# FUNCTION

## Sheet Solution (Leader)

# DO YOURSEF - 1

(1)

(a) $y = 1 - \log_{10} x$

Domain of $y$ : $x > 0$

(b) $y = \dfrac{1}{\sqrt{x^2 - 4x}}$

Domain : $x^2 - 4x > 0$

$\Rightarrow x(x-4) > 0$

$\Rightarrow x < 0$ or $x > 4$

(2) (a) $\log_4 \left| x + \frac{1}{x} \right|$

Domain : $x \neq 0$

Range : $\left| x + \frac{1}{x} \right| \geq 2$ by AM $\geq$ GM

so $\log_4 \left| x + \frac{1}{x} \right| \geq \log_4 2$

$\geq \frac{1}{2}$

(b) $f(x) = \sin(3x^2+1)$

Domain : $x \in R$

Range : $3x^2+1 \geq 1$

: $\sin(3x^2+1) \in [-1, 1]$

(c) $f(x) = 2\sin\left(2x + \frac{\pi}{4}\right)$

Domain : $x \in R$

Range : $2\sin(2x+\pi/4) \in [-2, 2]$

(D) $f(x) = \cos(2x + \pi/4)$

Domain : $x \in R$

Range : $\cos(2x+\pi/4) \in [-1, 1]$

# DO YOURSELF - 2

Q 1

(A) $[x^2] \geq 4$

$\Rightarrow x^2 \geq 4$

$\Rightarrow x^2 - 4 \geq 0$

$\Rightarrow x \in (-\infty, -2] \cup [2, \infty)$

(B) $[x]^2 - 5[x] + 6 = 0$

$\Rightarrow I^2 - 5I + 6 = 0$

$\Rightarrow (I-2)(I-3) = 0$

$\Rightarrow I = 2$ or $I = 3$

$\Rightarrow [x] = 2$ or $[x] = 3$

$\Rightarrow x \in [2, 3)$ or $x \in [3, 4)$

or $x \in [2, 4)$

(C) $x = [x]$

$\Rightarrow [x] + \{x\} = [x]$

$\Rightarrow \{x\} = 0$

$\Rightarrow x \in I$

(D) $[x] < -5$

$\Rightarrow [x] \leq -6$

$\Rightarrow x < -5$

# DO YOURSELF - 3

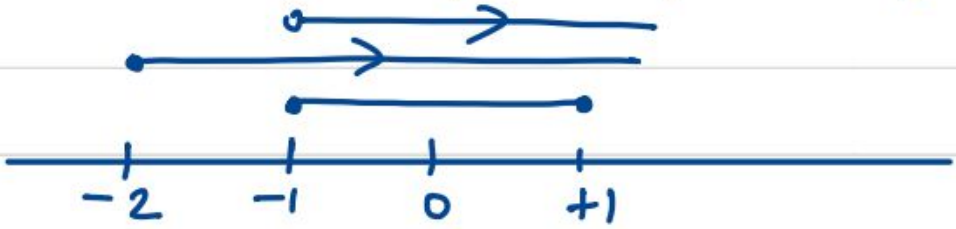
(1)

(a) $f(x) = \sin\left(\sqrt{1-x^2}\right) + \sqrt{x+2} + \dfrac{1}{\log_{10}(x+1)}$

Domain: $1-x^2 \geq 0$, $x+2 \geq 0$, $x+1 > 0$

$x+1 \neq 1$

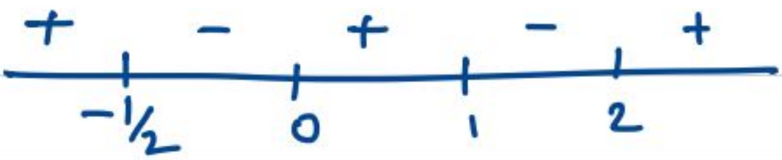
$: -1 \leq x \leq 1$, $x \geq -2$, $x > -1$, $x \neq 0$



$\therefore x \in [-1, 1] - \{0\}$

(b) $f(x) = \sqrt{\dfrac{(2x+1)}{x(x^2-3x+2)}}$

$= \sqrt{\dfrac{(2x+1)}{x(x-2)(x-1)}}$




$x \in (-\infty, -\tfrac{1}{2}] \cup (0,1) \cup (2, \infty)$

(11) (a)

$f(x) = \log_2(\log_{1/2}(x^2+4x+4))$

for Domain

$\Rightarrow \log_{1/2}(x^2+4x+4) > 0$

$\Rightarrow 0 < x^2+4x+4 < 1 \quad (x \neq -2)$

which satisfy so, $y \in R$

(b) $f(x) = \dfrac{1}{2-\cos 3x}$

$\Rightarrow 2-\cos 3x > 0$

$\because -1 \leq \cos 3x \leq 1, \; x \in R$

$\therefore \; 1 \leq 2-\cos 3x \leq 3$

$\therefore \; \dfrac{1}{3} \leq \dfrac{1}{2-\cos 3x} \leq 1$

Date: __/__/__

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Subject

Do your self (4)

Are the following functions identical?

(A) $f(x) = \frac{x}{x^2}$ & (B) $\phi(x) = \frac{x^2}{x}$

Domain of $f(x) = x \in R - \{0\}$

Domain of $\phi(x) = x \in R - \{0\}$

$f(x) = \frac{1}{x}$ & $\phi(x) = x$

$\therefore$ Not identical

(B) $f(x) = x$ & $\phi(x) = \sqrt{x^2}$

Domain of $f(x) =$ ~~$x \in R$~~ Range $(-\infty, \infty)$

Domain of $\phi(x) = |x|$ $\therefore$ ~~$x \in R^+$~~ Range $(0, \infty)$

$\therefore$ Not identical

(C) $f(x) = \log_{10} x^2$ & $\phi(x) = 2\log_{10} |x|$

Identical $\because$ Domain equal

$\because$ Range equal

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Subject ____________

## Do your self (5)

(i) Find the boundness of function

$$f(x) = \frac{x^2}{x^4+1}$$

$$y = \frac{x^2}{x^4+1}$$

$$y = \frac{1}{x^2 + \frac{1}{x^2}} \rightarrow \geq 2$$ By A.M. & G.M.

$$y = \left[0, \frac{1}{2}\right]$$

Date : __/__/____

MON TUE WED THU FRI SAT SUN

Subject ____________

(ii) Which of the following function is implicit function ?

(A) $xy - \cos(x+y) = 0$ (B) $y = x^3$

(C) $y = \log(x^2+x+1)$ (D) $y = |x|$

By definition of implicit function

$\boxed{A}$

Date: __/__/__

MON TUE WED THU FRI SAT SUN

Subject

(iii) Convert the implicit form into the explicit function:-

(a) $xy = 1$ (b) $x^2y = 1$

(a) $xy = 1$

$y = \frac{1}{x}$

(b) $x^2y = 1$

$y = \frac{1}{x^2}$

Date : __/__/____

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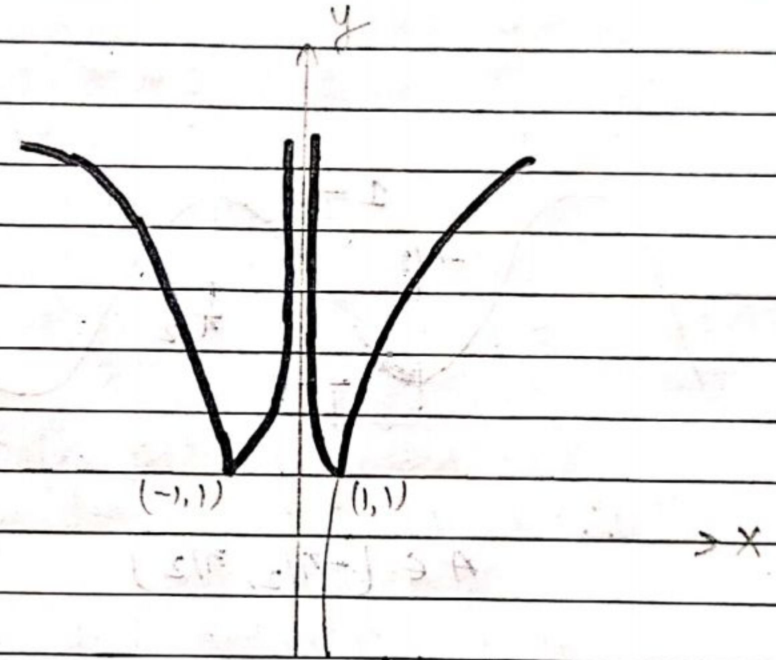
## Do your self (6)

(1) Draw graph of following function :-

(a) $y = |\ln|x|| + 1$

(1) Domain $x \neq 0$




**Note** (1) graph of $|f(x)|$ would be image of $y = f(x)$ in x-axis

(2) graph of $f(|x|)$ would be symmetric about the y-axis.

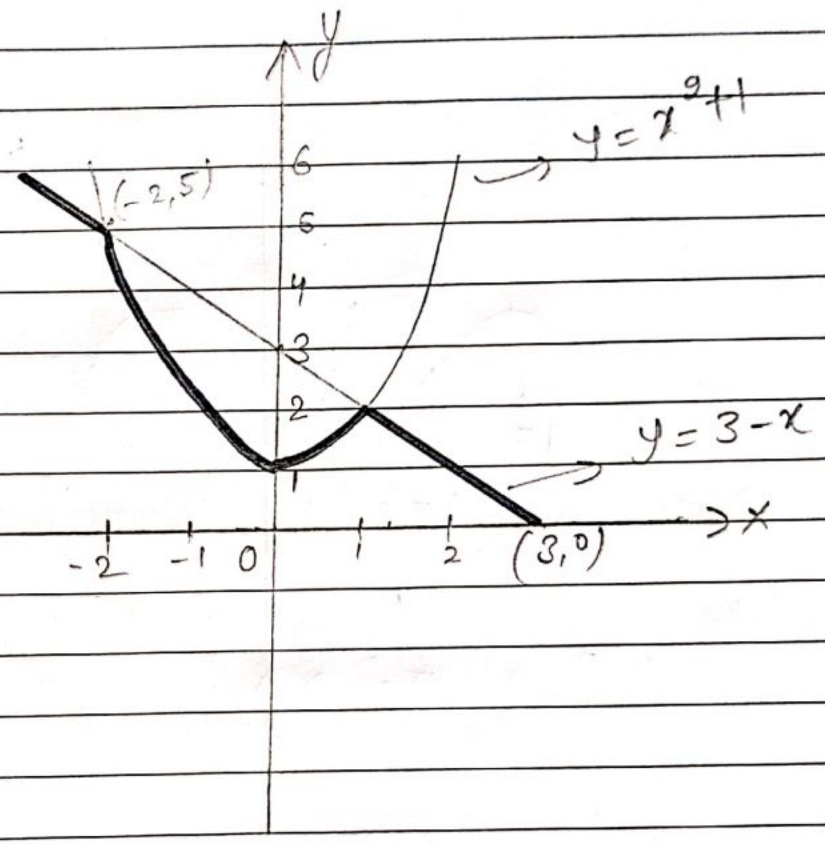
Date: __/__/____

MON TUE WED THU FRI SAT SUN

Subject

① ⓑ Draw graph of following function :-

$$y = \min\{x^2+1,\ 3-x\}$$



Date: __/__/____

Subject ______

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## Do your self (7)

(1) IS the function $f: N \to N$ [the set of natural numbers] defined by $f(x) = 2x+3$ Surjective?

$f(x) = 2x+3$

$f'(x) = 2 > 0$

$\therefore \ x \in N$

$f(x)$ is injective

| $f: N \to N$ |
|---|
| Domain $= N$ |
| CoDomain $= N$ |
| M-II $f(x_1) = f(x_2)$ |
| $2x_1 + 3 = 2x_2 + 3$ |
| $x_1 = x_2$ |
| $\therefore f(x)$ is injective |

Now $f(x) = 2x+3$

$y = 2x+3$

$x = \dfrac{y-3}{2}$

at $y = 4$ $\quad x = \dfrac{1}{2}$ Not Natural No.

$\therefore$ $f(x)$ is Injective Not ~~Surjective~~. onto.

Date : __/__/__

Subject ____________ MON TUE WED THU FRI SAT SUN

(2) Let $A = R - \{3\}$, $B = R - \{1\}$ and let $f : A \to B$ defined by $f(x) = \dfrac{x-2}{x-3}$.

check Whether the function $f(x)$ is bijective or not.

~~to check one-one let $f(x)$ many-one~~

$$f(x) = f(y)$$

$$\frac{x-2}{x-3} = \frac{y-2}{y-3}$$

$$\frac{x-3+1}{x-3} = \frac{y-3+1}{y-3}$$

$$1 + \frac{1}{x-3} = 1 + \frac{1}{y-3} \Rightarrow \frac{1}{x-3} = \frac{1}{y-3}$$

only possible when $\boxed{x = y}$

$\therefore$ function is one-one

for on to Range = Codomain

Here Codomain $R - \{1\}$

$$y = \frac{x-2}{x-3}$$

$$xy - 3y = x - 2$$

$$x = \frac{3y-2}{y-1} \quad y \in R - \{1\}$$

$\therefore$ Range = codomain

$\therefore$ $f(x)$ is Bijective.

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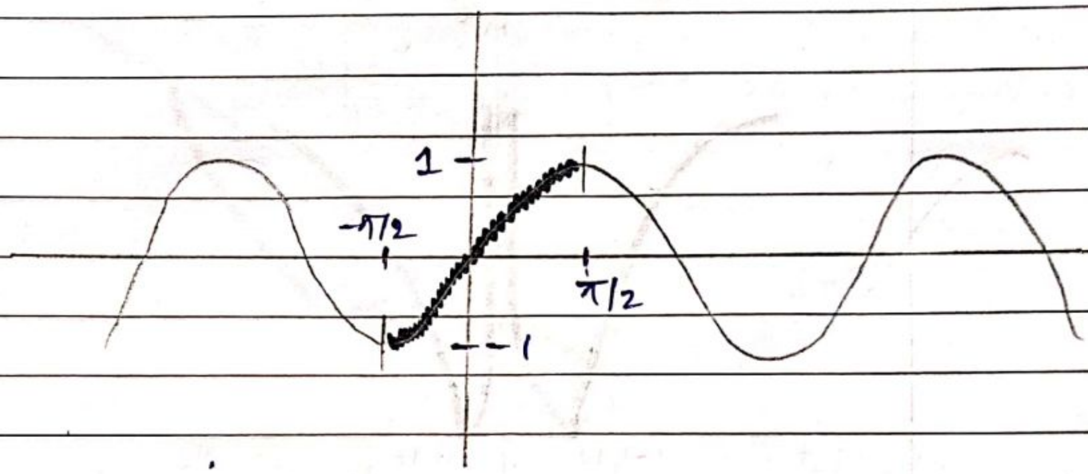
Subject ________

(3) A mapping $f: A \to [-1,1]$ defined by $f(x) = \sin x$ $\forall$ $x \in R$, where A is a subset of R (the set of all real Numbers) is one-one and onto if A is the interval, then A is belongs to = ?




$\therefore$

$A \in [-\pi/2, \pi/2]$

Date : ___/___/___

MON TUE WED THU FRI SAT SUN

Subject ____________

## Do your self (8)

(1) $f(x) = x^3 - x$

$g(x) = \sin 2x$ then find

(A) $f(f(1))$

$f(0) = \boxed{0}$

(B) $f(f(-1)) \Rightarrow f(-1+1)$

$f(0) = \boxed{0}$

(C) $f(g(\pi/2)) \Rightarrow f(\sin 2 \times \frac{\pi}{2})$

$f(0) = \boxed{0}$

(D) $f(g(\pi/4)) \Rightarrow f(\sin 2 \times \pi/4)$

$f(1) = (1)^3 - 1 = \boxed{0}$

(E) $g(f(1)) \Rightarrow g(1-1)$

$g(0) = \boxed{0}$

(F) $g\left(g\left(\frac{\pi}{2}\right)\right) \Rightarrow g\left(\sin 2 \times \frac{\pi}{2}\right)$

$g(0) = \boxed{0}$

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Subject ________ MON TUE WED THU FRI SAT SUN

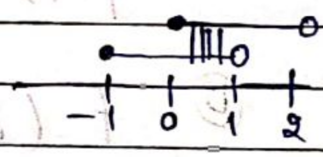
(2) If $f(x) = \begin{cases} x+1 & 0 \le x < 2 \\ |x| & 2 \le x < 3 \end{cases}$

then find $f \circ f(x)$.

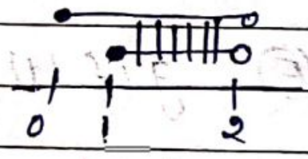
$|x|$ in $2 \le x < 3$ always true +ve

$\therefore f(x) = \begin{cases} x+1 & 0 \le x < 2 \\ x & 2 \le x < 3 \end{cases}$

$$f \circ f(x) = \begin{cases} (x+1)+1 & 0 \le x+1 < 2 \\ & -1 \le x < 1 \\ x+1 & 2 \le x+1 < 3 \\ & 1 \le x < 2 \\ x & 2 \le x < 3 \end{cases}$$









$\therefore f \circ f(x) \begin{cases} x+2 & 0 \le x < 1 \\ x+1 & 1 \le x < 2 \\ x & 2 \le x < 3 \end{cases}$

# Do yourself - 9

(1)
$$y = x|x| = \begin{cases} x^2, & 0 \le x \le 1 \\ -x^2 & -1 \le x < 0 \end{cases}$$

(I) for $0 \le x \le 1$

$$y = x^2$$

$\Big[$ or $x = \sqrt{y} \Rightarrow f^{-1}(x) = \sqrt{x}$

and $x = -\sqrt{y}$ (rejected as $x \geq 0$)

(II) for $-1 \le x < 0$

$$y = -x^2$$

or $x^2 = -y$ $(x<0,\ y<0)$

or $x = \pm\sqrt{-y}$

or $x = -\sqrt{-y}$,

or $f^{-1}(x) = -\sqrt{-x}$ ( as $x < 0$, $y < 0$

and $f^{-1}(x) = \sqrt{-x}$ (rejected as $x < 0$)

(ii)

$$f(x) = 1 + \ln(x+2)$$

$$\Rightarrow \quad y = 1 + \ln(x+2)$$

$$\Rightarrow \quad \ln(x+2) = (y-1)$$

$$\Rightarrow \quad x + 2 = e^{y-1}$$

$$\Rightarrow \quad x = -2 + e^{y-1}$$

$$\Rightarrow \quad \underline{f^{-1}(x) = -2 + e^{x-1}}$$

**Do yourself - 10 :**

**(i)** Which of the following functions is (are) even, odd or neither :

(a) $f(x) = x^3 \sin 3x$

(b) $f(x) = \dfrac{e^{x^2} + e^{-x^2}}{2x}$

(c) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(d) $f(x) = x^2 + 2^x$

Sol. (a) $f(x) = x^3 \sin 3x$

$f(-x) = (-x^3)(-\sin 3x)$

$= x^3 \sin 3x = f(x)$

$=$ even

(b) $f(x) = \dfrac{e^{x^2} + e^{-x^2}}{2x}$

$\Rightarrow f(-x) = \dfrac{e^{x^2} + e^{-x^2}}{-2x} = -f(x)$

$=$ odd

(c) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$f(-x) = \dfrac{e^{-x} - e^{x}}{e^x + e^{-x}} \Rightarrow -\dfrac{(e^x - e^{-x})}{(e^x + e^{-x})}$

$= -f(x) = \text{odd}$

(d) $f(x) = x^2 + 2^x$

$\Rightarrow f(-x) = (-x)^2 + 2^{-x}$

$= x^2 + \dfrac{1}{2^x}$

Neither even nor odd

**Do yourself - 11 :**

**(i)** Find the fundamental periods (if periodic) of the following functions.

(a) $f(x) = \ell n(\cos x) + \tan^3 x$.

