ELEMENTARY RACE

Find the antiderivative/primitive/integrals of the following by simple manipulation/simplifying and converting them into standard integrals.

1. $\int 2^x.e^x dx$

Solⁿ

$$\int (2e)^x dx$$

$$= \frac{(2e)^x}{\ln(2e)} + C$$

$$= \frac{2^x e^x}{\ln 2 + \ln e} + C$$

$$= \frac{2^x e^x}{1 + \ln 2} + C$$

2. $\int \frac{1+\cos^2 x}{1+\cos 2x}\, dx$

Sol^n $I = \int \frac{1+\cos^2 x}{1+\cos 2x}\, dx$

$1+\cos 2x = 2\cos^2 x$

$I = \int \frac{1+\cos^2 x}{2\cos^2 x}\, dx$

$I = \int \frac{\sec^2 x}{2} dx + \frac{1}{2}$

$I = \frac{1}{2}(\tan x + x) + C$

(3) $$\int \frac{1-\tan^2 x}{1+\tan^2 x}\,dx$$

Solⁿ

$$I = \int \frac{1-\tan^2 x}{1+\tan^2 x}\,dx$$

$$\frac{1-\tan^2 x}{1+\tan^2 x} = \cos 2x$$

$$I = \int \cos 2x\,dx$$

$$I = \frac{\sin 2x}{2} + C$$

4. $\int \frac{1+\tan^2 x}{1+\cot^2 x}\, dx$

Sol^n.

$$I = \int \frac{1+\tan^2 x}{1+\cot^2 x}\, dx$$

$$I = \int \frac{\sec^2 x}{\operatorname{cosec}^2 x}\, dx$$

$$I = \int \tan^2 x \, dx$$

$$I = \int (\sec^2 x - 1)\, dx$$

$$I = \tan x - x + C$$

5. $$\int \frac{e^{5\ln x} - e^{4\ln x}}{e^{3\ln x} - e^{2\ln x}}\, dx$$

Sol^n.

$$\boxed{e^{\ln(f(x))} = f(x)} \qquad f(x) > 0$$

$$I = \int \frac{e^{\ln x^5} - e^{\ln x^4}}{e^{\ln x^3} - e^{\ln x^2}}\, dx$$

$$I = \int \frac{x^5 - x^4}{x^3 - x^2}\, dx$$

$$I = \int \frac{(x^4)\cancel{(x-1)}}{(x^2)\cancel{(x-1)}}\, dx$$

$$I = \int x^2 = \frac{x^3}{3} + C$$

⑥ $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

Solⁿ

$$I = \int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$$

$$I = \int \frac{(\cos^2 x - \sin^2 x)}{\cos^2 x \sin^2 x} dx$$

$$I = \int \left(\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} \right) dx$$

$$I = \int \operatorname{cosec}^2 x \, dx - \int \sec^2 x \, dx$$

$$I = -\cot x - \tan x + C$$

⑦ $\int \left(\frac{1+2x^2}{x^2(1+x^2)}\right) dx$

Sol^n:

$$I = \int \frac{(1+2x^2)}{x^2(1+x^2)} dx$$

$$I = \int \frac{(1+x^2)+(x^2)}{x^2(1+x^2)} dx$$

$$I = \int \left\{ \frac{(1+x^2)}{x^2(1+x^2)} + \frac{(x^2)}{(x^2)(1+x^2)} \right\} dx$$

$$I = \int \left(\frac{1}{x^2} + \frac{1}{1+x^2} \right) dx$$

$$I = \frac{x^{-2+1}}{-2+1} + \tan^{-1}x + C$$

$$I = \frac{-1}{x} + \tan^{-1}x + C$$

(8) $\int (3\sin x \cos^2 x - \sin^3 x)\, dx$

Solⁿ

$$I = \int (3\sin x \cos^2 x - \sin^3 x)\, dx$$

$$I = \int \left(3\sin x (1-\sin^2 x) - \sin^3 x\right) dx$$

$$I = \int (3\sin x - 4\sin^3 x) \cdot dx$$

formula of $\sin 3x$

$$I = \int \sin(3x)\, dx$$

$$I = -\frac{\cos 3x}{3} + C$$

(9) $\int \frac{(1+x)^2}{x(1+x^2)}\,dx$

Solⁿ:

