

(If 11ve keys were there)?

Hash Function -

HT

M=10

0	250		
1			
2		H A S H	
3	100043	E U N	250 100043
4	64	G	699
5		B	758
6	10526		10526
7			
8	758		
9	699		

Hash address

keys

to retrieve 758

again  $\% 10 = 8(758)$

is stored at

8.

it will check for one's place of the given key i.e. ( $\% 10$ ) we get the (0-9) key just store it

as 699, 699% 10

9

∴ Again time complexity = O(1)

Hash function take more time compared to Hash table due to function calculation.

Hash function is better as it also save the space.

in if  $n < 10 \rightarrow \text{slot} = 0 \text{ to } 9$

$n > 10 \rightarrow \text{slot} = 0 \text{ to } 19$

Q. Collision - If two keys mapped to same hash address by hash function then it is called collision.

i.e. Ex- 699 759

both get collided at 9 pos^n

12345

YII CIS DEPARTMENT

Hash function is better if no. of collision is as min. as pos.

Types of Hash function-

1. Division Modulo Method-

2. Digit Extraction Method

3. Mid-Square Method

4. Folding Method -

(i) Fold Boundary Method.

(ii) Fold Shifting Method.

1. Division Modulo Method.

ex1- size of table m = 1000 (0 to 999)

key = 1 2 3 4 5 6 7 8 9

Acc to Division Modulo Method

key Hash-function  $f(key) = \text{key Modulo } m$

$$= 123456789 \% 1000$$

$$= 789$$

0	
..	..
789	123456789
..	..

ex2- size of table m = 8 ( $2^3$ )

(take LSB 3 bit

key = 1010010101100101 here)

$H_f(key) = 1010010101100101 \bmod 2^3$

$$= 101$$

if  $m = 2^k$

then  $H_f(key)$

this method do not bother about the whole

data to how to store just take last  $k^{th}$  bits-

( $k$  bits)  
of LSB

Ex 1010101111111111 mod 2<sup>4</sup> = 1111

Ex 3  $m = 2^k$

key = 101010010101111011101 mod  $2^k$  = LSB k bits

if  $k = 8$

$H_f \notin \text{key} = 11011101$

Hash function of

- if M is in power of 2, key will be LSB only that lead to more collision.
- So do not choose M as a power of 2.

Ex 101010010101111011101  $k=7$

1010101111111011101 LSB

$H_f(\text{key}_1) = 1011101$

$H_f(\text{key}_2) = 1011101$

1. So, do not pick M as exact power of 2 because if  $M = 2^k$  then Hash function of key = LSB k bits always.  
but if M is not power of 2 then less chances of collision

2. Pick the M value which is a prime no. but it cannot be close to power of 2.

Ques - Which M value can be chosen for better result:

(a) 61 (c) 729

(b) 523 (d) 1024

729 is chosen as it is not power of 2 & not close to powers

Steps - (1) Assume all are prime no

(2) choose one which is not close to power of 2.

encoding - decoding  
store - retrieve

## 2. Digit Extraction Method - M = 1000

key = 789123456

hash function(key) =  $\sum_{i=0}^{n-1} key_i \cdot 10^i$

	715	789123456

extract some digits

$$= 715 \quad \text{extract digit - } (1, 4, 8)$$

the digits to be extracted are defined in question. (you cannot extract 14 digits as size of stack is from 0-999)

: Here no consideration is done to storage time.

- When you are extracting 3 digit, may lead to one digit or two digit address (if one of them become zero)

- Collision may also occur here as if

7 7 7 1 2 3 4 5 9  
1 2 3 4 5 6 7 8 9

but it is not producing that much less collision i.e no of collision is more here.

- Digit Extraction Method produces more collision as compared to Division Modulo Method (as for Division modulo, you require  $M = 2^k$ ) but here only some bits have to be changed.

- We extract 3 digit address as 3 digit addresses >> 2 digit add

### 3. Mid-Square Method-

$$M = 1000$$

$$\text{key} = 84925$$

Step 1-

1. Square the given key

2. check middle of obtained key (take 3 bit from mid as it will have majority of 3-bit addresses)

Making counter counter here will be bit difficult, so ill lead to less counter & less collision. Because If key a larger no produces a more larging no.

$$\text{Hash function} = (84925)^2$$

$$= 721225625$$

225    255... as no exact mid possible

225	72184925

store at any one address

To Retrieve - (i) Square the key

(ii) take the mid, go to mid & extract the key.

Counter is not easy because of large each & every no (digit) participate in the calculation of square.

Folding Method -  $M = 1000(0-999)$   
 key = 123456789

(i) Fold Boundary Method

- fold the boundary  
 (take k bit from the key from boundary)
- $k \geq$  no of digit which have more address

for 3 bits - more address

123 456 789  
 123  
 789  
 913

If it lead to

1912	191	folding boundary again
913	193	9 again
key stored	912	123456789

123 45  
 123  
 45  
 168

Counter example = 123 89 789  
 913  
 ↓ again

counter is bit tougher than collision  
 bit extraction as 6 digit participate

(ii) Fold Shifting Method

- Fold it according to fold boundary but fold every bit

123 456 789

123

456

789

13 5 8

136

8

144 - 12345678

144

∴ fold shift is better than fold boundary as it require participation of 9 bit i.e all the bits.

• Counter is also possible:

Ex 789 456 123

↓ since addition satisfies commutative prop

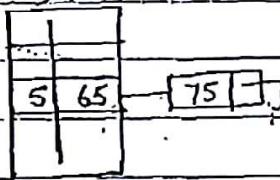
## Collision Resolution Technique

1. chaining

2. open addressing

chaining

- outside (add linked list)



(If more than m keys are to be kept then chaining is used)

Open Addressing

- when only m keys are to be kept
- inside (go to other slots)

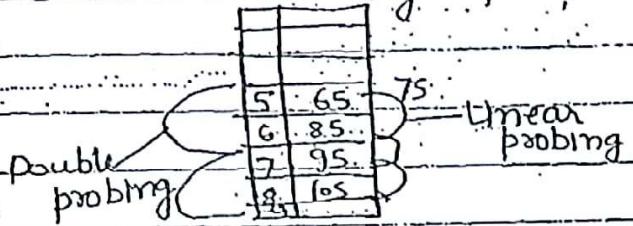
which are empty

• Linear probing if 75: check

for 5 fill 6 → fill and

continuous checking is done

when no empty space



- Quadratic probing, where random jumps are made known as quadratic probing
- Double hashing, when two values are directly jumped

Chaining Process -  $m=10$  (0-9)

key = (55, 98, 63, 75, 99, 73, 60, 95, 44, 29, 35, 73, 25, 108)

$$hf(key) = key \bmod m$$

Collision Resolution Technique = chaining

0	60			
1				
2				
3	63	73		Slot = 10
4	44			key = 14
5	55	75	35	25
6				
7				7
8	98	108		(Not present)
9	99	29	39	

0	6000	60 Null	1 length
1	Null	0	
2	Null	0 5000	500 1
3	5000	63 5001	73 5002 173 Null 3
4	Null	0 4000	
5	4000	44 Null	1
6	1000	55 1001	75 1002 95 1003 25 Null
7	Null	1000	1001 1002 1003
8	2000	2000 98 2001	2001 108 Null 2
9	3000	3000 99 3001	3001 29 3002 39 Null

Length of Longest chain = 4

Min. chain length = 1

Here searching time increases.

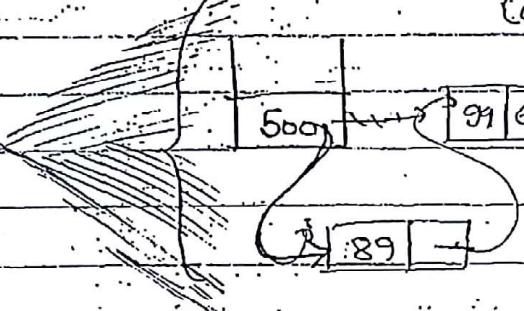
∴ in Worst case,  $TG = O(n)$  as same if all element occur in 1 slot.

but if you be using better hashing function then it can be reduced as  $O(1)$ .

It is also

## drawbacks

1. In chaining, space is wasted in the form of linked list even the space available inside.
2. The length of the longest chain possible is  $n$ . that lead to Worst case searching time =  $O(n)$ .  
Best Case =  $O(1)$   
Avg. Case =  $O(n/2) \approx O(n)$
3. the greatest advantage with the chaining is infinite no. of collision can be handled by the infinite no. of keys can be stored because of linked list.
4. Insertion of one key will take  $O(1)$  time (BC, WC, AC)  
(as insertion is done in linked list only)



... n elements

insertion will take  $O(1)$  time.

5. Deletion time will take - Best case =  $O(1)$   
Worst case =  $O(n)$  (as it may be at last so you have to visit each node)

If address of node which has to be deleted is given then

time =  $O(1)$  (BC: ~~W.C.~~)

$O(n)$  - (WC)

b [since you cannot go back so you have traversed by start]

## Solve More (K)

is to be deleted & its address is provided with do  
+ then time to delete = O(1)  
= O(1)

exists in existence as in 1 slot you can store 100  
cell.

Addressing - &

probing - M = 10 (0-9)

Hashing) Keys = {55, 99, 60, 75, 89, 42, 58, 69, 25}

hash function (key) = k mod m

C.R.T = Linear probing

simg (key, i) = (h<sub>f</sub>(key) + i) Mod m       $\rightarrow$  attempt  
 $\forall i = \{0, 1, 2, \dots, m-1\}$       no

0	60	$LP(55, 0) = (h_f(55) + 0) \text{ mod } 10$ here sign $= 5 \text{ mod } 10$	0	here sign 1st attempt
1	89			
2	42			
3	69	$LP(99, 0) = 9$	— (0)	N <sub>a</sub>
4		$LP(60, 0) = 0$	— (0)	inc
5	55	$LP(75, 0) = 5$ (fail collision)	— (0)	cc
6	75	$LP(75, 1) = 6$ (pass)	→ (2)	
7	25	$LP(89, 0) = 9$ (fail)		
8	58	$LP(89, 1) = 0$ (fail)	→ 2	
9	99	$LP(89, 2) = 8$ (pass)		
		$LP(42, 0) = 2$	— (0)	
		$LP(58, 0) = 8$	— (0)	
		$LP(69, 0) = 9$ (fail)		
		$LP(69, 1) = 0$ (fail)	$LP(69, 4) = 3$ (cc)	
		$LP(69, 2) = 1$ (fail)		
		$LP(69, 3) = 2$ (fail)	(4)	

Total collision = 9

5

You can store only  $m$  elements in  $M$ -slots.

To Retrieve same for 75 → goto 5 if there Yes  
else  $+1$

Quadratic

Double Hashing - two hash functions -  $h_1$  and  $h_2$

Quadratic probing (probing)

Quadratic Probing :  $M = 10$

Key { 25, 39, 46, 55, 89, 23, 68, 70, 94 } (56)

$H_f(\text{key}) = \text{key mod } m$        $c_1 = c_2 = 1$  (78)

CRT = QP

$QP(\text{key}, i) = (H_f(\text{key}) + c_1 \cdot i + c_2 \cdot i^2) \bmod m$

quadratic  
pattern

0	70	$\forall i = \{0, \dots, M-1\}$
1	89	$QP(25, 0) = 5$ (0) collision
2	23	$QP(39, 0) = 9$ (0)      Total collision = 2
3	23	$QP(46, 0) = 6$ (0)
4	94	$QP(55, 0) = 5$ fail
5	25	$QP(55, 1) = 7$ (pass) (0)
6	46	$QP(89, 0) = 9$ fail (i)
7	55	$QP(89, 1) = 1$ Pass
8	68	$QP(23, 0) = 3$ (0) $QP(56, 0) = 6$ (0)
9	39	$QP(68, 0) = 8$ (0) $QP(56, 1) = 6+2=8$
		$QP(70, 0) = 0$ (0) $QP(56, 2) = 6+5=11$
		$QP(94, 0) = 4$ (0) $QP(56, 3) = 6+1=7$

$$\begin{array}{ll}
 QP(78, 0) = 8 \text{ (fail)} & QP(78, 3) = \text{fail} \\
 QP(78, 1) = 0 \text{ (fail)} & QP(78, 4) = 8 \text{ fail} \\
 QP(78, 2) = 14 \text{ (fail)} & QP(78, 5) = 8 \text{ fail} \\
 \hline
 QP(78, 6) = 0 \text{ (fail)} & QP(78, 7) = 4 \text{ (fail)} \\
 QP(78, 8) = \cancel{2} \cancel{10} \text{ fail} & \cancel{2} \cancel{10} QP(78, 9) = \text{fail}
 \end{array}
 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{to collide}$$

$$\therefore \text{Total collision} = 12$$

In Worst case, m-1 to 0 is covered  $\therefore O(m)$  you have to check.

$$\boxed{\text{Worst case} = O(m)}$$

Quadratic probing is faster than linear probing. But it will generate inaccurate result (as gap may be there but you are not able to keep the element).

