



Gateway Classes



BCS402 Theory of Automata and Formal Languages

Unit-1

Introduction to Basic Concepts and Automata Theory

Syllabus

Basic Concepts and Automata Theory: Introduction to Theory of Computation- Automata, Computability and Complexity, Alphabet, Symbol, String, Formal Languages, Deterministic Finite Automaton (DFA)- Definition, Representation, Acceptability of a String and Language, Non Deterministic Finite Automaton (NFA), Equivalence of DFA and NFA, NFA with ϵ -Transition, Equivalence of NFA's with and without ϵ -Transition, Finite Automata with output- Moore Machine, Mealy Machine, Equivalence of Moore and Mealy Machine, Minimization of Finite Automata.



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What is Automata?

- Automata are like little machines that follow a set of rules. Imagine a vending machine: you put in money, press a button, and it gives you a snack. The vending machine can be in different states, such as idle, accepting coins, dispensing a product) and rules (if you put in enough money and press the right button, it gives you a snack).
- In computer science, automata are used to understand how computers process information. They have states (like "start" and "end") and rules (if certain conditions are met, move from one state to another). Automata help us understand how computers can solve problems, recognize patterns, and process data.
- Automata, in computer science, refer to abstract machines that can perform specific tasks or computations. They are often used to model and analyze the behavior of computational systems.

1. SYMBOL

- A symbol (often also called a character) is the smallest building block, which can be any alphabet, letter etc.
- a,b,c,0,1,2,A,B,C...

2. Alphabets (Σ):

- Alphabets are a set of symbols, which are always *finite*.
- Collection of symbols

$\Sigma = \{0,1\}$ is an alphabet of binary digit

$\Sigma = \{0,1,2,3,\dots,9\}$ an alphabet of decimal digit

$\Sigma = \{a, b, c\}$

$\Sigma = \{A, B, C, D, \dots, Z\}$

Σ (SIGMA)

3.String:

- A string is a finite sequence of symbols from some alphabet. A string is generally denoted as w and the length of a string is denoted as $|w|$.

Collection of symbol over alphabet

Empty string is the string with zero occurrence of symbols, represented as ϵ .(epsilon)

$\Sigma = \{a, b\}$ possible string over the alphabet $w = a$
 $a, b, ab, ba, aa, bb \dots$ $|w| = 1$

Number of Strings (of length 2) that can be generated over the alphabet {a, b}: - -

aa
bb
ab
ba

- Length of String $|w| = 2$
- Number of Strings = 4
- For alphabet {a, b} with length n, number of strings can be generated = 2^n

SOME BASIC TERMINOLOGY

➤ NOTE

The set of strings, including the empty string, over an alphabet Σ is denoted by Σ^* .

For $\Sigma = \{0, 1\}$ we have set of strings as $\Sigma^* = \{\epsilon, 0, 1, 01, 10, 00, 11, 10101, \dots\}$.

And $\Sigma^0 = \{\epsilon\}$, $\Sigma^1 = \{0\}$, $\Sigma^2 = \{00, 01, 10, 11\}$,
 $\Sigma^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$
 $\Sigma^4 = \{0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111\}$
 $\Sigma^5 = \{00000, 00001, 00010, 00011, 00100, 00101, 00110, 00111, 01000, 01001, 01010, 01011, 01100, 01101, 01110, 01111, 10000, 10001, 10010, 10011, 10100, 10101, 10110, 10111, 11000, 11001, 11010, 11011, 11100, 11101, 11110, 11111\}$
 $\Sigma^6 = \{000000, 000001, 000010, 000011, 000100, 000101, 000110, 000111, 001000, 001001, 001010, 001011, 001100, 001101, 001110, 001111, 010000, 010001, 010010, 010011, 010100, 010101, 010110, 010111, 011000, 011001, 011010, 011011, 011100, 011101, 011110, 011111, 100000, 100001, 100010, 100011, 100100, 100101, 100110, 100111, 101000, 101001, 101010, 101011, 101100, 101101, 101110, 101111, 110000, 110001, 110010, 110011, 110100, 110101, 110110, 110111, 111000, 111001, 111010, 111011, 111100, 111101, 111110, 111111\}$

➤ Σ^* contains an empty string ϵ . The set of non-empty string is denoted by Σ^+ . From this we get:

$$\Sigma^* = \Sigma^+ \cup \{\epsilon\} \quad \xrightarrow{\text{Positive}}$$

$$\Sigma = \{0, 1\}$$

$$\Sigma^* = \{\epsilon, 0, 1, 00, 01, 10, 11\}$$

$$\Sigma^+ = \{0, 1, 00, \dots\}$$

4. Language

- language is a set of string all of which are chosen from some Σ^* , where Σ is a particular alphabet. This means that language L is subset of Σ^* . An example is English language, where the collection of legal English words is a set of strings over the alphabet that consists of all the letters.

➤ For $\Sigma = \{0, 1\}$ we have set of strings as $\Sigma^* = \{\epsilon, 0, 1, 01, 10, 00, 11, 10101, \dots\}$.

✓ $L = \{ \text{set of all string end with 1 and start with 1} \}$

$$L = \{1, 101, 1111, 100001\}$$

$$L = \{1\}$$

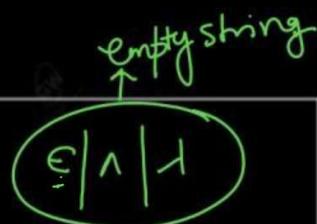
If Σ is an alphabet, and $L \subseteq \Sigma^*$

➤ A language that can be formed over ' Σ ' can be Finite or Infinite.

5. Kleene star Σ^* (universal set) INFINITE SET/ Kleene closure/ Sigma star

Representation: $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \dots \dots \Sigma = \{0, 1\}$ (let)

Set of all possible string of length p



$\Sigma^0 = \{\epsilon\}$ (LENGTH OF STRING MUST BE ZERO/NULL STRING/EMPTY STRING)

$\Sigma^1 = \{0, 1\}$

$\Sigma^2 = \{00, 01, 10, 11\}$

Example - If $\Sigma = \{a, b\}$, $\Sigma^* = \{\lambda, a, b, aa, ab, ba, bb, \dots\}$

Definition - The Kleene star, Σ^* , is a unary operator on a set of symbols or strings, Σ , that gives the infinite set of all possible strings of all possible lengths over Σ including λ OR ϵ

6. Kleene plus/kleene positive/positive clouser Σ^+

➤ **Representation**

$$\Sigma^+ = \Sigma^* - \{\epsilon\}$$

$$\Sigma^+ = \Sigma^* - \Sigma^0$$

$$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \dots$$

➤ **Example** – If $\Sigma = \{a, b\}$, $\Sigma^+ = \{a, b, aa, ab, ba, bb, \dots\}$

➤ **Definition** – The set Σ^+ is the infinite set of all possible strings of all possible lengths over Σ excluding λ .

Kleene closure & Kleene positive

- It is a mathematical model Used to describe computation, this model have discrete inputs, outputs, states and a set of transitions from state to state that occurs on input symbols from the alphabet Σ .



Finite automata is defined as a 5-tuples

$$M = (Q, \Sigma, \delta, q_0, F)$$

Symbol Meaning: Σ (sigma), δ (delta)

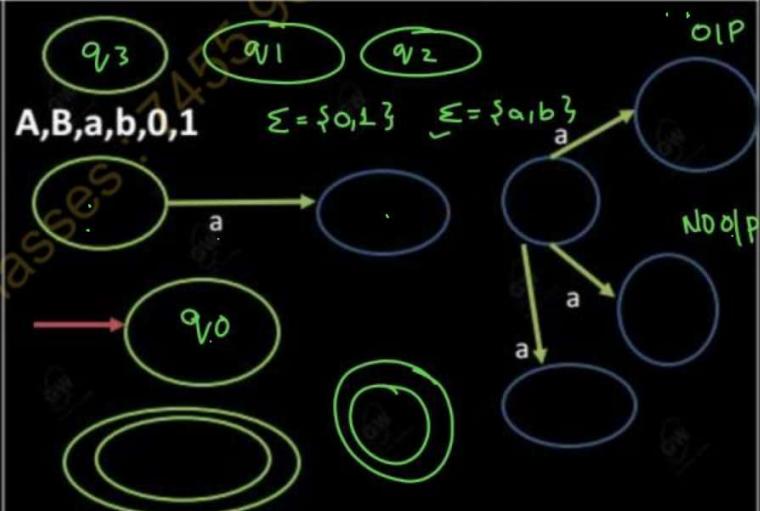
Q: Finite set called states. $\{q_0, q_1, \dots, q_n\}$ ----->

Σ : Finite set called alphabets. ----->

δ : is the transition function. ----->

$q_0 \in Q$ is the start or initial state. ----->

F: Final or accept state. $\{q_f\}$ ----->



DIFFERENT TYPES OF FINITE AUTOMATA

Finite Automata without output	Finite Automata with Output
<ul style="list-style-type: none">➤ Deterministic Finite Automata (<u>DFA</u>).➤ Non-Deterministic Finite Automata (<u>NFA</u> or <u>NDFA</u>).➤ Non-Deterministic Finite Automata with epsilon moves (<u>e-NFA</u> or <u>e-NDFA</u>).	<ul style="list-style-type: none">➤ Moore machine.➤ Mealy machine

Finite Automata Representation

- Graphical (Transition diagram)
- Tabular (Transition table)
- Mathematical (Transition function)

- The behavior of a DFA is entirely determined by its current state and the input symbol being processed.
- For any given state and input symbol, there is only one possible next state

➤ It has a finite number of states.

- An automaton is a mathematical model of a system that processes inputs according to a set of rules.
- In the case of DFAs, the system transitions between states based on the inputs it receives.

A DFA can be represented by a 5-tuple $(Q, \Sigma, \delta, q_0, F)$



- Q is a finite set of states. $\{q_0, q_1, q_3, q_4\}$
- Σ is a finite set of symbols called the alphabet.
- δ is the transition function where $\delta: Q \times \Sigma \rightarrow Q$
- q_0 is the initial state from where any input is processed ($q_0 \in Q$).
- F is a set of final state/states of Q ($F \subseteq Q$)

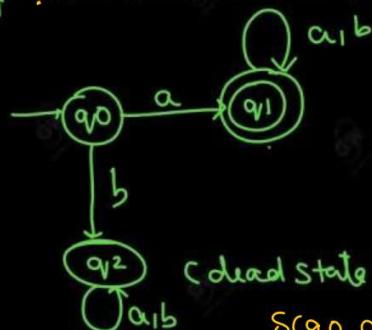
NOTE:

- *There can be many possible DFAs for a pattern.*

Minimum
Design a DFA/MDFA/FA $\Sigma = \{a, b\}$ such that string accepted must start with substring a

TRANSITION DIAGRAM

$L = \{a, ab, aab, aba, abbbabab, \dots\}$



$$\begin{aligned}\delta(q_0, a) &= q_1 & \delta(q_2, a) &= q_1 \\ \delta(q_0, b) &= q_2 & \delta(q_2, b) &= q_2 \\ \delta(q_1, a) &= q_1 & & \\ \delta(q_1, b) &= q_2 & &\end{aligned}$$

q_1 - final state
 $*q_L$ | (q_1)

Wrong String

$\times ba$

$\times babca$

$bbbabbb$

$baaa$

TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_1	q_2
$*q_1$ (q_1)	q_1	q_1
q_2	q_2	q_2

{Q, Σ , δ , q_0 , F}

{ q_0, q_1, q_2 , {a, b}, δ, q_0, q_1 }

δ transition function is defined by the transition table

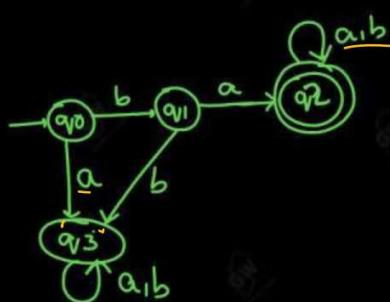
Note:

- Dead state is that state where there is no way to go back to final state

Design a DFA/MDFA $\Sigma = \{a, b\}$ such that string accepted must start with substring ba

TRANSITION DAIGRAM

$L = \{ba, baaaaa, babbb, babababab, \dots\}$



TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_3	q_1
q_1	q_2	q_3
$*q_2$	q_2	q_2
q_3	q_3	q_3

$\{Q, \Sigma, \delta, q_0, F\}$

$\{\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, q_2\}$

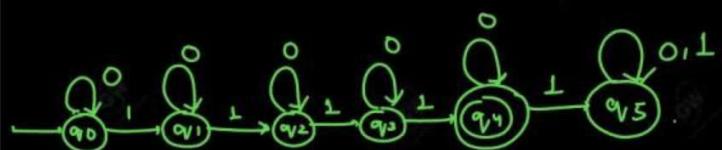
Note

Minimum state = minimum string length + 2

Design the finite automata which accept set of string containing exactly four 1's in every string
 $\Sigma = \{0, 1\}$

TRANSITION DAIGRAM

$L = \{1111, 001111, 01010101, 110011, 11110000, 111010\}$



$$\delta(q_0, 0) = q_0$$

$$\delta(q_0, 1) = q_1$$

$$\delta(q_1, 0) = q_1$$

$$\delta(q_1, 1) = q_2$$

Wrong string

001101011

1111010

1010101110

TRANSITION TABLE

	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_1	q_2
q_2	q_2	q_3
q_3	q_3	q_4
$*q_4$	q_4	q_5
q_5	q_5	q_5

$\{Q, \Sigma, \delta, q_o, F\}$

$\{\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{0, 1\}, \delta, q_o, q_4\}$

- **DNA Sequence Analysis:** Finite automata can be used to search for patterns in DNA sequences.
- **Robotics:** Finite automata can be used to model and control robot behavior in certain scenarios.
- **Pattern Recognition:** Finite automata can be used in image and signal processing for pattern recognition tasks.

LIMITATION OF FINITE AUTOMATA

- **Limited Memory:** Finite automata have a finite number of states,
- **Limited Expressiveness:** Finite automata can only recognize regular languages,

➤ **Difficulty with Complex Patterns:** While finite automata are good at recognizing simple pattern

What do you mean by Kleene closure of set A ?

- The Kleene closure of a set A, denoted A^*
- It includes the empty string ϵ all single elements of A, all possible pairs of elements from A, and so on.

A is the set $\{0,1\}$ then $A^* =$

$\{\epsilon, 0, 1, 00, 01, 10, 11, 000, \dots\}$,

Q1 ✓	Define alphabet and string in automata theory?	AKTU 2021-22
Q2 ✓	Give the definition of DFA.	AKTU 2021-22 AKTIU 2011-12
Q3 ✓	Write down the application of finite automata.	AKTU 2018-19
Q4 ✓	Define alphabet string and language.	AKTU 2017-18
Q5 ✓	Write down the application and limitation of finite automata.	AKTU 2017-18
Q6 ✓	Define and give the difference between positive closure and Kleene closure.	AKTU 2016-17
Q7 ✓	What do you mean by the Kleene closure of set A?	AKTU 2008-09
Q8 ✓	Design the finite automata which accept set of string containing exactly four 1's in every string $\Sigma=[0,1]$.	AKTU 2014-15
Q9 ✓	Differentiate between L^* and L^+	AKTU 2013-14

DPP

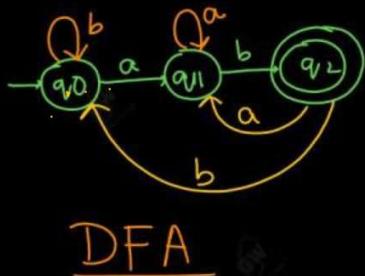
Q1 Draw the DFA over $\Sigma = \{0,1\}$ Start with the 10.

Q2 Draw the DFA | MDFA | FA over $\Sigma = \{0,1\}$ Start with 00

Design a DFA/MDFA/FA $\Sigma = \{a, b\}$ such that every string accepted must end with a string $w=ab$

TRANSITION DAIGRAM

$L = \{ab, bab, aaabbbaab, aaaaab, bbbabaaab, \dots\}$



DFA

TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_2
$*q_2$	q_1	q_0

Wrong string

$\times ababaaa$

$\times bababb$

$\times Abababbabb$

$w = ab$

$|w| = 2$

Minimum number
of states = $2 + 1 = 3$

$\{Q, \Sigma, \delta, q_0, F\}$

$\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, q_2\}$

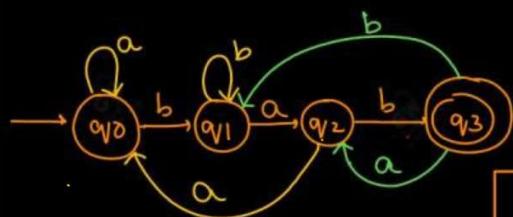
Note no dead state

Number of state = length of minimum string + 1

Design a DFA/MDFA $\Sigma = \{a, b\}$ such that string accepted must end with substring bab

TRANSITION DIAGRAM

$L = \{bab, aaaabab, aaabbabbab, bbabaabab, \dots\}$



Min length string = bab (W)

$$|W| \leq 3$$

$$\text{Minimum States} = 3 + 1 = 4$$

TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_1
q_2	q_0	q_3
$*q_3$	q_2	q_1

$\{Q, \Sigma, \delta, q_0, F\}$

$\{\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, q_3\}$

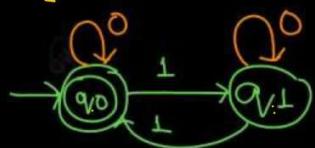
Wrong strings

- aaabaaa
- aababaa
- aaabbabbab
- bbbbaba

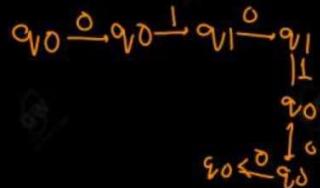
Design a DFA/MDFA $\Sigma = \{0, 1\}$ such that accept the string which contain even number of 1s

TRANSITION DAIGRAM

$L = \{0, 00000, 11, 0101, 110011, 0101011, 1100110011, \dots\}$



NOTE 0 | 0100



TRANSITION TABLE

	0	1
$\xrightarrow{*} q_0$	q_0	q_1
q_1	q_1	q_0

Wrong string

~ 111000
 $\sim 000|000$
 ~ 111111000

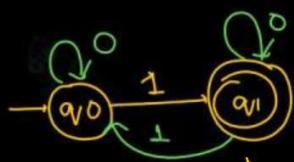
$\{Q, \Sigma, \delta, q_0, F\}$

$\{\{q_0, q_1\}, \{0, 1\}, \delta, q_0, q_0\}$

Design a DFA/MDFA $\Sigma = \{0, 1\}$ such that accept the string which contain Odd number of 1s

TRANSITION DAIGRAM

$L = \{1, 10, 01, 1110, 10000, 1110000, 00001111, 000110100, 001110, \dots\}$



TRANSITION TABLE

	0	1
$\rightarrow q_0$	q_0	q_1
q_1	$*_{q_1}$	q_1

$\{Q, \Sigma, \delta, q_o, F\}$

$\{\{q_0, q_1\}, \{0, 1\}, \delta, q_o, q_1\}$

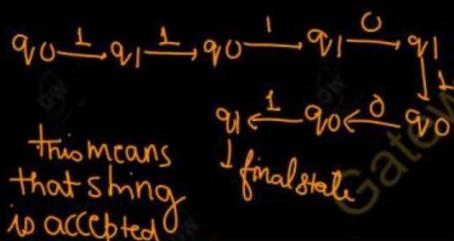
Non accepting string

111100 ✓

0001111

1010101

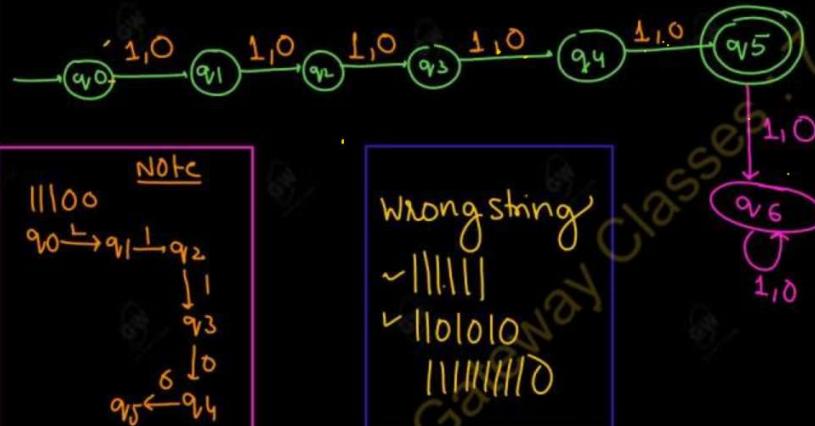
NOTE → 1110101



Construct a DFA that accept all the string of 1's and 0's where length of string is exactly 5.

