Exercise S1

EXERCISE (S-1)

1. In a box, there are 8 alphabets cards with the letters: S, S, A,A,A,H,H,H. Find the probability that the word 'ASH' will form if :

(i) the three cards are drawn one by one & placed on the table in the same other that they are drawn.

(ii) the three cards are drawn simultaneously.

Solⁿ $2S, 3A, 3H$

(i) $P(\text{I card A}) \cdot P\left(\begin{array}{l}\text{II card S} \\ \text{given I} \\ \text{card is} \\ \text{A}\end{array}\right) \cdot P\left(\begin{array}{l}\text{III card H} \\ \text{given I card} \\ \text{is A and II} \\ \text{card is S}\end{array}\right)$

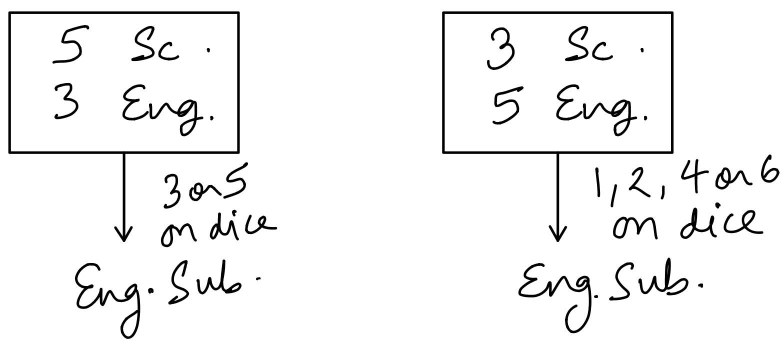
$$= \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{3}{6} = \frac{3}{56} \text{ (Ans.)}$$

(ii) When three cards are drawn simultaneously, the drawn cards must contain one each of A, S and H

$$\therefore \text{Reqd prob.} = \frac{{}^3C_1 \cdot {}^2C_1 \cdot {}^3C_1}{{}^8C_3} = \frac{9}{28} \text{ (Ans)}$$

2. There are 2 groups of subjects one of which consists of 5 science subjects & 3 engg. subjects & other consists of 3 science & 5 engg. subjects. An unbiased die is cast . If the number 3 or 5 turns up a subject is selected at random from first group, otherwise the subject is selected from 2nd group. Find the probability that an engg. subject is selected.

Soln



$$P\left(\begin{matrix}\text{Eng. Subject}\\ \text{selected}\end{matrix}\right) =$$

$$P\left(\begin{matrix}3 \text{ or } 5\\ \text{on dice}\end{matrix}\right) \times P\left(\begin{matrix}\text{Eng. sub.}\\ \text{from I}\\ \text{grp.}\end{matrix}\right) + P\left(\begin{matrix}1,2,4\\ \text{or } 5\\ \text{on}\\ \text{dice}\end{matrix}\right) \times P\left(\begin{matrix}\text{Eng. sub.}\\ \text{from I}\\ \text{grp.}\end{matrix}\right)$$

$$= \frac{2}{6} \times \frac{3}{8} + \frac{4}{6} \times \frac{5}{8} = \frac{26}{48} = \frac{13}{24} \text{ (Ans)}$$

3. A pair of fair dice is tossed. Find the probability that the maximum of the two numbers is greater than 4.

Soln

P(Max. of two numbers is greater than 4)

$= 1 - P\begin{pmatrix}\text{outcome is} \\ \text{less than or} \\ \text{equal to} \\ \text{4 on I dice}\end{pmatrix} \cdot P\begin{pmatrix}\text{outcome is} \\ \text{less than or} \\ \text{equal to} \\ \text{4 on II dice}\end{pmatrix}$

$= 1 - \frac{4}{6} \cdot \frac{4}{6} = 1 - \frac{4}{9} = \frac{5}{9}$ (Ans.)

4 A covered basket of flowers has some lilies and roses. In search of rose, Sweety and Shweta alternately pick up a flower from the basket but puts it back if it is not a rose. Sweety is 3 times more likely to be the first one to pick a rose. If sweety begin this 'rose hunt' and if there are 60 lilies in the basket, find the number of roses in the basket.

Sol^n

Let there are x roses.

$\boxed{\begin{array}{l} x \text{ roses} \\ 60 \text{ lilies} \end{array}}$

Let S denotes the probability of picking a rose and f denotes the probability of picking a lily.

$$S = \frac{x}{60+x} \quad ; \quad f = \frac{60}{60+x}$$

$$P(\text{Sweety wins}) = P(S \cup ffS \cup ffffS \ldots)$$
$$= S + ffS + ffffS + \ldots \infty$$
$$= \frac{S}{1-f^2}$$

$$P(\text{Shweta wins}) = P(fS \cup fffS \cup fffffS \ldots)$$
$$= fS + f^3S + f^5S + \ldots \infty$$
$$= \frac{fS}{1-f^2}$$

$\because$ P(Sweety wins) = 3 P(Shweta wins)

$$\Rightarrow \frac{S}{1-f^2} = \frac{3fS}{1-f^2} \Rightarrow f = 1/3$$

$$\Rightarrow \frac{60}{60+x} = \frac{1}{3} \Rightarrow x = 120 \text{ (Ans)}$$

5 A certain drug, manufactured by a Company is tested chemically for its toxic nature. Let the event "THE DRUG IS TOXIC" be denoted by H and the event "THE CHEMICAL TEST REVEALS THAT THE DRUG IS TOXIC" be denoted by S. Let P(H) = a, P(S/H) = $P(\bar{S}/\bar{H})$ = 1 − a. Then show that the probability that the drug is not toxic given that the chemical test reveals that it is toxic is free from 'a'.

Sol^n

H: Drug is toxic

S: Chemical test reveals drug is toxic

Given: $P(H) = a$; $P(S/H) = P(\bar{S}/\bar{H}) = 1-a$

To find: $P(\bar{H}/S)$

Now, $P(S/H) = \dfrac{P(S\cap H)}{P(H)} = 1-a$

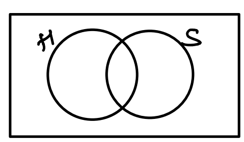
$\Rightarrow \quad P(S\cap H) = a(1-a)$ ——— (I)

Also, $P(\bar{S}/\bar{H}) = \dfrac{P(\bar{S}\cap\bar{H})}{P(\bar{H})} = 1-a$

$\Rightarrow \quad \dfrac{P(\overline{S\cup H})}{1-P(H)} = 1-a \Rightarrow 1-P(S\cup H) = (1-a)^2$

$\Rightarrow P(S\cup H) = 2a-a^2$ ——— (II)

$\because \; P(S\cup H) = P(S) + P(H) - P(S\cap H)$



$\Rightarrow 2a-a^2 = P(S) + a - a + a^2$

$\Rightarrow \quad P(S) = 2a - 2a^2 = 2a(1-a)$

Now $\quad P(\bar{H}/S) = \dfrac{P(\bar{H}\cap S)}{P(S)} = \dfrac{P(S)-P(H\cap S)}{P(S)}$

$\Rightarrow P(\bar{H}/S) = 1 - \dfrac{P(H\cap S)}{P(S)} = 1 - \dfrac{a(1-a)}{2a(1-a)} = 1-\dfrac{1}{2} = 1/2$ Ans

6 Players A and B alternately toss a biased coin, with A going first. A wins if A tosses a Tail before B tosses a Head; otherwise B wins. If the probability of a head is p, find the value of p for which the game is fair to both players.

Solⁿ

$$P(H) = p \;;\; P(T) = 1-p$$

$$P(A \text{ wins}) = P(T) + P(HTT) + P(HTHTT) + \text{-------}$$

$$= 1-p + p(1-p)^2 + p^2(1-p)^3 + \text{-------}\infty$$

$$\Rightarrow P(A \text{ wins}) = \frac{1-p}{1-p(1-p)}$$

For game to be fair to both players:

$$P(A \text{ wins}) = P(B \text{ wins}) = 1/2$$

$$\Rightarrow \frac{1-p}{1-p(1-p)} = \frac{1}{2} \Rightarrow p^2+p-1=0$$

$$\Rightarrow p = \frac{-1+\sqrt{5}}{2} \text{ or } \underbrace{p = \frac{-1-\sqrt{5}}{2}}_{\text{reject}}$$

$$\Rightarrow p = \frac{\sqrt{5}-1}{2} \text{ (Ans)}$$

7. The entries in a two-by-two determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ are integers that are chosen randomly and independently, and , for each entry, the probability that the entry is odd is p. If the probability that the value of the determinant is even is 1/2, then find the value of p.

Soln

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \underbrace{ad}_{} - \underbrace{bc}_{} \longrightarrow \text{Even}$$

CI O O
CII E E } possibilities

Case I: $\underbrace{ad}_{\text{odd}} - \underbrace{bc}_{\text{odd}}$ {Product xy is odd only when x, y are both odd}

$(p \cdot p) \cdot (p \cdot p)$

$$\therefore \boxed{\text{Probability} = p^4}$$

Case II: $\underbrace{ad}_{E} - \underbrace{bc}_{E}$ {Product xy is Even if none of x and y is odd}

$1-p^2 \qquad 1-p^2$

$1-(P(a \text{ odd}) \cdot P(d \text{ odd})) \qquad 1-(P(b \text{ odd}) \cdot P(c \text{ odd}))$

$$\therefore \boxed{\text{Probability} = (1-p^2)^2}$$

$\therefore$ by cases (1) and (11):

$$p^4 + (1-p^2)^2 = 1/2$$

$$\Rightarrow 4p^4 - 4p^2 + 1 = 0 \Rightarrow (2p^2-1)^2 = 0 \Rightarrow p = 1/\sqrt{2} \text{ (Ans)}$$

8 There are 4 urns. The first urn contains 1 white & 1 black ball, the second urn contains 2 white & 3 black balls, the third urn contains 3 white & 5 black balls & the fourth urn contains 4 white & 7 black balls. The selection of each urn is not equally likely. The probability of selecting i^{th} urn is $\frac{i^2+1}{34}$ (i = 1,2,3,4). If we randomly select one of the urns & draw a ball, then the probability of ball being white is p/q where p and q $\in$ N are in their lowest form. Find (p+q).

Sol^n

I	II	III	IV
$\boxed{\begin{matrix}1W\\1B\end{matrix}}$	$\boxed{\begin{matrix}2W\\3B\end{matrix}}$	$\boxed{\begin{matrix}3W\\5B\end{matrix}}$	$\boxed{\begin{matrix}4W\\7B\end{matrix}}$
$2/34$	$5/34$	$10/34$	$17/34$

$$\underbrace{\frac{2}{34} \quad \frac{5}{34} \quad \frac{10}{34} \quad \frac{17}{34}}_{\text{Probability of selection}}$$

P(drawing a white ball)

$$= \frac{2}{34}\cdot\frac{1}{2} + \frac{5}{34}\cdot\frac{2}{5} + \frac{10}{34}\cdot\frac{3}{8} + \frac{17}{34}\cdot\frac{4}{11}$$

$$= \frac{569}{1496} = \frac{p}{q}$$

$$\therefore p+q = 569 + 1496 = 2065 \text{ (Ans)}$$

9 A room has three electric lamps. From a collection of 10 electric bulbs of which 6 are good 3 are selected at random & put in the lamps. Find the probability that the room is lighted.

Soln

$$10 \text{ bulbs} \begin{cases} 6G \\ 4D \end{cases}$$

Room is considered lighted if atleast one good bulb is selected

$$\therefore P\left(\begin{matrix}\text{room is}\\ \text{lighted}\end{matrix}\right) = 1 - P\left(\begin{matrix}\text{All D bulbs}\\ \text{are selected}\end{matrix}\right)$$

$$= 1 - \frac{{}^{4}C_3}{{}^{10}C_3}$$

$$= \frac{29}{30} \text{ (Ans)}$$

10 Find the minimum number of tosses of a pair of dice so that the probability of getting the sum of the digits on the dice equal to 7 on at least one toss is greater than 0.95.
($\log_{10}2 = 0.3010$; $\log_{10}3 = 0.4771$)

Sol^n

$$P(\text{Sum 7 on pair of dice}) = \frac{6}{36} = 1/6$$

We call getting sum 7 as success and not getting sum 7 as failure.

