

(If five keys were there)?

Hash Function -

HT

M=10

		keys
0	250	
1		
2		H A S H
3	100043	
4	64	E U N
5		
6	10526	T
7		
8	758	
9	699	

Hash address

if we check for one's place of the given key ie ( $\% 10$ ) we get the (0-9) key just store it

to retrieve 758

as 699, 699, 6

again  $\% 10 = 8(758)$

is stored at

8.

∴ Again time complexity =  $O(1)$

Hash function take more time compared to Hash table due to function calculation.

Hash function is better as it also save the space.

In if  $n < 10 \rightarrow$  slot = 0 to 9

$n > 10 \rightarrow$  slot = 0 to 10

Q. Collision : If two keys mapped to same hash address by hash function then it is called collision.

i.e fix- 699 759

both get collided at 9 pos

Hash function is better if no. of collision is as min. as pos.

### Types of Hash function-

1. Division Modulo Method-

2. Digit Extraction Method

3. Mid Square Method

4. Folding Method -

(i) Fold Boundary Method

(ii) Fold Shifting Method

#### 1. Division Modulo Method-

Ex1- size of table  $m = 1000$  (0 to 999)

key = 1 2 3 4 5 6 7 8 9

Acc to Division Modulo Method

$$\text{key Hash-function (key)} = \text{key Modulo } m$$

$$= 123456789 \% 1000$$

$$= 789$$

6	
:	:
789	123456789
:	:
6	

Ex2- size of table  $m = 8 (2^3)$

(take LSB 3 bit

key = 1010010101100101 here)

$$H_f(\text{key}) = 1010010101100101 \bmod 2^3$$

$$= 101$$

if  $m = 2^k$

then  $H_f(\text{key})$

this method don't bother about the whole

data to how to store just take last  $k^{th}$  bits-

$(k \text{ bits})$   
of LSB

$$\text{Ex } 1010101111111111 \bmod 2^4 = \underline{1111}$$

Ex 3  $m = 2^k$

key = 10100010101111011101 mod  $2^k$  = LSB k bits

if  $k = 8$

$H_f(key) = 11011101$

Hash function of

- if M is in power of 2, key will be LSB only that lead to more collision.
- So do not choose M as a power of 2.

Ex 1010001010111101101  $k=7$

1010001010111101101 LSB

$H_f(key_1) = 1011101$

$H_f(key_2) = 1011101$

1. So, do not pick M as exact power of 2 because if  $M = 2^k$  then Hash function of key = LSB k bits always.  
but if M is not power of 2 then less chances of collision

2. Pick the M value which is a prime no but it cannot be close to power of 2.

Ques- Which M value can be chosen for better result

(a) 61 (b) 729

(c) 523 (d) 1024

729 is chosen as it is not power of 2 & not close to powers

Steps - (1) Assume all are prime no

(2) choose one which is not close to power of 2.

encoding-decoding  
store - retrieve

## 2. Digit Extraction Method- M=1000

key = 789123456

hashfunction(key) = (7 8 9 1 2 3 4 5 6  
                      1 2 3 4 5 6 7 8 9)


extract some digits

= 715 . extract digit-(124, 8)

the digits to be extracted are defined

in question. (you cannot extract 4 digits)

as size of stack is from 0-999

: Here no consideration is done to storage time.

- When you are extracting 3 digit, may lead to one digit or two digit address (if one of them become zero)

• Collision may also occur here as if

7 7 7 1 2 3 4 5 9  
1 2 3 4 5 6 7 8 9

but it is not producing that much less collision i.e no of collision is more here.

- Digit Extraction Method produces more collision as compared to Division Modulo Method (as for division modulus, you require  $M = 2^k$ ) but here only some bits have to be changed.

• We extract 3-digit address as 3 digit addresses >> 2 digit ad.

### 3. Mid-Square Method-

$M = 1000$

$\text{key} = 84925$

#### Step 1-

1. Square the given key
2. check middle of obtained key (take 3 bit from mid as it will have majority of 3-bit addresses.

Making counter counter there will be bit difficult, so it lead to less counter & less collision. Because itself key a larger no produces a more larging no.

$$\text{Hash function} = (84925)^2$$

$$= 7212(2.5)5625$$

225	72124925

225    255... (as no exact mid possible)

store at any one address

To Retrieve - (1) Square the key

(2) take the mid, go to mid & extract the key.

Counter is not easy because of larger each & every no (digit) participate in the calculation of square.

Folding Method -  $M = 1000$  (0-999)

key = 123456789

(i) Fold Boundary Method

- fold the boundary  
(take k bit from the key from boundary)

$k \geq$  no of digit which have more address

for 3 bits - more address

123 456 789

123

789

912

If it lead to

(191)	
912	191
1 or 2	193

folding bnm  
clarity again  
again

key stated 912 123456789

123 45 123  
95  
169

Counter example = 123 89 789

912  
↓ again

counter is bit tougher than  
bit extraction as 6 digit participate

(ii) Fold Shifting Method

- Fold it according to fold boundary but fold every bit

123 456 789

123

456

789

13 58

136

8

144 - 12345678

144

∴ fold shift is better than fold boundary as it require participation of 9 bit ie all the bits.

• Counter is also possible:

Ex. 789 456 123

↓ since addn satisfies commutative prop

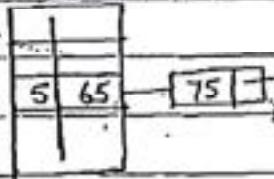
## Collision Resolution Technique

1. chaining

2. open addressing

chaining

• outside (add linked list)



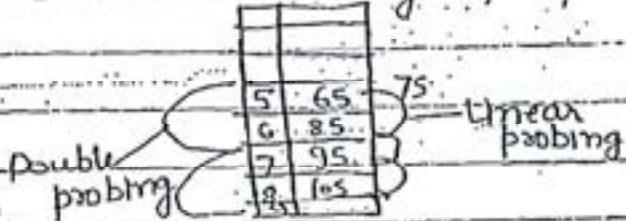
(If more than m keys are to be kept then chaining is used)

Open Addressing

- When only m keys are to be kept
- Inside (go to other slots)

which are empty

- Linear probing if 75 check for 5 fill 6 → fill and continuous checking is done when no empty space



- Quadratic probing where random jumps are made known as quadratic probing
- Double hashing when two values are directly mapped

Chaining Process -  $m=10$  (0-9)

key = (55, 98, 63, 75, 99, 73, 60, 95, 49, 29, 39, 73, 25, 108)

$$hf(key) = key \bmod m$$

Collision Resolution Technique = chaining

0	60							
1								
2								
3	63	73					Slot = 10	
4	44						key = 14	
5	55	75	95	25				
6								
7							7	
8	98	108					(Not part)	
9	99	29	39					

0	6000	6 Null	1 length					
1	Null	0						
2	Null	0 5000	5001	5002				
3	5000	63	73	73 Null	173 Null	3		
4	Null	0 4000						
5	4000	44 Null	1					
6	1000	55 1001	75 1002	95 1003	25 1004	Null		
7	Null	2000	2001				Length of Longest chain = 4	
8	2000	98 2001	108 Null	2				
9	3000	99 3001	29 3002	39 Null	Mim. chain length = 1			

Here searching time increases.

Avg =  $\frac{1}{10}$

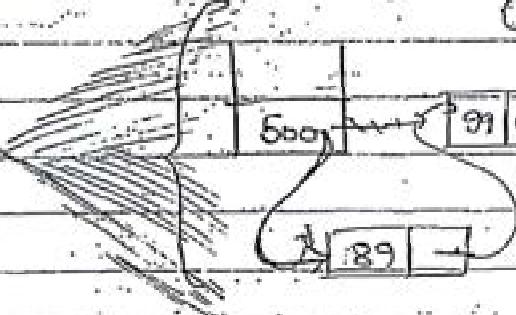
∴ in Worst case,  $TG = O(n)$ : as same if all element occur in 1 slot.

but if you be using better hashing function then it can be reduced as  $O(1)$ .

It is also

## drawbacks

1. In chaining, space is wasted in the form of linked list even the space available inside.
2. The length of the longest chain possible is  $n$ . that lead to worst case searching time =  $O(n)$ .  
Best Case =  $O(1)$   
Avg. Case =  $O(n/2) \approx O(n)$
3. the greatest advantage with the chaining is infinite no. of collision can be handled by it i.e infinite no. of keys can be stored because of linked list.
4. Insertion of one key will take  $O(1)$  time (BC, WC, AC)  
(as insertion is done in linked list only)



n elements  
insertion will take  $O(1)$  time.

5. Deletion time will take - Best case =  $O(1)$   
Worst case =  $O(n)$  (as it may be at last so you have to visit each node)

If address of node which has to be deleted is given then

Time =  $O(1)$  (BC) ~~W.C.~~

$O(n)$  - (WC)

b [since you cannot go back so you have traversed by start]

Solve More(k)

is to be deleted & its address is provided with do  
+ then time to delete =  $O(1)$   
 $= O(1)$

done in existence as in 1 slot you can store 100  
well.

Addressing - ~~an~~ it

probing -  $M = 10(0-9)$

Hashing) Keys = {55, 99, 60, 75, 89, 42, 58, 69, 25}

hash function (key) =  $k \bmod m$

C: R, T = Linear probing

img (key, i) =  $(h_f(\text{key}) + i) \bmod m$  → attempt  
no  
 $\forall i = \{0, 1, 2, \dots, M-1\}$

0	60	$LP(55, 0) = (h_f(55) + 0) \bmod 10$ here sign $= 5 \bmod 10$	0 here sign 1st attempt
1	89	$LP(99, 0) = 9$	→ (0) <span style="float: right;">N/A</span>
2	42	$LP(60, 0) = 0$	→ (0) <span style="float: right;">N/A</span>
3	69	$LP(75, 0) = 5$ (fail collision)	
4	75	$LP(75, 1) = 6$ (pass)	→ (2)
5	25	$LP(89, 0) = 9$ (fail)	
6	58	$LP(89, 1) = 0$ (fail)	→ 2
7	99	$LP(89, 2) = 2$ (pass)	

$LP(42, 0) = 2$	→ (0)
$LP(58, 0) = 8$	→ (0)
$LP(69, 0) = 9$ (fail)	
$LP(69, 1) = 0$ (fail)	$LP(69, 4) = 3$ (fail)
$LP(69, 2) = 1$ (fail)	
$LP(69, 3) = 2$ (fail)	(4) (4)

Total Collision = 9

5

You can store only  $m$  elements in  $M$ -slots.

To Retrieve - same for 75  $\rightarrow$  goto 5 if there Yes  
else (f1)

Quadratic

Double Hashing - two hash functions -  $h_1$  and  $h_2$

Quadratic probing (probing) -  $h_1$  and  $h_2$

Quadratic Probing  $M = 10$

key { 25, 39, 46, 55, 89, 23, 68, 70, 94 },  $h_1$

$h_1(\text{key}) = \text{key mod } m$        $c_1 = c_2 = 1$       78

CRT = QP

$$QP(\text{key}, i) = (h_1(\text{key}) + c_1 \cdot i + c_2 \cdot i^2) \bmod m$$

quadratic  
probing

i	75	$\forall i = \{0, \dots, M-1\}$
0	75	$QP(25, 0) = 5$ (0) collision
1	89	$QP(39, 0) = 9$ (0)      Total collisions = 2
2	23	$QP(46, 0) = 6$ (0)
3	94	$QP(55, 0) = 5$ fail
4	25	$QP(55, 1) = 7$ (pass) (0)      if we choose 6 probe = 4 will
5	46	$QP(89, 0) = 9$ fail (1)      i.e. collision de
6	55	$QP(89, 1) = 1$ Pass
7	68	$QP(23, 0) = 3$ (0) $QP(56, 0) = 6$
8	39	$QP(68, 0) = 8$ (0) $QP(56, 1) = 6 + 2 = 8$
9	70	$QP(70, 0) = 0$ (0) $QP(56, 2) = 6 + 5 = 11$
		$QP(94, 0) = 4$ (0) $QP(56, 3) =$

$$\begin{array}{ll}
 QP(78,0) = 8 \text{ (fail)} & QP(78,3) = fail \\
 QP(78,1) = 0 \text{ (fail)} & QP(78,4) = 8 \text{ fail} \\
 QP(78,2) = 14 \text{ (fail)} & QP(78,5) = 8 \text{ fail} \\
 \\ 
 Q(78,6) = 0 \text{ (fail)} & Q(78,7) = 4 \text{ (fail)} \\
 \\ 
 \cancel{Q(78,8) = 2 \cancel{\text{pass}}} \text{ fail} & \cancel{Q(78,9) = fail}
 \end{array}
 \quad \left. \right\} \text{to collie}$$

$\therefore$  Total collision = 12

In Worst case, M-1 to 0 is covered  $\therefore O(m)$  you have to check.

$\therefore$  Worst case =  $O(m)$

- Quadratic probing is faster than linear probing. But it will generate inaccurate result (as gap may be there but you are not able to keep the element)

If Linear Probing is used, if an empty slot is present, element will surely get stored.

