a → Terminals
A → variables

where x is any input. Repeat this step until no more marking can be made.

4. Combine all the unmark pair & make them a single state in minimize DFA.

Module 2

16/Sep/2019
Monday

Grammar: It is a set of rule ~~It is~~ and given by 4 tupples i.e.

$$G = (V_N, \Sigma, P, S)$$

where

V_N = set of ~~terminals~~ variables
Σ = set of elements called terminals
P = set of elements called production rule $\alpha \rightarrow \beta$
S = special variable called start symbol

validation rule:

(1) If $S \rightarrow BC$ is a production it means that S can be replaced by BC but BC cannot be replaced by S i.e. reverse substitution is not possible. (2) If $S \rightarrow BC$ is a given production then it is not necessary that $BC \rightarrow S$ is a production i.e. no inversion operation is permitted.

- multiple language can belongs to same grammar

Grammar → Language

Ques: If $G = (\underbrace{\{S\}}_{V_N}, \underbrace{\{0,1\}}_{\Sigma}, \underbrace{\{S \rightarrow 0S1, S \rightarrow \wedge\}}_{P}, \underbrace{S}_{S})$ find $L(G)$.

PAGE NO
DATE:

$S \to \wedge \in L(G)$

$S \to 0S1$

$S \to 0\wedge 1$

$\to 01 \in L(G)$

$$\boxed{L(G) = \{0^n 1^n \mid n \geq 0\}}$$

$S \to 00S11$

$\to 0^2 S 1^2$

$\to 0011 \in L(G)$

$V_n \quad \Sigma \quad P \quad S \to 0^n 1^n$ Generalized

Ques: If $G = (\{S\}, \{a\}, \{S \to SS\}, S)$ Give $L(G)$

$S \to SS$

aa

$L(G) = \{aa \mid n \geq 0\}$ ✗

$L(G) = \phi$

because there is no terminal generated through the production rule.

Ques: $G = (\{S, C\}, \{a, b\}, P, S)$ where P consist of $S \to aCa$, $C \to aCa \mid b$ Find $L(G)$

$L(G) = \{a^n b a^n \mid n \geq 1\}$

Language to grammar

M/C ★ Chomsky classification on Grammar

M/C	Type	Grammar	
TM	0	UR	All grammar
LBA	1	CSG	$\phi A \psi \to \phi \alpha \psi$
PDA	2	CFG	$A \to \alpha$, $A \in V_N$, $\alpha \in (V_N \cup \Sigma)^*$
FA	3	Regular Grammar	$A \to a$ or $A \to aB$ where $A, B \in V_N$, $a \in \Sigma$

CFG: A grammar in the form $A \to \alpha$; $A \in V_N$, $\alpha \in (V_N \cup \Sigma)^*$ is called type 2 grammar. The m/c that accept type 2 grammar (~~CFG~~ CFG) is called PDA

Q) i) LMD
ii) RMD
iii) Parse Tree
iv) Is this grammar ambiguous?
2 marks

CFG: A grammar that follow type 2 production

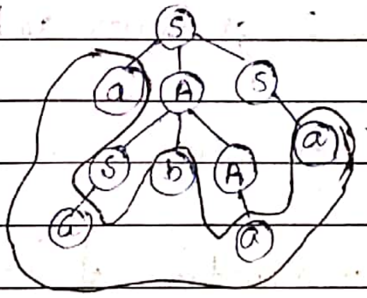
① Derivation Tree or Parse Tree: A derivation tree or parse tree for a CFG $G = (V_N, \Sigma, P, S)$ is a tree satisfying the following condition

1. Every vertex has a label which is a variable or terminal or null.
2. The root has label S
3. The label of an internal vertex is a variable.
4. If the vertices $n_1, n_2 \ldots n_k$ written with the label $x_1, x_2 \ldots x_k$ are the sons of vertex n with label A Then $A \rightarrow x_1, x_2 \ldots x_k$ is a production in P
5. A vertex n is a leaf if its label $a \in \Sigma$ or Λ

Eg- $G = (\{S, A\}, \{a, b\}, P, S)$

$S \rightarrow aAS \mid a \mid SS$ aabaa

$A \rightarrow SbA \mid ba \mid a$



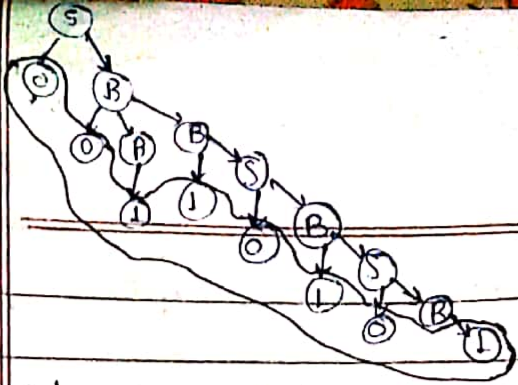
$S \rightarrow aAS$
$\rightarrow aSbAS$
$\rightarrow aSbAa$
$\rightarrow aabAa$
$\rightarrow aabaa$

RMD Right most Derivative
LMD: Left most derivative

Ques Given the grammar $S \rightarrow 0B \mid 1A$; $A \rightarrow 0 \mid 0S \mid 1AA$; $B \rightarrow 1 \mid 1S \mid 0BB$; for string 00110101 Find LMD, RMD, Parse Tree?