Let there be n tosses

$$P(\text{Atleast one } S) = 1-(P(F))^n$$

$$= 1 - \left(\frac{5}{6}\right)^n > 0.95$$

$$\Rightarrow \quad .05 > (5/6)^n$$

$$\Rightarrow \quad (5/6)^n < 1/20$$

Taking log on base 10 on both sides:

$$\Rightarrow \quad n(\log 5 - \log 6) < -(1 + \log 2)$$

$$\Rightarrow \quad n(1 - \log 2 - \log 2 - \log 3) < -1 - \log 2$$

$$\Rightarrow \quad n(2\log 2 + \log 3 - 1) > 1 + \log 2$$

$$\Rightarrow n > \frac{1+\log 2}{2\log 2 + \log 3 - 1} \rightarrow \frac{1 + .3010}{2(.3010) + .4771 - 1}$$

$$\Rightarrow \quad n > 16.5 \quad \Rightarrow \quad n = 17 \text{ (Ans)}$$

11. The probability that a person will get an electric contract is $2/5$ and the probability that he will not get plumbing contract is $4/7$. If the probability of getting at least one contract is $2/3$, what is the probability that he will get both?

Solⁿ

$$P(\text{Electric}) = 2/5$$

$$P(\text{Plumbing}) = 1 - 4/7 \Rightarrow P(\text{Plumbing}) = 3/7$$

$$P(\text{Electric} \cup \text{Plumbing}) = 2/3$$

$$P(\text{Electric} \cap \text{Plumbing}) = \frac{2}{5} + \frac{3}{7} - \frac{2}{3}$$

$$= \frac{42 + 45 - 70}{105}$$

$$P(\text{Electric} \cap \text{Plumbing}) = \frac{17}{105} \quad \text{(Ans)}$$

12. Five horses compete in a race. John picks two horses at random and bets on them. Find the probability that John picked the winner. Assume no dead heat.

Solⁿ

No Dead Heat means no two or more horses crosses the finish line simultaneously

Let A and B are the horses picked up by John

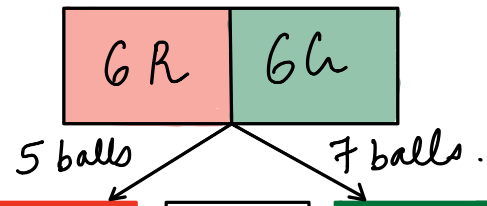
$$\therefore P(A \text{ or } B \text{ wins}) = P(A \text{ wins}) + P(B \text{ wins})$$

$$= \frac{1}{5} + \frac{1}{5}$$

$$= \frac{2}{5} \quad \text{(Ans)}$$

13 There are 6 red balls and 6 green balls in a bag. Five balls are drawn out at random and placed in a red box. The remaining seven balls are put in a green box. If the probability that the number of red balls in the green box plus the number of green balls in the red box is not a prime number, is $\frac{p}{q}$ where p and q are relatively prime, then find the value of $(p+q)$

Soln



Red box: R	Red box: G		Green balls in red box + Red balls in green box.		Green box: R	Green box: G
0	5	→	11	←	6	1
1	4	→	9	←	5	2
2	3	→	7	←	4	3
3	2	→	5	←	3	4
4	1	→	3	←	2	5
5	0	→	1	←	1	6

$$\text{Reqd. Prob.} = \frac{\underbrace{{}^6C_1 \cdot {}^6C_4}_{\text{1 Red and 4 green ball chosen to put in red box}} + \underbrace{{}^6C_5 \cdot {}^6C_0}_{\text{5 Red and No green ball chosen to put in red box}}}{{}^{12}C_5}$$

$$= \frac{90+6}{11\cdot 9\cdot 8} = \frac{4}{33} = \frac{p}{q}$$

$$\therefore p+q = 37 \quad \text{(Ans)}$$

14 A lot contains 50 defective & 50 non defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as :

A = { the first bulb is defective}; B = { the second bulb is non defective}

C = { the two bulbs are both defective or both non defective}

Determine whether (i) A,B,C are pair wise independent (ii) A,B,C are independent

Solⁿ

$\boxed{50D \quad 50ND} \longrightarrow$ 2 Bulbs drawn one at a time with replacement

$$P(A) = \frac{50}{100} = 1/2$$

$$P(B) = P(\text{I}D\ \text{II}ND + \text{I}ND\ \text{II}ND)$$

$$= \frac{50}{100}\cdot\frac{50}{100} + \frac{50}{100}\cdot\frac{50}{100} = 1/2$$

$$P(C) = P(\text{I}D\,\text{II}D + \text{I}ND\,\text{II}ND)$$

$$= \frac{50}{100}\cdot\frac{50}{100} + \frac{50}{100}\cdot\frac{50}{100} = 1/2$$

$\boxed{(i)}$ Pair wise independence

(a) A and B

Check: $\underbrace{P(A\cap B)}_{P(\text{I}D\ \text{II}ND)} = \underbrace{P(A)}_{1/2}\cdot\underbrace{P(B)}_{1/2}$

$$= \frac{50}{100}\cdot\frac{50}{100}$$

$\therefore$ A and B are Pairwise independent

(b) B and C

$B \cap C$: (II bulb ND) $\cap$ (both bulbs D or both ND)

$\Rightarrow$ $B \cap C$: both bulbs are ND

check: $\underbrace{P(B \cap C)}_{\frac{50}{100} \cdot \frac{50}{100}} = \underbrace{P(B)}_{1/2} \cdot \underbrace{P(C)}_{1/2}$

$\therefore$ B and C are Pairwise independent

(c) C and A

$C \cap A$: (I bulb is D) $\cap$ (both bulbs D or ND)

$\Rightarrow$ $C \cap A$: both bulbs D

check: $\underbrace{P(C \cap A)}_{\frac{50}{100} \cdot \frac{50}{100}} = \underbrace{P(C)}_{\frac{1}{2}} \cdot \underbrace{P(A)}_{\frac{1}{2}}$

$\therefore$ C and A are Pairwise independent

$\boxed{\text{(ii)}}$ For independence of A, B, C :

(I) A, B, C should be pairwise independent
(Done in (i) part)

(II) $\underbrace{P(A \cap B \cap C)}_{A \cap B \cap C \text{ is } \phi} = \underbrace{P(A)}_{1/2} \cdot \underbrace{P(B)}_{1/2} \cdot \underbrace{P(C)}_{1/2}$

$\therefore P(A \cap B \cap C) = 0$

$\therefore$ All 3 events are are <u>not</u> independent

15. The chance of one event happening is the square of the chance of a 2nd event, but odds against the first are the cubes of the odds against the 2nd. Find the chances of each (assume that both events are neither sure nor impossible).

Solⁿ

let prob. of 2nd event $= x$

$\therefore$ Prob. of 1st event $= x^2$

Odds against 1st $= \dfrac{1-x^2}{x^2}$

Odds against 2nd $= \dfrac{1-x}{x}$

$$\therefore \quad \frac{1-x^2}{x^2} = \left(\frac{1-x}{x}\right)^3$$

$$\Rightarrow \quad \frac{\cancel{(1-x)}(1+x)}{\cancel{x^2}} = \frac{(1-x)^{\cancel{3}\,2}}{x^{\cancel{3}}}$$

$$\Rightarrow \quad x(1+x) = (1-x)^2$$

$$\Rightarrow \quad x + x^2 = 1 + x^2 - 2x \Rightarrow x = 1/3$$

$\therefore$ $P(1^{st}\text{ Event}) = 1/3$; $P(2^{nd}\text{ Event}) = 1/9$ (Ans)

16. In a batch of 10 articles, 4 articles are defective. 6 articles are taken from the batch for inspection. If more than 2 articles in this batch are defective, the whole batch is rejected Find the probability that the batch will be rejected.

Sol^n

$$\underbrace{10 \text{ Articles}}_{\downarrow} \begin{cases} 4 \text{ Defective} \\ 6 \text{ Good.} \end{cases}$$

Select 6

(Reject if more than 2 are defective)

$$\therefore P\begin{pmatrix}\text{Batch is} \\ \text{rejected}\end{pmatrix} = P(3G\,3D) + P(2G\,4D)$$

$$= \frac{{}^4C_3 \cdot {}^6C_3}{{}^{10}C_6} + \frac{{}^4C_4 \cdot {}^6C_2}{{}^{10}C_6}$$

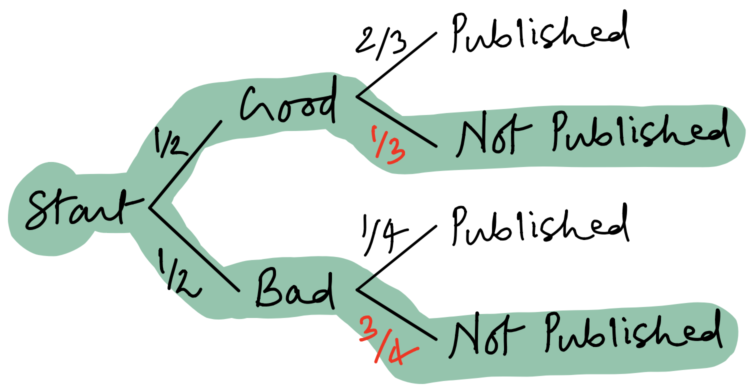
$$= \frac{4 \cdot 20}{210} + \frac{1 \cdot 15}{210}$$

$$= \frac{95}{210}$$

$$= \frac{19}{42} \text{ Ans.}$$

17. An author writes a good book with a probability of 1/2. If it is good it is published with a probability of 2/3. If it is not, it is published with a probability of 1/4. Find the probability that he will get atleast one book published if he writes two.

Soln



$$P\begin{pmatrix}\text{Atleast one}\\ \text{book published}\\ \text{if he writes}\\ \text{two}\end{pmatrix} = 1 - P\begin{pmatrix}\text{I book}\\ \text{not}\\ \text{published}\end{pmatrix} P\begin{pmatrix}\text{II book}\\ \text{not}\\ \text{published}\end{pmatrix}$$

$$= 1 - \Big(\underbrace{\tfrac{1}{2}\cdot\tfrac{1}{3}}_{\text{Good book Not published}} + \underbrace{\tfrac{1}{2}\cdot\tfrac{3}{4}}_{\text{Bad book Not published}}\Big)\underbrace{\Big(\tfrac{1}{2}\cdot\tfrac{1}{3} + \tfrac{1}{2}\cdot\tfrac{3}{4}\Big)}$$

$$= 1 - \left(\frac{13}{24}\right)^2$$

$$= \frac{407}{576} \quad \text{(Ans)}$$

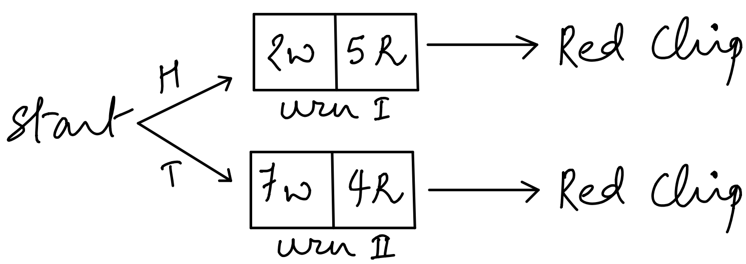
18. A biased coin which comes up heads three times as often as tails is tossed. If it shows heads, a chip is drawn from urn-I which contains 2 white chips and 5 red chips. If the coin comes up tail, a chip is drawn from urn-II which contains 7 white and 4 red chips. Given that a red chip was drawn, what is the probability that the coin came up heads ?

Solⁿ

Let $P(T) = x \Rightarrow P(H) = 3x$

$\because x + 3x = 1 \Rightarrow x = 1/4$

$\therefore P(T) = 1/4 \; ; \; P(H) = 3/4$



$P(Red/H) = 5/7$

$P(Red/T) = 4/11$

According to Baye's Theorem:

$$P(H/Red) = \frac{P(Red/H) \times P(H)}{P(Red/H) \times P(H) + P(Red/T) \times P(T)}$$

$$= \frac{5/7 \times 3/4}{5/7 \times 3/4 + 4/11 \times 1/4} = \frac{15/28}{15/28 + 1/11}$$

$$= \frac{165}{193} \text{ (Ans)}$$

19. A normal coin is continued tossing unless a head is obtained for the first time. Find the probability that

(a) number of tosses needed are atmost 3.

