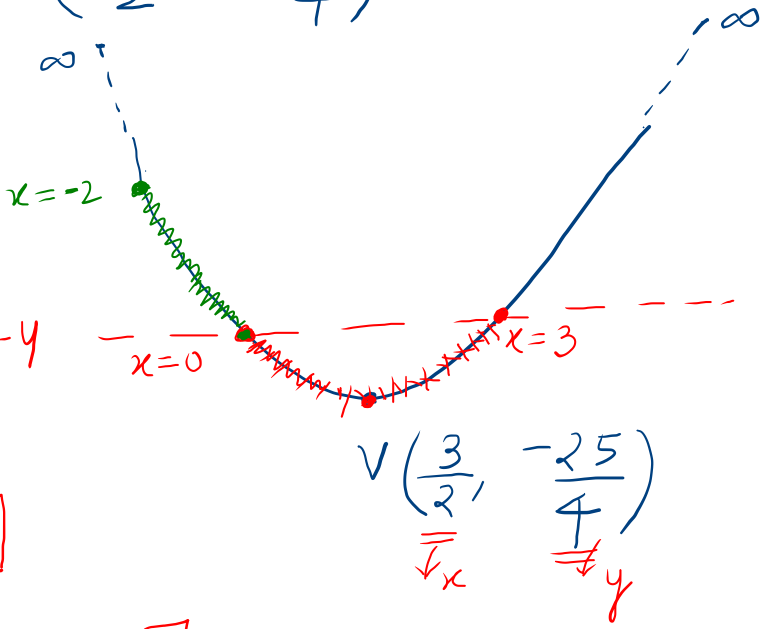
③ $x \in [-2, 0]$

Sol^n

$f(x) = y = x^2 - 3x - 4$

$\frac{-D}{4a} = \frac{-(9+16)}{4} = \frac{-25}{4}$

vertex : $\left(\frac{3}{2}, \frac{-25}{4}\right)$



① $x \in R$

$y \in \left[-\frac{25}{4}, \infty\right)$

$f(0) = -4$

$f(3) = \cancel{9} - \cancel{9} - 4 = -4$

② $x \in [0, 3]$

$y \in \left[-\frac{25}{4}, -4\right]$ Ans

③ $x \in [-2, 0]$

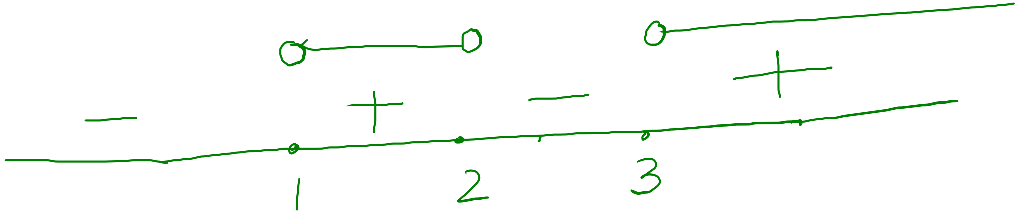
$f(-2) = (-2)^2 - 3(-2) - 4 = 6.$

$f(0) = -4$

$y \in [-4, 6]$ Ans

Inequalities :-

(1) $(x-1)(x-2)(x-3) > 0.$

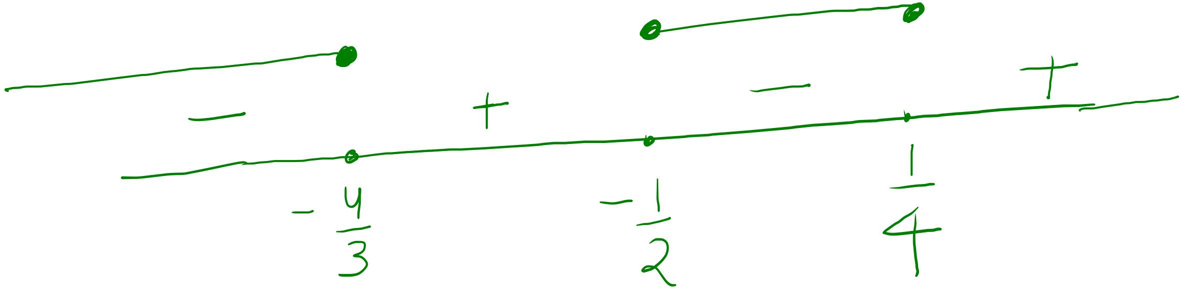


$x \in (1, 2) \cup (3, \infty)$

(2) $(2x+1)(3x+4)(1-4x) \geq 0.$

$2\left(x+\frac{1}{2}\right)3\left(x+\frac{4}{3}\right)(-4)\left(x-\frac{1}{4}\right) \geq 0.$

$\left(x+\frac{1}{2}\right)\left(x+\frac{4}{3}\right)\left(x-\frac{1}{4}\right) \leq 0.$

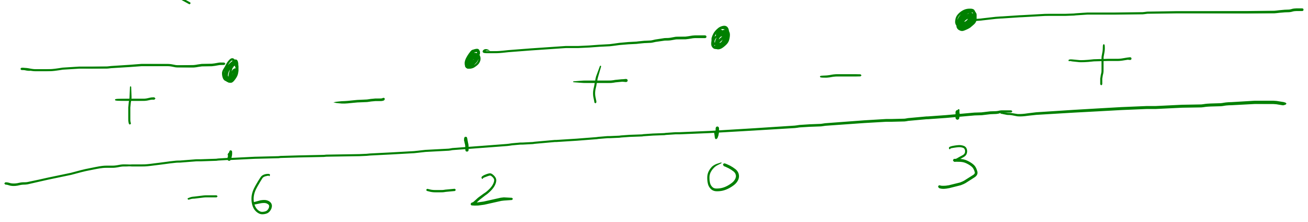


$x \in \left(-\infty, -\frac{4}{3}\right] \cup \left[-\frac{1}{2}, \frac{1}{4}\right]$

Ans

(3) $(x^2-x-6)(x^2+6x) \geq 0.$

$(x-3)(x+2)\,x\,(x+6) \geq 0.$

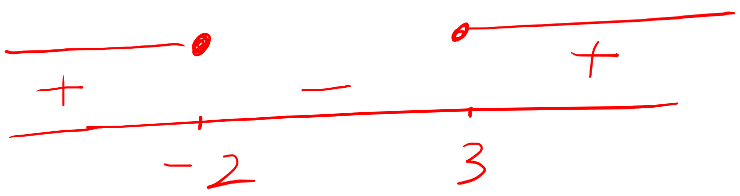


$x \in (-\infty,-6] \cup [-2,0] \cup [3,\infty)$

(4) $\underbrace{(x^2-x-6)}(x^2+x+1) \geq 0.$

a>0 and D<0 (always +ve)

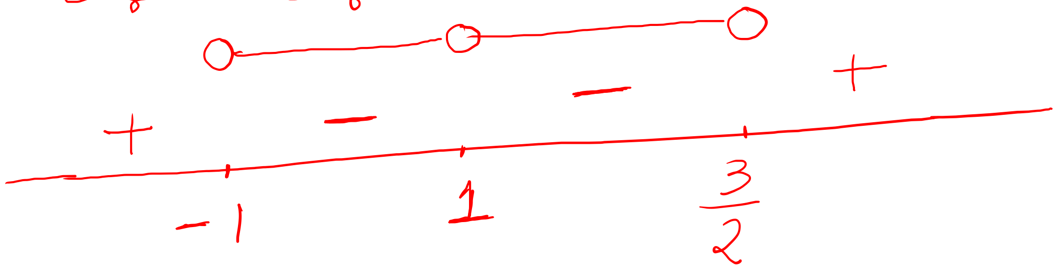
$(x-3)(x+2) \geq 0$



$x \in (-\infty,-2] \cup [3,\infty)$

(5) $\underbrace{(x-1)^{2}}\,\underbrace{(x+1)^{3}}\,\underbrace{(2x-3)^{5}} < 0.$

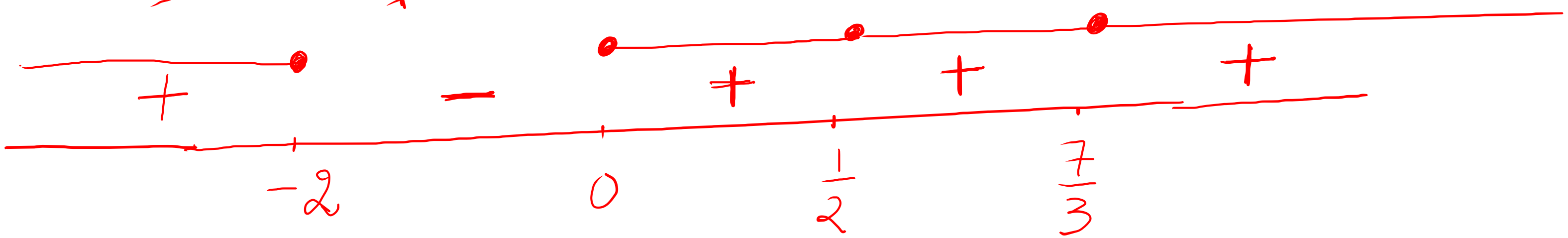
even



$x \in (-1,1) \cup (1, 3/2)$ OR $x \in \left(-1, \frac{3}{2}\right) - \{1\}$

(6)

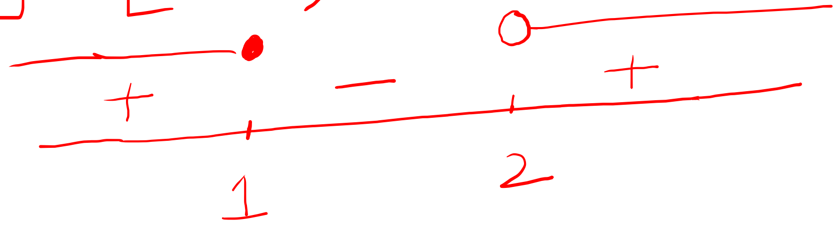
$$(x+2)^{101}(x)^{53}(2x-1)^{60}(3x-7)^{70} \geq 0.$$



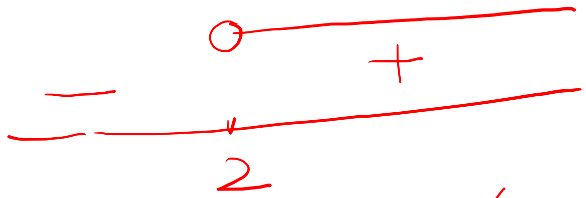
$$x \in (-\infty, -2] \cup [0, \infty).$$

