1. $\int \sin x d(\sin x)$

Sol:- $\int \sin x \, d(\sin x) = \frac{(\sin x)^{1+1}}{1+1} = \frac{\sin^2 x}{2} + C$

ALITER:-

$\int \sin x \, d(\sin x) = \int \sin x \frac{d(\sin x)}{dx} . dx$

$= \int \sin x \cos x \, dx$

$= \frac{1}{2} \int \sin 2x \, dx$

$= \frac{1}{2} \frac{(-)\cos 2x}{2} + c'$

$= \frac{-1}{4} (1 - 2\sin^2 x) + c'$

$= \frac{\sin^2 x}{2} - \frac{1}{4} + c'$

$\int \sin x \, d(\sin x) = \frac{\sin^2 x}{2} + c$

2. $\int \tan^3 x d(\tan x)$

Sol:- $= \dfrac{(\tan x)^{3+1}}{3+1} + c \quad \left[\int x^n dx = \dfrac{x^{n+1}}{n+1} + c\right]$

$= \dfrac{\tan^4 x}{4} + c$

3. $\int \frac{d(1+x^2)}{\sqrt{1+x^2}}$

Sol:- $\int \frac{f'(x)}{\sqrt{f(x)}} = 2\sqrt{f(x)} + c$

$= 2\sqrt{1+x^2} + c$

4. $\int (x+1)^{15}\,dx$

Sol:-

$$= \frac{(x+1)^{15+1}}{15+1} + c$$

$$= \frac{(x+1)^{16}}{16} + c$$

5. $\int \frac{dx}{(2x-3)^5}$

Sol:- $= \int (2x-3)^{-5} dx$

$$= \frac{\frac{(2x-3)^{-5+1}}{-5+1}}{2}$$

$$= -\frac{1}{8}(2x-3)^{-4} + c$$

⑥ $\int \sqrt[5]{(8-3x)^6}\,dx$

Sol:-

$$= \int (8-3x)^{6/5}\,dx$$

$$= \frac{\dfrac{(8-3x)^{6/5+1}}{\frac{6}{5}+1}}{-3} + c$$

$$= \frac{(8-3x)^{11/5}}{\frac{11}{5}(-3)} + c$$

$$= -\frac{5}{33}(8-3x)^{11/5} + c$$

⑦ $\int \sqrt{8-2x}dx$

Sol :- $= \int (8-2x)^{1/2} dx$

$$= \frac{\frac{(8-2x)^{\frac{1}{2}+1}}{\frac{1}{2}+1}}{-2} + c$$

$$= \frac{(8-2x)^{3/2}}{\frac{3}{2}(-2)} + c$$

$$= \frac{-1}{3}(8-2x)^{3/2} + c$$

(8) $\int 2x\sqrt{x^2+1}\,dx$

Sol :- Put $x^2+1=t$

$\Rightarrow 2x\,dx = dt$

$$\Rightarrow I = \int \sqrt{t}\,dt$$

$$= \frac{t^{3/2}}{\frac{3}{2}} + c$$

$$I = \frac{2}{3}(x^2+1)^{3/2} + c$$

(9) $\int x\sqrt{1-x^2}\,dx$

Sol:- Put $1-x^2 = t$

$\Rightarrow -2x\,dx = dt$

$\Rightarrow I = -\int \frac{1}{2}\sqrt{t}\,dt$

$= \frac{-1}{2}\frac{t^{3/2}}{\frac{3}{2}} + c$

$= \frac{-1}{3}t^{3/2} + c$

$I = \frac{-1}{3}(1-x^2)^{3/2} + c$

(10) $\int x^2 \sqrt[5]{x^3+2}\, dx$

Sol:- Put $x^3+2 = t \Rightarrow 3x^2 dx = dt$

$\Rightarrow I = \int \frac{1}{3} t^{1/5} dt$

$= \frac{1}{3} \frac{t^{6/5}}{\frac{6}{5}} + c$

$I = \frac{5}{18} (x^3+2)^{6/5} + c$

(11) $$\int \frac{xdx}{\sqrt{x^2+1}}$$

Sol:- Put $x^2+1 = t \Rightarrow 2xdx = dt$

$$I = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \frac{1}{2}\, 2\sqrt{t} + c$$

