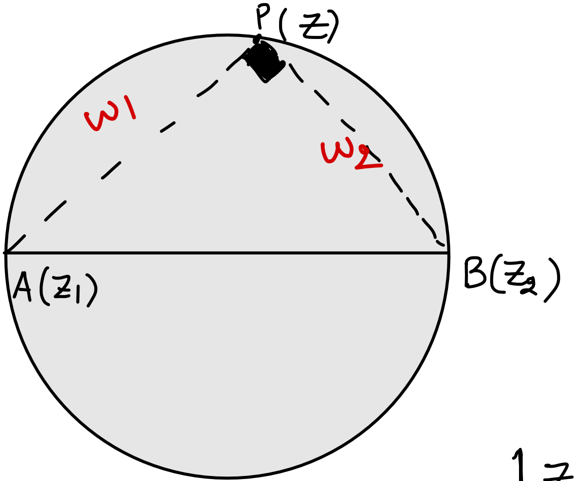
Diametrical form of Circle :-

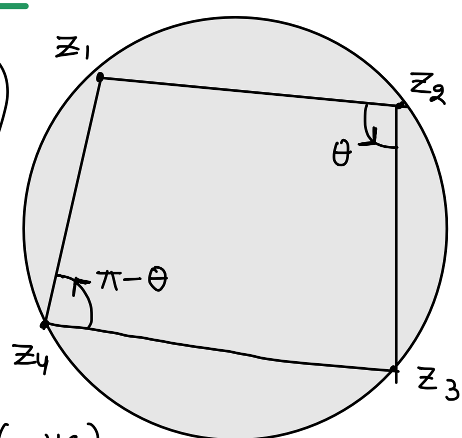


$$\frac{z-z_1}{\bar{z}-\bar{z}_1} + \frac{z_2-z}{\bar{z}_2-\bar{z}} = 0.$$

$$\frac{z-z_1}{\bar{z}-\bar{z}_1} + \frac{z-z_2}{\bar{z}-\bar{z}_2} = 0.$$

$$|z-z_1|^2 + |z-z_2|^2 = |z_1-z_2|^2.$$

Condition for 4 points to be con-cyclic :-

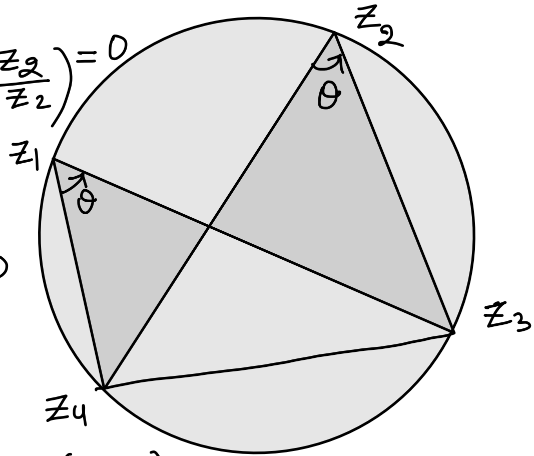


$$\arg\left(\frac{z_3-z_2}{z_1-z_2}\right) + \arg\left(\frac{z_1-z_4}{z_3-z_4}\right) = \pi$$

$$\arg\left(\frac{z_3-z_2}{z_1-z_2} \cdot \frac{z_1-z_4}{z_3-z_4}\right) = \pi$$

$\dfrac{z_3-z_2}{z_1-z_2} \cdot \dfrac{z_1-z_4}{z_3-z_4}$ is P.R (−ve) —(1)—

$$\frac{z_3-z_2}{z_1-z_2} \cdot \frac{z_1-z_4}{z_3-z_4} = \frac{\bar{z}_3-\bar{z}_2}{\bar{z}_1-\bar{z}_2} \cdot \frac{\bar{z}_1-\bar{z}_4}{\bar{z}_3-\bar{z}_4}.$$



$$\arg\left(\frac{z_3-z_1}{z_4-z_1}\right) - \arg\left(\frac{z_3-z_2}{z_4-z_2}\right) = 0$$

