Q. A positive divisor of N = 75600 is randomly chosen. Find the probability that

(i) divisor N is divisible by 12 but not by 30.
(ii) divisor N is neither divisible by 12 nor 30.

Soln $N = 75600 = 2^4 \cdot 3^3 \cdot 5^2 \cdot 7^1$

Total no. of positive divisors of N $= 5 \times 4 \times 3 \times 2 = 120$

$n(S) = 120$

A : divisors of 'N' which are divisible by 12.

$12 = 2^2 \cdot 3^1$

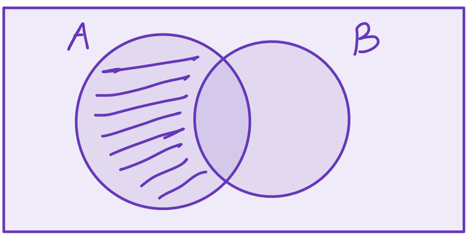
$n(A) = 3 \times 3 \times 3 \times 2 = 54$

B : divisors of 'N' which are divisible by 30

$30 = 2^1 \cdot 3^1 \cdot 5^1$

$n(B) = 4 \times 3 \times 2 \times 2 = 48$

$P(A) = \frac{n(A)}{n(S)} = \frac{54}{120}$; $P(B) = \frac{n(B)}{n(S)} = \frac{48}{120}$



(i) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

$LCM(12, 30) = 60$

$60 = 2^2 \cdot 3^1 \cdot 5^1$

$n(A \cap B) = 3 \times 3 \times 2 \times 2$
$= 36$

$P(A \cap B) = \frac{36}{120}$

(ii) $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$
$= 1 - (P(A) + P(B) - P(A \cap B))$

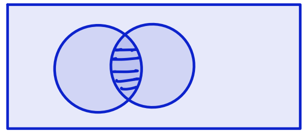
Q. Consider a set S = { 1,2,3,.......10 } . Three numbers are randomly chosen. Find the probability that their minimum is 4 and max. is 8.

Sol^n

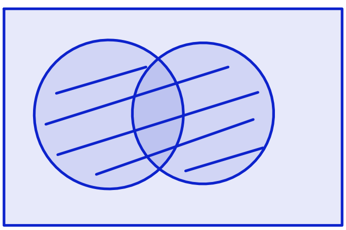
$n(S) = {}^{10}C_3$

$n(A) = {}^{3}C_1$

Req. prob $= \dfrac{{}^{3}C_1}{{}^{10}C_3}$



Q. Consider a set S = { 1,2,3,.......10 } . Three numbers are randomly chosen. Find the probability that their minimum is 4 or max. is 8.



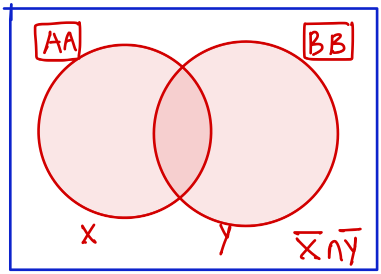
Req. prob $= \dfrac{{}^{6}C_2 + {}^{7}C_2 - {}^{3}C_1}{{}^{10}C_3}$

Q. By using all the letters of the word ' GABBAR ' , words are formed randomly. Find the probability in a randomly chosen word no two alike letters are together.

Sol^n GABBAR

Total No. of words $= \frac{6!}{2!\,2!}$

$= 180.$



G, B, B, R, $\boxed{AA}$

$$n(X) = \frac{5!}{2!} = 60 \Rightarrow P(X) = \frac{60}{180}$$

$$n(Y) = \frac{5!}{2!} = 60 \Rightarrow P(Y) = \frac{60}{180}$$

G, $\boxed{BB}$, R, A, A

$\boxed{AA}$, $\boxed{BB}$, G, R

$$n(X \cap Y) = 4! = 24 \Rightarrow P(X \cap Y) = \frac{24}{180}.$$

$$P(\overline{X} \cap \overline{Y}) = P(\overline{X \cup Y}) = 1 - P(X \cup Y)$$

$$= 1 - \big(P(X) + P(Y) - P(X \cap Y)\big)$$

CONDITIONAL PROBABILITY : / Restricted sample space (Reduced)

Let A and B be two events such that P (A) >0. Then P(B/A) denote the conditional probability of B given that A has occurred. Since A is known to have occurred, it becomes the new sample space replacing the original S. From this we led to the definition.

