INDEFINITE INTEGRATION

RACE (4)

Date: __/__/____

MON TUE WED THU FRI SAT SUN

Subject ____________

(1) $\int x e^{-x} \, dx$

Solⁿ

$$\int \underset{I}{x} \, \underset{II}{e^{-x}} \, dx$$

$$I = -x e^{-x} + \int 1 \cdot e^{-x} \, dx$$

$$I = -x e^{-x} - e^{-x} + C$$

$$I = \boxed{-e^{-x}[x+1] + C}$$

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(29) $\int x^n \cdot \ln x \, dx \quad (n \neq -1)$

Sol^n

$$\int \underset{II}{x^n} \cdot \underset{I}{\ln x} \, dx$$

$$I = \ln x \cdot \int x^n dx - \int \frac{1}{x} \cdot \frac{x^{n+1}}{n+1} dx$$

$$I = \frac{x^{n+1}}{n+1} \cdot \ln x - \int \frac{1}{n+1} \cdot x^n \, dx$$

$$I = \frac{x^{n+1}}{n+1} \cdot \ln x - \frac{1}{n+1} \int x^n dx$$

$$I = \frac{x^{n+1}}{n+1} \cdot \ln x - \frac{1}{n+1} \left\{ \left[\frac{x^{n+1}}{n+1} \right] + c \right\}$$

$$I = \frac{x^{n+1}}{n+1} \cdot \ln x - \frac{1}{(n+1)^2} x^{n+1} + c$$

$$I = \boxed{\frac{x^{n+1}}{n+1} \left[\ln x - \frac{1}{n+1} \right] + c}$$

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(3) $\int \arctan\sqrt{x}\,dx$

let $\sqrt{x} = t$

$\frac{1}{2\sqrt{x}}\,dx = dt$

$dx = 2t\,dt$

$\therefore \int \arctan t \; 2t\,dt$

$2\int \underset{II}{t}\;\underset{I}{\tan^{-1}t}\,dt$

$2\left[\tan^{-1}t\cdot\frac{t^2}{2} - \int \frac{1}{1+t^2}\cdot\frac{t^2}{2}\,dt\right]$

$t^2.\tan^{-1}t - \int \frac{t^2+1-1}{t^2+1}\,dt$

$t^2\cdot\tan^{-1}t - \int\left[1 - \frac{1}{1+t^2}\right]dt$

(4) $t^2.\tan^{-1}t - t + \int \frac{1}{1+t^2}\,dt$

$t^2.\tan^{-1}t - t + \tan^{-1}t + c$

$\therefore \boxed{x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + c}$

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(4) $$\int \frac{x \arctan x}{\sqrt{1+x^2}}\,dx$$

Sol^n

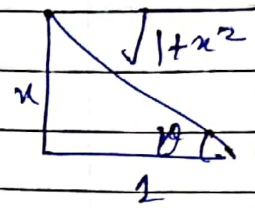
$$\int \frac{\tan\theta \tan^{-1}\tan\theta \sec^2\theta\, d\theta}{\sec\theta}$$

put $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$\tan^{-1}x = \theta$

$$\int \underset{I}{\theta}\ \underbrace{\tan\theta \sec\theta}_{II}\, d\theta$$



$$\theta \cdot \sec\theta - \int 1 \cdot \sec\theta\, d\theta$$

$$\theta \sec\theta - \ln\left[|\sec\theta + \tan\theta|\right] + c$$

$$\boxed{\tan^{-1}x \cdot \sqrt{1+x^2} - \ln\left[\sqrt{x^2+1} + x\right] + c}$$

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5 $$\int \frac{x^2}{(x^2+1)^2}\,dx$$