(b) $f(x) = e^{x-[x]}$, [.] denotes greatest integer function

(c) $f(x) = \left|\sin\frac{x}{2}\right| + \left|\cos\frac{x}{2}\right|$

(a) $f(x) = \ln(\cos x) + \tan^3 x$

period of $\cos x = 2\pi = T_1$

period of $\tan x = \pi = T_2$

min value of at which

$f(x+T) = f(x)$ is $2\pi$

$f(x+2\pi) = \ln \cos(x+2\pi) + \tan^3(x+2\pi)$

$= \ln \cos x + \tan^3 x = f(x)$

$(\cos(\pi + x) = -\cos x)$

(b) $f(x) = e^{x-[x]} = e^{\{x\}}$

period of $\{x\}$ is 1

so period of $f(x)$ is 1

(c) $f(x) = \left|\sin x/_2\right| + \left|\cos x/_2\right|$

period of $\sin x/_2 = 4\pi$ $(T_1)$

period of $\cos x/_2 = 4\pi$ $(T_2)$

L.C.M of $(T_1, T_2) = 4\pi$

$$\left.\begin{aligned} & f(x + 4\pi) = f(x) \\ \text{again } & f(x + 2\pi) = f(x) \\ \text{again } & f(x + \pi) = f(x) \end{aligned}\right\} \underline{\text{check}}$$

So min value of $T = \pi$

# EXERCISE (O-1)

**1. Find the domain of definition of the given functions :**

(i) $y=\sqrt{-px}\,(p>0)$ (ii) $y=\dfrac{1}{x^2+1}$ (iii) $y=\dfrac{1}{x^3-x}$

(iv) $y=\dfrac{1}{\sqrt{x^2-4x}}$ (v) $y=\sqrt{x^2-4x+3}$ (vi) $y=\dfrac{x}{\sqrt{x^2-3x+2}}$

(vii) $y=\sqrt{1-|x|}$ (viii) $y=\log_x 2.$ (ix) $y=\dfrac{1}{\log_{10}(1-x)}+\sqrt{x+2}$

(x) $y=\dfrac{1}{\sqrt{\sin x}}+\sqrt[3]{\sin x}$

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**Sol:-**

(i) $y=\sqrt{-px}$ $(P>0)$

$-px \geq 0$

$x \leq 0$

(ii) $y=\dfrac{1}{x^2+1}$

$x^2+1 \neq 0$

always true

$x \in R$

(iii) $y=\dfrac{1}{x^3-x}$

$x^3-x \neq 0$

$\Rightarrow x(x-1)(x+1) \neq 0$

$\Rightarrow x \in R-\{0,1,-1\}$

(v) $y=\sqrt{x^2-4x+3}$

$x^2-4x+3 \geq 0$

$(x-3)(x-1) \geq 0$

$+ \quad - \quad +$

$1 \quad 3$

$x \in (-\infty,1] \cup [3,\infty)$

(vi) $y=\dfrac{x}{\sqrt{x^2-3x+2}}$

$x^2-3x+2 > 0$

$(x-1)(x-2) > 0$

$+ \quad - \quad +$

$1 \quad 2$

(iv) $y = \dfrac{1}{\sqrt{x^2-4x}}$

$x^2 - 4x > 0$

$x(x-4) > 0$

$+ \quad - \quad +$

$0 \quad 4$

$x \in (-\infty, 0) \cup (4, \infty)$

(vii) $y = \sqrt{1-|x|}$

$1 - |x| \geq 0$

$|x| \leq 1$

$x^2 \leq 1$

$x^2 - 1 \leq 0$

$(x-1)(x+1) \leq 0$ ↳ inside roots

$x \in [-1, 1]$

(viii) $y = \log_x 2$

$x > 0$, & $x \neq 1$

So $x \in (0,1) \cup (1, \infty)$

(ix) $y = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

$\rightarrow x + 2 \geq 0$

$x \geq -2$

$\rightarrow 1 - x > 0$

$x < 1$

$\rightarrow \log_{10}(1-x) \neq 0$

$1 - x \neq 10^0$

$x \neq 0$

Taking intersection

$x \in [-2, 1) - \{0\}$

(x) $y = \dfrac{1}{\sqrt{\sin x}} + \sqrt[3]{\sin x}$

$\sqrt[3]{\sin x}$ ↓ no issue $x \in R$

$\dfrac{1}{\sqrt{\sin x}}$ ↓ $\sin x > 0$

$x \in 2k\pi + (0, \pi)$

Taking intersection

$x \in 2k\pi + (0, \pi)$

$\Rightarrow x \in (2k\pi, (2k+1)\pi)$

$\underline{\underline{k \in I}}$

**2. Find the range of the following functions :**

(i) $f(x)=\dfrac{x-1}{x+2}$ (ii) $f(x)=\dfrac{2}{x}$ (iii) $f(x)=\dfrac{1}{x^2-x+1}$

(iv) $f(x)=e^{(x-1)^2}$ (v) $f(x)=x^3-x^2+x+1$ (vi) $f(x)=\sin^2x-2\sin x+4$

(vii) $f(x)=\sin(\log_2 x)$ (viii) $f(x)=2^{x^2}+1$

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(i) $f(x)=\dfrac{x-1}{x+2}$

$y=\dfrac{x-1}{x+2}$

$xy+2y=x-1$

$x(y-1)=\underline{-1-2y}$

$x=\dfrac{1+2y}{1-y}$

$\underline{y\neq 1}$

Range $=(-\infty,\infty)-\{1\}$

Alter

$y=\dfrac{ax+b}{cx+d}$

$y\in R-\left\{\dfrac{a}{c}\right\}$

So $y\in R-\left\{\dfrac{1}{1}\right\}$

(ii) $f(x)=\dfrac{2}{x}$

$y=\dfrac{0(x)+2}{x}$

$y\in R-\left\{\dfrac{0}{1}\right\}$

$y\in R-\{0\}$

(iii) $y=\dfrac{1}{x^2-x+1}$

$x^2-x+1$ always +ve

$\underline{D<0}$

min max $=+\infty$

$\dfrac{-D}{4a}=\dfrac{3}{4}$

$R_f=\left(0,\dfrac{4}{3}\right]$

(iv) $y = e^{(x-1)^2}$

$0 \le (x-1)^2 < \infty$

So $1 \le e^{(x-1)^2} < \infty$

$R_f = [1, \infty)$

(v) $y = x^3 - x^2 + x + 1$ $\hookrightarrow$ odd degree Polynomial

$R_f = \underline{\underline{R}}$

(vi) $y = \sin^2 x - 2\sin x + 4$

$y = (\sin x - 1)^2 + 3$

Range $= [3, 7]$

(vii) $f(x) = \sin(\log_2 x)$

$-\infty < \log_2 x < \infty$

So
$-1 \le \sin(\log_2 x) \le 1$

$R_f = [-1, 1]$

(viii) $y = 2^{x^2} + 1$

$x^2 \geq 0$

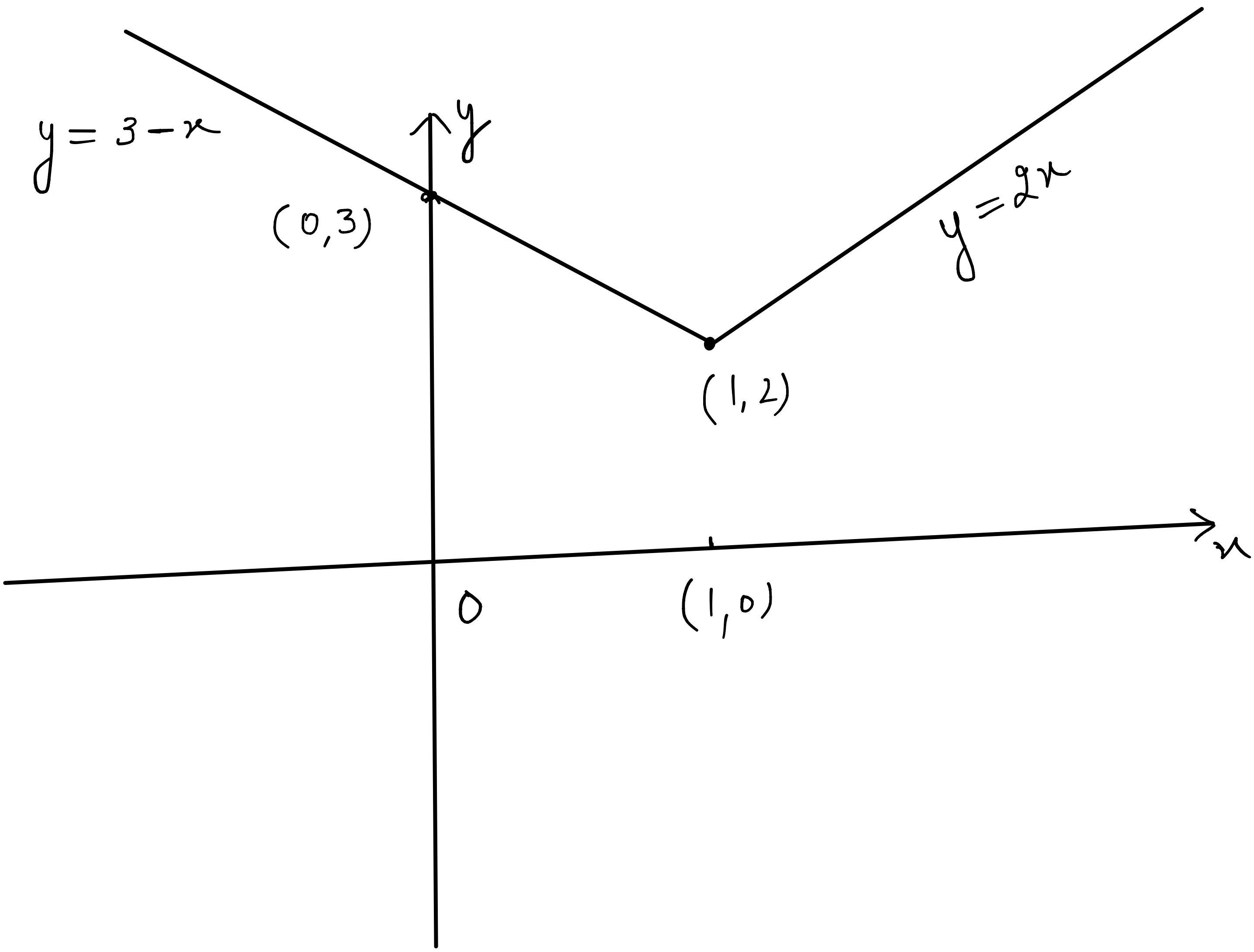
$2^{x^2} \geq 1$

$2^{x^2} + 1 \geq 2$

$R_f = [2, \infty)$

**3.** Graph the function $F(x) = \begin{cases} 3 - x, & x \le 1 \\ 2x, & x > 1 \end{cases}$

Sol$^n$



④

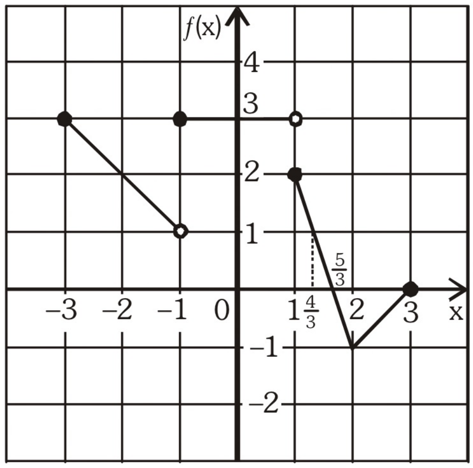
Solve the following inequalities using graph of f(x) :

(a) $0 \le f(x) \le 1$

(b) $-1 \le f(x) \le 2$

(c) $2 \le f(x) \le 3$

(d) $f(x) > -1$ & $f(x) < 0$




Sol:-

(a) $\left[\frac{4}{3}, \frac{5}{3}\right] \cup \{3\}$

(b) $[1, 3] \cup [-2, -1)$

(c) $[-3, -2] \cup [-1, 1]$

(d) $\left(\frac{5}{3}, 3\right) - \{2\}$

5

## Straight Objective Type

. If [x] and {x} denotes the greatest integer function less than or equal to x and fractional part function respectively, then the number of real x, satisfying the equation (x–2)[x] = {x} – 1, is-

(A) 0 (B) 1 (C) 2 (D) infinite

Sol:-

$$(x-2)[x] = \{x\} - 1$$

$$(x-2)[x] = x-[x]-1$$

$$(x-2+1)[x] = (x-1)$$

$$\cancel{(x-1)}[x] = \cancel{(x-1)}$$

$x-1=0$ or $[x]=1$ → infinite sol$^n$s

$x=1$

6

The range of the function $f(x) = \operatorname{sgn}\left(\dfrac{\sin^2 x + 2\sin x + 4}{\sin^2 x + 2\sin x + 3}\right)$ is (where sgn(.) denotes signum function)-

(A) $\{-1,0,1\}$ (B) $\{-1,0\}$ (C) $\{1\}$ (D) $\{0,1\}$

Sol:-

$$f(x) = \operatorname{Sgn}\left(\frac{\sin^2 x + 2\sin x + 4}{\sin^2 x + 2\sin x + 3}\right)$$

$$y = \operatorname{Sgn}\left(1 + \frac{1}{(\sin x + 1)^2 + 2}\right)$$

always +ve

$y = 1$ always

$R_f = \{1\}$ (C)

7

If $2f(x) - 3f\left(\frac{1}{x}\right) = x^2$, x is not equal to zero, then f(2) is equal to-

(A) $-\frac{7}{4}$ (B) $\frac{5}{2}$ (C) $-1$ (D) none of these

Sol:-

$$2f(x) - 3f\left(\frac{1}{x}\right) = x^2 \qquad f(2) = ?$$

$$x \longrightarrow \frac{1}{x}$$

$$2f\left(\frac{1}{x}\right) - 3f(x) = \frac{1}{x^2}$$

$$2f(x) - 3f\left(\frac{1}{x}\right) = x^2$$

Solving both equation

$$f(x) \Rightarrow \frac{-1}{5}\left(\frac{3}{x^2} + 2x^2\right)$$

$$f(2) = \frac{-1}{5}\left(\frac{35}{4}\right)$$

$$\Rightarrow \frac{-7}{4} \qquad \text{A}$$

8

The number of integers lying in the domain of the function $f(x) = \sqrt{\log_{0.5}\left(\dfrac{5-2x}{x}\right)}$ is -

(A) 3 (B) 2 (C) 1 (D) 0

Sol:-

$$f(x) = \sqrt{\log_{(0.5)}\left(\frac{5-2x}{x}\right)}$$

$\dfrac{5-2x}{x} > 0$ & $\dfrac{5-2x}{x} \leq 1$

$\dfrac{2x-5}{x} < 0$ $\qquad$ $\dfrac{5-3x}{x} \leq 0$

$\qquad\qquad\qquad\qquad$ $\dfrac{x-5/3}{x} \geq 0$

$+ \quad - \quad +$

$0 \quad 5/2$

Integer $(1, 2)$

$+ \quad - \quad +$

$0 \quad 5/3$

Intersection

$x \in \left[\dfrac{5}{3}, \dfrac{5}{2}\right) \Rightarrow \underline{x = 1}$ $\quad$ (C)

9

The range of the function $f : \mathbb{N} \to \mathbb{Z};\ f(x) = (-1)^{x-1}$, is -

(A) $[-1, 1]$ (B) $\{-1, 1\}$ (C) $\{0, 1\}$ (D) $\{0, 1, -1\}$

Sol:-

$f : N \to Z$ $f(x) = (-1)^{x-1}$ Range = ?

$(-1)^{\text{even}} = 1$ $(-1)^{\text{odd}} = -1$

$\{-1, 1\}$ B

(10)

The range of the function $f(x) = e^{-x} + e^{x}$, is -

(A) $f(x) \geq 1$ (B) $f(x) \leq 1$ (C) $f(x) \geq 2$ (D) $f(x) \leq 2$

Sol:-

$f(x) = e^{x} + e^{-x}$ Range = ?

$e^{x} \geq 0$

So $e^{x} + \frac{1}{e^{x}} \geq 2$ (A.M $\geq$ G.M)

$f(x) \geq 2$

11

If $f(x) = \dfrac{4^x}{4^x + 2}$, then $f(x) + f(1 - x)$ is equal to-

(A) 0 (B) –1 (C) 1 (D) 4

Sol:-

$$f(x) = \frac{4^x}{4^x + 2}$$

$$f(1-x) = \frac{4^{1-x}}{4^{1-x} + 2} = \frac{\frac{4}{4^x}}{\frac{4}{4^x} + 2}$$

$$\Rightarrow \frac{2}{4^x + 2}$$

$$f(x) + f(1-x) = \frac{4^x + 2}{4^x + 2} = 1$$

C

12

A function $f$ has domain $[-1, 2]$ and range $[0, 1]$. The domain and range respectively of the function $g$ defined by $g(x) = 1 - f(x+1)$ is

(A) $[-1, 1]$ ; $[-1, 0]$ (B) $[-2, 1]$; $[0, 1]$ (C) $[0, 2]$ ; $[-1, 0]$ (D) $[1, 3]$ ; $[-1, 0]$

Sol:-

$f : [-1, 2] \longrightarrow [0, 1]$

$g(x) = 1 - \boxed{f(x+1)}$

$(x+1)$ $\qquad\qquad$ $[0, 1]$

$-1 \le x+1 \le 2$

$-2 \le x \le 1$

$D_g \; [-2, 1]$ $\qquad\qquad$ $R_g = [0, 1]$ $\qquad$ B

(13)

If $f:\mathbb{R}\to\mathbb{R}$ & $f(x)=\dfrac{\sin([x]\pi)}{x^2+2x+3}+2x-1+\sqrt{x(x-1)+\dfrac{1}{4}}$ (where [x] denotes integral part of x), then f(x) is -

(A) one-one but not onto

(B) one-one & onto

(C) onto but not one-one

(D) neither one-one nor onto

( ) $f(x)=\dfrac{\sin([x]\pi)}{x^2+2x+3}+2x-1+\sqrt{x(x-1)+\dfrac{1}{4}}$

$x^2+2x+3 \rightarrow D<0$ {So no issue of Domain}

$[x]=$ Integer

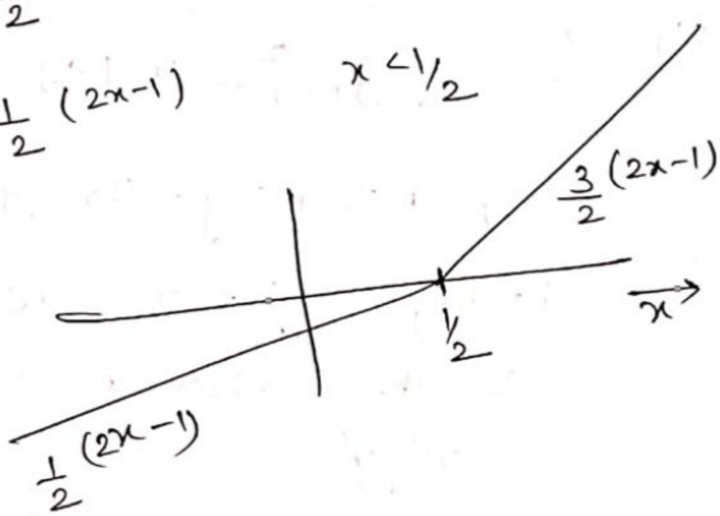
$\sin(\text{Integer})\pi=0$

$f(x)=2x-1+\sqrt{x^2-x+\frac{1}{4}}$

$f(x)=2x-1+\sqrt{(x-1/2)^2}$

$f(x)=(2x-1)+\left|\frac{2x-1}{2}\right|$

$$f(x)=\begin{cases}\frac{3}{2}(2x-1) & x\geq 1/2\\ \frac{1}{2}(2x-1) & x<1/2\end{cases}$$




One-one
onto
(B)

(14)

Which of the following function is surjective but not injective

(A) $f:\mathbb{R}\to\mathbb{R}$ $f(x) = x^4 + 2x^3 - x^2 + 1$

(B) $f:\mathbb{R}\to\mathbb{R}$ $f(x) = x^3 + x + 1$

(C) $f:\mathbb{R}\to\mathbb{R}^+$ $f(x) = \sqrt{1+x^2}$

(D) $f:\mathbb{R}\to\mathbb{R}$ $f(x) = x^3 + 2x^2 - x + 1$

$\underbrace{\text{Surjective}}_{\downarrow\ \text{onto}}$ but $\underbrace{\text{not injective}}_{\downarrow\ \text{many one}}$

(A) $f(x) = x^4 + 2x^3 - x^2 + 1$

$f: R \to R$ → even degree polynomial (into)

$\lim_{x\to\pm\infty} (\to\infty)$ many one

will necessarily have curvature.

(B) $f: R\to R$ $f(x) = x^3 + x + 1$

odd degree polynomial

✓ onto

$f'(x) = 3x^2+1 > 0$ increasing

so one-one

(C) $f: R \to R^+$ $f(x) = \sqrt{1+x^2}$ → min value $=(1)$

into function

many one → (even function)

✓(D) $f: R\to R$ $f(x) = x^3 + 2x^2 - x + 1$

odd degree polynomial → onto

$f'(x) = 3x^2 + 4x - 1$ → $D > 0$

So can't be one-one

(D)

15

If $f(x) = x|x|$ then $f^{-1}(x)$ equals-

(A) $\sqrt{|x|}$ (B) $(\text{sgn } x).\sqrt{|x|}$ (C) $-\sqrt{|x|}$ (D) Does not exist

( ) $f(x) = x|x|$ $f^{-1}(x) = ?$

$$f(x) = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$$

$y = x^2$

$x = \pm\sqrt{y}$

(−) reject (by Domain)

$y = -x^2$

$x = \pm\sqrt{-y}$

+ reject (by Domain)

So $f^{-1}(x) = (\text{sgn } x).\sqrt{|x|}$

B

16

**16.** If $f : (-\infty, 3] \to [7, \infty)$ ; $f(x) = x^2 - 6x + 16$, then which of the following is true -

(A) $f^{-1}(x) = 3 + \sqrt{x-7}$

(B) $f^{-1}(x) = 3 - \sqrt{x-7}$

(C) $f^{-1}(x) = \dfrac{1}{x^2 - 6x + 16}$

(D) $f$ is many-one

(16) $f : (-\infty, 3] \to [7, \infty)$

$$y = x^2 - 6x + 16$$

$$x^2 - 6x + 16 - y = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4(16 - y)}}{2}$$

$$x = 3 \pm \sqrt{y - 7}$$

$$x \leftrightarrow y$$

$$y = 3 \pm \sqrt{x - 7}$$

+ sign will be reject by function Domain

$$f^{-1}(x) = 3 - \sqrt{x - 7}$$

(B)

17

$f : \mathbb{R} \to \mathbb{R}$ such that $f(x) = \ell n\left(x+\sqrt{x^2+1}\right)$. Another function g(x) is defined such that $gof(x) = x \ \forall \ x \in R$. Then g(2) is -

(A) $\frac{e^2+e^{-2}}{2}$ (B) $e^2$ (C) $\frac{e^2-e^{-2}}{2}$ (D) $e^{-2}$

$f : R \to R$

$f(x) = \ln\left(x+\sqrt{x^2+1}\right)$

$g(2) = ?$

$gof(x) = x \quad \forall \ x \in R$

$g(x) = f^{-1}(x)$

$y = \ln_e\left(x+\sqrt{x^2+1}\right)$

$(e^y - x) = \sqrt{x^2+1}$

$e^{2y} + x^2 - 2xe^y = x^2+1$

$x = \frac{e^{2y}-1}{2e^y}$

$f^{-1}(x) = \frac{e^{2x}-1}{2e^x}$

$g(2) = \frac{e^4-1}{2e^2} \Rightarrow \frac{e^2-e^{-2}}{2}$ (C)

18

Let P (x) = $kx^3 + 2k^2x^2 + k^3$. The sum of all real numbers k for which (x – 2) is a factor of P(x), is

(A) 4 (B) 8 (C) – 4 (D) –8

$P(x) = Kx^3 + 2K^2x^2 + K^3$

$(x-2)$ is a factor of $P(x)$

factor theorm

$K(8) + 2K^2(4) + K^3 = 0$

$K(K^2 + 8K + 8) = 0$

$K = 0$ | $K^2 + 8K + 8 = 0$

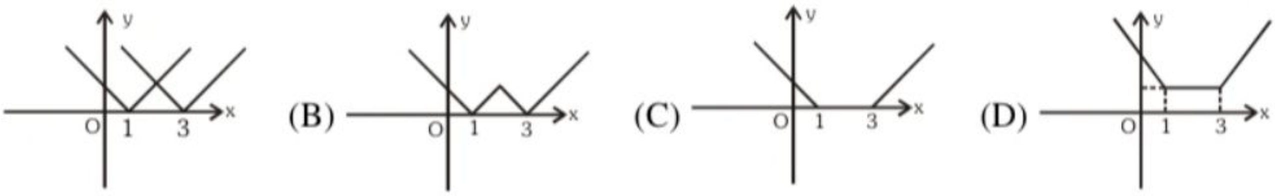
$\hookrightarrow D > 0$

Sum $= -8$ (D)

(19)

Which of the following is the graph of y = |x – 1| + |x – 3| ?

(A) (B) (C) (D)





**Sol:-**

$y = |x-1| + |x-3|$ Graph = ?