(7)

$$\frac{x-1}{x-2} \geq 0$$



$$x \in (-\infty, 1] \cup (2, \infty)$$



(8)

$$\frac{x-1}{x-2} \geq 1. \Rightarrow \frac{x-1}{x-2} - 1 \geq 0.$$

$$x \in (2, \infty)$$

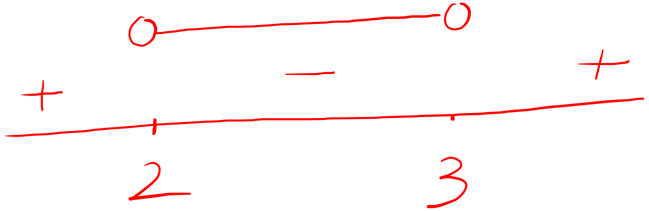
Don't cross-multiply :- $\dfrac{(x-1)-(x-2)}{x-2} \geq 0 \Rightarrow \dfrac{1}{x-2} \geq 0$

Q $$\underbrace{\frac{x^2-5x+6}{x^2+x+1}}_{}<0 \Rightarrow x^2-5x+6<0.$$

$$\underbrace{x^2+x+1}_{\text{always +ve}}$$

$a>0$ & $D<0$

$$(x-2)(x-3)<0.$$



$$x\in(2,3).\ \text{Ans}$$

Q $$\frac{x+1}{x-1}\geq\frac{x+5}{x+1}$$

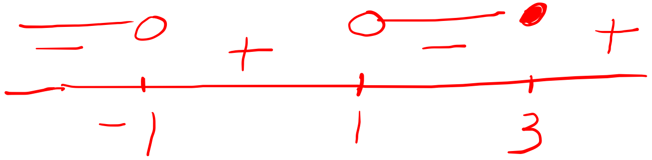
$$\frac{x+1}{x-1}-\frac{x+5}{x+1}\geq 0$$

$$\frac{(x+1)^2-(x-1)(x+5)}{(x-1)(x+1)}\geq 0.$$

$$\frac{\cancel{x^2}+2x+1-(\cancel{x^2}+4x-5)}{(x-1)(x+1)}\geq 0$$

$$\frac{-2x+6}{(x-1)(x+1)}\geq 0 \Rightarrow \frac{-2(x-3)}{(x-1)(x+1)}\geq 0.$$

$$\frac{(x-3)}{(x-1)(x+1)}\leq 0.$$



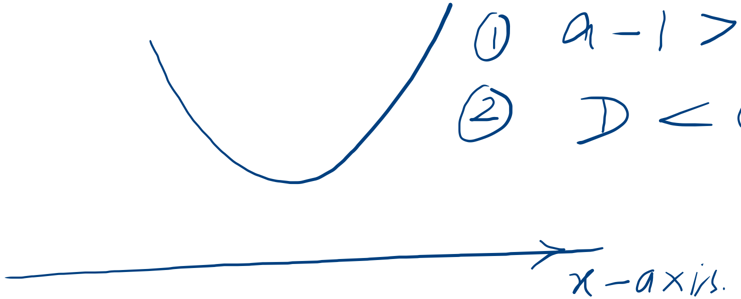
$$x\in(-\infty,-1)\cup(1,3]\ \text{Ans}$$

Q. Find the set of value(s) of 'a' for which the quadratic inequality

$$(a-1)x^2-(a-1)x+(a+1)>0 \quad \forall x\in \mathbb{R}.$$

Sol^n $\boxed{a-1\neq 0}$ *

$$y=(a-1)x^2-(a-1)x+(a+1).$$



$\left.\begin{array}{l} ① \ a-1>0 \\ ② \ D<0 \end{array}\right\} \cap$

① $a-1>0 \Rightarrow a>1$ — ① —

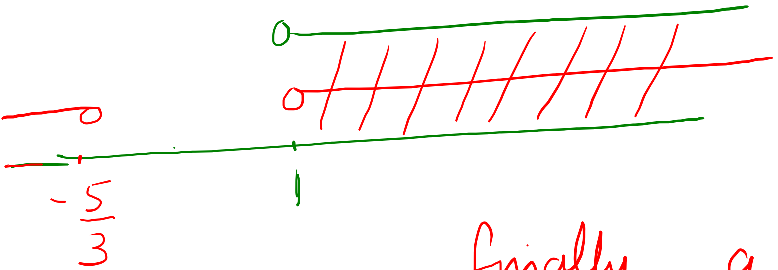
② $D<0 \Rightarrow (a-1)^2-4(a-1)(a+1)<0.$

$$(a-1)\left(a-1-4(a+1)\right)<0.$$

$$(a-1)(-3a-5)<0.$$

$$(a-1)(3a+5)>0. \quad —②—$$

① ∩ ②.



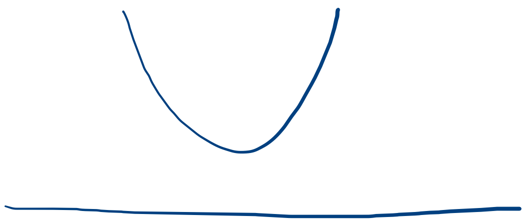
finally $a\in(1,\infty)$ Ans

Q2 Find the set of value(s) of 'a' for which inequality

$$(a-1)x^2-(a-1)x+(a+1)>0 \quad \forall x \in R \ ?$$

Sol^n

C – I $\qquad a-1 \neq 0$



same as previous problem

$$\boxed{a \in (1,\infty)}$$

C – II $\qquad a-1=0 \Rightarrow \boxed{a=1}$

$$\underbrace{(1-1)}x^2-\underbrace{(1-1)}x+2>0 \quad \text{which is } \underline{\text{TRUE}}$$

$\therefore \boxed{a=1}$ is also acceptable

from C – I & C – II

$$a \in [1,\infty) \quad \text{Ans}$$

Q. If the equation $x^2+16y^2-3x+2=0$ is satisfied by real value(s) of x and y then find range of x & y ?

Sol^n

$$x^2+16y^2-3x+2=0.$$

$$x^2-3x+(16y^2+2)=0$$

Since $x \in \mathbb{R}$

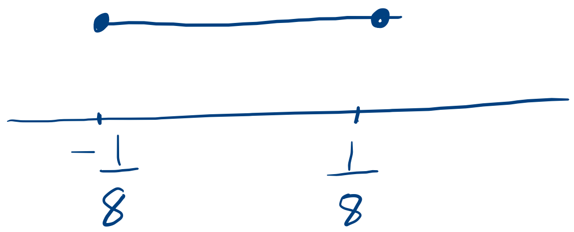
$$D \geq 0.$$

$$9-4(1)(16y^2+2) \geq 0.$$

$$9-64y^2-8 \geq 0.$$

$$64y^2-1 \leq 0.$$

$$(8y-1)(8y+1) \leq 0$$



$y \in \left[-\frac{1}{8}, \frac{1}{8}\right]$ Ans

$$16y^2+0y+(x^2-3x+2)=0$$

Since $y \in \mathbb{R}$; $D \geq 0.$

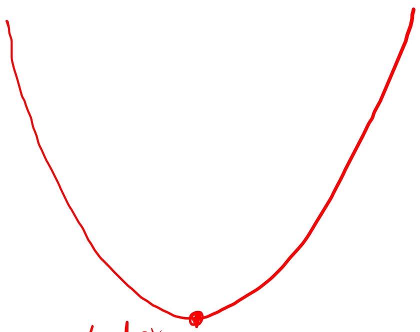
$$0-4.16\,(x^2-3x+2) \geq 0.$$

$$x^2-3x+2 \leq 0$$

$x \in [1,2]$ Ans

Q Let $P(x) = ax^2 + bx + 8$ is a quadratic poly. If the minimum value of $P(x)$ is 6 when $x = 2$ then find $a+b$?

Sol^n



Vertex $(x=2,\ y=6)$

vertex :

$$x = \frac{-b}{2a} = 2$$

$$b = -4a \quad -① -$$

$$y = \frac{-D}{4a} = 6$$

$$\frac{-(b^2 - 32a)}{4a} = 6$$

$$-b^2 + 32a = 24a$$

$$b^2 = 8a \quad -② -$$

From ① & ②

$$\boxed{a = 1/2 \ ; \ b = -2}$$

$$\boxed{a + b = -\frac{3}{2}}$$ Ans

Note

Find range of $y = \dfrac{\text{Quad poly}_1}{\text{Quad poly}_2}$.

(1) If x is real then find range of $y = \dfrac{x^2+14x+9}{x^2+2x+3}$?

Sol^n

$$y = \frac{x^2+14x+9}{\underbrace{x^2+2x+3}_{D<0}}.$$

Check for common factors in N^r & D^r.

No common factor

$$x^2y + 2xy + 3y - x^2 - 14x - 9 = 0.$$

$$x^2(y-1) + (2y-14)x + 3y - 9 = 0.$$

C-I $\boxed{y-1 \neq 0}$ and $x \in R \Rightarrow D \geq 0 \Rightarrow (2y-14)^2 - 4(y-1)3(y-3) \geq 0.$

$$\cancel{4}(y^2+49-14y) - \cancel{4}(3(y^2-4y+3)) \geq 0$$

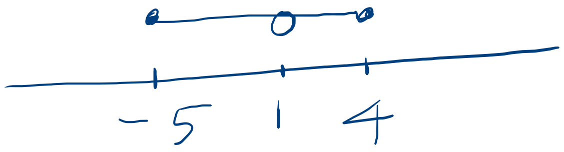
$$-2y^2 - 2y + 40 \geq 0 \Rightarrow y^2+y-20 \leq 0$$

$$y^2+y-20 \le 0$$

$$y^2+5y-4y-20 \le 0 \Rightarrow (y+5)(y-4) \le 0 \Rightarrow y \in [-5,4]-\{1\}$$

— 1 —

**



C-II

$$y-1=0 \Rightarrow y=1$$ *

$$y = \frac{x^2+14x+9}{x^2+2x+3}$$

$$1 = \frac{x^2+14x+9}{x^2+2x+3} \Rightarrow \cancel{x^2}+2x+3 = \cancel{x^2}+14x+9$$

$$12x = -6$$

$$x = -\frac{1}{2} \quad \text{which is real.}$$

$\therefore$ $y=1$ is acceptable — 2 —

C-I Union C-II

finally $y \in [-5,4]$ Ans

Range of $y = \dfrac{\text{Linear poly}_1}{\text{Linear poly}_2}$.

① $y = \dfrac{x-2}{2x+1} \rightarrow y \in R - \left\{\frac{1}{2}\right\}$

Sol^n $\quad x \neq -\frac{1}{2}$

$x \in R - \left\{-\frac{1}{2}\right\}$

$2xy + y = x - 2$

$2xy - x = -y - 2$

$\boxed{x = \dfrac{-y-2}{2y-1}} \rightarrow y \in R - \left\{\frac{1}{2}\right\}$

② $y = \dfrac{3x+1}{7x-4}$

$y \in R - \left\{3/7\right\}$

③ $y = \dfrac{x+1}{3x+3}$.

$y \in \left\{\frac{1}{3}\right\}$

Q3 If $x \in R$ then find range of $y = \dfrac{x^2 - 7x + 10}{x^2 - 9x + 14}$?

