Mathematical Reasoning

STATEMENT : OR Mathematical statement.

A sentence which is either true or false but cannot be both are called a statement. A sentence which is an exclamatory or a wish or an imperative or an interrogative can not be a statement.

If a statement is true then its truth value is T and if it is false then its truth value is F

For ex.

(i) "New Delhi is the capital of India", a true statement

(ii) "$3 + 2 = 6$", a false statement

(iii) "Where are you going ?" not a statement beasuse it connot be defined as true or false

Note : A statement cannot be both true and false at a time

SIMPLE STATEMENT :

Any statement whose truth value does not depend on other statement are called simple statement

For ex. (i) "$\sqrt{2}$ is an irrational number" (ii) "The set of real number is an infinite set"

COMPOUND STATEMENT :

A statement which is a combination of two or more simple statements are called compound statement Here the simple statements which form a compound statement are known as its sub statements

For ex.

(i) "If x is divisible by 2 then x is even number"

(ii) "ΔABC is equilatral if and only if its three sides are equal"

LOGICAL CONNECTIVES :

The words or phrases which combined simple statements to form a compound statement are called logical connectives.

In the following table some possible connectives, their symbols and the nature of the compound statement formed by them

S.N.	Connectives	symbol	use	operation
1.	and	$\wedge$	$p \wedge q$	conjunction
2.	or	$\vee$	$p \vee q$	disjunction
3.	not	$\sim$ or $'$	$\sim p$ or p'	negation
4.	If .... then	$\Rightarrow$ or $\rightarrow$	$p \Rightarrow q$ or $p \rightarrow q$	Implication or conditional
5.	If and only if (iff)	$\Leftrightarrow$ or $\leftrightarrow$	$p \Leftrightarrow q$ or $p \leftrightarrow q$	Equivalence or Bi-conditional

Explanation :

(i) $p \wedge q \equiv$ statement p and q

($p \wedge q$ is true only when p and q both are true otherwise it is false)

(ii) $p \vee q \equiv$ statement p or q

($p \vee q$ is true if at least one from p and q is true i.e. $p \vee q$ is false only when p and q both are false)

(iii) $\sim p \equiv$ not statement p

($\sim p$ is true when p is false and $\sim p$ is false when p is true)

(iv) $p \Rightarrow q \equiv$ statement p then statement q

($p \Rightarrow q$ is false only when p is true and q is false otherwise it is true for all other cases)

(v) $p \Leftrightarrow q \equiv$ statement p if and only if statement q

($p \Leftrightarrow q$ is true only when p and q both are true or false otherwise it is false)

TRUTH TABLE :

A table which shows the relationship between the truth value of compound statement S(p, q, r ....) and the truth values of its sub statements p, q, r, ... is said to be truth table of compound statement S

If p and q are two simple statements then truth table for basic logical connectives are given below

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

p	$(\sim p)$
T	F
F	T

Conditional

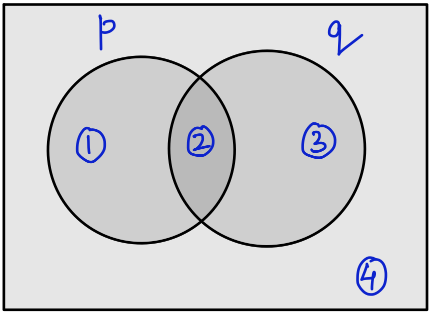
p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$ or $p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Note : If the compound statement contain n sub statements then its truth table will contain 2^n rows.

Use of set theory :-



$p \vee q \rightarrow$ 1, 2, 3

$p \wedge q \rightarrow$ 2

$p \rightarrow q \rightarrow$ 2, 3, 4

$(p \rightarrow q) \wedge (q \rightarrow p)$

Region	p	q	$p \vee q$	$p \wedge q$	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
1	T	F	T	F	F	T	F
2	T	T	T	T	T	T	T
3	F	T	T	F	T	F	F
4	F	F	F	F	T	T	T