TRANSITION DAIGRAM

$L = \{00000, 11111, 10101, 01011, 00011, 11000, 10101 \dots\}$



TRANSITION TABLE

	0	1
$\rightarrow q_0$	q_1	q_1
q_1	q_2	q_2
q_2	q_3	q_3
q_3	q_4	q_4
q_4	q_5	q_5
$*q_5$	q_6	q_6
q_6	q_6	q_6

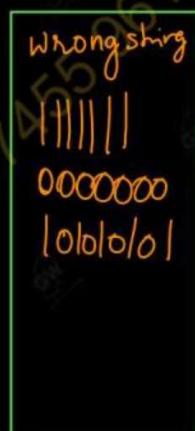
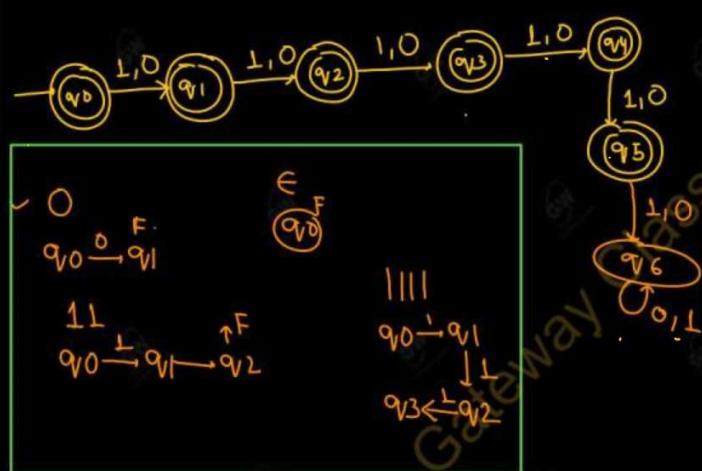
$\{Q, \Sigma, \delta, q_0, F\}$

$\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{0, 1\}, \delta, q_0, q_5\}$

Construct a DFA that accept all the string of 1's and 0's where length of string is at most 5

TRANSITION DAIGRAM

$L = \{\epsilon, 1, 0, 11, 01, 10, 111, 0111, 01010, 10101, \dots\}$



TRANSITION TABLE

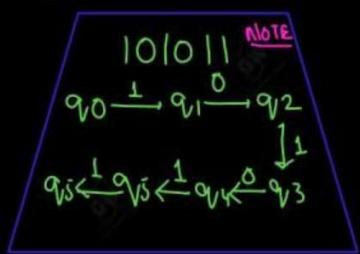
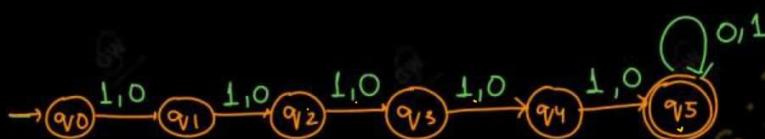
	0	1
$\rightarrow *q_0$	q_1	q_1
$*q_1$	q_2	q_2
$*q_2$	q_3	q_3
$*q_3$	q_4	q_4
$*q_4$	q_5	q_5
$*q_5$	q_6	q_6
q_6	q_6	q_6

$\{Q, \Sigma, \delta, q_0, F\} = \{\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{0, 1\}, \delta, q_0, \{q_0, q_1, q_2, q_3, q_4, q_5\}\}$

Construct a DFA that accept all the string of 1's and 0's where length of string is at least 5

TRANSITION DAIGRAM

$L = \{00000, 11111, 11111111, 1010100011, 010111, 0001000, 1100000, 101010 \dots\}$



TRANSITION TABLE

	0	1
$\rightarrow q_0$	q_1	q_1
q_1	q_2	q_2
q_2	q_3	q_3
q_3	q_4	q_4
q_4	q_5	q_5
$*q_5$	q_5	q_5

Non-acceptable String

|||
0|0
|||
|||0

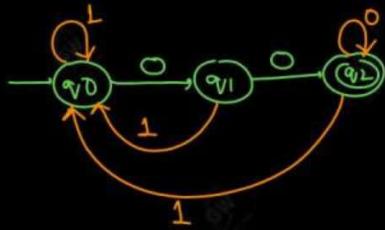
$\{Q, \Sigma, \delta, q_0, F\}$

$\{\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{0, 1\}, \delta, q_0, F\}$

Design a DFA/MDFA/FA $\Sigma=\{0,1\}$ such that every string accepted must end with a string w=00

TRANSITION DAIGRAM

$L=\{00, 100, 1100, 110100, 0000\}$



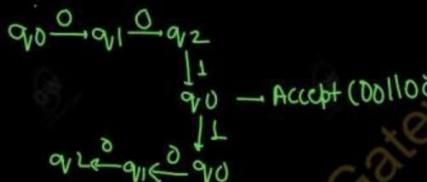
TRANSITION TABLE

	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_0
$*q_2$	q_2	q_0

$\{Q, \Sigma, \delta, q_0, F\}$
 $\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, q_2\}$

Non accepting string

111
1110
0010



Design a DFA/MDFA/FA $\Sigma = \{0, 1\}$ such that every string accepted does not end with a string $w=01$

TRANSITION DIAGRAM

$$L = \{\epsilon, 0, 1, 1111, 000000, 0101111, \dots\}$$

(i) end with 01



- Convert final state to non-final
- Convert all non-final states to final

DFA that will accept $(01)^*$



TRANSITION TABLE

	0	1
$\rightarrow *q_0$	q_1	q_0
$*q_1$	q_1	q_2
q_2 (Non-final state)	q_1	q_0

$\{Q, \Sigma, \delta, q_0, F\}$
 $\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0,$
 $\{q_0, q_1\}\}$

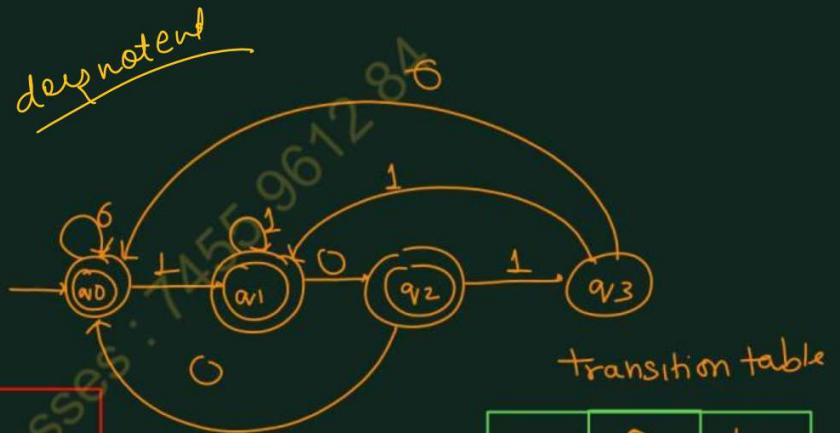
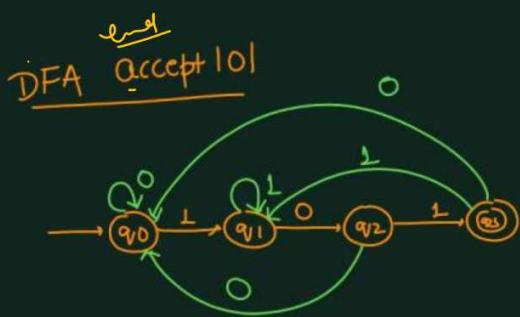
ϵ (Right String)

Wrong string

- 000001
- 111101
- 1100001
- 01

0
1
0
1
0
1
0
1

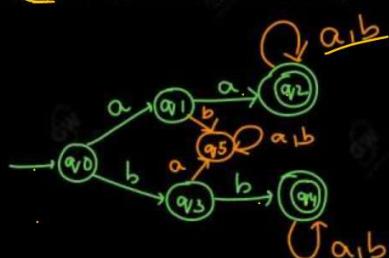
DFA Accept every string except 101 $\Sigma = \{0, 1\}$



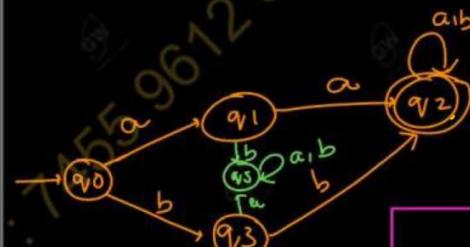
$Q, \Sigma, \delta, q_0, F$
 $\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta,$
 $q_0, \{q_0, q_1, q_2\}$

transition table

	0	1
$*q_0$	q_0	q_1
$*q_1$	q_2	q_1
$*q_2$	q_0	q_3
q_3	q_0	q_1

TRANSITION DAIGRAM $L=\{aa, bb, aabab, bbbaba, aabbba.....\}$ 

Minimize

 q_5 (dead stat.)

Wrong string
~ abab
~ baba
~ b

(Right string)
aabb
aababc
bbaa

Design a DFA/MDFA/FA $\Sigma = \{a, b\}$ such that every string accepted that start with either aa or bb

TRANSITION TABLE

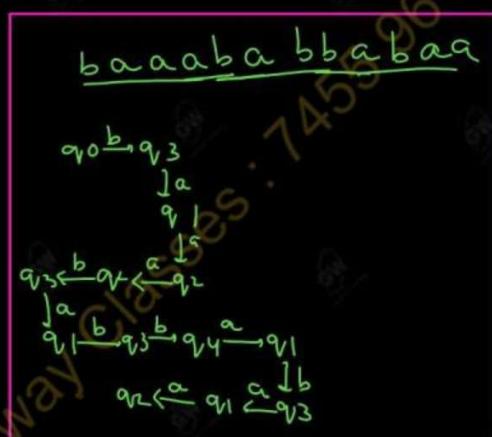
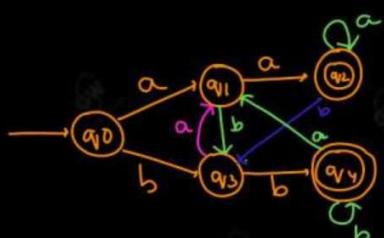
	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_2	q_5
$*q_2$	q_2	q_2
q_3	q_5	q_2
q_5	q_5	q_5

$\{Q, \Sigma, \delta, q_0, F\}$

$\{\{q_0, q_1, q_2, q_3, q_5\}, \{a, b\}, \delta, q_0, q_2\}$

TRANSITION DAIGRAM

$L = \{aa, bb, baaababbabaa, ababbbaaa, babababb, ababbabb, abababbbbaa, \dots\}$



Wrong String

→ abab

→ buba

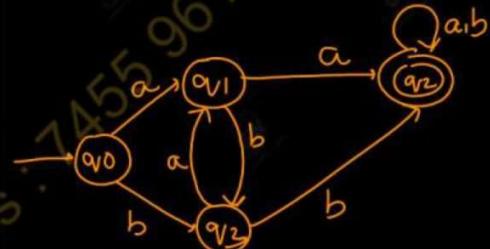
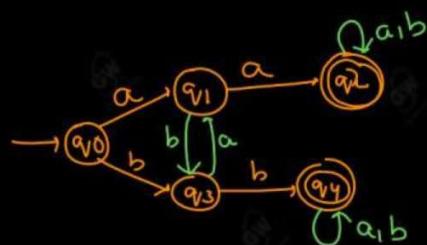
TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_2	q_3
$*q_2$	q_2	q_3
q_3	q_1	q_4
$*q_4$	q_1	q_5

{Q, Σ, δ, q₀, F}{ {q₀, q₁, q₂, q₃, q₄}, {a, b}, δ, q₀, {q₂, q₄} } }

TRANSITION DAIGRAM

$L=\{aa, bb, aababab, bbbbabab, bababaabbab, bbabaaaabb, abababbbabab\}$



TRANSITION TABLE

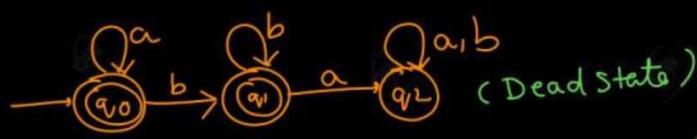
	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_2	q_3
$*q_2$	q_2	q_2
q_3	q_1	q_2

{Q, Σ, δ, q₀, F}{ {q₀, q₁, q₂, q₃ }, {a, b}, δ, q₀, q₂ } }

TRANSITION DAIGRAM $L = \{\epsilon, a, aa, aaaa, aaaa, \dots\}$ **TRANSITION TABLE**

	a
$\rightarrow *q_0$	q_0

 $\{Q, \Sigma, \delta, q_o, F\}$ $\{\{q_0\}, \{a\}, \delta, q_o, q_0\}$

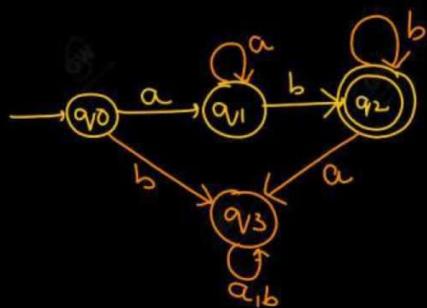
TRANSITION DAIGRAM $L = \{\epsilon, a, aaaa, b, bbbb, aabbb, \dots\}$ 

<u>aba</u>	ϵ (Acceptable String)
abaaaa	<u>a</u>
babbba	<u>aaaaa</u>
(Non-acceptable String)	<u>b</u>
	bbbbbbb
	bab
	aabbb

TRANSITION TABLE

	a	b
$\rightarrow^* q_0$	q_0	q_1
$* q_1$	q_2	q_1
q_2	q_2	q_2

 $\{Q, \Sigma, \delta, q_0, F\}$ $\{\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_1, q_2\}\}$

TRANSITION DAIGRAM $L = \{ab, aaabb, abbb, aaaaab\}$ 

Wrong strings

- ✓ aaba
- ✓ baba
- ✓ bba
- ✓ a
- ✓ b

TRANSITION TABLE

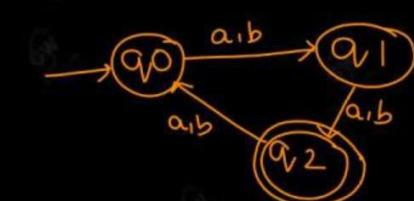
	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_1	q_2
$*q_2$	q_3	q_2
q_3	q_3	q_3

 $\{Q, \Sigma, \delta, q_0, F\}$ $\{\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_2\}\}$

TRANSITION DIAGRAM

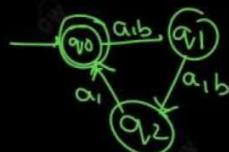
 $L = \{ aa, bb, aaabb, aabbb, aabbaabb, bbbbbaaaa, \dots \}$

$|w| \equiv 2 \pmod{3}$



(Multiple of 3), divisible by 3

$|w| \equiv 0 \pmod{3}$



	a	b
$\rightarrow q_0$	q_1	q_1
$\rightarrow q_1$	q_2	q_2
$\rightarrow q_2$	q_0	q_0
q_0	q_1	q_1
q_1	q_2	q_2
q_2	q_0	q_0

TRANSITION TABLE $|w| \equiv 2 \pmod{3}$

	a	b
$\rightarrow q_0$	q_1	q_1
q_1	q_2	q_2
q_2	q_0	q_0

{Q, Σ, δ, q₀, F}

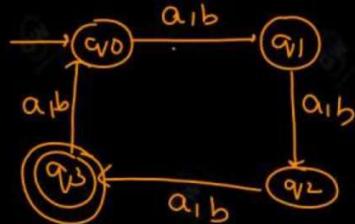
{ {q₀, q₁, q₂}, {a, b}, δ, q₀, q₂ }

NOTE

W=aa $|w|=2$ (length)

TRANSITION DAIGRAM

$L = \{ \text{aaa, bbb, abababa, bababab, ababababbbb.....} \}$
 $|w|: 3 \quad 3 \quad 7 \quad 7 \quad 11$

NOTE

	Initial State	Final State
$0 \pmod{4}$ / Multiple of 4	q_0	q_0
$1 \pmod{4}$	q_0	q_1
$2 \pmod{4}$	q_0	q_2

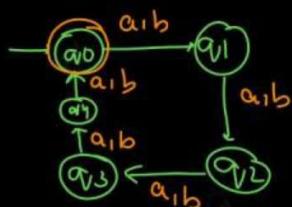
TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_1	q_1
q_1	q_2	q_2
q_2	q_3	q_3
* q_3	q_0	q_0

$$\{Q, \Sigma, \delta, q_0, F\} = \{\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_3\}\}$$

TRANSITION DAIGRAM

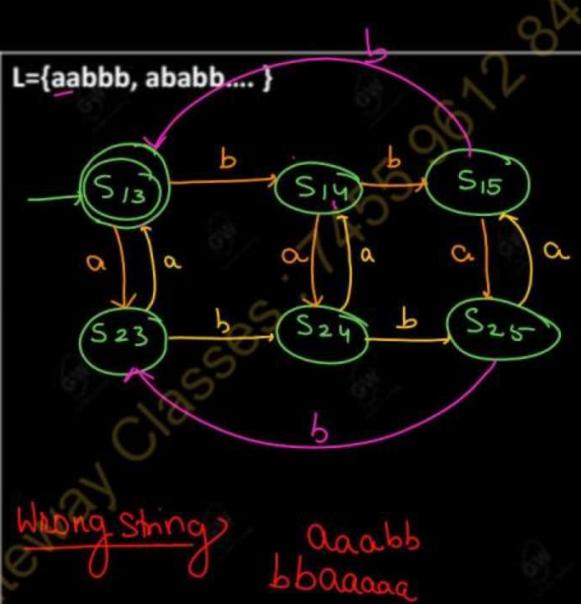
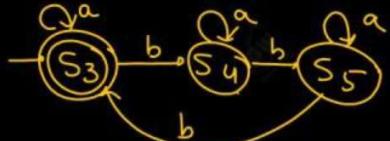
$L = \{\epsilon, \text{aaaaa}, \text{bbbbbb}, \text{ababababab}, \text{bababababa} \dots\}$



TRANSITION TABLE

	a	b
$\rightarrow^* q_0$	q_1	q_1
q_1	q_2	q_2
q_2	q_3	q_3
q_3	q_4	q_4
q_4	q_0	q_0

$\{Q, \Sigma, \delta, q_o, F\} = \{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_o, \{q_o\}$

$a \text{ is multiple of } 2$ $a \text{ is divisible by } 2 \mid |w|_a \equiv 0 \pmod{2}$
 $a \text{ is even}$  $b \text{ is divisible by } 3 \mid \text{Multiple of } 3 \quad |w|_b = 0 \pmod{3}$ 

Construct a DFA that accept all the string over={a,b} in which a is even(a is divisible by 2) and b is divisible by 3

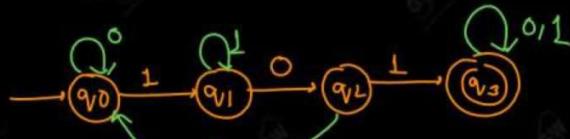
TRANSITION TABLE

	a	b
$\rightarrow *S_{13}$	S_{23}	S_{14}
S_{14}	S_{24}	S_{15}
S_{15}	S_{25}	S_{13}
S_{23}	S_{13}	S_{24}
S_{24}	S_{14}	S_{25}
S_{25}	S_{15}	S_{23}

$$\{Q, \Sigma, \delta, q_0, F\} = \{\{S_{13}, S_{14}, S_{15}, S_{23}, S_{24}, S_{25}\}, \{a, b\}, \delta, S_{13}, S_{13}\}$$

DFA accept 101 as substring

$L = \{101, 10111, 1010010, 1110100 \dots\}$



Complement of $L(\bar{L})$

$\bar{L} = \{\epsilon, 0, 1, 11, 1000, 001, 1111, 100 \dots\}$
(this will not accept 101 as substring)

Wrong string
 $\times 101 \quad 000101$
 11011

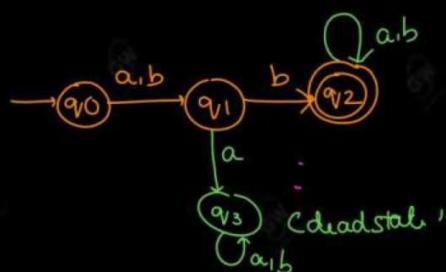
TRANSITION TABLE

	0	1
$\rightarrow^* q_0$	q_0	q_1
$* q_1$	q_2	q_1
$* q_2$	q_0	q_3
q_3	q_3	q_3

{Q, Σ, δ, q₀, F}{ {q₀, q₁, q₂, q₃}, {0, 1}, δ, q₀, {q₀, q₁, q₂} }

TRANSITION DAIGRAM

$L = \{ab, bb, abbbbabab, bbababab, \dots\}$



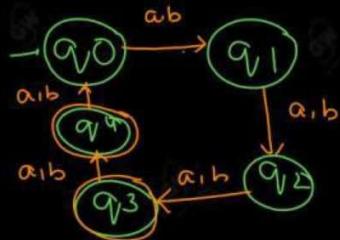
Wrong string
aaba
baba

TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_1	q_1
q_1	q_3	q_2
$*q_2$	q_2	q_2
q_3	q_3	q_3

$\{Q, \Sigma, \delta, q_0, F\}$

$\{\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_2\}\}$

TRANSITION DAIGRAM $L = \{aaa, baba, babababa, \dots\}$ 

$w = ababbbb$
 $|w| = 7$

$w = ababa$
 $|w| = 5$

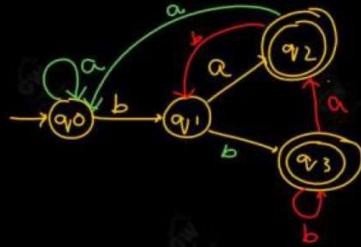
$w = aba$
 $|w| = 3$

TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_1	q_1
q_1	q_2	q_2
q_2	q_3	q_3
$*q_3$	q_4 ✓	q_4 ✓
$*q_4$	q_0	q_0

