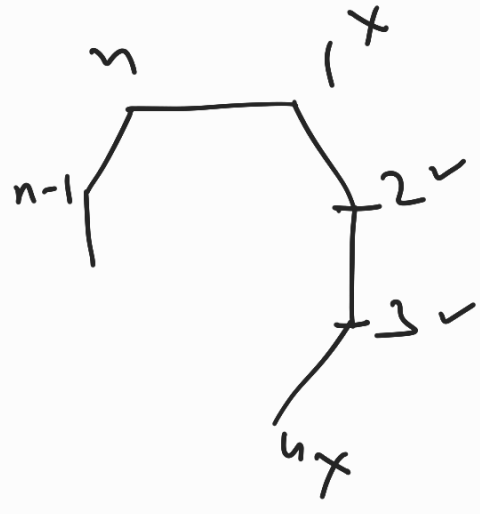
Ex:- find number of triangles formed by joining of vertices of convex polygon of n sided. out of these Δ's also find

(1) Number of Δ's having exactly one side common to polygon

(2) Number of Δ's having exactly two sides common to polygon

(3) Number of Δ's having no side common to the polygon

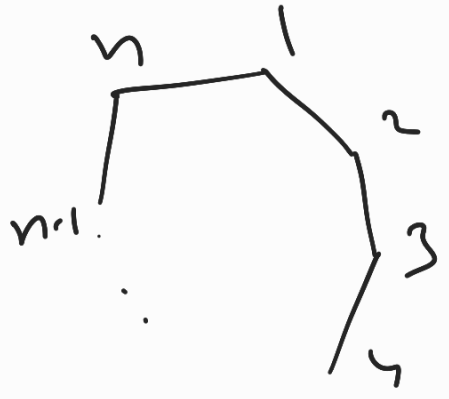


Soln ⇒ TOTal no. of Δ's $= {}^{n}C_3$

(1) ${}^{n}C_1 \times {}^{n-4}C_1 = n(n-4)$

↑ one side out of 'n' sides

↑ 3rd vertex



(2) $123, 234, 345, \ldots$

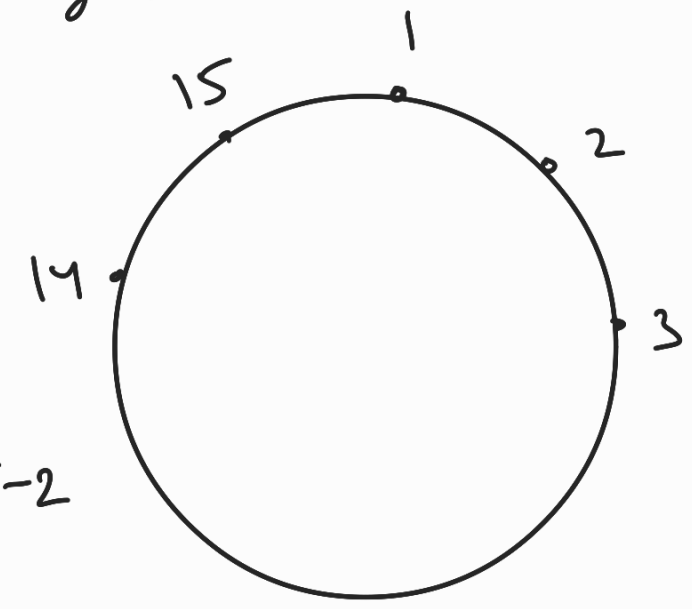
$\ldots (n-2)(n-1)n, (n-1)n1, n12$

$= n$

(3) ${}^{n}C_3 - n(n-4) - n = {}^{n-2}C_3 - {}^{n-4}C_1$

TOTal − one common side − two Common Side

Ex:- How many hexagon can be constructed joining of vertices of quindecagon (15 sides) if none of the side of hexagon is also the side of quindecagon.