$$I = \sqrt{x^2+1} + c$$

(12) $\int \frac{x^4 dx}{\sqrt{4+x^5}}$

Sol:- Put $x^5+4=t \Rightarrow 5x^4dx=dt$

$$\Rightarrow I = \int \frac{dt}{5\sqrt{t}}$$

$$= \frac{1}{5} 2\sqrt{t} + c$$

$$I = \frac{2}{5}\sqrt{x^5+4} + c$$

(13) $\int \frac{x^3 dx}{\sqrt[3]{x^4+1}}$

Sol: Put $x^4+1 = t \Rightarrow 4x^3 dx = dt$

$$I = \int \frac{dt}{4\, t^{1/3}}$$

$$= \frac{1}{4} \int t^{-1/3} dt$$

$$= \frac{1}{4} \frac{t^{2/3}}{\left(\frac{2}{3}\right)} + c$$

$$I = \frac{3}{8} (x^4+1)^{2/3} + c$$

(14) $\int \frac{(6x-5)dx}{2\sqrt{3x^2-5x+6}}$

Sol :- Put $3x^2-5x+6 = t$

$\Rightarrow (6x-5)\,dx = dt$

$$I = \int \frac{dt}{2\sqrt{t}}$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt$$

$$= \frac{1}{2}\,2\sqrt{t} + c$$

$$I = \sqrt{3x^2-5x+6} + c$$

(15) $\int \sin^3 x \cos x dx$

Sol:- Put $\sin x = t \Rightarrow \cos x dx = dt$

$$I = \int t^3 dt$$

$$= \frac{t^4}{4} + c$$

$$= \frac{\sin^4 x}{4} + c$$

(16) $\int \frac{\sin x \, dx}{\cos^2 x}$

Sol:- Put $\cos x = t \Rightarrow -\sin x \, dx = dt$

$$I = -\int t^{-2} \, dt$$

$$= -\frac{t^{-2+1}}{-2+1} + c$$

$$= \frac{1}{t} + c$$

$$= \frac{1}{\cos x} + c$$

$$I = \sec x + c$$

ALITER:- $I = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \, dx$

$$= \int \tan x \sec x \, dx$$

$$I = \sec x + c$$

(17) $\int \cos^3 x \sin 2x dx$

Sol:- $I = \int \cos^3 x \, 2\sin x \cos x dx$

$I = 2\int \sin x \cos^4 x dx$

Put $\cos x = t \Rightarrow -\sin x dx = dt$

$= -2\int t^4 dt$

$= -2\frac{t^5}{5} + c$

$I = -\frac{2}{5}(\cos x)^5 + c$

(18) $\int \frac{\sqrt{\ell nx}}{x} dx$

Sol :- Put $\ln x = t \Rightarrow \frac{1}{x} dx = dt$

$$I = \int \sqrt{t}\, dt$$

$$= \frac{2t^{3/2}}{3} + c$$

$$I = \frac{2}{3} (\ln x)^{3/2} + c$$

(19) $$\int \frac{(\arctan x)^2 \, dx}{1+x^2}$$

Sol :- Put $\tan^{-1} x = t$

$$\Rightarrow \frac{1}{1+x^2} dx = dt$$

$$I = \int t^2 dt$$

$$= \frac{t^3}{3} + c$$

$$I = \frac{(\tan^{-1} x)^3}{3} + c$$

(20) $\int \frac{dx}{\cos^2 x\sqrt{1+\tan x}}$

Sol:- $I = \int \frac{\sec^2 x\, dx}{\sqrt{1+\tan x}}$

Put $1+\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$I = \int \frac{dt}{\sqrt{t}}$

