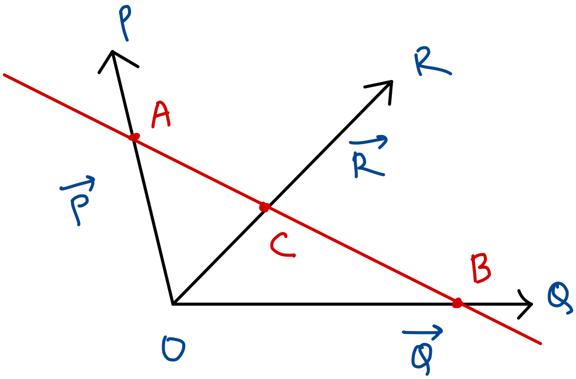
Q
HW

Vectors $\vec{P}$, $\vec{Q}$, act at 'O' (origin) have a resultant $\vec{R}$. If any transversal cuts their line of action at A, B, C respectively, then show that $\frac{OP}{OA}+\frac{OQ}{OB}=\frac{OR}{OC}$.



$$\vec{P}+\vec{Q}=\vec{R}.$$

$$\vec{OP}+\vec{OQ}=\vec{OR}$$

$$|\vec{OP}|\,\hat{OP}+|\vec{OQ}|\,\hat{OQ}=|\vec{OR}|\,\hat{OR}$$

$$(OP)\,\hat{OA}+(OQ)\,\hat{OB}-(OR)\,\hat{OC}=0$$

$$(OP)\frac{\vec{OA}}{OA}+(OQ)\frac{\vec{OB}}{OB}-(OR)\frac{\vec{OC}}{OC}=0$$

Since A, B, C are Collinear

hence

$$\frac{OP}{OA}+\frac{OQ}{OB}+\left(\frac{-OR}{OC}\right)=0.$$

(H.P.)

Q Two vectors $\vec{e}_1$ and $\vec{e}_2$ with $|\vec{e}_1| = 2$ and $|\vec{e}_2| = 1$ and angle between $\vec{e}_1$ and $\vec{e}_2$ is $60°$. The angle between $(2t\vec{e}_1 + 7\vec{e}_2)$ and $(\vec{e}_1 + t\vec{e}_2)$ belongs to the interval $(90°, 180°)$. Find the range of t.

Sol^n $\quad \vec{e}_1 \cdot \vec{e}_2 = |\vec{e}_1||\vec{e}_2| \cos 60° = (2)(1)\cdot\left(\frac{1}{2}\right) = 1. \quad — ① —$

$\vec{V}_1 = 2t\vec{e}_1 + 7\vec{e}_2$ and $\vec{V}_2 = \vec{e}_1 + t\vec{e}_2$

$\vec{V}_1 \cdot \vec{V}_2 < 0$ ** $\qquad \theta \in \left(\frac{\pi}{2}, \pi\right]$

$(2t\vec{e}_1 + 7\vec{e}_2) \cdot (\vec{e}_1 + t\vec{e}_2) < 0$

$2t e_1^2 + 2t^2 \vec{e}_1 \cdot \vec{e}_2 + 7\vec{e}_1 \cdot \vec{e}_2 + 7t e_2^2 < 0.$

$8t + 2t^2 + 7 + 7t < 0.$

$2t^2 + 15t + 7 < 0 \Rightarrow (2t+1)(t+7) < 0$

$\therefore t \in \left(-7, -\frac{1}{2}\right) \quad — ② —$

Now check for angle $180°$

$\vec{V}_1 \parallel \vec{V}_2$

$\frac{2t}{1} = \frac{7}{t} \Rightarrow t^2 = \frac{7}{2} \Rightarrow t = \pm\sqrt{\frac{7}{2}}$

$\therefore$ finally $\quad t \in \left(-7, -\frac{1}{2}\right) - \left\{-\sqrt{\frac{7}{2}}\right\}$