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)},}$$ Rem

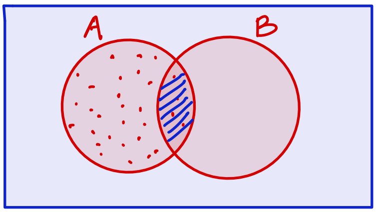
which is called conditional probability of B given A

$$P(B/A) = P(B|A) = P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$P(A/B) = P(A|B) = P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$P(B/A) = \frac{n(A \cap B)}{n(A)}$$

$$= \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(A)}{n(S)}} = \frac{P(A \cap B)}{P(A)}$$



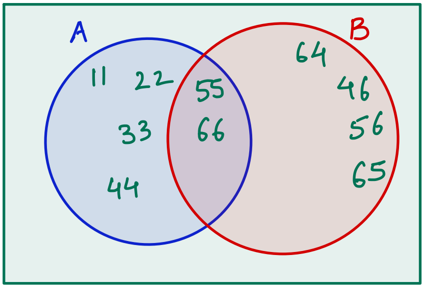
eg:-

Exp. : Throwing a pair of dice

Event : A : getting a doublet

B : getting a sum of 10 or more

P(A/B) = conditional prob. of occurrence of A given B has occurred



$$P(A/B) = \frac{2}{6}$$

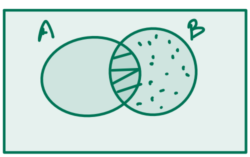
$$\frac{P(A \cap B)}{P(B)} = \frac{2/\cancel{36}}{6/\cancel{36}} = \frac{2}{6}$$

Note:- (i) $P(A/B) + P(\bar{A}/B) = 1.$

(ii) If B is subset of A then

$$P(B/A) = \frac{P(B)}{P(A)} \quad \text{and} \quad P(A/B) = 1.$$

(i) $P(A/B) + P(\bar{A}/B) = \dfrac{P(A \cap B)}{P(B)} + \dfrac{P(\bar{A} \cap B)}{P(B)} = \dfrac{P(B)}{P(B)} = 1$



(ii)

A (i) $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(B)}{P(B)} = 1.$

(ii) $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{P(B)}{P(A)}$

Q Given P(A) = 1/2 ; P(B) = 1/3 and P(A $\cap$ B) = 1/4. Find

(i) P(A/B) (ii) P(B/A) (iii) P(A $\cup$ B) (iv) P(A^C/B^C) (v) P(B^C/A^C)

Sol^n (i) $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

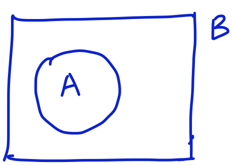
(ii) $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(iv) $P(\bar{A}/\bar{B}) = \dfrac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \dfrac{P(\overline{A \cup B})}{P(\bar{B})} = \dfrac{1 - P(A \cup B)}{1 - P(B)}$

Q Find P(B/A) if (i) A is subset of B (ii) A and B are disjoint

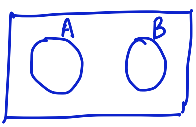
(i)



(i) $P(A \cap B) = P(A)$

$P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{\cancel{P(A)}}{\cancel{P(A)}} = 1$

(ii)

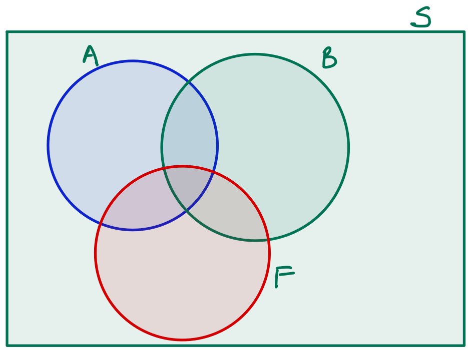


$P(B/A) = \dfrac{P(B \cap A)}{P(A)} = 0$

$A \cap B = \phi \quad \therefore P(A \cap B) = 0$

Q If A and B are any two events of a sample space S and F is an event of S such that $P(F) \neq 0$. Prove that $P((A \cup B)/F) = P(A/F) + P(B/F) - (P(A \cap B)/F)$

Sol^n



$$P((A \cup B)/F) = \frac{P((A \cup B) \cap F)}{P(F)}$$

$$= \frac{P(A \cap F) + P(B \cap F) - P(A \cap B \cap F)}{P(F)}$$

$$= \frac{P(A \cap F)}{P(F)} + \frac{P(B \cap F)}{P(F)} - \frac{P(A \cap B \cap F)}{P(F)}$$

$$= P(A/F) + P(B/F) - P((A \cap B)/F)$$

(H.P.)

