Q. The proposition $p \rightarrow \sim(p \wedge \sim q)$ is equivalent to:

(1) $(\sim p) \vee q$ (2) q (3) $(\sim p) \wedge q$ (4) $(\sim p) \vee (\sim q)$

Sol[n]

$\sim p \vee \sim(p \wedge \sim q)$

$\sim p \vee (\sim p \vee q)$

$(\sim p \vee \sim p) \vee q$

$\sim p \vee q \longrightarrow [1]$

Q Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively :

(1) T, F, T (2) F, T, F (3) T, T, F (4) T, T, T

Q Given the following two statements :

(S_1) : $(q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.

(S_2) : $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy.

Then :

(1) only (S_1) is correct.

(2) both (S_1) and (S_2) are correct.

(3) both (S_1) and (S_2) are not correct.

(4) only (S_2) is correct.

[3]

Q If P and Q are two statements, then which of the following compound statement is a tautology ?

(1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

(2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$

(3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$

(4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

Soln

$((P \rightarrow Q) \wedge \sim Q)$

$(\sim P \vee Q) \wedge \sim Q$

$(\underbrace{\sim P \wedge \sim Q}) \vee (\underbrace{Q \wedge \sim Q}_{C})$

$\sim P \wedge \sim Q \equiv \sim (P \vee Q)$

[2] option

Q If the Boolean expression $(p \wedge q) \circledast (p \otimes q)$ is a tautology, then $\circledast$ and $\otimes$ are respectively given by

(1) $\rightarrow, \rightarrow$ (2) $\wedge, \vee$ (3) $\vee, \rightarrow$ (4) $\wedge, \rightarrow$

Sol^n (1) $\rightarrow, \rightarrow$

$(p \wedge q) \rightarrow (p \rightarrow q)$

$\downarrow$

$\sim(p \wedge q) \vee (p \rightarrow q)$

$(\sim p \vee \sim q) \vee (\sim p \vee q)$

$\sim p \vee \underbrace{(\sim q \vee q)}_{t} \equiv t$ ✓ [1]

Q If the Boolean expression (p ⇒ q) ⇔ (q ∗ (~p)) is a tautology, then the Boolean expression p ∗ (~q) is equivalent to :

(1) q ⇒ p (2) ~q ⇒ p (3) p ⇒ ~q (4) p ⇒ q

Sol^n

$\because \quad p \to q \equiv \sim p \vee q$

$* \equiv \vee$

[1]

Q The statement among the following that is a tautology is :

(1) $A \vee (A \wedge B)$

(2) $A \wedge (A \vee B)$

(3) $B \rightarrow [A \wedge (A \rightarrow B)]$

(4) $[A \wedge (A \rightarrow B)] \rightarrow B$

Sol^n

$$A \wedge (A \rightarrow B)$$

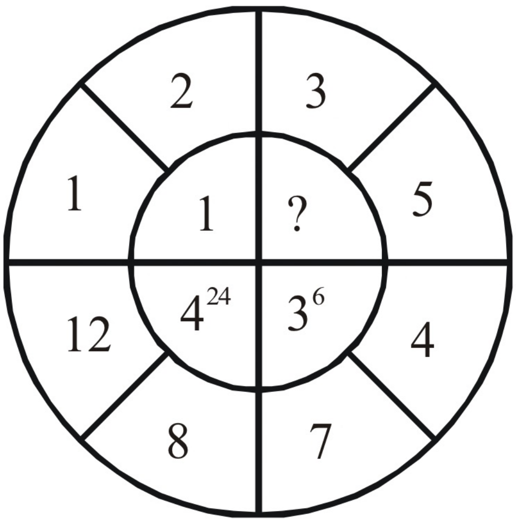
$$A \wedge (\sim A \vee B)$$

$$\underbrace{(A \wedge \sim A)} \vee (A \wedge B)$$

$$C \vee (A \wedge B) \equiv A \wedge B$$

[4]

Q The missing value in the following figure is



$$? = (5-3)^{2!} = 2^{2!} = 4.$$

HEIGHT AND DISTANCE

1. INTRODUCTION :

One of the important application of trigonometry is in finding the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

2. ANGLES OF ELEVATION AND DEPRESSION :

Let OP be a horizontal line in the vertical plane in which an object R is given and let OR be joined.

