Allen Career Institute
Kota

3-D Geometry
(Solutions)

For Leader Batch 2020-21

Do your self ①

(i) Find the distance between the points P(3,4,5) & Q(−1,2,−3).

Sol^n

P ———————— Q

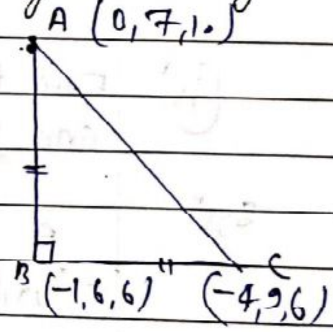
(3,4,5) (−1,2,−3)

Distance B/w PQ $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$= \sqrt{(-1-3)^2+(2-4)^2+(-3-5)^2}$

$= \sqrt{84} \Rightarrow \boxed{2\sqrt{21}}$

(ii) Show that the points A(0,7,10) B(−1,6,6) C(−4,9,6) are vertices of an isosceles Right Angled triangle.



Sol^n We Have to prove AB = BC

& ∠ABC = 90°

in △ABC

$AB = \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}$

$= \sqrt{1+1+16} \Rightarrow \sqrt{18}$

$BC = \sqrt{(-4+1)^2+(9-6)^2+(6-6)^2} \Rightarrow \sqrt{18}$

$AC = \sqrt{(-4-0)^2+(9-7)^2+(6-10)^2} \Rightarrow \sqrt{36}$

Now $AC^2 = BC^2 + AB^2$

$\boxed{36 = 18 + 18}$

Hence proved

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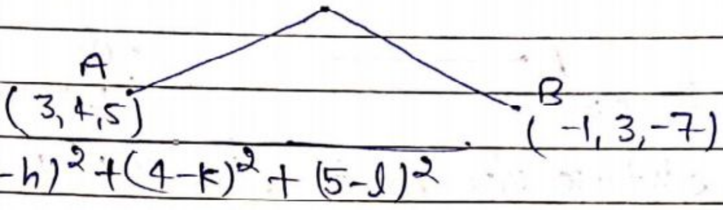
(iii) Find the locus of a point s.t. the difference of the square of its distance from the points A(3,4,5) B(−1,3,−7) is equal to $2k^2$.

Solⁿ let the point be $P(h,k,m)$



$$PA = \sqrt{(3-h)^2+(4-k)^2+(5-l)^2}$$

$$PB = \sqrt{(-1-h)^2+(3-k)^2+(-7-l)^2}$$

Now $\because$ $(PA)^2 - (PB)^2 = \pm 2k^2$

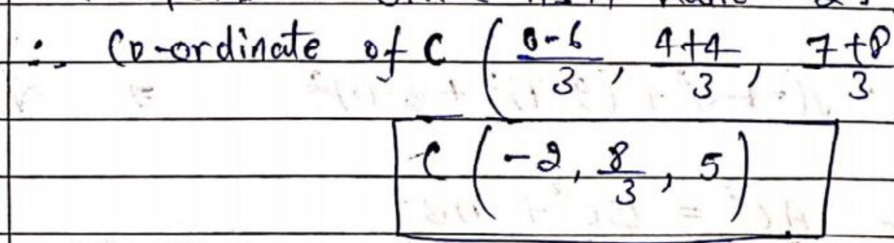
$\therefore (3-h)^2+(4-k)^2+(5-l)^2 - \left[(-1-h)^2+(3-k)^2+(-7-l)^2\right]$

$\Rightarrow -9-8h-2k-24l = \pm 2k^2$

locus be $\boxed{8x+2y+24z+9 \pm 2k^2=0}$

(iv) Find the co-ordinates of points which trisects the line joining the points A(−3,2,4) B(0,4,7).

Solⁿ



Here point C divide AB in Ratio 1 : 2

& point D divide AB in Ratio 2 : 1

$\therefore$ Co-ordinate of C $\left(\frac{0-6}{3}, \frac{4+4}{3}, \frac{7+8}{3}\right)$

$$\boxed{C\left(-2, \frac{8}{3}, 5\right)}$$

Co-ordinate of D $\left(\frac{0-3}{3}, \frac{8+2}{3}, \frac{14+4}{3}\right)$

$$\boxed{D\left(-1, \frac{10}{3}, 6\right)}$$

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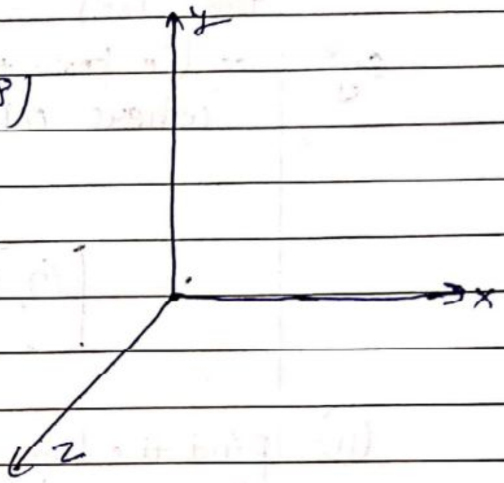
(v) Find the ratio in Which the planes (a) xy (b) yz divide the line Joining the points P(−2,4,7) and Q(3,−5,8).

Solⁿ

$$\underset{(-2,4,7)}{P} \xrightarrow{\lambda : 1} \underset{(3,-5,8)}{Q}$$



(A) in xy plane $z = 0$

coordinate of (x,y,z) be

$$\left(\frac{3\lambda-2}{\lambda+1}, \frac{-5\lambda+4}{\lambda+1}, \frac{8\lambda+7}{\lambda+1}\right)$$

$$\therefore \frac{8\lambda+7}{\lambda+1} = 0 \Rightarrow \boxed{\lambda = -\frac{7}{8}}$$

$\therefore$ 7 : 8 Externally

(B) in yz plane $x = 0$

$$\therefore \frac{3\lambda-2}{\lambda+1} = 0 \Rightarrow \boxed{\lambda = \frac{2}{3}}$$

$\therefore$ 2 : 3 Internally

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Do your self (2)

(i) Find the length of projections of the line segment Joining the origin O to the point $P(3, 2, -5)$ on the axes.

Solⁿ ∵ We know that projection of OP on x-axis $= |x|$ where O is origin $(0,0,0)$ is $= |3|$

on y-axis $= |2|$

on z-axis $= |-5|$

∴ $\boxed{(3, 2, 5)}$

(ii) Find the length of projections of the line Joining the points $P(3, 2, 5)$ and $Q(0, -2, 8)$ on the axes.

Solⁿ Projection's of line Joining $P(3, 2, 5)$ $Q(0, -2, 8)$

$|x_2 - x_1|$, $|y_2 - y_1|$, $|z_2 - z_1|$

$|0 - 3|$, $|-2 - 2|$, $|8 - 5|$

$\boxed{3, 4, 3}$

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(iii) Find the direction Ratios & direction Cosines of the line Joining the points $O(0,0,0)$ and $P(2,3,4)$.

Sol^n d.r.'s of OP are x_2-x_1, y_2-y_1, z_2-z_1

d.c.'s of OP are $\frac{1}{r}(x_2-x_1)$, $\frac{1}{r}(y_2-y_1)$, $\frac{1}{r}(z_2-z_1)$

Where $r = \sqrt{\Sigma (x_2-x_1)^2}$

$\therefore$ d.r.'s of op are $(2-0)$, $(3-0)$, $(4-0)$

$\boxed{2, 3, 4}$

$\therefore$ d.c.'s of OP are $\frac{1}{r}(2)$, $\frac{1}{r}(3)$, $\frac{1}{r}(4)$

Where $r = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$\sqrt{4+9+16}$

$\therefore r = \sqrt{29}$

$\therefore$ $\boxed{\frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}}$

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Do your self (3)

(i) Find the angle between the lines whose direction ratios are $1, -2, 1$ and $4, 3, 2$.

Sol^n ∵ θ be angle b/w two lines whose dr's are given is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{(1)(4)+(-2)(3)+(1)(2)}{\sqrt{1+4+1}\sqrt{16+9+4}} = 0$$

$$\therefore \boxed{\theta = \pi/2}$$

(ii) If a line makes α, β, γ angle with axes, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Sol^n

d.c's $= \cos\alpha, \cos\beta, \cos\gamma$

∵ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore 1-\sin^2\alpha + 1-\sin^2\beta + 1-\sin^2\gamma = 1$ $\left\{ \because \sin^2\theta + \cos^2\theta = 1,\ \therefore \cos^2\theta = 1-\sin^2\theta \right\}$

$3 - \sin^2\alpha - \sin^2\beta - \sin^2\gamma = 1$

$$\boxed{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2}$$

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(iii) Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to $(1,-2,-2)$ and $(0,2,1)$.

Solⁿ: let d.c.'s of line which is $\perp^r$ to line is (a,b,c)

$(a_1, b_1, c_1) \perp^r (a_2, b_2, c_2)$ then

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\therefore \quad a - 2b - 2c = 0 \quad \text{—— (1)}$$

$$\& \quad 0a + 2b + c = 0 \quad \text{—— (2)}$$

$$\frac{a}{-2+4} = \frac{-b}{1+0} = \frac{c}{2+0}$$

$$\frac{a}{2} = \frac{-b}{1} = \frac{c}{2}$$

d.r.'s is $(2,-1,2)$

$\therefore$ dc's is $\frac{2}{\sqrt{9}}, \frac{-1}{\sqrt{9}}, \frac{2}{\sqrt{9}}$

$$\boxed{\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}}$$

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Do your self (1)

(i) Eq^n of a plane is $3x + 4y + 5z = 7$

(a) Find the direction cosines of its Normal.
(b) Find the points where it intersects the axes.
(c) Find its Intercept form
(d) Find its eq^n in Normal form
(in Cartesian as well as in vector form).

Sol^n (a) $\because \vec{r} \cdot \vec{n} = d$

$$\vec{r} \cdot (3\hat{i} + 4\hat{j} + 5\hat{k}) = 7$$

$$|\vec{n}| = \sqrt{(3)^2 + (4)^2 + (5)^2} \Rightarrow \sqrt{50}$$

$$\hat{n} = \frac{3}{\sqrt{50}}\hat{i} + \frac{4}{\sqrt{50}}\hat{j} + \frac{5}{\sqrt{50}}\hat{k}$$

$\therefore$ d.c.'s (l, m, n) are $\boxed{\frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}}$

(b) Intercept at x-axis is $(\frac{7}{3}, 0, 0)$

______ y-axis — $(0, 7/4, 0)$

______ z-axis — $(0, 0, 7/5)$

(c) Eq^n of a plane in intercept form is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Given eq^n. in cartesian form $3x + 4y + 5z = 7$

$$\therefore \boxed{\frac{x}{7/3} + \frac{y}{7/4} + \frac{z}{7/5} = 1}$$

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(d) In Cartesian form $lx + my + nz = p$

$$\frac{3x}{5\sqrt{2}} + \frac{4y}{5\sqrt{2}} + \frac{5z}{5\sqrt{2}} = \frac{7}{5\sqrt{2}}$$

in vector form $\vec{r} \cdot \hat{n} = d$

$$\vec{r} \cdot \left[\frac{3}{5\sqrt{2}}\hat{i} + \frac{4}{5\sqrt{2}}\hat{j} + \frac{5}{5\sqrt{2}}\hat{k}\right] = \frac{7}{5\sqrt{2}}$$

from sol^n (P) (a)

(ii) Find the eq^n of a plane passing through the points $(2,3,1)$, $(3,0,2)$, $(-1,2,3)$.

Sol^n The General eq^n of a plane passing through $(2,3,1)$ is

$$a(x-2) + b(y-3) + c(z-1) = 0 \quad \text{— (1)}$$

It Will pass through $(3,0,2)$ & $(-1,2,3)$ if

$$a(3-2) + b(0-3) + c(2-1) = 0$$

$$a - 3b + c = 0 \quad \text{— (2)}$$

& $a(-1-2) + b(2-3) + c(3-1) = 0$

$$-3a - b + 2c = 0 \quad \text{— (3)}$$

from (2) & (3) $\dfrac{a}{-6+1} = \dfrac{-b}{2+3} = \dfrac{c}{-1-9}$

$$\frac{a}{-5} = \frac{b}{-5} = \frac{c}{-10} = \lambda \text{ say}$$

$\therefore a = -5\lambda, \quad b = -5\lambda, \quad c = -10\lambda$

from (1) $-5\lambda(x-2) - 5\lambda(y-3) - 10\lambda(z-1) = 0$

$$-5x + 10 - 5y + 15 - 10z + 10 = 0$$

$$5x + 5y + 10z - 35 = 0$$

$$\boxed{x + y + 2z = 7}$$

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Do your self (5)

(i) prove that the planes $3x-2y+z+17=0$ & $4x+3y-6z-25=0$ are perpendicular.

Solⁿ Two planes are perpendicular if

$$aa'+bb'+cc'=0 \quad \text{—— (1)}$$

Here $a=3$, $b=-2$, $c=1$, $d=17$

$a'=4$, $b'=3$, $c'=-6$, $d'=-25$

$\therefore$ from (1) $(3)(4)+(-2)(3)+(1)(-6)$

$12-6-6$

$\boxed{0}$

Hence planes $3x-2y+z+17=0$ & $4x+3y-6z-25=0$ are perpendicular.

(ii) Find the angle b/w the planes $3x+4y+z+7=0$ and $-x+y-2z-5=0$

Solⁿ $\because$ We know that the angle b/w the planes are given by

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{(3)(-1)+(4)(1)+(1)(-2)}{\sqrt{9+16+1}\sqrt{1+1+4}}$$

$$\cos\theta = \frac{-1}{\sqrt{26}\sqrt{6}} \Rightarrow \frac{-1}{\sqrt{156}}$$

$$\boxed{\theta = \cos^{-1}\frac{1}{\sqrt{156}}}$$

Do yourself - 6 :

(i) Find the perpendicular distance of the point P(1, 2, 3) from the plane 2x + y + z + 1 = 0.

(ii) Find the equation of the plane passing through the line of intersection of the planes x + y + z = 5 and 2x + 3y + z + 5 = 0 and passing through the point (0, 0, 0).

DY-6

(1)

$$p = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{(a^2 + b^2 + c^2)}}$$

Perpendicular distance $= \left| \frac{2 \cdot 1 + 2 + 3 + 1}{\sqrt{2^2 + 1^2 + 1^2}} \right|$

$= \boxed{\frac{8}{\sqrt{6}}}$

(2) A plane passing through point of intersection of planes $x + y + z = 5$ an $2x + 3y + z + 5 = 0$ can be written as

$$x + y + z - 5 + \lambda(2x + 3y + z + 5) = 0$$

$$(P_1 + \lambda P_2) = 0$$

Passes through (0, 0, 0) so

$$-5 + \lambda(5) = 0$$

$$\boxed{\lambda = 1}$$

eq^n of plane is

$$x + y + z - 5 + (2x + 3y + z + 5) = 0$$

$$\boxed{3x + 4y + 2z = 0}$$

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Do yourself - 7 :

(i) Find the position of the point P(2, –2, 1), Q(3, 0, 1) and R(–12, 1, 8) w.r.t. the plane $2x - 3y + 4z - 7 = 0$.

(ii) Two given planes are $-2x + y - 2z + 5 = 0$ and $6x - 2y + 3z - 7 = 0$. Find

(a) equation of plane bisecting the angle between the planes.

(b) equation of a plane parallel to the plane bisecting the angle between both the two planes and passing through the point (3, 2, 0).

(c) specify which plane is acute angle bisector and which one is obtuse angle bisector.

DY-7

(i) $2x - 3y + 4z - 7 = 0$

for P(2, –2, 1) ⇒ $4 + 6 + 4 - 7 = 7 > 0$

for Q(3, 0, 1) ⇒ $6 - 0 + 4 - 7 = 3 > 0$

for R(–12, 1, 8) ⇒ $-24 - 3 + 32 - 7 = -2 < 0$

so P, Q are same side and R is opposite side

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ii)(a) eqn of given planes

$-2x + y - 2z + 5 = 0$

and $-6x + 2y - 3z + 7 = 0$

eqn of planes Bisecting the angle b/w the given planes is

$$\frac{-2x + y - 2z + 5}{\sqrt{2^2 + 1^2 + 2^2}} = \pm\left(\frac{-6x + 2y - 3z + 7}{\sqrt{6^2 + 2^2 + 3^2}}\right)$$

$$\frac{-2x + y - 2z + 5}{3} = \pm\left(\frac{-6x + 2y - 3z + 7}{7}\right)$$

(+) (–)

$\boxed{4x + y - 5z + 14 = 0}$ and $\boxed{32x - 13y + 23z - 56 = 0}$

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(b) eq^n of Parallel Planes can be written as

$$4x + y - 5z + \lambda = 0$$

and $\quad 32x - 13y + 23z + K = 0$

Passes through $(3, 2, 0)$

$$12 + 2 - 0 + \lambda = 0$$
$$\lambda = -14$$

$$96 - 26 + K = 0$$
$$K = -70$$

so eq^n of Planes are

$$\boxed{4x + y - 5z - 14 = 0}$$

and $\boxed{32x - 13y + 23z - 70 = 0}$

(c) Planes are
$$\left.\begin{array}{l} -2x + y - 2z + 5 = 0 \\ -6x + 2y - 3z + 7 = 0 \end{array}\right\} d_1, d_2 > 0$$

value of $a_1a_2 + b_1b_2 + c_1c_2 = 12 + 2 + 6 = 20 > 0$

(+)ve sign gives obtuse angle Bisecting plane

which is $\boxed{4x + y - 5z + 14 = 0}$

(−)ve sign gives acute angle Bisecting plane

which is $\boxed{32x - 13y + 23z - 56 = 0}$

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Do yourself - 8 :

(i) Find the equation of the line passing through the point (4, 2, 3) and having direction ratios 1, −1, 2

(ii) Find the symmetrical form of the line $x - y + 2z = 5$, $3x + y + z = 6$.

(iii) Find the angle between the plane $3x + 4y + 5 = 0$ and the line $\frac{x-1}{2} = \frac{y-2}{0} = \frac{z-1}{1}$.

(iv) Prove that the line $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ is parallel to the plane $4x + 4y - 5z + 2 = 0$.

DY-8

(i) eq^n of line will be

$$\boxed{\frac{x-4}{1} = \frac{y-2}{-1} = \frac{z-3}{2}}$$

ii) Let d.r. of lines are (a, b, c)

then $a - b + 2c = 0$

and $3a + b + c = 0$

$$\frac{a}{\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix}}$$

$$\frac{a}{-1-2} = \frac{b}{6-1} = \frac{c}{1+3}$$

$$\frac{a}{-3} = \frac{b}{5} = \frac{c}{4}$$

$z = 0 \Rightarrow x - y = 5$

$3x + y = 6$

$x = \frac{11}{4}, \quad y = -\frac{9}{4}$

line is

$$\boxed{\frac{x - 11/4}{-3} = \frac{y + 9/4}{5} = \frac{z - 0}{4}}$$

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iii)

$$\sin\theta = \frac{al + bm + cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$$

$$= \frac{3\times 2 + 4\times 0 + 0\times 1}{\sqrt{3^2+4^2+0^2}\sqrt{2^2+0^2+1^2}}$$

$$\sin\theta = \frac{6}{5\sqrt{5}}$$

$$\boxed{\theta = \sin^{-1}\left(\frac{6}{5\sqrt{5}}\right)}$$

(iv) value of $al + bm + cn$

$$= 4(2) + 4(3) + (-5)4$$

$$= 0$$

$$\sin\theta = 0$$

$$\theta = 0^\circ$$

so line and plane are parallel.

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Do yourself - 9 :

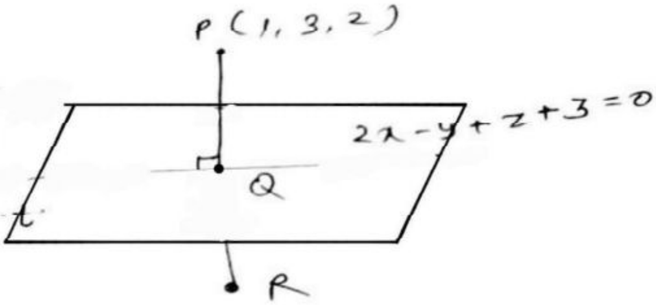
(i) Find the image of point P(1, 3, 2) in the plane $2x - y + z + 3 = 0$ as well as the foot of the perpendicular drawn from the point (1, 3, 2).

(ii) Find the distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$

(iii) Prove that $\frac{x+1}{-2} = \frac{y+2}{3} = \frac{z+5}{4}$ lies in the plane $x + 2y - z = 0$.

Dy (9)

(i) Let Q, R are foot of Perpendicular and Image of P w r t the given plane.



Line is Parallel to normal of the plane

so d.r. of line will be (2, -1, 1)

eq^n of line $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-2}{1} = r$

coordinates of Q $\equiv (2r+1, -r+3, r+2)$

which lies on the plane

$$2(2r+1) - (-r+3) + r + 2 + 3 = 0$$

$$6r = -4$$

$$r = -\frac{2}{3}$$

point $\boxed{Q \equiv \left(-\frac{1}{3}, \frac{11}{3}, \frac{4}{3}\right)}$

point $\boxed{R \equiv \left(-\frac{5}{3}, \frac{13}{3}, \frac{2}{3}\right)}$ (By mid point theorem)

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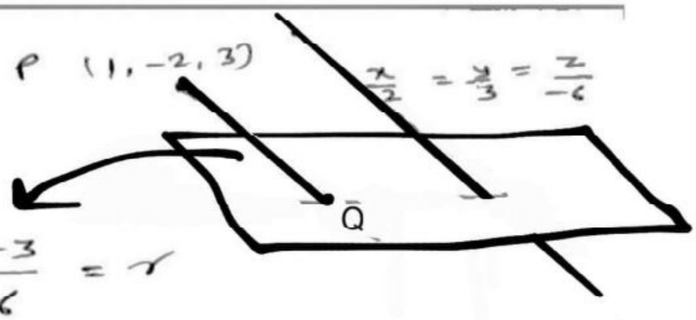
(ii)

eq^n of PQ



$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r$$

coordinates of $Q \equiv (2r+1,\ 3r-2,\ -6r+3)$
which lies on the given plane.