If Linear Probing is used, if an empty slot is present, element will surely get stored.

In Linear Probing, Worst case searching time = O(m)

Best case = O(1)

Average case = O(m)

## Double Hashing

$$M = 10$$

keys = (25, 39, 46, 55, 89, 23, 68, 70, 94, 78)

$$h_{j_1}(\text{key}) = \text{key mod } m$$

$$h_{j_2}(\text{key}) = 1 + (\text{key mod } (m-1))$$

CRT = Double hashing

$$DH(\text{key}, i) = (h_{j_1}(\text{key}) + i \cdot h_{j_2}(\text{key}))$$

$$\forall i = (0 \text{ to } M-1)$$

0	70	$DH(25, 0) = 5$
1	89	$DH(39, 0) = 9 + 0 \cdot 8 = 9$
2	78	$DH(46, 0) = 6 + 0 \cdot 8 = 6$
3	55	$DH(55, 0) = 5 + 0 = 5 \text{ fail}$
4	94	(4) $DH(55, 1) = 5 + 1 \cdot h_{j_2}(55) = 5 + (1 + 55 \bmod 8) = 5 + 8 = 13 \bmod 10 = 3$
5	25	
6	46	$D(89, 0) = 9 \text{ (fail)}$
7	23	(4) $D(89, 1) = 9 + (1 + 89 \bmod 8) = 11 \bmod 10 = 1$
8	68	
9	39	$D(23, 0) = 0 \text{ (fail)}$
		$D(23, 1) = 3 + (1 + 23 \bmod 8) = 11 \bmod 10 = 1 \text{ (full)}$

$$D(23, 3) = 7$$

$$D(23, 2) = 3 + 2(8) = 19 = 9$$

$$D(68, 0) = 8 \text{ (pass)} \quad (0)$$

$$D(70, 0) = 0 \text{ (pass)} \quad (0)$$

$$D(94, 0) = 4 \text{ (pass)} \quad (0)$$

$$D(78, 0) = 8 \text{ (fail)}$$

$$(2) D(78, 1) = 8 + 1(1 + 6) = 15 \bmod 10 =$$

$$D(78, 2) = 8 + 2(7) = 22 \bmod 10 =$$

$$= 2 \text{ (pass)}$$

Total collision = 1

Problem 1 Apply all 3 open addressing strategies-

$$m=10$$

key = 25, 15, 90, 75, 69, 59, 44, 58, 85

- if you want to find any element. And if gap occur then element is not present in table.

- 1. We are not wasting space outside in the form of linked list
  - 2. Searching time =  $O(1)$  for Best case  
 $O(m)$  for Worst case
  - 3. Insertion time =  $O(1)$  for Best case  
 $O(m)$  for Worst case
  - 4. Deletion time =  $O(1)$  for Best case  
 $O(m)$  for Worst case
  - 5. Whenever delete a particular element, place \$ otherwise at a time of searching gap occurs & it may lead to miss searching.

~~if your table~~

- ⑤ If you delete one element, it will create trouble to other elements but we can manage with help of \$ symbol if more dollar symbol than rehash again.

### Quadratic Probing -

1. No space is wasted outside in form of linked list.

2. Search time  $\Rightarrow O(1) = BC$   
 $O(m) = WC$

3. Insertion time  $\Rightarrow O(1) = BC$   
 $O(m) = WC$

3) deletion time  $\Rightarrow O(1) = BC$   
 $O(m) = WC$

4. deletion of one element will trouble to other element but can manage with \$, if more deletion, then rehash again.

### Double Hashing -

0	90	$DH(25, 0) = 5$
1	69	$DH(15, 0) = 5$ fail
2	15	$DH(15, 1) = 5 + 8 = 13 = 3$
3	15	$DH(90, 0) = 90$
4	44	$DH(75, 0) = 5$ fail
5	25	$DH(75, 1) = 5 + 11$
6		
7	59	
8	55	
9	75	



y using Quadratic Probing

$$O \Delta = 1.$$

$$\Delta =$$

0	60	$QP(80, 0) = 0$ (fail)
1	91	$QP(80, 1) = 0 + 1 + 1$ (fail)
2	82	$QP(80, 2) = 0 + 2 + 2^2 = 6$ (true)
3	53	
4		
5		$QP(90, 0) = 0$ (fail)
6	86	$QP(90, 1) = 0 + 1 + 1$ (fail)
7		$QP(90, 2) = 0 + 2 + 2^2 = 6$ (fail)
8	90	$QP(90, 3) = 0 + 3 + 3^2 = 12 = 2$ (fail)
9		$QP(90, 4) = 0 + 20 = 0$ (fail)
		and so $QP(90, 5) = 0$ (fail)
		on $QP(90, 6) = 2$ (fail)
		$QP(90, 7) = 8$ (true)

Here primary clustering occurs as what path is followed by

1 element will definitely followed by 2 element

but here Avg. case deg decrease due to large jump

Here clustering is known as secondary clustering

Secondary Clustering: if two key contain same starting hash address, they both will follow same path unnecessarily in quadratic manner, because of this reason avg. searching time increases (it is less than linear search probing time), this problem is known as secondary clustering

Quadratic probing is suffering with secondary clustering

Avg case =  $O(m)$  but actually it is less than  $(m+1)$

(Asymptotically same but mathematically less)

## Double Hashing:

		80	
0	60		$DH(80, 0) = 0$
1	91		$DH(80, 1) = 1$
2	82		$DH(80, 2) = 2$
3	53		$DH(80, 3) = 3$
4	80		$DH(80, 4) = 4$
5			
6	90		$DH(90, 0) = 0$
7			$DH(90, 1) = 3$
8	80		$DH(90, 2) = 0 + 6$
9	80		

Here path is diff with initially address (same).

Here diff path is carried out with same initial address. No path will be same.

$$DH(100, 0) = 0$$

$$DH(100, 0) = 0 + 5 = 5 \text{ (True)}$$

$$\text{Avg} = \frac{3 + 5 + 7 + \dots + 2}{m} \quad DH$$

as a particular slot is covered by single element only

(It might be possible that there must be some cases which lead to redundancy)

there in avg case, we have to consider for all the given case so on an avg it is said that it is repeating one time).

In Double Hashing, Even though two key contain same starting hash address, they both will follow diff path because second level hash fn will give diff value.

because of this reason 99% of redundancy is eliminated,  
 slot covered by I element

$$\text{Avg case} = 3 + 5 + 7 + \dots + 2$$

↓

slot covered

by II element

∴ on an avg

$$= \frac{m}{m} \Rightarrow cm = O(1)$$

$$\text{Avg. case} = O(1)$$



(as only few key will be repeating only)

let 4 key repeated

$$\text{total} = \frac{4m}{m}$$

$$= O(4)$$

∴ In double hashing, secondary clustering problem is present littlebit (1/.)

Note - Using Perfect Hashing, you will get Worst case Searching time =  $O(1)$ .

in hashing if collision occur to a, b, c at slot 5, they will go to 5. which have another  $h_5$  which is best  $f^n$  and a hash table to store those three element then collision is removed perfectly.

0	1	2	3	4	5	6	7	8	9

a	b	c
0	1	2

$$\text{space} = O(m^2)$$

You cannot decide of size so use m size else use linked list

so space increases but time decreases

$$\text{Total table} = m+1$$

$$\text{Total } f^n = M+1$$

$$\therefore \text{Worst case} = O(1)$$

m-slots n-keys

m slots - will store n keys

1-slot will store  $\frac{n}{m}$  keys



Load factor - no of keys getting stored in 1 slot

$$\text{Load factor } (\alpha) = \frac{n}{m} \text{ keys per slot}$$

Note 1 - the expected no of probes (attempt) in an unsuccessful search of an open addressing technique is

$$\left[ \frac{1}{1-\alpha} \right]$$

Note 2 - the expected no of probes in successful search of an open addressing technique is  $= \left[ \frac{1}{\alpha} \log_{\frac{1}{\alpha}} \frac{1}{1-\alpha} \right]$

(Cormen)

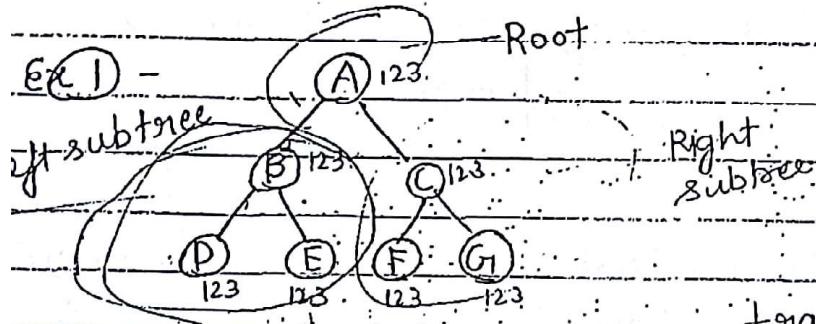
## TREE AND GRAPH TRAVERSALS (2 Marks)

### B Tree Traversals -

1. Preorder (Root LST. RST.)

2. Postorder (LSTRST Root)

3. Inorder (LST Root RST)

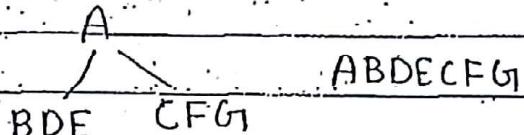


Tree is best solved by recursion as of having almost sym quality.

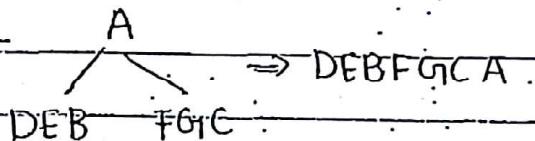
Traversal - visiting each node one by one in given tree.

Preorder Left & Right tree cannot be changed i.e. left will always come before Right subtree  
if changed  $\rightarrow$  3!