$\{Q, \Sigma, \delta, q_0, F\}$

$\{\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_0, \{q_3, q_4\}\}$

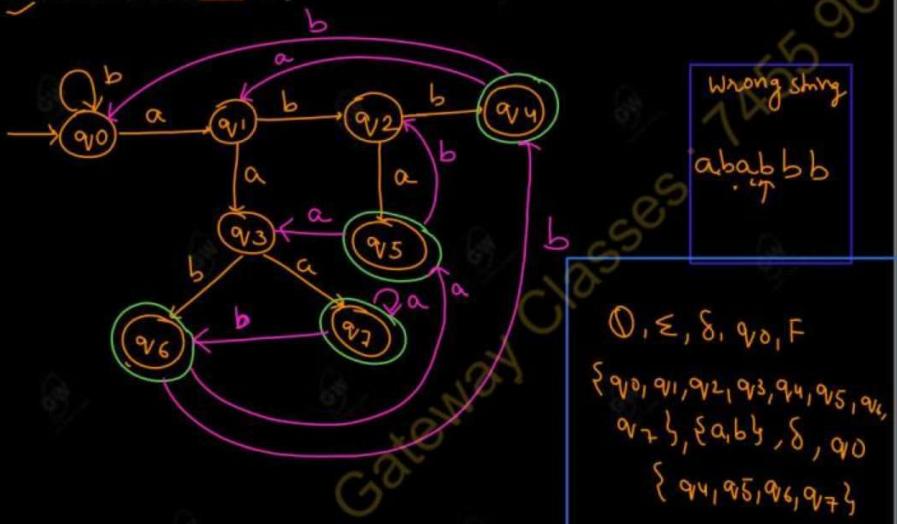
TRANSITION DAIGRAM $L = \{ba, bb, aba, aaaaba, \dots\}$ **TRANSITION TABLE**

	a	b
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_3
$*q_2$	q_0	q_1
$*q_3$	q_2	q_3

Number of states 2^n n (position of symbol from right)

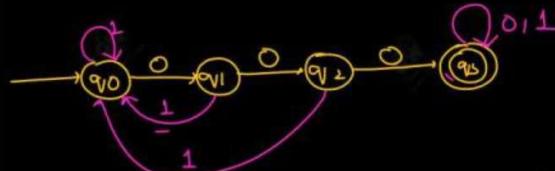
$\{Q, \Sigma, \delta, q_0, F\}$

$\{\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_2, q_3\}\}$

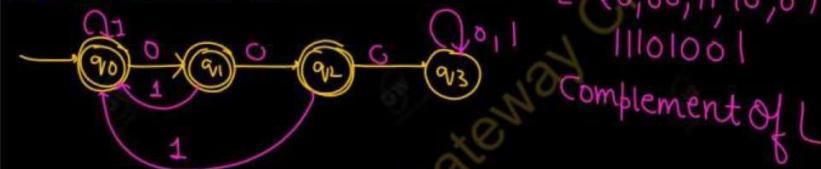
TRANSITION DAIGRAM $L = \{aaa, baaa, bbbaba, \dots\}$ **TRANSITION TABLE**

	b	a
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_5
q_3	q_6	q_7
$*q_4$	q_0	q_1
$*q_5$	q_2	q_3
$*q_6$	q_4	q_5
$*q_7$	q_6	q_7

TRANSITION DIAGRAM

 $L = \{000, 11010000, 000111, 101000111, \dots\}$ 

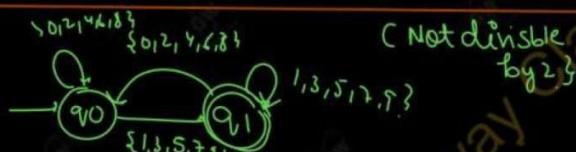
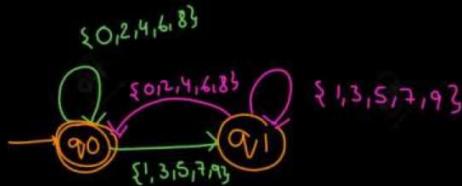
Does not contain consecutive three zeros



TRANSITION TABLE

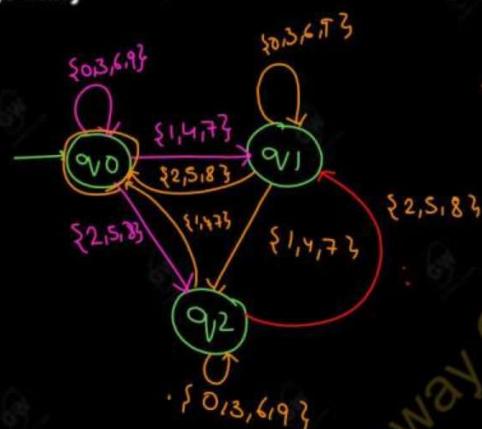
	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_0
q_2	q_3	q_0
$*q_3$	q_3	q_3

 $\{Q, \Sigma, \delta, q_0, F\}$ $\{\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta, q_0, \{q_3\}\}$

TRANSITION DAIGRAM $L = \{2, 4, 8, 10, 12, 14, 20, 22, 24, \dots\}$ **TRANSITION TABLE**

	$\{0, 2, 4, 6, 8\}$	$\{1, 3, 5, 7, 9\}$
$\rightarrow^* q_0$	q_0	q_1
q_1	q_0	q_1

$$\begin{array}{c} \{Q, \Sigma, \delta, q_0, F\} \\ \xleftarrow{\quad \quad \quad} \{ \{q_0, q_1\}, \delta, q_0, \{q_0\} \} \\ \xrightarrow{\quad \quad \quad} \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \end{array}$$

TRANSITION DAIGRAM $L = \{0, 3, 6, 9, \dots\}$ **TRANSITION TABLE**

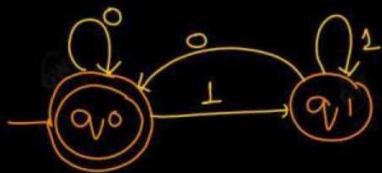
	$\{0, 3, 6, 9\}$	$\{1, 4, 7\}$	$\{2, 5, 8\}$
$\rightarrow^* q_0$	q_0	q_1	q_2
q_1	q_1	q_2	q_0
q_2	q_2	q_0	q_1

 $\{Q, \Sigma, \delta, q_o, F\}$ $\{\{q_0, q_1, q_2\}, \{0, 1, 2, \dots, 9\}, \delta, q_o, \{q_0\}\}$

Construct a DFA for the which accept the set of all string over {0,1} which interpreted as binary number divisible by 2

TRANSITION DAIGRAM

$L = \{00, 10, 000, 010, 100, \dots\}$

**TRANSITION TABLE**

	0	1
$\rightarrow^* q_0$	q_0	q_1
q_1	q_0	q_1

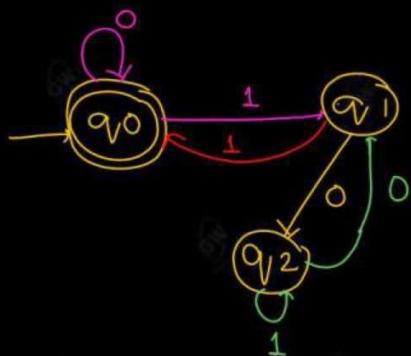
$\{Q, \Sigma, \delta, q_o, F\}$

$\{\{q_0, q_1\}, \{0, 1\}, \delta, q_o, \{q_0\}\}$

Construct a DFA for the which accept the all string over{0,1} which interpreted as binary number divisible by 3

TRANSITION DAIGRAM

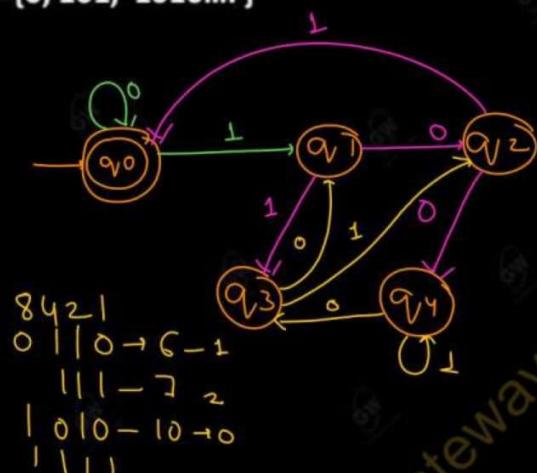
$$L = \{0, 11, 110, \dots\}$$

**TRANSITION TABLE**

	0	1
$\xrightarrow{*} q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

$$\{Q, \Sigma, \delta, q_o, F\}$$

$$\{\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_o, \{q_0\}\}$$

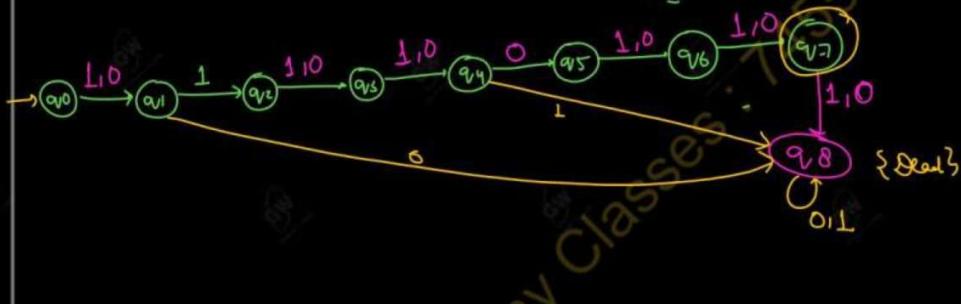
TRANSITION DAIGRAM $L = \{0, 101, 1010, \dots\}$ **TRANSITION TABLE**

	0	1
$\rightarrow^* q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

 $\{Q, \Sigma, \delta, q_0, F\}$ $\{\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta, q_0, \{q_0\}\}$

Construct a DFA for the which accept the string of all string over {0,1} having length 7 provided 2 digit 1 from left and 3rd digit from right is 0

TRANSITION DAIGRAM



$$\{Q, \Sigma, \delta, q_0, F\}$$

$$\{(q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8), \{0,1\}, \delta, q_0, \{q_8\}\}$$

TRANSITION TABLE

	0	1
$\rightarrow q_0$	q_1	q_1
q_1	q_8	q_2
q_2	q_3	q_3
q_3	q_4	q_4
q_4	q_5	q_8
q_5	q_6	q_6
$*q_7$	q_8	q_8
q_8	q_8	q_8

➤ The process of elimination of state whose presence or absence does not affect the language accepting capability of DFA is called minimization of DFA and the result is minimal deterministic finite automata commonly known as MFA

NOTE : MFA is unique for a language

DEAD STATE

It is basically created to make the system complete, can be defined as a state from which there is no transition possible to the final state

NOTE : IN DFA , there can be more than one state but logically always one dead state is sufficient to complete the functionality

➤ **UNREACHABLE STATE**

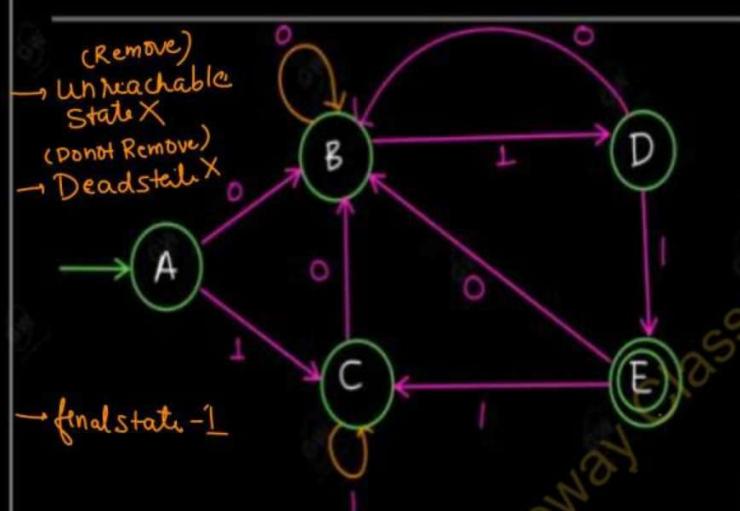
Unreachable states are the states that are not reachable from the initial state of the DFA, for any input string.

➤ **EQUIVALENT STATE**

In the minimization of a DFA, equivalent states are states that can be merged without changing the language recognized by the DFA.

➤ **EQUIVALENCE CLASS**

An equivalence class (or equivalence class) refers to a grouping of states that are equivalent to each other. This means that all states within an equivalence class behave identically with respect to the language recognized by the DFA.

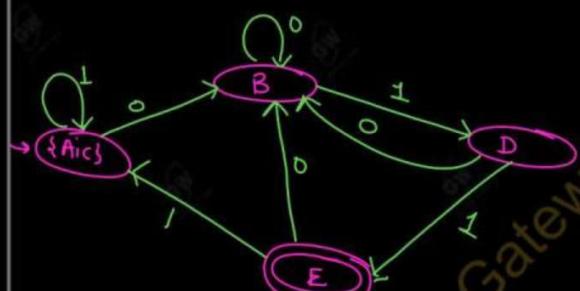


TRANSITION TABLE

	0	1
→A	B	C
B	B	D
C	B	C
D	B	E
*E (E)	B	C

EQUIVALENCE CLASS

$\pi_0 = \{A, B, C, D\} \setminus \{E\}$ 0-equivalence class
 $\pi_1 = \{A, B, C\} \setminus \{D\} \setminus \{E\}$ 1-equivalence class
 $\pi_2 = \{A, C\} \setminus \{B\} \setminus \{D\} \setminus \{E\}$ 2-equivalence class
 $\pi_3 = \{A, C\} \setminus \{B\} \setminus \{D\} \setminus \{E\}$ 3-equivalence



TRANSITION TABLE

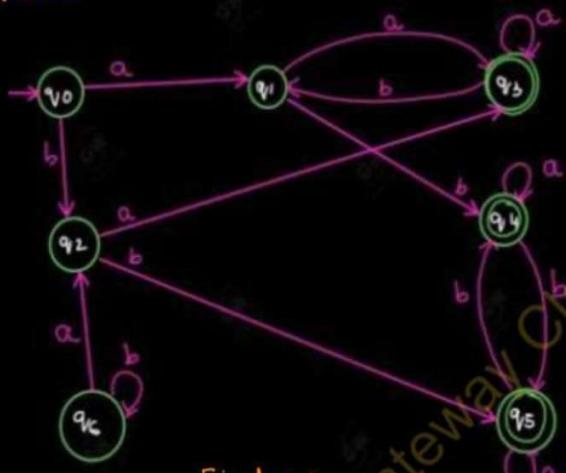
	0	1
$\rightarrow A$	B	C
B	B	D
C	B	C
D	B	E
$\ast E$	B	C

MINIMIZED TRANSITION TABLE

	0	1
$\rightarrow \{A, C\}$	B	{A, C}
B	B	D
D	B	E
$\ast E$	B	{A, C}

2. MINIMIZATION OF DFA

→ Unreachable state (Removed)
→ Dead State X



Final stat - 3

TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_1	q_2
q_1	q_3	q_4
q_2	q_3	q_5
$*q_3$	q_3	q_1
$*q_4$	q_4	q_5
$*q_5$	q_5	q_4
q_6	q_2	q_6

q_6 unreachable state

2. MINIMIZATION OF DFA

EQUIVALENCE CLASS

$\pi_0 = \{q_0, q_1, q_2\}$ $\{q_3, q_4, q_5\}$ 0-equivalence class

q_0 q_1

$\pi_1 = \{q_0\}$, $\{q_1, q_2\}$, $\{q_3\}$, $\{q_4, q_5\}$

q_1 q_2

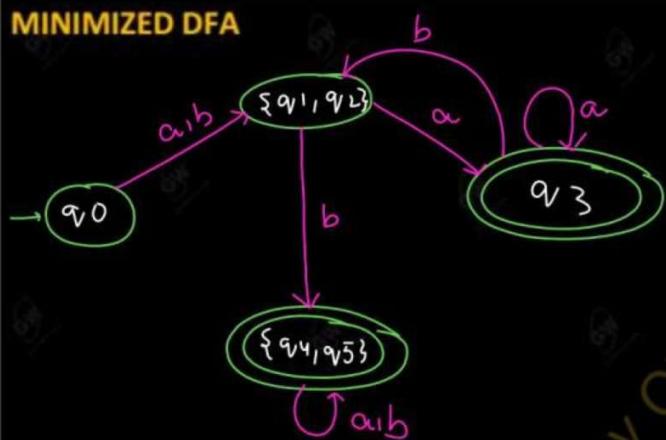
$\pi_2 = \{q_0\}$, $\{q_1, q_2\}$, $\{q_3\}$, $\{q_4, q_5\}$

(1-equivalence class)

q_1
Class

TRANSITION TABLE

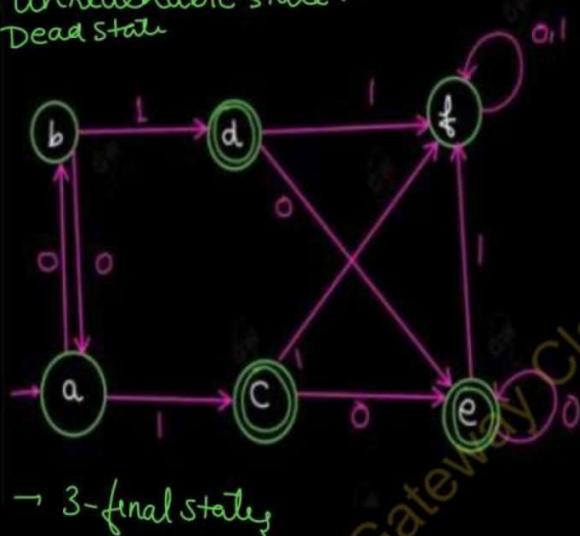
	a	b
$\rightarrow q_0$	q_1	q_2
q_1	q_3	q_4
q_2	q_3	q_5
$*q_3$	q_3	q_1
$*q_4$	q_4	q_5
$*q_5$	q_5	q_4

MINIMIZED DFA**TRANSITION TABLE**

	a	b
$\rightarrow q_0$	$\{q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	q_3	$\{q_4, q_5\}$
$*q_3 \{q_3\}$	q_3	$\{q_1, q_2\}$
$*\{q_4, q_5\} \{q_4, q_5\}$	$\{q_4, q_5\}$	$\{q_4, q_5\}$

3 MINIMIZATION OF DFA

→ Unreachable state X
→ Dead state



→ 3-final states

TRANSITION TABLE

	0	1
→ a	b	c
b	a	d
*c (c)	e	f
*d (d)	e	f
*e (e)	e	f
f —	f	f

f dead state

EQUIVALENCE CLASS

$\pi_0 = \{a, b, f\} \{c, d, e\}$ 0-equivalence class

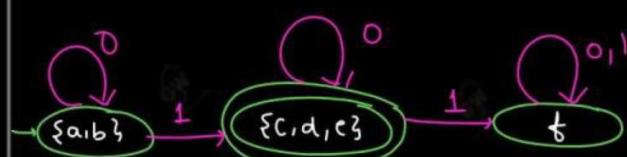
$\pi_1 = \{a, b\} \{f\} \{c, d, e\}$ 1-equivalence class

$\pi_2 = \{a, b\}, \{f\} \{c, d, e\}$ 2-equivalence Class

TRANSITION TABLE

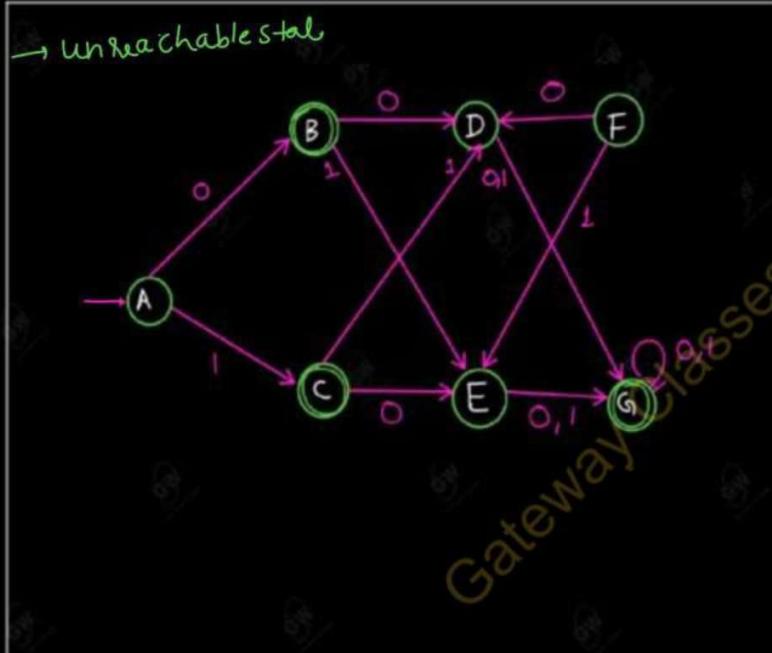
	0	1
$\rightarrow a$	b	c
b	a	d
*c	e	f
*d	e	f
*e	e	f
f	f	f