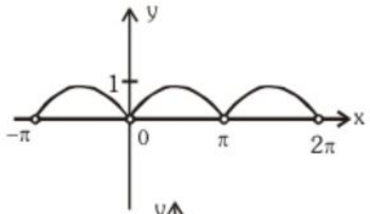
Critical point 1, 3

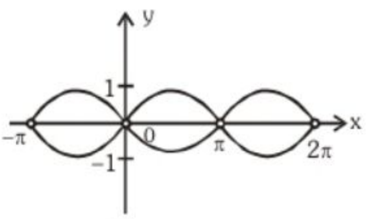
$$f(x) = \begin{cases} 4-2x & x \le 1 \\ 2 & 1 < x < 3 \\ 2x-4 & x \geq 3 \end{cases}$$

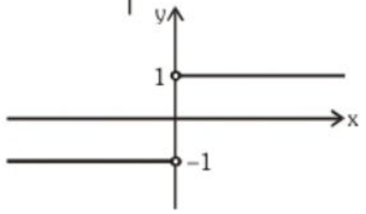
clearly (D)

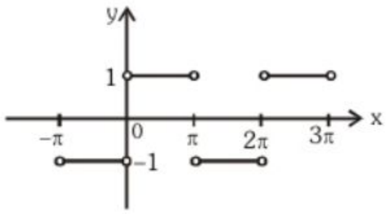
20

Which of the following is the graph of $y = \dfrac{|\sin x|}{\sin x}$ ?

(A)



(B)



(C)



(D)


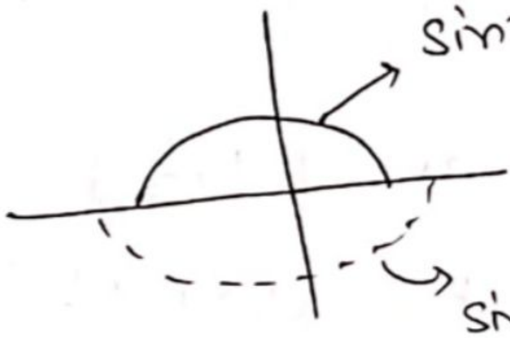

Sol:-

Graph of $y = \dfrac{|\sin x|}{\sin x}$

first $\sin x \neq 0$ so $x \neq n\pi$

$$y = \begin{cases} 1 & 2k\pi < x < (2k+1)\pi \\ -1 & (2k+1)\pi < x < 2k\pi \end{cases}$$

Clearly (D)



sinx +ve

sinx -ve

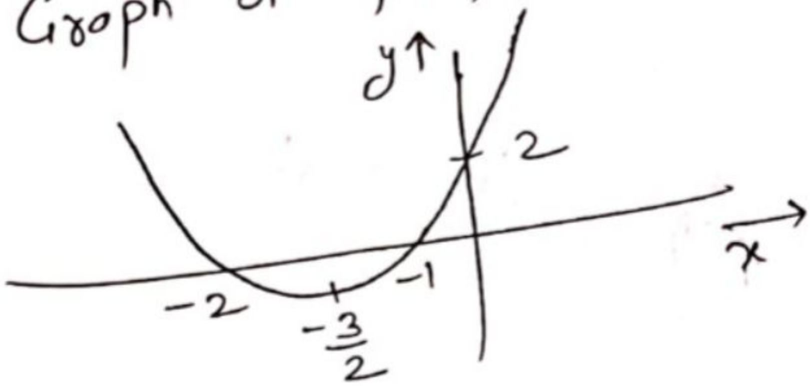
21

Let $f(x) = x^2 + 3x + 2$, then number of solutions to -

(A) $f(|x|) = 2$ is 1 (B) $f(|x|) = 2$ is 3 (C) $|f(x)| = 0.125$ is 4 (D) $|f(|x|)| = 0.125$ is 8

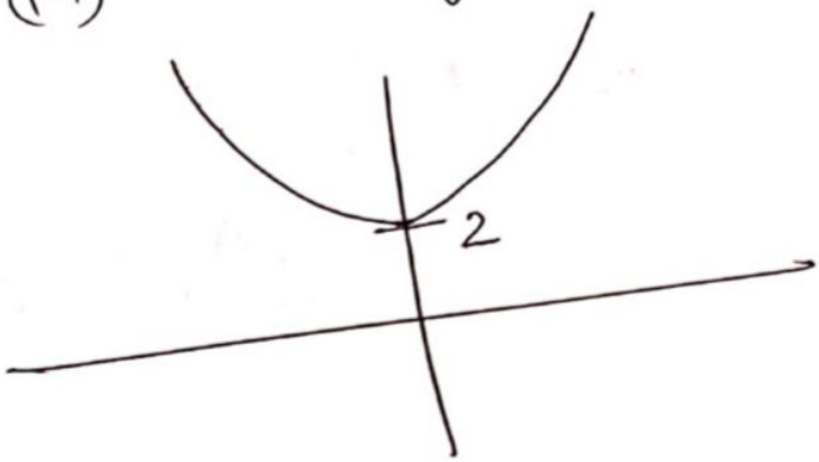
$f(x) = x^2 + 3x + 2$

Graph of $f(x)$




Graph of $f(|x|)$

$f(|x|) \rightarrow$ even function $\rightarrow$ Sym. about y axis



(22)

Which of the following pair(s) of function have same graphs?

(A) $f(x) = \dfrac{\sec x}{\cos x} - \dfrac{\tan x}{\cot x}$, $g(x) = \dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\operatorname{cosec} x}$

(B) $f(x) = \operatorname{sgn}(x^2 - 4x + 5)$, $g(x) = \operatorname{sgn}\left(\cos^2 x + \sin^2\left(x + \dfrac{\pi}{3}\right)\right)$ where sgn denotes signum function.

(C) $f(x) = e^{\ln(x^2+3x+3)}$, $g(x) = x^2 + 3x + 3$

(D) $f(x) = \dfrac{\sin x}{\sec x} + \dfrac{\cos x}{\operatorname{cosec} x}$, $g(x) = \dfrac{2\cos^2 x}{\cot x}$

<u>Same Graphs</u>

(A) $f(x) = \dfrac{\sec x}{\cot x} - \dfrac{\tan x}{\cot x}$ $\quad x \neq (2n+1)\frac{\pi}{2}$

$x \neq n\pi$

$x \neq \frac{n\pi}{2}$

$\Rightarrow \dfrac{1}{\cos^2 x} - \dfrac{\sin^2 x}{\cos^2 x}$

$\Rightarrow 1$

$g(x) = \dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\operatorname{cosec} x}$ $\quad x \neq n\pi$

$x \neq (2n+1)\frac{\pi}{2}$

$g(x) = \cos^2 x + \sin^2 x$ $\quad x \neq \frac{n\pi}{2}$

$= 1$

22 Continue....

(B) $f(x) = \text{Sgn}(\underbrace{x^2+4x+5}_{D<0}) = 1$

$g(x) = \text{Sgn}\left(\underbrace{\cos^2 x + \sin^2\left(x+\frac{\pi}{3}\right)}_{>0}\right) = 1$

$D_f = x \in R$ , $D_g \Rightarrow x \in R$

(C) $f(x) = e^{\ln(x^2+3x+3)}$

$\underbrace{x^2+3x+3}_{D<0} > 0$ (By Domain)

So always

$f(x) = x^2+3x+3 = g(x)$

(D) $f(x) = \dfrac{\sin x}{\sec x} + \dfrac{\cos x}{\operatorname{cosec} x}$ $\qquad g(x) = \dfrac{2\cos^2 x}{\cot x}$

$x \neq (2n+1)\frac{\pi}{2}$ , $n\pi$

so

$x \neq \frac{n\pi}{2}$

$f(x) = \sin x \cos x + \cos x \sin x$

$= \sin 2x$

$g(x) = \dfrac{2\cos^2 x \sin x}{\cos x}$ $\qquad x \neq n\pi$ , $(2n+1)\frac{\pi}{2}$

so $x \neq \frac{n\pi}{2}$

$= \sin 2x$

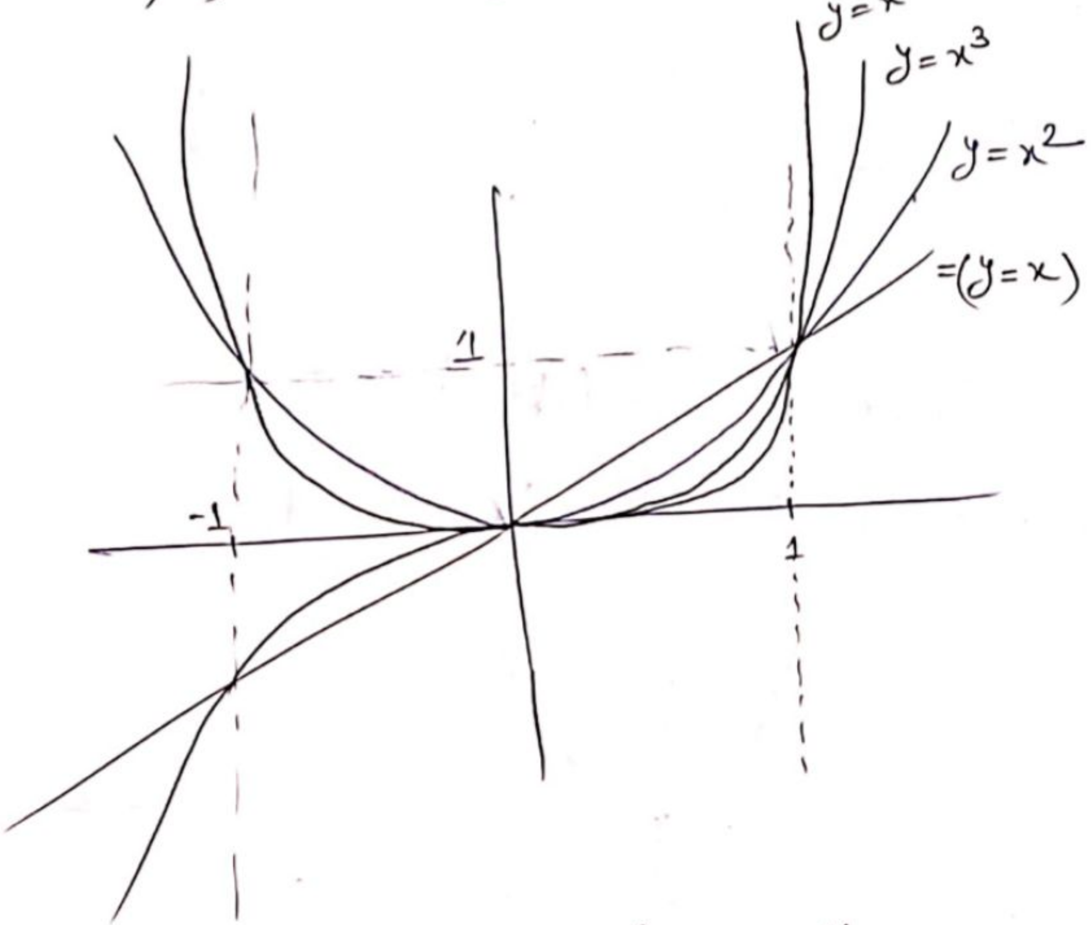
A B C D

23

For each real x, let $f(x) = \max\{x, x^2, x^3, x^4\}$, then $f(x)$ is -

(A) $x^4$ for $x \le -1$ (B) $x^2$ for $-1 < x \le 0$ (C) $f\left(\frac{1}{2}\right) = \frac{1}{2}$ (D) $f\left(\frac{1}{2}\right) = \frac{1}{4}$

$f(x) = \max\{x, x^2, x^3, x^4\}$




$f(x) = \max\{A_1, A_2\}$ means which ever graph is ~~upward~~ above.

Clearly

A, B, C.

24

Let $f(x) = \sin^6 x + \cos^6 x$, then -

(A) $f(x) \in [0, 1]\ \forall\ x \in \mathbb{R}$

(B) $f(x) = 0$ has no solution

(C) $f(x) \in \left[\frac{1}{4}, 1\right] \forall x \in \mathbb{R}$

(D) $f(x)$ is an injective function

$f(x) = \sin^6 x + \cos^6 x$

$f(x) = 1 - \frac{3}{4}\sin^2 2x \rightarrow [0,1]$

Range of $f(x) = \left[\frac{1}{4}, 1\right]$  A, C

25

Let $f(x) = \begin{cases} x^2 - 3x + 4 & ; \quad x < 3 \\ x + 7 & ; \quad x \geq 3 \end{cases}$ and $g(x) = \begin{cases} x + 6 & ; \quad x < 4 \\ x^2 + x + 2 & ; \quad x \geq 4 \end{cases}$, then which of the following is/are true -

(A) $(f + g)(1) = 9$ (B) $(f - g)(3.5) = 1$ (C) $(f\,g)(0) = 24$ (D) $\left(\frac{f}{g}\right)(5) = \frac{8}{3}$

(A) $(f+g)(1) = 9$

$f(1) = 2, \; g(1) = 7$

(B) $(f-g)(3.5) = 1$

$f(3.5) = 10.5 \quad , \quad g(3.5) = 9.5$

(C) $(fg)(0) = 24$

$f(0) = 4 \quad , \quad g(0) = 6$

(D) $\left(\frac{f}{g}\right)(5) = \frac{8}{3}$

$f(5) = 12 \quad , \quad g(5) = 32$

$\left(\frac{f}{g}\right)(5) = \frac{12}{32} \neq \frac{8}{3}$

A, B, C

(26)

## Matrix Match Type

Match the functions given in column-I correctly with mappings given in column-II.

| Column-I | Column-II |
|---|---|
| (A) $f:\left[-\frac{1}{2},\frac{1}{2}\right]\to\left[\frac{4}{7},\frac{4}{3}\right]$ | (P) Injective mapping |
| $f(x)=\dfrac{1}{x^2+x+1}$ | (Q) Non-injective mapping |
| (B) $f:[-2, 2]\to[-1, 1]$ | (R) Surjective mapping |
| $f(x) = \sin x$ | (S) Non-surjective mapping |
| (C) $f:\mathbb{R}-I\to\mathbb{R}$ | (T) Bijective mapping |
| $f(x) = \ell n\{x\}$, (where $\{.\}$ represents fractional part function) | |
| (D) $f:(-\infty, 0]\to[1, \infty)$, $f(x)=(1+\sqrt{-x})+(\sqrt{-x}-x)$ | |

(A) $\Rightarrow$ $f(x) = \dfrac{1}{x^2+x+1}$ , $f:\left[-\frac{1}{2},\frac{1}{2}\right]\to\left[\frac{4}{7},\frac{4}{3}\right]$

$f'(x) = \dfrac{-(2x+1)}{(x^2+x+1)^2}$ always negative in its domain

So Range $\left[f\left(\frac{1}{2}\right), f\left(-\frac{1}{2}\right)\right] \Rightarrow \left[\frac{4}{7},\frac{4}{3}\right]$

P R T

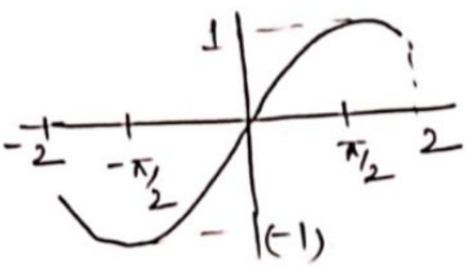
(B) $f:[-2,2]\to[-1,1]$

$f(x) = \sin x$

$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ & $[-2,2]$

onto

many - one



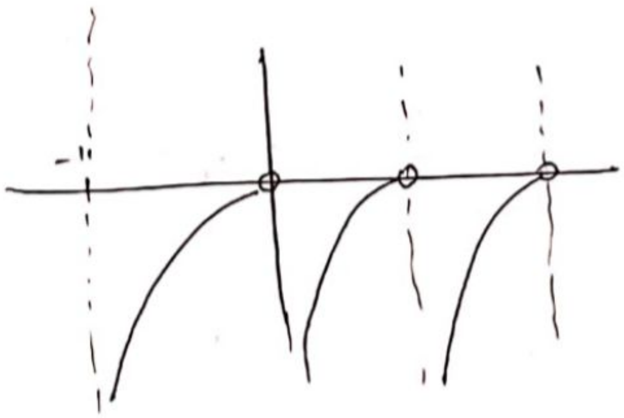

Q, R

(26) Continue....

(C) $f: R - I \rightarrow R$

$f(x) = \ln \{x\}$ → Periodic

$\{x\}$ → $[0, 1)$




Q, S

(D) $f: (-\infty, 0] \rightarrow [1, \infty)$

$f(x) = (1 + \sqrt{-x}) + (\sqrt{-x} - x)$

P, R, T

$f(-\infty) \rightarrow \infty$

$f(0) \rightarrow 1$

$f'(x) = \dfrac{1(-1)}{2\sqrt{-x}} + \dfrac{1(-1)}{2\sqrt{-x}} - 1 = -\left(1 + \dfrac{1}{\sqrt{-x}}\right) \underline{\underline{< 0}}$

Decreasing $f^n$

## Linked Comprehension Type

**Paragraph for Question**

Let $f(x) = \begin{cases} x & ; \quad x < 0 \\ 1-x & ; \quad x \geq 0 \end{cases}$ & $g(x) = \begin{cases} x^2 & ; \quad x < -1 \\ 2x+3 & ; \quad -1 \leq x \leq 1 \\ x & ; \quad x > 1 \end{cases}$

On the basis of above information, answer the following questions :

Range of $f(x)$ is -

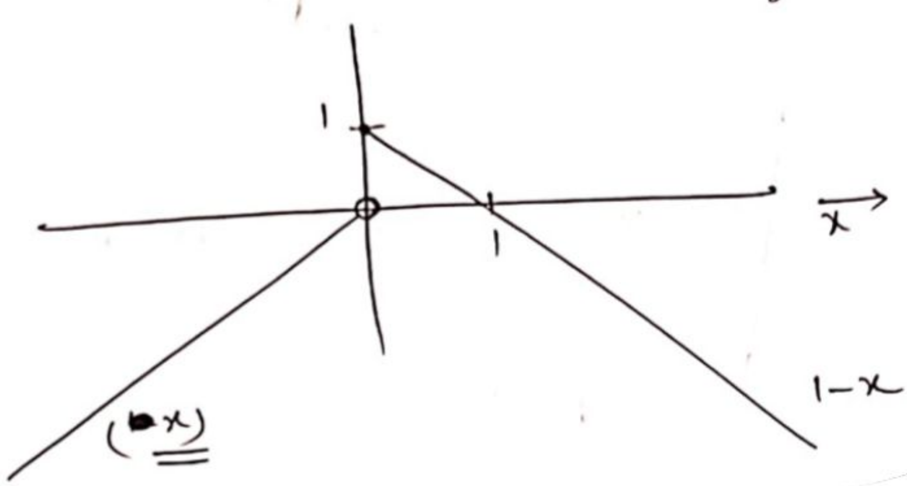
27 (A) $(-\infty, 1]$ (B) $(-\infty, \infty)$ (C) $(-\infty, 0]$ (D) $(-\infty, 2]$

27/28 Paragraph

$f(x) = \begin{cases} x & x < 0 \\ 1-x & x \geq 0 \end{cases}$

$g(x) = \begin{cases} x^2 & x < -1 \\ 2x+3 & -1 \leq x \leq 1 \\ x & x > 1 \end{cases}$

Graph of $f(x)$




Range of $f(x)$

$\underline{\underline{(-\infty, 1]}}$

A

28. Range of $g(f(x))$ is -

(A) $(-\infty,\infty)$ (B) $[1,3)\cup(3,\infty)$ (C) $[1,\infty)$ (D) $[0,\infty)$

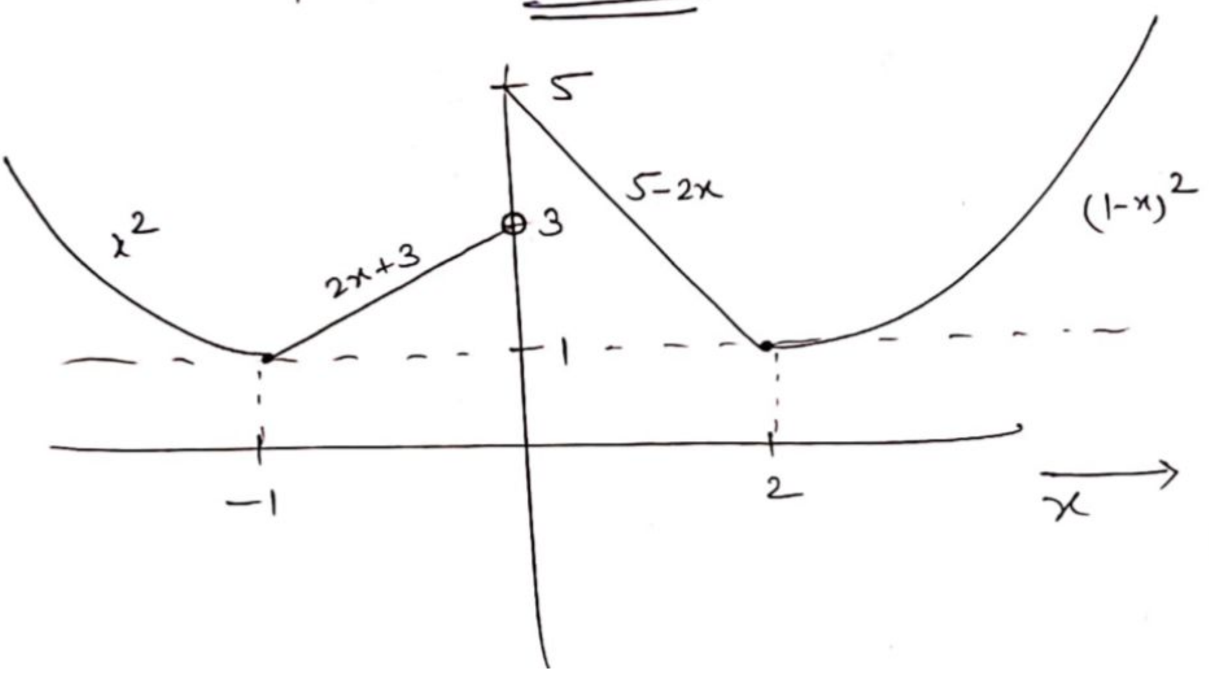
$$g(f(x)) = \begin{cases} (f(x))^2 & f(x) < -1 \\ 2(f(x))+3 & -1 \le f(x) \le 1 \\ f(x) & f(x) > 1 \end{cases}$$

$$g(f(x)) = \begin{cases} x^2 & x < -1 \\ (1-x)^2 & x > 2 \\ 2x+3 & -1 \le x < 0 \\ 2(1-x)+3 & 0 \le x \le 2 \end{cases}$$

Clearly Range of $g(f(x)) = [1,\infty)$

C

Graph of $g(f(x))$



1

# EXERCISE (O-2)

The number of integral values of x satisfying the inequality $[x-5][x-3]+2<[x-5]+2[x-3]$ (where [.] represents greatest integer function) is -

(A) 0 (B) 1 (C) 2 (D) 3

Sol. We have $[x-5][x-3]+2<[x-5]+2[x-3]$

$\Rightarrow [x-3]([x-5]-2)+(2-[x-5])<0$

$\Rightarrow (2-[x-5])([x-3]-1)>0$

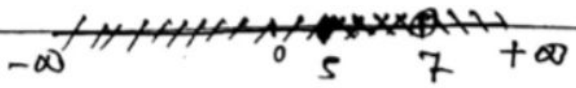
$\Rightarrow 2-[x-5]>0$ & $[x-3]-1>0$ (case I)

$\Rightarrow [x-5]<2$ $\quad [x-3]\geq 1$

$\Rightarrow [x]-5<2$ $\quad [x]-3\geq 2$

$\Rightarrow [x]<7$ $\quad [x]\geq 5$

$\Rightarrow x<7$ $\quad x\geq 5$




$\Rightarrow \boxed{x\in\{5,6\}}$ ✓

$2-[x-5]<0$ & $[x-3]-1<0$ (case II)

$\Rightarrow [x-5]\geq 3$ & $[x-3]<1$

$\Rightarrow x\geq 8$ $\quad x<4$

$\Rightarrow x\in\phi$ $\quad \therefore$ (C) answer.