Sol^n

$$\boxed{y = \frac{(x-2)(x-5)}{(x-2)(x-7)}} \; ; \; \boxed{x \neq 2 ; 7}$$

common factor

$$y = \frac{x-5}{\underbrace{x-7}_{x \neq 7}} \rightarrow \cancel{y \in R - \{1\}}$$

Since $x \neq 2$ therefore $y \neq \dfrac{2-5}{2-7} \Rightarrow y \neq \dfrac{3}{5}$

finally $y \in R - \left\{1, \dfrac{3}{5}\right\}$ Ans

Q If $x \in R$ then find range of

$$y = \frac{x^2+2x-11}{2(x-3)}.$$

Sol^n

$$y = \frac{x^2+2x-11}{2(x-3)}$$

"no common factor"

$$2xy - 6y = x^2 + 2x - 11.$$

$$x^2 + (2-2y)x + 6y - 11 = 0$$

Since $x \in R$ $\therefore D \geq 0.$

$$(2-2y)^2 - 4(6y-11) \geq 0.$$

$$(1+y^2-2y) - 6y + 11 \geq 0$$

$$y^2 - 8y + 12 \geq 0$$

$$(y-2)(y-6) \geq 0$$

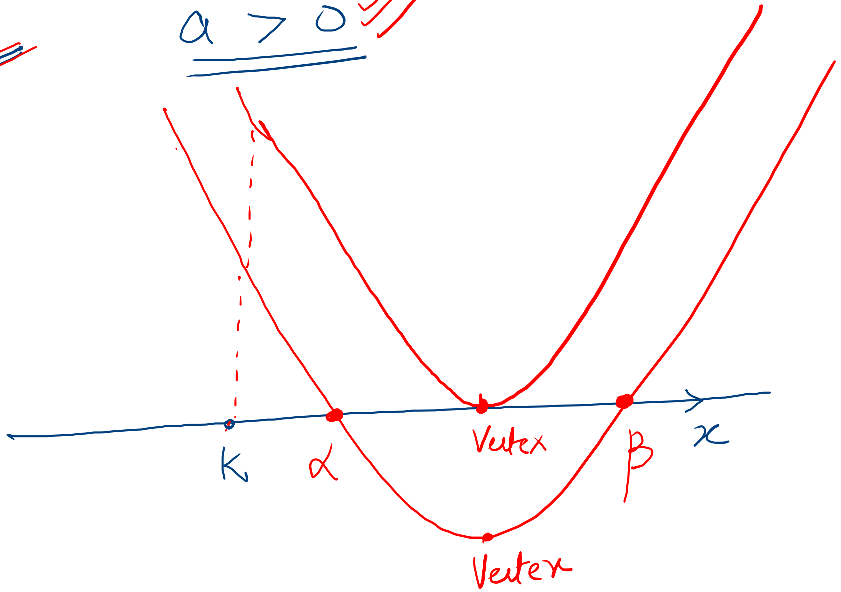
$y \in (-\infty, 2] \cup [6, \infty)$ Ans

Location of Roots :-

(I) Both roots of quadratic equation $ax^2+bx+c=0$ are greater than a specified number say 'K'.

$$\boxed{f(x) = ax^2+bx+c}$$

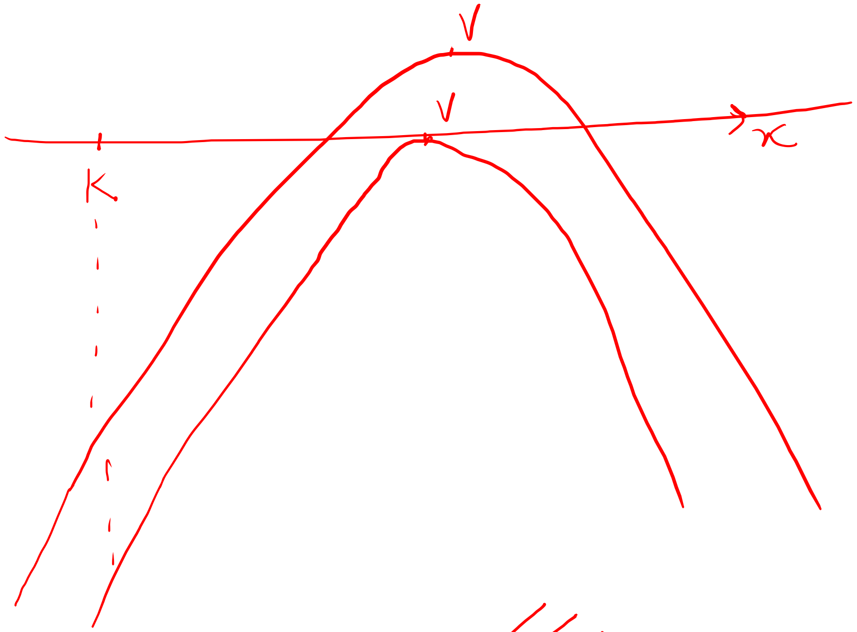
C–I $a>0$



Necessary & Sufficient condition

$$\left.\begin{array}{l} ① \ D \geq 0. \\ ② \ \dfrac{-b}{2a} > K. \\ ③ \ f(K) > 0. \end{array}\right\} \Omega$$

C-II $a < 0$



① $D \geq 0$
② $\frac{-b}{2a} > K$
③ $f(K) < 0$

Ω

Combining 2 cases :-

① $D \geq 0$
② $\frac{-b}{2a} > K$
③ $a\, f(K) > 0$

Ω

Best Method :-

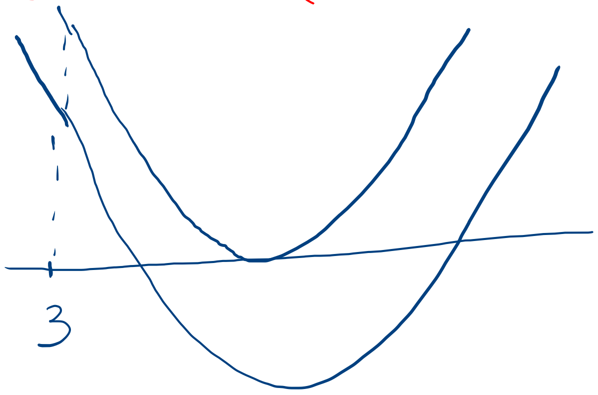
divide by coeff of x^2

(provided quadratic is mention).

Q Find all value(s) of 'K' for which both roots of equation

$x^2 - 6Kx + (2 - 2K + 9K^2) = 0$ exceed '3' ?

Sol^n



$f(x) = x^2 - 6Kx + (2 - 2K + 9K^2)$

vertex: $x = \frac{6K}{2} = 3K$.

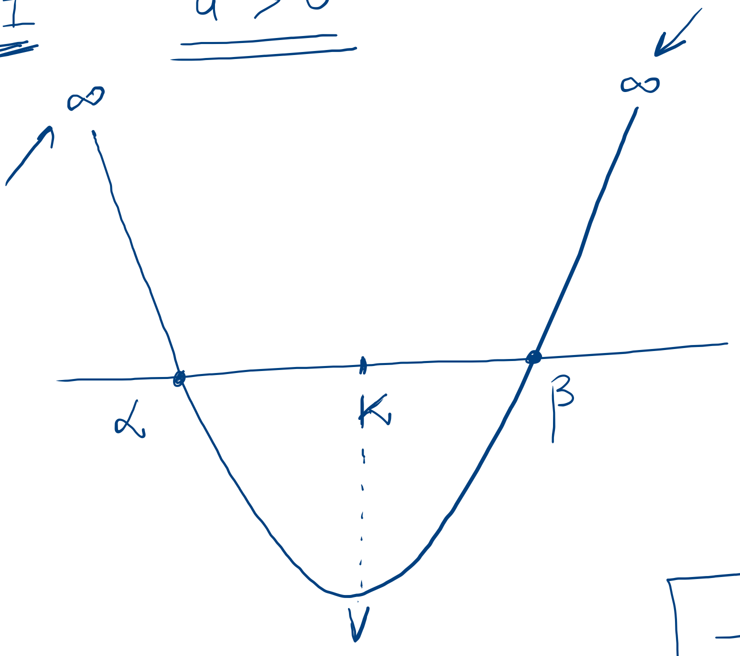
$$\left.\begin{array}{l} ① \ D \geq 0. \\ * ② \ 3K > 3. \\ ③ \ f(3) > 0. \end{array}\right\} \Omega$$

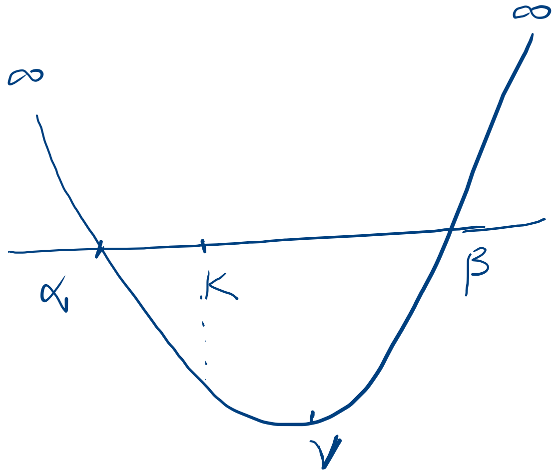
finally $K \in \left(\frac{11}{9}, \infty\right)$ Ans

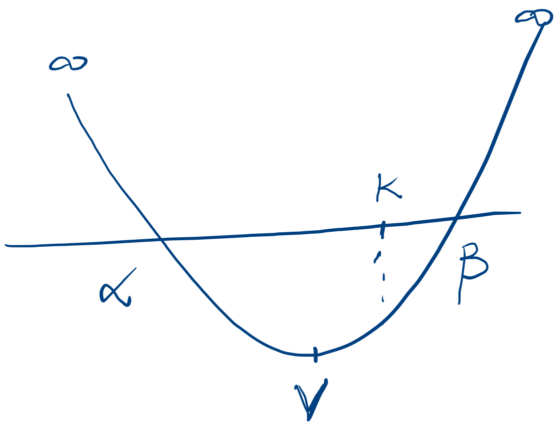
T-2 Both roots lie on either side of fixed number 'K'.

$$f(x) = ax^2 + bx + c$$

C-I $a > 0$





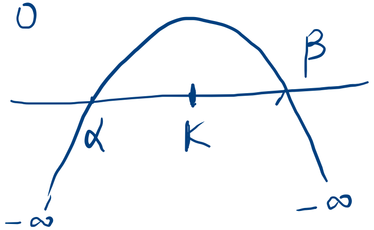


$$\boxed{f(K) < 0}$$

Combining 2 cases:

$$\boxed{a.f(K) < 0}$$

C-II $a < 0$

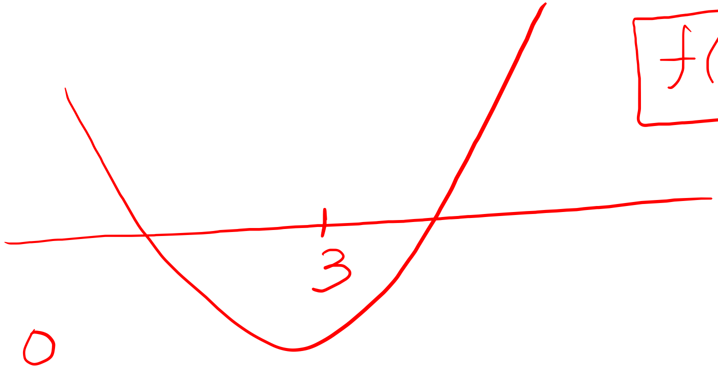


$$\boxed{f(K) > 0}$$

Q. Find the set of value(s) of 'a' for which zeroes of quadratic polynomial $(a^2+a+1)x^2+(a-1)x+a^2$ are located on either side of '3'.

