28/03/22

Ex:- $N = 75600 = 2^4 3^3 5^2 7^1$

Q① find sum of all positive divisors

Q② How many different ways in which 'N' can be resolved as a product of two positive divisors

Q③ Number of ways in which N can be resolved as a product of 2 divisors which are relatively prime.

Solⁿ ① $N = 2^4 3^3 5^2 7^1$ (Total divisors = 120)

Sum of all divisors =

$$\underbrace{(2^0+2^1+2^2+2^3+2^4)}_{G.P.}\underbrace{(3^0+3^1+3^2+3^3)}_{G.P.}\underbrace{(5^0+5^1+5^2)}_{G.P.}\underbrace{(7^0+7^1)}_{G.P.}$$

$$= (31) \quad (40) \quad (31) \quad (8)$$

Solⁿ ② $N = 2^4 3^3 5^2 7^1$

Total Divisors = 120

$ab = N$

$a, b = +ve$

$$\boxed{\frac{120}{2} = 60 \text{ Ans.}}$$

$N = 36$ (let)

$N = 2^2 3^2$

Total Divisors $= (2+1)(2+1) = 9$

$ab = N$ $a, b = +ve$

1×36

2×18

3×12

4×9

6×6

$\text{Ans} = \frac{9+1}{2} = 5$

9 divisors = 1, 2, 3, 4, 6, 9, 12, 18, 36

NOTE If Total of divisors of a Number $= n$ then number of ways in which Number can be resolved as product of two positive divisors is

$$= \begin{cases} \frac{n}{2} & \text{if } n = \text{even} \\ \frac{n+1}{2} & \text{if } n = \text{odd} \end{cases}$$

$\boxed{\text{Sol}^n \ (3)}$

$$N = 2^4 3^3 5^2 7^1$$

$$N = a \times b \qquad \boxed{HCF(a, b) = 1}$$

(1)	2^4	$3^3 5^2 7^1$
(2)	3^3	$2^4 5^2 7^1$
(3)	5^2	$2^4 3^3 7^1$
(4)	7^1	$2^4 3^3 5^2$
(5)	$2^4 3^3$	$5^2 7^1$

6	$2^4 5^2$	$3^3 7^1$
7	$2^4 7$	$3^3 5^2$
8	1	$2^4 3^3 5^2 7^1$

$\boxed{Ans = 8}$

$$N = 2^4\, 3^3\, 5^2\, 7^1$$

1 2 3 4

$p = 4$

$$2^{p-1} = 2^3 = 8$$

Ex:- $N = 2^8\, 3^3\, 5^5\, 7^2\, 11^1\, 13^{11}\, 17^3\, 19^5$

1 2 3 4 5 6 7 8

$ab = N$

$HCF(a, b) = 1$

$$Ans = 2^{8-1} = 2^7$$
$$= 128$$

NOTE:- Number of ways in which

N can be resolved as product of 2 divisors which are relatively prime $= 2^{(p-1)}$

where p is number of primes involved in prime factorisation of N

DISTRIBUTION OF ALIKE OBJECTS

(I) Number of ways in which 'n' identical objects can be distributed among 'r' persons if each can receive any no. of things $= {}^{n+r-1}C_{r-1}$

Ex:- How many different ways in which 6 identical coin can be distributed among 3 persons

$\underbrace{0\ 0\ 0\ 0\ 0\ 0}_{\text{6 Coins identical}}\ \underbrace{|\,|}_{2}$ partitions