**2.** Range of $f(x)=\dfrac{\sec x+\tan x-1}{\tan x-\sec x+1};\ x\in\left(0,\dfrac{\pi}{2}\right)$ is-

(A) (0,1) (B) $(1,\infty)$ (C) (–1,0) (D) $(-\infty,-1)$

Sol. $\dfrac{(\sec x+\tan x-1)}{(\tan x-\sec x+1)}=\dfrac{(\sec x+\tan x-1)}{-\left(\dfrac{(\sec x-\tan x)\times(\sec x+\tan x)}{(\sec x+\tan x)}-1\right)}$

$\Rightarrow \dfrac{(\sec x+\tan x-1)}{-\left(\dfrac{\sec^2 x-\tan^2 x}{\sec x+\tan x}-1\right)}=\dfrac{(\sec x+\tan x-1)}{-\left(\dfrac{1}{\sec x+\tan x}-1\right)}$

$\Rightarrow \dfrac{\sec x+\tan x-1}{-\dfrac{(1-\sec x-\tan x)}{(\sec x+\tan x)}}=\dfrac{(\cancel{(\sec x+\tan x)-1})(\sec x+\tan x)}{(\cancel{(\sec x+\tan x)-1})}$

$\Rightarrow f(x)=\sec x+\tan x$

$\Rightarrow f(x)=\dfrac{1+\sin x}{\cos x}$

Now let us check its monotonicity

$f'(x)=\dfrac{1+\sin x}{\cos^2 x}>0 \quad \because x\in(0,\pi/2)$

So

$f(x)$ at $_{x=0}=1.$ & $f(x)$ at $_{x=\frac{\pi}{2}}=\dfrac{2}{0}\to\infty$

$\Rightarrow f(x)\in(1,\infty)$ (B)

**3.** If $f(x,y) = \max(x,y) + \min(x,y)$ and $g(x,y) = \max(x,y) - \min(x,y)$, then the value of $f\left(g\left(-\frac{2}{3},-\frac{3}{2}\right), g(-3,-4)\right)$ is greater than -

(A) 1 (B) 2 (C) 3 (D) 4

Sol.

$g\left(\frac{-2}{3}, \frac{-3}{2}\right) = \frac{-2}{3} - \left(\frac{-3}{2}\right) = \frac{-2}{3} + \frac{3}{2} = \frac{5}{6}$

$g(-3,-4) = -3-(-4) = -3+4 = 1$

$\Rightarrow f\left(\frac{5}{6}, 1\right) = 1 + \frac{5}{6} = \frac{11}{6} = 1.8$

$\because$ $1.8 > 1$, so (A)

4

(0-2)

If functions $f(x)$ and $g(x)$ are defined on $\mathbb{R} \rightarrow \mathbb{R}$ such that $f(x)=\begin{cases} x+3 & , \quad x \in \text{rational} \\ 4x & , \quad x \in \text{irrational} \end{cases}$,

$g(x)=\begin{cases} x+\sqrt{5} & , \quad x \in \text{irrational} \\ -x & , \quad x \in \text{rational} \end{cases}$ then $(f-g)(x)$ is-

(A) one-one & onto

(B) neither one-one nor onto

(C) one-one but not onto

(D) onto but not one-one

Let. $f-g = h(x)$

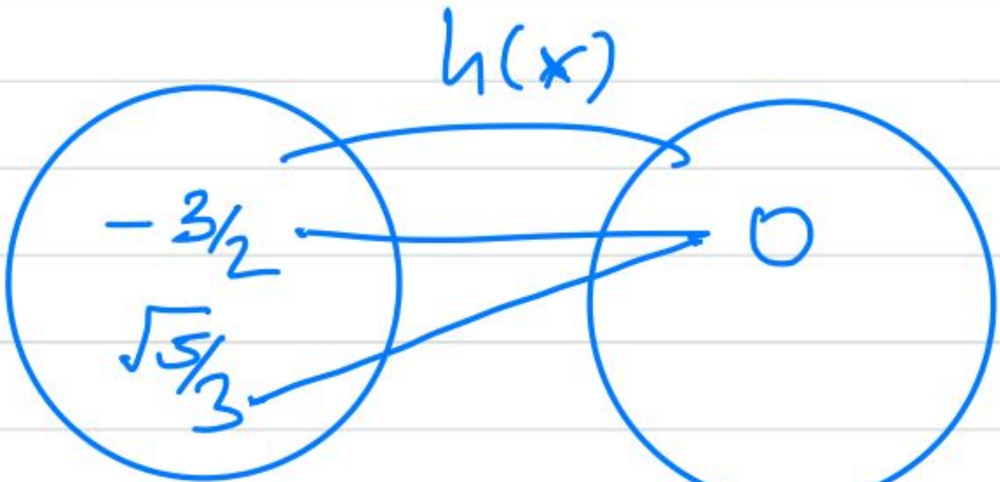
Sol.

$$h(x) = \begin{cases} (2x+3) & x \in Q \\ (3x-\sqrt{5}) & x \notin Q \end{cases}$$

at $x = -3/2$ , $h(-3/2) = 0$

at $x = \sqrt{5}/3$ , $h(\sqrt{5}/3) = 0$

&




So function is many one

4 Continue...

again if $h(x) = -\sqrt{5}$

is possible for $x = 0$

as $3x - \sqrt{5} = -\sqrt{5}$

at $x = 0$ (which is not irrational)

So Range $\neq R$

$\therefore$ function is many one-Into

5

Let $f$: A $\rightarrow$ B be an onto function such that $f(x)=\sqrt{x-2-2\sqrt{x-3}}-\sqrt{x-2+2\sqrt{x-3}}$ , then set 'B' is-

(A) [–2,0] (B) [0,2] (C) [–3,0] (D) [–1,0]

Sol. We have

$$y = f(x) = \underbrace{\sqrt{x-2-2\sqrt{(x-3)}}}_{\text{smaller}} - \underbrace{\sqrt{x-2+2\sqrt{(x-3)}}}_{\text{bigger}} \Rightarrow f(x) \le 0 \quad \text{---} (*) \qquad (1)$$

and $f(x)$ to be defined $x-3 \geq 0 \Rightarrow x \geq 3$

& $x-2-2\sqrt{(x-3)} \geq 0$

$\Rightarrow x-2 \geq 2\sqrt{(x-3)}$ squaring both sides

$(x-2)^2 \geq (2\sqrt{(x-3)})^2$

$\Rightarrow x^2+4+(4x) \geq 4(x-3)$

$\Rightarrow x^2-8x+16 \geq 0$

$\Rightarrow (x-4)^2 \geq 0 \Rightarrow x \in R$

Squaring (1) we get

$y^2 = (x-2-2\sqrt{(x-3)}) + (x-2+2\sqrt{(x-3)}) - 2\sqrt{(x-2)^2-(2\sqrt{(x-3)})^2}$

$\Rightarrow 2x-4-2\sqrt{x^2+4-4x-4x+12}$

$\Rightarrow 2x-4-2\sqrt{x^2-8x+16} \Rightarrow 2x-4-2\sqrt{(x-4)^2}$

$\Rightarrow 2x-4-2(x-4) = \cancel{2x}-4-\cancel{2x}+8 = 4$

$\Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

$\Rightarrow y = -2 \quad [\because (*)\ f(x) \le 0]$

$\Rightarrow y \in [-2, 0]$ ✓ Hence result (A)

———x———

(6)

Let $f : \mathbb{R} \to \mathbb{R}$ and $f(x) = x^3 + ax^2 + bx - 8$. If $f(x) = 0$ has three real roots & $f(x)$ is a bijective function, then $(a + b)$ is equal to

(A) 0 (B) 6 (C) –6 (D) 12

Sol.

We have $f: R \to R$, $f(x) = x^3 + ax^2 + bx - 8$

$f(x) = 0$ has three real roots

$\Rightarrow$ 3 real values of $x$ will give same values of $y$

But $f(x)$ is given bijective (ie one-one and onto both)

$\Rightarrow$ All three roots should be equal

ie $\alpha = \beta = \gamma$

$\Rightarrow \alpha + \beta + \gamma = 3\alpha = -a$

$\Rightarrow \alpha = \dfrac{-a}{3}$ —— (1)

Now

$\Sigma\alpha\beta = \alpha\beta + \beta\gamma + \gamma\alpha = 3\alpha^2 = b$ —— (2)

$\Sigma\alpha\beta\gamma = \alpha^3 = 8 \Rightarrow \alpha = 2$ —— (3)

From (2) & (3)

$b = 12$

& From (1) & (3) $a = -6$

$\therefore a + b = -6 + 12 = 6$ (B) is correct answer

x

7

Which of the following functions is an odd function ?

(A) $|x-2|+(x+2)\,\mathrm{sgn}(x+2)$ (B) $\dfrac{1}{x(e^x-1)}+\dfrac{1}{2x}$

(C) $\log\left(\sin x+\sqrt{1+\sin^2 x}\right)$ (D) $e^{-4x}\left(e^{2x}-1\right)^4$

(where sgn(x) denotes signum function of x)

(A) $|x-2|+(x+2)\,\mathrm{sgn}(x+2)$

Case I : if $x-2>0 \Rightarrow x>2$

$\Rightarrow f(x)= x-2+(x+2)\,\mathrm{sgn}(x+2)$ $\quad \because \mathrm{sgn}(x+2)=1$ if $x+2>1$

$= x-\cancel{2}+x+\cancel{2}=2x$

$f(x)=2x$

$f(-x)=2(-x)=-2x$

$f(-x)=-f(x)$

Case II if $x-2\le 0$ $x\le 2$

$f(x)=-(x-2)+x+2$ . if $0<x\le 2$

$=-\cancel{x}+2+\cancel{x}+2=4$ ($\because$ Constant function is an even fun)

Case III if $x<0$ Case III $x=0$

$f(x)=-(x-2)+(x+2)(-1)$ $\quad f(0)=|-2|+(0+2)\cdot 1$

$=-x+\cancel{2}-x-\cancel{2}=-2x$ $\quad =+2+0+2=4$

$f(x)=-2x$ ($\because$ Constant $f^n$ is an even function)

$f(-x)=-2(-x)=2x$

~~$\Rightarrow f(-x)=-f(x) \Rightarrow$ odd function~~

$\therefore f(x)$ is not an

$\because f(x)$ is not an odd function throughout its interval

$\therefore$ it not an odd function

—x—

(7) Continue...

(B) $f(x) = \frac{1}{x(e^x-1)} + \frac{1}{2x}$

$\Rightarrow \frac{2}{2x(e^x-1)} + \frac{e^x - 1}{2x(e^x-1)}$

$\Rightarrow \frac{2 + e^x - 1}{2x(e^x-1)} = \boxed{\frac{(1+e^x)}{2x(e^x-1)} = f(x)}$

Now

$f(x) = \frac{1+e^x}{2x(e^x-1)}$

$f(-x) = \frac{1+e^{-x}}{2(-x)(e^{-x}-1)} = \frac{e^x+1}{2(-x)\cancel{e^x}(1-e^x)}$

$f(-x) = -\frac{(e^x+1)}{2x(1-e^x)} = +\frac{(e^x+1)}{2x(e^x-1)}$

$f(-x) = f(x)$ $\therefore$ $f(x)$ is not an odd function

Q 14 (C) $f(x) = \log(\sin x + \sqrt{1+\sin^2 x})$

$f(-x) = \log(\sin(-x) + \sqrt{1+\sin^2(-x)})$

$f(-x) = \log(-\sin x + \sqrt{1+\sin^2 x})$

$f(-x) = \log\left[\frac{(\sqrt{1+\sin^2 x} - \sin x) \times (\sqrt{1+\sin^2 x}) + \sin x}{(\sqrt{1+\sin^2 x} + \sin x)}\right]$

$\Rightarrow \log\left(\frac{1+\cancel{\sin^2 x} - \cancel{\sin^2 x}}{\sqrt{1+\sin^2 x} + \sin x}\right)$

7 Continue ...

$f(-x) = -\log(\sqrt{1+\sin^2 x} + \sin x)$

$f(-x) = -f(x)$, $\therefore$ it is an odd function

x

(D) Let $f(x) = e^{-4x}(e^{2x}-1)^4$

$f(-x) = e^{-4(-x)}(e^{2(-x)}-1)^4$

$= e^{4x}(e^{-2x}-1)^4$

$= e^{4x}\dfrac{(1-e^{2x})^4}{e^{8x}} = e^{-4x}(1-e^{2x})^4$

$= e^{-4x}(e^{2x}-1)$

$f(-x) = f(x)$, $\therefore$ it is not an odd function, it is an even function.

x

8

Period of $f(x)=\{x\}+\left\{x+\frac{1}{3}\right\}+\left\{x+\frac{2}{3}\right\}$ is equal to (where $\{.\}$ denotes fractional part function)

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Sol. We have

$$f(x)=\underset{(\mathrm{I})}{\{x\}}+\underset{(\mathrm{II})}{\left\{x+\frac{1}{3}\right\}}+\underset{(\mathrm{III})}{\left\{x+\frac{2}{3}\right\}}$$

$$f\left(x+\frac{1}{3}\right)=\left\{x+\frac{1}{3}\right\}+\left\{x+\frac{1}{3}+\frac{1}{3}\right\}+\left\{x+\frac{1}{3}+\frac{2}{3}\right\}$$

$$=\left\{x+\frac{1}{3}\right\}+\left\{x+\frac{2}{3}\right\}+\{x+1\}$$

$$=\left\{x+\frac{1}{3}\right\}+\left\{x+\frac{2}{3}\right\}+\{x\}=f(x)$$

$\therefore$ Period of $f(x)$ is $\frac{1}{3}$.

$$f\left(x+\frac{2}{3}\right)=\left\{x+\frac{2}{3}\right\}+\left\{x+\frac{2}{3}+\frac{1}{3}\right\}+\left\{x+\frac{2}{3}+\frac{2}{3}\right\}$$

$$=\left\{x+\frac{2}{3}\right\}+\{x+1\}+\left\{x+\frac{4}{3}\right\}$$

$$=\left\{x+\frac{2}{3}\right\}+\{x\}+\left\{x+\frac{1}{3}\right\}=f(x)\quad(\because \{\cdot\}\text{ fractional part function})$$

so period of $f(x)$ is $\frac{1}{3}$

x

9

Let $f(x) = 2x - \left\{\frac{x}{\pi}\right\}$ and $g(x) = \cos x$, where {.} denotes fractional part function, then period of $gof(x)$ is -

(A) $\frac{\pi}{2}$ (B) $\pi$ (C) $\frac{3\pi}{2}$ (D) $\frac{\pi}{4}$

Sol. We have $f(x) = 2x - \left\{\frac{x}{\pi}\right\}$ & $g(x) = \cos x$

$gof(x) = g(f(x)) = \cos\left(2x - \left\{\frac{x}{\pi}\right\}\right)$

As $x$ is changing, so $\left\{\frac{x}{\pi}\right\}$ is changing and $\cos\left(2x - \left\{\frac{x}{\pi}\right\}\right)$ is also changing

$\therefore$ we need to consider period of $\left\{\frac{x}{\pi}\right\}$ and $\cos\left(2x - \left\{\frac{x}{\pi}\right\}\right)$

Period of $\left\{\frac{x}{\pi}\right\}$ is $\frac{1}{\left|\frac{1}{\pi}\right|} = \pi$ & period of $\cos\left(2x - \left\{\frac{x}{\pi}\right\}\right)$ is $\frac{2\pi}{|2|} = \pi$

$\therefore$ Period of $g(f(x))$ is $\pi$.

X

10

# Multiple Correct Answer Type

Let $f(x)=\begin{cases} x^2 & ; \quad 0<x<2 \\ 2x-3 & ; \quad 2\le x<3 \\ x+2 & ; \quad x\ge 3 \end{cases}$ Then :-

(A) $f\left\{f\left(f\left(\frac{3}{2}\right)\right)\right\}=f\left(\frac{3}{2}\right)$

(B) $1+f\left\{f\left(f\left(\frac{5}{2}\right)\right)\right\}=f\left(\frac{5}{2}\right)$

(C) $f\{f(1)\} = f(1) = 1$

(D) none of these

Sol.

(A) $f\left\{f\left(f\left(\frac{3}{2}\right)\right)\right\} = f\left\{f\left(\frac{9}{4}\right)\right\}$

$\frac{3}{2} \in 0<x<2$

$2 \le \frac{9}{4} < 3$ so $f\left\{\cancel{2}\left(\frac{9}{\cancel{4}_2}\right) - \frac{3\times 2}{2}\right\}$

$= f\left(\frac{3}{2}\right) = \frac{9}{4}$ ✓ hence true

(B) $1+f\left\{f\left(f\left(\frac{5}{2}\right)\right)\right\}$

$\Rightarrow 1+f\left\{f\left(\cancel{2}\times\frac{5}{\cancel{2}}-3\right)\right\}$

$\Rightarrow 1+f\{f(2)\}$

$\Rightarrow 1+f(2\times 2-3)$

$1+f(1) = 1+1^2 = 2 = f\left(\frac{5}{2}\right)$

$= \cancel{2}\times\frac{5}{\cancel{2}} - 3 = 2$

$2=2$, Hence true

10 Continue...

(c) $f\{f(1)\} = f(1) = 1$

$f\{1^2\} = f(1) = 1$

$1^2 = 1^2 = 1$ ✓ true

(A), (B) & (C) true

(11)

## Multiple Correct Answer Type

The range of the function $f(\theta)=\sqrt{8\sin^2\theta+4\cos^2\theta-8\sin\theta\cos\theta}$ is -

(A) $\left[\sqrt{5}-1,\ \sqrt{5}+1\right]$

(B) $\left[0,\ \sqrt{5}+1\right]$

(C) $\left[\sqrt{6-\sqrt{20}},\ \sqrt{6+\sqrt{20}}\right]$

(D) none of these

Sol.

We have $f(\theta)=\sqrt{8\sin^2\theta+4\cos^2\theta-8\sin\theta\cos\theta}$

$\Rightarrow \sqrt{4(\sin^2\theta+\cos^2\theta)+4\sin^2\theta-8\sin\theta\cos\theta}$

$\Rightarrow \sqrt{4+2(1-\cos 2\theta)-4\sin 2\theta}$

$\Rightarrow \sqrt{4+2-2\cos 2\theta-4\sin 2\theta}$

$\Rightarrow \sqrt{6-2\cos 2\theta-4\sin 2\theta}$

$\Rightarrow \sqrt{2\times 3-2(\cos 2\theta+2\sin 2\theta)}$

$\Rightarrow \sqrt{2}\sqrt{3-(\cos 2\theta+2\sin 2\theta)}$

$f(\theta)$ will be min. when $(\cos 2\theta+\sin 2\theta)$ is maximum

& $f(\theta)$ will be max. when $(\cos 2\theta+\sin 2\theta)$ is minimum

$\therefore f(\theta)_{min}=\sqrt{2}\sqrt{3-\sqrt{1^2+2^2}} = \sqrt{6-2\sqrt{5}} = \sqrt{6-\sqrt{20}}$

$f(\theta)_{max}=\sqrt{2}\sqrt{3+\sqrt{1^2+2^2}} = \sqrt{6+2\sqrt{5}}\quad \sqrt{6+\sqrt{20}}$

Or

$\sqrt{6-2\sqrt{5}} = \sqrt{(\sqrt{5})^2+(1)^2-2\sqrt{5}.1} = \sqrt{(\sqrt{5}-1)^2} = (\sqrt{5}-1)$

& $\sqrt{6+2\sqrt{5}} = \sqrt{(\sqrt{5})^2+(1)^2+2\sqrt{5}.1} = \sqrt{(\sqrt{5}+1)^2} = \sqrt{5}+1$

$\therefore$ (A) & (C) are correct answer.

—×—

(12)

For the function $f(x) = |x+3| - |x+1| - |x-1| + |x-3|$, identify correct option(s)

(A) Range of $f(x)$ is $(-\infty, 4]$

(B) maximum value of $f(x)$ is 4

(C) $f(x) = 4$ has infinite solutions

(D) $f(x) = 0$ has infinite solutions

**Sol.** We have

$f(x) = |x+3| - |x+1| - |x-1| + |x-3|$

$x = -3, -1, 1$ & $3$

$-\infty \quad -3 \quad -1 \quad 0 \quad 1 \quad 3 \quad +\infty$

I $\quad$ II $\quad$ III $\quad$ IV $\quad$ V

Case (I): $x \le -3$

$f(x) = -(x+3) + (x+1) + (x-1) - (x-3)$

$= \cancel{-x} \cancel{-3} + \cancel{x} + \cancel{1} + \cancel{x} \cancel{-1} - \cancel{x} + \cancel{3} = 0$

$f(x) = 0$

Case (II): $-3 < x \le -1$

$f(x) = x + 3 + x + \cancel{1} + \cancel{x} \cancel{-1} - \cancel{x} + 3$

$f(x) = 2x + 6$

$f(-1) = -2 + 6 = 4$ Maximum value of $f(x)$ is 4

Case (III): $-1 < x \le 1$

$f(x) = (\cancel{x}+3) - (\cancel{x}+1) + \cancel{x} - 1 - \cancel{x} + 3$

$f(x) = 4$ Maximum value of $f(x)$ is 4.

