California_housing

March 21, 2024

Prediction of Housing Prices

Introduction

Due to the large growth of the urban population, housing resources seem to be relatively inadequate (Jiang & Qiu, 2022). Therefore, studying trends in housing prices has become a topic of great concern among economists and the government, due to the importance of real estate valuation on the general condition of the population's economic state (Heldak & Pluciennik, 2019). Given that accurate valuation of houses is needed by both renters and owners, it is vital to use prediction models based on the right characteristics. The factors that can influence the median housing price in a block include (not limited to): median income; the number of bedrooms; population; proximity to water bodies. Therefore we are going to predict what is the expected median house price (response variable) in a block in California based on certain values of those factors (explanatory variables), and determine the best subset of factor(s) that produce the most accurate prediction model. We will use the following "Housing" dataset https://www.kaggle.com/datasets/camnugent/california-housing-prices. It contains information about 20640 blocks in California with their median house price, along with 10 descriptive variables.

Analysis Question

What is the best regression model to predict the median house price of a block given the aforementioned set of characteristics?

Data Loading and Processing

We begin by loading essential R libraries for data manipulation, visualization, and modeling.

```
[2]: library(glmnet)
    library(tidyverse)
    library(broom)
    library(repr)
    library(mltools)
    library(leaps)
    library(car)
    library(GGally)
    library(fastDummies)
    library(corrplot)
```

The dataset, "housing 3.csv", contains detailed information about housing in California, including the location (latitude and longitude), median age of the houses, total rooms, bedrooms, population,

and more. Initial exploration with functions like head() and glimpse() provides us with a preliminary understanding of the data structure and the types of variables present.

```
[3]: data = read.csv("housing 3.csv")
```

[4]: head(data)

		longitude	latitude	housing_median_age	$total_rooms$	$total_bedrooms$	populat
A data.frame: 6×10		<dbl></dbl>	<dbl $>$	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl $>$
	1	-122.23	37.88	41	880	129	322
	2	-122.22	37.86	21	7099	1106	2401
	3	-122.24	37.85	52	1467	190	496
	4	-122.25	37.85	52	1274	235	558
	5	-122.25	37.85	52	1627	280	565
	6	-122.25	37.85	52	919	213	413

[5]: glimpse(data)

```
Rows: 20,640
Columns: 10
                      <dbl> -122.23, -122.22, -122.24,
$ longitude
-122.25, -122.25, -122.2...
                      <dbl> 37.88, 37.86, 37.85, 37.85, 37.85,
$ latitude
37.85, 37.84, 37...
$ housing_median_age <dbl> 41, 21, 52, 52, 52, 52, 52, 52,
42, 52, 52, 52, 52,...
                      <dbl> 880, 7099, 1467, 1274, 1627, 919,
$ total_rooms
2535, 3104, 2555,...
$ total_bedrooms
                      <dbl> 129, 1106, 190, 235, 280, 213,
489, 687, 665, 707, ...
$ population
                      <dbl> 322, 2401, 496, 558, 565, 413,
1094, 1157, 1206, 15...
$ households
                      <dbl> 126, 1138, 177, 219, 259, 193,
514, 647, 595, 714, ...
                      <dbl> 8.3252, 8.3014, 7.2574, 5.6431,
$ median_income
3.8462, 4.0368, 3.6...
$ median_house_value <dbl> 452600, 358500, 352100, 341300,
342200, 269700, 299...
                      <chr> "NEAR BAY", "NEAR BAY", "NEAR
$ ocean proximity
BAY", "NEAR BAY", "NE...
```

In the preprocessing phase, rows with missing values were removed from the housing dataset to ensure data integrity, and a new feature, rooms_household, was calculated as the ratio of total rooms to households.

```
[6]: data = na.omit(data)
```