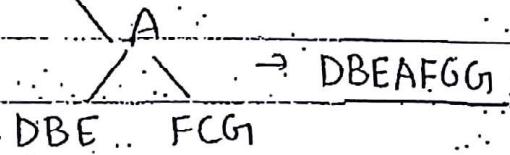
Preorder - ABDEC~~F~~G

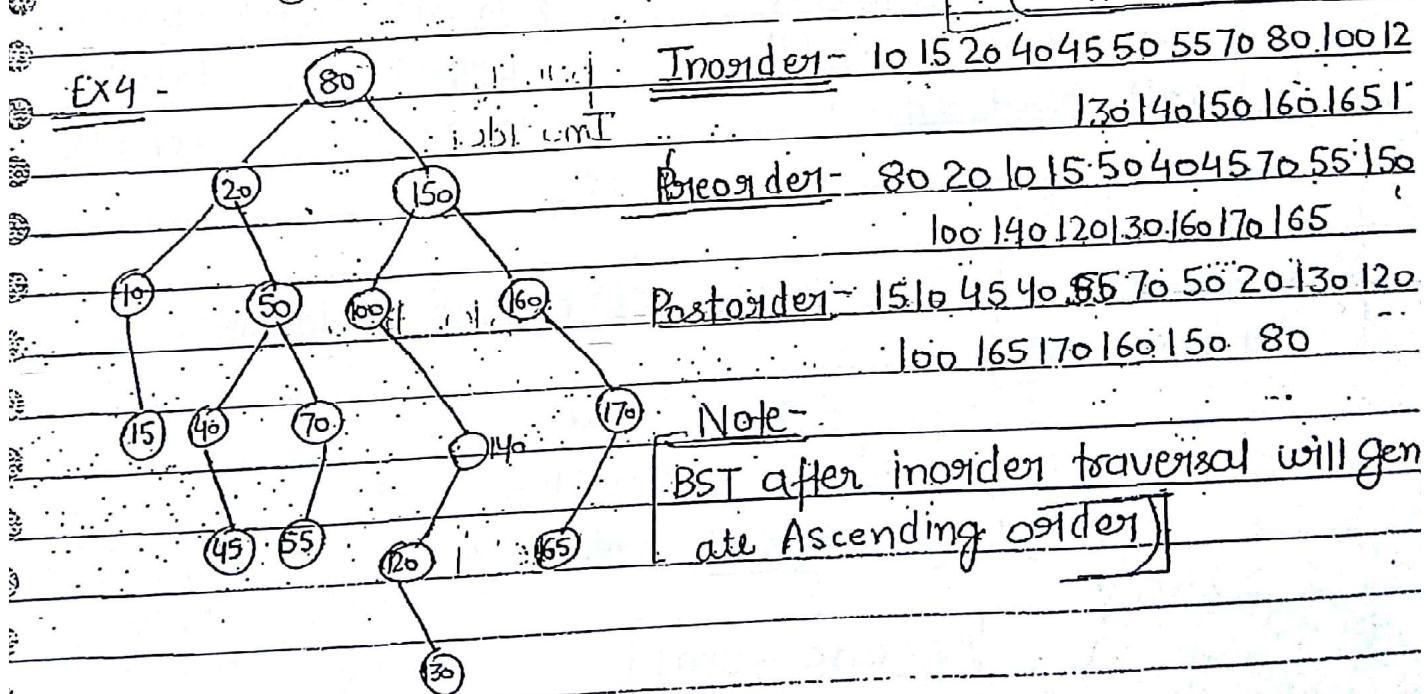
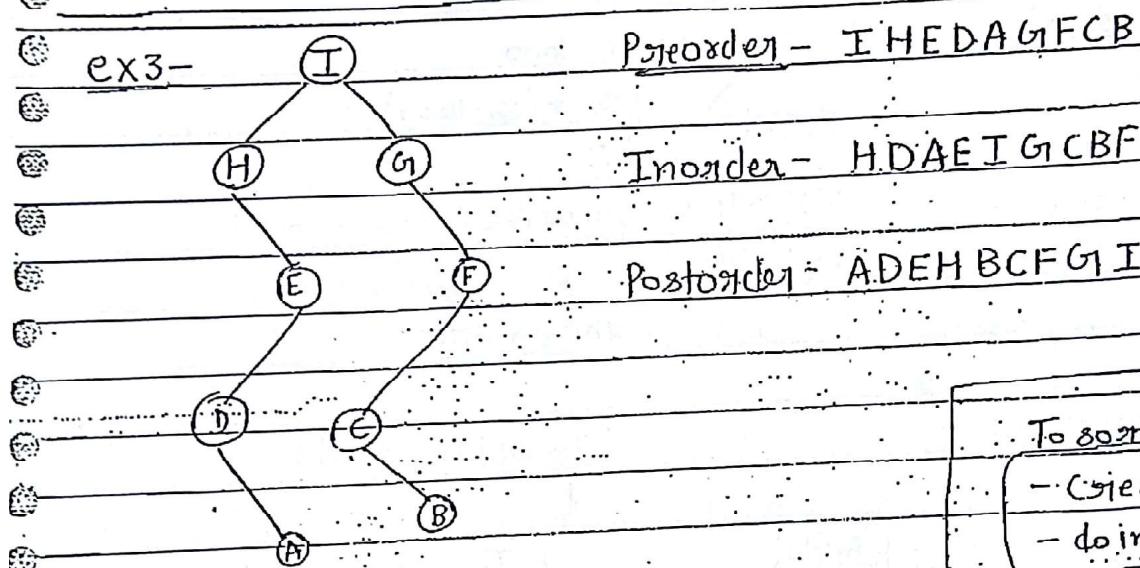
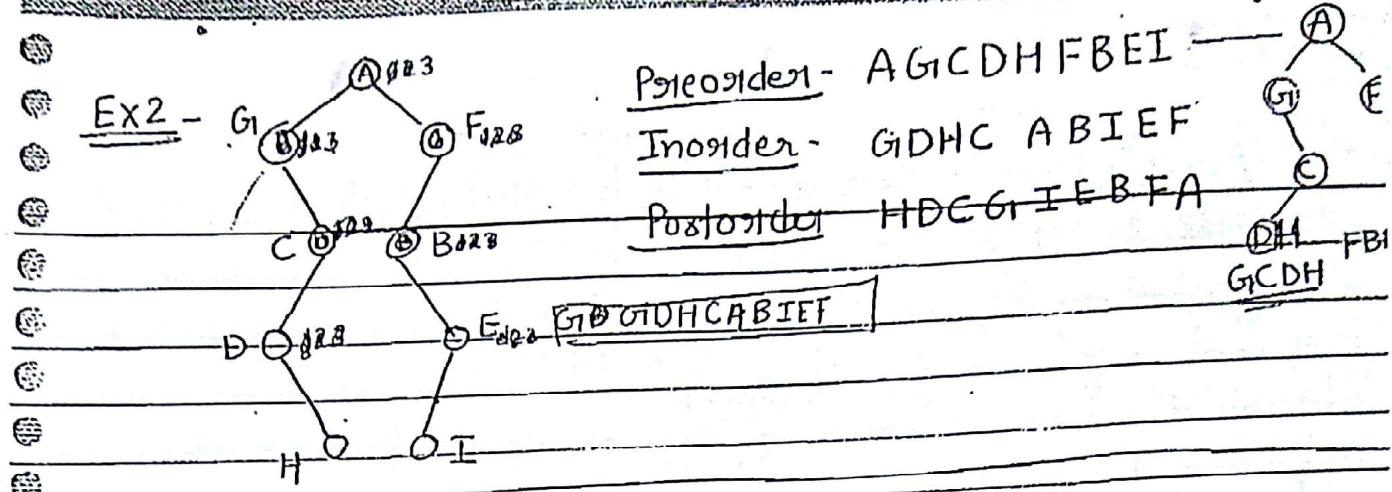


Postorder - DEBF~~G~~CA

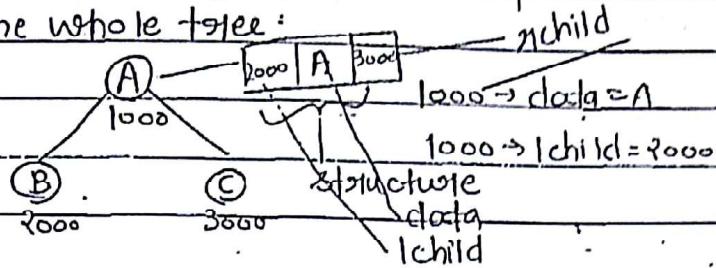


Inorder - DBEAFC~~G~~





Whenever a tree is given, a root address is provided which can be used to traverse the whole tree:



Preorder(root)

{

```
printf("root->data");
preorder(root->1child);
```

$\Rightarrow O(1)$

} A

$\Rightarrow$  preorder( $\Rightarrow 1000 \rightarrow 1\text{child}$ );

preorder( $\Rightarrow 1000 \rightarrow 1\text{child}$ );

$\Rightarrow$  preorder(2000)

and so on.

postorder(root)

$\Rightarrow$  postorder( $\Rightarrow \text{root} \rightarrow 1\text{child}$ );  
 postorder( $\Rightarrow \text{root} \rightarrow 2\text{child}$ );  
 printf("root->data");

Inorder(root)

$\Rightarrow$  Inorder( $\Rightarrow \text{root} \rightarrow 1\text{child}$ );  
 printf("root->data");  
 Inorder( $\Rightarrow \text{root} \rightarrow 2\text{child}$ );

[for preorder  
of a binary tree]

$$T(n) = 2T\left(\frac{n}{2}\right) + O(1) \text{ (by master theorem)}$$

$= O(n)$  (Best case)

Worst partition -

$$T(n) = T(n-1) + T(0) + O(1)$$

$= O(n)$  (Worst case)

[for preorder,  
postorder,  
Inorder]

Avg case =  $O(n)$

WC, BC, AC same as each node has to visit once

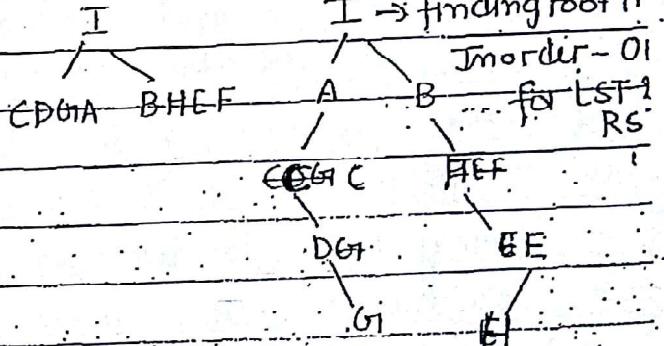
Recurrence relation can also tell about stack space  
 If  $T(n/2) \rightarrow \log n$   
 $T(n-1) = n$

Note -

- Preorder Inorder Postorder will take  $O(n)$  time on  $n$ -node binary tree ( $WC, AC, BC$ ), because each node has to be visited once.
- Stack space.  
 Space complexity =  $O(\log n)$  for balanced tree (Best case)  
 $O(n)$  for unbalanced tree (Worst case)
- Ques - if a BST is given, time to get sorted order -  $O(n)$

- Ques - if a BST is given time to sum element 20 to 50  
 $\rightarrow O(n)$  - Worst case  
 $O(1)$  - Best case (start from 20 when 20 came & add 50)

- Ques - Consider the following binary tree data -  
 $\rightarrow$  finding root  
 Preorder: I A C D G I B F E H  
 Inorder: C D G I A I B H E F  
 Postorder: G I D C A H E F B T



Inorder traversal cannot judge for pre-root because it cannot be balanced always  
 Preorder can give root as well as postorder  
 Inorder is used to give Left subtree & right subtree

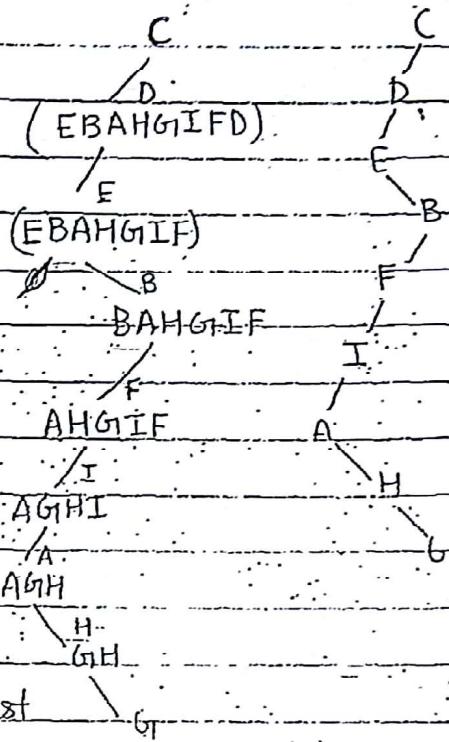
$$\therefore \text{Total complexity} = n \times O(n) \left[ \begin{array}{l} \text{at 1 level} \\ \text{(for LST & RST)} \end{array} \right] \\ = O(n^2)$$

Ques - Consider the following binary Tree data -

Postorder - G H A I F B E D C

Inorder E B A H G I F D C

Preorder - C D E B F I A H G I



X = sometimes possible)

To find out directly any order

find out root

find its pos<sup>n</sup> in inorder case

the digit or no. of element in inorder

try to find them in postorder just

reverse the order and vice versa

a shortcut Not applicable everywhere

Ex - Here postorder & Inorder given, C its position in inorder  
will most so C has empty right ST.

then go to postorder get the element in LST of C in Inorder

take them & reverse them to obtain Post order.