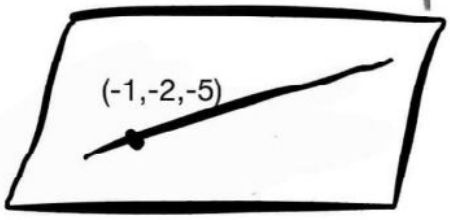
$$x - y + z = 5$$
$$2r+1 - 3r+2 - 6r+3 = 5$$
$$-7r = -1$$
$$r = 1/7$$

Point $Q = \left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$

$$PQ = \sqrt{(2r)^2 + 9r^2 + (-6r)^2}$$
$$= 7|r|$$

$$\boxed{PQ = 1}$$

(iii)



$al + bm + cn$
$= -2\times1 + 3\times2 + 4(-1)$
$= 0$

so line is parallel to the plane

$$x + 2y - z = 0$$
$$-1 + 2(-2) + 5 = 0$$

Point $(-1,-2,-5)$ lies on the plane.
so the given line lies in the plane

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Do yourself - 10 :

(i) Prove that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ are coplanar. Find their point of intersection.

(ii) Find the area of the triangle whose vertices are the points (1, 2, 3), (–2, 1, –4), (3, 4, –2).

DY 10

(i)

$(-1,-3,-5)\ ;\ (2,4,6)$

$$\begin{vmatrix} -1-2 & -3-4 & -5-6 \\ 3 & 5 & 7 \\ 1 & 3 & 5 \end{vmatrix}$$

$$-3(25-21)+7(15-7)-11(9-5)$$

$$-12+56-44=0$$

so lines are coplanar.

$$\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}=r_1$$

Point on the line $(3r_1-1,\ 5r_1-3,\ 7r_1-5)$

$$\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}=r_2$$

Point on the line $(r_2+2,\ 3r_2+4,\ 5r_2+6)$

for Point of intersection

$3r_1-1=r_2+2$ and $5r_1-3=3r_2+4$

$r_2=3r_1-3$ $\qquad 5r_1-7=3(3r_1-3)$

$r_1=1/2$

so Point of intersection $\equiv \boxed{\left(-\frac{1}{2},\ \frac{-1}{2},\ \frac{-3}{2}\right)}$

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(ii) Find the area of the triangle whose vertices are the points (1, 2, 3), (–2, 1, –4), (3, 4, –2).

Soln

$A(1,2,3)$; $B(-2,1,-4)$; $C(3,4,-2)$

$\vec{AB} = -3\hat{i} - \hat{j} - 7\hat{k}$; $\vec{AC} = 2\hat{i} + 2\hat{j} - 5\hat{k}$

$$\Delta = \frac{1}{2}\left|\vec{AB} \times \vec{AC}\right|$$

$$\Rightarrow \Delta = \frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -1 & -7 \\ 2 & 2 & -5 \end{vmatrix}\right|$$

$$= \frac{1}{2}\left|19\hat{i} - 29\hat{j} - 4\hat{k}\right|$$

$$= \frac{1}{2}\sqrt{19^2 + 29^2 + 4^2}$$

$$= \frac{1}{2}\sqrt{1218}$$ Sq. units (Ans)

Exercise 01

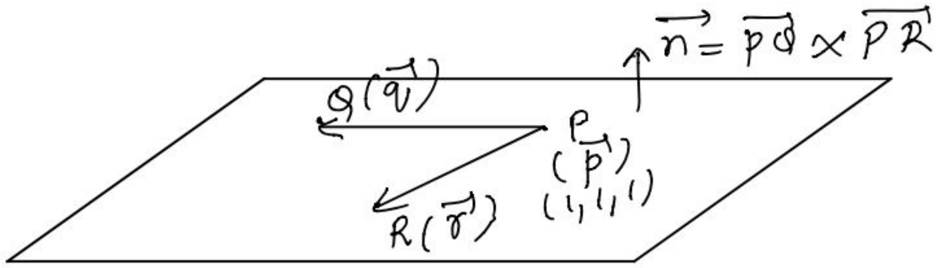
EXERCISE (O-1)

[STRAIGHT OBJECTIVE TYPE]

1. Consider three vectors $\vec{p}=\hat{i}+\hat{j}+\hat{k}$, $\vec{q}=2\hat{i}+4\hat{j}-\hat{k}$ and $\vec{r}=\hat{i}+\hat{j}+3\hat{k}$. If $\vec{p}$, $\vec{q}$ and $\vec{r}$ denotes the position vector of three non-collinear points then the equation of the plane containing these points is

(A) $2x - 3y + 1 = 0$ (B) $x - 3y + 2z = 0$ (C) $3x - y + z - 3 = 0$ (D) $3x - y - 2 = 0$

Solⁿ



$$\vec{PQ} = \vec{q} - \vec{p} = \hat{i} + 3\hat{j} - 2\hat{k}$$
$$\vec{PR} = \vec{r} - \vec{p} = 2\hat{k}$$

$$\vec{n} = \vec{PQ} \times \vec{PR} = 6\hat{i} - 2\hat{j} = 2(3\hat{i} - \hat{j})$$

$\therefore$ Eqⁿ. of plane :

$$3(x-1) - 1(y-1) + 0(z-1) = 0$$

$$\Rightarrow \quad 3x - y - 2 = 0 \quad (\text{Ans. D})$$

2

If the distance between the planes

$$8x + 12y - 14z = 2$$

and

$$4x + 6y - 7z = 2$$

can be expressed in the form $\frac{1}{\sqrt{N}}$ where N is natural then the value of $\frac{N(N+1)}{2}$ is

(A) 4950 (B) 5050 (C) 5150 (D) 5151

Sol

$P_1 : \quad 4x + 6y - 7z = 1$

$P_2 : \quad 4x + 6y - 7z = 2$

$$\text{Distance} = \left| \frac{2-1}{\sqrt{16+36+49}} \right| = \frac{1}{\sqrt{101}}$$

$$\therefore N = 101 \Rightarrow \frac{N(N+1)}{2} = 5151 \text{ (Ans D)}$$

3

A plane passes through the point P(4, 0, 0) and Q(0, 0, 4) and is parallel to the y-axis. The distance of the plane from the origin is

(A) 2 (B) 4 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

Soln

Eqn. of plane parallel to y axis: $ax+bz+1=0$

Pass thru (4,0,0) : $4a+1=0 \Rightarrow a=-1/4$

Pass thru (0,0,4) : $4b+1=0 \Rightarrow b=-1/4$

$\therefore$ Eqn. of plane: $-\frac{1}{4}x-\frac{1}{4}z+1=0$

$\Rightarrow x+z-4=0$

Distane from origin $=\left|\frac{-4}{\sqrt{1+1}}\right|=\frac{4}{\sqrt{2}}$

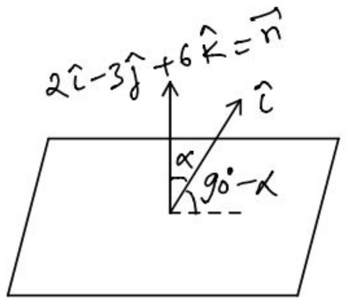
$=2\sqrt{2}$ units.

(Ans D)

4 If the plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(k)$ with x-axis, then k is equal to

(A) $\sqrt{3}/2$ (B) 2/7 (C) $\sqrt{2}/3$ (D) 1

Solⁿ



$$90^\circ - \alpha = \sin^{-1} K$$
$$\Rightarrow \alpha = \cos^{-1} K$$
$$\Rightarrow \cos\alpha = K$$

Now $$\cos\alpha = \left| \frac{\hat{i}\cdot(2\hat{i}-3\hat{j}+6\hat{k})}{1\cdot\sqrt{4+9+36}} \right|$$

$$\Rightarrow \cos\alpha = \frac{2}{7} = K \quad (\text{Ans } B)$$

5. The plane XOZ divides the join of (1, –1, 5) and (2, 3, 4) in the ratio $\lambda : 1$, then λ is

(A) – 3 (B) – 1/3 (C) 3 (D) 1/3

Sol^n

A $\quad \lambda : 1 \quad$ P $\quad$ B

(1, −1, 5) $\qquad$ (2, 3, 4)

$$\left(\frac{2\lambda+1}{\lambda+1}, \frac{3\lambda-1}{\lambda+1}, \frac{4\lambda+5}{\lambda+1} \right) \qquad \lambda+1$$

$\because$ P lies on xoz plane

$\therefore$ y coordinate $= 0$

$$\Rightarrow \frac{3\lambda-1}{\lambda+1} = 0 \Rightarrow \lambda = 1/3 \quad \underline{\text{Ans}}\ D$$

6

Equation of the line which passes through the point with p. v. (2, 1, 0) and perpendicular to the plane containing the vectors $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is

(A) $\vec{r} = (2, 1, 0) + t(1, -1, 1)$

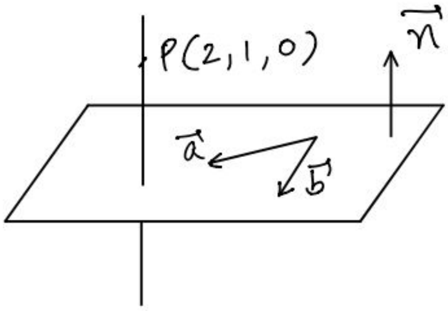
(B) $\vec{r} = (2, 1, 0) + t(-1, 1, 1)$

(C) $\vec{r} = (2, 1, 0) + t(1, 1, -1)$

(D) $\vec{r} = (2, 1, 0) + t(1, 1, 1)$

where t is a parameter

Sol^n



$\vec{n} = \vec{a}\times\vec{b}$

$\vec{a} = \hat{i}+\hat{j}$

$\vec{b} = \hat{j}+\hat{k}$

Vector ⊥ to given plane:

$$\vec{n} = (\hat{i}+\hat{j})\times(\hat{j}\times\hat{k}) = \hat{k}-\hat{j}+\hat{i}$$

$$\Rightarrow \vec{n} = \hat{i}-\hat{j}+\hat{k}$$

∴ Eq^n of required line:

$$\vec{r} = 2\hat{i}+\hat{j} + \lambda(\hat{i}-\hat{j}+\hat{k}),\ \lambda \in R$$

(Ans A)

7 Which of the following planes are parallel but not identical?

$P_1 : 4x - 2y + 6z = 3$

$P_2 : 4x - 2y - 2z = 6$

$P_3 : -6x + 3y - 9z = 5$

$P_4 : 2x - y - z = 3$

(A) P_2 & P_3 (B) P_2 & P_4 (C) P_1 & P_3 (D) P_1 & P_4

Soln

$P_1 : 2x - y + 3z = 3/2$

$P_2 : 2x - y - z = 3$

$P_3 : 2x - y + 3z = -5/3$

$P_4 : 2x - y - z = 3$

$\therefore$ P_1 & P_3 are parallel distinct planes.

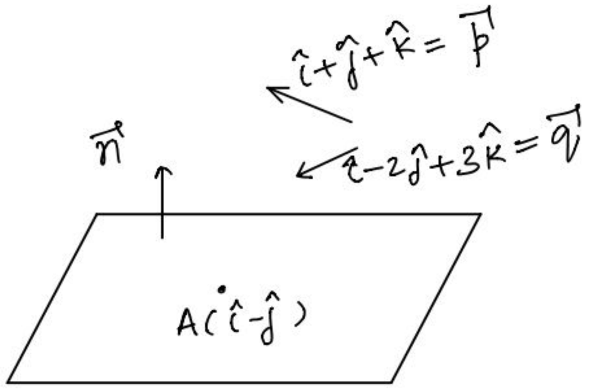
(Ans C)

8. Vector equation of the plane $\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$ in the scalar dot product form is

(A) $\vec{r}.(5\hat{i} - 2\hat{j} + 3\hat{k}) = 7$ (B) $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 7$

(C) $\vec{r}.(5\hat{i} - 2\hat{j} - 3\hat{k}) = 7$ (D) $\vec{r}.(5\hat{i} + 2\hat{j} + 3\hat{k}) = 7$

Solⁿ



$$\vec{n} = \vec{p} \times \vec{q}$$
$$= (\hat{i}+\hat{j}+\hat{k}) \times (\hat{i}-2\hat{j}+3\hat{k})$$

$$\Rightarrow \vec{n} = 5\hat{i} - 2\hat{j} - 3\hat{k}$$

$\therefore$ Eqⁿ. of plane :

$$(\vec{r} - (\hat{i}-\hat{j}))\cdot(5\hat{i}-2\hat{j}-3\hat{k}) = 0$$

$$\Rightarrow \vec{r}\cdot(5\hat{i}-2\hat{j}-3\hat{k}) - (5+2) = 0$$

$$\Rightarrow \vec{r}\cdot(5\hat{i}-2\hat{j}-3\hat{k}) = 7$$ (Ans C)

9. The vector equations of the two lines L_1 and L_2 are given by

$L_1 : \vec{r} = 2\hat{i}+9\hat{j}+13\hat{k}+\lambda(\hat{i}+2\hat{j}+3\hat{k})$; $L_2 : \vec{r} = -3\hat{i}+7\hat{j}+p\hat{k}+\mu(-\hat{i}+2\hat{j}-3\hat{k})$

then the lines L_1 and L_2 are

(A) skew lines for all $p \in R$

(B) intersecting for all $p \in R$ and the point of intersection is $(-1, 3, 4)$

(C) intersecting lines for $p = -2$

(D) intersecting for all real $p \in R$

Sol^n M1 $L_1: r = 2\hat{i}+9\hat{j}+13\hat{k}+\lambda(\hat{i}+2\hat{j}+3\hat{k})$

Point $A \equiv (2+\lambda, 9+2\lambda, 13+3\lambda)$ lie on L_1

$L_2: r = -3\hat{i}+7\hat{j}+p\hat{k}+\mu(-\hat{i}+2\hat{j}-3\hat{k})$

Point $B \equiv (-3-\mu, 7+2\mu, p-3\mu)$ lie on L_2

Now Point A and Point B are same (if possible)

solve $2+\lambda = -3-\mu$ & $9+2\lambda = 7+2\mu \Rightarrow \mu = -2, \lambda = -3$

Satisfy $13+3\lambda = p-3\mu \Rightarrow \boxed{p=-2}$

$\Rightarrow$ Point of intersection of lines $\equiv (2+\lambda, 9+2\lambda, 13+3\lambda)$
$= (-1, 3, 4)$

or

M2 $\because$ L_1 & L_2 are not parallel $\Rightarrow$ if lines are coplanar (lines will intersect)

lines coplanar if $\begin{vmatrix} 5 & 2 & 13-p \\ 1 & 2 & 3 \\ -1 & 2 & -3 \end{vmatrix} = 0 \Rightarrow \boxed{p=-2}$

$\Rightarrow$ line intersect if $\boxed{p=-2}$

10 Consider the plane $(x, y, z) = (0, 1, 1) + \lambda(1, -1, 1) + \mu(2, -1, 0)$. The distance of this plane from the origin is

(A) 1/3 (B) $\sqrt{3}/2$ (C) $\sqrt{3/2}$ (D) $2/\sqrt{3}$

Soln $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a} = 0\hat{i} + \hat{j} + \hat{k}$

$\vec{b} = \hat{i} - \hat{j} + \hat{k}$

$\vec{c} = 2\hat{i} - \hat{j} + 0\hat{k}$

Eqn of plane given by $[\vec{r}\ \vec{b}\ \vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]$

$$\begin{vmatrix} x & y & z \\ 1 & -1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & -1 & 0 \end{vmatrix}$$

$$x(0+1) - y(0-2) + z(-1+2) = 0 - 1(0-2) + 1(-1+2)$$

$$x + 2y + z = 3 \text{ —— } (1)$$

$\perp^{lar}$ distance from $(0,0,0)$ to (1) $= \left| \dfrac{-3}{\sqrt{1+4+1}} \right|$

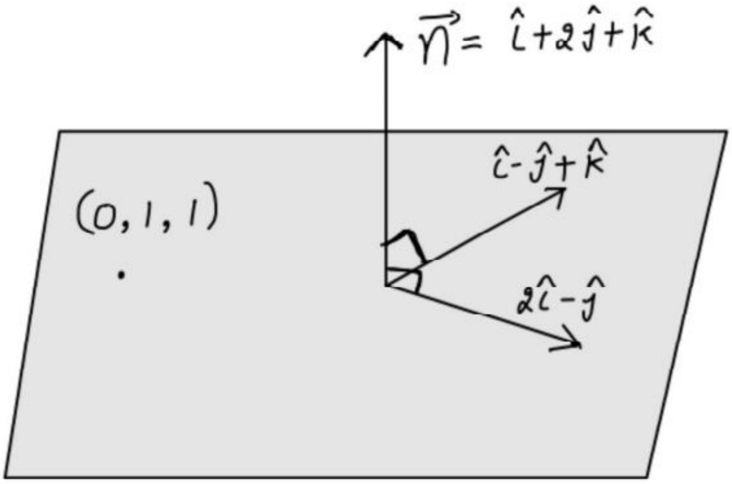
$$= \sqrt{\frac{3}{2}}$$

(10) Continues...

Method II.

$$(x, y, z) = (0, 1, 1) + \lambda(1, -1, 1) + \mu(2, -1, 0)$$

parametric form of plane



$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & -1 & 0 \end{vmatrix}$$

$$= \hat{i}+2\hat{j}+\hat{k}$$

$\therefore$ eq^n of plane $\Rightarrow$ $x + 2y + z = d$

$$\left\{ \begin{array}{l} \text{pass through } (0, 1, 1) \\ \therefore\ 0+2+1 = d \\ \Rightarrow\ d = 3 \end{array} \right\}$$

Plane $\Rightarrow$ $\boxed{x + 2y + z = 3}$

$$\therefore \text{ Distance from Origin} = \frac{|-3|}{\sqrt{1+4+1}} = \boxed{\sqrt{\frac{3}{2}}}$$

11

The value of 'a' for which the lines $\frac{x-2}{1}=\frac{y-9}{2}=\frac{z-13}{3}$ and $\frac{x-a}{-1}=\frac{y-7}{2}=\frac{z+2}{-3}$ intersect, is

(A) – 5 (B) – 2 (C) 5 (D) – 3

Soln

Let the two lines be

$$\frac{x-\alpha_1}{\ell_1}=\frac{y-\beta_1}{m_1}=\frac{z-\gamma_1}{n_1} \quad \text{.......... (i)}$$

and $$\frac{x-\alpha_2}{\ell_2}=\frac{y-\beta_2}{m_2}=\frac{z-\gamma_2}{n_2} \quad \text{.......... (ii)}$$

These lines will coplanar if $\begin{vmatrix} \alpha_2-\alpha_1 & \beta_2-\beta_1 & \gamma_2-\gamma_1 \\ \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \end{vmatrix}=0$ (It is condition for intersection of two lines)

$$\Rightarrow \begin{vmatrix} 2-a & 9-7 & 13+2 \\ 1 & 2 & 3 \\ -1 & 2 & -3 \end{vmatrix}=0$$

$$(2-a)(-6-6)-2(-3+3)+15(2+2)=0$$

$$-24+12a+60=0 \Rightarrow \boxed{a=-3}$$

12

Given A (1, –1, 0) ; B(3, 1, 2) ; C(2, –2, 4) and D(–1, 1, –1) which of the following points neither lie on AB nor on CD?

(A) (2, 2, 4) (B) (2, –2, 4) (C) (2, 0,1) (D) (0, –2, –1)

Solⁿ Eqⁿ of line through points (x_1, y_1, z_1) & $(x_2\ y_2\ z_2)$ is given by

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

⇒ Eqⁿ of Line AB $\frac{x-1}{3-1} = \frac{y+1}{1-(-1)} = \frac{z-0}{2-0}$

$$\Rightarrow \quad x-1 = y+1 = z \quad \text{——} \ (1)$$

Eqⁿ of line CD

$$\frac{x-2}{2-(-1)} = \frac{y+2}{-2-1} = \frac{z-4}{4-(-1)}$$

$$\Rightarrow \quad \frac{x-2}{3} = \frac{y+2}{-3} = \frac{z-4}{5} \quad \text{——} \ (2)$$

Clearly (2, 2, 4) neither lie on Eq (1) nor on (2)

(13)

For the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$, which one of the following is incorrect?