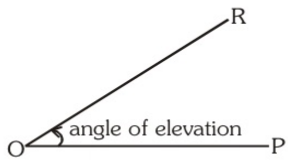


Fig. (a)

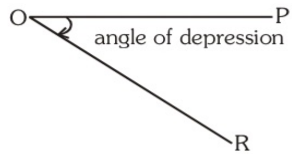


Fig. (b)

In Fig. (a), where the object R is above the horizontal line OP, the angle POR is called the angle of elevation of the object R as seen from the point O. In Fig. (b) where the object R is below the horizontal line OP, the angle POR is called the angle of depression of the object R as seen from the point O.

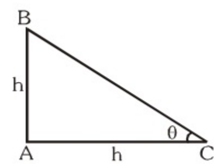
Remark :

Unless stated to the contrary, it is assumed that the height of the observer is neglected, and that the angles of elevation are measured from the ground.

Ex.1 Find the angle of elevation of the sum when the length of shadow of a vertical pole is equal to its height.

Sol. Let height of the pole AB = h and

length of the shadow of the Pole (AC) = h



In ΔABC $\tan\theta = \frac{AB}{AC} = \frac{h}{h} = 1 \Rightarrow \tan\theta = 1$

$$\Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°$$

Ex.2 The shadow of the tower standing on a level ground is found to be 60 metres longer when the sun's altitude is 30° than when it is 45°. The height of the tower is-

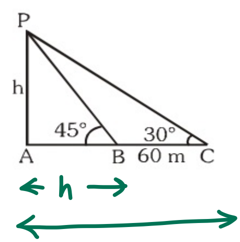
(1) 60 m (2) $30(\sqrt{3}-1)$m (3) $60\sqrt{3}$ m (4) $30(\sqrt{3}+1)$ m.

Sol. $AC = h\cot 30° = \sqrt{3}\,h$

$AB = h\cot 45° = h$

$\therefore\ BC = AC - AB = h(\sqrt{3}-1) \Rightarrow 60 = h(\sqrt{3}-1)$

$\therefore\ h = \frac{60}{\sqrt{3}-1} = \frac{60(\sqrt{3}+1)}{3-1} = 30(\sqrt{3}+1)$



Q AB is a vertical tower. The point A is on the ground and C is the mid point of AB. The part CB subtend an angle α at a point P on the ground. If $AP = n\,AB$ then the correct relation is

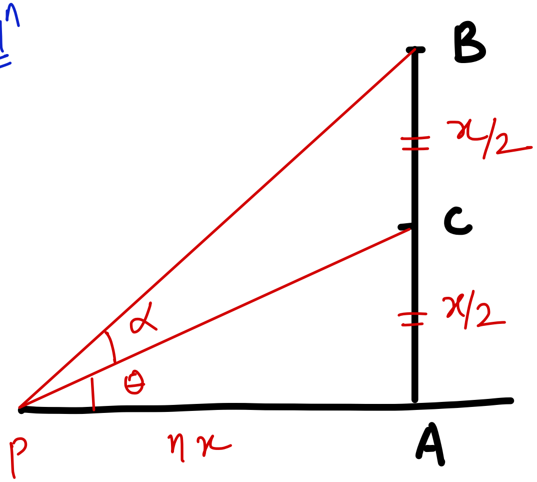
(A) $n = (n^2+1)\tan\alpha$.

(B) $n = (2n^2-1)\tan\alpha$.

(C) $n^2 = (2n^2+1)\tan\alpha$.