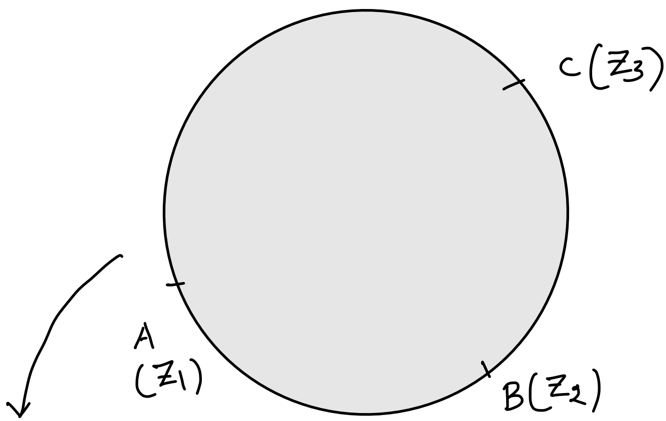
$\downarrow$

$$\arg\left(\frac{z_3-z_1}{z_4-z_1}\cdot\frac{z_4-z_2}{z_3-z_2}\right) = 0$$

$$\left(\frac{z_3-z_1}{z_4-z_1}\cdot\frac{z_4-z_2}{z_3-z_2}\right) \text{ is P.R (+ve)} \quad \text{— (2) —}$$

$\downarrow$

$$\boxed{\frac{z_3-z_1}{z_4-z_1}\cdot\frac{z_4-z_2}{z_3-z_2} = \frac{\bar{z}_3-\bar{z}_1}{\bar{z}_4-\bar{z}_1}\cdot\frac{\bar{z}_4-\bar{z}_2}{\bar{z}_3-\bar{z}_2}.}$$



Eqn of this Circle

Replace z_4 by z in the above relation

NOTE :– ★★

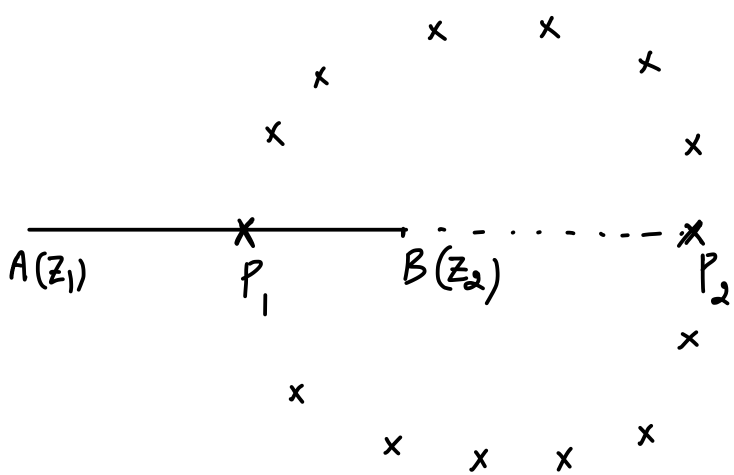
A point P(z) moves in argand plane in such a way that its distances from two fixed points $A(z_1)$ & $B(z_2)$ are such that $\frac{PA}{PB} = k$

$$\Rightarrow \left|\frac{z - z_1}{z - z_2}\right| = k \quad (k > 0)$$

(i) $k = 1$: locus is the perpendicular bisector of the line joining z_1 & z_2.