Note - To construct Binary tree for the given Preorder Inorder  
or Postorder Inorder will take  $O(n^2)$  time (n times linear  
search to find LST & RST in given Inorder)

$$(n^2 + n + \text{O}(n))$$

↓

to find LST to find root  
RST n time

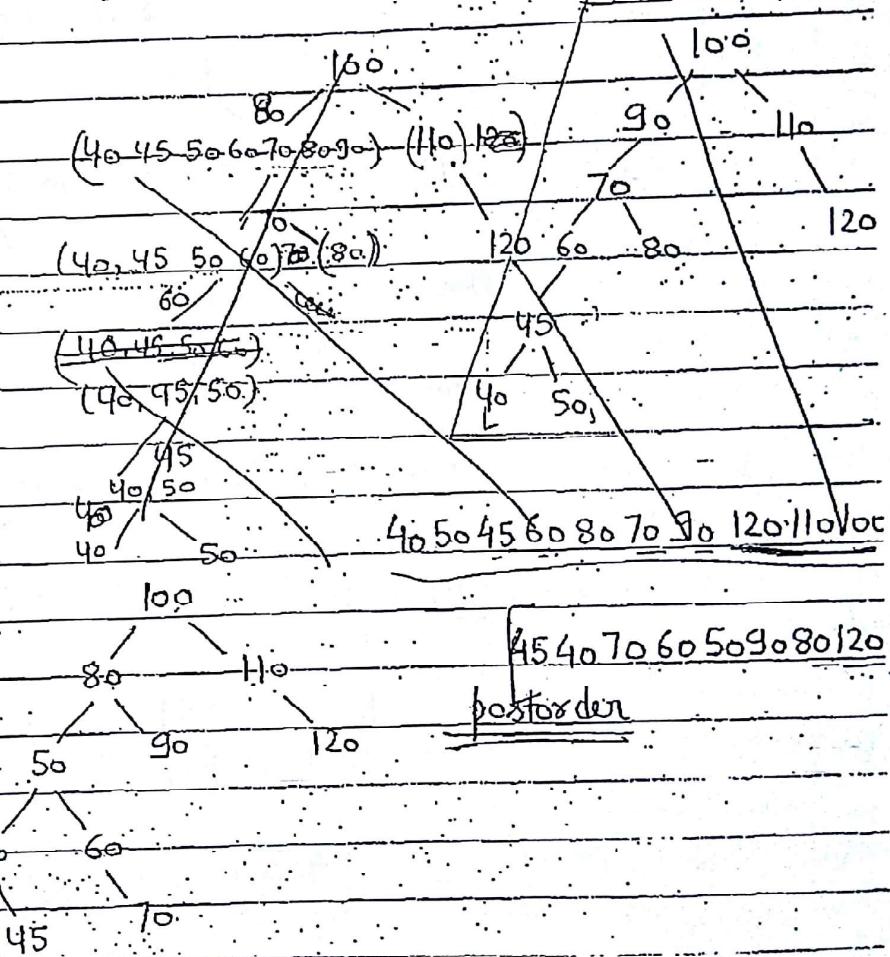
We cannot apply binary search as given sequence is not sorted but if BST it will take  $n \log n$  time.

Ques - Consider the following BST data-

Page 100 (80 50 40 45 60 70 90 110 120)

Post: 90 70 60 45 40 50 80 120 110 100

Im- (40 45 50. 60 70 80 90 100 110 120 (A.O))



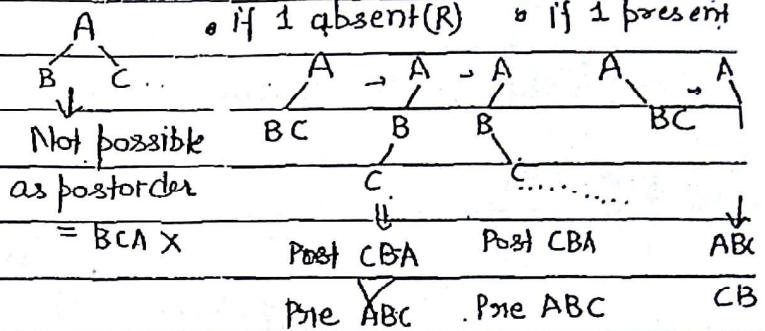
Note → for the given preorder inorder for postorder inorder to construct equivalent BT require  $(n \log n)$  time if inorder is already sorted.

preorder: ABC  
NLR

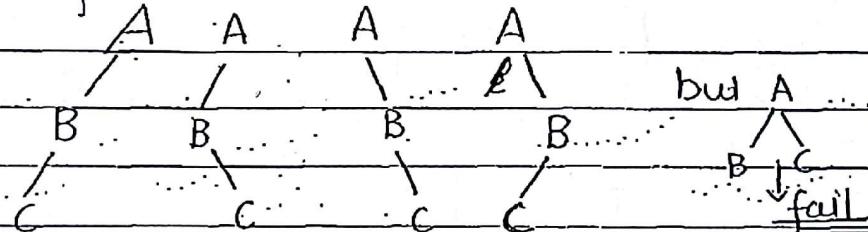
root = A

• Let both RST & LST present

postorder: CBA  
RLN



∴ possible trees are



Note

If Preorder Inorder given - unique binary tree ~~posh~~ Binary tree  
postorder Inorder given - Unique Binary tree Unique BT  
if preorder, postorder given - more than one binary tree  
↳ Binary tree posh but not.

To construct unique binary tree, Inorder is required (compulsory)

When pre, post is given, manual checking is done To each node two possibilities either going to left or right so total possibility for n nodes =  $O(2^n)$

Upper bound as root will have only one possibility.

To find out cycle initially  
 flag = 0 then if visited  
 flag = 1 if again count<sup>th</sup>  
 flag = 1 then it is a cycle.

## Graph Traversal

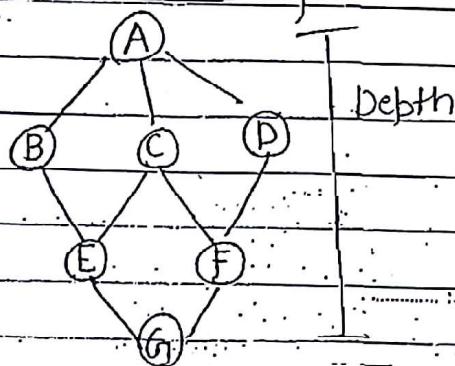
1. BFT (Breadth First Traversal)

2. DFT (Depth First Traversal)

Total Graph Traversal time =  $O(V+E)$

Breadth

By visiting each vertex & each edge



We do not consider edge in tree & edge is less than 2 vertices

$$\text{Tree} = (V+E) \cdot 1$$

$$V + V - 1$$

$$= 2V - 1 \approx O(V)$$

Breadth first traversal - ABCDEFG

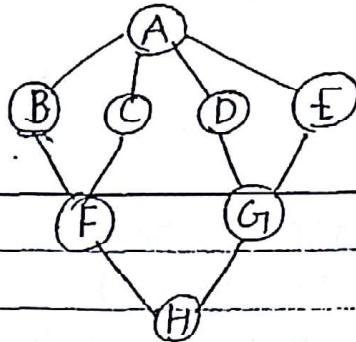
Depth " " ABEGFCFD

Breadth first Traversal - Tree traversal are unique but graph traversal not.

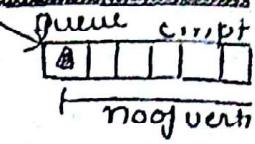
- Visited is required as graph contains cycle and there is no mean of visiting a vertex again & again.

- Queue is used in BFT as we need FIFO structure.

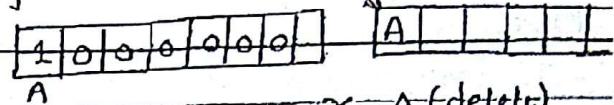
while loop is covering vertex & for loop is covering for edges.



Initially  
Visited = 0 0 0 0 0 0 0 0  
Starting with A



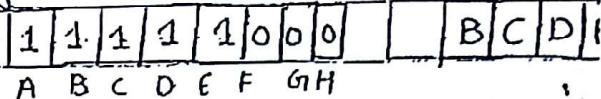
Ex



$x = A$  (delete)

$w = B C D E$

Visited



B deleted

$w = A F$

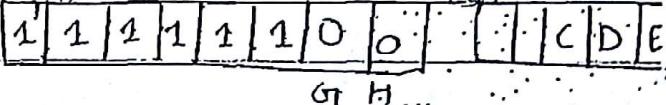
↓ add

Visited[V] = 1

add(V, Q);

while (Q is not empty)

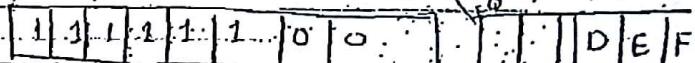
Visited



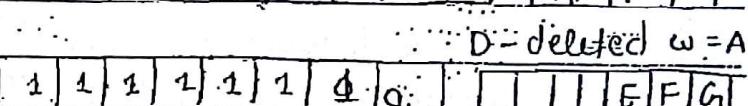
Q not added again

$x = \text{delete}(Q)$

print(Q)

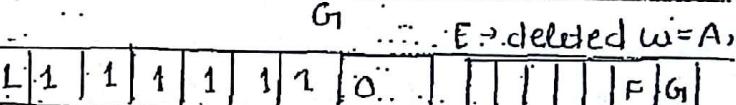


for all w adjacent to x

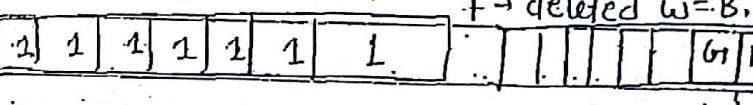


if (visited[w] = 0)

visited[w] = 1



add(w, Q)



G

E → deleted  $w = A$

F → deleted  $w = B$

G → deleted  $w = C$

b.

O/P → A B C D E F G H

(Order of deletion of element)

empty

loop end

Note -

- To implement BFT, we use queue data structure.