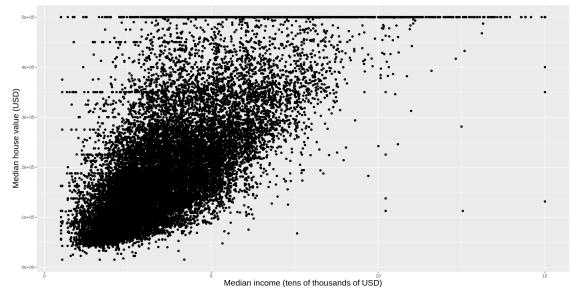
[7]: data\$rooms_household <- data\$total_rooms / data\$households

Exploratory Data Analysis (EDA)

Our primary goal in the EDA is to visually explore how different predictor variables, such as median income, geographical location, house age, and proximity to the ocean, among others, correlate with the median house value. By examining these relationships, we aim to understand better which features play a pivotal role in determining house prices.

We first take a look at how median income of the block affects median house value.

Median house value vs Median income

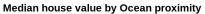


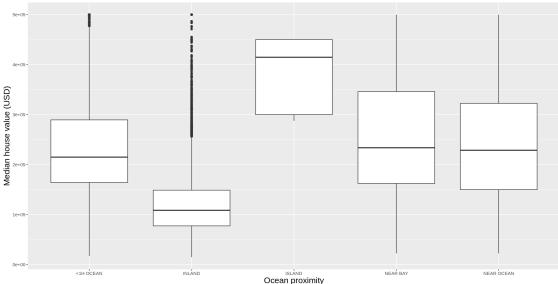
The median income of the block shows a stong positive correlation with the median house value.

We now look at how median house value differs based on different levels of ocean proximity.

```
[9]: options(repr.plot.width=15, repr.plot.height=8)
ggplot(data, aes(x = ocean_proximity, y = median_house_value)) +
    geom_boxplot() +
    labs(x = "Ocean proximity", y = "Median house value (USD)") +
    ggtitle("Median house value by Ocean proximity") +
    theme(
```

```
plot.title = element_text(size = 20, face = "bold"),
axis.title.x = element_text(size = 16),
axis.title.y = element_text(size = 16))
```





There appears to be differences in median house value based on Ocean proximity. Average median house value is the highest in Islands followed by close proximity to water bodies and then Inland. This can clearly be seen in the table below.

```
[10]: mean_values <- summarise(group_by(data, ocean_proximity), `Mean value` = 

→mean(median_house_value, na.rm = TRUE))
mean_values
```

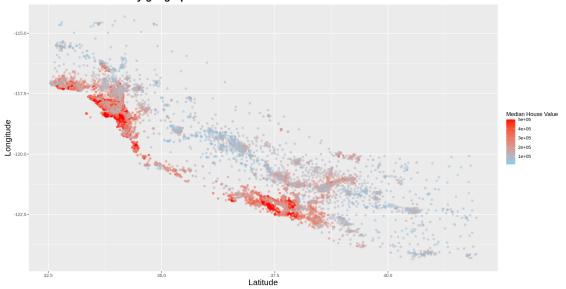
	ocean_proximity	Mean value
	<chr $>$	<dbl $>$
	<1H OCEAN	240268.0
A tibble: 5×2	INLAND	124896.9
	ISLAND	380440.0
	NEAR BAY	259279.3
	NEAR OCEAN	249042.4

Finally, we plot the geographical map of the California region and colour the locations by median house value to see how the median house value differs based on location.

```
[11]: options(repr.plot.width=15, repr.plot.height=8)
    ggplot(data, aes(x = latitude, y = longitude, color = median_house_value)) +
        geom_point(alpha = 0.5) +
        scale_color_gradient(low = "skyblue", high = "red") +
        labs(x = "Latitude", y = "Longitude", color = "Median House Value") +
        ggtitle("Median house value by geographical location") +
```

```
theme(
  plot.title = element_text(size = 20, face = "bold"),
  axis.title.x = element_text(size = 16),
  axis.title.y = element_text(size = 16))
```