(ii) $k \neq 1$, locus is a circle whose diametric ends (P_1 & P_2) are the points which divides line joining z_1 and z_2 internally & externally in the ratio $k : 1$

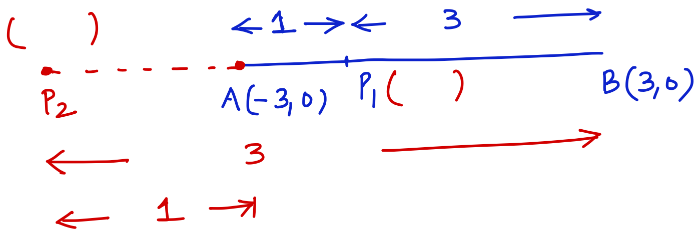
$\therefore$ Centre of the circle = mid point of P_1P_2



Q $\left|\frac{z-3}{z+3}\right| = 3.$

alt: $z = x+iy$

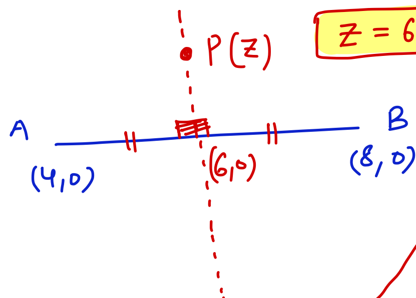
CIRCLE



**

Q If $\frac{|z-12|}{|z-8i|} = \frac{5}{3}$ and $\left|\frac{z-4}{z-8}\right| = 1$, then the value of z is

$|z-4| = |z-8|$



$\boxed{z = 6+\lambda i}$; $\lambda \in \mathbb{R}$

Sol^n

$3|z-12| = 5|z-8i|$

$3|6+\lambda i-12| = 5|6+\lambda i-8i|$

$3|-6+\lambda i| = 5|6+(\lambda-8)i|$

$3\sqrt{6^2+\lambda^2} = 5\sqrt{6^2+(\lambda-8)^2}$

↓

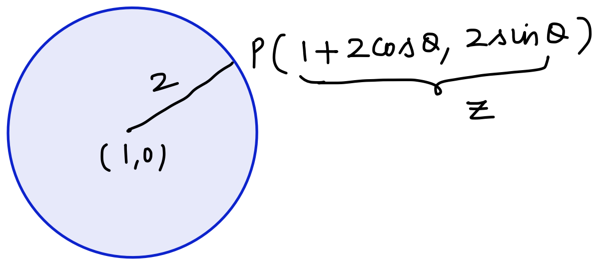
get value of 'λ'

Q. Let P(z) is a point in complex plane which satisfies $|z-1| = 2$

Locus of point Q(w) which satisfies $w = z + \dfrac{2}{z-1}$ is a conic, whose eccentricity is

Sol^n

$|z-1| = 2$



$$w = z + \frac{2}{z-1}$$

$$h+ik = (1+2\cos\theta)+i(2\sin\theta) + \frac{\cancel{2}}{\cancel{2}\cos\theta + i(\cancel{2}\sin\theta)} \times \frac{\cos\theta - i\sin\theta}{\cos\theta - i\sin\theta}$$

$$h+ik = (1+2\cos\theta)+i(2\sin\theta) + \frac{\cos\theta - i\sin\theta}{1}$$

$$h = 3\cos\theta + 1 \quad ; \quad K = \sin\theta.$$

$$\left(\frac{h-1}{3}\right)^2 + K^2 = \cos^2\theta + \sin^2\theta = 1.$$

$$\frac{(x-1)^2}{9} + \frac{y^2}{1} = 1.$$

Locus

$$|z - z_1| + |z - z_2| = K \text{ (constant)}$$

z_1 & $z_2 \rightarrow$ fixed Complex No.

(1) If $K > |z_1 - z_2|$ then locus of 'z' is an ellipse whose foci are z_1 & z_2 and length of major axis is 'K'

$$\boxed{e = \frac{|z_1 - z_2|}{K}}$$

(2) If $K = |z_1 - z_2|$ then locus of 'z' will be line segment joining z_1 & z_2

(3) If $K < |z_1 - z_2|$ then NO LOCUS.

Q1 $|z - 4i| + |z + 4i| = 10 \rightarrow K = 10$

A $(0, 4)$ B $(0, -4)$ $AB = 8$ $K > AB$

locus of 'z' will be an ellipse whose $e = \frac{8}{10} = \frac{4}{5}$.

