Report Reinforcement Learning

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Question 1

Q-Learning

Q-Learning is an off-policy Reinforcement Learning algorithm. It uses table to store Q-Values of all possible state-action possible pairs. Table is updated using Bellman equation while action selection can be done using $\epsilon - greedy$ policy or some other policy.

We will work with a grid world environment consisting of H × W tiles laid out in a 2-dimensional grid. An agent acts by moving up, down, left or right in the grid-world. This corresponds to the following Markov decision process:

State Space : S = {(x,y|x ϵ {1 , …. , H} , y ϵ {1,…..,H}} Action Space : A = {up,down,left,right}

Additionally, we assume state space to be fully observable. The reward function is a deterministic function of the state and does not depend on the actions taken by the agent. We assume the agent gets the reward as soon as it moves to a state. The transition model is defined by the agent moving in the direction chosen with probability (1−β). The agent might also slip and end up moving in the direction to the left or right of its chosen action, each with probability $\beta$$/2. The transition model is unknown to the agent, forcing us to resort to model-free solutions. The environment is episodic and all states with a non-zero reward are terminal. Throughout this lab we use integer representations of the different actions: Up=1, right=2, down=3 and left=4

Environment A

For our first environment, we will use H = 5 and W = 7. This environment includes a reward of 10 in state (3,6) and a reward of -1 in states (2,3), (3,3) and (4,3). We specify the rewards using a reward map in the form of a matrix with one entry for each state. States with no reward will simply have a matrix entry of 0. The agent starts each episode in the state (3,1). The function vis_environment is used to visualize the environment and learned action values and policy.

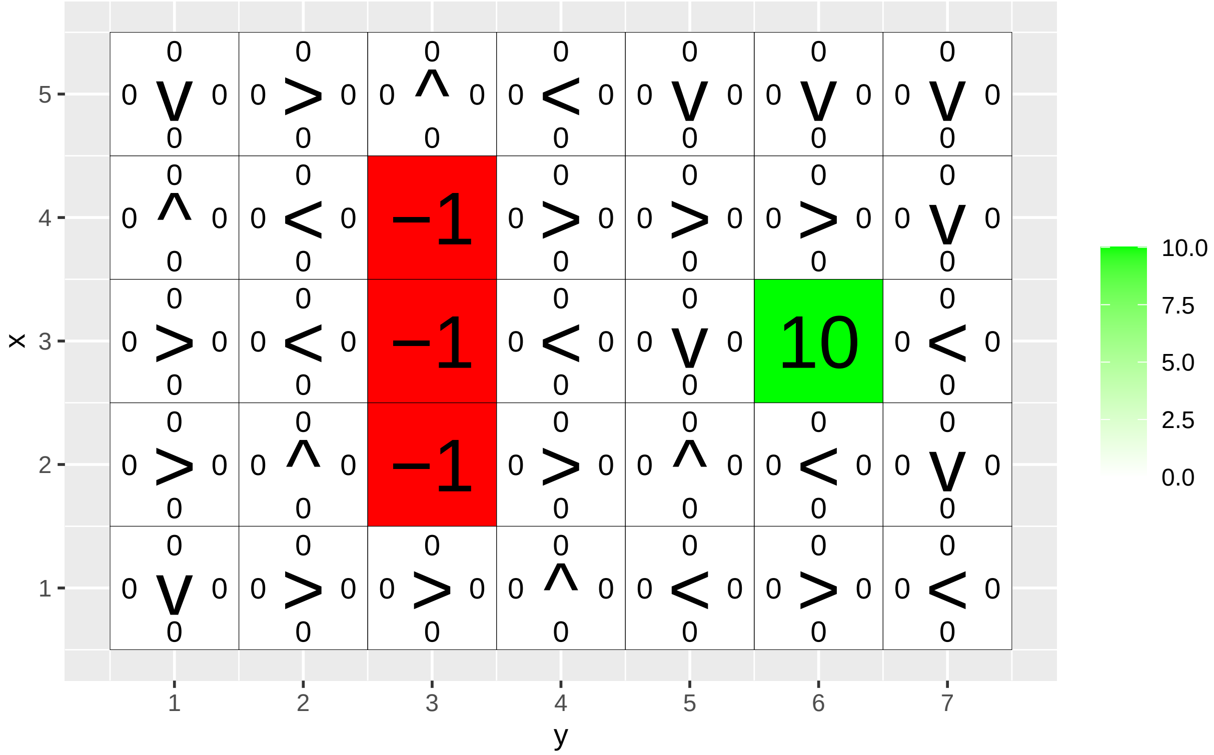
Q-Learning is off-policy, it evaluates one policy (target policy) while observing behavior policy. Finding optimal action value function Q_* under arbitrary behavior policy is achieved by policy iteration. Q converges to optimal action value function $Q_*(S, A)$ and its greedy converges to optimal policy under appropriate choice of learning rate (α) over time.

$$Q_*(S, A) = Q(S, A) + \alpha(R + \gamma(max_a Q(S', a)) - Q(S, A))$$

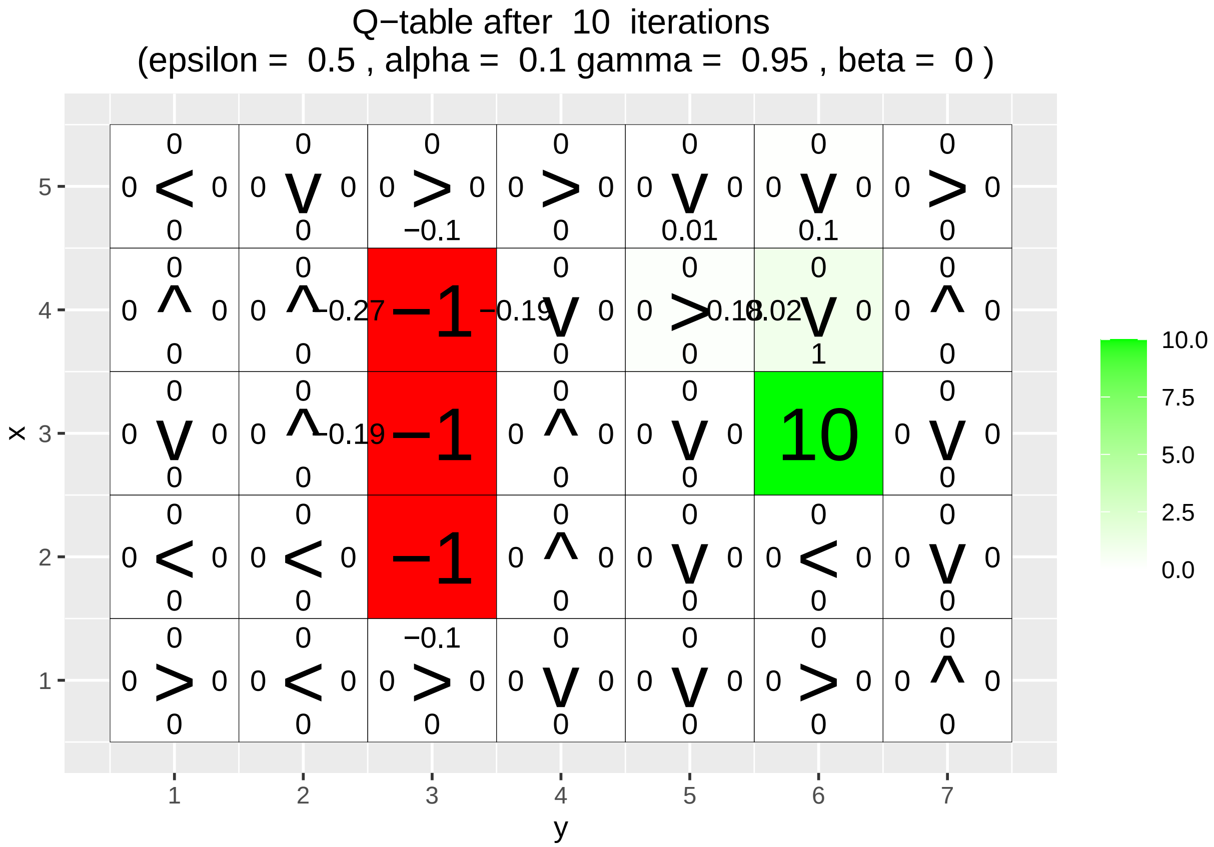
Here :

S : is current state A : Action chosen by ϵ-Greedy policy S': Transition state based on current state and chosen action A R : is Reward for moving to transition state S' a : Action chosen by Greedy Policy α :

Learning Rate γ : Discount factor, ranges between (0 , 1) which control importance of future rewards in comparison to current one.

