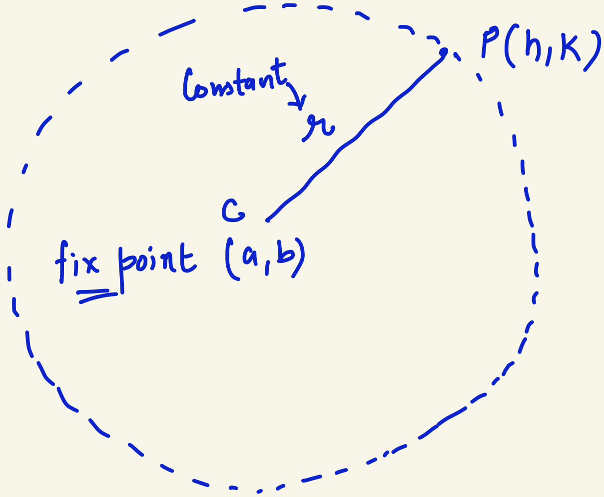
CIRCLE



$$PC = r \Rightarrow \sqrt{(h-a)^2+(K-b)^2} = r$$

$$\boxed{(x-a)^2+(y-b)^2 = r^2}$$

$$x^2+y^2-2xa-2yb+a^2+b^2-r^2 = 0.$$

$$\underbrace{a \to -g}\ ;\ \underbrace{b \to -f}\ ;\ \underbrace{a^2+b^2-r^2 = c}$$

Rem

$$\boxed{x^2+y^2+2gx+2fy+c=0}$$

General eqn of Circle.

Note:- Coeff of x^2 = coeff of y^2 = 1

Centre : $\left(-\frac{1}{2}\text{coeff of } x,\ -\frac{1}{2}\text{coeff of } y\right)$

Radius of Circle $= \boxed{r = \sqrt{g^2 + f^2 - c}}$ Rem

Q. Find the centre & radius of $2x^2 + 2y^2 - 3x - 5y - 2 = 0$.

$$x^2 + y^2 - \frac{3}{2}x - \frac{5}{2}y - 1 = 0$$

$$C\left(\frac{3}{4}, \frac{5}{4}\right) \quad ; \quad r = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{5}{4}\right)^2 - (-1)}$$

$$r = \sqrt{\frac{9}{16} + \frac{25}{16} + 1}$$

Q

If $x^2 + y^2 + 2x - 4y - k = 0$ is midway between circles $x^2 + y^2 + 2x - 4y - 4 = 0$ & $x^2 + y^2 + 2x - 4y - 20 = 0$, then $k = ?$

$C_1(-1, 2)$

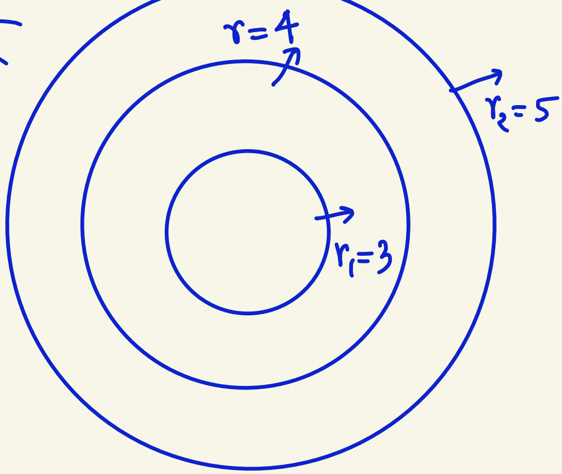
$r_1 = \sqrt{1 + 2^2 + 4} = 3$

Sol^n $C_3(-1, 2)$

$r = 4 = \sqrt{1 + 4 + K}$

$16 = 5 + K$

$\therefore \boxed{K = 11}$



$C_2 : (-1, 2)$

$r_2 = \sqrt{1 + 4 + 20}$

$= 5$

Note :

(i) Necessary and sufficient condition for general equation of degree 2 i.e. $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to represent a circle is

(a) coefficient of x^2 = coefficient of y^2 (not necessarily unity) $\neq 0$ and

(b) coefficient of $xy = 0$

(ii) The general equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ contains 3 independent arbitrary constants g, f and c that a unique circle passes through 3 non-collinear points hence 3 points on a circle or 3 tangents to a circle or 2 tangents to a circle and a point etc. must be given to determine the unique equation of the circle.

Nature of circle :

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\text{where } r = \sqrt{g^2 + f^2 - c}$$

If $g^2 + f^2 - c > 0$ (Real circle with finite radius)	If $g^2 + f^2 - c = 0$ (point circle)	If $g^2 + f^2 - c < 0$ (imaginary circle)

Q If $ax^2 + by^2 + (a + b - 4)xy - ax - by - 20 = 0$ represent a circle, then its radius is ?

$a = b$ & $a + b - 4 = 0 \Rightarrow a = b = 2$

$2x^2 + 2y^2 - 2x - 2y - 20 = 0$

$x^2 + y^2 - x - y - 10 = 0.$

Q

Find the equation of the circle

(a) through 3 non-collinear points (0, 1), (2, 3) and (–2, 5)

$\pi r^2 = 154$

$2x - 3y = 5$
$3x - 4y = 7$
Centre

(b) having lines 2x –3y = 5 and 3x – 4y = 7 as its diameter/Normal/ longest chord and whose area is 154 sq. units (Take π = 22/7).

(c) which is concentric with $3x^2 + 3y^2 - 5x - 6y - 14 = 0$ and perimeter of its semicircle is 36 (Take π = 22/7).

$x^2 + y^2 - \frac{5}{3}x - 2y - \frac{14}{3} = 0$

$\pi r + 2r = 36$

$r = \frac{36}{\pi + 2}$ **

πr

r r

(d) Centre is on the line y = 2x and passing through (–1, 2) and (3, –2).

(e) passes through (2, 3) and centre on the x-axis radius being 5.

(f) which has its centre as the point (4, 3) and touching the line 5x – 12y – 10 = 0.

(a) $x^2 + y^2 + 2gx + 2fy + c = 0$

(0,1) (2,3) (–2,5)

$0 + 1 + 0 + 2f + c = 0$ — ① —

$4 + 9 + 4g + 9f + c = 0$ — ② —

$4 + 25 - 4g + 10f + c = 0$ — ③ —

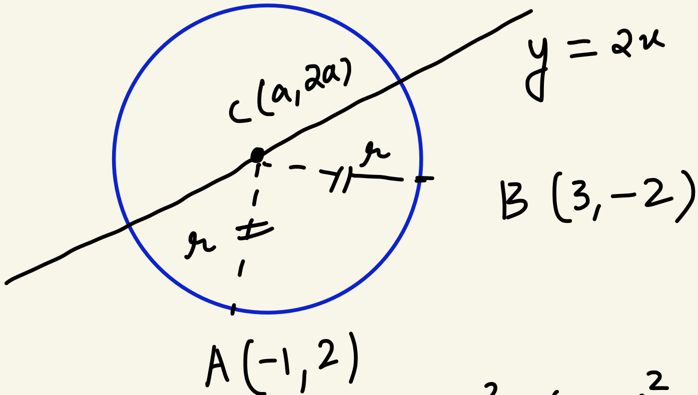
solve to get g, f, c

(d) Centre is on the line $y = 2x$ and passing through $(-1, 2)$ and $(3, -2)$.

(e) passes through $(2, 3)$ and centre on the x-axis radius being 5.

(f) which has its centre as the point $(4, 3)$ and touching the line $5x - 12y - 10 = 0$.

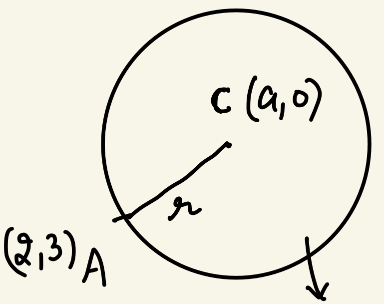
(d)



$AC = BC$

$$(a+1)^2 + (2a-2)^2 = (a-3)^2 + (2a+2)^2$$

(e)

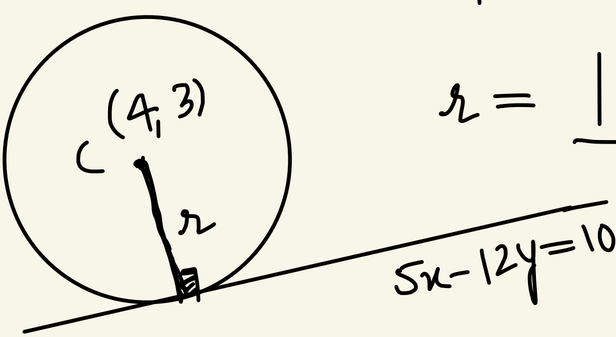


$r = 5$

$$AC = r. \Rightarrow \sqrt{(a-2)^2 + 3^2} = 5$$

$$(x-a)^2 + (y-0)^2 = 5^2$$

(f)



$$r = \frac{|5(4) - 12(3) - 10|}{13}$$

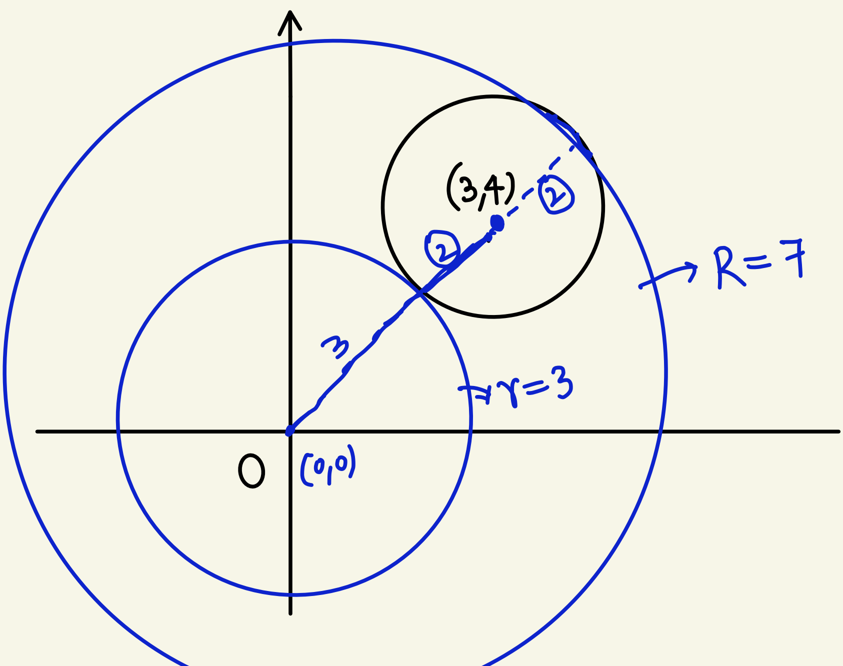
Q Find equation of circle whose centre is at origin and which touches $x^2+y^2-6x-8y+21=0$?

Sol^n $x^2+y^2-6x-8y+21=0$

$C:(3,4)$

$r=\sqrt{9+16-21}$

$\boxed{r=2}$



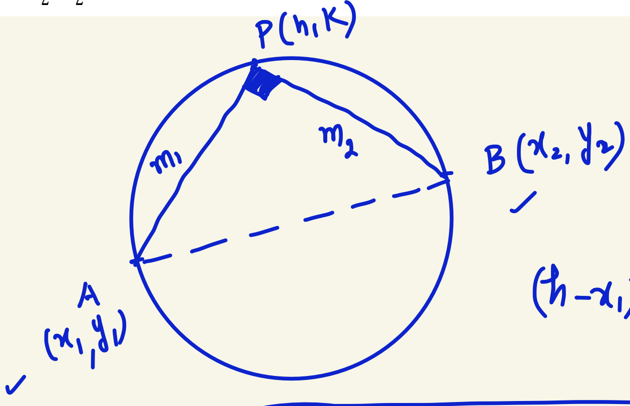
$\left.\begin{array}{l} x^2+y^2=9 \\ x^2+y^2=49 \end{array}\right\}$ Ans

DIAMETRICAL FORM OF CIRCLE :

Equation of circle with (x_1, y_1) and (x_2, y_2) as its diameter is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0 \quad \text{—(i)—}$$

Note that Equation (i) is also the equation of the circle with least radius or area or perimeter passes through (x_1, y_1) and (x_2, y_2).

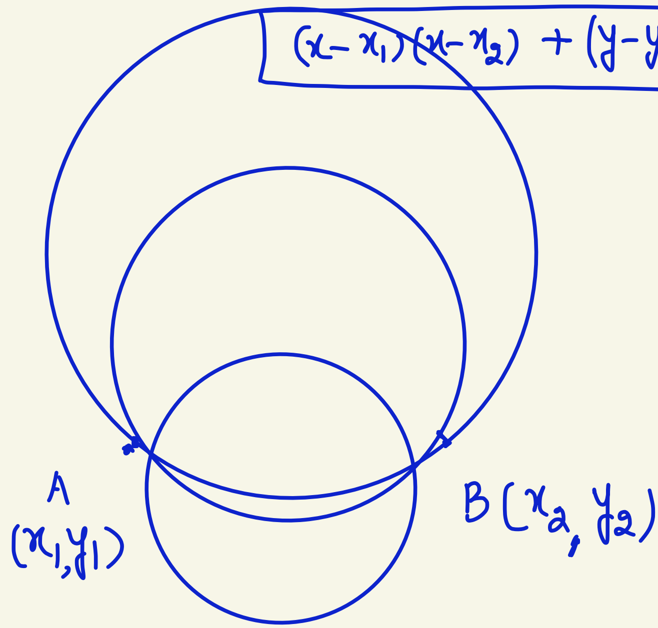


$m_1 m_2 = -1$

$$\left(\frac{h-x_1}{K-y_1}\right)\left(\frac{h-x_2}{K-y_2}\right) = -1$$

$$(h-x_1)(h-x_2) + (K-y_1)(K-y_2) = 0$$

$$\boxed{(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0}$$



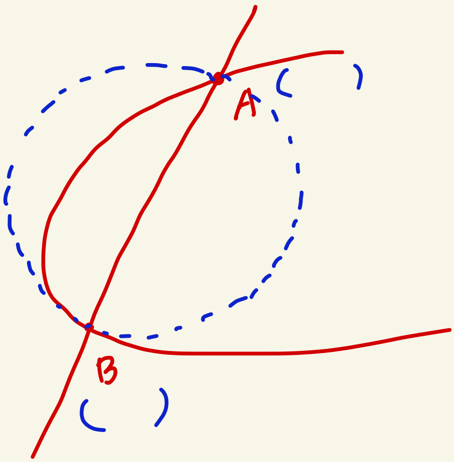
Q1

Line y = mx cuts the curve $y^2 = 4ax$ at A and B. Find the equation of circle with AB as diameter.

Q2

Find the equation of a circle circumscribing the quadrilateral formed by the lines $2x + 3y = 2$; $3x - 2y = 4$; $x + 2y = 3$; $2x - y = 3$.

1



$y = mx$ & $y^2 = 4ax$

$(mx)^2 = 4ax$

$\boxed{m^2x^2 - 4ax = 0}$ — ① —

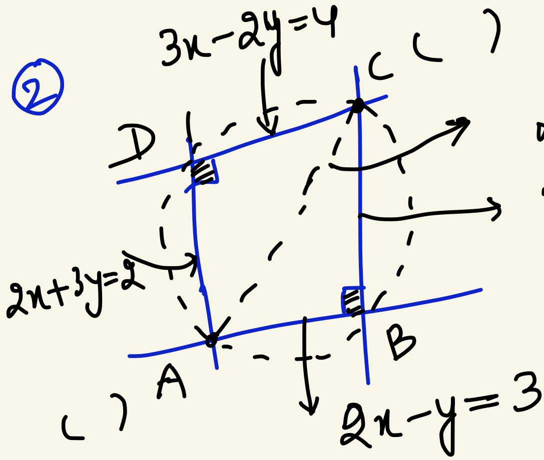
$y^2 = 4a\left(\frac{y}{m}\right)$

$my^2 - 4ay = 0$ — ② —

$\boxed{m^2y^2 - 4amy = 0}$ — ③ —

Add ① & ③ to obtain req. eqn.

2



Diameter

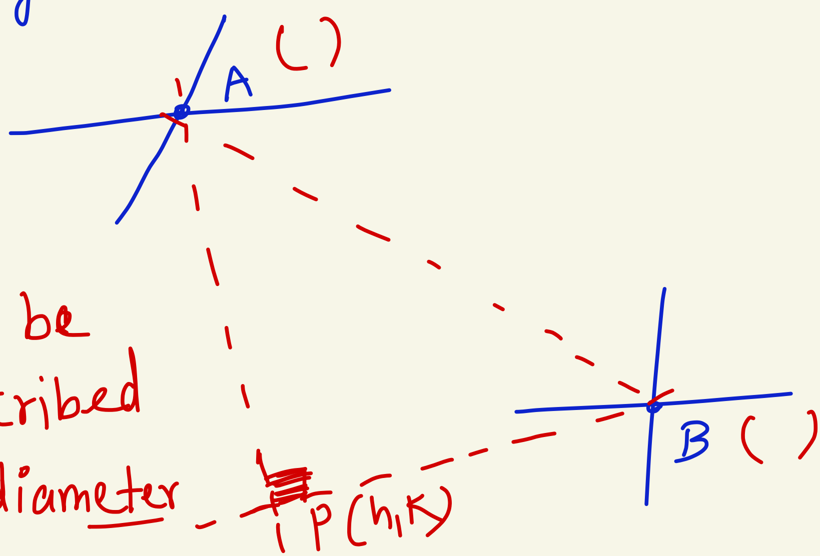
$x+2y=3$

Q

Consider the family of lines $(x - y - 6) - \lambda(2x + y + 3) = 0$ and $(x + 2y - 4) - \mu(3x - 2y - 4) = 0$. If the lines of these 2 families are at right angle to each other than the locus of their point of intersection is

$$\left.\begin{aligned} x - y &= 6 \\ 2x + y &= -3 \end{aligned}\right\} \rightarrow A(\quad)$$

$$\left.\begin{aligned} x + 2y &= 4 \\ 3x - 2y &= 4 \end{aligned}\right\} \rightarrow B(\quad)$$



Locus will be circle described on AB as diameter

INTERCEPT (MADE BY THE CIRCLE) :

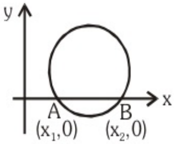
Intercept made by a circle on the coordinate axes, is the distance between the 2 points where the circle cuts the axis of x and axis of y

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

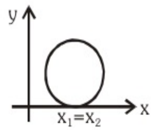
For x-intercept : put y = 0

$$x^2 + 2gx + c = 0 \; ; \; x_1 + x_2 = -2g \; ; \; x_1x_2 = c$$

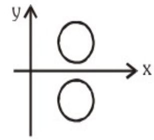
$$\boxed{|x_1 - x_2| = 2\left|\sqrt{g^2 - c}\right|}$$ Rem



(i) If $g^2 - c > 0 \Rightarrow$ circle cuts the x-axis at 2 distinct points.



(ii) If $g^2 - c = 0 \Rightarrow$ circle touches the x-axis

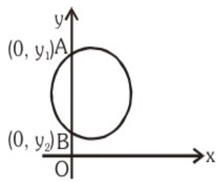


(iii) If $g^2 < c \Rightarrow$ circle lies completely above or below the x-axis.

Similarly **For y-intercept** : put x = 0

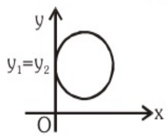
$$y^2 + 2fy + c = 0$$

$$\boxed{|y_1 - y_2| = 2\left|\sqrt{f^2 - c}\right|}$$ Rem



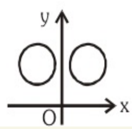
(i) If $f^2 - c > 0$

$\Rightarrow$ circle cuts the y-axis at 2 distinct points.



(ii) If $f^2 = c$

$\Rightarrow$ circle touches the y-axis



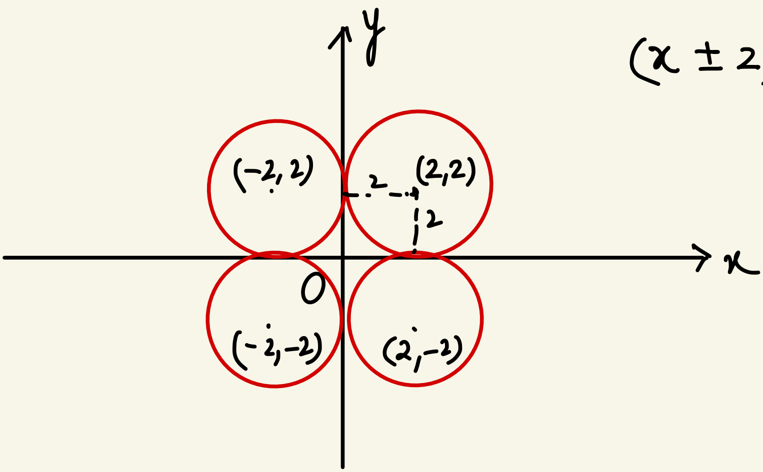
(iii) If $f^2 < c$

$\Rightarrow$ circle lies completely either on right or left of y-axis.

① Find the equation of a circle which touches the coordinate axes and whose radius = 2.

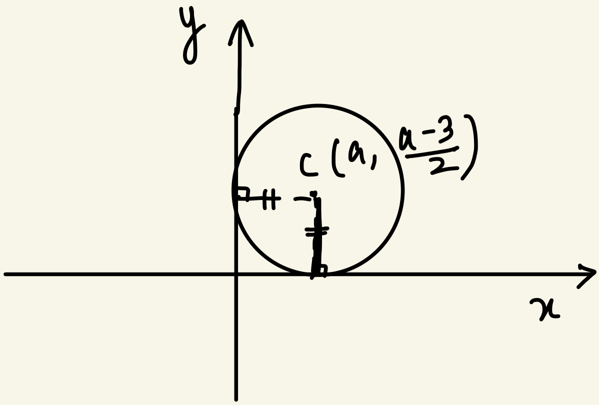
② Find the equation of a circle touching both axes & whose centre lies on $x - 2y = 3$.