LMD
$S \rightarrow 0B$
$\rightarrow 00BB$
$\rightarrow 001B$
$\rightarrow 0011S$
$\rightarrow 00110B$
$\rightarrow 001101S$
$\rightarrow 0011010B$
$\rightarrow 00110101$

RMD
$S \rightarrow 0B$
$\rightarrow 00BB$
$\rightarrow 00B1S$
$\rightarrow 00B10B$
$\rightarrow 00B101S$
~~$\rightarrow 00B1010B$~~
$\rightarrow 00B10101$
$\rightarrow 00110101$



PAGE NO
DATE:

If more then one left most derivative exist for a given string then grammar is said to be ambiguous.

Ques: $G = (\{S\}, \{a, b, +, *\}, P, S)$
$S \rightarrow S+S \mid S*S \mid a \mid b$ construct a parse tree for $a+a*b$. ② Is this grammar ambiguous
③ LMD, RMD ?

$S \rightarrow S+S$
$\rightarrow a+S*S$
$\rightarrow a+a*S$
$\rightarrow a+a*b$

Ques: $S \rightarrow aB \mid bA$ for string aaabbabbba
$A \rightarrow a \mid aS \mid bAA$ find (i) LMD (ii) RMD
$B \rightarrow b \mid bS \mid aBB$ (iii) Parse Tree
(iv) Is grammar ambiguous?

$S \rightarrow$

★ Simplification of ~~Gram~~ CFG

(i) Reduction of CFG / construction of Reduced Grammar

Theorem: 1 If G is CFG such that $L(G) \neq \phi$, we can find an equivalent grammar G' such that each variable in G' derives some terminal string.

Proof: $G = (V_N, \Sigma, P, S)$
$G' = (V_N', \Sigma, P', S)$ as follows

ⓐ Construction of V_N'

We define $W_i \subseteq V_N$ by recursion

$W_1 = \{A \in V_N \mid$ where there exist a production $A \to w$ where $w \in \Sigma^*\}$

$W_{i+1} = W_i \cup \{A \in V_N \mid$ there exist some production $A \to \alpha$ with $\alpha \in (\Sigma \cup W_i)^*\}$

(b) Construction of P'

$P' = \{A \to \alpha \mid A, \alpha \in (V_N' \cup \Sigma)^*\}$

Eg.

$S \to AB$	$W_1 = \{A, B, E\}$	
$A \to a$	$W_2 = \{S, A, B, E\}$	$W_3 = W_2$
$B \to b$	$W_3 = \{S, A, B, E\}$	$V_N' = \{S, A, B, E\}$
$B \to C$		
$E \to c$		

$S \to AB$
$A \to a$
$B \to b$
$E \to c$

Tuesday 24/09/2019

Theorem-2

For every CFG $G = (V_N, \Sigma, P, S)$ we can construct an equivalent grammar $G' = (V_N', \Sigma', P', S)$ such that every symbol in $V_N \cup \Sigma'$ appears in some sentencial form

We construct $G' = (V_N', \Sigma', P', S)$ as follows

(a) Construction of W_i for $i \geq 1$

$W_1 = \{S\}$

$W_{i+1} = W_i \cup \{X \in V_N \cup \Sigma \mid$ where there exist a production $A \to \alpha$ with $A \in W_i$ and α containing the symbol $X\}$

PAGE NO
DATE:

(b) construction of V_N', Σ', P'

$V_N' = V_N \cap W_k$ $\qquad P' = \{A \to \alpha \mid A \in W_k\}$

$\Sigma' = \Sigma \cap W_k$

Ques: Find a reduced grammar equivalent to the grammar G whose productions are

G : $S \to AB \mid CA$ $\qquad A \to a$

$B \to BC \mid AB$ $\qquad C \to aB \mid b$

Step 1: $W_1 = \{A, C\}$

$W_2 = \{A, C\} \cup \{S\} = \{S, A, C\}$

$W_3 = \{S, A, C\} \cup \phi$

G' $V_N' = W = \{S, A, C\}$

P' $\boxed{S \to CA \\ A \to a \\ C \to \cancel{aB} \mid b}$

$\Sigma' = \{a, b\}$

$G' = (\{S, A, C\}, \{a, b\}, P', S)$

step 2: apply theorem 02

$W_1 = \{S\}$

$W_2 = \{S\} \cup \{C, A\} = \{S, C, A\}$

$W_3 = \{S, C, A\} \cup \{a, b\}$

$W_4 = \{S, C, A, a, b\} \cup \phi$

$V_N'' = \{S, C, A\}$ $\qquad S \to CA$

$\Sigma'' = \{a, b\}$ $\qquad A \to a$

$C \to b$

Ques: Construct a reduced grammar equivalent to the given grammar $S \to aAa$,

$A \to Sb \mid bCC \mid DaA$

$C \to abb \mid DD$ $\qquad W_1 = \{C\}$

$E \to aC$ $\qquad W_2 = \{C, A, E\}$

$D \to aDA$ $\qquad W_3 = \{S, C, A, E\}$

$S \to aAa$

$A \to Sb \mid bCC$

$C \to abb$

$E \to aC$

step 2

$W_1 = \{S\}$

$W_2 = \{S\} \cup \{a, A\} = \{S, A, a\}$

$W_3 = \{S, A, a\} \cup \{b, C\} = \{S, A, C, a, b\}$

$W_4 = \{S, A, C, a, b\}$

$V_N'' = \{S, A, C\}$ $\Sigma'' = \{a, b\}$

P'' $S \to aAa$
$A \to Sb \mid bCC$
$C \to abb$

+2 marks

Simplification:

Elimination of Null production

$S \to ABAC$	$S \to ABAC \mid BAC \mid ABC \mid BC \mid AAC \mid AC \mid C$
$A \to aA \mid \Lambda$	$A \to aA \mid a$
$B \to bB \mid \Lambda$	$B \to bB \mid b$
$C \to d$	$C \to d$

$C \to d$ included in P'

$A \to aA$ gives rise to $A \to aA \mid a$

$B \to bB \longrightarrow B \to bB \mid b$

$L(G') = L(G) - \{\Lambda\}$

$G' = (\{S, A, B, C\}, \{a, b, d\}, P', S)$

Q1 $S \to ABCd$
$A \to BC$
$B \to bB \mid \Lambda$
$C \to aC \mid \Lambda$

$S \to ABCd \mid ACd \mid ABd \mid Ad$
$A \to BC \mid C \mid B$
$B \to bB \mid b$
$C \to aC \mid a$

Q2 $S \to ASA \mid aB \mid b$
$A \to B$
$B \to b \mid \Lambda$
$S \to A$

Unit production

($A \to B$ ✓ $A \to a$ ✗ $A \to Ba$ ✗ $A \to aB$ ✗)

eliminate unit production

$S \to AB$	$S \to AB$	$W(S) = S$
$A \to a$	$A \to a$	$W(A) = A$
$B \to C \mid b$	$B \to a \mid b$	$W_0(B) = \{B, C, D, E\}$ $W_1(B) -$
$C \to D$	$C \to a$	$W(C) = \{C, D, E\}$
$D \to E$	$D \to a$	$W(D) = \{D, E\}$
$E \to a$	$E \to a$	$W(E) = E$

PAGE NO
DATE:

*Q.

$E \to E+T \mid T$
$T \to T*F \mid F$
$F \to (E) \mid a$

$E \to E+T \mid a T*F \mid (E) \mid a$
$T \to T*F \mid a(E) \mid a$
$F \to (E) \mid a$

30/09/2019

If certain restriction are apply on a CFG G then the grammar is said to be in a normal form.

(3 marks or 2 marks)

Normal Form
- CNF (Chomsky Normal form)
- GNF (Greibach Normal form)

CNF

Def 1: A production in the form of $A \to a$ or $A \to BC$ is said to be in chomsky normal form.

$\boxed{A \to a}$
$\boxed{A \to BC}$

Def 2: A CFG G is in CNF if every production is of the form $A \to a$ or $A \to BC$ and $S \to \Lambda$ is in G if $\Lambda \in L(G)$. When Λ is in $L(G)$, we assume that S does not appear on the R.H.S. of any production.

Step 1: Elimination of null production & unit production

Step 2: Elimination of terminals on RHS

Step 3: Restrict the no. of variable on RHS

Ques: Convert the given CFG in CNF.

$S \to aAD$
$A \to aB \mid bAB$
$B \to b$
$D \to d$

R.H.S = Right hand side

step 1 There is no null and unit production in the given grammar so proceed to step 2

step 2 Elimination of Terminal

$S \rightarrow aAD$ give rise to, Let $G' = (V_N', \Sigma, P', S)$

$\boxed{S \rightarrow C_a AD, \quad C_a \rightarrow a}$

$A \rightarrow aB$ give rise to

$\boxed{A \rightarrow C_a B}$

$A \rightarrow bAB$ give rise to

$\boxed{A \rightarrow C_b AB, \quad C_b \rightarrow b}$

$\boxed{B \rightarrow b}$ included into P'

$\boxed{D \rightarrow d}$ included into P'

step 3 restrict the no. of variables

$B \rightarrow b$ $\qquad S \rightarrow C_a AD$ give rise to

$D \rightarrow d$ $\qquad \boxed{S \rightarrow C_a C_1, \quad C_1 \rightarrow AD}$

$C_a \rightarrow a$ $\qquad A \rightarrow C_b AB$ give rise to

$C_b \rightarrow b$ $\qquad \boxed{A \rightarrow C_b C_2, \quad C_2 \rightarrow AB}$

$A \rightarrow C_a B$

$G' = (\{S, A, B, D, C_a, C_b, C_1, C_2\}, \{a, b, d\}, P', S)$

where P' is

$B \rightarrow b$, $C_a \rightarrow a$, $A \rightarrow C_a B$, $C_1 \rightarrow AD$ $\quad C_2 \rightarrow AB$

$D \rightarrow d$, $C_b \rightarrow b$, $S \rightarrow C_a C_1$ $\quad A \rightarrow C_b C_2$

All production are in chomsky normal form.

Ques. Find a grammar in CNF equivalent to a the grammar $S \rightarrow \sim S \mid [S \supset S] \mid p \mid q$ where S being only variable.

$S \rightarrow aS \mid bScSd \mid p \mid q$ $\qquad$ renaming the terminals

PAGE NO
DATE:

1/10/2019

GNF

A CFG G is in GNF if every production is in the form of $A \to a\alpha$ where $a \in \Sigma$ and $\alpha \in V_N^*$

Lemma-1

Let $G = (V_N, \Sigma, P, S)$ be CFG. Let $A \to B\gamma$ be an A-production in P. Let B-production be:
$B \to \beta_1 | \beta_2 | \cdots | \beta_s$ then $P_1 = ((P - \{A \to B\gamma\}) \cup \{A \to \beta_i \gamma\})$ where $1 \le i \le s$

