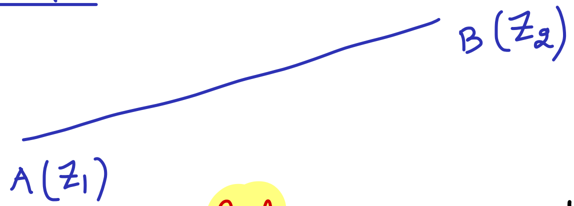
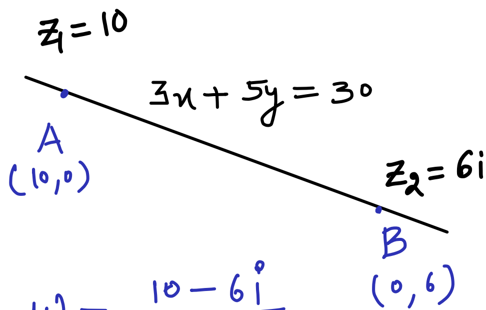
Complex Slope :-



Rem

$$\boxed{\omega = \frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2}}$$

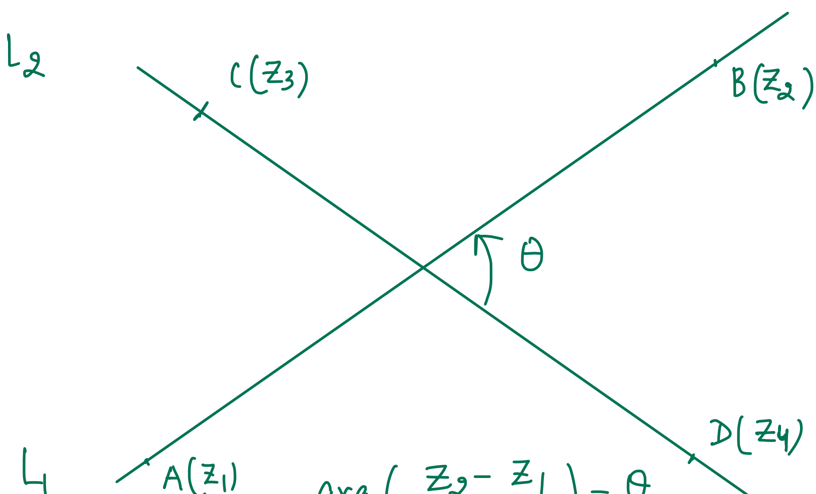
Complex slope



$$\omega = \frac{10 - 6i}{10 + 6i}$$

Complex slope

Condition for lines to be parallel or perpendicular :-



$$\arg\left(\frac{z_2 - z_1}{z_4 - z_3}\right) = \theta$$

① If $L_1 \parallel L_2$ then $\theta = 0$ or π.

$$\arg\left(\frac{z_2 - z_1}{z_4 - z_3}\right) = 0 \text{ or } \pi.$$

$$\Rightarrow \quad \frac{z_2 - z_1}{z_4 - z_3} \text{ is P.R.}$$

$$\Rightarrow \quad \frac{z_2 - z_1}{z_4 - z_3} = \left(\frac{\bar{z}_2 - \bar{z}_1}{\bar{z}_4 - \bar{z}_3}\right)$$

$$\Rightarrow \quad \frac{z_2 - z_1}{\bar{z}_2 - \bar{z}_1} = \frac{z_4 - z_3}{\bar{z}_4 - \bar{z}_3} \Rightarrow \boxed{\omega_1 = \omega_2}$$

Rem

② If $L_1 \perp L_2$ then $\theta = \pm \pi/2$.

$$\arg\left(\frac{z_2 - z_1}{z_4 - z_3}\right) = \pm\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{z_2 - z_1}{z_4 - z_3} \text{ is P.I}$$

$$\Rightarrow \quad \frac{z_2 - z_1}{z_4 - z_3} + \frac{\bar{z}_2 - \bar{z}_1}{\bar{z}_4 - \bar{z}_3} = 0.$$