(A) it lies in the plane $x-2y+z=0$

(B) it is same as line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

✓(C) it passes through (2, 3, 5)

(D) it is parallel to the plane $x-2y+z-6=0$

Soln

$$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}=\lambda \quad \text{--- (1)}$$

General Point on line (1) $\equiv (1+\lambda,\ 2+2\lambda,\ 3+3\lambda)$

(A) above Point lies in Plane $x-2y+z=0$

$\Rightarrow$ line (1) lies in plane $x-2y+z=0$

(B)

$$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$$

$$\Rightarrow \frac{x}{1}-1=\frac{y}{2}-1=\frac{z}{3}-1 \Rightarrow \frac{x}{1}=\frac{y}{2}=\frac{z}{3}$$

(C) Point (2, 3, 5) does not lie on (1)

(D) Dr's of line $<1,2,3> = <l_1, m_1, n_1>$ (say)

and Dr's of normal of plane $x-2y+z-6=0$

$<1,-2,1> = <l_2, m_2, n_2>$

$l_1l_2+m_1m_2+n_1n_2 = 1-4+3=0$

$\Rightarrow$ line is parallel to plane $x-2y+z-6=0$

14

Given planes $P_1 : cy + bz = x \longrightarrow$ $x - cy - bz = 0$

$P_2 : az + cx = y$ $\quad cx - y + az = 0$

$P_3 : bx + ay = z$ $\quad bx + ay - z = 0$

P_1, P_2 and P_3 pass through one line, if

(A) $a^2 + b^2 + c^2 = ab + bc + ca$

(B) $a^2 + b^2 + c^2 + 2abc = 1$

(C) $a^2 + b^2 + c^2 = 1$

(D) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca + 2abc = 1$

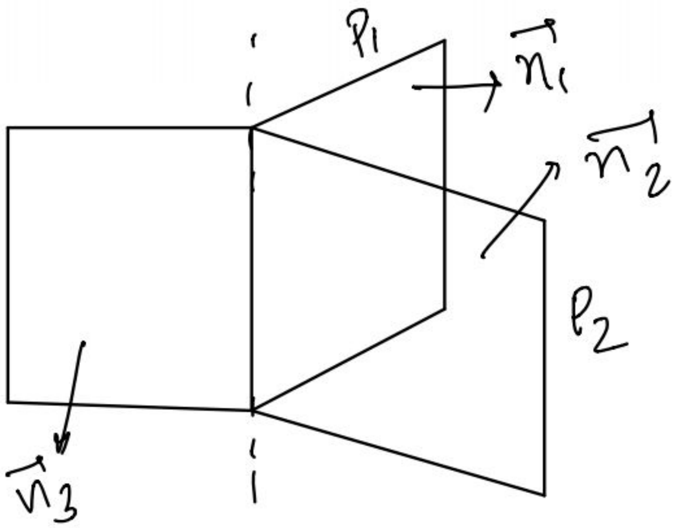
Soln P_1, P_2 and P_3 pass through one line, if they have infinite solution. or {Normals of P_1, P_2, P_3 are coplanar}

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$-1(1-a^2) - c(-c-ab) + b(ac+b) = 0$$

$$\boxed{a^2+b^2+c^2+2abc=1}$$



15

The line $\frac{x-x_1}{0}=\frac{y-y_1}{1}=\frac{z-z_1}{2}$ is

(A) parallel to x-axis
✓(B) perpendicular to x-axis
(C) perpendicular to YOZ plane
(D) parallel to y-axis

Sol^n

$$\frac{x-x_1}{0} = \frac{y-y_1}{1} = \frac{z-z_1}{2}$$

Dr/s of line $\langle 0\ 1\ 2 \rangle = \langle l_1\ m_1\ n_1 \rangle$

DCs of x axis $\langle 1\ 0\ 0 \rangle \equiv \langle l_2\ m_2\ n_2 \rangle$

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$\Rightarrow$ line is $\perp^{lar}$ to x axis

16

The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

(A) $k = 0$ or -1 (B) $k = 1$ or -1 (C) $k = 0$ or -3 (D) $k = 3$ or -3

Soln $(2, 3, 4)$; $(1, 4, 5)$

lines are coplanar if $\begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

$1(1+2k)+1(1+k^2)-1(2-k)=0$

$k^2+3k=0$

$\Rightarrow \boxed{k=0 \text{ or } k=-3}$

17

The line which contains all points (x, y, z) which are of the form $(x, y, z) = (2, -2, 5) + \lambda(1, -3, 2)$ intersects the plane $2x - 3y + 4z = 163$ at P and intersects the YZ plane at Q. If the distance PQ is $a\sqrt{b}$ where $a, b \in N$ and $a > 3$ then $(a + b)$ equals

(A) 23 (B) 95 (C) 27 (D) none

Soln $\because$ line intersect the plane at Point

$P \equiv (2+\lambda, -2-3\lambda, 5+2\lambda)$ (P is General point on line)

$\Rightarrow$ Point P lie on Plane $2x-3y+4z=163$

$2(2+\lambda) - 3(-2-3\lambda) + 4(5+2\lambda) = 163$

$19\lambda = 133 \Rightarrow \lambda = 7$

$\Rightarrow P \equiv (9, -23, 19)$

$\because$ line intersect the yz plane at Q

$\Rightarrow Q \equiv (2+\lambda, -2-3\lambda, 5+2\lambda)$

$2+\lambda = 0 \Rightarrow \boxed{\lambda = -2}$ ($\because$ in yz plane $x=0$)

$\Rightarrow Q \equiv (0, 4, 1)$

$\Rightarrow PQ = \sqrt{(9-0)^2 + (-23-4)^2 + (19-1)^2} = \sqrt{1134}$

$= 9\sqrt{14} = a\sqrt{b}$

$\Rightarrow a = 9, b = 14$

$\boxed{a+b = 23}$

18

Let L_1 be the line $\vec{r}_1 = 2\hat{i}+\hat{j}-\hat{k}+\lambda(\hat{i}+2\hat{k})$ and let L_2 be the line $\vec{r}_2 = 3\hat{i}+\hat{j}+\mu(\hat{i}+\hat{j}-\hat{k})$. Let Π be the plane which contains the line L_1 and is parallel to L_2. The distance of the plane Π from the origin is

(A) 1/7 (B) $\sqrt{2/7}$ (C) $\sqrt{6}$ (D) none

Solⁿ Eqⁿ of plane containing L_1 is given by

$\Pi: a(x-2)+b(y-1)+c(z+1)=0$ —— (1)

where $a.1+b.0+c.2=0 \Rightarrow a+2c=0$ —— (2)

$\because$ plane is parallel to $L_2: \vec{r}_2 = 3\hat{i}+\hat{j}+\mu(\hat{i}+\hat{j}-\hat{k})$

$\Rightarrow a.1+b.1+c(-1)=0$ —— (3)

from Eqⁿ (2) Put $a=-2c$ in Eqⁿ (3)

$-2c+b-c=0 \Rightarrow \boxed{b=3c}$

Put $a=-2c$, $b=3c$ in Eqⁿ (1)

$\Pi: -2c(x-2)+3c(y-1)+c(z+1)=0$

$-2x+4+3y-3+z+1=0$

$\Pi: 2x-3y-z-2=0$

The distance of plane Π from origin

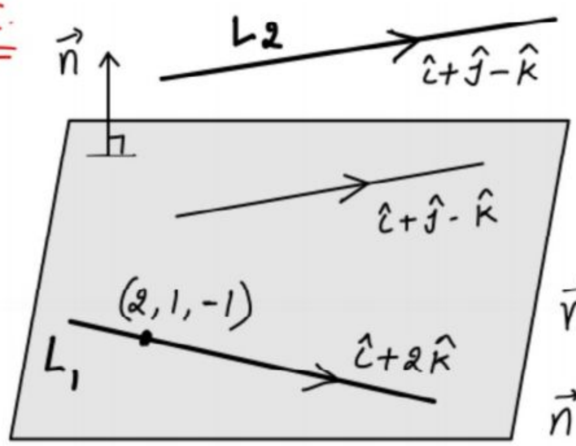
$$= \left|\frac{2\times0-3\times0-0-2}{\sqrt{4+9+1}}\right| = \sqrt{\frac{2}{7}}$$

18 Continues...

Method II.

$L_1: \vec{r}_1 = 2\hat{i}+\hat{j}-\hat{k}+\lambda(\hat{i}+2\hat{k})$

$L_2: \vec{r}_2 = 3\hat{i}+\hat{j}+\mu(\hat{i}+\hat{j}-\hat{k})$



$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 0 & 2 \end{vmatrix}$$

$$\vec{n} = 2\hat{i}-3\hat{j}-\hat{k}$$

$\therefore$ Plane $\Rightarrow$ $2x-3y-z=d$

$$\left\{\begin{array}{l} \text{passes through } (2,1,-1) \\ \therefore\ 4-3+1=d \\ \Rightarrow\ d=2 \end{array}\right\}$$

So, Plane $\Rightarrow$ $\boxed{2x-3y-z-2=0}$

$\therefore$ Distance of Plane from Origin $= \dfrac{|0-0-0-2|}{\sqrt{4+9+1}}$

$$= \boxed{\sqrt{\frac{2}{7}}}$$

(19)

The distance of the point (–1, –5, – 10) from the point of intersection of the line $\frac{x-2}{2}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x - y + z = 5$ is

(A) $2\sqrt{11}$ (B) $\sqrt{126}$ (C) 13 (D) 14

Soln

$$\frac{x-2}{2} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda \text{ (let)} \quad \text{—— (1)}$$

General Point on line (1) $P \equiv (2+2\lambda, -1+4\lambda, 2+12\lambda)$

$\because$ line (1) intersect the plane $x-y+z=5$

$\Rightarrow$ Point P lie. in the plane $x-y+z=5$

$\Rightarrow$ $(2+2\lambda) - (-1+4\lambda) + (2+12\lambda) = 5$

$10\lambda = 0 \Rightarrow \lambda = 0$

$\Rightarrow P \equiv (2, -1, 2)$ & $Q \equiv (-1, -5, -10)$ given

$\Rightarrow$ distance b/w Point P&Q is

$$= \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$$

$$= \underline{\underline{13}}$$

20

[MATRIX MATCH TYPE]

Consider the following four pairs of lines in **column-I** and match them with one or more entries in **column-II**.

	Column-I		Column-II
(A)	$L_1 : x = 1 + t,\ y = t,\ z = 2 - 5t$ $L_2 : \vec{r} = (2,1,-3) + \lambda(2, 2, -10)$	(P)	non coplanar lines
(B)	$L_1 : \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1}$ $L_2 : \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$	(Q)	lines lie in a unique plane
(C)	$L_1 : x = -6t,\ y = 1 + 9t,\ z = -3t$ $L_2 : x = 1 + 2s,\ y = 4 - 3s,\ z = s$	(R)	infinite planes containing both the lines
(D)	$L_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ $L_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$	(S)	lines are not intersecting at a unique point

(A) $L_1: x = 1 + t,\quad y = t,\quad z = 2 - 5t$

$$\frac{x-1}{1} = \frac{y-0}{1} = \frac{z-2}{-5}$$

$$L_2: \frac{x-2}{2} = \frac{y-1}{2} = \frac{z+3}{-10}$$

L_1 Passing through $(x_1, y_1 z_1) \equiv (1, 0, 2)$

& Dr's $\langle a, b, c \rangle \equiv \langle 1, 1, -5 \rangle$

L_2: Passing through $(x_2, y_2, z_2) \equiv (2, 1, -3)$

and Dr's $\langle a', b', c' \rangle \equiv \langle 2, 2, -10 \rangle$

Here $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$ and L_1 passing through point $(2, 1, -3)$ $\Rightarrow$ L_1 & L_2 Lines are Coincident line $\Rightarrow$ infinite planes possible passing through both L_1 & L_2

$\because$ Lines are coincident $\Rightarrow$ they intersect at infinite points

$\Rightarrow \boxed{R, S}$

(20) Continues...

(B) $L_1 : \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1} = \lambda$

$L_2 : \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3} = \mu$

Now Point on line $L_1 \equiv (1+2\lambda, 3+2\lambda, 2-\lambda)$

Point on line $L_2 \equiv (2+\mu, 6-\mu, -2+3\mu)$

$\underbrace{1+2\lambda = 2+\mu \quad | \quad 3+2\lambda = 6-\mu}_{\text{solve}}$

$\lambda = 1$
$\mu = 1$ which satisfy the z co-ordinate

ie $2-\lambda = -2+3\mu$

$\Rightarrow$ L_1 & L_2 intersect at Point $(3, 5, 1)$

and lines are not parallel

$\Rightarrow$ lines lie in an unique plane. (Q)

(C) $L_1 : x = -6t, \; y = 1 + 9t, \; z = -3t$
$L_2 : x = 1 + 2s, \; y = 4 - 3s, \; z = s$

$L_1 : \frac{x}{-6} = \frac{y-1}{9} = \frac{z}{-3} = t$

$L_2 : \frac{x-1}{2} = \frac{y-4}{-3} = \frac{z}{1} = s$

L_1 & L_2 are parallel lines but not coincident

($\because$ Point $(0, 1, 0)$ does not lie on L_2)

$\Rightarrow$ $\boxed{Q, S}$

(20) Continues...

(D) $L_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

$L_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$ lines are not parallel.

$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda \Rightarrow P \equiv (\lambda, 1+2\lambda, 2+3\lambda)$

$\frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2} = \mu \Rightarrow Q \equiv (3-4\mu, 2-3\mu, 1+2\mu)$

$\Rightarrow \lambda = 3-4\mu$ & $1+2\lambda = 2-3\mu$

Solve

$\mu = 1$

$\lambda = -1$

$\because \quad 2+3\lambda \neq 1+2\mu$ at $\lambda = -1$, $\mu = 1$

$\Rightarrow$ L_1 & L_2 are skew lines

$\Rightarrow \boxed{P, S}$

Exercise 02

EXERCISE (O-2)

[MULTIPLE OBJECTIVE TYPE]

1

Consider Lines $L_1 : \frac{x-\alpha}{1} = \frac{y}{-2} = \frac{z+\beta}{2}$, $L_2 : x = \alpha, \frac{y}{-\alpha} = \frac{z+\alpha}{2-\beta}$, plane $P : 2x+2y+z+7=0$. Let line L_2 lies in plane P, then

(A) $\alpha = -7$

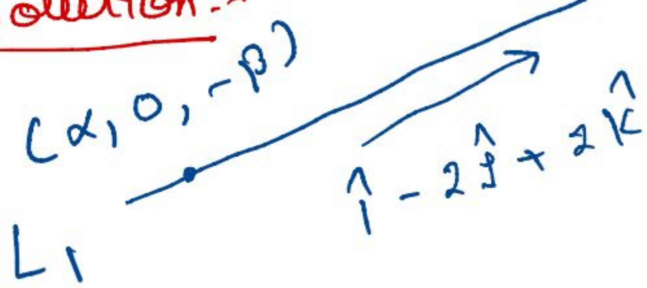
(B) $\alpha = 7$

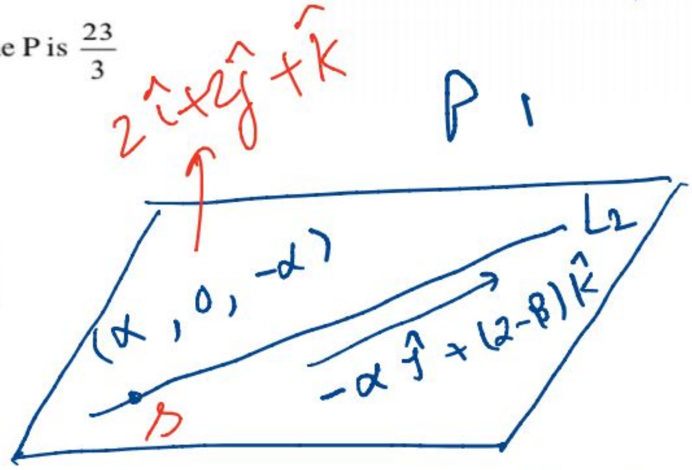
(C) minimum distance between line L_1 and plane P is 11.

(D) minimum distance between line L_1 and plane P is $\frac{23}{3}$

$\hookrightarrow \frac{x-\alpha}{0} = \frac{y-0}{-\alpha} = \frac{z+\alpha}{2-\beta}$

Solution:-





$2x+2y+z+7=0$

L_2 lies on $P_1 \Rightarrow (2\ 2\ 1)\cdot(0\ -\alpha\ 2-\beta) = 0$

$2(0) + 2(-\alpha) + 1(2-\beta) = 0$

& Point satisfy the Plane

$2\alpha - \alpha + 7 = 0 \Rightarrow \boxed{\alpha = -7}$ $\boxed{\beta = 16}$

Basically L_1 is $\parallel$ to P_1 as

$(1\ -2\ 2)\cdot(2\ 2\ 1) = 0$

minimum distance $\Rightarrow$ Perpendicular dista

$\left|\frac{2\alpha + 0 - \beta + 7}{3}\right| \Rightarrow \frac{23}{3}$ A, D

2

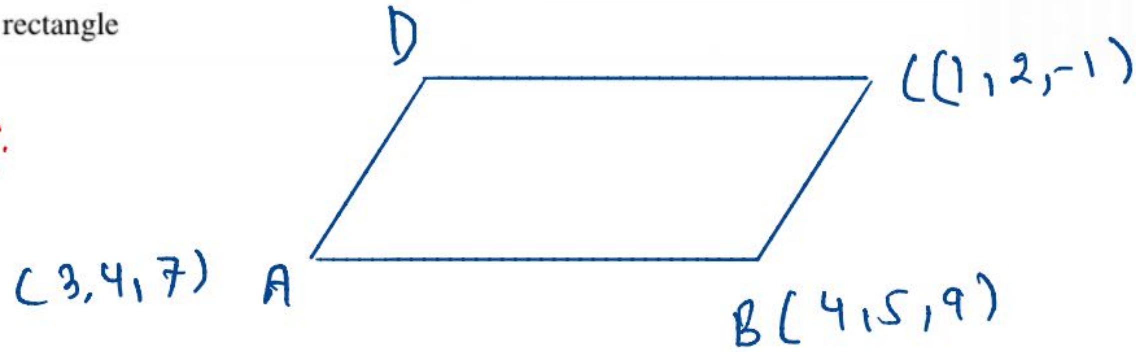
The point $\vec{A}\,(3,4,7)$, $\vec{B}\,(4,5,9)$ and $\vec{C}\,(1,2,-1)$ are three vertices of a parallelogram ABCD, then -

(A) vector equation of line AB is $\vec{r} = 4\hat{i}+5\hat{j}+9\hat{k}+\lambda\left(\hat{i}+\hat{j}+2\hat{k}\right)$

(B) cartesian equation of line BC is $\frac{x-4}{3}=\frac{x-5}{3}=\frac{z-9}{10}$

(C) coordinates of D are $(0,1,-3)$

(D) ABCD is rectangle



Solution:

mid Point of AC & BD are Same

$\Rightarrow$ D is $(0,1,-3)$

$\overline{AB} = (4,5,9) + \lambda(1,1,2)$

$\overline{BC} = (4,5,9) + \mu(3,3,10)$

$\equiv \frac{x-4}{3} = \frac{y-5}{3} = \frac{z-9}{10}$

$\overline{AB} \cdot \overline{BC} = (1\ 1\ 2)\cdot(3\ 3\ 10)$

$\neq 0 \Rightarrow$ not a Rectangle

A, B, C

3

(D) ABCD is rectangle

Let two planes $P_1 : 2x - y + z - 2 = 0$ and $P_2 : x + 2y - z - 3 = 0$ are given then-

(A) The equation of the plane through line of intersection of $P_1 = 0$ and $P_2 = 0$ and the point (3,2,1) is $x - 3y + 2z + 1 = 0$

(B) The equation of the plane through line of intersection of $P_1 = 0$ and $P_2 = 0$ and the point (3,2,1) is $3x - y + 2z - 9 = 0$

(C) The equation of acute angle bisector plane of $P_1 = 0$ and $P_2 = 0$ is $x - 3y + 2z + 1 = 0$

(D) The equation of acute angle bisector plane of $P_1 = 0$ and $P_2 = 0$ is $x + 3y + 2z + 2 = 0$

Solution:-

Plane Passing thru intersection line of $P_1 = 0$ & $P_2 = 0$ is $P_1 + \lambda P_2 = 0$

$$(2x - y + z - 2) + \lambda(x + 2y - z - 3) = 0$$

Passing thru $(3,2,1) \Rightarrow 3 + \lambda \cdot 3 = 0 \Rightarrow \lambda = -1$

$$\Rightarrow x - 3y + 2z + 1 = 0 \qquad \text{(A)}$$

$P_1 : 2x - y + z - 2 = 0$

$P_2 : x + 2y - z - 3 = 0$

check $a_1 a_2 + b_1 b_2 + c_1 c_2 = 2 - 2 - 1 < 0$

$\Rightarrow$ + Angle bisector bisects Acute $\angle$

$$\frac{2x - y + z - 2}{\sqrt{6}} = + \frac{x + 2y - z - 3}{\sqrt{6}}$$

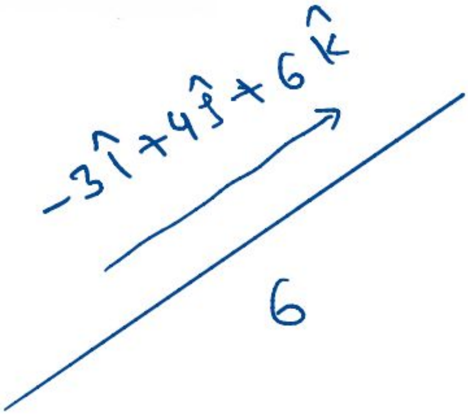
$$\Rightarrow x - 3y + 2z + 1 = 0 \qquad \text{(C)}$$

4. A line segment has length 6 and direction ratios are –3,4,6, then the component of the line vector are-

(A) $\frac{-18}{\sqrt{61}}, \frac{24}{\sqrt{61}}, \frac{36}{\sqrt{61}}$ (B) 27,–18,–54 (C) 27,–18,54 (D) $\frac{18}{\sqrt{61}}, \frac{-24}{\sqrt{61}}, \frac{-36}{\sqrt{61}}$

A D

Solution:-



Component of line:-

$$\pm\left(\frac{-3\hat{i}+4\hat{j}+6\hat{k}}{\sqrt{61}}\right)\cdot 6$$

$$\pm\left(\frac{-18}{\sqrt{61}}, \frac{24}{\sqrt{61}}, \frac{36}{\sqrt{61}}\right)$$

A, D

5. Which of the following is (are) correct -

(A) If two lines in space are not intersecting, then they must be skew lines.

(B) If two lines are parallel to a plane 'P', then their direction ratios will be proportional

(C) If two lines are perpendicular to a plane 'P', then their direction ratios will be proportional

(D) Equation $\frac{x+1}{a}=\frac{y-1}{b}=\frac{z}{c}$, where a, b, c are real parameters, represents a family of concurrent lines in space

Solution:-

(A) Not intersecting non skew

Parallel lines

$\Rightarrow$ false

(B) two lines || to a Plane need not to be Parallel ex: x-axis & y-axis are || to a Plane z=5 but not || hence DCs will not be surely Proportional $\Rightarrow$ false

(C) two lines $\perp$ to a Plane must be || hence DCs will be Proportional $\Rightarrow$ true

(D) $\frac{x+1}{a}=\frac{y-1}{b}=\frac{z}{c}$ all such lines always Passes thru a point $(-1, 1, 0)$ irrespective of values of a, b, c $\Rightarrow$ lines are Concurrent $\Rightarrow$ true (C, D)

6. Given the equations of the line $3x - y + z + 1 = 0,\ 5x + y + 3z = 0$.

Then which of the following is correct ?

(A) Symmetrical form of the equations of line is $\dfrac{x}{2} = \dfrac{y - \frac{1}{8}}{-1} = \dfrac{z + \frac{5}{8}}{1}$

(B) symmetrical form of the equations of line is $\dfrac{x + \frac{1}{8}}{1} = \dfrac{y - \frac{5}{8}}{1} = \dfrac{z}{-2}$

(C) equation of the plane through (2, 1, 4) and prependicular to the given lines is $2x - y + z - 7 = 0$

(D) equation of the plane through (2, 1, 4) and prependicular to the given lines is $x + y - 2z + 5 = 0$

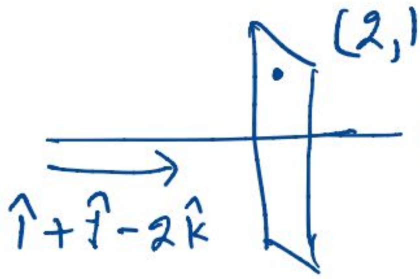
B, D

Solution:- $3x - y + z = -1$

$5x + y + 3z = 0$

Let $z = \lambda \Rightarrow 3x - y = -1 - \lambda$
$5x + y = -3\lambda$ $\Rightarrow x = \dfrac{-4\lambda - 1}{8},\ y = \dfrac{-4\lambda + 5}{8}$

$\Rightarrow \dfrac{8x + 1}{-4} = \dfrac{8y - 5}{-4} = \dfrac{z}{1} \Rightarrow \dfrac{x + \frac{1}{8}}{-\frac{1}{2}} = \dfrac{y - \frac{5}{8}}{-\frac{1}{2}} = \dfrac{z}{1}$ (B)



$\Rightarrow$ Eq of Plane:

$(x - 2)1 + (y - 1)1 + (z - 4)(-2) = 0$

$x + y - 2z + 5 = 0$ (D)

⑦

A line L passing through the point P(1,4,3), is perpendicular to both the lines

$\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-2}{4}$ and $\frac{x+2}{3}=\frac{y-4}{2}=\frac{z+1}{-2}$.