Sol^n $f(x) = (\underbrace{a^2+a+1}_{\text{always +ve}})x^2 + (a-1)x + a^2$.

$D < 0$ & leading coeff $= 1$



$\boxed{f(3) < 0}$

$f(3) = 9(a^2+a+1) + 3(a-1) + a^2 < 0$

$= 10a^2 + 12a + 6 < 0.$

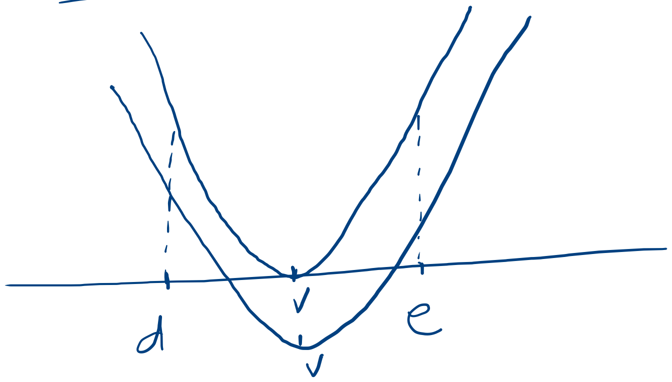
$\Rightarrow \underbrace{5a^2 + 6a + 3}_{\text{always +ve}} < 0 \Rightarrow a \in \phi.$

$a > 0$ & $D < 0$.

T-3 When both roots are confined between 'd' & 'e' (where $d<e$)

$f(x) = ax^2+bx+c$

C-I $a>0$



① $D \geq 0$

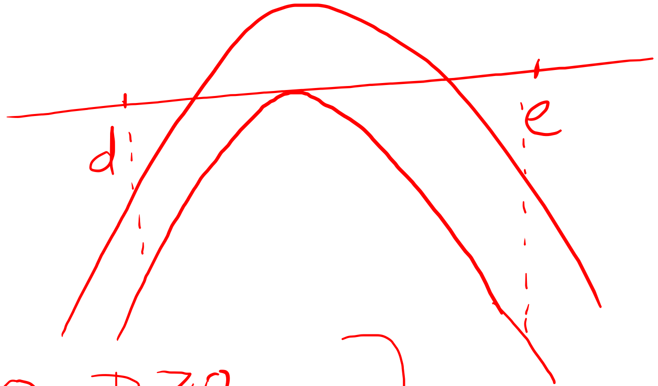
② $d < \frac{-b}{2a} < e$

③ $f(d) > 0$

④ $f(e) > 0$

Ω

C-II $a<0$



① $D \geq 0$

② $d < -\frac{b}{2a} < e$

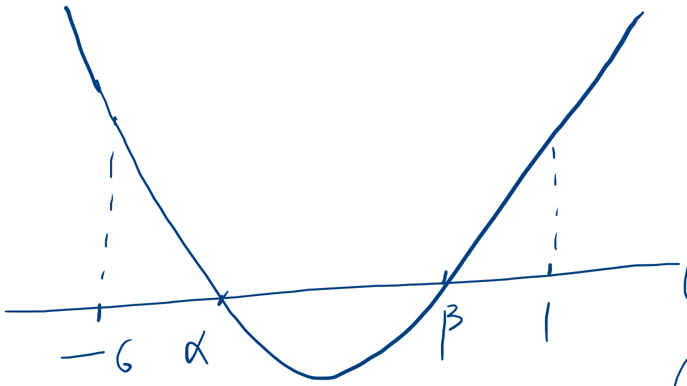
③ $f(d) < 0$

④ $f(e) < 0$

Ω

Q If α, β are the roots of quadratic equation $x^2+2(k-3)x+9=0$ $(\alpha \neq \beta)$. and $\alpha, \beta \in (-6, 1)$ then find k ?

Soln



$$f(x) = x^2 + 2(k-3)x + 9$$

Vertex :

$$x = \frac{-2(k-3)}{2}$$

① $D > 0$

② $-6 < -(k-3) < 1$

③ $f(-6) > 0$

④ $f(1) > 0.$

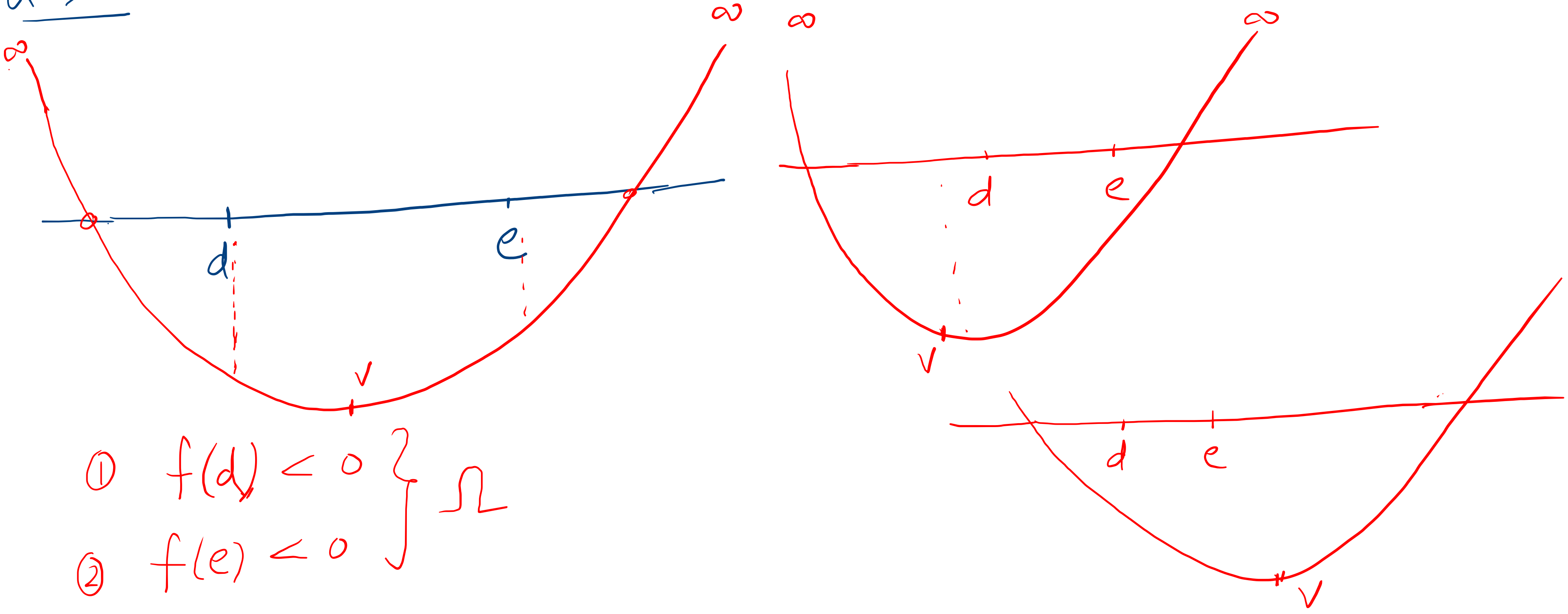
Ω

finally $k \in \left(6, \frac{27}{4}\right)$ Ans

T-4 One root greater than 'e' and other root is less than 'd' where $d<e$

$$f(x) = ax^2+bx+c.$$

C-I $a>0$

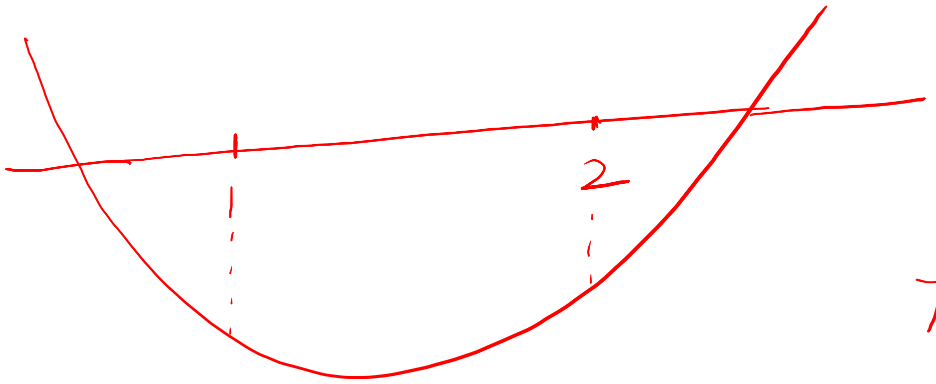


① $f(d)<0$
② $f(e)<0$
$\Big\}\ \Omega$

Q. Find 'k' for which one root of QE $(k-5)x^2 - 2kx + k-4 = 0$ is smaller than 1 and other root exceed '2'?

Sol^n

$$f(x) = x^2 - \left(\frac{2k}{k-5}\right)x + \left(\frac{k-4}{k-5}\right)$$



$$\left.\begin{array}{l} f(1) < 0 \\ f(2) < 0 \end{array}\right\} \Omega$$

finally $k \in (5, 24)$

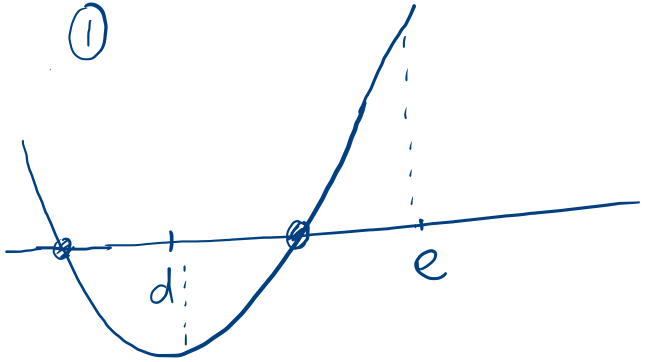
Ans

T-5 **Exactly one root lies in (d, e)**

$f(x) = ax^2 + bx + c$.