Case (IV): $1 < x \le 3$

$f(x) = (x+3) - x - 1 - x + 1 - x + 3$

$= -2x + 6$

$f(3) = -6 + 6 = 0$

$f(1) = -2 + 6 = 4$

Maximum value of $f(x)$ is 4

Case (V): $x > 3$

$f(x) = x + 3 - (x+1) - (x-1) + (x-3) = 0$

$\Rightarrow f(x) = 0$

$\therefore$ Answers are

(B) Maximum value of $f(x)$ is 4

(C) $f(x) = 4$ has infinite solns

(D) $f(x) = 0$ has infinite solutions

—X—

13

Which of the following statement(s) is(are) correct ?

(A) If f is a one-one mapping from set A to A, then f is onto.

(B) If f is an onto mapping from set A to A, then f is one-one

(C) Let f and g be two functions defined from $\mathbb{R} \to \mathbb{R}$ such that gof is injective, then f must be injective.

(D) If set A contains 3 elements while set B contains 2 elements, then total number of functions from A to B is 8.

1

**Solution:**

(A) If f is a One-One mapping from set A to A then f is on to

Sol. ~~If~~ let $f : N \to N$ & $f(x) = x+1$, then f is one-one but not on to

let $y = x+1$

$x = y-1$

$f^{-1}(x) = x-1$, there is no element correspond to 1 in the natural no. ∵ 0 is not a natural no.

so it is $\boxed{\text{Not correct}}$

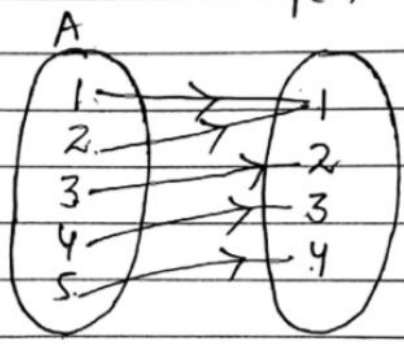
(B) If f is an on to mapping from set A to A then f is one-one

Sol.

let $f : N \to N$

$f(x) = x-1 \; ; \; x \geq 3$ &

$f(1) = f(2) = 1$




∵ function is on to

& f is on to but many-one

∴ Not correct

## 13 Continue...

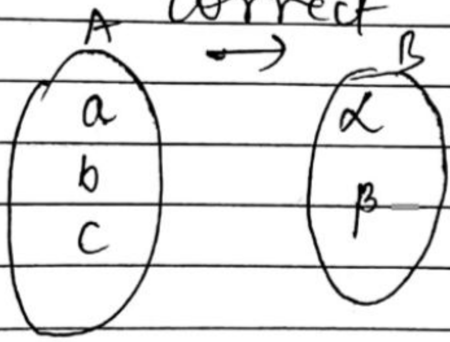
(C) Let $f$ & $g$ be two functions defined from $R \to R$ such that $g \circ f$ is injective, then $f$ must be injective.

Sol. Composition of two injective function is injective, so if $g \circ f$ is injective then $f$ must be injective.

---

(D) If set A consist 3 element while set B contains 2 elements, then total no of functions from A to B is 8

Correct




$2^3 = 8$

(14)

If $f(x) = ax + b$ and $f(f(f(x))) = 27x + 13$ where a and b are real numbers, then-

(A) $a + b = 3$ (B) $a + b = 4$ (C) $f'(x) = 3$ (D) $f'(x) = -3$

**Sol:** we have $f(x) = ax+b$

$f(f(f(x))) =$ ~~a~~

$f(f(x)) = a(ax+b)+b = a^2x+ab+b$

$f(f(f(x))) = a(a^2x+ab+\cancel{a}b)+b$

$= a^3x+a^2b+ab+b = 27x+13$

$\Rightarrow a^3 = 27 \Rightarrow a = 3$ & $a^2b+ab+b = 13$

$b(a^2+a+1) = 13$

$b(3^2+3+1) = 13$

$\Rightarrow b = 1$

$\therefore a+b = 1+3 = 4$

(B) is correct

$f(x) = ax+b = 3x+1$

$f'(x) = 3$ ✓ (C) is correct

**15.** Let $f(x)=\begin{cases} x^2-4 & \text{if } |x|\le 3 \\ 5\,\text{sgn}|x-3| & \text{if } |x|>3 \end{cases}$

and $g(x) = 2\tan^{-1}(e^x) - \frac{\pi}{2}$ for all $x \in R$, then which of the following is wrong ?

(where sgn(x) denotes signum function of x)

(A) $f(x)$ is an even function

(B) $gof(x)$ is an even function

(C) $g(x)$ is an odd function

(D) $fof(x)$ is an odd function

$\boxed{\text{Answer: D}}$

**Solution:**

$$f(x)=\begin{cases} x^2-4 \ ; & |x|\le 3 \\ 5 \ ; & |x|>3 \end{cases}$$

as $\text{sgn}|x-3| = \begin{cases} 0 \ ; & x=3 \\ 1 \ ; & x\neq 3 \end{cases}$

$\therefore \ f(-x) = f(x) \Rightarrow$ Even function.

$$g(x)+g(-x) = 2\tan^{-1}e^x - \frac{\pi}{2} + 2\tan^{-1}e^{-x} - \frac{\pi}{2}$$

$$= 2\left(\tan^{-1}e^x + \tan^{-1}\frac{1}{e^x}\right) - \pi$$

$$= 2\left(\tan^{-1}e^x + \cot^{-1}e^x\right) - \pi$$

$$= 2\left(\frac{\pi}{2}\right) - \pi = 0 \Rightarrow g(x) \text{ is odd}$$

Hence, $gof(x)$ Even & $fof(x)$ Even

A, B, C are correct, So $\boxed{D}$ is wrong

16

## Matrix Match Type

| Column-I Number of integers in | | Column-II | |
|---|---|---|---|
| (A) | Domain of $f(x) = \ln\{x\}$ | (P) | 0 |
| (B) | Domain of $f(x) = \sqrt{\sec(\sin x)} + \sqrt{\left[x + \frac{1}{x}\right]} + \sqrt{10 - [x]^2}$ | (Q) | 2 |
| (C) | Range of $f(x) = x^2 - 2x + 2,\ x \in [0,2]$ | (R) | 3 |
| (D) | Range of $f(x) = \sqrt{25 - [x]^2}$ | (S) | less than 3 |
| | | (T) | more than 3 |

(where [.] and {.} denote greatest integer function and fractional part function respectively)

**Solutions:**

Column(I)

(A) Domain of $f(x) = \ln\{x\}$

$f(x)$ to be defined $\{x\} > 0$

$\Rightarrow x \neq 0$ & $x \neq 1$

$\therefore$ domain consists of no integer i.e. (P) 0 & (S) less than 3 are correct match.

(B) Domain of $f(x) = \underbrace{\sqrt{\sec(\sin x)}}_{\mathrm{I}} + \underbrace{\sqrt{\left[x + \frac{1}{x}\right]}}_{\mathrm{II}} + \underbrace{\sqrt{10 - [x]^2}}_{\mathrm{III}}$

(I) $\sqrt{\sec(\sin x)}$ to be defined

$\sec(\sin x) > 0$ $\qquad \because -1 \le \sin x \le 1, x \in R$ & $\sec(-\theta) = \sec\theta$

$\therefore x \in R$

(II) $\sqrt{\left[x + \frac{1}{x}\right]}$ to be defined

$x + \frac{1}{x} \geq 0, \quad x + \frac{1}{x} \geq 2 \quad x \in R^+$

(16) Continues. .

(III) $\sqrt{10-[x]^2}$ to be defined

$10-[x]^2 \geq 0$

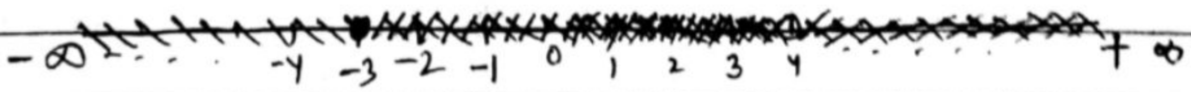
$\Rightarrow [x]^2-10 \leq 0$

$\Rightarrow [x]^2 \leq 10$

$\Rightarrow |[x]| \leq \pm\sqrt{10} \Rightarrow [x] \in [-\sqrt{10}, +\sqrt{10}]$

$\Rightarrow [x] = \{-3,-2,-1,0,1,2,3\}$

$\Rightarrow x \in [-3, 4)$



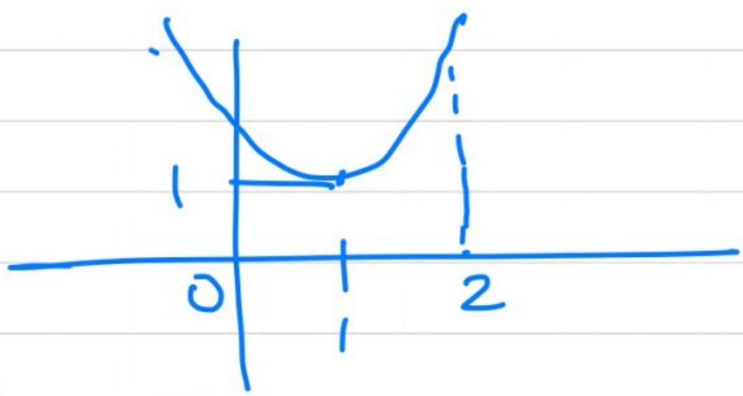

$x \in \{1,2,3\}$, Hence

(R) is correct option

(C)

(C) $f(x) = x^2 - 2x + 2$

or $y = (x-1)^2 + 1$




By graph $f(x)_{min} = 1$ at $x = 1$

$f(x)_{max} = 2$ at $x = 0$ or $x = 2$

So Range of $f(x) \in [1, 2]$ and

No. of integers are $\underline{2}$

(16) Continues...

(D)

Range of $f(x) = \sqrt{25-[x]^2}$

$f(x)$ to be defined $25-[x]^2 \geq 0$

$\Rightarrow [x]^2 - 25 \leq 0$

$[x]^2 \leq 25$

$|[x]| \leq \pm 5$

$[x] \in [-5, +5]$

$\Rightarrow [x] \in \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

$\Rightarrow y \in [0, 5]$

$\Rightarrow y \in \{0, 1, 2, 3, 4, 5\}$

In this also ~~P~~ 'T' is correct option.

17

Match the function mentioned in column-I with the respective classification given in column-II. (where [.] and {.} denote greatest integer function and fractional part function respectively)

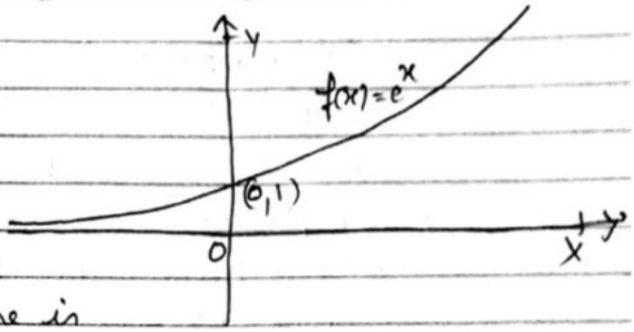
| Column-I | Column-II |
|---|---|
| (A) $f:\mathbb{R}\to\mathbb{R}^+$ $f(x)=\left(e^{[x]}\right)\left(e^{\{x\}}\right)$ | (P) one-one |
| (B) $f:(-\infty,-2)\cup(0,\infty)\to\mathbb{R}$ $f(x) = \ell n(x^2 + 2x)$ | (Q) many-one |
| (C) $f : [-2,2] \to [-1,1]$ $f(x) = \sin x$ | (R) onto |
| (D) $f : \mathbb{R} \to \mathbb{R}$ $f(x) = x^3 - 3x^2 + 3x - 7$ | (S) periodic |
| | (T) aperiodic |

Column I

(A) $f: R\to R^+$ $f(x) = \left(e^{[x]}\right)\left(e^{\{x\}}\right)$

$f(x) = e^{[x]} \cdot e^{x-[x]} = e^{\cancel{[x]}+x-\cancel{[x]}}$

$= e^{x}$




$\therefore f(x)$ is one-one function and

$\because$ for $\forall x \in R$ there is a unique $y \in R^+$ therefore $f(x)$ is on-to also and it is an aperiodic (P)(R)(T)

(B) $f:(-\infty,-2)\cup(0,\infty)\to R$, $f(x) = \ln(x^2+2x)$

Let $x_1$ & $x_2$ are two arbitrary points in $f(x)$'s domain for which, where, $x_1 \neq x_2$

$f(x_1) = f(x_2)$

$\Rightarrow \ln(x_1^2+2x_1) = \ln(x_2^2+2x_2)$

$\Rightarrow x_1^2+2x_1 = x_2^2+2x_2$

$x_1^2-x_2^2+2(x_1-x_2)=0$

$(x_1-x_2)(x_1+x_2+2)=0$

$x_1-x_2=0$ or $x_1+x_2+2=0$ $\quad\because x_1\neq x_2$

$\Rightarrow x_1 = -(2+x_2)$

$\therefore$ for such relation ship $f(x)$ is many-one function i.e. Q is correct option,

Now $f(x) = \ln(x^2+2x)$, $f(x)$ to be defined

$x^2+2x>0 \Rightarrow 0<x(x+2)<\infty$ taking log both sides

$\ln 0 < \ln x(x+2) < \ln\infty$ $\quad\left[\because e>1,\ \log_e\infty\to\infty,\ \log_e 0\to-\infty\right.$

$-\infty < \ln x(x+2) < \infty$

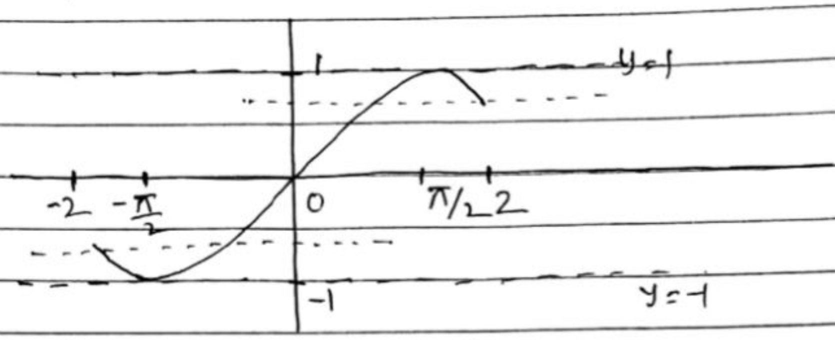
17 Continues...

$\Rightarrow f(x) \in R \Rightarrow f(x)$ is onto as for all values of $x$ in its domain gives all $f(x) \in R$, $\therefore$ it is aperiodic

(41)

$\therefore$ (Q)(R)(T) are correct options

(C) $f : [-2, 2] \to [-1, 1]$, $f(x) = \sin x$




$\therefore$ it is clear that $f(x)$ is many one onto and and aperiodic function (Q)(R)(T) are correct.

~~~x~~~

(D) $f : R \to R$ $f(x) = x^3 - 3x^2 + 3x - 7$

We have $f'(x) = 3x^2 - 6x + 3$

$\Rightarrow 3(x^2 - 2x + 1)$

$\Rightarrow 3(x-1)^2$

$\Rightarrow f'(x) \geq 0 \ \forall \ x \in R$

$\therefore$ it is an increasing function so it is one-one

$\because$ it is an odd degree polynomial so it's range is $\in R$

~~~x~~~

# EXERCISE (S-1)

**1.** Find the domains of definitions of the following functions :

(Read the symbols [*] and {*} as greatest integers and fractional part functions respectively.)

(i) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

(ii) $f(x) = \log_7 \log_5 \log_3 \log_2(2x^3 + 5x^2 - 14x)$

(iii) $f(x) = \ln\left(\sqrt{x^2 - 5x - 24} - x - 2\right)$

(iv) $y = \log_{10} \sin(x - 3) + \sqrt{16 - x^2}$

(v) $f(x) = \sqrt{x^2 - |x|} + \dfrac{1}{\sqrt{9 - x^2}}$

Solutions of S-1

S-1

(1)

(i) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

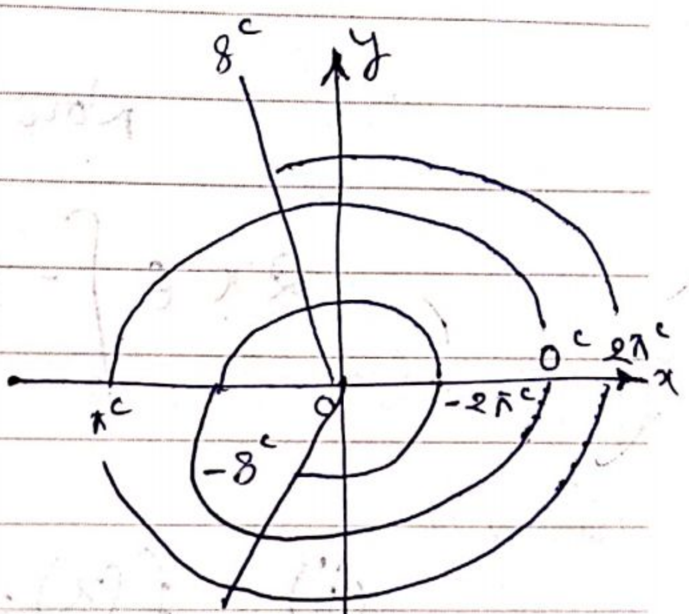
here $16 - x^2 \geq 0$ and $\cos 2x \geq 0$

$\Rightarrow x^2 - 16 \leq 0$

$\Rightarrow (x-4)(x+4) \leq 0$

$\Rightarrow x \in [-4, 4]$

$\Rightarrow 2x \in [-8, 8]$




we can see that $\cos 2x$ is positive if $2x$ lies in Ist and IV$^{th}$ quadrant as well as in $[-8, 8]$

so required interval is

$$2x \in \left[-\frac{5\pi}{2}, -\frac{3\pi}{2}\right] \cup \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$$

$$\Rightarrow x \in \left[-\frac{5\pi}{4}, -\frac{3\pi}{4}\right] \cup \left[-\pi/4, \pi/4\right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$$

alternate:-

here $2x \in [-8, 8]$

and $\cos 2x$ is positive if

$$2x \in \left[+2n\pi - \pi/2,\ 2n\pi + \pi/2\right]$$

Now put $n = -1, 0, 1$

$$2x \in \left[-\frac{5\pi}{2}, -\frac{3\pi}{2}\right] \cup \left[-\pi/2, \pi/2\right] \cup \left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$$

(ii) $f(x) = \log_7 \log_5 \log_3 \log_2 (2x^3 + 5x^2 - 14x)$

$\Rightarrow \log_5 \log_3 \log_2 (2x^3 + 5x^2 - 14x) > 0$

$\Rightarrow \log_3 \log_2 (2x^3 + 5x^2 - 14x) > 1$

$\Rightarrow \log_2 (2x^3 + 5x^2 - 14x) > 3$

$\Rightarrow 2x^3 + 5x^2 - 14x > 8$

$\Rightarrow 2x^3 + 5x^2 - 14x - 8 > 0$

$\Rightarrow (x-2)(x+4)(x+1/2) > 0$

$-\quad +\quad -\quad +$

$-4 \quad -1/2 \quad 2$

$\Rightarrow x \in (-4, -1/2) \cup (2, \infty)$

(iii) $f(x) = \ln\left(\sqrt{x^2-5x-24} - x - 2\right)$

here $x^2-5x-24 \geq 0$

$\Rightarrow x^2-8x+3x-24 \geq 0$

$\Rightarrow x(x-8)+3(x-8) \geq 0$

$\Rightarrow (x+3)(x-8) \geq 0$

$\Rightarrow$ $+ \quad - \quad +$ (at $-3$ and $8$)

$\Rightarrow x \in (-\infty, -3] \cup [8, \infty)$ —— (1)

Again

$\sqrt{x^2-5x-24} - x - 2 > 0$

$\Rightarrow \sqrt{x^2-5x-24} > x+2$

Now if $x+2 \geq 0$ then squarring both sides

$\Rightarrow x^2-5x-24 > x^2+4x+4$

$\Rightarrow 9x < -28$

$\Rightarrow x < -\frac{28}{9}$ —— (2)

so from (1) & (2)

$x \in \left(-\infty, -\frac{28}{9}\right)$ —— (3)

but $x+2$ must be positive so reject (3)

Now if $x+2 \leq 0$ —— (4)

then $\sqrt{x^2-5x-24} > x+2$

is always true

from (1) & (4)

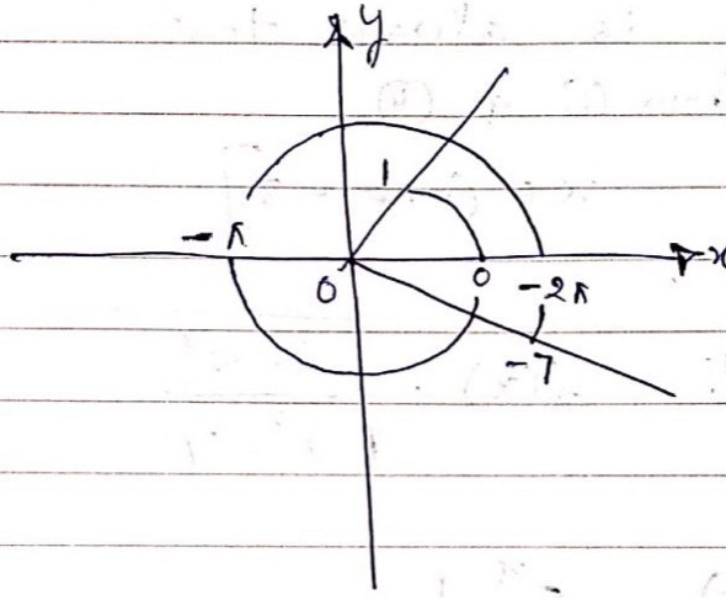
$x \in (-\infty, -3]$

(iv). $y = \log_{10} \sin(x-3) + \sqrt{16-x^2}$

here $16-x^2 \geq 0 \Rightarrow x \in [-4,4]$ —— (1)

if $x \in [-4,4] \Rightarrow x-3 \in [-7,1]$

but $\sin(x-3)$ must be +ve (i.e Ist & IInd quad)




so $(x-3) \in (-2\pi, -\pi) \cup (0,1]$

$\Rightarrow x \in (-2\pi+3, -\pi+3) \cup (3,4]$

[alternatively:- solve by using graph]

(v) $f(x) = \sqrt{x^2 - |x|} + \dfrac{1}{\sqrt{9 - x^2}}$

here $9 - x^2 > 0 \Rightarrow x \in (-3, 3)$ —— (1)

and $x^2 - |x| \geq 0 \Rightarrow x^2 \geq |x|$

$\Rightarrow x^4 - x^2 \geq 0 \Rightarrow x^2 (x^2 - 1) \geq 0$

$\Rightarrow x^2 (x^2 - 1) \geq 0 \Rightarrow x \in (-\infty, -1] \cup [1, \infty) \cup \{0\}$ —— (2)

but from (1) & (2)