①



$$(x \pm 2)^2 + (y \pm 2)^2 = 2^2$$

②



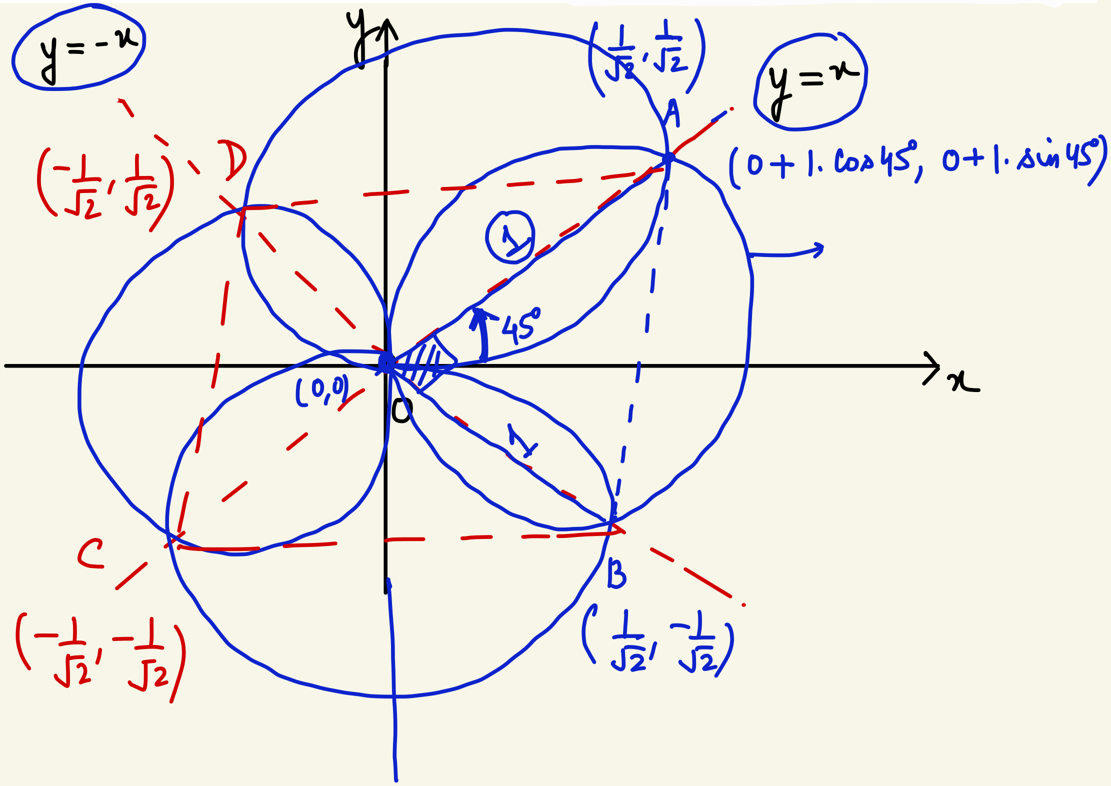
$$\boxed{x - 2y = 3}$$

$$|a| = \left|\frac{a-3}{2}\right|$$

$$a = \pm\left(\frac{a-3}{2}\right)$$

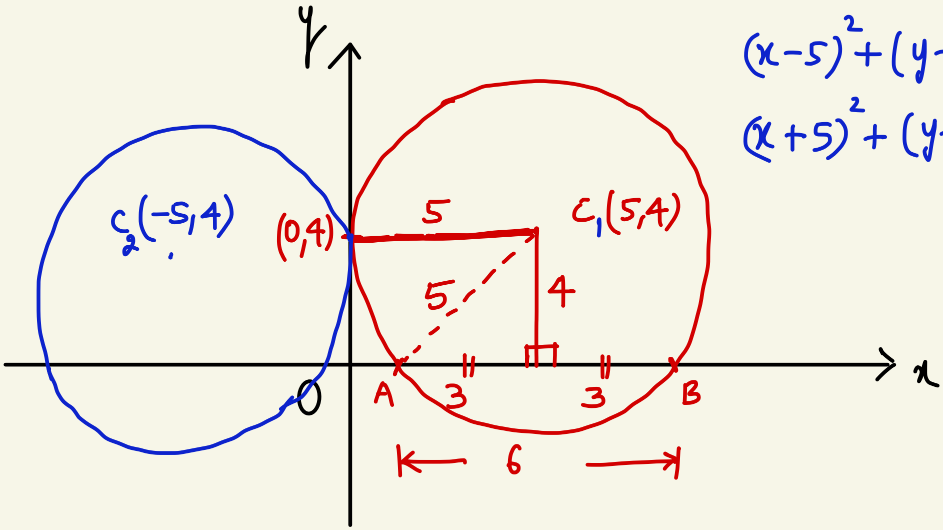
Q

Find the equation of a circle passing through origin cutting off intercept equals to unity on the lines $y^2 - x^2 = 0$.



$$\boxed{\left(x \pm \frac{1}{\sqrt{2}}\right)\left(x \pm \frac{1}{\sqrt{2}}\right) + \left(y \pm \frac{1}{\sqrt{2}}\right)\left(y \pm \frac{1}{\sqrt{2}}\right) = 0}$$

Q Find the equation of a circle which touches the positive axis of y at a distance of 4 unit from origin and cuts off an intercept of 6 units from the axis of x.



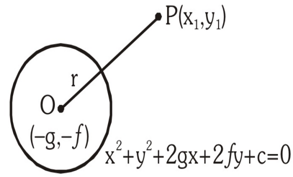
$$\left.\begin{aligned}(x-5)^2+(y-4)^2&=5^2\\(x+5)^2+(y-4)^2&=5^2.\end{aligned}\right\}$$

Q HW

Find the equation of the locus of the centre of a circle which touches the positive y-axis and having intercept on x-axis equal to 2ℓ.

POSITION OF A POINT W.R.T A CIRCLE :

Point 'P' lies outside the circle according as :



$OP > r$

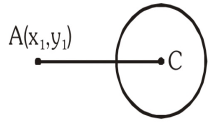
i.e. $\quad (x_1 + g)^2 + (y_1 + f)^2 > r^2$

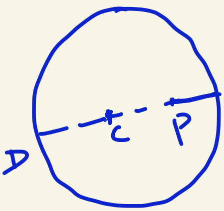
$$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c > 0$$

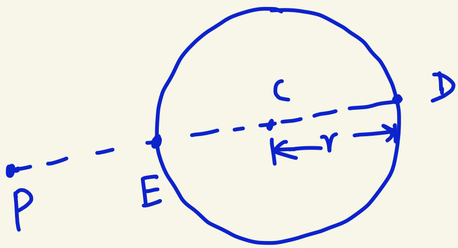
or $\quad S_1 > 0,\ \text{where } S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

Point 'P' lies inside the circle, then $S_1 < 0$

Note : Greatest and least distance of a point $A(x_1, y_1)$ from a circle with centre 'C' and radius 'r' is $|AC + r|$ and $|AC - r|$







Greatest = PC + r

Least = |PC − r|

Q. $S_1 = x^2 + y^2 - 4x + 6y - 3 = 0;\ S_2 = x^2 + y^2 + 4x - 6y - 3 = 0$, then point (1, 2) lies -

(A) inside $S_1 = 0$ and inside $S_2 = 0$

(B) outside $S_1 = 0$ and outside $S_2 = 0$

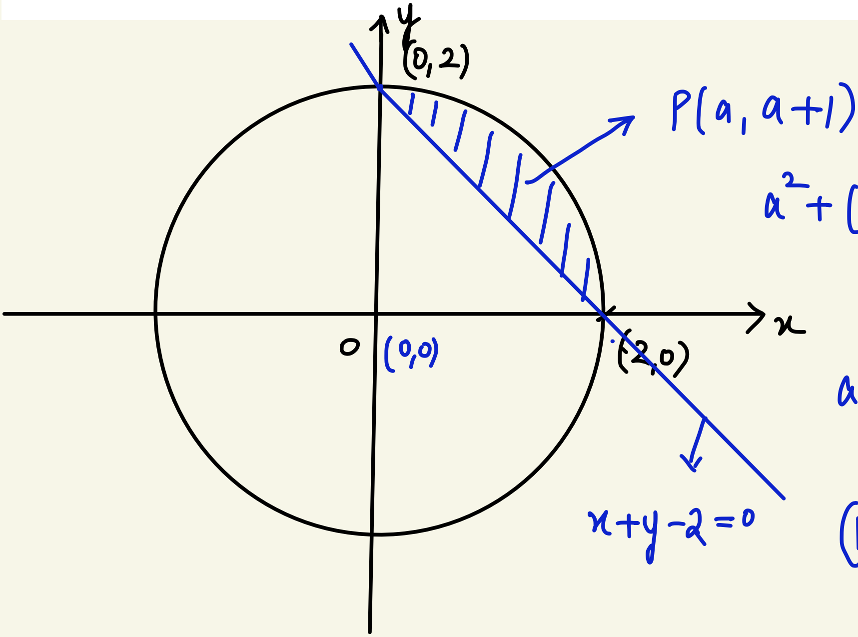
(C) inside $S_1 = 0$ and outside $S_2 = 0$

(D) outside $S_1 = 0$ and inside $S_2 = 0$

$S_1(1,2) = 1^2 + 2^2 - 4 + 6(2) - 3 > 0$ Outside

$S_2(1,2) = 1^2 + 2^2 + 4 - 6(2) - 3 < 0.$ inside.

Q For what value of 'a' the point $(a, a+1)$ lies inside the region bounded by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ in the first quadrant.



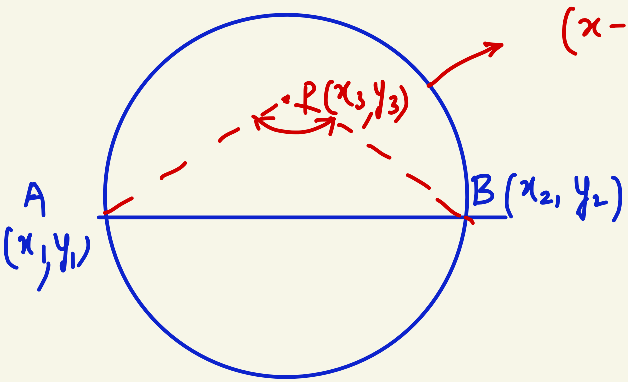
$$a^2 + (a+1)^2 - 4 < 0 \quad —①—$$

$$a + (a+1) - 2 > 0 \quad —②—$$

$$① \cap ②.$$

Q If the join of (x_1, y_1) & (x_2, y_2) makes an obtuse angle at (x_3, y_3) then comment upon on sign of $(x_3-x_1)(x_3-x_2)+(y_3-y_1)(y_3-y_2)$?

Negative



$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

(x_3, y_3)

PARAMETRIC EQUATION OF A CIRCLE :

$$\boxed{\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r}$$ (Distinguish it with the parametric equation of line, carefully)

$x = x_1 + r\cos\theta$

and $y = y_1 + r\sin\theta$

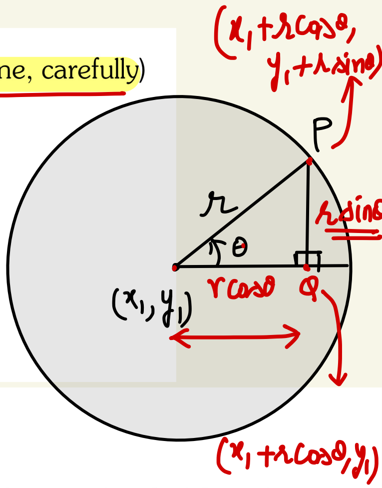
$(x_1, y_1) \rightarrow$ fixed centre

$r \rightarrow$ fixed radius and $\theta \in [0, 2\pi)$ is a parameter.

coordinates of any point on the circle is $(x_1 + r\cos\theta, y_1 + r\sin\theta)$

if $(x_1, y_1) \equiv (0, 0) \Rightarrow x = r\cos\theta$ & $y = r\sin\theta$

which are the parametric coordinates for the circle $x^2 + y^2 = r^2$



1. $x^2 + y^2 - 6x + 4y - 3 = 0$. convert into parametric form

2. If $A(\cos\theta_1, \sin\theta_1)$; $B(\cos\theta_2, \sin\theta_2)$; $C(\cos\theta_3, \sin\theta_3)$ are the vertices of a triangle ABC, then find the orthocentre of the triangle.

3. For the condition $x^2 + y^2 - 4x + 3 = 0$, find
   (a) maximum & minimum value of $x^2 + y^2$
   (b) maximum value of $x^2 + y^2 + 2y$
   (c) maximum & minimum value of $3x + 4y$

$C : (2, 0)$; $r = \sqrt{4-3} = 1$

$$\left.\begin{aligned} x &= 2 + \cos\theta \\ y &= 0 + \sin\theta \end{aligned}\right\}$$

1. $x^2 + y^2 - 6x + 4y - 3 = 0$; $C(3, -2)$

$r = \sqrt{3^2 + 2^2 + 3} = 4$

$$\left.\begin{aligned} x &= 3 + 4\cos\theta \\ y &= -2 + 4\sin\theta \end{aligned}\right\} \theta \in \text{parameter}$$

(a)

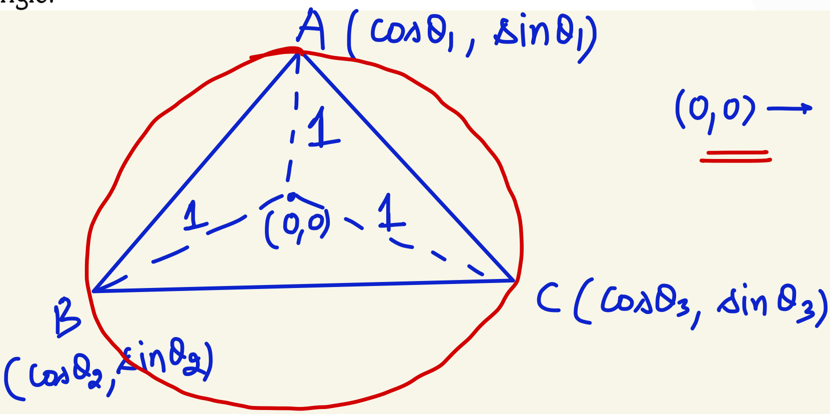
$x^2 + y^2 = 4 + \cos^2\theta + 4\cos\theta + \sin^2\theta$

$(x^2+y^2)_{max} = 9$

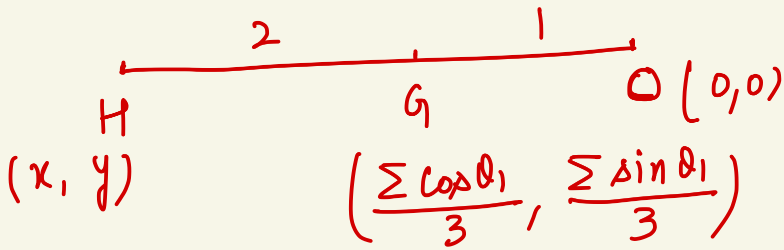
$(x^2+y^2)_{min} = 1.$

$$\boxed{x^2 + y^2 = 5 + 4\cos\theta}$$ $\theta \in [0, 2\pi)$

(2) If A($\cos\theta_1$, $\sin\theta_1$); B($\cos\theta_2$, $\sin\theta_2$); C($\cos\theta_3$, $\sin\theta_3$) are the vertices of a triangle ABC, then find the orthocentre of the triangle.

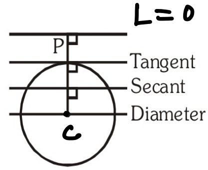


$\underline{\underline{(0,0)}} \rightarrow$ Circumcentre



LINE & A CIRCLE :

Let $L = 0$ be a line & $S = 0$ be a circle. If 'r' is the radius of the circle and 'p' is the length of the perpendicular from the centre on the line, then :



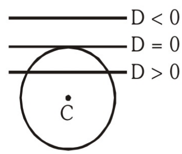
(i) If $p > r \Rightarrow$ line is neither secant nor tangent ; passes out side the circle

(ii) If $p = r \Rightarrow$ line is tangent to the circle.

(iii) If $p < r \Rightarrow$ line is a secant.

(iv) If $p = 0 \Rightarrow$ line is a diameter.

Alternatively : solve the line with the circle and if



(i) $D > 0 \Rightarrow$ line is a Secant.

(ii) $D = 0 \Rightarrow$ line is a Tangent

(iii) $D < 0 \Rightarrow$ line passes outside the circle.

This is true for a time with any 2nd degree curve

Q If $4l^2 - 5m^2 + 6l + 1 = 0$ then show that the line $lx + my + 1 = 0$ touches a fixed circle. Find the centre and radius of the circle.

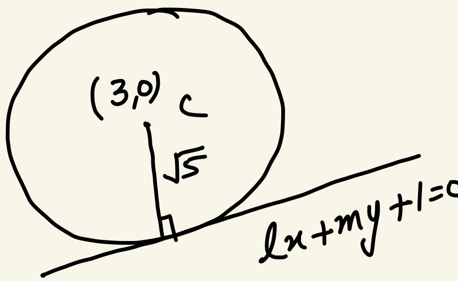
$$4l^2 + 6l + 1 = 5m^2$$

$$9l^2 + 6l + 1 = 5m^2 + 5l^2$$

$$(3l+1)^2 = 5(m^2 + l^2)$$

$$|3l+1| = \sqrt{5}\sqrt{l^2+m^2}$$

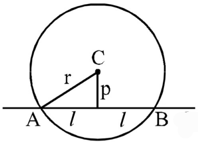
$$\boxed{\frac{|3l+1|}{\sqrt{l^2+m^2}} = \sqrt{5}}$$



Note: Length of the chord intercepted on the circle by a given line

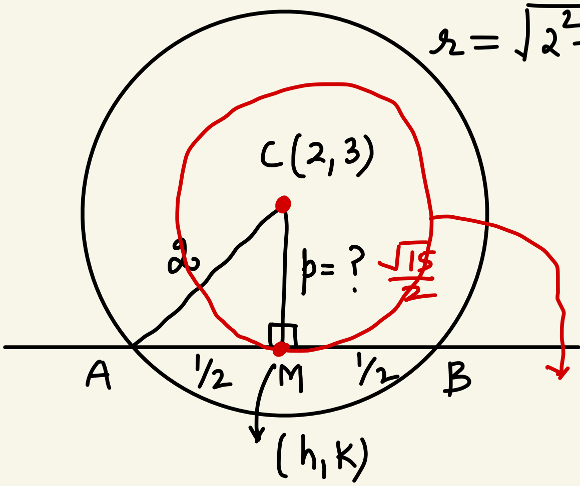
$$l = \sqrt{r^2 - p^2}$$

$$\therefore \quad \boxed{AB = 2l = 2\sqrt{r^2 - p^2}}$$



Q Find the locus of mid points of chords of the circle $x^2 + y^2 - 4x - 6y + 9 = 0$ which are of unit length ?

Sol^n



$$r = \sqrt{2^2 + 3^2 - 9} = 2$$

$$p = \sqrt{4 - \frac{1}{4}}$$

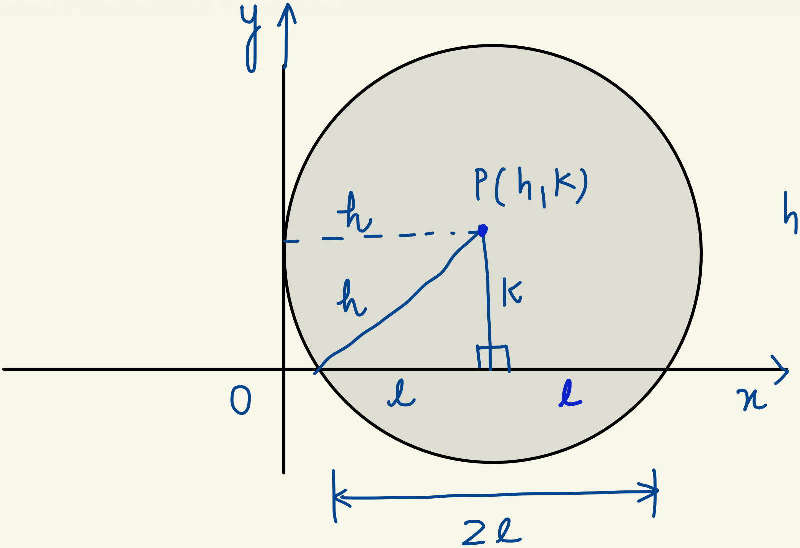
$$p = \frac{\sqrt{15}}{2}$$

$$(x-2)^2 + (y-3)^2 = \frac{15}{4}$$

Ans

Q HW

Find the equation of the locus of the centre of a circle which touches the positive y-axis and having intercept on x-axis equal to 2ℓ.



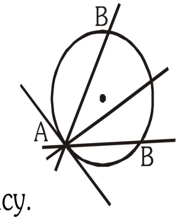
$h^2 = k^2 + \ell^2$

$h^2 - k^2 = \ell^2$

$x^2 - y^2 = \ell^2$

Ans

TANGENT AND NORMAL :



Tangent : Tangent is the limiting case of the secant as the point $B \to A$

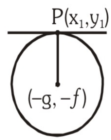
Normal : Normal is a line perpendicular to the tangent passing through the point of tangency.

In case of circle normal always passes through centre.

Equation of the tangent drawn to the circle in various forms :

(a) Tangent drawn to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at its point (x_1, y_1) is given by

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.$$



If circle is $x^2 + y^2 = a^2$

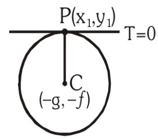
then tangent is $xx_1 + yy_1 = a^2$ (Cartesian from)

Note :

* (i) Equation of tangent drawn to any second degree curve at $P(x_1, y_1)$ on it can be obtained by replacing $x^2 \to xx_1$; $y^2 \to yy_1$; $2x \to x + x_1$; $2y \to y + y_1$; $2xy \to xy_1 + yx_1$

(ii) **Point of Tangency :**

For P : either solve tangent and normal to get P or compare the equation of tangent at (x_1, y_1) with the given tangent to get point of tangency.



(b) Parametric form : $\left.\begin{aligned} x_1 &= a\cos\alpha \\ y_1 &= a\sin\alpha \end{aligned}\right\}$ For $x^2+y^2=a^2$.

$\therefore$ equation of tangent is $x\cos\alpha + y\sin\alpha = a$.

(c) Slope form : $y = mx + c$ is tangent to the circle $x^2 + y^2 = a^2$

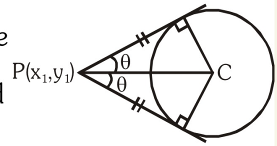
if $c^2 = a^2(1 + m^2)$ (condition of tangency).

$\therefore$ $y = mx \pm a\sqrt{1+m^2}$ is the equation of the tangent to $x^2 + y^2 = a^2 \ \forall\ m \in R$.

Note :

(i) For a unique value of m there will be 2 tangents which are parallel to each other.

* (ii) From an external point 2 tangents can be drawn to the circle which are equal in length and are equally inclined to the line joining the point and the centre of the circle.



(iii) Tangents from external point (x_1, y_1)

Step-1 : Assume equation of tangents $(y - y_1) = m(x - x_1)$

Step-2 : Use p = r to find out 'm'.