Angle betⁿ 2 lines :-

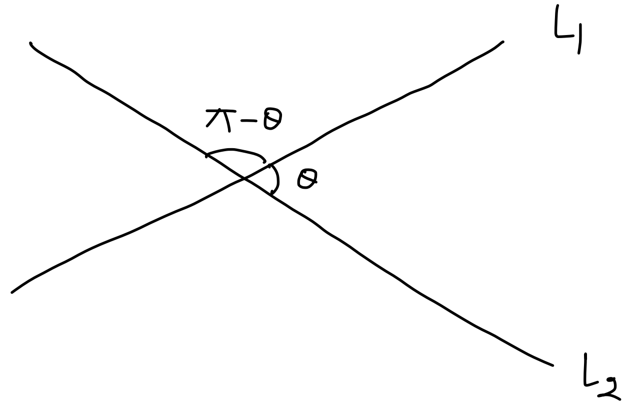
$L_1 : \vec{r} = \vec{a} + \lambda \vec{b}$

$L_2 : \vec{r} = \vec{p} + \mu \vec{q}$

$$\boxed{\vec{b} \cdot \vec{q} = |\vec{b}|\,|\vec{q}| \cos\theta.}$$

θ . ✓

$\pi - \theta$ ✓



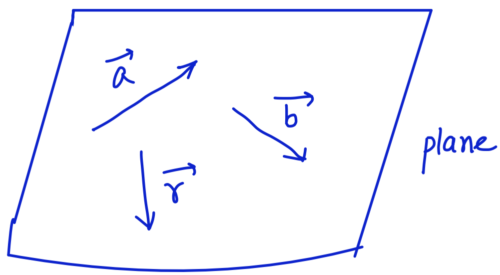
Linear combination

A vector $\vec{r}$ is said to be a linear combination of the vectors $\vec{a}, \vec{b}, \vec{c}$......
if $\exists$ scalars x,y,z, ... such that $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c} + ...$

V.imp

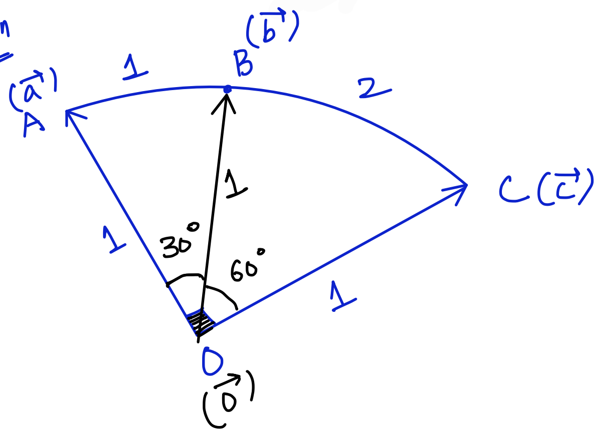
Theorem in plane

If $\vec{a}$ and $\vec{b}$ are two non zero non collinear vetors then any vector $\vec{r}$ coplanar with them can be expressed as a linear combination $\vec{r} = x\vec{a} + y\vec{b}$.



(1) Arc AC of the quadrant of a circle with centre as origin and radius unity subtends a right angle at the origin. Point B divides the arc AC in the ratio 1 : 2. Express the vector $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.

