Permutation and Combination

Arrangement   Selection

① Fundamental principle of Counting

If I^{st} event can occur in m ways

and 2^{nd} ——— " ——— n ways

then

Addition Thm ⇒ Exactly one even

can occur $= (n+m)$ Ways

↓

Either I^{st} event $\boxed{\text{or}}$ 2^{nd} event

⇓

Add

Multiplication Thm If I^{st} event can

occur in m ways, 2^{nd} —— " ——

—— n ways then total number

of way of Simultenously occurnce of both event in a definite order

$= (mn)$ ways

I^{st} event $\boxed{\text{and}}$ 2^{nd} event $= mn$ ways

multiply

Ex:-

20 Boys
15 girls

Class

$\rightarrow$ $\boxed{1 \text{ monitor}}$

no. of ways $= 35$

Either $\underbrace{\text{a boy}}_{20}$ or $\underbrace{\text{a girl}}_{15}$

$20 + 15$

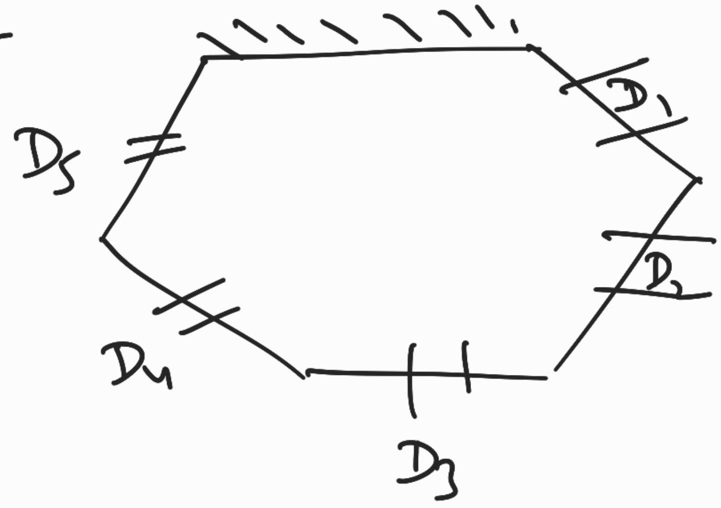
2 monitors = $\underbrace{1 \text{ boy}}$ & $\underbrace{1 \text{ girl}}$

$20 \times 15 = 300$

B_1	G_1
B_2	G_2
$\vdots$	$\vdots$
B_{20}	G_{15}

$$\left.\begin{array}{l} B_1 \rightarrow 15 \\ B_2 \rightarrow 15 \\ \vdots \\ B_{20} \rightarrow 15 \end{array}\right\} 300$$

Ex:-



Find no. of ways ⇒ ① Entery = 5

② exit = 5

③ Entry and Exit = 5 × 5

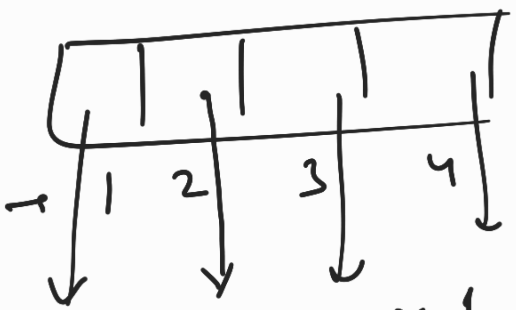
④ Entry and exit from different gates

= 5 × 4 = 20

$D_1, D_2, D_3, D_4, \cancel{D_5}$

Ex:-

Places →



$\underbrace{P, C, M, B}_{\text{4 Books}}$

4 × 3 × 2 × 1 = 24

Exn Find number of 3 digit numbers

using by 1, 2, 3, 4, 5

Case I → without repetition

Ans. $\boxed{60} = 5 \times 4 \times 3$

Case II → with repetition

Ans = $\boxed{125} = 5 \times 5 \times 5$

Ex:- find number of 3 digit numbers which has atleast one 7.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Total without any condition $\Big\{ 9 \times 10 \times 10 = 900$

Total numbers without 7 ⇒ $\underbrace{0, 1, 2, 3, 4, 5, 6, 8, 9}$

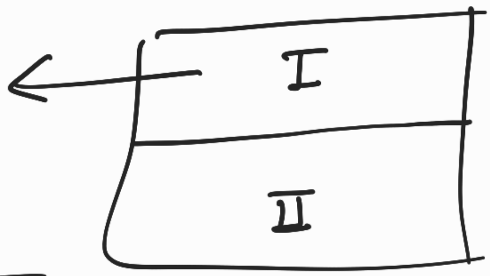
$8 \times 9 \times 9 = 648$

atleast one 7 = Total − none of digit is 7

$= 900 - 648 = 252$

atleast one = Total − none

Ex:- 7 flags of different colours. number of different signals that can be transmitted by using 2 flags of one above the other.



$$7 \times 6 = 42$$

7 different

use 2

Ex2 10 T/F questions.