Rem

** $\boxed{p \rightarrow q \equiv \sim p \vee q}$

$p \rightarrow q \equiv \sim 1 \equiv \sim(p \wedge \sim q)$

$\Downarrow$

$\sim p \vee q$

De-morgan's law :-

$\overline{A \cap B} \equiv \bar{A} \cup \bar{B}$

$\overline{A \cup B} \equiv \bar{A} \cap \bar{B}$

$\sim(A \cap B) \equiv \sim A \cup \sim B$

$\sim(A \cup B) \equiv \sim A \cap \sim B$

LOGICAL EQUIVALENCE :

Two compound statements $S_1(p, q, r...)$ and $S_2(p, q, r ....)$ are said to be logically equivalent or simply equivalent if they have same truth values for all logically possibilities

Two statements S_1 and S_2 are equivalent if they have identical truth table i.e. the entries in the last column of their truth table are same. If statements S_1 and S_2 are equivalent then we write $S_1 \equiv S_2$

For ex. The truth table for $(p \rightarrow q)$ and $(\sim p \vee q)$ given as below

p	q	$(\sim p)$	$p \rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

We observe that last two columns of the above truth table are identical hence compound statements $(p \rightarrow q)$ and $(\sim p \vee q)$ are equivalent

i.e. $\boxed{p \rightarrow q \equiv \sim p \vee q}$

TAUTOLOGY AND CONTRADICTION :

(i) Tautology : A statement is said to be a tautology if it is true for all logical possibilities i.e. its truth value always T. it is denoted by t.

For ex. the statement $p \vee \sim (p \wedge q)$ is a tautology

p	q	$p \wedge q$	$\sim (p \wedge q)$	$p \vee \sim (p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Clearly, The truth value of $p \vee \sim (p \wedge q)$ is T for all values of p and q. so $p \wedge \sim (p \wedge q)$ is a tautology

(ii) Contradiction : A statement is a contradiction if it is false for all logical possibilities. i.e. its truth value always F. It is denoted by c.

Contradiction = Fallacy.

For ex. The statement $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction

p	q	$\sim p$	$\sim q$	$p \vee q$	$(\sim p \wedge \sim q)$	$(p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

Clearly, then truth value of $(p \vee q) \wedge (\sim p \wedge \sim q)$ is F for all value of p and q. So $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

Note : The negation of a tautology is a contradiction and negation of a contradiction is a tautology

DUALITY :

Two compound statements S_1 and S_2 are said to be duals of each other if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$

If a compound statement contains the special variable t (tautology) and c (contradiction) then obtain its dual we replaced t by c and c by t in addition to replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

Note :

(i) the connectives $\wedge$ and $\vee$ are also called dual of each other.

(ii) If $S^*(p, q)$ is the dual of the compound statement $S(p, q)$ then

(a) $S^*(\sim p, \sim q) \equiv \sim S(p, q)$ (ii) $\sim S^*(p, q) \equiv S(\sim p, \sim q)$

For ex. The duals of the following statements

(i) $(p \wedge q) \vee (r \vee s)$ (ii) $(p \vee t) \wedge (p \vee c)$

(iii) $\sim(p \wedge q) \vee [p \wedge \sim(q \vee \sim s)]$

are as given below

(i) $(p \vee q) \wedge (r \wedge s)$

(ii) $(p \wedge c) \vee (p \wedge t)$

(iii) $\sim(p \vee q) \wedge [p \vee \sim(q \wedge \sim s)]$

CONVERSE, INVERSE AND CONTRAPOSITIVE OF THE CONDITIONAL STATEMENT $(p \rightarrow q)$:

(i) **Converse** : The converse of the conditional statement $p \rightarrow q$ is defined as $q \rightarrow p$

same

(ii) **Inverse** : The inverse of the conditional statement $p \rightarrow q$ is defined as $\sim p \rightarrow \sim q$

(iii) **Contrapositive** : The contrapositive of conditional statement $p \rightarrow q$ is defined as $\sim q \rightarrow \sim p$

same

NEGATION OF COMPOUND STATEMENTS :

If p and q are two statements then

(i) **Negation of conjunction :** $\boxed{\sim(p \wedge q) \equiv \sim p \vee \sim q}$

p	q	$\sim p$	$\sim q$	$(p \wedge q)$	$\sim(p \wedge q)$	$(\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