Sol^n



Theorem in plane

$$\vec{c} = x\vec{a} + y\vec{b} \quad -①-$$

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

① dot with $\vec{a}$

$$\vec{a}.\vec{c} = x a^2 + y\, \vec{a}.\vec{b}$$

$$0 = x(1) + y(1)(1)\cos 30^\circ$$

$$0 = x + y.\frac{\sqrt{3}}{2} \quad -②-$$

(1) dot with $\vec{c}$:-

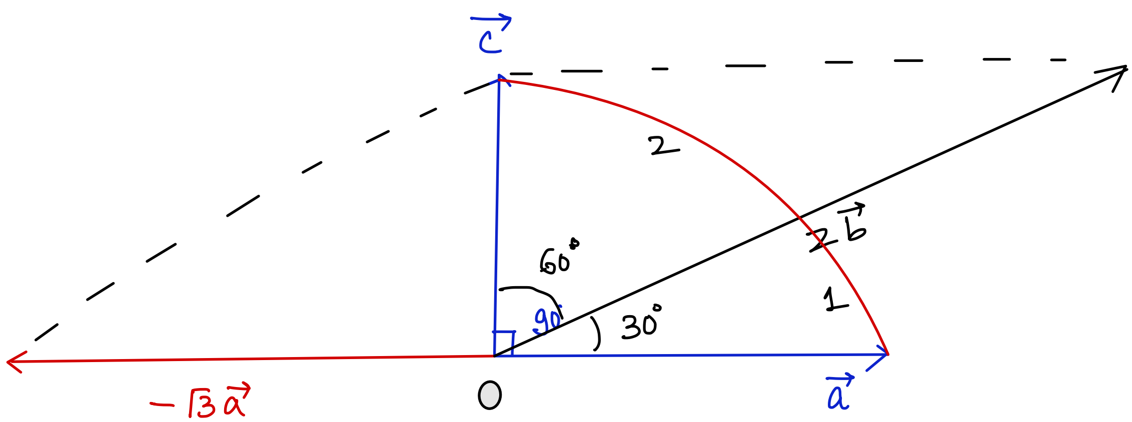
$$1 = x\,\vec{a}\cdot\vec{c} + y\,\vec{b}\cdot\vec{c}$$

$$1 = 0 + y\cdot(1)(1)\left(\frac{1}{2}\right)$$

$\therefore$ $\boxed{y = 2}$ put in (2)

$$x = -\sqrt{3}$$

$$\therefore \quad \vec{c} = -\sqrt{3}\,\vec{a} + 2\vec{b} \quad \text{Ans}$$



Q Given that $\vec{a} = \hat{i} - \hat{j}$ and $\hat{b} = \hat{i} + 2\hat{j}$ are two vectors. Find a unit vector coplanar with $\vec{a}$ and $\vec{b}$ and perpendicular to $\vec{a}$.

Sol^n $\vec{c} = x\vec{a} + y\vec{b}$; $|\vec{c}| = 1$ and $\vec{c} \cdot \vec{a} = 0$.

$$\vec{c} = x(\hat{i} - \hat{j}) + y(\hat{i} + 2\hat{j})$$

$$\vec{c} = (x+y)\hat{i} + (-x+2y)\hat{j}$$

$$|\vec{c}| = \sqrt{(x+y)^2 + (-x+2y)^2} = 1$$

$$x^2 + y^2 + 2xy + x^2 + 4y^2 - 4xy = 1.$$

$$2x^2 + 5y^2 - 2xy = 1 \quad \text{— (1) —}$$

$x = $

$y = $

$\vec{c} \cdot \vec{a} = 0 \Rightarrow$

$$\vec{c} \cdot \vec{a} = x a^2 + y\, \vec{a} \cdot \vec{b}$$

$$0 = x(2) + y(1-2) \Rightarrow \boxed{2x - y = 0} \quad \text{— (2) —}$$

Q If $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar vectors, prove that
$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = 0$$

Sol^n $\vec{c} = x\vec{a} + y\vec{b}$

$$\begin{vmatrix} \vec{a} & \vec{b} & x\vec{a} + y\vec{b} \\ \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & x\vec{a} \cdot \vec{a} + y\,\vec{a} \cdot \vec{b} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & x\vec{a} \cdot \vec{b} + y\,\vec{b} \cdot \vec{b} \end{vmatrix}$$

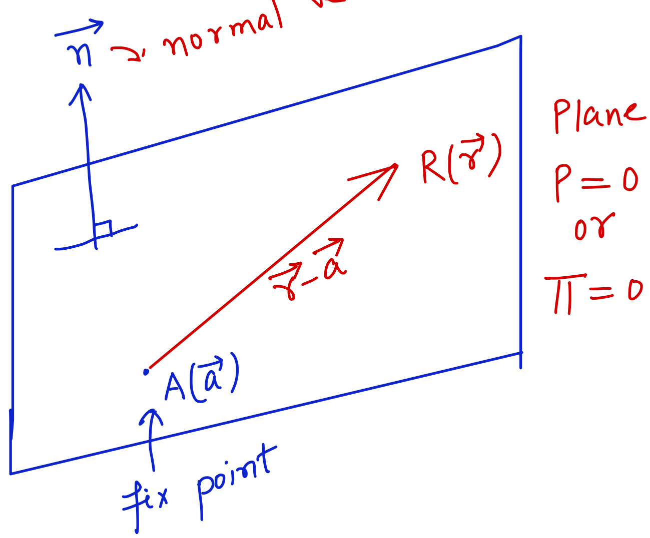
same $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ same