$x \in (-3, -1] \cup [1, 3) \cup \{0\}$

**2.** Find the domain & range of the following functions.

(i) $y = \log_{\sqrt{5}}\left(\sqrt{2}(\sin x - \cos x) + 3\right)$

(ii) $y = \dfrac{2x}{1+x^2}$

(iii) $f(x) = \dfrac{x^2 - 3x + 2}{x^2 + x - 6}$

(iv) $f(x) = \dfrac{x}{1+|x|}$

(v) $f(x) = \dfrac{\sqrt{x+4} - 3}{x-5}$

**Solution:**

(i) $y = \log_{\sqrt{5}}\left(\sqrt{2}(\sin x - \cos x) + 3\right)$

$\because \sqrt{2}(\sin x - \cos x) + 3 > 0$

$\Rightarrow \sin x - \cos x > \dfrac{-3}{\sqrt{2}}$

but $\sin x - \cos x \in [-\sqrt{2}, \sqrt{2}]$

so $\sin x - \cos x$ is always greater that $-3/\sqrt{2}$

so domain $x \in R$

again $\sin x - \cos x \in [-\sqrt{2}, \sqrt{2}]$

$\Rightarrow \sqrt{2}(\sin x - \cos x) \in [-2, 2]$

$\Rightarrow \sqrt{2}(\sin x - \cos x) + 3 \in [1, 5]$

$\Rightarrow \log_{\sqrt{5}}\left(\sqrt{2}(\sin x - \cos x) + 3\right) \in [0, 2]$

<u>2(ii)</u> $y = \dfrac{2x}{1+x^2}$

it is obvious $x \in R$

$y = \dfrac{2}{\frac{1}{x} + x}$

but $x + \frac{1}{x} \in (-\infty, -2] \cup [2, \infty)$

$\Rightarrow \dfrac{2}{x + \frac{1}{x}} \in [-1, 1]$

alternate :- $1 + x^2 \geq 2|x|$

$\Rightarrow \dfrac{2|x|}{1+x^2} \leq 1 \Rightarrow \dfrac{2x}{1+x^2} \in [-1, 1]$

2(iii) $f(x) = \dfrac{x^2-3x+2}{x^2+x-6}$

$f(x) = \dfrac{(x-1)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$ but $x \neq 2, -3$

$y = \dfrac{x-1}{x+3}$ where $x \neq 2$

$\Rightarrow y \neq \dfrac{1}{5}$

$\Rightarrow xy + 3y = x - 1$

$\Rightarrow x(y-1) = -1-3y$

$\Rightarrow x = \dfrac{-1-3y}{y-1} \Rightarrow y \neq 1$

alternate:-

if $y = \dfrac{ax-b}{cx-d} \Rightarrow y \neq \dfrac{a}{c}$

where $c \neq 0$

2(iv) $f(x) = \dfrac{x}{1+|x|}$

$\Rightarrow x \in R$

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(iv) $f(x) = \frac{x}{1+|x|}$

C-1 : when $x \geq 0$

$$y = \frac{x}{1+x} = 1 - \frac{1}{(1+x)}$$

at $x = 0$, $y = 0$,

& $x \to \infty$ $y \to 1$

So $y \in [0, 1)$

C-2 : $x < 0$

$$y = \frac{x}{1-x} = -1 + \frac{1}{(1-x)}$$

$x \to -\infty$, $y \to -1$

So $x < 0$ then

$y \in (-1, 0)$

so final range is

$(-1, 1)$

---

2(v)

$$f(x) = \frac{\sqrt{x+4} - 3}{\left(\sqrt{x+4}\right)^2 - (3)^2}$$

for domain $x+4 \geq 0 \Rightarrow x \geq -4$

and $x-5 \neq 0 \Rightarrow x \neq 5$

$x \in [-4, \infty) - \{5\}$

again

$$f(x) = \frac{\sqrt{x+4} - 3}{\left(\sqrt{x+4} - 3\right)\left(\sqrt{x+4} + 3\right)}$$

$$= \frac{1}{\sqrt{x+4} + 3}$$

but $x \in [-4, \infty) - \{5\}$

$\Rightarrow \quad x+4 \in [0, \infty) - \{9\}$

$\Rightarrow \sqrt{x+4} \in [0, \infty) - \{3\}$

$\Rightarrow \sqrt{x+4} + 3 \in [3, \infty) - \{6\}$

$\Rightarrow \dfrac{1}{\sqrt{x+4} + 3} \in \left(0, \tfrac{1}{3}\right] - \left\{\tfrac{1}{6}\right\}$

3

The sum of integral values of the elements in the domain of $f(x)=\sqrt{\log_{\frac{1}{2}}|3-x|}$ is

**Solution:**

$$f(x)=\sqrt{\log_{1/2}|3-x|}$$

$$\log_{1/2}|3-x| \geq 0$$

$$\Rightarrow \quad 0<|3-x|\leq 1$$

$$\Rightarrow \quad x-3 \in [-1,1]-\{0\}$$

$$\Rightarrow \quad x \in [2,4]-\{3\}$$



so possible integral values of $x$ are
2 & 4

(4)

Number of integers in range of $f(x) = x(x+2)(x+4)(x+6)+7,\ x \in [-4,2]$ is

**Solution:**

$$f(x) = x(x+2)(x+4)(x+6)+7$$

$$\Rightarrow f(x) = (x^2+6x)(x^2+6x+8)+7$$

Now let $x^2+6x+4 = t$

$$\Rightarrow g(t) = (t-4)(t+4)+7$$

$$\Rightarrow g(t) = t^2-9 \quad \text{---} \ (1)$$

But $x \in [-4,2]$

Now $t = (x+3)^2-5$

so $x+3 \in [-1,5]$

$$\Rightarrow (x+3)^2 \in [0,25]$$

$$\Rightarrow (x+3)^2-5 \in [-5,20]$$

$$\Rightarrow t \in [-5,20]$$

Now $g(t) = t^2-9 \Rightarrow t^2 \in [0,400]$

$$g(t) = t^2-9 \in [-9,391]$$

so number of integers are 401

5

Identify the pair(s) of functions which are identical ?
(where [x] denotes greatest integer and {x} denotes fractional part function)

(i) $f(x) = sgn(x^2 - 3x + 4)$ and $g(x) = e^{[\{x\}]}$ (ii) $f(x) = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$ and $g(x) = \tan x$

(iii) $f(x) = ln(1 + x) + ln(1 - x)$ and $g(x) = ln(1 - x^2)$ (iv) $f(x) = \frac{\cos x}{1-\sin x}$ and $g(x) = \frac{1+\sin x}{\cos x}$

**Solution:**

(i) $f(x) = sgn(x^2 - 3x + 4)$

$D < 0$

$= sgn(+ve) = 1$

$g(x) = e^{[\{x\}]} = e^{[0 \text{ to } 1)} = e^0$

$= 1$

identical because same domain & range (for each value of $x$)

(ii) $f(x) = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}} = \sqrt{\frac{2\sin^2 x}{2\cos^2 x}} = |\tan x|$

$g(x) = \tan x$

different range (Not identical)

(iii) $f(x) = ln(1+x) + ln(1-x)$

here $1+x > 0 \Rightarrow x > -1$

and $1-x > 0 \Rightarrow x < 1$

$x \in (-1, 1)$

(5) Continues...

if $x \in (-1,1)$

$f(x)$ becomes $\ln(1-x^2)$

but $g(x) = \ln(1-x^2)$

$\Rightarrow 1-x^2 > 0$

$\Rightarrow x^2-1 < 0$

$\Rightarrow x \in (-1,1)$

so identical

(iv) $f(x) = \dfrac{\cos x}{1-\sin x} \times \dfrac{1+\sin x}{1+\sin x}$

$\Rightarrow \dfrac{1+\sin x}{\cos x}$

but $1-\sin x \neq 0$

$\Rightarrow \sin x \neq 1$

$\Rightarrow x \neq \{2n\pi + \pi/2\}$

$g(x) = \dfrac{1+\sin x}{\cos x}$

but $\cos x \neq 0 \Rightarrow x \neq n\pi + \pi/2$

different domains (Not identical)

6

Classify the following functions f(x) defined in $\mathbb{R} \to \mathbb{R}$ as injective, surjective, both or none.

(a) $f(x) = \dfrac{x^2+4x+30}{x^2-8x+18}$ (b) $f(x) = x^3 - 6x^2 + 11x - 6$ (c) $f(x) = (x^2+x+5)(x^2+x-3)$

Solution:

(a)

$$f(x) = \frac{x^2+4x+30}{x^2-8x+18}$$

$f(x) = \dfrac{\text{quadratic}}{\text{quadratic}}$ with no common factor

so many-one

again $y = \dfrac{x^2+4x+30}{x^2-8x+18}$

$yx^2 - 8xy + 18y = x^2 + 4x + 30$

$\Rightarrow (y-1)x^2 - (8y+4)x + 18y - 30 = 0$

as $x \in R \Rightarrow D \geq 0$

$(8y+4)^2 - 4(y-1)(18y-30) \geq 0$

$\Rightarrow 4(2y+1)^2 - 2(y-1)(9y-15) \geq 0$

$\Rightarrow 2(2y+1)^2 - (9y^2 - 24y + 15) \geq 0$

$\Rightarrow 2(4y^2+4y+1) - 9y^2 + 24y - 15 \geq 0$

(6) Continues...

$-y^2+32y-13 \geq 0$

$\Rightarrow y^2-32y+13 \leq 0$

obvious $y \notin R$

so into

(b). $f(x) = x^3-6x^2+11x-6$
polynomial function with odd power $\Rightarrow$ range is $R$
$\Rightarrow$ onto

Now $f'(x) = 3x^2-12x+11$
$(D>0)$

$f'(x)$ may be +ve or -ve according to $x$ so many-one.

(c) $f(x) = (x^2+x+5)(x^2+x-3)$
is a polynomial of degree 4
so manyone and into (because range can not be $R$)

⑦

Solve the following problems from (a) to (d) on functional equation :

(a) The function f(x) defined on the real numbers has the property that $f(f(x)).(1+f(x))=-f(x)$ for all x in the domain of f. If the number 3 is in the domain and range of f, compute the value of f(3).

(b) Suppose f is a real function satisfying $f(x + f(x)) = 4f(x)$ and $f(1) = 4$. Find the value of f(21).

(c) Let f be function defined from $\mathbb{R}^+ \to \mathbb{R}^+$. If $[f(xy)]^2 = x(f(y))^2$ for all positive numbers x and y and $f(2) = 6$, find the value of f(50).

(d) Let f be a function such that $f(3) = 1$ and $f(3x) = x + f(3x - 3)$ for all x. Then find the value of f(300).

Soln.

(a)

$f(f(x)) \cdot (1+f(x)) = -f(x)$

put $f(x) = x$

$f(x) \cdot (1+x) = -x$

$f(x) = \dfrac{-x}{1+x}$

$f(3) = \dfrac{-3}{4}$

(b) $f(x+f(x)) = 4f(x)$

put $x=1$

$f(1+f(1)) = 4f(1)$

$\Rightarrow f(5) = 16$

Now put $x=5$

$f(5+f(5)) = 4f(5)$

$f(21) = 4 \times 16 = 64$

⑦ Continues...

(c)

$$f^2(xy) = x f^2(y)$$

put $x = 25$ and $y = 2$

$$f^2(50) = 25 \times 36$$

$$f(50) = 5 \times 6 = 30$$

(d)

$$f(3) = 1$$

$$\cancel{f(3x)} + \quad f(3x) = x + f(3x-3)$$

$$\Rightarrow f(3x) - f(3x-3) = x$$

put $x = 2$ $\quad f(6) - f(3) = 2$

put $x = 3$ $\quad f(9) - f(6) = 3$

put $x = 4$ $\quad f(12) - f(9) = 4$

$\vdots$

put $x = 100$ $\quad f(300) - f(297) = 100$

adding $\quad f(300) - f(3) = 2+3+4+\cdots+100$

$$\Rightarrow f(300) = 1+2+3+\cdots+100 = \frac{100 \times 101}{2}$$

$$= 5050.$$

8

Suppose f(x) = sinx and g(x) = 1 – $\sqrt{x}$ . Then find the domain and range of the following functions.

(a) fog (b) gof (c) fof (d) gog

Solution:

$f(x) = \sin x \quad g(x) = 1-\sqrt{x}$

(a) $fog(x) = \sin(1-\sqrt{x}) \in [-1, 1]$

and $x \geq 0$

(b) $gof(x) = 1-\sqrt{\sin x}$

$\sqrt{\sin x} \in [0, 1]$

$-\sqrt{\sin x} \in [-1, 0]$

$1-\sqrt{\sin x} \in [0, 1]$ Range

for domain $\sin x \geq 0$

$x \in [2n\pi, (2n+1)\pi]$

(c) $fof(x) = \sin(\sin x)$

$x \in R$

for range $fof(x) = \sin(-1^c \text{ to } 1^c)$

$\Rightarrow fof(x) \in [-\sin 1, \sin 1]$

8 Continues...

(d). $g\circ g(x) = 1 - \sqrt{1-\sqrt{x}}$

$x \geq 0$ & $1-\sqrt{x} \geq 0$

$\sqrt{x} \leq 1$

$x \leq 1$

$\Rightarrow x \in [0,1]$

$\sqrt{x} \in [0,1]$

$-\sqrt{x} \in [-1,0]$

$1-\sqrt{x} \in [0,1]$

$\sqrt{1-\sqrt{x}} \in [0,1]$

$-\sqrt{1-\sqrt{x}} \in [-1,0]$

$1-\sqrt{1-\sqrt{x}} \in [0,1]$

9

A function $f : \mathbb{R} \to \mathbb{R}$ is such that $f\left(\frac{1-x}{1+x}\right) = x$ for all $x \neq -1$. Prove the following.

(a) $f(f(x)) = x$ (b) $f(1/x) = -f(x)$, $x \neq 0$ (c) $f(-x-2) = -f(x) - 2$.

**Solution:**

$$f\left(\frac{1-x}{1+x}\right) = x$$

let $\frac{1-x}{1+x} = y$

$\Rightarrow 1 - x = xy + y$

$\Rightarrow x(y+1) = 1 - y$

$\Rightarrow x = \frac{1-y}{1+y}$



$\Rightarrow f(y) = \frac{1-y}{1+y}$

(a)

$\Rightarrow f(f(x)) = \dfrac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}} = \dfrac{(1+x)-(1-x)}{(1+x)+(1-x)}$

$= x$

(b) $f(1/x) = \dfrac{1 - 1/x}{1 + 1/x} = \dfrac{x-1}{x+1} = -f(x)$

(c) $f(-x-2) = \dfrac{1-(-x-2)}{1+(-x-2)} = \dfrac{x+3}{-x-1}$

$= -\left(\frac{1-x}{1+x}\right) - 2$ hence proved

(10)

$f(x) = \begin{cases} 1-x & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$ and $g(x) = \begin{cases} -x & \text{if } x < 1 \\ 1-x & \text{if } x \ge 1 \end{cases}$ find (fog)(x) and (gof)(x).

Solution

$$f(x) = \begin{cases} 1-x & x \le 0 \\ x^2 & x > 0 \end{cases}$$

$$g(x) = \begin{cases} -x & x < 1 \\ 1-x & x \ge 1 \end{cases}$$

for fog(x)

$g(x)$

$-x > -1 \Leftarrow x < 1$

$-1 < -x \le 0$ | $-x > 0$

$\boxed{0 \le x < 1}$ | $\boxed{x < 0}$

$\boxed{1-x \le 0} \Leftarrow x \ge 1$

$$\Rightarrow fog(x) = \begin{cases} (-x)^2 & x < 0 \\ 1-(-x) & 0 \le x < 1 \\ 1-(1-x) & x \ge 1 \end{cases}$$

for gof(x)

$f(x)$

$1-x \ge 1 \Leftarrow \boxed{x \le 0}$

$x^2 > 0 \Leftarrow x > 0$

$0 < x^2 < 1$ | $x^2 \ge 1$

$\boxed{0 < x < 1}$ | $\boxed{x \ge 1}$

$$gof(x) = \begin{cases} 1-(1-x) & x \le 0 \\ -(x^2) & 0 < x < 1 \\ 1-x^2 & x \ge 1 \end{cases}$$

**11.** Find whether the following functions are even or odd or none :

(a) $f(x) = \log\left(x+\sqrt{1+x^2}\right)$ (b) $f(x) = \dfrac{x(a^x+1)}{a^x-1}$ (c) $f(x) = \sin x + \cos x$

(d) $f(x) = x\sin^2 x - x^3$ (e) $f(x) = \sin x - \cos x$ (f) $f(x) = \dfrac{(1+2^x)^2}{2^x}$

(g) $f(x) = [(x + 1)^2]^{1/3} + [(x - 1)^2]^{1/3}$

**Solution :**

(a)

$$y = f(x) = \log\left(x+\sqrt{1+x^2}\right)$$

Now. $f(-x) = \log\left(-x+\sqrt{1+x^2}\right)$

$$= \log \frac{\left(\sqrt{1+x^2}-x\right)\left(\sqrt{1+x^2}+x\right)}{\left(\sqrt{1+x^2}+x\right)}$$

$$= \log\left(\frac{1}{x+\sqrt{1+x^2}}\right)$$

$$= -\log\left(x+\sqrt{1+x^2}\right) = -f(x)$$

odd function

(b) $f(x) = \dfrac{x\left(a^x+1\right)}{\left(a^x-1\right)}$

$$\Rightarrow f(-x) = \frac{-x\left(a^{-x}+1\right)}{\left(a^{-x}-1\right)} \times \frac{a^x}{a^x}$$

$$= \frac{-x\left(1+a^x\right)}{1-a^x} = \frac{x\left(a^x+1\right)}{a^x-1}$$

$= f(x) \Rightarrow$ even function

(c) $f(x) = \sin x + \cos x$

11 → $f(-x) = -\sin x + \cos x$

$\neq f(x)$ & $-f(x)$

so none

(d) $f(x) = x\sin^2 x - x^3$

$f(-x) = -x\sin^2 x + x^3$

$= -\left(x\sin^2 x - x^3\right) = -f(x)$

odd function

(e) $f(x) = \sin x - \cos x$

$f(-x) = -\sin x - \cos x$

$\neq -f(x)$ OR $f(x)$

so none

(f) $f(x) = \dfrac{(1+2^x)^2}{2^x}$

$f(-x) = \dfrac{(1+2^{-x})^2}{2^{-x}} \times \dfrac{2^{2x}}{2^{2x}}$

(11) Continues...

$$f(-x) = \frac{(2^x+1)^2}{2^x} = f(x)$$

even function

(g). $f(x) = (x+1)^{2/3} + (x-1)^{2/3}$

$\Rightarrow f(-x) = (x-1)^{2/3} + (x+1)^{2/3}$

$= f(x)$

$\therefore$ is an even function.

12

(i) Write explicitly, functions of y defined by the following equations and also find the domains of definition of the given implicit functions :

(a) $10^x + 10^y = 10$ (b) $x + |y| = 2y$

(ii) The function f(x) is defined on the interval [0, 1]. Find the domain of definition of the functions.

(a) f(sinx) (b) f(2x + 3)

(iii) Given that $y = f(x)$ is a function whose domain is [4,7] and range is [−1, 9]. Find the range and domain of

(a) $g(x) = \frac{1}{3} f(x)$ (b) $h(x) = f(x - 7)$

**Solution:**

(i) (a) $10^x + 10^y = 10$

$\Rightarrow 10^y = 10 - 10^x$

$\Rightarrow y = \log_{10}(10 - 10^x)$

$10 - 10^x > 0 \Rightarrow 10^x < 10 \Rightarrow x < 1$

similarly $y < 1$

(b). $x + |y| = 2y$

if $y \geq 0 \Rightarrow x + y = 2y$

$\Rightarrow y = x$

and if $y \leq 0 \Rightarrow x - y = 2y$

$\Rightarrow y = x/3$

(12) Continues...