Q① How many sequences of answers are possible

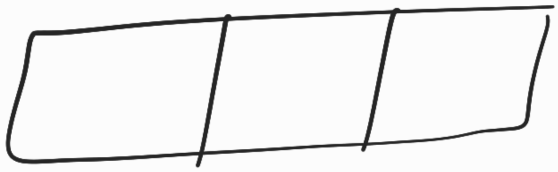
$2 \times 2 \times 2 \times \ldots$

$2 \times 2 = 2^{10}$

Q(2) How many sequences of atleast one wrong Answer $= 2^{10} - 1 = 1023$ Ans

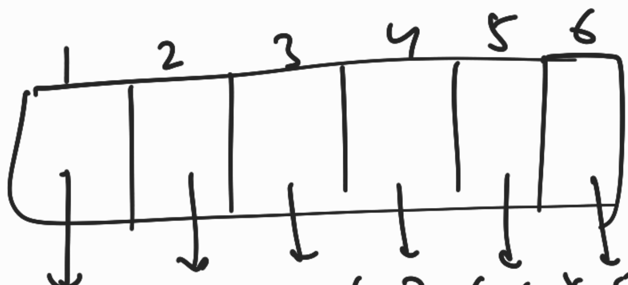
All Answers are correct

Ex:- 10 Students compete in a race In how many ways can they occupy first 3 positions.



$10 \times 9 \times 8 = 720$

Ex:- 10 persons, 6 chairs



$10 \times 9 \times 8 \times 7 \times 6 \times 5$

Ex:- 6 persons, 10 chairs

1st person → 10 choices

2nd → 9

3rd → 8

4th → 7

5th → 6

6th → 5

Ans ⇐ $10 \times 9 \times 8 \times 7 \times 6 \times 5$

Ex:- 6 rings & 4 fingers.

Total ways $\Rightarrow 4^6$

Ex:- 6 rings & 4 fingers

one ring in one finger.

$\underline{\ \ } \ \underline{\ \ } \ \underline{\ \ } \ \underline{\ \ }$

$6 \times 5 \times 4 \times 3$

Ex:- m monkeys, n - persons

Distribute m monkeys in to n persons

$$= n^m$$

$$\begin{array}{ccccc} M_1 & M_2 & M_3 & \cdots & M_m \\ \downarrow & \downarrow & \downarrow & & \downarrow \\ n & n & n & & n = n^m \end{array}$$

__Ex:-__ 5 letters, 10 post offices

no. of ways $= 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

__Ex2__ Number of words which can formed from All letters of word machine if

Q① Vowels may occupy odd positions

Q② ———— " ———— even positions

$$\boxed{\begin{array}{ccccccc} M & A & C & H & I & N & E \\ \overline{1} & \overline{2} & \overline{3} & \overline{4} & \overline{5} & \overline{6} & \overline{7} \end{array}}$$

given

Sol^n ① $\Rightarrow$ $\boxed{\overline{1}}$ $\overline{2}$ $\boxed{\overline{3}}$ $\overline{4}$ $\boxed{\overline{5}}$ $\overline{6}$ $\boxed{\overline{7}}$

M A
C I
H E
N

$$4 \times 3 \times 2$$

vowel → 1st 2nd 3rd

1, 3, 5, 7

$\boxed{2\ 4\ 6}$

Cons. ⇒ $4 \times 3 \times 2 \times 1$

Ans:- $(4 \times 3 \times 2) \times (4 \times 3 \times 2 \times 1) = 576$

Solⁿ ②

$\_1\_ \quad \_2\_ \quad \_3\_ \quad \_4\_ \quad \_5\_ \quad \_6\_ \quad \_7\_$

M A
C I
H E
N

$$\underbrace{(3 \times 2 \times 1)}_{\text{Vowels}} \times \underbrace{(4 \times 3 \times 2 \times 1)}_{C}$$

$$= 6 \times 24 = 144$$

Ex:- If All letters of 'TOUGH' are written in all possible ways and arranged as in a dictionary then find rank of TOUGH.