(b) number of tosses are even.

Sol^n

$$P(H) = 1/2 \quad ; \quad P(T) = 1/2$$

(a) Reqd prob. $= P(H + TH + TTH)$

$= P(H) + P(TH) + P(TTH)$

$= 1/2 + (1/2)(1/2) + (1/2)(1/2)(1/2)$

$= 1/2 + 1/4 + 1/8 = 7/8$ (Ans.)

(b) Reqd prob. $= P(TH + TTTH + TTTTTH + \ldots)$

$= (1/2)1/2 + (1/2)^3(1/2) + (1/2)^5(1/2) + \ldots$

$= (1/2)^2 + (1/2)^4 + (1/2)^6 + \ldots$

$= \dfrac{(1/2)^2}{1-(1/2)^2} = \dfrac{1/4}{1-1/4}$

$= 1/3$ (Ans)

20. A is one of the 6 horses entered for a race, and is to be ridden by one of two jockeys B or C. It is 2 to 1 that B rides A, in which case all the horses are equally likely to win; if C rides A, his chance is trebled, what are the odds against his winning?

Soln

Chance is trebled means chance becomes three times.

A → Horse ; B, C → Jockeys.

$P(B \text{ rides } A) = 2/3$; $P(C \text{ rides } A) = 1/3$

$P(A \text{ wins}/B \text{ rides } A) = 1/6$ ($\because$ 6 horses, each is equally likely to win)

$P(A \text{ wins}/C \text{ rides } A) = 1/6 \times 3 = 3/6$
($\because$ chances trebled)

$\therefore P(A \text{ wins}) = P(B)\cdot P(A/B) + P(C)\cdot P(A/C)$

$= 2/3 \cdot 1/6 + 1/3 \cdot 3/6$

$= 2/18 + 3/18$

$\Rightarrow P(A \text{ wins}) = 5/18$

$\therefore$ Odds against A winning = 13 to 5 (Ans)

Exercise S2

1. N fair coins are flipped once. The probability that at most 2 of the coins show up as heads is $\frac{1}{2}$. Find the value of N.

Soln P(atmost 2 Coin Show head)

$= P(\text{'0' head}) + P(\text{'1' head}) + P(\text{'2' head})$

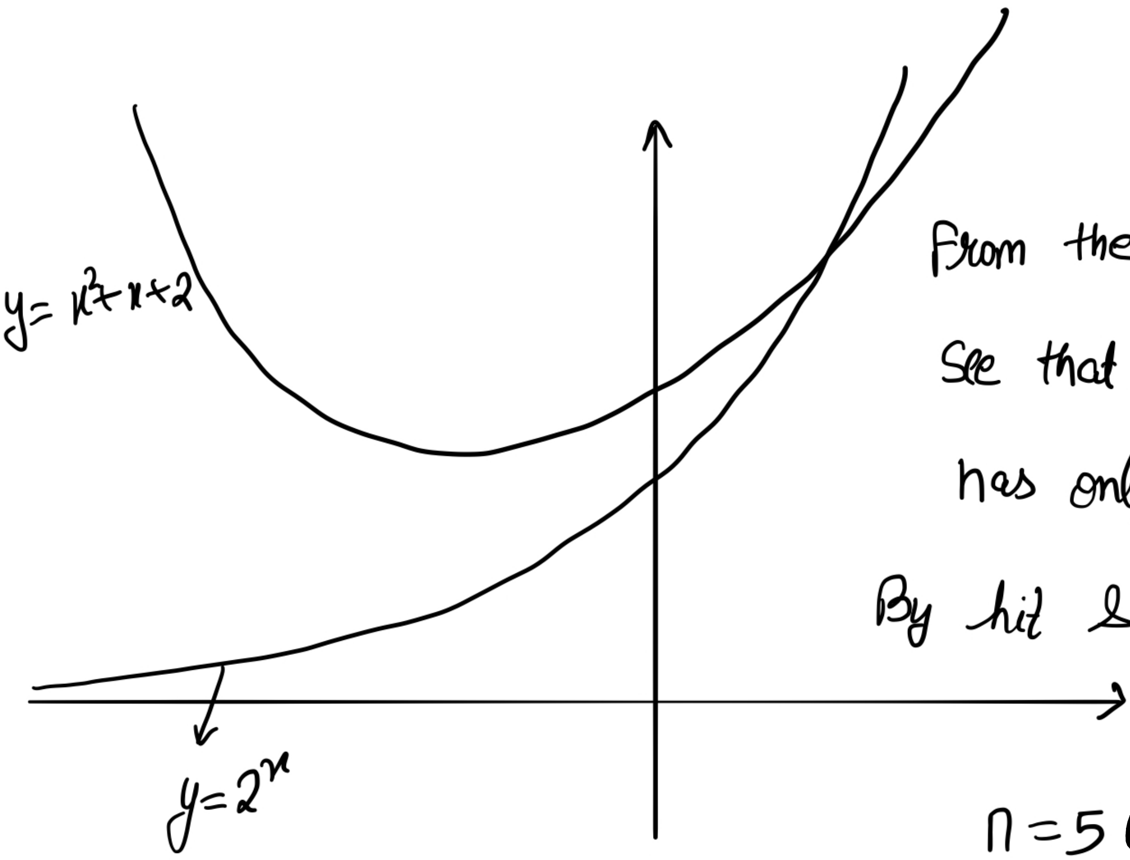
$= \left(\frac{1}{2}\right)^n + {}^nC_1\left(\frac{1}{2}\right)^{n-1}\left(\frac{1}{2}\right) + {}^nC_2\left(\frac{1}{2}\right)^{n-2}\left(\frac{1}{2}\right)^2 = \frac{1}{2}$

$\Rightarrow \left(\frac{1}{2}\right)^n\left(1 + n + \frac{n(n-1)}{2}\right) = \frac{1}{2}$

$\Rightarrow 2^{n-1} = \frac{n^2+n+2}{2}$

$\Rightarrow n(n+1) = 2^n - 2 \Rightarrow n^2+n+2 = 2^n$

$(n \in N)$



From the graph we see that the equation has only one solution. By hit & trial we see that

$n = 5$ (Ans)

2. A box contains three coins two of them are fair and one two – headed. A coin is selected at random and tossed. If the head appears the coin is tossed again, if a tail appears, then another coin is selected from the remaining coins and tossed.

(i) Find the probability that head appears twice.

(ii) If the same coin is tossed twice, find the probability that it is two headed coin.

(iii) Find the probability that tail appears twice.

Solⁿ

$$\boxed{\begin{array}{l} 2\text{-fair} \\ 1\text{-DH} \end{array}} \qquad P(\text{Fair}) = \frac{2}{3}, \quad P(DH) = \frac{1}{3}$$

(i) P(Head appear twice) = P(fair coin) + P(Double head)

$$= \frac{2}{3} \cdot \underset{H}{\frac{1}{2}} \cdot \underset{H}{\frac{1}{2}} + \frac{1}{3} \cdot \underset{H}{1} \cdot \underset{H}{1} = \frac{1}{2} \quad (\text{Ans})$$

(iii) (Tail appear twice) = P(fair coin) + P(Double head)

$$= \underset{F}{\frac{2}{3}} \cdot \underset{T}{\frac{1}{2}} \cdot \left(\underset{F}{\frac{1}{2}} \cdot \overset{T}{\frac{1}{2}} + \underset{DH}{\frac{1}{2} \cdot \overset{T}{0}} \right) + \frac{1}{3} \cdot 0 = \frac{1}{12} \quad (\text{Ans})$$

$$\boxed{1F + 1\text{-}DH} \rightarrow \text{Box}$$

(ii) P(Same coin tossed twice) = P(Fair coin) + P(DH)

$$= \underset{F}{\frac{2}{3}} \Big[\underbrace{\underset{H}{\frac{1}{2}} \cdot \left(\overset{H}{\frac{1}{2}} + \overset{T}{\frac{1}{2}} \right) + \underset{T}{\frac{1}{2}} \cdot 0}_{x} \Big] + \underset{DH}{\frac{1}{3}} \cdot \underbrace{[1 \cdot 1]}_{y} = \underset{x}{\frac{1}{3}} + \underset{y}{\frac{1}{3}}$$

$$P = \frac{fav}{Total} = \frac{y}{x+y} = \frac{1/3}{2/3} = \frac{1}{2} \quad (Ans)$$

3 Eight players P_1 , P_2 , P_3 ,..........P_8 play a knock out tournament. It is known that whenever the players P_i and P_j play , the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the players P_4 reaches the final ?

Sol^n P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8

P_i & P_j play . P_i will win if $i<j$

I^{st} round → P_4 should be pair with any one of P_5, P_6, P_7, P_8 ($^4C_1$ ways)

(after I^{st} round 4 players play and any one of P_5 to P_8 should also reach II^{nd} round so. one pair should be from remaining P_5 to P_8 ($^3C_2$ ways)

only one pairing possible

II^{nd} round : P_4 and one from P_5 to P_8 (3 ways)

(2 players left)

III^{rd} round : P_4 & other.

$$\text{fav. Pairing} = {}^4C_1 \times {}^3C_2 \times {}^3C_2 = 36$$

$$\text{Total pairing} = \frac{8!}{(2!)^4 \cdot 4!} \times \frac{4!}{(2!)^2 \cdot 2!} \times 1 = 45 \times 7$$

$$P = \frac{36}{45 \times 7} = \frac{4}{35} \text{ (Ans)}$$

4. (a) Two natural numbers x and y are chosen at random. Find the probability that $x^2 + y^2$ is divisible by 10.

(b) Two numbers x & y are chosen at random from the set {1,2,3,4,....3n}. Find the probability that $x^2 - y^2$ is divisible by 3.

Solⁿ (a) $x^2 + y^2 = 10\lambda$

let $x = 10\lambda_1 + r_1$ & $y = 10\lambda_2 + r_2$

$(x^2+y^2) = 10\lambda_3 + (r_1^2 + r_2^2) \rightarrow$ should be divisible by 10

$r_1, r_2 \in \{0, 1, \ldots, 9\} \rightarrow$ fix one by one

$$P = \frac{fav}{Total} = \frac{(0,0)\to 1 + (1,3)\to 2 + (1,7)\to 2 + (2,4)\to 2 + (2,6)\to 2 + (3,9)\to 2 + (4,8)\to 2 + (5,5)\to 1 + (6,8)\to 2 + (7,9)\to 2}{100}$$

Total $= 10 \times 10$

(r_1 & r_2 be selected in 10 ways)

$$P = \frac{18}{100} = \frac{9}{50} \text{ (Ans)}$$

(b) $x, y \in \{3\lambda, 3\lambda+1, 3\lambda+2\}$

Numbers of category 3λ, $3\lambda+1$, $3\lambda+2$ are each equal to 'n'

$(x^2-y^2) = (x-y)(x+y)$ is divisible by 3

C-1 x & y are of same category

No. of ways $= {}^3C_1 \cdot {}^nC_2 \rightarrow$ select 2 nos.

