Ex:- Consider word 'DAUGHTER'. How many four letter word can be formed from letters of given word If each word include G.

Soln DAUGHTER = 8 letters

$$\underset{8}{\_} \times \underset{7}{\_} \times \underset{6}{\_} \times \underset{5}{\_}$$

$8 \times 7 \times 6 \times 5$ = Total without condition

$7 \times 6 \times 5 \times 4$ = Total words without G

DAUHTER

Ans $= (8 \times 7 \times 6 \times 5) - (7 \times 6 \times 5 \times 4) =$

M②

$$\underbrace{\text{DAUHTER}}_{7}$$

G

_ _ _ _

$^7C_3 \times 4!$

Ex:- 10 different Consonants

4 different vowels are given

find number of words using 3 Consonants and 2 vowels and begin with a consonant.

Sol^n

$$ {}^{10}C_3 \; {}^{4}C_2 \times \underset{3}{\underset{\uparrow}{\_}} \times \underbrace{\_\;\_\;\_\;\_}_{4!} $$

3 'C' out of 10 'C' | 2 'V' out of 4 V

$\boxed{M\#2}$

$\boxed{10C,\ 4V} \longrightarrow 3C\ 2V \longrightarrow \boxed{9C,\ 4V} \longrightarrow 2C\ 2V$

$\boxed{C}$ _ _ _ _

$$ {}^{10}C_1 \quad {}^{9}C_2 \quad {}^{4}C_2 \quad 4! $$

Ex:- 4 Boys and 4 Girls are to be seated in a line. find number of ways they can be seated So that

Q(1) No two girls are together

$\boxed{B_1 \; B_2 \; B_3 \; B_4 \quad G_1 \; G_2 \; G_3 \; G_4}$

$/B_1/B_2/B_3/B_4/$ 5 gaps

$\underbrace{4!}_{\text{Boys}} \; \underbrace{{}^5C_4}_{\text{4 gaps}} \; \underbrace{4!}_{\text{Girls}}$

Q(2) All girls are together

$\underbrace{B_1 \; B_2 \; B_3 \; B_4}_{\text{4 Boys}} \quad \underbrace{\boxed{G_1 \, G_2 \, G_3 \, G_4}}_{\text{1 Box}}$

$= \underbrace{5!}_{\text{(4 Boys 1 Box)}} \; \underbrace{4!}_{\text{Girls inside box}}$

Q③ Atleast one girl is separated

OR NOT all girls are together

$$\text{Total} - \underbrace{\text{none is separated}}_{\text{All are together}}$$

$$= 8! - 5!\,4!$$

$$\boxed{\text{Atleast one} = \text{Total} - \text{none}}$$ *

Q④ Boys and girls are Alternate

$\boxed{\text{given } 4 \text{ Boys } 4 \text{ girls}}$

$\uparrow B_1 \uparrow B_2 \uparrow B_3 \uparrow B_4$

BGBGBGBG

OR

GBGBGBGB

$2 \cdot 4!\,4!$

$4!$ ${}^{2}C_1$ $4!$

$\uparrow$ Boys, $\uparrow$ Black or Blue, $\uparrow$ Girls

Q5 Boys and girls are alternate and a particular boy and a particular girl are never together

$B_1 B_2 B_3 G_1 G_2 G_3$ $\boxed{B_4 G_4}$

$\boxed{\text{Total} - \text{Both are together}}$

Alternate

$\text{Total} = 2 \cdot 4! \, 4!$

$\text{Both are together} = \underbrace{2 \cdot 3! \, 3!}_{} \; {}^{7}C_1$

Alternate $\rightarrow B_1 B_2 B_3 G_1 G_2 G_3$ $\boxed{B_4 G_4}$

Ans $= 2\cdot 4!\,4! - 2\cdot 3!\,3!\,({}^{7}C_{1})$

Ex:- use A, B, C, D, E, F, G, H

How many 8 letter words can be formed if 'A appears before G' and 'D appears before F'.

Sol^n Total $= 8!$

$\Rightarrow$ ABCDEFGH ✓
GBCDEFAHX

$2 \to 1$

$1 \to \frac{1}{2}$

$8! \to \frac{8!}{2!}$

$\boxed{\text{A before G} = \frac{8!}{2!}}$

A B G
A G B
B A G
B G A X
G A B X
G B A X

A befor G & D before f $= \dfrac{8!}{2!\,2!}$

✓ A B C D E F G H

A B C F E D G H ✗

$2 \longrightarrow 1$

$1 \longrightarrow \frac{1}{2}$

$\frac{8!}{2!} \longrightarrow \frac{1}{2} \cdot \frac{8!}{2!}$

Ex:- A before D, D before F $\boxed{ABCDEFGH}$

✓ ADF
✗ AFD
✗ FAD
✗ FDA
✗ DAF
✗ DFA

$\left.\right\}$ 3!