$$I = \int \frac{(1+x)^2}{x(1+x^2)}\,dx$$

$$I = \int \frac{1+x^2+2x}{x(1+x^2)}\,dx$$

$$I = \int \left(\frac{(1+x^2)}{x(1+x^2)} + \frac{2x}{x(1+x^2)} \right) dx$$

$$I = \int \left(\frac{1}{x} + \frac{2}{1+x^2} \right) dx$$

$$I = \int \frac{1}{x}\,dx + 2\int \frac{1}{1+x^2}\,dx$$

$$I = \ln|x| + 2\tan^{-1}x + C$$

(10) $\int \frac{x}{2x+1} dx$

Sol^n

$$I = \int \frac{x}{2x+1} dx$$

$$I = \frac{1}{2} \int \frac{2x+1-1}{2x+1} dx$$

$$I = \frac{1}{2} \int \frac{(2x+1)}{2x+1} - \frac{1}{(2x+1)} dx$$

$$I = \frac{1}{2} \left\{ \int dx - \int \frac{1}{(2x+1)} dx \right\}$$

$$I = \frac{1}{2} \left(x - \frac{\ln(2x+1)}{2} \right) + C$$

$$I \Rightarrow \frac{x}{2} - \frac{1}{4} \ln(2x+1) + C$$

(11) $\int \left(\frac{\sec 2x - 1}{\sec 2x + 1}\right) dx$

Solⁿ

$$I = \int \frac{\sec 2x - 1}{\sec 2x + 1} dx$$

$$I = \int \frac{\sec 2x + 1 - 2}{\sec 2x + 1} dx$$

$$I = \int 1\, dx - 2\int \frac{1}{\sec 2x + 1} dx$$

$$I = x - 2\int \frac{\sec 2x - 1}{(\sec 2x + 1)(\sec 2x - 1)} dx$$

$$I = x - 2\int \frac{\sec 2x - 1}{\tan^2 2x} dx$$

$$I = x - 2\int (\cot 2x . \operatorname{cosec} 2x - \cot^2 2x)\, dx$$

$$I = x - 2\left\{\left[-\frac{\operatorname{cosec} 2x}{2}\right] - \int (\operatorname{cosec}^2 2x - 1)\, dx\right\}$$

$$I = x + \operatorname{cosec} 2x + 2\left(-\frac{\cot 2x}{2} - x\right)$$

$$I = \operatorname{cosec} 2x - \cot 2x - x + C$$

$$I = \frac{1 - \cos 2x}{\sin 2x} - x + C$$

$$I = \tan x - x + C$$

(12) $\int \left(\frac{2x-1}{x-2}\right) dx$

Sol^n $I = \int \frac{2x-1}{x-2}\, dx$

$I = \int \frac{2x-4+3}{(x-2)}\, dx$

$I = \int \left(\frac{(2x-4)}{(x-2)} + \frac{3}{(x-2)}\right) dx$

$I = \int 2\, dx + 3\int \frac{1}{(x-2)}\, dx$

$I = 2x + 3\, \frac{\ln(x-2)}{1} + C$

(13) $\int \frac{e^{2x}-1}{e^x}\,dx$

<u>Sol^n</u>

$$I = \int \left(\frac{e^{2x}-1}{e^x}\right) dx$$

$$I = \int (e^x - e^{-x})\,dx$$

$$I = \int e^x\,dx - \int e^{-x}\,dx$$

$$I = e^x - \frac{e^{-x}}{(-1)} + C$$

$$I = e^x + e^{-x} + C$$

(14) $\displaystyle\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$

Sol^n

$$I = \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}\, dx$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$I = \int \frac{2\cos^2 x - 1 - (2\cos^2\alpha - 1)}{\cos x - \cos\alpha}\, dx$$

$$I = 2\int \frac{\cos^2 x - \cos^2\alpha}{\cos x - \cos\alpha}\, dx$$

$$I = 2\int \frac{(\cos x + \cos\alpha)\cancel{(\cos x - \cos\alpha)}}{\cancel{(\cos x - \cos\alpha)}}\, dx$$

$$I = 2\sin x + 2\cos\alpha\,(x) + C$$

(15) $\int \frac{x^6-1}{x^2+1}\,dx$

Soln M1

$$I = \int \frac{x^6-1}{x^2+1}\,dx$$

Divide (x^6-1) by (x^2+1)