$\downarrow$

select one category

C-2 x & y is of $(3\lambda+1)$ & $(3\lambda+2)$

category

No of ways $= {}^nC_1 \cdot {}^nC_1$

$$fav = {}^3C_1 \cdot {}^nC_2 + {}^nC_1 \cdot {}^nC_1$$

$$fav = \frac{3n(n-1)}{2} + n^2 = \frac{(5n-3)n}{2}$$

$$Total = {}^{3n}C_2 = \frac{3n(3n-1)}{2}$$

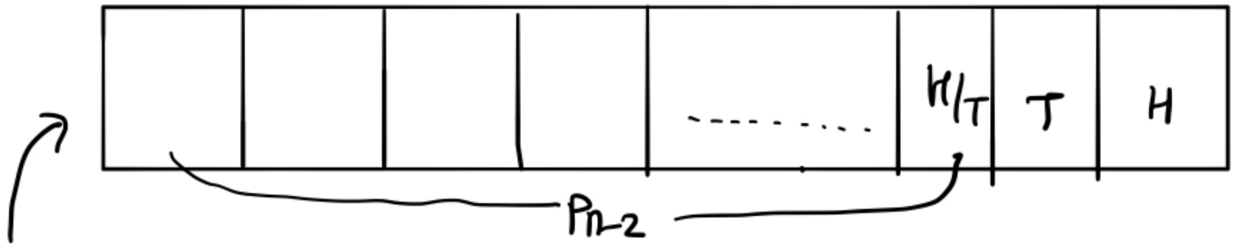
$$P = \frac{fav}{Total} = \frac{5n-3}{9n-3} \quad \text{(Ans)}$$

5. A coin has probability 'p' of showing head when tossed. It is tossed 'n' times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that,

$$p_1 = 1,\ p_2 = 1 - p^2 \ \&\ p_n = (1-p)p_{n-1} + p(1-p)p_{n-2}, \text{ for all } n \geq 3.$$

Sol^n $P(H) = P$, $P(T) = 1-P$

P_n = Probablity that no two (or more) consecutive head occour



C-1 when head occur in last position

C-2 when Tail occour in the last position

So from both the cases

$$P_n = \underset{(C-1)}{P_{n-2}\cdot(1-P)\cdot P} + \underset{(C-2)}{P_{n-1}}\cdot$$

P_1 : $\boxed{T/H}$ $= P(T) + P(H) = 1-P+P = 1$

P_2 : $\cancel{\boxed{H \mid H}}$ $= 1 - P(HH) = 1 - P^2$

HP

16. A pair of students is selected at random from a probability class. The probability that the pair selected will consist of one male and one female student is $\frac{10}{19}$. Find the maximum number of students the class can contain.

Sol^n^ male $= x$, female $= y$, No. of Student $= x+y$

$$P(1M+1F) = \frac{{}^{x}C_1 \, {}^{y}C_1}{{}^{x+y}C_2} = \frac{10}{19}$$

$$\Rightarrow 19xy = 10\,\frac{(x+y)(x+y-1)}{2}$$

$$\Rightarrow 5(x+y)^2 - 5(x+y) - 19xy = 0 \Rightarrow xy = \frac{5t^2 - 5t}{19}$$

$\Rightarrow$ let $x+y = t$, $\quad AM \geq GM$

$$\Rightarrow \frac{x+y}{2} \geq \sqrt{xy} \Rightarrow \frac{t^2}{4} \geq \frac{5t^2 - 5t}{19}$$

$$\Rightarrow 19t^2 \geq 20t^2 - 20t \Rightarrow -t^2 \geq -20t$$

$$\Rightarrow t \leq 20$$

$$(x+y)_{max} = 20 \quad \text{(Ans)}$$

7. 3 students {A, B, C} tackle a puzzle together and offers a solution upon which majority of the 3 agrees. Probability of A solving the puzzle correctly is p. Probability of B solving the puzzle correctly is also p. C is a dumb student who randomly supports the solution of either A or B. There is one more student D, whose probability of solving the puzzle correctly is once again, p. Out of the 3 member team {A, B, C} and one member team {D}, which one is more likely to solve the puzzle correctly.

Solⁿ $P(A) = P$, $P(B) = P$, $P(D) = P$

$P(C) =$ Randomly support either A or B

P(A,B,C correctly solve the puzzle correctly)

$= P(ABC) + P(A\bar{B}\underline{C{-}A}) + P(\bar{A}B\underline{C{-}B})$

$= p^2 + p(1-p)\cdot\frac{1}{2} + p(1-p)\cdot\frac{1}{2}$

$= p^2 + p(1-p) = p$

P(A,B,C solve the puzzle correctly) $= p$

$P(D) = p$

So both are equally likely (Ans)

8. During a power blackout, 100 persons are arrested on suspect of looting. Each is given a polygraph test. From past experience it is known that the polygraph is 90% reliable when administered to a guilty person and 98% reliable when given to some one who is innocent. Suppose that of the 100 persons taken into custody, only 12 were actually involved in any wrong doing. If the probability that a given suspect is innocent given that the polygraph says he is guilty is a/b where a and b are relatively prime, find the value of (a + b).

[Sol. A: polygraph says person is guility

B_1 : person is innocent $P(B_1) = 0.88$

B_2 : person is guility $P(B_2) = 0.12$

$P(A/B_1) = 0.02;$ $P(A/B_2) = 0.90$

$$P(B_1/A) = \frac{P(B_1)\cdot P(A/B_1)}{P(B_1)\cdot P(A/B_1) + P(B_2)\cdot P(A/B_2)} = \frac{0.88\times 0.02}{0.88\times 0.02 + 0.12\times 0.90}$$

$$= \frac{88\times 2}{88\times 2 + 12\times 90} = \frac{176}{1256} = \frac{22}{157} \quad \Rightarrow \quad a + b = 179$$ **Ans.**]

Exercise JM

1. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals **[AIEEE-2009]**

(1) 5/14 (2) 1/50 (3) 1/14 (4) 1/7

Sol.

S = { 00, 01, 02, …., 49 }

Let A be the even that sum of the digits on the selected ticket is 8 then

A = { 08, 17, 26, 35, 44 }

Let B be the event that the product of the digits is zero

B = { 00, 01, 02, 03, …., 09, 10, 20, 30, 40 }

$A \cap B = \{8\}$

$$\text{Required probability} = P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{50}}{\frac{14}{50}} = \frac{1}{14}$$

2. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than **[AIEEE-2009]**

(1) $\frac{9}{\log_{10}4 - \log_{10}3}$ (2) $\frac{4}{\log_{10}4 - \log_{10}3}$ (3) $\frac{1}{\log_{10}4 - \log_{10}3}$ (4) $\frac{1}{\log_{10}4 + \log_{10}3}$

Sol: (1)

$$1 - q^n \geq \frac{9}{10} \Rightarrow \left(\frac{3}{4}\right)^n \leq \frac{1}{10} \Rightarrow n \geq -\log_{\frac{3}{4}} 10 \Rightarrow n \geq \frac{1}{\log_{10}{}^4 - \log_{10}{}^3}$$

3. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have difference colours is :- **[AIEEE-2010]**

(1) $\frac{1}{3}$ (2) $\frac{2}{7}$ (3) $\frac{1}{21}$ (4) $\frac{2}{23}$

Sol.

$n(S) = {}^9C_3$

$n(E) = {}^3C_1 \times {}^4C_1 \times {}^2C_1$

$$\text{Probability} = \frac{3\times4\times2}{{}^9C_3} = \frac{24\times3!}{9!}\times6! = \frac{24\times6}{9\times8\times7} = \frac{2}{7}.$$

4. Four numbers are chosen at random (without replacement) from the set (1, 2, 3, ..., 20).

Statement–1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$

Statement–2 : In the four chosen numbers form an AP, then the set of all possible values of common difference is $\{\pm1, \pm2, \pm3, \pm4, \pm5\}$. **[AIEEE-2010]**

(1) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(2) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1.

(3) Statement–1 is true, Statement–2 is false.

(4) Statement–1 is false, Statement–2 is true.

Sol:

$N(S) = {}^{20}C_4$

Statement-1: common difference is 1; total number of cases = 17
common difference is 2; total number of cases = 14
common difference is 3; total number of cases = 11
common difference is 4; total number of cases = 8
common difference is 5; total number of cases = 5
common difference is 6; total number of cases = 2

$$\text{Prob.} = \frac{17+14+11+8+5+2}{{}^{20}C_4} = \frac{1}{85}.$$

5. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is :- **[AIEEE-2011]**

(1) $P(C|D) < P(C)$ (2) $P(C|D) = \frac{P(D)}{P(C)}$ (3) $P(C|D) = P(C)$ (4) $P(C|D) \geq P(C)$

Sol:-

$$\because P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} \qquad \left\{ \begin{array}{l} \because C \subset D \\ \therefore P(C \cap D) = P(C) \end{array} \right\}$$

$$\Rightarrow \boxed{P\left(\frac{C}{D}\right) = \frac{P(C)}{P(D)}} \text{ --- } (1)$$

$$\because P(D) \leq 1$$

$$\Rightarrow 1 \leq \frac{1}{P(D)}$$

$$\Rightarrow P(C) \leq \frac{P(C)}{P(D)} \quad \{\text{from } (1)\}$$

$$\Rightarrow \boxed{P(C) \leq P\left(\frac{C}{D}\right)}$$

6. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval :- **[AIEEE-2011]**

(1) $\left[0,\frac{1}{2}\right]$ (2) $\left(\frac{11}{12},1\right]$ (3) $\left(\frac{1}{2},\frac{3}{4}\right]$ (4) $\left(\frac{3}{4},\frac{11}{12}\right]$

Sol. $\because$ Probability of atleast one failure

$= 1 - P(\text{no failure})$

$= 1 - {}^{n}C_{5}\cdot(p)^{5}$

Now, $1 - p^{5} \geq \frac{31}{32}$ (according to Qus)

$\Rightarrow \quad p \leq \frac{1}{2}$

$\therefore \boxed{p \in \left[0, \frac{1}{2}\right]}$

7. Let A, B, C be pairwise independent events with P(C)> 0 and $P(A\cap B\cap C) = 0$. Then $P(A^c\cap B^c|C)$ is equal to: **[AIEEE-2011]**

(1) $P(A^c) - P(B)$ (2) $P(A) - P(B^c)$ (3) $P(A^c) + P(B^c)$ (4) $P(A^c) - P(B^c)$

Sol:-

$$\because \quad P\left(\frac{\bar{A}\cap\bar{B}}{C}\right) = \frac{P((\bar{A}\cap\bar{B})\cap C)}{P(C)}$$

$$= \frac{P(C) - P(A\cap C) - P(B\cap C)}{P(C)}$$

{∵ A, B, C are pairwise independent}

$$= \frac{P(C) - P(A).P(C) - P(B).P(C)}{P(C)}$$

$$= 1 - P(A) - P(B)$$

$$= \boxed{P(\bar{A}) - P(B)}$$

option(A).

8. Three numbers are chosen at random without replacement from $\{1, 2, 3, \ldots, 8\}$. The probability that their minimum is 3, given that their maximum is 6, is : **[AIEEE-2012]**

(1) $\frac{2}{5}$ (2) $\frac{3}{8}$ (3) $\frac{1}{5}$ (4) $\frac{1}{4}$

Sol:- $\{1, 2, 3, 4, 5, 6, 7, 8\}$

* let A be the event that maximum is 6.
* let B be the event that minimum is 3.

$\therefore\ P(A) = \dfrac{{}^5C_2}{{}^8C_3}$ {the number < 6 are 5}

& $P(B) = \dfrac{{}^5C_2}{{}^8C_3}$ {the number > 3 are 5}

$\therefore\ P(A \cap B) = \dfrac{{}^2C_1}{{}^8C_3}$ {Number either 4 or 5}

So, Required probability is

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\left(\dfrac{{}^2C_1}{{}^8C_3}\right)}{\left(\dfrac{{}^5C_2}{{}^8C_3}\right)} = \frac{{}^2C_1}{{}^5C_2}$$

$$= \boxed{\frac{1}{5}}$$

9. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is : **[JEE-MAIN 2013]**

(1) $\frac{17}{3^5}$ (2) $\frac{13}{3^5}$ (3) $\frac{11}{3^5}$ (4) $\frac{10}{3^5}$

Sol: $\because$ Probability of Correct ans $= \frac{1}{3}$

$\therefore$ Probability of Incorrect ans $= \frac{2}{3}$

There are Two Cases

4 answers are Correct

5 answers are Correct

$$= {}^5C_4 \cdot \left(\frac{1}{3}\right)^4 \cdot \left(\frac{2}{3}\right)^1 + {}^5C_5 \cdot \left(\frac{1}{3}\right)^5$$

$$= \frac{10}{3^5} + \frac{1}{3^5}$$

$$= \boxed{\frac{11}{3^5}}$$

10. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are : **[JEE(Main)-2014]**

(1) mutually exclusive and independent.
(2) equally likely but not independent.
(3) independent but not equally likely.
(4) independent and equally likely.

