29/03/22

0-1

14

A	B	C
4	4	4
1	1	3
1	3	1
3	1	1
1	2	2
2	1	2
2	2	1

$$({}^4C_1\, {}^4C_1\, {}^4C_3)\,3$$

$$+\,({}^4C_1\, {}^4C_2\, {}^4C_2)\,3$$

15 $0, 1, 2, 3, \ldots 8, 9 = 10$ digits

$$m = {}^{10}C_5 \cdot 1 \qquad n = {}^9C_5 \cdot 1$$

$$m - n = {}^{10}C_5 - {}^9C_5 = {}^9C_4 = {}^9C_{9-4}$$

16

$$\underset{\text{4 out of 5}}{{}^5C_4}\ \underbrace{{}^2C_1\ {}^2C_1\ {}^2C_1\ {}^2C_1}_{\text{1 from each pair}}$$

(21) $P_n = {}^{n-2}C_3$

$P_{n+1} = {}^{n+1-2}C_3$

$\Rightarrow P_{n+1} - P_n = 15$

(11)

$${}^{10}C_2$$

$\underline{5 I_M}$ $\underline{5 I_W}$ $5 A_M$ $\underline{5 A_W}$

$${}^{10}C_2$$

$${}^{10}C_2$$

(17) $\boxed{AB}$ C D E F G H

$$\frac{7!\ 2!}{2!}$$

(24) cyphers = number of zeros at end

$${}^{2002}C_{1001} = \frac{(2002)!}{\underbrace{(1001)!}\ \underbrace{(1001)!}}$$

find exponent of 10

$\Downarrow$

$a-2b$

$$\frac{10^a}{10^b\ 10^b} = 10^{a-2b}$$

(27) $x_1 + x_2 + x_3 = 5$ $\qquad {}^{5-1}C_{3-1} = {}^4C_2$

$$\frac{4!}{1!\,1!\,2!\;2!} \qquad 3!$$

28

	India	Pakistan	Total match
$^5C_5$	5	0	5
$^5C_4$	6th match is 5th win of India	$\underbrace{1\,2\,3\,4\,5}_{4I,\ 1P}$ $6 \uparrow$ 5I	6
$^6C_4$	7th match ↓ 5th win	$\underbrace{1\,2\,3\,4\,5\,6}_{4I,\ 2P}$ $7 \downarrow$ 5I	7
$^7C_4$	8th match ↓ 5th win	$\underbrace{1 \ldots 7}_{4I,\ 3P}$ 8th ↓ 5th I	8
$^8C_4$	9th math ↓ 5th win	$\underbrace{1\,2\,3\,4\,5\,6\,7\,8}_{4I,\ 4P}$ $9 \downarrow$ 5I	9
		$\cancel{\underbrace{1\,2\,3\,4 \ldots 9}_{4I,\ 5P}}$	$\cancel{10}$

33 A $\quad {}^{n}C_m \cdot 1$

$\{a_1, \dots a_n\} \to$ m select

30 $\quad (x_1+x_2+x_3)(y_1+y_2)=77 \quad$ $x_i \in N$, $y_i \in N$

$x_1+x_2+x_3=11$ OR $x_1+x_2+x_3=7$

$y_1+y_2=7 \qquad y_1+y_2=11$

$${}^{11-1}C_{3-1} \cdot {}^{7-1}C_{2-1} + {}^{7-1}C_{3-1} \cdot {}^{11-1}C_{2-1}$$

32 $\quad 5! - \underbrace{(0 \text{ wrong})}_{\text{All correct}} - \underbrace{(1 \text{ wrong})}_{\text{NOT possible}}$