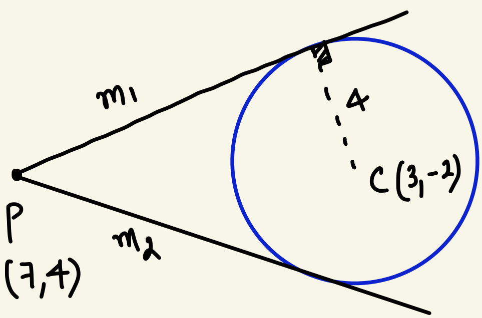
Q

(i) Find the equation of the tangent drawn to the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ from the point (7, 4).

(ii) . Also find the point of contact.

Sol^n

$S_1 : 7^2 + 4^2 - 6(7) + 4(4) - 3 > 0$ Pt (7,4) lies outside Circle



$r = \sqrt{3^2 + 2^2 + 3} = 4$

$y - 4 = m(x - 7)$

$mx - y - 7m + 4 = 0$

$$\frac{|m(3) - (-2) - 7m + 4|}{\sqrt{m^2+1}} = 4$$

$|-\cancel{4}^{2}m + \cancel{6}^{3}| = \cancel{4}^{2}\sqrt{m^2+1}$

$(3 - 2m)^2 = 4(m^2+1)$

$0m^2 - 12m + 5 = 0 \leftarrow 9 + 4m^2 - 12m = 4m^2 + 4$

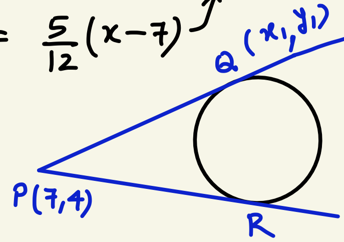
$m = 5/12$ & $m \to \infty$

$T_1 : \; x = 7$

$5x - 35 = 12y - 48$

$5x - 12y = -13$

$T_2 : \; y - 4 = \frac{5}{12}(x - 7)$



$x^2 + y^2 - 6x + 4y - 3 = 0$

$\downarrow (x_1, y_1)$

$T: \; xx_1 + yy_1 - 3(x + x_1) + 2(y + y_1) - 3 = 0$

$$x(x_1 - 3) + y(y_1 + 2) - 3x_1 + 2y_1 - 3 = 0.$$

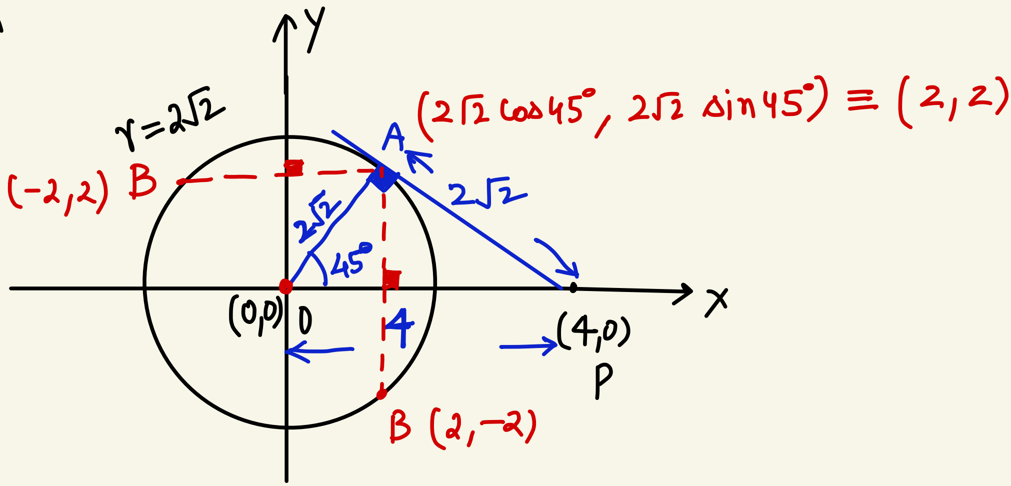
$$5x \quad - 12y \quad + 13 \quad = 0.$$

Compare :

$$\frac{x_1 - 3}{5} = \frac{y_1 + 2}{-12} = \frac{-3x_1 + 2y_1 - 3}{13}.$$

Q. Tangent is drawn from the point P(4, 0) to the circle $x^2 + y^2 = 8$ touches it at the point A in the 1st quadrant. Find the coordinates of another point B on the circle such that AB = 4.

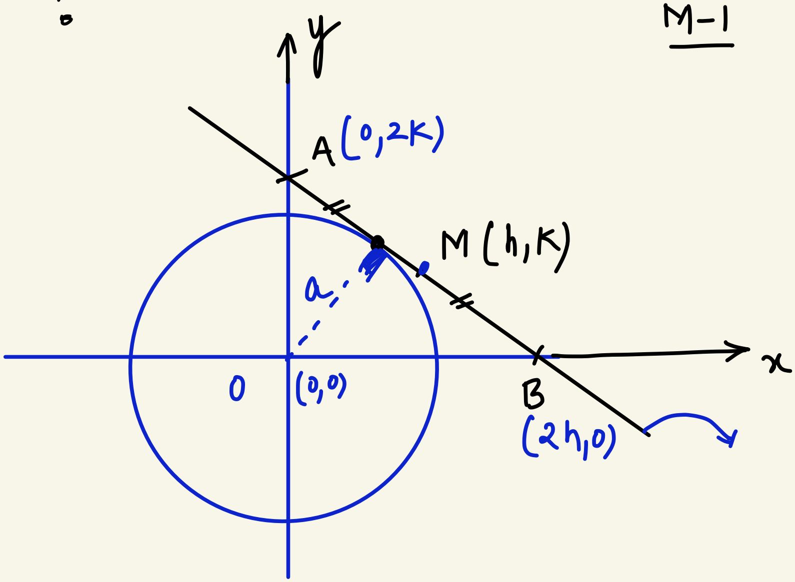
Sol^n



Q Find the locus of the middle point of the portion of tangent to $x^2+y^2=a^2$ terminated by co-ordinate axes ?

Sol^n

M-1



$$\frac{x}{2h}+\frac{y}{2K}=1.$$

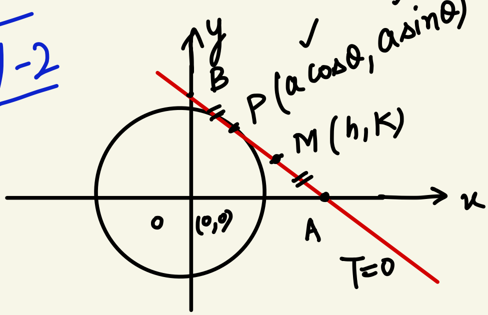
$$\Big\downarrow \; p=r$$

$$\left|\frac{0+0-1}{\sqrt{\frac{1}{4h^2}+\frac{1}{4K^2}}}\right|=a$$

$$\Leftarrow \quad 1=a^2\left(\frac{1}{4h^2}+\frac{1}{4K^2}\right)$$

$$\boxed{4=\frac{a^2}{x^2}+\frac{a^2}{y^2}} \text{ Ans}$$

M-2



T: $x(\cancel{a}\cos\theta)+y(\cancel{a}\sin\theta)=a^{\cancel{2}}$

T: $x\cos\theta+y\sin\theta=a.$

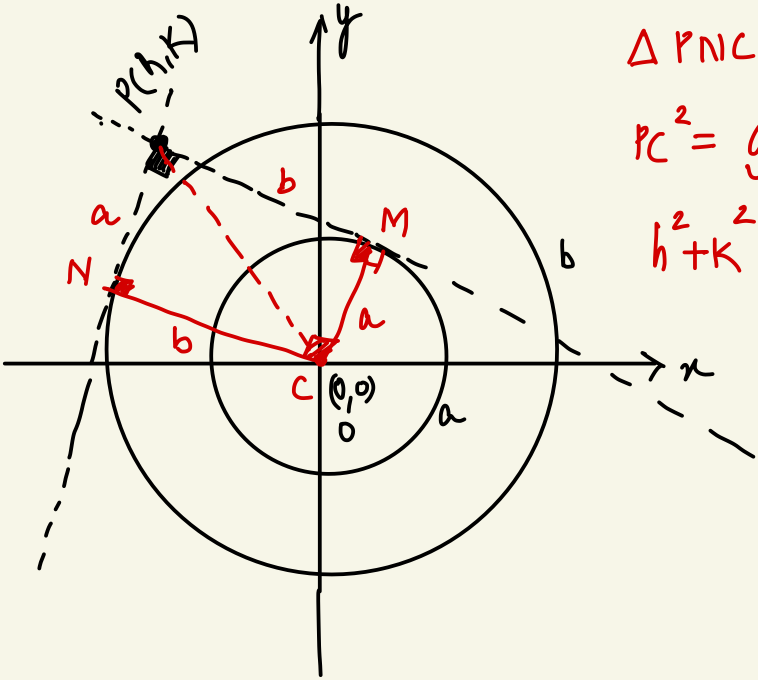
$\downarrow$

$A(a\sec\theta, 0)$; $B(0, a\operatorname{cosec}\theta)$

$2h=a\sec\theta$; $2K=a\operatorname{cosec}\theta.$

$\cos\theta=\frac{a}{2h}$; $\sin\theta=\frac{a}{2K}.$

Q Tangents are drawn to two concentric circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ at right angles to each other then find the locus of their point of intersection



ΔPNC:

$PC^2 = \underbrace{a^2+b^2}$.

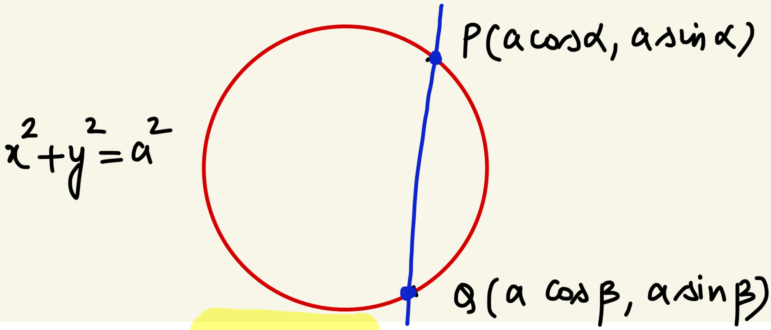
$h^2+k^2=a^2+b^2$

$\boxed{x^2+y^2=a^2+b^2}$

Ans

HW

Q Find equation of circle whose radius is 3 units & which touches the circle $x^2+y^2-4x-6y-12=0$ internally at point $(-1,-1)$?



Rem

Equation of chord of circle $x^2 + y^2 = a^2$ joining two points $P(\alpha)$ & $Q(\beta)$ on the circle is

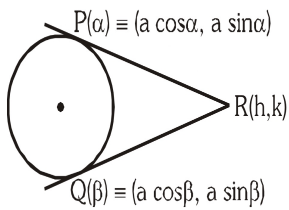
$$\boxed{x\cos\left(\frac{\alpha+\beta}{2}\right)+y\sin\left(\frac{\alpha+\beta}{2}\right)=a\cos\left(\frac{\alpha-\beta}{2}\right)}$$

Note :

Point of intersection of the tangents drawn to the circle $x^2 + y^2 = a^2$

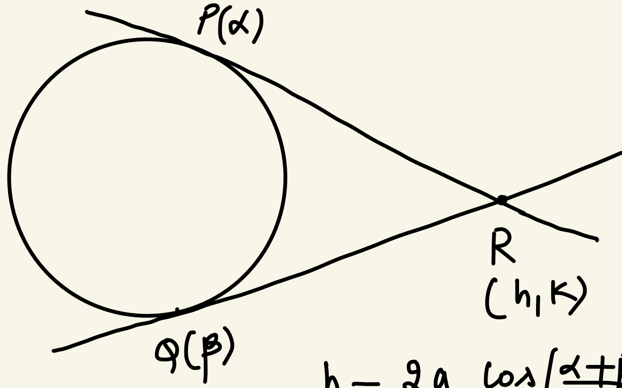
Rem

at the point $P(\alpha)$ and $Q(\beta)$ is $\boxed{h=\dfrac{a\cos\dfrac{\alpha+\beta}{2}}{\cos\dfrac{\alpha-\beta}{2}};\ k=\dfrac{a\sin\dfrac{\alpha+\beta}{2}}{\cos\dfrac{\alpha-\beta}{2}}}$



Q Find the locus of the point of intersection of the pair of tangents drawn to a circle $x^2 + y^2 = a^2$ at $P(\alpha)$ and $Q(\beta)$, where $|\alpha - \beta| = 120°$.

Sol



$|\alpha - \beta| = 120°$

$$h = \frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$K = \frac{a\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$\left.\begin{aligned} h &= 2a\cos\left(\tfrac{\alpha+\beta}{2}\right) \\ K &= 2a\sin\left(\tfrac{\alpha+\beta}{2}\right) \end{aligned}\right\}$$ square & add

$$\boxed{h^2 + K^2 = 4a^2}$$

DIRECTOR CIRCLE :

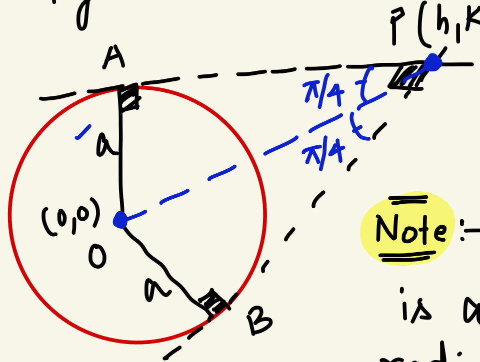
Director circle is a name given to a special locus. Locus of a point 'P' which moves in such a way such that the pair of tangents drawn from 'P' to a given curve makes an anlge of 90°.

OR

Locus of the point of intersection of two mutually perpendicular tangents to a given curve is called the director circle of the given curve.

Director Circle of Circle :-

$S: x^2 + y^2 = a^2$



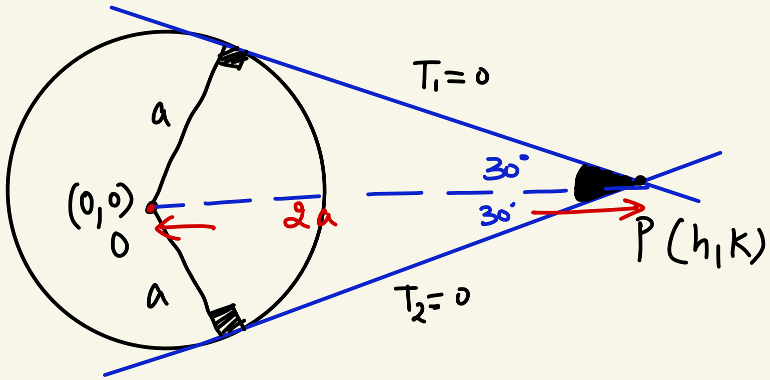
$OP = \sqrt{2}a.$

$\sqrt{h^2 + K^2} = \sqrt{2}a$

$$\boxed{x^2 + y^2 = 2a^2}$$

Note :- Director Circle of Circle is a Concentric Circle whose radius is $\sqrt{2}$ times radius of given circle.

Q Locus of point 'P' which moves such that the angle made by the pair of tangents drawn to the circle $x^2 + y^2 = a^2$ is 60°.

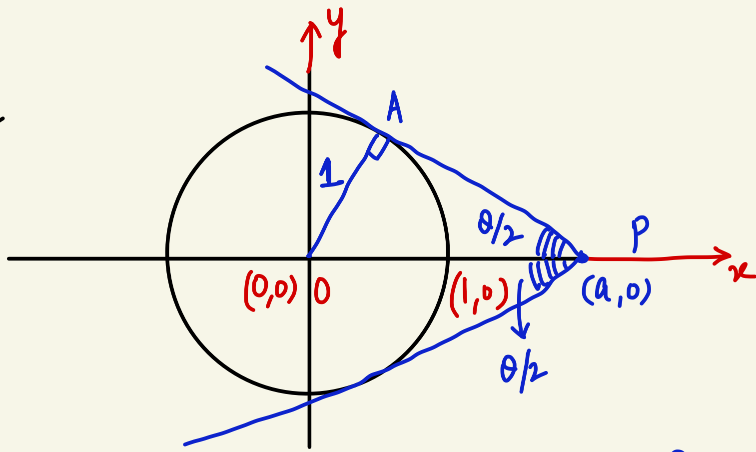


$OP = 2a \Rightarrow \sqrt{h^2 + k^2} = 2a \Rightarrow h^2 + k^2 = 4a^2$

$$\boxed{x^2 + y^2 = (2a)^2}$$

Q Find the range of values of 'a' such that the angles 'θ' between the pair of tangents drawn from the point (a, 0) to the circle $x^2 + y^2 = 1$ satisfies $\frac{\pi}{2} < \theta < \pi$.

M-I



$OP = |a|$

$\frac{\theta}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\sin\frac{\theta}{2} = \frac{1}{|a|}$

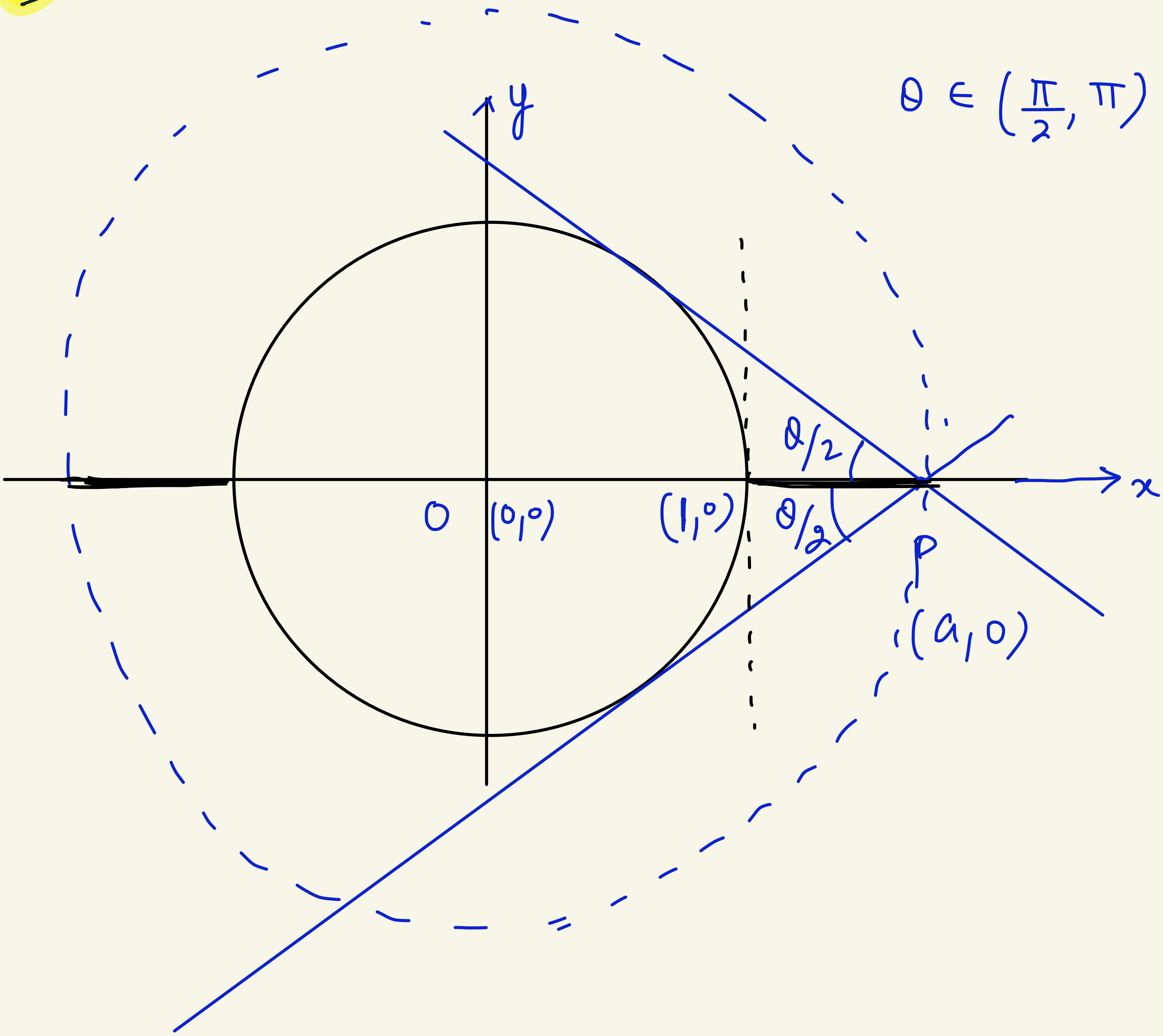
$|a| = \operatorname{cosec}\left(\frac{\theta}{2}\right)$

$\operatorname{cosec}\frac{\theta}{2} \in (1, \sqrt{2})$

$\therefore |a| \in (1, \sqrt{2})$

$a \in (-\sqrt{2}, -1) \cup (1, \sqrt{2})$ Ans

M-2



$\theta \in \left(\frac{\pi}{2}, \pi\right)$

$a \in (-\sqrt{2}, -1) \cup (1, \sqrt{2})$ Ans

Length of Tangent & power of a point.

"Length of the tangent from an external point $P(x_1, y_1)$ to a given circle."

$$S = x^2 + y^2 + 2gx + 2fy + c = 0$$

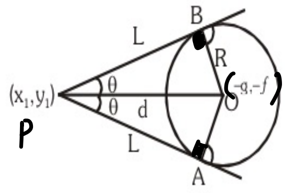
$$L^2 + R^2 = d^2$$

$$L^2 = d^2 - R^2 = (x_1 + g)^2 + (y_1 + f)^2 - (g^2 + f^2 - c)$$

$$L^2 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$$

Rem

$$\therefore \quad L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} = \sqrt{S_1}$$



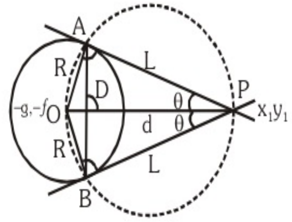
Note : All these formulae are applicable when coefficient of x^2 & y^2 is unity

(i) Area of Quad PAOB = $2\Delta POA = 2 \cdot \frac{1}{2}RL = RL$

(ii) Find AB i.e. length of chord of contact

$AB = 2L\sin\theta$ where $\tan\theta = \frac{R}{L}$

$$= \frac{2RL}{\sqrt{R^2 + L^2}}$$



(iii) Area of ΔPAB (Δ formed by pair of Tangent & corresponding C.O.C.)

$$\Delta PAB = \frac{1}{2}AB \times PD = \frac{1}{2}(2L\sin\theta)(L\cos\theta) = L^2\sin\theta\cos\theta = \frac{RL^3}{R^2 + L^2}$$

(iv) Angle between the pair of Tangent $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2RL^2}{L(L^2 - R^2)}$

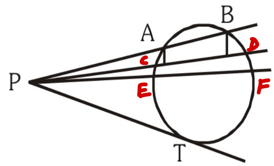
$$2\theta = \tan^{-1}\left(\frac{2RL}{L^2 - R^2}\right)$$

* (v) Equation of the circle circumscribing the ΔPAB. (One such circle described on OP as diameter)

i.e. $(x - x_1)(x + g) + (y - y_1)(y + f) = 0$

Power of a point :

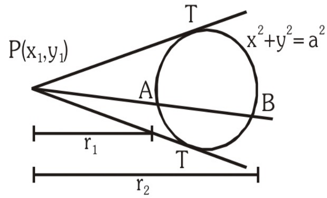
Square of the length of the tangent from the point P is called power of the point P w.r.t. a given circle i.e. $PT^2 = S_1$



Note : Power of a point remains constant w.r.t. a circle

$PA \cdot PB = PT^2 = PC \times PD = PE \times PF$

Analytical proof :



$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$; substituting $x = x_1 + r\cos\theta$

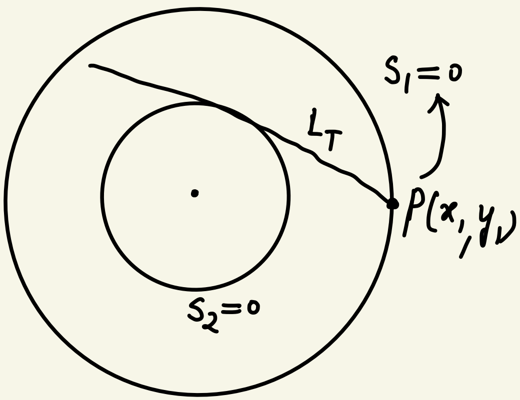
and $y = y_1 + r\sin\theta$ in $x^2 + y^2 = a^2$,

we get, $r^2 + 2r(x_1\cos\theta + y_1\sin\theta) + x_1^2 + y_1^2 - a^2 = 0$

$r_1r_2 = x_1^2 + y_1^2 - a^2 = \text{constant} = (PT)^2$

Note : Power of a point is positive/0 (zero)/negative according as point 'P' lies outside/on/inside the circle.