If the position vector of point Q on L is(a_1,a_2,a_3) such that $(PQ)^2 = 357$, then $(a_1 + a_2 + a_3)$ can be-

(A) 16 (B) 15 (C) 2 (D) 1

Sol :- L is $\perp$ar to lines with parallel vectors $2\hat{i}+\hat{j}+4\hat{k}$, $3\hat{i}+2\hat{j}-2\hat{k}$

$\Rightarrow$ L is $\parallel$ to cross product of $2\hat{i}+\hat{j}+4\hat{k}$ and $3\hat{i}+2\hat{j}-2\hat{k}$

$$\begin{vmatrix} i & j & k \\ 2 & 1 & 4 \\ 3 & 2 & -2 \end{vmatrix} = -10\hat{i}+16\hat{j}+\hat{k}$$

$\Rightarrow$ drs of L are $-10\hat{i}+16\hat{j}+\hat{k}$

$\Rightarrow$ dc's of L are

$$\frac{-10}{\sqrt{357}}, \frac{16}{\sqrt{357}}, \frac{1}{\sqrt{357}}$$

$PQ = \sqrt{357}$

$\Rightarrow Q = P \pm PQ \cdot \left(\frac{-10}{\sqrt{357}}, \frac{16}{\sqrt{357}}, \frac{1}{\sqrt{357}}\right)$

$= P \pm (-10, 16, 1)$

$Q = (1,4,3) \pm (-10,16,1)$

7 Continues...

$\Rightarrow \quad P = (-9, 20, 4)$

$\quad (or) \quad (11, -12, 2)$

$\Rightarrow \quad a_1 + a_2 + a_3 = -9 + 20 + 4 = 15$

$\quad (or) \quad 11 - 12 + 2 = 1$

8

[MATRIX MATCH TYPE]

P(0, 3, – 2); Q(3, 7, – 1) and R(1, – 3, – 1) are 3 given points. Let L_1 be the line passing through P and Q and L_2 be the line through R and parallel to the vector $\vec{V} = \hat{i} + \hat{k}$.

	Column-I			Column-II
(A)	perpendicular distance of P from L_2 (R)		(P)	$7\sqrt{3}$
(B)	shortest distance between L_1 and L_2 (Q)		(Q)	2
(C)	area of the triangle PQR (P)		(R)	6
(D)	distance from (0, 0, 0) to the plane PQR (S)		(S)	$\frac{19}{\sqrt{147}}$

Sol:- dr's of L_1 are $(3-0, 7-3, -1+2)$

$= (3, 4, 1)$

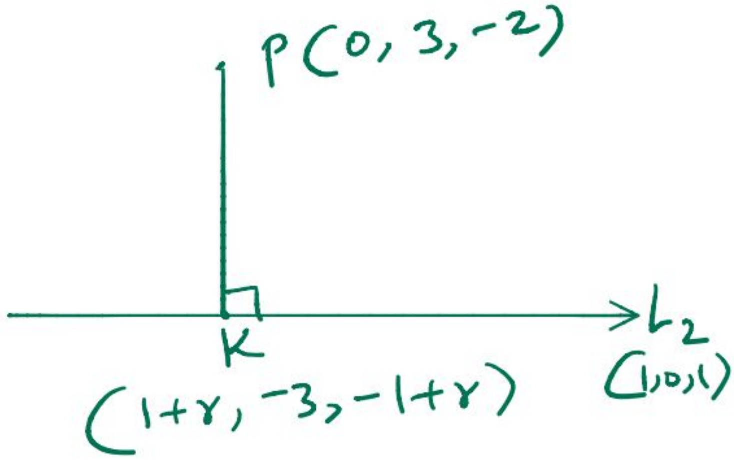
$\because$ L_1 passes through P

$\Longrightarrow$ $L_1 : \dfrac{x}{3} = \dfrac{y-3}{4} = \dfrac{z+2}{1}$

$L_2 : \dfrac{x-1}{1} = \dfrac{y+3}{0} = \dfrac{z+1}{1}$

A) Let arbitary point K on L_2 be

$k = (1+r, -3, -1+r)$



PK $\perp$ar L_2

$\Longrightarrow \left[(1+r)\hat{i} - 6\hat{j} + (1+r)\hat{k}\right] \cdot (\hat{i} + \hat{k}) = 0$

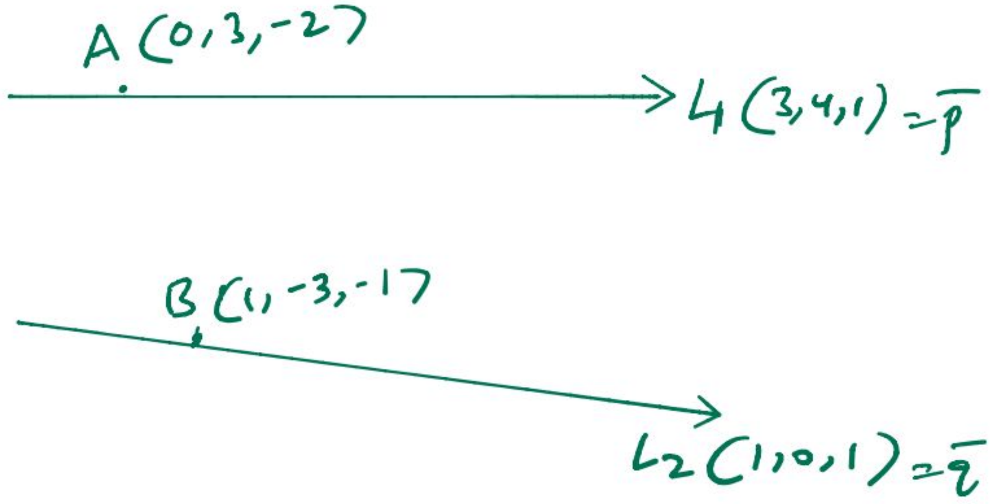
$\Longrightarrow 2(1+r) = 0 \Longrightarrow r = -1$

$\Longrightarrow K = (0, -3, -2)$

⑧ Continues...

$\Rightarrow \; PK = \sqrt{0^2+6^2+0^2} = 6$

B)



Shortest distance

$$d = \frac{\overline{AB} \cdot (\bar{p} \times \bar{q})}{|\bar{p} \times \bar{q}|}$$

$$\bar{p} \times \bar{q} = \begin{vmatrix} i & j & k \\ 3 & 4 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$\bar{p} \times \bar{q} = 4\hat{i} - 2\hat{j} - 4\hat{k}$$

$$\Rightarrow |\bar{p} \times \bar{q}| = \sqrt{4^2+2^2+4^2} = 6$$

$$\overline{AB} = \hat{i} - 6\hat{j} + \hat{k}$$

$$\Rightarrow d = \frac{1 \times 4 + (-6) \times (-2) + 1 \times (-4)}{6}$$

$$d = \frac{4+12-4}{6} = 2$$

8 Continues...

c) $P = (0,3,-2)$ $Q = (3,7,-1)$

$R = (1,-3,-1)$

$\overline{PQ} = 3\hat{i} + 4\hat{j} + \hat{k}$

$\overline{PR} = \hat{i} - 6\hat{j} + \hat{k}$

$$\overline{PQ} \times \overline{PR} = \begin{vmatrix} i & j & k \\ 3 & 4 & 1 \\ 1 & -6 & 1 \end{vmatrix}$$

$\overline{PQ} \times \overline{PR} = 10\hat{i} - 2\hat{j} - 22\hat{k}$

Area of $\triangle PQR = \frac{1}{2} |\overline{PQ} \times \overline{PR}|$

$= \frac{1}{2}\sqrt{10^2 + 2^2 + 22^2}$

$= 7\sqrt{3}$

D) Normal to plane PQR is

$\overline{PQ} \times \overline{PR} = 10\hat{i} - 2\hat{j} - 22\hat{k}$

$\Rightarrow$ Equation of PQR is

$10x - 2y - 22z = k_1$

$\Rightarrow 5x - y - 11z = k$

$\Rightarrow 5x - y - 11z = 5(0) - 3 - 11(-2)$

$\Rightarrow 5x - y - 11z = 19$

$\perp$ar distance from o to PQR is

$\frac{|19|}{\sqrt{5^2 + 1^2 + 11^2}} = 19/\sqrt{147}$

Exercise S1

EXERCISE (S-1)

1. Find the angle between the two straight lines whose direction cosines ℓ, m, n are given by $2\ell + 2m - n = 0$ and $mn + n\ell + \ell m = 0$.

Sol:
$2l + 2m - n = 0 \Rightarrow n = 2(l+m)$

$mn + nl + lm = 0$

$\Rightarrow n(m+l) + lm = 0$

$\Rightarrow 2(l+m)^2 + lm = 0$

$\Rightarrow 2l^2 + 5lm + 2m^2 = 0$

$\Rightarrow 2\left(\frac{l}{m}\right)^2 + 5\left(\frac{l}{m}\right) + 2 = 0$

$\Rightarrow \frac{l}{m} = -2, \frac{-1}{2}$

$\Rightarrow l = -2m,$

$n = 2(l+m)$

$= 2(-2m+m)$

$n = -2m$

dr's are proportional to

$= (-2m, m, -2m)$

$= (-2, 1, -2)$

$l = -\frac{m}{2}$

$n = 2\left(-\frac{m}{2} + m\right)$

$n = m$

dr's are proportional to

$= \left(-\frac{m}{2}, m, m\right)$

$= (-1, 2, 2)$

$$\cos\theta = \frac{(-2)(-1) + 1(2) + (-2)(2)}{\sqrt{(-2)^2 + 1^2 + (-2)^2}\,\sqrt{(-1)^2 + 2^2 + 2^2}}$$

$\cos\theta = 0 \Rightarrow \theta = 90^\circ$

2. The plane denoted by $\Pi_1 : 4x + 7y + 4z + 81 = 0$ is rotated through a right angle about its line of intersection with the plane $\Pi_2 : 5x + 3y + 10z = 25$. If the plane in its new position be denoted by Π, and the distance of this plane from the origin is $\sqrt{k}$ where $k \in N$, then find k.

Sol:- $\Pi : \Pi_1 + \lambda \Pi_2 = 0$

$\Rightarrow \Pi : 4x + 7y + 4z + 81 + \lambda(5x + 3y + 10z - 25) = 0$

$\Rightarrow \Pi : x(4+5\lambda) + y(7+3\lambda) + z(4+10\lambda) + 81 - 25\lambda = 0$

Π is $\perp$ar Π_1

$\Rightarrow 4(4+5\lambda) + 7(7+3\lambda) + 4(4+10\lambda) = 0$

$\Rightarrow 81\lambda + 81 = 0 \Rightarrow \lambda = -1$

$\Rightarrow \Pi : -x + 4y - 6z + 106 = 0$

$\Rightarrow \Pi : x - 4y + 6z - 106 = 0$

distance of Π from origin,

$$d = \frac{|0 - 0 + 0 - 106|}{\sqrt{1^2 + (-4)^2 + 6^2}}$$

$$= \frac{106}{\sqrt{53}}$$

$$d = 2\sqrt{53} = \sqrt{4 \times 53} = \sqrt{212}$$

$\Rightarrow k = 212$

3. Find the equations of the straight line passing through the point (1,2,3) to intersect the straight line $x + 1 = 2(y - 2) = z + 4$ and parallel to the plane $x + 5y + 4z = 0$.

Sol:- Required line is parallel to plane $x + 5y + 4z = 0$ and passes through $(1,2,3)$

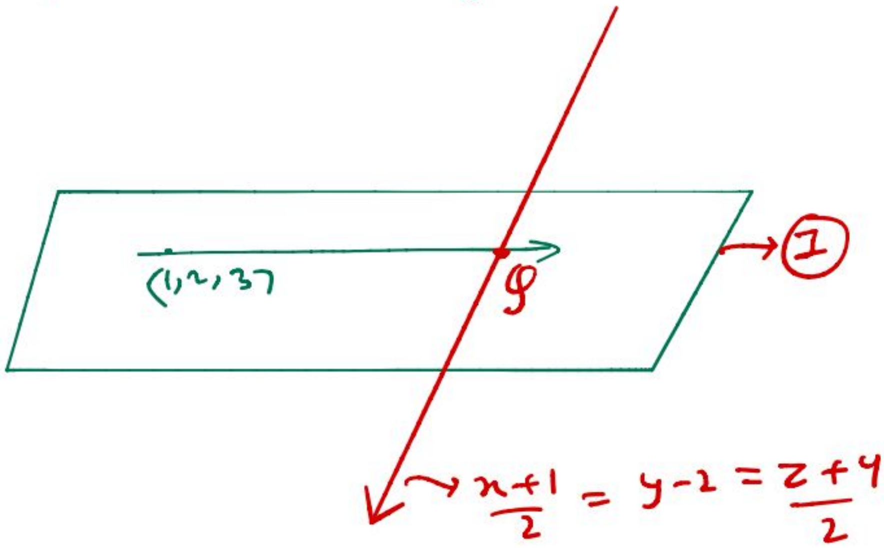
$\Rightarrow$ Line lies in plane

$$x + 5y + 4z = 1 + 5(2) + 4(3)$$

$$x + 5y + 4z = 23 \quad \text{——— (1)}$$

Also line intersects $\frac{x+1}{2} = y - 2 = \frac{z+4}{2}$

$\Rightarrow$ It will intersect given line at the point where given line intersects (1)



Any point P on $\frac{x+1}{2} = y - 2 = \frac{z+4}{2}$ is of the form,

$$P = (-1 + 2\lambda,\ 2 + \lambda,\ -4 + 2\lambda)$$

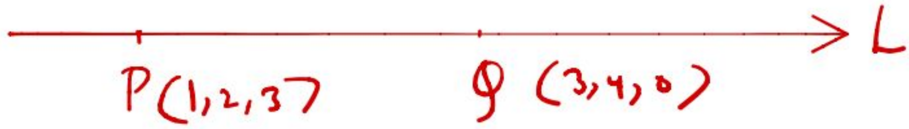
③ Continues...

$\because$ Q lies on $x+5y+4z=23$,

$-1+2\lambda+5(2+\lambda)+4(-4+2\lambda)=23$

$\Rightarrow \quad 15\lambda = 30 \quad \Rightarrow \quad \lambda = 2$

$\Rightarrow \quad Q = (3, 4, 0)$



Dr's of required line, L are

$(3-1, 4-2, 0-3) = (2, 2, -3)$

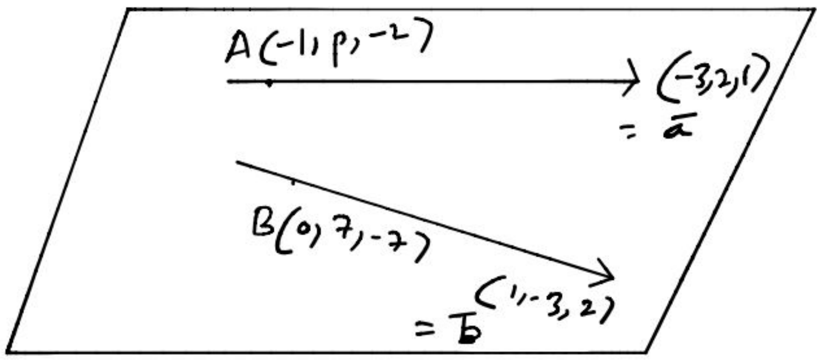
$\Rightarrow$ Equation of line L is

$$L: \frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{-3}$$

4

Find the value of p so that the lines $\frac{x+1}{-3} = \frac{y-p}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are in the same plane. for this value of p, find the coordinates of their point of intersection and the equation of the plane containing them.

Sol:-



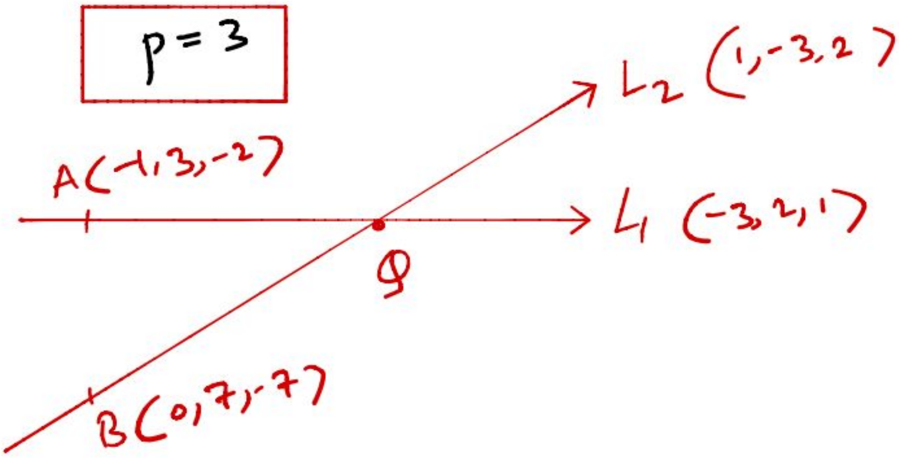
For the lines to be in same plane, $\overline{AB}$, $\bar{a}$, $\bar{b}$ should be in same plane $\Rightarrow [\overline{AB}\ \bar{a}\ \bar{b}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 7-p & -5 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 7 + (7-p)(7) + (-5)(7) = 0$$

$$\Rightarrow 21 = 7p$$

$$\Rightarrow \boxed{p = 3}$$



④ Continues...

Let $\mathcal{P}$ on L_1 be

$$\mathcal{P} = (-1-3\lambda,\ 3+2\lambda,\ -2+\lambda)$$

Let $\mathcal{P}$ on L_2 be

$$\mathcal{P} = (\mu,\ 7-3\mu,\ -7+2\mu)$$

$$-1-3\lambda = \mu$$
$$3+2\lambda = 7-3\mu$$
$$-2+\lambda = -7+2\mu$$

$$\Rightarrow \quad 3+2\lambda = 7-3(-1-3\lambda)$$
$$\Rightarrow \quad 3+2\lambda = 10+9\lambda$$
$$\Rightarrow \quad \lambda = -1$$
$$\Rightarrow \quad \boxed{\mathcal{P} = (2, 1, -3)}$$

Equation of plane containing L_1 and L_2

is
$$\begin{vmatrix} x & y-7 & z+7 \\ 1 & -3 & 2 \\ -3 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad x(-\cancel{7}) + (y-7)(-\cancel{7}) + (z+7)(-\cancel{7}) = 0$$

$$\Rightarrow \quad \boxed{x+y+z = 0}$$

3

Let L be the line given by $\vec{r} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ and let P be the point $(2,-1,1)$. Also suppose that E be the plane containing three non collinear points $A(0,1,1)$; $B(1,2,2)$ and $C(1,0,1)$.

Find

(a) Distance between the point P and the line L.

(b) Equation of the plane E.

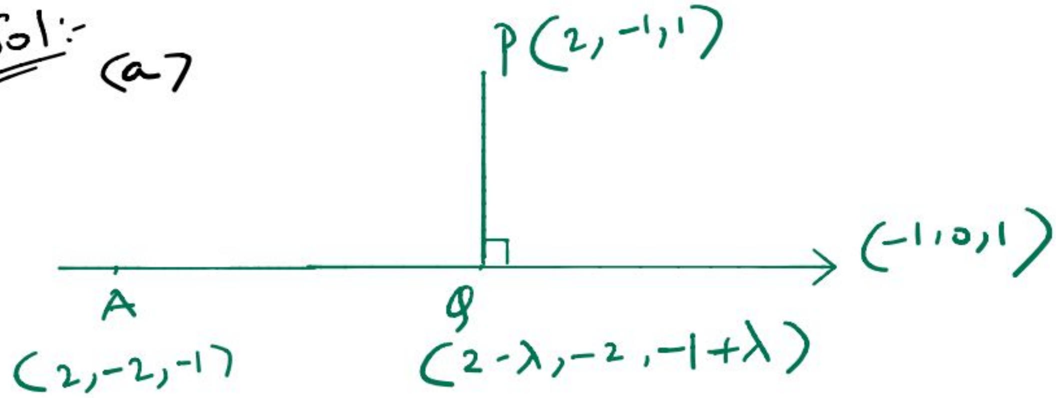
(c) Equation the plane F containing the line L and the point P.

(d) Acute angle between the plane E and F.

(e) Volume of the parallelopiped by A,B,C and the point $D(-3,0,1)$.

Sol:-

(a)



$$\overline{PQ} = -\lambda\hat{i} - \hat{j} + (\lambda - 2)\hat{k}$$

$$\overline{PQ} \perp\text{ar } -\hat{i} + \hat{k}$$

$$\Rightarrow \lambda + \lambda - 2 = 0 \Rightarrow \lambda = 1$$

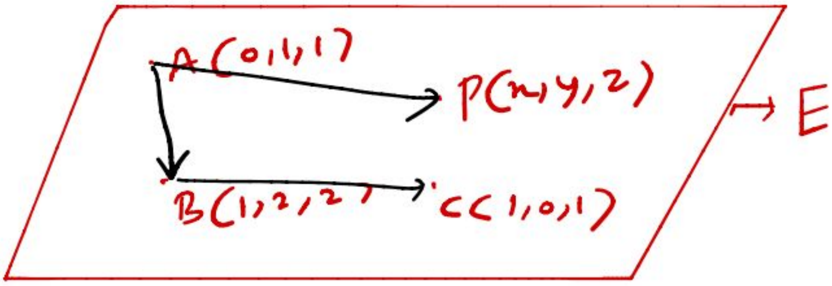
$$\Rightarrow Q = (1,-2,0)$$

$$\Rightarrow PQ = \sqrt{(2-1)^2 + (-1+2)^2 + (1-0)^2}$$

$$PQ = \sqrt{1+1+1} = \sqrt{3}$$

5 Continues...

(b)



Equation of plane is

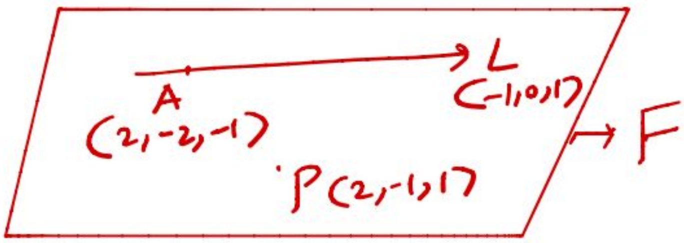
$$[\overline{AP} \quad \overline{AB} \quad \overline{BC}] = 0$$

$$\Longrightarrow \begin{vmatrix} x & y-1 & z-1 \\ 1 & 1 & 1 \\ 0 & -2 & -1 \end{vmatrix} = 0$$

$$\Longrightarrow x + (y-1)(1) + (z-1)(-2) = 0$$

$$\Longrightarrow x + y - 2z + 1 = 0$$

(c)



Equation of plane F is

$$\begin{vmatrix} x-2 & y+1 & z-1 \\ 2-2 & -2+1 & -1-1 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

5 Continues...

$$\Rightarrow \begin{vmatrix} x-2 & y+1 & z-1 \\ 0 & -1 & -2 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(-1) + (y+1)(2) + (z-1)(-1) = 0$$

$$\Rightarrow -x+2y-z+5=0$$

$$\Rightarrow x-2y+z-5=0$$

(d) $E: \; x+y-2z+1=0$

$F: \; x-2y+z-5=0$

$$\cos\theta = \frac{|1(1) + 1(-2) + (-2)(1)|}{\sqrt{1^2+1^2+(-2)^2}\sqrt{1^2+(-2)^2+1^2}}$$

$$= \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \theta = 60^\circ$$

(e) $A = (0,1,1)$

$B = (1,2,2)$

$C = (1,0,1)$

$D = (-3,0,1)$

Volume of parallelopiped =

$$\left| [\overline{AB} \quad \overline{AC} \quad \overline{AD}] \right|$$

$\boxed{5}$ Continues...

