

## XOR LOGIC

Outputs true (1) only when inputs differ. This is expressed as:

$$A \text{ XOR } B = (A \text{ AND NOT } B) \text{ OR } (\text{NOT } A \text{ AND } B)$$

$$0 \text{ XOR } 0 = (0 \text{ AND NOT } 0) \text{ OR } (\text{NOT } 0 \text{ AND } 0) = (0 \text{ AND } 1) \text{ OR } (1 \text{ AND } 0) = 0 \text{ OR } 0 = 0$$

## MATRIX COMPUTATION OF OR GATE

Input matrix logic\_inputs:

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Weights vector W1:

$$\begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

In layer:

inputs \* weights + bias then apply activation function to result

z = inputs \* weights + bias then sigmoid(z)

z = logic\_inputs \* weights + bias

$$z = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 20 \end{bmatrix} + (-10) = \begin{bmatrix} 0 \cdot 20 + 0 \cdot 20 \\ 0 \cdot 20 + 1 \cdot 20 \\ 1 \cdot 20 + 0 \cdot 20 \\ 1 \cdot 20 + 1 \cdot 20 \end{bmatrix} - 10 = \begin{bmatrix} 0 \\ 20 \\ 20 \\ 40 \end{bmatrix} - 10 = \begin{bmatrix} -10 \\ 10 \\ 10 \\ 30 \end{bmatrix}$$

$$\text{sigmoid}\left(\begin{bmatrix} -10 \\ 10 \\ 10 \\ 30 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$