HW

Q. Let x and y are 2 randomly selected integers.

Find the probability that :

(i) $(x+y)$ is divisible by '7'.

$$\cancel{7} \times \left(\frac{1}{\cancel{7}} \times \frac{1}{7}\right) = \frac{1}{7}$$

$7I \longrightarrow p = 1/7$

$7I+1 \longrightarrow$ "

$7I+2$ "

$7I+3$ "

$7I+4$ "

$7I+5$ "

$7I+6$ "

(ii) (x^2+y^2) is divisible by '3'

$x, y \longrightarrow \begin{cases} 3I \\ 3I+1 \\ 3I+2 \end{cases}$

$(3I)^2 = 9I^2 = 3\lambda$

$(3I+1)^2 = 9I^2 + 1 + 6I$

$= (3\lambda_1 + 1)$

$(3I+2)^2 = 9I^2 + 4 + 12I$

$4 \to 3+1$

$(3\lambda_2 + 1)$

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}.$$

(iii) (x^2+y^2) is divisible by 5.

$$x, y \to \begin{cases} 5I \\ 5I+1 \\ 5I+2 \\ 5I+3 \\ 5I+4 \end{cases} \qquad \begin{aligned} 25I^2 &= 5\lambda. \\ &= 5\lambda_1+1. \\ &= 5\lambda_2+4. \\ &= 5\lambda_3+4. \\ &= 5\lambda_4+1. \end{aligned}$$

$$9 \times \left(\frac{1}{5} \times \frac{1}{5}\right) = \frac{9}{25}$$

alt :-

$n(S) = 10 \times 10$
$= 100$

Unit digit in I	Unit digit in I^2
0	0
1	1
2	4
3	9
4	6
5	5
6	6
7	9
8	4
9	1

$\underbrace{(0,0), (0,5), (5,0), (5,5)}$, $(1,4), (4,1), (1,9), (9,1)$, $(4,6), (6,4), (9,6), (6,9)$,

$$n(A) = \underbrace{1+1+1+1}\ + 8 \times 4 = 36$$

Q When 3 normal dice are thrown, find the probability that

(a) sum on the faces is less than 11.

(b) " " " " is 6.

(c) " " " " is 9.

Sol^n $n(S) = 216$

(b) $x + y + z = 6$

$1 \le x \le 6$, $1 \le y \le 6$, $1 \le z \le 6$

$X + Y + Z = 3$

$n(A) = {}^5C_2 = 10$

Req. prob $= \frac{10}{216}$.

(c) $x + y + z = 9$

$1 \le x \le 6$, $1 \le y \le 6$, $1 \le z \le 6$

$X + Y + Z = 6$

6	0	0
0	6	0
0	0	6

3 cases

${}^8C_2 - 3 = 25$ cases.

Req. prob $= \frac{25}{216}$.

(a)

$P(S=3)$	$P(S=18)$
$P(S=4)$	$P(S=17)$
$P(S=5)$	$P(S=16)$
$P(S=6)$	$P(S=15)$
$P(S=7)$	$P(S=14)$
$P(S=8)$	$P(S=13)$
$P(S=9)$	$P(S=12)$
$P(S=10)$	$P(S=11)$

$$\underbrace{P(\text{sum} \le 10)}_{P} + \underbrace{P(\text{sum} \ge 11)}_{P} = 1$$

$$P + P = 1 \Rightarrow P = \frac{1}{2}$$

Q Find the prob that in a random throw of 12 dice, each face occurs twice ?

1, 1, 2, 2, 3, 3, 4, 4
5, 5, 6, 6

Sol^n

$$\text{Req. prob} = \frac{\left(\dfrac{12!}{(2!)^6}\right)}{6^{12}}$$

Throwing of a coin say 4 times :

S = {HTTT, HTTH, THTT, THTH, HHTT, HHTH, TTTT, TTTH, HTHT, HTHH, THHT, THHH, HHHT, HHHH, TTHT, TTHH}

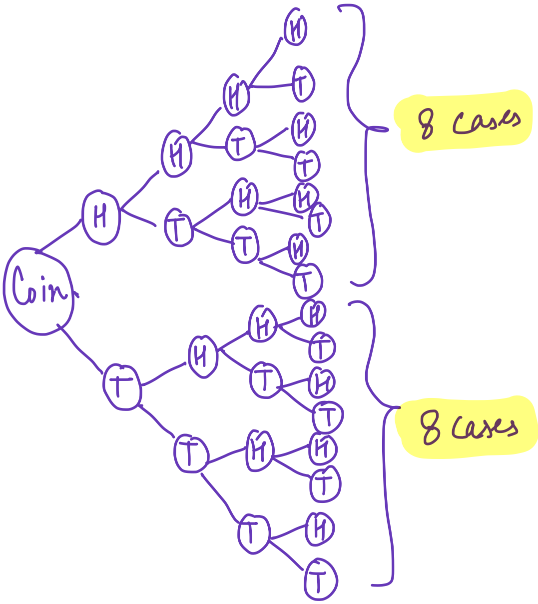
Find the probability of the following events

A : H and T come alternately

B : number of H occurring is more than the number of T.