(ii)

$x \in [0,1]$

(a) for $y = f(\sin x)$

$\Rightarrow \sin x \in [0,1]$

$\Rightarrow x \in [2n\pi, (2n+1)\pi]$

(b) for $y = f(2x+3)$

$2x+3 \in [0,1] \Rightarrow 2x \in [-3,-2]$

$x \in [-3/2, -1]$

12 Continues...

(iii). for $y = f(x)$

$x \in [4, 7]$

and $f(x) \in [-1, 9]$

(a) Now $g(x) = \frac{1}{3} f(x)$

domain $x \in [4, 7]$

and range $g(x) \in [-1/3, 3]$

(b) $h(x) = f(x-7)$

$x - 7 \in [4, 7]$

$x \in [11, 14]$

but $h(x) \in [-1, 9]$

(13)

Compute the inverse of the functions :

(a) $f(x) = \ell n\left(x+\sqrt{x^2+1}\right)$ (b) $f(x) = 2^{\frac{x}{x-1}}$ (c) $y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$

**Solution:**

(a) $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$

$$x = \ln\left(y + \sqrt{y^2+1}\right)$$

$$\Rightarrow e^x = y + \sqrt{y^2+1} \quad \text{— (1)}$$

$$x = \ln\left(\frac{1}{\sqrt{y^2+1} - y}\right)$$

$$\Rightarrow e^{-x} = \sqrt{y^2+1} - y \quad \text{— (2)}$$

$$(1) - (2) \Rightarrow e^x - e^{-x} = 2y$$

$$\Rightarrow y = \frac{e^x - e^{-x}}{2}$$

(b)

$$y = 2^{\frac{x}{x-1}}$$

interchange $x$ + $y$

$$x = 2^{\frac{y}{y-1}}$$

$$\log_2 x = \frac{y}{y-1}$$

(13) Continues...

$$y \log_2 x - \log_2 x = y$$

$$y(\log_2 x - 1) = \log_2 x$$

$$\Rightarrow y = \frac{\log_2 x}{\log_2 x - 1}$$

(c)

$$y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}} \times \frac{10^x}{10^x}$$

$$y = \frac{10^{2x} - 1}{10^{2x} + 1}$$

$$y \cdot 10^{2x} + y = 10^{2x} - 1$$

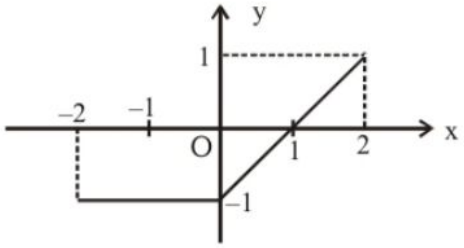
$$10^{2x}(y - 1) = -y - 1$$

$$10^{2x} = \frac{1+y}{1-y} \Rightarrow 2x = \log_{10}\left(\frac{1+y}{1-y}\right)$$

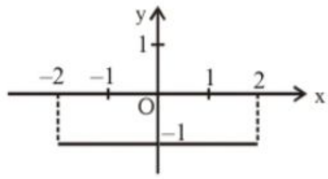
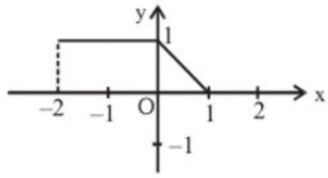
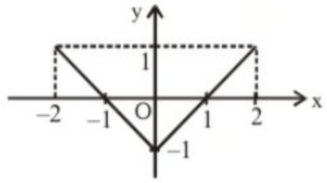
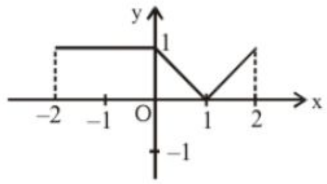
$$\Rightarrow f^{-1}(x) = \frac{1}{2} \log_{10}\left(\frac{1+x}{1-x}\right)$$

14

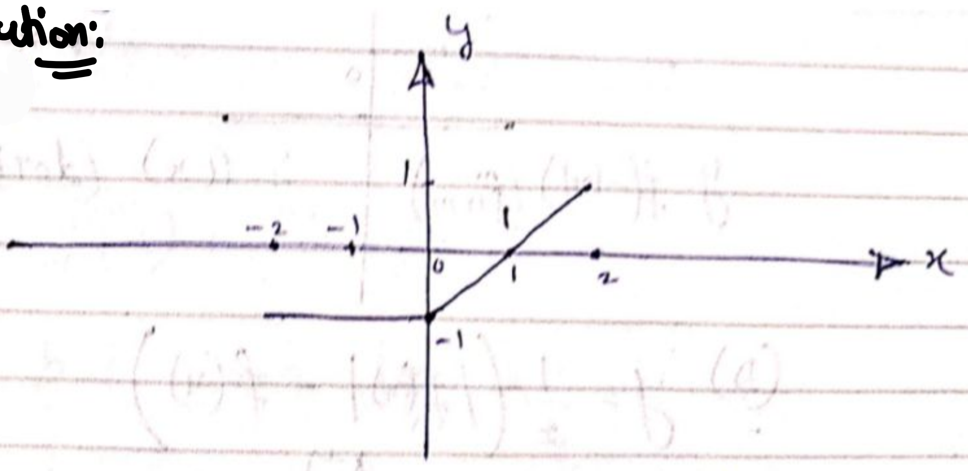
The graph of the function $y = f(x)$ is as follows :



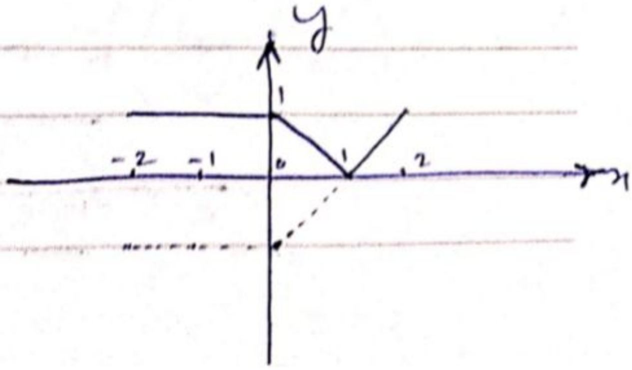
Match the function mentioned in **Column-I** with the respective graph given in **Column-II**.

| **Column-I** | | **Column-II** | |
|---|---|---|---|
| (A) | $y = \lvert f(x)\rvert$ | (P) |  |
| (B) | $y = f(\lvert x\rvert)$ | (Q) |  |
| (C) | $y = f(-\lvert x\rvert)$ | (R) |  |
| (D) | $y = \frac{1}{2}(\lvert f(x)\rvert - f(x))$ | (S) |  |

**Solution:**

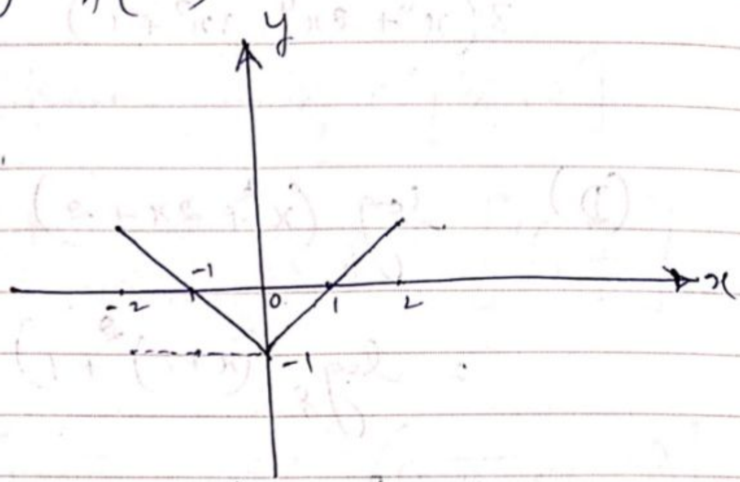


(A) $y = |f(x)|$

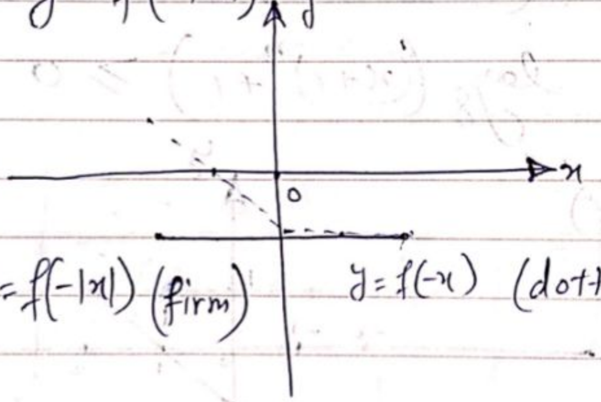


(14) Continues...

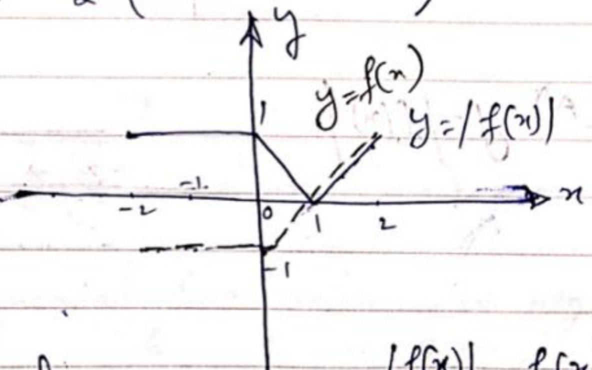
(B) $y = f(|x|)$



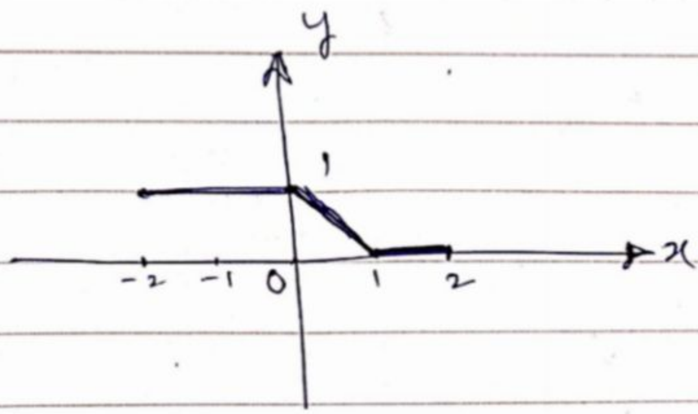
(C) $y = f(-|x|)$




(D) $y = \frac{1}{2}\left(|f(x)| - f(x)\right)$




so final curve for $y = \frac{|f(x)| - f(x)}{2}$ is



(15)

If $f(x) = a\log\left(\frac{1+x}{1-x}\right) + bx^3 + c\sin x + 5$ and $f(\log_3 2) = 4$, then $f\left(\log_3\left(\frac{1}{2}\right)\right)$ is equal to

**Solution:**

$$f(x) = \underbrace{a\log\left(\frac{1+x}{1-x}\right) + bx^3 + c\sin x}_{\substack{\text{odd function} \\ (\text{say } h(x))}} + \underbrace{5}_{\text{even part}}$$

$$f(-x) = -h(x) + 5$$

$$\Rightarrow f(x) + f(-x) = 10$$

put $x = \log_3 2$

$$\Rightarrow f(\log_3(2)) + f\left(\log_3\left(\tfrac{1}{2}\right)\right) = 10$$

$$\Rightarrow f\left(\log_3\left(\tfrac{1}{2}\right)\right) = 6$$

(16)

If $f(x)=\begin{cases}0 & x<1\\ 2x-2 & x\geq 1\end{cases}$; then the number of solutions of the equation $f(f(f(x)))=x$ is

**Solution:**

$$f(x)=\begin{cases}0 & x<1\\ 2x-2 & x\geq 1\end{cases}$$

$$f(f(f(x)))=x$$

$f(x)$

$0<1 \Leftarrow \boxed{x<1}$

$$f(f(x))=\begin{cases}0 & x<1\\ 0 & 1\leq x<3/2 \Leftarrow\\ 2(2x-2)-2 & x\geq 3/2\\ =4x-6 & \end{cases}$$

Tuesday 27

$2x-2\geq 0 \Leftarrow x\geq 1$

$0\leq 2x-2<1$ | $2x-2\geq 1$

$\boxed{1\leq x<3/2}$ | $\boxed{x\geq 3/2}$

$$\Rightarrow f\circ f(x)=\begin{cases}0 & x<3/2\\ 4x-6 & x\geq 3/2\end{cases}$$

$$f\circ f\circ f(x)=\begin{cases}0 & x<3/2\\ 0 & 3/2\leq x<7/4 \Leftarrow\\ 2(4x-6)-2 & x\geq \frac{7}{4}\\ =8x-14 & \end{cases}$$

$f\circ f(x)$

$0<1 \Leftarrow \boxed{x<3/2}$

$f\circ f(x)$

$4x-6\geq 0 \Leftarrow x\geq 3/2$

$1>4x-6\geq 0$ | $4x-6\geq 1$

$\boxed{3/2\leq x<7/4}$ | $\boxed{x\geq 7/4}$

⑯ Continues...

if $x < 7/4$

$\Rightarrow fofof(x) = \boxed{0 = x}$

if $x \geq 7/4$

$fofof(x) = 8x - 14 = x$

$\Rightarrow 7x - 14 = 0$

$\Rightarrow \boxed{x = 2}$

17

Let '$f$' be an even periodic function with period '4' such that $f(x) = 2^x - 1,\ 0 \le x \le 2$.

The number of solutions of the equation $f(x) = 1$ in $[-10, 20]$ are

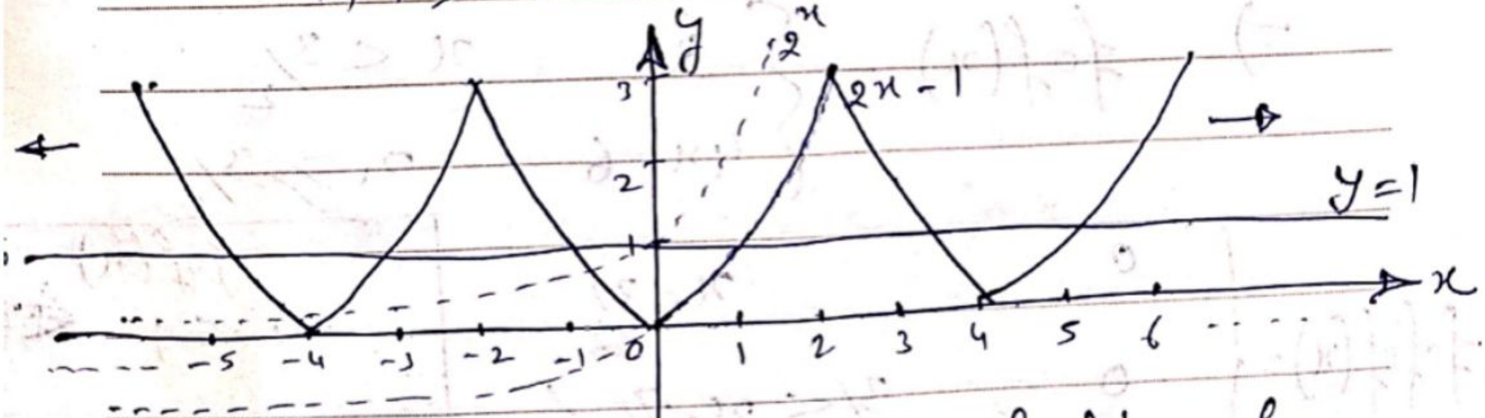
4

**Solution:**

$f(-x) = f(x)$

$T = 4$

$f(x) = 2^x - 1 \quad 0 \le x \le 2$




we are getting one solution for $[0, 2]$ OR $[2, 4]$ OR $[4, 6]$ — —

so number of solutions in $[-10, 20]$ are 15

18

Let $f(x) = \dfrac{2x-1}{x+3}$. If $f^{-1} = \dfrac{ax+b}{c-x}$, then a + b + c is

Solution:

$$f(x) = \frac{2x-1}{x+3}$$

interchange $x$ & $y$

$$x = \frac{2y-1}{y+3} \Rightarrow xy + 3x = 2y - 1$$

$$\Rightarrow y(x-2) = -3x - 1$$

$$\Rightarrow y = \frac{3x+1}{2-x}$$

$$\Rightarrow a = 3,\ b = 1,\ c = 2$$

$$\Rightarrow a + b + c = 6$$

19

Let $f(x)$ be a periodic function with period 'p' satisfying $f(x) + f(x + 3) + f(x + 6) + \ldots + f(x + 42)$ = constant $\forall$ x $\in \mathbb{R}$, then sum of digits of 'p' is

**Solution:**

$$f(x) + f(x+3) + f(x+6) + \_\_\_ + f(x+42) = K$$

put $x \Rightarrow x+3$

$$f(x+3) + f(x+6) + \_\_\_\_ + f(x+45) = K$$

subtract

$$f(x) - f(x+45) = 0$$

$$\Rightarrow f(x) = f(x+45)$$

period $T = 45$

# EXERCISE (S-2)

1 (a) Let $P(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ be a polynomial such that $P(1) = 1; P(2) = 2; P(3) = 3; P(4) = 4; P(5) = 5$ and $P(6) = 6$, then find the value of $P(7)$.

Solution:

here (by Remainder theorem)

(a) $P(x) = (x-1)(x-2)(x-3)(x-4)(x-5)(x-6) + x$

Now solve.

1

(b) Let a and b be real numbers and let $f(x) = a\sin x + b\sqrt[3]{x} + 4, \forall x \in R$. If $f(\log_{10}(\log_3 10))$ $= 5$ then find the value of $f(\log_{10}(\log_{10} 3))$.

(b) $f(x) = a\sin x + b\,{}^{2}\sqrt{x} + 4 \quad \forall\ x \in R$

$f(-x) = -a\sin x - b\sqrt[3]{x} + 4$

$f(x) + f(-x) = 8$

$f(\log_{10}(\log_3 10)) = 5$

$f(\log_{10}(\log_{10} 3)) = f(-\log_{10}\log_3 10) = ?$

$\therefore \quad f(-\log_{10}\log_3 10) = 3$

2

Suppose p(x) is a polynomial with integer coefficients. The remainder when p(x) is divided by x – 1 is 1 and the remainder when p(x) is divided by x – 4 is 10. If r(x) is the remainder when p(x) is divided by (x –1)(x – 4), find the value of r(2006).

**Solution:**

Let $p(x) = A(x-1)(x-4) + (ax+b)$

(linear) $= r(x)$

Now $p(1) = 1$ & $p(4) = 10$ (Given)

So $\left.\begin{aligned} a+b &= 1 \\ 4a+b &= 10 \end{aligned}\right\}$ $a = 3,\ b = -2$

$r(x) = (3x - 2)$

or $r(2006) = 3 \times 2006 - 2$

$= 6016$

③

A function f, defined for all x, y $\in \mathbb{R}$ is such that $f(1) = 2$ & $f(x + y) - kxy = f(x) + 2y^2$, where k is some constant. Find f(x) & show that :

$$f(x+y)f\left(\frac{1}{x+y}\right) = k \text{ for } x + y \neq 0.$$

Solution:

$f(1) = 2, \quad f(2) = 8$

↓

put $x = 2, y = 0$

again put $x = 1 = y$

$f(2) = k + f(1) + 2$

$8 = k + 2 + 2$

$\Rightarrow k = 4$

$\therefore f(x+y) = 4xy + f(x) + 2y^2$

put $y = -x$

$f(0) = 2x^2 + f(x) - 4x^2$

$f(0) = f(x) - 2x^2$

put $x = 1, y = -1$

$f(0) = 0$

or $f(x) = 2x^2$

$\therefore f(x+y) = 2(x+y)^2$

$f\left(\frac{1}{x+y}\right) = \frac{2}{(x+y)^2}$

$\therefore f(x+y) \cdot f\left(\frac{1}{x+y}\right) = 4$

4

If $f(x) = -1 + |x - 2|,\ 0 \le x \le 4$

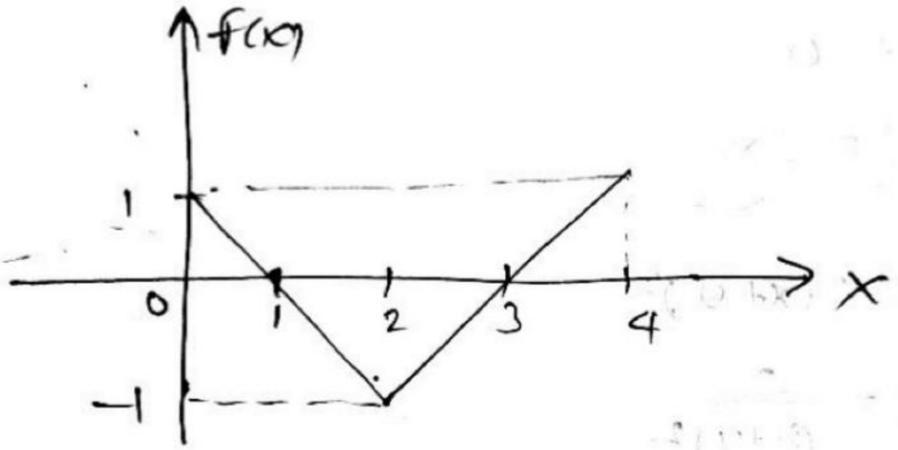
$g(x) = 2 - |x|,\ -1 \le x \le 3$

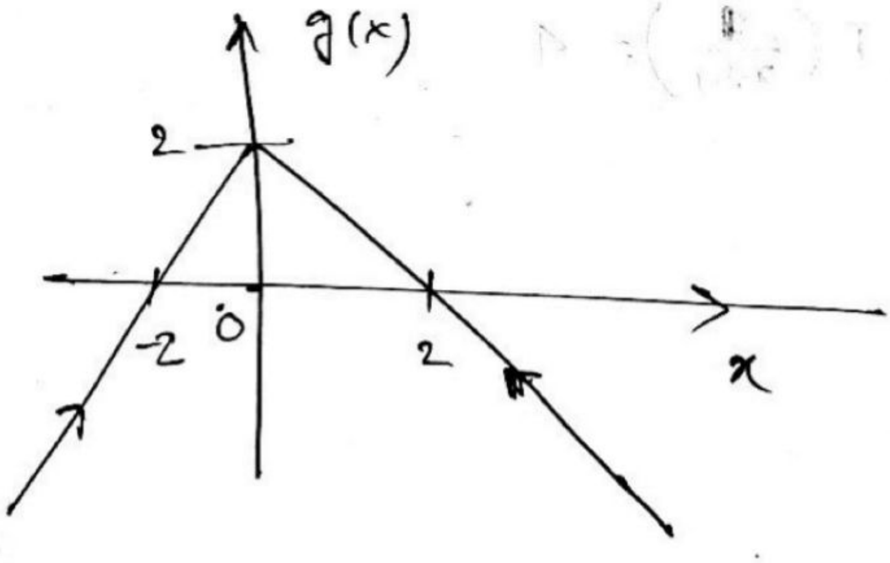
Then find fog(x) & gof(x). Draw rough sketch of the graphs of fog(x) & gof(x).

**Solution:**

$f(x) = -1 + |x-2|, \quad 0 \le x \le 4$

$g(x) = 2 - |x|, \quad -1 \le x \le 3$



$\boxed{4}$ Continues...

$$f(x) = \begin{cases} (1-x) , & x < 2 \\ (x-3) & x \geq 2 \end{cases}$$

$$g(x) = \begin{cases} 2+x & -1 \leq x < 0 \\ 2-x & 0 \leq x < 3 \end{cases}$$

<u>Now</u>

$$f\circ g = \begin{cases} 1-g , & g < 2 \\ g-3 , & g \geq 2 \end{cases}$$

C-1 If $g < 2$

$-1 \leq x < 0$ $\qquad$ $0 \leq x < 3$

$g = 2+x$ $\qquad$ $g = 2-x$

$$\therefore f\circ g = \begin{cases} 1-(2+x) , & -1 \leq x < 0 \\ 1-(2-x) , & 0 \leq x < 3 \end{cases}$$

C-2 $g > 2$ not possible

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4 Continues...

$$gof = \begin{cases} 2+f & -1 \leq f < 0 \\ 2-f & 0 \leq f < 3 \end{cases}$$

C-1 $\quad -1 \leq f < 0$

$f = 1 - x$

$1 \leq x < 2$

$f = x - 3$

$2 \leq x \leq 3$

C-2 $\quad 0 \leq f \leq 1$

$f = 1 - x$

$0 \leq x < 1$

$f = x - 3$

$3 < x \leq 4$

$\boxed{4}$ Continues...

$$gof = \begin{cases} 2+(1-x) & , \; 1 \le x < 2 \\ 2+(x-3) & \; 2 \le x \le 3 \\ 2-(1-x) & \; 0 \le x < 1 \\ 2-(x-3) & \; 3 < x \le 4 \end{cases}$$

$$gof = \begin{cases} 3-x & , \; 1 \le x < 2 \\ -1+x & , \; 2 \le x \le 3 \\ 1+x & , \; 0 \le x < 1 \\ 5-x & , \; 3 < x \le 4 \end{cases}$$

5

Let $f(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$. If $f(x)$ is divided by $x^3 - x$ then the remainder is some function of x say g(x). Find the value of g(10).