Lemma-2

Let $G = (V_N, \Sigma, P, S)$ be CFG. Let set of A-production be $A \to A\alpha_1 | A\alpha_2 | \ldots | A\alpha_r | \beta_1 | \beta_2 | \cdots | \beta_s$. Let Z be a new variable. Let $G_1 = (V_N \cup \{Z\}, \Sigma, P_1, S)$ where P_1 defined as

i) the set of A-productions in P_1 are

$A \to \beta_1 | \beta_2 | \cdots | \beta_s$

$A \to \beta_1 Z | \beta_2 Z | \cdots | \beta_s Z$

ii) the Z-productions in P_1 are

$Z \to \alpha_1 | \alpha_2 | \cdots | \alpha_r$

$Z \to \alpha_1 Z | \alpha_2 Z | \cdots | \alpha_r Z$

4/10/2019

3 marks

$S \to AA | a$
$A \to SS | b$

Step 1 Renaming of variable

$A_1 \to A_2 A_2 | a$
$A_2 \to A_1 A_1 | b$

As there is no null & unit production in the grammar so proceed further

$A \to B$ Lemma 1 (max. 3 variable in GNF in exam)

$A \to A$ Lemma 2

Step A_1 productions are in required form.

$\boxed{A_2 \to b}$ is in GNF

$A_2 \to A_1 A_1$ ∵ $A \to B\gamma$ (Lemma 1)

$\boxed{A_2 \to aA_1 \mid A_2A_2A_1 \mid b}$

$A_2 \to A_2A_2A_1$ $A \to A\alpha$ (Lemma 2)

$$\boxed{\begin{array}{l} A_2 \to aA_1 \mid b \\ A_2 \to aA_1Z \mid bZ \\ Z \to A_2A_1 \mid A_2A_1Z \end{array}}$$

$$\boxed{\begin{array}{l} Z \to aA_1A_1 \mid bA_1 \\ Z \to aA_1A_1Z \mid bA_1Z \end{array}}$$

✓ $A_1 \to A_2A_2 \mid a$

$\boxed{A_1 \to a}$

$A_1 \to A_2A_2$ $A \to B\gamma$ (Lemma 1)

$\boxed{A_1 \to aA_1A_2 \mid bA_2 \mid aA_1ZA_2 \mid bZA_2 \mid a}$

Ques: $S \to AB$
$A \to BS \mid b$
$B \to SA \mid a$

(2) $E \to E+T \mid T$
$T \to T*F \mid F$
$F \to (E) \mid a$

convert the given grammar into GNF

Q1 Given ~~In~~ a CFG for the language $L = \{x \in (0,1)^* \mid$ where x starts and end with different symbol.

Q2 Consider the production of the grammar $S \to AA$ and $A \to aa \mid bb$ the language is specified

i) $L = (x \in (a,b)^* \mid$ where x consist of all strings having aa or bb as a substring.

ii) $L = (x \in (a,b)^* \mid$ where x consist of all strings starting with aa or bb

PAGE NO
DATE:

iii) L = { aaaa, aabb, bbaa, bbbb }

iv) L = {abab, abaa, aaab, baaa}

(3) Which 1 of these rules are not in GNF

$S \rightarrow AS \mid SBB \mid a$

$A \rightarrow bAA \mid b$

$B \rightarrow ab \mid Ba$

(4) Convert the following grammar into CNF

$S \rightarrow bA \mid aB$

$A \rightarrow bAA \mid aS \mid a$

$B \rightarrow aBB \mid bS \mid b$

(5) Eliminate null and unit productions from the following grammar.

$S \rightarrow aXbX$

$X \rightarrow aY \mid bY \mid \Lambda$

$Y \rightarrow X \mid d$

(1) $S \rightarrow 0A1 \mid 1A0 \quad A \rightarrow 0A \mid 1A \mid \Lambda$

null production — unit production

(5)

null production	unit production
$S \rightarrow aXbX \mid abX \mid aXb \mid ab$	$S \rightarrow aXbX \mid abX \mid aXb \mid ab$
$X \rightarrow aY \mid bY \mid a \mid b$	$X \rightarrow aY \mid bY \mid a \mid b$
$Y \rightarrow d$	$Y \rightarrow aY \mid bY \mid a \mid b \mid d$

(4) ~~$S \rightarrow \theta$~~

$A \rightarrow a$

$B \rightarrow b$

$S \rightarrow C_b A \qquad C_b \rightarrow b$

$S \rightarrow C_a B \qquad C_a \rightarrow a$

$A \rightarrow C_a S$

$B \rightarrow C_b S$

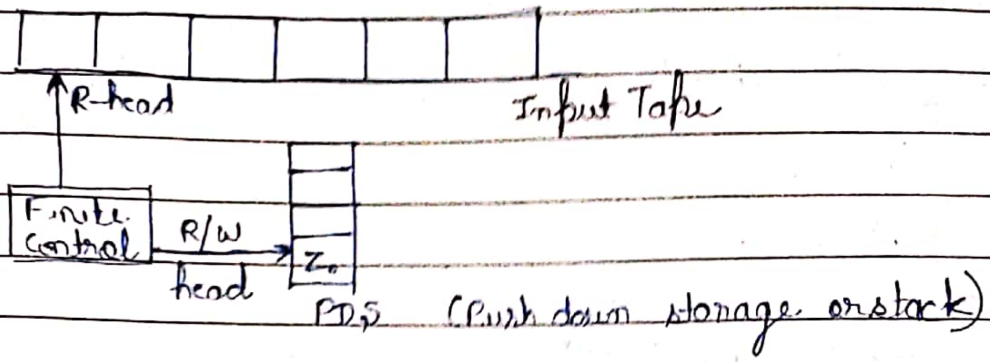
$A \rightarrow C_b AA \qquad B \rightarrow C_a BB$

$\boxed{A \rightarrow C_b C_1 \quad C_1 \rightarrow AA} \qquad \boxed{B \rightarrow C_a C_2 \quad C_2 \rightarrow BB}$

Ans :- Blck diag. tuple, transition fn,

7/10/2019

PDA (Push Down Automata):



Z_0: stack symbol is mark to bottom of stack or initial stack symbol.