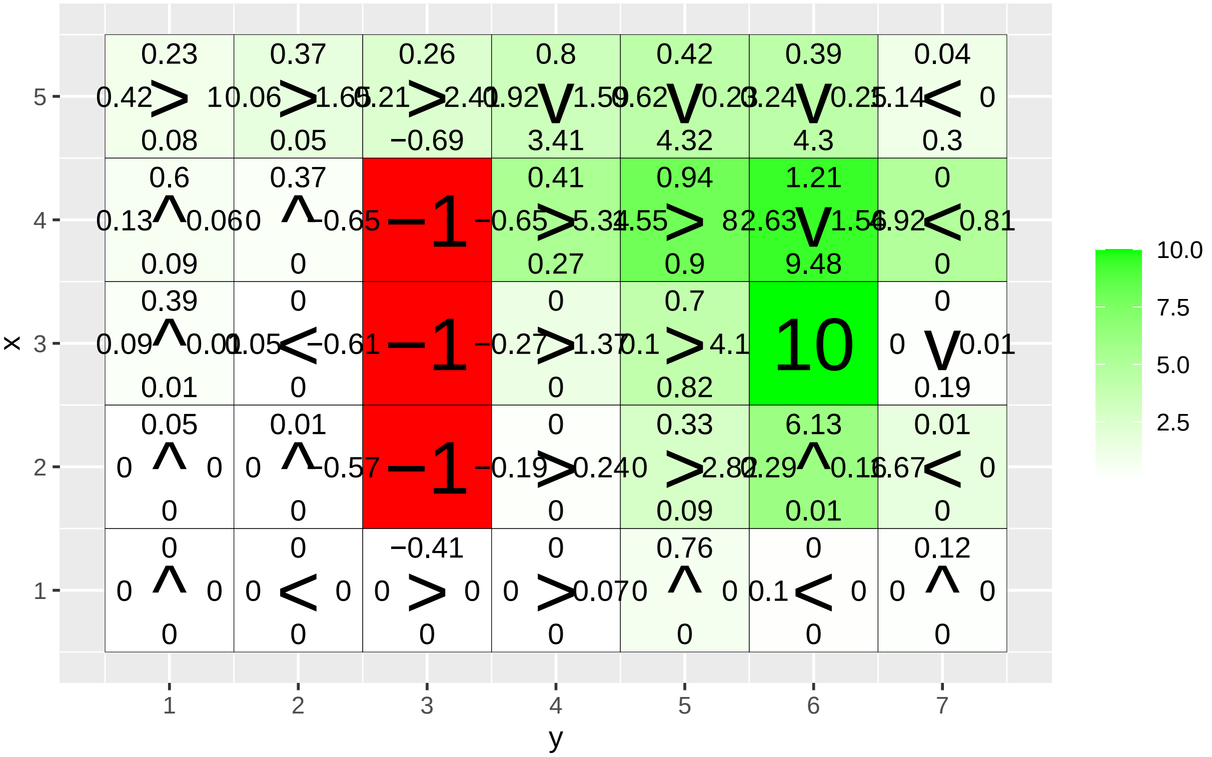


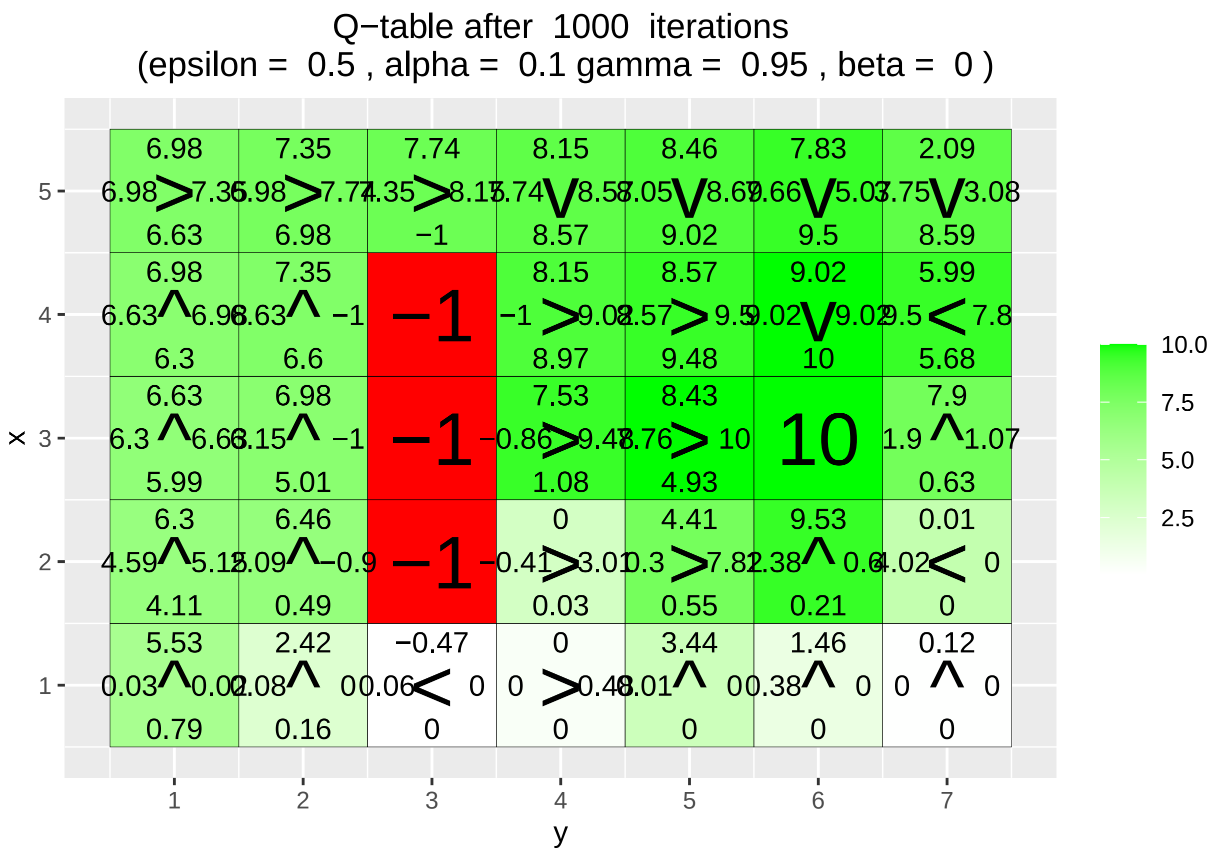
Q−table after 0 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)

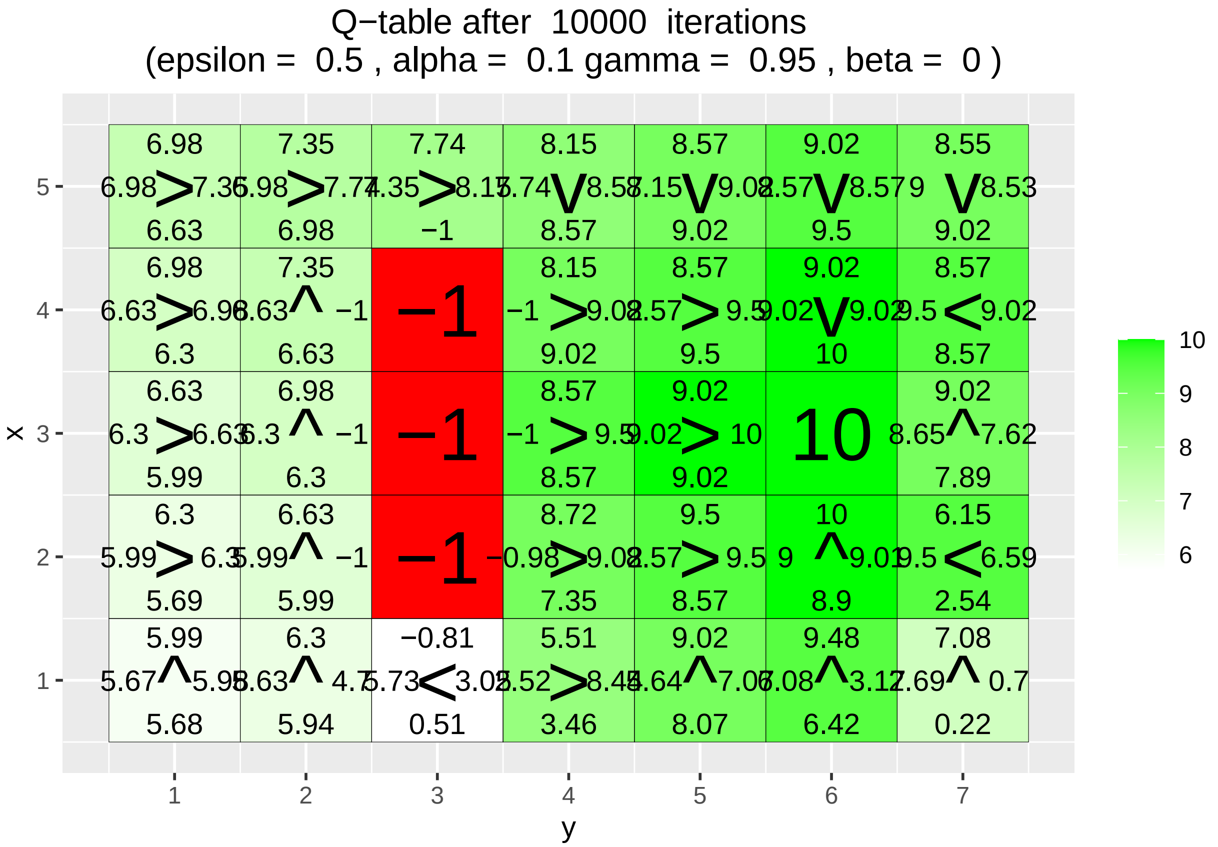


Q−table after 10 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)

Q−table after 100 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)




Q−table after 1000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)
x
y
1
2
3
4
5
6
7
10.0
7.5
5.0
2.5
6.98 6.98 > 7.36 6.63
7.35 6.98 > 7.74 6.98
7.74 7.35 > 8.15 −1
8.15 7.74 V 8.57 8.57
8.46 8.05 V 8.69 9.02
7.83 7.66 V 5.03 9.5
2.09 3.75 V 3.08 8.59
6.98 6.63 ^ 6.98 6.3
7.35 6.63 ^ −1 6.6
−1
8.15 −1 > 9.02 8.97
8.57 8.57 > 9.5 9.48
9.02 9.02 V 9.02 10
5.99 9.5 < 7.8 5.68
6.63 6.3 ^ 6.66 5.99
6.98 6.15 ^ −1 5.01
−1
7.53 −0.86 > 9.48 1.08
8.43 8.76 > 10 4.93
10
7.9 1.9 ^ 1.07 0.63
6.3 4.59 ^ 5.15 4.11
6.46 5.09 ^ −0.9 0.49
−1
0 −0.41 > 3.01 0.03
4.41 0.3 > 7.82 0.55
9.53 2.38 ^ 0.6 0.21
0.01 4.02 < 0 0
5.53 0.03 ^ 0.02 0.79
2.42 2.08 ^ 0 0.16
−0.47 0.06 < 0 0
0 0 > 0.48 0
3.44 8.01 ^ 0 0
1.46 0.38 ^ 0 0
0.12 0 ^ 0 0



What has the agent learned after the first 10 episodes ?

From the above Q-Table named "Q-Table after 10 Iterations" with ten iterations, we have a small number of episodes and, the agent is not able to learn optimal paths towards maximum reward location as it is in the exploring stage, so there is no significant learning in terms of optimal path but agent started to learn to avoid negative reward available in grid 2,3 , 3,3 and 3,4.

Is the final greedy policy (after 10000 episodes) optimal? Why / Why not ?

From plot "Q-Table after 10000 Iterations", the model has learned multiple paths towards the maximum reward position state in 10000 episodes, and thus when agent start from grid position of 3,1, it will converge to the maximum reward position.

The final greedy policy is optimal in terms of reaching to highest reward grid position when we have a high number of episodes in the learning phase.

Does the agent learn that there are multiple paths to get to the positive reward ? If not, what could be done to make the agent learn this ?

In "Q-Table after 10000 Iterations", looking at maximum rewards for each grid, it is observed that the agent did learn multiple paths to reach to maximum reward position in the grid based on where it started i.e. 3,1.

Now if we want to explore more paths during during learning, we can set ϵ parameter (Exploration probability) with much more higher value. It is explained below in more details.

Environment B

Investigate how the ϵ and γ parameters affect the learned policy by running 30000 episodes of Q-learning with ϵ = 0.1, 0.5, γ = 0.5, 0.75, 0.95, β = 0 and α = 0.1

Exploration vs Exploitation

Exploration allows an agent to improve its current knowledge about each action, hopefully leading to long-term benefit. Improving the accuracy of the estimated action-values, enables an agent to make more informed decisions in the future.

Exploitation on the other hand, chooses the greedy action to get the most reward by exploiting the agent's current action-value estimates. But by being greedy with respect to action-value estimates, may not actually get the most reward and lead to sub-optimal behavior.

When an agent explores, it gets more accurate estimates of action-values. And when it exploits, it might get more reward. It cannot, however, choose to do both simultaneously.

Use of ϵ parameter:

ϵ-Greedy is a simple method to balance exploration and exploitation by choosing between exploration and exploitation randomly.

Now with probability of 1-ϵ we decide to exploit and with probability of ϵ we decide to explore.