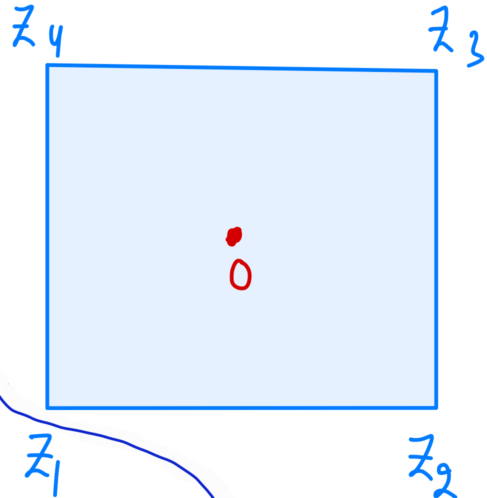
$$\Rightarrow \quad \frac{z_2 - z_1}{\bar{z}_2 - \bar{z}_1} + \frac{z_4 - z_3}{\bar{z}_4 - \bar{z}_3} = 0$$

$$\boxed{\omega_1 + \omega_2 = 0}$$

Rem

Q

If z_1, z_2, z_3, z_4 in order are the vertices of the square taken in order then which of the following is(are) TRUE?



(a) $\dfrac{z_1 - z_3}{z_2 - z_4}$ is purely imaginary

(b) $\dfrac{z_1 - z_2}{z_3 - z_4}$ is purely real

(c) $\dfrac{z_4 - z_3}{z_2 - z_3}$ is purely real.

(d) $\dfrac{z_2 + z_4}{z_1 + z_3}$ is purely imaginary

(e) $$\frac{z_1 - z_3}{z_2 - z_4} + \frac{\bar{z}_1 - \bar{z}_3}{\bar{z}_2 - \bar{z}_4} = 0.$$

(f) $$\frac{z_4 - z_3}{\bar{z}_4 - \bar{z}_3} + \frac{z_2 - z_3}{\bar{z}_2 - \bar{z}_3} = 0.$$

$$\frac{\cancel{\left(\frac{z_2 + z_4}{2}\right)}}{\cancel{\left(\frac{z_1 + z_3}{2}\right)}} = 1$$

$\downarrow$

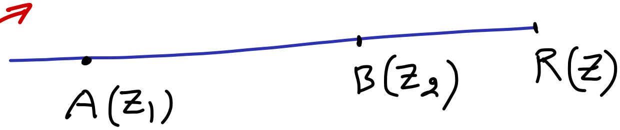
P.R

Equation of straight line in various forms :

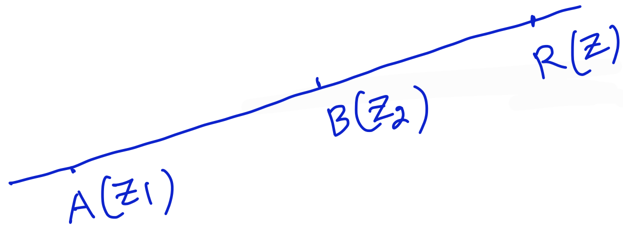
(1) Two point form :-

$w_1 = w_2$

$$\boxed{\frac{z - z_1}{\bar{z} - \bar{z}_1} = \frac{z_2 - z_1}{\bar{z}_2 - \bar{z}_1}}$$



(2) Parametric form :-



$$\boxed{z = z_1 + \lambda(z_2 - z_1)} ; \ \lambda \in \mathbb{R}$$

$$\frac{z - z_1}{z_2 - z_1} = \lambda$$ P.R

$$\Rightarrow \frac{z - z_1}{z_2 - z_1} = \frac{\bar{z} - \bar{z}_1}{\bar{z}_2 - \bar{z}_1} \Rightarrow \frac{z - z_1}{\bar{z} - \bar{z}_1} = \frac{z_2 - z_1}{\bar{z}_2 - \bar{z}_1}$$