Sol^n $$\int \frac{x^2+1-1}{(x^2+1)^2}\,dx$$

$$\int \left[\frac{1}{1+x^2} - \frac{1}{(x^2+1)^2}\right]dx$$

$$\tan^{-1}x - \int \frac{1}{(x^2+1)^2}\,dx \quad \text{—— (1)}$$

$\hookrightarrow I_1$ let

$$\therefore I_1 = \int \frac{1}{(x^2+1)^2}\,dx$$

Put $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$$\int \frac{1}{\sec^4\theta}\sec^2\theta\, d\theta$$

$$\int \frac{1}{\sec^2\theta}\,d\theta \Rightarrow \int \cos^2\theta\, d\theta$$

$$\int \frac{1+\cos 2\theta}{2}\,d\theta \Rightarrow \int \frac{1}{2}\,d\theta + \int \frac{\cos 2\theta}{2}\,d\theta$$

$$\frac{1}{2}\theta + \frac{1}{2}\frac{\sin 2\theta}{2} + c$$

$$\frac{1}{2}\left[\tan^{-1}x + \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}}\right] + c$$

$$\frac{1}{2}\left[\tan^{-1}x + \frac{x}{\sqrt{x^2+1}} \times \frac{1}{\sqrt{x^2+1}}\right] + c \quad \text{—— (2)}$$

from (1) & (2) $$\tan^{-1}x - \frac{1}{2}\left[\tan^{-1}x + \frac{x}{x^2+1}\right] + c$$

$$\boxed{\frac{1}{2}\tan^{-1}x - \frac{1}{2}\frac{x}{1+x^2} + c}$$

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(6) $$\int \frac{x^3}{\sqrt{1+x^2}}\,dx$$

Sol^n

$$I = \int \frac{x^3}{\sqrt{1+x^2}}\,dx$$

$1+x^2 = t$

$2x\,dx = dt$

$x\,dx = \frac{1}{2}dt$

$$I = \int \frac{x^2 \cdot x}{\sqrt{1+x^2}}\,dx$$

$$\therefore \quad I = \int \frac{1}{\sqrt{t}}(t-1)\frac{1}{2}\,dt$$

$$I = \frac{1}{2}\int \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)dt$$

$$I = \frac{1}{2}\left[\frac{t^{3/2}}{3/2} - \frac{t^{1/2}}{1/2}\right] + c$$

$$I = \frac{1}{3}t^{3/2} - t^{1/2} + c$$

$$I = \frac{1}{3}t^{1/2}\left[t - 3\right] + c \qquad \because t = 1+x^2$$

$$I = \frac{1}{3}\sqrt{1+x^2}\left[1+x^2-3\right] + c$$

$$I = \frac{1}{3}\sqrt{1+x^2}\,(x^2-2) + c$$

$$I = \boxed{\frac{1}{3}x^2 \cdot \sqrt{x^2+1} - \frac{2}{3}\sqrt{1+x^2} + c}$$

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(7) $\int \frac{\ln^3 x}{x^2} dx$

Solⁿ $I = \int \frac{\ln^3 x}{x^2} dx$

let $\ln x = t$

$\frac{1}{x} dx = dt$

$x = e^t$

$\int \underset{I}{t^3} \underset{II}{e^{-t}} dt$

$-t^3 e^{-t} + \int 3t^2 e^{-t} dt$

$-e^{-t} t^3 + 3 \int \underset{I}{t^2} \underset{II}{e^{-t}} dt$

$-e^{-t} t^3 + 3 \left[-t^2 e^{-t} + \int 2t e^{-t} dt \right]$

$-e^{-t} t^3 - 3t^2 e^{-t} + 6 \int \underset{I}{t} \underset{II}{e^{-t}} dt$

$-e^{-t} t^3 - 3t^2 e^{-t} + 6 \left[-t \cdot e^{-t} - \int -e^{-t} dt \right]$

$\left[-e^{-t} t^3 - 3t^2 e^{-t} - 6t e^{-t} - 6e^{-t} \right] + c$

$\boxed{\frac{-1}{x} \left[\ln^3 x + 3\ln^2 x + 6 \ln x + 6 \right] + c}$

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8

$$\int \sin(\ln x)\,dx$$

Solⁿ

$$\int \underset{\text{II}}{1}\cdot \underset{\text{I}}{\sin(\ln x)}\,dx$$

$$I = \sin(\ln x)\cdot x - \int \cos(\ln x)\times\frac{1}{\cancel{x}}\cdot\cancel{x}\,dx$$