Q Find the length of the Tangent from any point on the circle $S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + \lambda = 0$ to the circle $S_2 \equiv x^2 + y^2 + 2g_1x + 2f_1y + \mu = 0$ is



$L_T = \sqrt{S_2}$

$L_T = \sqrt{x_1^2+y_1^2+2g_1x_1+2f_1y_1+\mu}$

$L_T = \sqrt{-\lambda+\mu}$

$L_T = \sqrt{\mu-\lambda}$ Ans

$\underbrace{x_1^2+y_1^2+2g_1x_1+2f_1y_1}_{0}+\lambda=0$ —①—

Q Find the value of 'p' for which the power of a point P(2, 5) is negative w.r.t a circle $x^2 + y^2 - 8x - 12y + p = 0$ and the circle neither touches nor intersects the coordinate axis.

Sol^n

$(2)^2 + (5)^2 - 8(2) - 12(5) + p < 0$ — ① —

$g^2 - c < 0 \Rightarrow 16 - p < 0$ — ② —

$f^2 - c < 0 \Rightarrow 36 - p < 0$ — ③ —

$g = -4$

$f = -6$

$c = p$

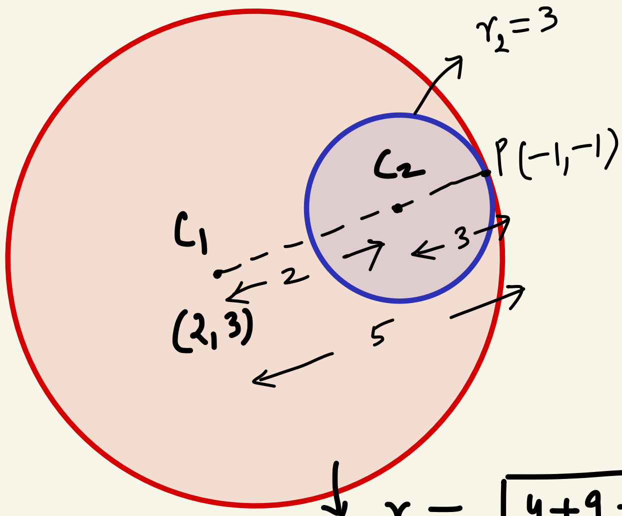
① ∩ ② ∩ ③

HW

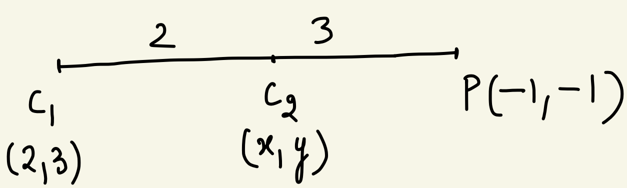
Q Find the locus of a point the tangents from which to the circles $4x^2 + 4y^2 - 9 = 0$ and $9x^2 + 9y^2 - 16 = 0$ are in the ratio 3 : 4.

HW

Q Find equation of circle whose radius is 3 units & which touches the circle $x^2+y^2-4x-6y-12=0$ internally at point $(-1,-1)$?



$$r_1 = \sqrt{4+9+12} = 5$$



$$x = \frac{2(-1)+3(2)}{5} \quad \& \quad y = \frac{2(-1)+3(3)}{5}$$

$$x = \frac{4}{5} \quad \& \quad y = \frac{7}{5}$$

S_2: $\left(x-\frac{4}{5}\right)^2 + \left(y-\frac{7}{5}\right)^2 = 3^2$ Ans

Q Find the locus of a point the tangents from which to the circles $4x^2 + 4y^2 - 9 = 0$ and $9x^2 + 9y^2 - 16 = 0$ are in the ratio $3 : 4$.

$$S_1 : x^2 + y^2 - \frac{9}{4} = 0$$

$$S_2 : x^2 + y^2 - \frac{16}{9} = 0.$$

$P(h, k)$

$$\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{3}{4}$$

$$4\sqrt{S_1} = 3\sqrt{S_2}$$

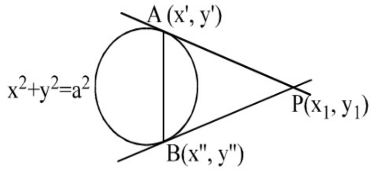
$$16\left(x^2 + y^2 - \frac{9}{4}\right) = 9\left(x^2 + y^2 - \frac{16}{9}\right)$$

$$16x^2 + 16y^2 - 36 = 9x^2 + 9y^2 - 16.$$

$$\boxed{7x^2 + 7y^2 = 20}$$ Ans

CHORD OF CONTACT : C·O·C

Definition : It is defined as the straight line joining the point of contact of the pair of tangents drawn from an external point P (x_1, y_1) to a circle.



Equation of chord of contact AB :

Tangent at 'A' $\quad x\,x' + y\,y' = a^2$

passes through (x_1, y_1) $\quad \Rightarrow x_1\,x' + y_1\,y' = a^2$ ....(1)

|||ly at 'B' $\quad x\,x'' + y\,y'' = a^2$

passes through (x_1, y_1) $\quad \Rightarrow x_1\,x'' + y_1\,y'' = a^2$ ....(2)

from (1) and (2) equation if chord of contact can be written by replacing x', x" by x and y', y" by y.

hence, $\boxed{x\,x_1 + y\,y_1 = a^2}$ i.e. $\boxed{T=0}$ ....(3) (same as that of tangent)

(i) Equation (3) is a linear relation in x and y

$\therefore$ it represent a line.

(ii) Point 'A' and 'B' satisfy equation (3) $\qquad x^2+y^2+2gx+2fy+c=0$

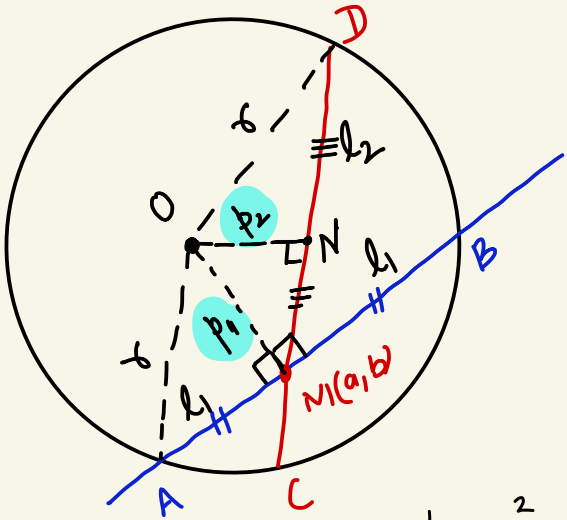
$\therefore$ it represent the equation of chord of contact. For general equation of circle chord of contact $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

i.e. $\boxed{T=0}$

EQUATION OF THE CHORD WITH A GIVEN MIDDLE POINT ($T = S_1$) :

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point $M(x_1, y_1)$ is $\mathbf{y - y_1 = -\dfrac{x_1 + g}{y_1 + f}(x - x_1)}$. This on simplication can be put in the form $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $\boxed{\mathbf{T = S_1}}$.

Note : Of all the chords which passes through a given point M(a, b) inside the circle the shortest chord is one whose middle point is (a, b)



$l_1 = \sqrt{r^2 - p_1^2}$

$l_2 = \sqrt{r^2 - p_2^2}$

$p_1 > p_2$

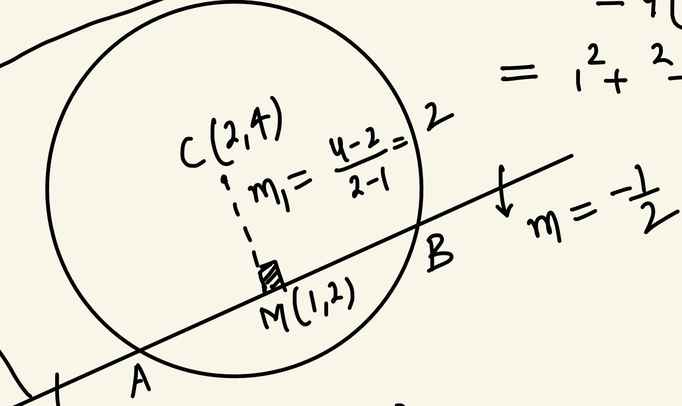
$\Rightarrow l_2 > l_1$

Q Find equation of chord to circle $x^2 + y^2 - 4x - 8y + 1 = 0$ whose middle point is M(1, 2) ?

$\boxed{T = S_1}$

$x(1) + y(2) - 2(x+1) - 4(y+2) + 1$

$= 1^2 + {}^2 - 4 - 8(2) + 1$

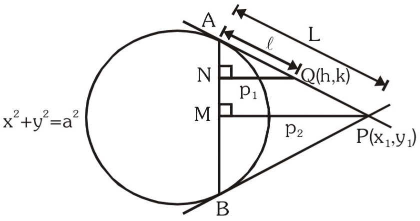


AB: $y - 2 = \frac{-1}{2}(x - 1)$

PAIR OF TANGENTS :

Combined equation of the pair of tangents drawn from an external point 'P' to a given circle is $\boxed{SS_1 = T^2}$, where $S \equiv x^2 + y^2 - a^2$; $S_1 \equiv x_1^2 + y_1^2 - a^2$.

$S \equiv x^2 + y^2 - a^2$; $S_1 \equiv x_1^2 + y_1^2 - a^2$.



Proof : Equation of COC AB is $xx_1 + yy_1 = a^2$

triangles AQN and APM are similar

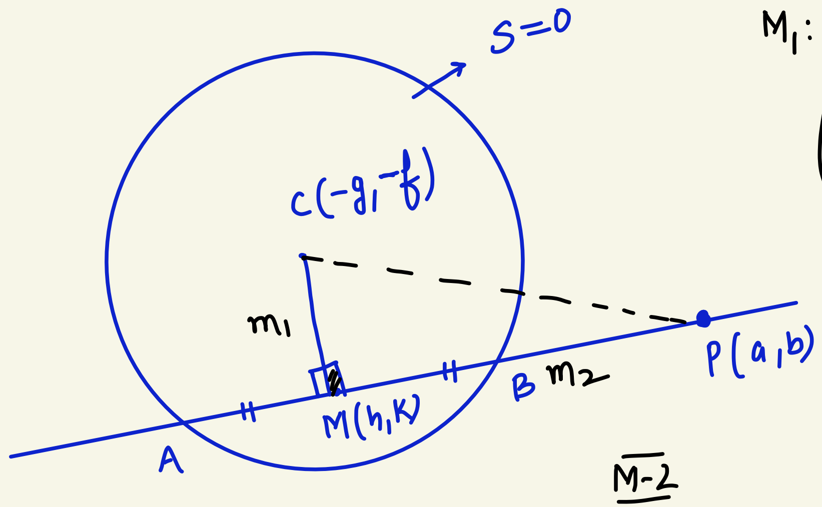
$$\frac{AQ}{AP} = \frac{QN}{PM} \Rightarrow \frac{\ell}{L} = \frac{P_1}{P_2}$$

$$\Rightarrow \quad \frac{\sqrt{h^2 + k^2 - a^2}}{\sqrt{x_1^2 + y_1^2 - a^2}} = \frac{hx_1 + ky_1 - a^2}{x_1^2 + y_1^2 - a^2}$$

$$\therefore \quad \underbrace{(x^2 + y^2 - a^2)}_{S}\underbrace{(x_1^2 + y_1^2 - a^2)}_{S_1} = \underbrace{(xx_1 + yy_1 - a^2)^2}_{=T^2}$$

Q Locus of the middle point of the chords of the circles $x^2 + y^2 + 2gx + 2fy + c = 0$ which passes through a fixed point (a, b) lying outside the circle.

Sol^n

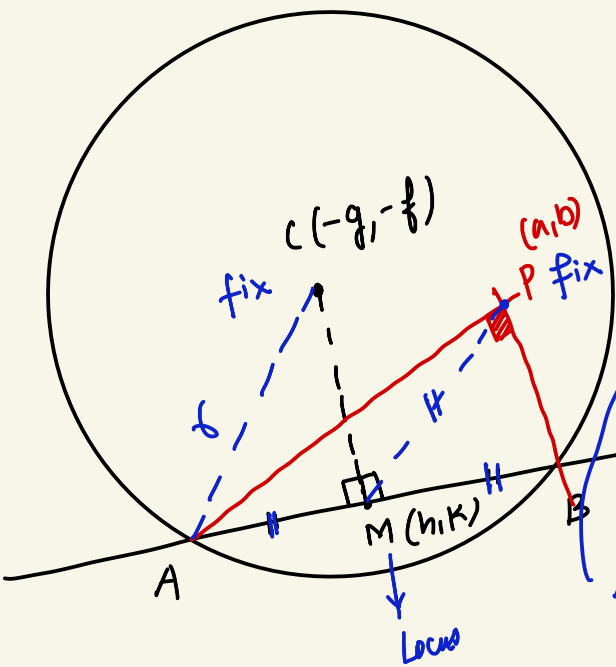


M_1: $m_1 m_2 = -1$

$$\left(\frac{K+f}{h+g}\right)\left(\frac{K-b}{h-a}\right) = -1$$

M-2 : Circle described on PC as diameter.

Q. Find the equation to the locus of the middle point of the chord of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which subtends right angle at a given point P (a, b)



In Δ ACM :

$AC^2 = AM^2 + CM^2$

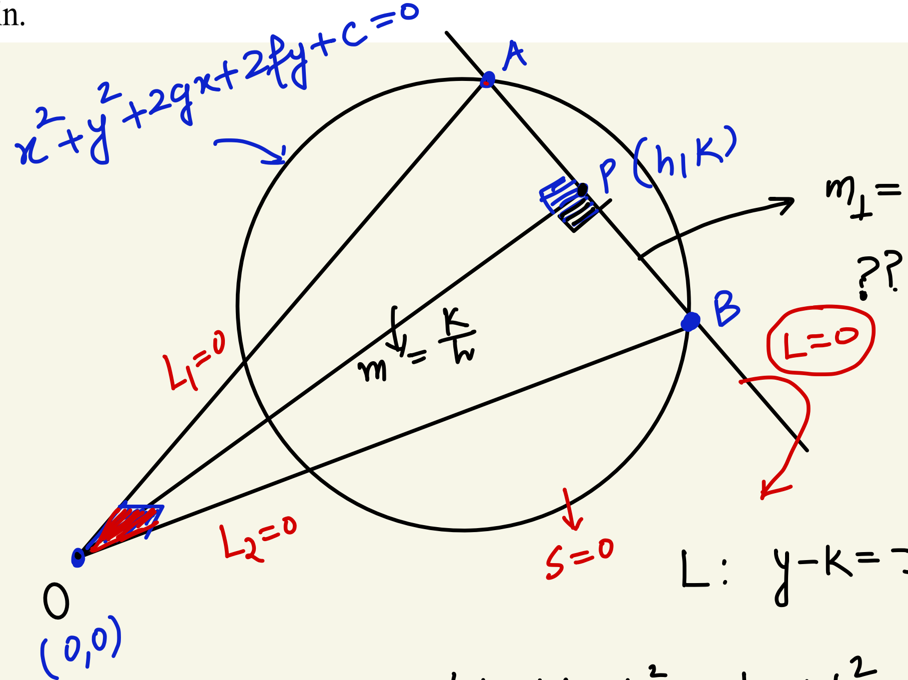
$g^2 + f^2 - c = \underbrace{AM^2}$ +

$(h+g)^2 + (k+f)^2$

AM = MB = MP

$AM = \sqrt{(h-a)^2 + (k-b)^2}$.

Q Find the equation to the locus of the feet of the perpendicular drawn from the origin upon a variable chord of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which subtends right angle at origin.



$L: \; y-K = \frac{-h}{K}(x-h)$

$L: \; yK - K^2 = -hx + h^2$

$hx + yK = h^2 + K^2$

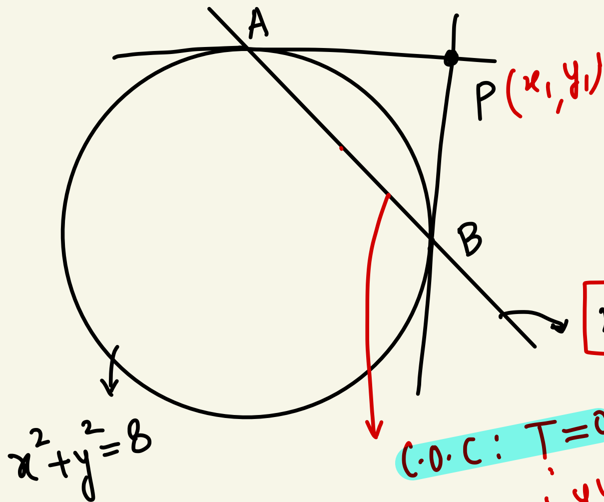
$\frac{hx+yK}{h^2+K^2} = 1 \quad — (1)$

Homogenise Circle with (1) :—

$x^2+y^2+(2gx+2fy)\left(\frac{hx+yK}{h^2+K^2}\right)+c\left(\frac{hx+yK}{h^2+K^2}\right)^2 = 0.$

Coeff of x^2 + Coeff of $y^2 = 0.$

Q Tangents are drawn to the circle $x^2 + y^2 = 8$ at the points where the line $x + 2y = 4$ intersect the circle. Find the coordinates of the point of intersection of the tangents.



$\boxed{x+2y=4}$ —①—

C.O.C : T=0

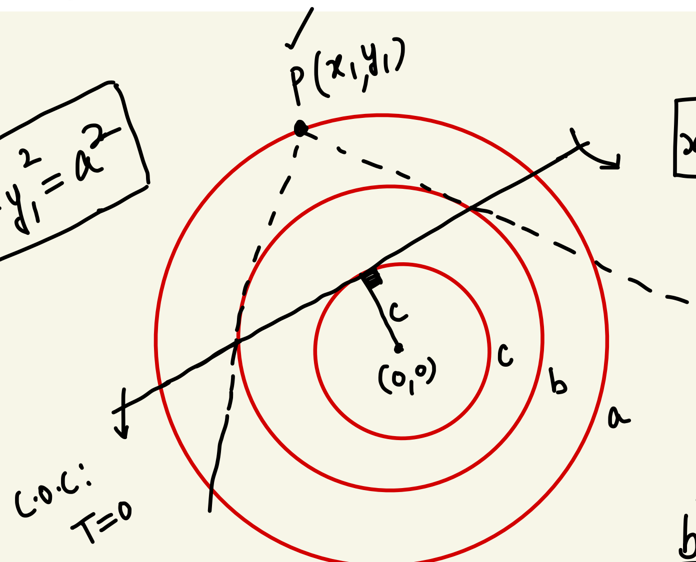
$xx_1 + yy_1 = 8$ —②—

Compare:—

$$\frac{x_1}{1} = \frac{y_1}{2} = \frac{8}{4} \Rightarrow \left.\begin{matrix} x_1 = 2 \\ y_1 = 4 \end{matrix}\right\}$$

Q Chord of contact of the tangent drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$. Prove that a, b, c are in G.P. (a, b, c > 0)

$\boxed{x_1^2 + y_1^2 = a^2}$



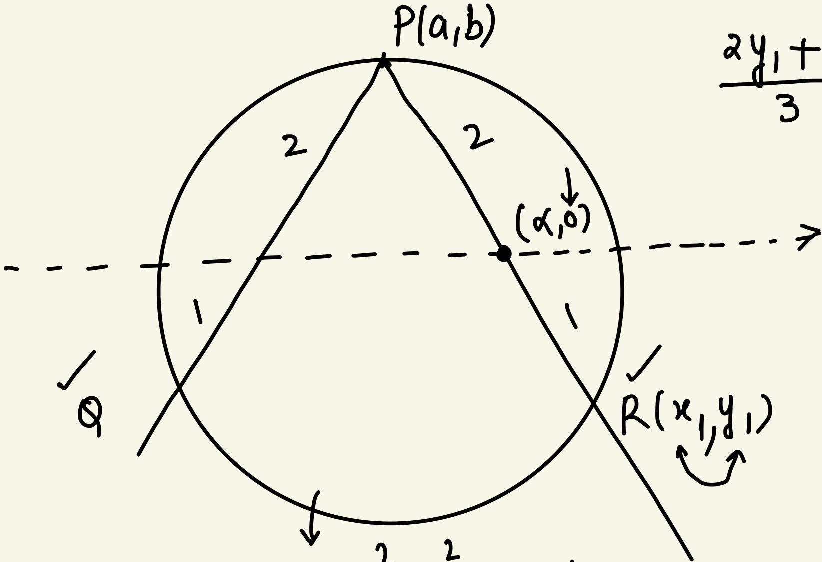
$\boxed{xx_1 + yy_1 = b^2}$

$$\frac{|0+0-b^2|}{\sqrt{x_1^2+y_1^2}} = c$$

$$b^2 = c\sqrt{x_1^2+y_1^2}$$

$$\boxed{b^2 = ca}$$

If 2 distinct chords of the circle $x^2 + y^2 - ax - by = 0$ drawn from the point (a, b) is divided by the x-axis in the ratio 2 : 1 then prove that $a^2 > 3b^2$.