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ -3 & -1 & 0 \end{vmatrix}$$

$$= 4$$

6

find the equation of the line which is reflection of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane $3x-3y+10z = 26$.

Solⁿ

D.R of line $(9,-1,-3)$

normal of plane $(3,-3,10)$

Dot product $\left(9\times 3 + (-1)(-3) + (-3)(10)\right)$

$\Rightarrow \underline{\underline{0}}$

So line is parallel to the plane.

* reflected line will have Same D.R as given line.

Image of point $(1,2,-3)$ in plane

$$\frac{x-1}{3} = \frac{y-2}{-3} = \frac{z+3}{10} = \frac{-2(3-6-30-26)}{9+9+100}$$

$x = 4,\ y = -1,\ z = 7$

eqⁿ of reflected line

$$\boxed{\frac{x-4}{9} = \frac{y+1}{-1} = \frac{z-7}{-3}}$$

*

7 Find the equation of the plane containing the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{2}$ & parallel to the line $\frac{x-3}{2} = \frac{y}{5} = \frac{z-2}{4}$.

Find also the S.D between the two lines.

Solⁿ normal of plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ 2 & 5 & 4 \end{vmatrix}$

$= 2(\hat{i} - 2\hat{j} + 2\hat{k})$

eq. of plane passing through $(1,0,0)$

point on the line which plane contains.

$$\boxed{x - 2y + 2z = 1}$$

lines in vector form

$\bar{r} = \hat{i} + \lambda(2\hat{i} + 3\hat{j} + 2\hat{k}) = \vec{a} + \lambda\bar{b}$

$\bar{r} = 3\hat{i} + 2\hat{k} + \mu(2\hat{i} + 5\hat{j} + 4\hat{k}) = \bar{c} + \mu\bar{d}$

S.D = Projection of $(\bar{c} - \bar{a})$ on $\bar{b} \times \bar{d}$

$$= \left| \frac{(2\hat{i} + 2\hat{k}) \cdot (\hat{i} - 2\hat{j} + 2\hat{k})}{|\hat{i} - 2\hat{j} + 2\hat{k}|} \right|$$

$$\Rightarrow \frac{|2+4|}{\sqrt{1+4+4}} = \frac{6}{3} = 2.$$

8 The equation of the plane which has the Property that the point Q (5, 4, 5) is the reflection of point (P) (1, 2, 3) through that plane, is $ax + by + cz = d$ where $a, b, c, d \in N$ find the least value of $(a+b+c+d)$

Solⁿ

$$\vec{PQ} = 4\hat{i} + 2\hat{j} + 2\hat{k}$$
$$= 2(2\hat{i} + \hat{j} + \hat{k})$$

normal of plane is propotional to $\vec{PQ}$

$$\frac{a}{4} = \frac{b}{2} = \frac{c}{2} \Rightarrow \frac{a}{2} = \frac{b}{1} = \frac{c}{1}$$

$\therefore \boxed{a = 2, \ b = 1, \ c = 1}$ (least & natural number)

mid point of PQ will lie on the plane

$$m\left(\frac{\vec{PQ}}{2}\right) = (3, 3, 4)$$

m satisfy plane

$$2x + y + z = d$$
$$\therefore 3(2) + 3(1) + 4(1) = d$$
$$d = 13$$

$a+b+c+d \Rightarrow 2+1+1+13 \Rightarrow 17$

(9) Find the point where the line of intersection of the planes $x-2y+z=1$ & $x+2y-2z=5$, intersects the plane $2x+2y+z+6=0$

<u>Sol^n</u>

$$x-2y = 1-z$$
$$x+2y = 5+2z$$
$$\overline{2x = 6+z}$$
$$x = 3+z/2$$

$$y = 1+\frac{3z}{4}$$

$$\frac{x-3}{(1/2)} = \frac{y-1}{(\frac{3}{4})} = z$$

$$\frac{x-3}{2} = \frac{y-1}{3} = \frac{z}{4}$$

let a point on line $(3+2\lambda, 1+3\lambda, 4\lambda)$

Satisfy plane

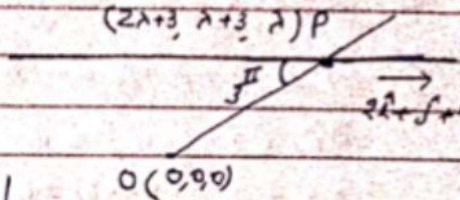
$$2x+2y+z+6=0$$
$$2(3+2\lambda)+2(1+3\lambda)+4\lambda+6=0$$
$$\lambda=-1$$

then Point $(1,-2,-4)$

Exercise S2

EXERCISE (S-2)

1. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle of $\frac{\pi}{3}$.



$$\cos\frac{\pi}{3} = \left|\frac{\vec{OP}\cdot(2\hat{i}+\hat{j}+\hat{k})}{|\vec{OP}|\cdot\sqrt{6}}\right|$$

$$\Rightarrow \frac{1}{2} = \left|\frac{2(2\lambda+3)+(\lambda+3)+\lambda}{\sqrt{(2\lambda+3)^2+(\lambda+3)^2+\lambda^2}\cdot\sqrt{6}}\right|$$

$$\Rightarrow \sqrt{6\lambda^2+18\lambda+18}\,\sqrt{6} = 2(6\lambda+9)$$

$$\Rightarrow 6(6\lambda^2+18\lambda+18) = 36[4\lambda^2+12\lambda+9]$$

$$\Rightarrow \lambda^2+3\lambda+3 = 4\lambda^2+12\lambda+9$$

$$\Rightarrow 3\lambda^2+9\lambda+6=0 \quad \begin{cases} \lambda=-1 \Rightarrow P(1,2,-1) \\ \lambda=-2 \Rightarrow P(-1,1,-2) \end{cases}$$

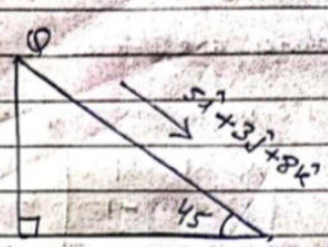
$\Rightarrow$ two possible equations of lines are

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}\ ,\quad \frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$$

②

The line $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). Find the equation of the remaining sides.

$$\cos 45^\circ = \left|\frac{\vec{AP}\cdot(5\hat{i}+3\hat{j}+8\hat{k})}{|\vec{AP}|\sqrt{98}}\right|$$



A (7, 2, 4)   P $(5\lambda-6, 3\lambda-10, 8\lambda-14)$

$$\Rightarrow |\vec{AP}|\cdot 7 = \vec{AP}\cdot(5\hat{i}+3\hat{j}+8\hat{k})$$

$$\Rightarrow 49\left[(5\lambda-13)^2+(3\lambda-12)^2+(8\lambda-18)^2\right]=\left[(5\lambda-13)\cdot 5+(3\lambda-12)\cdot 3+(8\lambda-18)8\right]^2$$

$$49(98\lambda^2-490\lambda+637)=(98\lambda-245)^2$$

$$98\lambda^2-490\lambda+637=49(2\lambda-5)^2 \Rightarrow 98\lambda^2-490\lambda+588$$

$$\Rightarrow \lambda = 3, 2$$

reqd lines are

$$\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6}, \quad \frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}$$

3

PAGE NO.: Icon

Planes P_1', P_2', P_3' are drawn parallel to the planes $P_1 : x + y + z = 3$, $P_2 : x - y + z = 1$ & $P_3 : x + y - z = 2$ respectively from the point (2,2,3). If d_1, d_2, d_3 are distances of P_1', P_2', P_3' from (1,1,2) respectively then $\left(d_1^2 + \frac{1}{d_2^2} + \frac{1}{d_3^2}\right)$ is equal to

eqn of $P_1' : x + y + z = d$

it passes through $(2,2,3) \Rightarrow d = 7$

$\Rightarrow \boxed{P_1' : x + y + z = 7}$

||ly $P_2' : x - y + z = 3$, $P_3' : x + y - z = 1$

Now, d_1 = distance of $x+y+z=7$ from (1,1,2)

$\Rightarrow d_1 = \left|\frac{3}{\sqrt{3}}\right| = \sqrt{3}$

||ly $d_2 = \frac{1}{\sqrt{3}}$, $d_3 = \frac{1}{\sqrt{3}}$

$\Rightarrow d_1^2 + \frac{1}{d_2^2} + \frac{1}{d_3^2} = 9$.

④ let the equation of the plane containing the line $x-y-z-4=0=x+y+2z-4$ and is parallel to the line of intersection of the planes $2x+3y+z=1$ & $x+3y+2z=2$

be $x+Ay+Bz+C=0$

Compute the value of $|A+B+C|$

Solⁿ

Let the plane be

$$P = P_1 + \lambda P_2 = 0$$

$$= (x-y-z-4) + \lambda(x+y+2z-4)$$

$$P = (1+\lambda)x + (\lambda-1)y + (2\lambda-1)z - 4\lambda - 4 = 0$$

The plane 'P' is parallel to line of Intersection of P_3 & P_4

L = D.R of line (L) of intersection of P_3 & P_4

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix} = 3(\hat{i}-\hat{j}+\hat{k})$$

normal of plane 'P' & D.R of Line (L) are at Right angle.

④ Continues...

$$(1+\lambda) - (\lambda-1) + (2\lambda-1) = 0 \quad \text{(Dot product)}$$

$$2\lambda + 1 = 0$$

$$\lambda = -1/2$$

Plane "p" $= \left(1-\frac{1}{2}\right)x + \left(-\frac{1}{2}-1\right)y$

$$+ \left(2\left(-\frac{1}{2}\right)-1\right)z - 4\left(-\frac{1}{2}\right) - 4 = 0$$

$$\Rightarrow \frac{x}{2} - \frac{3y}{2} - 2z = 2$$

$$\Rightarrow x - 3y - 4z - 4 = 0$$

$A = -3$, $B = -4$, $C = -4$

$$A + B + C = -11$$

$$|A+B+C| = 11$$

5

Consider the plane

$E: \quad \overline{r} = (-1,1,1) + \lambda(1,2,0) + \mu(1,0,1)$

let F be the plane containing the point A (4, 2, 2) and parallel to E.

Suppose the point B is on the plane E such that B has a minimum distance from the point A. If C(−3,0,4) lies in the plane F. Find the Area of the triangle ABC.

Solⁿ: normal of $E = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix}$

$\Rightarrow \quad 2\hat{i} - \hat{j} - 2\hat{k}$

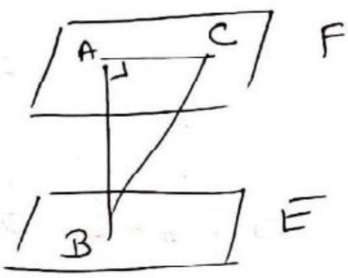
eqⁿ of plane E

$$2x - y - 2z = -2 - 1 - 2$$
$$2x - y - 2z = -5$$

eqⁿ of plane F

$$2x - y - 2z = -8 - 2 - 4$$
$$2x - y - 2z = -14$$

$\text{Area}(ABC) = \frac{1}{2} |\vec{AB}| \, |\vec{AC}|$



$\Rightarrow \quad \frac{1}{2} \; \frac{|14 - 5|}{\sqrt{4+1+4}} \; |\hat{i} - 2\hat{j} + 2\hat{k}|$

$\Rightarrow \quad 9/2$

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6. Find the equation of the line passing through the point $(4,-14,4)$ and intersecting the line of intersection of the planes :
$3x+2y-z=5$ & $x-2y-2z=-1$ at right angle.

Soln

let $z=k$

$$3x+2y-z=5$$
$$x-2y-2z=-1$$

$\Rightarrow$

$$3x+2y=5+k$$
$$x-2y=-1+2k$$

$$x=1+\frac{3k}{4}$$

$$y=1-\frac{5k}{8} \quad \& \quad z=k$$

$$\frac{x-1}{(3/4)}=\frac{y-1}{(-5/8)}=z=k$$

$$\boxed{\frac{x-1}{6}=\frac{y-1}{-5}=\frac{z}{8}}$$

let point on line $A(1+6\lambda, 1-5\lambda, 8\lambda)$
$P(4,-14,4)$

$PA=(6\lambda-3, -5\lambda-15, 8\lambda-4)$

$\vec{PA}\cdot(6\hat{i}-5\hat{j}+8\hat{k})=0$

$$36\lambda-18+25\lambda+75+64\lambda-32=0$$
$$125\lambda+25=0$$
$$\lambda=-1/5$$

⑥ Continues...

$$PA\left(6\left(-\tfrac{1}{5}\right)-3,\ -5\left(-\tfrac{1}{5}\right)-15,\ 8\left(-\tfrac{1}{5}\right)-4\right)$$

$$PA\left(-\frac{21}{5},\ -14,\ -\frac{28}{5}\right)$$

eq^n of line

$$\boxed{\frac{x-4}{-21/5} = \frac{y+14}{-14} = \frac{z-4}{-28/5}}$$

$$\boxed{\frac{x-4}{3} = \frac{y+14}{10} = \frac{z-4}{4}}$$

7. Feet of the perpendicular drawn from the point P(2,3,-5) on the axes of Cordinates are A, B and C.
Find the equation of the plane Passing through their feet and the area of ABC.

Sol^n

A (2,0,0)

B (0,3,0)

C (0,0,-5)

eq^n of plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{-5} = 1$

$$\text{Area} = \frac{1}{2} |\vec{AB} \times \vec{BC}|$$

$$= \frac{1}{2} |(-2\hat{i} + 3\hat{j}) \times (-3\hat{j} - 5\hat{k})|$$

$$\Rightarrow \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & 0 \\ 0 & -3 & -5 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2} |-15\hat{i} - 10\hat{j} + 6\hat{k}|$$

$$\Rightarrow \frac{1}{2} \sqrt{225 + 100 + 36}$$

$$\Rightarrow \frac{1}{2} \sqrt{361} = \frac{19}{2}$$

8 Find the equation of the plane containing the straight line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$ and perpendicular to the plane

$$x - y + z + 2 = 0$$

Sol^n:

line : $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$

Plane : $x - y + z + 2 = 0$

D.R of normal of required plane

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 5 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\Rightarrow 2\hat{i} + 3\hat{j} + \hat{k}$$

Passing through point $(1, -2, 0) \rightarrow$ line contained by plane.

eq^n of plane

$$2x + 3y + z = 2 - 6$$

$$\boxed{2x + 3y + z + 4 = 0}$$

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Exercise JM

EXERCISE (JM)

1. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals

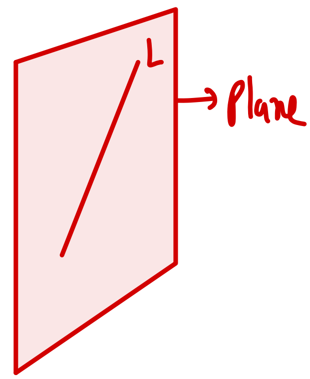
[AIEEE-2009]

(1) $(5, -15)$ (2) $(-5, 5)$ (3) $(6, -17)$ (4) $(-6, 7)$

Solution: Line passes through $(2, 1, -2)$.

and D.R.'s of line are $(3, -5, 2)$

Plane is $x + 3y - \alpha z + \beta = 0$



So, $2 + 3(1) - \alpha(-2) + \beta = 0$

and $1\cdot 3 + 3\cdot(-5) + (-\alpha)(2) = 0$

$\Rightarrow 2\alpha + \beta = -5$ and $3 - 15 - 2\alpha = 0$

$\Rightarrow 2\alpha + \beta = -5$ and $2\alpha = -12$

$\Rightarrow \alpha = -6$ and $\beta = 7$

$\Rightarrow (\alpha, \beta) \equiv (-6, 7)$

2. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are :- **[AIEEE-2009]**

(1) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$ (2) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (3) 6, –3, 2 (4) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$

Solution:

$$D.C's \equiv \pm\left(\frac{6\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{6^2+(-3)^2+2^2}}\right)$$

$$\equiv \pm\left(\frac{6\hat{i}-3\hat{j}+2\hat{k}}{7}\right)$$

$$\equiv \pm\left(\frac{6}{7}\hat{i}-\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}\right)$$

3. **Statement–1 :** The point A(3, 1, 6) is themirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$. **[AIEEE-2010]**

Statement–2 : The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

(1) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(2) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for statement–1.

(3) Statement–1 is true, Statement–2 is false.

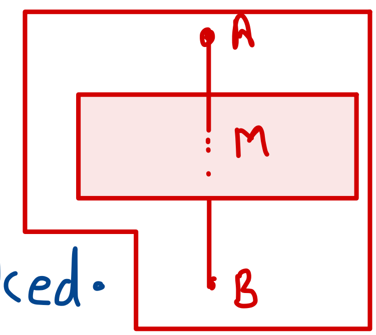
(4) Statement–1 is false, Statement–2 is true.

Solution: 1 Mid-point of AB is $(2, 2, 5)$.

and it is satisfying the plane $x-y+z=5$.

2 D.R's of AB $\equiv (2, -2, 2)$ or $(1, -1, 1)$

which are same as D.R.'s of normal of the given plane.



So, two things should be checked.

Statement-2 is verifying only 1, so it is not complete justification for statement 1

So, Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for statement–1.

4. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals:- **[AIEEE-2011]**

(1) $\frac{2}{5}$ (2) $\frac{5}{3}$ ~~(3)~~ $\frac{2}{3}$ (4) $\frac{3}{2}$

Solution:

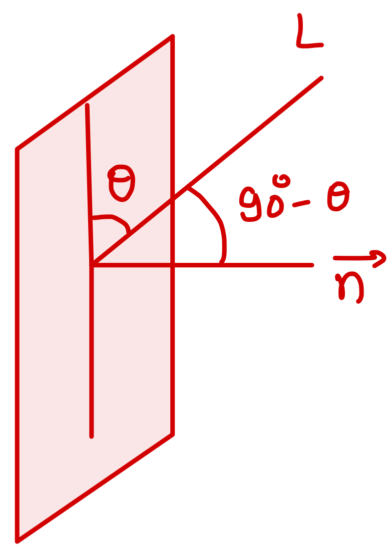
$$\cos(90^\circ - \theta) = \frac{1\cdot 1 + 2\cdot 2 + \lambda\cdot 3}{\sqrt{1^2+2^2+\lambda^2}\,\sqrt{1^2+2^2+3^2}}$$

$$\Rightarrow \sin\theta = \frac{3\lambda+5}{\sqrt{\lambda^2+5}\,\sqrt{14}} = \frac{3}{\sqrt{14}}$$

$$\Rightarrow \frac{3\lambda+5}{\sqrt{\lambda^2+5}} = 3 \Rightarrow (3\lambda+5)^2 = 9(\lambda^2+5)$$

$$\Rightarrow 30\lambda + 25 = 45$$

$$\Rightarrow \boxed{\lambda = \frac{2}{3}}$$



5. **Statement-1 :** The point A(1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.

Statement-2 : The line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining A (1, 0, 7) and B(1, 6, 3). **[AIEEE-2011]**

(1) Statement-1 is true, Statement-2 is false.

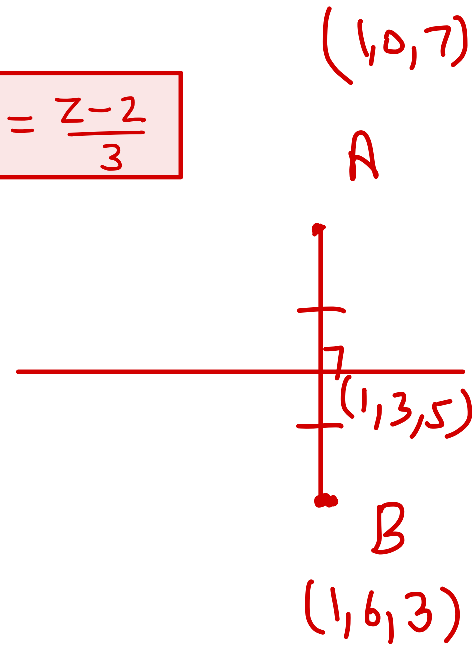
(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

Solution: ① Mid-point of AB ≡ (1, 3, 5)

which is satisfying the line, $\boxed{\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}}$



② D.R.'s of AB ≡ (0, 6, −4)

DR's of line ≡ (1, 2, 3)

See, $0\times1+6\times2+(-4)\times(3)=0$

So, AB ⊥r to line L.

So, two things should be checked.

Statement-2 is verifying only ①, so it is not complete justification for statement ①

So, Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for statement–1.