Q2 $|z-2| + |z+2| = 4.$

locus is line segment

Q3 $|z-1| + |z+1| = 1$

NO LOCUS

$$\big|\, |z - z_1| - |z - z_2| \,\big| = K \text{ (Constant)}$$; z_1 & z_2 are fixed Complex No.

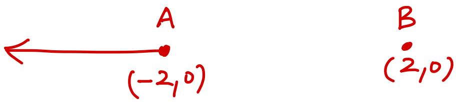
(1) If $K < |z_1 - z_2|$ then locus of 'z' will be hyperbola whose foci are z_1 & z_2 and length of T.A. is equal to 'K'

$$\boxed{e = \frac{|z_1 - z_2|}{K}}$$

(2) If $K > |z_1 - z_2|$ then NO LOCUS

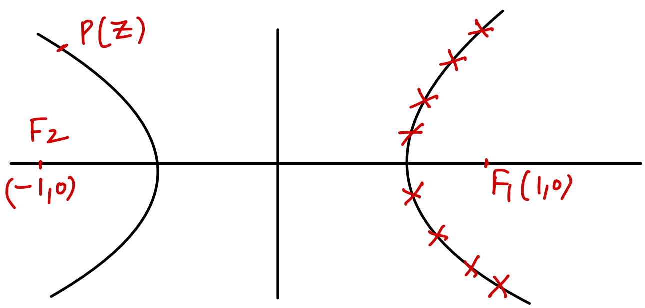
(3) If $K = |z_1 - z_2|$ then locus of 'z' is pair of rays emanating from z_1 & z_2 and moving away from it

① $|z-2| - |z+2| = 4.$

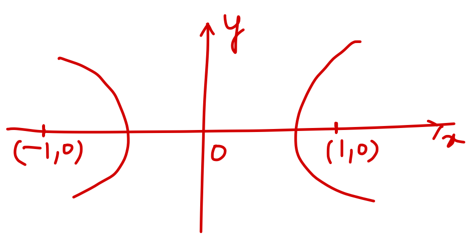


$||z-2| - |z+2|| = 4$

② $|z-1| - |z+1| = 1$



$||z-1| - |z+1|| = 1$

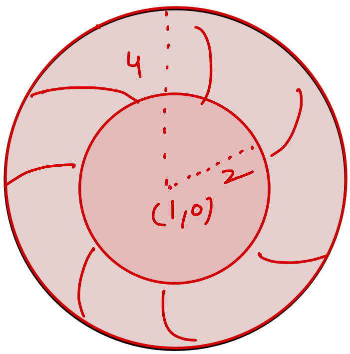


③ $|z-4i| - |z+4i| = 10$

NO LOCUS

$K = 10$
$AB = 8$
$K > AB$

Q $2 < |z-1| \le 4$

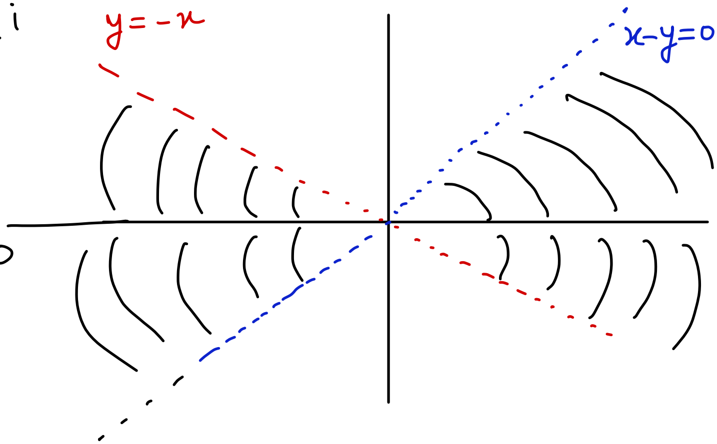


'z' will be lying between 2 concentric circles including periphery of bigger circle but excluding periphery of smaller circle