x-axis

$$\frac{2y_1 + b}{3} = 0$$

$$y_1 = \frac{-b}{2}$$

$$S: x^2 + y^2 - ax - by = 0.$$

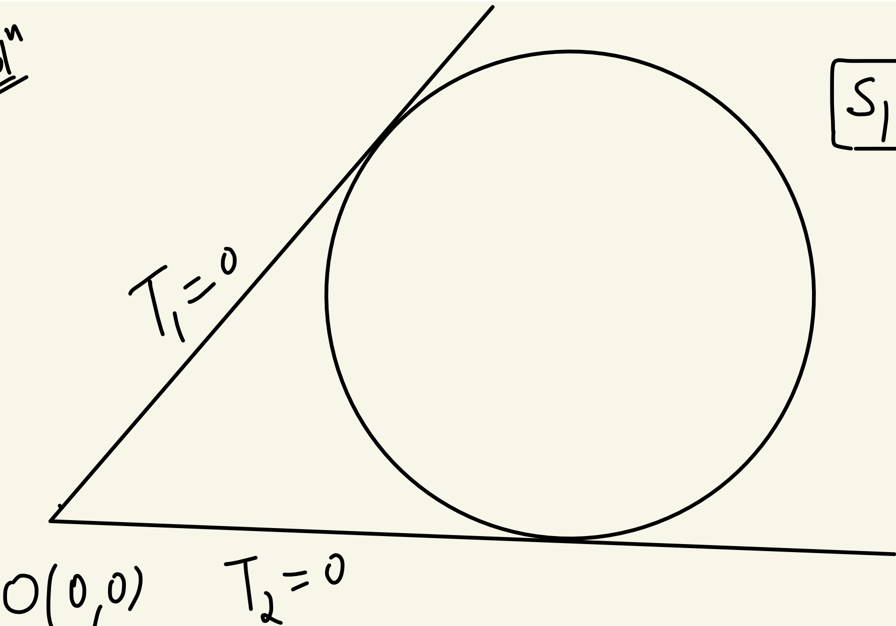
$$x_1^2 + y_1^2 - ax_1 - by_1 = 0$$

$$x_1^2 + \frac{b^2}{4} - ax_1 + \frac{b^2}{2} = 0$$

$$x_1^2 - ax_1 + \frac{3b^2}{4} = 0. \xrightarrow{D>0} a^2 - \cancel{4}\cdot\left(\frac{3b^2}{\cancel{4}}\right) > 0$$

Q. Show that the equation to the pair of tangents drawn from the origin to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $(gx + fy)^2 = c(x^2 + y^2)$

Sol^n



$$\boxed{S_1 = c}$$

$$\boxed{SS_1 = T^2}$$

$$(x^2+y^2+2gx+2fy+c).c = \left(x(0)+y(0)+g(x+0)+f(y+0)+c\right)^2$$

$$(x^2+y^2+2gx+2fy+c)\,c = (\underbrace{gx+fy}+c)^2$$

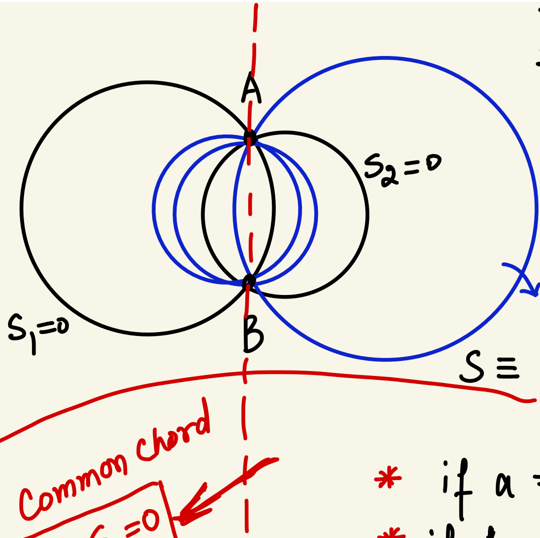
$$(x^2+y^2)c+\cancel{(2gx+2fy)c}+\cancel{c^2} = (gx+fy)^2+\cancel{c^2}+\cancel{2c(gx+fy)}$$

FAMILY OF CIRCLES :

Type-1 : Equation of the family of circles which passes through the points of intersection of two circles $S_1 = 0$ and $S_2 = 0$ is

$S_1 : x^2+y^2+2g_1x+2f_1y+c_1=0$

$S_2 : x^2+y^2+2g_2x+2f_2y+c_2=0$



$S_1-S_2 = 2(g_1-g_2)x+2(f_1-f_2)y+c_1-c_2=0.$

Common chord $\boxed{S_1-S_2=0}$

$S \equiv \boxed{aS_1+bS_2=0}$

* if $a=0$ then $S_2=0$.

* if $b=0$ then $S_1=0$.

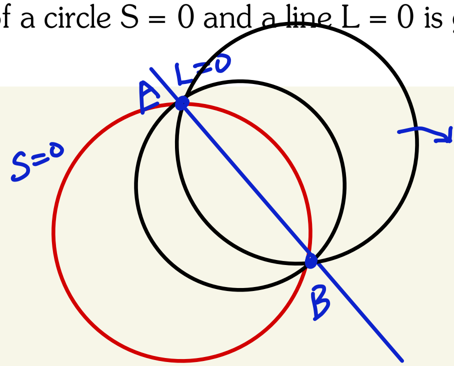
* if $a=-b$ then $S_1-S_2=0$. → Eqn of line AB.

if $a\neq 0$

$\boxed{S_1+\lambda S_2=0}$; $\lambda\in R$ & $\lambda\neq -1$ → Family of Circle passing thru intersection of $S_1=0$ & $S_2=0$.

Type-2 : Equation of the family of circles passes through the point of intersection of a circle S = 0 and a line L = 0 is given by $S + \lambda L = 0$.

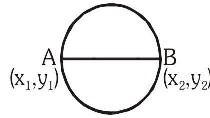
$\lambda \in R$



Type-3 : Equation of the family of circles passes through two given points $A(x_1, y_1)$ & $B(x_2, y_2)$ is :

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+\lambda\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0 \Rightarrow \boxed{S+\lambda L=0}$$

Note : Fixed circle of this family is circle described on AB as diameter.



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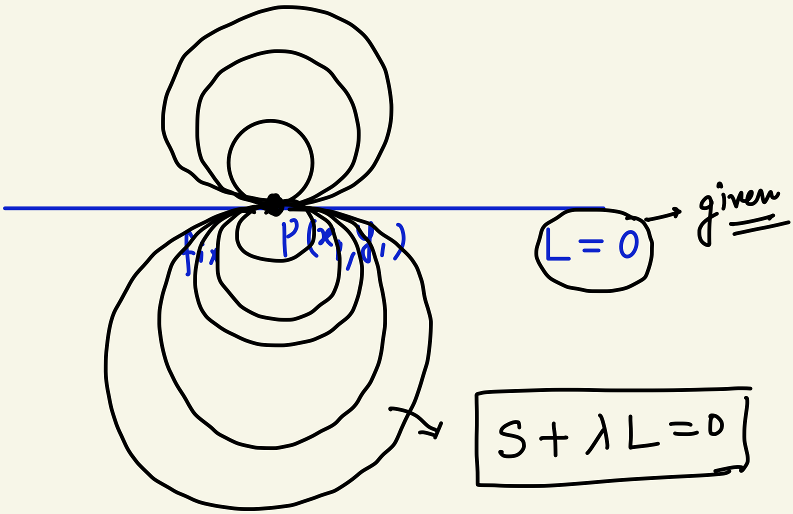
Type-4 : Equation of family of circle touching a line L at its fixed point (x_1, y_1) is



$$\boxed{S+\lambda L=0}$$

$$\boxed{\underbrace{(x-x_1)^2+(y-y_1)^2}_{0}+\lambda L=0}$$

Q Find the equation of a circle which passes through the point of intersection of $S_1 = 0$ and $S_2 = 0$.

$S_1 : x^2 + y^2 - 4x + 6y - 3 = 0$

$S_2 : x^2 + y^2 + 4x - 6y + 12 = 0$

whose : (i) radius is 5 units.

(ii) which passes through $(0,1)$.

(iii) whose centre lies on x-axis.

Sol^n $S_1 + \lambda L = 0$ where $L \equiv S_1 - S_2 = 0$

$L : 8x - 12y + 15 = 0$

$$(x^2+y^2-4x+6y-3) + \lambda(8x-12y+15) = 0$$

$$S : x^2 + y^2 + (8\lambda-4)x + (6-12\lambda)y + 15\lambda - 3 = 0$$

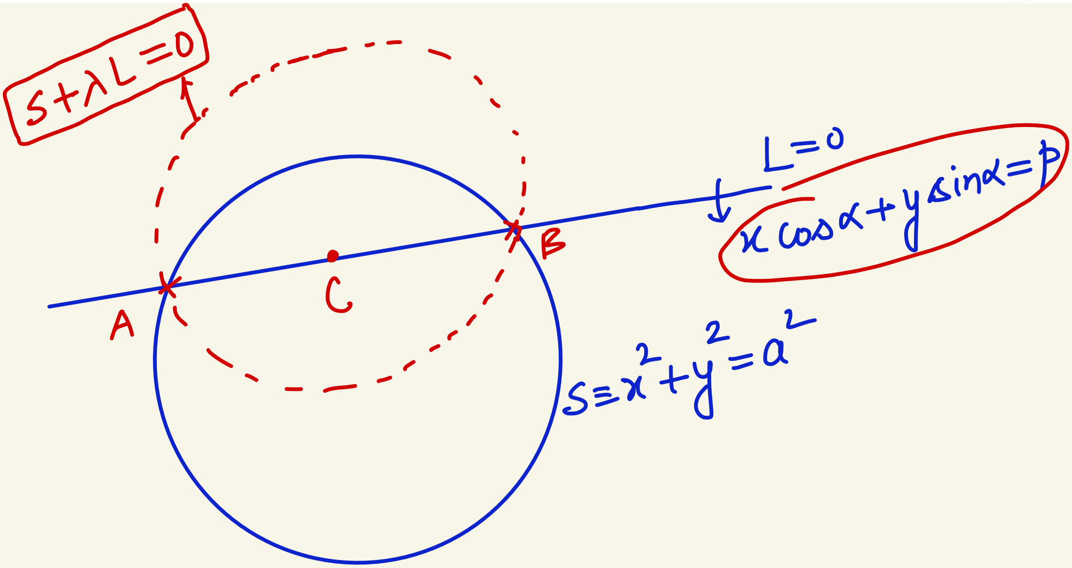
(ii) $(0,1)$

$$1 + 6 - 12\lambda + 15\lambda - 3 = 0$$

$$3\lambda + 4 = 0 \Rightarrow \boxed{\lambda = -\frac{4}{3}}$$

Q

Find the equation of a circle drawn on the chord x cos α + y sin α = p of the circle $x^2 + y^2 = a^2$ as its diameter.



$$S+\lambda L \equiv (x^2+y^2-a^2)+\lambda(x\cos\alpha+y\sin\alpha-p)=0.$$

Centre : $\left(-\frac{\lambda}{2}\cos\alpha ; -\frac{\lambda}{2}\sin\alpha\right)$

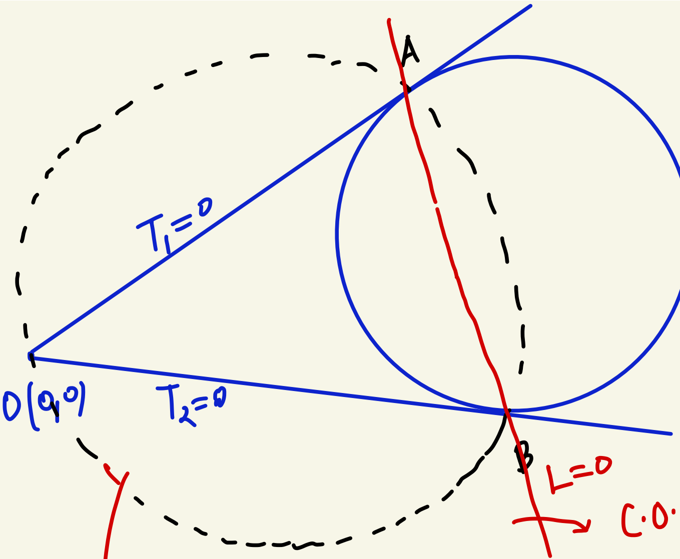
lies on $L=0$

$$-\frac{\lambda}{2}\cos^2\alpha-\frac{\lambda}{2}\sin^2\alpha=p \Rightarrow \lambda=-2p$$

Q Find the equation of a circle which passes through origin and through the point of contact of the tangents drawn from the origin to the circle $x^2 + y^2 - 11x + 13y + 17 = 0$.

Sol^n

M-1



$S: x^2+y^2-11x+13y+17=0$

C.O.C : $T=0$

$-\frac{11}{2}(x+0)+\frac{13}{2}(y+0)+17=0$

$L \equiv -11x+13y+34=0$

$S+\lambda L=0$

$$\boxed{x^2+y^2-11x+13y+17 + \lambda(-11x+13y+34)=0}$$

(0,0)

$17+\lambda(34)=0 \Rightarrow \lambda = -\frac{1}{2}$

M-2

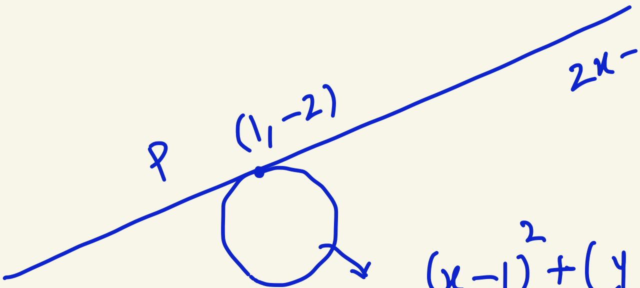
This will be circle described on (0,0) & $\left(\frac{11}{2}, -\frac{13}{2}\right)$ as diameter.

Q. Find the equation of a circle which touches the line $2x - y = 4$ at the point $(1, -2)$ and

(a) Passes through $(3, 4)$

(b) Radius = 5

Sol^n



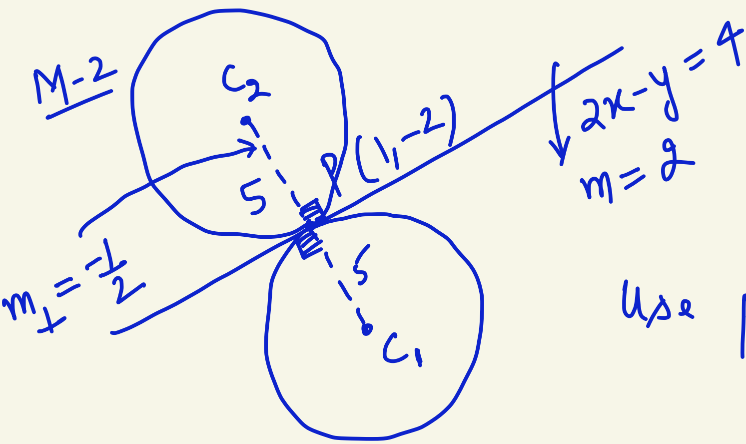
$$(x-1)^2 + (y+2)^2 + \lambda(2x-y-4) = 0$$

(i) $(3,4)$

$$(3-1)^2 + (4+2)^2 + \lambda(2(3)-4-4) = 0$$

(ii) Rad = 5

M-2



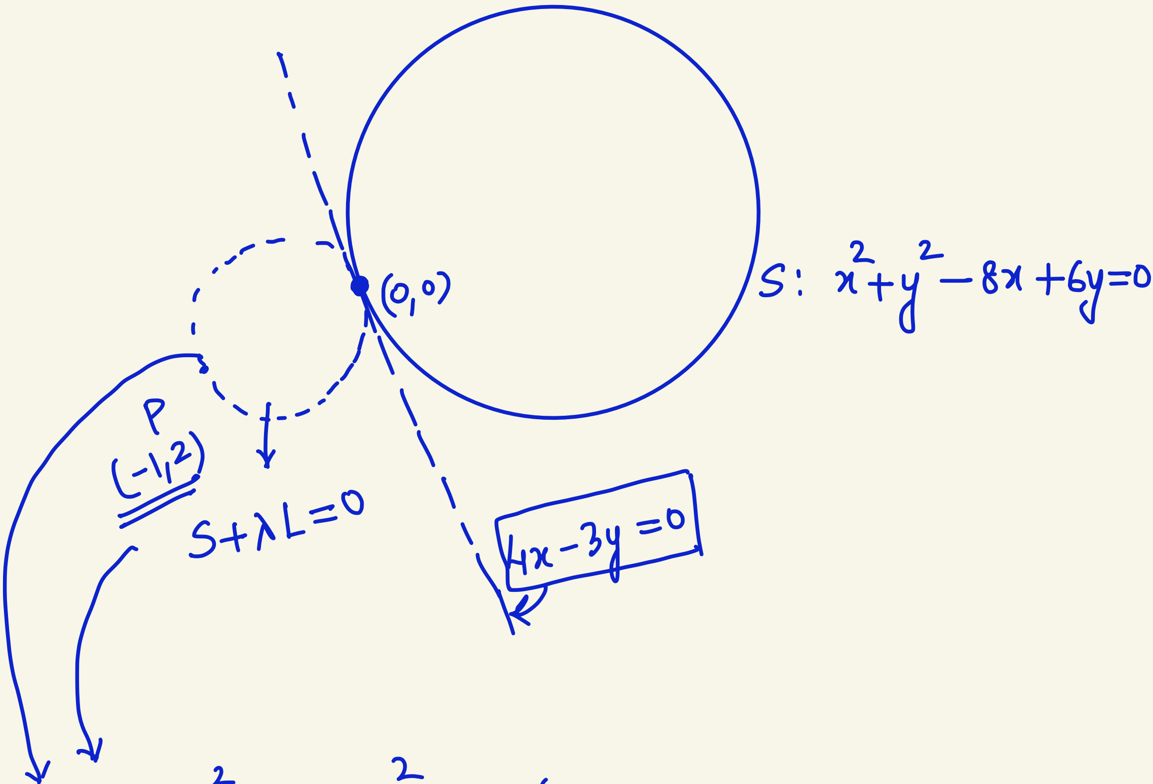
$\tan\theta = -\frac{1}{2}$

$\sin\theta = 1/\sqrt{5}$

$\cos\theta = -2/\sqrt{5}$

Use parametric to get C_1 & C_2

Q Find the equation of the circle which passes through the point (–1, 2) & touches the circle $x^2 + y^2 - 8x + 6y = 0$ at origin.



$$(x-0)^2+(y-0)^2+\lambda(4x-3y)=0$$

$$(-1)^2+(2)^2+\lambda(-4-6)=0.$$

Q Find the equation of the circles to which the line $4x + 3y = 10$ is a common tangent at $(1, 2)$ and radius of each of the circle is 5.

HW

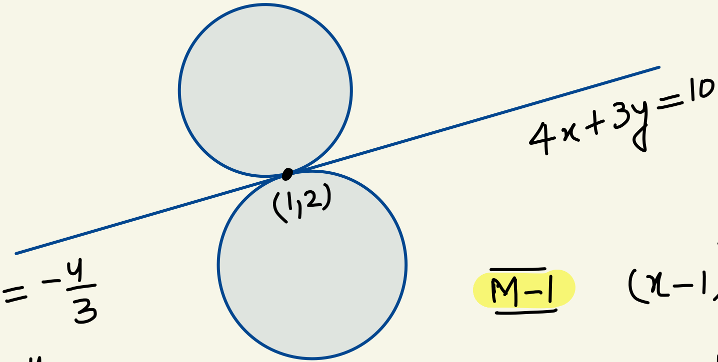
HW

Q Tangents are drawn to unit circle centered at origin from every point on $2x+y=4$. then

(i) P.T the chord passes through a fixed point

(ii) P.T equation of locus of middle point of chord of contact is $4x^2+4y^2-2x-y=0$.

Q Find the equation of the circles to which the line 4x + 3y = 10 is a common tangent at (1, 2) and
HW radius of each of the circle is 5.



$m = -\frac{4}{3}$

$\tan\theta = -4/3$

$\sin\theta = 4/5$

$\cos\theta = -3/5$

M-1 $(x-1)^2 + (y-2)^2 + \lambda(4x+3y-10) = 0$

Equate radius = 5

M-2 use parametric

$$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = \pm 5$$

get centre of circles.

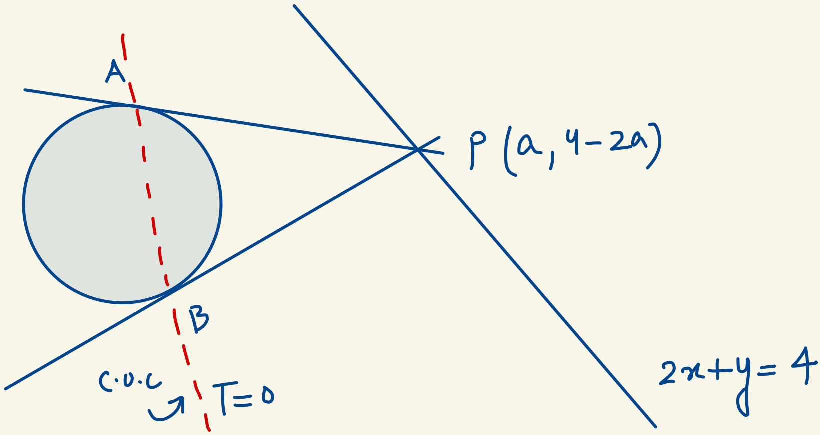
HW

Q. Tangents are drawn to unit circle centered at Origin from every point on $2x+y=4$. then

(i) P.T the chord passes through a fixed point

(ii) P.T equation of locus of middle point of chord of contact is $4x^2+4y^2-2x-y=0$.

Sol^n



$$x(a)+y(4-2a)=1 \quad \text{— ① —}$$

$$(4y-1)+a(x-2y)=0$$

$$L_1 + a\,L_2 = 0 \Rightarrow \quad 4y-1=0 \ \& \ x-2y=0$$

$$y=\frac{1}{4}\ ;\ x=\frac{1}{2}.$$

(ii) $M(h,k)$ (Middle point)

$$T=S_1 \Rightarrow \quad xh+yk-\cancel{1}=h^2+k^2-\cancel{1}$$

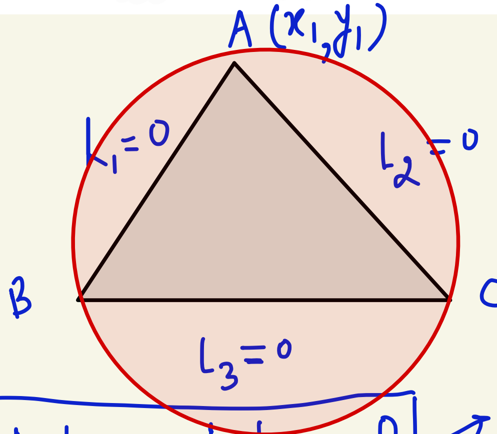
$$xh+yk=h^2+k^2 \quad \text{— ② —}$$

Compare ① & ② : $\quad \underbrace{\frac{h}{a}=\frac{k}{4-2a}}=\frac{h^2+k^2}{1}$

eliminate 'a' to get req. locus.