Use of γ (discount factor) parameter:

The discount factor essentially determines how much the reinforcement learning agents cares about rewards in the distant future relative to those in the immediate future. If $\gamma = 0$, the agent will be completely shortsighted and only learn about actions that produce an immediate reward. If $\gamma = 1$, the agent will evaluate each of its actions based on the sum total of all of its future rewards.

```
## Epsilon set as 0.5
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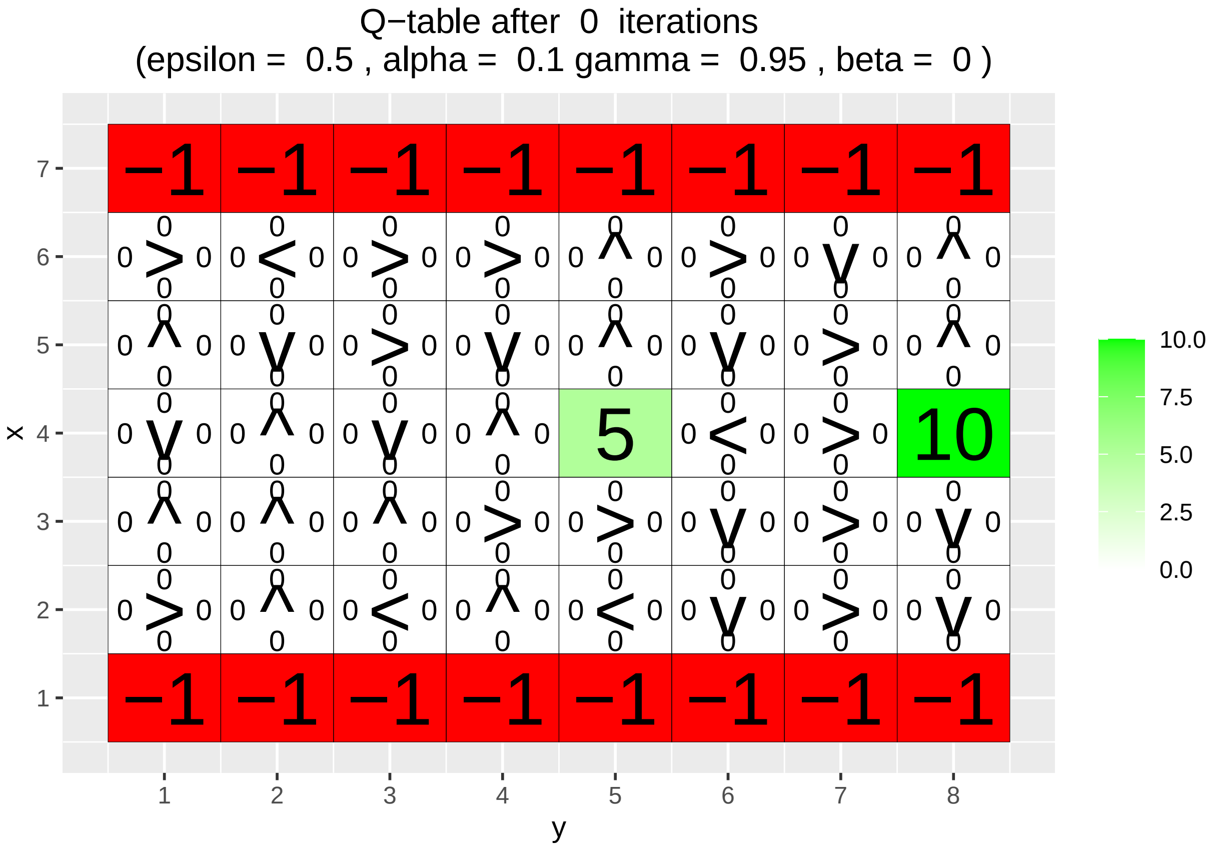
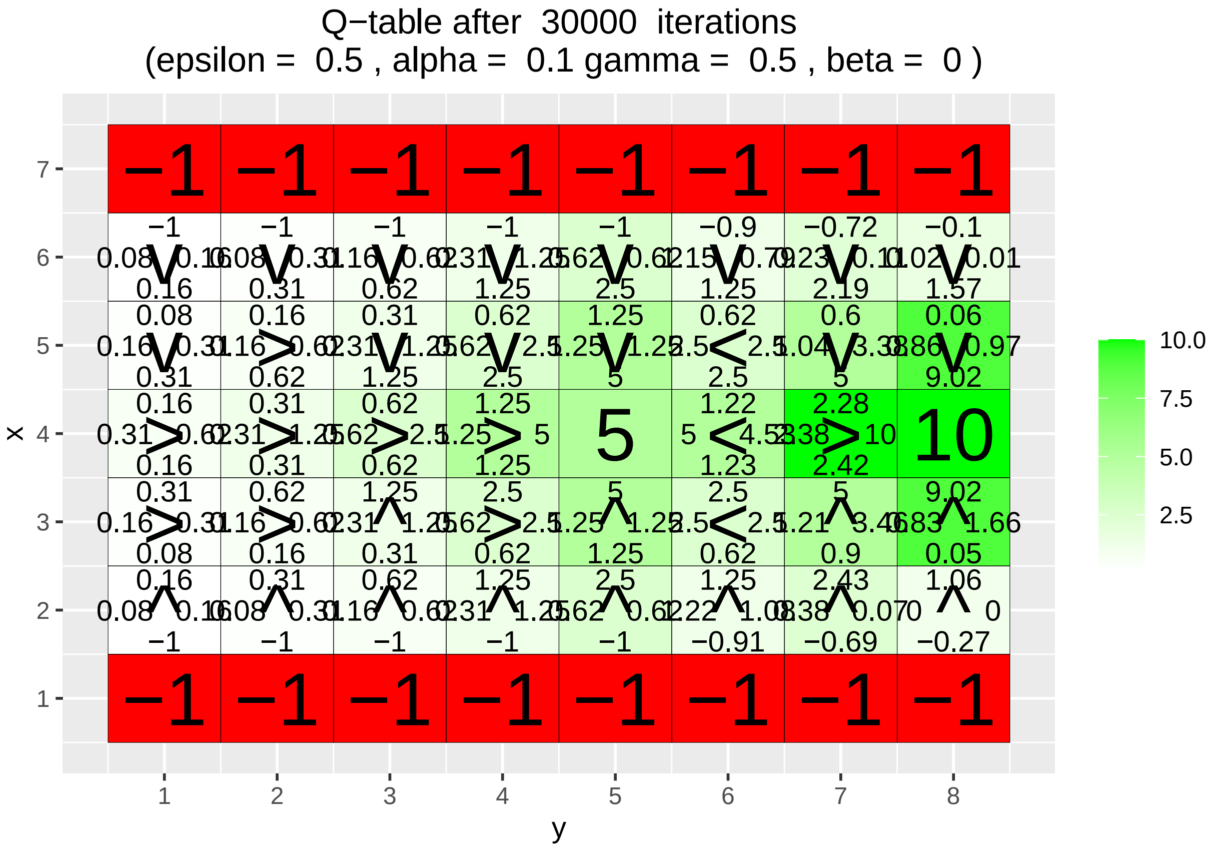
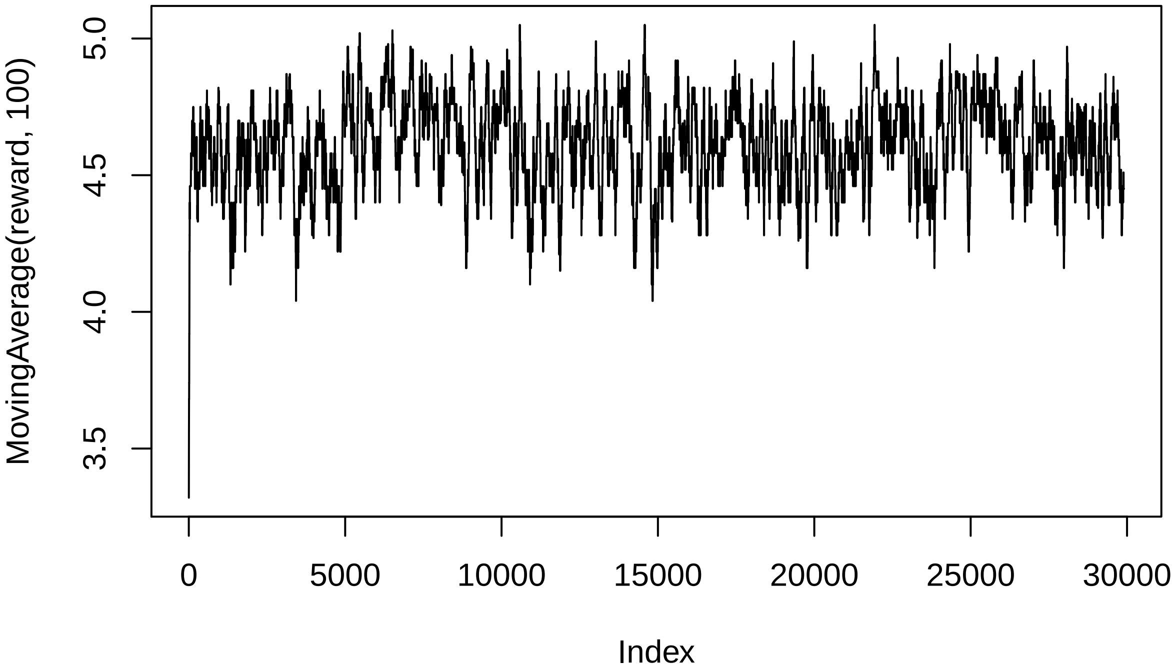
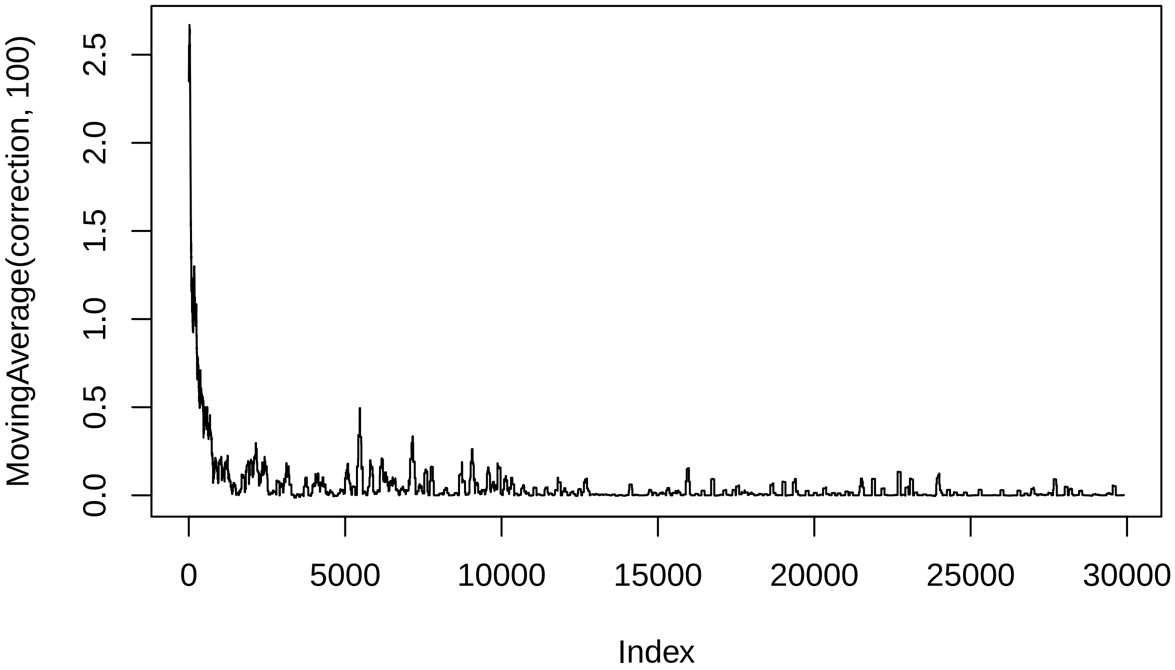
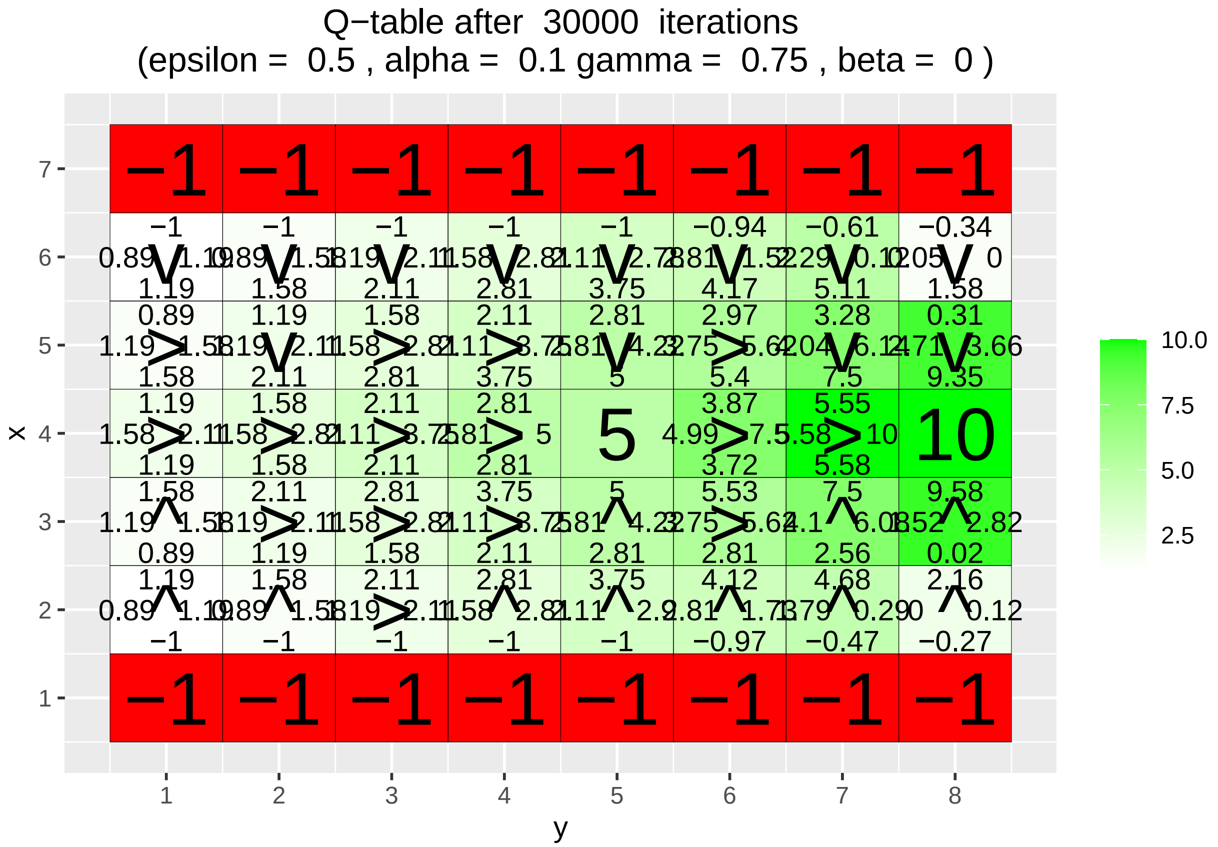


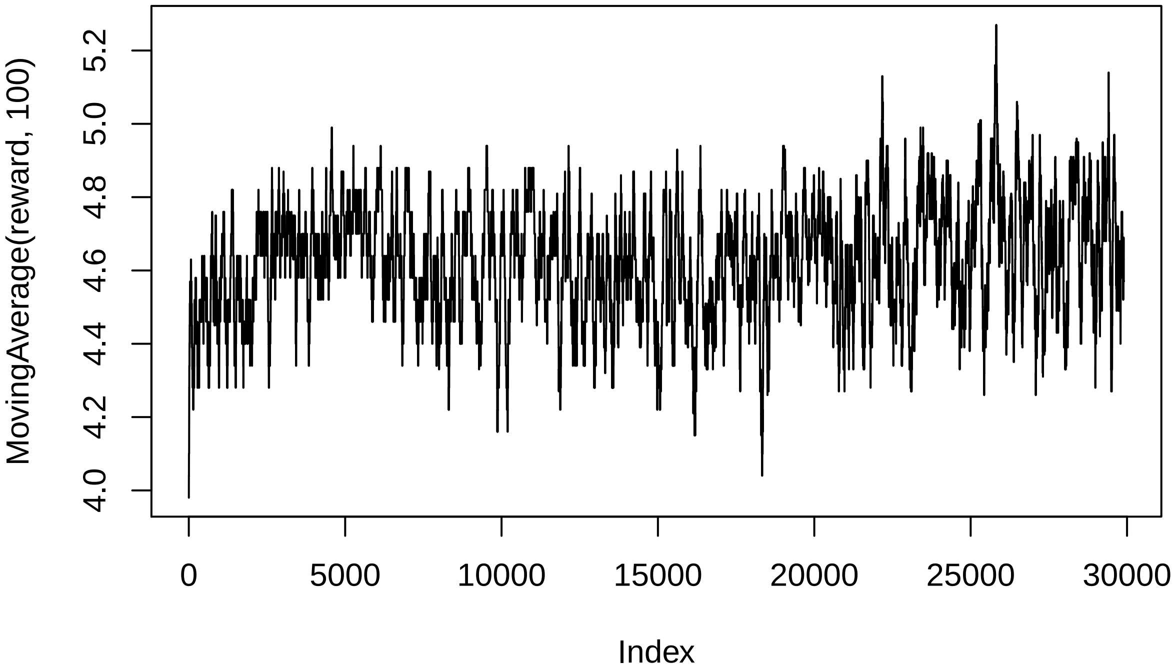


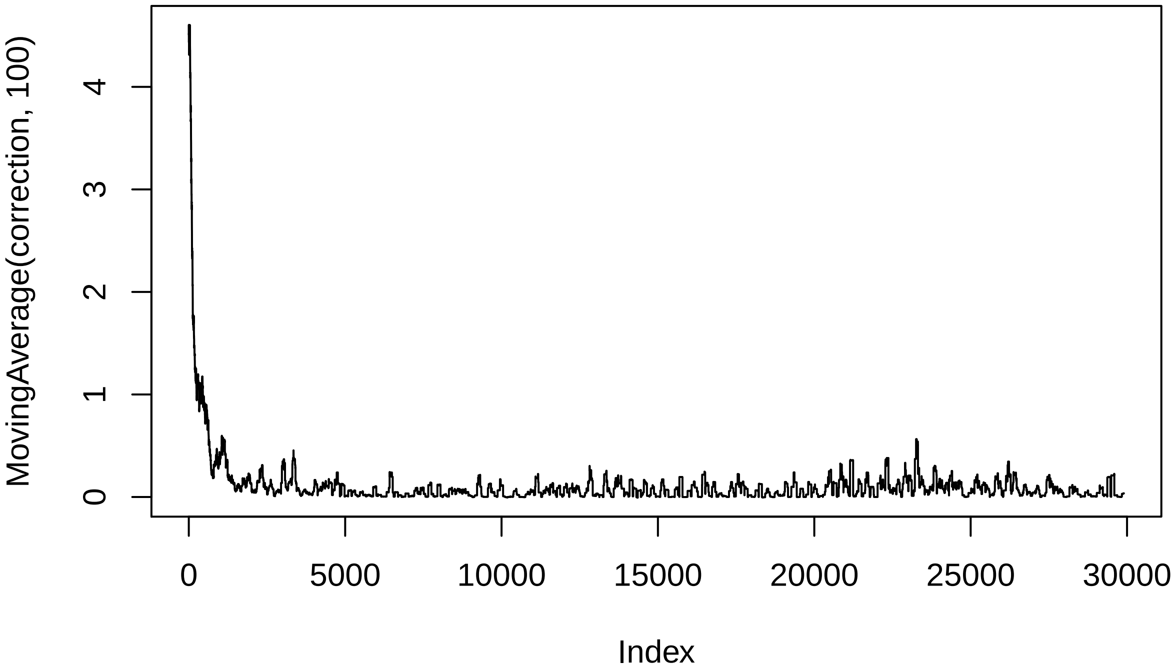
Q−table after 30000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.5 , beta = 0)
x
y
1
2
3
4
5
6
7
8
10.0
7.5
5.0
2.5
−1
5
10
−1
−1
−1
−1
−1
−0.9
−0.72
−0.1
0.16
0.31
0.62
1.25
2.5
1.25
2.19
1.57
0.08
0.16
0.31
0.62
1.25
0.62
0.6
0.06
0.31
0.62
1.25
2.5
5
2.5
5
9.02
0.16
0.31
0.62
1.25
1.22
2.28
0.16
0.31
0.62
1.25
1.23
2.42
0.31
0.62
1.25
2.5
5
2.5
5
9.02
0.08
0.16
0.31
0.62
1.25
0.62
0.9
0.05
0.16
0.31
0.62
1.25
2.5
1.25
2.43
1.06
−1
−1
−1
−1
−1
−0.91
−0.69
−0.27


MovingAverage(reward, 100)
3.5
4.0
4.5
5.0
0
5000
10000
15000
20000
25000
30000
Index

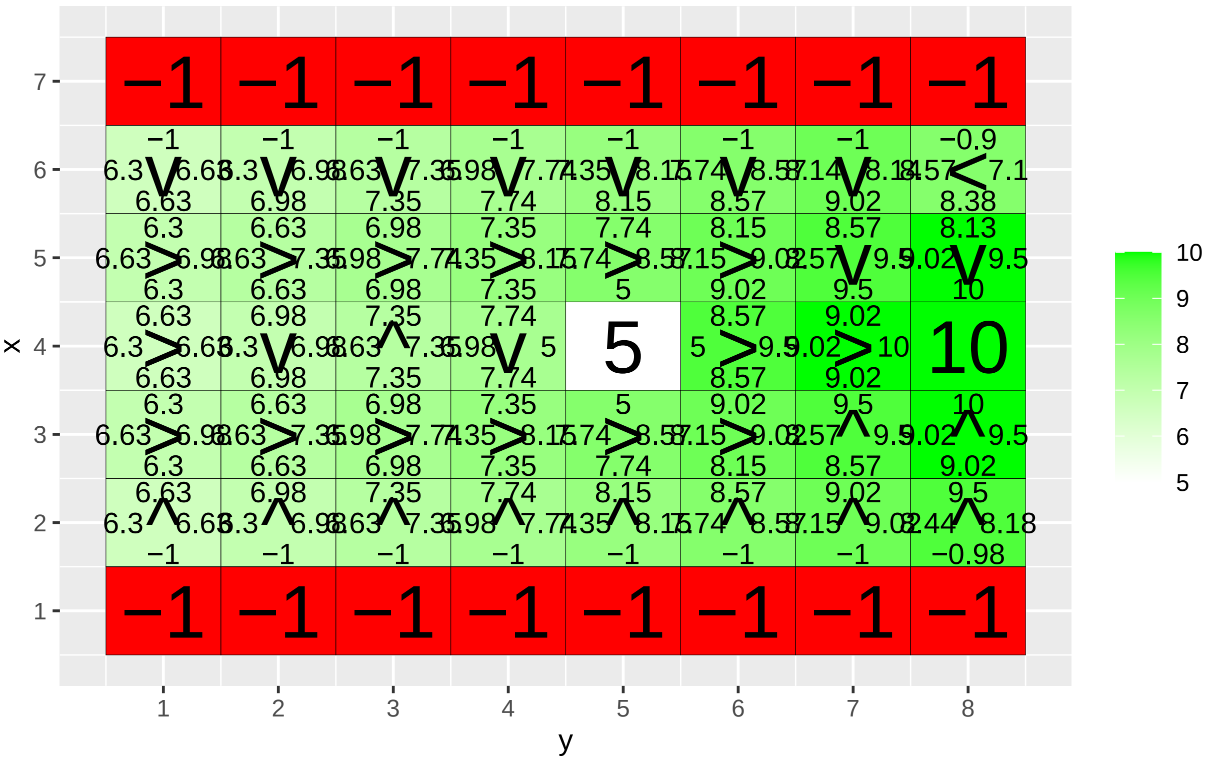

MovingAverage(correction, 100)
0.0
0.5
1.0
1.5