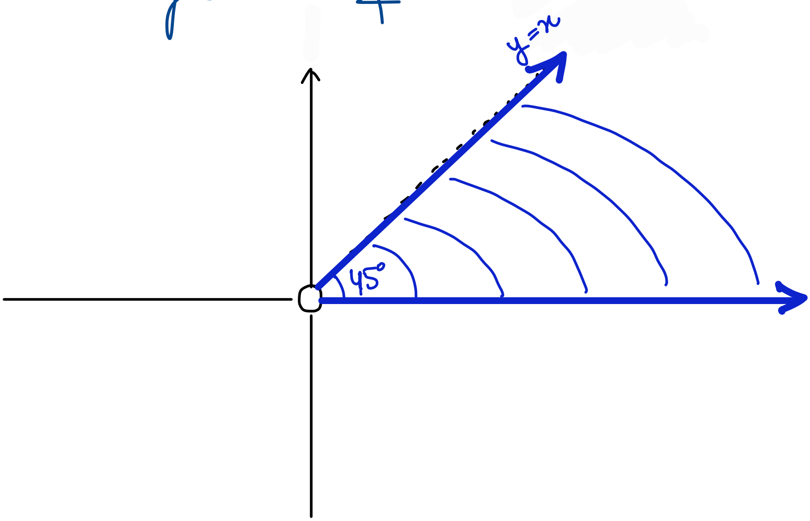
Q $\quad Re(z^2) > 0$.

$$z = x + iy$$
$$z^2 = x^2 - y^2 + 2xyi$$
$$Re(z^2) = x^2 - y^2$$

$$x^2 - y^2 > 0$$
$$(x-y)(x+y) > 0$$



Q $\quad 0 \le Arg(z) \le \frac{\pi}{4}$



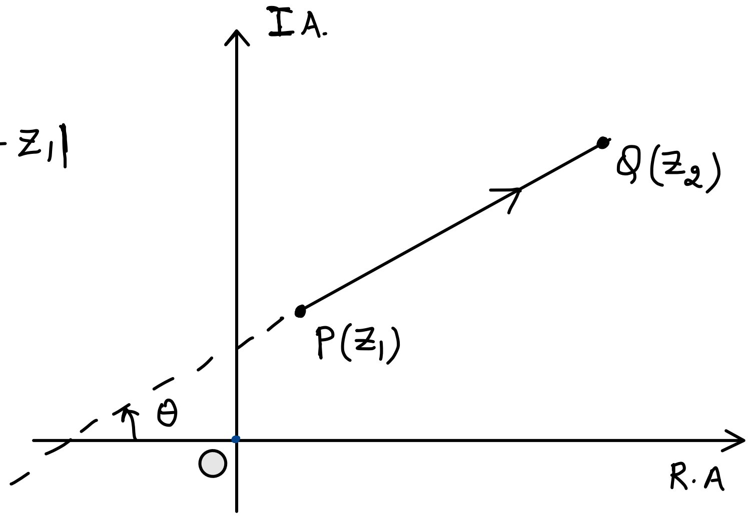
Vectorial representation of complex number

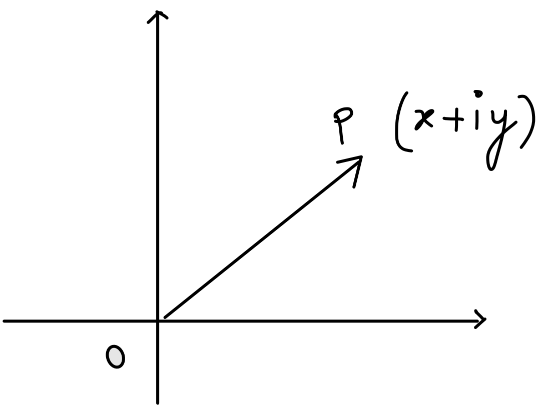
Complex Number 'z_1' corresponding to point 'p' can be treated as the position vector of point 'p'.

$$\boxed{\overrightarrow{PQ} = z_2 - z_1}$$

Such that $PQ = |z_2 - z_1|$

& $\text{Arg}(z_2 - z_1) = \theta$.