- Total complexity =  $O(V+E)$  (Time) (BC, WC, AC)

[If adj matrix is used  $TC = O(V^2)$ ]

Space  $\rightarrow$  I/P + extra

$\downarrow$        $\downarrow$       if graph is adjacency list  
 $(V+E)$      $(V+E)$   
queue      visited array

$$= (V+E+2V)$$

$$= 3V+E$$

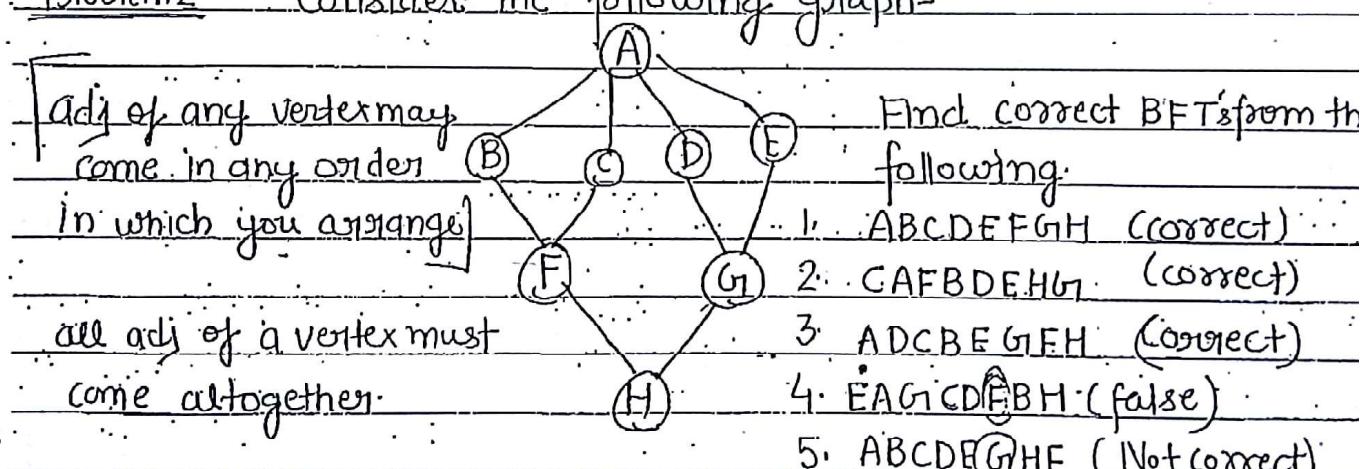
$$= O(V+E)$$

$= O(V^2)$       if adj. matrix is used ]

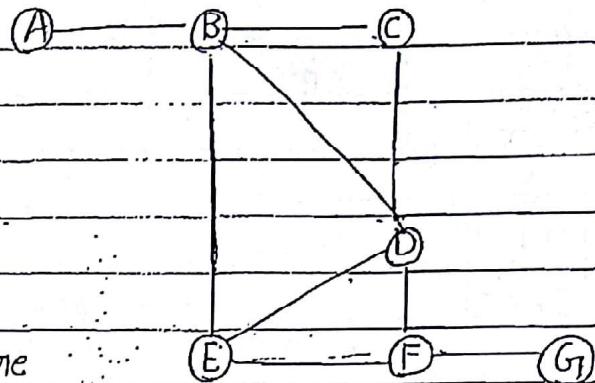
- BFT is also known as Level order traversal i.e level by level printing of nodes. (means adjacency lists)

A  $\rightarrow$  next level (adj of A)

Problem 2 - Consider the following graph-



Ques - Consider the following graph.



Whether these are

correct BFTs or not

(a) EBD<sup>D</sup>CAG (Correct)

(b) GFCEDBA (incorrect)

(c) CBDAEFG (Correct)

(d) DBCEFA<sup>G</sup> (Correct)

### Application of BFT -

Ques - if a graph is given, how to check whether it is connected or not?

• Draw & Apply BFT

• Check visited array

• if any 0 in visited array

↓ to check for 0's

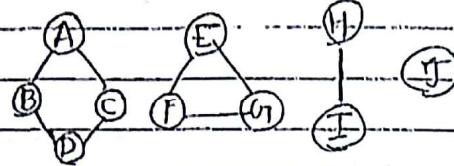
• then graph disconnected

$O(V+E) + V$

if a linked list is given, to check whether it is connected?

a linked list is also a graph but directed so apply the same procedure ~

We can verify given graph is connected or not.



1	1	1	1	0	0	0	0	0
A	B	C	D	E	F	G	H	I

starting from A

no of disconnected component  
Count = 1 (initially zero)

When you get 1 zero, just stop & again apply the D.F.T again and so on, you can find disconnected components can be find out

again

1	1	1	1	1	1	1	1	1
A	B	C	D	E	F	G	H	I

Count = 2



again

1	1	1	1	1	1	1	1	1
A	B	C	D	E	F	G	H	I

Count = 3



Count = 4

No of connected components = No of time BFT applied

T.C to find connected comp = O(V+E)

Using BFT we can find out connected components in the given graph.

If a null graph  $G(V, E)$  - no of connected component =  $|V|$

3. to check for cycle in a given graph

- Apply BFT

→ If  $\text{visited}[i] = 1$  & it encounters again then there is a cycle.

Time complexity =  $O(V+E)$  (as BFT work)

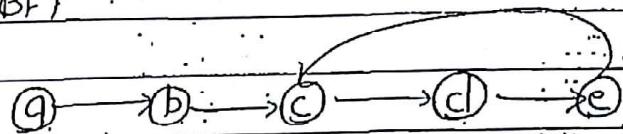
4. Whenever Kruskal is going on edges are added to MST we have to check for occurrence of cycle too as no cycle must occur, then by side BFT also working to check for  $\text{visited}[i]$  → it takes constant time (for 1 edge)

$$T(n) = E \log E + (V+E)$$

↓ for BFT cycle check

3. Using BFT, we can check given graph contain cycle or not

if a linked list contain cycle, it can be checked easily by applying BFT

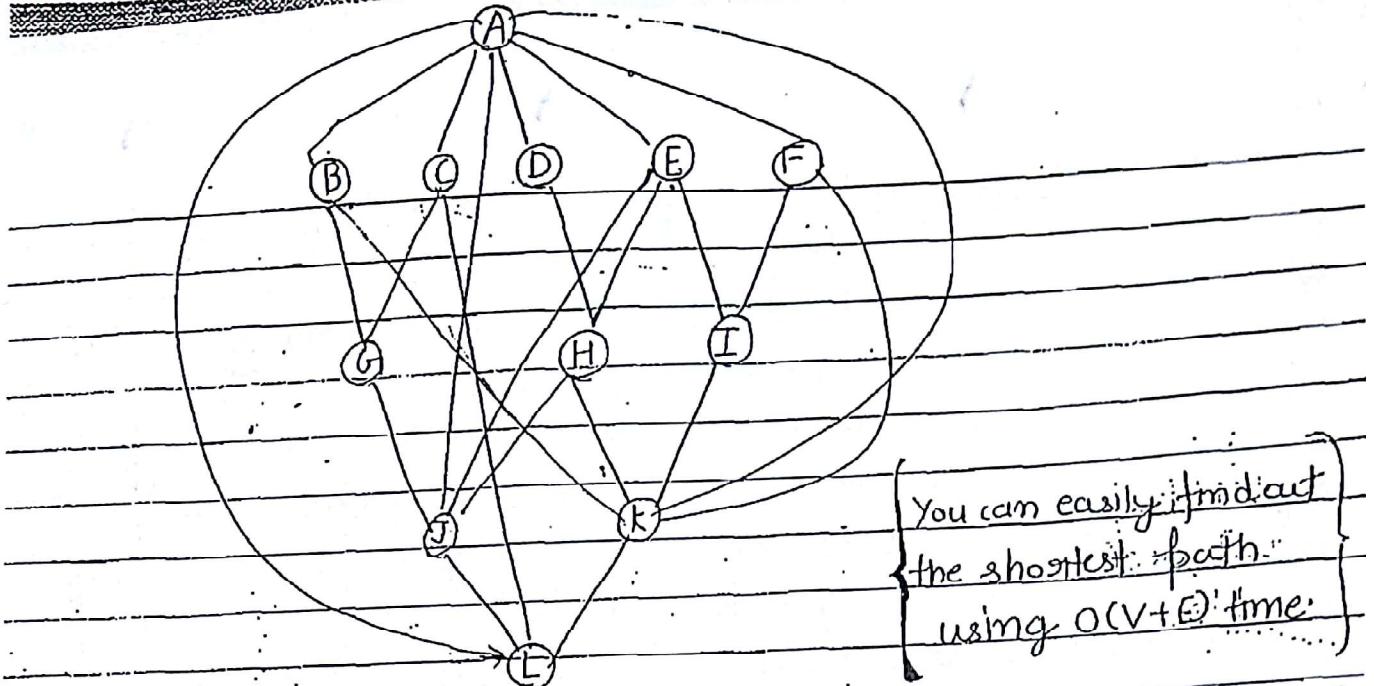


A B C D E

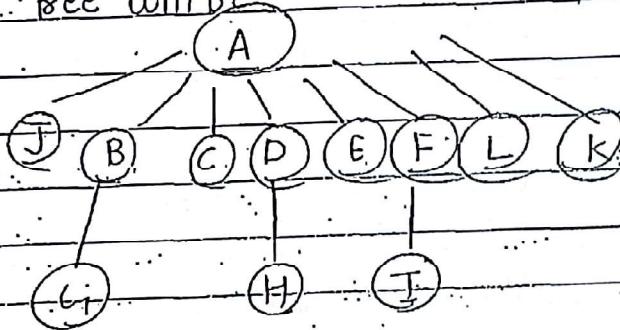
↑ diagram occurs ∴ cycle exists

4. If an unweighted graph is given & single source shortest path is not to be found, then BFT is the efficient one algorithm as it will find the shortest path.

∴ Using BFT, we can find out shortest path from given source to every vertex in given unweighted graph or the graph having equal edge weight (true)



the BFT tree will be



in weighted graph  
default weight

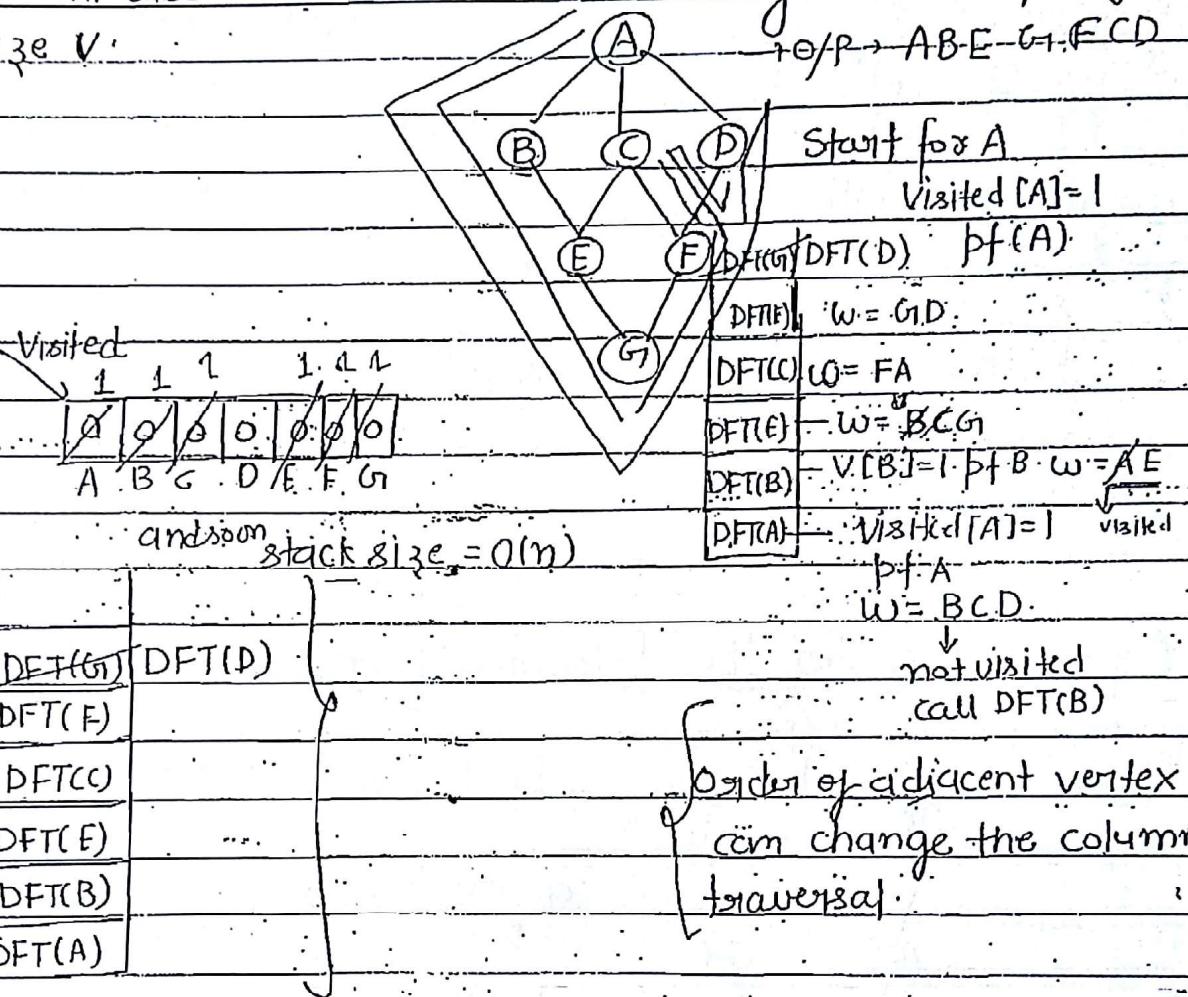
Ques- if a unweighted graph is given; the shortest path to be calculated from given source. which data structure is used?