2.0
2.5
0
5000
10000
15000
20000
25000
30000
Index

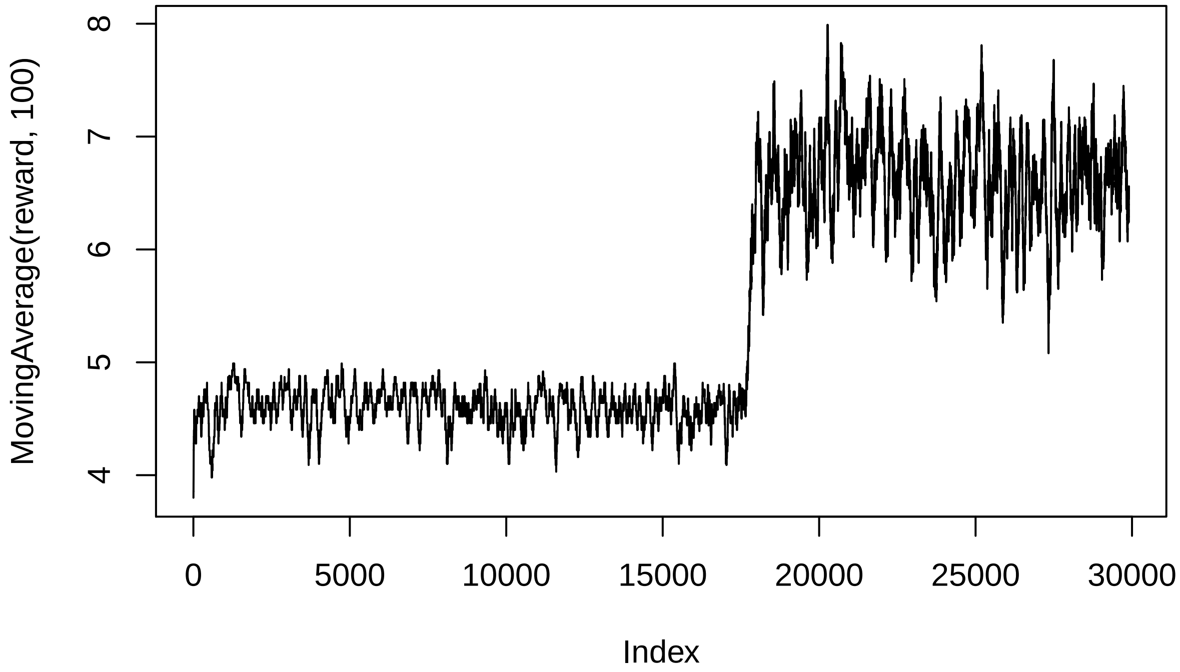

Q−table after 30000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.75 , beta = 0)
−1 −1 −1 −1 −1 −1 −1 −1
−1 −1 −1 −1 −1 −0.94 −0.61 −0.34
0.89 V 1.19 0.89 V 1.58 1.19 V 2.11 1.58 V 2.81 2.11 V 2.78 2.81 V 1.52 2.29 V 0.12 0.205 V 0
1.19 1.58 2.11 2.81 3.75 4.17 5.11 1.58
0.89 1.19 1.58 2.11 2.81 2.97 3.28 0.31
1.19 > 1.58 1.19 V 2.11 1.58 > 2.81 2.11 > 3.75 2.81 V 4.23 2.75 > 5.64 2.04 V 6.12 4.71 V 3.66
1.58 2.11 2.81 3.75 5 5.4 7.5 9.35
1.19 1.58 2.11 2.81 3.87 5.55
1.58 > 2.11 1.58 > 2.81 2.11 > 3.75 2.81 > 5 5 4.99 > 7.5 5.58 > 10 10
1.19 1.58 2.11 2.81 3.72 5.58
1.58 2.11 2.81 3.75 5 5.53 7.5 9.58
1.19 ^ 1.58 1.19 > 2.11 1.58 > 2.81 2.11 > 3.75 2.81 ^ 4.23 2.75 > 5.64 4.1 ^ 6.08 3.52 ^ 2.82
0.89 1.19 1.58 2.11 2.81 2.81 2.56 0.02
1.19 1.58 2.11 2.81 3.75 4.12 4.68 2.16
0.89 ^ 1.19 0.89 ^ 1.58 1.19 > 2.11 1.58 ^ 2.81 2.11 ^ 2.9 2.81 ^ 1.71 3.79 ^ 0.29 0 ^ 0.12
−1 −1 −1 −1 −1 −0.97 −0.47 −0.27
−1 −1 −1 −1 −1 −1 −1 −1
x
7
6
5
4
3
2
1
y
1 2 3 4 5 6 7 8
10.0
7.5
5.0
2.5

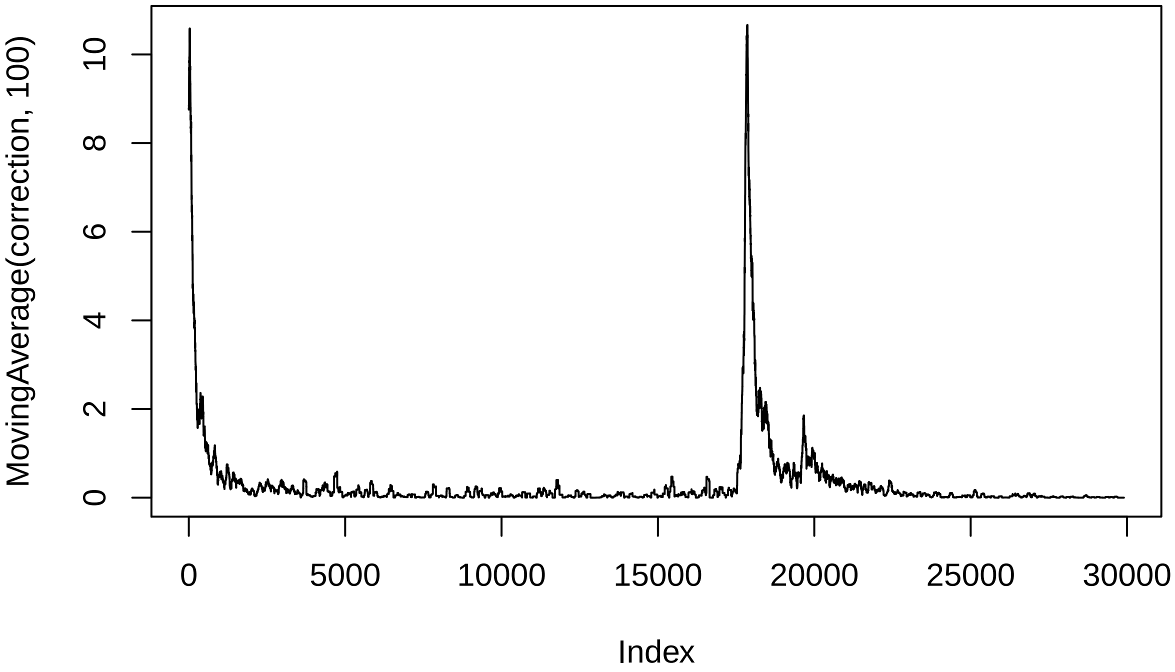

MovingAverage(reward, 100)
4.0
4.2
4.4
4.6
4.8
5.0
5.2
0
5000
10000
15000
20000
25000
30000
Index


MovingAverage(correction, 100)
0
1
2
3
4
0
5000
10000
15000
20000
25000
30000
Index

Q−table after 30000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)




MovingAverage(reward, 100)
4
5
6
7
8
0
5000
10000
15000
20000
25000
30000
Index