(D) $n = (2n^2+1)\tan\alpha$. ✓

Sol^n



$AP = n\,AB$

$AP = nx$

$$\tan\theta = \frac{x/2}{nx} = \frac{1}{2n}.$$

$$\tan(\theta+\alpha) = \frac{x}{nx} = \frac{1}{n}.$$

$$\frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha} = \frac{1}{n}$$

$$\frac{\frac{1}{2n}+\tan\alpha}{1-\frac{1}{2n}\cdot\tan\alpha} = \frac{1}{n}.$$

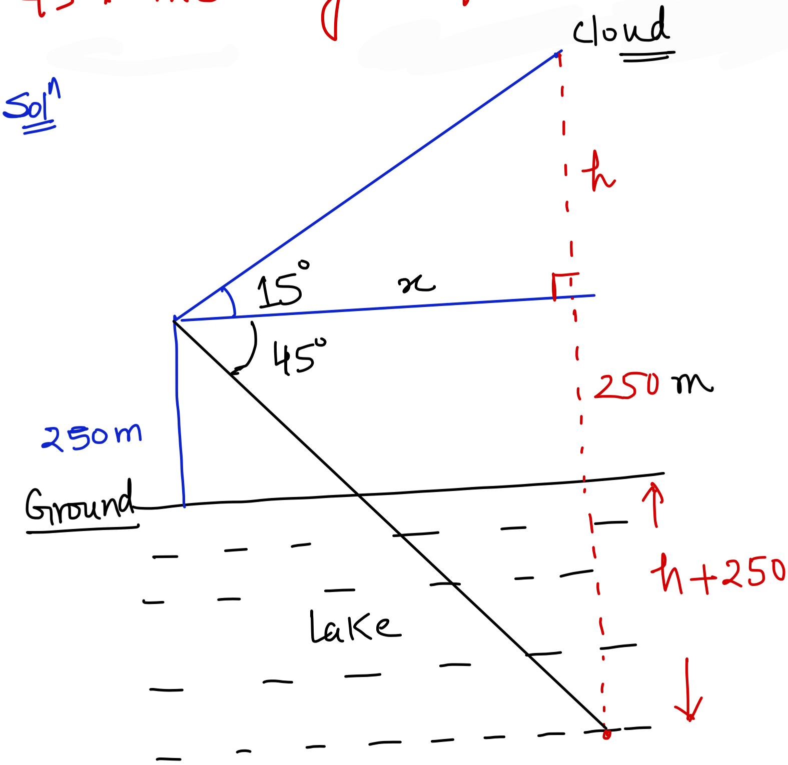
$$\frac{1+2n\tan\alpha}{2n-\tan\alpha} = \frac{1}{n}$$

$\Leftarrow$

$$n+2n^2\tan\alpha = 2n-\tan\alpha$$

Q The angle of elevation of a cloud from a point 250 m above a lake is 15° and angle of depression of its reflection in the lake is 45°. The height of cloud is

Sol^n



Height of cloud $= h + 250$.

$$\tan 15^\circ = \frac{h}{x}$$

$$\tan 45^\circ = \frac{500+h}{x}$$

$$\frac{\tan 15^\circ}{\tan 45^\circ} = \frac{h}{500+h}$$

$$(2-\sqrt{3})(h+500) = h$$

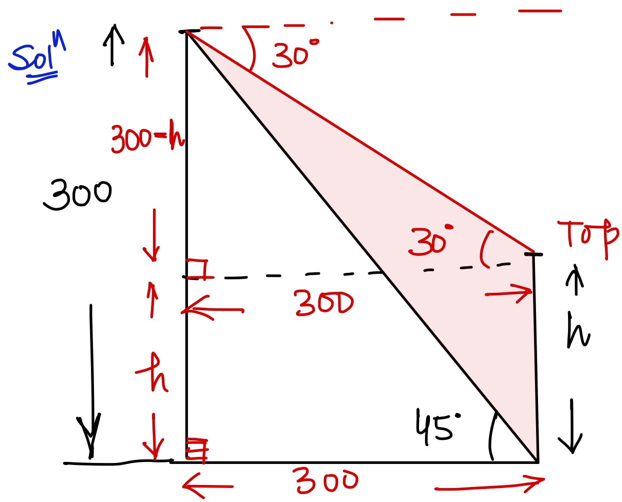
$$h = 250(\sqrt{3}-1)$$

$$h + 250 = 250\sqrt{3}$$

Ans

Q From the top of cliff 300 m high, the top of a tower was observed at an angle of depression 30° and from the foot of the tower the top of the cliff was observed at an angle of elevation of 45°. The height of tower is ______