Median house value by geographical location

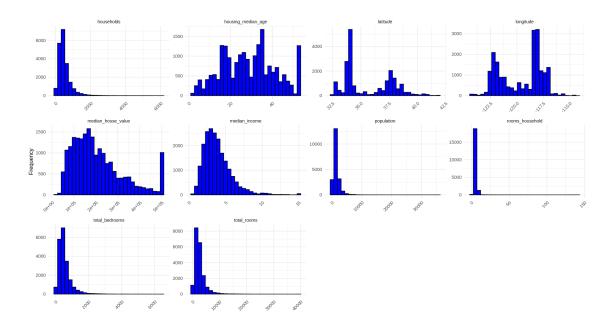


The median house value is highist near the coast and decreases as we move Inland. This aligns with our previous results based on ocean proximity.

Model Assumptions

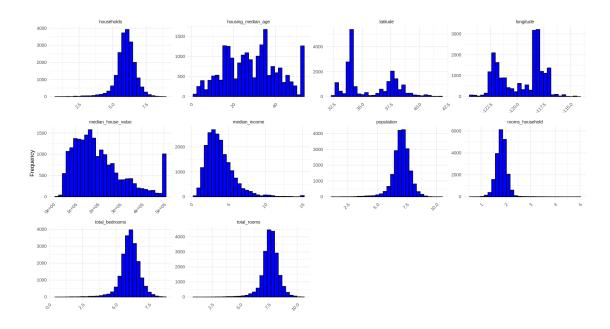
Normality Assumption

In regression modeling, adherence to the normality assumption for predictor variables enhances the reliability of parameter estimates and significance tests. We plot the predictors to check if they follow the normality assumption.



To address deviations from this assumption, particularly skewed distributions, we apply logarithmic transformations to skewed predictors, thereby approximating normality and optimizing the model's performance.

```
[13]: data = mutate(data,
          total_rooms = log(total_rooms + 1),
          total_bedrooms = log(total_bedrooms + 1),
          population = log(population + 1),
          households = log(households + 1),
          rooms_household = log(rooms_household + 1))
```

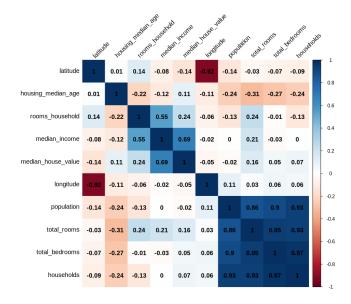


The predictor variables are now approximately normal.

Multicollinearity

Multicollinearity occurs when predictor variables in a regression model are highly correlated, potentially undermining the reliability of the model's coefficient estimates. To mitigate this issue, we evaluate the degree of multicollinearity through Variance Inflation Factor (VIF) and remove highly correlated predictors, thereby stabilizing the model's estimates.

We perform a correlation analysis to identify potentially multicollinear variables, which could affect model performance.



We can see that some of our predictor variables are highly correlated with each other, for example households and population, raising possible concerns of multicollinearity.

We calculate the Variance Inflation Factor (VIF) for each predictor to quantify multicollinearity.

		GVIF	Df	$GVIF^(1/(2*Df))$
	longitude	18.622870	1	4.315422
	latitude	20.762686	1	4.556609
	housing_median_age	1.296838	1	1.138788
	$total_rooms$	2705.570451	1	52.015098
A matrix: 10×3 of type dbl	$total_bedrooms$	58.105079	1	7.622669
	population	8.356116	1	2.890695
	households	3090.213564	1	55.589689
	$median_income$	2.881180	1	1.697404
	ocean_proximity	4.250048	4	1.198255
	rooms_household	391.527815	1	19.787062

Based on VIF values, we decide to remove variables with high multicollinearity, such as latitude, longitude, total rooms, and households, to improve model accuracy and interpretation.

```
[17]: data <- subset(data, select = -c(latitude, longitude, total_rooms, households))
```

		GVIF	Df	$GVIF^(1/(2*Df))$
	housing_median_age	1.278324	1	1.130630
	$total_bedrooms$	5.973744	1	2.444124
A matrix: 6×3 of type dbl	population	6.059447	1	2.461594
	$median_income$	1.872268	1	1.368308
	ocean_proximity	1.472897	4	1.049595
	$rooms_household$	1.956831	1	1.398868

The VIF scores of all predictors are now stable and not inflated.

