Q A bag contains 5 balls of unknown colours. A ball is drawn twice with replacement from the bag found to be red on both the occasions. The contents of the bag were replenished. If now two ball are drawn simultaneously from the bag, find the probabilities that they will be both red. Assume all number of red balls in the bag to be equally likely.

$$\frac{\frac{1}{6}\left(\frac{5}{5}\right)^2 \frac{{}^5C_2}{{}^5C_2} + \frac{1}{6}\left(\frac{4}{5}\right)^2 \frac{{}^4C_2}{{}^5C_2} + \frac{1}{6}\left(\frac{3}{5}\right)^2 \frac{{}^3C_2}{{}^5C_2} + \left(\frac{1}{6}\right)\left(\frac{2}{5}\right)^2 \frac{{}^2C_2}{{}^5C_2} + 0 + 0}{\frac{1}{6}\left(\frac{5}{5}\right)^2 + \frac{1}{6}\left(\frac{4}{5}\right)^2 + \frac{1}{6}\left(\frac{3}{5}\right)^2 + \frac{1}{6}\left(\frac{2}{5}\right)^2 + \frac{1}{6}\left(\frac{1}{5}\right)^2 + 0}$$

E(2) Box A contains nine cards numbered 1 through 9 and box B contains five cards numbered I through 5. A box is chosen at random and a card drawn; if the card shows an even number, another card is drawn from the same box. If the card shows an odd number, a card is drawn from the other box ;

(a) What is the probability that both cards show even numbers ?

(b) What is the probability that both cards show odd numbers ?

(a) $\frac{1}{2}\left(\frac{4}{9}\right)\left(\frac{3}{8}\right)+\frac{1}{2}\left(\frac{2}{5}\right)\left(\frac{1}{4}\right)$

(b) $\frac{1}{2}\left(\frac{5}{9}\right)\left(\frac{3}{5}\right)+\frac{1}{2}\left(\frac{3}{5}\right)\left(\frac{5}{9}\right)$

Probability Distribution :-

(FOR JEE MAINS ONLY)

Rem (a) Mean or expectation of any probability distribution of a random variable is given by : (x_i)

$$\mu = \sum p_i x_i \quad (\text{Since } \Sigma p_i = 1)$$

Rem (b) Variance of a random variable is given by, $\sigma^2 = \Sigma (x_i - \mu)^2 \cdot p_i$

$$\sigma^2 = \Sigma p_i x_i^2 - \mu^2 \qquad (\text{Note that SD} = +\sqrt{\sigma^2})$$

Rem (c) Standard deviation $= \sigma = \sqrt{\text{variance}}$

$$\sigma^2 = \Sigma (x_i - \mu)^2 p_i$$

$$= \Sigma (x_i^2 p_i + \mu^2 p_i - 2 x_i \mu p_i)$$

$$= \Sigma p_i x_i^2 + \mu^2 \Sigma p_i - 2\mu \Sigma x_i p_i$$

$$= \Sigma p_i x_i^2 + \mu^2 - 2\mu(\mu)$$

$$\boxed{\sigma^2 = \Sigma p_i x_i^2 - \mu^2} \quad \text{H.P}$$

Binomial probability distribution :

If there are n Bernoulli trials & Probability of success = p

Probability of failure = q, then

(i) Rem Mean of BPD = np ;

(ii) Rem variance of BPD = npq ;

(iii) Rem SD = $+\sqrt{npq}$

x_i	$P(x_i)$
0	—
1	—
⋮	⋮
r	${}^nC_r \cdot p^r q^{n-r}$
⋮	⋮
n	—

mean $= \mu = \sum x_i p(x_i)$

$$\mu = \sum_{r=0}^{n} r \cdot {}^nC_r \, p^r \cdot q^{n-r}$$

$$= \sum_{r=0}^{n} \cancel{r} \cdot \frac{n}{\cancel{r}} \cdot {}^{n-1}C_{r-1} \, p^r \cdot q^{n-r}$$

$$= np \sum_{r=1}^{n} {}^{n-1}C_{r-1} \, p^{r-1} q^{n-r}$$

$$= np \, (p+q)^{n-1}$$

$$\boxed{\mu = np}$$

8 A pair of fair dice is thrown. Let X be the random variable which denotes the minimum of the two numbers which appear. Find the probability distribution, mean and variance of X.

Sol^{n}

x_i	$P(x_i)$	$x_i P(x_i)$	x_i^2	$x_i^2 P(x_i)$
1	11/36	11/36	1	11/36
2	9/36	18/36	4	36/36
3	7/36	21/36	9	63/36
4	5/36	20/36	16	80/36
5	3/36	15/36	25	75/36
6	1/36	+ 6/36	36	36/36
		$\mu = 91/36$		$\Sigma x_i^2 P(x_i) =$?

$$\sigma^2 = \Sigma x_i^2 P(x_i) - \mu^2$$

↓ Variance

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66.