In Linear Probing, Worst case searching time = O(m)

Best case = O(1)

Average Case =  $O(m)$

Double Hashing  $M = 10$

keys = (25, 39, 46, 55, 89, 23, 68, 70, 94, 78)

$h_{j_1}(\text{key}) = \text{key mod } m$

$h_{j_2}(\text{key}) = 1 + (\text{key mod } (m-1))$

CRT = Double hashing

$$DH(\text{key}, i) = (h_{j_1}(\text{key}) + ih_{j_2}(\text{key}))$$

$\forall i = (0 \text{ to } M-1)$

0	70	$DH(25, 0) = 5$
1	89	$DH(39, 0) = 9 + 0 \cdot (1) = 9$
2	78	$DH(46, 0) = 6 + 0 \cdot (2) = 6$
3	55	$DH(55, 0) = 5 + 0 = 5 \text{ (full)}$
4	94	(4) $DH(55, 1) = 5 + 1 \cdot h_{j_2}(55) = 5 + 1 + 55 \bmod 8$ $= 5 + 8 = 13 \bmod 10 = 3$
5	25	
6	46	$D(89, 0) = 9 \text{ (full)}$
7	23	(4) $D(89, 1) = 9 + (1 + 89 \bmod 8) = 11 \bmod 10$ $= 1$
8	68	
9	39	$D(23, 0) = 3 \text{ (full)}$
		$D(23, 1) = 3 + (1 + 23 \bmod 8)$ $= 11 \bmod 10 = 1 \text{ (full)}$

$$D(23, 2) = 3 + 2(8) = 19 = 9$$

$$D(68, 0) = 8 \text{ (pass)} \quad (o)$$

$$D(70, 0) = 0 \text{ (pass)} \quad (o)$$

$$D(94, 0) = 4 \text{ (pass)} \quad (o)$$

$$D(78, 0) = 8 \text{ (full)}$$

$$(4) D(78, 1) = 8 + 1(1 + 6) = 15 \bmod 10 =$$

$$D(78, 2) = 8 + 2(7) = 22 \bmod 10$$

$$= 2 \text{ (pass)}$$

Total collision = 1

Problem 1 Apply all 3 open addressing strategies.

$m = 10$

key = 25, 15, 90, 75, 69, 59, 44, 58, 85

	0	90	(Linear probing)	Quadratic probing
	1	59		
	2	85	Total collision = $1+2+2$ = 5	0 90
	3			1 75
	4	44		2
	5	25	(in linear probing, order of occurrence in sequence matters.)	3 25 T collision
	6	15		4 44 = 1 + 2 + 1
	7	75		5 25 + 15
	8	58	If 25 came first it occupy	6
	9	69	the first position 5	7 15 R3
				8 58 3 + 1
				9 69
<ul style="list-style-type: none"> <li>If you want to find any element and if gap occur then element is not present in table.</li> </ul>				

- 1. We are not wasting space outside in the form of linked list
  - 2. Searching time =  $O(1)$  for Best case  
 $O(m)$  for Worst case
  - 3. Insertion time =  $O(1)$  for Best case  
 $O(m)$  for Worst case
  - 4. Deletion time =  $O(1)$  for Best case  
 $O(m)$  for Worst case
  - 5. Whenever delete a particular element, place \$ otherwise at a time of searching, gap occurs & it may lead to mis. searching.

if your table

- ⑨ If you delete one element, it will create trouble to other elements but we can manage with help of \$ symbol if more dollar symbol than rehash again.

### Quadratic Probing

1. No space is wasted outside in form of linked list.

2. Search time  $\Rightarrow O(1) = BC$   
 $O(m) = WC$

3) Insertion time  $\Rightarrow O(1) = BC$   
 $O(m) = WC$

3) Deletion time  $\Rightarrow O(1) = BC$   
 $O(m) = WC$

4. deletion of one element will trouble to other element but can manage with \$, if more deletion, then rehash again.

### Double Hashing

0	90	$DH(25,0) = 5$
1	69	$DH(15,0) = 5$ fail
2	15	$DH(15,1) = 5 + 8 = 13 = 3$
3	15	$DH(90,0) = 90$
4	44	$DH(75,0) = 5$ fail
5	25	$DH(75,1) = 5 + 1 = 6$
6		
7	59	
8	55	
9	75	

Ex3 -  $m=10$

keys = 53, 60, 80, 82, 91, 80, 90

$$h_f(k) = k \bmod m$$

CRT = linear probing

0	60	$LP(60, 0) = 0+0 = 0$
1	91	$LP(91, 1) = 1+1 = 2$
2	82	$LP(82, 2) = 2+2 = 4$
3	53	$LP(53, 3) = 3+3 = 6$
4	80	$LP(80, 4) = 4+4 = 8$
5	90	$LP(90, 5) = 5+5 = 10$
6		$LP(90, 6) = 6+6 = 12$
7		$LP(90, 7) = 7+7 = 14$
8		$LP(90, 8) = 8+8 = 16$
9		$LP(90, 9) = 9+9 = 18$

1 person                      m+1 person

Avg -  $\frac{1+2+3+\dots+m}{m} = \frac{m(m+1)/2}{m} = \frac{m+1}{2}$

If Primary Clustering - if two keys same initial hash address they both will follow same path unnecessarily. In linear fashion because of this avg. time increases, this problem is known as primary clustering.

Linear probing is suffering with primary clustering.

Avg Case =  $1+2+3+\dots+m$

$$= \frac{m(m+1)}{2} = \frac{m+1}{2}$$
 as it suffers with the problem of primary clustering

as 2 will suffer with problem of 1.

using Quadratic Probing

$$O(k^2)$$

0	60	$QP(80, 0) = 0 \text{ (fail)}$
1	91	$QP(80, 1) = 0 + 1 + 1 = 2 \text{ (fail)}$
2	92	$QP(80, 2) = 0 + 2 + 2^2 = 6 \text{ (true)}$
3	53	
4		
5		$QP(90, 0) = 0 \text{ (fail)}$
6	86	$QP(90, 1) = 0 + 1 + 1 = 2 \text{ (fail)}$
7		$QP(90, 2) = 0 + 2 + 2^2 = 6 \text{ (fail)}$
8	90	$QP(90, 3) = 0 + 3 + 3^2 = 12 = 2 \text{ (fail)}$
9		$QP(90, 4) = 0 + 20 = 0 \text{ (fail)}$
and		$QP(90, 5) = 0 \text{ (fail)}$
on		$QP(90, 6) = 2 \text{ (fail)}$
		$QP(90, 7) = 8 \text{ (true)}$

Here primary clustering occurs as what path is followed by  
 1 element will definitely followed by 2 element  
 but here Avg. case deg decrease due to large jump  
 Here clustering is known as secondary clustering

Secondary Clustering: If two key contain same starting hash address, they both will follow same path unnecessarily in quadratic manner, because of this avg. searching time increases (it is less than linear search probing time), this problem is known as secondary clustering.

Quadratic probing is suffering with secondary clustering

Avg case =  $O(m)$  but actually it is less than  $(m+1)/2$   
 (Asymptotically same but mathematically less)

## Double Hashing:

		80	
0	60		$DH(80, 0) = 0$
1	91		$DH(80, 1) = 1$
2	82		$DH(80, 2) = 2$
3	53		$DH(80, 3) = 3$
4	80		$DH(80, 4) = 4$
5			
6	90		$DH(90, 0) = 0$
7			$DH(90, 1) = 3$
8	80		$DH(90, 2) = 0 + 6$
9	80		

Here path is diff with initially address (same).

Here diff path is carried out with same initial address... no path will be same.

$$DH(100, 0) = 0$$

$$DH(100, 0) = 0 + 5 = 5 \text{ (True)}$$

$$\text{Avg} = \frac{3 + 5 + 7 + \dots + 2}{m - \text{acl}} \quad \begin{aligned} &\text{as a particular slot is covered} \\ &\text{by single element only} \end{aligned}$$

(It might be possible that there must be some cases which lead to redundancy)

there in avg. case, we have to consider for all the given case so it on an avg. it is said that it is repeating one time).

In Double Hashing, Even though two key contain same starting hash address, they both will follow diff. slot because second level hash fn will give diff. value.

because of this reason 99% of redundancy is eliminated.  
 slot covered by I element

Avg case =  $3 + 5 + 7 + \dots + 2$

$\downarrow$   
 slot covered

by II element

∴ on an avg

$$= \frac{m}{m} \Rightarrow cm = O(1)$$

Avg case =  $O(1)$

(as only few key will be repeating only)

let 4 key repeated

$$\text{total } = \frac{4m}{m}$$

$$= O(4)$$

In double hashing, secondary clustering problem is present littlebit (1%).

Note - Using Perfect Hashing, you will get Worst case Searching time =  $O(1)$ .

In hashing if collision occur to a, b, c at slot 5, they will go to 5 which have another his which is best  $f^n$  and a hash table to store those three element their collision is removed perfectly.

0			
1			
2			
3			
4			
5	hs		
6			
7			
8			
9			

a	b	c
0	1	2

$$\text{Space} = O(m^2)$$

You cannot decide of size so use m size else use linked list

so space increases but time decreases



m-slots n-keys

m slots - will store n keys

1-slots will store  $\frac{n}{m}$  keys

↓

Load factor - no of keys getting stored in 1 slot

Load factor ( $\alpha$ ) =  $\frac{n}{m}$  keys per slot

Note 1 - the expected no of probes (attempt) in an unsuccessful search of an open addressing technique is

$$\left[ \frac{1}{1-\alpha} \right]$$

Note 2 - the expected no of probes in successful search of an open addressing technique is  $= \left[ \frac{1 + \log_e \frac{1}{1-\alpha}}{\alpha} \right]$

(Cormen)

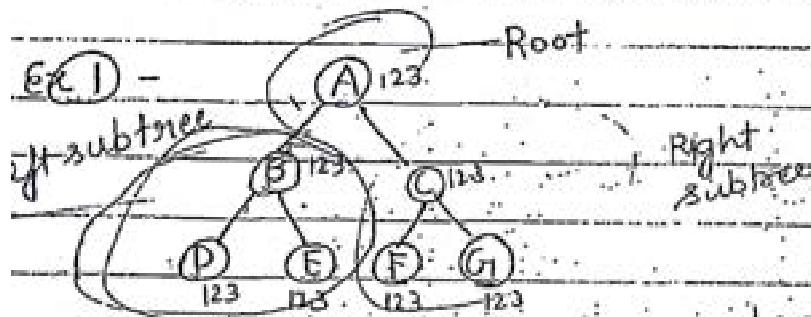
## TREE AND GRAPH TRAVERSALS (2 Marks)

### B Tree Traversals -

1. Preorder (Root LST. RST.)

2. Postorder (LSTRST Root)

3. Inorder (LST Root RST)

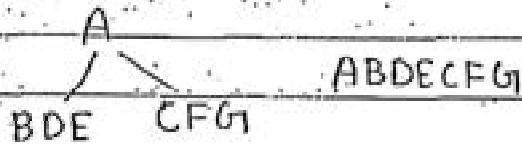


tree is best solved  
by recursion as of  
having almost same  
quality;

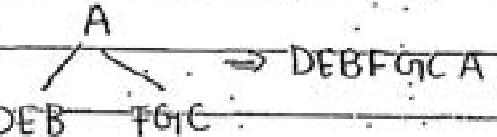
Traversal - visiting each node one by one in given tree.

Point - Left & Right tree cannot changed i.e. left will always come before Right subtree  
if changed  $\rightarrow$  3!