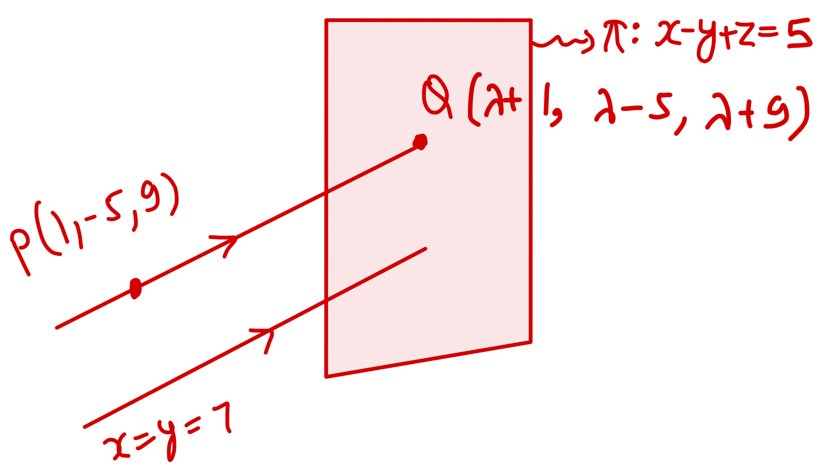
6. The distance of the point (1, –5, 9) from the plane x – y + z = 5 measured along a straight line x = y = z is : **[AIEEE-2011]**

(1) $3\sqrt{5}$ (2) $10\sqrt{3}$ (3) $5\sqrt{3}$ (4) $3\sqrt{10}$

Solution:

Let equation of line passing through (1, -5, 9) and parallel to $x=y=z$ is

$$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda \text{ (say)}$$



Coordinate of $Q \equiv (\lambda+1, \lambda-5, \lambda+9)$ which will lie on $\Pi: x-y+z=5$

$$\Rightarrow (\lambda+1) - (\lambda-5) + (\lambda+9) = 5 \Rightarrow \lambda = -10$$

$$\therefore PQ = \sqrt{\lambda^2+\lambda^2+\lambda^2} = \boxed{10\sqrt{3}}$$

EXERCISE (JM)

7 An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is : **[AIEEE-2012]**

(1) $x - 2y + 2z + 5 = 0$ (2) $x - 2y + 2z - 3 = 0$

(3) $x - 2y + 2z + 1 = 0$ (4) $x - 2y + 2z - 1 = 0$

Sol. Equation of plane parallel to $x - 2y + 2z - 5 = 0$ is $x - 2y + 2z + k = 0$ ...... (1)
perpendicular distance from O(0, 0, 0) to (1) is 1

$$\frac{|k|}{\sqrt{1+4+4}} = 1 \quad \Rightarrow |k| = 3 \quad \Rightarrow k = \pm 3 \quad \therefore x - 2y + 2z - 3 = 0$$

8

If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to :

[AIEEE-2012]

(1) 0 (2) – 1 (3) $\frac{2}{9}$ (4) $\frac{9}{2}$

Sol. Any point on $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=t$ is $(2t+1, 3t-1, 4t+1)$

And any point on $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}=s$ is $(s+3, 2s+k, s)$

Given lines are intersecting $\Rightarrow t=-\frac{3}{2}$ and $s=-5$ $\therefore k=\frac{9}{2}$

9

Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is :-

[JEE-MAIN 2013]

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{7}{2}$ (4) $\frac{9}{2}$

Sol:-

$2x + y + 2z - 8 = 0$ ...(P_1)

$2x + y + 2z + \frac{5}{2} = 0$ ...(P_2)

Distance between P_1 and P_2 = $\left|\frac{-8-\frac{5}{2}}{\sqrt{2^2+1^2+2^2}}\right|=\frac{7}{2}$

10

If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have :

[JEE-MAIN 2013]

(1) any value

(2) exactly one value

(3) exactly two values

(4) exactly three values.

Sol :-

$[a - c, b, d] = 0$

$$\begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(1 + 2k) + (1 + k^2) - (2 - k) = 0$

$\Rightarrow k^2 + 2k + k = 0$

$\Rightarrow k^2 + 3k = 0$

$\Rightarrow k = 0, -3$

11

A vector $\vec{n}$ is inclined to x-axis at 45°, to y –axis at 60° and at an acute angle to z-axis. If $\vec{n}$ is a normal to a plane passing through the point ($\sqrt{2}$, –1, 1), then the equation of the plane is :

[JEE-MAIN Online 2013]

(1) $\sqrt{2}x - y - z = 2$

(2) $\sqrt{2}x + y + z = 2$

(3) $3\sqrt{2}x - 4y - 3z = 7$

(4) $4\sqrt{2}x + 7y + z = 2$

Sol:- $l = C_{45^\circ}$; $m = C_{60^\circ}$; $n = C_\theta$

$\because l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{4} + n^2 = 1 \Rightarrow n^2 = \frac{1}{4}$

$\Rightarrow \boxed{n = \frac{1}{2}}$

$\therefore \left.\begin{aligned} l &= \frac{1}{\sqrt{2}} \\ m &= \frac{1}{2} \\ n &= \frac{1}{2} \end{aligned}\right\}$ so, $\vec{n} = l\hat{i} + m\hat{j} + n\hat{k}$

$\boxed{\vec{n} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}}$

eq^n of plane $\Rightarrow \frac{x}{\sqrt{2}} + \frac{y}{2} + \frac{z}{2} = d$

pass through $(\sqrt{2}, -1, 1)$

$1 - \frac{1}{2} + \frac{1}{2} = d \Rightarrow \boxed{d = 1}$

so eq^n of plane $\Rightarrow \boxed{\sqrt{2}x + y + z = 2}$ 2.

12

The acute angle between two lines such that the direction cosines ℓ, m, n of each of them satisfy the equations $\ell + m + n = 0$ and $\ell^2 + m^2 - n^2 = 0$ is :- **[JEE-MAIN Online 2013]**

(1) 30° (2) 45° ✓(3) 60° (4) 15°

12 $\quad l + m + n = 0 \quad$ and $\quad n^2 = l^2 + m^2$

$(l+m)^2 = l^2 + m^2 \Rightarrow lm = 0$

$(l, m, n) \equiv \underbrace{(0, m, -m)}_{=\vec{P}}$ and $(l, m, n) \equiv \underbrace{(l, 0, -l)}_{=\vec{q}}$

$$\cos\theta = \frac{\vec{P}\cdot\vec{q}}{|\vec{P}||\vec{q}|} = \left(\frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}\right)\cdot\left(\frac{1}{\sqrt{2}}\hat{i} - \frac{\hat{k}}{\sqrt{2}}\right)$$

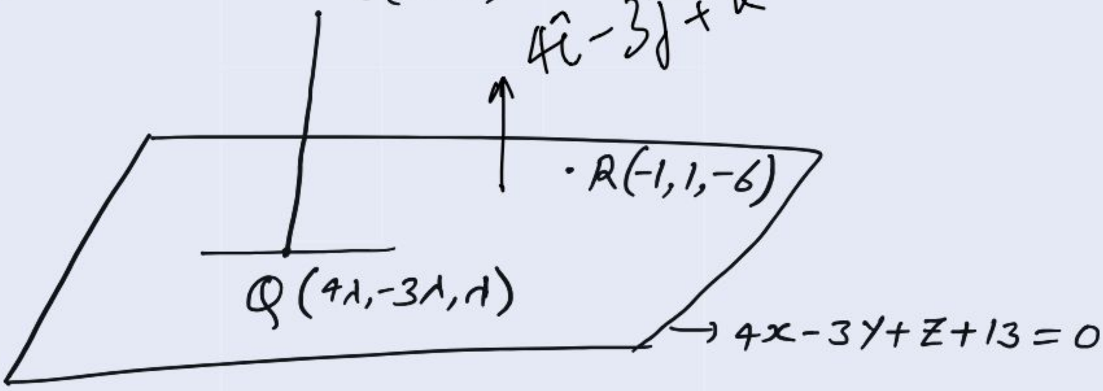
$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \pi/3$$

13

Let Q be the foot of perpendicular from the origin to the plane $4x - 3y + z + 13 = 0$ and R be a point $(-1, 1, -6)$ on the plane. Then length QR is :- **[JEE-MAIN Online 2013]**

(1) $3\sqrt{\frac{7}{2}}$ (2) $\sqrt{14}$ (3) $\sqrt{\frac{19}{2}}$ (4) $\frac{3}{\sqrt{2}}$

Sol:-



line OQ : $\frac{x}{4} = \frac{y}{-3} = \frac{z}{1} = \lambda$

$\because$ $Q(4\lambda, -3\lambda, \lambda)$ lie on Plane $4x - 3y + z + 13 = 0$

$\therefore$ $16\lambda + 9\lambda + \lambda + 13 = 0$

$\Rightarrow$ $\boxed{\lambda = -1/2}$

$\therefore$ Point $Q\left(-2, \frac{3}{2}, \frac{-1}{2}\right)$

$\therefore$ $QR = \sqrt{(-2+1)^2 + \left(\frac{3}{2} - 1\right)^2 + \left(\frac{-1}{2} + 6\right)^2}$

$= \boxed{3\sqrt{\frac{7}{2}}}$ Ans.

14

If the projections of a line segment on thex, y and z-axes in 3-dimensional space are 2, 3 and 6 respectively, then the length ofthe line segment is : **[JEE-MAIN Online 2013]**

(1) 7 (2) 9 (3) 12 (4) 6

Sol:- let $\vec{AB} = x\hat{i} + y\hat{j} + z\hat{k}$

* $\because$ Projection at X-axis = 2

$\Rightarrow \left|\dfrac{\vec{AB}\cdot\hat{i}}{|\hat{i}|}\right| = 2 \Rightarrow \boxed{x = 2}$

similarly $\boxed{y = 3}$ & $\boxed{z = 6}$

$\therefore \vec{AB} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

so $|\vec{AB}| = \sqrt{4+9+36} = 7$ Ans.

15

If two lines L_1 and L_2 in space, are defined by **[JEE-MAIN Online 2013]**

$L_1 = \{x = \sqrt{\lambda}\, y + (\sqrt{\lambda} - 1)$
$z = (\sqrt{\lambda} - 1)\, y + \sqrt{\lambda}\}$ and

} l_1 line is intersecting line of both plane

$L_2 = \{x = \sqrt{\mu}\, y + (1 - \sqrt{\mu})$

$z = (1 - \sqrt{\mu})\, y + \sqrt{\mu}\}$, then L_1 is perpendicular to L_2, for all non-negative reals λ and μ, such that :

(1) $\lambda = \mu$ (2) $\lambda \neq \mu$ (3) $\sqrt{\lambda} + \sqrt{\mu} = 1$ (4) $\lambda + \mu = 0$

Sol: * if $\vec{n}_1$ is dirⁿ of L_1

then $\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -\sqrt{\lambda} & 0 \\ 0 & \sqrt{\lambda}-1 & -1 \end{vmatrix} = \sqrt{\lambda}\,\hat{i} + \hat{j} + (\sqrt{\lambda}-1)\hat{k}$

* if $\vec{n}_2$ is dirⁿ of L_2

then $\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -\sqrt{\mu} & 0 \\ 0 & 1-\sqrt{\mu} & -1 \end{vmatrix} = \sqrt{\mu}\,\hat{i} + \hat{j} + (1-\sqrt{\mu})\hat{k}$

$\because$ if $l_1 \perp l_2$

then $\vec{n}_1 \cdot \vec{n}_2 = 0$

$\Rightarrow \sqrt{\lambda\mu} + 1 + (\sqrt{\lambda}-1)(1-\sqrt{\mu}) = 0$

9

If two lines L_1 and L_2 in space, are defined by **[JEE-MAIN Online 2013]**

$L_1 = \{x = \sqrt{\lambda}\, y + (\sqrt{\lambda} - 1)$
$z = (\sqrt{\lambda} - 1)\, y + \sqrt{\lambda}\}$ and

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(1) $\lambda = \mu$ (2) $\lambda \neq \mu$ (3) $\sqrt{\lambda} + \sqrt{\mu} = 1$ (4) $\lambda + \mu = 0$

15 Continues...

M2 L_1: $$\left.\begin{aligned} x &= \sqrt{\lambda}\, y + \sqrt{\lambda} - 1 \\ z &= (\sqrt{\lambda} - 1) y + \sqrt{\lambda} \end{aligned}\right\}$$

$$\Rightarrow \frac{x - (\sqrt{\lambda} - 1)}{\sqrt{\lambda}} = y = \frac{z - \sqrt{\lambda}}{\sqrt{\lambda} - 1}$$

$\therefore L_1$ is $\parallel$ to $\sqrt{\lambda}\,\hat{i} + \hat{j} + (\sqrt{\lambda} - 1)\hat{k}$

L_2: $$\begin{aligned} x &= \sqrt{\mu}\, y + 1 - \sqrt{\mu} \\ z &= (1 - \sqrt{\mu}) y + \sqrt{\mu} \end{aligned}$$

$$\Rightarrow \frac{x - (1 - \sqrt{\mu})}{\sqrt{\mu}} = y = \frac{z - \sqrt{\mu}}{1 - \sqrt{\mu}}$$

$\therefore L_2$ is $\parallel$ to $\sqrt{\mu}\,\hat{i} + \hat{j} + (1 - \sqrt{\mu})\hat{k}$

$\because L_1 \perp L_2$

$$\Rightarrow \sqrt{\lambda} \cdot \sqrt{\mu} + 1 + (\sqrt{\lambda} - 1)(1 - \sqrt{\mu}) = 0$$

$$\Rightarrow \sqrt{\lambda\mu} + 1 + \sqrt{\lambda} - \sqrt{\lambda\mu} - 1 + \sqrt{\mu} = 0$$

$$\Rightarrow \sqrt{\lambda} + \sqrt{\mu} = 0$$

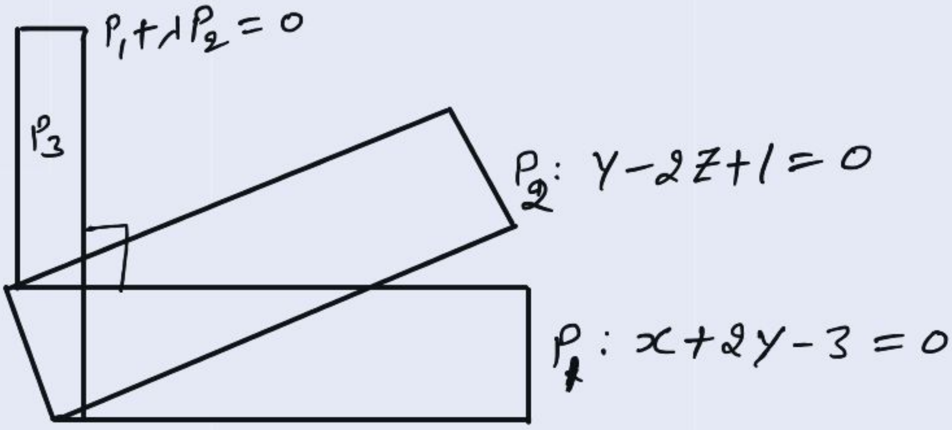
This is possible only when $\lambda = \mu = 0$

16

The equation of a plane through the line of intersection of the planes $x+2y=3$, $y-2z+1=0$, and perpendicular to the first plane is : **[JEE-MAIN Online 2013]**

(1) $2x-y+7z=11$ (2) $2x-y+10z=11$ (3) $2x-y-9z=10$ (4) $2x-y-10z=9$

Sol:-



$\therefore \quad P_1+\lambda P_2=0$

$P_3: \boxed{x+(2+\lambda)y-2\lambda z+(\lambda-3)=0}$ — (1)

$\because \quad P_3 \perp P_1$

$\therefore \quad 1+2\cdot(2+\lambda)-(2\lambda)\cdot 0=0$

$\Rightarrow \quad \lambda=-5/2$ put in (1)

$\therefore \quad x+\left(2-\frac{5}{2}\right)y-2\left(\frac{-5}{2}\right)z+\left(\frac{-5}{2}-3\right)=0$

$\Rightarrow \quad x-\frac{1}{2}y+5z-\frac{11}{2}=0$

$\Rightarrow \quad \boxed{2x-y+10z=11}$ 2.

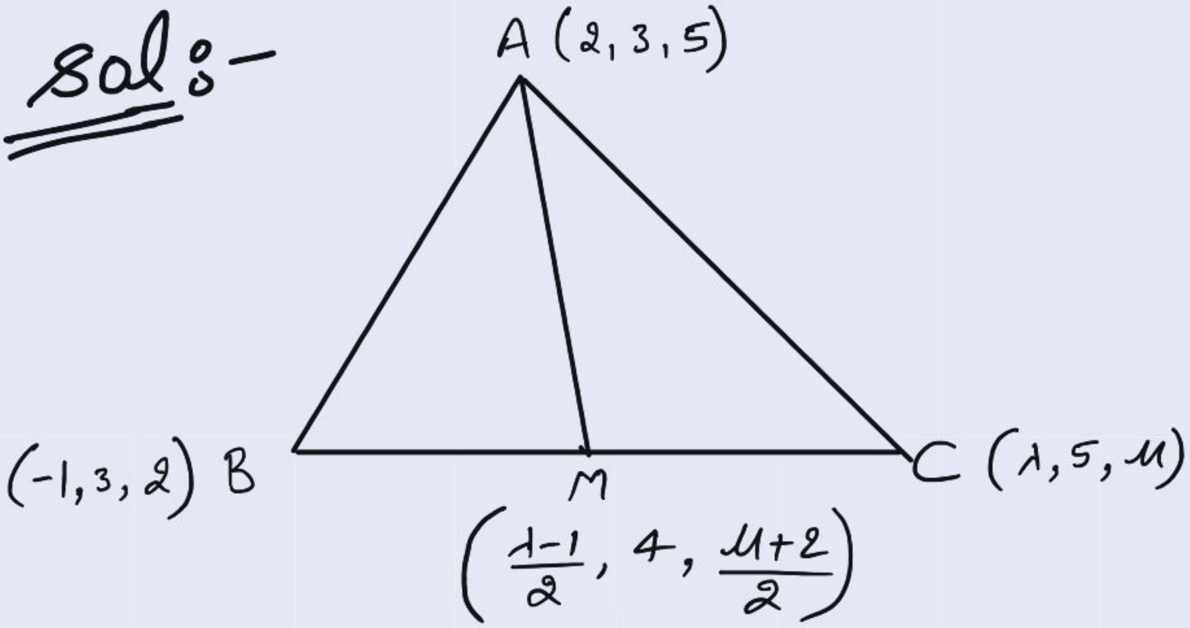
17

Let ABC be a triangle with vertices at points A (2, 3, 5), B (-1, 3, 2) and C (λ, 5, μ) in three dimensional space. If the median through A is equally inclined with the axes, then (λ, μ.) is equal to :

[JEE-MAIN Online 2013]

(1) (10, 7) (2) (7.5) (3) (7, 10) (4) (5,7)

Sol:-



$$\therefore \quad \overrightarrow{AM} = \left(\frac{\lambda-1}{2} - 2\right)\hat{i} + \hat{j} + \left(\frac{\mu+2}{2} - 5\right)\hat{k} \quad \text{—— (1)}$$

$\because$ $\overrightarrow{AM}$ is equally inclined with Co-ordinate axes.

$$\therefore \quad \frac{\lambda-1}{2} - 2 = 1 = \frac{\mu+2}{2} - 5$$

$$\Rightarrow \quad \lambda = 7 \; ; \; \mu = 10 \qquad (\text{Ans } 3)$$

18

The angle between the lines whose direction cosines satisfy the equations $\ell + m + n = 0$ and $\ell^2 = m^2 + n^2$ is : **[JEE-MAIN 2014]**

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

Sol:-

$\ell + m + n = 0;$ $\ell^2 = m^2 + n^2$

$\Rightarrow \ell = -(m + n)$

$\Rightarrow m^2 + n^2 + 2mn = m^2 + n^2$

$\Rightarrow mn = 0$

$\Rightarrow m = 0$ or $n = 0$

Case I	Case II
$m = 0$	$n = 0$
$\ell + n = 0$	$\ell + m = 0$
$\Rightarrow \ell = k$	$\ell = k$
$m = 0$	$m = -k$
$n = -k$	$n = 0$
$\ell = 1/\sqrt{2}$	$\ell = 1/\sqrt{2}$
$m = 0$	$m = -1/\sqrt{2}$
$n = -1/\sqrt{2}$	$n = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \pi/3.$

19

The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x-y+z+3=0$ is the line : **[JEE-MAIN 2014]**

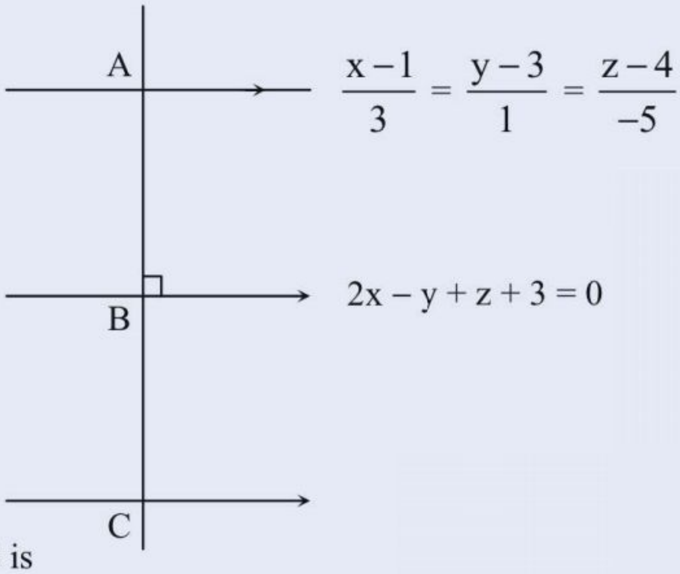
(1) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

(2) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

(3) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

(4) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

Sol:-



Equation of AB is

$$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}=\lambda$$

Co-ordinate of point B is

$\Rightarrow$ $x = 1 + 2\lambda$ point satisfy the equation of plane

$y = 3 - \lambda$

$z = 4 + \lambda$

$2(1 + 2\lambda) - (3 - \lambda) + (4 + \lambda) + 3 = 0$

$\Rightarrow$ $\lambda = -1$

$\Rightarrow$ Co-ordinate of point B$(-1, 4, 3)$

$\Rightarrow$ Co-ordinate of point C$(-3, 5, 2)$

$\Rightarrow$ equation of line passing through 'C' is

$$\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$$

20

The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is : **[JEE(Main)-2015]**

(1) $x + 3y + 6z = 7$

(2) $2x + 6y + 12z = -13$

(3) $2x + 6y + 12z = 13$

(4) $x + 3y + 6z = -7$

Sol:-

Let equation of plane is $(2x - 5y + z - 3) + \lambda (x + y + 4z - 5) = 0$

As plane is parallel to $x + 3y + 6z - 1 = 0$

$$\frac{2+\lambda}{1} = \frac{\lambda-5}{3} = \frac{1+4\lambda}{6}$$

$\Rightarrow 6 + 3\lambda = \lambda - 5$

$11 = -2\lambda$

$$\lambda = -\frac{11}{2}$$

Also, $6\lambda - 30 = 3 + 12\lambda$

$-6\lambda = 33$

$$\lambda = -\frac{11}{2}.$$

so the equation of required plane is

$(4x - 10y + 2z - 6) - 11 (x + y + 4z - 5) = 0$

$\Rightarrow -7x - 21y - 42z + 49 = 0$

$\Rightarrow x + 3y + 6z - 7 = 0.$

(21)

The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x - y + z = 16$, is : **[JEE(Main)-2015]**

(1) $3\sqrt{21}$ (2) 13 (3) $2\sqrt{14}$ (4) 8

Sol:-

Let the point of intersection be $(2 + 3\lambda, 4\lambda - 1, 12\lambda + 2)$

$(2 + 3\lambda) - (4\lambda - 1) + 12\lambda + 2 = 16$

$11\lambda = 11$

$\lambda = 1$

$\Rightarrow$ point of intersection is (5, 3, 14)

$\Rightarrow$ distance $= \sqrt{(5-1)^2+9+12^2}$

$= \sqrt{16+9+144} = 13.$

(22)

The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is : **[JEE(Main)-2016]**

(1) $\frac{20}{3}$ (2) $3\sqrt{10}$ (3) $10\sqrt{3}$ (4) $\frac{10}{\sqrt{3}}$

Sol:-

Let Q (1, –5, 9)

$$\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-0}{1}$$

Line is $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=r$ (say)

Any pt on line we can take P $(r+1, r-5, r+9)$

So, Pt satisfy Plane

$\Rightarrow (r+1)-(r-5)+(r+9)=5$

$r=-10$

So, Point $P=(-9,-15,-1)$

Distance is $PQ=\sqrt{(10)^2+(10)^2+(10)^2}=10\sqrt{3}$

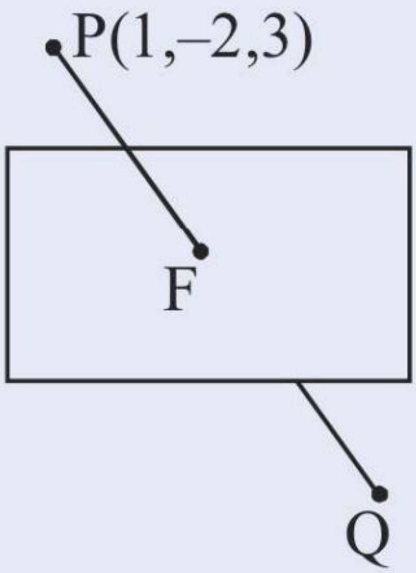
23

If the image of the point P(1, –2, 3) in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to :- **[JEE(Main)-2017]**

(1) $6\sqrt{5}$ (2) $3\sqrt{5}$ (3) $2\sqrt{42}$ (4) $\sqrt{42}$

Sol. Line PQ ; $\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}$

Let $F(\lambda + 1, 4\lambda - 2, 5\lambda + 3)$



F lies on the plane

$2(\lambda + 1) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$

$\Rightarrow -6\lambda + 6 = 0 \Rightarrow \lambda = 1$

F (2, 2, 8)

$PQ = 2\ PF = 2\sqrt{42}$.