MINIMIZED DFA



TRANSITION TABLE

	0	1
$\rightarrow \{a, b\}$	$\{a, b\}$	$\{c, d, e\}$
$*\{c, d, e\}$	$\{c, d, e\}$	f
f	f	f



TRANSITION TABLE

	0	1
→ A	B	C
* B	D	E
* C	E	D
D	G	G
E	G	G
F	D	E
* G	G	G

F is unreachable state

4 MINIMIZATION OF DFA

EQIVALENCE CLASS

$\pi_0 = \{A, D, E\}$ $\{B, C, G\}$ 0-equivalence class

$\pi_1 = \{A, D, E\}$ $\{B, C\}$ $\{G\}$ 1-equivalence class

$\pi_2 = \{A\}$, $\{D, E\}$, $\{B, C\}$, $\{G\}$
2-equivalence class

$\pi_3 = \{A\}, \{D, E\}, \{B, C\}, \{G\}$
3-equivalence class

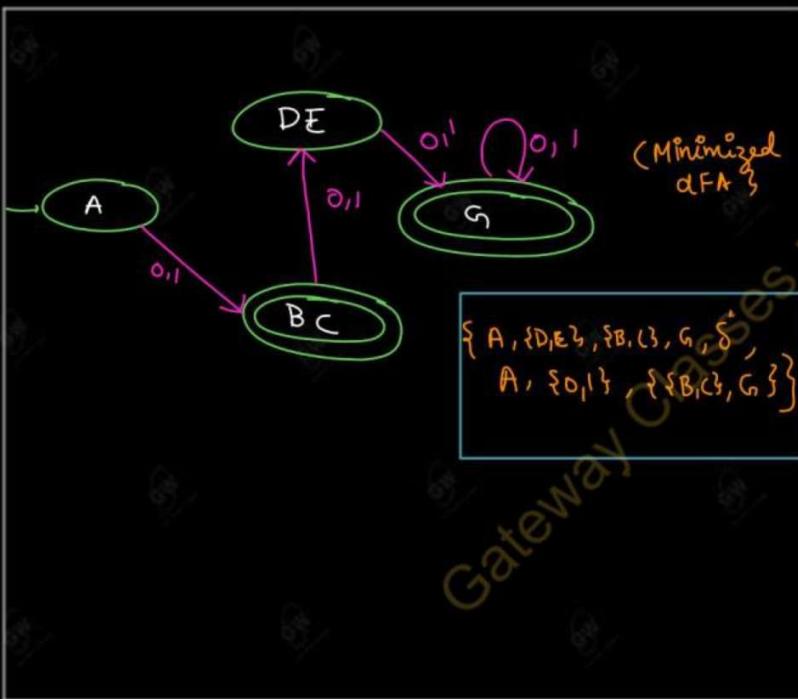
TRANSITION TABLE

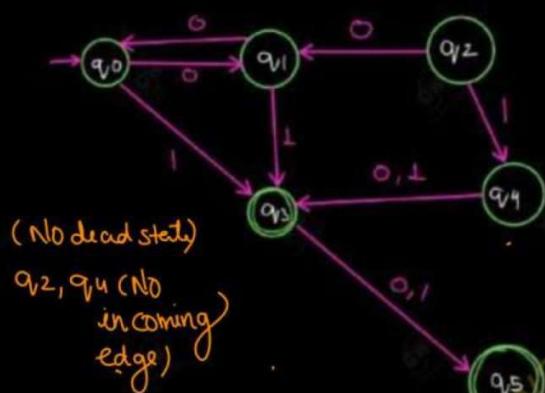
	0	1
$\rightarrow A$	B	C
*B	D	E
*C	E	D
D	G	G
E	G	G
*G	G	G

4 MINIMIZATION OF DFA

(Minimized)
TRANSITION TABLE

	0	1
$\rightarrow A$	{B,C}	{B,C}
{D,E}	G	G
*{B,C} $\bar{B,C}$	{D,E}	{D,E}
*G \bar{G}	G	G





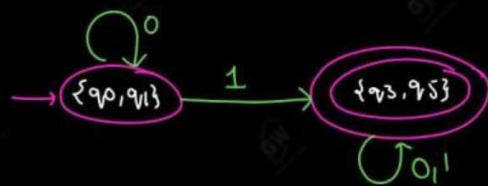
TRANSITION TABLE

	0	1
→ q ₀	q ₁	q ₃
q ₁	q ₀	q ₃
q ₂	q ₁	q ₄
* q ₃ (q ₂)	q ₅	q ₅
q ₄	q ₃	q ₃
* q ₅ (q ₁)	q ₅	q ₅

Noteq₂, q₄ are unreachable state

EQUIVALENCE CLASS

$\pi_0 = \{q_0, q_1\} \{q_3, q_5\}$ 0-equivalence class
 $\pi_1 = \{q_0, q_1\}, \{q_3, q_5\}$ 1-equivalence class



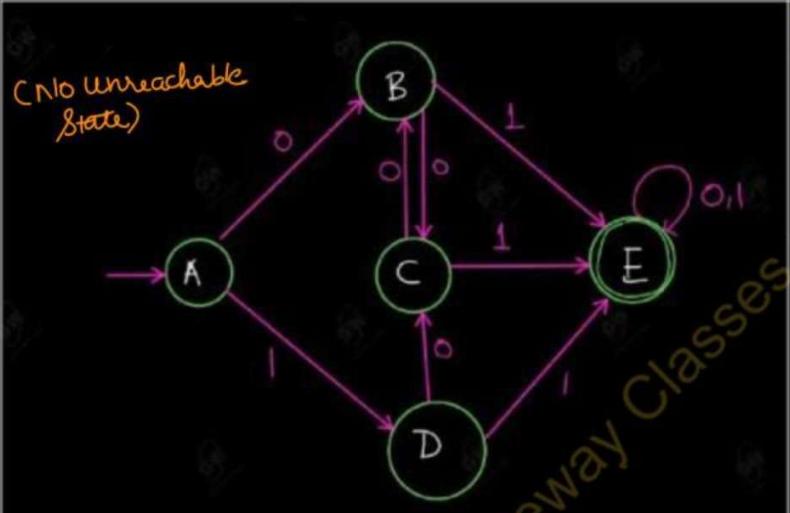
$\left\{ \{q_0, q_1\} \{q_3, q_5\} \right\}, \delta, \{0, 1\} \left\{ \{q_0, q_1\} \right\}, \left\{ \{q_3, q_5\} \right\}$

TRANSITION TABLE

	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_0	q_3
$*q_3$	q_5	q_5
$*q_5$	q_5	q_5

MINIMIZED TRANSITION TABLE

	0	1
$\rightarrow \{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_3, q_5\}$
$\{q_3, q_5\}$	$\{q_3, q_5\}$	$\{q_3, q_5\}$



TRANSITION TABLE

	0	1
→A	B	D
B	C	E
C	B	E
D	C	E
*E	E	E

EQUIVALENCE CLASS

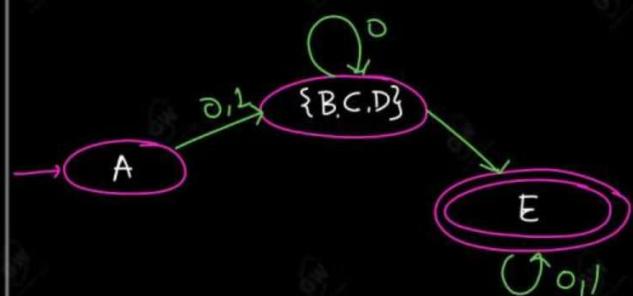
$\pi_0 = \{A, B, C, D\} \{E\}$ 0-equivalence class

$\pi_1 = \{A\}, \{B, C, D\}, \{E\}$ 1-equivalence class

$\pi_2 = \{A\}, \{B, C, D\}, \{E\}$ 2-equivalence class

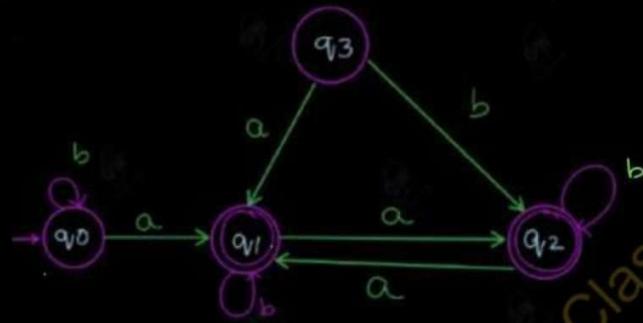
TRANSITION TABLE

	0	1
$\rightarrow A$	B	D
B	C	E
C	B	E
D	C	E
*E	E	E

MINIMIZED DFA**MINIMIZED TRANSITION TABLE**

	0	1
→ A	{B,C,D}	{B,C,D}
{B,C,D}	{B,C,D}	E
*E (E)	E	E

unreachable state



TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_1	q_0
$*q_1$	q_2	q_1
$*q_2$	q_1	q_2
q_3	q_1	q_2

q_3 is unreachable state

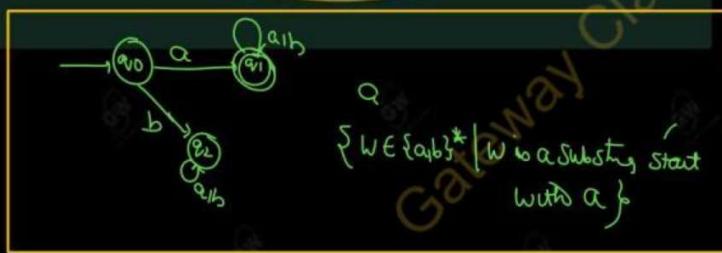
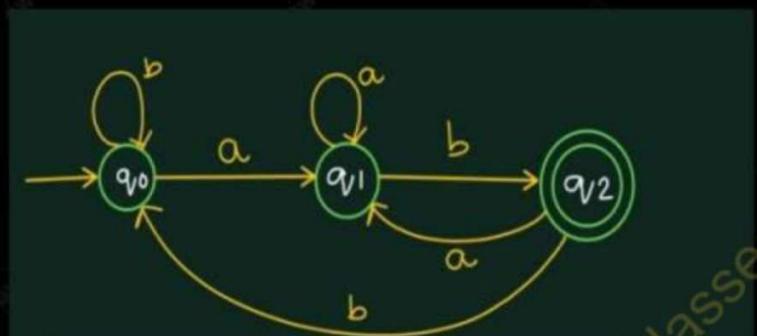
$\pi_0 = \{q_0\} \quad \{q_1, q_2\}$ 0-equivalence class
 $\pi_1 = \{q_0\}, \{q_1, q_2\}$ 1-equivalence class



TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_1	q_0
$*q_1$	q_2	q_1
$*q_2$	q_1	q_2

	a	b
$\rightarrow q_0$	$\{q_1, q_2\}$	q_0
$*\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$



$\{w \in \{a,b\}^* \mid w \text{ is a substring starting with } ab\}$

$\{w \in \{a,b\}^* \mid w \text{ is ending with } ab\}$

Explanation

\in belongs to

$w \rightarrow ab$ $w \rightarrow abab$ $w \rightarrow abaabb$

dead state \rightarrow NO \rightarrow Starting with ab X

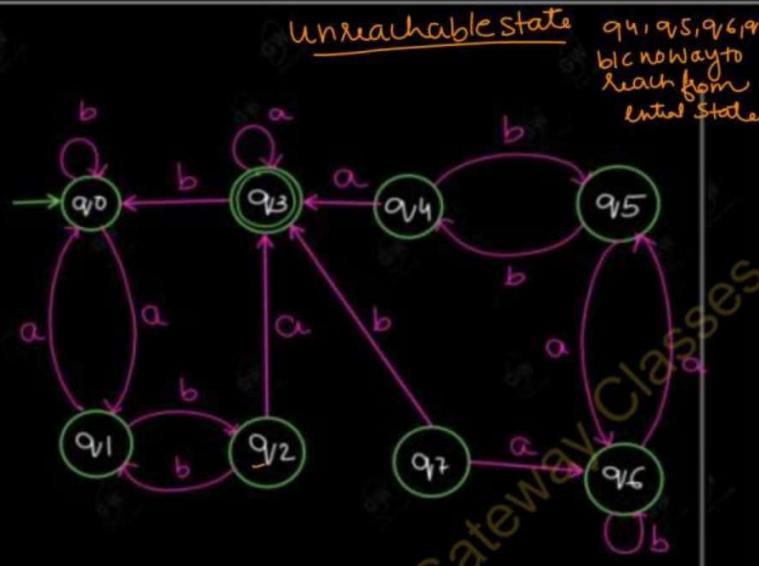
✓ ending with ab

✓ Substring ab \rightarrow X q_2 loop a, b

$$\Sigma = \{a, b\}$$

$$\Sigma^* = \{\epsilon, a, b, aa, bb, ab, ba, bbb, \dots\}$$

$$\{a, b\}^* = \{\epsilon, a, b, aa, bb, ab, ba, bbb, \dots\}$$



TRANSITION TABLE

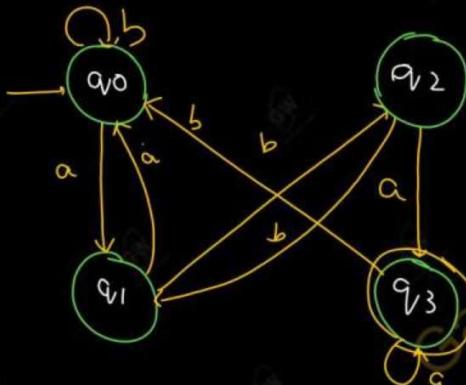
	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
* q_3	q_3	q_0
q_4	q_3	q_5
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_6	q_3

EQUIVALENCE CLASS

$\pi_0 = \{q_0, q_1, q_2\}, \{q_3\}$ 0-equivalence class
 $\pi_1 = \{q_0, q_1\}, \{q_2\}, \{q_3\}$ 1-equivalence class

$\pi_2 = \{q_0\}, \{q_1\}, \{q_2\}, \{q_3\}$ 2-equivalence class

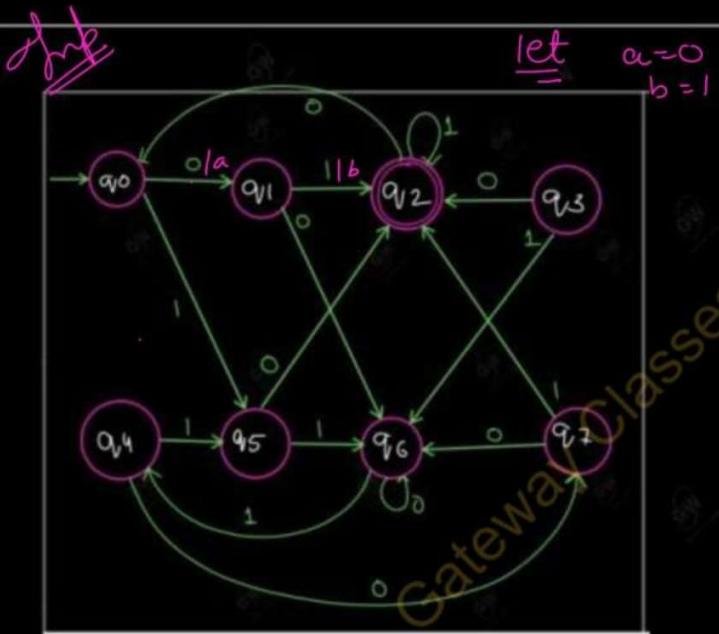
$\pi_3 = \{q_0\}, \{q_1\}, \{q_2\}, \{q_3\}$ 3-equivalence class

**TRANSITION TABLE**

	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
$*q_3$	q_3	q_0

MINIMIZED TRANSITION TABLE

	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
$*q_3$	q_3	q_0



TRANSITION TABLE

	a 0	b 1
$\rightarrow q_0$	q_1	q_5
q_1	q_6	q_2
$*q_2$	q_0	q_2
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

EQUIVALENCE CLASS

$$\pi_0 = \{q_0, q_1, q_4, q_5, q_6, q_7\} \quad \{q_2\}$$

$$\pi_1 = \{q_0, q_4, q_6\} \quad \{q_1, q_7\} \quad \{q_5\} \quad \{q_2\}$$

$$\pi_2 = \{q_0, q_4, q_6\} \quad \{q_6\} \quad \{q_1, q_7\} \quad \{q_5\} \quad \{q_2\}$$

$$\pi_3 = \{q_0, q_4\} \quad \{q_6\} \quad \{q_1, q_7\} \quad \{q_5\} \quad \{q_2\}$$

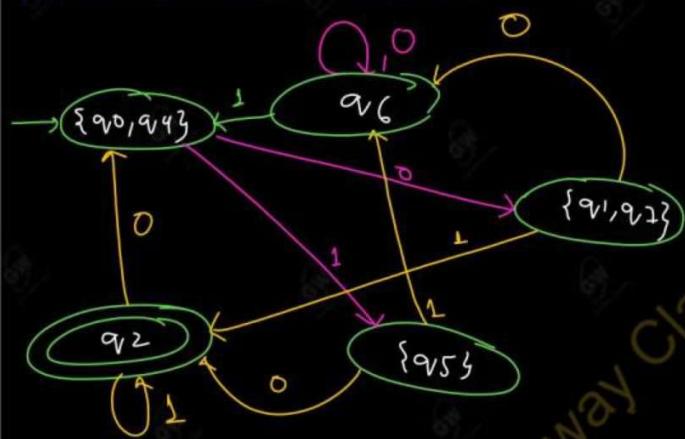
NOTE - If DFA is given first make DFA

from table Why?

So that you can find unreachable states

TRANSITION TABLE

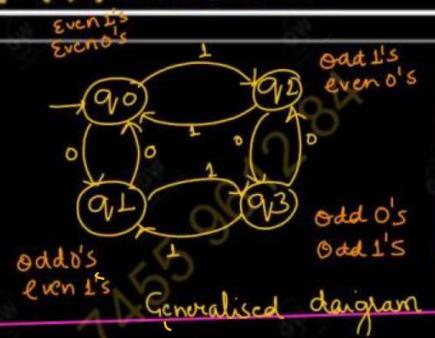
	a / 0	b / 1
$\rightarrow q_0$	q_1	q_5
q_1	q_6	q_2
$*q_2$	q_0	q_2
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

MINIMIZED TRANSITION Diagram**MINIMIZED TRANSITION TABLE**

	0 a	1 b
$\rightarrow \{q_0, q_4\}$	$\{q_1, q_7\}$	q_5
q_6	q_6	$\{q_0, q_4\}$
$\{q_1, q_7\}$	q_6	q_2
q_5	q_2	q_6
* q_2	$\{q_0, q_4\}$	q_2

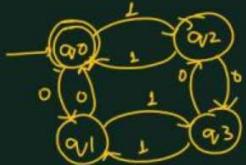
1. Even number of 0 and even number of 1 $\{q_0\}$
2. Odd number of 0 and even number of 1 $\{q_1\}$
3. Even number of 0 or even number of 1 $\{q_0, q_1, q_2\}$
4. Odd number of 0 or even number of 1 $\{q_0, q_1, q_3\}$
5. either odd number of 0 or even number of 1
But not both together $\{q_0, q_3\}$
6. odd number of 0 and odd number of 1 $\{q_3\}$
7. Odd number of 0 or odd number of 1 $\{q_1, q_2, q_3\}$

final

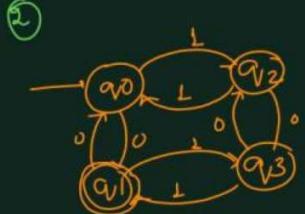


Generalised transition table

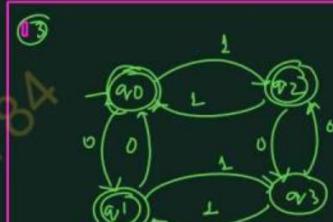
	0	1
$\rightarrow q_0$	q_1	q_2
q_1	q_0	q_3
q_2	q_3	q_0
q_3	q_2	q_1



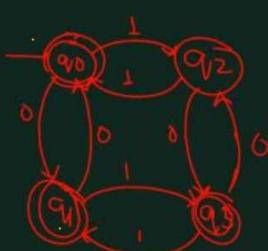
$M = \emptyset, \delta, \alpha_{V0}, F, \Sigma$
 $\{\{q_0, q_1, q_2, q_3\}, \delta, q_0, q_0, \{0, 1\}\}$



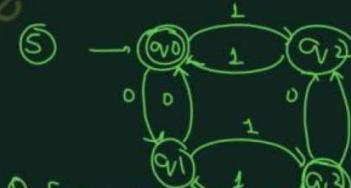
$M = \emptyset, \delta, \alpha_{V0}, F, \Sigma$
 $\{\{q_0, q_1, q_2, q_3\}, \delta, q_0, q_0, \{0, 1\}\}$