$$\begin{vmatrix} \vec a & \vec b & x\vec a \\ \vec a\cdot\vec a & \vec a\cdot\vec b & x\vec a\cdot\vec a \\ \vec b\cdot\vec a & \vec b\cdot\vec b & x\vec a\cdot\vec b \end{vmatrix} + \begin{vmatrix} \vec a & \vec b & y\vec b \\ \vec a\cdot\vec a & \vec a\cdot\vec b & y\vec a\cdot\vec b \\ \vec b\cdot\vec a & \vec b\cdot\vec b & y\vec b\cdot\vec b \end{vmatrix}$$

$x \qquad\qquad\qquad\qquad\qquad\qquad y$

$$0 + 0 = 0.$$

Equation of Plane :-



Eqn of plane

$$\boxed{(\vec r - \vec a)\cdot\vec n = 0}$$

$$\vec r\cdot\vec n = \underbrace{\vec a\cdot\vec n} = K$$

$$\boxed{\vec r\cdot\vec n = K}$$

$$\vec r\cdot\hat n = \frac{K}{|\vec n|} = \lambda.$$

$$\boxed{\vec r\cdot\hat n = \lambda}$$

Angle betn 2 planes :-

P_1 : $\qquad \vec r\cdot\vec n_1 = K_1$

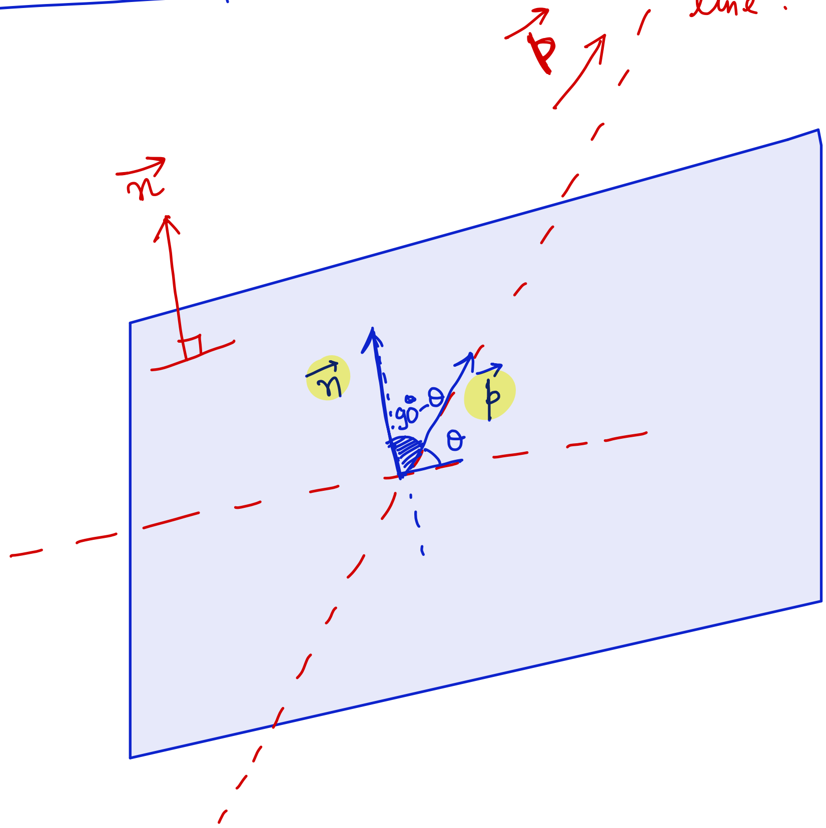
P_2 : $\qquad \vec r\cdot\vec n_2 = K_2$