$$(x^6-1) = (x^2+1)(x^4-x^2+1) - 2$$

$$I = \int \left(x^4-x^2+1 + \frac{(-2)}{x^2+1}\right) dx$$

$$I = \frac{x^5}{5} - \frac{x^3}{3} + x - 2\tan^{-1}x + C$$

M2

$$\int \frac{x^6-1}{x^2+1}\,dx = \int \frac{x^6+1-2}{x^2+1}\,dx$$

$$\int \frac{x^6+1}{x^2+1} - \frac{2}{x^2+1}\,dx = \int \frac{(x^2)^3+1}{x^2+1}\,dx - 2\cdot\int \frac{dx}{1+x^2}$$

$$= \int \frac{\cancel{(x^2+1)}(x^4-x^2+1)}{\cancel{x^2+1}}\,dx - 2\cdot\tan^{-1}x + C$$

$$= \frac{x^5}{5} - \frac{x^3}{3} + x - 2\tan^{-1}x + C$$

(16) $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\, dx$

Sol^n

$$I = \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\, dx$$

$$I = \int \left(\frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x} \right) dx$$

$$I = \int (\tan x . \sec x + \cot x \operatorname{cosec} x)\, dx$$

$$I = \int (\tan x . \sec x)\, dx + \int \cot x \operatorname{cosec} x\, dx$$

$$I = \sec x - \operatorname{cosec} x + C$$

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(17) $$\int \frac{x^4+x^2+1}{2(1+x^2)}\,dx$$

Sol^n

$$I = \int \frac{x^4+x^2+1}{2(1+x^2)}\,dx$$

$$I = \frac{1}{2}\int \frac{x^2(1+x^2)+1}{(1+x^2)}\,dx$$

$$I = \frac{1}{2}\int \left(\frac{x^2\cancel{(1+x^2)}}{\cancel{(1+x^2)}} + \frac{1}{1+x^2}\right)dx$$

$$I = \frac{1}{2}\left[\int x^2\,dx + \int \frac{1}{1+x^2}\,dx\right]$$

$$I = \frac{1}{2}\left[\frac{x^3}{3} + \frac{1}{2}\tan^{-1}x\right] + C$$

(18) $$\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x.\cos^2 x} dx$$

Solⁿ
$$\int \frac{(\sin^2 x)^3 + (\cos^2 x)^3}{\sin^2 x \cdot \cos^2 x} dx$$

$$= \int \frac{(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x)\, dx}{\sin^2 x \cos^2 x}$$

$$= \int \frac{\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x} dx - \int dx$$

$$= \int \tan^2 x \, dx + \int \cot^2 x \, dx - \int dx$$

$$= \int \sec^2 x - 1 \, dx + \int \operatorname{cosec}^2 x - 1 \, dx - \int dx$$

$$= \tan x - x + (-\cot x) - x - x + C$$

$$= \tan x - \cot x - 3x + C$$ Ans

(19) $\int \left(\sin^2\left(\frac{9\pi}{8}+\frac{x}{4}\right) - \sin^2\left(\frac{7\pi}{8}+\frac{x}{4}\right)\right) dx$

Sol^n

$$I = \int \left(\sin^2\left(\frac{9\pi}{8}+\frac{x}{4}\right) - \sin^2\left(\frac{7\pi}{8}+\frac{x}{4}\right)\right) dx$$

$\sin^2\alpha - \sin^2\beta = \sin(\alpha+\beta) \cdot \sin(\alpha-\beta)$

$$I = \int \sin\left(2\pi + \frac{x}{2}\right) \sin\left(\frac{\pi}{4}\right) dx$$

$$I = \int \sin\left(\frac{x}{2}\right) \cdot \frac{1}{\sqrt{2}} dx$$

$$I = \frac{1}{\sqrt{2}} \left(\frac{-\cos\left(\frac{x}{2}\right)}{\left(1/2\right)} \right) + C$$