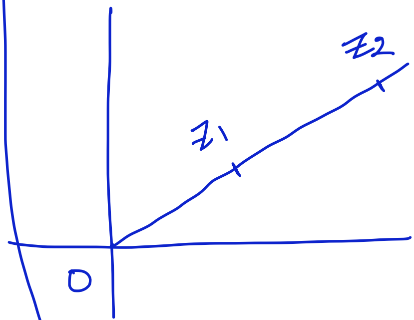
$$\overrightarrow{OP} = x\hat{i} + y\hat{j}$$

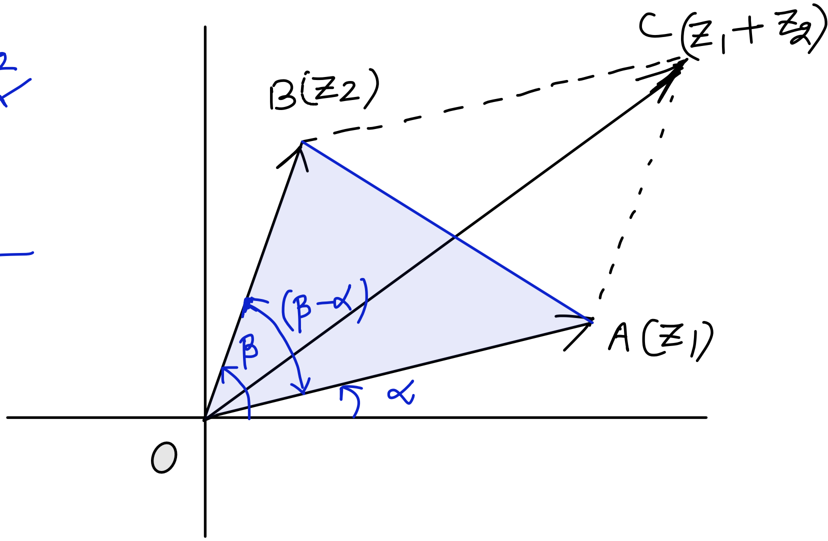
Note :-

$$|z_1+z_2|^2 = \underbrace{|z_1|^2+|z_2|^2+2|z_1||z_2|\cos(\beta-\alpha)}_{E}.$$

$$E_{max} = (|z_1|+|z_2|)^2$$ when $\cos(\beta-\alpha)=1$

$\beta-\alpha = 2n\pi\,;\ n\in I$



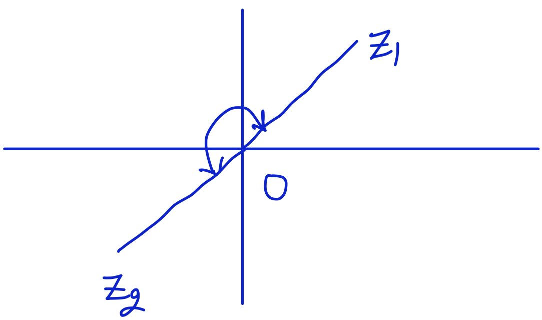


$$|z_1+z_2| \le |z_1|+|z_2|$$

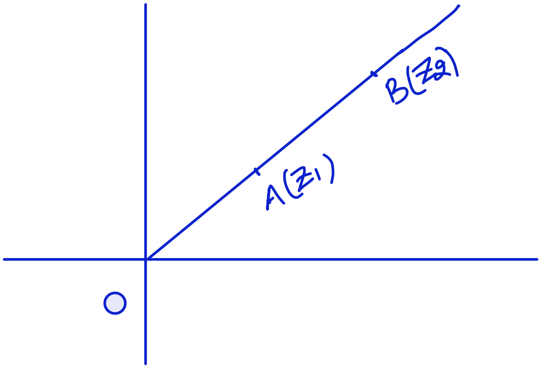
$$E_{min} = (|z_1|-|z_2|)^2$$ when $\cos(\beta-\alpha) = -1$