TOTAL = $\underset{5}{\_}\ \underset{4}{\_}\ \underset{3}{\_}\ \underset{2}{\_}\ \underset{1}{\_} = 120$

$\boxed{\text{TOUGH}}$ GHOTU

$G\ \underbrace{4 \times 3 \times 2 \times 1}_{\text{HOTU}} = \boxed{24}$

$H\ \underbrace{4 \times 3 \times 2 \times 1}_{\text{GOTU}} = \boxed{24}$

$O\ \underset{\text{GHTU}}{4 \times 3 \times 2 \times 1} = \boxed{24}$

$\boxed{T}\ G\ \underbrace{3 \times 2 \times 1}_{\text{HOU}} = \boxed{6}$ GHO~~T~~U

$\boxed{T}\ H\ \underset{\text{GOU}}{3 \times 2 \times 1} = \boxed{6}$

$\boxed{T}\ \boxed{O}\ G\ \underbrace{2 \times 1}_{\text{HU}} = \boxed{2}$ GH~~O~~~~T~~U

$\boxed{T}$ $\boxed{O}$ $\underline{H}$ $\underbrace{\_\_}_{G\ U}\ \frac{2 \times 1}{} = \boxed{2}$

~~GHOTU~~

$\boxed{T}$ $\boxed{O}$ $\boxed{U}$ $\boxed{G}$ $\boxed{H} = \boxed{1}$

TOTAL $= 89$

Meaning of ${}^nC_r$ & ${}^nP_r$

① Factorial notation $n \in N$

$$n! = 1 \cdot 2 \cdot 3 \cdot \ldots (n-1) \cdot n$$

$$\underbrace{n! = \lfloor n = \text{factorial } n}$$

Product of first n natural numbers

$$\Rightarrow n! = \underbrace{1 \cdot 2 \cdot 3 \ldots (n-1)} \cdot n$$

$$\underbrace{n! = (n-1)!\ n} = (n-2)!\,(n-1)\,n$$

$$n=1 \Rightarrow 1! = 0! \cdot 1 \Rightarrow 0! = 1$$

$$\boxed{\angle -ve = 0}$$

$2! = 2$, $3! = 1\cdot 2\cdot 3 = 6$

$4! = 24$, $5! = 120$

Exponent of Prime number in $n!$

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \cdots$$

$\cdots$ up to zero

$[\cdot] =$ G.I.F.

$$10! = 2^a\, 3^b\, 5^c\, 7^d \qquad E_5(10!) = 2 + 0 = 2$$

$$E_2(10!) = \left[\frac{10}{2}\right] + \left[\frac{10}{4}\right] + \left[\frac{10}{8}\right] + \left[\frac{10}{16}\right] + \cdots$$

$$= 5 + 2 + 1 + 0 = 8$$

Ex:- $E_5(100!) = 20 + 4 + 0 = 24$

Ex:- $E_{10}(100!) = ??$

$$E_2(100!) = 50 + 25 + 12 + 6 + 3 + 1 + 0$$

$$= 97$$

$$100! = \underbrace{2^{97} \quad 3^{b} \quad 5^{24}} \quad 7^{c} \, 11^{d} \cdots$$

$$\underbrace{(2^{24} 5^{24})}_{10^{24}} (2^{73} \, 3^{b} 7^{c} 11^{d} \cdots)$$

$$10^{24} \left(\qquad \qquad \right)$$

$$E_{30}(100!) = ??$$

$$E_{24}(100!) =$$

$$E_3(100!) = 33 + 11 + 3 + 1 = 47$$

$$100! = 2^{97} 3^{47} 5^{24} 7^{c} 11^{d} \cdots$$

$$100! = 2\underbrace{(2^3)^{32}\,3^{47}}\,5^{24} \ldots$$

$$= 2\,(2^3 \cdot 3)^{32} \cdot 3^{15}\,5^{24} \ldots$$

$\boxed{\text{H.W.}}$ S-1 $\boxed{\text{Q 1 to Q 10}}$