C-I $\quad a > 0$

1



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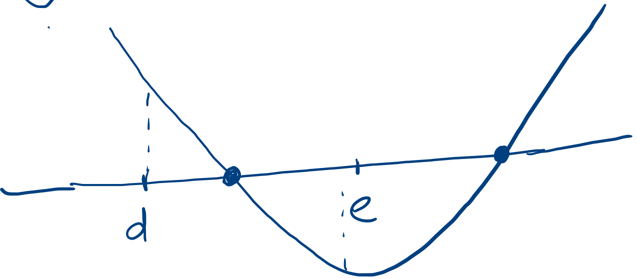


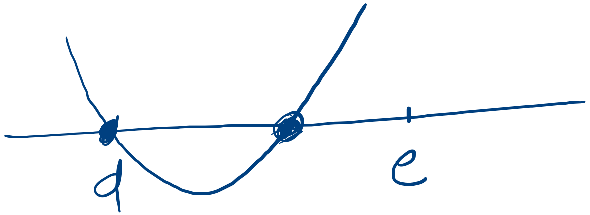
$$\boxed{f(d)\cdot f(e) < 0}$$

** *

CHECK

For "kahi yeh won toh Nahi"

3



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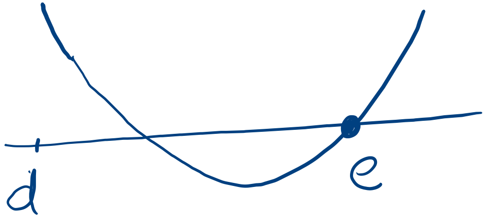
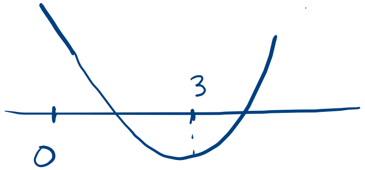
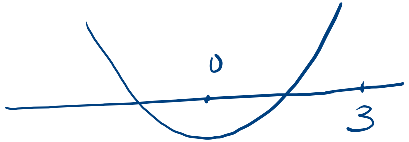


Q. Find all the possible value(s) of 'a' for which exactly one root of QE $x^2-(a+1)x+2a=0$ lie in $(0,3)$?

Sol^n $\boxed{f(x)=x^2-(a+1)x+2a}$





$\boxed{f(0)=2a}$

$f(3)=9-3a-3+2a$

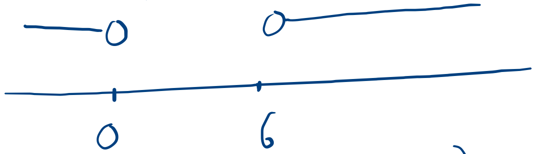
$\boxed{f(3)=(6-a)}$

$f(0)\cdot f(3)<0$.

$2a(6-a)<0$.

$-2a(a-6)<0$

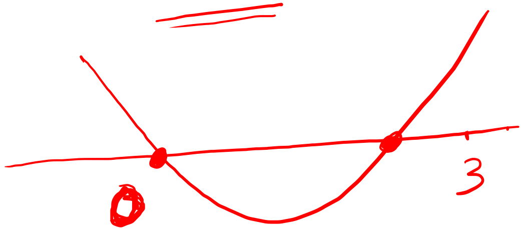
$a(a-6)>0$.



$a\in(-\infty,0)\cup(6,\infty)$ — ①

NOW CHECK !!!

③



$f(0)=0 \Rightarrow 2a=0 \Rightarrow \boxed{a=0}$ ✓

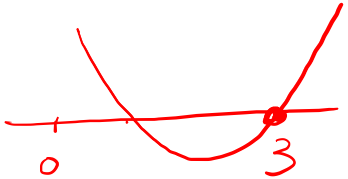
QE when $a=0$ ✓

$x^2-x=0$ ✓

$\boxed{x=0;1}$

$\therefore$ $a=0$ is acceptable

④



$f(3)=0 \Rightarrow 6-a=0$

$\boxed{a=6}$

QE when $a=6$.

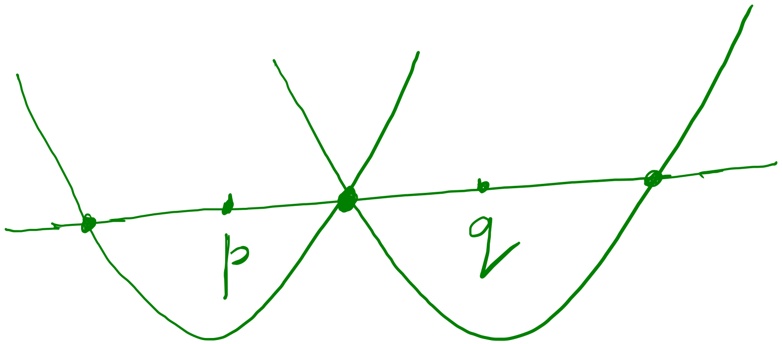
$x^2-7x+12=0$

$x=3,4$ ✗✗

Finally

$a\in(-\infty,0]\cup(6,\infty)$ Ans

Note :- For $y = f(x) = ax^2 + bx + c$ if there exist 'p' and 'q' $\in R$ such that $f(p)$ and $f(q)$ are of opposite sign ie $\boxed{f(p) \cdot f(q) < 0}$ then equation $ax^2 + bx + c = 0$ has one root lying between p and q.



Q1 The coefficients of QE : $ax^2+bx+c=0$ $(a \neq 0)$ satisfy the inequality $(a+b+c)(4a-2b+c) < 0$ then Comment upon nature of roots of QE?

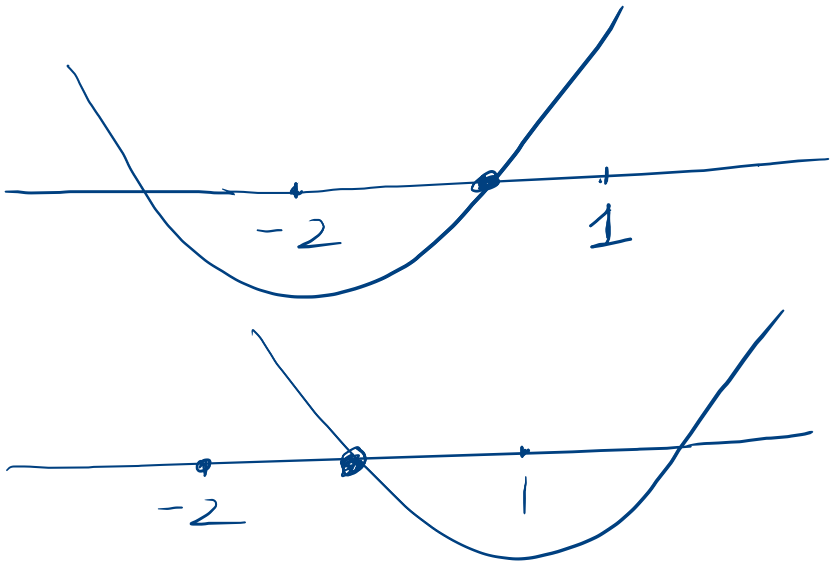
Solⁿ

$$f(x) = ax^2+bx+c.$$

$$f(1) = a+b+c.$$

$$f(-2) = 4a-2b+c.$$

$$f(1) \cdot f(-2) < 0 \quad \text{(Given)}$$

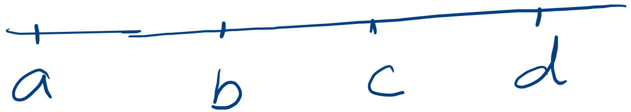


Real & distinct roots.

Q2 If $a < b < c < d$, then show that QE
$(x-a)(x-c)+\lambda(x-b)(x-d)=0$ has real roots for all real values of λ?

Sol^n

$$f(x) = (x-a)(x-c) + \underline{\underline{\lambda}}\,\underbrace{(x-b)}\underbrace{(x-d)}.$$



$$f(b) = \underbrace{(b-a)}_{+}\underbrace{(b-c)}_{-} < 0.$$

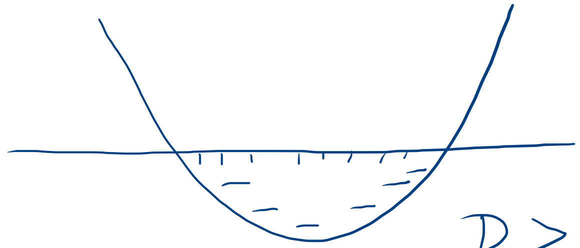
$$f(d) = \underbrace{(d-a)}_{+}\underbrace{(d-c)}_{+} > 0.$$

$$\boxed{f(b)\cdot f(d) < 0.}$$

Q Find 'a' for which the inequality $(a^2+3)x^2 + (\underbrace{\sqrt{5a+3}})x - \frac{1}{4} < 0$ is satisfied for <u>atleast one real x</u> ?

Sol^n $a^2+3 > 0$ (always)

$$f(x) = (a^2+3)x^2 + (\sqrt{5a+3})x - \frac{1}{4}$$



D > 0

$$D = 5a+3 - 4(a^2+3)\left(-\frac{1}{4}\right)$$

$$\left.\begin{array}{l} D > 0 \quad \text{—①—} \\ 5a+3 \geq 0 \quad \text{—②—} \end{array}\right\} \cap$$

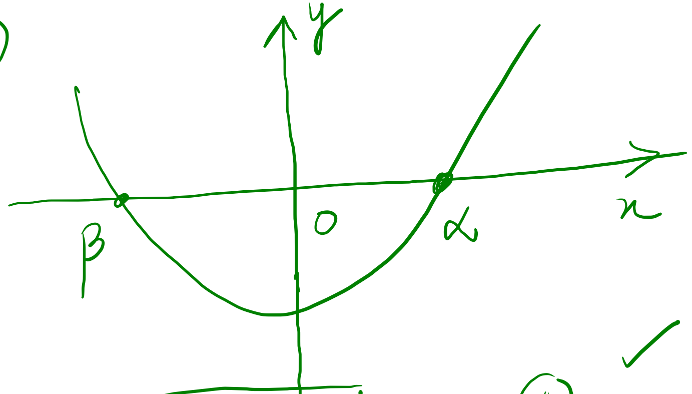
finally $a \in \left[-\frac{3}{5}, \infty\right)$ Ans