$= 2\sqrt{t} + c$

$I = 2\sqrt{1+\tan x} + c$

(21) $\int (\cos\alpha - \cos 2x)\, dx$

Sol :-

$$= \cos\alpha \int 1\, dx - \int \cos 2x\, dx$$

$$I = \cos\alpha\, x - \frac{\sin 2x}{2} + c$$

(22) $\int e^{x}(\sin e^{x})\,dx$

Sol :- Put $e^{x} = t \Rightarrow e^{x}dx = dt$

$= \int \sin t \, dt$

$= -\cos t + c$

$= -\cos e^{x} + c$

(23) $\int \frac{d(1+x^2)}{1+x^2}$

Sol :-

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + c$$

$$\Rightarrow I = \ln(1+x^2) + c$$

24) $\int \frac{d(\arcsin x)}{\arcsin x}$

Sol:- $= \ln|\sin^{-1}x| + c$

$\left[\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + c \right]$

(25) $\int \frac{(2x-3)dx}{x^2-3x+8}$

Sol:- Put $x^2-3x+8 = t$

$\Rightarrow (2x-3)dx = dt$

$= \int \frac{dt}{t}$

$= \ln t + c$

$= \ln(x^2-3x+8) + c$

(26) $$\int \frac{xdx}{x^2+1}$$

Sol :- Put $x^2+1=t \rightarrow 2xdx = dt$

$$= \int \frac{dt}{2t}$$

$$= \frac{1}{2} \ln t + c$$

$$I = \frac{1}{2} \ln(x^2+1) + c$$

(27) $\int \frac{x^2 dx}{x^3+1}$

Sol :- Put $x^3+1=t \Rightarrow 3x^2 dx = dt$

$$I = \int \frac{dt}{3t}$$

$$= \frac{1}{3} \ln t + c$$

$$I = \frac{1}{3} \ln |x^3+1| + c$$

(28) $\int \frac{e^x dx}{e^x + 1}$

Sol^r Put $e^x + 1 = t \Rightarrow e^x dx = dt$

$$I = \int \frac{dt}{t}$$

$$= \ln t + c$$

$$I = \ln(e^x + 1) + c$$

(29) $\int \frac{e^{2x}dx}{e^{2x}+a^2}$

Sol:- Put $e^{2x}+a^2=t \Rightarrow 2e^{2x}dx=dt$

$$I = \int \frac{dt}{2t}$$

$$= \frac{1}{2}\ln t + c$$

$$I = \frac{1}{2}\ln(e^{2x}+a^2)+c$$

(30) $\int \tan 3x \, dx$

Sol:

$$I = \frac{\ln|\sec 3x|}{3} + c$$

(31) $\int \frac{\sin 2x}{1+\cos^2 x} dx$

Sol:- Put $1+\cos^2 x = t$

$\Rightarrow \quad 2\cos x \, (-\sin x)\, dx = dt$

$\Rightarrow \quad -\sin 2x \, dx = dt$

$I = \int \frac{-dt}{t}$

$= -\ln t + c$

$I = -\ln(1+\cos^2 x) + c$

(32) $\int \frac{dx}{x\ell nx}$

Sol :- Put $\ln x = t \Rightarrow \frac{1}{x} dx = dt$

$$I = \int \frac{dt}{t}$$

$$= \ln t + c$$

$$I = \ln|\ln x| + c$$

(33) $\int e^{\sin x}\, d(\sin x)$

Sol:- $\int e^{x}\,dx = e^{x} + c$

$\Rightarrow \int e^{\sin x}\, d(\sin x) = e^{\sin x} + c$

(34) $\int e^{\sin x} \cos x dx$

Sol:- Put $\sin x = t \Rightarrow \cos x dx = dt$

$$I = \int e^t dt$$

$$= e^t + c$$

$$I = e^{\sin x} + c$$

ALITER:-

$$I = \int e^{\sin x} \cos x \, dx$$

$$= \int e^{\sin x} d(\sin x)$$

$$I = e^{\sin x} + c$$

(35) $\int e^{-3x+1}\,dx$

Sol:- $I = \int e^{-3x+1}\,dx$

$$= \frac{e^{-3x+1}}{-3} + c$$

$$I = -\frac{1}{3} e^{-3x+1} + c$$

(36) $\int e^{x^2} x \, dx$

Sol:- Put $x^2 = t \Rightarrow 2x\,dx = dt$

$$I = \int \frac{e^t \, dt}{2}$$

$$= \frac{1}{2} e^t + c$$

$$I = \frac{1}{2} e^{x^2} + c$$

(37) $\int e^{-x^3} x^2 dx$

Sol:- Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$$I = \int \frac{e^{-t} dt}{3}$$

$$= \frac{1}{3} \frac{e^{-t}}{-1} + c$$

$$= \frac{-1}{3} e^{-x^3} + c$$

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$$\int \frac{d\left(\frac{x}{3}\right)}{\sqrt{1-\left(\frac{x}{3}\right)^2}}$$