C : No two consecutive Heads occur

A → {HTHT, THTH}



(i) $P(A) = \frac{2}{16}$

(ii) $P(B) = \frac{5}{16}$

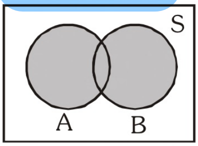
(iii) $P(C) = \frac{8}{16}$

VENN DIAGRAMS :

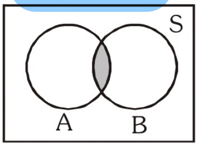
A diagram used to illustrate relationships between sets. Commonly, a rectangle represents the universal set and a circle within it represents a given set (all members of the given set are represented by points within the circle). A subset is represented by a circle within a circle and union and intersection are indicated by overlapping circles.

Let S is the sample space of an experiment and A, B, C are three events corresponding to it :

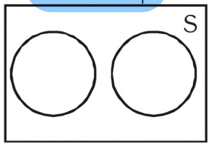
$A \cup B$ or $A+B$



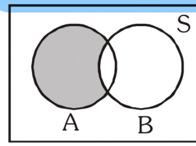
$A \cap B$ or AB



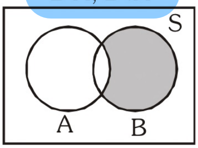
$A \cap B = \phi$



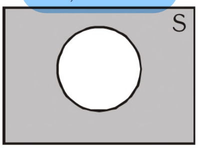
$A-B$ or $A \setminus B$ or $A \cap \bar{B}$ or only A.



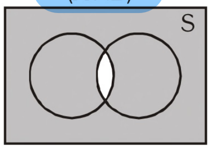
$B-A$, $B \setminus A$



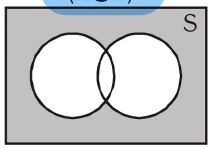
A', A^C or $\bar{A}$



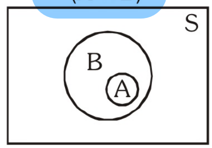
$(A \cap B)'$



$(A \cup B)'$



$(A \subset B)$

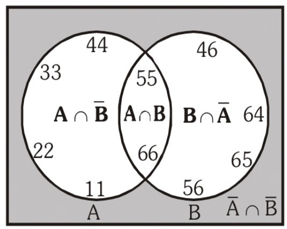


Example : Let us now conduct an experiment of tossing a pair of dice.

Two events defined on the experiment are

A : getting a doublet {11, 22, 33, 44, 55, 66}

B : getting total score of 10 or more {64, 46, 55, 56, 65, 66}



Note :

(i) If A & B are two subsets of a universal set U, then

(a) $(A \cup B)^c = A^c \cap B^c$ **(b)** $(A \cap B)^c = A^c \cup B^c$ **(DE MORGAN'S LAW)**

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

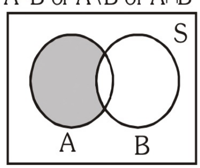
(iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iv) $(A \setminus B) \rightarrow$ read as A minus B

$(A \setminus B) = (A \cap B^c)$

$P(A \setminus B) = P(A) - P(A \cap B)$

$A-B$ or $A \setminus B$ or $A \cap \bar{B}$

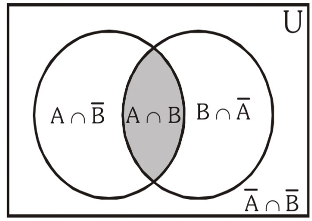


ADDITION THEOREM :

$A \cup B = A + B =$ A or B denotes occurrence of at least A or B.

For 2 events A & B :

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$



Note :

(i)

$$\left.\begin{array}{l} P(A \cup B) \\ P(A + B) \\ P(A \text{ or } B) \\ P(\text{occurence of atleast one A or B}) \\ P(\text{either A or B}) \end{array}\right] = \begin{array}{l} P(A) + P(B) - P(A \cap B) \text{ (This is known as generallised addition theorem)} \\ P(A) + P(B \cap \bar{A}) \\ P(B) + P(A \cap \bar{B}) \\ P(A \cap \bar{B}) + P(A \cap B) + P(B \cap \bar{A}) \\ 1 - P(A^C \cap B^C) \\ 1 - P(A \cup B)^C \end{array}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$P(A \cup B \cup C) = \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$$

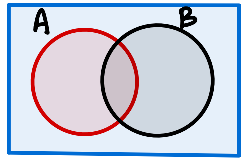
Probability of neither A nor B $= P(\bar{A} \cap \bar{B})$

$$P(\overline{A \cup B}) = 1 - P(A \cup B)$$

Probability of neither A nor B $= 1 - P(\text{either A or B})$.