$120 - 1 - 0 = 119$

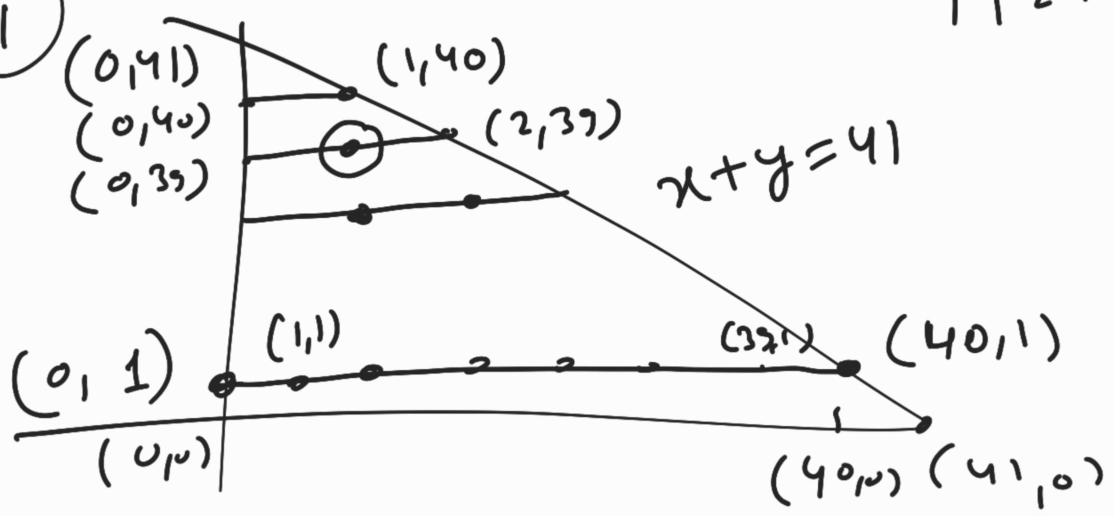
33 C $\quad |1|2|-|\dots (m-1)|$ m gaps

${}^{m}C_n$

JM

$1+2+\cdots+39=$

①



$\Rightarrow \quad x+y<41 \qquad x\in N \quad y\in N$

$x+y+t=41 \qquad t\in N$

$\uparrow$

Dummy $\qquad {}^{41-1}C_{3-1} = {}^{40}C_2$

⑧ $\sum_{k=2}^{13}(7k+2)+\sum_{k=1}^{13}(7k+5)$

⑨ $S=\{1,2,3,\ldots 100\}$ $\qquad$ 50 odd

$\text{Total nonempty subsets} = (2^{100}-1)$

ϕ

product odd