(ii) Negation of disjunction : $\boxed{\sim(p \vee q) \equiv \sim p \wedge \sim q}$

p	q	$\sim p$	$\sim q$	$(p \vee q)$	$(\sim p \vee q)$	$(\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

(iii) Negation of conditional : $\boxed{\sim(p \rightarrow q) \equiv p \wedge \sim q}$

p	q	$\sim q$	$(p \rightarrow q)$	$\sim(p \rightarrow q)$	$(p \wedge \sim q)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

(iv) Negation of biconditional : $\boxed{\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)}$

we know that $\quad p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$$\therefore \quad \sim(p \leftrightarrow q) \equiv \sim[(p \rightarrow q) \wedge (q \rightarrow p)]$$

$$\equiv \sim(p \rightarrow q) \vee \sim(q \rightarrow p)$$

$$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$$

Note : The above result also can be proved by preparing truth table for $\sim(p \leftrightarrow q)$ and $(p \wedge \sim q) \vee (q \wedge \sim p)$

ALGEBRA OF STATEMENTS :

If p, q, r are any three statements then the some law of algebra of statements are as follow

(i) Idempotent Laws :

(a) $p \wedge p \equiv p$ (b) $p \vee p \equiv p$

i.e. $p \wedge p \equiv p \equiv p \vee p$

p	$(p \wedge p)$	$(p \vee p)$
T	T	T
F	F	F

(ii) Comutative laws :

(a) $p \wedge q \equiv q \wedge p$ (b) $p \vee q \equiv q \vee p$

p	q	$(p \wedge q)$	$(q \wedge p)$	$(p \vee q)$	$(q \vee p)$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	F	F	T	T
F	F	F	F	F	F

(iii) Associative laws :

(a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

p	q	r	$(p \wedge q)$	$(q \wedge r)$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Similarly we can proved result (b)

(iv) Distributive laws :

(a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ (c) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge (p \wedge r)$

(b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (d) $p \vee (q \vee r) \equiv (p \vee q) \vee (p \vee r)$

p	q	r	$(q \vee r)$	$(p \wedge q)$	$(p \wedge r)$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Similarly we can prove result (b), (c), (d)

(v) De Morgan Laws : (a) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

(b) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$(p \wedge q)$	$\sim(p \wedge q)$	$(\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Similarly we can proved resulty (b)

(vi) Involution laws (or Double negation laws) : $\sim(\sim p) \equiv p$

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

(vii) Identity Laws : If p is a statement and t and c are tautology and contradiction respectively then

(a) $p \wedge t \equiv p$ (b) $p \vee t \equiv t$ (c) $p \wedge c \equiv c$ (d) $p \vee c \equiv p$

p	t	c	$(p \wedge t)$	$(p \vee t)$	$(p \wedge c)$	$(p \vee c)$
T	T	F	T	T	F	T
F	T	F	F	T	F	F

(viii) Complement Laws :

(a) $p \wedge (\sim p) \equiv c$ (b) $p \vee (\sim p) \equiv t$ (c) $(\sim t) \equiv c$ (d) $(\sim c) \equiv t$

p	$\sim p$	$(p \wedge \sim p)$	$(p \vee \sim p)$
T	F	F	T
F	T	F	T

(ix) Contrapositive laws : $\boxed{p \to q \equiv \sim q \to \sim p}$

p	q	~ p	~ q	p → q	~ q →~ p
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

QUANTIFIED STATEMENTS AND QUANTIFIERS :

The words or phrases "All", "Some", "None", "There exists a" are examples of quantifiers.

A statement containing one or more of these words (or phrases) is a quantified statement.

E.g. (1) All dogs are poodles

(2) Some books have hard covers

(3) There exists an odd number which is prime.

Note : Phrases "There exists a" and "Atleast one" and the word "some" have the same meaning.

NEGATION OF QUANTIFIED STATEMENTS :

(1) '**None**' is the negation of '**at least one**' or '**some**' or '**few**'

Statement : Some dogs are poodles.

Negation : No dogs are poodles.