$M = (Q, \Sigma, \delta, q_0, F, Z_0, \Gamma)$ Γ - Tau

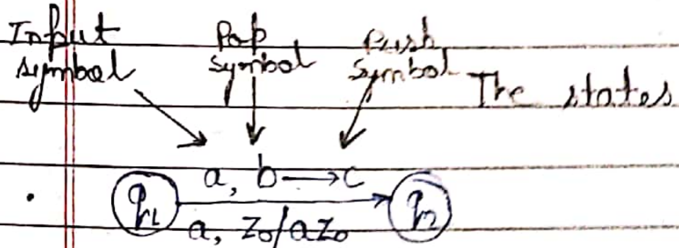
where Z_0 - A starting stack symbol

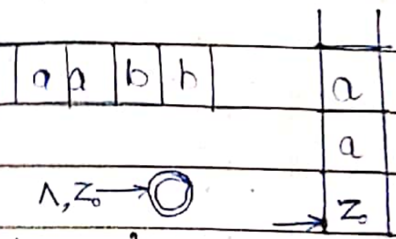
Γ = A finite stack alphabet

transition fn

$\delta : Q \times (\Sigma \cup \{\Lambda\}) \times \Gamma \longrightarrow$ finite subsets of $Q \times \Gamma^*$

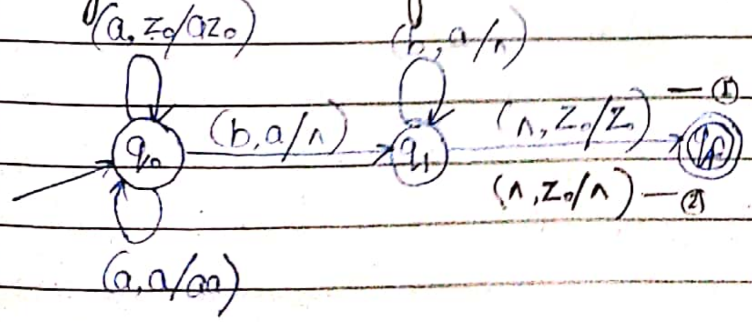
$\delta : Q \times (\Sigma \cup \{\Lambda\}) \times \Gamma \longrightarrow$ finite subsets of $2^{(Q \times \Gamma^*)}$ (NPDA)





Construction of PDA

Ques-1 Design a PDA for $L = \{a^n b^n \mid n \geq 1\}$



$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \Lambda)$

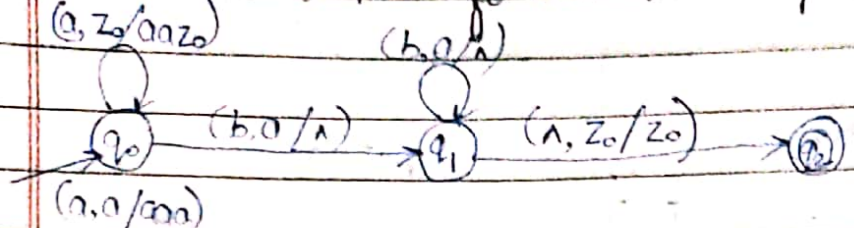
$\delta(q_1, b, a) = (q_1, \Lambda)$

$\delta(q_1, \Lambda, Z_0) = (q_f, Z_0)$ // by final state ①

or

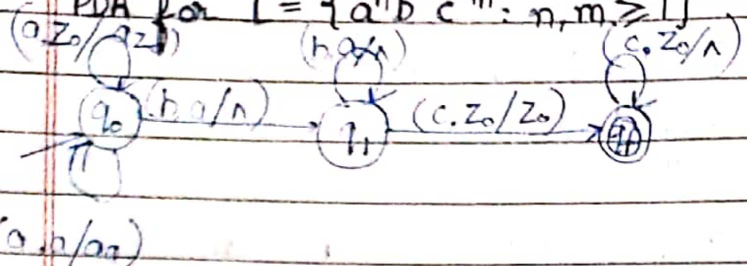
$\delta(q_1, \Lambda, Z_0) = (q_1, \Lambda)$ // by empty stack