Imp **MULTIPLICATION THEOREM :**

Rem

$$\boxed{\begin{aligned} &P(A \cap B) = P(A)\,P(B/A),\ P(A) \neq 0 \\ \text{or} \quad &P(A \cap B) = P(B)\,P(A/B),\ P(B) \neq 0 \end{aligned}}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(B)\cdot P(A/B)$$
$$= P(A)\cdot P(B/A)$$

(simultaneous occur)

which is called compound probability or multiplication theorem. It says the probability that both A and B occur is equal to the probability that A occur times the probability that B occurs given that A has occurred.

For three events A,B & C:

$$P(ABC) = P(\underbrace{A \cap B}_{D} \cap C) = P(A).P(B/A).\ P(C/A \cap B)$$

$$P(D \cap C) = P(D)\cdot P(C/D)$$
$$= P(A \cap B)\cdot P(C/A \cap B)$$
$$= P(A)\cdot P(B/A)\cdot P(C/A \cap B)$$

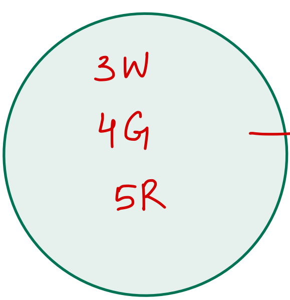
Q An urn contains 3 W, 4 G, 5 R balls. Three balls are drawn one by one at random with replacement. Find the prob. that

(i) first two balls are white & third is red.

(ii) first white, second green & third red.

(iii) all are of different colours.

(iv) two of them are white & other is red.



(i) WWR

$$\left(\frac{3}{12}\right)\left(\frac{3}{12}\right)\left(\frac{5}{12}\right)$$

(ii) WGR

$$\left(\frac{3}{12}\right)\left(\frac{4}{12}\right)\left(\frac{5}{12}\right)$$

(iii) one is red, one is white & one is green.

6 cases
- RWG $\longrightarrow$ $\left(\frac{5}{12}\right)\left(\frac{3}{12}\right)\left(\frac{4}{12}\right)$
- RGW $\longrightarrow$ "
- WRG $\longrightarrow$ "
- WGR $\longrightarrow$ $\left(\frac{3}{12}\right)\left(\frac{4}{12}\right)\left(\frac{5}{12}\right)$
- GWR $\longrightarrow$ "
- GRW $\longrightarrow$ "

$$6 \times \left(\frac{5}{12} \times \frac{3}{12} \times \frac{4}{12} \right)$$

(iv) two are white & one is red.

WWR $\longrightarrow$ $\left(\frac{3}{12}\right)\left(\frac{3}{12}\right)\left(\frac{5}{12}\right)$
WRW $\longrightarrow$ "
RWW $\longrightarrow$ "

$$3 \times \left(\frac{3}{12} \times \frac{3}{12} \times \frac{5}{12} \right)$$

Q An urn contains 3 W, 4 G, 5 R balls. Three balls are drawn one by one at random without replacement. Find the prob. that

(i) first two balls are white & third is red.

(ii) first white, second green & third red.

(iii) all are of different colours.

(iv) two of them are white & other is red.