Similarly negation of '**some**' is '**none**'

(2) The negation of "**some A are B**" or "**There exist A which is B**" is "**No A are (is) B**" or "**There does not exist any A which is B".**

Statement-1 : Some boys in the class are smart

Statement-2 : There exists a boy in the class who is smart

Statement-3 :Alteast one boy in the class is smart

All the three above statements have same meaning as they all indicate "**existence**" of smart boy in the class.

Negation of these statements is

No boy in the class is smart.

or

There does not exist any boy in the class who is smart.

(3) Negation of "**All A are B**" is "**Some A are not B**".

Statement : All boys in the class are smart.

Negation : Some boys in the class are not smart.

or

There exists a boy in the class who is not smart.

Q Contrapositive of the statement
"If two numbers are not equal, then their squares are not equal." is :-

(1) If the squares of two numbers are equal, then the numbers are equal.

(2) If the squares of two numbers are equal, then the numbers are not equal.

(3) If the squares of two numbers are not equal, then the numbers are equal.

(4) If the squares of two numbers are not equal, then the numbers are not equal.

Contrapositive of $p \to q$ is $\sim q \to \sim p$

Q The expression ~(~p→q) is logically equvalent to :

(1) ~ p ^ ~ q (2) p ^ q

(3) ~ p ^ q (4) p ^ ~ q

$p \to q \equiv \sim p \vee q$

$\sim p \to q \equiv \sim(\sim p) \vee q \equiv p \vee q$

$\sim(\sim p \to q) \equiv \sim(p \vee q)$

$\equiv \sim p \wedge \sim q$

Q The negation of the boolean expression

$\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) r (2) $s \wedge r$

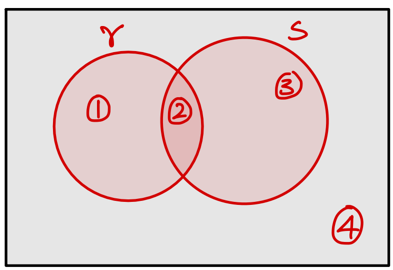
(3) $s \vee r$ (4) $\sim s \wedge \sim r$

$\sim r \wedge s \to$ ③

$\sim s \to$ ①, ④

$\sim s \vee (\sim r \wedge s) \to$ ①, ③, ④

$\sim(\sim s \vee (\sim r \wedge s)) \to$ ②



Q The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to :

(1) $(\sim p) \Rightarrow q$

(2) $p \vee q$

(3) $q \Rightarrow \sim p$

(4) $p \wedge q$ ✓

$\sim(p \rightarrow (\sim q))$

$\Downarrow$

$\sim(\sim p \vee \sim q)$

$p \wedge q$ ✓

Q If the truth value of the statement $p \rightarrow (\sim q \vee r)$ is false(F), then the truth values of the statements p, q, r are respectively :

(1) F, T, T

(2) T, F, F

(3) T, T, F ✓

(4) T, F, T

$p \rightarrow q \Rightarrow$ False

T F

$p \rightarrow \underbrace{(\sim q \vee r)}_{F} \Rightarrow$ False

$\downarrow$

T

Q The boolean expression

$((p\wedge q)\vee(p\vee\sim q))\wedge(\sim p\vee q)$ is equivalent to

(A) $p\leftrightarrow q$ (B) $p\rightarrow q$

(C) $\sim p\vee q$ (D) $p\wedge\sim q$

p	q	$\overbrace{p\wedge q}^{m}$	$\overbrace{p\vee\sim q}^{n}$	$\overbrace{\sim p\vee q}^{\alpha}$	$\overbrace{m\vee n}^{\beta}$	$\alpha\wedge\beta$
T	T	T	T	T	T	T
T	F	F	T	F	T	F
F	T	F	F	T	F	F
F	F	F	T	T	T	T