$\boxed{\text{no. of persons} - 1}$

6 Coins 2 par. = Arrange $= \dfrac{8!}{6!\,2!}$

$\underbrace{0}_{I}\,|\,\underbrace{0\ 0}_{II}\,|\,\underbrace{0\ 0\ 0}_{III}$

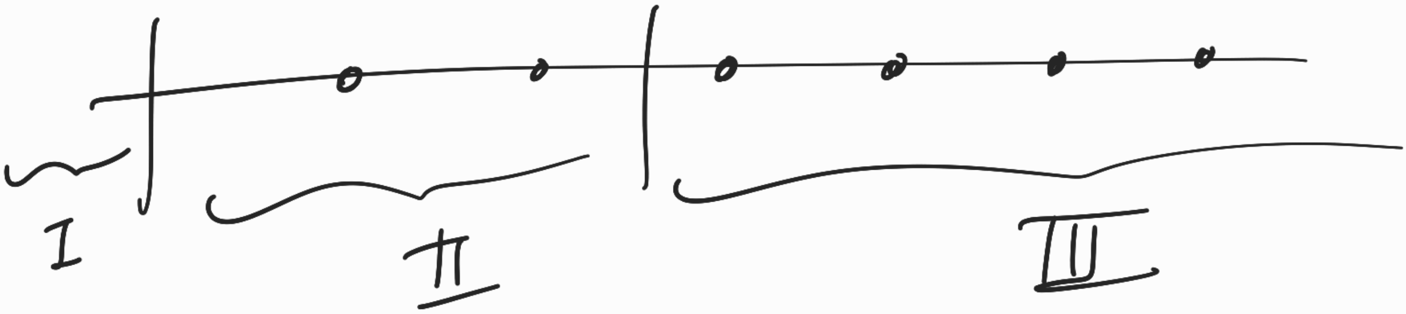
$\underbrace{\ }_{I}\,|\,\underbrace{0\,0\,0\,0\,0\,0}_{II}\,|\,\underbrace{\ }_{III}$

$\underbrace{\ }_{I}\,|\,\underbrace{\ }_{II}\,|\,\underbrace{0\,0\,0\,0\,0\,0}_{III}$

$\underbrace{0\,0\,0\,0\,0\,0}_{I}\,|\,\underbrace{\ }_{II}\,|\,\underbrace{\ }_{III}$

$\underbrace{0\ 0}_{I}\,|\,\underbrace{0\ 0}_{II}\,|\,\underbrace{0\ 0}_{III}$

6	0	0	→ 3
1	5	0	→ 6
1	1	4	→ 3
2	4	0	→ 6
2	2	2	→ 1
1	2	3	→ 6
3	3	0	→ 3



$\# \quad \underbrace{\circ\circ\circ\cdots\circ}_{n \text{ identical}} \quad \underbrace{|||\cdots|}_{(r-1)\text{ partitions}}$

$$\text{Total} = n + (r-1)$$

$$\text{Arrange} = \frac{(n+r-1)!}{n!\,(r-1)!} = {}^{n+r-1}C_{r-1}$$

Ex :- 12 identical coins distribute in to 6 persons each can get any no. of things

$$\frac{(12+6-1)!}{(12)!\,(6-1)!} = {}^{17}C_5$$

II Number of non-negative integral

Sol^n of Eq^n

$$\boxed{x_1 + x_2 + x_3 + \cdots \cdots + x_r = n}$$

$x_i \in W$ are $= {}^{n+r-1}C_{r-1}$

Ex: $x+y+z=6$, $\boxed{x, y, z \in W}$

find triplet $(x, y, z) = {}^8C_2 = 28$

$(x+y)^2 = x^2 + 2xy + y^2$

$\underline{x}\ \underline{x} = 1 \cdot x^2$

$\underline{y}\ \underline{y} = 1 \cdot y^2$

$\left\{ \begin{array}{l} \underline{x}\ \underline{y} = xy \\ \underline{y}\ \underline{x} = xy \end{array} \right] \Rightarrow 2xy$

Arrange $(\underline{x}\ \underline{y}$ or $\underline{y}\ \underline{x}) = 2$

$$(x+y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$$

$$\underline{x}\ \underline{x}\ \underline{x} = 1 \cdot x^3$$

$$\underbrace{\underline{x}\ \underline{y}\ \underline{y}}_{\text{Arrange}} = \frac{3!}{2!}\ xy^2$$