Ques-2 Construct a PDA for $L = \{a^n b^{2n} \mid n \geq 1\}$

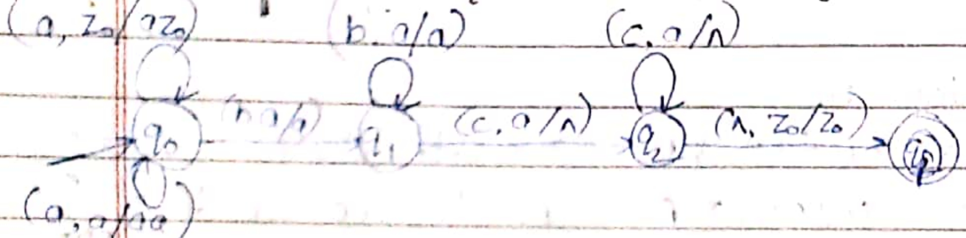


~~2marks Grammar to PDA~~

Ques: PDA for $L = \{a^n b^n c^m : n, m \geq 1\}$

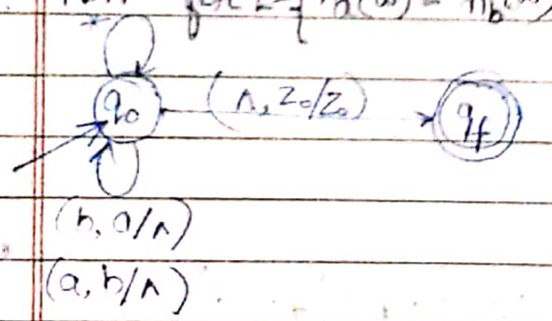


Ques: PDA for $L = \{a^n b^m c^n : n, m \geq 1\}$



Ques: PDA for $L = \{n_a(w) = n_b(w)\}$

b, b/bb
a, a/aa
b, Z0/bZ0
a, Z0/aZ0



+ 2marks Grammar to PDA

construction of PDA through Grammar

$\delta(q, \wedge, A) = \{(q, \alpha) \mid A \to \alpha \text{ in } P\}$

$\delta(q, t, t) = \{(q, \wedge)$ for every t in $\Sigma\}$

where t = terminal

$\{wcw^R \mid w \in (a,b)^+\}$
$\{wcw^R \mid w \in (a,b)^+\}$

Ques: $S \rightarrow 0 \mid 1 \mid 1A$ Grammar $\longrightarrow$ PDA
$A \rightarrow 0 \mid 1S$

$\delta(q, \Lambda, S) = \{(q,0), (q,1), (q,1A)\}$
$\delta(q, \Lambda, A) = \{(q,0), (q,1S)\}$

$\delta(q,0,0) = \{(q,\Lambda)\}$
$\delta(q,1,1) = \{(q,\Lambda)\}$

Ques: Convert the following CFG to PDA
$S \rightarrow aAA$
$A \rightarrow aS \mid bS \mid a$

$m = (\{q\}, \{0,1\}, \delta, \{q\}, \{q\}, Z_0, \{S, A, 0, 1\})$ where $\delta =$

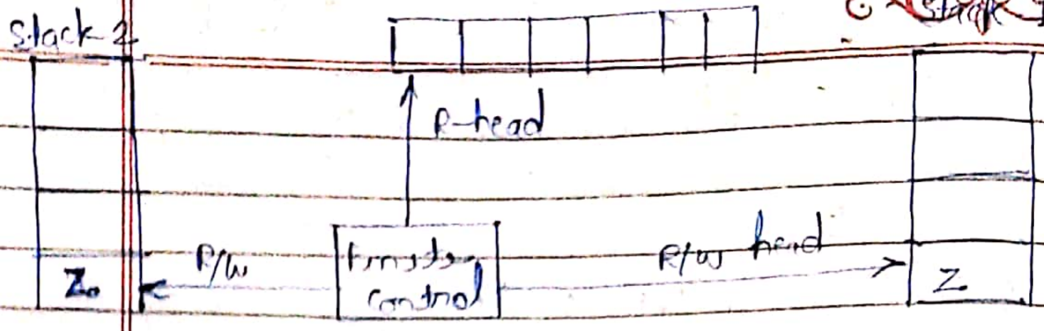
Ques: Find the PDA equivalent to the given CFG
$S \rightarrow 0BB$
$B \rightarrow 0S \mid 0 \mid 1S$

$\delta(q, \Lambda, S) = \{(q, 0BB)\}$
$\delta(q, \Lambda, B) = \{(q, 0S), (q, 0), (q, 1S)\}$

$\delta(q,0,0) = \{(q,\Lambda)\}$
$\delta(q,1,1) = \{(q,\Lambda)\}$ $m = (\{$

2PDA (Two stack PDA) {shortnote}

$Q \times \Sigma \cup \{\Lambda\} \times \Gamma$ (stack1/stack2) $\longrightarrow Q \times \Gamma^*$ (stack1/stack2)



$\{a^m b^m c^n / m, n \geq 1\}$ specific type of problem

Imp 5 mark

Q1 $L = \{wcw^R / w \in \{a,b\}^+\}$ R = reverse

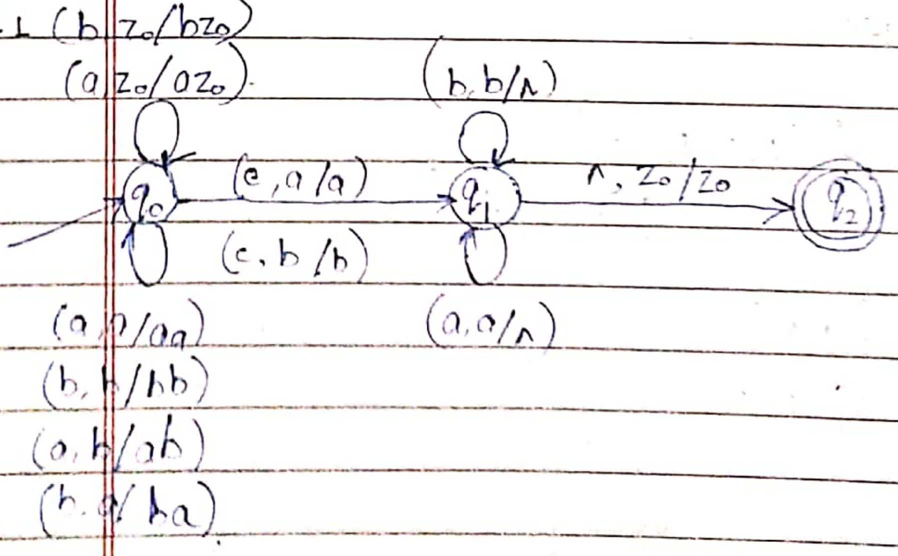
baa c aab
bbaa aabb
abab baba

Q2 $L = \{ww^R / w \in \{a,b\}^+\}$ — ~~NDPA~~ NPDA

$\underbrace{ab}_{w}, \underbrace{ba}_{w^R}$

abba abba

Sol 1



PDA for $L = \{ww^R : w \in (a,b)^+\}$

$ww^R = ww^T$

Ques: Design a PDA for the language $L = a^i b^j c^k$ where $i, j, k \geq 0$ and if $k = j$