Model Evaluation and Selection

We begin by one-hot encoding the categorical levels of ocean proximity column into individual binary columns. This is done because feature selection algorithms like forward selection and LASSO do not work well with categorical predictors with multiple levels.

```
[20]: data <- fastDummies::dummy_cols(data, select_columns = "ocean_proximity", u eremove_selected_columns = TRUE)
```

Our project aims to build three types of models:

- A full additive regression model including all predictors.
- A reduced model selected through forward selection, optimizing for predictors that contribute most significantly to the outcome.
- A LASSO regression model to enforce regularization, automatically selecting important features and shrinking coefficients of less important ones.

We will then compare their performance using RMSE to evaluate the best model.

Since predictive models are built and trained to predict new observations, We first divide the data into two datasets: a training set (70% of the data in the training_housing dataset) and a test set (30% of the data in the testing_housing dataset).

```
[21]: set.seed(321) #for reproducibility

data$ID <- rownames(data)
   training_housing <- sample_n(
        data,
        size = nrow(data) * 0.70,
        replace = FALSE)</pre>
```

```
testing_housing <- anti_join(
    data,
    training_housing,
    by = "ID")

training_housing <- select(training_housing, -"ID")

testing_housing <- select(testing_housing, -"ID")

head(training_housing)

nrow(training_housing)

head(testing_housing)

nrow(testing_housing)</pre>
```

		housing_median_age	$total_bedrooms$	population	$median_income$	median_hou
		<dbl></dbl>	<dbl $>$	<dbl $>$	<dbl $>$	<dbl></dbl>
A data.frame: 6×11	1	52	6.431331	7.080868	2.3887	337500
	2	30	7.034388	7.847372	3.5817	369300
	3	14	6.448889	7.170888	2.6023	221100
	4	32	6.391917	8.157944	2.7250	450000
	5	34	5.968708	7.174724	4.0379	172800
	6	23	6.315358	6.905753	3.6620	211900

14303

		housing_median_age	total_bedrooms	population	median_income	median_hou
		<dbl></dbl>	<dbl></dbl>	<dbl $>$	<dbl $>$	<dbl></dbl>
	1	41	4.867534	5.777652	8.3252	452600
A data.frame: 6×11	2	52	6.194405	6.998510	3.6591	299200
A data. Hame: 0×11	3	52	6.562444	7.347300	3.6912	261100
	4	52	6.163315	7.002156	3.0750	213500
	5	50	6.122493	6.898715	1.9911	158700
	6	42	5.908083	6.835185	1.7135	159800

6130

Focusing on the training_housing dataset, we estimate a full additive MLR and obtain the out-of-sample predictions from the test set by using the predict() function.

```
1 418616.148401584 2 263848.24340151 3 269299.195669519 4 229624.11241877 5 183503.459448093 6 157090.062103791
```

Now, we select a smaller subset of variables by using the forward selection algorithm

Using regsubsets() function to perform the forward selection, we construct a tibble 'housing_fwd_summary_df' that stores different evaluation metrics including RSS, BIC, and Cp from the best models chosen by forward selection for each n (number of predictors).

```
housing_forward_sel <- regsubsets(
    x = median_house_value ~ .,
    nvmax = 9,
    data = training_housing,
    method = "forward")

housing_fwd_summary <- summary(housing_forward_sel)

housing_fwd_summary_df <- tibble(
    n_input_variables = 1:9,
    RSS = housing_fwd_summary$rss,
    BIC = housing_fwd_summary$bic,
    Cp = housing_fwd_summary$cp)

housing_fwd_summary_df
summary(housing_forward_sel)</pre>
```

Warning message in leaps.setup(x, y, wt = wt, nbest = nbest, nvmax = nvmax, force.in = force.in, :

"1 linear dependencies found"