(3) Determinant form :-

A(z_1) B(z_2) R(z)

$$\begin{vmatrix} 1 & 1 & 1 \\ z & z_1 & z_2 \\ \bar{z} & \bar{z}_1 & \bar{z}_2 \end{vmatrix} = 0$$

$$(z_1\bar{z}_2 - \bar{z}_1 z_2) - 1(z\bar{z}_2 - \bar{z} z_2) + (z\bar{z}_1 - \bar{z} z_1) = 0$$

$$z(\bar{z}_1 - \bar{z}_2) + \bar{z}(z_2 - z_1) + z_1\bar{z}_2 - \bar{z}_1 z_2 = 0$$

$$iz(\bar{z}_1 - \bar{z}_2) + \bar{z}(z_2 - z_1)i + i(z_1\bar{z}_2 - \bar{z}_1 z_2) = 0$$

$$i(z_1 - z_2) = \alpha$$

$$-i(\bar{z}_1 - \bar{z}_2) = \bar{\alpha}$$

$$z\bar{\alpha} + \bar{z}\alpha - i(z_1\bar{z}_2 - \bar{z}_1 z_2) = 0$$

Rem

$$\boxed{z\bar{\alpha} + \bar{z}\alpha + \beta = 0}$$

β is P.R.

α is Complex No.

Q Find x-intercept of line

$$z(3-4i) + \bar{z}(3+4i) + 5 = 0$$

Sol^n $z = x+iy$

$$(x+iy)(3-4i) + (x-iy)(3+4i) + 5 = 0.$$

$$6x+8y+5=0 \longrightarrow x_{int} = -\frac{5}{6}.$$

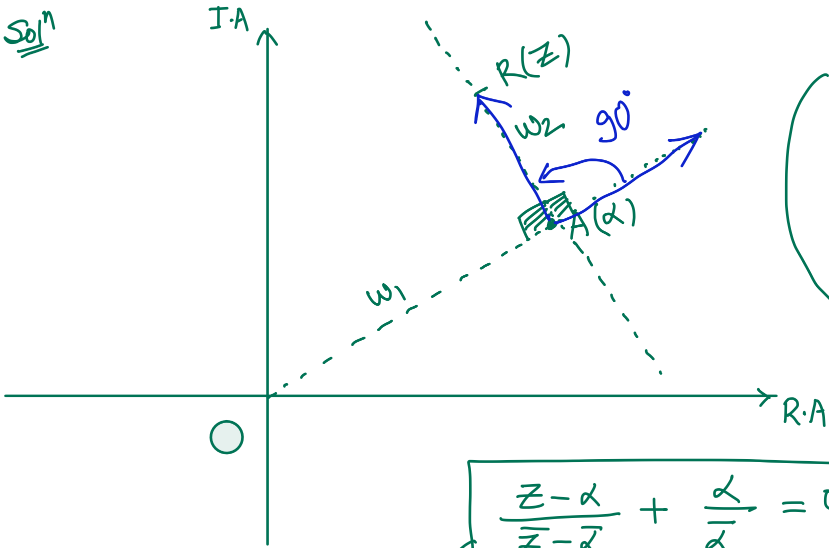
M-2 $z = \bar{z}$ (on R.A.)

$$z(3-4i) + z(3+4i) + 5 = 0$$

$$6z+5=0 \Rightarrow z = -\frac{5}{6}.$$

Q Find the equation of a line on complex plane which passes through a point A denotes by complex number α and is perpendicular to the vector $\overrightarrow{OA}$.

Sol^n



$$\omega_1 + \omega_2 = 0.$$

$$\omega_1 = \frac{\alpha - 0}{\bar{\alpha} - 0}$$

$$\omega_2 = \frac{z-\alpha}{\bar{z}-\bar{\alpha}}$$

$$\boxed{\frac{z-\alpha}{\bar{z}-\bar{\alpha}} + \frac{\alpha}{\bar{\alpha}} = 0}$$

$$\left(\frac{z-\alpha}{\alpha}\right) + \left(\frac{\bar{z}-\bar{\alpha}}{\bar{\alpha}}\right) = 0$$

$$\Downarrow$$

$\dfrac{z-\alpha}{\alpha}$ is P.I. i.e $\dfrac{z-\alpha}{\alpha} = \lambda i$; $\lambda \in \mathbb{R}$