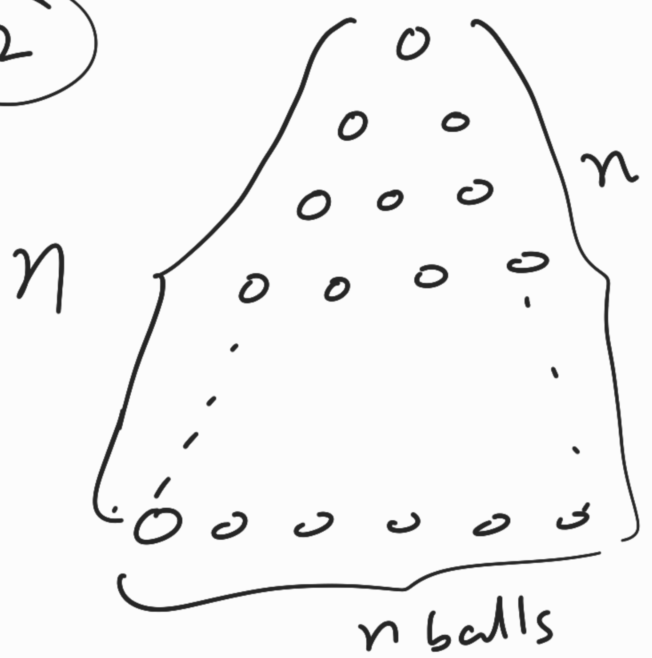
$\underbrace{}$

non empty subsets $=2^{50}-1$

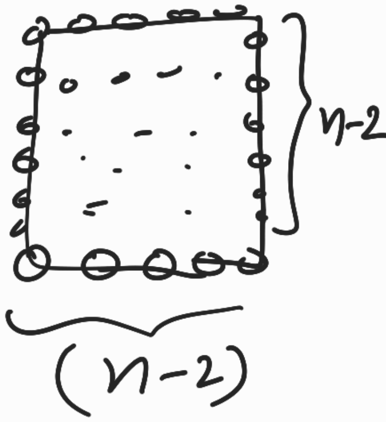
ϕ

Ans = $(2^{100}-1)-(2^{50}-1)$

12



$+ 99 =$



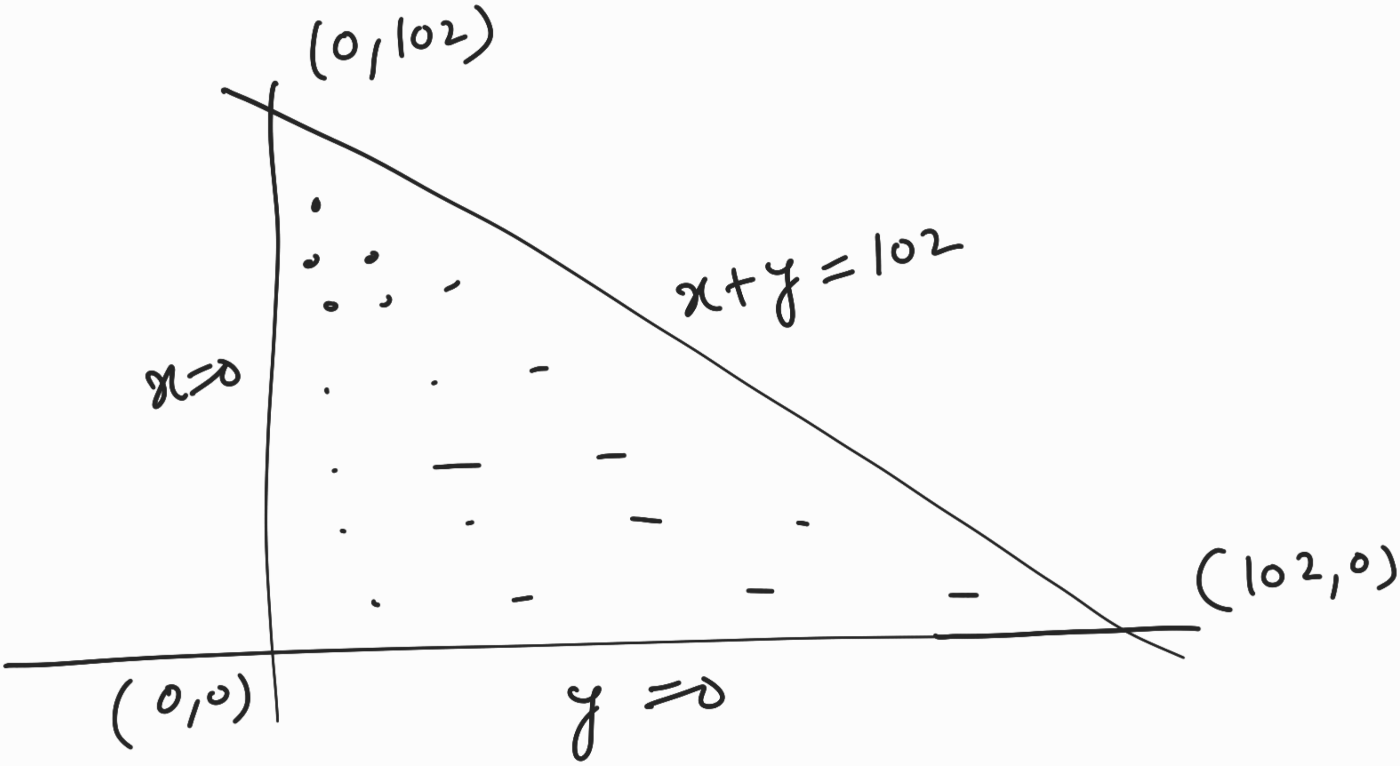
$$\left(1+2+3+\dots+n\right)+99=(n-2)^2$$

$$\frac{n(n+1)}{2}+99=(n-2)^2$$

$n=19$

H.W. = S-1, JA

Ex2 find number of points with integral co-ordinates that lie inside a triangle with vertices (0,0) (0,102) (102,0)