	$n_{input_variables}$	RSS	BIC	Cp
	<int></int>	<dbl $>$	<dbl $>$	<dbl $>$
	1	9.920103e+13	-9265.963	7966.068244
	2	7.817853e + 13	-12662.702	3249.696394
	3	7.595656e + 13	-13065.539	2752.988479
A tibble: 9×4	4	7.421491e + 13	-13387.750	2364.085911
	5	6.607259e + 13	-15040.349	538.591951
	6	6.391183e+13	-15506.349	55.622013
	7	6.378403e + 13	-15525.410	28.937863
	8	6.367745e + 13	-15539.761	7.016896
	9	6.367737e + 13	-15530.210	9.000000

Subset selection object

Call: regsubsets.formula(x = median_house_value ~ ., nvmax = 9, data = $_{\sqcup}$ $_{\to}$ training_housing,

method = "forward")

10 Variables (and intercept)

	Forced in	Forced out
housing_median_age	FALSE	FALSE
total_bedrooms	FALSE	FALSE
population	FALSE	FALSE
median_income	FALSE	FALSE
rooms_household	FALSE	FALSE
`ocean_proximity_<1H OCEAN`	FALSE	FALSE

```
FALSE
                                          FALSE
ocean_proximity_INLAND
                               FALSE
                                          FALSE
ocean_proximity_ISLAND
`ocean_proximity_NEAR BAY`
                               FALSE
                                          FALSE
`ocean_proximity_NEAR OCEAN`
                               FALSE
                                          FALSE
1 subsets of each size up to 9
Selection Algorithm: forward
        housing_median_age total_bedrooms population median_income
                                         11 11
                                                    "*"
  (1)""
                           11 11
                                         11 11
2
  (1)""
                                                    "*"
  (1)"*"
                           11 11
                                         11 11
                                                    "*"
3
                                         11 11
4
  (1)"*"
                           "*"
                                                    "*"
5
  (1)"*"
                           "*"
                                         "*"
                                                    "*"
                                         "*"
  (1)"*"
                           "*"
                                                    11 11 11
6
  (1)"*"
                           "*"
                                         "*"
7
                                                    "*"
                                         "*"
  (1)"*"
                           "*"
                                                    "*"
8
                           "*"
                                         "*"
                                                    "*"
  (1)"*"
        rooms_household `ocean_proximity_<1H OCEAN` ocean_proximity_INLAND
                                                   11 11
  (1)""
                        11 11
                                                   "*"
2
  (1)""
  (1)""
                                                   "*"
3
  (1)""
                        11 11
                                                   "*"
5
  (1)""
                                                   11 * 11
6
  (1)"*"
                                                   "*"
7
  (1)"*"
                                                   "*"
8
  (1)"*"
                        11 11
                                                   "*"
                        "*"
  (1)"*"
        ocean_proximity_ISLAND `ocean_proximity_NEAR BAY`
  (1)""
                               11 11
2
  (1)""
3
  (1)""
4
  (1)""
5
  (1)""
  (1)""
6
7
  (1)""
  (1)"*"
8
                               11 11
  (1)"*"
        `ocean_proximity_NEAR OCEAN`
  (1)""
1
2
  (1)""
  (1)""
3
  (1)""
4
  (1)""
5
6
  (1)""
7
  (1)"*"
  (1)"*"
8
  (1)"*"
```

The Mallows's Cp values show a decreasing trend with more variables in the model (until 8 predic-

tors), and the lowest Cp is 7.016896 with 8 input variables, which indicates that the model (with the 8 input variables) may be relatively precise (has small variance) in predicting future responses.

We then set a regression model by using the 8 input variables which are selected using forward selection, and predict the new data with this reduced model.