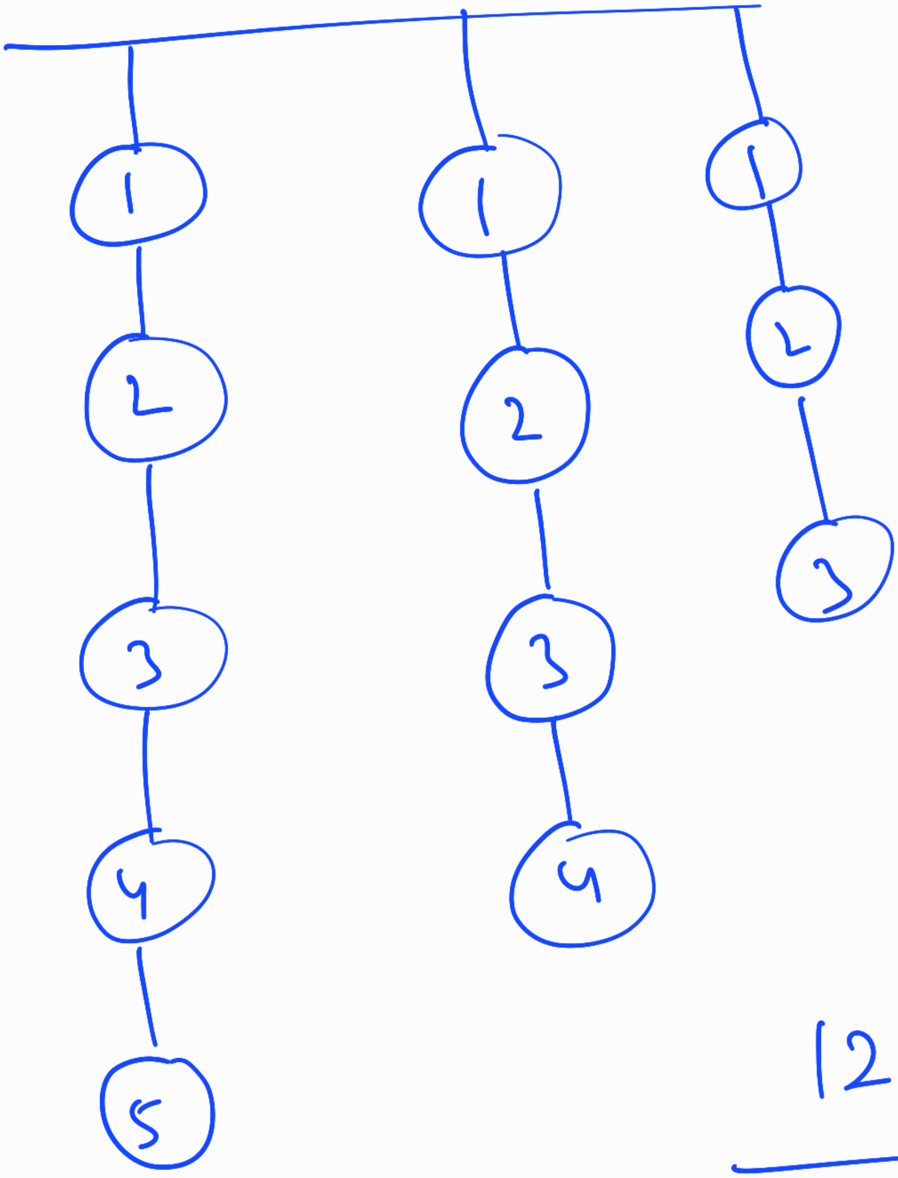
$$\frac{8!}{3!}$$

— A — D — F — —

$${}^{8}C_{3} \cdot 5! = \frac{8!}{3!}$$

Ex:-

different



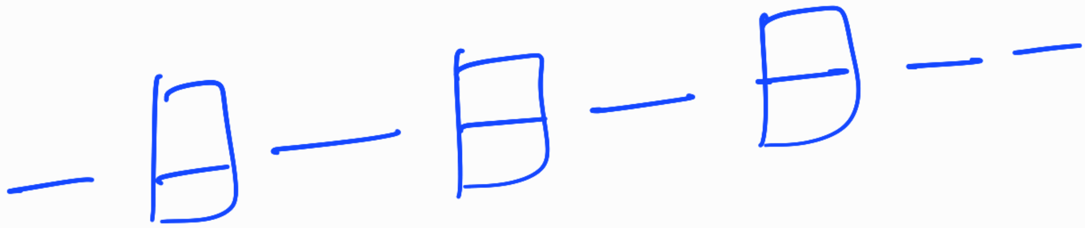
$$\frac{12!}{5!\,4!\,3!}$$

Ex:- use A, B, C, D, E, F, G, H

How many 8 letter words can be formed if 'A appears before D' and 'D appears after F'.