$x>0$, $y>0$ $\quad$ $x+y<102$

$x \in N$

$y \in N$

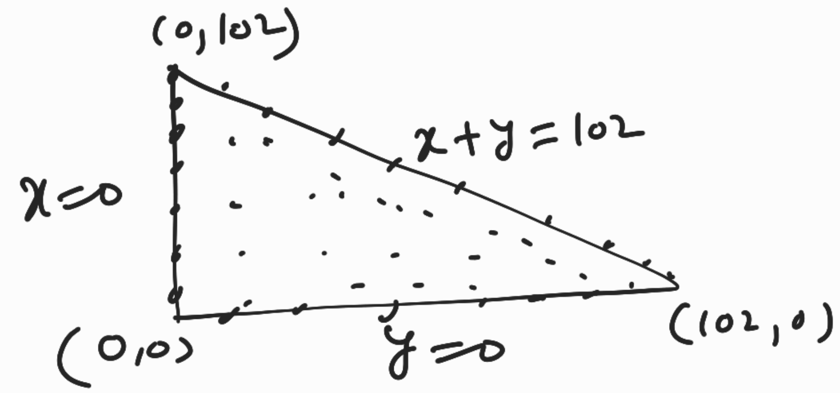
$\boxed{x+y = 2, \; - - - - \; 101}$

$x+y+t = 102$ $\quad$ $\boxed{(x+y)_{Max} = 101 \quad t_{min} = 1}$

$t \in N$

natural sol^n $= {}^{102-1}C_{3-1} = {}^{101}C_2$

\# points [lie on triangle + inside triangle]



$x \geq 0$

$y \geq 0$

$x, y \in W$

$x+y \leq 102$

$$\boxed{\begin{aligned}(x+y)_{max} &= 102 \\ t_{min} &= 0\end{aligned}}$$

$x+y+t = 102$

$x, y \in W$ $\quad$ $t \in W$

$\underline{Ans} = {}^{102+3-1}C_{3-1} = {}^{104}C_2$

MISCELLANEOUS

① Maximum value of ${}^nC_r$

at
$$\boxed{\begin{array}{ll} r = \frac{n}{2} & \text{if } n \text{ is even} \\ \hline r = \frac{n-1}{2} & \\ \text{or} & \text{if } n \text{ is odd} \\ r = \frac{n+1}{2} & \end{array}}$$

$\boxed{\text{Proof}}$ $\quad$ ${}^nC_r > {}^nC_{r-1}$ & ${}^nC_r > {}^nC_{r+1}$

$$\left({}^{20}C_r\right)_{max} = {}^{20}C_{10}$$

$$\left({}^{25}C_r\right)_{max} = {}^{25}C_{12} = {}^{25}C_{13}$$

Ex:- There are $2n$ letters which are either 'a' or 'b'. find max^m number of arrangements of these letters

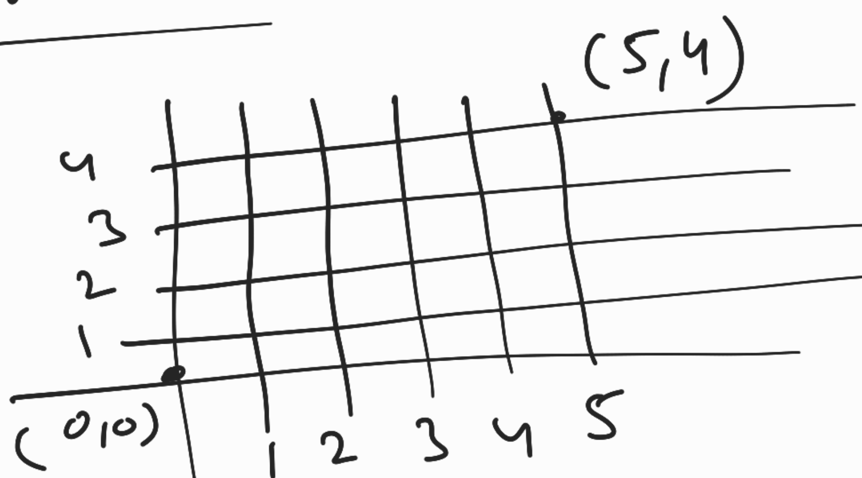
Soln Let $\underbrace{a, a, a \ldots a}_{x}\ \underbrace{b, b \ldots b}_{(2n-x)}$

$$\text{Arrange.} = \frac{(2n)!}{x!\,(2n-x)!} = {}^{2n}C_x$$

$$\left({}^{2n}C_x\right)_{Max} = {}^{2n}C_{\left(\frac{2n}{2}\right)} = {}^{2n}C_n$$