Ans:

Q1 Write the grammar for $L = \{ wcw^R \mid \text{where } w \in (a,b)^* \}$

Q2 Write the grammar for generating a string as a pallindrome

$S \rightarrow aSa \mid bSb \mid c$

Ques: Write a grammar for a language $L = (\underbrace{\{0^n 1^n \mid n \geq 0\}}_{S_1} \cup \underbrace{\{1^n 0^n \mid n \geq 0\}}_{S_2})$

$S \rightarrow S_1 / S_2$

$S_1 \rightarrow 0S_11 \mid \wedge$

$S_2 \rightarrow 1S_20 \mid \wedge$

$S \rightarrow S_1 / S_2$

Ques: Write a CFG for a language $L = \{a^n b^n c^m d^m / n, m \geq 1\}$

$S \rightarrow XY$

$X \rightarrow aXb \mid ab$

$Y \rightarrow cYd \mid cd$

Ques: Write a CFG for set of all palindrome

$S \rightarrow aSa \mid bSb \mid a \mid b \mid \wedge$

$S \rightarrow aSa \mid bSb \mid \wedge$ Even

$S \rightarrow aSa \mid bSb \mid a \mid b$ odd

18/10/19

PDA to CFG

~~Define~~ ~~$S \rightarrow 0AB \mid 1A \mid 1B \mid 0A \mid 0$~~

Let the grammar $G = (V_N, \Sigma, P, S)$ where

$V_N = \{S\} \cup \{[q, z, q'] / \text{where } q, q' \in Q \ z \in \Gamma\}$ ie

PAGE NO
DATE:

any element of V_N is either the new symbol S or an ordered triple whose Ist and IIIrd elements are states and the IInd element is a push down symbol.

The productions in P are induced by moves of PDA as

R_1: S-productions are given by

$$S \rightarrow [q_0, Z_0, q] \text{ for every } q \text{ in } Q$$

R_2: each move erasing a push down symbol given by

$$\delta(q, a, Z) = (q', \Lambda)$$ induces the production $[q, Z, q'] \rightarrow a$

R_3: Each move not erasing a push down symbol is given by

$$\delta(q, a, Z) = (q_1, Z_1 Z_2 \ldots Z_m)$$

induces many productions of the form

$$[q, Z, q'] \rightarrow a[q_1, Z_1, q_2][q_2, Z_2, q_3] \cdots [q_m, Z_m, q']$$

+3marks

Ques: construct a CFG G which accept PDA (A) where $A = (\{q_0, q_1\}, \{a, b\}, \{Z_0, Z\}, \delta, q_0, Z_0, \phi)$

state terminal stack symbol

when δ is defined as

i) $\{\delta(q_0, b, Z_0) = (q_0, ZZ_0)$

ii) $\delta(q_0, \Lambda, Z_0) = (q_0, \Lambda)$

iii) $\delta(q_0, b, Z) = (q_0, ZZ)$

iv) $\delta(q_0, a, Z) = (q_1, Z)$

v) $\delta(q_1, b, Z) = (q_1, \Lambda)$

vi) $\delta(q_1, a, Z_0) = (q_0, Z_0)\}$

[- - -]

state, stack symbol, state

$$\boxed{\begin{array}{l} S \rightarrow [q_0, Z_0, q_0] \\ S \rightarrow [q_0, Z_0, q_1] \end{array}}$$

$\delta(q_0, \wedge, Z_0) = (q_0, \wedge)$ It is erasing from stack

ii) $[q_0, Z_0, q_0] \longrightarrow \wedge [q_0]$ $(2^0 = 1)$

$$\boxed{[q_0, Z_0, q_0] \longrightarrow \wedge}$$

no. of symbol push = a
productions = 2^a

(iv) $\delta(q_0, a, Z) = (q_1, Z)$

$$\boxed{\begin{array}{l} [q_0, Z, q_0] \longrightarrow a [q_1, Z, q_0] \\ [q_0, Z, q_1] \longrightarrow a [q_1, Z, q_1] \end{array}}$$ $\because (2^1 = 2)$

(i) $\delta(q_0, b, Z_0) = (q_0, ZZ_0)$ $\because (2^2 = 4)$

$[q_0, Z_0, q_0] \longrightarrow b [q_0, Z, q_0] [q_0, Z_0, q_0]$
$[q_0, Z_0, q_1] \longrightarrow b [q_0, Z, q_0] [q_0, Z_0, q_1]$
$[q_0, Z_0, q_0] \longrightarrow b [q_0, Z, q_1] [q_1, Z_0, q_0]$
$[q_0, Z_0, q_1] \longrightarrow b [q_0, Z, q_1] [q_1, Z_0, q_1]$

iii) $\delta(q_0, b, Z) = (q_0, ZZ)$ $\because (2^2 = 4)$

$[q_0, Z, q_0] \longrightarrow b [q_0, Z, q_0] [q_0, Z, q_0]$
$[q_0, Z, q_1] \longrightarrow b [q_0, Z, q_0] [q_0, Z, q_1]$
$[q_0, Z, q_0] \longrightarrow b [q_0, Z, q_1] [q_1, Z, q_0]$
$[q_0, Z, q_1] \longrightarrow b [q_0, Z, q_1] [q_1, Z, q_1]$

v) $\delta(q_1, b, Z) = (q_1, \wedge)$

PAGE NO
DATE:

(1) $\delta(q_0, b, Z) = (q_0, bZ)$
$\delta(q_0, c, Z) = (q_0, \Lambda)$

(2) $\delta(q_0, b, b) =$

(3) $\delta(q_0, c, b) = (q_1, b)$
$\delta(q_0, b, b) = (q_0, bb)$

Ans-1
$[q_0, Z, q_0] \longrightarrow b[q_0, b, q_0][q_0, b, q_0]$
$[q_0, Z, q_1] \longrightarrow b[q_0, b, q_0][q_0, b, q_1]$
$[q_0, Z, q_0] \longrightarrow b[q_0, b, q_1][q_1, b, q_0]$
$[q_0, Z, q_1] \longrightarrow b[q_0, b, q_1][q_1, b, q_1]$

(1) Design a PDA to recognize a language
$S \rightarrow S+S \mid S*S \mid 4 \mid 2$

(2) Convert the given grammar into CNF

$$\left.\begin{array}{l} S \rightarrow PQP \\ P \rightarrow 0P \mid \Lambda \\ Q \rightarrow 1Q \mid \Lambda \end{array}\right\} \text{CNF}$$

(3)

$$\left.\begin{array}{l} S \rightarrow AACD \\ A \rightarrow aAb \mid \Lambda \\ C \rightarrow aC \mid a \\ A \rightarrow aDa \mid bDb \mid \Lambda \end{array}\right\} \text{CNF}$$

(4)

$$\left.\begin{array}{l} S \rightarrow ABCd \\ A \rightarrow BC \\ B \rightarrow bB \mid \Lambda \\ C \rightarrow aC \mid \Lambda \end{array}\right\} \text{Remove null}$$

(5) Design a PDA for $L = WCW^R$ where $W \in (0,1)^*$
(6) " " " $L = WW^R$ where $W \in (0,1)^*$
" " PDA for $L = a^{2n}b^n$ where $n \geq 1$

★ Pumping Lemma for CFG

Let L be a CFL. Then we can find a natural number n such that

(i) every $z \in L$ with $|z| \geq n$ can be written as $uvwxy$ for some string u, v, w, x, y

(ii) length of $|vx| \geq 1$

(iii) length of $|vwx| \leq n$

(iv) $uv^k wx^k y \in L$ for all $k \geq 0$

1. Construct a PDA for balanced parenthesis
2. Consider the following grammar

$S \rightarrow 1S1 \mid T$

$T \rightarrow 1X1 \mid X$

$X \rightarrow 0X0 \mid 1$

a) Write 4 string of L(G)

b) Give an example of a string $w \in (0,1)^+$ such that $|w| > 7$ and $w \notin L(G)$

3. A PDA is given as $A = (\{q_0, q_1\}, \{a, b\}, \{a, z_0\}, \delta, q_0, z_0, \phi)$ where δ is defined as

$R_1: \delta(q_0, a, z_0) = (q_0, az_0)$ $\quad R_4: \delta(q_1, b, a) = (q_1, a)$

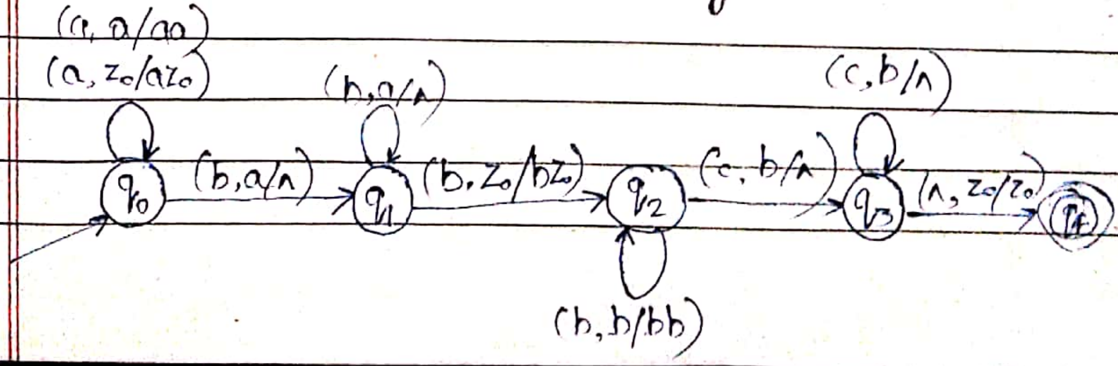
$R_2: \delta(q_0, a, a) = (q_0, aa)$ $\quad R_5: \delta(q_1, a, a) = (q_1, \wedge)$

$R_3: \delta(q_0, b, a) = (q_1, a)$ $\quad R_6: \delta(q_1, \wedge, z_0) = (q_1, \wedge)$

find the CFG corresponding to this PDA

4. Write an algorithm to convert an CFG into CNF or GNF

Ques: $L = (a^i b^j c^k \,/\, i, j, k > 0 \text{ and } i + k = j)$



Construct a PDA for $L = \{a^{2n} b^n / n \geq 1\}$.