Rem

$$\boxed{\vec n_1\cdot\vec n_2 = |\vec n_1|\,|\vec n_2|\cos\theta}$$

Angle betⁿ line & plane :-

line : $\vec{r} = \vec{a} + \lambda \vec{b}$



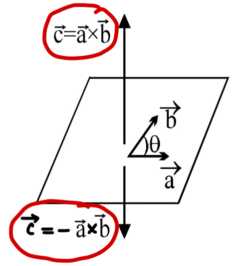
P : $\vec{r} \cdot \vec{n} = d$

Angle betⁿ line & plane is the complement of angle betⁿ normal vector of plane & direction vector of line.

$$\boxed{\vec{b} \cdot \vec{n} = |\vec{b}|\,|\vec{n}| \cos(90° - \theta)}$$

Vector product (Cross Product)

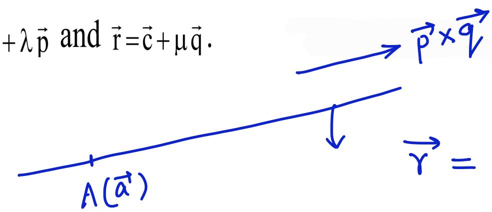
(1) $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$ where $\hat{n}$ is the unit vector perpendicular to the plane containing the vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a}$ and $\vec{b}$ and $\hat{n}$ forms a right handed screw system.



In general $\vec{a}\times\vec{b}\neq\vec{b}\times\vec{a}$

Q Equation of a line which passes through the point with p.v. $\vec{a}$ and perpendicular to the lines $\vec{r}=\vec{b}+\lambda\vec{p}$ and $\vec{r}=\vec{c}+\mu\vec{q}$.

Sol^n



$\vec{r}=\vec{a}+t(\vec{p}\times\vec{q})$.

$t\in$ scalar

Note :-

① Unit vector perpendicular to the plane of $\vec{a}$ & $\vec{b}$ is $\pm\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$ & a vector of magnitude 'r' perpendicular to the plane of $\vec{a}$ & $\vec{b}$ = $\pm\dfrac{r(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

② If θ is the angle between $\vec{a}$ & $\vec{b}$ then $\sin\theta=\pm\dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}$

Langrange's Identity :

$|\vec{a} \times \vec{b}|$ is very frequently needed for which Lagranges identity is used.

i.e. $\left|\vec{a} \times \vec{b}\right|^2 = \vec{a}^{\,2}\vec{b}^{\,2} - (\vec{a}.\vec{b})^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} \end{vmatrix}$.

Proof: $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2\sin^2\theta$

$= a^2b^2(1-\cos^2\theta)$

$= a^2b^2 - a^2b^2\cos^2\theta$

$= (\vec{a}.\vec{a})(\vec{b}.\vec{b}) - (\vec{a}.\vec{b})^2$

Q For any vector $\vec{a}$, $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2 = \lambda\vec{a}^2$ then $\lambda = ?$ 2 Ans

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{a}.\hat{i} = a_1$

Sol^n $|\vec{a}\times\hat{i}|^2 = a^2\hat{i}^2 - (\vec{a}.\hat{i})^2$

$= a^2(1) - (a_1)^2$ — 1 —

$||^{ly}$ $|\vec{a}\times\hat{j}|^2 = a^2(1) - (a_2)^2$ — 2 —

$|\vec{a}\times\hat{k}|^2 = a^2(1) - (a_3)^2$ — 3 —

Add 1, 2, 3

$= 3a^2 - (a_1^2 + a_2^2 + a_3^2)$

$= 3a^2 - a^2$

$= 2a^2$

Note : If $\vec{c}.\vec{a} = \vec{c}.\vec{b} = 0$ where $\vec{a}, \vec{b}$ vector are non-zero, non-parallel vectors, then $\vec{c} = \lambda(\vec{a} \times \vec{b})$.