Q. Find the value(s) of 'a' for which the equation $x^2+2(a-1)x+a+5=0$ has atleast one positive root?

Sol^n

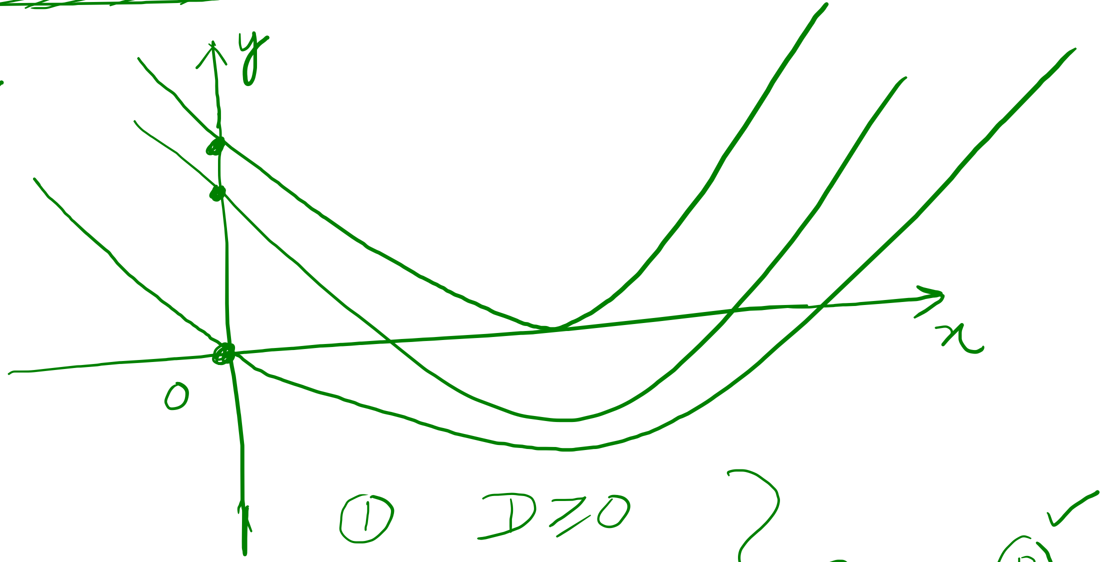
$$f(x) = x^2+2(a-1)x+a+5.$$

(I)



$\boxed{f(0) < 0}$ — (A) —

(II)



$$\left.\begin{array}{ll} ① & D \geq 0 \\ ② & \dfrac{-b}{2a} > 0 \\ ③ & f(0) \geq 0. \end{array}\right\} \cap \text{ — (B) —}$$

(A) Union (B) $\longrightarrow a \in (-\infty, -1]$ Ans

Theory of equations :-

$$ax^3+bx^2+cx+d=0 \begin{matrix}\alpha\\ \beta\\ \gamma\end{matrix}$$

α, β, γ

$$ax^3+bx^2+cx+d = a(x-\alpha)(x-\beta)(x-\gamma).$$

$$ax^3+bx^2+cx+d = a\left(x^3 - x^2(\alpha+\beta+\gamma) + x(\alpha\beta+\beta\gamma+\gamma\alpha) - \alpha\beta\gamma\right)$$

Compare coeff of x^2 both sides :

$$b = -a(\alpha+\beta+\gamma) \Rightarrow \underbrace{\alpha+\beta+\gamma}_{\text{Sum of roots taken one at a time.}} = \boxed{\Sigma\alpha = -\frac{b}{a} = -\frac{\text{Coeff of } x^2}{\text{Coeff of } x^3}}$$

Coeff of x :

$$c = a(\alpha\beta+\beta\gamma+\gamma\alpha) \Rightarrow \underbrace{\overbrace{\alpha\beta+\beta\gamma+\gamma\alpha}^{\Sigma\alpha\beta}}_{\text{Sum of roots taken two at a time}} = \frac{c}{a} = \frac{\text{Coeff of } x}{\text{Coeff of } x^3}$$

Coeff of Constant term :

$$d = -a(\alpha\beta\gamma) \Rightarrow \underbrace{\alpha\beta\gamma}_{\text{P.O.R}} = -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coeff of } x^3}$$

P.O.R

OR

Sum of roots taken all at a time.

General :

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0 = 0$$

$x_1, x_2, \ldots, x_n$

$$\Sigma x_1 = -\left(\frac{a_{n-1}}{a_n}\right)$$

$$\Sigma x_1 x_2 = \left(\frac{a_{n-2}}{a_n}\right)$$

$$\Sigma x_1 x_2 x_3 = -\left(\frac{a_{n-3}}{a_n}\right)$$

$$x_1 x_2 \ldots x_n = (-1)^n \cdot \frac{a_0}{a_n}$$

eg: $3x^3 + x - 1 = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$

$3x^3 + 0x^2 + x - 1 = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$

$\sum \alpha = -\frac{0}{3}$; $\sum \alpha\beta = \frac{1}{3}$

$\alpha\beta\gamma = \frac{-(-1)}{3}$

eg: $7x^5 - 4x^2 + 5 = 0 \begin{cases} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{cases}$

$7x^5 + 0x^4 + 0x^3 - 4x^2 + 0x + 5 = 0.$

$\sum x_1 = -\frac{0}{7}$; $\sum x_1 x_2 = \frac{0}{7}$

$\sum x_1 x_2 x_3 = -\frac{(-4)}{7}$; $\sum x_1 x_2 x_3 x_4 = \frac{0}{7}$

$x_1 x_2 x_3 x_4 x_5 = \frac{-(5)}{7}$

Q.1 Find ① sum of squares
② sum of cubes of the roots of cubic equation $x^3 - px^2 + qx - r = 0$.

Sol^n $x^3 - px^2 + qx - r = 0$ $\begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}$ $\Sigma\alpha = p$; $\Sigma\alpha\beta = q$; $\alpha\beta\gamma = r$.

① $\alpha^2 + \beta^2 + \gamma^2 = ?$

$$(\alpha + \beta + \gamma)^2 = \underbrace{\alpha^2 + \beta^2 + \gamma^2}_{E} + 2\underbrace{(\alpha\beta + \beta\gamma + \gamma\alpha)}_{q}$$

$$\boxed{p^2 - 2q = \Sigma\alpha^2}$$

② $\alpha^3 + \beta^3 + \gamma^3 = ?$

$$\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = (\alpha + \beta + \gamma)\underbrace{(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha)}$$ ①

$$= (\alpha + \beta + \gamma)\left((\alpha + \beta + \gamma)^2 - 3(\alpha\beta + \beta\gamma + \gamma\alpha)\right).$$ ②

$$= \left(\frac{\alpha + \beta + \gamma}{2}\right)\underbrace{\left(\underbrace{(\alpha - \beta)}^2 + \underbrace{(\beta - \gamma)}^2 + \underbrace{(\gamma - \alpha)}^2\right)}.$$ ③

Note: $\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha=0. \Rightarrow \boxed{\alpha=\beta=\gamma}$

$$\frac{1}{2}\left(\underbrace{(\alpha-\beta)^2}_{0}+\underbrace{(\beta-\gamma)^2}+\underbrace{(\gamma-\alpha)^2}\right)=0$$

$$\alpha^3+\beta^3+\gamma^3=3\alpha\beta\gamma+(\alpha+\beta+\gamma)(\underbrace{\alpha^2+\beta^2+\gamma^2}_{0}-\alpha\beta-\beta\gamma-\gamma\alpha).$$

$$=3r+(p)(p^2-2q-q)$$

$$=p^3-3pq+3r.$$

Q2 If a, b, c are roots of cubic $x^3 - x^2 + 1 = 0$ then find value of $(a^{-2} + b^{-2} + c^{-2}) = ?$

Sol^n

$$x^3 - x^2 + 0x + 1 = 0 \quad a, b, c$$

$\Sigma a = 1 \qquad abc = -1.$

$\Sigma ab = 0$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = ? \qquad \frac{1}{a} = A; \quad \frac{1}{b} = B; \quad \frac{1}{c} = C.$$

$$A^2 + B^2 + C^2 = (A+B+C)^2 - 2(AB + BC + CA)$$

$$= \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)^2 - 2\left(\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}\right)$$

$$= \cancel{\left(\frac{\overset{0}{bc + ac + ab}}{abc}\right)^2} - 2\left(\frac{\overbrace{c + a + b}^{1}}{\underbrace{abc}_{-1}}\right) = -2\left(\frac{1}{-1}\right) = 2$$

Ans

Q3 Find cubic each of whose roots is greater by unity than root of equation

$x^3 - 5x^2 + 6x - 3 = 0$?

Sol^n $x^3 - 5x^2 + 6x - 3 = 0$ $\begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$ $\Sigma\alpha = 5$; $\Sigma\alpha\beta = 6$; $\alpha\beta\gamma = 3$.

$$x^3 - x^2(\underbrace{\alpha+\beta+\gamma}) + x(\underbrace{\alpha\beta+\beta\gamma+\gamma\alpha}) - \underbrace{\alpha\beta\gamma} = 0.$$

$x_1 + x_2 + x_3 = \Sigma\alpha + 3 = 8.$

Cubic eqn $= 0$ $\begin{cases} \alpha+1 = x_1 \\ \beta+1 = x_2 \\ \gamma+1 = x_3. \end{cases}$

$\Sigma x_1 x_2 = 19$

$x_1 x_2 x_3 = 15.$

$$x^3 - x^2(x_1+x_2+x_3) + x(x_1x_2 + x_2x_3 + x_3x_1) - x_1x_2x_3 = 0.$$

$$\boxed{x^3 - 8x^2 + 19x - 15 = 0}$$ Required cubic

$x^3 - 5x^2 + 6x - 3 = 0$. α, β, γ

$x = \alpha$

Cubic eqn $= 0$ — $\alpha+1$, $\beta+1$, $\gamma+1$

Transformation of roots

$$\boxed{x = \alpha + 1} \Rightarrow \boxed{\alpha = x - 1}$$

$$(x-1)^3 - 5(x-1)^2 + 6(x-1) - 3 = 0.$$

$$x^3 - 1 - 3x(x-1) - 5(x^2 - 2x + 1) + 6x - 6 - 3 = 0$$

$$\boxed{x^3 - 8x^2 + 19x - 15 = 0.}$$ Ans

** whose roots are double the roots of given cubic.

cubic eqn = 0 $\begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$

$x = \alpha$

My cubic eqn = 0 $\begin{cases} 2\alpha \\ 2\beta \\ 2\gamma \end{cases}$

$x = 2\alpha$

$\alpha = \frac{x}{2}$

Transformation

Q Form a cubic whose roots are the cubes of roots of $x^3 + 3x^2 + 2 = 0$?

Sol^n

$x^3 + 3x^2 + 2 = 0$ $\begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$

cubic eqn = 0 $\begin{cases} \alpha^3 \\ \beta^3 \\ \gamma^3 \end{cases}$

$x = \alpha^3 \Rightarrow \alpha = (x)^{1/3}$

$(x^{1/3})^3 + 3(x^{1/3})^2 + 2 = 0.$

$x + 3x^{2/3} + 2 = 0.$

$(x+2)^3 = (-3x^{2/3})^3$

$x^3 + 8 + 6x(x+2) = -27x^2.$

Q. Consider biquadratic $x^4 - kx - 3 = 0$ then find the equation whose roots are $\alpha = \frac{a+b+c}{d^2}$, $\beta = \frac{a+b+d}{c^2}$, $\gamma = \frac{a+c+d}{b^2}$, $\delta = \frac{b+c+d}{a^2}$ where a, b, c, d are roots of given biquadratic equation?