**Solution:**

Let

$$f(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$$

and if $f(x)$ is divisible by $x(x+1)(x-1)$

then Remainders are

$f(0) = 1$, $f(1) = 3$, $f(-1) = -1$

$$x^{135} + x^{125} - x^{115} + x^5 + 1 = A(x(x+1)(x-1)) + ax^2 + bx + c \quad \text{(1)}$$

by eqn (1)

$$\left.\begin{aligned} c &= 1 \\ a+b+c &= 3 \\ a-b+c &= -1 \end{aligned}\right\} \quad a = 0,\ b = 2,\ c = 1.$$

$\therefore$ $g(x) = 2x + 1$

or $g(10) = 21$

6

Let {x} & [x] denote the fractional and integral part of a real number x respectively. Solve 4{x} = x + [x].

$4\{x\} = x + [x]$

**Solution:**

we know $x = [x] + \{x\} = I + f$

$\therefore \Rightarrow 4f = I + f + I$

$\Rightarrow 2I = 3f$

$\Rightarrow f = \frac{2I}{3}$ $\quad \therefore 0 \le f < 1$

or $0 \le \frac{2I}{3} < 1$

$0 \le I < 3/2$

$\therefore I = 0, 1$ (only integers)

for $I = 0$, $f = 0$ $\quad \therefore x = 0$

an for $I = 1$, $f = 2/3$ $\quad x = 5/3$

7

Let $f(x)=\frac{9^x}{9^x+3}$ then find the value of the sum $f\left(\frac{1}{2006}\right)+f\left(\frac{2}{2006}\right)+f\left(\frac{3}{2006}\right)+\ldots\ldots.f\left(\frac{2005}{2006}\right)$

**Solution:**

Let $f(x) = \frac{9^x}{9^x+3}$

or $f(1-x) = \frac{9^{1-x}}{9^{1-x}+3} = \frac{3}{9^x+3}$

$\therefore$ $f(1-x) + f(x) = 1$

$f\left(\frac{1}{2006}\right) + f\left(\frac{2005}{2006}\right) = 1$

$(2006, 2005)$, $(2, 2004)$, .... $= 1002$ pairs

for $f\left(\frac{1003}{2006}\right) = f\left(\frac{1}{2}\right) = f(x) = \frac{1}{2}$

$\therefore$ Total sum $= 1002 + \frac{1}{2} = \underline{1002.5}$

8

Let f(x) = (x + 1)(x + 2)(x + 3)(x + 4) + 5 where x ∈ [–6, 6]. If the range of the function is [a, b] where a, b ∈ N then find the value of (a + b).

**Solution:**

$f(x) = (x+1)(x+2)(x+3)(x+4) + 5$

for $x \in [-6, 6]$

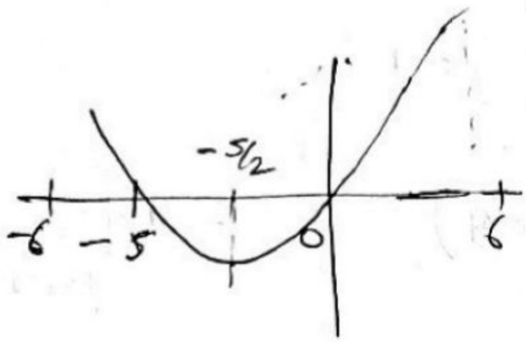
$\therefore f(x) = (x^2+5x+4)(x^2+5x+6)+5$

again let $x^2+5x = t$

$f(x) = (t+4)(t+6) = t^2+10t+24+5$

for $x \in [-6, 6]$ $\quad x^2+5x = t$

$x(x+5) = t$




$t_{min} = -25/4$ (at $x = -5/2$)

$t_{MAX} = 66$ ($x = 6$)

$\therefore$ for $t^2+10t+29$

$f(x) = (t+5)^2 + 4$

$f(x)_{min} = 4$ at $t = -5$

$f(x)_{MAX} = (66+5)^2 + 4 = 5045$

9

The set of real values of 'x' satisfying the equality $\left[\frac{3}{x}\right]+\left[\frac{4}{x}\right]=5$ (where [ ] denotes the greatest integer function) belongs to the interval $\left(a, \frac{b}{c}\right]$ where a, b, c $\in$ N and $\frac{b}{c}$ is in its lowest form. Find the value of a + b + c + abc.

Solution:

$$\left[\frac{3}{x}\right]+\left[\frac{4}{x}\right]=5$$

Since $\left[\frac{3}{x}\right]$ and $\left[\frac{4}{x}\right]$ are integers.

so possible cases are

(i) $\left[\frac{3}{x}\right]=0, \quad \left[\frac{4}{x}\right]=5$

(ii) $\left[\frac{3}{x}\right]=1 \quad \left[\frac{4}{x}\right]=4$

(iii) $\left[\frac{3}{x}\right]=2 \quad \left[\frac{4}{x}\right]=3$

(iv) $\left[\frac{3}{x}\right]=3 \quad \left[\frac{4}{x}\right]=2$

(v) $\left[\frac{3}{x}\right]=4 \quad \left[\frac{4}{x}\right]=1$

(vi) $\left[\frac{3}{x}\right]=5 \quad \left[\frac{4}{x}\right]=0$

9 Continues...

Case (1) $\left[\frac{3}{x}\right]=0$ $\left[\frac{4}{x}\right]=5$

$0\le \frac{3}{x}<1$ & $5\le\frac{4}{x}<6$

or $x>3$ & $\frac{2}{3}<x\le\frac{4}{5}$

which is not possible

Hence only possible solution cases are

$\left[\frac{3}{x}\right]=2$ and $\left[\frac{4}{x}\right]=3$

$2\le\frac{3}{x}<3$ $3\le\frac{4}{x}<4$

$1<x\le\frac{3}{2}$ and $1<x\le\frac{4}{3}$

$\therefore x\in\left(1,\frac{4}{3}\right]$

$\therefore a=1,\ b=4,\ c=3$

$\therefore a+b+c+abc=\underline{20}$

10

$f(x)$ and $g(x)$ are linear function such that for all x, $f(g(x))$ and $g(f(x))$ are Identity functions. If $f(0) = 4$ and $g(5) = 17$, compute $f(2006)$.

10

Solⁿ

If $f(g(x))$ & $g(f(x))$ are Identity function then $f(x)$ & $g(x)$ are inverse of each other

Let $f(x) = ax + 4$ ($\because f(0) = 4$ Given)

$\therefore f^{-1}(x) = \frac{x-4}{a} = g(x)$

or $g(5) = 17$

$\frac{5-4}{a} = 17 \Rightarrow a = \frac{1}{17}$

$\therefore f(x) = \frac{1}{17}x + 4$

$f(2006) = \frac{1}{17}(2006) + 4$

$= 122$

(11)

The function $f(x)$ has the property that for each real number x in its domain, 1/x is also in its domain and $f(x) + f(1/x) = x$. Find the largest set of real numbers that can be in the domain of $f(x)$ ?

Soln:

$$f(x) + f(1/x) = x \rightarrow (1)$$

put $x \rightarrow 1/x$

$$f(1/x) + f(x) = 1/x \rightarrow (2)$$

by (1) & (2)

$$x = \frac{1}{x} \Rightarrow x = \pm 1$$

1

# EXERCISE (JM)

If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$, and **[JEE(Main)-2016]**

$S = \{x \in \mathbb{R} : f(x) = f(-x)\}$; then S :

(1) contains more than two elements. (2) is an empty set.

(3) contains exactly one element (4) contains exactly two elements

Sol:- * $f(x) + 2 \cdot f\left(\frac{1}{x}\right) = 3x$ — 1

* $f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$ — 2

solve 1 & 2

$$\boxed{f(x) = \frac{2}{x} - x}$$

$\because f(x) = f(-x)$

$\Rightarrow \frac{2}{x} - x = x - \frac{2}{x}$

$\Rightarrow \frac{4}{x} = 2x$

$\Rightarrow x^2 = 2$

$\Rightarrow \boxed{x = \pm\sqrt{2}}$

2

The function $f : \mathbb{R} \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is : **[JEE(Main)-2017]**

(1) neither injective nor surjective.

(2) invertible.

(3) injective but not surjective.

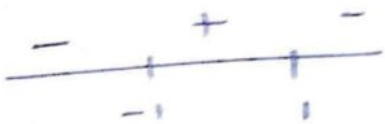
(4) surjective but not injective

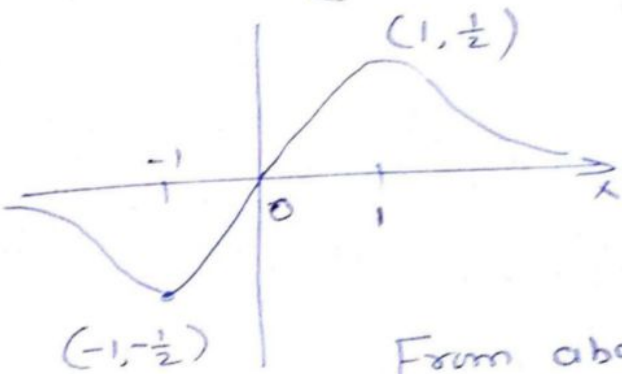
Sol:

$f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$

$f(x) = \frac{x}{1+x^2} \quad \forall\, x \in R$

$f'(x) = -\frac{(x+1)(x-1)}{(1+x^2)^2}$




From above diagram of f(x), f(x) is surjective but not injective.

Ans → 4

③

Let $S = \{x \in \mathbb{R} : x \geq 0 \text{ and } 2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S : **[JEE(Main)-2018]**

(1) contains exactly one element.

(2) contains exactly two elements.

(3) contains exactly four elements.

(4) is an empty set.

Sol:-

$$2|\sqrt{x}-3| + \sqrt{x}(\sqrt{x}-6) + 6 = 0$$

I) $x \in (0, 9)$

$$2(3-\sqrt{x}) + x - 6\sqrt{x} + 6 = 0$$

$$x - 8\sqrt{x} + 12 = 0$$

$$\sqrt{x} = 4, 2$$

$$x = 16, 4$$

16 ↓ Rej

$\underline{x = 4}$

II) $x \in [9, \infty)$

$$2(\sqrt{x}-3) + x - 6\sqrt{x} + 6 = 0$$

$$x - 4\sqrt{x} = 0$$

$$x = 16, 0$$

0 ↓ Rej

$x = 4, 16$

Ans → (2)

4

For $x \in \mathbb{R} - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then J(x) is equal to :- **[JEE(Main)-2019]**

(1) $f_3(x)$ (2) $f_1(x)$ (3) $f_2(x)$ (4) $\frac{1}{x} f_3(x)$

Sol:-

$f_1(x) = \frac{1}{x}$ $f_2(x) = 1 - x$ $f_3(x) = \frac{1}{1-x}$

$(f_2 o J o f_1)(x) = f_3(x)$

$f_2\left(J\left(\frac{1}{x}\right)\right) = \frac{1}{1-x}$

$1 - J\left(\frac{1}{x}\right) = \frac{1}{1-x}$

$J\left(\frac{1}{x}\right) = \frac{x}{x-1}$ Ans → 1

$J(x) = \frac{1/x}{\frac{1}{x} - 1}$

$= \frac{1}{1-x}$

$= f_3(x)$

5

Let $\mathbb{N}$ be the set of natural numbers and two functions f and g be defined as $f,g : \mathbb{N} \to \mathbb{N}$

such that : $f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$ and $g(n) = n-(-1)^n$. The fog is : **[JEE(Main)-2019]**

(1) Both one-one and onto

(2) One-one but not onto

(3) Neither one-one nor onto

(4) onto but not one-one

**Solution:**

$$g(n) = n - (-1)^n$$

$$g(n) = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n-1 & \text{if } n \text{ is even} \end{cases}$$

$$f(g(n)) = \begin{cases} \frac{n-1+1}{2} = \frac{n}{2} & \text{if } n \text{ is even} \\ \frac{n+1}{2} & \text{if } n \text{ is odd} \end{cases}$$

many one onto

Ans → 4

6

Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \dfrac{x}{1+x^2}$, $x \in \mathbb{R}$. Then the range of f is : **[JEE(Main)-2019]**

(1) $(-1, 1) - \{0\}$ (2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (3) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (4) $R - [-1, 1]$

**Solution:**

$f(0) = 0$ and $f(x)$ is odd.

Further, if $x > 0$ then

$$f(x) = \frac{1}{x + \frac{1}{x}} \in \left(0, \tfrac{1}{2}\right]$$

Hence $f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Ans $\rightarrow$ 2

7

The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ......, 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is :- **[JEE(Main)-2019]**

(1) $(15)! \times 6!$ (2) $5^6 \times 15$ (3) $5! \times 6!$ (4) $6^5 \times (15)!$

**Solution:**

$f(k) = 3m$ (3, 6, 9, 12, 15, 18)

For $k = 4, 8, 12, 16, 20$

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$ ways

For rest no's $15!$ ways

Total ways $= 6!(15)!$

Ans → 1

8

If $f(x)=\log_e\left(\frac{1-x}{1+x}\right), |x|<1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to : **[JEE(Main)-2019]**

(1) $2f(x)$ (2) $2f(x^2)$ (3) $(f(x))^2$ (4) $-2f(x)$

**Solution:**

$$f(x) = \log_e\left(\frac{1-x}{1+x}\right) \qquad |x|<1$$

$$f\left(\frac{2x}{1+x^2}\right) = \log_e\left(\frac{1-\frac{2x}{1+x^2}}{1+\left(\frac{2x}{1+x^2}\right)}\right)$$

$$= \log_e\left(\frac{(x-1)^2}{(x+1)^2}\right)$$

$$= 2\log_e\left|\frac{x-1}{x+1}\right|$$

$$= 2f(x)$$

Ans $\longrightarrow$ 1

9

Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$, where the function $f$ satisfies $f(x+y) = f(x)f(y)$ for all natural numbers x, y and $f(1) = 2$. then the natural number 'a' is **[JEE(Main)-2019]**

(1) 4 (2) 3 (3) 16 (4) 2

**Solution:**

$$f(x) = 2^x \quad \forall x \in N$$

$$2^{a+1} + 2^{a+2} + 2^{a+3} \cdots + 2^{a+10} = 16(2^{10}-1)$$

$$2^a(2 + 2^2 + 2^3 \cdots + 2^{10}) = 16(2^{10}-1)$$

$$\Rightarrow \quad 2^{a+1}\left(\frac{\cancel{2^{10}-1}}{2-1}\right) = 16(\cancel{2^{10}-1})$$

$$2^{a+1} = 2^4 \Rightarrow \boxed{a = 3}$$

Ans $\rightarrow$ 2

10

Le $f(x) = x^2$, $x \in \mathbb{R}$. For any $A \subseteq R$, define $g(A) = \{x \in \mathbb{R}, f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true ? **[JEE(Main)-2019]**

(1) $f(g(S)) \neq f(S)$ (2) $f(g(S)) = S$ (3) $g(f(S)) = g(S)$ (4) $g(f(S)) \neq S$

**Solution:**

$g(S) = [-2, 2]$

So $f(g(S)) = [0, 4] = S$

And $f(S) = [0, 16] \Rightarrow f(g(S)) \neq f(S)$

Also $g(f(S)) = [-4, 4] \neq g(S)$

So $g(f(S)) \neq S$

Ans $\longrightarrow$ 3

11. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is : **[JEE(Main)-2019]**

(1) 2 (2) 3 (3) 4 (4) 1

**Solution:**

11

$$5 + |2^x - 1| = 2^x(2^x - 2)$$

Let $2^x = t$

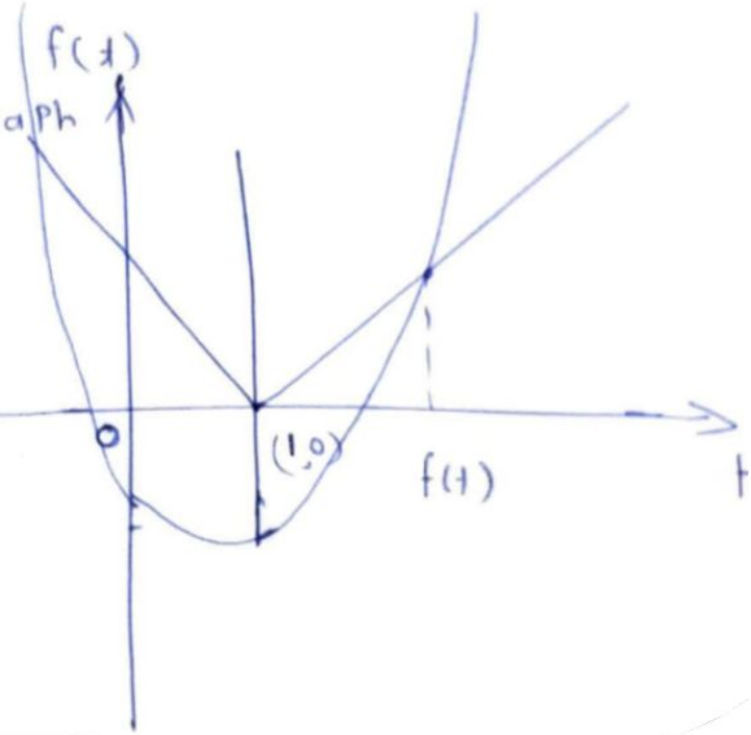
$$5 + |t - 1| = t^2 - 2t$$

$$\Rightarrow |t - 1| = t^2 - 2t - 5$$

$\downarrow$ $\downarrow$

$g(t)$ $f(t)$

From the graph




$\because t > 0$

So number of real root is 1

Ans $\rightarrow$ 4

12

For $x \in \left(0, \frac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi = \left(\frac{\pi}{3}\right)$ is equal to : **[JEE(Main)-2019]**

(1) $\tan\frac{\pi}{12}$ (2) $\tan\frac{7\pi}{12}$ (3) $\tan\frac{11\pi}{12}$ (4) $\tan\frac{5\pi}{12}$

**Solution.**

$$fog(x) = \sqrt{\tan x}$$

$$h(fog)(x) = \frac{1-\tan x}{1+\tan x}$$

$$= \tan\left(\frac{\pi}{4} - x\right)$$

$$= \phi(x)$$

$$\phi\left(\frac{\pi}{3}\right) = -\tan\frac{\pi}{12}$$

$$= \tan\left(\pi - \frac{\pi}{12}\right)$$

$$= \tan\left(\frac{11\pi}{12}\right)$$

Ans $\rightarrow$ 3

13. For $x \in \mathbb{R}$, let $[x]$ denote the greatest integer $\leq x$, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\ldots\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right] \text{ is}$$

**[JEE(Main)-2019]**

(1) $-153$ (2) $-133$ (3) $-131$ (4) $-135$

**Solution:**

$$\underbrace{\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\ldots}$$

$$+\underbrace{\left[-\frac{1}{3}-\frac{66}{100}\right]+\left[-\frac{1}{3}-\frac{67}{100}\right]+\ldots}$$

$$+\left[-\frac{1}{3}-\frac{99}{100}\right]$$

$$=(-1)67-2(33)$$

$$=-67-66$$

$$=-133$$

Ans $\rightarrow$ 2

# EXERCISE (JA)

1. If functions f (x) and g (x) are defined on $\mathbb{R} \rightarrow \mathbb{R}$ such that

$f(x)=\begin{cases}0, & x \in \text{rational} \\ x, & x \in \text{irrational}\end{cases}$, $g(x)=\begin{cases}0, & x \in \text{irrational} \\ x, & x \in \text{rational}\end{cases}$, then $(f-g)(x)$ is -

(A) one-one and onto
(B) neither one-one nor onto
(C) one-one but not onto
(D) onto but not one-one **[JEE 2005 (Scr.)]**

**Ans.** **A**

Sol. Let $h(x)=f(x)-g(x)=\begin{cases}x\,; & x \in \text{irrational} \\ -x\,; & x \in \text{rational}\end{cases}$

$\Rightarrow$ the function h(x) is one-one and onto.

2. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in \mathbb{R}$. Then the set of all x satisfying $(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is- **[JEE 2011, 3, (–1)]**

(A) $\pm\sqrt{n\pi}, n \in \{0,1,2,....\}$

(B) $\pm\sqrt{n\pi}, n \in \{1,2,...\}$

(C) $\frac{\pi}{2} + 2n\pi, n \in \{....,-2,-1,0,1,2,.....\}$

(D) $2n\pi, n \in \{....,-2,-1,0,1,2,.....\}$

***Sol.*** **(A)**

$(fogogof)(x) = \sin^2(\sin x^2)$

$(gogof)(x) = \sin(\sin x^2)$

$\therefore \sin^2(\sin x^2) = \sin(\sin x^2)$

$\Rightarrow \sin(\sin x^2)[\sin(\sin x^2) - 1] = 0$

$\Rightarrow \sin(\sin x^2) = 0 \text{ or } 1$

$\Rightarrow \sin x^2 = n\pi \text{ or } 2m\pi + \pi/2$, where $m, n \in I$

$\Rightarrow \sin x^2 = 0$

$\Rightarrow x^2 = n\pi \Rightarrow x = \pm\sqrt{n\pi}, n \in \{0, 1, 2, \ldots\}$.

**3.** The function $f : [0,3] \to [1,29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is : **[JEE 2012, 3, (–1)]**

(A) one-one and onto

(B) onto but not one-one

(C) one-one but not onto

(D) neither one-one nor onto

***Sol.*** **(B)**

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6)$

$= 6(x - 2)(x - 3)$

$f(x)$ is increasing in [0, 2] and decreasing in [2, 3]

$f(x)$ is many one

$f(0) = 1$

$f(2) = 29$

$f(3) = 28$

Range is [1, 29]

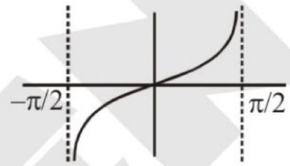
Hence, $f(x)$ is many-one-onto

**4.** Let $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to\mathbb{R}$ be given by $f(x) = (\log(\sec x + \tan x))^3$ Then - **[JEE(Advanced)-2014, 3]**

(A) $f(x)$ is an odd function
(B) $f(x)$ is a one-one function
(C) $f(x)$ is an onto function
(D) $f(x)$ is an even function

**Sol. Ans. (A,B,C)**

$$f(x) = (\ell n(\sec x + \tan x))^3$$




$$f'(x) = \frac{3(\ell n(\sec x + \tan x))^2(\sec x \tan x + \sec^2 x)}{(\sec x + \tan x)} > 0$$

$f(x)$ is an increasing function

$$\lim_{x\to -\frac{\pi}{2}} f(x) \to -\infty \quad \& \quad \lim_{x\to \frac{\pi}{2}} f(x) \to \infty$$

Range of $f(x)$ is R and onto function

$$f(-x) = (\ell n(\sec x - \tan x))^3 = \left(\ell n\left(\frac{1}{\sec x + \tan x}\right)\right)^3$$

$f(-x) = -(\ell n(\sec x + \tan x))^3$

$f(x) + f(-x) = 0 \Rightarrow f(x)$ is an odd function.