Sol^n



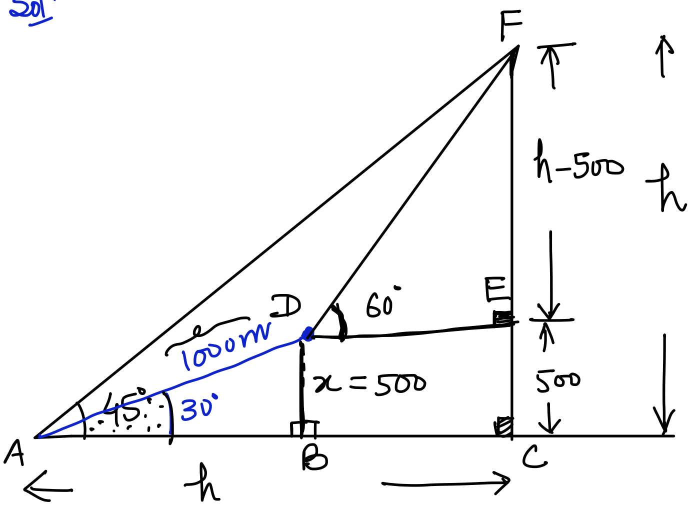
$$\tan 30^\circ = \frac{300-h}{300}$$

$$\frac{1}{\sqrt{3}} = \frac{300-h}{300}$$

$$\boxed{h = 100(3-\sqrt{3})}$$

Q At the foot of a mountain the elevation of its summit is 45° after ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. The height of mountain is ________

Sol^n



$\frac{x}{1000} = \sin 30° = \frac{1}{2}$

$x = 500$

$AB = 1000 . \cos 30°$
$= 1000 \frac{\sqrt{3}}{2} = 500\sqrt{3}$

$BC = h - 500\sqrt{3} = DE$

Δ DEF :

$\tan 60° = \frac{h-500}{h-500\sqrt{3}}$

$\sqrt{3}h - 1500 = h - 500$

$(\sqrt{3}-1)h = 1000$

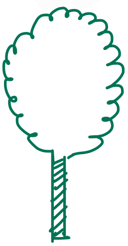
$h = \frac{1000}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

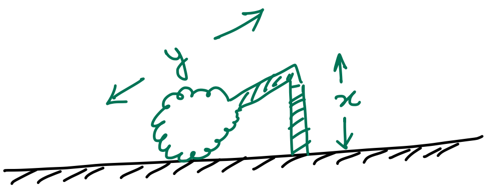
$h = 500(\sqrt{3}+1)$

Ans

Q The upper part of a tree broken by wind makes an angle of 30° with the ground and the distance from the root (bottom) to the point where the top of the tree touches the ground is 10 m. The height of the tree is ——

Sol^n



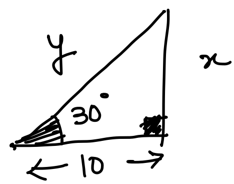


Height of tree $= x+y$

$= \frac{30}{\sqrt{3}}$

$= 10\sqrt{3}$

Ans



$x = \frac{10}{\sqrt{3}}$

$y = \frac{20}{\sqrt{3}}$

Representation of Data :-

- Ungrouped data

60, 55, 70, 50, 51.

- Group with single class value

200

Age	No. of students (frequency)
16	50
17	120
18	30

- Group with class-value interval.

Height	No. of students (frequency)
(141 – 150) cm	5
(151 – 160) cm	58
(161 – 170) cm	65
(171 – 180) cm	10

STATISTICS

MEASURES OF CENTRAL TENDENCY :

An average value or a central value of a distribution is the value of variable which is representative of the entire distribution, this representative value are called the measures of central tendency. Generally the following five measures of central tendency.