Sol:- * $P(\overline{A \cup B}) = \frac{1}{6}$

$\Rightarrow 1 - P(A \cup B) = \frac{1}{6}$

$\Rightarrow \boxed{P(A \cup B) = \frac{5}{6}}$

* $\boxed{P(A \cap B) = \frac{1}{4}}$

* $\because P(\bar{A}) = \frac{1}{4} \Rightarrow \boxed{P(A) = \frac{3}{4}}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$

$\Rightarrow \boxed{P(B) = \frac{1}{3}}$

Now, $\because \underbrace{P(A \cap B)}_{\Downarrow \ \frac{1}{4}} = \underbrace{P(A)}_{\Downarrow \ \frac{3}{4}} \cdot \underbrace{P(B)}_{\Downarrow \ \frac{1}{3}}$

$\therefore$ $\boxed{\text{A \& B are Independent}}$ & $\boxed{\because \text{P(A) \& P(B) are different, so not equally likely}}$

17. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is **NOT true** ? **[JEE(Main)-2016]**

(1) E_1, E_2 and E_3 are independent.
(2) E_1 and E_2 are independent.
(3) E_2 and E_3 are independent.
(4) E_1 and E_3 are independent.

Sol:- A ⇒ 1, 2, 3, 4, 5, 6
B ⇒ 1, 2, 3, 4, 5, 6

$* \; P(E_1) = \frac{1}{6}$

$* \; P(E_2) = \frac{1}{6}$

$* \; P(E_3) = \frac{1}{2}$ $\left\{ P(E_3) = \frac{9+9}{36} \right\}$

A	1	1	1	3	3	3	5	5	5
B	2	4	6	2	4	6	2	4	6

A	2	4	6	2	4	6	2	4	6
B	1	1	1	3	3	3	5	5	5

$* \; P(E_1 \cap E_2) = \frac{1}{36}$

$* \; P(E_2 \cap E_3) = \frac{1}{12}$

$* \; P(E_3 \cap E_1) = \frac{1}{12}$

$* \; P(E_1 \cap E_2 \cap E_3) = 0$

$\boxed{\therefore E_1, E_2, E_3 \text{ are not Independent}}$

12. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one–by–one, with replacement, then the variance of the number of green balls drawn is :- **[JEE(Main)-2017]**

(1) $\frac{6}{25}$ (2) $\frac{12}{5}$ (3) 6 (4) 4

Sol:-

$* \; p = \frac{15}{25} = \frac{3}{5}$

$* \; q = \frac{2}{5}$

We can apply binomial distribution

Variance $= n \cdot p q$

$= 10 \cdot \frac{3}{5} \cdot \frac{2}{5}$

$= \boxed{\frac{12}{5}}$

13. If two different numbers are taken from the set {0, 1, 2, 3, ..., 10), then the probability that their sum as well as absolute difference are both multiple of 4, is :- **[JEE(Main)-2017]**

(1) $\frac{7}{55}$ ~~(2)~~ $\frac{6}{55}$ (3) $\frac{12}{55}$ (4) $\frac{14}{45}$

Sol:- let $A = \{0, 1, 2, 3, \ldots\ldots, 10\}$

$\therefore \; n(S) = {}^{11}C_2$

* let E be the given event

$\therefore E = \{(0,4), (0,8), (2,6), (2,10), (4,8), (6,10)\}$

so, $n(E) = 6$

$\therefore \; P(E) = \frac{n(E)}{n(S)} = \frac{6}{{}^{11}C_2} = \boxed{\frac{6}{55}}$

14. For three events A, B and C, P(Exactly one of A or B occurs) = P(Exactly one of B or C occurs) = P(Exactly one of C or A occurs) = $\frac{1}{4}$ and P(All the three events occur simultaneously) = $\frac{1}{16}$. Then the probability that at least one of the events occurs, is :- **[JEE(Main)-2017]**

$A \cap B \cap C$

(1) $\frac{3}{16}$ (2) $\frac{7}{32}$ ~~(3)~~ $\frac{7}{16}$ (4) $\frac{7}{64}$

Sol:- * P(Exactly one of A or B occurs) = 1/4

$$\Rightarrow P(A) + P(B) - 2P(A \cap B) = \frac{1}{4} \quad \text{--- (1)}$$

* P(Exactly one of B or C occurs) = 1/4

$$\Rightarrow P(B) + P(C) - 2P(B \cap C) = \frac{1}{4} \quad \text{--- (2)}$$

* P(Exactly one of C or A occurs) = 1/4

$$\Rightarrow P(C) + P(A) - 2P(C \cap A) = \frac{1}{4} \quad \text{--- (3)}$$

Add (1) + (2) + (3) ⇒

$$\Rightarrow \boxed{\sum P(A) - \sum P(A \cap B) = \frac{3}{8}}$$

* $\boxed{P(A \cap B \cap C) = \frac{1}{16}}$ Given

Now,

$$\therefore P(A \cup B \cup C) = \underbrace{\sum P(A) - \sum P(A \cap B)}_{3/8} + \underbrace{P(A \cap B \cap C)}_{1/16}$$

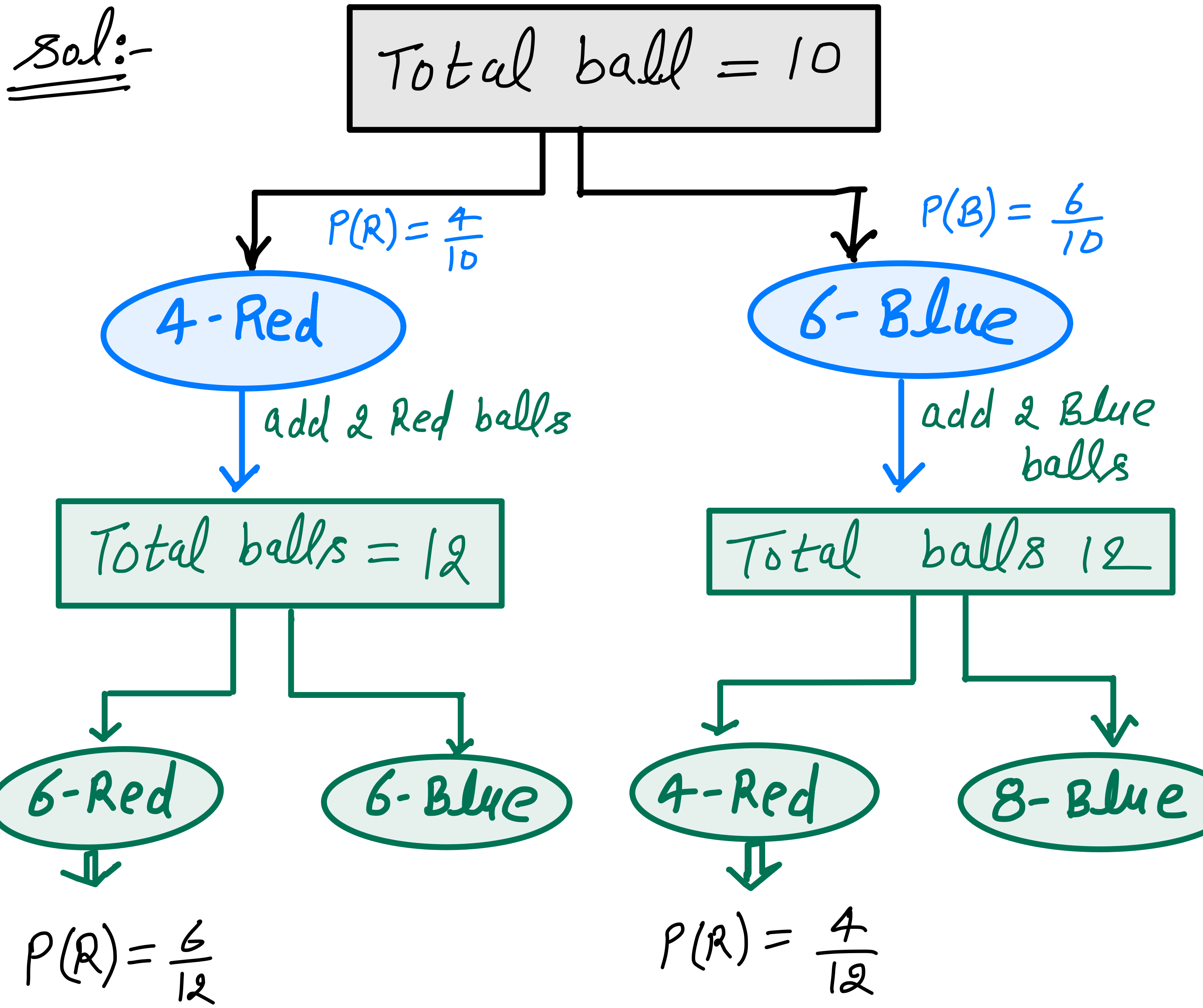
$$\boxed{P(A \cup B \cup C) = \frac{7}{16}}$$ Ans.

15. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

[JEE(Main)-2018]

(1) $\frac{2}{5}$ ✓ (2) $\frac{1}{5}$ (3) $\frac{3}{4}$ (4) $\frac{3}{10}$

Sol:-



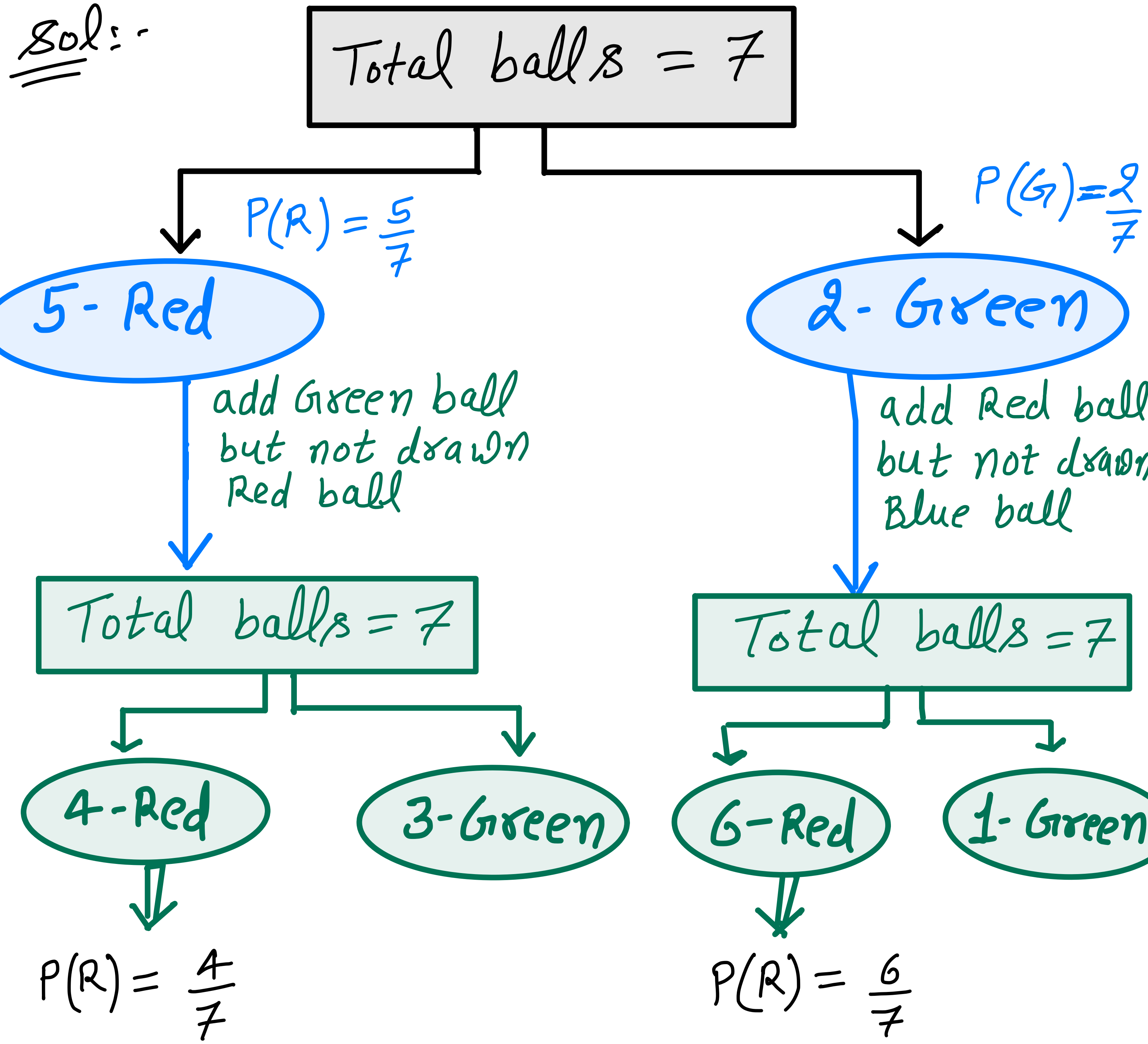
So, required Probability $= \left(\frac{4}{10}\cdot\frac{6}{12}\right)+\left(\frac{6}{10}\cdot\frac{4}{12}\right)$

$= \boxed{\frac{2}{5}}$ Ans.

16. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is : **[JEE(Main)-2019]**

(1) $\frac{26}{49}$ (2) $\frac{32}{49}$ (3) $\frac{27}{49}$ (4) $\frac{21}{49}$

Sol:-



So, required probability $= \left(\frac{5}{7}\cdot\frac{4}{7}\right) + \left(\frac{2}{7}\cdot\frac{6}{7}\right)$

$= \boxed{\frac{32}{49}}$ Ans.

17. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1,2,3,...,9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is : **[JEE(Main)-2019]**

(1) $\frac{13}{36}$ (2) $\frac{19}{36}$ (3) $\frac{19}{72}$ (4) $\frac{15}{72}$

Sol:- Results when a pair of dice thrown

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

$\Rightarrow$ sum = 7 {6 case}

$\Rightarrow$ sum = 8 {5 case}

Total = 11 case

START

$P(H) = 1/2$ → H ⟶ sum 7 or 8 $\Rightarrow P = \frac{11}{36}$

$P(T) = \frac{1}{2}$ → T ⟶ Number 7 or 8 $\Rightarrow P = \frac{2}{9}$

$$\therefore P(E) = \left(\frac{1}{2}\cdot\frac{11}{36}\right) + \left(\frac{1}{2}\cdot\frac{2}{9}\right)$$

$$= \boxed{\frac{19}{72}}$$

Ans.

18. If the probability of hitting a target by a shooter, in any shot, is 1/3, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is : **[JEE(Main)-2019]**

(1) 6 (2) 5 (3) 4 (4) 3

Sol:- $\because p = \frac{1}{3}$ (Probability of success)

& $q = \frac{2}{3}$ (Probability of failure)

$\because P\left(\begin{matrix}\text{hitting the Target} \\ \text{atleast once}\end{matrix}\right) > \frac{5}{6}$

$\Rightarrow 1 - P\left(\begin{matrix}\text{do not hit} \\ \text{any Target}\end{matrix}\right) > \frac{5}{6}$

$\Rightarrow 1 - {}^{n}C_0 \cdot p^0 q^n > \frac{5}{6}$

$\Rightarrow 1 - 1 \cdot \left(\frac{1}{3}\right)^0 \cdot \left(\frac{2}{3}\right)^n > \frac{5}{6}$

$\Rightarrow \left(\frac{2}{3}\right)^n < \frac{1}{6} \Longrightarrow 0.1666$

$\therefore \boxed{n_{min.} = 5}$ Ans.

19. Let S = {1, 2, ... , 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is :- [JEE(Main)-2019]

(1) $\frac{6}{2^{20}}$ (2) $\frac{5}{2^{20}}$ (3) $\frac{4}{2^{20}}$ (4) $\frac{7}{2^{20}}$

Sol :-

$\because S = \{\underbrace{1, 2, 3, \ldots\ldots, 20}_{\text{sum} = 210}\}$

$\because$ required sum = 203

Cases of subset B

* Include all the Numbers from 1 to 20 except Number 7, so sum will be 203.
* Include all the Numbers from 1 to 20 except Number 1, 6, so sum will be 203
* Similarly Except Numbers 2 & 5
* Except Numbers 3 & 4
* Except Numbers 1, 2 & 4.

$\therefore$ required Probability $= \frac{\text{required subsets}}{\text{Total subsets}}$

$= \boxed{\frac{5}{2^{20}}}$ Ans.

20 In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to : **[JEE(Main)-2019]**

(1) $\frac{150}{6^5}$ (2) $\frac{175}{6^5}$ (3) $\frac{200}{6^5}$ (4) $\frac{225}{6^5}$

Sol:-

*

6	6	6	6	6

$n(S) = 6^5$ (sample space)

required Cases

*

			4	4

$\underbrace{\quad\quad\quad}_{1,2,3,5,6}$ $\Rightarrow 5.5.5 = 125$

*

4			4	4

$\underbrace{\quad\quad}_{1,2,3,5,6}$ $\Rightarrow 5.5 = 25$

*

	4		4	4

$\underbrace{\quad\quad}_{1,2,3,5,6}$ $\Rightarrow 5.5 = 25$

Total $= \boxed{175}$

$\therefore$ required Probability $= \frac{n(E)}{n(S)} = \boxed{\frac{175}{6^5}}$ Ans.

27. In a game, a man wins Rs. 100 if he gets 5 of 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is : [JEE(Main)-2019]

(1) $\frac{400}{3}$ gain (2) $\frac{400}{3}$ loss (3) 0 (4) $\frac{400}{9}$ loss

Sol:-

* If W denotes probability that outcome 5 or 6

$$\left(W = \frac{2}{6} = \frac{1}{3}\right)$$

* If L denotes probability that outcome 1, 2, 3, 4

$$\left(L = \frac{4}{6} = \frac{2}{3}\right)$$

$\therefore$ Expected Loss/Gain

$$= W \times 100 + LW(-50+100) + L^2W(-50-50+100) + L^3(-150)$$

$$= \frac{1}{3} \times 100 + \frac{2}{3} \cdot \frac{1}{3}(50) + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)(0) + \left(\frac{2}{3}\right)^3(-150)$$

$$= \boxed{0}$$ Ans.

22. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is : **[JEE(Main)-2019]**

(1) $\frac{1}{11}$ (2) $\frac{1}{17}$ (3) $\frac{1}{10}$ (4) $\frac{1}{12}$

Sol :- $P(B) = P(G) = \frac{1}{2}$

$\because$ Required Probability $= \dfrac{\text{All 4 Girls}}{(\text{All 4-G}) + (\text{3-G \& 1-B}) + (\text{2-G \& 2-B})}$

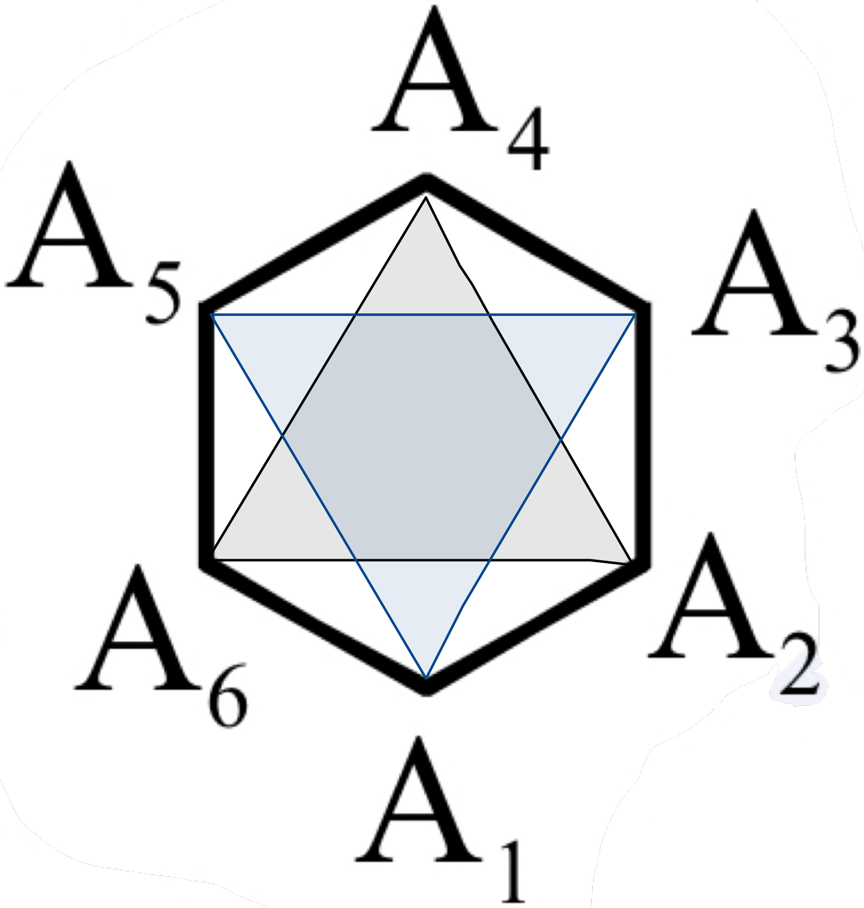
$$= \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^4 + {}^4C_3 \cdot \left(\frac{1}{2}\right)^4 + {}^4C_2 \cdot \left(\frac{1}{2}\right)^4}$$

$$= \boxed{\frac{1}{11}}$$ Ans.

23. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is : [JEE(Main)-2019]

(1) $\frac{3}{10}$ (2) $\frac{1}{10}$ (3) $\frac{3}{20}$ (4) $\frac{1}{5}$

Sol:-



Only Two equilateral Triangle are possible $\Rightarrow \Delta A_1A_3A_5$ & $\Delta A_2A_4A_6$

$\therefore$ Required Probability $= \frac{n(E)}{n(S)}$

$$= \frac{2}{{}^6C_3} = \frac{2}{20} = \boxed{\frac{1}{10}}$$

Ans.

Exercise JA

1 (a) Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is -

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Sol. $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$; r_1, r_2, r_3 are to be selected form {1, 2, 3, 4, 5, 6}

As we know that $1 + \omega + \omega^2 = 0$

$\therefore$ form r_1, r_2, r_3, one has remainder 1, other has remainder 2 and third has remainder 0 when divided by 3.

$\therefore$ we have to select r_1, r_2, r_3 form (1, 4) or (2, 5) or (3, 6) which can be done in ${}^2C_1 \times {}^2C_1 \times {}^2C_1$ ways value of r_1, r_2, r_3 can be interchanged in 3! ways.