$$I = x\cdot\sin(\ln x) - \underbrace{\int \cos(\ln x)\,dx}_{\text{let } I_1} \quad\text{———— } (1)$$

$$\therefore\ I_1 = \int \underset{\text{II}}{1}\cdot\underset{\text{I}}{\cos(\ln x)}\,dx$$

$$I_1 = \cos(\ln x)\cdot x - \int -\sin(\ln x)\cdot\frac{1}{\cancel{x}}\cdot\cancel{x}\,dx$$

$$I_1 = x\cdot\cos(\ln x) + I \quad\text{———— } (2)$$

$\therefore$ from (1) & (2)

$$I = x\cdot\sin(\ln x) - x\cos(\ln x) - I$$

$$I = \frac{x\cdot\sin(\ln x) - \cos(\ln x)\cdot x}{2}$$

$$\boxed{I = \frac{x}{2}\left[\sin(\ln x) - \cos(\ln x)\right] + c}$$

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9 $\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

Solⁿ

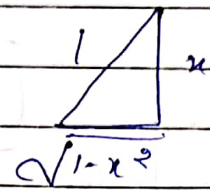
$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx$$

Put $x = \sin\theta$

$dx = \cos\theta\, d\theta$

$\theta = \sin^{-1} x$

$$\int \frac{\sin^2\theta\ \cancel{\cos\theta}\, d\theta}{\cancel{\cos\theta}}$$



$$\int \sin^2\theta\, d\theta$$

$$\int \frac{1-\cos 2\theta}{2}\, d\theta$$

$$\frac{\theta}{2} - \frac{\sin 2\theta}{4} + c$$

$$\frac{\sin^{-1}x}{2} - \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{4}\,2} + c$$

$$\boxed{\frac{\sin^{-1}x}{2} - \frac{x}{2}\sqrt{1-x^2} + c}$$

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10

$$\int \ln\left(x+\sqrt{1+x^2}\right) dx$$

Sol^n

$$I = \int 1 \cdot \ln\left(x+\sqrt{1+x^2}\right) dx$$

$$I = \ln\left(x+\sqrt{1+x^2}\right)\cdot x - \int \frac{1}{x+\sqrt{1+x^2}}\left(1+\frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}\right)\cdot x\, dx$$

$$I = x\cdot\ln\left(x+\sqrt{1+x^2}\right) - \int \frac{x}{\cancel{x+\sqrt{1+x^2}}}\left(\frac{\cancel{x+\sqrt{1+x^2}}}{\sqrt{1+x^2}}\right) dx$$

$$I = x\ln\left(x+\sqrt{1+x^2}\right) - \underbrace{\int \frac{x}{\sqrt{1+x^2}}\, dx}_{\text{let } I_1} \quad \text{--- (1)}$$

$$\therefore I_1 = \int \frac{x}{\sqrt{1+x^2}}\, dx$$

put $1+x^2 = t^2$

$\cancel{2}x\, dx = \cancel{2}t\, dt$

$x\, dx = t\, dt$

$$I_1 = \int \frac{\cancel{t}}{\cancel{t}}\, dt$$

$$I_1 = t$$

$$\therefore I_1 = \sqrt{1+x^2} \quad \text{--- (2)}$$

from eq^n (1) & (2)

$$\boxed{I = x\ln\left[x+\sqrt{1+x^2}\right] - \sqrt{1+x^2} + c}$$

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11

$$\int \frac{dx}{1+\sqrt{x+1}}$$

Sol^n

$$\int \frac{dx}{1+(x+1)^{1/2}}$$

let $(x+1)^{1/2} = t$

$\frac{1}{2}(x+1)^{-\frac{1}{2}} dx = dt$

$\frac{dx}{2t} = dt$

$$\therefore \int \frac{1}{1+t} 2t \, dt$$

$$\int 2\left[1 - \frac{1}{1+t}\right] dt$$

$$2\left[t - \ln(1+t)\right] + c$$

$$\boxed{2\left[\sqrt{x+1} - \ln(1+\sqrt{x+1})\right] + c}$$

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(12) $$\int \frac{\sqrt{x}}{\sqrt{x}-\sqrt[3]{x}}\,dx$$