RMSE of the full additive regression model and reduced regression model are computed from the test set to indicate which one has a better prediction performance.

```
[24]: housing red <- lm(median_house_value ~ housing_median_age + total_bedrooms +
                         population + median_income + rooms_household +
                         ocean_proximity_INLAND + ocean_proximity_ISLAND +
                         `ocean_proximity_NEAR OCEAN`,
          data = training_housing)
      housing_test_pred_red <- predict(housing_red, newdata = testing_housing[, -5])</pre>
      head(housing test pred red)
      housing_R_MSE_models <- rbind(</pre>
          tibble(
              Model = "Full Additive Regression Model",
              R_MSE = rmse(
              preds = housing_test_pred_full_add,
              actuals = testing_housing$median_house_value)),
          tibble(
              Model = "Reduced Regression Model",
              R_MSE = rmse(
              preds = housing_test_pred_red,
              actuals = testing_housing$median_house_value)))
      housing_R_MSE_models
```

```
1
     418795.453953122 2
                            264013.773885811 3
                                                   269463.519919427 4
                                                                          229792.055818742 5
183673.823546989 6
                                                157276.894579737
                                                R MSE
                Model
                <chr>
                                                <dbl>
A tibble: 2 \times 2 -
                Full Additive Regression Model
                                                70017.91
                Reduced Regression Model
                                                70017.34
```

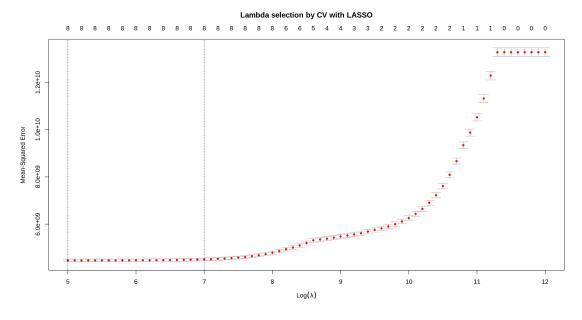
RMSE (Root Mean Square Error) is a standard way to measure the error of a model in prediction. The Reduced regression Model has a smaller RMSE, which is 70017.34.

Moving on, we compare the predictive performance of these models relative to a LASSO regression.

We use the cv.glmnet() function to find an optimal value of , but glmnet requires matrices with input variables and a vector of responses, so I first convert 'training_housing' and 'testing_housing' into matrices.

A plot is made to visualize how the estimated test-MSE changes for different values of s.

```
[25]: model_matrix_X_train <- model.matrix(</pre>
          object = median_house_value ~ .,
          data = training_housing)[, -1]
      matrix_Y_train <- as.matrix(</pre>
          training_housing$median_house_value,
          ncol = 1)
      model_matrix_X_test <- model.matrix(</pre>
          object = median_house_value ~ .,
          data = testing_housing)[, -1]
      matrix_Y_test <- as.matrix(</pre>
          testing_housing$median_house_value,
          ncol = 1)
      Housing_cv_lambda_LASSO <- cv.glmnet(</pre>
          x = model_matrix_X_train,
          y = matrix_Y_train,
          alpha = 1,
          lambda = \exp(seq(5, 12, 0.1)))
      plot(Housing_cv_lambda_LASSO, main = "Lambda selection by CV with LASSO\n\n")
```



In the plot, the two vertical dotted lines corresponding to two values of : $\hat{}$ min (e $\hat{}$ 5=148.41) provide the minimum MSE in the grid. $\hat{}$ 1SE (around e $\hat{}$ 7) is the largest value of lambda such that the corresponding MSE is within 1 standard error of that of the minimum.

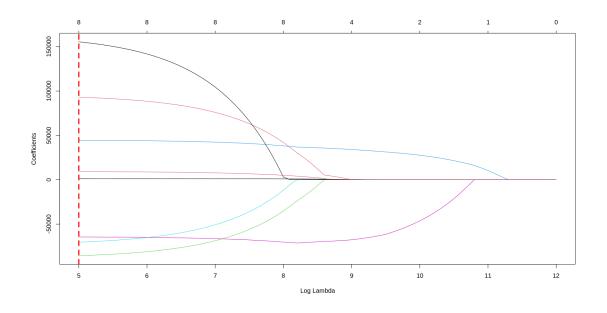
Then, we compute the lambda value with the least MSE, and visualize the estimated regression coefficients.

```
[26]: Housing_lambda_min_MSE_LASSO <- round(Housing_cv_lambda_LASSO$lambda.min, 4)
    round(Housing_lambda_min_MSE_LASSO,2)
    round(log(Housing_lambda_min_MSE_LASSO),2)

plot(Housing_cv_lambda_LASSO$glmnet.fit, "lambda")
    abline(v = log(Housing_lambda_min_MSE_LASSO), col = "red", lwd = 3, lty = 2)</pre>
```