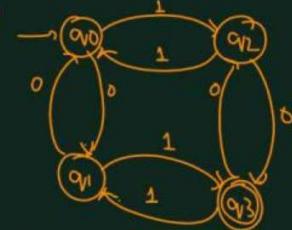
$M = \emptyset, \delta, \alpha_{V0}, F, \Sigma$
 $\{\{q_0, q_1, q_2, q_3\}, \delta, q_0, q_0, \{0, 1\}\}$



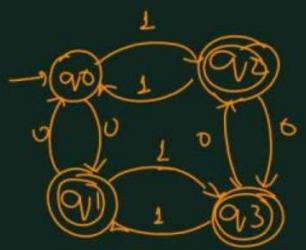
$M = \emptyset, \delta, \alpha_{V0}, F, \Sigma$
 $\{\{q_0, q_1, q_2, q_3\}, \delta, q_0, q_0, \{0, 1\}\}$



$M = \emptyset, \delta, \alpha_{V0}, F, \Sigma$
 $\{\{q_0, q_1, q_2, q_3\}, \delta, q_0, q_0, \{0, 1\}\}$



$M = \emptyset, \delta, \alpha_{V0}, F, \Sigma$
 $\{\{q_0, q_1, q_2, q_3\}, \delta, q_0, q_0, \{0, 1\}\}$



$M = \{Q, \Sigma, \delta, q_0, F\}$

$\Sigma = \{l, r, u, d\}$

Gateway Classes : 7455 9612 84

Construct a DFA that contain odd number of 0 and 1 is multiple of 3



Construct a DFA that contain odd number of 0 and 1 is multiple of 3 over {0,1}

	0	1
$\rightarrow S_{13}$	$S_{23} \checkmark$	$S_{14} \checkmark$
S_{14}	$S_{24} \checkmark$	$S_{15} \checkmark$
S_{15}	$S_{25} \checkmark$	S_{13}
$*S_{23}$	$S_{13} \checkmark$	$S_{24} \checkmark$
S_{24}	$S_{14} \checkmark$	$S_{25} \checkmark$
S_{25}	$S_{15} \checkmark$	$S_{23} \checkmark$

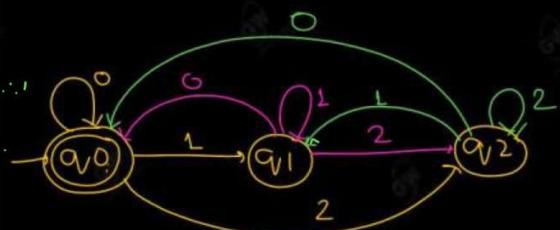
{Q, Σ, δ, q₀, F}

{S₁₃, S₁₄, S₁₃, S₂₃, S₂₄, S₂₅}, {0,1}, δ, S₁₃, {S₂₃} }

Construct the finite automata that accept all the string containing 11 and 010 as substring

$$\Sigma = \{0, 1\}$$



Construct a DFA for ternary number divisible by 3 $\Sigma = \{0, 1, 2\}$ **TRANSITION TABLE**

	0	1	2
$\rightarrow^* q_0$	q_0	q_1	q_2
q_1	q_0	q_2	q_0
q_2	q_0	q_1	q_2

 $\{Q, \Sigma, \delta, q_o, F\}$ $\{\{q_0, q_1, q_2\}, \{0, 1, 2\}, \delta, q_0, \{q_0\}\}$

Construct a DFA for ternary number divisible by 4

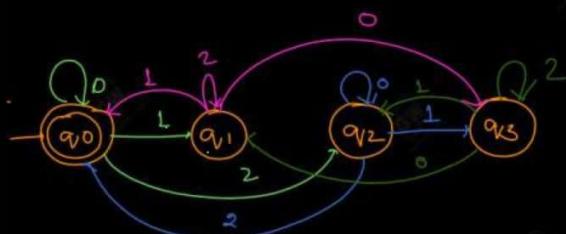
$\Sigma = \{0, 1, 2\}$

TRANSITION TABLE

	0	1	2
$\rightarrow^* q_0$	q_0	q_1	q_2
q_1	q_3	q_0	q_1
q_2	q_2	q_3	q_0
q_3	q_1	q_2	q_3

$\{Q, \Sigma, \delta, q_0, F\}$

$\{\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta, q_0, \{q_0\}\}$



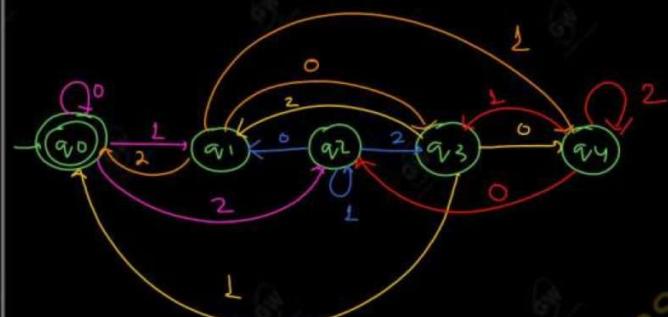
Construct a DFA for ternary number divisible by 5.

TRANSITION TABLE

	0	1	2
→*q ₀	q ₀	q ₁	q ₂
q ₁	q ₃	q ₄	q ₀
q ₂	q ₁	q ₂	q ₃
q ₃	q ₄	q ₀	q ₁
q ₄	q ₂	q ₃	q ₄

{Q, Σ, δ, q₀, F}

{{{q₀, q₁, q₂, q₃, q₄}, {0,1}, δ, q₀}, {q₀}}



- It is defined as a 5 tuple, $M=(Q, \Sigma, \delta, q_0, F)$

Q: Finite set of states

Σ : Finite set of the input symbol

q_0 : Initial state

F: Final state $F \subseteq Q$

δ : Transition function: $Q \times \Sigma \rightarrow 2^Q$

NOTE

- In NFA, given the current state there could be multiple next state
- The next state may be chosen at random
- All next state may be chosen in parallel

GW Construct the NFA over $\{0,1\}$ accept all set of string end with zero

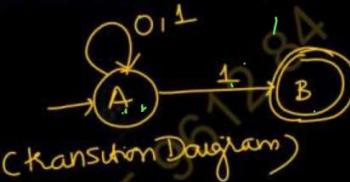


transition table

	0	1
$\rightarrow A$	{A,B}	A
$*B$	--	--

$$M = (Q, \Sigma, \delta, q_0, F) = \{\{A, B\}, \{0, 1\}, \delta, A, B\}$$

GW Construct the NFA over $\{0,1\}$ accept all set of string end with 1

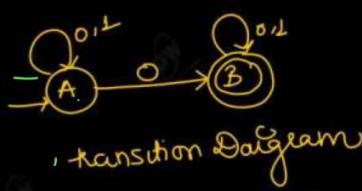


transition table

	0	1
$\rightarrow A$	A	{A,B}
$*B$	--	--

$$M = (Q, \Sigma, \delta, q_0, F) = \{\{A, B\}, \{0, 1\}, \delta, A, B\}$$

Construct the NFA over $\{0,1\}$ accept all set of string containing zero



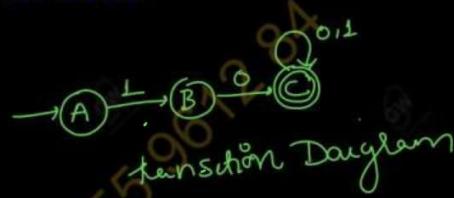
transition table

	0	1
A	$\{\underline{A,B}\}$ A,B	{A} ✓
*B	{B} / B	{B}

$$M = (Q, \Sigma, \delta, q_0, F)$$

$$\{\{A,B\}, \{0,1\}, \delta, A, B\}$$

Construct the NFA over $\{0,1\}$ accept all set of string start with 10



transition table

	0	1
$\rightarrow A$	--	B
B	C	--
*C	C	C

$$M = (Q, \Sigma, \delta, q_0, F) = \{\{A,B,C\}, \{0,1\}, \delta, A, C\}$$

Construct the NFA over {0,1} accept all set of string containing 01 as a substring



Construct the NFA over {0,1} accept all set of string end with 11



	0	1
$\rightarrow A$	A,B	A
B	--	C
*C	C	C

$M = (Q, \Sigma, \delta, q_0, F)$

$\{\{A,B,C\}, \{0,1\}, \delta, A, C\}$

	0	1
$\rightarrow A$	{A}	A,B
B	--	C
*C	--	--

$M = (Q, \Sigma, \delta, q_0, F) = \{\{A,B,C\}, \{0,1\}, \delta, A, C\}$

Construct a NFA for the language which accept all the string in which third symbol from the right end is always a over{a , b}



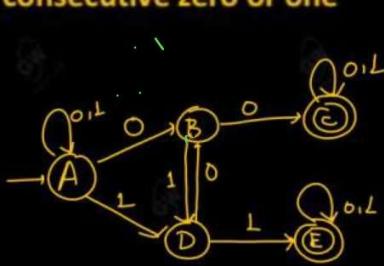
- ① NFA
- ② NFA to DFA conversion - DFA \rightarrow 90 - 95%
- ③ Minimization of DFA \rightarrow 90%
Construct NFA \rightarrow DFA (Minimization) 20 - 30%

TRANSITION TABLE

	a	b
A	A, B	A
B	C	C
C	D	D
*D	-	-

$$M = (Q, \Sigma, \delta, q_0, F) = (\{A, B, C, D\}, \{0, 1\}, \delta, A, D)$$

Construct a NFA for the language which accept all the string over {0,1} that have at least two consecutive zero or one

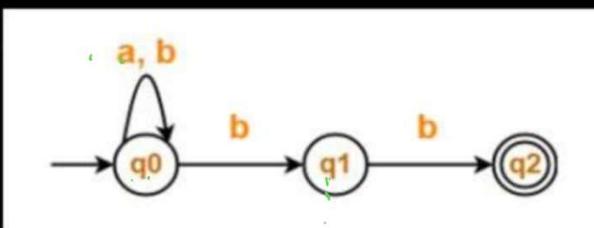


NFA → Incomplete

TRANSITION TABLE

	0	1
A	A,B	A,D
B	C	D
C	C	C
D	B	E
E	E	E

$$M = (Q, \Sigma, \delta, q_0, F) = \{\{A, B, C, D, E\}, \{0, 1\}, \delta, A, \{E, C\}\}$$



(NFA)

Step 1
TRANSITION TABLE OF NFA

State / Alphabet	a	b
$\rightarrow q_0$	q_0	q_0, q_1
q_1	-	$*q_2$
$*q_2$	-	-

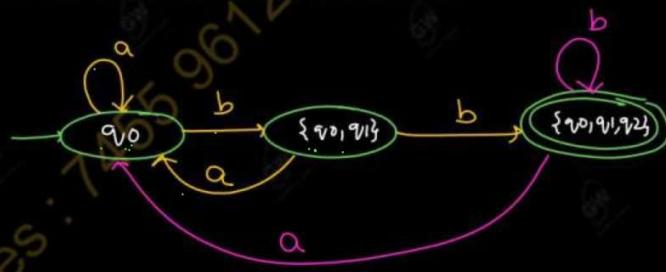
TRANSITION TABLE FOR DFA

State / Alphabet	a	b
$\rightarrow q_0$	q_0	$\{q_0, q_1\}$
$\{q_0, q_1\}$	q_0	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_2\}$	q_0	$\{q_0, q_1, q_2\}$

TRANSITION TABLE FOR DFA

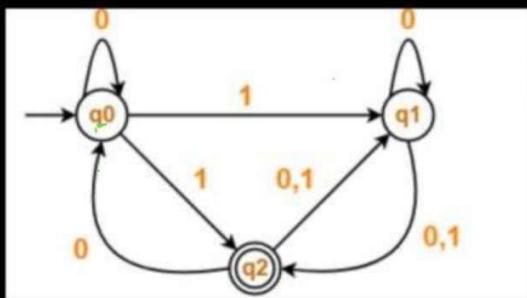
State / Alphabet	a	b
$\rightarrow q_0$ A	q_0	$\{q_0, q_1\}$
$\{q_0, q_1\}$ B	q_0	$\{q_0, q_1, q_2\}$
* $\{q_0, q_1, q_2\}$ C	q_0	$\{q_0, q_1, q_2\}$

TRANSITION DIAGRAM FOR DFA



	a	b
A	A	B
B	A	C
C	A	C

$\{A, B\} \cap \{C\} = \emptyset_0$
 $\{A\} \cap \{B\} \cap \{C\} = \emptyset_1$
 $\{A\} \cap \{B\} \cap \{C\} = \emptyset_2$



(NFA)

TRANSITION TABLE FOR DFA

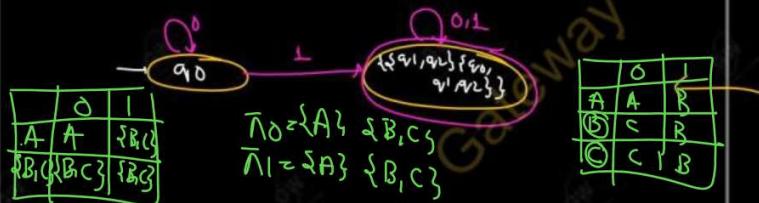
State / Alphabet	0	1
$\rightarrow q_0$	q_0	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$

TRANSITION TABLE OF NFA

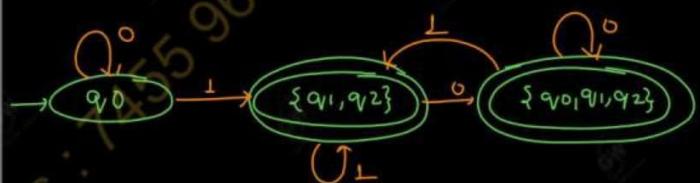
State / Alphabet	0	1
$\rightarrow q_0$	q_0	q_1, q_2
q_1	q_1, q_2	q_2
$*q_2$	q_0, q_1	q_1

TRANSITION TABLE FOR DFA

State / Alphabet	0	1
$\rightarrow q_0$ A	q_0 A	$\{q_1, q_2\}$ B
$*\{q_1, q_2\}$ B	$*\{q_0, q_1, q_2\}$ C	$\{q_1, q_2\}$ B
$*\{q_0, q_1, q_2\}$ C	$\{q_0, q_1, q_2\}$ C	$\{q_1, q_2\}$ B



TRANSITION DIAGRAM FOR DFA

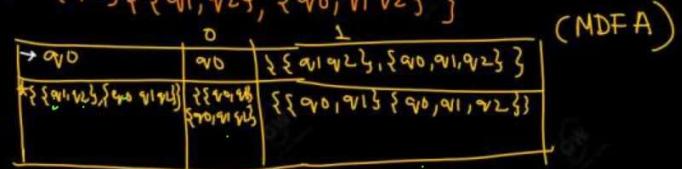


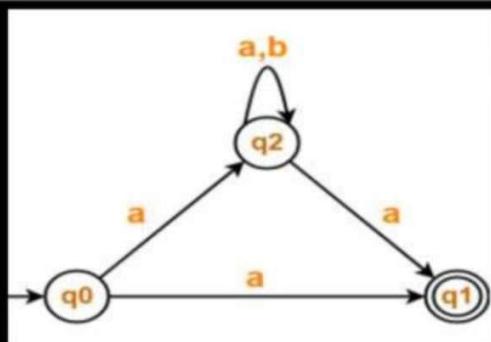
Minimization DFA ($NFA \rightarrow \text{MinDFA}$)

$Q_0 = \{q_0\} \setminus \{q_1, q_2\}, \{q_0, q_1, q_2\}$

$Q_1 = \{q_0\} \setminus \{q_1, q_2\}, \{q_0, q_1, q_2\}$

① $NFA \rightarrow \text{DFA}$
 $\text{DFA} \rightarrow \text{MDFA}$





(NFA)

TRANSITION TABLE FOR DFA

State / Alphabet	a	b
$\rightarrow q_0$	$\{q_1, q_2\}$	\emptyset
$\{q_1, q_2\}$	$\{q_1, q_2\}$	q_2
q_2	$\{q_1, q_2\}$	q_2
\emptyset	\emptyset	\emptyset

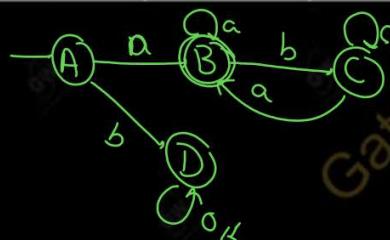
 \emptyset (Dead State)

TRANSITION TABLE OF NFA

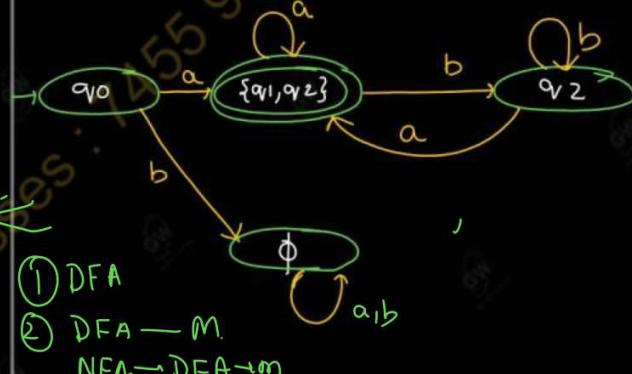
State / Alphabet	a	b
$\rightarrow q_0$	q_1, q_2	-
$*q_1$	-	-
q_2	q_1, q_2	q_2

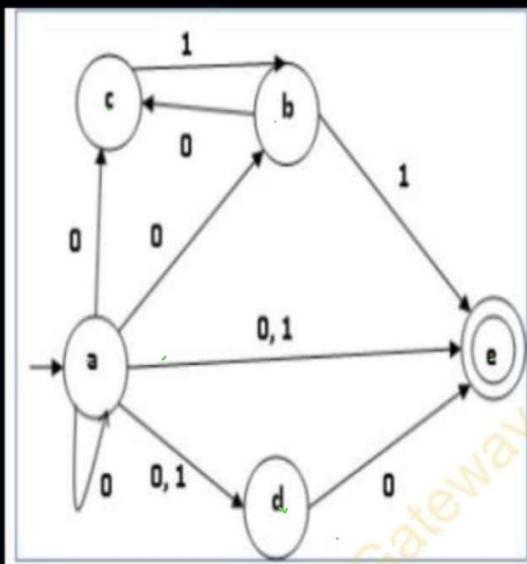
TRANSITION TABLE FOR DFA

State / Alphabet	a	b
$\rightarrow q_0 A$	$\{q_1, q_2\} B$	$\emptyset D$
$*\{q_1, q_2\} B$	$\{q_1, q_2\} B$	$q_2 C$
$q_2 C$	$\{q_1, q_2\} B$	$q_2 C$
$\emptyset D$	$\emptyset D$	$\emptyset D$



TRANSITION DIAGRAM FOR DFA





(NFA)

TRANSITION TABLE FOR NDFA

State / Alphabet	0	1
a	a,b,c,d,e	d,e
b	c	e
c	Ø	b
d	e	Ø
*e	Ø	Ø

Transition table for DFA

State / Alphabet	0	1
$\rightarrow a$	$\{a, b, c, d, e\}$	$\{d, e\}$
$\{a, b, c, d, e\}$	$\{a, b, c, d, e\}$	$\{b, d, e\}$
$\{d, e\}$	$\{e\}$	\emptyset
$\{b, d, e\}$	$\{c, e\}$	$\{e\}$
$\{e\}$	\emptyset	$\{b\}$
$\{c, e\}$	\emptyset	$\{b\}$
$\{b\}$	$\{c\}$	$\{e\}$
$\{c\}$	\emptyset	$\{b\}$
\emptyset	\emptyset	\emptyset

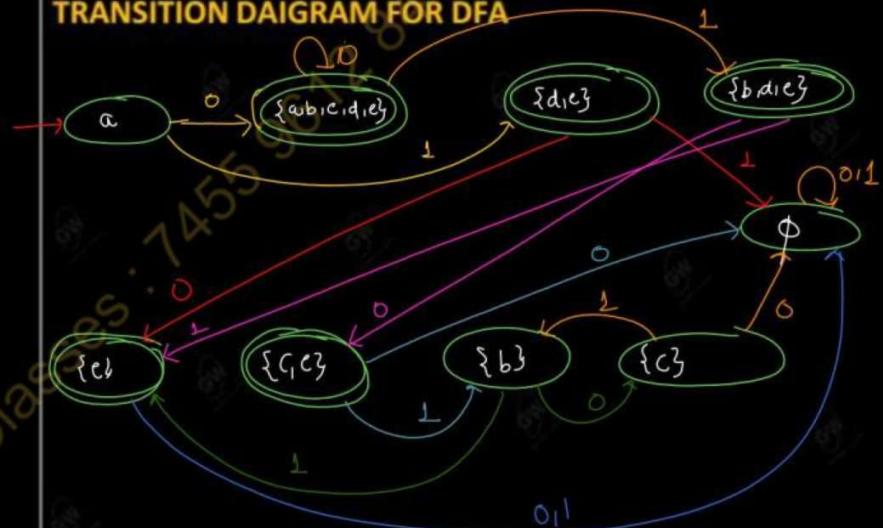
TRANSITION TABLE FOR NDFA

State / Alphabet	0	1
$\rightarrow a$	$\{a, b, c, d, e\}$	$\{d, e\}$
b	c	e
c	\emptyset	b
d	e	\emptyset
e	\emptyset	\emptyset