Properties of cross product :

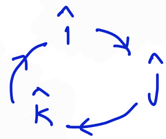
(i) $\vec{a}\times\vec{b} = \vec{0} \Rightarrow \vec{a} = \lambda\vec{b}(\vec{a}\neq 0;\ \vec{b}\neq 0)$ i.e. $\vec{a}$ and $\vec{b}$ are Collinear/Linearly dependent however if $\vec{a}\times\vec{b} = \vec{0}$ and $\vec{a}.\vec{b} = 0 \Rightarrow \vec{a} = 0$ or $\vec{b} = 0$

(ii) $\vec{a}\times\vec{b}\neq\vec{b}\times\vec{a}$ (in general) (not commutative)

(iii) $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$ (distributive to be proved later using triple product)

(iv) $(\vec{a}\times\vec{b})\times\vec{c}\neq\vec{a}\times(\vec{b}\times\vec{c})$ (in general) (vector product is not associative).

(v) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$ and $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$



(vi) $(\vec{a}\times\vec{b}).\vec{a} = 0,\ (\vec{a}\times\vec{b}).\vec{b} = 0$

Expression for $\vec{a}\times\vec{b}$, where $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ & $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ then $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$

Q1 Find the equation of the line through the point with p.v. $2\hat{i}+3\hat{j}$ and perpendicular to the vectors $\vec{A} = \hat{i}+2\hat{j}+3\hat{k}$ and $\vec{B} = 3\hat{i}+4\hat{j}+5\hat{k}$.

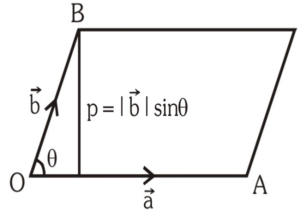
$\vec{r} = (2\hat{i}+3\hat{j})+\lambda(\vec{A}\times\vec{B})$

Q2 If $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k}) = \vec{0}$ find λ and μ.

$2\hat{i}+6\hat{j}+27\hat{k}$ must be Collinear with $\hat{i}+\lambda\hat{j}+\mu\hat{k}$

$$\frac{2}{1} = \frac{6}{\lambda} = \frac{27}{\mu} \Rightarrow \left.\begin{matrix}\lambda = 3\\ \mu = \frac{27}{2}\end{matrix}\right\}$$

Geometrical interpretation : $|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}|\sin\theta$ denotes the area of parallelogram whose two adjacent sides are the vectors $\vec{a}$ & $\vec{b}$.



Note :

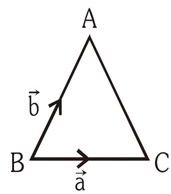
(i) Area of a parallelogram / quad. if diagonal vectors $\vec{d}_1$ & $\vec{d}_2$ are known is given by $\frac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right|$.

Q If $\vec{d}_1 = 2\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{d}_2 = 3\hat{i} - 4\hat{j} - \hat{k}$, then find area of parallelogram.

Vector area of a plane triangle :

(i) Area of the triangle (in terms of sides)

$$= \frac{|\vec{a} \times \vec{b}|}{2} = \frac{1}{2}\left|\overrightarrow{BA} \times \overrightarrow{BC}\right| = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$$

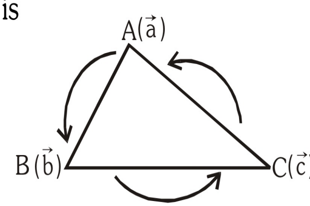


(ii) If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors then the vector area of ΔABC is

$$\vec{\Delta} = \frac{1}{2}[(\vec{c} - \vec{b}) \times (\vec{a} - \vec{b})]$$

Rem

$$\boxed{\vec{\Delta} = \frac{1}{2}[(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a})]}$$



$\overrightarrow{BC} = \vec{c} - \vec{b}$

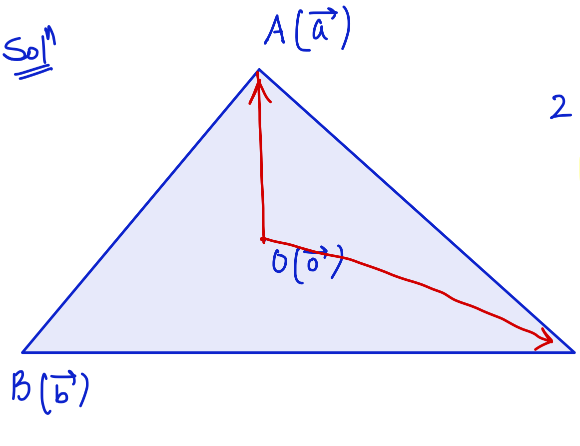
$\overrightarrow{BA} = \vec{a} - \vec{b}$