(a) Mathematical average

(i) Arithmetic mean (ii) Geometric mean (iii) Harmonic mean

(b) Positional average

(i) Median (ii) Mode

1. ARITHMETIC MEAN :

(i) For ungrouped dist. : If $x_1, x_2, ... x_n$ are n values of variate x_i then their A.M. $\bar{x}$ is defined as

OR

μ

$$\bar{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$\Rightarrow$ $\Sigma x_i = n\bar{x}$

(ii) For ungrouped and grouped freq. dist. : If $x_1, x_2, ... x_n$ are values of variate with corresponding frequencies $f_1, f_2, ... f_n$ then their A.M. is given by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + ... + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{N}, \quad \text{where } N = \sum_{i=1}^{n} f_i$$

Ex.1 Find the A.M. of the following freq. dist.

x_i	5	8	11	14	17
f_i	4	5	6	10	20

assumed mean = a

Sol. Here $N = \Sigma f_i = 4 + 5 + 6 + 10 + 20 = 45$

$\Sigma f_ix_i = (5 \times 4) + (8 \times 5) + (11 \times 6) + (14 \times 10) + (17 \times 20) = 606$

$\therefore \quad \bar{x} = \frac{\Sigma f_ix_i}{N} = \frac{606}{45} = 13.47$

(iii) By short method : If the value of x_i are large, then the calculation of A.M. by using previous formula is quite tedious and time consuming. In such case we take deviation of variate from an arbitrary point a.

Let $d_i = x_i - a$

$\therefore \quad \bar{x} = a + \frac{\Sigma f_id_i}{N}$, where a is assumed mean

(iv) By step deviation method : Sometime during the application of short method of finding the A.M. If each deviation d_i are divisible by a common number h(let)

Let $u_i = \frac{d_i}{h} = \frac{x_i - a}{h}$

$\therefore \quad \bar{x} = a + \left(\frac{\Sigma f_iu_i}{N}\right)h$

Ex.2 Find the mean of the following freq. dist.

x_i	5	15	25	35	45	55
f_i	12	18	27	20	17	6

Sol. Let assumed mean a = 35, h = 10

here $N = \Sigma f_i = 100$, $u_i = \dfrac{(x_i - 35)}{10}$

$\therefore \quad \Sigma f_i u_i = (12 \times -3) + (18 \times -2) + (27 \times -1) + (20 \times 0) + (17 \times 1) + (6 \times 2) = -70$

$\therefore \quad \bar{x} = a + \left(\dfrac{\Sigma f_i u_i}{N}\right) h = 35 + \dfrac{(-70)}{100} \times 10 = 28$

(v) Weighted mean : If $w_1, w_2, \ldots\ldots w_n$ are the weights assigned to the values $x_1, x_2, \ldots\ldots x_n$ respectively then their weighted mean is defined as

$$\boxed{\text{Weighted mean} = \frac{w_1x_1 + w_2x_2 + \ldots.. + w_nx_n}{w_1 + \ldots.. + w_n} = \frac{\sum_{i=1}^{n} w_ix_i}{\sum_{i=1}^{n} w_i}}$$

Ex.3 Find the weighted mean of first n natural numbers when their weights are equal to their squares respectively

Sol. $\text{Weighted Mean} = \dfrac{1.1^2 + 2.2^2 + \ldots. + n.n^2}{1^2 + 2^2 + \ldots. + n^2} = \dfrac{1^3 + 2^3 + \ldots.. + n^3}{1^2 + 2^2 + \ldots. + n^2} = \dfrac{[n(n+1)/2]^2}{[n(n+1)(2n+1)/6]} = \dfrac{3n(n+1)}{2(2n+1)}$

(vi) Combined mean : If $\bar{x}_1$ and $\bar{x}_2$ be the means of two groups having n_1 and n_2 terms respectively then the mean (combined mean) of their composite group is given by

$$\text{combined mean} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

If there are more than two groups then, $\boxed{\text{combined mean} = \dfrac{n_1\bar{x}_1 + n_1\bar{x}_2 + n_3\bar{x}_3 + \ldots.}{n_1 + n_2 + n_3 + \ldots.}}$

Ex.4 The mean income of a group of persons is Rs. 400 and another group of persons is Rs. 480. If the mean income of all the persons of these two groups is Rs. 430 then find the ratio of the number of persons in the groups.