* **If A & B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$.**

* For any two events A & B, P(exactly one of A, B occurs)



$= P(A \cap \bar{B}) + P(B \cap \bar{A}) = P(A) + P(B) - 2P(A \cap B)$

$= P(A \cup B) - P(A \cap B) = P(A^c \cup B^c) - P(A^c \cap B^c)$

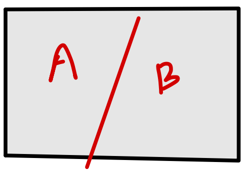
** **If $E_1, E_2 \ldots\ldots E_n$ are mutually exclusive & exhaustive, then $P(E_1 \cup E_2 \ldots\ldots \cup E_n) = 1$**

$\Rightarrow \quad \mathbf{P(E_1) + P(E_2) \ldots\ldots + P(E_n) = 1}$

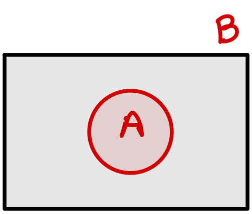
$P(A \cap B) \leq P(A)$ or $P(B) \leq P(A \cup B) \leq P(A) + P(B)$

If A and B are M.E and exhaustive, then

$$\boxed{P(A \cup B) = P(A) + P(B) = 1.}$$



* A implies B $(A \Rightarrow B)$ i.e occurance of A ensures occurance of B.



E(1) A card is drawn from a pack of 52 cards

$A \rightarrow$ red card drawn; $\quad B \rightarrow$ face card drawn

find the following :

1. $P(A) =$

2. $P(B) =$

3. $P(A \cap B) =$

4. $P(A \cup B) = P(A) + P(B) - P(A \cap B) =$

5. $P(A^c) = 1 - P(A) =$

6. $P(A \setminus B) = P(A) - P(A \cap B) =$

7. $P(B \cap A^c) = P(B \setminus A) = P(B) - P(A \cap B) =$

8. $P(A^c \cup B^c) = P(A \cap B)^c = 1 - P(A \cap B) =$

9. $P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B) =$

10. $P(A^c \setminus B^c) = P(A^c) - P(A^c \cap B^c) =$

Sol^n

1) $P(A) = \frac{26}{52} = \frac{1}{2}$

2) $P(B) = \frac{12}{52} = \frac{3}{13}$

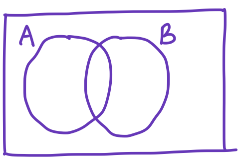
3) $P(A \cap B) = \frac{6}{52} = \frac{3}{26}$.

4) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

5) $P(A^c) = 1 - P(A) = 1 - \frac{1}{2}$
$= \frac{1}{2}$

6) $P(A \setminus B) = P(A - B)$
$= P(A) - P(A \cap B)$

7) $P(B \cap \bar{A}) = P(\text{only } B)$
$= P(B) - P(A \cap B)$



8) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$
$= 1 - P(A \cap B)$

9) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

10) $P(\bar{A} \setminus \bar{B}) = P(\bar{A}) - P(\bar{A} \cap \bar{B})$

E(2) For any two events A and B.

(a) Let $P(A \cup B) = 7/8$; $P(A \cap B) = 1/4$ and $P(A^c) = 5/8$.

Find P(A), P(B) and $P(A \setminus B)$

HW

(b) If A and B are any two events with $P(A) = 3/8$; $P(B) = 1/2$ and $P(A \cap B) = 1/4$. Find

(i) $P(A \cup B)$ (ii) $P(A^c)$ and $P(B^c)$ (iii) $P(A^c \cap B^c)$ (iv) $P(A^c \cup B^c)$

(v) $P(A \cap B^c)$ (vi) $P(B \cap A^c)$

[**Ans.** (i) 5/8; (ii) 5/8 & 1/2; (iii) 3/8 ; (iv) 3/4 ; (v) 1/8 ; (vi) 1/4]

(a) $P(A \cup B) = 7/8$; $P(A \cap B) = 1/4$; $P(\bar{A}) = 5/8$

$\Downarrow$

$\boxed{P(A) = \frac{3}{8}}$

$P(B) = ?$; $P(A \setminus B) = ?$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{7}{8} = \frac{3}{8} + P(B) - \frac{1}{4} \Rightarrow P(B) = \checkmark$$

$$P(A \setminus B) = P(A - B) = P(A) - P(A \cap B)$$

HW Prob. Sheet

Ex 0-1 Part - 1 only