3W
4G
5R

(i) WWR

$$\left(\frac{3}{12}\right)\left(\frac{2}{11}\right)\left(\frac{5}{10}\right)$$

(ii) WGR

$$\left(\frac{3}{12}\right)\left(\frac{4}{11}\right)\left(\frac{5}{10}\right)$$

(iii) 6 cases $\left\{ \text{WGR} \right.$ $\rightarrow$ $\left(\frac{3}{12}\right)\left(\frac{4}{11}\right)\left(\frac{5}{10}\right)$

$$6 \times \left(\frac{3}{12} \times \frac{4}{11} \times \frac{5}{10}\right)$$

(iv) $\left\{ \begin{array}{l} \text{WWR} \\ \text{WRW} \\ \text{RWW} \end{array} \right.$

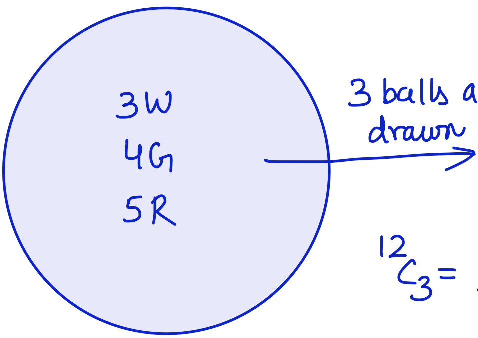
$$3\left(\frac{3}{12} \cdot \frac{2}{11} \cdot \frac{5}{10}\right)$$

Q An urn contains 3 W, 4 G, 5 R balls. Three balls are drawn simultaneously at random. Find the prob. that:

(i) All of them are of different colours.

(ii) two of them are white & other is red.

Sol^n



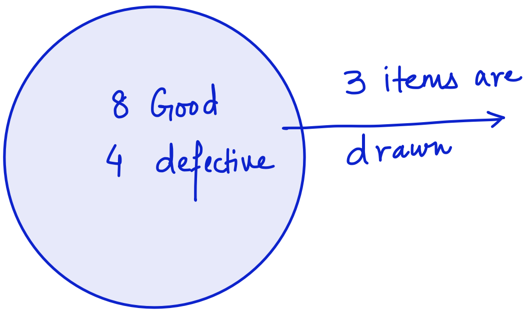
$$^{12}C_3 = \frac{12 \times 11 \times 10}{6}$$

(i) $$\frac{^3C_1 \cdot {^4C_1} \cdot {^5C_1}}{^{12}C_3}$$

$$= \left(\frac{3}{12} \cdot \frac{4}{11} \cdot \frac{5}{10}\right) \times 6$$

(ii) $$\frac{^3C_2 \cdot {^5C_1}}{^{12}C_3}$$

Q A lot contains 12 items of which 4 are defective. 3 items are drawn one after the other without replacement find the probability that all 3 are non defective.



$$\left(\frac{{}^{8}C_3}{{}^{12}C_3}\right) = \text{Prob.}$$

Alt:

$$\left(\frac{8}{12}\right)\cdot\left(\frac{7}{11}\right)\left(\frac{6}{10}\right)$$

Imp

INDEPENDENT EVENTS :

If A & B are two events such that P(A/B) = P(A) and P(B/A) = P(B) then they are called independent events.

$$P(A/B) = P(A) \quad ; \quad P(A/\bar{B}) = P(A)$$

$$\frac{P(A\cap B)}{P(B)} = P(A) \Rightarrow \boxed{P(A\cap B) = P(A)\cdot P(B)} \ **.$$

2 Events A and B are said to be independent if occurrence or non-occurrence of one does not affect the probability of occurrence or non-occurrence of the other.

Exp. Drawing a card

$E_A \rightarrow$ The card is spade ; $E_B \rightarrow$ card is an ace

$P(A) = 1/4$; $P(B) = 1/13$; $P(A \cap B) = 1/52$

$P(B/A) = 1/13$ Here A & B are independent events.

Rem

Note :

(i) $P(A \cap B) = P(A) \,.\, P(B)$ is generally used as a defining equation for independent events.

(ii) If A and B are independent then $P(A \cup B) = P(A) + P(B) - P(A) \,.\, P(B)$

(iii) If A and B are independent then $\bar{A}$ and $\bar{B}$; A and $\bar{B}$; $\bar{A}$ and B are also independent.

** [**Proof :** (iii) Given A and B are independent $\Rightarrow$ $P(A \cap B) = P(A) \,.\, P(B)$

T.P.T. $\bar{A}$ and $\bar{B}$ are also independent i.e $P(\bar{A} \cap \bar{B}) = P(\bar{A}) \,.\, P(\bar{B})$

R.H.S. $[1 - P(A)][1 - P(B)] = 1 - P(A) - P(B) + P(A) \,.\, P(B)$

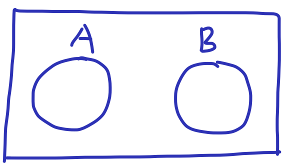
$= 1 - [P(A) + P(B) - P(A) \,.\, P(B)]$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A \cup B) = P(\overline{A \cup B})$

$= P(\bar{A} \cap \bar{B}) =$ L.H.S.]