$$I = -\sqrt{2} \cos\left(\frac{x}{2}\right) + C$$

(20) $$\int \frac{\cos 4x + 1}{\cot x - \tan x}\, dx$$

Sol^n

$$I = \int \frac{\cos 4x + 1}{\cot x - \tan x}\, dx$$

$$I = \int \frac{2\cos^2 2x}{\frac{\cos^2 x - \sin^2 x}{\cos x \sin x}}\, dx$$

$$I = \int \frac{2\cos x \sin x \cos^{\cancel{2}} 2x}{\cancel{\cos 2x}}\, dx$$

$$I = \int \sin 2x\, (\cos 2x)\, dx$$

$$\cancel{I = \frac{-\cos 2x}{2} + C}$$

$$I = \int \frac{\sin 4x}{2}$$

$$I = -\frac{\cos 4x}{8} + C$$

(21) $$\int \frac{\sin 2x + \sin 5x - \sin 3x}{\cos x + 1 - 2\sin^2 2x}\,dx$$

Sol^n

$$I = \int \frac{\sin 2x + \sin 5x - \sin 3x}{\cos x + 1 - 2\sin^2 2x}\,dx$$

$$I = \int \frac{\sin 2x + 2\sin x\cos 4x}{\cos x + \cos 4x}\,dx$$

$$I = \int \frac{2\sin x\cos x + 2\sin x\cos 4x}{\cos x + \cos 4x}\,dx$$

$$I = \int 2\sin x \frac{\cancel{(\cos x + \cos 4x)}}{\cancel{(\cos x + \cos 4x)}}\,dx$$

$$I = -2\cos x + C$$

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(22) $\int \frac{2x^3+3x^2+4x+5}{2x+1} dx$

Soln

$$I = \int \frac{2x^3+3x^2+4x+5}{2x+1} dx$$

$$I = \int \frac{(2x^3+x^2)+(2x^2+x)+(3x+5)}{2x+1} dx$$

$$I = \int \left(\frac{x^2(2x+1)}{(2x+1)} + \frac{x(2x+1)}{(2x+1)} + \left(\frac{3x+5}{2x+1}\right)\right) dx$$

$$I = \frac{x^3}{3} + \frac{x^2}{2} + \int \frac{3x+5}{2x+1} dx$$

$$I = \frac{x^3}{3} + \frac{x^2}{2} + \frac{3}{2} \int \frac{2x + 5\left(\frac{2}{3}\right)}{2x+1} dx$$

$$I = \frac{x^3}{3} + \frac{x^2}{2} + \frac{3}{2}\left[\int \left(\frac{2x+1}{2x+1} + \frac{\frac{10}{3}-1}{2x+1}\right) dx\right.$$

$$I = \frac{x^3}{3} + \frac{x^2}{2} + \frac{3}{2}\left[x + \frac{7}{3}\frac{\ln(2x+1)}{2}\right] + C$$

$$I = \frac{x^3}{3} + \frac{x^2}{2} + \frac{3x}{2} + \frac{7}{4}\ln(2x+1) + C$$

(23) $\int \frac{(x^2+\sin^2 x)\sec^2 x}{(1+x^2)}\,dx$

Solⁿ

$$I = \int \frac{(x^2+\sin^2 x)(\sec^2 x)}{(1+x^2)}\,dx$$

$$I = \int \frac{(x^2+1-\cos^2 x)(\sec^2 x)}{(1+x^2)}\,dx$$

$$I = \int\left(\frac{(x^2+1)\sec^2 x - \cos^2 x \cdot \sec^2 x}{1+x^2}\right)dx$$

$$I = \int\left(\sec^2 x - \frac{1}{1+x^2}\right)dx$$

$$I = \tan x - \tan^{-1} x + C$$

(24) $\int \frac{dx}{\sqrt{9-16x^2}}$

Solⁿ

$$I = \int \frac{dx}{\sqrt{9-16x^2}}$$

$$I = \frac{1}{3}\int \frac{dx}{\sqrt{1-\left(\frac{4}{3}x\right)^2}}$$

$$I = \frac{1}{\cancel{3}} \sin^{-1}\left(\frac{\frac{4x}{3}}{1}\right) \cdot \frac{1}{4/\cancel{3}} + C$$

$$I = \frac{1}{4}\sin^{-1}\left(\frac{4x}{3}\right) + C$$

25) $\int \frac{dx}{25+4x^2}$

Sol^n

$$\frac{1}{25}\int \frac{dx}{1+\frac{4x^2}{25}} = \frac{1}{25}\cdot\int \frac{dx}{1+\left(\frac{2x}{5}\right)^2}$$

$$= \frac{1}{25}\cdot\frac{\tan^{-1}\left(\frac{2x}{5}\right)}{\frac{2}{5}} + C$$