Sol. Here $\bar{x}_1 = 400$, $\bar{x}_2 = 480$, $\bar{x} = 430$

$\because \quad \bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \Rightarrow 430 = \dfrac{400n_1 + 480n_2}{n_1 + n_2}$

$\Rightarrow \quad \dfrac{n_1}{n_2} = \dfrac{5}{3}$

(vii) Properties of Arithmetic mean :

- Sum of deviations of variate from their A.M. is always zero i.e. $\Sigma(x_i - \bar{x}) = 0$, $\Sigma f_i(x_i - \bar{x}) = 0$
- Sum of square of deviations of variate from their A.M. is minimum i.e. $\Sigma(x_i - \bar{x})^2$ is minimum
- If $\bar{x}$ is the mean of variate x_i then A.M. of $(x_i + \lambda) = \bar{x} + \lambda$
  A.M. of $(\lambda x_i) = \lambda\bar{x}$
  A.M. of $(ax_i + b) = a\bar{x} + b$ (where λ, a, b are constant)
- A.M. is independent of change of assumed mean i.e. it is not effected by any change in assumed mean.

2. MEDIAN :

The median of a series is the value of middle term of the series when the values are written in ascending order. Therefore median, divided an arranged series into two equal parts.

Formulae of median :

(i) For ungrouped distribution : Let n be the number of variate in a series then

$$\text{Median} = \begin{cases} \left(\dfrac{n+1}{2}\right)^{th} \text{term}, \ (\text{when } n \text{ is odd}) \\ \text{Mean of} \left(\dfrac{n}{2}\right)^{th} \text{and} \left(\dfrac{n}{2}+1\right)^{th} \text{terms}, \ (\text{when } n \text{ is even}) \end{cases}$$

(ii) For ungrouped freq. dist. : First we prepare the cumulative frequency (c.f.) column and Find value of N then

$$\text{Median} = \begin{cases} \left(\dfrac{N+1}{2}\right)^{th} \text{term}, \ (\text{when } N \text{ is odd}) \\ \text{Mean of} \left(\dfrac{N}{2}\right)^{th} \text{and} \left(\dfrac{N}{2}+1\right)^{th} \text{terms}, \ (\text{when } N \text{ is even}) \end{cases}$$

(iii) For grouped freq. dist : Prepare c.f. column and find value of $\dfrac{N}{2}$ then find the class which contain value of c.f. is equal or just greater to N/2, this is median class

$$\therefore \ \text{Median} = \ell + \frac{\left(\dfrac{N}{2}-F\right)}{f} \times h$$

where
ℓ — lower limit of median class
f — freq. of median class
F — c.f. of the class preceeding median class
h — Class interval of median class

Ex.5 Find the median of following freq. dist.

class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
f	8	30	40	12	10

Sol.

class	f_i	c.f.
0 – 10	8	8
10 – 20	30	38
20 – 30	40	78
30 – 40	12	90
40 – 50	10	100

Here $\dfrac{N}{2} = \dfrac{100}{2} = 50$ which lies in the value 78 of c.f. hence corresponding class of this c.f. is 20-30 is the median class, so $\ell = 20$, $f = 40$, $F = 38$, $h = 10$

$$\therefore \ \text{Median} = \ell + \frac{\left(\dfrac{N}{2}-F\right)}{f} \times h = 20 + \frac{(50-38)}{40} \times 10 = 23$$

Q Find mean :

92, 98, 101, 102, 97, 106 $\qquad a = 100$

Sol^n

$$\bar{x} = 100 + \frac{(-8-2+1+2+(-3)+6)}{6}$$

$$= 100 - \frac{4}{6} = 100 - 0.666$$

Q Find median

95, 102, 97, 99, 101, 105, 92, 100

Sol^n $\boxed{n=8}$ ie even

$\frac{n}{2} = 4$

$\frac{n}{2}+1 = 5$

92, 95, 97, 99, 100, 101, 102, 105

$$\text{Median} = \frac{99+100}{2} = \frac{199}{2}$$