Put $x = t^6$

$dx = 6t^5\,dt$

Sol^n

$$\int \frac{\sqrt{x}}{\sqrt{x}-(x)^{1/3}}\,dx$$

$$\int \frac{t^3 \times 6t^5}{t^3 - t^2}\,dt$$

$$6\int \frac{t^8}{t^2(t-1)}\,dt \Rightarrow 6\int \frac{t^6}{t-1}\,dt$$

$$6\int \frac{(t-1)(t^5+t^4+t^3+t^2+t)+t}{(t-1)}$$

$$6\int t^5+t^4+t^3+t^2+t+\frac{t-1+1}{t-1}\,dt$$

$$6\left[\frac{t^6}{6}+\frac{t^5}{5}+\frac{t^4}{4}+\frac{t^3}{3}+\frac{t^2}{2}+t+\ln(t-1)\right]$$

$$t^6+\frac{6}{5}t^5+\frac{3}{2}t^4+2t^3+3t^2+6t+6\ln(t-1)$$

$$\boxed{x+\frac{6}{5}\sqrt[6]{x^5}+\frac{3}{2}\sqrt[3]{x^2}+2\sqrt{x}+3\sqrt[3]{x}+6\sqrt[6]{x}+6\ln\left|\sqrt[6]{x}-1\right|+c}$$

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13

$$\int \frac{\sqrt{1+x^2}}{x^4}\,dx$$

Solⁿ

$$\int \frac{x\sqrt{\frac{1}{x^2}+1}}{x^4}\,dx$$

let $1+\frac{1}{x^2} = t^2$

$-\frac{\cancel{2}}{x^3}\,dx = \cancel{2}t\,dt$

$\frac{dx}{x^3} = -t\,dt$

$$\int \frac{1}{x^3}\sqrt{1+1/x^2}\,dx$$

$$\int t(-t)\,dt$$

$$\int -t^2\,dt$$

$$-\frac{t^3}{3}+c$$

$$-\frac{1}{3}\left[1+\frac{1}{x^2}\right]^{3/2}+c$$

$$\boxed{-\frac{1}{3}\frac{(x^2+1)^{3/2}}{x^3}+c}$$

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(14) $$\int \frac{dx}{\sqrt{(a^2+x^2)^3}}$$

put $x = a\tan\theta$

$dx = a\sec^2\theta \, d\theta$

Sol^n

$$\int \frac{dx}{\sqrt{(a^2+a^2\tan^2\theta)^3}}$$

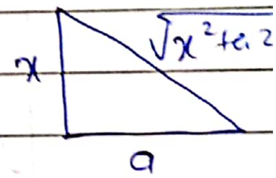
$$\int \frac{dx}{a^3\sqrt{(1+\tan^2\theta)^3}} \Rightarrow \int \frac{a\sec^2\theta \, d\theta}{a^3\sqrt{(\sec^2\theta)^3}}$$

$$\int \frac{1}{a^2} \frac{\sec^2\theta}{\sec^2\theta^{3/2}} \, d\theta$$

$$\frac{1}{a^2}\int \frac{1}{\sec\theta} \, d\theta \Rightarrow \frac{1}{a^2}\int \cos\theta \, d\theta$$

$$\frac{1}{a^2} \sin\theta + C$$

$\theta = \tan^{-1}\frac{x}{a}$



$$\frac{1}{a^2} \sin \sin^{-1} \frac{x}{\sqrt{x^2+a^2}} + c$$

$$\boxed{\frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}} + c}$$

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15

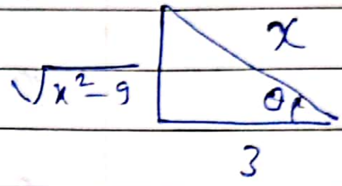
$$\int \frac{dx}{x^2\sqrt{x^2-9}}$$

Sol^n

put $x = 3\sec\theta$

$dx = 3\sec\theta\tan\theta\, d\theta$

$\theta = \sec^{-1}\left(\frac{x}{3}\right)$



$$\int \frac{3\sec\theta\tan\theta\, d\theta}{9\sec^2\theta\sqrt{9\sec^2\theta-9}}$$