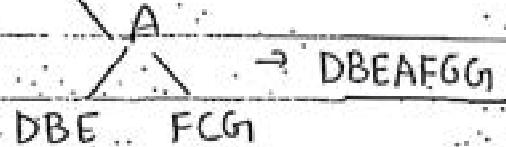
Preorder - ABDEC<sub>FG</sub>

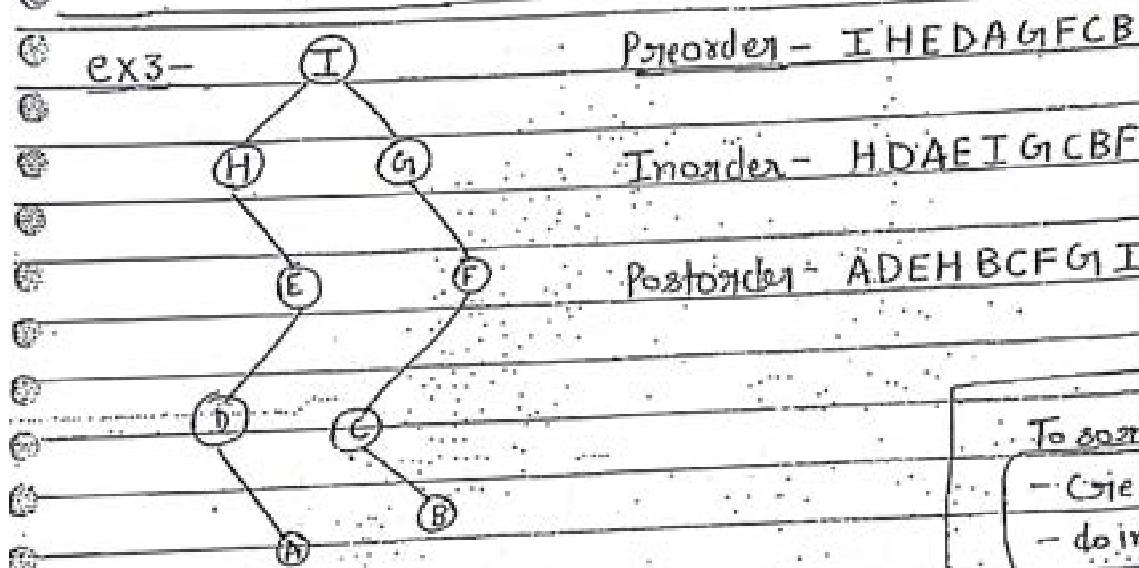
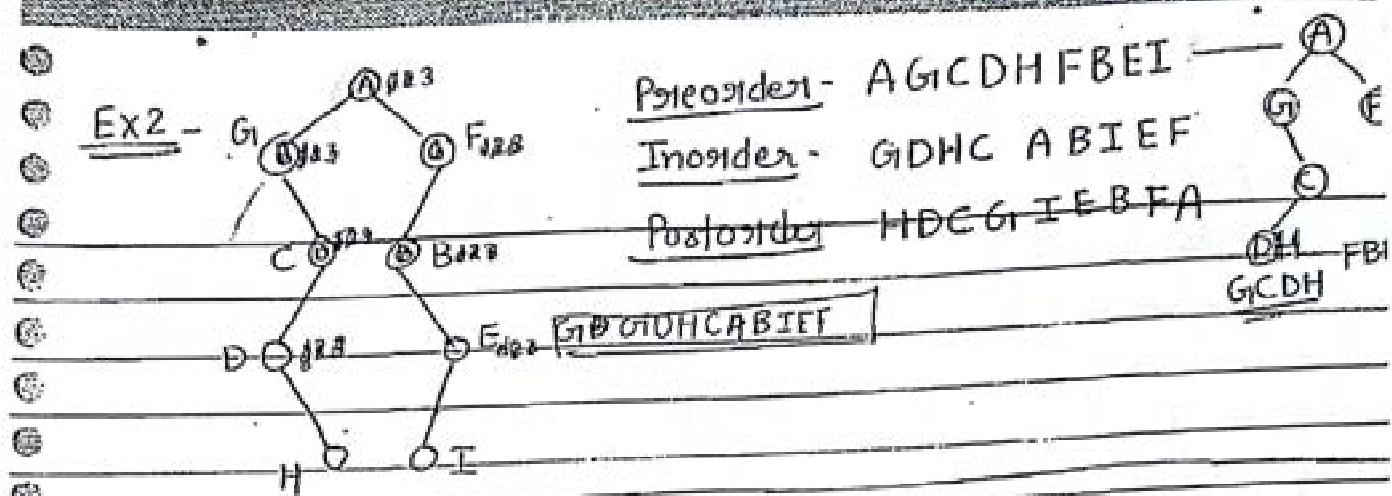


Postorder - D<sub>E</sub>B F<sub>G</sub>C A



Inorder - D B E A F C G

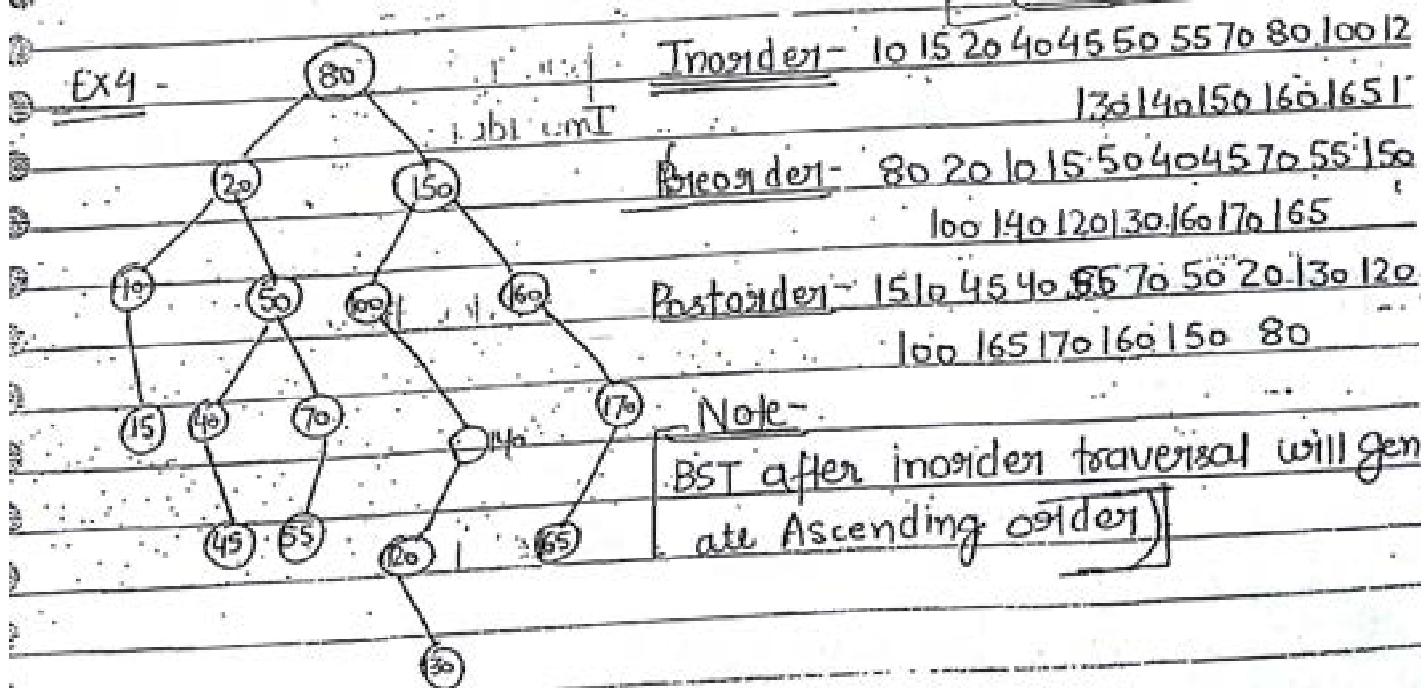




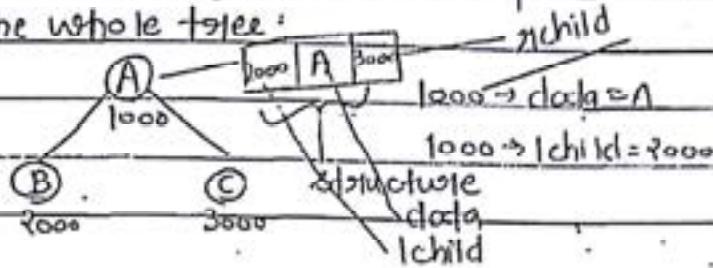
To sort an array

- Create BST

- do inorder traversal



Whenever a tree is given, a  $\text{root}$  is provided which can be used to traverse the whole tree.



Preorder( $\text{root}$ )

$\text{print}(\text{root} \rightarrow \text{data})$ ;  $\text{preorder}(\text{root} \rightarrow \text{lchild})$

$\text{preorder}(\text{root} \rightarrow \text{lchild}), \neg T(n)$   $\text{preorder}(\text{root} \rightarrow \text{lchild})$

$\text{preorder}(\text{root} \rightarrow \text{lchild}), \neg T(n)$

$\text{root} = 1000$

$\text{preorder}(1000)$   
A

$\text{preorder}(2000)$

and so on.

postorder( $\text{root}$ )

$\text{postorder}(\text{root} \rightarrow \text{lchild})$ ;

$\text{postorder}(\text{root} \rightarrow \text{rchild})$ ;

$\text{print}(\text{root} \rightarrow \text{data})$ ;

Inorder( $\text{root}$ )

$\text{Inorder}(\text{root} \rightarrow \text{lchild})$ ;

$\text{print}(\text{root} \rightarrow \text{data})$ ;

$\text{Inorder}(\text{root} \rightarrow \text{rchild})$ ;

[for preorder  
of a binary tree]

$$T(n) = 2T\left(\frac{n}{2}\right) + O(1) \text{ (by master theorem)}$$

$= O(n)$  (Best case)

Worst partition-

$$T(n) = T(n-1) + T(0) + O(1)$$

$= O(n)$  (Worst case)

for preorder,  
postorder,  
Inorder

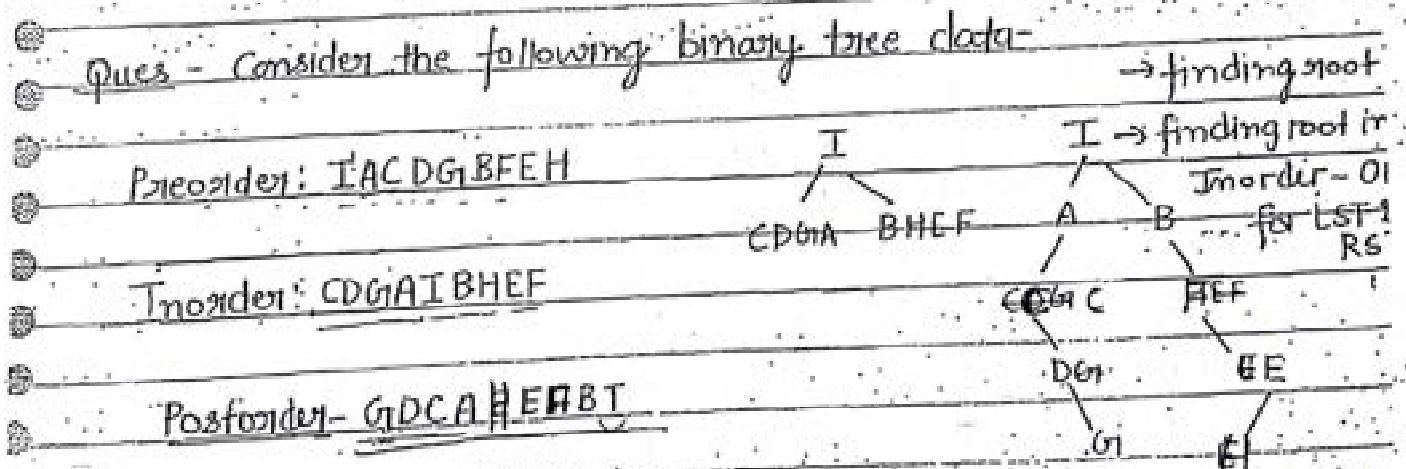
Avg case =  $O(n)$

WC, BC, AC same as each node has to visit once

Recurrence relation can also tell about stack space  
 $T(n/2) + \log n$   
 $T(n-1) = n$

Note -

- Preorder Inorder Postorder will take  $O(n)$  time on  $n$ -node binary tree (WC, AC, BC), because each node has to be visited once.
- Stack complexity =  $O(\log n)$  for balanced tree (Best case)  
 $O(n)$  for unbalanced tree (Worst case)
- Ques - if a BST is given, time to get sorted order -  $O(n)$
- Ques - if a BST is given, time to sum element  $\rightarrow O(n)$  - Worst case  
 $O(1)$  - Best case (start from 20 when 20 came & add 50)



Preorder can give root as well as postorder.  
Inorder traversal cannot judge for preroot because it cannot tell are not balanced always.