Q The statement $(p\rightarrow q)\vee(q\rightarrow p)$ is

(A) Tautology

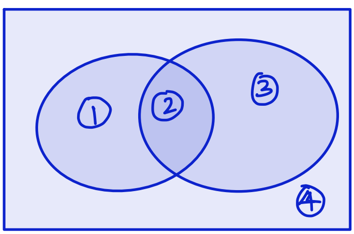
(B) Contradiction

(C) Neither tautology nor contradiction

(D) None of these

$(p\rightarrow q)\equiv\sim p\vee q\rightarrow$ 2, 3, 4

$(q\rightarrow p)\equiv\sim q\vee p\rightarrow$ 1, 2, 4



$(p\rightarrow q)\vee(q\rightarrow p)\Rightarrow$ 1, 2, 3, 4

Q The negation of the statement

"If I become a teacher, then I will open a school", is :

(1) I will not become a teacher or I will open a school.

(2) I will become a teacher and I will not open a school.

(3) Either I will not become a teacher or I will not open a school.

(4) Neither I will become a teacher nor I will open a school.

p : I become teacher

q : I will open school.

$\sim(p \to q)$

$\sim(\sim p \vee q) \equiv p \wedge \sim q$

Q Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is :

(1) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

(2) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

(3) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

(4) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$

Sol^n

Contrapositive of $p \to q$ is $\sim q \to \sim p$.

$(A \subseteq B) \wedge (B \subseteq D) \longrightarrow (A \subseteq C)$

$\sim(A \subseteq C) \longrightarrow \sim(A \subseteq B) \vee \sim(B \subseteq D)$

$A \not\subseteq C \longrightarrow (A \not\subseteq B) \vee (B \not\subseteq D)$

[2] Ans

Q The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

(1) p (2) q (3) ~p (4) ~q

Sol^n

$(\sim p \vee q) \wedge (\sim q \vee \sim p)$

$(\sim p \vee q) \wedge (\sim p \vee \sim q)$

$\sim p \vee \underbrace{(q \wedge \sim q)}_{c}$

$\sim p \vee c \equiv \sim p$

[3]

Q. Which of the following statements is a tautology?

(1) $\sim(p \vee \sim q) \rightarrow p \vee q$ ✓

(2) $\sim(p \wedge \sim q) \rightarrow p \vee q$

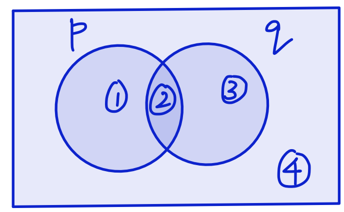
(3) $\sim(p \vee \sim q) \rightarrow p \wedge q$

(4) $p \vee (\sim q) \rightarrow p \wedge q$

Sol^n

(1) $\sim(p \vee \sim q) \rightarrow p \vee q$

$\underbrace{(\sim p \wedge q)}_{} \rightarrow \underbrace{p \vee q}_{} \Rightarrow$ Tautology



$(\sim p \wedge q) \rightarrow 3$

$(p \vee q) \rightarrow 1, 2, 3$

Q Which one of the following is a tautology ?

(1) $P \wedge (P \vee Q)$

(2) $P \vee (P \wedge Q)$

(3) $Q \rightarrow (P \wedge (P \rightarrow Q))$

(4) $(P \wedge (P \rightarrow Q)) \rightarrow Q$ ✓

(1) $P \wedge (P \vee Q) \equiv P$

(2) $P \vee (P \wedge Q) \equiv P$

(3) $Q \rightarrow (P \wedge (P \rightarrow Q))$

$Q \rightarrow (P \wedge (\sim P \vee Q))$

$Q \rightarrow (P \wedge Q)$

$\sim Q \vee (P \wedge Q) \equiv P \vee (\sim Q)$

(4) $(P \wedge (P \rightarrow Q)) \rightarrow Q$

$(\underbrace{P \wedge (\sim P \vee Q)}) \rightarrow Q$

$(P \wedge Q) \rightarrow Q \Rightarrow$ Tautology

$\sim (P \wedge Q) \vee Q$

$(\sim P \vee \sim Q) \vee Q$

$(\sim P) \vee (\underbrace{\sim Q \vee Q}_{t}) \equiv t$

Q If $p \rightarrow (p \wedge \sim q)$ is false, then the truth values of p and q are respectively :

(1) F, T
(2) T, T
(3) F, F
(4) T, F

Soln

$p \rightarrow T$

$(p \wedge \sim q) \longrightarrow F$

$T \wedge F$

Q Negation of the statement :

$\sqrt{5}$ is an integer or 5 is irrational is :

(1) $\sqrt{5}$ is irrational or 5 is an integer.
(2) $\sqrt{5}$ is not an integer and 5 is not irrational.
(3) $\sqrt{5}$ is an integer and 5 is irrational.
(4) $\sqrt{5}$ is not an integer or 5 is not irrational.