$$\int \frac{\cancel{3}\sec\theta\tan\theta\, d\theta}{9\sec^2\theta\ \cancel{3}\sqrt{\sec^2\theta-1}}$$

$$\int \frac{1}{9}\frac{1}{\sec\theta}\frac{\cancel{\tan\theta}}{\cancel{\tan\theta}}\, d\theta$$

$$\int \frac{1}{9}\cos\theta\, d\theta$$

$$\frac{1}{9}\sin\theta + c$$

$$\frac{1}{9}\sin\sin^{-1}\frac{\sqrt{x^2-9}}{x} + c$$

$$\boxed{\frac{1}{9}\frac{\sqrt{x^2-9}}{x} + c}$$

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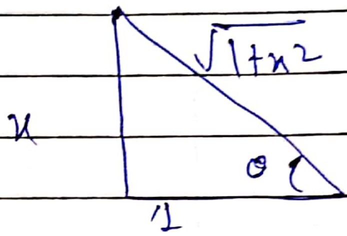
(16) $\int \frac{dx}{x\sqrt{1+x^2}}$

put $x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

Sol^n

$\int \frac{\sec^{\cancel{2}}\theta \, d\theta}{\tan\theta \cdot \cancel{\sec\theta}}$



$\int \frac{1}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{\sin\theta} \, d\theta$

$\int \operatorname{cosec}\theta \, d\theta$

$\ln|\operatorname{cosec}\theta - \cot\theta| + c$

$\ln|\operatorname{cosec}\tan^{-1}x - \cot\tan^{-1}x| + c$

$\ln\left|\operatorname{cosec}\frac{\sqrt{1+x^2}}{x} - \cot\frac{1}{x}\right| + c$

$$\boxed{\ln\frac{|x|}{1+\sqrt{x^2+1}} + c}$$

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17

$$\int \frac{dx}{\sqrt{x-x^2}}$$

Sol^n

$$\int \frac{dx}{\sqrt{-x^2+2\times\frac{1}{2}x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$

$$\int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2-\left[x^2-\frac{1}{2}\times 2\times x+\left(\frac{1}{2}\right)^2\right]}}$$

$$\int \frac{dx}{\sqrt{\frac{1}{4}-\left[x-\frac{1}{2}\right]^2}}$$

$$\int \frac{dx}{\sqrt{\frac{1}{4}-\frac{1}{4}(2x-1)^2}}$$

let $2x-1=t$

$2dx=dt$

$$\int \frac{2dx}{\sqrt{1-(2x-1)^2}}$$

$$\int \frac{dt}{\sqrt{1-t^2}} \Rightarrow \sin^{-1}t+c$$

$$\therefore \boxed{\sin^{-1}(2x-1)+c}$$

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18

$$\int \frac{x+1}{x(1+xe^{x})}\,dx$$

sol^n

$$\int \frac{x+1}{x(1+xe^{x})}\,dx$$

put $xe^{x} = t$

$(xe^{x} + e^{x})\,dx = dt$

$e^{x}(x+1)\,dx = dt$

$\frac{t}{x}(x+1)\,dx = dt$

$\frac{(x+1)}{x}\,dx = \frac{dt}{t}$

$$\int \frac{1}{t(1+t)}\,dt$$

$$\int \left(\frac{1}{t} - \frac{1}{(t+1)}\right) dt$$

$$\ln t - \ln(1+t) + c$$

$$\ln\left(\frac{t}{1+t}\right) + c$$

$$\boxed{\ln\left(\frac{xe^{x}}{1+xe^{x}}\right) + c}$$

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19 $$\int \frac{dx}{x^4\sqrt{x^2+4}}$$

Sol^n

$$\int \frac{dx}{x^4\sqrt{\frac{x^2+(2)^2}{x^2}\times x^2}}$$

put $1+\frac{(2)^2}{x^2} = t$

$$\int \frac{dx}{x^5\sqrt{1+\frac{(2)^2}{x^2}}} \Rightarrow \int \frac{dx}{x^3 \cdot x^2\sqrt{1+\frac{(2)^2}{x^2}}}$$