$$\underline{x}\ \underline{x}\ \underline{y} = \frac{3!}{2!}\ x^2y$$

$$\underline{y}\ \underline{y}\ \underline{y} = 1 \cdot y^3$$

Ex:- find coeff of $x^3 y^4 z^7$ in expansion of $(x+y+z)^{14}$

$$\underline{\ \ }_1\ \underline{\ \ }_2\ \underline{\ \ }_3\ \cdots\ \underline{\ \ }_{14}$$

$$x\,x\,x \qquad y\,y\,y\,y \qquad z\,z\,z\,z\,z\,z\,z$$

$$\frac{14!}{3!\,4!\,7!} = \text{Coeff}^{nt}$$

Ex:- find total no. of terms in

$$(x+y+z)^{15}$$

$$\underbrace{\overline{1}\ \overline{2}\ \overline{3}\ -\ -}_{r_1 \text{ times } x}\ \underbrace{-\ -\ -}_{r_2 \text{ times } y}\ \underbrace{\overline{15}}_{r_3 \text{ times } z}$$

$$r_1 + r_2 + r_3 = 15 \qquad r_i \in W$$

$$^{15+3-1}C_{3-1} = {}^{17}C_2$$

(III) Number of ways in which 'n' identical coins can be distributed into 'r' persons if each can receive atleast one $= {}^{n-1}C_{r-1}$

(IV) number of $\underbrace{\text{positive integral}}_{\text{Natural}}$ soln

of eqn $\boxed{x_1 + x_2 + x_3 + \cdots + x_r = n}$

$x_i \in N$

are $\boxed{{}^{n-1}C_{r-1}}$

$x_1 + x_2 + \cdots + x_r = n$

$x_1 = 1 + x_1'$

$x_2 = 1 + x_2'$ $\boxed{x_i' \in W}$

$\vdots$

$x_r = 1 + x_r'$

$x_1' + x_2' + x_3' + \cdots + x_r' = n - r$

Distribute $(n-r)$ identical coins

in to 'r' persons each gets any no. of coins

$$\underbrace{oo\ldots\ldots o}_{(n-r)} \underbrace{||||\ldots|}_{(r-1)\text{ partitions}}$$

$$\text{Arrange} = \frac{(n-r+r-1)!}{(n-r)!\,(r-1)!} = {}^{n-1}C_{r-1}$$

<u>Ex:-</u> In how many ways 30 marks can be allotted to 8 questions if atleast 2 marks are to be given of each question?