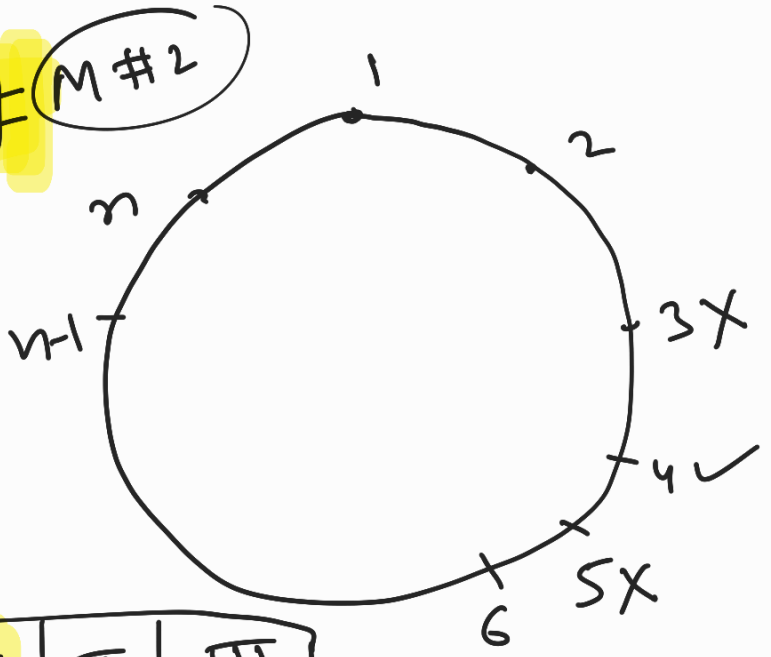


$^{15-6+1}C_6$ = linear

Ans. = $^{15-6+1}C_6 - {}^{15-6+1-2}C_{6-2}$

$= {}^{10}C_6 - {}^{8}C_4 = 140$ Ans.

#M#2



n = persons

select 3

no two of them are consecutive

I	II	III
4	10	25
→ 10	4, 25	
→ 25	4, 10	

Remaining = n-3

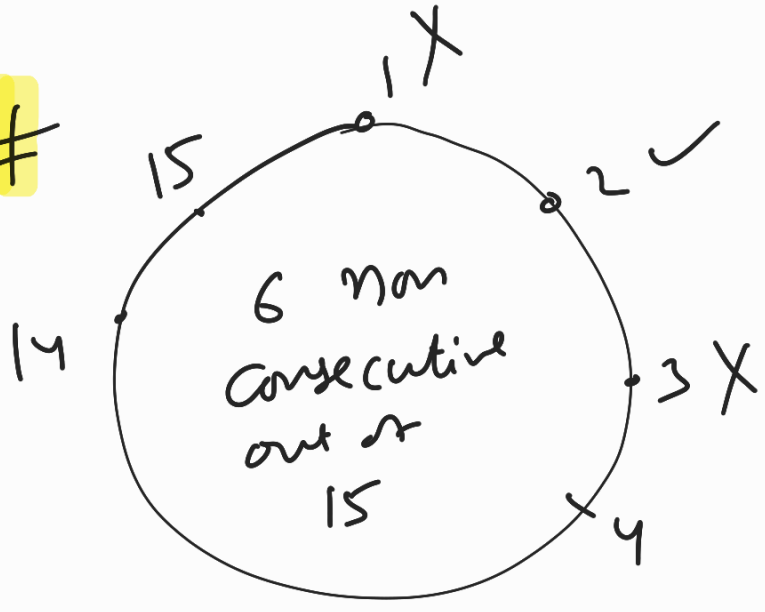
1, 2, 6, ... n = live

$$Am = n_{C_1} \; {}^{(n-3)-2+1}C_2 \; / 3$$

I st selection out of n

Select 2 non consecutive out of (n-3) (linearly)

#



$$15_{C_1} \; {}^{12-5+1}C_5$$

$$= 6$$

$$= 140$$

4 5 6 ..... 14, 15

12 linear

$$\# \text{ linear} = {}^{n-r+1}C_r$$

$$\text{Circle} = {}^{n-r+1}C_r - {}^{n-r-1}C_{r-2}$$

n - given ; select r non consecutive

Total number of combination (selection)