```
## Epsilon set as 0.1
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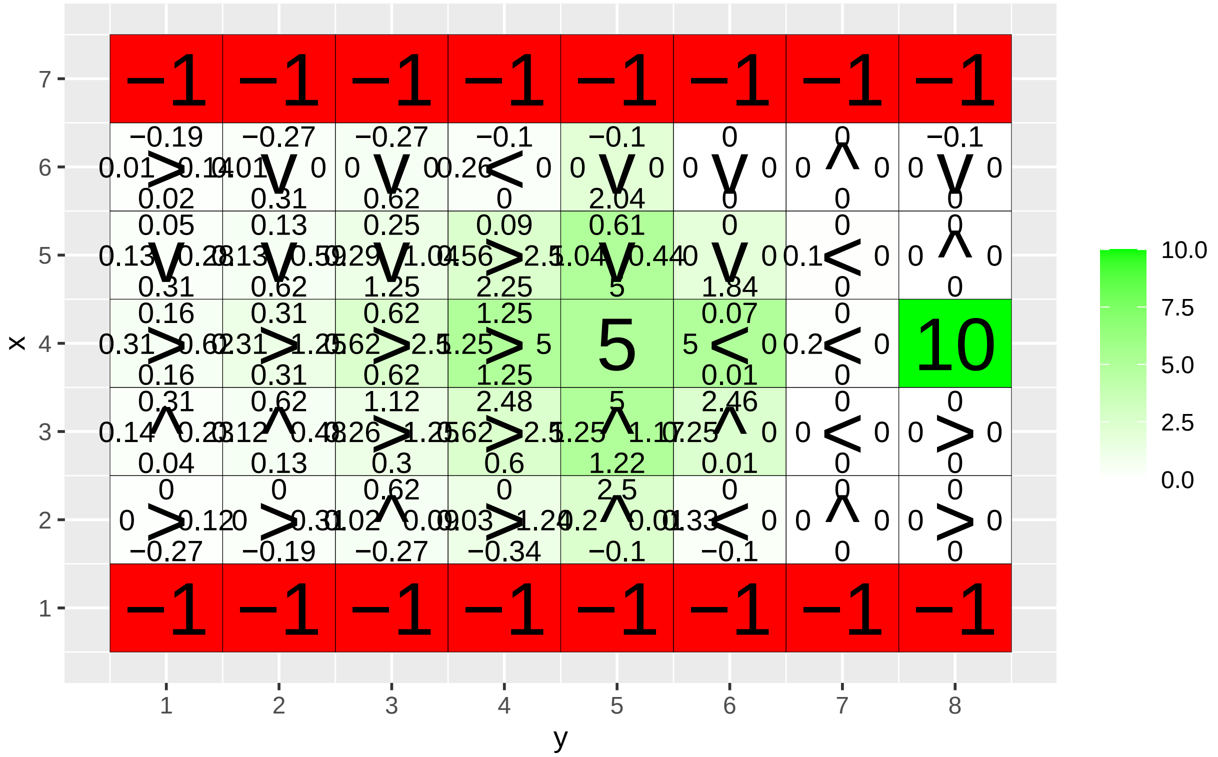
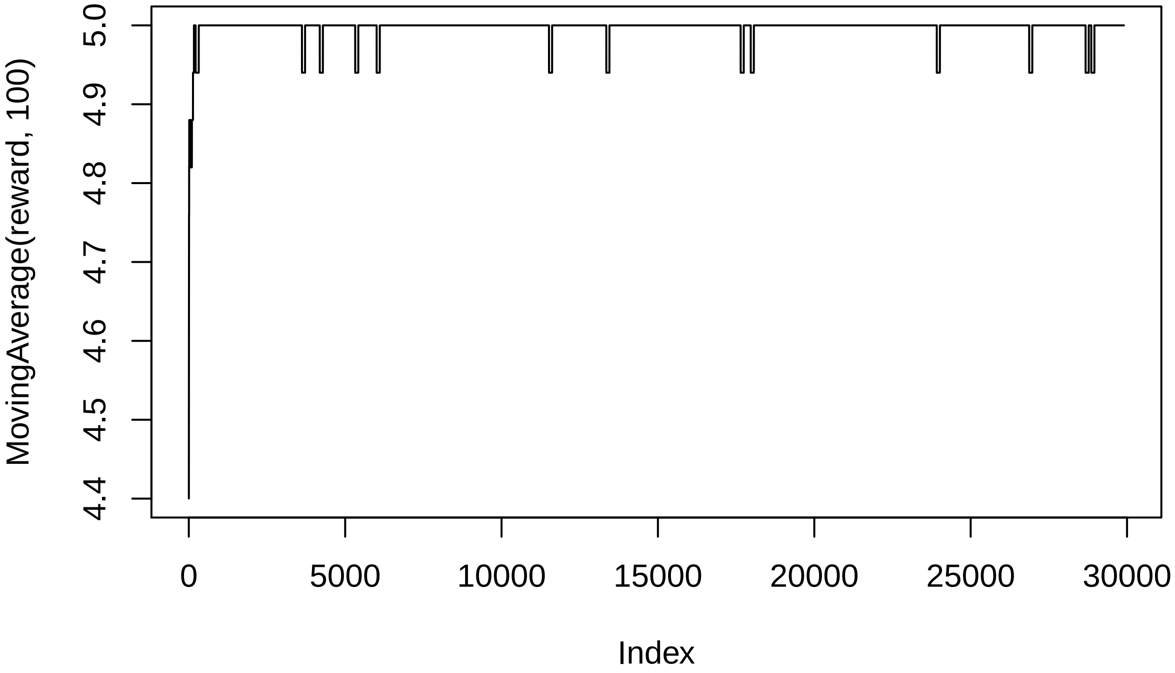
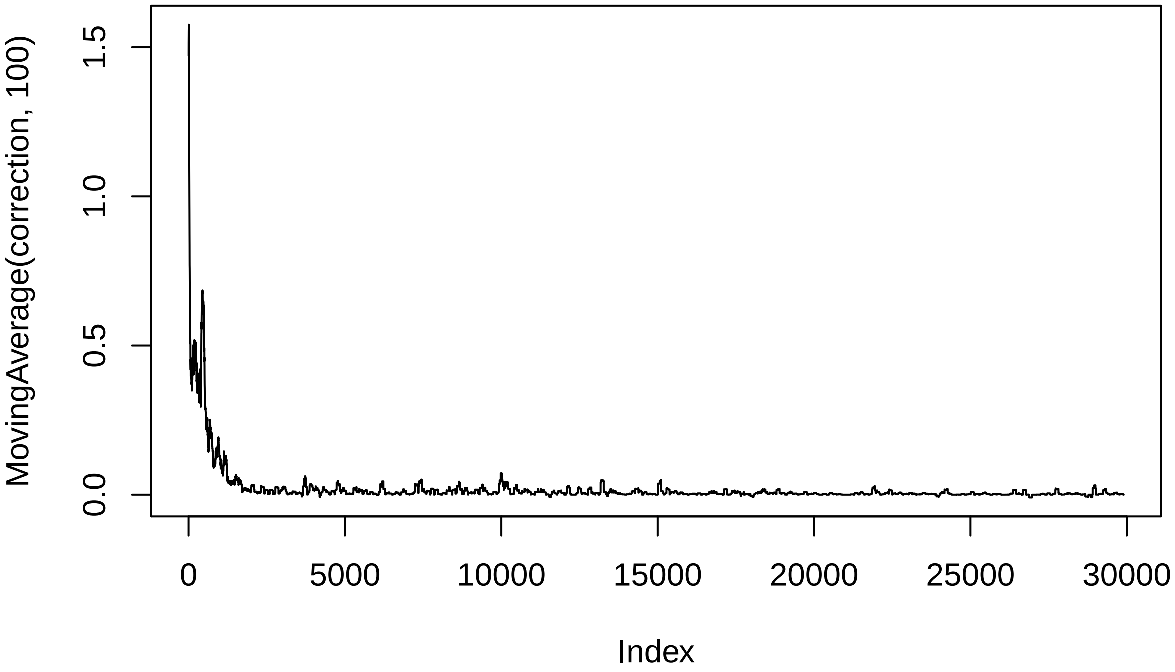
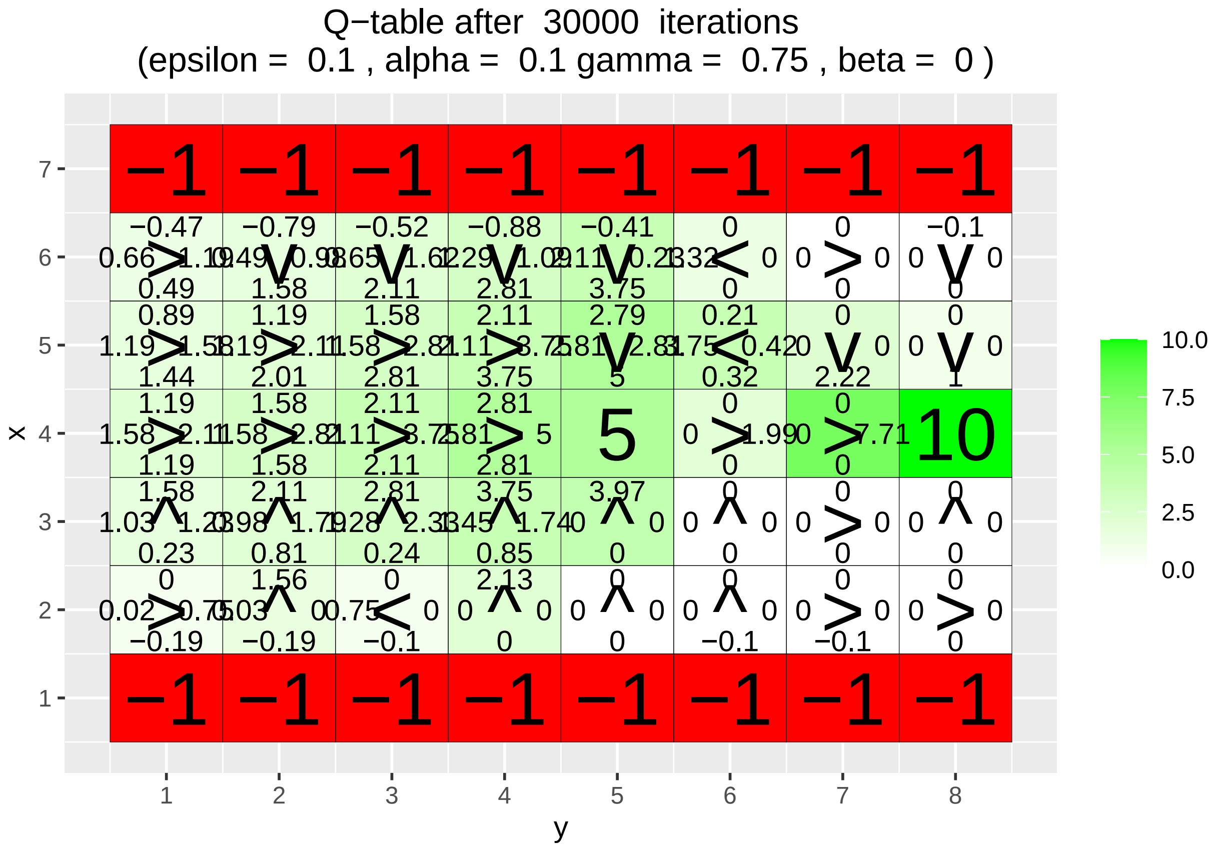
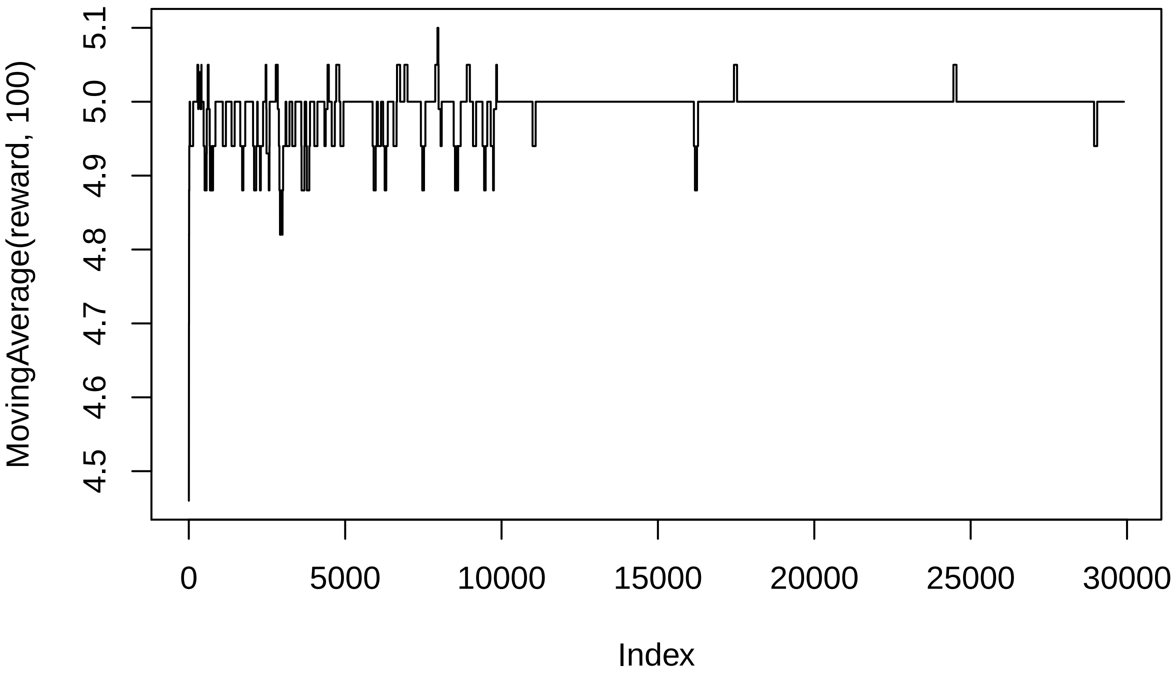
Q−table after 30000 iterations
(epsilon = 0.1 , alpha = 0.1 gamma = 0.5 , beta = 0)