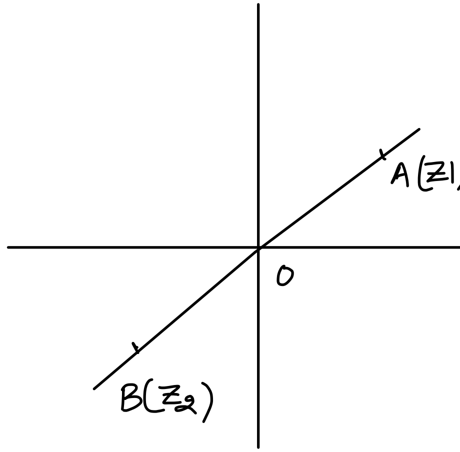
i.e $\beta-\alpha = (2n+1)\pi$

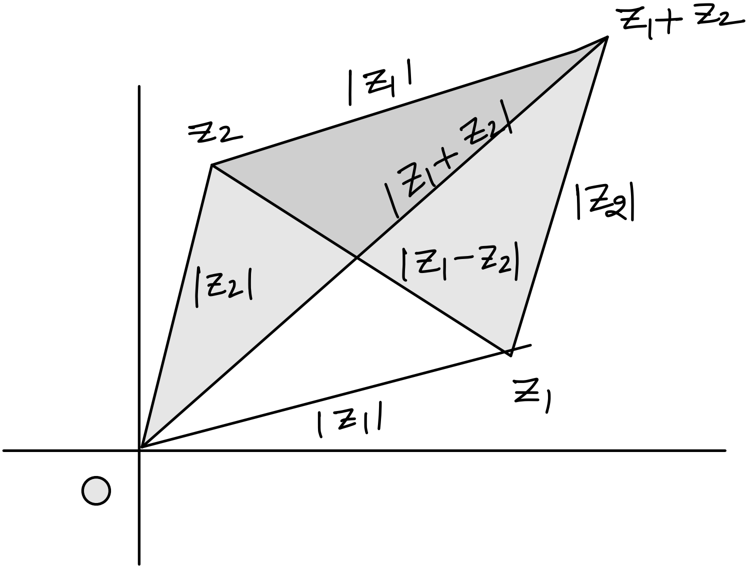
$n\in I$



$$|z_1+z_2| \geq \big||z_1|-|z_2|\big|$$


B(z2)
A(z1)
O


A(z1)
O
B(z2)


z1 + z2
|z1|
z2
|z1 + z2|
|z2|
|z1 − z2|
|z2|
z1
|z1|
O

$$\big||z_1|-|z_2|\big| \le |z_1+z_2| \le |z_1|+|z_2|$$ [TRIANGLE INEQUALITY]

Imp

$$\big||z_1|-|z_2|\big| \le |z_1-z_2| \le |z_1|+|z_2|$$

$\underline{Q_1}$ Find least value of $|z|+|1-z|$?