Inorder is used to give Left subtree & right subtree

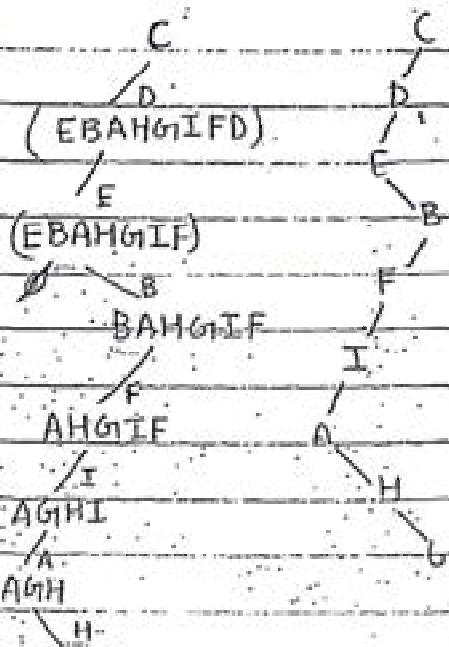
$$\therefore \text{Total complexity} = n \times O(n) \left[ \begin{array}{l} \text{for LST & RST} \\ = O(n^2) \end{array} \right]$$

Ques - Consider the following binary Tree data -

Postorder - GHAIFBEDC

Inorder EBAHGIIFDC

breaden - CDEBFIAHGI



(X = sometimes possible)

To find out directly any order

find out root

find its pos<sup>n</sup> in inorder case

the digit at pos<sup>n</sup> element in inorder

try to find them in postorder just

reverse the order and vice versa

a shortcut Not applicable everywhere

Ex - Here postorder & Inorder given, C its position in inorder  
right most so C has empty right ST.

then go to postorder get the element in LST of C in Inorder

take them & reverse them to obtain Post order

Note - To construct Binary tree for the given preorder inorder  
or postorder inorder will take O( $n^2$ ) time (n times linear  
search to find LST & RST in given inorder)

$$(n^2 + n + \Theta)$$

↓

to find LST & RST      to find root      n times

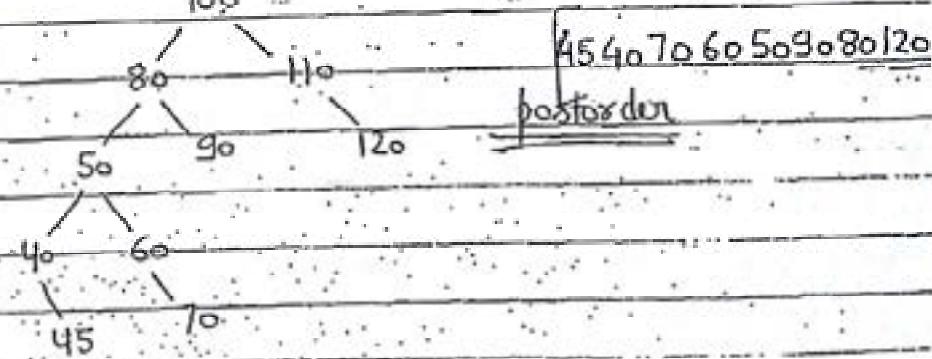
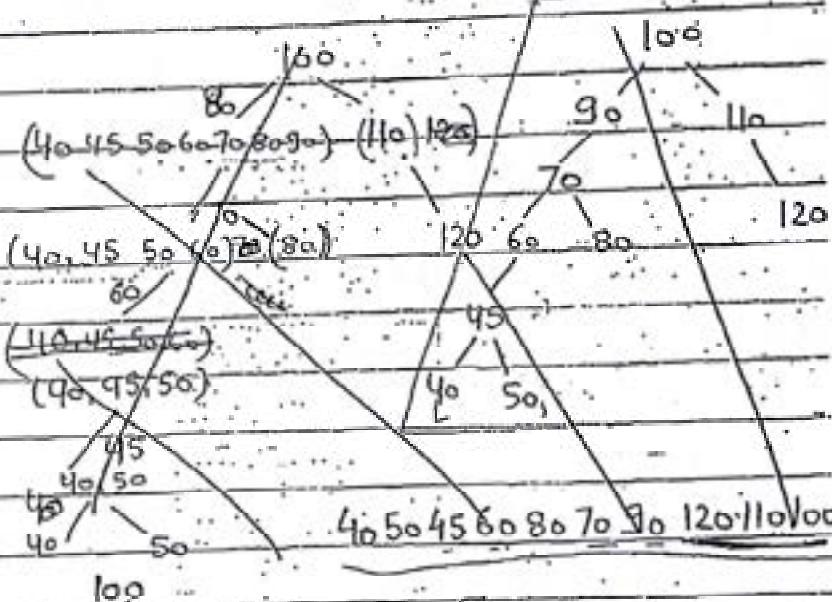
We cannot apply binary search as given sequence is not sorted but if BST it will take  $n \log n$  time.

Ques - Consider the following BST data-

Pre : 100 (80 50 40 45 60 70 90 110 120)

Post : 90 70 60 110 40 50 80 120 100

In- (40 45 50 60 70 80 90 100 110 120) (A.O.)



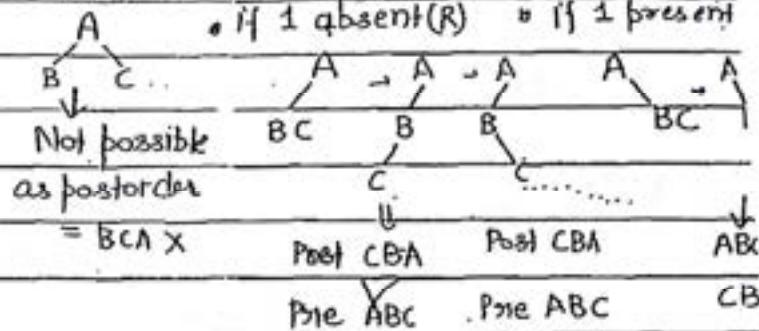
Note - for the given preorder inorder for postorder inorder to construct equivalent BT require  $(n \log n)$  time if inorder is already sorted.

Preorder: ABC  
NLR

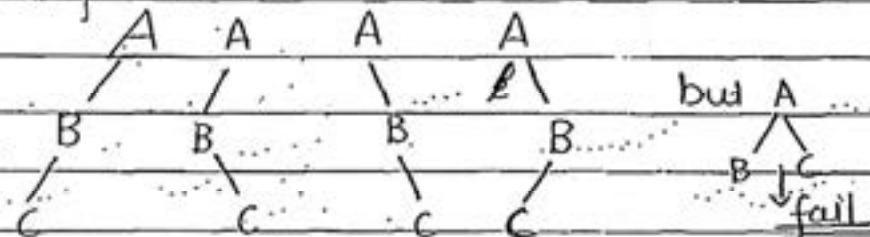
Root = A

• Let both RST & LST present

Postorder: CBA  
R LN



∴ possible trees are



Note

If Preorder Inorder given - unique binary tree  $\rightarrow$  Binary tree  
postorder Inorder given - Unique Binary tree  $\rightarrow$  Unique BT  
if Preorder postorder given - more than one binary tree  $\rightarrow$  Binary tree pos<sup>n</sup> but not.

To construct unique binary tree, Inorder is required (compulsion)

When pre, post is given, manual checking is done To each node two possibilities either going to left or right so total possibility for n node =  $O(2^n)$

Upper bound as root will have only one possibility.

To find out cycle initially flag initially 0 if visited flag = 1 if again count flag = 1 if again count then it is a cycle.

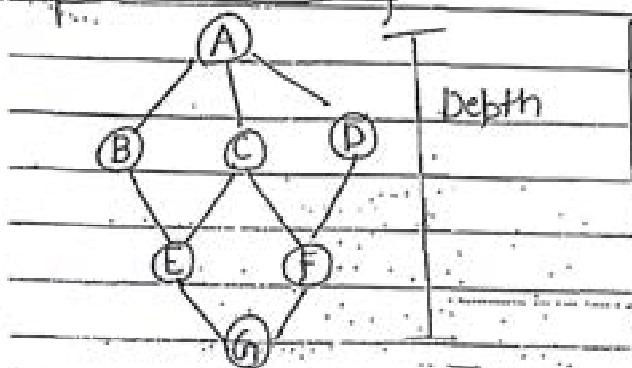
## Graph Traversal

1. BFT (Breadth First Traversal)

2. DFT (Depth First Traversal)

Total Graph Traversal time =  $O(V+E)$

Breadth



By visiting each vertex & each edge

We do not consider edge in tree & edge is less than  $E$  vertices

$$Tree = (V+E) \cdot 1$$

$$V+E = 1$$

$$= 2V-1 = O(V)$$

Breadth first traversal - ABCDEFG

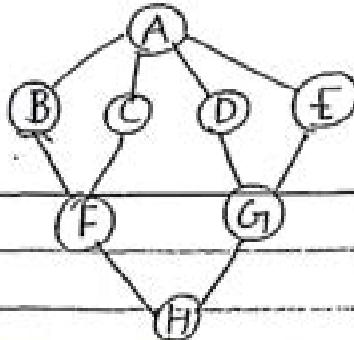
Depth " " ABEGFCFD

Breadth first Traversal - Tree traversal are unique but graph traversal not.

Visited is required as graph contains cycle and there is no mean of visiting a vertex again & again.

Queue is used in BFT as we need FIFO structure

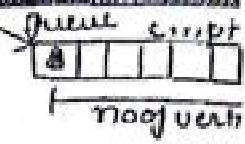
While loop is covering vertex & for loop is caring for edges.



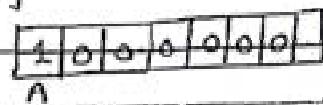
Initially

Visited = 0 0 0 0 0 0 0 0

Starting with A



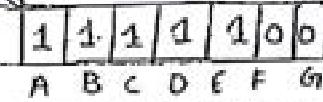
Ex



$x = A$  (delete)

$w = B \ C \ D \ E$

Visited



BFT(v)

B deleted

$w = A \ F$

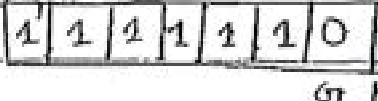
$q$  not added again

Visited[v] = 1

add(v, q);

while (q is not empty)

Visited



$q$  not added again

$x = \text{delete}(q)$

C deleted  $w = A$

print(q)



$q$  not added again

for all w adjacent to x



D deleted  $w = A$

if (visited[w] = 0)

G

E deleted  $w = A$

visited[w] = 1



F deleted  $w = A$

add(w, q)



G deleted  $w = A$

b



O/P → A B C D E F G H

(Order of deletion of element)



H deleted  $w = A$



G deleted  $w = A$



F deleted  $w = A$



E deleted  $w = A$



D deleted  $w = A$



C deleted  $w = A$



B deleted  $w = A$



A deleted  $w = A$



empty

loop end

Note -

- To implement BFT, we use queue data structure.

- Total complexity =  $O(V+E)$  (Time) (Bc, wc, nc)

If adj matrix is used  $TC = O(V^2)$

Space  $\rightarrow$  i/P + extop

$$\frac{V+E}{V+V}$$

If graph is adjacency list

↓ Queen visited array

$$= (V + E + 2W)$$

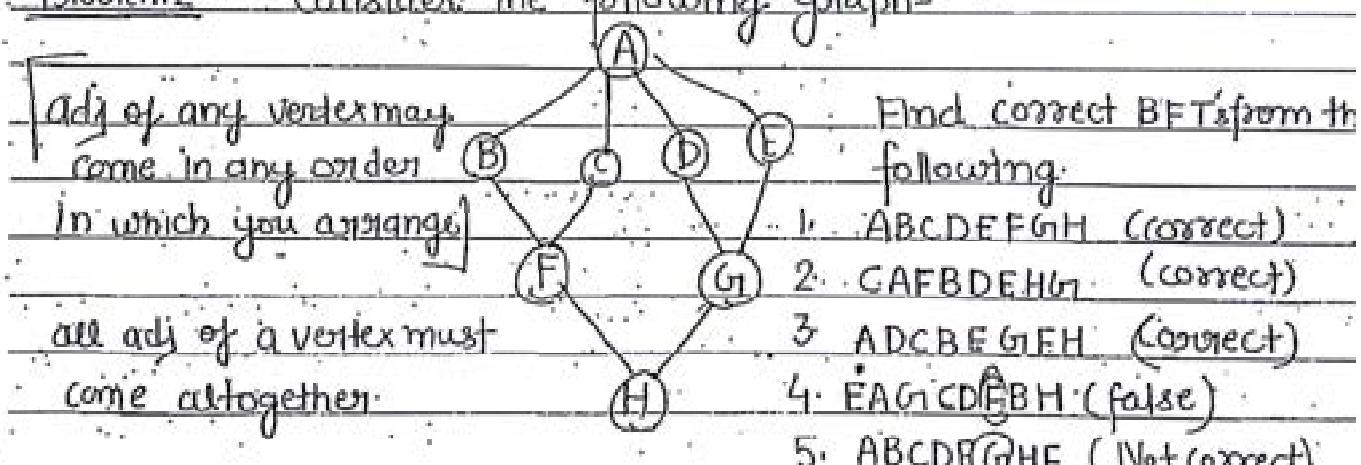
$$= 3y + \sqrt{y}$$

$= O(V+E)$

$= O(v^2)$  if adj. matrix is used

- BFT is also known as Level order traversal i.e level by level printing of nodes. (means adjacency list)

Problem 2 – Consider the following graph:



Ques - Consider the following graph-

(A) — (B) — (C)

(D)  
|  
(E) — (F) — (G)

Whether these are

correct BFTs or not.

(a) FBDCAF (correct)

(c) CBDAEFG (correct)

(b) GFCEDBA (incorrect)

(d) DBCFEAG (correct)

## Application of BFT

Ques - If a graph is given, how to check whether it is connected or not?

• Draw & Apply BFT

• Check visited array

• If any 0 in visited array

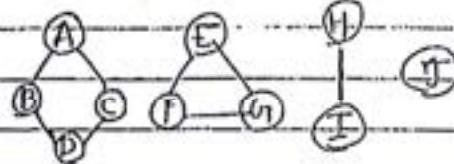
to check for 0

• Then graph disconnected:  $O(V+E)+V$

If a linked list is given, to check whether it is connected?

A linked list is also a graph but directed so apply the same procedure.