⇓

Atleast one selection

(I) Selection of 'r' diff^{nt} things out of 'n' diff^{nt} things $= {}^nC_r$ $\boxed{0 \le r \le n}$

(II) Total no. of selection =

$${}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n = \underbrace{2^n - 1}$$

⇓ n diff^{nt} things

Atleast one selection = Total − none

⇓

we select any no. of things or nothing

n identical things are given ⇓

(I) Number of ways of selection of 'r' identical things out of 'n' identical things $= 1$ $\boxed{0 \le r \le n}$

(II) Total number of selections out

of n identical things $= n$

selection of 0 thing $= 1$
—"— 1 —"— $= 1$
—"— 2 —"— $= 1$
⋮
—"— n —"— $= 1$

Select any no. of things or nothing out of n identical $= (n+1)$

(III) p things are alike of 1st kind
q —"— 2nd kind
t = diff^{nt}

TOTAL $= p+q+t$

TOTAL no. of selections = no. of ways of atleast one selection

$= (p+1)(q+1)2^t - 1$

Ex:- 5 pen → Blue

8 balls = Black

1 marker, 1 Apple, 1 orange, 1 pencil

1 eraser

Q①

Number of ways of selection of

at least one thing = ??

$$\underbrace{(5+1)(8+1)\,2^5}_{\text{when we select any no. of things or nothing}} - 1 \leftarrow \text{nothing}$$

Q② number of selection atleast one thing of each kind

$$= 5 \times 8 = 40$$

$$= (5+1-1)(8+1-1)(2-1)$$

$$(2-1)(2-1)(2-1)(2-1)$$

Ex:- M_1, M_2, M_3 → 3 MATH

$P_1, P_2, P_3, P_4 \rightarrow$ 4 Physics

$C_1, C_2, C_3, C_4, C_5 \rightarrow$ 5 Chemistry

find number of selection of

Q① one book on each subject

$$^3C_1 \; ^4C_1 \; ^5C_1 = 60$$

Q② atleast one book on each subject

MATH	Phy.	Chem.
(2^3-1)	(2^4-1)	(2^5-1)

Q③ atleast one book $= 2^{12} - 1$

Q④ only one book $= {}^{12}C_1$

Ex:- 2 mangoes, 4 Apples, 5 oranges

case① fruits of same species are different

find no. of ways of selection

of ① atleast one fruit

$= 2^{11} - 1 = 2047$

---

② at least one fruit of each species

Mango. Apple orange

$(2^2 - 1)(2^4 - 1)(2^5 - 1) = 3 \times 15 \times 31$

Case II Fruits of same species are identical

MM, AAAA, OOOOO given

① ⇒ atleast one fruit

$(2+1)(4+1)(5+1) - 1$

$= 3 \times 5 \times 6 - 1 = 89$ Ans

Q(2) ⇒ atleast one fruit of each species

$$(2+1-1)(4+1-1)(5+1-1)$$

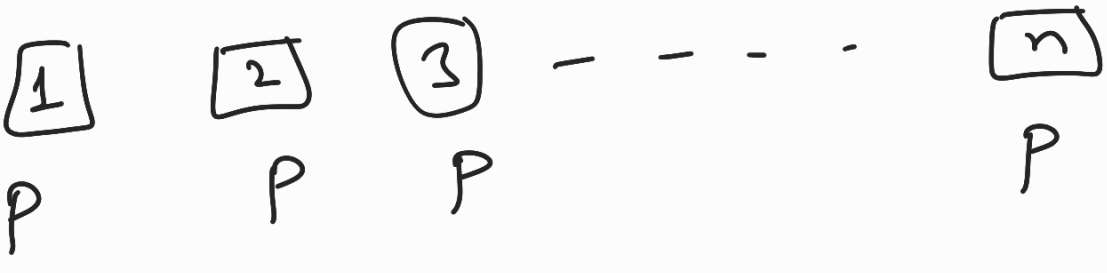
$$= 2 \times 4 \times 5 = 40 \text{ Ans}$$

Ex:- There are 10 bulbs in a hall find no. of ways in which hall can be illuminated

$$(2^{10}-1)$$

Ex:- A shopkeeper has 'n' diff^{nt} books, each having 'p' copies. find no. of ways of different selections that can be made.