Q A lot contain 8 items of which 5 are good and 3 defective. Getting a defective item is considered as success. If 3 are randomly drawn. Find the probability distribution, mean and S.D. of defective item.

$X_i \rightarrow$ defective item

X_i	$P(X_i)$	$X_i P(X_i)$	X_i^2	$X_i^2 P(X_i)$
0	$\frac{{}^5C_3}{{}^8C_3}$			
1	$\frac{{}^5C_2 \cdot {}^3C_1}{{}^8C_3}$			
2	$\frac{{}^5C_1 \cdot {}^3C_2}{{}^8C_3}$			
3	$\frac{{}^3C_3 \cdot {}^5C_0}{{}^8C_3}$			

Q If the mean and SD of a binomial variate X are 9 and 3/2 respectively. Find the probability that X takes a value greater than one.

Sol^n

$np = 9$; $\sqrt{npq} = \frac{3}{2}$; $p+q=1$

$\Downarrow$

$npq = \frac{9}{4}$

$q = \frac{1}{4}$; $p = \frac{3}{4}$

$n = \frac{9}{p} = 12$

$P(X=2) + P(X=3) + \cdots + P(X=12)$

$1 - P(X=0) - P(X=1) = 1 - \left({}^{12}C_0 \left(\frac{1}{4}\right)^{12} + {}^{12}C_1 \left(\frac{3}{4}\right)^1 \left(\frac{1}{4}\right)^{11} \right)$

Q Find the mean (expected) number of dots when a dice is thrown once.

$x_i \rightarrow$ No. of dots

x_i	$P(x_i)$	$x_i P(x_i)$
1	1/6	1/6
2	1/6	2/6
3	1/6	3/6
4	1/6	4/6
5	1/6	5/6
6	1/6	Add 6/6
		μ = ?

Mathematical expectation :-

It is worthwhile indicating that if p represents a person's chance of success in any venture and M the sum of money which he will receive in case of success, then the sum of money denoted by pM is called his expectation.

Q Four players P_i, i = 1, 2, 3, 4 take part in a tournament, where there is only one winner & chance of P_i being the winner is proportinal to i. If the prize money is Rs. 10,000, then find the amount which the players can expect.

Sol^n

$P_i \propto i \Rightarrow P_i = Ki$

$P_1 = K$; $P_2 = 2K$; $P_3 = 3K$; $P_4 = 4K$

$P_1 + P_2 + P_3 + P_4 = 1 \Rightarrow 10K = 1 \Rightarrow \boxed{K = 1/10}$

Money expected by P_1 = $K \times 10,000$ = 1000 Rs

" " " P_2 = 2000 Rs

" " " P_3 = 3000 Rs

" " " P_4 = 4000 Rs

Q. In a game , a man wins Rs 100 if he gets 5 or 6 on a throw of a fair die and loses Rs 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws , then his expected gain/ lose (in rupees) is

Sol^{n} X : expected gain/loss in Rupees

$P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$; $P(\overline{5 \text{ or } 6}) = \frac{4}{6} = \frac{2}{3}$

One throw : $(5 \text{ or } 6) \longrightarrow$ Rs 100

Two throws : $(\overline{5 \text{ or } 6})(5 \text{ or } 6) \longrightarrow -50 + 100 =$ Rs 50

Three " : $(\overline{5 \text{ or } 6})(\overline{5 \text{ or } 6})(5 \text{ or } 6) \longrightarrow -50 - 50 + 100 =$ Rs 0

$(\overline{5 \text{ or } 6})(\overline{5 \text{ or } 6})(\overline{5 \text{ or } 6}) \longrightarrow -50 - 50 - 50 =$ Rs (-150)

X_i	$P(X_i)$	$X_i P(X_i)$
100	$\frac{1}{3}$	$\frac{100}{3}$
50	$\frac{2}{3} \times \frac{1}{3}$	$\frac{100}{9}$
0	—	0
-150	$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$	$-\frac{150 \times 8}{27}$
		$\Sigma X_i P(X_i) = 0$

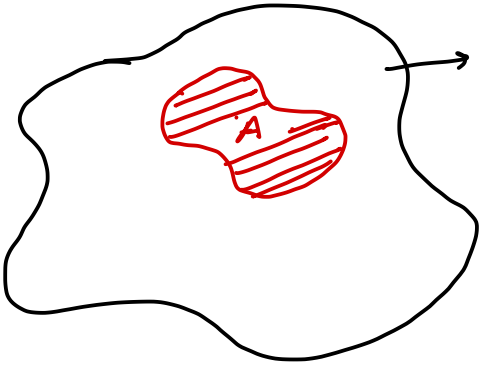
Ans

GEOMETRICAL PROBABILITY :

(i) If a point is randomly taken on area S & area A is included in area S then the probability that the point lies in area A is $\frac{A}{S}$