---

② Grid problem



find number of Shortest path from $(0,0)$ to $(5,4)$

one step = rightward or upward by 1 unit

Sol^n = $(0,0) \longrightarrow (5,4)$

5 unit in $+x$ direction

4 unit in $+y$ direction

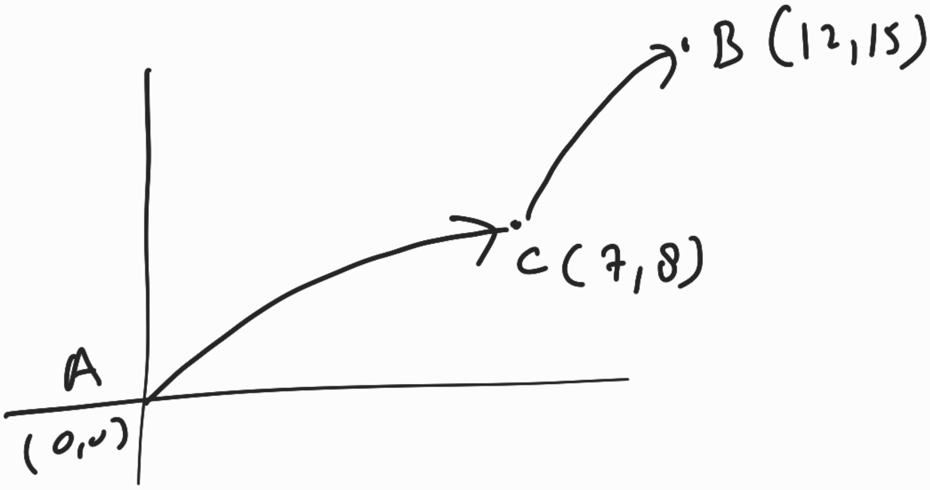
5 steps in $+x$ dir^n

4 steps in $+y$ dir^n

$$\text{Ans.} = \frac{9!}{5!\,4!}$$

Ex:- $A(0,0)$ to $B(12,15)$ via $C(7,8)$

find total no. of Shortest Path



(A to C) & (C to B)

$$\frac{15!}{7!\,8!} \times \frac{12!}{5!\,7!}$$

A (0, 0)

7 steps + 8 steps = 15 total $\equiv \frac{15!}{7!\,8!}$

C (7, 8)

5 steps + 7 steps = 12 total $\equiv \frac{12!}{5!\,7!}$

B (12, 15)

---

③ Summation of numbers

Ex:- $\boxed{1, 2, 3, 7, 6, 9}$ find sum of all 6 digit numbers

No Repetition

___ ___ ___ ___ ___ ___ TOTAL $= 6!$
$= 720$

1×120 ___ ___ ___ ___ ___ 1×120

2×120 ___ ___ ___ ___ ___ 2×120

3×120 ___ ___ ___ ___ ___ 3×120

6×120 ___ ___ ___ ___ ___ 6×120

7×120 ___ ___ ___ ___ ___ 7×120

9×120 ___ ___ ___ ___ ___ 9×120

3360	3360	3360	3360	3360	3360
373	373	372	369	336	
	3733	3732	3729	3696	
3733	3	2	9	6	0

373332960 = Sum

123

132
231
213
312
321

1332

1x2	1x2	1x2 = 2
2x2	2x2	2x2 = 4
3x2	3x2	3x2 = 6
12	12	12
1	1	
13	13	
13	3	2

$$
\begin{array}{r}
5768 \\
+\ 3589 \\
\hline
8\ \ 12\ \ 14\ \ 17 \\
1\ \ \ 1\ \ \ 1\ \ \ \ \ \\
1\ \ 13\ \ 15\ \ \ \ \ \\
\hline
9\ \ 3\ \ 5\ \ 7
\end{array}
$$

$$
\begin{array}{r}
5768 \\
3589 \\
\hline
9357
\end{array}
$$

$\boxed{1\ 2\ 2\ 3} \Rightarrow$ _ _ _ _

4 digit numbers

$$\frac{4!}{2!} = 12 = \text{numbers}$$

1×3	1×3	1×3	1×3	2, 2, 3
2×6	2×6	2×6	2×6	1 2 3
3×3	3×3	3×3	3×3	1 2 2
24	24	24	24	
2	2	2		
26	26	26		

$\boxed{2\,6 \quad 6 \quad 6 \quad 4}$ Ans