Sol:-

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + c$$

$$\Rightarrow \quad I = \sin^{-1}\left(\frac{x}{3}\right) + c$$

(39) $\int \frac{dx}{\sqrt{1-25x^2}}$

Sol:- $I = \int \frac{dx}{\sqrt{1-(5x)^2}}$

$I = \frac{\sin^{-1}(5x)}{5} + c$

(40) $\int \frac{dx}{1+9x^2}$

Sol:- $I = \int \frac{dx}{1+(3x)^2}$

$I = \frac{\tan^{-1}(3x)}{3} + c$

41 $\displaystyle\int \frac{dx}{\sqrt{4-x^2}}$

Sol:- $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

$$I = \int \frac{dx}{\sqrt{2^2-x^2}}$$

$$I = \sin^{-1}\left(\frac{x}{2}\right) + c$$

(42) $\int \frac{dx}{2x^2+9}$

Sol :-

$$I = \int \frac{dx}{2\left(x^2+\frac{9}{2}\right)}$$

$$= \frac{1}{2} \int \frac{dx}{x^2+\left(\frac{3}{\sqrt{2}}\right)^2}$$

$$= \frac{1}{2} \frac{1}{\frac{3}{\sqrt{2}}} \tan^{-1}\left(\frac{x}{3/\sqrt{2}}\right) + c$$

$$I = \frac{1}{3\sqrt{2}} \tan^{-1}\left(\frac{\sqrt{2}x}{3}\right) + c$$

(43) $\int \frac{dx}{\sqrt{4-9x^2}}$

Sol:

$$I = \int \frac{dx}{\sqrt{2^2-(3x)^2}}$$

$$I = \frac{\sin^{-1}\left(\frac{3x}{2}\right)}{3} + c$$

(44) $\int \frac{xdx}{x^4+1}$

Sol:- $I = \int \frac{x\,dx}{(x^2)^2+1}$

Put $x^2 = t \Rightarrow 2x\,dx = dt$

$I = \int \frac{dt}{2(t^2+1)}$

$= \frac{1}{2}\tan^{-1} t + c$

$I = \frac{1}{2}\tan^{-1}(x^2) + c$

(45) $\int \frac{x\,dx}{\sqrt{a^2 - x^4}}$

Sol:- $I = \int \frac{x\,dx}{\sqrt{a^2-(x^2)^2}}$

Put $x^2 = t \Rightarrow 2x\,dx = dt$

$I = \int \frac{dt}{2\sqrt{a^2-t^2}}$

$= \frac{1}{2} \sin^{-1}\left(\frac{t}{a}\right) + c$

$I = \frac{1}{2} \sin^{-1}\left(\frac{x^2}{a}\right) + c$

(46) $\int \frac{x^2 dx}{x^6 + 4}$

Sol:- $I = \int \frac{x^2 dx}{(x^3)^2 + 4}$

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$I = \int \frac{dt}{3(t^2 + 2^2)}$

$= \frac{1}{3} \int \frac{dt}{t^2 + 2^2}$

$= \frac{1}{3} \frac{1}{(2)} \tan^{-1}\left(\frac{t}{2}\right) + c$

$I = \frac{1}{6} \tan^{-1}\left(\frac{x^3}{2}\right) + c$

(47) $\int \frac{x^3 dx}{\sqrt{1-x^8}}$

Sol:- $I = \int \frac{x^3 dx}{\sqrt{1-(x^4)^2}}$

Put $x^4 = t \Rightarrow 4x^3 dx = dt$

$I = \int \frac{dt}{4\sqrt{1-t^2}}$

$I = \frac{1}{4} \sin^{-1}(t) + c$

$I = \frac{1}{4} \sin^{-1}(x^4) + c$

(48) $\int \frac{e^x dx}{e^{2x}+4}$

Sol:- Put $e^x = t \Rightarrow e^x dx = dt$

$$I = \int \frac{dt}{t^2+2^2}$$

$$= \frac{1}{2} \tan^{-1}\left(\frac{t}{2}\right) + c$$

$$I = \frac{1}{2} \tan^{-1}\left(\frac{e^x}{2}\right) + c$$

(49) $\int \frac{2^x dx}{\sqrt{1-4^x}}$

Sol :- $I = \int \frac{2^x dx}{\sqrt{1-(2^x)^2}}$

Put $2^x = t \Rightarrow 2^x \ln 2 \, dx = dt$

$$I = \int \frac{dt}{\ln 2 \sqrt{1-t^2}}$$

$$= \frac{1}{\ln 2} \sin^{-1}(t) + c$$

$$I = \frac{1}{\ln 2} \sin^{-1}(2^x) + c$$

(50) $\int \frac{e^{2x}-1}{e^{x}}dx$

Sol:- $I = \int (e^x - e^{-x})\,dx$

$= e^x - \frac{e^{-x}}{-1} + c$

$I = e^x + e^{-x} + c$

(51) $\int (e^x + 1)^3 \, dx$

Sol:

$$I = \int (e^{3x} + 3e^{2x} + 3e^{x} + 1) \, dx$$

$$I = \frac{e^{3x}}{3} + \frac{3}{2}e^{2x} + 3e^{x} + x + c$$

(52) $\int \frac{1+x}{\sqrt{1-x^2}}\,dx$

Sol^n $I = \underbrace{\int \frac{dx}{\sqrt{1-x^2}}}_{\sin^{-1}x} + \underbrace{\int \frac{x\,dx}{\sqrt{1-x^2}}}_{I_1}$

$1-x^2 = t$

$-2x\,dx = dt$

$\Rightarrow x\,dx = -\frac{dt}{2}$

$\therefore I_1 = \frac{-1}{2}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2}2\sqrt{t} = -\sqrt{t}$

$\therefore I = \sin^{-1}x - \sqrt{1-x^2} + c$

(53) $\int \frac{3x-1}{x^2+9}dx$

Sol: $I = \int \frac{3x}{x^2+9} - \int \frac{1}{x^2+9}dx$

$= \frac{3}{2}\int \frac{2x}{x^2+9} - \int \frac{1}{x^2+3^2}dx$

$I = \frac{3}{2}\ln(x^2+9) - \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+c$

(54) $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Sol:

$$I = \int \frac{1-x}{\sqrt{1-x^2}}\,dx$$

$$= \int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= \sin^{-1}x + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx$$

$$= \sin^{-1}x + \frac{1}{2}\,2\sqrt{1-x^2} + c$$

$$I = \sin^{-1}x + \sqrt{1-x^2} + c$$

(55) $\int \frac{x(1-x^2)}{1+x^4} dx$

Sol:- $I = \underbrace{\int \frac{x}{1+(x^2)^2} dx}_{I_1} - \underbrace{\int \frac{x^3}{1+x^4} dx}_{I_2}$

$I_1 = \frac{1}{2} \int \frac{2x}{1+(x^2)^2} dx$ put $x^2 = t$
$2x\,dx = dt$

$\Rightarrow I_1 = \frac{1}{2} \cdot \int \frac{dt}{1+t^2} = \frac{1}{2} \tan^{-1} t + C_1$

$\Rightarrow I_1 = \frac{1}{2} \tan^{-1} x^2 + C_1$

$I_2 = \frac{1}{4} \int \frac{4x^3}{1+x^4} dx$ put $1+x^4 = y$
$4x^3 dx = dy$

$\Rightarrow I_2 = \frac{1}{4} \cdot \int \frac{dy}{y} = \frac{1}{4} \ln y = \frac{1}{4} \ln(1+x^4) + C_2$

$\therefore I = I_1 - I_2$

$= \frac{1}{2} \tan^{-1} x^2 + C_1 - \frac{1}{4} \ln(1+x^4) + C_2$

$= \frac{1}{2} \tan^{-1} x^2 - \frac{1}{4} \ln(1+x^4) + C$

(56) $\int \frac{dx}{(x+\sqrt{x^2-1})^2}$

Sol:- Notice that

$$\frac{1}{x+\sqrt{x^2-1}} = x-\sqrt{x^2-1}$$

$$\Rightarrow I = \int (x-\sqrt{x^2-1})^2 dx$$

$$= \int (x^2+x^2-1-2x\sqrt{x^2-1})dx$$

$$= \int (2x^2-1-2x\sqrt{x^2-1})dx$$

$$= \frac{2x^3}{3} - x - \int \sqrt{x^2-1}\, d(x^2-1)$$

$$I = \frac{2x^3}{3} - x - \frac{2(x^2-1)^{3/2}}{3} + c$$

(57) $\int \frac{2x - \sqrt{\arcsin x}}{\sqrt{1-x^2}}\,dx$

Soln

$$I = \underbrace{\int \frac{2x}{\sqrt{1-x^2}}\,dx}_{} - \underbrace{\int \frac{\sqrt{\sin^{-1}x}}{\sqrt{1-x^2}}\,dx}_{}$$