Queue.

5. Using BFT we can verify a given graph is Bipartite graph?

## Depth First Traversal

- It uses stack as recursion occur here.
  - it will also maintain a visited array, to check for cycle.



1. To implement DFD we are using stacks

2. Time Complexity = no of functional calls + work done at each func call

$$= V + E \text{ (as if work is summed up)}$$

$$= O(V+E)$$

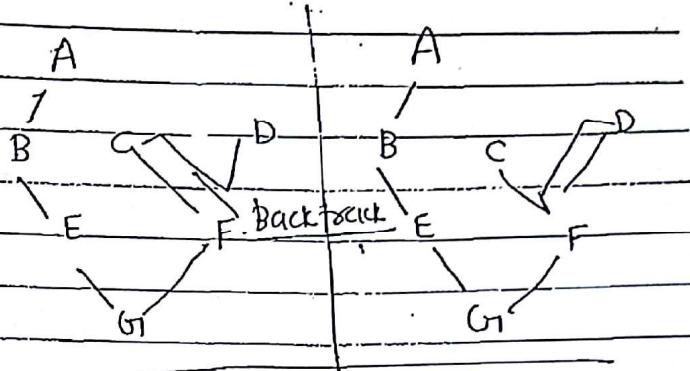
3. Space Complexity = i/p + extra

$$= V+2E \quad \downarrow \quad \text{stack visit} = O(V+E)$$

$V+2E$      $V$      $V$        $\downarrow$  for adjacency list

DFT(V)

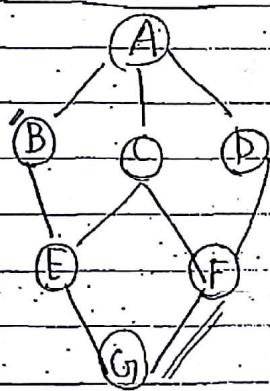
1. Visited[V] = 1
2. DFT(V);
3. for all w adj to V
  1. if (w is not visited)
    2. DFT(w);



In case of Adjacency Matrix =  $O(V^2)$

(space as well as time complexity)

Problem:- Consider the following graph-



Find Correct DFT's

(a) ABEGFCD

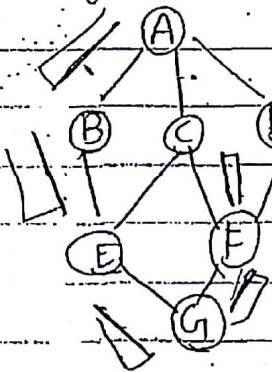
You can take  
any of the  
adjacent of  
a particular  
element.

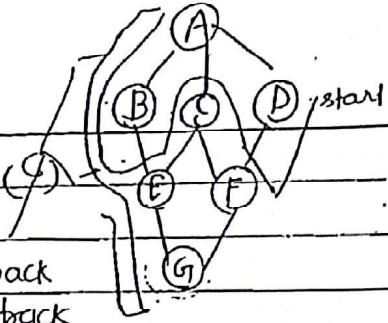
No of backtrack

(b) LABECFDG

backtrack  
occurs  
here

Total Backtrack = 6





DFCEBAG (correct)

• ? back  
• back

(d)

CFDABEG (No Backtrack)

(correct)

(Stack Size = n)

[max stack size = O(n)]

↓ When no back track occurs

⑥ (e) GIEGFABD: (Incorrect) correct - GIECEDAB

(f) ACFGEBD (correct)

(g) BACFDEG → (fail) Incorrect) correct - BACFDGIE

because at D when possibilities ended we back

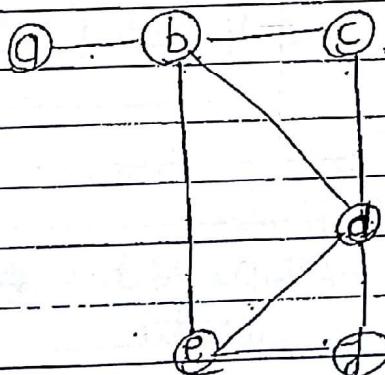
track to F & F has G to be done but it is  
first completing G.

B) [Backtracking is done when all possibilities ended]

Backtracking is nothing but popping

In DFT, if I print D after B, what is the relation b/w them  
No relation because of backtracking any thing  
possible in DFT but BFT have adjacency.

Ques - Consider the following graph -



check whether they are DFT  
or ~~not~~ or not.

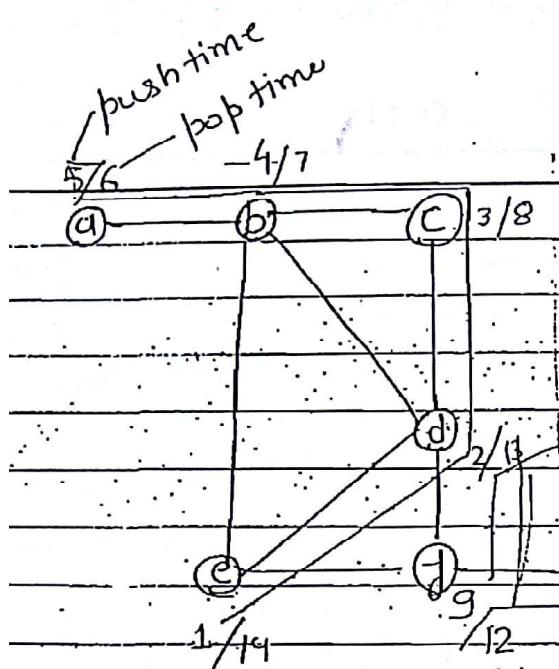
1) edcbafg (DFT)

2) fdebagc (incorrect)

3) abdfgce (incorrect)

4) dcbeafga (correct)

5) befgdca (DFT)



e d c b a f g

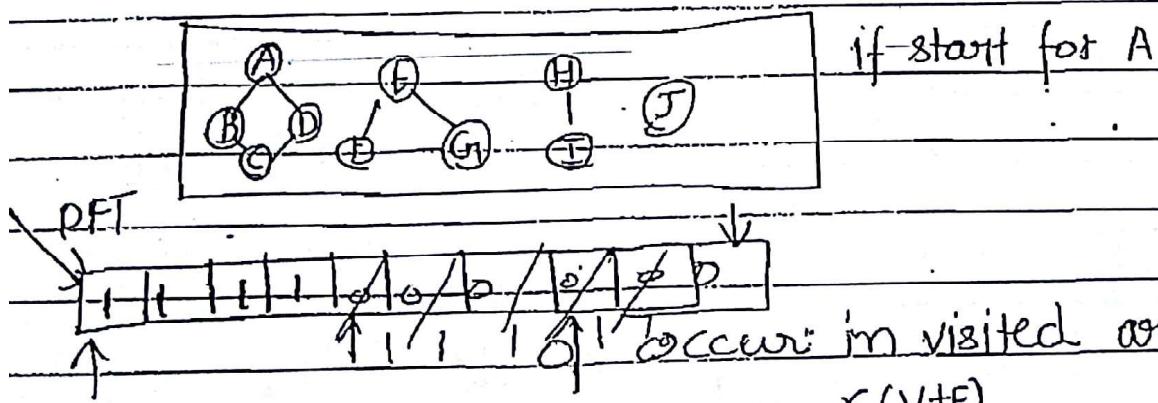
how many elements where  
the diff b/w push & pop  
time is 1 (immediate  
back track)

Total element = 2 (q, g)

1/14 2/13 3/12 4/9 5/8 6/7 10/11  
d c b e f g a  
1/14 2/11 3/10 4/5 6/9 7/8 12/13  
b e f g d c a

## Applications of DFT-

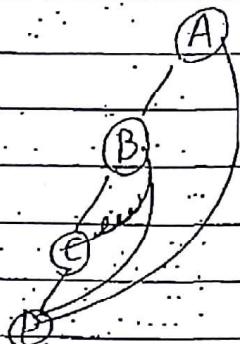
will check where graph is connected or not



3. We can verify given graph contain cycle or not  
 $O(V+E)$

3. We can find out no of connected components  
count = no of time DFT applied.

4



Here shortest path = (A+B) (1)

but if we take

A

B

C

D

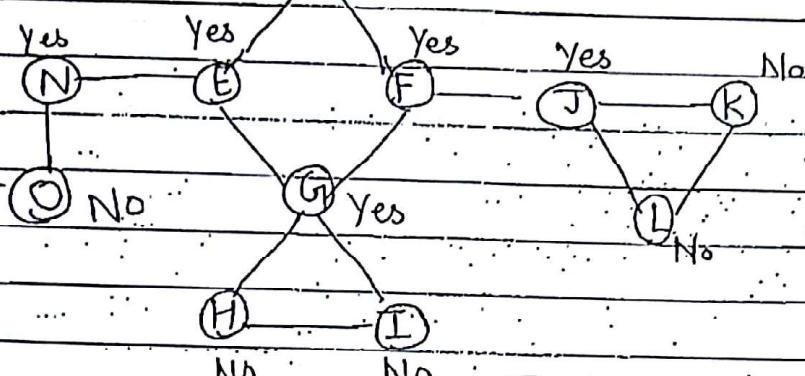
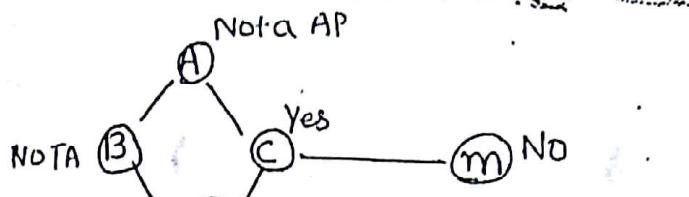
= 3 length path

So, DFT cannot (may or may not) work finding single source shortest path for given unweighted graph.

5. Using DFT, we can check given directed graph is strongly connected or not?

6. Using DFT we can verify a vertex is articulation point or not.

Articulation point - if the deletion of an edge ~~leads~~ vertex along with its edge lead to disconnection of graph known as articulation point.



To known whether a graph have any articulation point

1) Apply DFT on given graph

2) Delete that vertex & edge.

3) Again apply DFT

4) If Visited contain 1 for all except that delete vertex

then that is an articulation point

$$\text{Total Complexity} = O(V+E) + O(V+E)$$

$$= O(V+E)$$

## Bipartite Graph using BFT

first time visited in one set

second time another set

(level by level storing of elements)

strongly connected using DFT

finding strongly connected component

You can use the concept of strongly connected graph articulation point.

No of articulation point = No of strongly connected compn  
cnt + 1

$$\text{Total Complexity} = E(V+E+V+E)$$

for articulation point

$$= EV + E^2$$

$$= O(E^2)$$

## Programming - I

Array - 1-D Array

Ex - 1

`int a[10] = { 10, 20, 30, 40, ..., 100 }` a will store base address.

↳ declaration (memory creation)

`a 1000 1002 1004 1006 1008 1010 1012 1014 1016 1018`

10	20	30	40	50	60	70	80	90	100	1000	1002	1004	1006	1008	1010	1012	1014	1016	1018
0	1	2	3	4	5	6	7	8	9	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009

continuous

(0-19)

1 to 7 → how many elements =  $7 - 1 + 1 = 7$

Ex 2

-5 -4 -3 -2 -1 0 1 2 3 4       $4 - (-5) + 1 = 10$  elements

$2 - (-2) + 1 = 5$  elements

print `a = 1000` (base address)

5 element occupy

1010 1011

$$\text{Loc}[a[5]] = \text{base address} + \cancel{\text{size of data type}} * (5 - 0) \\ = 1010$$

\* 1010 → to access that particular M/M location

$$\text{Loc}[a[9]] = \text{base address} + \text{size of int} * (9 - 0) \\ = 1000 + 2 \times 9 = 1018$$

(only 9-0 bco  
you dont use  
for 10th  
element)

Ex 2 -

$$a[55 \dots 550] \quad \text{Total element} = 550 - 55 + 1 = 496$$

Base addres = 990

Size of an element = 10 byte (C)

$$\text{Loc}[a[450]] = \text{Base address} + C * [450 - 55]$$

$$= 990 + 10 * 395$$

$$= 3950 = \underline{\underline{4940}}$$

before 450 from  
55

- array must be continuous no matter whether indices are (+)ve or (-)ve.