$$Q_1 + Q_2 + \ldots + Q_8 = 30$$

$$Q_i \in N \qquad Q_i \geq 2$$

Distribute 2 marks to each question

$$Q_1' + Q_2' + \dots + Q_8' = 14$$

$$Q_i' \in W$$

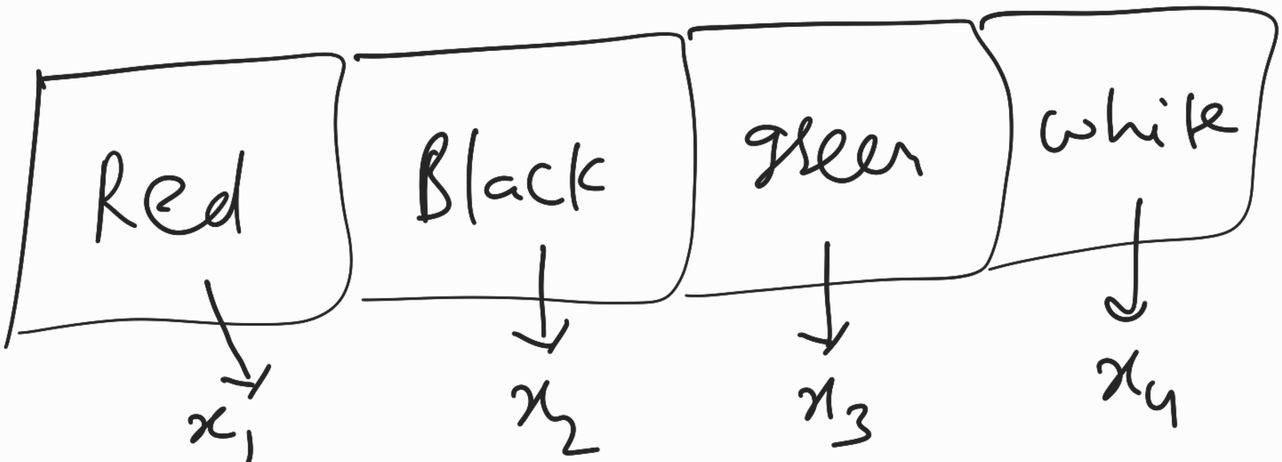
$$^{14+8-1}C_{8-1} = {}^{21}C_7$$

Ans

<u>Ex:-</u> There are unlimited balls of Red, Black, green, white.

(balls are identical except colour)

① Number of ways in which selection of 10 balls can be made



$$x_1 + x_2 + x_3 + x_4 = 10$$

$x_i \in W$ $\quad$ $\text{Ans} = {}^{10+4-1}C_{4-1}$

$$= {}^{13}C_3$$

(2) number of ways in which 10 balls can be selected contains all four diff^nt colours

$$x_1 + x_2 + x_3 + x_4 = 10$$

$\boxed{x_i \in N}$ $\quad$ ${}^{10-1}C_{4-1} = {}^{9}C_3$

X 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 X

$x_1+x_2+x_3+x_4=10$ $\quad x_i \in W$

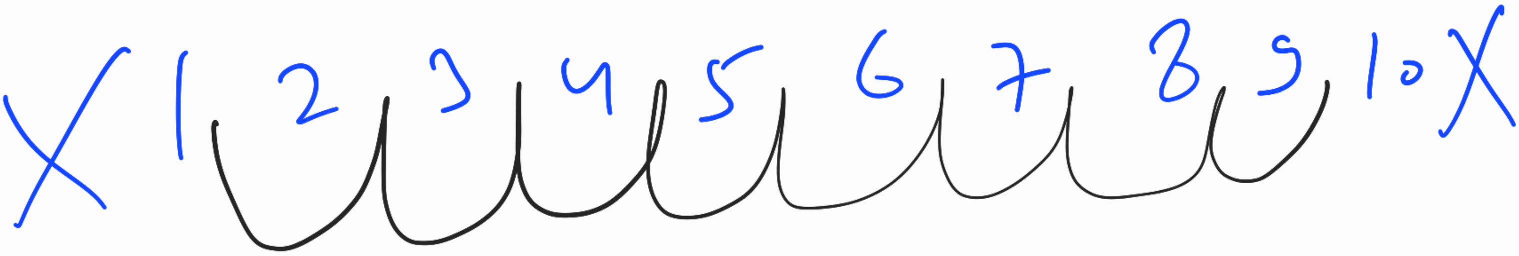
$$\underbrace{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10}_{10}\ \underbrace{|\ |\ |}_{3}$$

$$\text{Arrange} = \frac{13!}{10!\,3!} = {}^{13}C_3$$

$$\underbrace{\qquad}_{x_1}\ \Big|\ \underbrace{\qquad}_{x_2}\ \Big|\ \underbrace{\qquad}_{x_3}\ \Big|\ \underbrace{\qquad}_{x_4}$$

$x_1+x_2+x_3+x_4=10$

$x_i \in N$



3 out of 9 places $= {}^{9}C_{3}$

Ex:- $x+y+z=102$

$x, y, z \in N$

Sol^n $= {}^{102-1}C_{3-1} = {}^{101}C_{2}$

Ex:- $x+y+z \leq 30$

$x, y, z \in W$

Sol^n $x+y+z=0 \Rightarrow 1$

$x+y+z=1 \Rightarrow {}^{1+3-1}C_{3-1} = {}^{3}C_{1}$

$\vdots$

$x+y+z=30 \Rightarrow {}^{30+3-1}C_{3-1} = {}^{32}C_{2}$

$x+y+z \le 30$ | $x, y, z \in W$

$x+y+z+t=30$

Dummy

$t \in W$

$^{30+4-1}C_{4-1} = {}^{33}C_3$

$\boxed{P \& C} \Rightarrow$ O1, JM