We can verify given graph is connected or not.



skating from A

$\therefore \text{count} = 1$  (initially zero)

- When you get T zero, just stop & again apply the DFT again and so on, you can find ~~disconnected~~ connected components can be find out again

A B C D E F G 0 0 0 -- Count = 2

## ગુરુત્વ

A	B	C	D	E	F	G	H	I	J
---	---	---	---	---	---	---	---	---	---

count=3

County

$\therefore$  No of connected component = No of time BFT applied

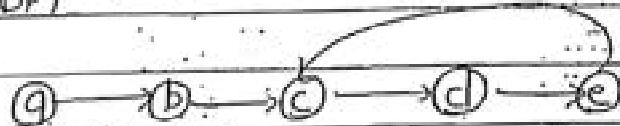
$\therefore$  TC to find connected comp =  $O(V+E)$

: Using BFT we can find out connected components in the given graph.

if a null graph  $G(v, E)$  - no of connected component =  $v$

- 3. to check for cycle in a given graph
  - Apply BFT
  - If  $\text{visited}[i] = 1$  & it encounters again then there is a cycle.
- Time complexity =  $O(V+E)$  (as BFT work)
- 4. Whenever Kruskal is going on edges are added to MST we have to check for occurrence of cycle too as no cycle must occur, then by side BFT also working to check for  $\text{visited}[i]$  → it take constant time (for 1 edge)
- $T(n) = E \log E + (V+E)$
- ↓ for BFT cycle check

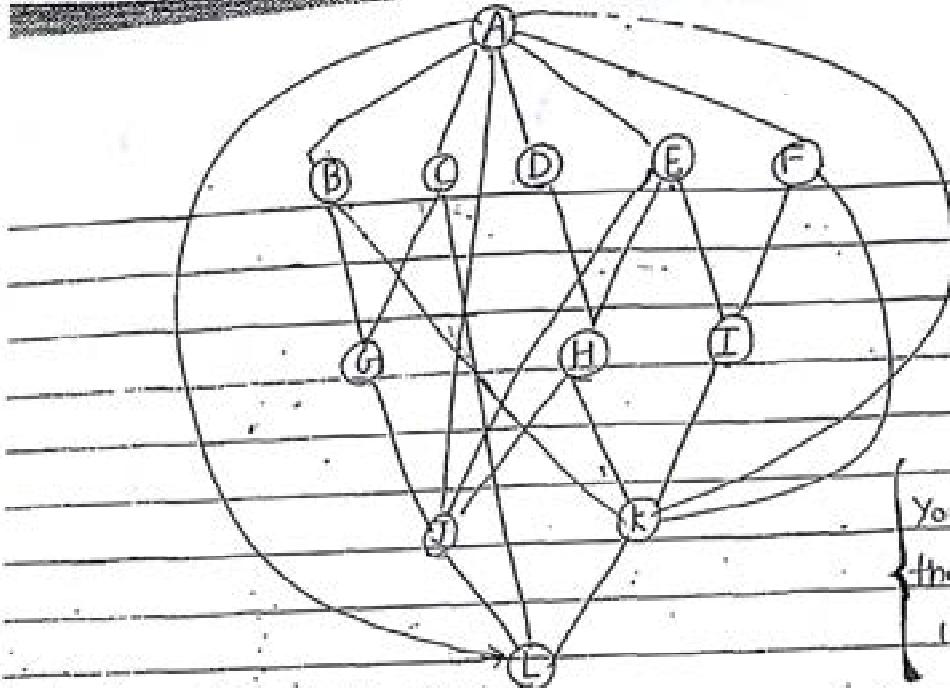
- 3. Using BFT, we can check given graph contain cycle or not
- If a linked list contain cycle, it can be checked easily by applying BFT



A <sup>V</sup> B <sup>V</sup> C <sup>V</sup> D <sup>V</sup> E <sup>V</sup>

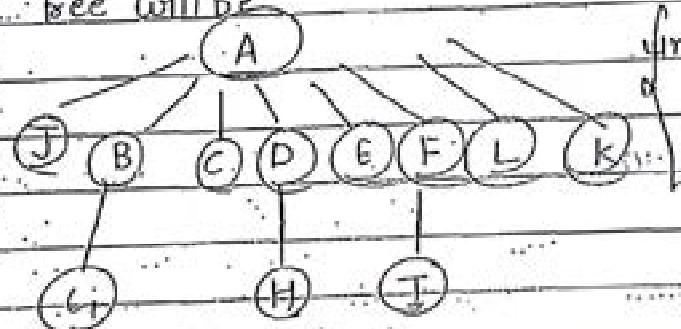
↑ diagram occur ∴ cycle exists

- 4. If an unweighted graph is given & single source ~~source~~<sup>shortest</sup> is not to be finded, then BFT is the efficient one algo as it will find the sh.
- ∴ Using BFT, we can find out shortest path from given source to every vertex in given unweighted graph or the graph having equal edge weight (true)



You can easily find out  
the shortest path  
using  $O(V+E)$  time

the BFT tree will be



unweighted graph  
default weight  
= 1

Ques - if a unweighted graph is given; the shortest path to be calculated from given source which data structure is used?

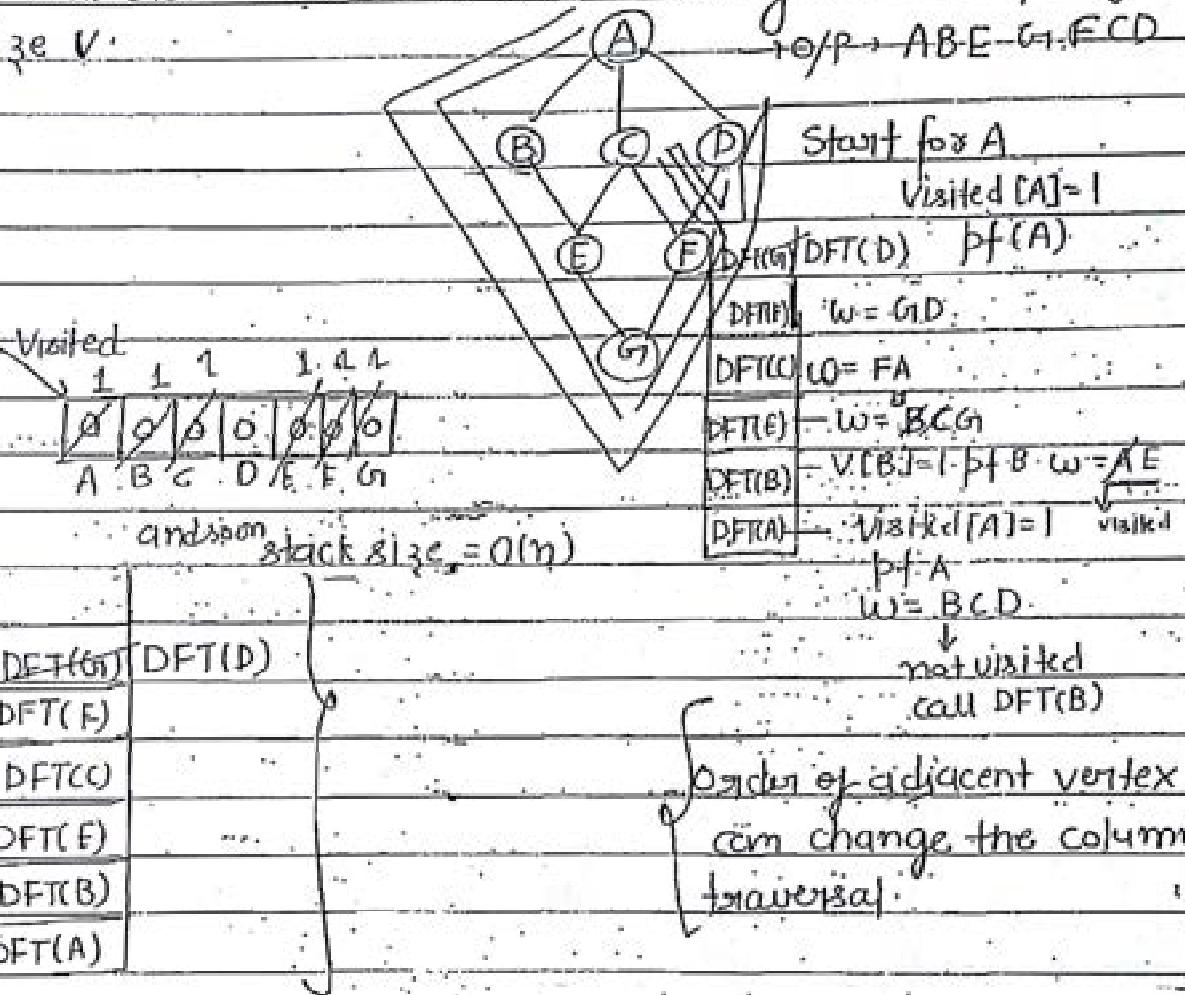
Queue

5. Using BFT we can verify a given graph is Bipartite graph?

## Depth First Traversal

- It uses stack as recursion occur here.
- it will also maintain a visited array to check for cycle.

size V



1. To implement DFT we are using stack.

2. Time Complexity = no of functional calls + work done at each func call

$$= V + E \text{ (as if work is summed up)}$$

$$= O(V+E)$$

3. Space Complexity = i/p + extra

$$= V+2E \downarrow \text{stack visit} = O(V+E)$$

$$V+2E \quad V \quad V$$

↓ for adjacency list

DFT(v)

{

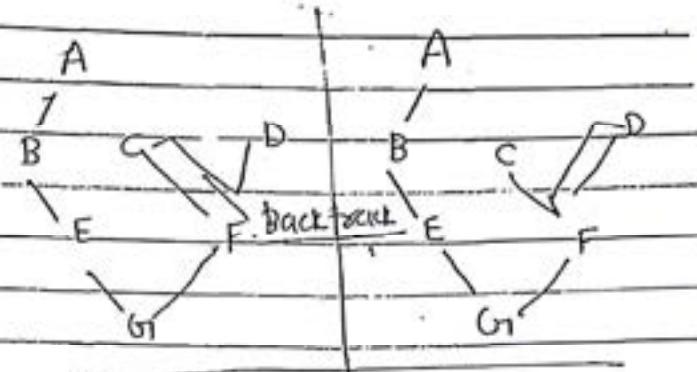
1. Visited[v] = 1

2.  $\text{df}(v);$

3. for all w adj to v

    1. if (w is not visited)

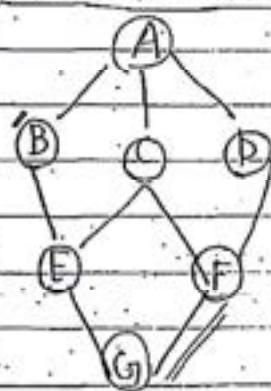
        2. DFT(w);



In case of Adjacency Matrix =  $O(V^2)$

(Space as well as time complexity)

Problem:- Consider the following graph-



Find Correct DFT's

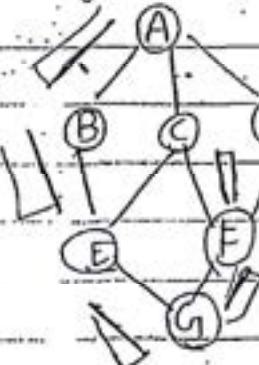
(a) ABEGFCD

You can take  
any of the  
adjacent of  
a particular  
element.

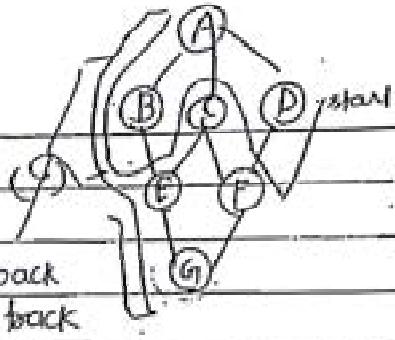
No of back track

(b) ABECFDG

backtrack  
occurred



Total Backtrack = 6



DFCEBAG<sub>1</sub> (correct)

• ? back  
back

(d) CFDABEG<sub>1</sub> (No Backtrack) (correct)  
(stack size = n)

[max stack size = O(n)]

↓ when no back track occurs

(e) GIEGFABD: (Incorrect) correct GFEDAB

(f) ACFGEBD (correct)

(g) BACEDEG<sub>1</sub> → (fail) Incorrect correct BACFDGE

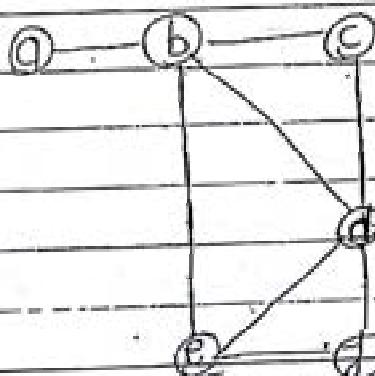
because at D when possibilities ended we back track to F & F has to be done but it is first computing by

Box { Backtracking is done when all possibilities ended }

Backtracking is nothing but popping

In DFT, if I print D after B, what is the relation b/w them  
No relation because of backtracking any thing possible in DFT but BFT have adjacency

Ques - Consider the following graph -



Check whether they are DFT or ~~BFT~~ or not.