Ques  $a[-55 \text{ to } 55]$  BA = 1000 C = 5

$$\begin{aligned}\text{Loc}[a[5]] &= \text{BA} + [5 - (-55)] \times 5 \\ &= 1000 + 60 \times 5 = 1000 + 300 \\ &= 1300\end{aligned}$$

ex4  $a[lb \dots ub]$  BA = Base address

lower bound      upper bound      size = c  
total element = ub - lb + 1

$$\boxed{\text{Loc}[a[i]] = \text{BA} + c * [i - lb]}$$

array index when started from 0, no need to do subtract at time to calculate location but when it start with 1, a subtract has to be done. So less time is required to calculate the value when starting from 0.

~~offset calculation~~

$$\begin{aligned}&\text{Loc}[a[1 \dots 10]] \quad \text{Loc}[a[5]] = \text{BA} + (5-1) * c \\ &\text{Loc}[a[0 \dots 10]] \quad \text{Loc}[a[5]] = \text{BA} + (5-0) * c\end{aligned}$$

• Array index is, by default, will start from 0 only because no need to calculate offset value

row  
↓  
(0=3)      (0=4)

column  
→ (5-1)+1 = 5 column

	1	2	3	4	5
int a[4][5]	a <sub>11</sub>	a <sub>12</sub>	a <sub>13</sub>	a <sub>14</sub>	a <sub>15</sub>
	a <sub>21</sub>	a <sub>22</sub>	a <sub>23</sub>	a <sub>24</sub>	a <sub>25</sub>
total row ✓	a <sub>31</sub>	a <sub>32</sub>	a <sub>33</sub>	a <sub>34</sub>	a <sub>35</sub>
(4-1)+1	a <sub>41</sub>	a <sub>42</sub>	a <sub>43</sub>	a <sub>44</sub>	a <sub>45</sub>
= 5 Row	a <sub>51</sub>	a <sub>52</sub>	a <sub>53</sub>	a <sub>54</sub>	a <sub>55</sub>
Total element = 20 element					

In m/m 40 bytes for continuous m/m (no separate row & colur is present)

1000	02	04	06	08	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	1000
a <sub>11</sub>	a <sub>12</sub>	a <sub>13</sub>	a <sub>14</sub>	a <sub>15</sub>	a <sub>21</sub>	a <sub>22</sub>	a <sub>23</sub>	a <sub>24</sub>	a <sub>25</sub>	a <sub>31</sub>	a <sub>32</sub>	a <sub>33</sub>	a <sub>34</sub>	a <sub>35</sub>	a <sub>41</sub>	a <sub>42</sub>	a <sub>43</sub>	a <sub>44</sub>	a <sub>45</sub>	1000
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	1000

Row Major Order - When elements are getting stored row by row

Ex -

a. C follows Row Major Order

$$\text{Loc}[a[4][3]] = 1000 + [4-1] \times 2 \quad 1000 \quad 4 \times 2 + 2 \quad 2(5 * (4-1) + (3-1)) \\ 19 + 2 \quad 21$$

$$= BA + C [ \text{Column} \times (\alpha - lb_1) + (\gamma - lb_2) ]$$

$$= BA + C [(ub_2 - lb_2 + 1)(\alpha - lb_1) + (\gamma - lb_2)]$$

$$\text{Loc } a[2][5] = BA + C [ 5 * [2-1] + (5-1) ]$$

$$= 1000 + 2 [ 5 + 4 ]$$

$$= 1018$$

$$\text{Loc } a[i][j] = BA + C [ (ub_2 - lb_2 + 1)(i - lb_1) + (j - lb_2) ]$$

lb<sub>1</sub> ub<sub>1</sub> lb<sub>2</sub> ub<sub>2</sub>

visit correct row.

visiting column

Ex-2

$$a[25 \dots 750][80 \dots 150] \quad c = 10 \text{ byte}$$

Raw Major Order BA = 1000

$$\text{Loc}(a[550][140]) = BA + c[(150-80+1) * (750-25)(550-25) + (140-80)]$$

$$= 1000 + 10[71(525) + 60]$$

$$= 1000 + 10[37335]$$

$$= 373350 + 1000$$

$$= 374350$$

$$51 \quad 101$$

Ex 3 :  $a[-25 \dots +25][-50 \dots 50]$ , BA = 0, c = 1 Byte : RMO

$$\text{Loc } a[20][30] = BA + c[(50+50+1)(20+25) + (30+50)]$$

$$= 0 + 1(101 * 45 + 80)$$

$$= 4545 + 80$$

$$= 4625$$

Ex 4 -  $a[lb_1 \dots ub_1, lb_2 \dots ub_2]$

$$\downarrow \quad \downarrow \\ ub_1 - lb_1 + 1 \quad ub_2 - lb_2 + 1$$

$19[47][3]$

BA, C

$\cup$  nc (no of column)

Row Major order  $a[i][j] = BA + c[(ub_2 - lb_2 + 1)(j - lb_1) + (i - lb_1)]$

Column Major order  $a[i][j] = BA + c[(ub_1 - lb_1 + 1)(j - lb_2) + (i - lb_1)]$   
(no of rows)

26-Mar

Column Major Order  $a[1..4][1..5]$

a) 1 2 3 4 5

1 |  $a_{11} \ a_{12} \ a_{13} \ a_{14} \ a_{15}$

2 |  $a_{21} \ a_{22} \ a_{23} \ a_{24} \ a_{25}$

3 |  $a_{31} \ a_{32} \ a_{33} \ a_{34} \ a_{35}$

4 |  $a_{41} \ a_{42} \ a_{43} \ a_{44} \ a_{45}$

$$BA = 1000 \quad C = 10$$

$a_{100}$	$a_{02}$	$a_{04}$	$a_{06}$	$a_{08}$	$a_{0D}$	$a_{12}$	$a_{14}$	$a_{16}$	$a_{18}$	$a_{20}$	$a_{22}$	$a_{24}$	$a_{26}$	$a_{28}$	$a_{30}$	$a_{32}$	$a_{34}$	$a_{36}$	$a_{38}$
$a_{11}$	$a_{21}$	$a_{31}$	$a_{41}$	$a_{12}$	$a_{22}$	$a_{32}$	$a_{42}$	$a_{13}$	$a_{23}$	$a_{33}$	$a_{43}$	$a_{14}$	$a_{24}$	$a_{34}$	$a_{44}$	$a_{15}$	$a_{25}$	$a_{35}$	$a_{45}$
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

, Column 1      Column 2      Column 3      Column 4      Column 5

$$\begin{aligned} \text{Loc}[a[4][3]] &= BA + C [(3-1) * (\text{no. of Rows}) + (4-1)] \\ &= 1000 + 10^2 (2 \times 4 + 3) \\ &= 11022 \end{aligned}$$

$$\begin{aligned} \text{Loc}[a[2][5]] &= 1000 + 10^2 [(5-1) * \text{no. of Rows} + (2-1)] \\ &= 1000 + 2 [4 \times 4 + 1] \\ &= 1034 \end{aligned}$$

both are same (Row Major, Column Major)

if no order is mentioned, go for Row Major (by default)

Ex2 -  $a[-75..125][80..150]$  BA=0 C=10 byte

CMO

$$\begin{aligned} \text{Loc}[a[15][130]] &= 0 + 10 [(130-80) * (125+75+1) + (15+19)] \\ &= 10(50 * 201 + 90) \\ &= 10(10050 + 90) = 100640 \\ &= 101400 \end{aligned}$$

$$nr = lb_1 - lb_1 + 1$$

Ex3 -   $a[lb_1 \dots ub_1][lb_2 \dots ub_2]$  BA, C, CMo

$$\downarrow$$

$$nc = ub_2 - lb_2 + 1$$

$$\left\{ \text{Loc}[a[i][j]] = BA + c[(j - lb_2) * nr + (i - lb_1)] \right\}$$

3-D Arrays - collection of 2-D arrays

$a[3 \dots 9][15 \dots 35][8 \dots 50]$

$$\begin{array}{ccccccc} & \downarrow & & \downarrow & & & \\ 9-3+1 & 35-15+1 & & & 50-8+1 & \Rightarrow & 7 \text{ table of size } 21 \times 43 \\ = 7 & & & & \downarrow & = 7 & [2-D \text{ array of size } 21 \times 43] \\ & & & & 43 & & \\ & & & & \downarrow & & \\ & & & & \text{no of rows} & & \text{no of column} \end{array}$$

BA, Base Address = 1000 C = 10, RMo

$$\begin{aligned} \text{Loc}[a[7][30][40]] &= BA + c[(7-3)(nc * nr) + (30-15) * nc + (40-8)] \\ &= 1000 + 10(4 * 21 * 43 + 15 * 43 * 32) \\ &= 43890 \end{aligned}$$

(Mo)

$$\begin{aligned} \text{Loc}[a[7][30][40]] &= BA + 10[(7-3) * nc * nr * (40-8) * nr + (30-15)] \\ &= BA + 10 \end{aligned}$$

$$\begin{aligned} &= 1000 + 10(4 * 21 * 43 + 8 * 21 * 15) \\ &= 1000 + 10(3612 + 168 + 15) \\ &= 1000 + 10(3795) \\ &= 38950 \end{aligned}$$

Loc

Ex  $a[lb_1, ub_2] \quad a[lb_1, ub_1, lb_2, ub_2, lb_3, ub_3]$   
 BA, C

$$n_1 = ub_2 - lb_2 + 1$$

$$n_c = ub_3 - lb_3 + 1$$

$$\text{no of } 2D = lb_1 - lb_1 + 1$$

(R, M, O)

$$\text{loc}[a[i][j][k]] = BA + C [(i-lb_1) * nc * n_1 + (j-lb_2) * nc * (k-lb_3)]$$

CMo

$$\text{loc}[a[i][j][k]] = BA + C [(i-lb_1) * nc * n_1 + (k-lb_3) * n_1 + (j-lb_2)]$$

Lower Triangular Matrix -

	1	2	3	4
1	$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$
2	$a_{21}$	$a_{22}$	$a_{23}$	$a_{24}$
3	$a_{31}$	$a_{32}$	$a_{33}$	$a_{34}$
4	$a_{41}$	$a_{42}$	$a_{43}$	$a_{44}$

$a_{11}$	0	0	0
$a_{21}$	$a_{22}$	0	0
$a_{31}$	$a_{32}$	$a_{33}$	0
$a_{41}$	$a_{42}$	$a_{43}$	$a_{44}$

↓ 16. element

$$G = \text{zero}$$

$a_{ij}$  belong to upper part if

$\{ j > i \}$

$(n_2)$  approx - zero

↓ to save spa

) return 0 at that

why we should store

time

zero

Here store in row Major order -

row - 1 will have 1 element -

row - 2 will have 2 element -

## on Major Order-

$a$	1000	100 <sup>2</sup>	04	06	02	40	12	14	16	.18
	911	921	912	931	932	933	941	942	943	944
	0	1	2	3	4	5	6	7	8	9

↓  
1001    1002    1003    1004    1005    1006    1007    1008    1009    10010

$$\begin{aligned}
 \text{Loc}(a[4][3]) &= BA + C \left[ (4+3)3+2+1 \right] (4-1)(3+2+1) + (3-1) \\
 &\quad \left. \begin{array}{l} \text{sum of 3 natural} \\ \text{at no.} \\ (4-1)\text{nat no} \end{array} \right] \\
 &= BA + C \left[ \text{sum of 3 natural no.} + (3-1) \right] \\
 &= 1000 + 2 [6+2] = 1016
 \end{aligned}$$

[All triangular Matrix are square Matrix]