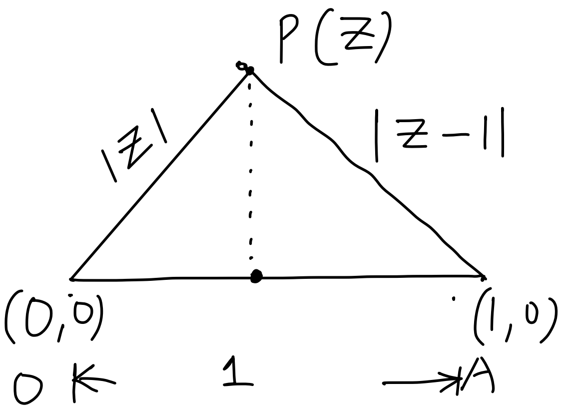
$$|z_1+z_2| \le |z_1|+|z_2|$$

$$|\cancel{z}+1-\cancel{z}| \le |z|+|1-z|$$

$$1 \le \underbrace{|z|+|1-z|}$$

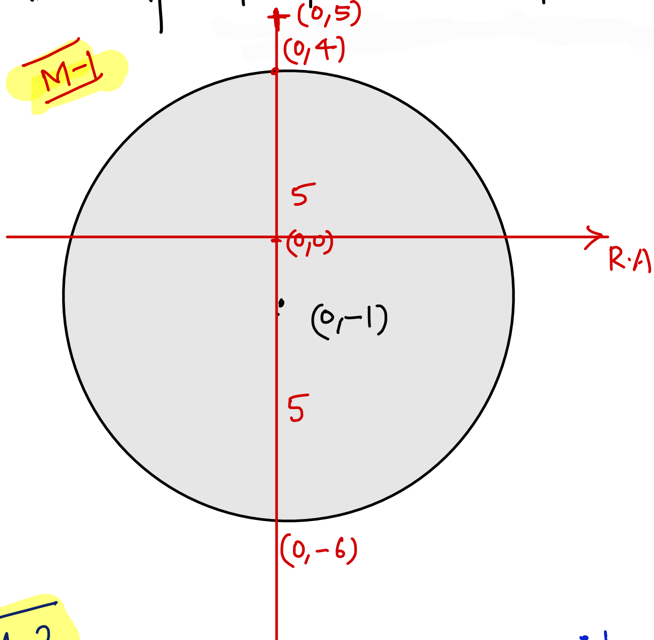
1 Ans

alt :



Q If $|z+i| = 5$ then find max & minimum value of $|z|$ and $|z-5i|$?

Sol^n **M-1**



$|z|_{min} = 4$
$|z|_{max} = 6.$ ✓

$|z-5i|$

min $= 1$ max $= 11$

M-2

① $$\big||z| - |i|\big| \le |z+i| \le |z| + |i|$$

$$\underbrace{\big||z| - 1\big| \le 5}_{} \quad \underbrace{\le |z| + 1}_{}$$

$$-5 \le |z| - 1 \le 5 \qquad |z| \ge 4$$

$$-4 \le |z| \le 6$$

$$\Downarrow$$

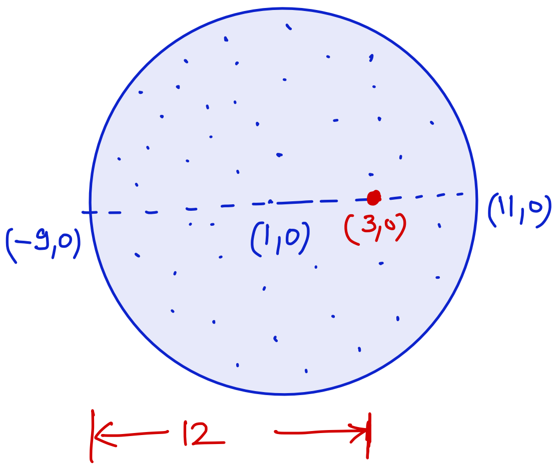
$$\boxed{4 \le |z| \le 6}$$

② $$\big||z-5i| - |6i|\big| \le |\underbrace{z-5i}_{} + \underbrace{6i}_{}| \le |z-5i| + |6i|$$

$$\big||z-5i| - 6\big| \le 5 \le |z-5i| + 6$$

Q If $|z-1| \le 10$ find $|z-3|_{max}$

$|z-3|_{min}$?



$|z-3|_{min} = 0$

$|z-3|_{max} = 12$

HW

Q

Let z_1 lies on $|z| = 1$ and z_2 lies on $|z| = 2$, then which of the following is/are true -

(A) maximum value of $|z_1 + z_2|$ is 3

(B) minimum value of $|2z_1 - z_2|$ is 0

(C) maximum value of $|2z_1 + z_2|$ is 4

(D) minimum value of $|2z_1 - 3z_2|$ is 5

HW

Q Let z is a complex number such that $|z| = 1$, then maximum value of $|z+1| + |z^2 - z + 1|$

No H·W Today

(Do revision)