put $1-x^2 = t$

$-2x\,dx = dt$

$$-\int \frac{dt}{t^{1/2}}$$

$$= -\int t^{-1/2}\,dt$$

$$= -\frac{t^{1/2}}{1/2} + C_1$$

$$= -2\cdot(1-x^2)^{1/2}$$

put $\sin^{-1}x = y$

$$\frac{1}{\sqrt{1-x^2}}\,dx = dy$$

$$\int \sqrt{y}\,dy$$

$$= \frac{y^{3/2}}{3/2} + C_2$$

$$= \frac{2}{3}(\sin^{-1}x) + C_2$$

$$\therefore \boxed{I = -2(1-x^2)^{1/2} - \frac{2}{3}\sin^{-1}x + C}$$

(58) $\int \frac{x+(\arccos 3x)^2}{\sqrt{1-9x^2}}\,dx$

Sol:-

$$\cos^{-1}3x = t$$

$$\Rightarrow \quad \frac{-1}{\sqrt{1-9x^2}}(3)\,dx = dt$$

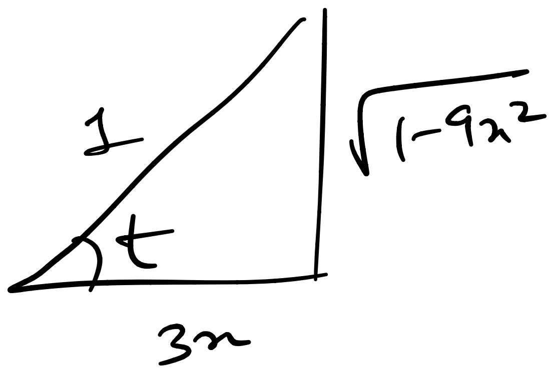
$$I = \int\left[\frac{\cos t}{3} + t^2\right]\frac{dt}{-3}$$

$$= -\frac{1}{9}\int \cos t\,dt \;-\frac{1}{3}\int t^2\,dt$$

$$= \frac{-1}{9}\sin t - \frac{1}{3}\frac{t^3}{(3)} + c$$

$$I = \frac{-1}{9}\sin(\cos^{-1}3x) - \frac{1}{9}(\cos^{-1}3x)^3 + c$$

$$I = \frac{-1}{9}\sqrt{1-9x^2} - \frac{1}{9}(\cos^{-1}3x)^3 + c$$



(M2: Solve like Q.67)

(59) $\int \frac{(1+x)^2}{x^2+1}dx$

Sol :- $I = \int \frac{x^2+2x+1}{x^2+1} dx$

$= \int 1 \, dx + \int \frac{2x}{x^2+1} dx$

$I = x + \ln(x^2+1) + c$

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$$\int \frac{x^4}{1-x} dx$$

Sol:- Put $1-x=t \Rightarrow -dx=dt$

$$I = -\int \frac{(1-t)^4}{t} dt$$

$$= -\int \frac{t^4-4t^3+6t^2-4t+1}{t} dt$$

$$= -\int \left(t^3-4t^2+6t-4+\frac{1}{t}\right) dt$$

$$= \frac{-t^4}{4} + \frac{4}{3}t^3 - \frac{6}{2}t^2 + 4t - \ln t + C$$

$$I = -\frac{(1-x)^4}{4} + \frac{4}{3}(1-x)^3 - 3(1-x)^2$$

$$+ 4(1-x) - \ln(1-x) + C$$

ALITER:- $\quad I = \displaystyle\int \frac{x^4}{1-x}\,dx$

$(1-x)\ \cancel{x^4}\ \big(-x^3 - x^2 - x - 1$

$\quad \underline{-\cancel{x^4} \underset{+}{-} x^3}$

$\qquad \cancel{x^3}$

$\qquad \underline{-\cancel{x^3} \underset{+}{-} x^2}$

$\qquad\quad \cancel{x^2}$

$\qquad\quad \underline{-\cancel{x^2} \underset{+}{-} x}$

$\qquad\qquad \cancel{x}$

$\qquad\qquad \underline{-\cancel{x} \underset{+}{-} 1}$

$\qquad\qquad\quad (1)$

$$\Rightarrow I = \int \left(-x^3 - x^2 - x - 1 + \frac{1}{1-x}\right) dx$$

$$I = -\frac{x^4}{4} - \frac{x^3}{3} - \frac{x^2}{2} - x - \ln|1-x| + c$$