Note :-

1. Equation of a circle circumscribing a triangle whose sides are given by $\ell_1 = 0$; $\ell_2 = 0$ & $\ell_3 = 0$ is given by

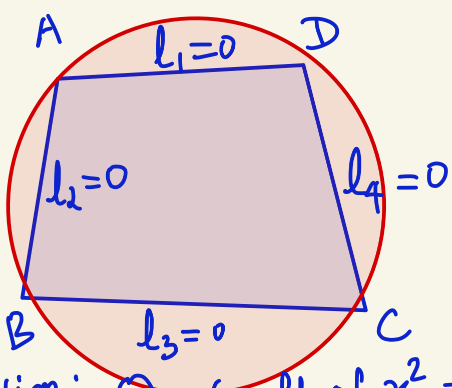


$$\boxed{L_1 L_2 + \lambda L_2 L_3 + \mu L_3 L_1 = 0} \rightarrow$$ 2nd degree curve passing thru A, B, C.

Req. Condition: ① Coeff of x^2 = Coeff of y^2 **and**

② Coeff of $xy = 0$

2. Equation of a circle circumscribing a quadrilateral whose sides in order are represented by the lines $\ell_1 = 0$; $\ell_2 = 0$; $\ell_3 = 0$; $\ell_4 = 0$ is given



$$\boxed{l_1 l_3 + \lambda l_2 l_4 = 0}$$

↓

2nd degree passing thru A, B, C & D

Req. Condition: ① Coeff of x^2 = Coeff of y^2 **and**

② Coeff of $xy = 0$

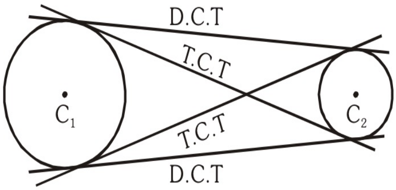
COMMON TANGENTS TO TWO CIRCLES :

Common tangents to two circles

(1) Direct Common Tangent (DCT)
(External common tangent)

(2) Transverse Common Tangent (TCT)
(Internal common tangent)

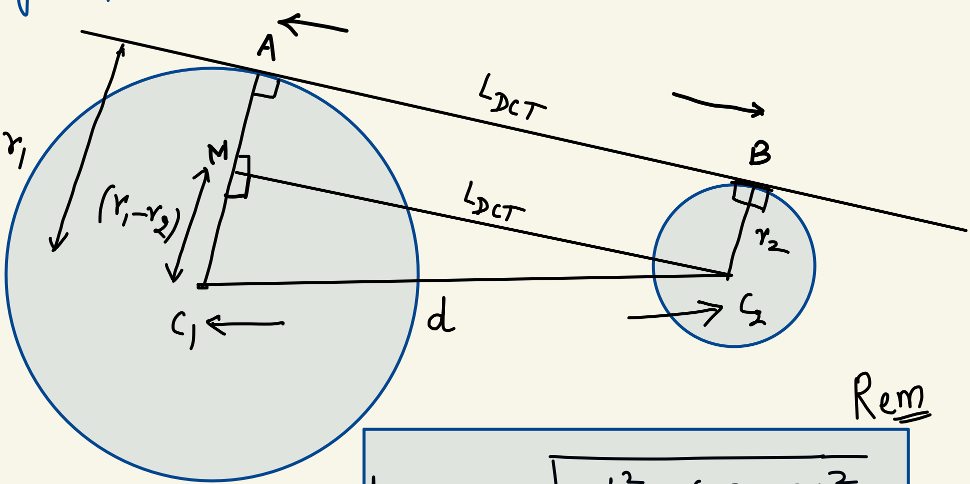
How do we distinguished between D.C.T. and T.C.T.



Direct Common Tangent (D.C.T.) : Here the centre of both the circles lies on the same side of the tangent line.

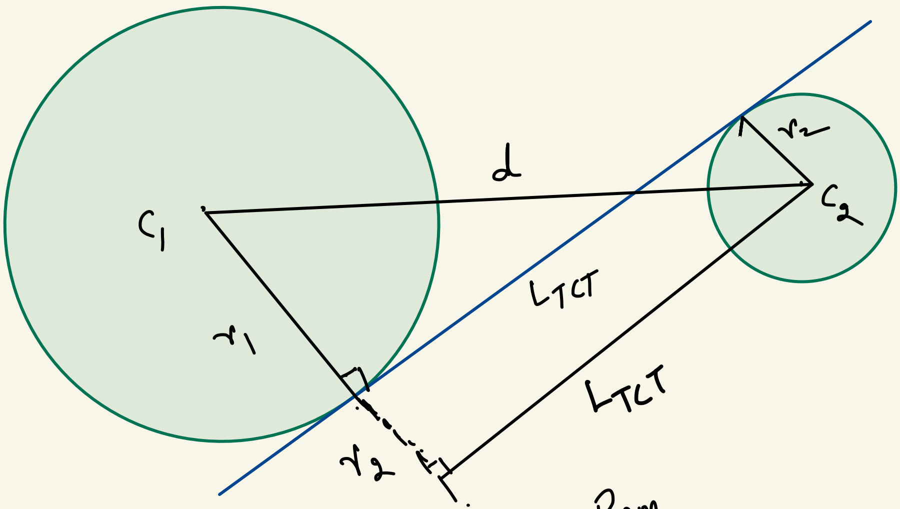
Transverse Common Tangent (T.C.T.) : Here the centre of both the circles lies on the opposite side of the tangent line.

Length of DCT & TCT :–



Rem

$$\boxed{L_{DCT} = \sqrt{d^2-(r_1-r_2)^2}}$$



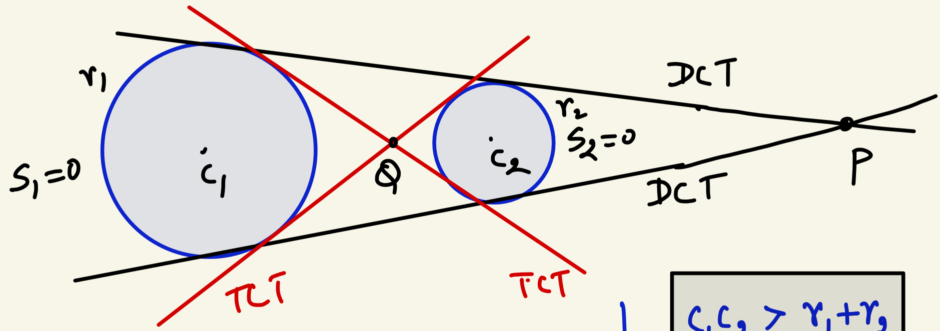
Rem

$$\boxed{L_{TCT} = \sqrt{d^2 - (r_1 + r_2)^2}}$$

* $L_{DCT} > L_{TCT}$

Position of Circles & No. of common tangents :-

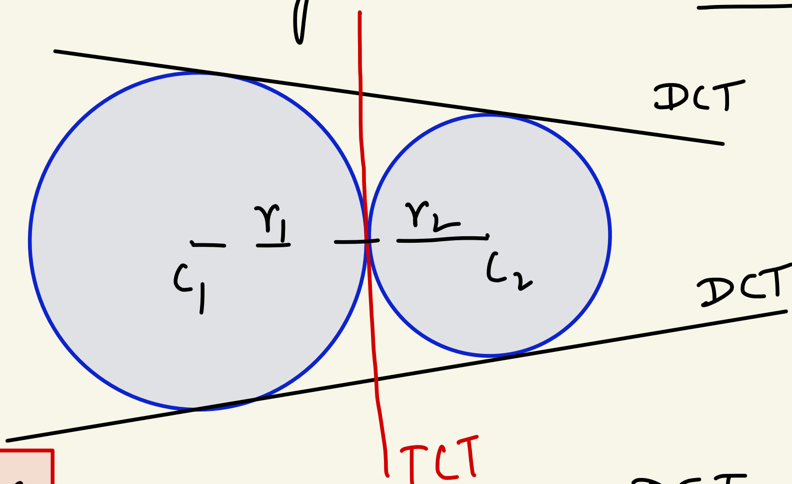
① Circles away from each other :



$\boxed{C_1C_2 > r_1+r_2}$

4 CT's → 2 DCT, 2 TCT.

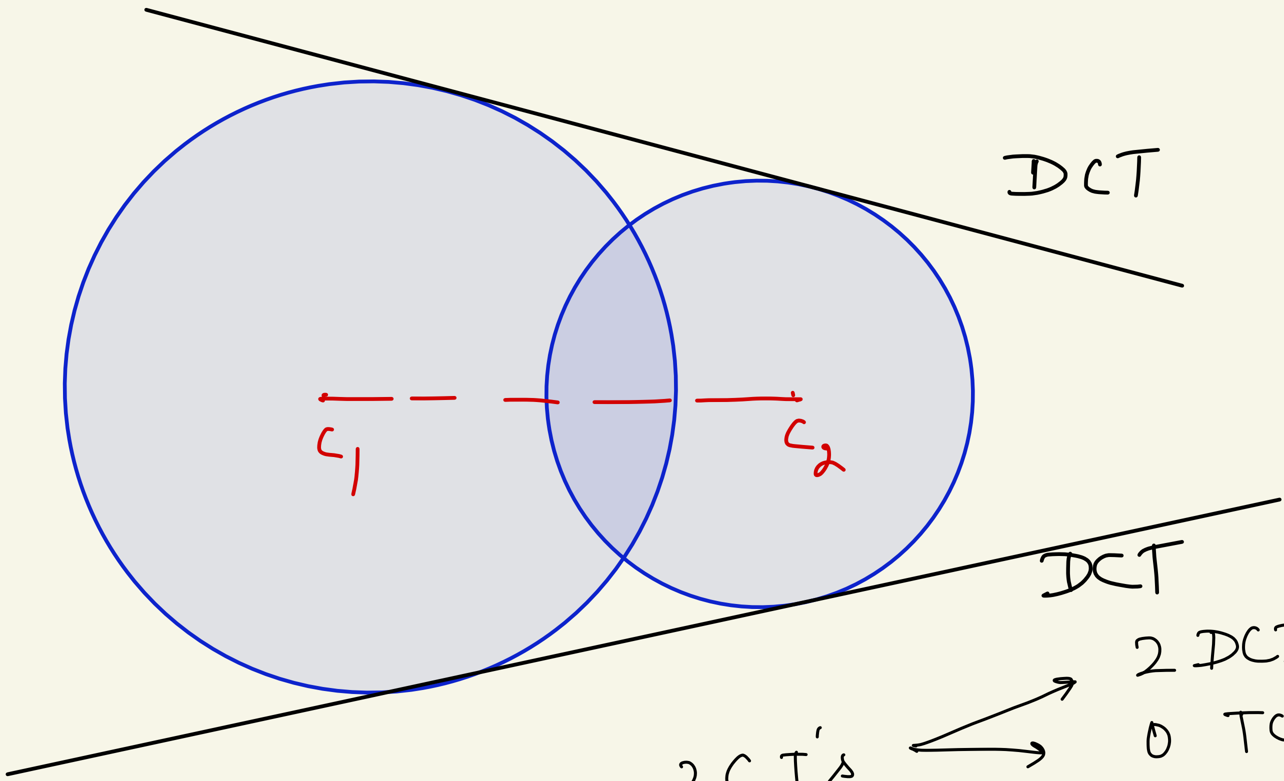
② Circles touching each other externally.



$\boxed{C_1C_2 = r_1+r_2}$

3 CT's → 2 DCT, 1 TCT.

③ Circles cutting each other at 2 distinct pts

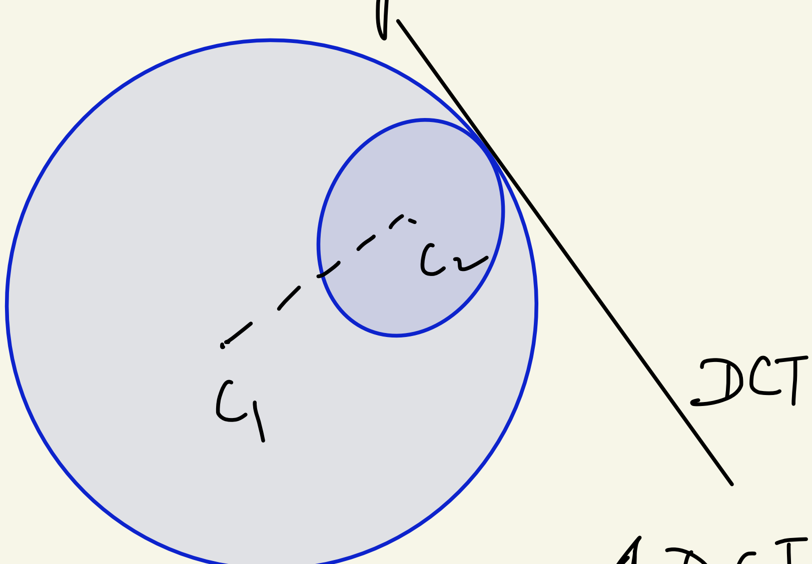


2 CT's → 2 DCT, 0 TCT.

$$\boxed{|r_1 - r_2| < C_1C_2 < r_1 + r_2}$$

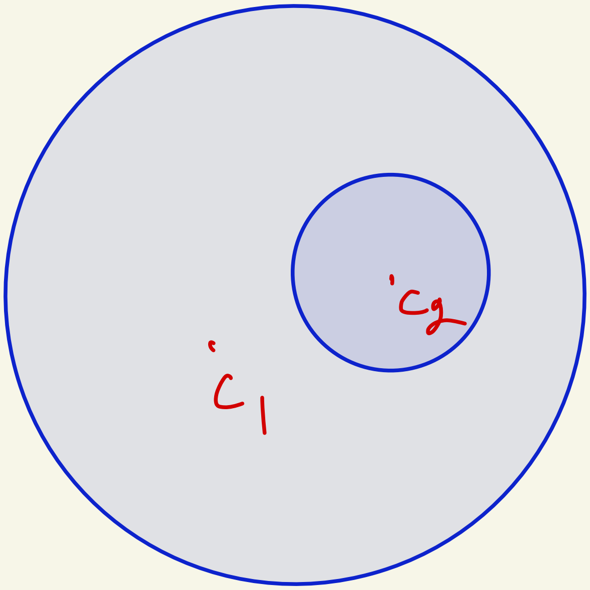
④ Circles touching each other internally.

$$\boxed{C_1C_2 = |r_1 - r_2|}$$



1 CT → 1 DCT, 0 TCT

⑤ One Circle completely in other Circle.



0 CT's

$$\boxed{C_1C_2 < |r_1 - r_2|}$$

Equation of D.C.T. and T.C.T.

Rem

(a) Direct Common Tangent (D.C.T.) meet at a point which divides the line joining the centres of circles externally in the ratio of their radii.

Rem

(b) Transverse Common Tangent (T.C.T.) meets at a point which divides the line joining the centres of circles internally in the ratio of their radii.

Proof : $\dfrac{PC_1}{PC_2} = \dfrac{r_1}{r_2}$

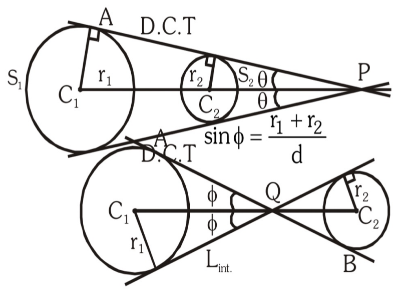
(since for $S_2 = 0$, C_2P is the angle bisector and also for $S_1 = 0$, C_1P is the angle bisector.

$\therefore$ C_1, C_2, P will lie in a same line

Similarly $\dfrac{C_1Q}{C_2Q} = \dfrac{r_1}{r_2}$

Note : C_1Q, C_1C_2, C_1P are in H.P.

$\sin\theta = \dfrac{|r_1 - r_2|}{d}$



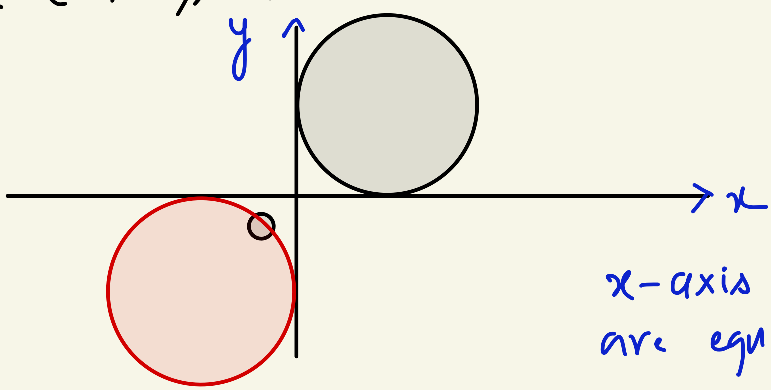
Q1 Find equation of TCT between 2 circles

$S_1 : x^2 + y^2 - 4x - 4y + 4 = 0$.

$S_2 : x^2 + y^2 + 6x + 6y + 9 = 0$.

Sol^n $C_1 : (2,2)$; $r_1 = \sqrt{2^2 + 2^2 - 4} = 2$

$C_2 : (-3,-3)$; $r_2 = \sqrt{9+9-9} = 3$



x-axis & y-axis are eqn TCT

Q Find all the common tangent to circles $x^2 + y^2 = 1$ and $(x-1)^2 + (y-3)^2 = 4$.

Sol^n

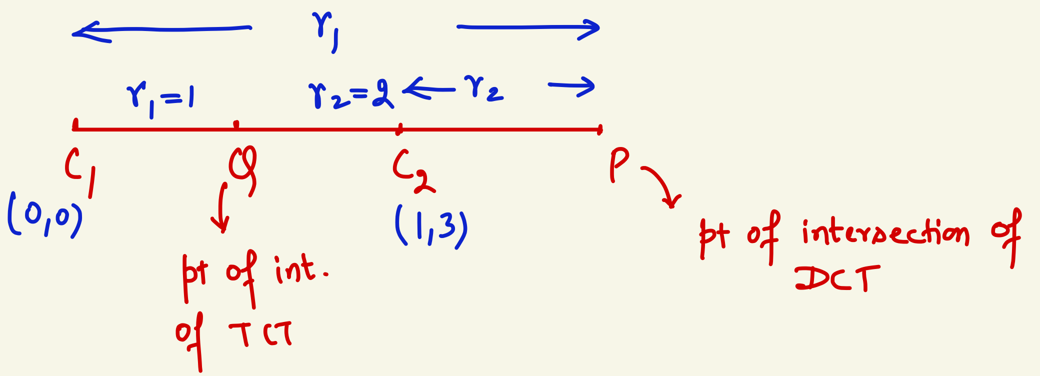
$C_1 : (0,0)\ ;\ r_1 = 1$

$C_2 : (1,3)\ ;\ r_2 = 2$

$C_1C_2 = \sqrt{1^2 + 3^2} = \sqrt{10}$

$r_1 + r_2 = 3.$

$\boxed{C_1C_2 > r_1 + r_2}$

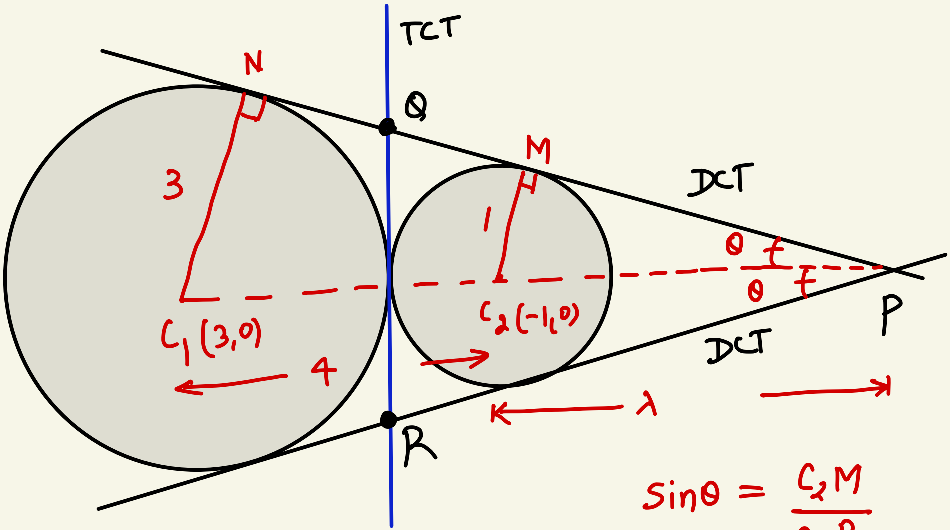


Q. Prove that the common tangents to the circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 + 2x = 0$ from an equilateral triangle.

Sol^n

$C_1 : (3,0)$; $r_1 = 3$ $\qquad$ $C_1C_2 = 4$

$C_2 : (-1,0)$; $r_2 = 1$ $\qquad$ $r_1 + r_2 = 4$



$$\sin\theta = \frac{C_2M}{C_2P} = \frac{C_1N}{C_1P}$$

$$\frac{C_2M}{C_1N} = \frac{C_2P}{C_1P} \Rightarrow \frac{C_2M}{C_2P} = \frac{C_1N}{C_1P}$$

$$\sin\theta = \frac{1}{\lambda} = \frac{3}{\lambda+4} \Rightarrow \sin\theta = \frac{2}{4} = \frac{1}{2}$$

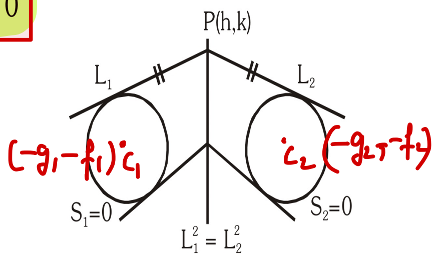
$$\boxed{\theta = 30^\circ}$$

Q HW

Find the co-ordinates of the point at which the circles $x^2 + y^2 - 4x - 2y + 4 = 0$ & $x^2 + y^2 - 12x - 8y + 36 = 0$ touch each other. Also find equation of common tangents touching the circle at distinct points.