Soln

$\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$

Q The negation of the Boolean expression x $\leftrightarrow$ ~y is equivalent to :

(1) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

(2) $(x \wedge \sim y) \vee (\sim x \wedge y)$

(3) $(x \wedge y) \vee (\sim x \wedge \sim y)$

(4) $(x \wedge y) \wedge (\sim x \vee \sim y)$

Sol^n

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

$$x \leftrightarrow \sim y \equiv (x \to \sim y) \wedge (\sim y \to x)$$

$\because$ $p \to q \equiv \sim p \vee q$

$$x \leftrightarrow \sim y \equiv (\sim x \vee \sim y) \wedge (y \vee x)$$

$$\sim(x \leftrightarrow \sim y) \equiv (x \wedge y) \vee (\sim y \wedge \sim x)$$

$\longrightarrow$ [3]

Q The statement $(p \to (q \to p)) \to (p \to (p \vee q))$ is:

(1) a contradiction

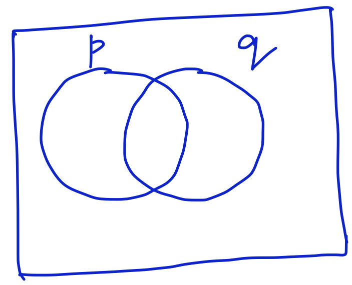
(2) equivalent to $(p \wedge q) \vee (\sim q)$

(3) a tautology

(4) equivalent to $(p \vee q) \wedge (\sim p)$

Sol^n

$p \to (p \vee q)$



$\sim p \vee (p \vee q)$

$(\sim p \vee p) \vee q \equiv$ Always true

$p \to (q \to p)$

$p \to (\sim q \vee p) \equiv \sim p \vee (\sim q \vee p)$

$(\sim p \vee \sim q) \vee p.$

Q The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to :

(1) $\sim p \vee \sim q$ (2) $\sim p \vee q$ (3) $\sim p \wedge \sim q$ (4) $p \wedge \sim q$

Solⁿ

$$p \vee (\sim p \wedge q) \equiv \underbrace{(p \vee \sim p)} \wedge (p \vee q)$$

$$\equiv T \wedge (p \vee q)$$

$$\equiv p \vee q$$

$$\sim (p \vee (\sim p \wedge q)) \equiv \sim (p \vee q) \equiv \sim p \wedge \sim q$$

[3]

Q Consider the statement :

"For an integer n, if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is :

(1) For an integer n, if $n^3 - 1$ is not even, then n is not odd.

(2) For an integer n, if n is even, then $n^3 - 1$ is odd.

(3) For an integer n, if n is odd, then $n^3 - 1$ is even.

(4) For an integer n, if n is even, then $n^3 - 1$ is even.

Do yourself

Q Which of the following is a tautology ?

(1) $(\sim p) \wedge (p \vee q) \rightarrow q$ (2) $(q \rightarrow p) \vee \sim(p \rightarrow q)$ (3) $(p \rightarrow q) \wedge (q \rightarrow p)$ (4) $(\sim q) \vee (p \wedge q) \rightarrow q$

(1) $\sim p \wedge (p \vee q) \rightarrow q$

$\underbrace{(\sim p \wedge p)}_{c} \vee (\sim p \wedge q) \rightarrow q$

$c \vee (\sim p \wedge q) \rightarrow q$

$(\sim p \wedge q) \rightarrow q$

$\sim(\sim p \wedge q) \vee q$

$(p \vee \sim q) \vee q \equiv (p \vee q) \vee \underbrace{(\sim q \vee q)}_{t} \equiv t$ [1]