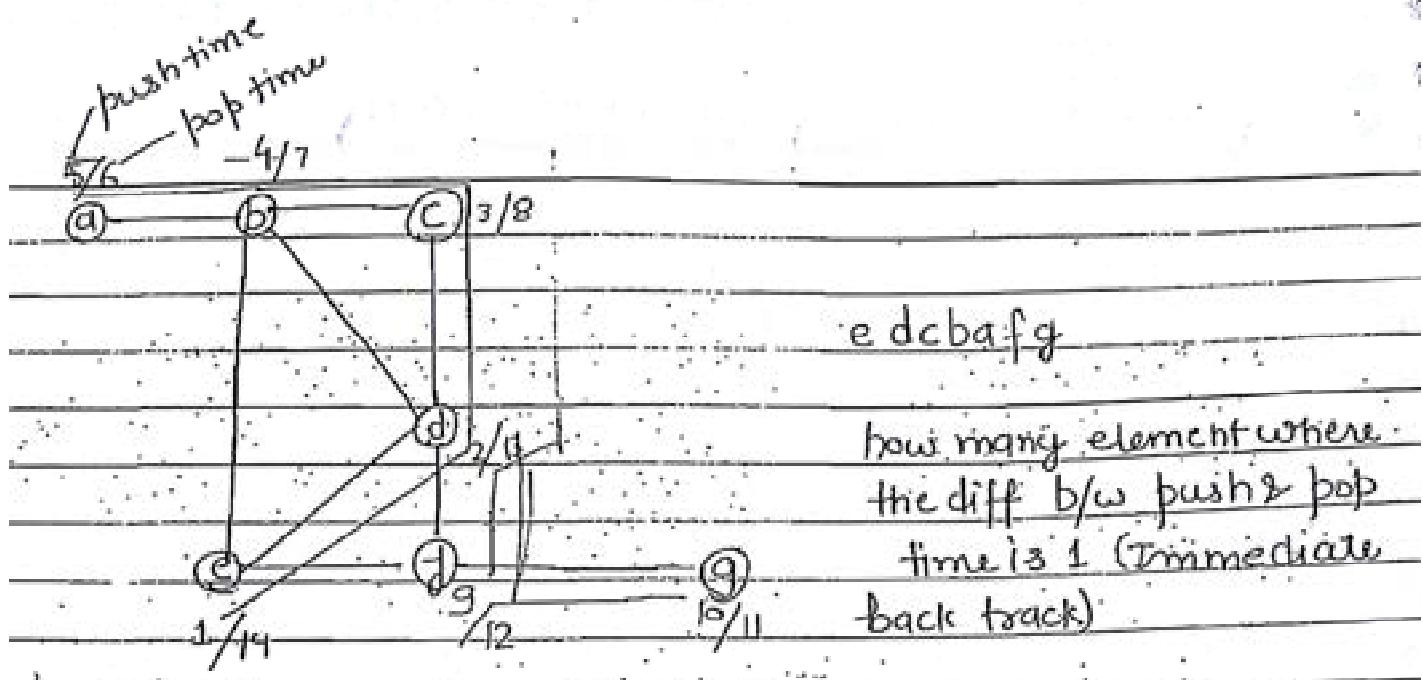
1) edcbafg (DFT)

2) jdebagc (incorrect)

3) abdjgec (incorrect)

4) dcabetga (correct)

5) befgdca (DFT)

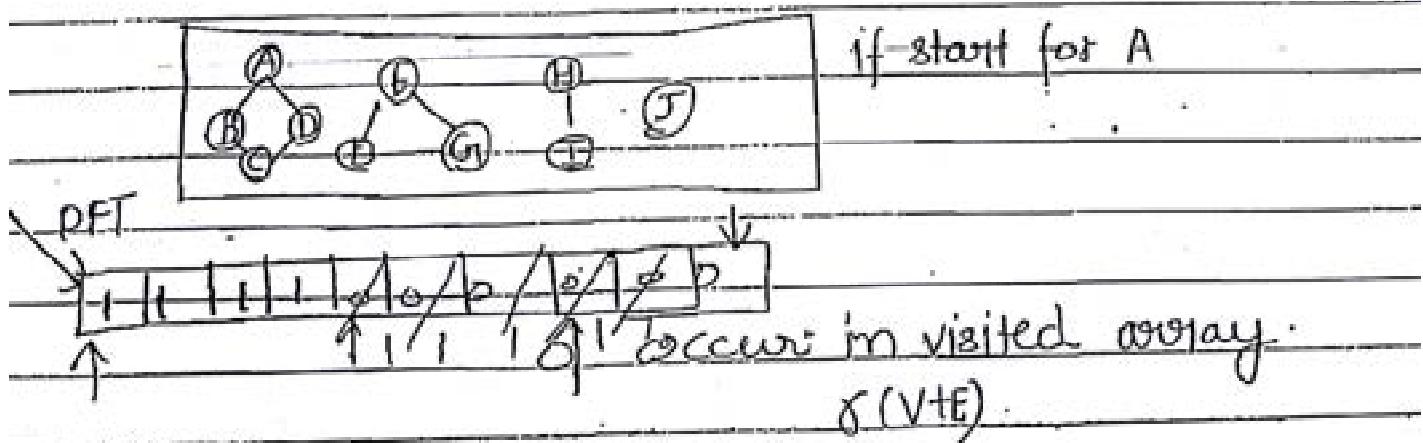


Total elements = 2 (q,g)

1) d c b e f g a  
 2) j/h 2/11 3/10 4/5 6/9 7/8 12/13  
 3) b e d g a c a.

## Applications of DFT-

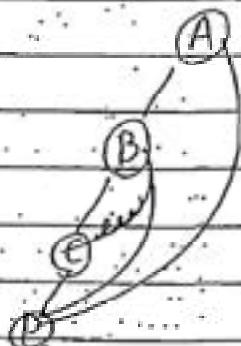
will check where graph is connected or not.



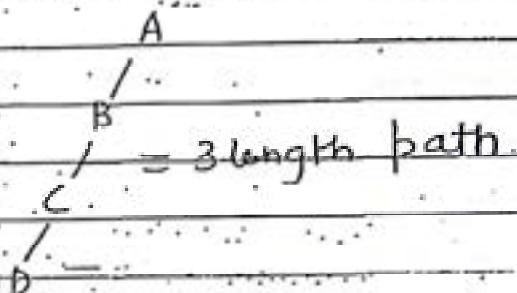
3) We can verify given graph contain cycle or not  
 $O(V+E)$

3. We can find out no of connected components  
Count = no of time DFT applied.

④



Here shortest path =  $A \rightarrow B \rightarrow (1)$   
but if we take

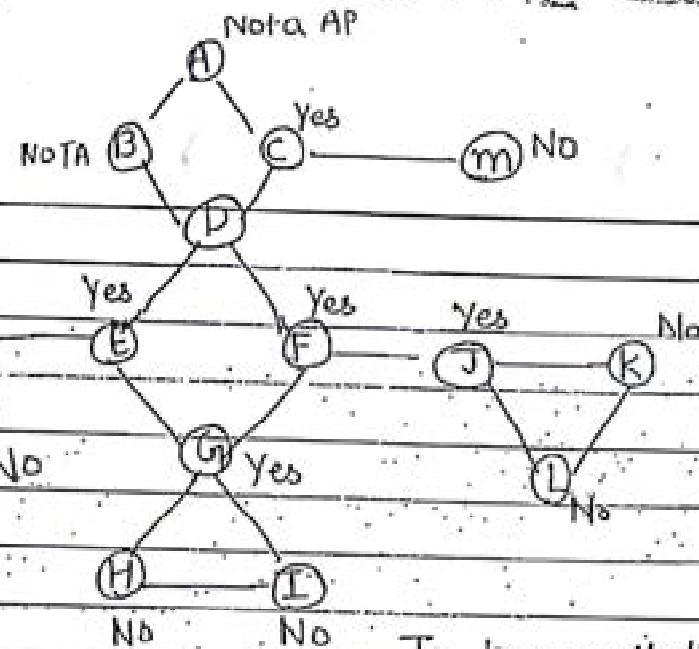


So, DFT cannot (may or may not) work finding single source shortest path for given unweighted graph.

5. Using DFT, we can check given directed graph is strongly connected or not?

6. Using DFT we can verify a vertex is articulation point or not.

Articulation point - if the deletion of an edge lead to vertex along with its edge lead to disconnection of graph known as articulation point.



To known whether a vertex is any articulation point-

b) Apply DFT on given graph

3) Delete that vertex & edge.

3) Again apply DFT

4) If Visited contain 1 for all except that delete vertex

then that is an articulation point

$$\text{Total Complexity} = O(V+E) + O(V+E)$$

$$= O(V+E)$$

## Bipartite Graph using BFT

first time visited in one set

second time another set

(level by level visiting of elements)

strongly connected using DFT

in finding strongly connected component

you can use the concept of strongly connected graph articulation point.

No of articulation point = No of strongly connected component + 1

$$\text{Total Complexity} = E(V+E+V+E)$$

for articulation point

$$= EV + E^2$$

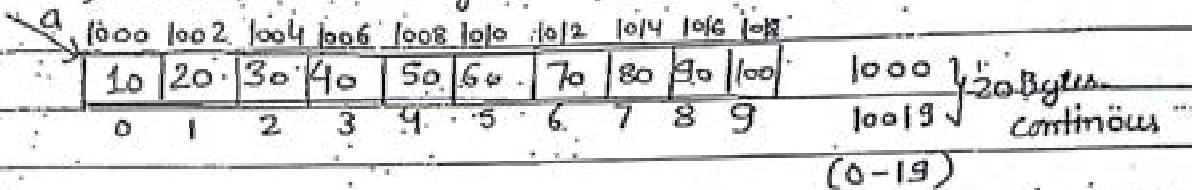
$$= O(E^2)$$

## Programming - I

Array - 1-D Array

Ex - 1

int q[10] = { 10, 20, 30, 40, ..., 100 }      q will store base address.  
 ↓ declaration (memory creation)



$$1 \text{ to } 7 \rightarrow \text{how many elements} = 7 - 1 + 1 = 7$$

$$\begin{array}{ccccccccccccc} -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & \dots & 4 - (-5) + 1 = 10 \text{ elements} \end{array}$$

$$2 - (-2) + 1 = 5 \text{ elements}$$

print q = 1000 (base address)

5 elements occupy

1000 to 1014

$$\text{Loc}[q[5]] = \text{base address} + \text{size of data type} * (5 - 0) \\ = 1000$$

\* 100 - to access that particular M/M location

$$\text{Loc}[q[9]] = \text{base address} + \text{size of int} * (9 - 0) \\ = 1000 + 2 * 9 = 1018$$

only 9-0 bco  
you don't use  
to access 9th  
element)

Ex 2 -

$$q[55 \dots 550] \quad \text{Total element} = 550 - 55 + 1 = 496$$

$$\text{Base address} = 990$$

size of an element = 10 byte (C)

$$\text{Loc}[q[450]] = \text{Base address} + C * [450 - 55]$$

$$= 990 + 10 * 395$$

$$= 3950 = \underline{\underline{4940}}$$

before 450 from  
55

- array must be continuous no matter whether indices are +ve or -ve.

Ques  $a[-55 \text{ to } 55]$   $BA = 1000$   $C = 5$

$$\begin{aligned} \text{Loc}[A[5]] &= BA + [5 - (-55)] \times C \\ &= 1000 + 60 \times 5 \\ &= 1000 + 300 \\ &= 1300 \end{aligned}$$

ex4  $a[lb \dots ub]$   $BA = \text{Base address}$

lower bound      upper bound      size = C  
total element =  $ub - lb + 1$

$$\boxed{\text{Loc}[a[i]] = BA + C * [i - lb]}$$

array index when started from 0, no need to do subtract at time to calculate location, but when it start with 1, a subtract has to be done. So less time is required to calculate the value when starting from 0.

~~$\text{Loc}[a[1 \dots 10]] = BA + (5-1) * C$~~  ~~offset calculation~~  
 ~~$a[0 \dots 10] = BA + (5-0) * C$~~

• Array index is, by default, will start from 0 only, because no need to calculate offset value.

row  
↓  
 $(0=3)$

column  
↑  
 $(0=1)$

$\Rightarrow (5-1)+1 = 5 \text{ column}$

	1	2	3	4	5
$\text{int } a[4][5]$	$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$	$a_{15}$
	$a_{21}$	$a_{22}$	$a_{23}$	$a_{24}$	$a_{25}$
total row	$a_{31}$	$a_{32}$	$a_{33}$	$a_{34}$	$a_{35}$
$(4-1)+1$	$a_{41}$	$a_{42}$	$a_{43}$	$a_{44}$	$a_{45}$
= 5 Row	$a_{51}$	$a_{52}$	$a_{53}$	$a_{54}$	$a_{55}$
Total element = 20 element					

In m/m 40 bytes for continuous m/m (no separate row & colur is present)

1000	02	04	06	08	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	1000
$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$	$a_{15}$	$a_{21}$	$a_{22}$	$a_{23}$	$a_{24}$	$a_{25}$	$a_{31}$	$a_{32}$	$a_{33}$	$a_{34}$	$a_{35}$	$a_{41}$	$a_{42}$	$a_{43}$	$a_{44}$	$a_{45}$	1000
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	1000
row 1	row 2	row 3	row 4		row 1	row 2	row 3	row 4		row 1	row 2	row 3	row 4		row 1	row 2	row 3	row 4		row 1

Row Major Order - When elements are getting stored row by row

Ex -

C follows Row Major Order

$$\text{Loc}[a[4][3]] = 1000 + [4-1]*2 + 1000 + 2(5*(4-1) + (3-1)) \\ = 1000 + 2[5+4] \\ = 1018$$

$$= BA + C [(ub_2 - lb_2 + 1)(x - lb_1) + (y - lb_2)]$$

$$= BA + C [(ub_2 - lb_2 + 1)(x - lb_1) + (y - lb_2)]$$

$$\text{Loc } a[2][5] = BA + C [5 * [2-0] + (5-1)] \\ = 1000 + 2[5+4] \\ = 1018$$

$$\text{Loc } a[i][j] = BA + C [(ub_2 - lb_2 + 1)(i - lb_1) + (j - lb_2)]$$

$\downarrow$  visit correct row.