RADICAL AXIS & RADICAL CENTRE :

The radical axis of 2 circles is the locus of a points whose powers w.r.t. the two circles are equal. The equation of radical axis of the two circles $S_1 = 0$ and $S_2 = 0$ is given by $\boxed{S_1 - S_2 = 0}$



i.e. $2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0$

$\Rightarrow \quad \cancel{h^2} + \cancel{k^2} + 2g_1h + 2f_1k + c_1 = \cancel{h^2} + \cancel{k^2} + 2g_2h + 2f_2k + c_2$

$\therefore \quad 2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0$

is the required equation of radical axis of $S_1 = 0$ and $S_2 = 0$.

$S_1: x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

$S_2: x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

$$m_{C_1C_2} = \frac{f_2 - f_1}{g_2 - g_1}$$

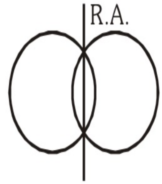
$$m_{RA} = \frac{-(g_1 - g_2)}{(f_1 - f_2)}$$

$$\boxed{m_{RA} \cdot m_{C_1C_2} = -1}$$

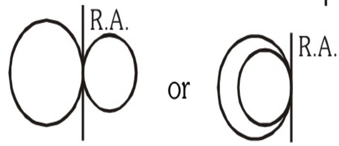
Rem

Note :

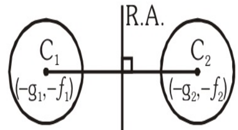
(i) If two circles intersect, then the radical axis is the common chord of the two circles.



(ii) If two circles touch each other then the radical axis is the common tangent of the two circles at the common point of contact.



(iii) Radical axis is always perpendicular to the line joining the centres of the two circles.

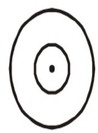


$2(g_1,-f_2)x+2(f_1-f_2)y + C_1-C_2=0$

(iv) Radical axis need not always pass through the mid point of the line joining the centres of the two circles.

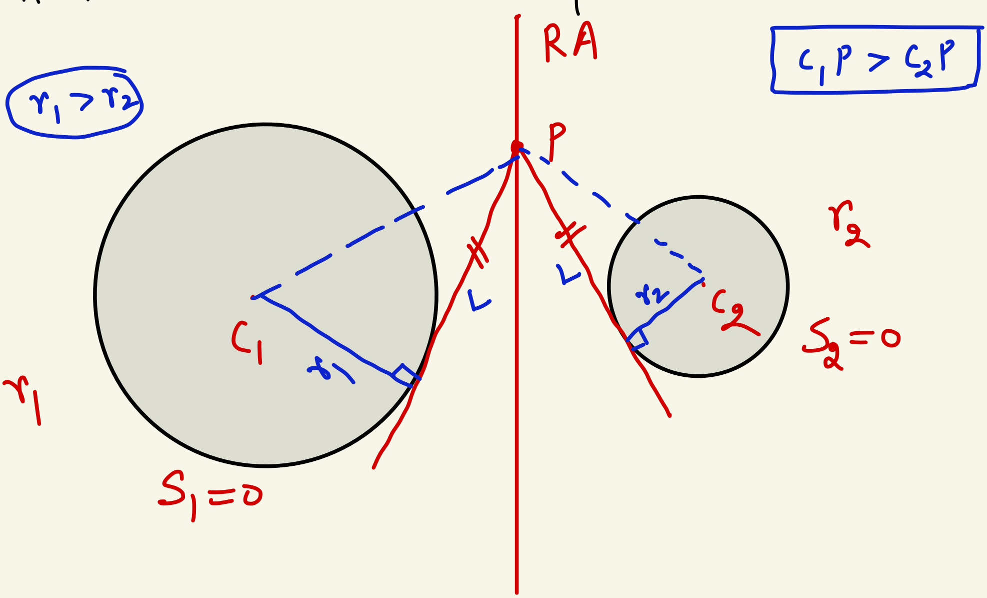
(v) Radical axis bisects a common tangent between the two circles.

(vi) Pairs of circles which do not have radical axis are concentric.



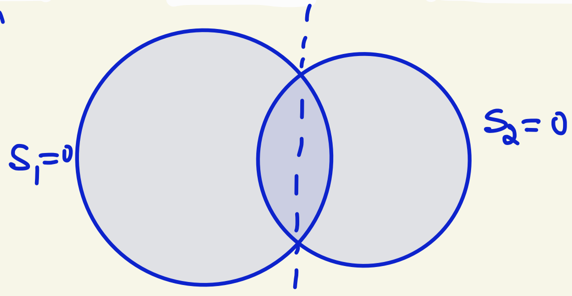
$$S_1 = x^2 + y^2 + 2gx + 2fy + c_1 = 0$$
$$S_2 = x^2 + y^2 + 2gx + 2fy + c_2 = 0$$

*
(vii) R.A is nearer to the centre of the smaller circle.



Q Number of real values of 'a' for which line $5x + by - a = 0$ passes through points of intersection of circles $x^2 + y^2 + 2ax + cy + a = 0$ & $x^2 + y^2 - 3ax + dy - 1 = 0$ $\boxed{\text{Ans} \to 0}$

Sol^n



$S_1 - S_2 = 0$

$5ax + (c-d)y + a + 1 = 0$ —②—

$5x + by - a = 0$ —①—

$S_1 - S_2 = 0$

Compare ① & ②

$$\frac{\cancel{5}a}{\cancel{5}} = \frac{c-d}{b} = \frac{a+1}{-a}$$

$$-a^2 = a + 1 \Rightarrow a^2 + a + 1 = 0$$

$D < 0$.

Q

Tangent are drawn to the circle $\underbrace{x^2 + y^2 = 12}_{S_1=0}$ at the points where it is met by the circle $\underbrace{x^2 + y^2 - 5x + 3y - 2 = 0}_{S_2=0}$. Find the point of intersection of the tangents.

Sol^n

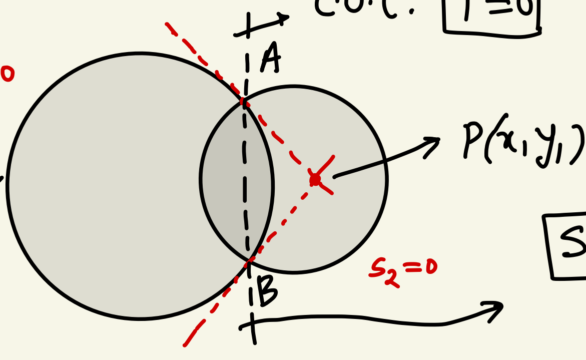
$C_1 : (0, 0)$; $r_1 = \sqrt{12}$ ✓

$C_2 : \left(\frac{5}{2}, -\frac{3}{2}\right)$; $r_2 = \sqrt{\frac{25}{4} + \frac{9}{4} + 2} = \sqrt{\frac{42}{4}} = \sqrt{\frac{21}{2}}$ ✓

C.O.C : $\boxed{T=0}$ —①—

$\boxed{xx_1 + yy_1 = 12}$

$\boxed{5x - 3y = 10}$

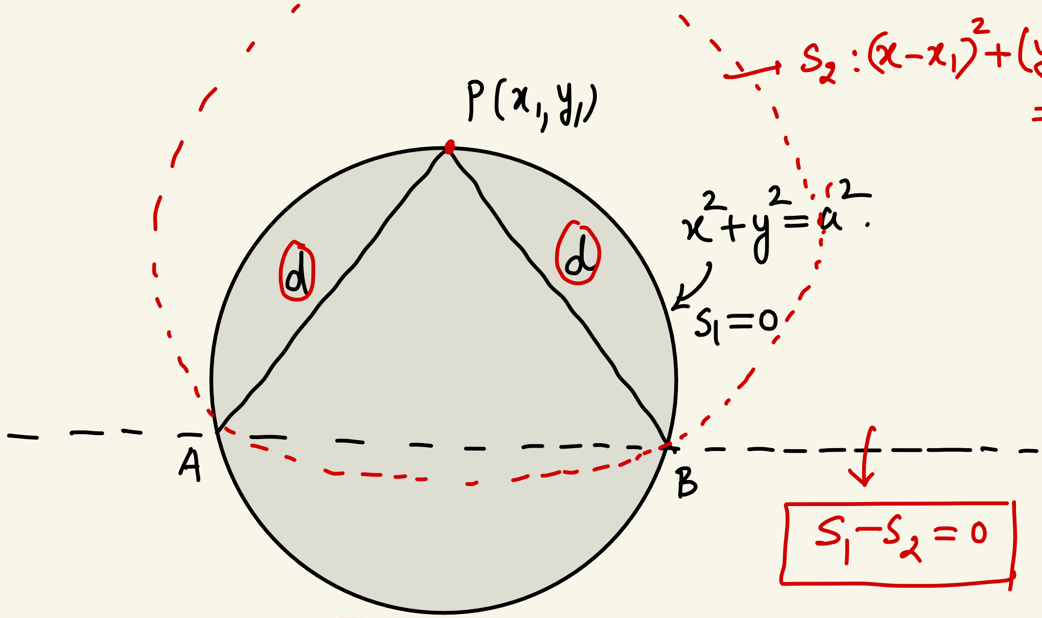


$$\frac{x_1}{5} = \frac{y_1}{-3} = \frac{12}{10}$$

$\boxed{S_1 - S_2 = 0}$ —②—

Q Find the equation of a straight line meeting the circle $x^2 + y^2 = a^2$ in 2 points at equal distance 'd' from the point (x_1, y_1) on its circumference is $xx_1 + yy_1 - a^2 + \frac{d^2}{2} = 0$.

Sol^n

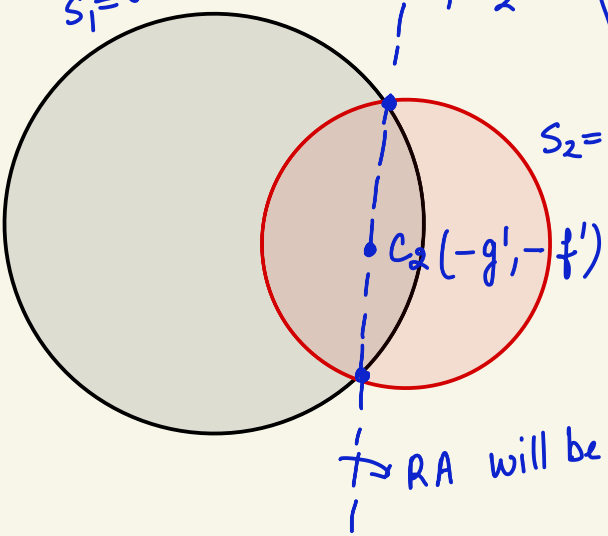


$S_2 : (x - x_1)^2 + (y - y_1)^2 = d^2$

$\boxed{S_1 - S_2 = 0}$

$\boxed{x_1^2 + y_1^2 = a^2.}$

Q Prove that the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ will bisect the circumference of the circle $x^2 + y^2 + 2g'x + 2f'y + c' = 0$ if $2g'(g - g') + 2f'(f - f') = c - c'$.



$S_1 - S_2 = 0 \Rightarrow 2(g-g')x + 2(f-f')y + c - c' = 0$

$(-g', -f')$ lies on it

$\Rightarrow$ RA will be diameter of smaller circle.

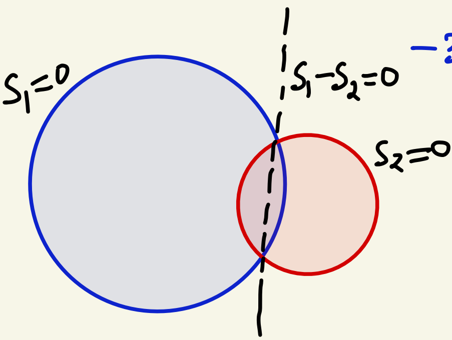
Q Consider a family of circles passing through two fixed points A(3, 7) and B(6, 5). Show that the chords in which the circles $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the member of the family are concurrent at a point.

Sol^n $S_1: (x-3)(x-6) + (y-7)(y-5) + \lambda L = 0$

$L: \quad y - 7 = \frac{7-5}{3-6}(x-3)$

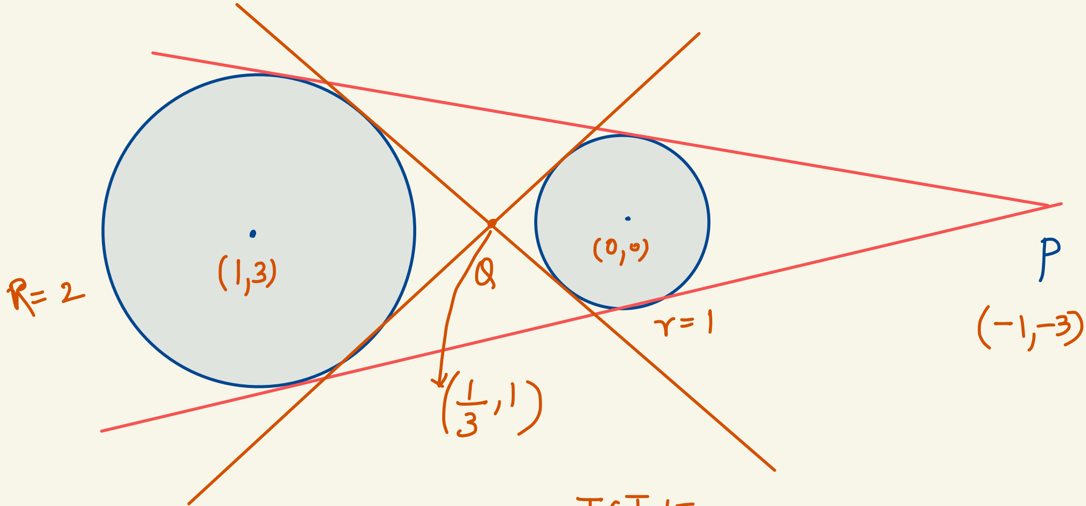
$-3y + 21 = 2x - 6 \Rightarrow \boxed{2x + 3y = 27}$

$S_1: x^2 - 9x + 18 + y^2 - 12y + 35 + \lambda(2x + 3y - 27) = 0$



$\boxed{S_1 - S_2 = 0}$

Q Find all the common tangent to circles $x^2 + y^2 = 1$ and $(x-1)^2 + (y-3)^2 = 4$.



DCT :

$y + 3 = m(x+1)$

Apply $p = r$

$$\left.\begin{array}{l} T_1 : \ x + 1 = 0 \\ T_2 : \ 4x - 3y - 5 = 0 \end{array}\right\}$$

TCT :-

$y - 1 = m(x - 1/3)$

Apply $p = r$

$$\left.\begin{array}{l} T_1 : \ y = 1 \\ T_2 : \ 3x + 4y = 5 \end{array}\right\}$$

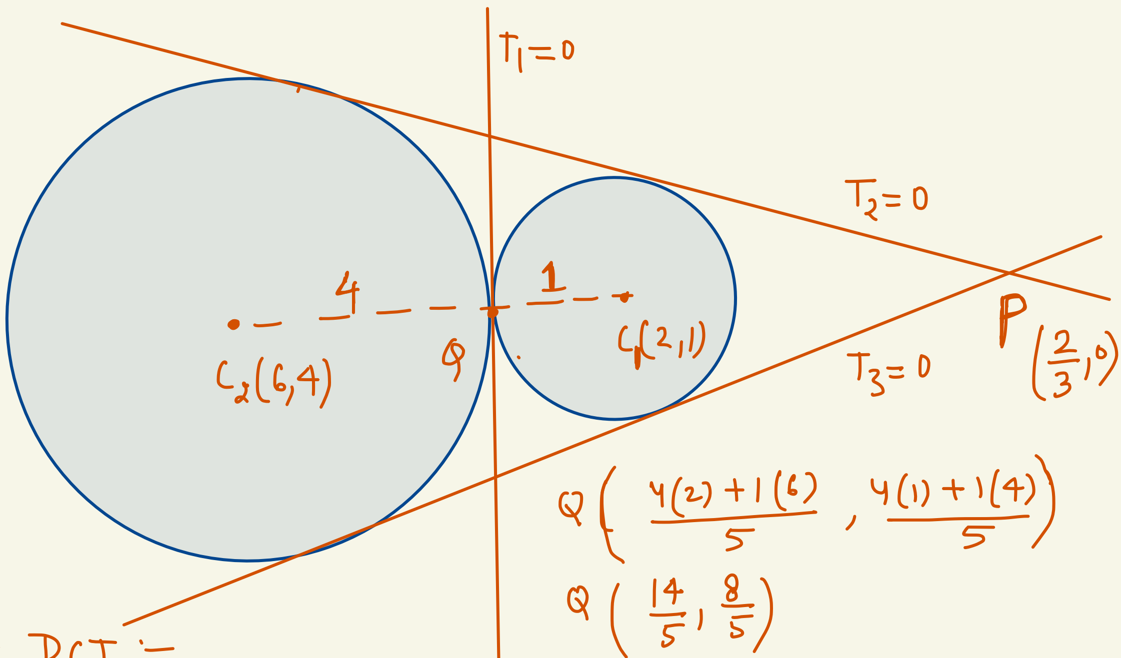
Q Find the co-ordinates of the point at which the circles $x^2 + y^2 - 4x - 2y + 4 = 0$ & $x^2 + y^2 - 12x - 8y + 36 = 0$ touch each other. Also find equation of common tangents touching the circle at distinct points.

Sol^n

$S_1 : x^2 + y^2 - 4x - 2y + 4 = 0$; $C_1(2, 1)$
$r_1 = 1$

$S_2 : x^2 + y^2 - 12x - 8y + 36 = 0$; $C_2(6, 4)$
$r_2 = 4.$

$C_1C_2 = 5$; $r_1 + r_2 = 5$



$Q\left(\frac{4(2) + 1(6)}{5}, \frac{4(1) + 1(4)}{5}\right)$

$Q\left(\frac{14}{5}, \frac{8}{5}\right)$

For DCT :-

$y - 0 = m\left(x - \frac{2}{3}\right)$

$\downarrow$ Apply $p = r$

get $m = 0$ & $m = \frac{24}{7}$.

$T_2 : y = 0$

$T_3 : 7y - 24x + 16 = 0$

Q Consider a family of circles passing through two fixed points A(3, 7) and B(6, 5). Show that the chords in which the circles $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the member of the family are concurrent at a point.

Sol^n S_1: $(x-3)(x-6) + (y-7)(y-5) + \lambda(2x+3y-27) = 0$

S_2: $x^2 + y^2 - 4x - 6y - 3 = 0$

$S_1 - S_2 = 0 \Rightarrow (-5x - 6y + 56) + \lambda(2x+3y-27) = 0$

$L_1 + \lambda L_2 = 0$

solve $\left.\begin{aligned} 5x+6y-56 &= 0 \\ 2x+3y-27 &= 0 \end{aligned}\right\} \rightarrow P\left(2, \frac{23}{3}\right)$

Ans

Find the locus of the centre of circles which bisect the circumference of the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 2x + 6y + 1 = 0$

Sol^n

$S_1 : x^2 + y^2 = 4$

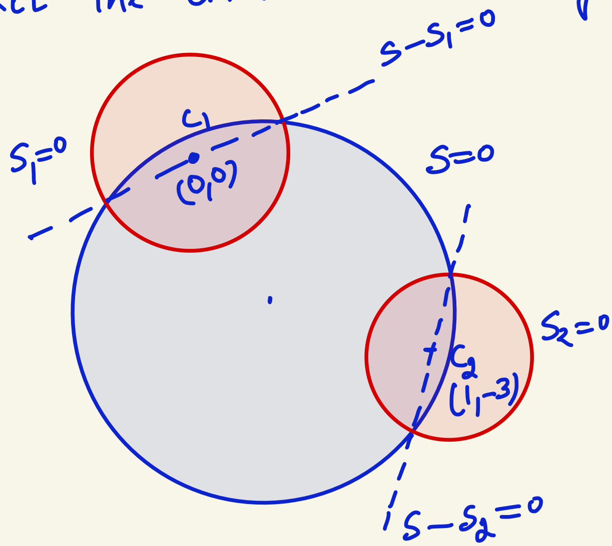
$S_2 : x^2 + y^2 - 2x + 6y + 1 = 0.$

Let the Circle be $S : x^2 + y^2 + 2gx + 2fy + c = 0.$

$\downarrow$

$C(-g, -f)$ **

(h, k)



$S - S_1 = 0$

$2gx + 2fy + c + 4 = 0$ —①—

$\downarrow$ $(0,0)$

$\boxed{c = -4}$

$S - S_2 = 0$

$2gx + 2fy + c + 2x - 6y - 1 = 0$

$\downarrow$ $(1, -3)$

$2g - 6f - 4 + 2 + 18 - 1 = 0$

Ans

$2(-x) - 6(-y) + 15 = 0 \Rightarrow \boxed{2x - 6y - 15 = 0}$

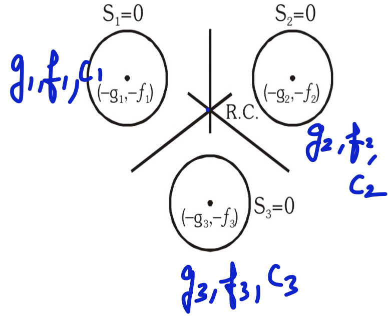
Q Find the equation of a circle which bisects the circumferences of the circles $x^2 + y^2 = 1$, $x^2 + y^2 + 2x = 3$ and $x^2 + y^2 + 2y = 3$.