Sol^n

$$x^4 + 0x^3 + 0x^2 - kx - 3 = 0 \quad a, b, c, d$$

$\Sigma a = 0.$

$\underbrace{a+b+c}+d = 0$

$$\alpha = \frac{\overbrace{a+b+c}^{e}}{d^2} = \frac{-d}{d^2} \Rightarrow \alpha = -1/d$$

$||^y$

$\beta = -1/c$

$\gamma = -1/b$

$\delta = -1/a$

$x \to -1/x$

$$\left(\frac{-1}{x}\right)^4 - k\left(\frac{-1}{x}\right) - 3 = 0$$

$$\boxed{3x^4 - kx^3 - 1 = 0}$$

Q. If the equation $x^3+10x^2+ax+b=0$. and $x^3+4x^2+ax+c=0$ have two roots in common then find product of uncommon roots of two equations?

Sol^n

$x^3+10x^2+ax+b=0$ — α, β, γ

$x^3+4x^2+ax+c=0$ — α, β, δ

$\boxed{\alpha+\beta+\gamma=-10}$

$\boxed{\alpha+\beta+\delta=-4}$

$\gamma\delta = ?$

$$\cancel{\alpha^3}+10\alpha^2+\cancel{a\alpha}+b=0.$$
$$\cancel{\alpha^3}+4\alpha^2+\cancel{a\alpha}+c=0.$$
$$- \quad - \quad - \quad -$$

$$6\alpha^2+(b-c)=0$$

$$6\alpha^2+0\alpha+b-c=0$$

$6x^2+0x+b-c=0$ — α, β

$\boxed{\alpha+\beta=0}$

$\gamma=-10$ & $\delta=-4$

$\boxed{\gamma\delta=40}$ Ans

Note

Condition under which $\boxed{ax^2+2hxy+by^2+2gx+2fy+c=0}$ Rem (general equation of degree-2) may be resolved as product of two linear factors is

$\Delta = 0$ where $\boxed{\Delta = abc + 2fgh - af^2 - bg^2 - ch^2.}$ Rem

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

Proof:

$$ax^2 + (2hy+2g)x + (by^2+2fy+c) = 0$$

QE in 'y'

$D = 0$

$$x = \frac{-\cancel{2}(hy+g) \pm \sqrt{\cancel{4}(hy+g)^2 - \cancel{4}\cdot a\cdot(by^2+2fy+c)}}{\cancel{2}a}$$

Q1 Prove that $2x^2+3xy+y^2+2y+3x+1=0$ can be <u>factorised</u> <u>into two linear factors</u> and find them?

$$\Big| \qquad \Big| = 0.$$

Soln

$$2x^2+3xy+y^2+2y+3x+1=0$$

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

$$a=2;\quad 2h=3;\quad b=1;\quad 2g=3;\quad 2f=2;\quad c=1.$$

$$\Delta = abc+2fgh-af^2-bg^2-ch^2$$

$$=(2)(1)(1)+\cancel{2}(1)\left(\frac{3}{\cancel{2}}\right)\left(\frac{3}{2}\right)-2(1)^2-1\left(\frac{3}{2}\right)^2-1\left(\frac{3}{2}\right)^2$$

$$=\cancel{2}+\frac{9}{2}-\cancel{2}\underbrace{-\frac{9}{4}-\frac{9}{4}}=0$$

$$2x^2 + (3y+3)x + y^2 + 2y + 1 = 0.$$

M-1

$$x = \frac{-(3y+3) \pm \sqrt{(3y+3)^2 - 4(2)(y^2+2y+1)}}{2(2)}$$

M-2

$$\underbrace{2x^2 + 3xy + y^2} + \underbrace{3x + 2y + 1} = 0.$$

$$2x^2 + 3xy + y^2 = 2x^2 + 2xy + xy + y^2$$

$$= 2x(x+y) + y(x+y).$$

$$= (2x+y)(x+y).$$

$$\underbrace{2x^2+3xy+y^2}+\underbrace{3x+2y+1} = (2x+y+c_1)(x+y+c_2)$$

Compare coeff of x:

$$3 = 2c_2 + c_1 \quad \text{—— (1) ——}$$

Compare coeff of y:

$$2 = c_2 + c_1 \quad \text{—— (2) ——}$$

Solve (1) & (2) $\quad c_1 = 1$ & $c_2 = 1$.

2 Linear factors : $\underbrace{(2x+y+1)}$ & $\underbrace{(x+y+1)}$

Q2 Find K for which the expression $y^2+2xy+2x+Ky-3=0$ can be resolved as product of 2 linear factors ?

Sol^n

$\boxed{\Delta = 0} \Rightarrow \boxed{K = -2}$ Ans

Note: If $f(\alpha,\beta) = f(\beta,\alpha)$ then $f(\alpha,\beta)$ denotes symmetric function of roots.

eg: $f(\alpha,\beta) = \alpha^2\beta + \beta^2\alpha$

$\alpha \to \beta$ & $\beta \to \alpha$

$f(\beta,\alpha) = \beta^2\cdot\alpha + \alpha^2\cdot\beta = f(\alpha,\beta)$ ✓✓

$\because \cos\theta = \cos(-\theta)$

eg: $f(\alpha,\beta) = \cos(\alpha-\beta)$

$f(\beta,\alpha) = \cos(\beta-\alpha) = f(\alpha,\beta)$ ✓✓

eg: $f(\alpha,\beta) = \sin(\alpha-\beta)$ ✗✗

$f(\beta,\alpha) = \sin(\beta-\alpha) \neq f(\alpha,\beta)$.

Note: For $ax^2+bx+c=0$; $a \neq 0$.

(1) If exactly one root of this QE is 0 then

$$ax^2+bx+c=0 \left< \begin{matrix} 0 \\ \alpha \end{matrix} \right.$$

$$P.O.R = 0 \Rightarrow \frac{c}{a} = 0 \Rightarrow \boxed{c=0}$$

(2) If both the roots of QE are 0 then

$$ax^2+bx+c=0 \left< \begin{matrix} 0 \\ 0 \end{matrix} \right.$$

$$P.O.R = 0 \Rightarrow \frac{c}{a} = 0 \Rightarrow$$

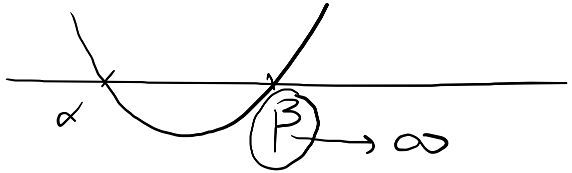
$$S.O.R = 0 \Rightarrow \frac{-b}{a} = 0 \Rightarrow$$

$$\boxed{\begin{matrix} c=0 \\ \& \\ b=0 \end{matrix}}$$

This QE will be $\boxed{x^2=0}$

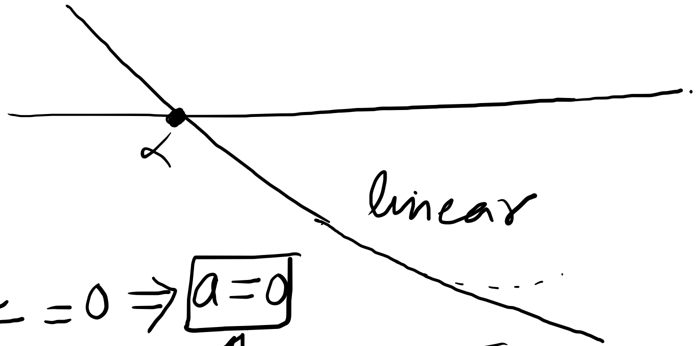
③ If exactly one root of QE is infinity

$$ax^2+bx+c=0 \quad \begin{cases} \alpha \\ \infty \end{cases}$$



$x \to \frac{1}{x}$

$$\frac{a}{x^2}+\frac{b}{x}+c=0 \Rightarrow cx^2+bx+a=0 \quad \begin{cases} \frac{1}{\alpha} \\ 0 \end{cases}$$



P.O.R $=0 \Rightarrow \frac{a}{c}=0 \Rightarrow \boxed{a=0}$

$\boxed{a \text{ tends } 0}$

but $\boxed{b \neq 0}$

④ If both roots of QE are infinite

$$ax^2+bx+c=0 \quad \begin{cases} \infty \\ \infty \end{cases}$$

$x \to \frac{1}{x}$

$$cx^2+bx+a=0 \quad \begin{cases} 0 \\ 0 \end{cases}$$

$\frac{-b}{c}=0$ & $\frac{a}{c}=0$

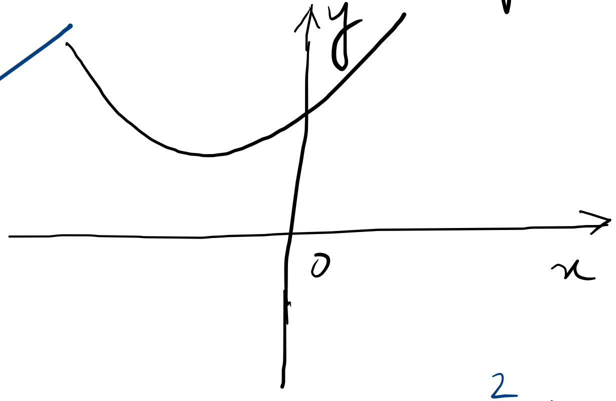
* $\boxed{b=a=0}$ but $c \neq 0$

Note :-

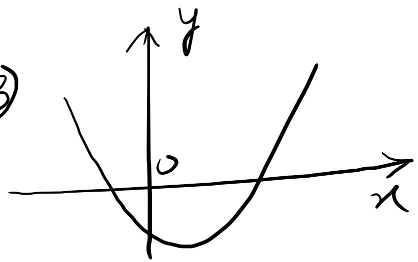
$$\left(x+\frac{1}{x}\right) \begin{cases} \leq -2 & \text{if } x<0 \text{. where equality holds when } \underline{\underline{x=-1}} \\ \geq 2 & \text{if } x>0 \quad \text{where equality holds when } \underline{\underline{x=1}} \end{cases}$$

Q Let $y=\left(\alpha+\frac{1}{\alpha}\right)x^2+x+3$ is given where $\alpha \in R-\{0\}$ be the graph of $y=f(x)$.
Then which of the following can

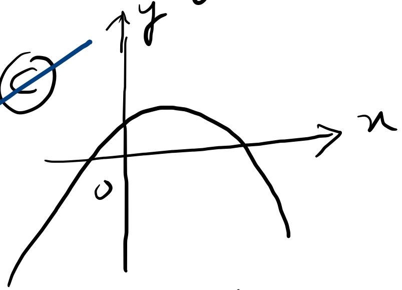
(A)