TRANSITION TABLE FOR DFA

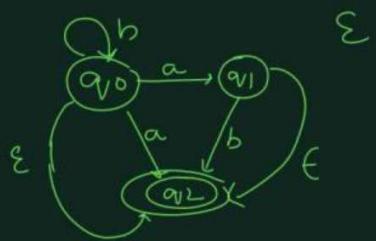
NFA to DFA
DFA to MDF
TMDTIO

	0	1
a	{a,b,c,d,e}	{d,e}
*{a,b,c,d,e}	{a,b,c,d,e}	{b,d,e}
*{d,e}	{e}	\emptyset
*{b,d,e}	{c,e}	{e}
*{e}	\emptyset	\emptyset
*{c, e}	\emptyset	{b}
{b}	{c}	{e}
{c}	\emptyset	{b}
\emptyset	\emptyset	\emptyset

TRANSITION DIAGRAM FOR DFA

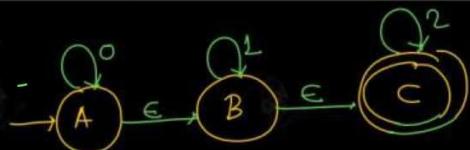


ε-NFA



$\Sigma = \{a, b\}$

Gateway Classes: 7455 9612 84



$\epsilon\text{-closure}(\delta(\epsilon\text{-closure}(A), 0))$
 $\epsilon\text{-closure}(\delta((A, B, C), 0))$
 $(\delta(A, 0) \cup \delta(B, 0) \cup \delta(C, 0))$
 $\epsilon\text{-closure}(A \cup \emptyset \cup \emptyset)$

TRANSITION TABLE OF ϵ NFA

State / Alphab et	0	1	2	ϵ
$\rightarrow A$	A	-	-	B
B	-	B	-	C
*C	-	-	C	-

EPSILON CLOSURE	
A	A, B, C
B	B, C
C	C

Step 1
 $\epsilon\text{-closure } A = \{A, B, C\}$
 $\delta(A, 0) = A$
 $\delta(B, 0) = \emptyset$
 $\delta(C, 0) = \emptyset$

	ϵ	0	ϵ
A	A B C	A — —	A B C

$A \cup \emptyset \cup \emptyset = \{A\}$
 $\epsilon\text{-closure } A = \{A, B, C\}$

State / Alphab et	0	1	2	ϵ
$\rightarrow A$	A	-	-	B
B	-	B	-	C
*C	-	-	C	-

	ϵ	1	ϵ
A	$A \xrightarrow{1} -$ $B \xrightarrow{1} B$ $C \xrightarrow{2} -$	-	B, C

epsilon closure(A)
 $\{A, B, C\}$

$$\delta(A, 1) = \emptyset$$

$$\delta(B, 1) = B$$

$$\delta(C, 1) = \emptyset$$

$$\emptyset \cup B \cup \emptyset$$

$$\{B\}$$

epsilon closure(B)
 $= B, C$

	ϵ	2 transition of A, B, C on 2	ϵ
A	A B C	- - C	C

epsilon closure(A)
 A, B, C

$$\delta(A, 2) = \emptyset$$

$$\delta(B, 2) = \emptyset$$

$$\delta(C, 2) = C$$

$$\emptyset \cup \emptyset \cup C$$

epsilon closure(C)
 C

State / Alphabet	0	1	2	ϵ
$\rightarrow A$	A	-	-	B
B	-	B	-	C
*C	-	-	C	-

	ϵ	0	2	ϵ
B	B $\xrightarrow{0}$ C	\emptyset	-	-

ϵ	1	ϵ
B	B $\xrightarrow{1}$ B, C C $\xrightarrow{1}$ -	-

ϵ	2	ϵ
B	B $\xrightarrow{2}$ - C $\xrightarrow{2}$ C	-

State / Alphab et	0	1	2	ϵ
$\rightarrow A$	A	-	-	B
B	-	B	-	C
*C	-	-	C	-

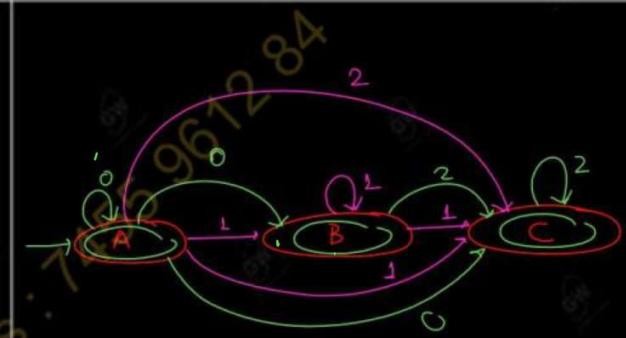
	ϵ	0	ϵ
C	C	\emptyset	

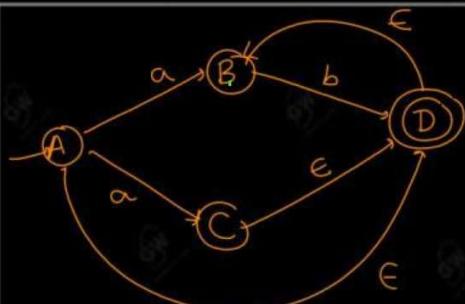
	ϵ	1	ϵ
C	C	\emptyset	-

	ϵ	2	ϵ
C	C	C	C

State / Alphabet	0	1	2
$\rightarrow^* A$	A, B, C	B, C	C
$\rightarrow^* B$	-	B, C	C
$\rightarrow^* C$	-	-	C

(NFA) transition table



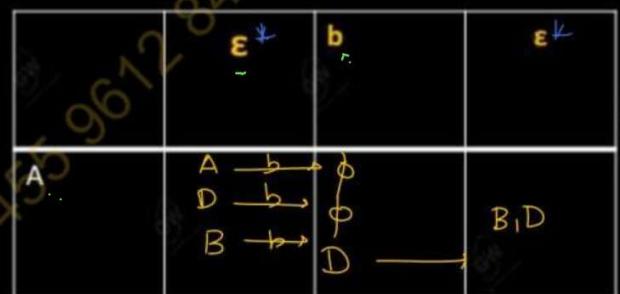


EPSILON CLOSURE	
A	A, D, B
B	B
C	C, D, B
D	D, B

	a	b	ε
→A	{B, C}	-	D
B	-	D	-
C	-	-	D
*D	-	-	B

State / Alphabet	a	b	ϵ
$\checkmark \rightarrow A$	{B,C}	-	D
B	-	D	-
C	-	-	D
*D	-	-	B

	ϵ	a	ϵ
A	$A \xrightarrow{a} B, C$ $D \xrightarrow{a} -$ $B \xrightarrow{a} -$	$B, C \rightarrow B, C, D$	-



	ϵ^*	a	ϵ^*
B	Closure	$B \xrightarrow{a} \emptyset$	-

State / Alphabet	a	b	ϵ
$\rightarrow A$	{B,C}	-	D
B	-	D	-
C	-	-	D
*D	-	-	B

	ϵ	b	ϵ
B	$B \xrightarrow{b} D \xrightarrow{\epsilon} B \cup D$		

ϵ Closure	a	ϵ Closure
C	$C \xrightarrow{a} \emptyset$	-
D	$D \xrightarrow{a} \emptyset$	-
B	$B \xrightarrow{a} \emptyset$	-

ϵ Closure	b	ϵ Closure
C	$C \xrightarrow{b} \emptyset$	-
D	$D \xrightarrow{b} \emptyset$	-
B	$B \xrightarrow{b} D \xrightarrow{\epsilon} D \cup B$	-

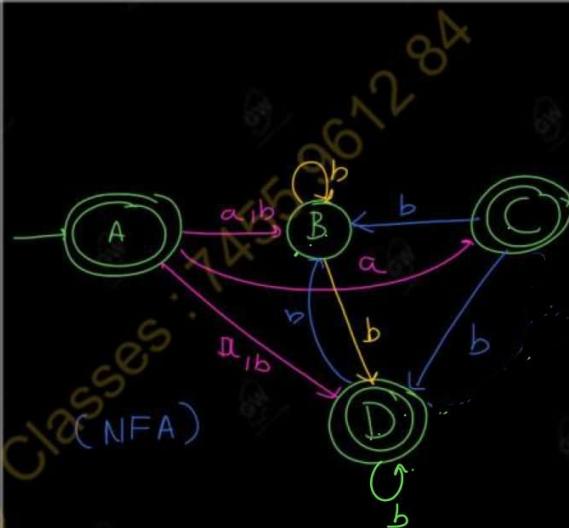
State / Alphabet	a	b	ϵ
$\rightarrow A$	{B,C}	-	D
B	-	D	-
C	-	-	D
*D	-	-	B

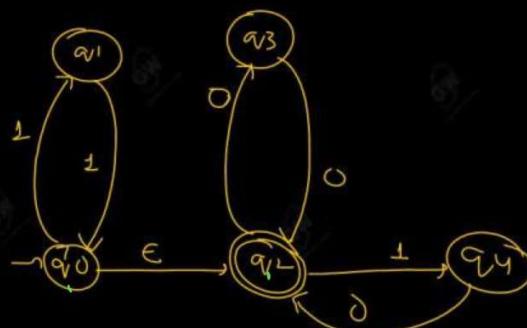
(1)	(2)	(3)
ϵ Closure	a	ϵ Closure

	ϵ	b	ϵ
D	D \xrightarrow{a} - B \xrightarrow{a} -	-	D $\xrightarrow{\epsilon}$ B,D

State / Alphabet	a	b
A	B,C,D	B,D
B	-	B,D
C	-	B,D
D	-	B,D

(NFA)





transition table of ϵ -NFA

	0	1	ϵ
q0	-	q1	q2
q1	-	q0	-
*q2	q3	q4	-
q3	q2	-	-
q4	q2	-	-

	Epsilon closure
q0	{q0,q2}
q1	q1
q2	q2
q3	q3
q4	q4

	0	1	ϵ
q0	-	q1	q2
q1	-	q0	-
*q2	q3	q4	-
q3	q2	-	-
q4	q2	-	-

	ϵ Closure	0	ϵ Closure
q0	q _{v0} q _{v2}	-	q _{v3} → q _{v3}

	ϵ Closure	1	ϵ Closure
q0	q _{v0} q _{v2}	q _{v1} q _{v4}	q _{v1} , q _{v4}

	0	1	ϵ
q0	-	q1	q2
q1	-	q0	-
*q2	q3	q4	-
q3	q2	-	-
q4	q2	-	-

	ϵ Closure	0	ϵ
q1	q_1	\emptyset	-

	ϵ Closure	1	ϵ Closure
q1	q_1	$q_0 \xrightarrow{1} q_0$	$\{q_0, q_2\}$

	0	1	ϵ
q0	-	q_1	q_2
q1	-	q_0	-
*q2	q_3	q_4	-
q3	q_2	-	-
q4	q_2	-	-

	ϵ	0	ϵ Closure
q2	$q_2 \xrightarrow{0} q_3$	q_3	

	ϵ	1	ϵ Closure
q2	$q_2 \xrightarrow{1} q_4$	q_4	q_4

	0	1	ϵ
q0	-	q1	q2
q1	-	q0	-
*q2	q3	q4	-
q3	q2	-	-
q4	q2	-	-

	ϵ	0	ϵ
q3	q_3^3	$q_1 \cup q_2$	$q_1 q_2$

	ϵ	1	ϵ
q3	q_3^3	-	-

	0	1	ϵ
q0	-	q1	q2
q1	-	q0	-
*q2	q3	q4	-
q3	q2	-	-
q4	q2	-	-

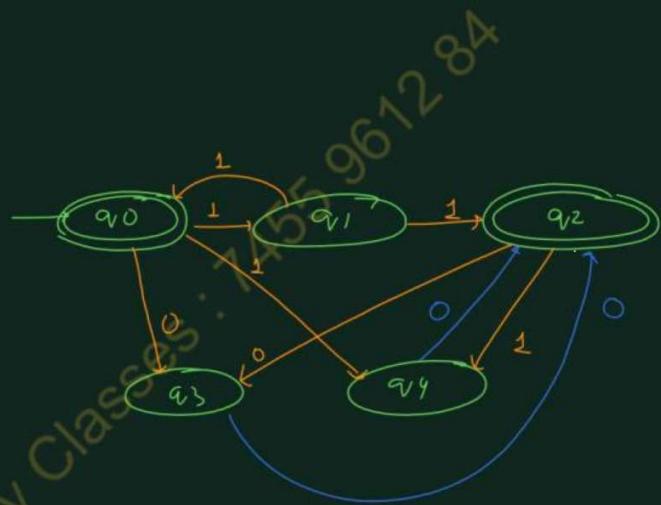
	ϵ Closure	0	ϵ Closure
q4	q_4	q_2	q_2

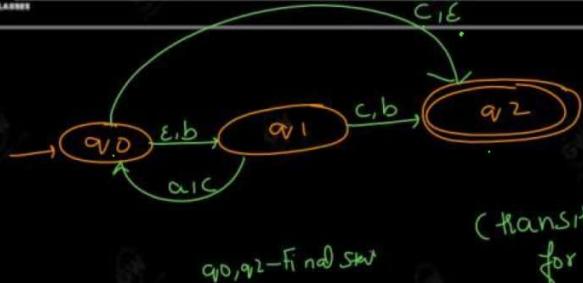
	ϵ Closure	1	ϵ Closure
q4	q_4	-	-

	0	1	ϵ
q0	-	q_1	q_2
q1	-	q_0	-
*q2	q_3	q_4	-
q3	q_2	-	-
q4	q_2	-	-

NFA

	0	1
q_0	q_3	q_1, q_4
q_1	-	q_0, q_2
q_2	q_3	q_4
q_3	q_2	-
q_4	q_2	-





(transition diagram)
for ϵ -NFA
 $\Sigma = \{a, b, c\}$

TRANSITION TABLE OF ϵ NFA

State / Alphab et	a	b	c	ϵ
$\rightarrow q_0$	-	q_1	q_2	q_1, q_2
q_1	q_0	q_2	q_0, q_2	-
$*q_2$	-	--	--	-

EPSILON CLOSURE	
q_0	q_0, q_1, q_2
q_1	q_1
q_2	q_2

TRANSITION TABLE OF ϵ NFA

	a	b	c	ϵ
$\rightarrow q_0$	-	q_1	q_2	q_1, q_2
q_1	q_0	q_2	q_0, q_2	-
$*q_2$	-	--	--	-

	ϵ closure	b	ϵ closure
q_0	$q_{v0} \xrightarrow{b} q_{v1}$ $q_{v1} \xrightarrow{b} q_{v2}$ $q_{v2} \xrightarrow{b} \emptyset$	$q_{v1} \xrightarrow{\epsilon \text{ close}} q_{v1}$ $q_{v2} \xrightarrow{\epsilon \text{ close}} q_{v2}$	$\{q_{v1}, q_{v2}\}$

	ϵ closure	a	ϵ closure
q_0	$q_{v0} \xrightarrow{a} \emptyset$ $q_{v1} \xrightarrow{a} q_{v0}$ $q_{v2} \xrightarrow{a} \emptyset$	$q_{v0} \cup q_{v1} \cup q_{v2}$	$q_{v0} \xrightarrow{\epsilon \text{ close}} q_{v0, v1, v2}$

	ϵ closure	c	ϵ closure
q_0	$q_{v0} \xrightarrow{c} \emptyset$ $q_{v1} \xrightarrow{c} \{q_{v0}, q_{v2}\}$ $q_{v2} \xrightarrow{c} \emptyset$	$q_{v0} \xrightarrow{\epsilon \text{ close}} q_{v0, v1, v2}$ $q_{v2} \xrightarrow{\epsilon \text{ close}} q_{v2}$	$q_{v0, v1, v2}$

TRANSITION TABLE OF ϵ NFA

	a	b	c	ϵ
$\rightarrow q_0$	-	q_1	q_2	q_1, q_2
q_1	q_0	q_2	q_0, q_2	-
$*q_2$	-	--	--	-

	ϵ closure	b	ϵ closure
q_1	$q_{v1} \xrightarrow{b} q_{v2}$		q_{v2}

	ϵ closure	a	ϵ closure
q_1	$q_{v1} \xrightarrow{a} q_{v0} \xrightarrow{\epsilon \text{ close}} \{q_0, q_{v1}, q_{v2}\}$		

	ϵ closure	c	ϵ closure
q_1	$q_{v1} \xrightarrow{c} q_{v0} \xrightarrow{\epsilon \text{ close}} \{q_0, q_{v1}, q_{v2}\}$		$q_{v2} \xrightarrow{\epsilon \text{ close}} q_{v2}$

TRANSITION TABLE OF ϵ NFA

	a	b	c	ϵ
$\rightarrow q_0$	-	q_1	q_2	q_1, q_2
q_1	q_0	q_2	q_0, q_2	-
$*q_2$	-	--	--	-

	ϵ closure	b	ϵ closure
q_2	q_1, q_2	ϕ	-

	ϵ closure	a	ϵ closure
q_2	q_1, q_2	ϕ	-

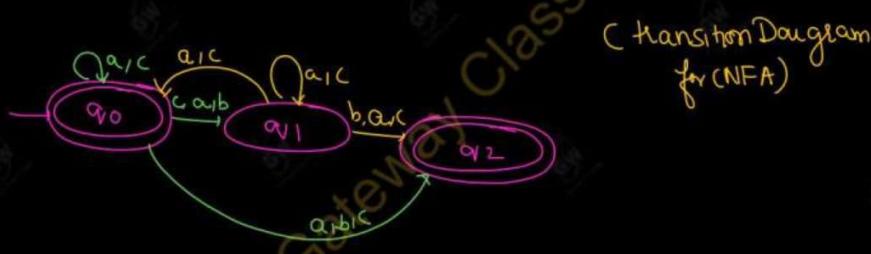
	ϵ closure	c	ϵ closure
q_2	q_1, q_2	ϕ	-

TRANSITION TABLE OF NFA

Input →	a	b	c
States ↓			
→ *q ₀	{q ₀ , q ₁ , q ₂ }	{q ₁ , q ₂ }	{q ₀ , q ₁ , q ₂ }
q ₁	{q ₀ , q ₁ , q ₂ }	q ₂	{q ₀ , q ₁ , q ₂ }
*q ₂	—	—	—

ε-NFA to MDFA

- ① ε NFA to NFA
- ② NFA to DFA
- ③ DFA to MDFA



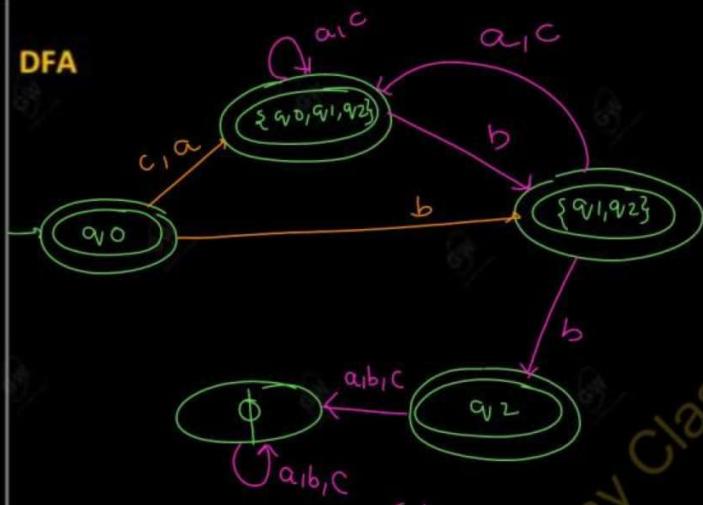
TRANSITION TABLE OF NFA

	a	b	c
$\rightarrow^* q_0$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
q_1	$\{q_0, q_1, q_2\}$	q_2	$\{q_0, q_1, q_2\}$
$*q_2$	-	--	--

DFA (NFA TO DFA)

	a	b	c
$\rightarrow^* q_0$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$	q_2	$\{q_0, q_1, q_2\}$
$*q_2$	\emptyset	\emptyset	\emptyset
\emptyset	\emptyset	q_2	\emptyset

DFA



DFA (NFA TO DFA)

(Constructible DFA)

	a	b	c
*q0	{q0,q1,q2}	{q1,q2}	{q0,q1,q2}
*{q0,q1,q2}	{q0,q1,q2}	{q1,q2}	{q0,q1,q2}
*{q1,q2}	{q0,q1,q2}	q2	{q0,q1,q2}
*q2	Φ	Φ	Φ
Φ	Φ	Φ	Φ

TRANSITION TABLE OF DFA

	a	b	c
$\rightarrow *A$	B	C	B
$*B$	B	C	B
$*C$	B	D	B
$*D$	Φ	Φ	Φ
Φ	Φ	Φ	Φ

DFA (NFA TO DFA)

	a	b	c
$\rightarrow *q_0 \text{ (A)}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$*\{q_0, q_1, q_2\} \text{ (B)}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$*\{q_1, q_2\} \text{ (C)}$	$\{q_0, q_1, q_2\}$	q_2	$\{q_0, q_1, q_2\}$
$*q_2 \text{ (D)}$	Φ	Φ	Φ
Φ	Φ	Φ	Φ