24

The distantce of the point $(1, 3, -7)$ from the plane passing through the point $(1, -1, -1)$, having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is :- **[JEE(Main)-2017]**

(1) $\frac{10}{\sqrt{74}}$ (2) $\frac{20}{\sqrt{74}}$ (3) $\frac{10}{\sqrt{83}}$ (4) $\frac{5}{\sqrt{83}}$

Sol. Normal vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i}+7\hat{j}+3\hat{k}$$

So plane is $5(x - 1) + 7(y + 1) + 3(z + 1) = 0$

$\Rightarrow 5x + 7y + 3z + 5 = 0$

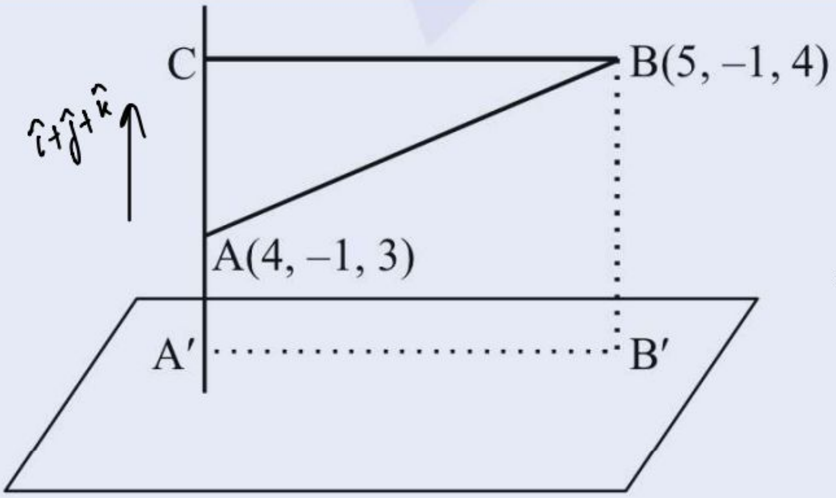
Distance $\frac{5+21-21+5}{\sqrt{25+49+9}} = \frac{10}{\sqrt{83}}$

25

The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, x + y + z = 7 is : **[JEE(Main)-2018]**

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\frac{2}{\sqrt{3}}$

Sol.



$\because$ AC → projection of AB on normal of plane.

$$AC = \overrightarrow{AB}.\widehat{AC}$$

$$= \left(\hat{i}+\hat{k}\right).\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$A'B' = BC = \sqrt{AB^2 - AC^2} = \sqrt{2-\frac{4}{3}} = \sqrt{\frac{2}{3}}$$

Length of projection $= \sqrt{\frac{2}{3}}$

26

If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2 is : **[JEE(Main)-2018]**

(1) $\frac{1}{3\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{4\sqrt{2}}$

Sol. Plane passes through line of intersectuion of first two planes is

$$(2x - 2y + 3z - 2) + \lambda(x - y + z + 1) = 0$$

$$x(\lambda + 2) - y(2 + \lambda) + z(\lambda + 3) + (\lambda - 2) = 0 \quad \text{..... (1)}$$

is having infinite number of solution with $x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$ then

$$\begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

Solving $\lambda = 5$

$$7x - 7y + 8z + 3 = 0$$

perpendicular distance from $(0, 0, 0)$

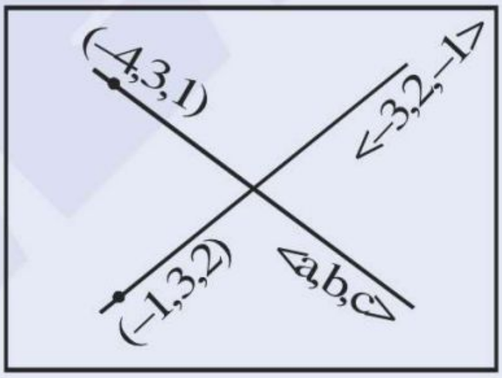
is $\frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$

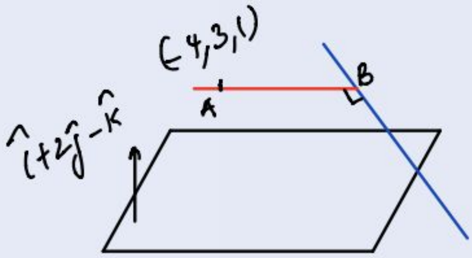
27

The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is : **[JEE(Main)-Jan 19]**

(1) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$ (2) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$ (3) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$ (4) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Sol.





Normal vector of plane containing two intersecting lines is parallel to vector.

$$(\vec{V}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}$$

$$= -2\hat{i} + 6\hat{k}$$

$\therefore$ Required line is parallel to vector

$$(\vec{V}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

$\Rightarrow$ Required equation of line is

$$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

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If the lines $x = ay + b$, $z = cy + d$ and $x=a'z + b'$, $y = c'z + d'$ are perpendicular, then :

[JEE(Main)-Jan 19]

(1) $cc' + a + a' = 0$

(2) $aa' + c + c' = 0$

(3) $ab' + bc' + 1 = 0$

(4) $bb' + cc' + 1 = 0$

Sol. Line $x = ay + b, z = cy + d \Rightarrow \dfrac{x-b}{a} = \dfrac{y}{1} = \dfrac{z-d}{c}$

Line $x = a'z + b'$, $y = c'z + d'$

$$\Rightarrow \frac{x-b'}{a'} = \frac{y-d'}{c'} = \frac{z}{1}$$

Given both the lines are perpendicular

$\Rightarrow aa' + c' + c = 0$

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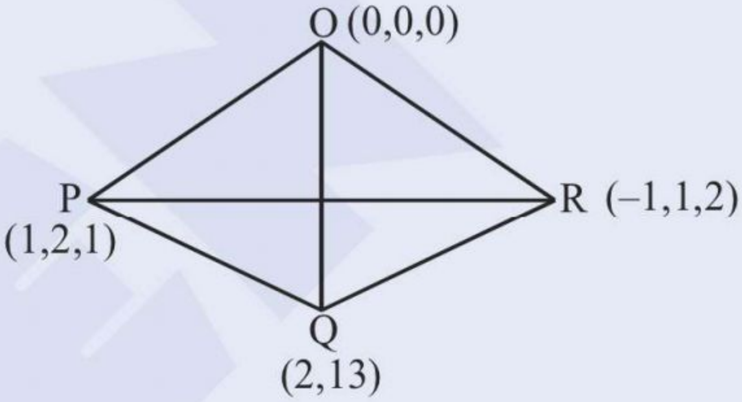
A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(–1,1,2) and O(0, 0, 0). The angle between the faces OPQ and PQR is : **[JEE(Main)-Jan 19]**

(1) $\cos^{-1}\left(\frac{9}{35}\right)$ (2) $\cos^{-1}\left(\frac{19}{35}\right)$ (3) $\cos^{-1}\left(\frac{17}{31}\right)$ (4) $\cos^{-1}\left(\frac{7}{31}\right)$

Sol. $\overrightarrow{OP}\times\overrightarrow{OQ}=(\hat{i}+2\hat{j}+\hat{k})\times(2\hat{i}+\hat{j}+3\hat{k})$

$5\hat{i}-\hat{j}-3\hat{k}$



$\overrightarrow{PQ}\times\overrightarrow{PR}=(\hat{i}-\hat{j}+2\hat{k})\times(-2\hat{i}-\hat{j}+\hat{k})$

$\hat{i}-5\hat{j}-3\hat{k}$

$$\cos\theta=\frac{5+5+9}{\left(\sqrt{25+9+1}\right)^2}=\frac{19}{35}$$

30

A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point : **[JEE(Main)-Apr 19]**

(1) (2,4,1) (2) (2, – 4, 1) (3) (1, 4, – 1) (4) (1, – 4, 1)

Sol. equation of bisector of angle

$$\frac{2x-y+2z-4}{3}=\pm\frac{x+2y+2z-2}{3}$$

(+) gives $x - 3y = 2$

(–) gives $3x + y + 4z = 6$

therefore option (ii) satisfy

Exercise JA

EXERCISE (JA)

1. (a) Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

(A) $x + 2y - 2z = 0$ (B) $3x + 2y - 2z = 0$

(C) $x - 2y + z = 0$ (D) $5x + 2y - 4z = 0$

Sol. Direction ratio of normal to plane containing the straight line

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\hat{i} - \hat{j} - 10\hat{k}$$

Then, Required plane $\begin{vmatrix} x-0 & y-0 & z-0 \\ 2 & 3 & 4 \\ 8 & -1 & -10 \end{vmatrix} = 0$

$\Rightarrow -26x + 52y - 26z = 0 \Rightarrow x - 2y + z = 0$

(b) If the distance of the point P(1,–2,1) from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is-

(A) $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$ (B) $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$ (C) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$ (D) $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

Sol. Distance of point $(1, -2, 1)$ from plane $x + 2y - 2z = \alpha$ is $5 \Rightarrow \alpha = 10$.

Equation of PQ $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-1}{-2}=t$

$Q \equiv (t + 1, 2t - 2, -2t + 1)$ and $PQ = 5 \Rightarrow t = \frac{5+\alpha}{9}=\frac{5}{3} \Rightarrow Q \equiv \left(\frac{8}{3}, \frac{4}{3}, \frac{-7}{3}\right)$.

(c) If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$, then $|d|$ is

Sol. Equation of plane is $\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$

$x - 2y + z = 0$ ....(1)

$Ax - 2y + z = d$ ....(2)

Compare $\frac{A}{1}=\frac{-2}{-2}=\frac{1}{1} \Rightarrow A = 1$

Distance between planes is $\left|\frac{d}{\sqrt{1+1+4}}\right| = \sqrt{6}$

$\Rightarrow |d| = 6$

(d) Match the statements in **Column-I** with the values in **Column-II**.

Column-I	Column-II
(A) A line from the origin meets the lines L_1 $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and L_2 $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(p) –4
(B) The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0
(C) Non-zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b}=0$, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$ and $2\left|(\vec{b}+\vec{c})\right|=\left|(\vec{b}-\vec{a})\right|$. $\quad |\vec{b}|^2+\vec{b}\cdot\vec{c}=\vec{a}\cdot\vec{c}$ If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	(r) 4
(D) Let f be the function on $[-\pi,\pi]$ given by $f(0) = 9$ and $f(x)=\sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for $x \neq 0$.	(s) 5
	(t) 6
The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ is	

Sol. A) $P(\lambda+2, 1-2\lambda, \lambda-1)$

$$Q\left(2\mu+\frac{8}{3}, -\mu-3, \mu+1\right)$$

$$|PQ|^2 = d^2 = \left(\lambda-2\mu-\frac{2}{3}\right)^2+(\mu-2\lambda+4)^2+(\lambda-\mu-2)^2$$

As $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are collinear $\frac{\lambda+2}{2\mu+\frac{8}{3}}=\frac{1-2\lambda}{-\mu-3}=\frac{\lambda-1}{\mu+1}$

(from last two)

$\lambda\mu-\lambda+2=0$ ...(i)

and $\lambda\mu-4\mu+\frac{5}{3}\lambda=\frac{14}{3}$ (from Ist and IIIrd) ...(ii)

from (i) and (ii) $2\lambda-3\mu=5$ ... (iii)

from (i) and (iii) $3\mu^2+2\mu-1=0$

$\therefore\ \mu=-1, \frac{1}{3}$

so, $\lambda = 1, 3$

Hence, $d^2=\frac{109}{9}$ or 6

(B) $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$

$\Rightarrow \quad \tan^{-1}(x+3)-\tan^{-1}(x-3)=\tan^{-1}\left(\frac{3}{4}\right)$

Let $\tan^{-1}(x+3)=\alpha$, $\tan^{-1}(x-3)=\beta$

$\Rightarrow \tan\alpha = x+3,\ \tan\beta = x-3$

$\tan(\alpha-\beta)=\frac{3}{4}$

$=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}=\frac{3}{4}$

$=\frac{(x+3)-(x-3)}{1+x^2-9}=\frac{3}{4}$

$=\frac{6}{x^2-8}=\frac{3}{4}$

$=x^2-8=8$

$=x^2=16$

$x=\pm 4$

C) $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$

Put $\vec{C}=\frac{\vec{a}-\mu\vec{b}}{4}$

$(\vec{b}-\vec{a}).\left(\vec{b}+\frac{\vec{a}-\mu\vec{b}}{4}\right)=0$

$(\vec{b}-\vec{a}).((4-\mu)\vec{b}+\vec{a})=0$

$(4-\mu)|b|^2-|a|^2=0$

$\vec{a}.\vec{b}=0$

$(4-\mu)|b|^2=|a|^2 \quad (\because\ \vec{a}.\vec{b}=0)$

Also $2|\vec{b}+\vec{c}|=|\vec{b}-\vec{a}|$ again put $\vec{C}=\frac{\vec{a}-\mu\vec{b}}{4}$

$2\left|\vec{b}+\frac{\vec{a}-\mu\vec{b}}{4}\right|=|\vec{b}-\vec{a}|$

$\frac{1}{2}|(4-\mu)\vec{b}+\vec{a}|=|\vec{b}-\vec{a}|$

$(4-\mu)^2|b|^2+|a|^2=4|b|^2+4|a|^2$ since $\vec{a}.\vec{b}=0$

$\left((4-\mu)^2-4\right)|b|^2=3|a|^2$

$\left((4-\mu)^2-4\right)|b|^2=3(4-\mu)|b|^2$

$(4-\mu)^2-4=12-3\mu$

$16+\mu^2-8\mu-4=12-3\mu$

$\mu^2-5\mu=0$

$\mu=0$ or 5. but $\mu=5$ is not satisfying so $\mu=0$.

(D) ***(r)***

$$I = \frac{2}{\pi}\int_{-\pi}^{\pi}\frac{\sin 9(x/2)}{\sin(x/2)}dx = \frac{2}{\pi}\times 2\int_{0}^{\pi}\frac{\sin 9(x/2)}{\sin(x/2)}dx$$

Even

x/2 = θ ⇒ dx = 2dθ

x = 0, θ = 0

x = π θ = π/2

$$I = \frac{8}{\pi}\int_{0}^{\pi/2}\frac{\sin 9\theta}{\sin\theta}d\theta$$

$$= \frac{8}{\pi}\int_{0}^{\pi/2}\frac{(\sin 9\theta - \sin 7\theta)}{\sin\theta} + \frac{(\sin 7\theta - \sin 5\theta)}{\sin\theta} + \frac{(\sin 5\theta - \sin 3\theta)}{\sin\theta} + \frac{(\sin 3\theta - \sin\theta)}{\sin\theta} + \frac{\sin\theta}{\sin\theta}d\theta$$

$$= \frac{16}{\pi}\int_{0}^{\pi/2}(\cos 8\theta + \cos 6\theta + \cos 4\theta + \cos 2\theta + 1)d\theta + \frac{8}{\pi}\int_{0}^{\pi/2}d\theta$$

$$= \frac{16}{\pi}\left[\frac{\sin 8\theta}{8} + \frac{\sin 6\theta}{6} + \frac{\sin 4\theta}{4} + \frac{\sin 2\theta}{2}\right] + \frac{8}{\pi}[\theta]_0^{\pi/2} = 0 + \frac{8}{\pi}\times\left[\frac{\pi}{2} - 0\right] = 4$$

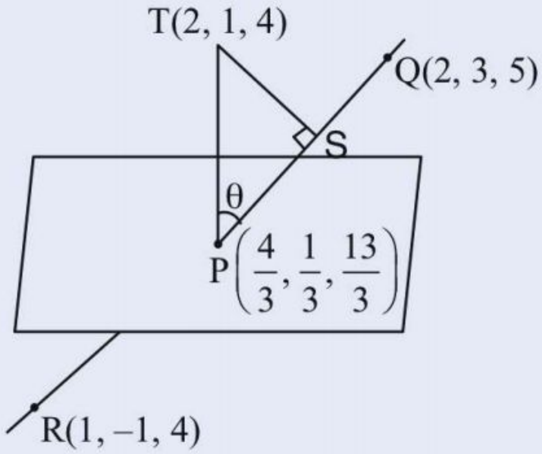
2. **(a)** The point P is the intersection of the straight line joining the points Q(2,3,5) and R(1,–1,4) with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point T(2,1,4) to QR, then the length of the line segment PS is -

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

Sol:

Equation of QR is

$$\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-5}{1}$$



Let $P \equiv (2 + \lambda, 3 + 4\lambda, 5 + \lambda)$

$$10 + 5\lambda - 12 - 16\lambda - 5 - \lambda = 1$$

$$-7 - 12\lambda = 1$$

$$\Rightarrow \quad \lambda = \frac{-2}{3}$$

then $P \equiv \left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$

Let $S = (2 + \mu, 3 + 4\mu, 5 + \mu)$

$$\vec{TS} = (\mu)\hat{i} + (4\mu + 2)\hat{j} + (\mu + 1)\hat{k}$$

$$\vec{TS} \cdot (\hat{i} + 4\hat{j} + \hat{k}) = 0$$

$$\mu + 16\mu + 8 + \mu + 1 = 0$$

$$\mu = -\frac{1}{2}$$

$$S = \left(\frac{3}{2}, 1, \frac{9}{2}\right)$$

$$PS = \sqrt{\left(\frac{4}{3} - \frac{3}{2}\right)^2 + \frac{4}{9} + \left(\frac{13}{3} - \frac{9}{2}\right)^2} = \sqrt{\frac{1}{36} + \frac{4}{9} + \frac{1}{36}} = \sqrt{\frac{1}{18} + \frac{4}{9}} = \sqrt{\frac{9}{18}} = \frac{1}{\sqrt{2}}$$

(b) The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is

(A) $5x - 11y + z = 17$

(B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) $x + y + z = \sqrt{3}$

(D) $x - \sqrt{2}y = 1 - \sqrt{2}$

Sol:-

Equation of required plane is

$$P \equiv (x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$$

$$\Rightarrow (1 + \lambda)x + (2 - \lambda)y + (3 + \lambda)z - (2 + 3\lambda) = 0$$

Its distance from (3, 1, –1) is $\frac{2}{\sqrt{3}}$

$$\Rightarrow \frac{2}{\sqrt{3}} = \frac{|3(1+\lambda) + (2-\lambda) - (3+\lambda) - (2+3\lambda)|}{\sqrt{(\lambda+1)^2 + (2-\lambda)^2 + (3+\lambda)^2}}$$

$$= \frac{4}{3} = \frac{(-2\lambda)^2}{3\lambda^2 + 4\lambda + 14} \Rightarrow 3\lambda^2 + 4\lambda + 14 = 3\lambda^2 \Rightarrow \lambda = -7/2$$

$$\Rightarrow P \equiv -5x + 11y - z + 17 = 0.$$

(c) If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

(A) $y + 2z = -1$ (B) $y + z = -1$ (C) $y - z = -1$ (D) $y - 2z = -1$

[JEE 2012, 3+3+4]

Sol. **(B, C)**

For given lines to be coplanar, we get

$$\begin{vmatrix} 2 & k & 2 \\ 5 & 2 & k \\ 2 & 0 & 0 \end{vmatrix} = 0 \Rightarrow k^2 = 4, k = \pm 2$$

For $k = 2$, obviously the plane $y + 1 = z$ is common in both lines

For $k = -2$, family of plane containing first line is $x + y + \lambda(x - z - 1) = 0$.

Point $(-1, -1, 0)$ must satisfy it

$-2 + \lambda(-2) = 0 \Rightarrow \lambda = -1$

$\Rightarrow y + z + 1 = 0.$

M-II. Apply Condition of Coplanarity then we get $K = \pm 2$

if $K=2$ then eq^n of plane $\begin{vmatrix} x-1 & y+1 & z \\ 2 & 2 & 2 \\ 5 & 2 & 2 \end{vmatrix} = 0$

$\Rightarrow \boxed{y - z = -1}$ Ans.

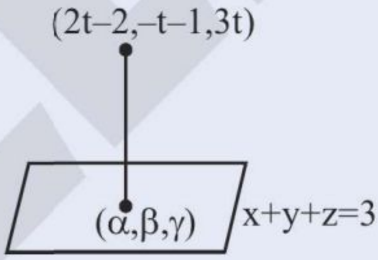
if $K=-2$ then eq^n of plane $\begin{vmatrix} x-1 & y+1 & z \\ 2 & -2 & 2 \\ 5 & 2 & -2 \end{vmatrix} = 0$

$\Rightarrow \boxed{y + z = -1}$ Ans.