$\downarrow$  visiting column

Ex-2

$a[25 \dots 750][80 \dots 150]$   $c = 10 \text{ byte}$

Row Major order  $BA = 1000$

$$\text{Loc}(a[550][140]) = BA + c[(150-80+1) * (140-25)(550-25) + (140-80)]$$

$$\begin{aligned} &= 1000 + 10[71(525) + 60] \\ &= 1000 + 10[373350] \\ &= 373350 + 1000 \\ &= 374350 \end{aligned}$$

Ex-3  $a[-25 \dots +25][-50 \dots 50]$ ,  $BA = 0$   $c = 1 \text{ Byte}$  RMO

$$\begin{aligned} \text{Loc } a[20][30] &= BA + c[(50+50+1)(20+25) + (30+50)] \\ &= 0 + 1[101 * 45 + 80] \\ &= 4545 + 80 \\ &= 4625 \end{aligned}$$

Ex-4  $a[lb_1 \dots ub_1, lb_2 \dots ub_2]$

$$\begin{array}{c} \downarrow \\ ub_1 - lb_1 + 1 \end{array} \quad \begin{array}{c} \downarrow \\ ub_2 - lb_2 + 1 \end{array}$$

$a[47][3]$

$BA, c$

$\downarrow n_c \text{ (no of column)}$

Row Major order  $a[i][j] = BA + c[(ub_2 - lb_2 + 1)(j - lb_1) + (j - lb_2)]$

Column Major order  $a[i][j] = BA + c[(ub_1 - lb_1 + 1)(j - lb_2) + (i - lb_1)]$   
(no of rows)

~~26-MOR~~

Column Major Order  $a[1..4][1..5]$

$a$	1	2	3	4	5	
1	$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$	$a_{15}$	
2	$a_{21}$	$a_{22}$	$a_{23}$	$a_{24}$	$a_{25}$	
3	$a_{31}$	$a_{32}$	$a_{33}$	$a_{34}$	$a_{35}$	
4	$a_{41}$	$a_{42}$	$a_{43}$	$a_{44}$	$a_{45}$	$BA = 1000 \quad C = 10$

$\begin{matrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \end{matrix}$

0 1 2 3 | 4 5 6 7 | 8 9 10 11 | 12 13 14 15 | 16 17 18 19 | Column 1 Column 2 Column 3 Column 4 Column 5

$$\begin{aligned} \text{Loc}[a[4][3]] &= BA + C[(3-1) \times (\text{no. of Rows}) + (4-1)] \\ &= 1000 + 10^2(2 \times 4 + 3) \\ &= 1022 \end{aligned}$$

$$\begin{aligned} \text{Loc}[a[2][5]] &= 1000 + 10^2[(5-1) \times \text{no. of Rows} + (2-1)] \\ &= 1000 + 2[4 \times 4 + 1] \\ &= 1034 \end{aligned}$$

both are same (Row Major, Column Major)

if no order is mentioned, go for Row Major (by default)

Ex2-  $a[-75..125][80..150] \quad BA = 0 \quad C = 10 \text{ byte}$

CMO

$$\begin{aligned} \text{Loc}[a[15][130]] &= 0 + 10[(130-80) \times (125+75+1) + (15+15)] \\ &= 10(50 \times 201 + 30) \\ &= 10(10050 + 30) = 100640 \\ &= 101400 \end{aligned}$$

$$n_2 = lb_1 - l_1 b_1 + 1$$

Ex3 -  $a[lb_1 \dots ub_1][lb_2 \dots ub_2]$  BA, C, CMo

$$\downarrow$$

$$nc = ub_2 - lb_2 + 1$$

$$\text{Loc}[a[i][j]] = BA + c[(j - lb_2) * n_2 + (i - lb_1)]$$

3-D Arrays - collection of 2-D arrays

$a[3 \dots 9][15 \dots 35][8 \dots 50]$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 9-3+1 & 35-15+1 & 50-8+1 \\ = 7 & 21 & 43 \\ 7 & \text{no of rows} & \text{no of columns} \end{array} \Rightarrow 7 \text{ table of size } 21 \times 43 \\ = 7 [2\text{-D array of size } 21 \times 43]$$

BA, Base Address = 1000 C = 10, RMo

$$\begin{aligned} \text{Loc}[a[7][30][40]] &= BA + c[(7-3)(nc * n_2) + (30-15) * nc + (40-8)] \\ &= 1000 + 10(4 * 21 * 43 + 15 * 43 * 32) \\ &= 43890 \end{aligned}$$

(Mo)

$$\begin{aligned} \text{Loc}[a[17][30][40]] &= BA + 10[(7-3) * nc * n_2 * (40-8) * n_2 + (30-15)] \\ &= BA + 10 * \end{aligned}$$

$$\begin{aligned} &= 1000 + 10(4 * 21 * 43 + 8 * 21 * 15) \\ &= 1000 + 10(3612 + 168 + 15) \\ &= 1000 + 10(3795) \\ &= 38950 \end{aligned}$$

Ans

Ex  $a[lb_1, ub_2] \quad a[lb_1 \dots ub_1, lb_2 \dots ub_2, lb_3 \dots ub_3]$

BA, C

$$n_1 = ub_2 - lb_2 + 1$$

$$n_2 = ub_3 - lb_3 + 1$$

$$n_3 \text{ of } 2D = ub_1 - lb_1 + 1$$

(R, M, O)

$$\text{loc}[a[i][j][k]] = BA + C [(i-lb_1) * nc * n_1 + (j-lb_2) * nc * (k-lb_3)]$$

CMO

$$\text{loc}[a[i][j][k]] = BA + C [(i-lb_1) * nc * n_1 + (k-lb_3) * n_2 + (j-lb_2)]$$

### Lower Triangular Matrix -

	1	2	3	4
1	$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$
2	$a_{21}$	$a_{22}$	$a_{23}$	$a_{24}$
3	$a_{31}$	$a_{32}$	$a_{33}$	$a_{34}$
4	$a_{41}$	$a_{42}$	$a_{43}$	$a_{44}$

$a_{11}$	0	0	0
$a_{21}$	$a_{22}$	0	0
$a_{31}$	$a_{32}$	$a_{33}$	0
$a_{41}$	$a_{42}$	$a_{43}$	$a_{44}$

↓ 16 element

G = zero

$a_{ij}$  belong to upper part if

$j > i$

( $n_2$ ) approx -2010

↓ to save spa

return 0 if that

why we should store

time

zero

Here store in row Major order -

row - 1 will have 1 element -

row - 2 will have 2 element -

Row Major Order -

$a$	1000	100 <sup>2</sup> 04	06 08 40	12 14 16 18
	a <sub>11</sub>	a <sub>21</sub>	a <sub>31</sub>	a <sub>41</sub>
	a <sub>12</sub>	a <sub>22</sub>	a <sub>32</sub>	a <sub>42</sub>

0 1 2 3 4 5 6 7 8 9

↓row1 ↓row2 ↓row3 ↓row4

visit 800

Column

visit 700

$$\text{Loc}(a[4][3]) = BA + c [ (4+3 \cdot 3+2+1) - (4+1)(3+2+1) + (3-1) ]$$

(sum of 3 natural  
atmos.  
(4-1) nat no)

$$= BA + c [ \text{sum of 3 natural no} + (3-1) ]$$

$$= 1000 + 2 [ 6+2 ] = 1016$$

[All triangular Matrix are square Matrix]

$\begin{matrix} 61 & \\ \nearrow & \searrow \end{matrix}$  (starting row no & column no is also same)

$$BA = 1000$$

$C = 10 \text{ Byte } RM_6 \text{ LTM (Lower Triangular Matrix)}$

$$\begin{aligned} \text{Loc}[a[60][55]] &= BA + c [ \text{sum of } (60-45) \text{ natural no} + (55-25) ] \\ &= 1000 + [ (60-25)-1 ] \\ &= BA + c [ \text{sum of } (60-25) \text{ natural no} + (55-25) ] \\ &= 1000 + c [ (60-25)(60-25+1) + 30 ] \\ &= 7600 \end{aligned}$$

formula

$$\text{Loc}[a[lb_1 \dots ub_1, lb_2 \dots ub_2]] = BA + c \quad RMO, LTM$$

$$\left\{ \text{Loc}[a[i][j]] = BA + c \left[ \frac{(j-lb_1)(i-lb_1+1)}{2} + (j-lb_2) \right] \right\}$$

~~$$a[4][3] = BA + c [ \text{sum of 3 natural no} + (\text{total rows} - (3-1)) ]$$~~
$$= 1000 + 10 [ 6 + 2 ]$$
$$= \underline{\underline{1016}}$$

~~$$a[25 \dots 85, 25, 85]$$~~ BA = 1000 C = 10 Byte~~CMO LTM  
Rows = 61 Column =~~~~$$\text{Loc } a[60, 55] = BA + c [ \text{sum of } (55-25) \text{ natural no} + \text{no of rows} - (55-1) ]$$~~

~~$$= 1000 + 10 [ 35 \times 18 + 61 - (55-25-1) ]$$~~~~$$= 1000 + 10 [ 63 @ \dots + 27 ]$$~~~~$$= \underline{\underline{7570}}$$~~

~~$$\text{CMO } (a[i][j] = BA + c [ \text{sum of } (j-1)b_1 \text{ no} + (\text{no of rows} - (j-1) \text{ natural} ) ]$$~~

~~extra~~

### Fibonacci heap

BA

$$\text{Loc}[a[4][3]] = 1000 + \left[ \frac{4 \times 5}{2} - 2 \times 3 + (4-3) \right] * 2$$

↓      ↓      → visiting 4th row  
crossing all column    crossing unnecessary column  
↳ 3 row and 3 column

$$= 1000 + 16 = 1016$$

~~$$\text{Loc}(a[4][1]) = \text{BA} + c \left[ \text{sum of 4 natural no} - \text{sum of } (4-4) \text{ no} \right]$$~~

if is used to cross all column      → visiting 11th row  
↳ 14mn      + (4-1) - 1      ↳ crossing unnecessary crossed column

$$= 1000 + 2 \left[ 10 - 10 + 3 \right]$$

$$= 1006$$

Ex 2 -  $A[25 \dots 100, 25, 100]$  BA = 1000 C = 10B LTM CM0

$$\text{Loc}(A[80][45]) = \text{BA} + c \left[ \text{sum of 76 natural} + \text{sum of } (100-45+1) \text{ no.} \right.$$

$$+ (80 - 45)]$$

$$= 1000 + 10 \left[ \frac{76 \times 77}{2} + \frac{31 \times 32}{2} + 10 \right]$$

↳ visiting both rows

5  
38  
77      31  
~~266~~      ~~16~~  
~~266~~      ~~31~~

$$= 1000 + 10 [38 \times 77 + 31 \times 16 + 10]$$

~~266~~      ~~16~~  
~~266~~      ~~31~~

$$= 1000 + 10 [2926 + 506 + 10]$$

$$= 1000 + 10 \left[ \frac{76 \times 77}{2} + \frac{56 \times 57}{2} + 35 \right]$$

$$= 151469$$

When you are at 15 2040 G.  
you are at 15 60-70 W.