* If A and B are M.E as well as independent events it would mean $P(A) = 0$ or $P(B) = 0$ or both zero.



$A \cap B = \phi$

$P(A \cap B) = 0$

$\Rightarrow P(A) \cdot P(B) = 0$

Hence atleast one of A or B must be impossible event.

Q If A and B are independent events such that $P(A) = p$, $P(B) = q$ where $p, q \in [0, 1]$ then

(i) $P(\overline{A}/\overline{B})$

(ii) $P(\overline{A} \setminus \overline{B})$

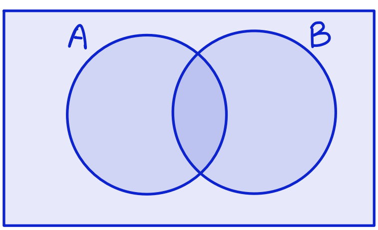
(iii) $P(\overline{A}/A \cup B)$

Sol^n

(i) $$P(\overline{A}/\overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{P(\overline{A}) \cdot \cancel{P(\overline{B})}}{\cancel{P(\overline{B})}} = 1 - p.$$

(ii) $$P(\overline{A} \setminus \overline{B}) = P(\overline{A}) - P(\overline{A} \cap \overline{B})$$
$$= P(\overline{A}) - \left(P(\overline{A}) \cdot P(\overline{B})\right) = P(\overline{A})\left(\underbrace{1 - P(\overline{B})}_{P(B)}\right)$$
$$= (1-p) \cdot q$$

(iii) $$P(\overline{A}/A \cup B) = \frac{P(\overbrace{\overline{A} \cap (A \cup B)})}{P(A \cup B)} \cdot \frac{P(\overline{A} \cap B)}{P(A \cup B)}$$



$$= \frac{P(\overline{A}) \cdot P(B)}{P(A) + P(B) - P(A) \cdot P(B)} .$$

Imp

If $p_1, p_2 \ldots\ldots\ldots p_n$ are probabilities of n independent events $E_1, E_2 \ldots\ldots E_n$ respectively, then

P(occurance of atleast one) $= P(E_1 \cup E_2 \cup \ldots\ldots E_n)$

$= 1 - P\left(\overline{E_1 \cup E_2 \cup \ldots\ldots E_n}\right)$

$= 1 - P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \ldots\ldots \bar{E}_n)$

$= 1 - P(\bar{E}_1) P(\bar{E}_2) \ldots\ldots P(\bar{E}_n)$

$= 1 - (1 - p_1)(1 - p_2) \ldots\ldots (1 - p_n)$

Q The probability that an antiaircraft gun can hit an enemy plane at the first, second and third shot are 0.6,0.7 and 0.1 respectively. Find the probability that the gun hits the plane.

Soln

$1 - P(\bar{1} \cap \bar{2} \cap \bar{3})$

$1 - P(\bar{1}) P(\bar{2}) P(\bar{3})$

$1 - (0.4)(0.3)(0.9)$

Note :-

If cards are dealt one by one from a well shuffled pack of 52 cards without replacement then the probability that a particular card is drawn at the nth draw ($1 \leq n \leq 52$) is independent of n and is equal to 1/52.

52 playing cards → Ace of spade

Ace of space in 1^{st} draw $= \frac{1}{52}$

" " " " 2^{nd} draw $= \left(\frac{\cancel{51}}{52}\right)\left(\frac{1}{\cancel{51}}\right) = \frac{1}{52}$

" " " " 5^{th} draw $= \left(\frac{\cancel{51}}{52}\right)\left(\frac{\cancel{50}}{\cancel{51}}\right)\left(\frac{\cancel{49}}{\cancel{50}}\right)\left(\frac{\cancel{48}}{\cancel{49}}\right)\left(\frac{1}{\cancel{48}}\right)$

$= \frac{1}{52}.$

HW

O-1 Part # 2.

S-1 Q 1, 2, 3.