Total = $8!$

$$\left.\begin{array}{l} \times \; ADF \\ \checkmark \; AFD \\ \times \; DAF \\ \times \; DFA \\ \checkmark \; FAD \\ \times \; FDA \end{array}\right\} 3! \longrightarrow 2$$



$$\text{Ans.} = \frac{8!}{3!} \cdot 2$$

Ex :- use A, B, C, D, E, F, G, H

How many 8 letter words can be formed if 'A appears before D' and 'D appears after B'

H.W.

STATION problem

There are n intermediate stations on a railway line. In how many ways can train stop at 3 of these intermediate stations if

ⓐ All three stations are consecutive

ⓑ Atleast two of stations are consecutive

ⓒ no two of these stations are consecutive

$\boxed{\text{Sol}^n}$ $S_1\ S_2\ S_3\ S_4 \; --- \; S_{n-2}\ S_{n-1}\ S_n$

ⓐ $S_1 S_2 S_3,\ S_2 S_3 S_4,\ S_3 S_4 S_5$

. . . . . $S_{n-2} S_{n-1} S_n$

$= (n-2)$ ways

(b) $S_1 S_2, S_2 S_3, S_3 S_4 \cdots, S_{n-1} S_n$

Two consecutive stations $= (n-1)$ ways

3^{rd} station $= {}^{(n-2)}C_1$

Let $\underbrace{S_3 S_4} S_5 = S_3 \underbrace{S_4 S_5}$

Repeat

All 3 consecutive

$S_8 \underbrace{S_9 S_{10}} = \underbrace{S_8 S_9} S_{10}$

part (a)

Ans $= (n-1)({}^{n-2}C_1) - (n-2)$

$$= (n-1)(n-2) - (n-2)$$

$$= (n-2)(n-1-1)$$

$$= (n-2)^2$$

(c) $^{n}C_3 - (n-2)^2$

(Total − Atleast two are consecutive)

$$= {}^{n-2}C_3$$

(b) Atleast 2 stations are consecutive

$S_1 S_2 \mid S_2 S_3 \mid S_3 S_4 \mid S_5 S_6 \mid \quad \mid S_{n-1} S_n$

Total $= {}^{(n-1)}C_1 \; {}^{n-2}C_1 = (n-1)(n-2)$

Repeat All 3 Consecutive

$S_3 S_4 S_5 = S_3 S_4 S_5$

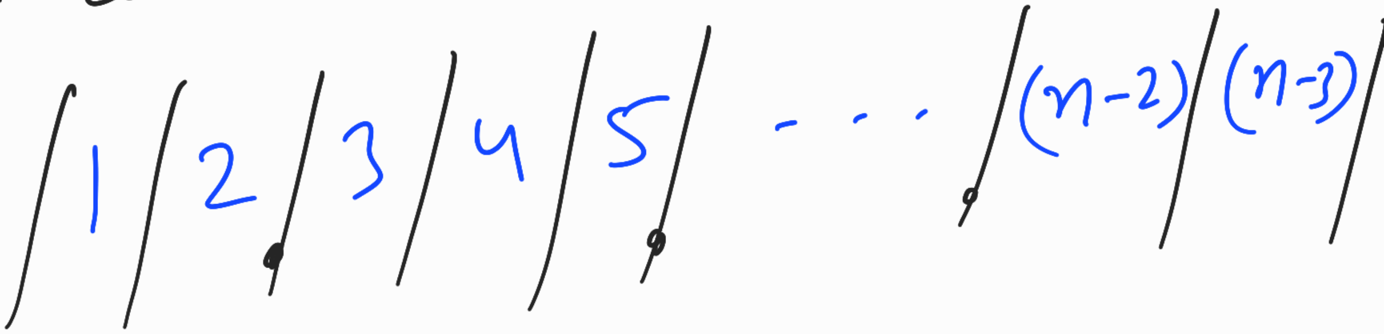
$\underbrace{S_3 S_4}_{1} \; \underbrace{S_5}_{2} \qquad \underbrace{S_3}_{2} \; \underbrace{S_4 S_5}_{1}$

Part (a)

Ans $= (n-1)(n-2) - (n-2)$

$= (n-2)^2$

Ex:- n persons are sitting in a line. In how many ways 3 person can be selected so that no two of them are consecutive.



$$\text{gaps} = {}^{(n-2)}C_3$$

n person in a line

Select r person so that no two of them are consecutive

Total $= n$

remove $= r$

remaining $= (n-r)$

gaps $= (n-r+1)$

$$\text{Ans.} = {}^{n-r+1}C_r$$

<u>Ex:</u> (12 Stations in a line), select 4 Stations non consecutive

$${}^{12-4+1}C_4 = {}^{9}C_4$$