Loc EAFS

101

$$A[-5_0 \dots 5_0, -5_0 \dots +5_0] \quad BA = 0 \quad C = 1 \text{ Byte, LCM, CM}$$

$$\text{Loc}(A[25][15]) = 0 + 1 \left[ \frac{\text{sum of 1st 101 nat no}}{\text{sum of 1st 50-15+ mat no}} \right]$$

$$= 101x \cancel{10^2} + 10 - 36x \cancel{37} \quad \uparrow \text{since you at 15}$$

$$\begin{array}{r} \cancel{3} \cancel{7} & \cancel{5} \cancel{1} & x & 0 \\ \cancel{1} \cancel{2} \cancel{9} \cancel{2} & \cancel{1} \cancel{0} \cancel{1} & = & 5161 - 666 \\ \cancel{1} \cancel{2} \cancel{9} \cancel{7} & \cancel{5} \cancel{0} \cancel{5} & & \\ \cancel{1} \cancel{6} & \cancel{5} & & \\ \hline & & 4495 & \end{array} \quad \text{(as 15 columns will start from}$$

15 rows as an element before  
15 cells: 2x10

Ex 4 A[lb<sub>1</sub>...ub<sub>1</sub>][lb<sub>2</sub>...ub<sub>2</sub>] BA, C, LTM, CMD

10

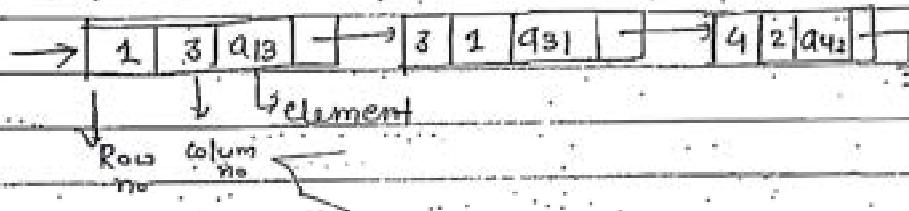
$$\text{Loc}[A[i][j]] = BA + C \left[ \sum_{\substack{\text{no} \\ \downarrow \\ \text{mat}}} (\text{ub}_2 - lb_2 + 1) \text{nat} - \sum_{\substack{\text{nat} \\ \text{no}}} (\text{ub}_2 - lb_2 + 1) \text{nat} + (i-j) \right] \sum_{\substack{\text{all} \\ \text{column}}} \text{sum}$$

Sparse Matrix - A very less element in array are 1 and other are zero & there is no relation in them

0	0	q13	0
0	0	0	0
q31	0	0	0
0	0	q42	0

⇒ so need to store in array

So we used linked list for its storage because if you use array lot of space is required.



Upper triangular Matrix

Z Matrix - all matrix except Z are zero



to exec  
to  
done

## Programming

### Scope of a variable

i) Static Scoping

ii) Dynamic Scoping

(data type followed by name of variab

1 - int a = 10;

(C)

- declarat

main()

{  
int a = 40;

for all f<sup>n</sup> call, a single stack is u

int a = 20;

DU;

B();

}

D()

(both static & 50 dynamic as no error problem)

B();

int a = 50;

pf(a);

D();

}

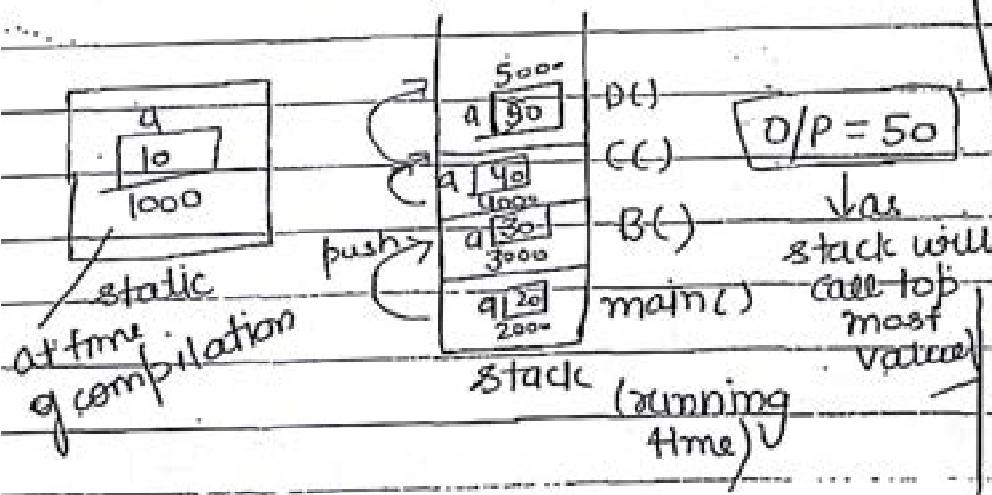
Whenever a function in memory to local var is created & its runn completed, m/m is deactivated but global variable, m

M/M

stack (for local f<sup>n</sup> variable)

static (at time of compilation)

heap (dynamic m/m)



the space taken by an any f<sup>n</sup> in static is equal to local variable

static by default - C

if both local & global data are not  
there, no mean (ambig.)

free variable - if a fn is using a variable which is not declared in it.

as if D()

then

D/P = {  
    a(static)      pf(a),  
    a(dynamic)}

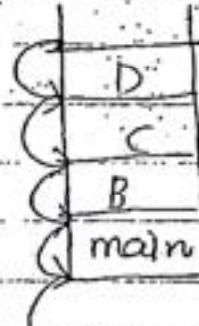
[no a is declared but is used]

[A variable which is not initialized  
can not be a free variable]

Whenever there is a free variable, compiler will identify it at time of compilation so, compiler will initialize it by global variable & this problem is known as scope problem. & this problem is resolved at time of compilation, known as static scoping because at that time only global variable was initialized. [compiler can't solve problem at run time]

but if compiler has restricted to solve scope problem at the time of compilation that it is not initialized by global data it will be resolved at run time. if 'a' is not in D then go to its calling function & use from there and so on. if no calling function has 'a' then use global variable 'a'.

Dynamic Scope  
will generate  
closer result  
as selecting  
the value  
from the  
closer fn



Dynamic is  
difficult to implement)

easier  
static scoping - problem solved  
at compile time  
dynamic scoping at run time  
↓ difficult

because In dynamic scoping  
Worst case - all fn  
to be passed to che

static scoping - by default

C, C++

static variable get m/m first & last  
deallocated at l  
re allocated b/c t  
completely b/c t

if a fn as D()

{ pf(a) and so on then static scoping - lo  
dynamic - go }

change done at compile time - static change

change done at run time - dynamic change (in dynamic  
changes are m  
to previous)

Ex2 int a=10, b=20;

main()

{

int a=5, b=6;

pf(a,b);

CC();

pf(a,b);

}

{ int b=3;

pf(a,b);

D(b);

pf(a,b);

a=3, b=4;

D(int a)

{

pf(a,b);

a=2;

b=3;

E();

pf(a,b);

a=1;

b=2;

F();

pf(a,b);

a=6;

b=7;

G();

pf(a,b);

a=6;

b=7;

H();

pf(a,b);

a=5;

b=6;

I();

pf(a,b);

a=4;

b=5;

J();

pf(a,b);

a=3;

b=4;

K();

pf(a,b);

a=2;

b=3;

L();

pf(a,b);

a=1;

b=2;

M();

pf(a,b);

a=0;

b=1;

N();

pf(a,b);

a=-1;

b=0;

O();

pf(a,b);

a=-2;

b=-1;

P();

pf(a,b);

a=-3;

b=-2;

Q();

pf(a,b);

a=-4;

b=-3;

R();

pf(a,b);

a=-5;

b=-4;

S();

pf(a,b);

a=-6;

b=-5;

T();

pf(a,b);

a=-7;

b=-6;

U();

pf(a,b);

a=-8;

b=-7;

V();

pf(a,b);

a=-9;

b=-8;

W();

pf(a,b);

a=-10;

b=-9;

X();

pf(a,b);

a=-11;

b=-10;

Y();

pf(a,b);

a=-12;

b=-11;

Z();

pf(a,b);

a=-13;

b=-12;

A();

pf(a,b);

a=-14;

b=-13;

B();

pf(a,b);

a=-15;

b=-14;

C();

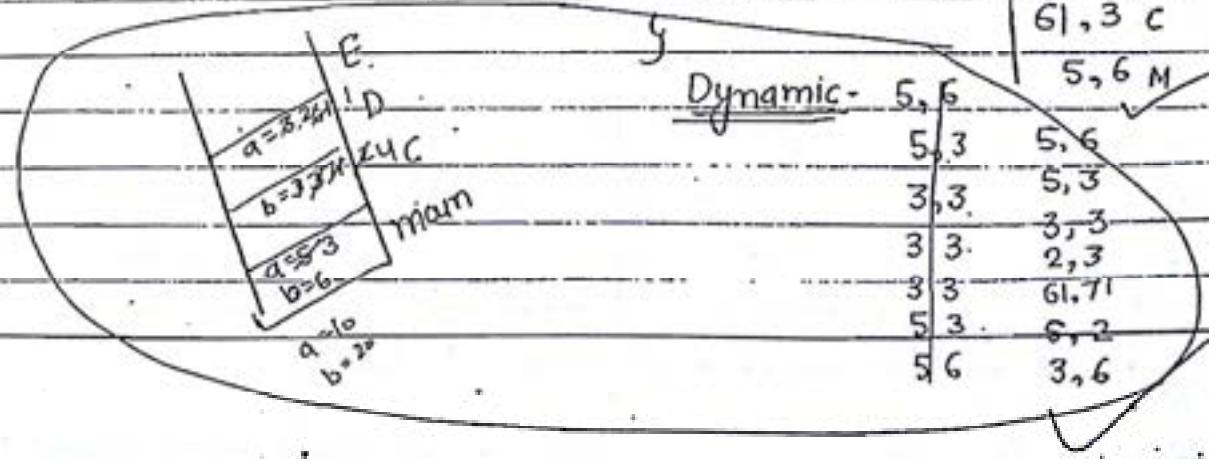
pf(a,b);

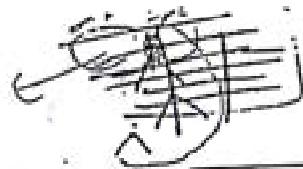
1000	2000
10 3	20 2
a	b
	static
	3

0 21	D
1 24	CC
2 1 5	main
3 6	
4	

5, 6 M	
10, 3 C	
3, 20 D	
10, 3 E	
2, 71 D	
61, 3 C	

5, 6 M	
5, 3	5, 6
3, 3	5, 3
3, 3	3, 3
3, 3	2, 3
3, 3	61, 71
5, 3	6, 2
5, 6	3, 6





-3 int a=1, b=2;  
main()

D (int a, int b)

{

b1(a, b)

a=1, b=2

{

E();

{

b1(a, b);

a=3, b=4;

C(int a)

}

E()

{

b1(a, b);

D(b, a)

b1(a, b);

a=6;

b=7;

b1(a, b);

a=3, b=7;

b1(a, b);

a=236;

b=7;

}

Static Scope - 1, 2

3, 4

4, 3

1, 4

3, 7

1, 2

3, 7

3, 7

1, 2

4, 3

1, 2

3, 7

3, 4

1, 7

a=k3

b=247

a=k3

b=324

a=236

b=7

E

D

C

B

A

m

main

Dynamic

1, 2

3, 4

4, 3

1, 2

3, 7

3, 4

1, 7

a=413230

b=3224

a=236

b=7

a=1

b=247

a=1

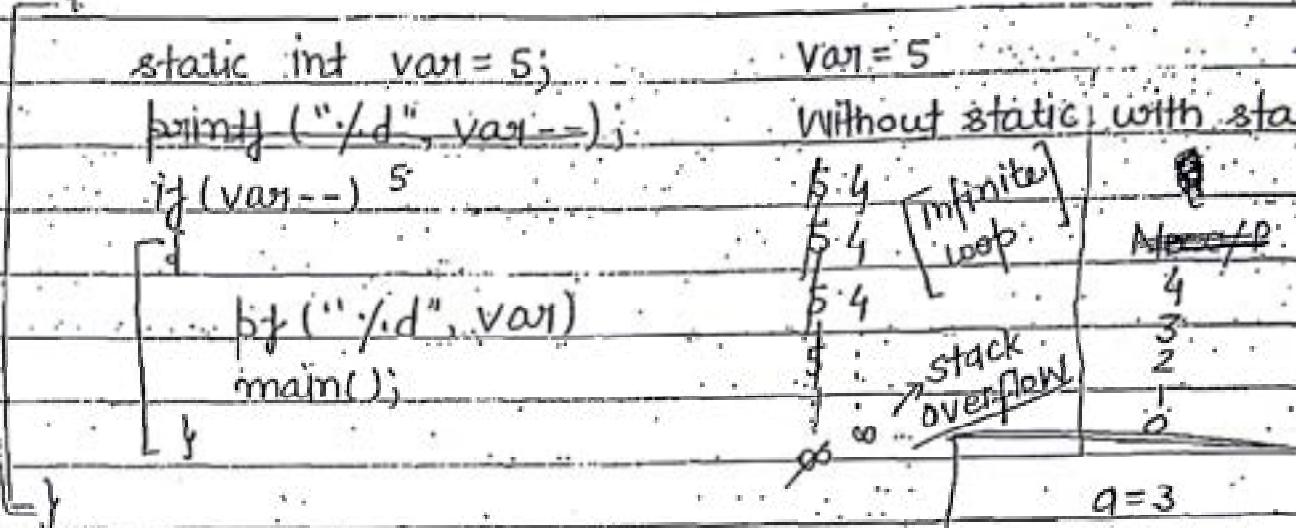
Static

m/m

## Static Variable

local var  
dynamic m/m > heap area  
in char - '10'  
int - '10'  
float '10.0'  
pointer 'Null'

ex: main()



- When local variable are declared with static, M/M will be created at Compile time.

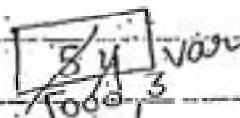
With static

main()

1. M/M already created

2. If (5)  
    ↓ print(4)

and so on  
no M/M created  
if (4)



by default global  
are static as  
M/M created at  
compile time.

O/P - 4 3 2 1 0  
as M/M is created  
only once.

at end of program

var = -1

- M/M created i.e. static is only one time as compilation is done one time only.

if '0' or Null or

if for