$\Delta = \frac{1}{2}\left|\overrightarrow{BC} \times \overrightarrow{BA}\right|$

Q Let 'O' be the interior point of ΔABC such that $2\vec{OA} + 5\vec{OB} + 10\vec{OC} = 0$. If the ratio of area of ΔABC to area of ΔAOC is λ (where O is origin) then find λ?

$\vec{OA} = \vec{a}$

Sol^n



$$2\vec{OA} + 5\vec{OB} + 10\vec{OC} = 0$$

$$2\vec{a} + 5\vec{b} + 10\vec{c} = 0. \quad -①-$$

$$\frac{\Delta ABC}{\Delta AOC} = \lambda.$$

$$\Delta ABC = \frac{1}{2}\left|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}\right|$$

$$\Delta AOC = \frac{1}{2}\left|\vec{c}\times\vec{a}\right|$$

$$(2\vec{a} + 5\vec{b} + 10\vec{c}) = \vec{0} \quad -①-$$

Cross with $\vec{c}$

$$2(\vec{a}\times\vec{c}) + 5(\vec{b}\times\vec{c}) + 10\underbrace{\vec{c}\times\vec{c}}_{\vec{0}} = \vec{0}$$

$$5(\vec{b}\times\vec{c}) = -2(\vec{a}\times\vec{c}) = 2(\vec{c}\times\vec{a})$$

$$\vec{b}\times\vec{c} = \frac{2}{5}(\vec{c}\times\vec{a})$$

① Cross with $\vec{a}$

$$\vec{0} + 5(\vec{a}\times\vec{b}) + 10(\vec{a}\times\vec{c}) = \vec{0}$$

$$5(\vec{a}\times\vec{b}) = -10(\vec{a}\times\vec{c}) = 10(\vec{c}\times\vec{a})$$

$$\vec{a}\times\vec{b} = 2(\vec{c}\times\vec{a})$$

$$\Delta ABC = \frac{1}{2}\left|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}\right|$$

$$= \frac{1}{2}\left|2(\vec{c}\times\vec{a}) + \frac{2}{5}(\vec{c}\times\vec{a}) + (\vec{c}\times\vec{a})\right|$$

$$\Delta ABC = \frac{1}{2}\left(2+\frac{2}{5}+1\right)\left|\vec{c}\times\vec{a}\right|$$

$$\Delta AOC = \frac{1}{2}\left|\vec{c}\times\vec{a}\right|$$

$$\frac{\Delta ABC}{\Delta AOC} = 3+\frac{2}{5} = \frac{17}{5}$$ Ans

Note :

(i) If 3 points with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are collinear then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$.

(ii) Unit vector perpendicular to the plane of the ΔABC when $\vec{a}, \vec{b}, \vec{c}$ are the p.v. of its angular point

$\hat{n} = \pm\left(\dfrac{\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}}{2\Delta}\right)$, where $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the angular points of the triangle ABC.

Q For a non zero vector $\vec{a}$, if $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$. Prove that $\vec{b}=\vec{c}$.

$\vec{a}\cdot\vec{b} - \vec{a}.\vec{c} = 0$ $\qquad\qquad$ $\vec{a}\times\vec{b} - \vec{a}\times\vec{c} = \vec{0}$

$\vec{a}.(\vec{b}-\vec{c}) = 0$ —①— $\qquad\qquad$ $\vec{a}\times(\vec{b}-\vec{c}) = \vec{0}$ —②—

from ① & ② $\qquad$ Since $\vec{a} \neq \vec{0}$

So $\vec{b}-\vec{c} = \vec{0} \Rightarrow \vec{b}=\vec{c}$.

(H.P.)