3. Perpendiculars are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane $x + y + z = 3$. The feet of perpendiculars lie on the line **[JEE-Advanced 2013, 2]**

(A) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$ (B) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$ (C) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$ (D) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

Sol. $\frac{\alpha-2t+2}{1}=\frac{\beta+t+1}{1}=\frac{\gamma-3t}{1}=k$



$\alpha = k + 2t - 2$

$\beta = k - t - 1$

$\gamma = k + 3t$

$\alpha + \beta + \gamma = 3$

$k=\frac{6-4t}{3}$

$\alpha=\frac{6-4t}{3}+2t-2=\frac{2t}{3}$

$\beta=\frac{6-4t}{3}-t-1=\frac{3-7t}{3}$

$\gamma=\frac{6-4t}{3}+3t=\frac{5t+6}{3}$

$\Rightarrow \frac{3\alpha}{2}=\frac{3\beta-3}{-7}=\frac{3\gamma-6}{5}$

$\Rightarrow \frac{x}{2}=\frac{y-3}{-7}=\frac{z-2}{5}$

4. A line ℓ passing through the origin is perpendicular to the lines

$\ell_1 : (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}, -\infty < t < \infty$

$\ell_2 : (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}, -\infty < s < \infty$

Then , the coordinate(s) of the point(s) on ℓ_2 at a distance of $\sqrt{17}$ from the point of intersection of ℓ and ℓ_1 is(are) - **[JEE-Advanced 2013, 4, (–1)]**

(A) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$ (B) (–1,–1,0) (C) (1,1,1) (D) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

Sol $\ell_1 : \vec{r} = (3, -1, 4) + (1, 2, 2)t$

$\ell_2 : \vec{r} = (3, 3, 2) + (2, 2, 1)s$

vector perpendicular to ℓ_1 and ℓ_2 :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i} + 3\hat{j} - 2\hat{k}$$

$\therefore$ Equation of line ℓ :

$\vec{r} = 0 + (-2, 3, -2)\lambda$

Point of intersection of ℓ_1 and ℓ :

$3 + t = -2\lambda$

$-1 + 2t = 3\lambda.$

$4 + 2t = -2\lambda.$

On solving we get $\lambda = -1$, $t = -1$

$\therefore$ Point of intersection of ℓ_1 & ℓ : P(2, –3, 2)

A point on ℓ_2 at distance of $\sqrt{17}$ from P :

$\Rightarrow (1 + 2s)^2 + (6 + 2s)^2 + s^2 = 17$

$\Rightarrow s = -\frac{10}{9}; s = -2$

for above s, point will be (B), (D)

5. Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then α can take value(s)

[JEE-Advanced 2013, 3, (–1)]

(A) 1 (B) 2 (C) 3 (D) 4

Sol. $L_1 : \frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2}$

$L_2 : \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$

for lines to be coplanar

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$\Rightarrow (5 - \alpha) ((3 - \alpha) (2-\alpha) - 2) = 0$

$\Rightarrow (5 - \alpha)(\alpha^2 - 5\alpha + 4) = 0$

$\Rightarrow \alpha = 1, 4, 5$

6. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}, L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 : 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2 and perpendicular to planes P_1 and P_2.

Match List-I with List-II and select the correct answer using the code given below the lists.

	List-I		List-II
P.	a =	1.	13
Q.	b =	2.	–3
R.	c =	3.	1
S.	d =	4.	–2

Codes :

	P	Q	R	S
(A)	3	2	4	1
(B)	1	3	4	2
(C)	3	2	1	4
(D)	2	4	1	3

[JEE-Advanced 2013, 3, (–1)]

Sol. For point of intersection of L_1 and L_2

$$\begin{cases} 2\lambda + 1 = \mu + 4 \\ -\lambda = \mu - 3 \\ \lambda - 3 = 2\mu - 3 \end{cases} \Rightarrow \begin{cases} \mu = 1 \\ \lambda = 2 \end{cases} \Rightarrow \text{point of intersction is } (5, -2, -1)$$

Now, vector normal to the plane is $\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix}$

$= -16(\hat{i} - 3\hat{j} - 2\hat{k})$

Let equation of required plane be $x - 3y - 2z = \alpha$

$\because$ it passes through $(5, -2, -1)$

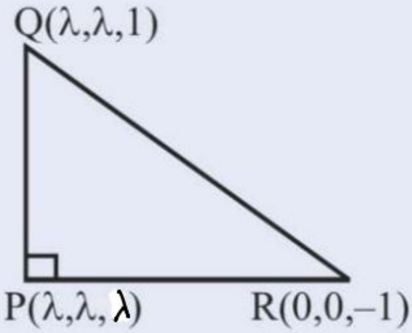
$\therefore$ $\alpha = 13 \Rightarrow$ equation of plane is $x - 3y - 2z = 13$

7. From a point $P(\lambda,\lambda,\lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y = x$, $z = 1$ and $y = -x$, $z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are)

(A) $\sqrt{2}$ (B) 1 (C) -1 (D) $-\sqrt{2}$

[JEE(Advanced)-2014, 3]

Sol. Ans. (C)



Line L_1 given by $y = x$; $z = 1$ can be expressed as

$$L_1: \frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = \alpha$$

Similarly $L_2(y = -x; z = -1)$ can be expressed as

$$L_2: \frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0} = \beta$$

Let any point $Q(\alpha,\alpha,1)$ on L_1 and $R(\beta,-\beta,-1)$ on L_2

Given that PQ is perpendicular to L_1

$\Rightarrow (\lambda - \alpha).1 + (\lambda - \alpha).1 + (\lambda - 1).0 = 0 \Rightarrow \lambda = \alpha$

$\therefore Q(\lambda,\lambda,1)$

Similarly PR is perpendicular to L_2

$(\lambda - \beta).1 + (\lambda + \beta)(-1) + (\lambda + 1).0 = 0 \Rightarrow \beta = 0$

$\therefore R(0,0,-1)$

Now as given

$\Rightarrow \overrightarrow{PR}.\overrightarrow{PQ} = 0$

$0.\lambda + 0.\lambda + (\lambda - 1)(\lambda + 1) = 0$

$\lambda \neq 1$ as P & Q are different points $\Rightarrow \lambda = -1$

8. In $\mathbb{R}^3$, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point $(0,1,0)$ from P_3 is 1 and the distance of a point (α,β,γ) from P_3 is 2, then which of the following relations is (are) true ? **[JEE 2015, 4M, –2M]**

(A) $2\alpha + \beta + 2\gamma + 2 = 0$ (B) $2\alpha - \beta + 2\gamma + 4 = 0$

(C) $2\alpha + \beta - 2\gamma - 10 = 0$ (D) $2\alpha - \beta + 2\gamma - 8 = 0$

Sol. Let $P_3 : (x + z - 1) + \lambda y = 0$

$x + \lambda y + z - 1 = 0$ ...(i)

distance of $(0,1,0)$ from P_3 is 1

$$\Rightarrow \frac{|\lambda - 1|}{\sqrt{2+\lambda^2}} = 1$$

$$\Rightarrow (\lambda - 1)^2 = 2 + \lambda^2$$

$$\Rightarrow \lambda = -\frac{1}{2}$$

$\therefore$ P_3 is $2x - y + 2z - 2 = 0$

distance from (α,β,γ) is $\left|\frac{2\alpha - \beta + 2\gamma - 2}{\sqrt{9}}\right| = 2$

$\therefore$ $2\alpha - \beta + 2\gamma - 2 = 6$ or $2\alpha - \beta + 2\gamma - 2 = -6$

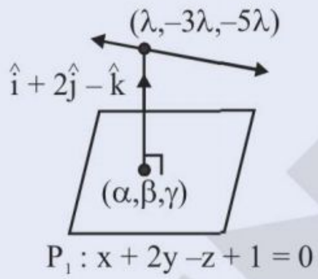
$2\alpha - \beta + 2\gamma - 8 = 0$ or $2\alpha - \beta + 2\gamma + 4 = 0$

9. In $\mathbb{R}^3$, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M ? **[JEE 2015, 4M, –2M]**

(A) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$ (B) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$ (C) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$ (D) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

Sol. Straight line 'L' is parallel to line of intersection of plane P_1 & plane P_2.

$\therefore$ Equation of line 'L' is



$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = \hat{i} - 3\hat{j} + 5\hat{k}$$

$$\frac{x}{1} = \frac{y}{-3} = \frac{z}{-5} = \lambda$$

$$\frac{\alpha - \lambda}{1} = \frac{\beta + 3\lambda}{2} = \frac{\gamma + 5\lambda}{-1} = k$$

$$\left.\begin{aligned} \alpha &= k + \lambda \\ \beta &= 2k - 3\lambda \\ y &= -k - 5\lambda \end{aligned}\right\} \quad ...(1)$$

satisfying in plane P_1

$k + \lambda + 4k - 6\lambda + k + 5\lambda + 1 = 0$

$6k = -1$

putting in (1) required locus is

$$x = -\frac{1}{6} + \lambda$$

$$y = -\frac{1}{3} - 3\lambda$$

$$z = \frac{1}{6} - 5\lambda$$

Now check the options.

10. Consider a pyramid OPQRS located in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP= 3. The point S is directly above the mid-point T of diagonal OQ such that TS = 3. Then-

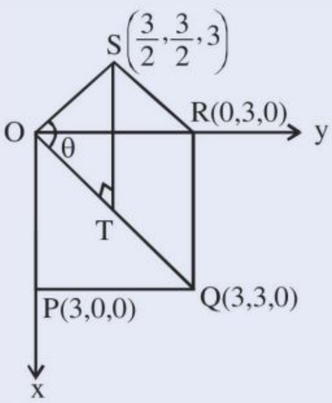
(A) the acute angle between OQ and OS is $\frac{\pi}{3}$

(B) the equaiton of the plane containing the triangle OQS is $x - y = 0$

(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Sol.



Given OP = OR = 3 and OPQR is a square

$\Rightarrow$ OQ $= 3\sqrt{2}$ $\Rightarrow$ OT $= \frac{3}{\sqrt{2}}$ and ST = 3

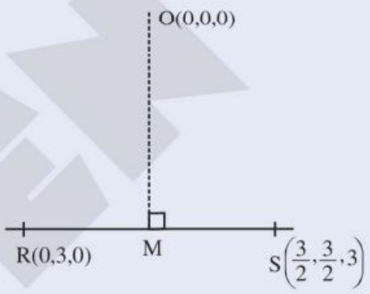
using ΔSOT, $\tan\theta = \frac{ST}{OT} = \sqrt{2} \Rightarrow \theta = \tan^{-1}\sqrt{2}$

clearly, equation of plane containing triangle OQS is $Y - X = 0$

Also, length of perpendicular from P to the plane containing the triangle OQS is PT $= \frac{3}{\sqrt{2}}$

Also equation of RS is $\overline{r} = 3\hat{j} + t\left(\frac{3}{2}\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right)$

$= \left(\frac{3t}{2}, 3 - \frac{3t}{2}, 3t\right)$



Let co-ordinates of $M = \left(\frac{3t}{2}, 3 - \frac{3t}{2}, 3t\right)$

$\because$ $\overrightarrow{OM} . \overrightarrow{RS} = 0$

$\Rightarrow \frac{9}{4}t - \frac{3}{2}\left(3 - \frac{3t}{2}\right) + 9t = 0 \quad \Rightarrow \frac{9t}{2} + 9t = \frac{9}{2} \quad \Rightarrow t = \frac{1}{3}$

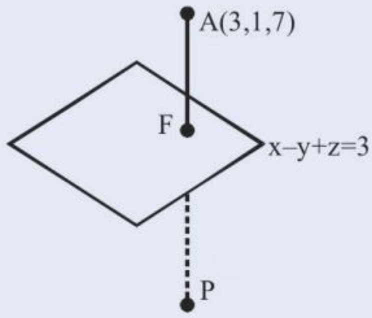
$\therefore$ $M = \left(\frac{1}{2}, \frac{5}{2}, 1\right) \quad \Rightarrow OM = \sqrt{\frac{1}{4} + \frac{25}{4} + 1} = \sqrt{\frac{30}{4}} = \sqrt{\frac{15}{2}}$

11. Let P be the image of the point (3, 1, 7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ is

[JEE(Advanced)-2016, 3(–1)]

(A) $x + y - 3z = 0$
(B) $3x + z = 0$
(C) $x - 4y + 7z = 0$
(D) $2x - y = 0$

Sol. Line AP : $\frac{x-3}{1}=\frac{y-1}{-1}=\frac{z-7}{1}=\lambda$



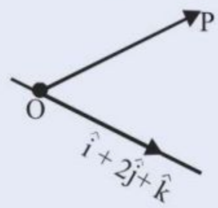
$\Rightarrow$ F$(3 + \lambda, 1 - \lambda, \lambda + 7)$ lies in the plane

$\therefore\ 3 + \lambda - (1 - \lambda) + \lambda + 7 = 3$

$3\lambda = -6 \Rightarrow \lambda = -2$

$\Rightarrow$ F(1,3,5)

$\Rightarrow$ P(–1,5,3)



so required plane is $\begin{vmatrix} x-0 & y-0 & z-0 \\ 1 & 2 & 1 \\ -1 & 5 & 3 \end{vmatrix}=0$

$\therefore\quad x - 4y + 7z = 0$

12. The equation of the plane passing through the point (1,1,1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is- **[JEE(Advanced)-2017]**

(A) $14x + 2y + 15z = 31$ (B) $14x + 2y - 15z = 1$

(C) $-14x + 2y + 15z = 3$ (D) $14x - 2y + 15z = 27$

Sol. The normal vector of required plane is parallel to vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = -14\hat{i} - 2\hat{j} - 15\hat{k}$$

$\therefore$ The equation of required plane passing through (1, 1, 1) will be

$-14(x - 1) - 2(y - 1) - 15(z - 1) = 0$

$\Rightarrow \boxed{14x + 2y + 15z = 31}$

$\therefore$ Option (A) is correct

Plane : $-14x - 2y - 15z = d$

$\underline{(1, 1, 1)}$

$-14 - 2 - 15 = d$

$d = -31$

$\therefore$ Plane $\Rightarrow -14x - 2y - 15z = -31$

$\Rightarrow \boxed{14x + 2y + 15z = 31}$

Ans.

13. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE ? **[JEE(Advanced)-2018, 4(–2)]**

(A) The line of intersection of P_1 and P_2 has direction ratios 1, 2, –1

(B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2

(C) The acute angle between P_1 and P_2 is 60°

(D) If P_3 is the plane passing through the point (4, 2, –2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_2 is $\frac{2}{\sqrt{3}}$

Sol. D.C. of line of intersection (a, b, c)

$\Rightarrow$ $2a + b - c = 0$

$a + 2b + c = 0$

$$\frac{a}{1+2} = \frac{b}{-1-2} = \frac{c}{4-1}$$

$\therefore$ D.C. is (1, –1, 1)

✗ A.) $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$

$= \underline{3\hat{i} - 3\hat{j} + 3\hat{k}}$

D.R is $\underline{(1, -1, 1)}$

✗ (B) $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$

$$\Rightarrow \quad \frac{x-4/3}{3} = \frac{y-1/3}{-3} = \frac{z}{3}$$

$\Rightarrow$ lines are parallel.

✓ (C) Acute angle between P_1 and P_2 = $\cos^{-1}\left(\frac{2\times1+1\times2-1\times1}{\sqrt{6}\sqrt{6}}\right)$

$$= \cos^{-1}\left(\frac{3}{6}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

✓ (D) Plane is given by $(x - 4) - (y - 2) + (z + 2) = 0$

$\Rightarrow$ $x - y + z = 0$

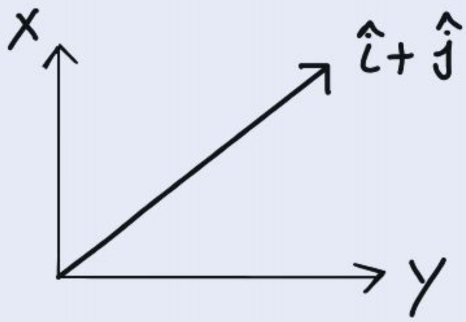
Distance of (2, 1, 1) from plane = $\frac{2-1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

14. Let P be a point in the first octant, whose image Q in the plane x + y = 3 (that is, the line segment PQ is perpendicular to the plane x + y = 3 and the mid-point of PQ lies in the plane x + y = 3) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is _____ . **[JEE(Advanced)-2018, 3(0)]**

Sol. Let $P(\alpha, \beta, \gamma)$

$Q(0, 0, \gamma)$ &

$R(\alpha, \beta, -\gamma)$



Now, $\overline{PQ} \parallel \hat{i}+\hat{j} \Rightarrow \left(\alpha\hat{i}+\beta\hat{j}\right) \parallel \left(\hat{i}+\hat{j}\right)$

$\Rightarrow \quad \alpha = \beta$

Also, mid point of PQ lies on the plane $\Rightarrow \dfrac{\alpha}{2}+\dfrac{\beta}{2}=3 \Rightarrow \alpha + \beta = 6 \Rightarrow \alpha = 3$

Now, distance of point P from X-axis is $\sqrt{\beta^2+\gamma^2}=5$

$\Rightarrow \quad \beta^2+\gamma^2=25 \;\Rightarrow\; \gamma^2=16$

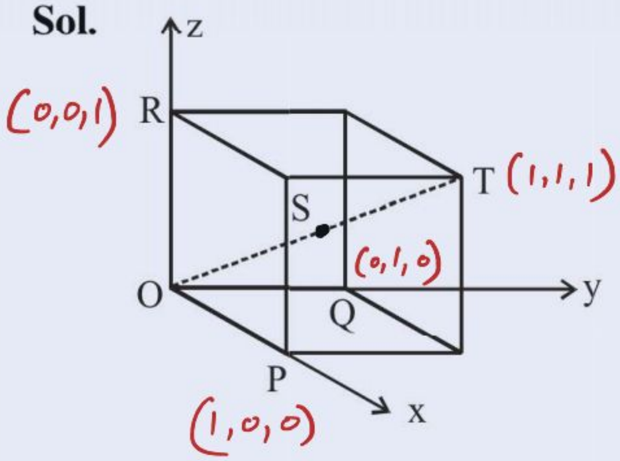
as $\beta = \alpha = 3$

as $\gamma = 4$

Hence, $PR = 2\gamma = 8$

15. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where O(0, 0, 0) is the origin. Let $S\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p}=\overrightarrow{SP}$, $\vec{q}=\overrightarrow{SQ}$, $\vec{r}=\overrightarrow{SR}$ and $\vec{t}=\overrightarrow{ST}$, then the value of $\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|$ is _____ .

Sol. **[JEE(Advanced)-2018, 3(0)]**



$S\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$

$$\vec{p}=\overrightarrow{SP}=\left(\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right)=\frac{1}{2}\left(\hat{i}-\hat{j}-\hat{k}\right)$$

$$\vec{q}=\overrightarrow{SQ}=\left(-\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)=\frac{1}{2}\left(-\hat{i}+\hat{j}-\hat{k}\right)$$

$$\vec{r}=\overrightarrow{SR}=\left(-\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right)=\frac{1}{2}\left(-\hat{i}-\hat{j}+\hat{k}\right)$$

$$\vec{t}=\overrightarrow{ST}=\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)=\frac{1}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|=\frac{1}{4}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -1\\ -1 & 1 & -1\end{vmatrix}\times\frac{1}{4}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & -1 & 1\\ 1 & 1 & 1\end{vmatrix}$$

$$=\frac{1}{16}\left|\left(2\hat{i}+2\hat{j}\right)\times\left(-2\hat{i}+2\hat{j}\right)\right|=\left|\frac{\hat{k}}{2}\right|=\frac{1}{2}$$

16. Let L_1 and L_2 denotes the lines

$$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in \mathbb{R}$$

$$\text{and} \quad \vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in \mathbb{R}$$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3 ? **[JEE(Advanced)-2019, 4(–1)]**

(1) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(2) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(3) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

(4) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in \mathbb{R}$

Sol. Points on L_1 and L_2 are respectively $A(1 - \lambda, 2\lambda, 2\lambda)$ and $B(2\mu, -\mu, 2\mu)$

So, $\overrightarrow{AB} = (2\mu + \lambda - 1)\hat{i} + (-\mu - 2\lambda)\hat{j} + (2\mu - 2\lambda)\hat{k}$

and vector along their shortest distance $= 2\hat{i} + 2\hat{j} - \hat{k}$.

D.R. of L_3

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{vmatrix} = 6\hat{i} + 6\hat{j} - 3\hat{k}$$

or $\boxed{2\hat{i} + 2\hat{j} - \hat{k}}$

Hence, $\dfrac{2\mu + \lambda - 1}{2} = \dfrac{-\mu - 2\lambda}{2} = \dfrac{2\mu - 2\lambda}{-1}$

$\Rightarrow \quad \lambda = \dfrac{1}{9}$ & $\mu = \dfrac{2}{9}$

Hence, $A \equiv \left(\dfrac{8}{9}, \dfrac{2}{9}, \dfrac{2}{9}\right)$ and $B \equiv \left(\dfrac{4}{9}, -\dfrac{2}{9}, \dfrac{4}{9}\right)$

$\Rightarrow$ Mid point of $AB \equiv \left(\dfrac{2}{3}, 0, \dfrac{1}{3}\right)$

17. Three lines are given by

$\vec{r} = \lambda\hat{i},\ \lambda \in \mathbb{R}$ $\longrightarrow$ $L_1: \frac{x}{1} = \frac{y}{0} = \frac{z}{0} = \lambda$

$\vec{r} = \mu(\hat{i}+\hat{j}),\mu \in \mathbb{R}$ and $\longrightarrow$ $L_2: \frac{x}{1} = \frac{y}{1} = \frac{z}{0} = \mu$

$\vec{r} = \nu(\hat{i}+\hat{j}+\hat{k}),\nu \in \mathbb{R}$. $\longrightarrow$ $L_3: \frac{x}{1} = \frac{y}{1} = \frac{z}{1} = \nu$

Let the lines cut the plane x + y + z = 1 at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals ___ **[JEE(Advanced)-2019, 3(0)]**

Sol. $A(1, 0, 0),\ B\left(\frac{1}{2},\frac{1}{2},0\right)\ \&\ C\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Hence, $\overrightarrow{AB} = -\frac{1}{2}\hat{i}+\frac{1}{2}\hat{j}\ \&\ \overrightarrow{AC} = -\frac{2}{3}\hat{i}+\frac{1}{3}\hat{j}+\frac{1}{3}\hat{k}$

So, $\Delta = \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right| = \frac{1}{2}\sqrt{\frac{1}{2}\times\frac{2}{3}-\frac{1}{4}}$

$$= \frac{1}{2\times 2\sqrt{3}}$$

$\Rightarrow\ (6\Delta)^2 = \frac{3}{4} = 0.75$