$$= \frac{1}{10}\tan^{-1}\left(\frac{2x}{5}\right) + C$$

(26) $\int \frac{2x+3}{(3x+2)} dx$

Soln

$$I = \int \frac{2x+3}{3x+2} dx$$

$$I = \frac{2}{3} \int \frac{3x + \frac{3}{2}\left(\frac{3}{2}\right)}{3x+2} dx$$

$$I = \frac{2}{3} \left[\int \left(\frac{3x+2}{3x+2} + \frac{\frac{9}{2} - 2}{(3x+2)} \right) dx \right.$$

$$I = \frac{2}{3} \left[\int 1 \, dx + \frac{5}{2} \int \frac{1}{3x+2} dx \right]$$

$$I = \frac{2x}{3} + \frac{2}{3} \cdot \frac{5}{2} \frac{\ln(3x+2)}{3} + C$$

$$I = \frac{2x}{3} + \frac{5}{9} \ln(3x+2) + C$$

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(27) $\int \frac{dx}{1+\sin x}$

Sol^n

$$I = \int \frac{dx}{1+\sin x}.$$

$$I = \int \frac{1-\sin x}{(1+\sin x)(1-\sin x)} dx$$

$$I = \int \frac{(1-\sin x)\, dx}{\cos^2 x}$$

$$I = \int (\sec^2 x - \tan x . \sec x)\, dx$$

$$I = \tan x - \sec x + C$$

(28) $$\int \frac{\cos 8x - \cos 7x}{1 + 2\cos(5x)}\,dx$$

Sol^n

$$I = \int \frac{\cos 8x - \cos 7x}{1 + 2\cos(5x)}\,dx$$

$$I = \int \frac{(\cos 8x - \cos 7x)}{(1 + 2\cos 5x)} \cdot \frac{\sin(5x)}{\sin(5x)}\,dx$$

$$\Rightarrow I = \int \frac{(\sin 5x)\left(2\sin\frac{15x}{2}\sin\left(-\frac{x}{2}\right)\right)}{\sin 5x + \underbrace{2\sin 5x\cos x}_{\sin 10x}}$$

$$I = \int \frac{(\sin 5x) \cdot \cancel{2}\sin\cancel{\left(\frac{15x}{2}\right)}\sin\left(-\frac{x}{2}\right)}{\cancel{2}\sin\cancel{\left(\frac{15x}{2}\right)}\cos\left(\frac{5x}{2}\right)}\,dx$$

$$I = \int \frac{2\sin\left(\frac{5x}{2}\right)\cancel{\cos\left(\frac{5x}{2}\right)}\sin\left(-\frac{x}{2}\right)}{\cancel{\cos\left(\frac{5x}{2}\right)}}\,dx$$

$$I = -\int \cos(2x) - \cos(3x) = \frac{\sin 3x}{3} - \frac{\sin 2x}{2} + C$$

(29) $$\int \frac{2+3x^2}{x^2(1+x^2)}\,dx$$

Sol^n

$$I = \int \frac{2+3x^2}{x^2(1+x^2)}\,dx$$

$$I = \int \frac{(2+2x^2+x^2)}{x^2(1+x^2)}\,dx$$

$$I = \int \left(\frac{2(1+x^2)}{x^2(1+x^2)} + \frac{x^2}{x^2(1+x^2)}\right)dx$$

$$I = \int \left(\frac{2}{x^2} + \frac{1\cancel{x^2}}{1+x^2}\right)dx$$

$$I = -2\left(\frac{1}{x}\right) + \tan^{-1}x + C$$

$$\cancel{I = \frac{-2}{x} + \frac{x^3}{3} + C}$$

$$I = \frac{-2}{x} + \tan^{-1}x + C$$

(30) $\int \sin x \cos x \cos 2x \cos 4x \, dx$

Sol^n

$$I = \int \sin x \cos x \cos 2x \cos 4x \, dx$$

$2 \sin x \cos x = \sin 2x$

$$\boxed{\left(\sin x \cos x = \frac{\sin 2x}{2}\right)}$$

$$I = \int \frac{\sin 2x}{2} \cos 2x \cos 4x \, dx$$

$$I = \frac{1}{2} \int \frac{\sin 4x}{2} \cos 4x \, dx$$

$$I = \frac{1}{4} \int \frac{\sin 8x}{2} \, dx$$

$$I = \frac{1}{8} \int \sin 8x \, dx$$

$$I = \frac{1}{8} \frac{-\cos 8x}{8} + C$$

$$I = \frac{-1}{64} \cos 8x + C$$