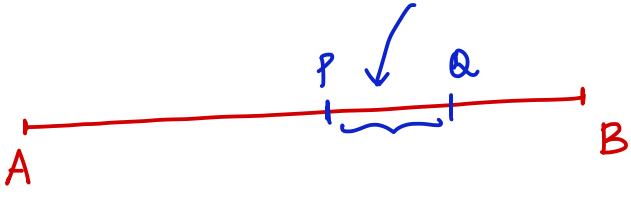
(ii) A point taken randomly on line segment AB lies on line segment PQ contained on it has probability $\frac{\ell(PQ)}{\ell(AB)}$

(i)



$Prob = \frac{A}{S}$

(ii)



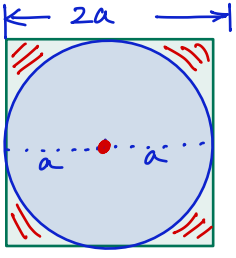
$Prob = \frac{l_{PQ}}{l_{AB}}$

E(1) A circle of radius 'a' is inscribed in a square of side 2a. Find the probability that a point chosen at random is inside the square but outside the circle.

E(2) A point is selected at random inside a circle. Find the probability that the point is closer to the circumference of the circle than to its centre.

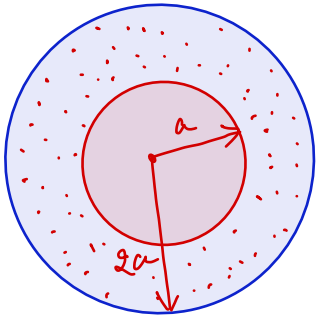
E(3) A point is selected at random inside the equilateral triangle of side 3. What is probability that a randomly selected point on it lies at a distance greater than 1 from any of the 3 corners.

(1)



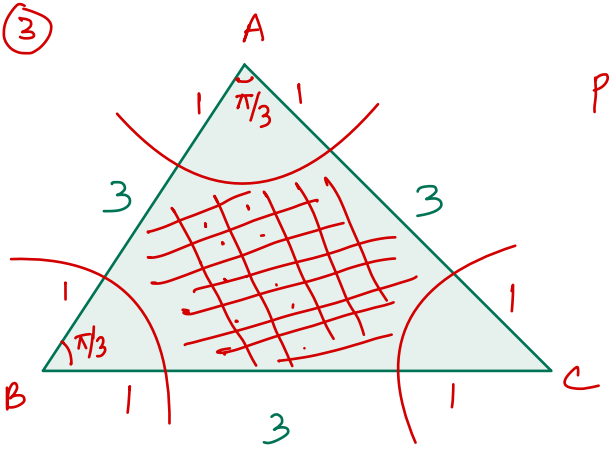
$$\text{Prob} = \frac{4a^2 - \pi a^2}{(2a)^2}$$

(2)



$$P = \frac{\pi (2a)^2 - \pi a^2}{\pi (2a)^2} = \frac{3}{4}.$$

(3)



$$P = \frac{\frac{\sqrt{3}}{4}(3)^2 - 3\left(\frac{1}{2}\cdot 1^2 \cdot \frac{\pi}{3}\right)}{\frac{\sqrt{3}}{4}(3)^2}$$

Q A line segment is divided into three parts, what is the chance that they from the sides of a possible triangle.

An Important Logic :-

n whole numbers taken at random are multiplied together, then the chance that the digit at the unit place of their product is

(a) 1, 3, 7 or 9 is $\left(\frac{2}{5}\right)^n$

(b) 2, 4, 6 or 8 is $\frac{4^n - 2^n}{5^n}$

(c) 5 is $\frac{5^n - 4^n}{10^n}$

(d) 0 is $\frac{10^n - 8^n - 5^n + 4^n}{10^n}$

eg: $31 \times 127 \times 79 =$ _____ 3

(i) $31 \times 123 \times 71 =$ _____ 3

$37 \times 123 \times 87 =$ _____ 7

Units digits → 0, 1, 2, ..., 9

fav. → 1, 3, 7 or 9

$N_1 \times N_2 \times N_3 = N$

$\left(\frac{4}{10}\right)\left(\frac{4}{10}\right)\left(\frac{4}{10}\right) = \left(\frac{4}{10}\right)^3$

(ii) Unit digit is 1, 3, 5, 7 or 9

eg: $35 \times 73 \times 97 =$ _____ 5

$N_1 \times N_2 \times N_3 = N$

$\left(\frac{5}{10}\right)\left(\frac{5}{10}\right)\left(\frac{5}{10}\right) = \left(\frac{5}{10}\right)^3$

(iii) Unit digit is 1, 2, 3, 4, 6, 7, 8, 9 → $\left(\frac{8}{10}\right)^n$

HW

JM Q6 to 23

O-2 Complete.