Sol

$$S: x^2 + y^2 + 2gx + 2fy + C = 0$$

Radical centre :

Definition : The common point of intersection of the radical axis of 3 circles taken 2 at a time is called the Radical Centre of three circles, from this



Radical Axis of S_1 & S_2 : $2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0$

Radical Axis of S_2 & S_3 : $2(g_2 - g_3)x + 2(f_2 - f_3)y + c_2 - c_3 = 0$

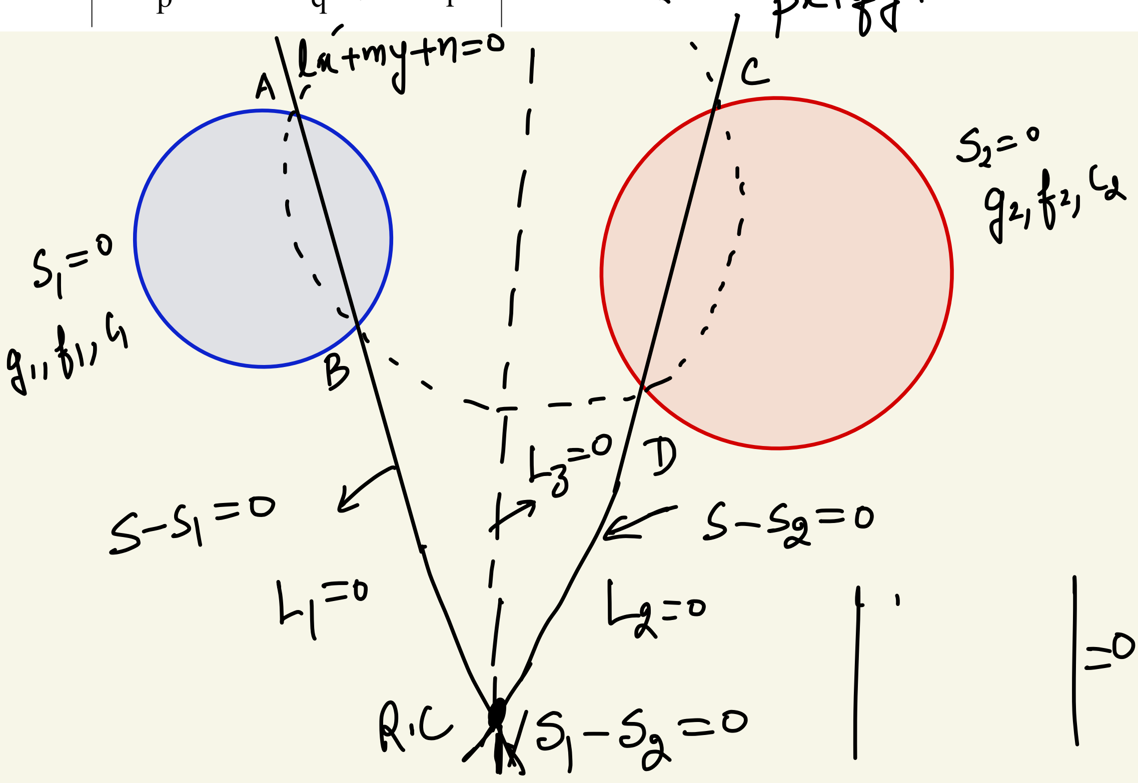
Radical Axis of S_3 & S_1 : $2(g_3 - g_1)x + 2(f_3 - f_1)y + c_3 - c_1 = 0$

$$D = \begin{vmatrix} g_1 - g_2 & f_1 - f_2 & c_1 - c_2 \\ g_2 - g_3 & f_2 - f_3 & c_2 - c_3 \\ g_3 - g_1 & f_3 - f_1 & c_3 - c_1 \end{vmatrix} = 0; \text{ Use } R_1 \to R_1 + R_2 + R_3 \Rightarrow D = 0$$

(solve any 2 radical axes to get radical centre)

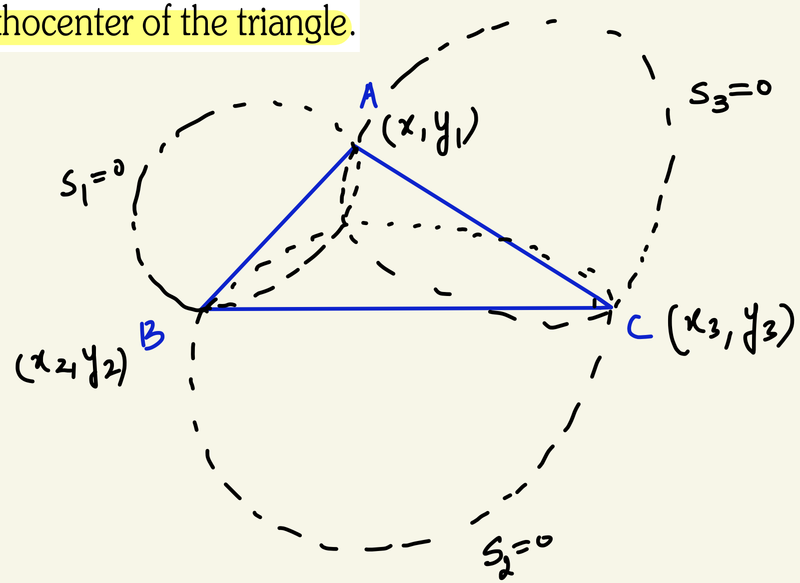
Q. Lines $lx + my + n = 0$; $px + qy + r = 0$ intersects the circle $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ at 4 points A and B ; C and D respectively. If these points are concyclic then prove that

$$\begin{vmatrix} 2(g_1 - g_2) & 2(f_1 - f_2) & c_1 - c_2 \\ l & m & n \\ p & q & r \end{vmatrix} = 0$$



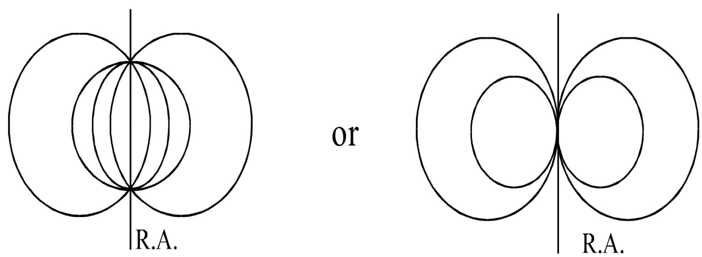
Note

Radical centre of 3 circles described on the sides of a triangle as diameter is the orthocenter of the triangle.



COAXIAL SYSTEM OF CIRCLES :

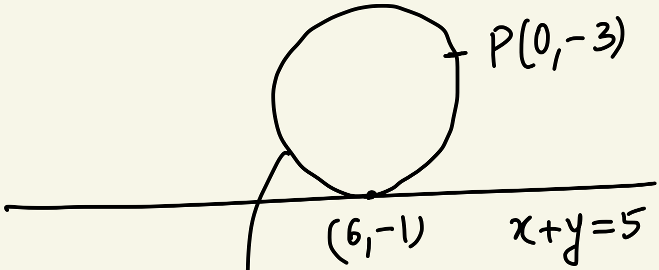
Definition : A system of circles, every 2 of which have the same radical axis, is called Coaxial system of circles.



Q. From a point P tangents drawn to the cirlces $x^2 + y^2 + x - 3 = 0$; $3x^2 + 3y^2 - 5x + 3y = 0$ and $4x^2 + 4y^2 + 8x + 7y + 9 = 0$ are of equal length. Find the equation of the circle passes through P and which touches the line $x + y = 5$ at $(6, -1)$

Sol^n P → Radical centre

$P(0,-3)$



$$(x-6)^2+(y+1)^2+\lambda(x+y-5)=0$$

$P(0,-3)$

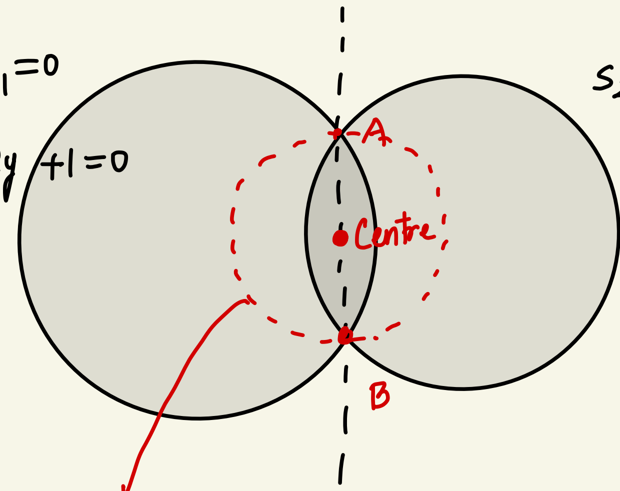
$$\boxed{\lambda=5}$$

Q Find the equation of a circle coaxial with $S_1 : x^2 + y^2 + 4x + 2y + 1 = 0$ and $S_2 : 2x^2 + 2y^2 - 2x - 4y - 3 = 0$ and the centre of the circle to be determined lies on the radical axes of these 2 circle.

$S_1 = 0$

$x^2 + y^2 + 4x + 2y + 1 = 0$

$S_2 = 0 : x^2 + y^2 - x - 2y - \frac{3}{2} = 0$



$\boxed{S_1 + \lambda L = 0.}$

Centre (   )

$S_1 - S_2 = 0 \Rightarrow 5x + 4y + \frac{5}{2} = 0$

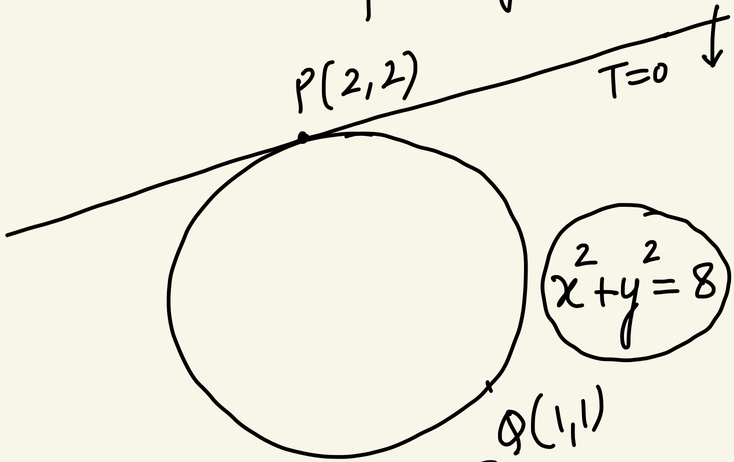
$\underbrace{L \equiv 10x + 8y + 5 = 0.}$

$\lambda = -\frac{23}{82}$

Q Find the equation of the circle passes through (1, 1) belonging to the system of coaxial circles which touches the locus of the point of intersection of 2 perpendiculars tangents to the circle $x^2 + y^2 = 4$ at (2, 2).

Sol^n

Director ⊙ of $x^2+y^2=4$ is $x^2+y^2=8$.



$$x(2)+y(2)=8$$

$$x+y=4$$

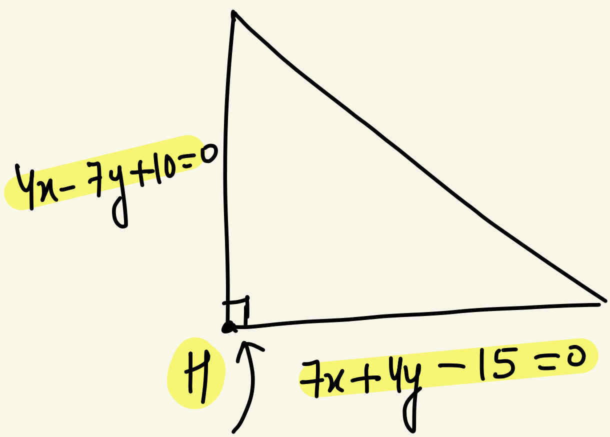
$$(x-2)^2+(y-2)^2+\lambda(x+y-4)=0 \Rightarrow \boxed{\lambda=1}$$

Q Find radical centre of circles described on sides of Δ as diameters.

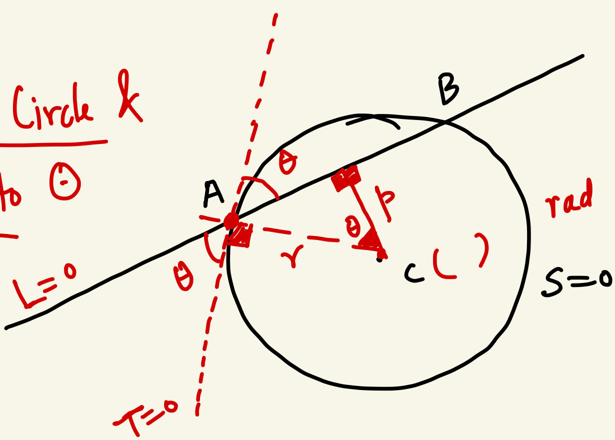
$4x - 7y + 10 = 0$; $x + y - 5 = 0$; $7x + 4y - 15 = 0$.

Sol^n

$m_1 = \frac{4}{7}$ $m_3 = -\frac{7}{4}$

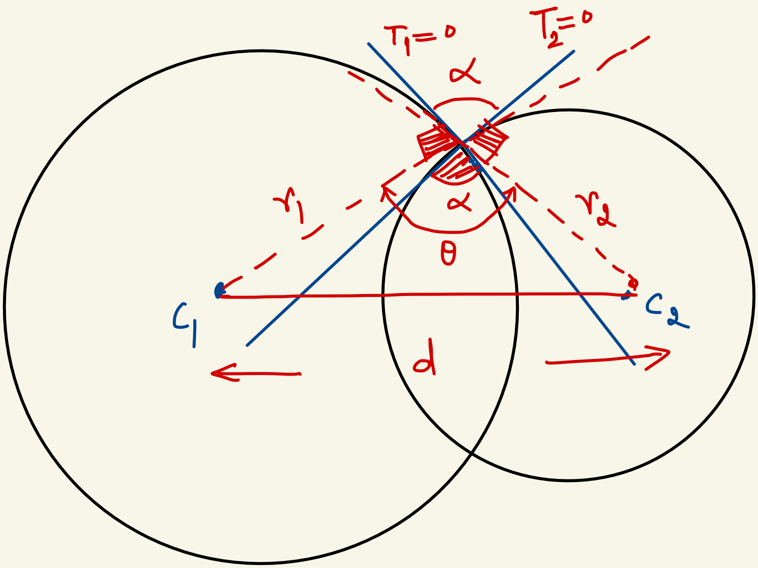


Angle bet^n Circle & Chord to $\odot$



$$\boxed{\cos\theta = \frac{p}{r}}$$

Angle of intersection of 2 circles :-



$$\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$$

$$\alpha + 90^\circ + 90^\circ + \theta = 360^\circ$$

$$\boxed{\theta = 180^\circ - \alpha}$$

$$\cos(180^\circ - \alpha) = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$$

$$\boxed{\cos\alpha = \left|\frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}\right|}$$ Rem

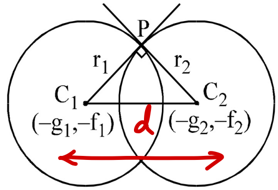
$\alpha \to$ acute.

$\pi - \alpha \to$ obtuse.

ORTHOGONALITY OF TWO CIRCLES :

Definition : Two curves are said to be orthogonal if they intersect each other at 90° wherever they intersects.

Condition for orthogonality of 2 circles : $r_1^2 + r_2^2 = d^2$



$$r_1^2 + r_2^2 = (g_1 - g_2)^2 + (f_2 - f_2)^2$$

$$\Rightarrow \quad (g_1^2 + f_1^2 - c_1) + (g_2^2 + f_2^2 - c_2) = (g_1 - g_2)^2 + (f_1 - f_2)^2$$

$$\therefore \quad \boxed{2 g_1 g_2 + 2 f_1 f_2 = c_1 + c_2} \qquad \text{(condition for orthogonality)}$$

Rem

Q If the circles, $S_1 : x^2 + y^2 + 2x + 2ky + 6 = 0$ and $S_2 : x^2 + y^2 + 2kx + k = 0$ intersects orthogonally then find k.

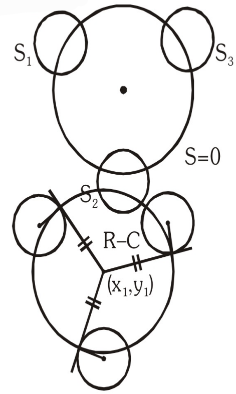
Sol^n

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

$$2(1)(K) + 2(K)(0) = 6 + K.$$

$$2K = 6 + K \Rightarrow \boxed{K = 6}$$

Circle orthogonal to 3 given circles :



Method-I : Let the equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ orthogonal to $S_1 = 0$; $S_2 = 0$; $S_3 = 0$ get 3 relation in g, f, c.

Method-II : Circle orthogonal to 3 given circles is one such circle whose centre is the radical centre of 3 given circles and radius equals to length of tangent.

[Solve any 2 radical axis to get radical centre and then compute $r = \alpha_T$]

Rem

Note : Locus of the centre of a variable circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which cuts the 2 given circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c = 0$ orthogonally is the radical axis of 2 given circles.

Sol^n

$S_1: \ x^2+y^2+2g_1x+2f_1y+c_1=0$

$S_2: \ x^2+y^2+2g_2x+2f_2y+c_2=0$

$S: \ x^2+y^2+2gx+2fy+C=0$

S & S_1 are orthogonal : $2gg_1+2ff_1 = C+C_1$

S & S_2 are " : $2gg_2+2ff_2 = C+C_2$

sub :—

$$\underset{(-x)}{2g}(g_1-g_2) + \underset{(-y)}{2f}(f_1-f_2) = C_1-C_2$$

$$-2x(g_1-g_2)-2y(f_1-f_2) = C_1-C_2$$

(H.P.)

Q Find the equation of a circle which passes through the point of intersection of $S_1 : x^2 + y^2 - 4x + 6y = 0$ and $S_2 : x^2 + y^2 - 6x + 4y = 0$ and cuts the circle $S_3 : x^2 + y^2 - 2x - 4y - 4 = 0$ orthogonally.

Sol^n

$S_1 + \lambda L = 0$ is required circle which is orthogonal to $S_3 = 0$

$L \equiv 2x + 2y = 0 \Rightarrow x + y = 0$

$$x^2 + y^2 - 4x + 6y + \lambda(x + y) = 0$$

$$x^2 + y^2 + (\lambda - 4)x + (6 + \lambda)y = 0.$$

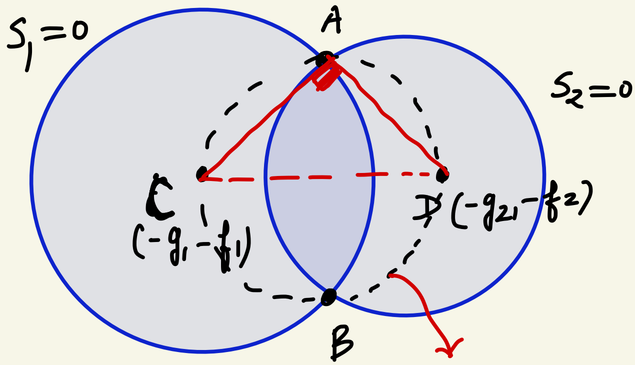
Condition of orthogonality:

$$2\left(\frac{\lambda - 4}{2}\right)(-1) + 2\left(\frac{6+\lambda}{2}\right)(-2) = 0 - 4$$

The circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ intersect orthogonally at A and B. If C, D are the centres of these circles, show that the equation of the circle passing through A, B, C, D is

$$2(x^2 + y^2) + 2(g_1 + g_2)x + 2(f_1 + f_2)y + c_1 + c_2 = 0.$$

Sol^n



Condition of orthogonality:

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$ —①—

$(x+g_1)(x+g_2) + (y+f_1)(y+f_2) = 0.$

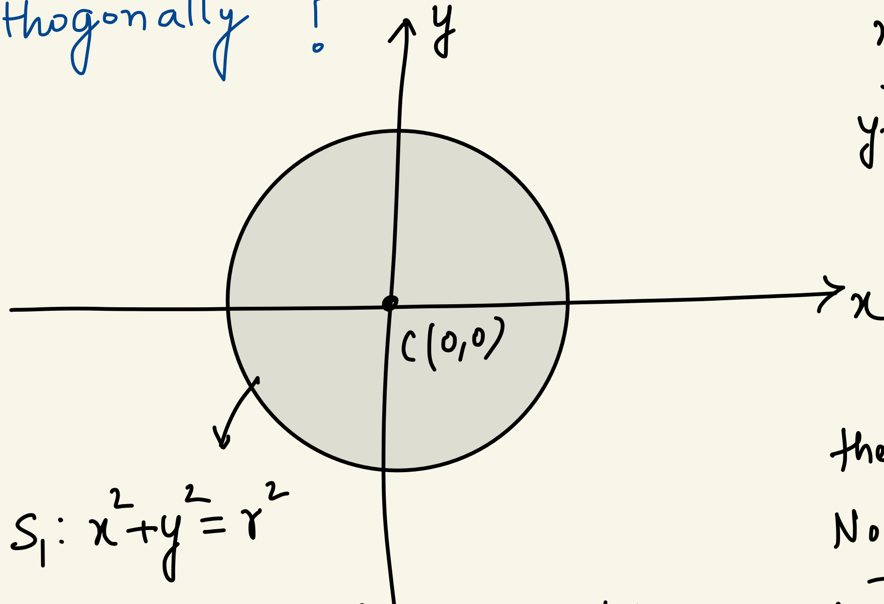
$$x^2 + y^2 + x(g_1+g_2) + y(f_1+f_2) + \underbrace{g_1g_2 + f_1f_2}_{①} = 0$$

$$x^2 + y^2 + x(g_1+g_2) + y(f_1+f_2) + \left(\frac{c_1+c_2}{2}\right) = 0$$

Q Find the equation of circle which cuts the circle $x^2+y^2-12x-16y+100=0$ & line pair $xy=0$ orthogonally ?

$xy=0 \to x=0 \to$ y-axis ; $y=0 \to$ x-axis

Solⁿ



$S_1: x^2+y^2=r^2$

Line is orthogonal to circle then it is Normal to ⊙ (i.e it is diameter)

Use Condition of orthogonality:

$0+0=100-r^2$

$\therefore \boxed{r=10}$

$S_1: \boxed{x^2+y^2=100}$ Ans

Q Let $S_1 = x^2 + y^2 - 1 = 0$; $S_2 \equiv x^2 + y^2 - 2x - 2y = 0$, P & Q be the points on S_1 & S_2. Now which of the following is true ?

- (A) Radical axis of $S_1 = 0$ & $S_2 = 0$ is $2x + 2y = 1$
- (B) The acute angle of intersection of $S_1 = 0$ & $S_2 = 0$ is $\cos^{-1}\frac{1}{2\sqrt{2}}$.
- (C) The maximum distance between P & Q is $1+2\sqrt{2}$

(D) The minimum distance between P & Q is 1.

Sol^n

$S_1 - S_2 = 0$

$2x + 2y - 1 = 0.$ (A)

$C_1(0,0)$

$C_2 : (1,1)$

$r_1 = 1$; $r_2 = \sqrt{2}$

$$\cos\theta = \left|\frac{r_1^2 + r_2^2 - d^2}{2r_1 r_2}\right|$$

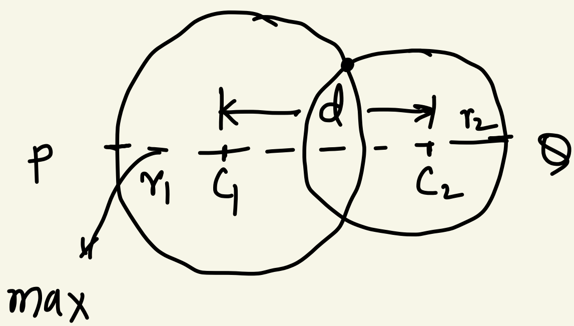
$\theta \rightarrow$ acute

$$= \left|\frac{1 + \cancel{2} - \cancel{2}}{2 \cdot 1 \cdot \sqrt{2}}\right| \Rightarrow \cos\theta = \frac{1}{2\sqrt{2}}$$

(B)

$C_1C_2 = \sqrt{2}$; $r_1 + r_2 = \sqrt{2} + 1$

min dist $= 0$



max dist $=$

$r_1 + r_2 + C_1C_2$

$\sqrt{2} + 1 + \sqrt{2}$ (C)