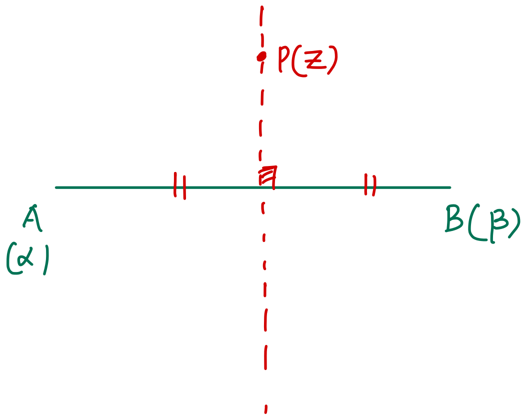
$$\boxed{z = \alpha + \alpha\lambda i} ;\ \lambda \in \mathbb{R}$$

alt :-

$$\operatorname{Arg}\left(\frac{z-\alpha}{\alpha}\right) = \frac{\pi}{2}$$

$\dfrac{z-\alpha}{\alpha}$ is P.I.

Note : It is to be noted that the equation $|z-\alpha| = |z-\beta|$ denotes the equation of the perpendicular bisector of the line joining the points α & β.



Q If $|z| = 1$ and $w = \dfrac{z-1}{z+1}$, then find locus of w ?

Sol

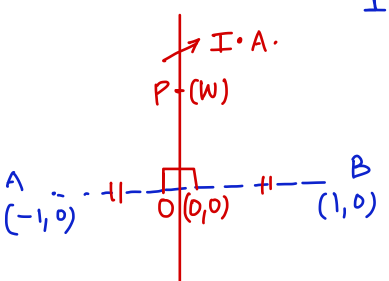
$$w = \frac{z-1}{z+1} \Rightarrow zw + w = z - 1$$

$$z = \frac{-1-w}{w-1} \Rightarrow z = \frac{w+1}{1-w}$$

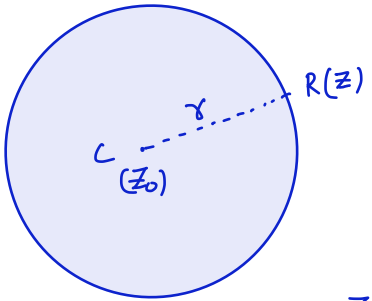
Since $|z| = 1$

$$|z| = \left|\frac{w+1}{1-w}\right| = \frac{|w+1|}{|w-1|}$$

$$1 = \frac{|w+1|}{|w-1|} \Rightarrow |w+1| = |w-1|$$



Equation of Circle :-



$$|\underset{\text{locus}}{z} - \overset{\text{fixed}}{z_0}| = r$$

$r \to$ radius

$$|z - z_0|^2 = r^2$$

$$(z - z_0)(\bar{z} - \bar{z}_0) = r^2$$

$$z\bar{z} - z\bar{z}_0 - z_0\bar{z} + \underbrace{z_0\bar{z}_0 - r^2}_{\beta} = 0$$

Real quantity

$-z_0 = \alpha$

$-\bar{z}_0 = \bar{\alpha}$

Rem

$$\boxed{z\bar{z} + \alpha\bar{z} + \bar{\alpha}z + \beta = 0}$$

Coeff of $z\bar{z}$ is 1

Centre is $-$ coeff of $\bar{z}$.

Radius $r = \sqrt{\alpha\bar{\alpha} - \beta}$

Q $2z\bar{z} + (6-8i)\bar{z} + (6+8i)z + 32 = 0$

Find centre & radius of Circle ?