$\therefore$ required probability = $\dfrac{({}^2C_1 \times {}^2C_1 \times {}^2C_1) \times 3!}{6 \times 6 \times 6} = \dfrac{2}{9}$

(b) A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is -

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

[JEE 2010, 3+5]

2(b) Event G = Original signal is green

E_1 = A receives the signal correct

E_2 = B receives the signal correct

E = signal received by B is green

P(signal received by B is green)

$$= P(GE_1E_2) + P(G\bar{E}_1\bar{E}_2) + P(\bar{G}E_1\bar{E}_2) + P(\bar{G}\bar{E}_1E_2)$$

$$= \left(\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}\right) + \left(\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}\right) + \left(\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}\right) + \left(\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}\right)$$

$$P(E) = \frac{46}{80}$$

$$P(G/E) = \frac{\left(\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}\right) + \left(\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}\right)}{(46/80)} = \boxed{\frac{20}{23}}$$

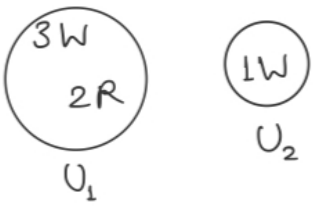
Paragraph for Question 2 and 3

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

2 The probability of the drawn ball from U_2 being white is -

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

Sol:-



Required probability

$$= P(H)\,P(W/H)\times P(W_2) + P(R/H)\,P(W_2) + P(T)\left[P\left(\frac{\text{both } W}{T}\right)P(W_2) + P\left(\frac{\text{both } R}{T}\right)P(W_2) + P\left(\frac{R_1 \,\&\, W_1}{T}\right)P(W_2)\right]$$

$$= \frac{1}{2}\left[\frac{3}{5}\times 1 + \frac{2}{5}\times\frac{1}{2}\right] + \frac{1}{2}\left[\frac{{}^3C_2}{{}^5C_2}\times 1 + \frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3} + \frac{{}^3C_1\times{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right]$$

$$= \frac{1}{2}\left[\frac{3}{5}+\frac{1}{5}\right] + \frac{1}{2}\left[\frac{3}{10}+\frac{1}{30}+\frac{2}{5}\right] = \frac{2}{5} + \frac{11}{30} = \boxed{\frac{23}{30}}$$

3 Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is -

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

[JEE 2011, 3+3]

Sol:-

Required probability

$$= \frac{P(H)\left[P\left(W_1/H\right)P(W_2) + P\left(R_1/H\right)P(W_2)\right]}{P(H)\left[P\left(W_1/H\right)P(W_2) + P\left(R_1/H\right)P(W_2)\right] + P(T)\left[P\left(\frac{\text{both } W}{T}\right)P(W_2) + P\left(\frac{\text{both } R}{T}\right)P(W_2) + P\left(\frac{R_1 \,\&\, W_1}{T}\right)P(W_2)\right]}$$

$$= \frac{\frac{1}{2}\left[\frac{3}{5}\times 1 + \frac{2}{5}\times\frac{1}{2}\right]}{23/30} = \boxed{\frac{12}{13}}$$

4 Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then -

(A) $P(E)=\frac{4}{5},\ P(F)=\frac{3}{5}$

(B) $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$

(C) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$

(D) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

Sol:-

$$P(E)[1-P(F)] + [1-P(E)]P(F) = \frac{11}{25}$$

$$P(E) + P(F) - 2P(E)P(F) = \frac{11}{25} \quad \text{—(1)}$$

$$(1-P(E))(1-P(F)) = \frac{2}{25}$$

$$1-P(E)-P(F)+P(E)P(F) = \frac{23}{25} \quad \text{—(2)}$$

from (1) & (2)

$$P(E)P(F) = \frac{12}{25}$$

& $P(E)+P(F) = \frac{7}{5}$

so either

$\boxed{P(E)=\frac{4}{5},\ P(F)=\frac{3}{5}}$ or $\boxed{P(E)=\frac{3}{5},\ P(F)=\frac{4}{5}}$

5 A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and X_1, X_2, X_3 denotes respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true?

[JEE 2012, 4M]

(A) $P\left[X_1^c \mid X\right] = \frac{3}{16}$

(B) P[Exactly two engines of ship are functioning $\mid X] = \frac{7}{8}$

(C) $P[X \mid X_2] = \frac{5}{16}$

(D) $P[X \mid X_1] = \frac{7}{16}$

Sol:-

$$P(X) = P(\bar{X}_1 \cap X_2 \cap X_3) + P(X_1 \cap \bar{X}_2 \cap X_3) + P(X_1 \cap X_2 \cap \bar{X}_3) + P(X_1 \cap X_2 \cap X_3)$$

$$P(X) = \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}$$

$$= \frac{1}{4}$$

$$P\left(\frac{\bar{X}_1}{X}\right) = \frac{P(\bar{X}_1 \cap X)}{P(X)} = \frac{P(\bar{X}_1 \cap X_2 \cap X_3)}{P(X)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$$

$$P\left(\frac{X}{X_2}\right) = \frac{P(X \cap X_2)}{P(X_2)}$$

$$= \frac{P(X_1)P(X_2)P(X_3) + P(X_1)P(X_2)P(\bar{X}_3) + P(\bar{X}_1)P(\bar{X}_2)P(X_3)}{P(X_2)}$$

$$= P(X_1)P(X_3) + P(X_1).P(\bar{X}_3) + P(\bar{X}_1)P(X_3)$$

$$= \frac{1}{2}\cdot\frac{1}{4}\cdot\ \frac{1}{2}\cdot\frac{3}{4} + \frac{1}{2}\cdot\frac{1}{4} = \frac{5}{8}$$

$$P\left(\frac{X}{X_1}\right) = P(X_2)P(X_3) = P(\bar{X}_2)P(X_3) + P(X_2)P(\bar{X}_3)$$

$$= \frac{1}{4}\cdot\frac{1}{4} + \frac{3}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{3}{4}$$

$$= \boxed{\frac{7}{16}}$$

$$P\left(\frac{\text{Exactly two engine are working}}{X}\right)$$

$$= \frac{P\left(\frac{\text{Exactly two engine are working}}{X} \cap X\right)}{P(X)}$$

$$= \frac{P(X_1)P(X_2)P(\bar{X}_3) + P(\bar{X}_1)P(X_2)P(X_3) + P(X_1)P(\bar{X}_2)P(X_3)}{P(X)}$$

$$= \frac{\frac{7}{32}}{\frac{1}{4}} = \boxed{\frac{7}{8}}$$

6 Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is -

[JEE 2012, 4M]

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

Sol:-

Required probability

$$= 1 - P(D_4 \text{ has diff.})$$

$$= 1 - \frac{(6.1.1.5 + {}^3C_2 .6.1.5.4 + 6.5.4.3)}{6^4}$$

$$= \boxed{\frac{91}{216}}$$

7 Let X and Y be two events such that $P(X|Y)=\frac{1}{2}, P(Y|X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$. Which of the following is(are) correct ? **[JEE 2012, 4M]**

(A) $P(X\cup Y)=\frac{2}{3}$

(B) X and Y are independent

(C) X and Y are not independent

(D) $P(X^C\cap Y)=\frac{1}{3}$

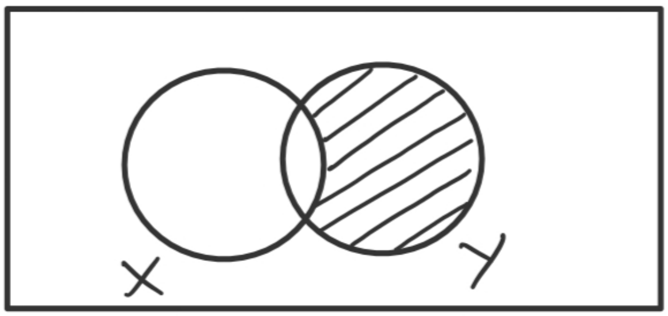
Sol:-

$$P\left(\frac{X}{Y}\right) = \frac{P(X\cap Y)}{P(Y)} = \frac{1}{2} \Rightarrow P(Y) = \frac{1}{3}$$

$$\frac{P(Y\cap X)}{P(X)} = \frac{1}{3} \Rightarrow P(X) = \frac{1}{2}$$

$$P(X\cup Y) = \frac{1}{2}+\frac{1}{3}-\frac{1}{6} = \frac{2}{3}$$

$P(X\cap Y) = P(X).P(Y)$ True



$(X^c\cap Y)$

$$P(X^c\cap Y) = P(Y) - P(X\cap Y)$$

$$= \frac{1}{3}-\frac{1}{6} = \frac{1}{6}$$

8. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is

[JEE(Advanced) 2013, 2M]

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$ (C) $\frac{3}{256}$ (D) $\frac{253}{256}$

Sol:-

$P(A) = \frac{1}{2}$, $P(B) = \frac{3}{4}$

$P(C) = \frac{1}{4}$ & $P(D) = \frac{1}{8}$

Required prob. $= 1 - P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{D})$

$= 1 - \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}$

$= \boxed{\frac{235}{256}}$

9. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)\, p = \alpha\beta$ and $(\beta - 3\gamma)\, p = 2\beta\gamma$. All the given probabilities are assumed of lie in the interval (0,1).

Then $\dfrac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$ **[JEE-Advanced 2013, 4, (–1)]**

Sol:-

Let probabilities of E_1, E_2 & E_3 be p_1, p_2 & p_3 respectively.

Given: $p_1(1-p_2)(1-p_3) = \alpha$,

$p_2(1-p_1)(1-p_3) = \beta$

and, $(1-p_1)(1-p_2)p_3 = \gamma$

Also, $(1-p_1)(1-p_2)(1-p_3) = p$

so $\dfrac{\alpha}{p} = \dfrac{p_1}{1-p_1}$, $\dfrac{\beta}{p} = \dfrac{p_2}{1-p_2}$, $\dfrac{\gamma}{p} = \dfrac{p_3}{1-p_3}$

also, given that

$(\alpha - 2\beta)p = \alpha\beta$

$\Rightarrow \dfrac{p}{\beta} - \dfrac{2p}{\alpha} = 1$ ——(1)

Also, $(\beta - 3\gamma)p = 2\beta\gamma$

$$\frac{p}{\gamma} - \frac{3p}{\beta} = 2 \qquad \text{— (2)}$$

from (1) & (2)

$$\frac{p}{\gamma} - 3 - \frac{6p}{\alpha} = 2$$

$$\frac{p}{\gamma} - \frac{6p}{\alpha} = 5$$

$$\frac{1-p_3}{p_3} - \frac{6(1-p_1)}{p_1} = 5$$

$$\Rightarrow \frac{p_1}{p_3} = \boxed{6}$$

Paragraph for Question 10 and 11

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

10 If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$ (C) $\frac{65}{181}$ (D) $\frac{55}{181}$

[JEE(Advanced) 2013, 3, (–1)]

Sol:-

Required prob.

$$P\left(\frac{B_2}{WR}\right) = \frac{\frac{1}{3}\times\frac{{}^2C_1 \times {}^3C_1}{{}^9C_2}}{\frac{1}{3}\times\frac{{}^1C_1\times{}^3C_1}{{}^6C_2} + \frac{1}{3}\times\frac{{}^2C_1\times{}^3C_1}{{}^9C_2} + \frac{1}{3}\times\frac{{}^3C_1\times{}^4C_1}{{}^{12}C_2}}$$

$$= \frac{1/6}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}} = \boxed{\frac{55}{181}}$$

11. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$ (C) $\frac{558}{648}$ (D) $\frac{566}{648}$

[JEE(Advanced) 2013, 3, (–1)]

Sol:-

$P(W)+P(R)+P(B)$

$= \frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}+\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}+\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}$

$= \boxed{\frac{82}{648}}$

12 Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is -

[JEE(Advanced)-2014, 3(–1)]

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Sol:-

3 boys & two girls

(1) B (2) B (3) B (4)

Girl can't occupy 4^{th} pos^n. Either girls can occupy 2 of 1, 2, 3 pos^n or they both can be at pos^n (1) or (2).

Hence, total number of ways in which girls can be seated is

$${}^3C_2 \times 2! \times 3! + {}^2C_1 \times 2! \times 3! = 36 + 24 = 60$$

Number of ways in which 3B & 2A can be seated = 5!

Hence required prob. $= \frac{60}{5!} = \boxed{\frac{1}{2}}$

Paragraph For Questions 13 and 14

Box 1 contains three cards bearing numbers, 1,2,3 ; box 2 contains five cards bearing numbers 1,2,3,4,5; and box 3 contains seven cards bearing numbers 1,2,3,4,5,6,7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, i = 1,2,3.