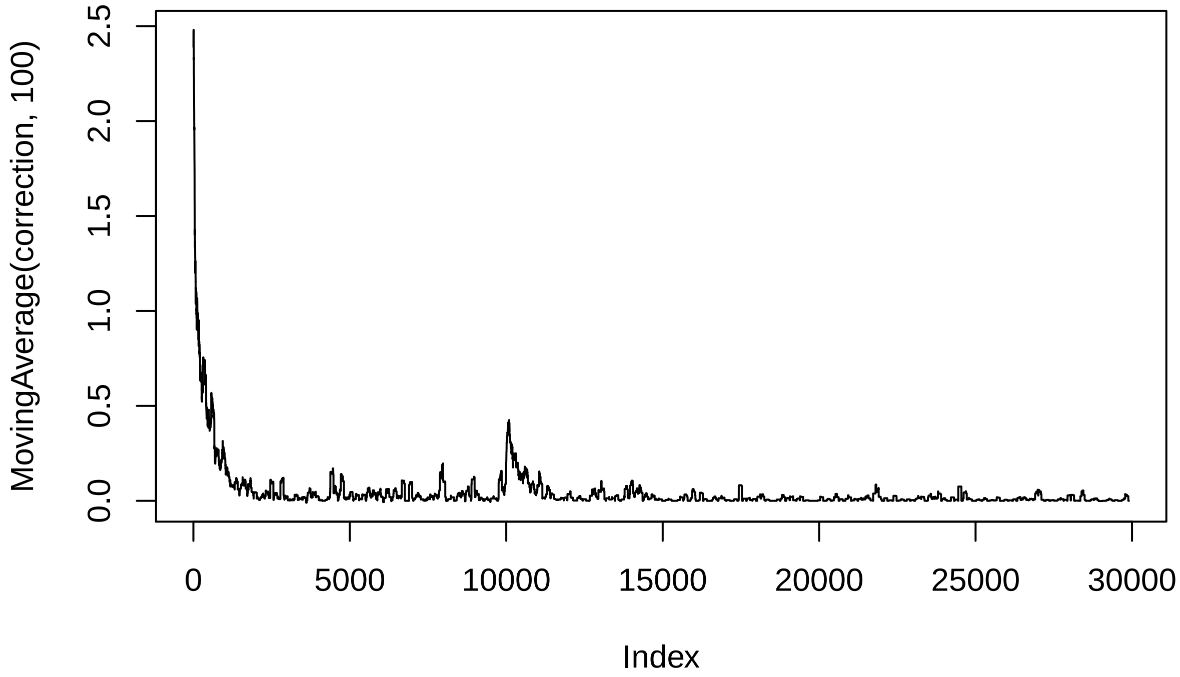



MovingAverage(reward, 100)
4.4
4.5
4.6
4.7
4.8
4.9
5.0
0
5000
10000
15000
20000
25000
30000
Index