Sol^n

$$z\bar{z} + (3-4i)\bar{z} + (3+4i)z + 16 = 0$$

$$\text{Centre} = -\text{Coeff of } \bar{z}$$

$$= -(3-4i) = -3+4i$$

$$\text{rad} = \sqrt{\alpha\bar{\alpha} - \beta} = \sqrt{(3-4i)(3+4i) - 16}$$

$$\text{rad} = \sqrt{25-16} = \sqrt{9} = 3$$

M-2

$z = x+iy$

and proceed to obtain eqn of circle in xy plane

↓

$$(x+3)^2 + (y-4)^2 = 9$$

Q Find Locus of 'z' where $|2z-3| = 2$

Solⁿ

$$\left|z - \frac{3}{2}\right| = 1$$

↓ Circle whose centre is $\left(\frac{3}{2}, 0\right)$ and rad $= 1$

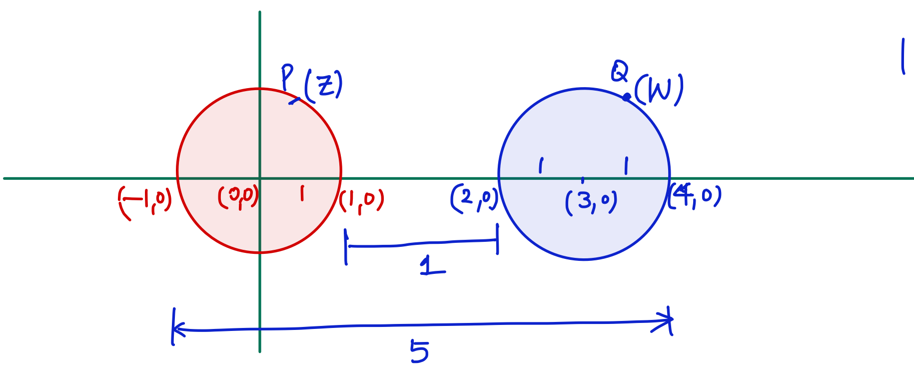
Q If $|z| = 1$ & $|w-3| = 1$ then find the maximum and minimum value of : ① $|z-w|$

② $|2z-3w|$

Solⁿ $|z| = 1$; $|w-3| = 1$

$|z-w|_{least} = 1$

$|z-w|_{greatest} = 5$



(b) $|z| = 1$ & $|w - 3| = 1$

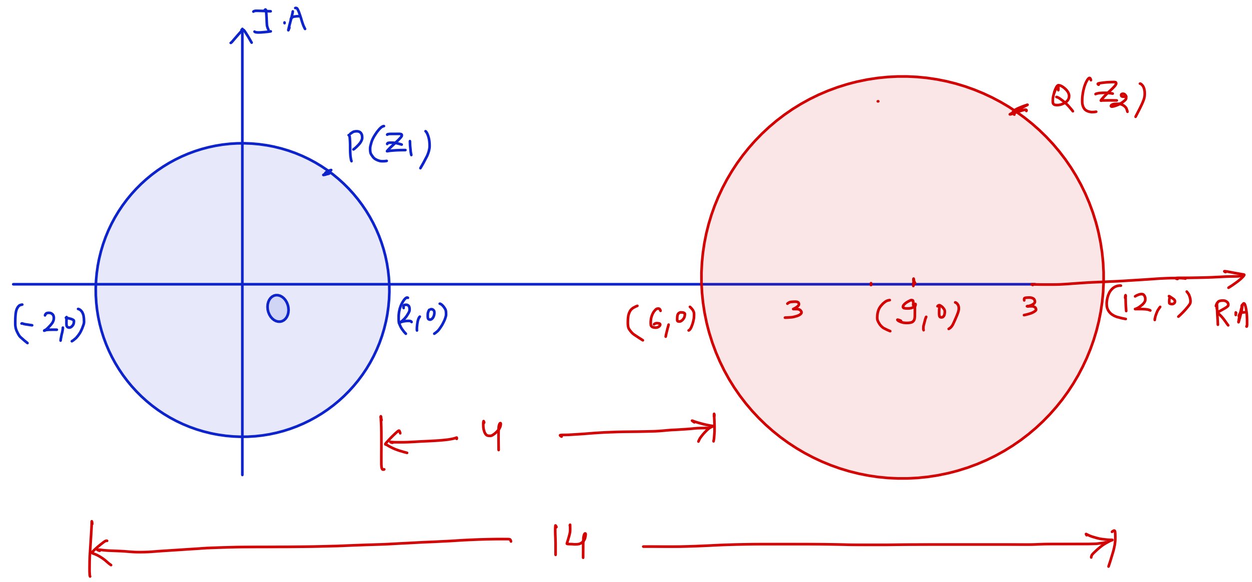
$|2z - 3w| \to L = ?,\ G = ?$

$\Downarrow$

$|z_1 - z_2| \to L = 4,\ G = 14$

$|\underbrace{2z}_{z_1}| = 2 \Rightarrow |z_1| = 2$

$|\underbrace{3w}_{z_2} - 9| = 3 \Rightarrow |z_2 - 9| = 3$



S-1 Q 6 to 10

S-2 Q 2, 3, 6, 8

JM Q 1, 2, 6 to 10, 13 to 15