13 The probability that $x_1 + x_2 + x_3$ is odd, is -

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$ (C) $\frac{57}{105}$ (D) $\frac{1}{2}$

[JEE(Advanced)-2014, 3(–1)]

Sol:-

$x_1 + x_2 + x_3$ is odd if

all three are odd or 2 are even & one is odd

(OOO) + (OEE) or (EOE) or (EEO)

$$\frac{2}{3} \times \frac{3}{5} \times \frac{4}{7} + \frac{2}{3} \times \frac{2}{5} \times \frac{3}{7} + \frac{1}{3} \times \frac{3}{5} \times \frac{3}{7} + \frac{1}{3} \times \frac{2}{5} \times \frac{4}{7}$$

$$= \boxed{\frac{53}{105}}$$

14 The probability that x_1, x_2, x_3 are in an arithmetic progression, is -

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$ (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

[JEE(Advanced)-2014, 3(–1

Sol:-

$2x_2 = x_1 + x_3$

If x_1 & x_3 both are odd

$2 \times 4 = 8$ Ways

x_1 & x_3 both are even

$1 \times 3 = 3$ ways

Total $= 8+3$

$= 11$ ways

Total $(x_1 x_2 x_3)$ triplets are $3 \times 5 \times 7 = 105$

$P = \boxed{\frac{11}{105}}$

15 The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is **[JEE 2015, 4M, –0M]**

Sol:-

Let coin is tossed n times

$$P(\text{atleast two heads}) = 1-\left(\frac{1}{2}\right)^n - {}^nC_2\left(\frac{1}{2}\right)^n \geq 0.96$$

$$\Rightarrow \frac{4}{100} \geq \frac{n+1}{2^n}$$

$$\Rightarrow \frac{n+1}{2^n} \leq \frac{1}{25} \Rightarrow \frac{2^n}{n+1} \geq 25$$

least value of n is $\boxed{8}$.

Paragraph For Questions 16 and 17

Let n_1 and n_2 be the number of red and black balls respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

16: One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

[JEE 2015, 4M, –0M]

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$

(B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$

(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$

(D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

Sol:-

Box-I < Red → n_1, Black → n_2

Box II < Red → n_3, Black → n_4

$$P(R) = \frac{1}{2}\cdot\frac{n_1}{n_1+n_2} + \frac{1}{2}\cdot\frac{n_3}{n_3+n_4}$$

$$R(II/R) = \frac{\frac{1}{2}\frac{n_3}{n_3+n_4}}{\frac{1}{2}\cdot\frac{n_1}{n_1+n_2}+\frac{1}{2}\frac{n_3}{n_3+n_4}} = \frac{\frac{n_3}{n_3+n_4}}{\frac{n_1}{n_1+n_2}+\frac{n_3}{n_3+n_4}}$$

by option $n_1=3$, $n_2=3$, $n_3=5$, $n_4=15$

$$P(II/R) = \frac{\frac{5}{20}}{\frac{3}{6}+\frac{5}{20}} = \frac{n_4}{\frac{1}{2}+\frac{1}{4}} = \frac{1}{4}\times\frac{4}{2+1}$$

$$= \boxed{\frac{1}{3}}$$

17 A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are) **[JEE 2015, 4M, –0M]**

(A) $n_1 = 4$ and $n_2 = 6$

(B) $n_1 = 2$ and $n_2 = 3$

(C) $n_1 = 10$ and $n_2 = 20$

(D) $n_1 = 3$ and $n_2 = 6$

Sol:-

Given, $\frac{n_1}{n_1+n_2} \cdot \frac{n_1-1}{n_1+n_2-1} + \frac{n_2}{n_1+n_2} \cdot \frac{n_1}{n_1+n_2-1} = \frac{1}{3}$

$3(n_1^2 - n_1 + n_1 n_2) = (n_1+n_2)(n_1+n_2-1)$

$3n_1(n_1+n_2-1) = n_1+n_2(n_1+n_2-1)$

$2n_1 = n_2$

18 A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

P(computer turns out to be defective given that is produced in plant T_1)

= 10P(computer turns out to be defective given that it is produced in plant T_2)

where P(E) denotes the probability of an event E. A computer produces in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is **[JEE(Advanced)-2016, 3(–1)]**

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{83}$

Sol:-

Let x = P(computer turns out to be defective given that it is produced in plant T_2)

$$\frac{7}{100} = \frac{1}{5}\times(10x) + \frac{4}{5}x$$

$$\Rightarrow 7 = 200x + 80x$$

$$x = \frac{7}{280}$$

$$P(\text{produced in } T_2 / \text{not defective}) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{4}{5}(1-x)}{\frac{1}{5}(1-10x) + \frac{4}{5}(1-x)} = \frac{\frac{4}{5}\left(\frac{273}{280}\right)}{\frac{1}{5}\left(\frac{280-70}{280}\right) + \frac{4}{5}\left(\frac{273}{280}\right)}$$

$$= \boxed{\frac{78}{93}}$$

Paragraph For Questions 19 and 20

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2, respectively, after two games

19 P(X > Y) is- **[JEE(Advanced)-2016, 3(0)]**

(A) $\frac{1}{4}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{7}{12}$

Sol:-

$$P(X>Y) = T_1T_1 + DT_1 + T_1D$$

$$= \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{6}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{6} = \boxed{\frac{5}{12}}$$

20 P(X = Y) is- **[JEE(Advanced)-2016, 3(0)]**

(A) $\frac{11}{36}$ (B) $\frac{1}{3}$ (C) $\frac{13}{36}$ (D) $\frac{1}{2}$

Sol:-

$$P(X=Y) = DD + T_1T_2 + T_2T_1$$

$$= \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{2}$$

$$= \frac{1}{36} + \frac{1}{3} = \boxed{\frac{13}{36}}$$

21 Let X and Y be two events such that $P(X)=\frac{1}{3}$, $P(X|Y)=\frac{1}{2}$ and $P(Y|X)=\frac{2}{5}$. Then

[JEE(Advanced)-2017, 4(–2)]

(A) $P(X'|Y)=\frac{1}{2}$

(B) $P(X\cap Y)=\frac{1}{5}$

(C) $P(X\cup Y)=\frac{2}{5}$

(D) $P(Y)=\frac{4}{15}$

Sol :-

$$\frac{P(X\cap Y)}{P(Y)}=\frac{1}{2}$$

$$\frac{P(Y\cap X)}{P(X)}=\frac{2}{5}$$

$$P(X\cap Y)=\frac{P(Y)}{2}=\frac{2}{5}P(x)=\frac{2}{5}\cdot\frac{1}{3}=\frac{2}{15} \Rightarrow P(Y)=\frac{4}{15}$$

$$\frac{P(\bar{X}\cap Y)}{P(Y)}=\frac{P(Y)-P(X\cap Y)}{P(Y)}=\frac{\frac{4}{15}-\frac{2}{15}}{\frac{4}{15}}=\frac{2}{4}=\frac{1}{2}$$

$$P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)=\frac{1}{3}+\frac{4}{15}-\frac{2}{15}=\boxed{\frac{7}{15}}$$

22 Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is **[JEE(Advanced)-2017, 3(–1)]**

(A) $\frac{36}{55}$ (B) $\frac{6}{11}$ (C) $\frac{5}{11}$ (D) $\frac{1}{2}$

Sol:-

$x+y+z=100$

Total number of non-negative solutions

$= {}^{100+3-1}C_{3-1} = {}^{12}C_2 = 66$

Not let $z=2n$

$x+y+2n = 10$; $n \geq 0$

Total number of non-negative solutions $= 11+9+7+5+3+1$

$= 36$

Req. prob $= \frac{36}{66} = \boxed{\frac{6}{11}}$

Paragraph For Questions 23 and 24

There are five students S_1, S_2, S_4 and S_5 in a music class and for them there are five sets R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, $i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

(*There are two questions based on Paragraph "A". the question given below is one of them*)

23 The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1 and **NONE** of the remaining students gets the seat previously allotted to him/her is -

[JEE(Advanced)-2018, 3(–1)]

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$ (C) $\frac{7}{40}$ (D) $\frac{1}{5}$

Sol:-

$$\text{Probability} = \frac{4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)}{5!} = \frac{9}{120} = \boxed{\frac{3}{40}}$$

24 For i = 1, 2, 3, 4, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is-

[JEE(Advanced)-2018, 3(–1)]

(A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

Sol:-

Total cases = 5!

favourable ways = 14

$\left.\begin{matrix} 1\ 3\ 5\ 2\ 4 \\ 1\ 4\ 2\ 5\ 3 \end{matrix}\right\} \rightarrow 2$

$5 \rightarrow 2$

$2\ 4\ 1\ \ldots\ \ldots \rightarrow 2$

$2\ 5\ 3\ 1\ 4 \rightarrow 1$

$4 \rightarrow 3$

$\left.\begin{matrix} 3\ 1\ 5\ 2\ 4 \\ 3\ 1\ 4\ 2\ 5 \end{matrix}\right\} 2$

$3\ 5\ \ldots\ \ldots\ \ldots \} \rightarrow 2$

$= 14$

Probability $= \frac{14}{120} = \boxed{\frac{7}{60}}$

26 There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls, Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}$, $\frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct ?

[JEE(Advanced)-2019, 4(–1)]

(1) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

(2) Probability that the chosen ball is green equals $\frac{39}{80}$

(3) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(4) Probability that the selected bag is B_3, given that the chosen balls is green, equals $\frac{5}{13}$

Sol :-

	Bag 1	Bag 2	Bag 3
Red balls	5	3	5
Green balls	5	5	3
Total	10	8	8

(1) $P(\text{Ball is green}) = P(B_1)P(G/B_1) + P(B_2)P(G/B_2) + P(B_3)P(G/B_3)$

$$= \frac{3}{10}\times\frac{5}{10} + \frac{3}{10}\times\frac{5}{8} + \frac{4}{10}\times\frac{3}{8} = \boxed{\frac{39}{80}}$$

(2) $P(\text{Ball chosen is green / Ball is from } 3^{rd} \text{ bag}) = \frac{3}{8}$

(3,4) $P(\text{Ball is from } 3^{rd} \text{ bag / Ball chosen is green})$

$$= \frac{P(B_3)P(G/B_3)}{P(B_1)P(G/B_1) + P(B_2)P(G/B_2) + P(B_3)P(G/B_3)} \quad \text{— (1)}$$

$$P(B_1) = \frac{3}{10}$$

$$P(B_2) = \frac{3}{10}$$

$$P(B_3) = \frac{4}{10}$$

So, from (1),

$$P = \frac{\frac{4}{10} \times \frac{3}{8}}{\frac{3}{10} \times \frac{5}{10} + \frac{3}{10} \times \frac{5}{8} + \frac{4}{10} \times \frac{3}{8}}$$

$$= \frac{4}{13}$$

26 Let S be the sample space of all 3×3 matrices with entries from the set $\{0, 1\}$. Let the events E_1 and E_2 be given by

$$E_1 = \{A \in S : \det A = 0\} \text{ and}$$
$$E_2 = \{A \in S : \text{sum of entries of } A \text{ is } 7\}.$$

If a matrix is chosen at random from S, then the conditional probability $P(E_1|E_2)$ equals ____

[JEE(Advanced)-2019, 3(0)]

Sol:-

E_2: Sum of elements of $A = 7$

$\Rightarrow$ There are 7 ones & 2 zeros

Number of such matrices $= {}^9C_2 = 36$

Out of all such matrices; E_1 will be those when both zeros lie in the same row or in the same column

eg: $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}$

$n(E_1 \cap E_2) = 2 \times {}^3C_2 \times {}^3C_2 = 18$

So $n(E_1/E_2) = \dfrac{n(E_1 \cap E_2)}{n(E_2)} = \dfrac{18}{36} = \boxed{\dfrac{1}{2}}$

27 Let $|X|$ denote the number of elements in set X. Let $S = \{1,2,3,4,5,6\}$ be a sample space, where each element is equally likely to occur. If A and B are indepenent events associated with S, then the number of ordered pairs (A,B) such that $1 \le |B| < |A|$, equals **[JEE(Advanced)-2019, 3(0)]**

Sol:-

A & B are independent events

$P(A)P(B) = P(A \cap B)$

$\Rightarrow \frac{a}{6} \times \frac{b}{6} = \frac{c}{6} \Rightarrow ab = 6c$

$|A| = a$, $|B| = b$, $|A \cap B| = c$

$(a,b,c) = (3,2,1)$ so ${}^6C_1\,{}^5C_2\,{}^3C_1 = 180$

$= (4,3,2)$ so ${}^6C_2\,{}^4C_2\,{}^2C_1 = 180$

$= (6,1,1)$ so ${}^6C_1 = 6$

$= (6,2,2)$ so ${}^6C_2 = 15$

$= (6,3,3)$ so ${}^6C_3 = 20$

$= (6,4,4)$ so ${}^6C_4 = 15$

$= (6,5,5)$ so ${}^6C_5 = 6$

Total $= 360 + 62 = \boxed{422}$