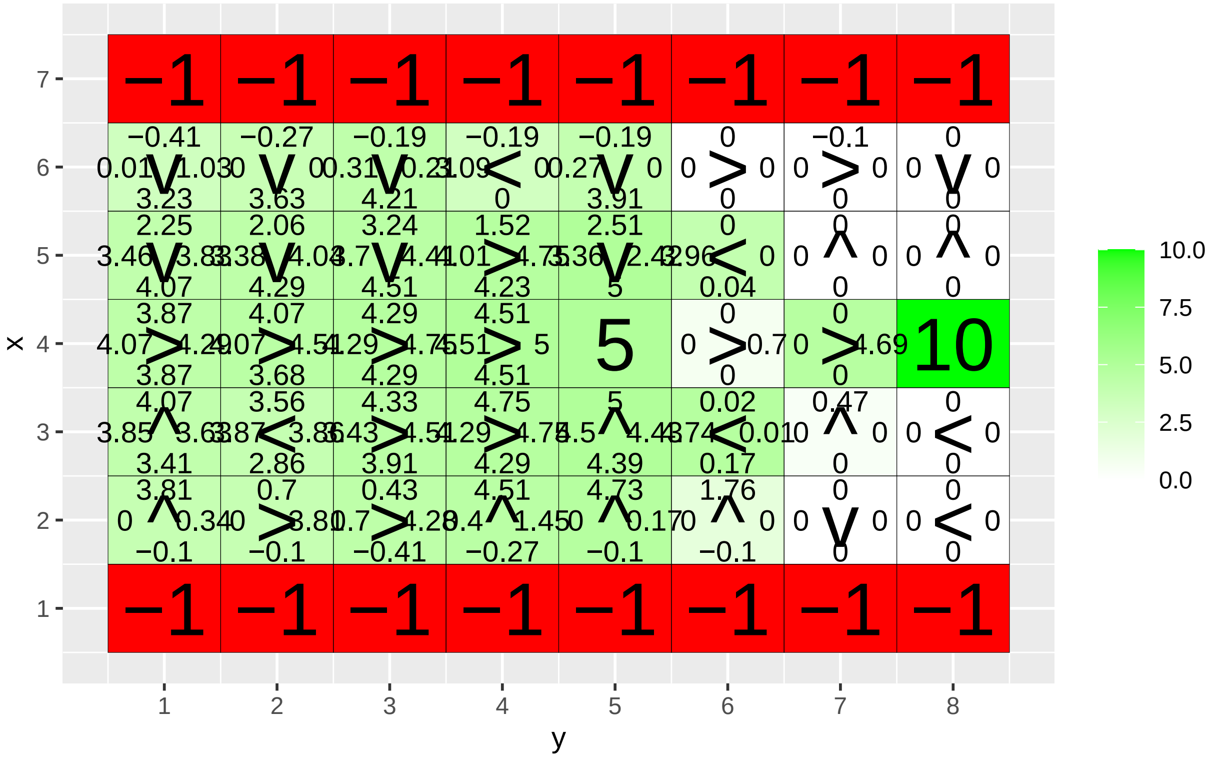

MovingAverage(correction, 100)
0.0
0.5
1.0
1.5
0
5000
10000
15000
20000
25000
30000
Index

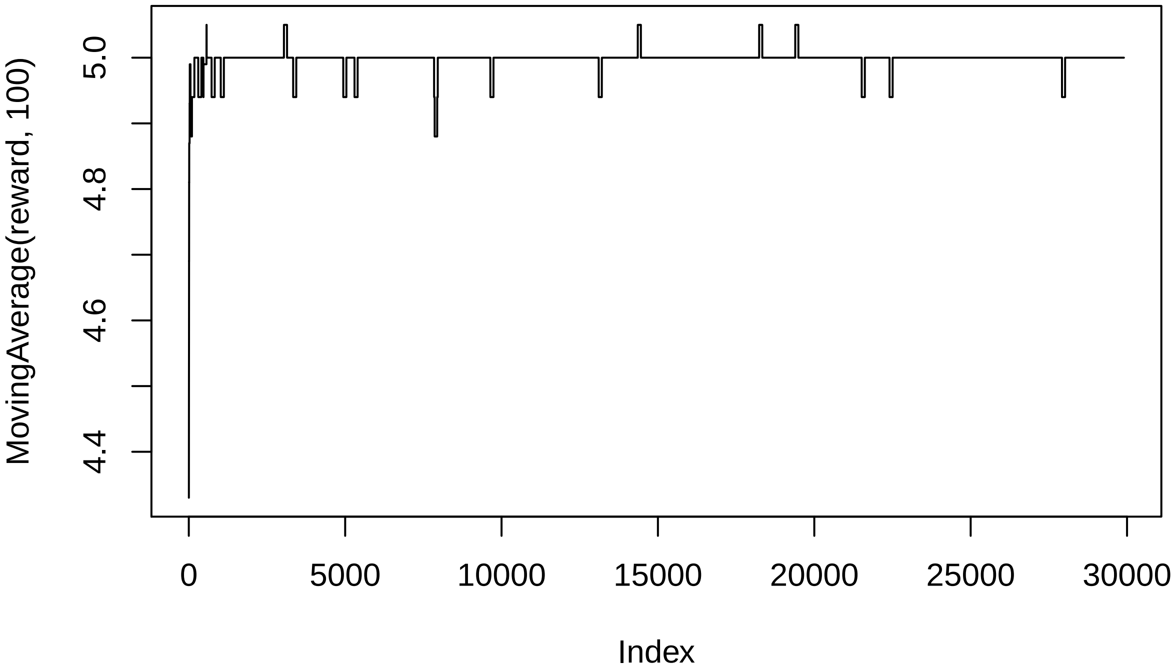
 Q−table after 30000 iterations
(epsilon = 0.1 , alpha = 0.1 gamma = 0.75 , beta = 0)
−1 −1 −1 −1 −1 −1 −1 −1
−0.47 −0.79 −0.52 −0.88 −0.41 0 0 −0.1
0.66 > 1.190.49 V 0.98.865 V 1.61.229 V 1.09.211 V 0.21.332 < 0 0 > 0 0 V 0
0.49 1.58 2.11 2.81 3.75 0 0 0
0.89 1.19 1.58 2.11 2.79 0.21 0 0
1.19 > 1.581.19 > 2.111.58 > 2.812.11 > 3.752.81 V 2.813.75 < 0.420 V 0 0 V 0
1.44 2.01 2.81 3.75 5 0.32 2.22 1
1.19 1.58 2.11 2.81 0 0
1.58 > 2.111.58 > 2.812.11 > 3.752.81 > 5 5 0 > 1.990 > 7.71 10
1.19 1.58 2.11 2.81 0 0
1.58 2.11 2.81 3.75 3.97 0 0 0
1.03 ^ 1.230.98 ^ 1.791.28 ^ 2.311.45 ^ 1.740 ^ 0 0 ^ 0 0 > 0 0 ^ 0
0.23 0.81 0.24 0.85 0 0 0 0
0 1.56 0 2.13 0 0 0 0
0.02 > 0.750.03 ^ 0 0.75 < 0 0 ^ 0 0 ^ 0 0 ^ 0 0 > 0 0 > 0
−0.19 −0.19 −0.1 0 0 −0.1 −0.1 0
−1 −1 −1 −1 −1 −1 −1 −1
x
7
6
5
4
3
2
1
y
1 2 3 4 5 6 7 8
10.0
7.5
5.0
2.5
0.0

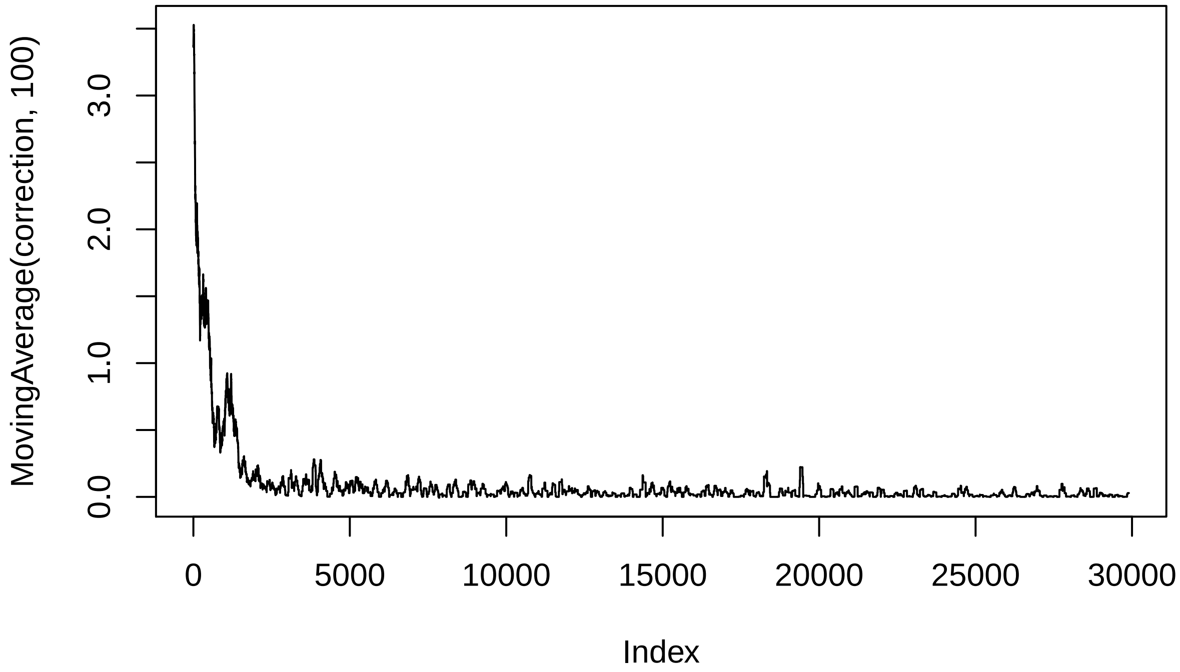

MovingAverage(reward, 100)
4.5
4.6
4.7
4.8
4.9
5.0
5.1
0
5000
10000
15000
20000
25000
30000
Index


MovingAverage(correction, 100)
0.0
0.5
1.0
1.5
2.0
2.5
0
5000
10000
15000
20000
25000
30000
Index

Q−table after 30000 iterations
(epsilon = 0.1 , alpha = 0.1 gamma = 0.95 , beta = 0)




MovingAverage(reward, 100)
4.4
4.6
4.8
5.0
0
5000
10000
15000
20000
25000
30000
Index



Epsilon Impact

When $\epsilon = 0.5$, we give equal probability to exploit and explore thus it can be seen that we can find multiple directions towards high reward stages while when we set $\epsilon = 0.1$, the system is more exploiting in nature and select path with only high rewards and thus we are not able to find multiple directions towards high reward stages which can be seen by less number of green grids when $\epsilon = 0.1$.