148.41

5



We set a LASSO model for minimum lambda and compare the coefficients of the selcted model with those of the full LS.

```
[27]: Housing_LASSO_min <- glmnet(
    x = model_matrix_X_train,
    y = matrix_Y_train,
    alpha = 1,
    lambda = Housing_lambda_min_MSE_LASSO)

Housing_LASSO_min.coef <- Housing_LASSO_min$beta

Housing_reg_coef <- round(cbind(
    Full_OLS = coef(housing_full_add),
    LASSO_min = c(
    Housing_LASSO_min$aO,</pre>
```

```
as.vector(Housing_LASSO_min.coef))), 4)
Housing_reg_coef
```

```
Full OLS
                                                                           LASSO min
                                                   (Intercept)
                                                               200846.756
                                                                           185331.848
                                         housing median age
                                                               1221.898
                                                                           1208.390
                                              total bedrooms
                                                               96013.417
                                                                           93331.291
                                                   population
                                                               -88444.769
                                                                           -85796.793
                                              median income
                                                               44895.289
                                                                           44570.610
A matrix: 11 \times 2 of type dbl
                                             rooms_household
                                                               -73569.296
                                                                           -70548.346
                              'ocean proximity <1H OCEAN'
                                                               -9348.238
                                                                           0.000
                                   ocean proximity INLAND
                                                               -73503.508
                                                                           -64402.789
                                    ocean proximity ISLAND
                                                               153975.182
                                                                           155423.067
                                'ocean proximity NEAR BAY'
                                                               -9599.230
                                                                           0.000
                            'ocean proximity NEAR OCEAN'
                                                               NA
                                                                           9181.403
```

We finally predict the new data from the LASSO model and compute the RMSE of LASSO Regression for the futhur comparison with the previous two models.

	Model	R_MSE
	<chr></chr>	<dbl $>$
A tibble: 3×2	Full Additive Regression Model	70017.91
	Reduced Regression Model	70017.34
	LASSO Regression with minimum MSE	69983.56

The RMSE of the LASSO Regression model is 69983.56, which is the least among the 3 models, meaning the LASSO model has the best prediction performance!

Results

We examined the variation in median house price of a block due to several descriptive variables like median income, ocean proximity and geographical coordinates using methods like boxplots and scatterplots. We saw that median house price of a block increases with increae in median income and also depends on the geographical location and proximity to ocean. It is higher in blocks close to the ocean and near the coast.

We then investigate the regression model assumptions. Using correlation plots and GVIF, we find that some of our predictors are heavily correlated leading to multicollinearity among them. We remove the predictors with a very high GVIF and this helps solve the issue as the new scores of the selected variables are low. We look at the distribution of predictors through histograms and logarithmic transform the varibales that deviated from normaloty assumption .

We move onto fitting the linear model and computing the RSME for prediction accuracy. Next, we performed feature selection. By using forward selection algorithm, we compared the Cp values of best regression models of different number of predictors. We found that the Mallows's Cp values show a decreasing trend with more variables in the model, and the lowest Cp is 7.016896 with 8 input variables. Therefore we fit an OLS model using the chosen 8 variables.

Finally, we fit a LASSO regression model, and used cross validation to choose the best lambda value (148.41), which minimizes MSE. Overall, we found that the RMSE of LASSO Regression (RMSE = 69983.56) is smaller than the RMSE of the other two models(Full Additive Regression Model RMSE: 70017.91 and Reduced Regression Model RMSE: 70017.34).

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