Soln



Total ways when we select any no of things or nothing =

$= (p+1)(p+1)(p+1)\cdots(p+1) = (p+1)^n$

Ans = $(p+1)^n - 1$ = Total no. of selection

Ex:- $N = 75600 = 2^4 3^3 5^2 7^1$

$2\cdot2\cdot2\cdot2 \quad 333 \quad 55 \quad 7$

Find ① Number of +ve divisors

$= (4+1)(3+1)(2+1)(1+1) = 120$

② Number of proper division

$=$ TOTAL $- 1 \quad = 119$

* 'N' itself is not a proper divisor

(3) Number of odd divisors =

$N = \cancel{2^4}\ 3^3\ 5^2\ 7^1$ | $(3+1)(2+1)(1+1)$

$= 24$

(4) Number of even divisors

= Total − odd divisors

$= 120 - 24$ $= 96$ Ans

$N = 2^4\ 3^3\ 5^2\ 7^1$

$N = 2\left[2^3\ 3^3\ 5^2\ 7^1\right]$

$(3+1)(3+1)(2+1)(1+1) = 96$

(5) number of divisors divisible

by (a) 5 (b) 12 (c) 30

(a) $N = 5\left[2^4 3^3 5^1 7^1\right]$

$(4+1)(3+1)(1+1)(1+1) = 80$

(b) $N = 2^2 3\left[2^2 3^2 5^2 7^1\right]$

$(2+1)(2+1)(2+1)(1+1) = 54$

(c) $N = 2^4 3^3 5^2 7^1$

$N = 2\times3\times5\left[2^3 3^2 5^1 7^1\right]$

$(3+1)(2+1)(1+1)(1+1)$

$= 48$

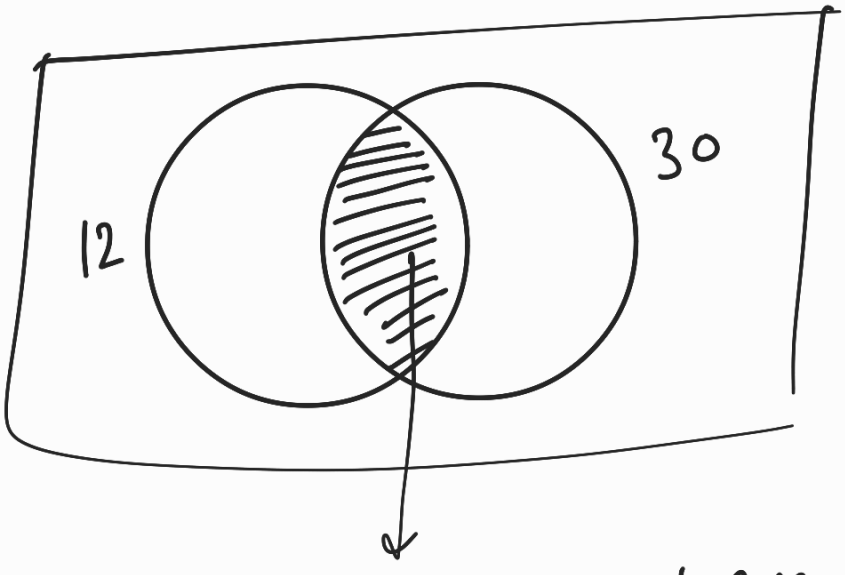
(6) Number of divisions divisible by 12 but not divisible by 30.

= no. of divisors divisible by 12

— no of divisors divisible by 60

↑↑

LCM(12,30)



Common = LCM (12,30) = 60

$N = 2^2 \cdot 3 \cdot 5 [2^2 3^2 5^1 7^1]$

$(2+1)(2+1)(1+1)(1+1)$

$= 36$

Ans $= 54 - 36 = 18$ Ans