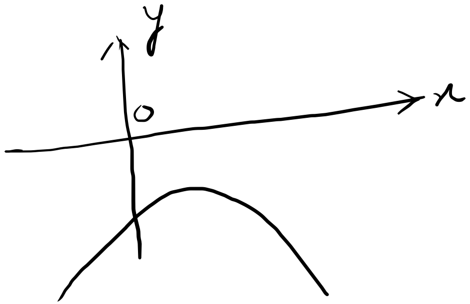
(B)



(C)



(D)



Sol^n

$$f(x)=\underbrace{\left(\alpha+\frac{1}{\alpha}\right)}\,x^2+x+3$$

C-I $\quad \alpha > 0$

$$\alpha + \frac{1}{\alpha} \geq 2$$

$$f(x) = \underbrace{\left(\alpha + \frac{1}{\alpha}\right)}_{\geq 2} x^2 + x + 3 \; ; \quad D = 1^2 - 4\underbrace{\left(\alpha + \frac{1}{\alpha}\right)}_{\geq 2}(3) < 0.$$

C-II $\quad \alpha < 0.$

$$\alpha + \frac{1}{\alpha} \leq -2$$

$$f(x) = \underbrace{\left(\alpha + \frac{1}{\alpha}\right)}_{\leq -2} x^2 + x + 3 \; ; \quad D = 1^2 - 4\underbrace{\left(\alpha + \frac{1}{\alpha}\right)}_{\leq -2} \cdot 3 > 0.$$

(A & C)

Q Let $P(x)$ be a polynomial of degree-4 with leading coefficient 1 and $P(1)=1$; $P(2)=3$; $P(3)=5$; $P(4)=7$ then find $P(5)$?

Sol^n $\cancel{P(5)=9}$

$$\left.\begin{array}{l} P(1)-1=0 \\ P(2)-3=0 \\ P(3)-5=0 \\ P(4)-7=0 \end{array}\right\} \Rightarrow \underline{\underline{P(x)-(2x-1)}}=0 \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array}$$

$$P(x)-(2x-1)=k(x-1)(x-2)(x-3)(x-4)$$

$$P(x)=\underbrace{k}_{1}(x-1)(x-2)(x-3)(x-4)+(2x-1)$$

$$P(x)=(x-1)(x-2)(x-3)(x-4)+(2x-1)$$

$$P(5)=4.3.2.1+9=33$$

Ans

eg:

$$\left.\begin{array}{l} P(1) = 1 \\ P(2) = 4 \\ P(3) = 9 \\ P(4) = 16. \end{array}\right\}$$

$$P(x) - x^2 = k\,(x-1)(x-2)(x-3)(x-4).$$

eg:

$$\left.\begin{array}{l} P(1) = 2 \\ P(2) = 5 \\ P(3) = 10. \\ P(4) = 17. \end{array}\right\}$$

$$P(x) - (x^2+1) = k\,(x-1)(x-2)(x-3)(x-4).$$

Sequence & Progression

Fredric Karl Gauss

Find sum of 1^{st} 100 natural nos ?

$$\text{my sum} = 1 + 2 + 3 + \cdots + 99 + 100.$$

$$\text{my sum} = 100 + 99 + 98 + \cdots + 2 + 1.$$

$$2(\text{my sum}) = 101 + 101 + 101 + \cdots + 101 + 101$$

$$\text{my sum} = \frac{101 \times 100}{2} = 5050. \checkmark$$

Sequence: A succession of terms which may be algebraic, real or complex nos written according to definite rule is sequence

$2, 3, 5, 7, 11, \ldots\ldots$ ✓

$1, -1, 1, -1, \ldots\ldots$ ✓

Progression :- Special case of sequence in which it is possible to express n^{th} term mathematically is called progression

$\underbrace{0, 7, 26,} \ldots\ldots \longrightarrow \boxed{T_n = n^3 - 1}$

minimum no. of terms shd be '3'

S_n = sum to n – terms of series

$$S_n = T_1 + T_2 + T_3 + \cdots + T_n$$

$$S_{n-1} = T_1 + T_2 + T_3 + \cdots + T_{n-1}$$

sub

$$S_n - S_{n-1} = T_n$$

Rem

$$\boxed{T_n = S_n - S_{n-1}}$$

n^{th} term of sequence

Q1 Find n^{th} term of sequence whose $S_n = 5n^2 + 2n$?

Sol^n

$$T_n = S_n - S_{n-1}$$

$$T_n = (5n^2 + 2n) - (5(n-1)^2 + 2(n-1))$$

$$T_n = (\cancel{5n^2} + \cancel{2n}) - (\cancel{5n^2} + 5 - 10n + \cancel{2n} - 2)$$

$$\boxed{T_n = (10n - 3)} \rightarrow \text{linear fns of } 'n'$$

$$\boxed{c.d = 10}$$

$T_1 = 7$

$T_2 = 17$

$T_3 = 27$

$\vdots \quad \vdots$

Common Sequences

- A.P
- G.P
- H.P.
- AGP
- Miscellaneous.

Arithmetic Progression (A.P.)

Standard appearance of AP:

$a, \; a+d, \; a+2d, \; a+3d, \; \ldots\ldots, \; \underbrace{a+(n-1)d}$.

last term $= l$

$a \rightarrow 1^{st}$ term. ✓

$d \rightarrow$ Common difference.

$n \rightarrow$ no. of terms.

Rem

$$\boxed{T_n = a+(n-1)d}$$

n^{th} term of AP

Note:

1. If $d > 0 \Rightarrow$ increasing AP ✓
2. If $d < 0 \Rightarrow$ decreasing AP. ✓
3. * If $d = 0 \Rightarrow$ Constant AP. ✓

Q1 If 6^{th} and 11^{th} term of AP are 17 and 32 respectively find 20^{th} term ?

Sol^n

$$T_6 = a + 5d = 17 \quad \text{—(1)—}$$

$$T_{11} = a + 10d = 32 \quad \text{—(2)—}$$

$$5d = 15$$

$$d = 3. \Rightarrow a = 2$$

$$T_{20} = a + 19d = ?$$

$$= 2 + 19 \times 3$$

$$= 59.$$

Q2 In an AP if $a_2 + a_5 - a_3 = 10$ and $a_2 + a_9 = 17$ then find a_1 ?

Sol^n

$$a_2 + a_5 - a_3 = 10.$$

$$(a_1+d) + (\cancel{a_1}+4d) - (\cancel{a_1}+2d) = 10$$

$$(a_1+d) + (a_1+8d) = 17.$$

$$\left.\begin{aligned} a_1 + 3d &= 10 \quad -(1)- \\ 2a_1 + 9d &= 17 \quad -(2)- \end{aligned}\right\}$$

$a_1 = 13$

Sum to n-terms of AP :-

$$S_n = a + (a+d) + (a+2d) + \cdots + (a+(n-1)d)$$

$$S_n = a+(n-1)d + a+(n-2)d + \cdots + a.$$

$$2S_n = \underbrace{(2a+(n-1)d)} + \underbrace{(2a+(n-1)d)} + \cdots + \underbrace{(2a+(n-1)d)}$$

$$2S_n = n \cdot (2a+(n-1)d) \Rightarrow \boxed{S_n = \frac{n}{2}(2a+(n-1)d)}$$

Rem

OR $S_n = \frac{n}{2}\left(a + \underbrace{(a+(n-1)d)}_{l}\right)$

* $a \to$ 1st term

$d \to$ common diff

$n \to$ no. of terms

Rem

$$\boxed{S_n = \frac{n}{2}(a+l)}$$

Note: S_n of an AP is quadratic function of 'n' with no constant term.

T_n of an AP is linear function of 'n'.

① Sum of 1^{st} 'n' natural nos ie $1+2+3+\cdots+n = \frac{n(n+1)}{2}$

② Sum of 1^{st} 'n' odd natural nos ie

$1+3+5+\cdots+(2n-1) = \frac{n}{2}(1+(2n-1)) = n^2$.

③ Sum of 1^{st} 'n' even natural nos ie

$2+4+6+\cdots+(2n) = \frac{n}{2}(2+(2n)) = n(n+1)$.

Q1 If n^{th} term of an A.P is $T_n = (a-2)n^2 + 3n + 5 \quad \forall\, n \in N$ then which of the following is(are) true (where S_n is sum to n terms of an AP).

(A) $T_4 = 17$ (B) $S_{10} = 215$ (C) $a + S_5 = 70$ (D) $T_6 = 23$.

Sol^n

$$T_n = (\underbrace{a-2}_{0})n^2 + 3n + 5 \Rightarrow T_n = 3n + 5$$

$$\therefore \boxed{a = 2} \qquad \boxed{C.d = 3}$$

$$T_1 = 1^{st} \text{ term} = 8$$

$$T_4 = T_1 + 3d = 8 + 3(3)$$

$$\boxed{T_4 = 17}$$

Q2 The sum of n terms of an AP is 153 and the common diff is 2 If first term is an integer and $n > 1$ then 'n' can have value

~~(A) 9~~ ~~(B) 17~~ ~~(C) 13~~ ~~(D) 51~~

Sol^n

$S_n = 153$; $d = 2$ $a \in I$

$S_n = \frac{n}{2}\left(2a + (n-1).d\right) \Rightarrow 153 = \frac{n}{\cancel{2}}\left(\cancel{2}a + (n-1)\cancel{2}\right)$

$\frac{153}{n} = \underbrace{a}_{} + (n-1). \Rightarrow a = \frac{153}{n} - \underbrace{(n-1)}_{\text{integer}}$

shd be integer

$153 = \underbrace{3^2 . 17}_{}$

divisors of 153 → 3 ✓, 3^2 ✓, 17 ✓, 3×17 ✓, $3^2 \times 17$ ✓

Q3 Find number of common terms in series

7, 10, 13, 16, ..., 307

11, 19, 27, ..., 731

?

$\boxed{13}$ Ans

Sol^n

7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43 ...., 307

$cd_1 = 3$

11, 19, 27, 35, 43, 51, ... 731

$cd_2 = 8$

19, 43, 67, ... l

$$\boxed{cd = LCM(cd_1, cd_2).}$$
$$= 24$$

$T_n \le 307 \Rightarrow 19 + (n-1) \cdot 24 \le 307.$

$(n-1)24 \le 288 \Rightarrow \boxed{n \le 13.}$

Note

$1, \overset{T_2}{4}, 7, \overset{T_4}{10}, \overset{T_5}{13}, 16 \rightarrow$ A.P.

17

17

17

$$T_2 + T_{\underbrace{6-2+1}}$$

Imp:-

In an A.P summation of k^{th} term from beginning and k^{th} term from last is always equal to sum of 1^{st} and last term:

$$\boxed{T_k + T_{\underbrace{n-k+1}} = \underbrace{a}_{1^{st}\text{ term}} + \underbrace{l}_{\text{last term}}}$$