$4(-2)x^{-3}\,dx = dt$

$\frac{dx}{x^3} = -\frac{1}{8}\,dt$

$$\int \left(-\frac{1}{8}\right)\left(\frac{1}{4}\right)\frac{(t-1)}{\sqrt{t}}\,dt$$

$\frac{1}{x^2} = \frac{t-1}{(2)^2}$

$$-\frac{1}{32}\int \frac{t-1}{\sqrt{t}}\,dt$$

$$-\frac{1}{32}\int\left[\sqrt{t}-\frac{1}{\sqrt{t}}\right]dt$$

$$-\frac{1}{32}\left[\frac{2}{3}t^{3/2} - 2t^{1/2}\right]+c$$

$$-\frac{1}{48}t^{3/2}+\frac{1}{16}t^{1/2}+c$$

$$-\frac{1}{48}\left(1+\frac{4}{x^2}\right)^{3/2}+\frac{1}{16}\left(1+\frac{4}{x^2}\right)^{1/2}+c$$

$$-\frac{1}{48}\frac{(x^2+4)^{3/2}}{x^3}+\frac{1}{16}\frac{(x^2+4)^{1/2}}{x}+c$$

$$\frac{(x^2+4)^{\frac{1}{2}}\left[-x^2-4+3x^2\right]}{48x^3}$$

$$\frac{\sqrt{x^2+4}\,\left[2x^2-4\right]}{48x^3} \Rightarrow \boxed{\frac{\sqrt{x^2+4}\,(x^2-2)}{24x^3}}$$

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20

$$\int \frac{dx}{x^4\sqrt{x^2-3}}$$

Solⁿ

$$\int \frac{1}{x^4\sqrt{x^2-3}}\,dx$$

put $x = \sqrt{3}\sec t$
$dx = \sqrt{3}\sec t \tan t\,dt$

$$\int \frac{\sqrt{3}\sec t\ \tan t\ dt}{9\sec^4 t\sqrt{3\sec^2 t-3}}$$

$\because t = \sec^{-1}\left(\frac{x}{\sqrt{3}}\right)$

$$\int \frac{\sqrt{3}\sec t\ \tan t\ dt}{9\sec^4 t\ \sqrt{3}\sqrt{\sec^2 t-1}}$$

$$\int \frac{\cancel{\sqrt{3}}\sec t\ \cancel{\tan t}\ dt}{9\sec^4 t\ \cancel{\sqrt{3}}\ \cancel{\tan t}}$$

$$\int \frac{1}{9\sec^3 t}\,dt \Rightarrow \frac{1}{9}\int \cos^3 t\,dt$$

$$\frac{1}{9}\int \cos t\cdot\cos^2 t\,dt$$

$$\frac{1}{9}\int \cos t\,[1-\sin^2 t]\,dt$$

put $\sin t = u$
$\cos t\,dt = du$

$$\frac{1}{9}\int (1-u^2)\,du$$

$$\frac{1}{9}\left[u-\frac{u^3}{3}\right]+c$$

$$\frac{1}{9}u-\frac{1}{27}u^3+c$$

$$\frac{1}{9}\sin t-\frac{1}{27}\sin^3 t+c \Rightarrow \frac{1}{9}\sin\sec^{-1}\left(\frac{x}{\sqrt{3}}\right)-\frac{\sin^3\sec^{-1}\frac{x}{\sqrt{3}}}{27}$$

$$\frac{1}{9}\sin\sin^{-1}\left[\frac{\sqrt{x^2-3}}{x}\right]-\sin^3\sin^{-1}\frac{\left[x^2-3\right]^{3/2}}{x^3\cdot 27}$$

$$\frac{\left[3x^2-(x^2-3)\right](x^2-3)^{\frac{1}{2}}}{27x^3}+c \Rightarrow \boxed{\frac{\sqrt{x^2-3}\,(2x^2+3)}{27x^3}+c}$$