$A[25 \cdot 85, 25 \cdot 85] \quad BA = 1000$  (starting row no & column no is also same)  
 $C = 10 \text{ Byte } RM6 \text{ LT.M (Lower Triangular Matrix)}$

$$\begin{aligned}
 \text{Loc}[a[60][55]] &= BA + c \left[ \text{sum of } (60-25) \text{ natural no} + (55-25) \right] \\
 &= 1000 + \frac{(60-25+1)}{2} \\
 &\stackrel{25}{=} BA + c \left[ \text{sum of } (60-25) \text{ natural no} + (55-25) \right] \\
 &= 1000 + c \left[ (60-25)(60-25+1) + 30 \right] \\
 &= 7600
 \end{aligned}$$

## formula

$$\text{Loc}[a[ub_1 \dots ub_j; ub_2 \dots ub_k]] = BA \rightarrow C \quad \text{RMO, LTM}$$

$$\text{Loc}[a[i][j]] = BA + c \left[ \frac{\sigma(i - lb_1)(i - lb_1 + 1)}{2} + (j - lb_2) \right]$$

$$a[4][3] = BA + c [ \text{sum of 3 natural no} + (\text{total rows} - (3-1)) ]$$
$$\dots = 1000 + 10 [ 6 + 2 ]$$
$$= \underline{1016}$$

~~o  $a[25 \dots 85, 25, 85]$  BA = 1000 C = 10 Byte~~

~~CMO LTM  
Rows = 61 Column =~~

~~Loc  $a[60, 55]$  = BA + c [ sum of (55-25) natural no~~

~~+ no of rows - (55-1)~~

$$35 \dots = 1000 + 10 [ 35 \times 18 + 61 - (55-25-1) ]$$
$$18 \dots = 1000 + 10 [ 63 @ \dots + 27 ]$$
$$8 \dots = \underline{7570}$$

CMO  $(a[i][j] = BA + c [ \text{sum of } (j-1)b_1 \text{ no} + (\text{no of rows} - (j-1) \text{ natural} ) ] )$

~~extra~~

### Fibonacci heap

BA

$$\text{Loc}[a[4][3]] = 1000 + \left[ \frac{4 \times 5}{2} - 2 \times 3 + (4-3) \right] * 2$$

↓      ↓      ↓

crossing all column    crossing unnecessary column    Visiting 4th row and 3 column

$$= 1000 + 16 = 1016$$

$$\text{Loc}(a[4][1]) = \text{BA} + c \left[ \text{sum of } 4 \text{ nat. no} - \text{sum of } (4-1) \text{ no} \right]$$

it is used to cross all column    Visiting 1st row

$$= 1000 + 2 \left[ 10 - 10 + 3 \right] = 1006$$

crossing unnecessary crossed column

ex 2 - A[25...100, 25, 100] BA=1000 C=10B LTM CM0

$$\text{Loc}(A[80][45]) = \text{BA} + c \left[ \text{sum of } 76 \text{ natural no} - \text{sum of } (100-45+1) \text{ no} \right]$$

$$= 1000 + 10 \left[ 76 \times 77 + \frac{31 \times 32}{2} + 10 \right] + (80 - 45)$$

$$= 1000 + 10 \left[ 76 \times 77 + \frac{31 \times 32}{2} + 10 \right] + (80 - 45)$$

Visiting 80th row

5

38

77

31

266

12

266

36

266

36

$$= 1000 + 10 [ 38 \times 77 + 31 \times 32 + 10 ]$$

$$= 1000 + 10 [ 2926 + ]$$

$$= 1000 + 10 \left[ \frac{76 \times 77}{2} + \frac{56 \times 57}{2} + 35 \right]$$

$$= 151469$$

when you are at 15 row col  
you are at 15 to row

Loc EAF5

$A[-50 \dots 50, -50 \dots +50]$  BA = 0 C = 1 Byte LTM, CM

$$\text{Loc}(A[25][15]) = 0 + 1 \left[ \frac{\text{sum of } 101 \text{ nat - sum of } [50-15+}}{\text{no. nat. no.}} \right.$$

$$+ (25-15) \left. \right]$$

$$= 101 \times 101 + 10 - 36 \times 37 \quad \begin{matrix} 31 \\ 18 \end{matrix} \uparrow \text{since you at 15} \\ \text{row}$$

$$= 5161 - 666 \quad \begin{matrix} 2 \\ 2 \end{matrix} \quad \begin{matrix} \text{as 15 column} \\ \text{will start from} \\ 15 rows as all} \end{matrix}$$

$$= 4495 \quad \begin{matrix} \text{element before} \\ 15 \text{ are 2010} \end{matrix}$$

Ex<sup>q</sup>  $A[lb_1 \dots ub_1][lb_2 \dots ub_2]$  BA, C, LTM, CMD

Loc E

$$\text{Loc}(A[i][j]) = BA + C \left[ \frac{\text{sum of } (ub_2 - lb_2 + 1) \text{ nat - sum of } (ub_2}}{\text{no. nat. no.}} \right.$$

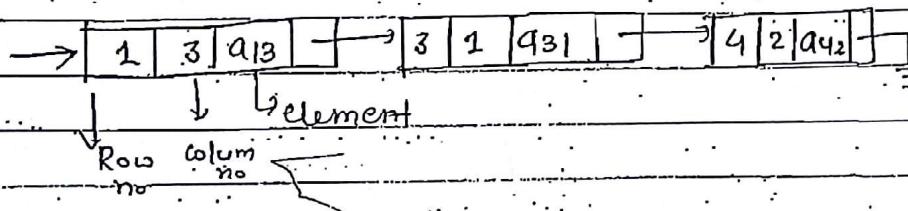
$$+ (j-i)] \quad \begin{matrix} \downarrow \\ \text{sum of all} \\ \text{column} \end{matrix}$$

Sparse Matrix - ~~very less element in array are 1, and other are zero & there is no relation in them~~

0	0	a <sub>13</sub>	0
0	0	0	0
a <sub>31</sub>	0	0	0
0	a <sub>42</sub>	0	0

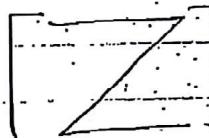
⇒ so need to store in array.

So we used linked list for its storage because if you use array lot of space is required.



Upper triangular Matrix

Z Matrix - all matrix except Z are zero



to exec  
to  
line

## Programming

### Scope of a variable

i) Static Scoping

ii) Dynamic Scoping

(data type followed by name of variable)

1 - int a = 10;

C()

- declaration

main()

{

int a = 20;  
B();

int a = 40;  
D();

for all f<sup>n</sup> call, a single stack is used

B()

D()

(both static & dynamic as no scope problem.)

int a = 30;  
D();

int a = 50;  
pf(a);

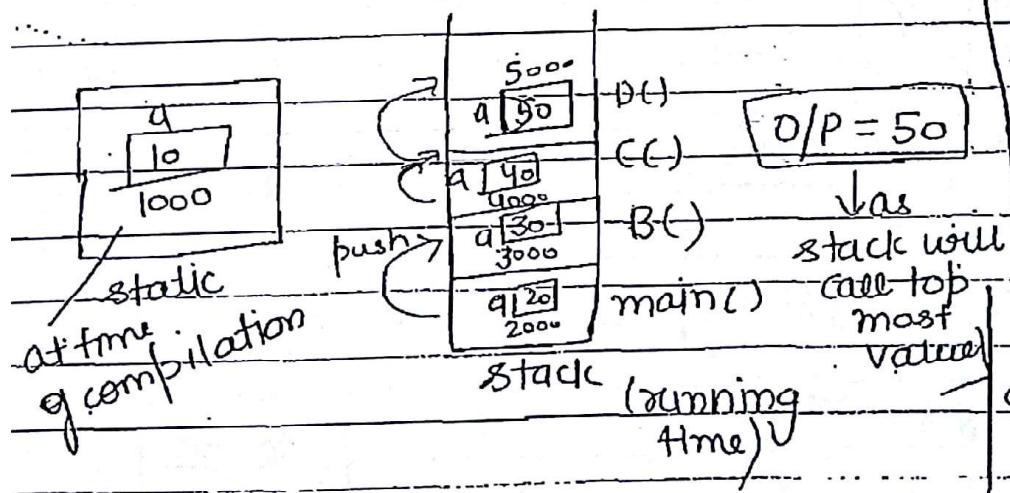
Whenever a function is created to local var is created & its m/m is completed; m/m is deactivated but global variable; m is created at time of compilation.

M/M

stack (for local f<sup>n</sup> variable)

static (at time of compilation)

heap (dynamic m/m)



static by default - C

if both local & global decls are not  
there, no mean ( $a = b + c$ )

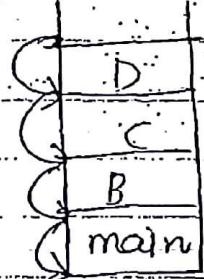
~~free~~ free variable - if a fn is using a variable which is not declared in it.

as if D()	no a is declared but is not initialized	A variable which can not be a does not exist
then	p(a)	[no a is declared but used]
O/P = f0(static)	f0(dynamic)	

whenever there is a free variable, compiler will identify it at time of compilation so, compiler will initialize it by global variable & this problem is known as scope problem. & this problem if resolved at time of compilation, known as static scoping because at that time only global variable was initialized by compiler can't solve problem at run time.

but if compiler has restricted to solve scope problem at the time of compilation that it is not initialized by global data it will be resolved at run time. If 'a' is not in D then go to its calling function & use from there and so on. if no calling function has 'a' then use global variable 'a'.

Dynamic Scope  
will generate closer scopes  
as selecting the values from the closer fn.



Dynamic is difficult to implement

static scoping - problem solved at compile time

dynamic scoping at run time

because In dynamic scoping worst case - all fn's to be passed to the

static scoping - by default

C, C++

static variable get m/m first & last  
deallocated at l  
re allocated b/t  
compilat

If a f<sup>n</sup> as D()

{ pf(a) and so on then static scoping - lo  
dynamic - go }

change done at compile time - static change

change done at run time - dynamic change (in dynamic  
change are m  
to previous)

Ex2 int a=10, b=20;

main()

int a=5, b=6;

pf(a,b);

C();

pf(a,b);

C();

{ int b=3;

pf(a,b);

D(b);

pf(a,b);

a=3, b=4;

D(int a)

{

pf(a,b);

a=2;

b=3;

E();

pf(a,b);

a=1;

b=2;

F();

g();

h();

i();

j();

k();

l();

m();

n();

o();

p();

q();

r();

s();

t();

u();

1000	2000
10, 3 5, 6	20, 8

a b

↓ static 3

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

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1000	2000
10, 3 5, 6	20, 8

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1000	2000
10, 3 5, 6	20, 8

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1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

1000	2000
10, 3 5, 6	20, 8

a b

5-3 int a=1, b=2;

main()

D (int a, int b)

bf(a, b);

c(b);

bf(a, b);

c(int a)

bf(a, b)

a=1, b=2

E();

bf(a, b);

a=3, b=4

a=3, b=4;

E()

bf(a, b);

D(b, a)

bf(a, b);

a=3, b=7

bf(a, b);

a=6;

b=7;

bf(a, b);

Static Scope - 1, 2

3, 4

4, 3

1, 4

3, 7

1, 2

3, 7

3, 7

4, 3

1, 2

3, 7

3, 7

4, 7

a=1, 3

b=2, 4, 7

a=4, 3

b=3, 2, 4

a=2, 3, 6

E

D

C

main

Dynamic

1, 2

3, 4

4, 3

1, 2

3, 7

3, 7

4, 7

a=4, 1, 3, 6, 3, 7

b=3, 2, 4, 7

a=2, 3, 6

b=1, 2, 4, 7

C

m

a=1

b=2, 4, 7

static

m/m

## Static Variable

local var  
dynamic m/m  $\rightarrow$  heap area  
in char - '10'  
int - '0'  
float '0.0'  
pointer 'Null'

ex: main()

static int var = 5;

printf ("%d", var);

if (var--) 5

} d

if ("%d", var)

main();

} b

Var = 5

Without static with sta

5 4 infinite

5 4 loop

5 4 stack

5 4 overflow

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