TRANSITION TABLE OF DFA

	a	b	c
→ *A	B	C	B
*B	B	C	B
*C	B	D	B
*D	— Φ	Φ	Φ
Φ	Φ	Φ	Φ

MINIMIZATION OF DFA

$$\pi_0 = \{A, B, C, D\} \quad \{ \phi \}$$

$$\pi_1 = \{A, B, C\} \{D\} \quad \{ \phi \}$$

$$\pi_2 = \{A, B\} \{C\}, \{D\} \quad \{ \phi \}$$

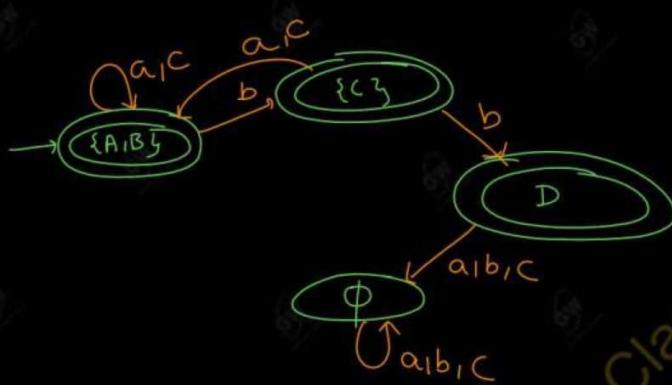
$$\pi_3 = \{A, B\} \{C\}, \{D\}, \{ \phi \}$$

TRANSITION TABLE OF DFA

	a	b	c
* A	B	C	B
* B	B	C	B
* C	B	D	B
* D	Φ	Φ	Φ
Φ	Φ	Φ	Φ

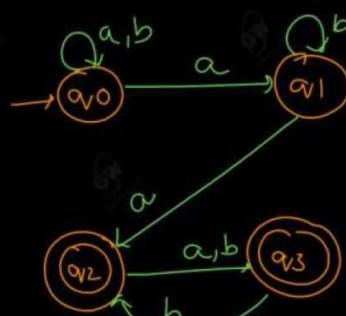
	a	b	c
* {A,B}	{A,B}	C	{A,B}
* C	{A,B}	D	{A,B}
* D	\emptyset	\emptyset	\emptyset
\emptyset	\emptyset	\emptyset	\emptyset

{minimized DFA table}

MINIMIZED DFA

	a	b	c
{A,B}	{A,B}	{C}	{A,B}
{C}	{A,B}	{D}	{A,B}
{D}	Φ	Φ	Φ
Φ	Φ	Φ	Φ

{minimized DFA table}



(transition
Diagram
for(NFA))

Transition table for NFA

	a	b
$\rightarrow q_0$	q_0, q_1	q_0
q_1	q_2	q_1
$*q_2$	q_3	q_3
$*q_3$	-	q_2

Transition table DFA

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$
$*\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_3\}$
$*\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$

Transition table for NFA

	a	b
$\rightarrow q_0$	q_0, q_1	q_0
q_1	q_2	q_1
$*q_2$	q_3	q_3
$*q_3$	-	q_2

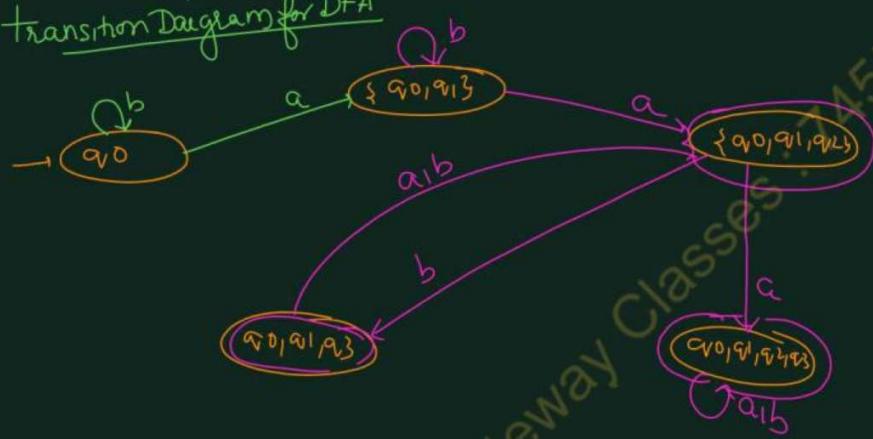
Transition table DFA

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$
$*\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_3\}$
$*\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$

Transition table for DFA

	a	b
$\rightarrow 0$	1	0
1	2	1
*		
*2	3	4
*3	3	3
*4	2	2

transition Diagram for DFA

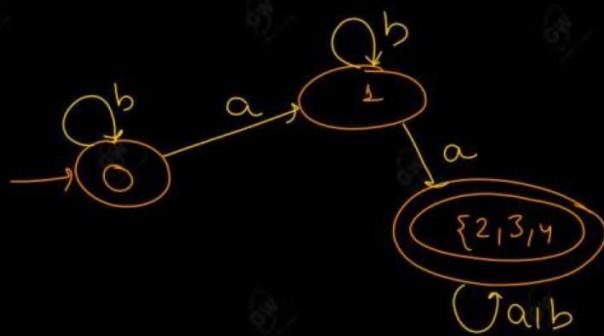


MINIMIZATION OF DFA

$\pi_0 = \{0, 1\}, \{2, 3, 4\}$ 0-equivalence class
 $\pi_1 = \{0\}, \{1\}, \{2, 3, 4\}$ 1-equivalence class
 $\pi_2 = \{0\}, \{1\}, \{2, 3, 4\}$ 2-equivalence class

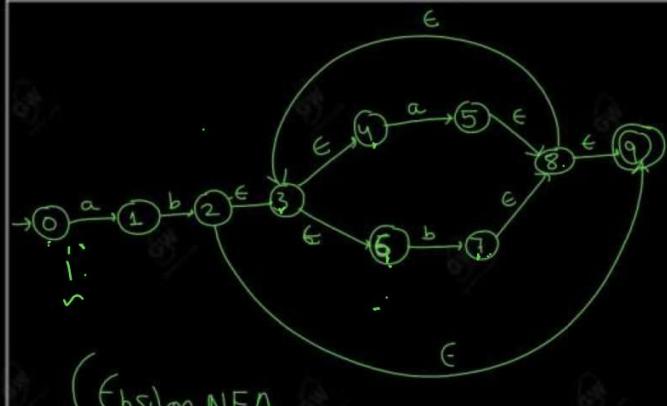
Transition table for DFA

	a	b
$\rightarrow 0$	1	0
1	2	1
*2	3	4
*3	3	3
*4	2	2

MINIMIZATION OF DFA (Transition Diagram.)

Transition table for DFA

	a	b
→ q0	1	0
q1	{2,3,4}	1
*{2,3,4}	{2,3,4}	{2,3,4}



(Epsilon NFA
transition Diagram)

	a	b	ϵ (Empty string)
→0	1	-	-
1	-	2	-
2	-	-	3,9
3	-	-	4,6
4	5	-	-
5	-	-	8
6	-	7	-
7	-	-	8
8			9,3
*9	-	-	-

Epsilon NFA table

	Epsilon closure
0	0
1	1
2	2,3,9,4,6
3	3,4,6
4	4
5	5,8,9,3,4,6
6	6
7	7,8,9,3,4,6
8	8,9,3,4,6
9	9

	a	b	ϵ
→0	1	-	-
1	-	2	-
2	-	-	3,9
3	-	-	4,6
4	5	-	-
5	-	-	8
6	-	7	-
7	-	-	8
8			9,3
*9	-	-	-

Initial State Of NFA	Epsilon closure	a	Epsilon closure	b	Epsilon closure
Q1	Q1	1	1	∅	-

	a	b	ε
→0	1	-	-
1	-	2	-
2	-	-	3,9
3	-	-	4,6
4	5	-	-
5	-	-	8
6	-	7	-
7	-	-	8
8			9,3
*9	-	-	-

State	a	Epsilon closure	b	Epsilon closure
1	∅	—	2	{2,3,9,4,6}
{2,3,9,4,6}	5	5,8,9,3,4,6	7	7,8,9,3,4,6
{5,8,9,3,4,6}	5	5,8,9,3,4,6	7	7,8,9,3,4,6
7,8,9,3,4,6	5	5,8,9,3,4,6	7,8	7,8,9,3,4,6
∅	—	—	—	—

	a	b	ε
→0	1	—	—
1	—	2	—
2	—	—	3,9
3	—	—	4,6
4	5	—	—
5	—	—	8
6	—	7	—
7	—	—	8
8	—	—	9,3
*9	—	—	—

	a	b
→ 0	1	∅
1	∅	{2, 3, 9, 4, 6}
*{2, 3, 9, 4, 6}	{5, 8, 9, 3, 4, 6}	{7, 8, 9, 3, 4, 6}
*{5, 8, 9, 3, 4, 6}	{5, 8, 9, 3, 4, 6}	{7, 8, 9, 3, 4, 6}
*{7, 8, 9, 3, 4, 6}	5, 8, 9, 3, 4, 6	{7, 8, 9, 3, 4, 6}
∅	∅	∅

 $\epsilon \text{NFA} \rightarrow ①$ $\text{DFA} \rightarrow \text{Final state - } \emptyset \text{ plus all}$

DFA table

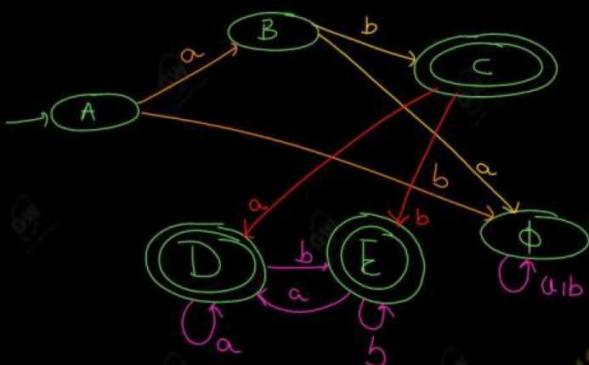
	a	b
→0	1	∅
1	∅	{2,3,9,4,6}
*{2,3,9,4,6}	{5,8,3,4,6,9}	{7,8,3,9,4,6}
*{5,8,3,4,6, 9}	{5,8,3,4,6,9}	{7,8,3,9,4,6}
*{7,8,3,9,4, 6}	{5,8,3,4,6,9}	{7,8,3,9,4,6}
∅	∅	∅

	a	b
→A	B	∅
B	∅	C
*C	D	E
*D	D	E
*E	D	E
∅	∅	∅

DFA Table



transition Diagram of DFA



	a	b
→A	B	φ
B	φ	C
*C	D	E
*D	D	E
*E	D	E
φ	φ	φ

Only if ENFA to minimized DFA Asked?

MINIMIZED DFA

$\pi_0 = \{A, B, \phi\}, \{C, D, E\}$ 0-equivalence class

$\pi_1 = \{A, \phi\}, \{B\}, \{C, D, E\}$ 1-equivalence class

$\pi_2 = \{A\}, \{B\} \cup \{\phi\}, \{C, D, E\}$ 2-equivalence class

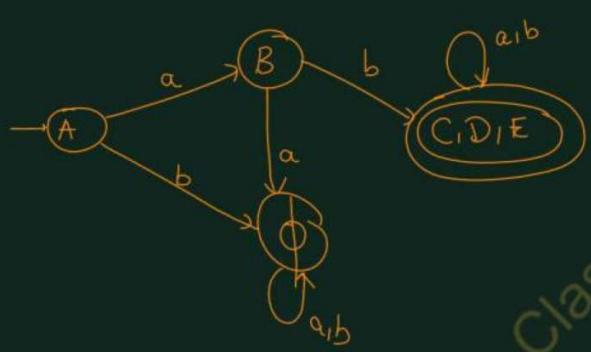
$\pi_3 = \{A\}, \{B\}, \{\phi\}, \{C, D, E\}$ 3-equivalence class

	a	b
$\rightarrow A$	B	ϕ
B	ϕ	C
*C	D	E
*D	D	E
*E	D	E
ϕ	ϕ	ϕ

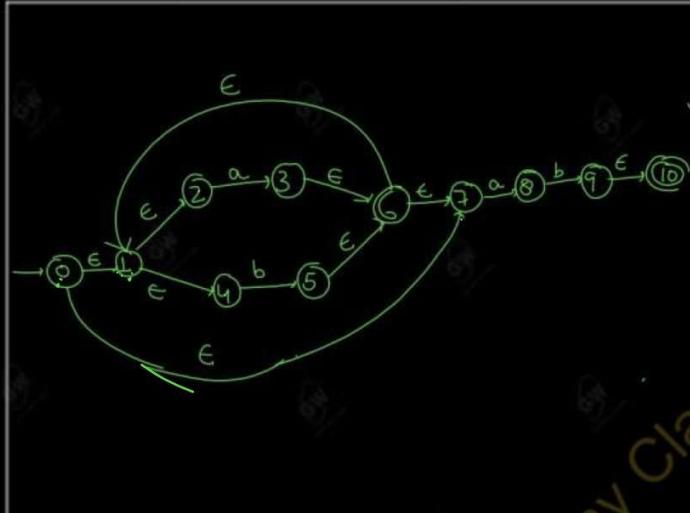
	a	b
→ A	B	C
B	∅	{C,D,E}
*{C,D,E}	{C,D,E}	{C,DE}
∅	∅	∅

Minimized DFA table

	a	b
→ A	B	∅
B	∅	C
*C	D	E
*D	D	E
*E	D	E
∅	∅	∅



Gateway Classes: 7455 9612 84



	a	b	ϵ
→0	-	-	1, 7
1	-	-	2, 4
2	3	-	-
3	-	-	6
4	-	5	-
5	-	--	6
6	-	-	1, 7
7	8	-	-
8	-	9	-
9	-	10	-
10	-	--	-

	EPSILON CLOSURE
0	0,1,2,4,7
1	1,2,4
2	2
3	3,6,1,2,4,7// 1,2,3,4,6,7
4	4
5	5,6,7,1,2,4//1,2,4,5,6,7
6	6,7,1,2,4//1,2,4,6,7
7	7
8	8
9	9
10	10

	a	b	ϵ
→0	-	-	1,7
1	-	-	2,4
2	3	-	-
3	-	-	6
4	-	5	-
5	-	--	6
6	-	-	1,7
7	8	-	-
8	-	9	-
9	-	10	-
10	-	--	-

Initial State Of NFA	Epsilon closure	a	Epsilon closure	b	Epsilon closure
Q	{0,1,2, 4,7}	3,8	1,2,3, 4,6,7,8	5	{1,2,4, 5,6,7}

1,2,3,4,6,7 U 8

	a	b	ϵ
0	-	-	1,2
1	-	-	2,4
2	3	-	-
3	-	-	6
4	-	5	-
5	-	--	6
6	-	-	1,7
7	8	-	-
8	-	9	-
9	-	10	-
10	-	--	-

State	a	b	Epsilon closure
(1,2,3, 4,6,7,8)	3,8	1,2,3,4, 6,7,8	5,9 1,2,4,5,6, 7,9
1,2,4, 5,6,7	3,8	1,2,3,4, 6,7,8	5 1,2,4,5,6, 7
1,2,4, 5,6,7, 9	3,8	1,2,3,4, 6,7,8	5,10 1,2,4,5,6, 7,10
(1,2,4, 5,6,7,8)	3,8	1,2,3,4, 6,7,8	5 1,2,4,5,6, 7,

	a	b	ϵ
→0	-	-	1,7
①	-	-	2,4
②	3	-	-
③	-	-	6
④	-	5	-
⑤	-	--	6
⑥	-	-	1,7
⑦	8	-	-
⑧	-	9	-
9	-	10	-
⑩	-	--	-

	a	b
$\rightarrow \{0, 1, 2, 4, 7\}$	$\{1, 2, 3, 4, 6, 7, 8\}$	$\{1, 2, 4, 5, 6, 7\}$
$\{1, 2, 3, 4, 6, 7, 8\}$	$\{1, 2, 3, 4, 6, 7, 8\}$	$\{1, 2, 4, 5, 6, 7, 9\}$
$\{1, 2, 4, 5, 6, 7\}$	$\{1, 2, 3, 4, 6, 7, 8\}$	$\{1, 2, 4, 5, 6, 7\}$
$\{1, 2, 4, 5, 6, 7, 9\}$	$\{1, 2, 3, 4, 6, 7, 8\}$	$\{1, 2, 4, 5, 6, 7, 10\}$
$* \{1, 2, 4, 5, 6, 7, 10\}$	$\{1, 2, 3, 4, 6, 7, 8\}$	$\{1, 2, 4, 5, 6, 7\}$

DFA table

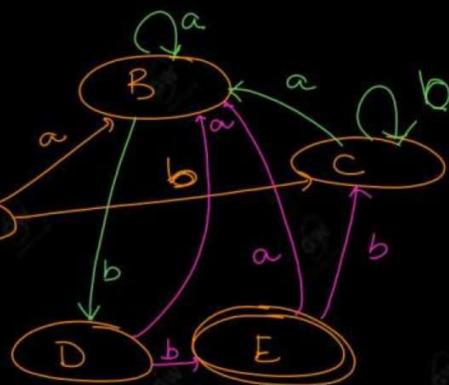
	a	b
- let A -> {0,1,2,4,7}	{1,2,3,4,6,7,8}	{1,2,4,5,6,7}
{1,2,3,4,6,7, ,8} B	{1,2,3,4,6,7,8}	{1,2,4,5,6,7,9}
{1,2,4,5,6,7} } C	{1,2,3,4,6,7,8}	{1,2,4,5,6,7}
{1,2,4,5,6,7, ,9} D	{1,2,3,4,6,7,8}	{1,2,4,5,6,7,10}
{1,2,4,5,6,7, ,10} E	{1,2,3,4,6,7,8}	{1,2,4,5,6,7}

DFA table

	a	b
→A	B	C
B	B	D
C	B	C
D	B	E
*E	B	C

DFA

DFA



	a	b
→A	B	C
B	B	D
C	B	C
D	B	E
*E	B	C

M^o minimized DFA
 $\pi_0 = \{A, B, C, D\}, \{E\}$

$\pi_1 = \{A, B, C\}, \{D\}, \{E\}$

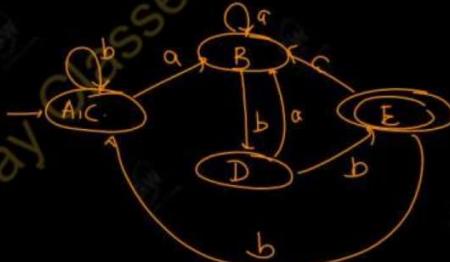
$\pi_2 = \{A, C\}, \{B\}, \{D\}, \{E\}$

$\pi_3 = \{A, C\}, \{B\}, \{D\}, \{E\}$

	a	b
{A,C}	B	{A,C}
B	B	D
D	B	F
*E	B	{A,C}

(Minimized
DFA
transition
table)

	a	b
A	B	C
B	B	D
C	B	C
D	B	E
*E	B	C



- Both Moore and mealy machine are the special case of DFA
- Both act like the output producers rather than the string acceptors
- In Moore and mealy machine no need to define the final state.
- No concept of final state and dead state
- Mealy and Moore machine is equivalent in power

Moore's machine is defined as a machine in the theory of computation whose output values are determined only by its current state. It has also 6 tuples

$$(Q, q_0, \Sigma, \Delta, \delta, \lambda)$$

Q is a finite set of states $\{q_0, q_1, q_2, q_3\}$

q₀ is the initial state q_0

Σ is the input alphabet $\{0, 1\}$

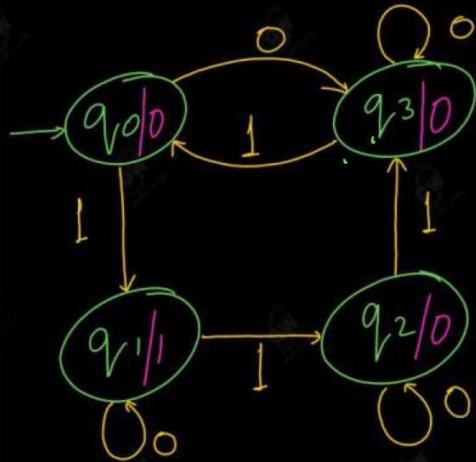
Δ is the output alphabet $\{0, 1\}$

δ is the transition function that maps $Q \times \Sigma \rightarrow Q$

λ is the output function that maps $Q \rightarrow \Delta$

NOTE

- In Moore machine for every state output is associated
- If length of input string is n, then length of output string will be n+1
- Moore machine response for empty string

$$\Sigma = \{0,1\}$$


Example

n 1110 - Output?