Q

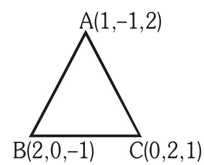
For the given figure find :

(a) A vector of magnitude $\sqrt{6}$ perpendicular to the plane ABC

(b) Area of triangle ABC

(c) Length ot the altitude from A (AB = AC = $\sqrt{11}$)

(d) Equation of the plane ABC



Sol^n

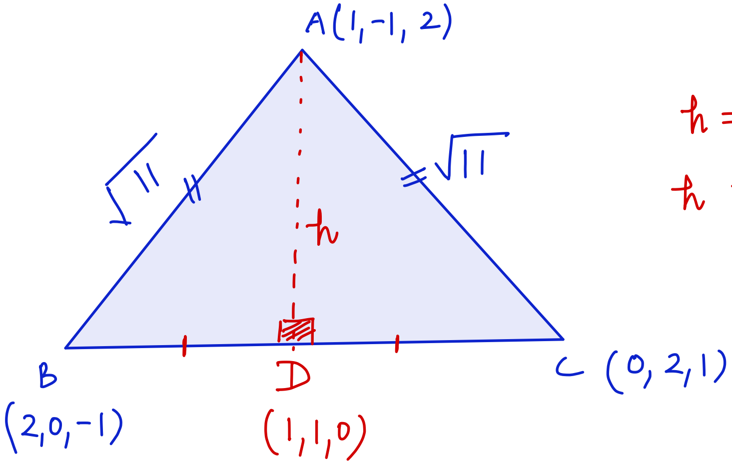
$\overrightarrow{BC} = -2\hat{i}+2\hat{j}+2\hat{k}$

$\overrightarrow{BA} = -\hat{i}-\hat{j}+3\hat{k}$

(d) $\left(\vec{r}-(2\hat{i}+0\hat{j}-\hat{k})\right)\cdot\vec{n} = 0$.

$\vec{n} = \overrightarrow{BC}\times\overrightarrow{BA}$

(a) $\pm\sqrt{6}\,\hat{n}$ $\qquad$ (b) $\frac{1}{2}\left|\overrightarrow{BC}\times\overrightarrow{BA}\right|$



$h = |\vec{AD}|$

$h = |2\hat{j} - 2\hat{k}| = \sqrt{4+4}$
$= 2\sqrt{2}$

*

Q. Find the unknown vector $\vec{R}$ satisfying $\vec{R}\times\vec{B} = \vec{C}\times\vec{B}$ and $\underbrace{\vec{R}.\vec{A} = 0}$, where $\vec{A} = 2\hat{i}+\hat{k}$; $\vec{B} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{C} = 4\hat{i}-3\hat{j}+7\hat{k}$

Sol^n

$\vec{R}\times\vec{B} - \vec{C}\times\vec{B} = \vec{0}$

$(\vec{R}-\vec{C})\times\vec{B} = \vec{0}$

(i) $\vec{R}-\vec{C} = \vec{0}$ OR
(ii) $\vec{B} = \vec{0}$ rejected OR
(iii) $\vec{R}-\vec{C}$ is Collinear with $\vec{B}$.

(i) If $\vec{R}-\vec{C} = \vec{0}$ then
$\vec{R} = \vec{C}$ when $\vec{R} = \vec{C}$ then $\vec{R}\cdot\vec{A} \neq 0$
but in question it is given that $\vec{R}\cdot\vec{A} = 0$
So this case is rejected.

(iii) $\vec{R}-\vec{C} = \lambda\vec{B}$; $\lambda \in$ scalar

$\vec{R} = \vec{C} + \lambda\vec{B} = (4\hat{i}-3\hat{j}+7\hat{k}) + \lambda(\hat{i}+\hat{j}+\hat{k})$ — (1)

$\vec{R} \cdot \vec{A} = 0$ to get λ.

$$2(4+\lambda) + 0 + (7+\lambda) = 0$$

$$\therefore \quad 15 + 3\lambda = 0$$

$\therefore \boxed{\lambda = -5}$ put in ① to get $\vec{R}$

HW Vector sheet :-

O-1 Q1 to 20