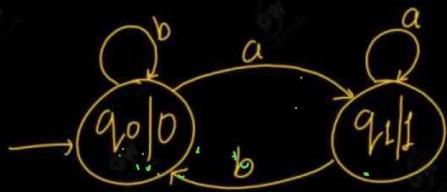
$q_0 \rightarrow q_1 \rightarrow q_2 \rightarrow q_3 \rightarrow q_0$

↓ ↓ ↓ ↓

0 1 0 0

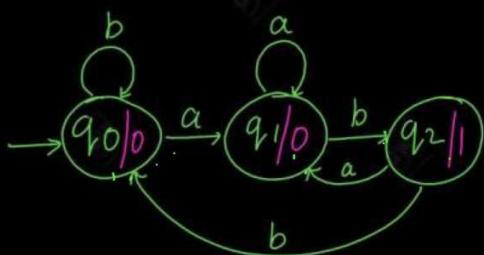
01000 n+1

Present state	Next state <u>a=0</u>	Next state <u>a=1</u>	Λ (output)
$\rightarrow q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0



Present state	a	b	$\Delta(\text{output})$
$\rightarrow q_0$	q1	q0	0
q1	q1	q0	1





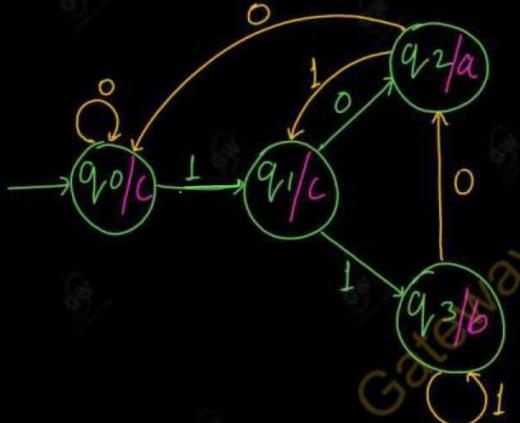
Present state	a	b	Δ (output)
$\rightarrow q_0$	q1	q0	0
q1	q1	q2	0
q2	q1	q0	1

Construct a Moore machine that should give output a if the input string end with 10 ,b if the input string ends with 11 $\Sigma = \{0,1\}$, $\Delta = \{a,b,c\}$

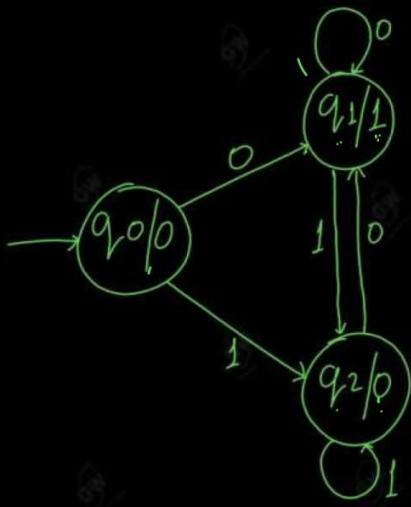
$10 \rightarrow a$

$11 \rightarrow b$

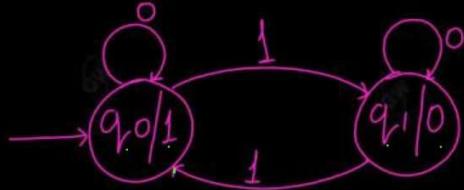
other than this c



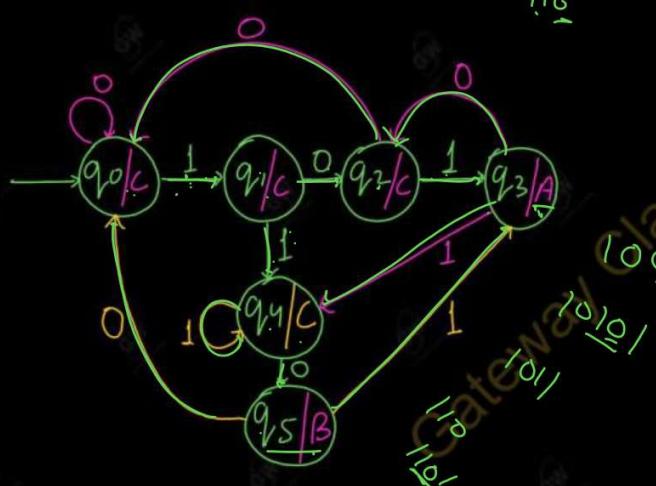
Present state	Next state 0	Next state 1	$\Delta(\text{output})$
$\rightarrow q_0$	q0	q1	c
q1	q2	q3	c
q2	q0	q1	a
q3	q2	q3	b



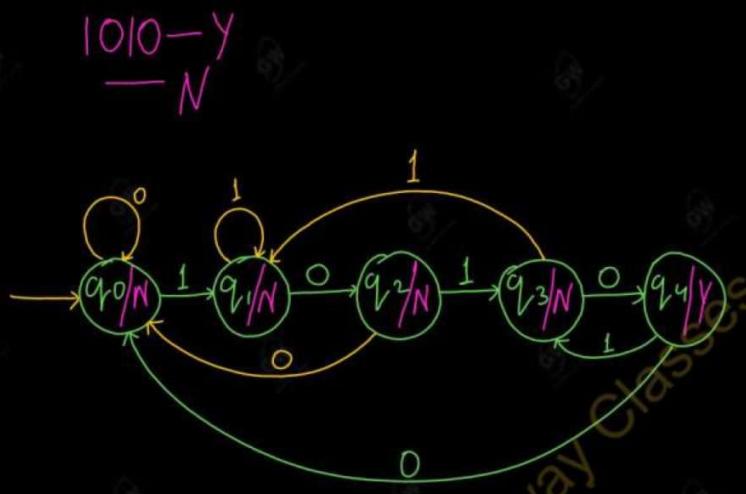
Present state	Next state 0	Next state 1	Λ (output)
$\rightarrow q_0$	q_1	q_2	0
q_1	q_1	q_2	1
q_2	q_1	q_2	0



Present state	Next state 0	Next state 1	A(output)
→ q0	q0	q1	1
q1	q1	q0	0

$\Sigma = \{0,1\}$ $101 \rightarrow A$ $110 \rightarrow B$ 

Present state	Next state		$\Lambda(\text{output})$
	0	1	
$\rightarrow q_0$	q_0	q_1	C
q_1	q_2	q_4	C
q_2	q_0	q_3	C
q_3	q_2	q_4	A
q_4	q_5	q_4	C
q_5	q_0	q_3	B



Present state	Next state 0	Next state 1	Λ (output)
$\rightarrow q_0$	q_0	q_1	N
q_1	q_2	q_1	N
q_2	q_0	q_3	N
q_3	q_4	q_1	N
q_4	q_0	q_3	Y

Mealy machines are also finite state machines with output value and their output depends on the present state and current input symbol. It can be defined as $(Q, q_0, \Sigma, O, \delta, \lambda')$ where:

Q is a finite set of states.

q_0 is the initial state.

Σ is the input alphabet.

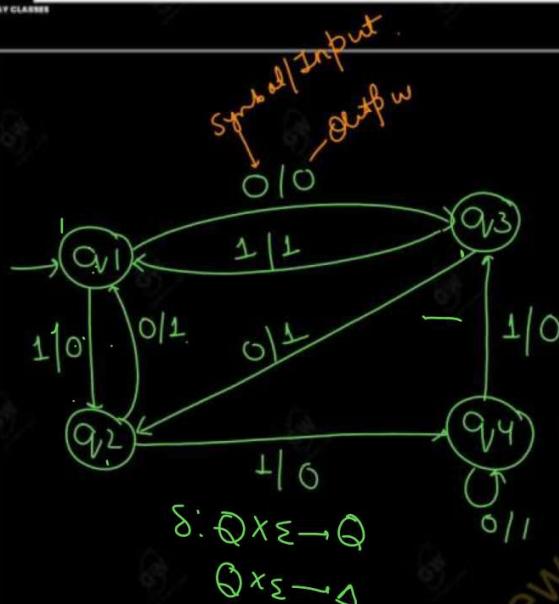
O is the output alphabet.

δ is the transition function which maps $Q \times \Sigma \rightarrow Q$.

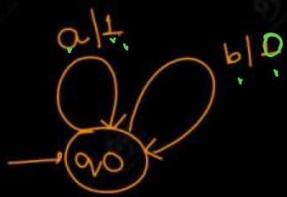
' λ' is the output function that maps $Q \times \Sigma \rightarrow O$.

NOTE:

- If the length of input string is n , then length of output string will be n
- Mealy machine do not response for empty string

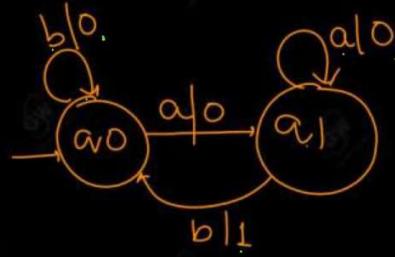


	0 State	output	1 State	output
q1	q3	0	q2	0
q2	q1	1	q4	0
q3	q2	1	q1	1
q4	q4	1	q3	0



	a State	output	b State	output
q0	q0	1	q0	0

aab
 $q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0$



	a State		b State	
	output		output	
->q0	q1	0	q0	0
q1	q1	0	q0	1

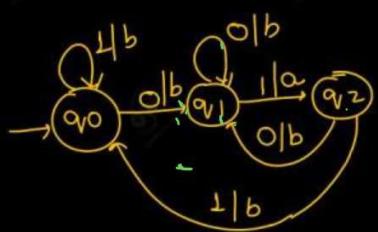


	0 State	output	1. State	output
->q0	q0	1	q0	0

Construct the Mealy machine that print a whenever the sequence 01 is encountered in any input binary string

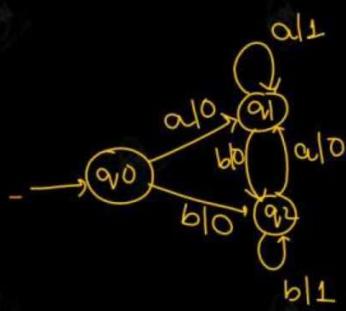
$$\Sigma = \{0, 1\}$$

Output a/b

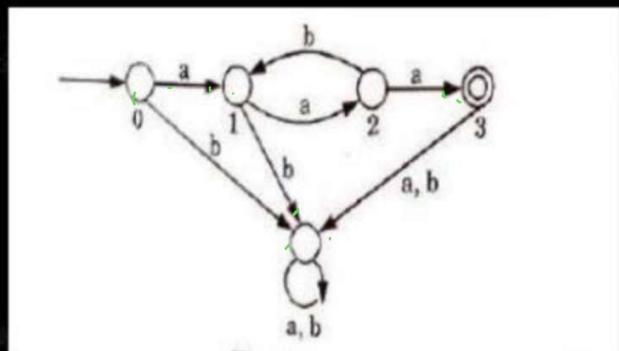


	0 State output		1 State output	
->q0	q1	b	q0	b
q1	q1	b	q2	a
q2	q1	b	q0	b

Construct the Mealy machine that accept language consisting having alphabet{a,b} and the string should end with aa or bb this will counted by 1



	a		b	
	State	output	State	output
$\rightarrow q_0$	q_1	0	q_2	0
q_1	q_1	1	q_2	0
q_2	q_1	0	q_2	1

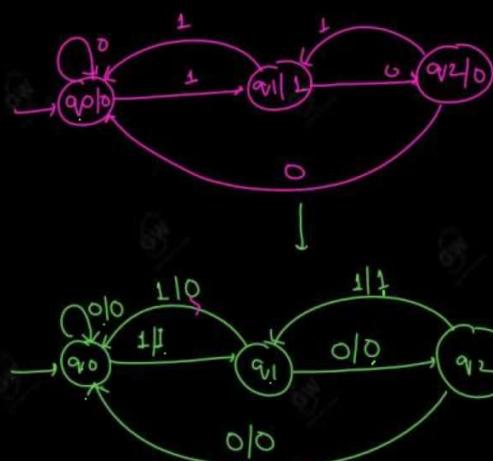


DFA that accept all string
that start & end with a

$L = \{W \in \{a, b\}^* \mid W \text{ accept all the string}$
 $\text{that start and end with a}\}$

Aspect	Moore Machines	Mealy Machines
✓ Output	Outputs depend only on the current state.	Outputs depend on the current state and input.
✓ Number of States	Tends to require more states due to separate output behavior.	Might require fewer states as outputs are tied to transitions.
✓ Response Time	Slower response to input changes as outputs update on state changes.	Faster response to input changes due to immediate output updates.
✓ Complexity	Can be simpler due to separation of output behavior.	Can be more complex due to combined state-input cases.

DFA	NFA
Epsilon move is not allowed in DFA	Epsilon move is allowed in NFA ϵ^NFA
DFA allows only one move for single input alphabet.	There can be choice (more than one move) for single input alphabet.
$\delta: Q \times \Sigma \rightarrow Q$ i.e. next possible state belongs to Q.	$\delta: Q \times (\Sigma \cup \epsilon) \rightarrow 2^Q$ i.e. next possible state belongs to power set of Q
All DFA are NFA.	Not all NFA are DFA.
DFA requires more space.	NFA requires less space than DFA.
In DFA, the next possible state is distinctly set.	In NFA, each pair of state and input symbol can have many possible next states.
DFA is more difficult to construct.	NFA is easier to construct.
DFA cannot use Empty String transition.	NFA can use Empty String transition.



(Moore Machine)



(Mealy Machine)

Present state	Next state 0	Next state 1	(output)
$\rightarrow q_0$	q_0	q_1	0
q_1	q_2	q_0	1
q_2	q_0	q_1	0

Moore machine transition table

	0		1	
	State	output	State	output
->q0	q0	0	q1	1
q1	q2	0	q0	0
q2	q0	0	q1	1

Mealy machine transition table

Present state	0	1	(output)
->q0	q0	q1	0
q1	q2	q0	1
q2	q0	q1	0

Moore transition table

	0		1	
	State	output	State	output
$\rightarrow q_0$	q_1	0	q_2	1
q_1	q_3	1	q_2	0
q_2	q_1	1	q_1	0
q_3	q_0	1	q_2	0

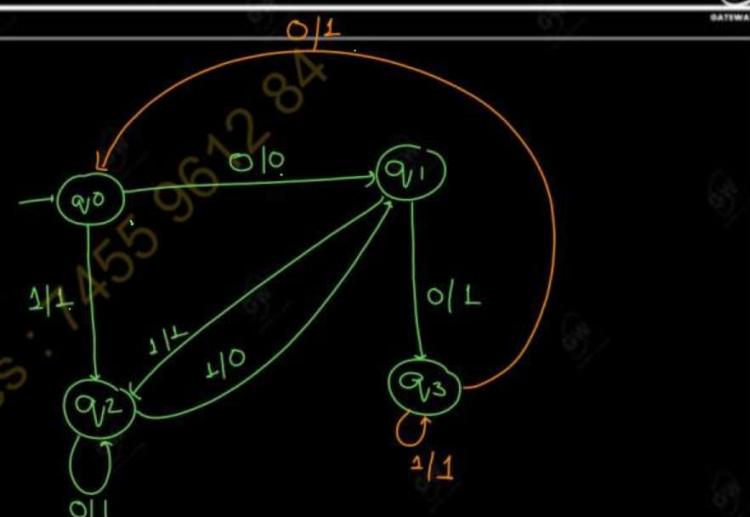
Mealy machine transition table

Present state	Next state 0	Next state 1	(output)
$\rightarrow q_0$	q_1	q_2	1
q_1	q_3	q_2	0
q_2	q_2	q_1	1
q_3	q_0	q_3	1

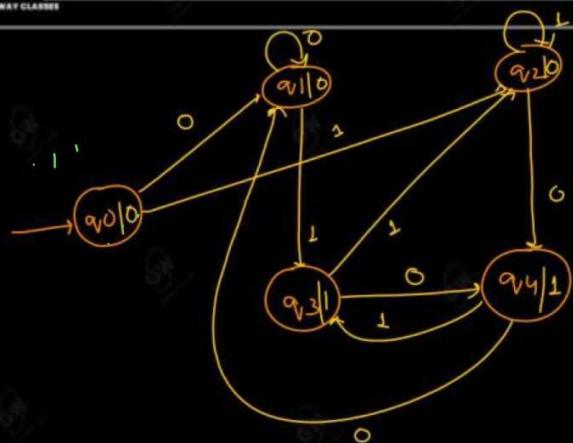
Moore transition table

	0 output		1 output	
	State	output	State	output
$\rightarrow q_0$	q_1	0	q_2	1
q_1	q_3	1	q_2	1
q_2	q_2	1	q_1	0
q_3	q_0	1	q_3	1

Mealy machine transition table



Mealy transition diagram



Present state	Next state		(output)
	0	1	
$\rightarrow q_0$	q1	q2	0
q1	q1	q3	0
q2	q4	q2	0
q3	q4	q2	1
q4	q1	q3	1

Moore machine transition table

	0 State output		1 State output	
q_0	q_1	0	q_2	0
q_1	q_1	0	q_3	1
q_2	q_4	1	q_2	0
q_3	q_4	1	q_2	0
q_4	q_1	0	q_3	1

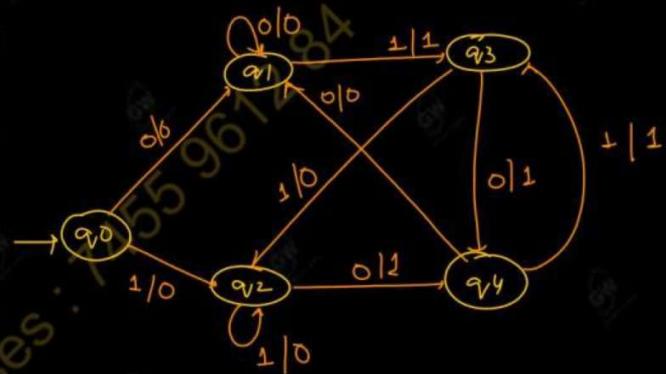
Mealy machine transition table

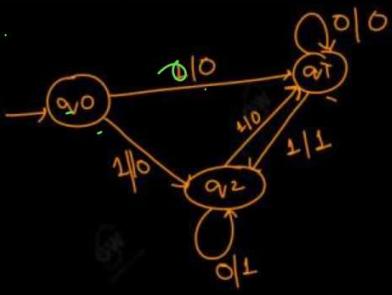
Present state	Next state 0	Next state 1	(output)
$\rightarrow q_0$	q_1	q_2	0
q_1	q_1	q_3	0
q_2	q_4	q_2	0
q_3	q_4	q_2	1
q_4	q_1	q_3	1

Moore transition table

	0 output		1 output	
	State	output	State	output
$\rightarrow q_0$	q_1	0	q_2	0
q_1	q_1	0	q_3	1
q_2	q_4	1	q_2	0
q_3	q_4	1	q_2	0
q_4	q_1	0	q_3	1

Mealy machine transition table

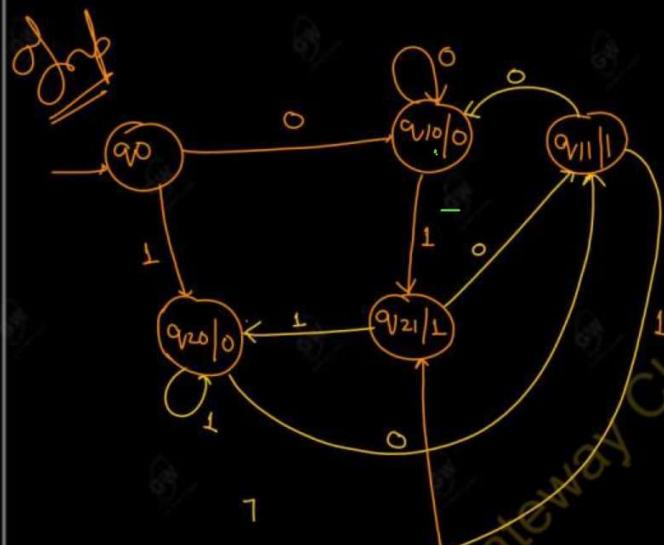




(Mealy Machine)

(Moore table)

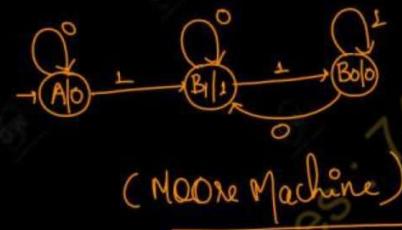
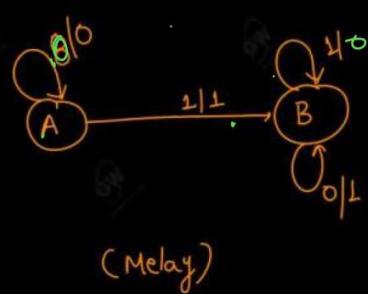
	0 State	output	1 State	output
-> q_0	q_1	0	q_2	0
q_1	q_1	0	q_2	1
q_2	q_2	1	q_1	0



	0 State	output	1 State	output
$\rightarrow q_0$	q_1	0	q_2	0
q_1	q_1	0	q_2	1
q_2	q_1	1	q_2	0

Present state	Next state		(output)
	0	1	
$\rightarrow q_0$	q_{10}	q_{20}	-
q_{10}	q_{10}	q_{21}	0
q_{11}	q_{10}	q_{21}	1
q_{20}	q_{11}	q_{20}	0
q_{21}	q_{11}	q_{20}	1

Moore transition table



	0 State	output	1 State	output
->A	A	0	B	1
B	B	1	B	0

Present state	Next state		(output)
	0	1	
→A	A	B1	0
B1	B1	B0	1
B0	B1	B0	0

Moore transition table



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