Gamma Impact

With different values of γ (discount factor) it can be seen that correction is low for $\gamma = 0.5$ while since it is looking for a high reward in near future only and when we increase this to $\gamma = 0.75$ we can see the higher variance in correction (when ϵ is 0.5) as now we are more interested in long term rewards than a short term one and when we increase it $\gamma = 0.95$, we are giving very very high value to future rewards and thus the system is more tuned for future rewards. Which can be seen in case of Gamma = 0.95 and Epsilon = .5 system is ignoring Reward of 5 at grid position of 4,5 as system is looking for higher reward available at grid position of 4,8.

Combined Impact of Epsilon and Gamma

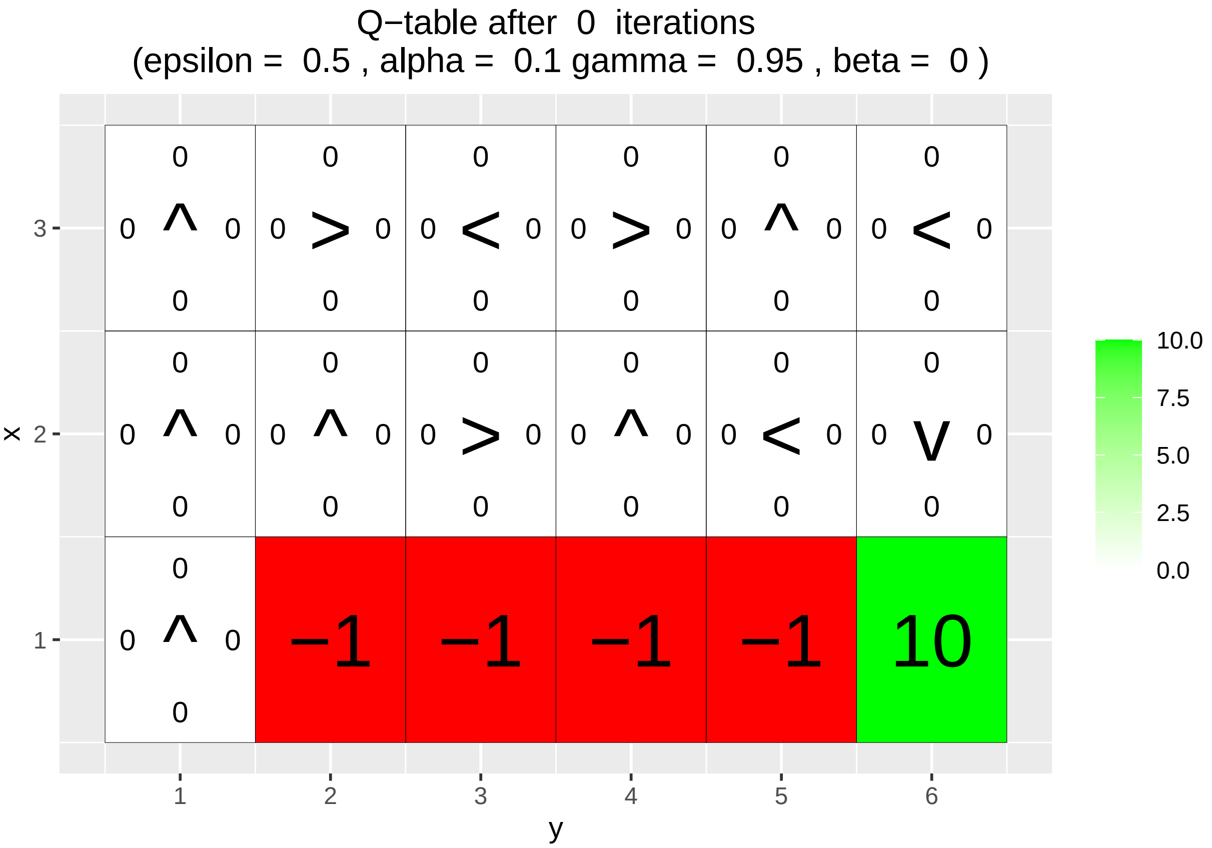
When we have ϵ as 0.5 and we keep on increasing γ, since model have balance exploration and exploitation and with every increase in γ we can see that system is giving more and more value to future rewards thus we get large number of episodes converging at max reward in grid which is 10.

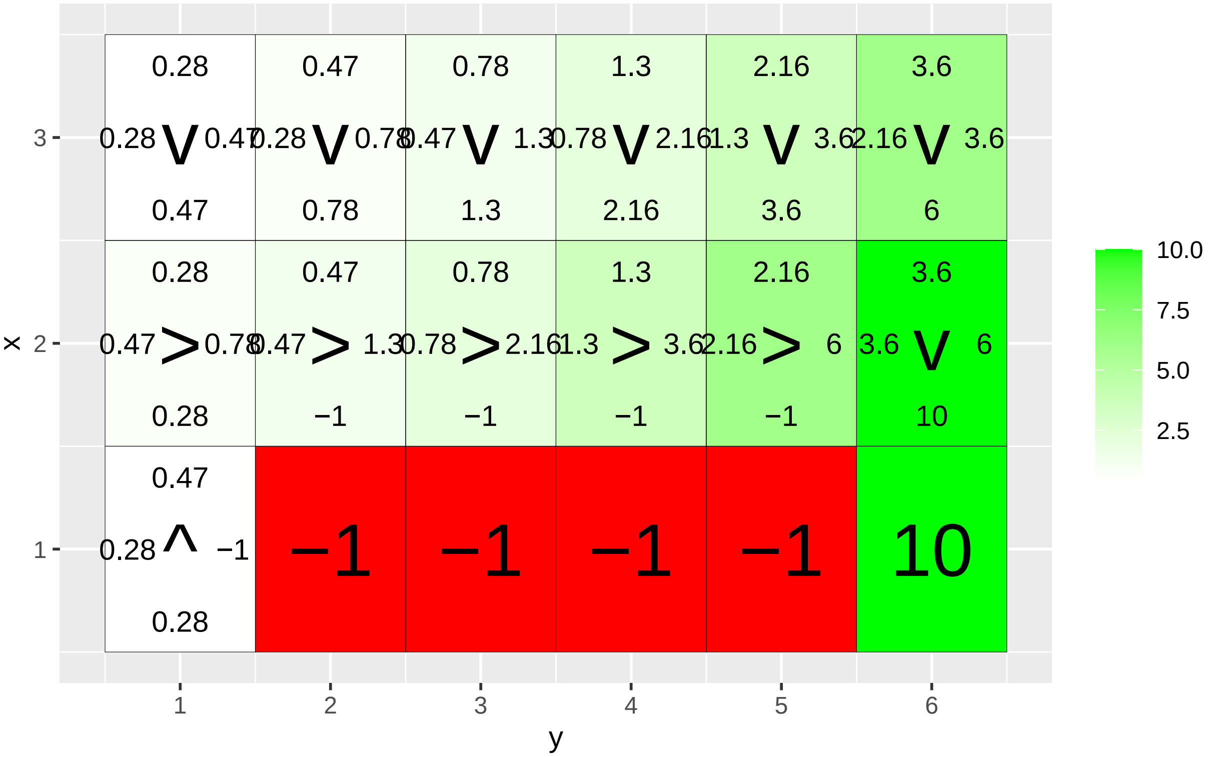
But when we have ϵ as 0.1 and even when we increase γ, system being exploiting (1-ϵ is high) and not exploring more we can see higher number of episodes converging at reward grid 5 which is not global optimal reward.

Environment C (the effect of beta)

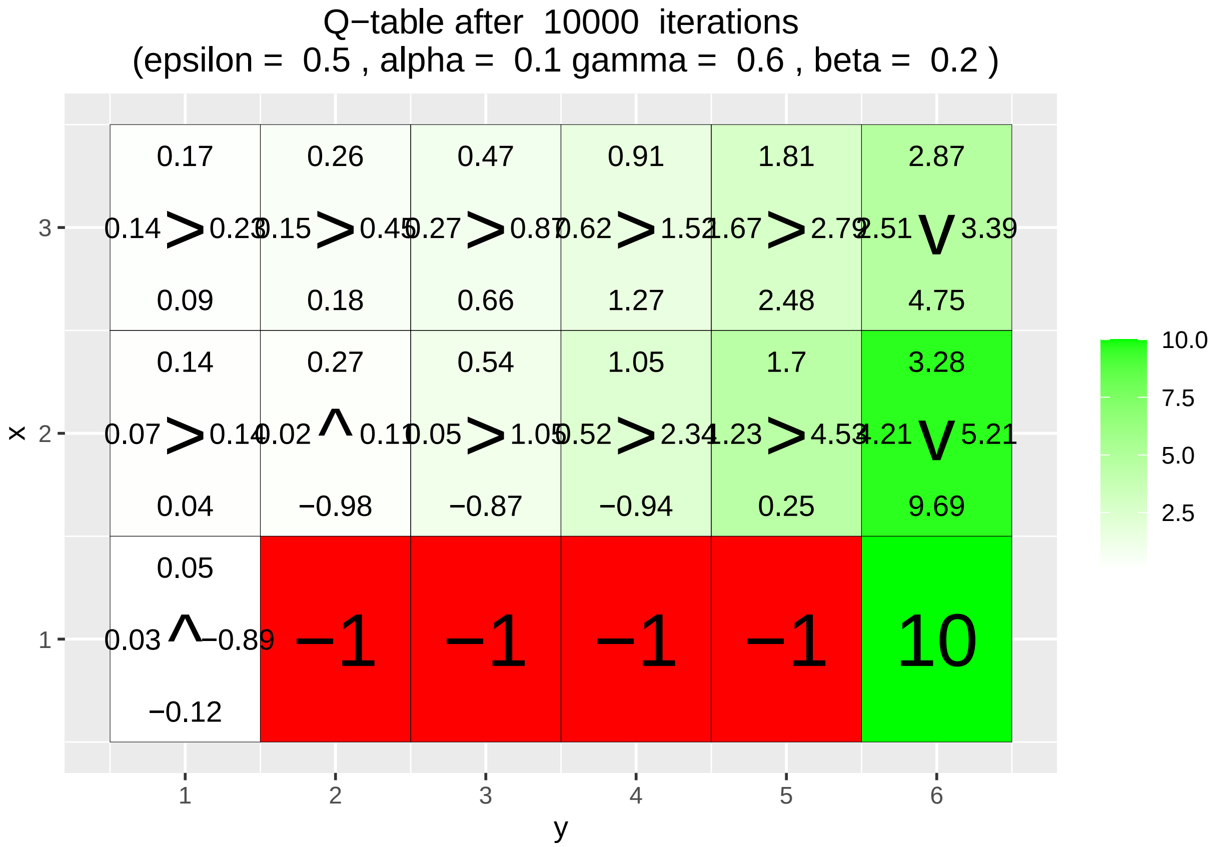
Your task is to investigate how the β parameter affects the learned policy by running 10000 episodes of Q-learning with $\beta = 0$, 0.2, 0.4, 0.66, $\epsilon = 0.5$, $\gamma = 0.6$ and $\alpha = 0.1$

Since the transition model is defined by the agent moving in the direction chosen with probability $(1-\beta)$. The agent might also slip and end up moving in the direction to the left or right of its chosen action, each with probability $\beta/2$

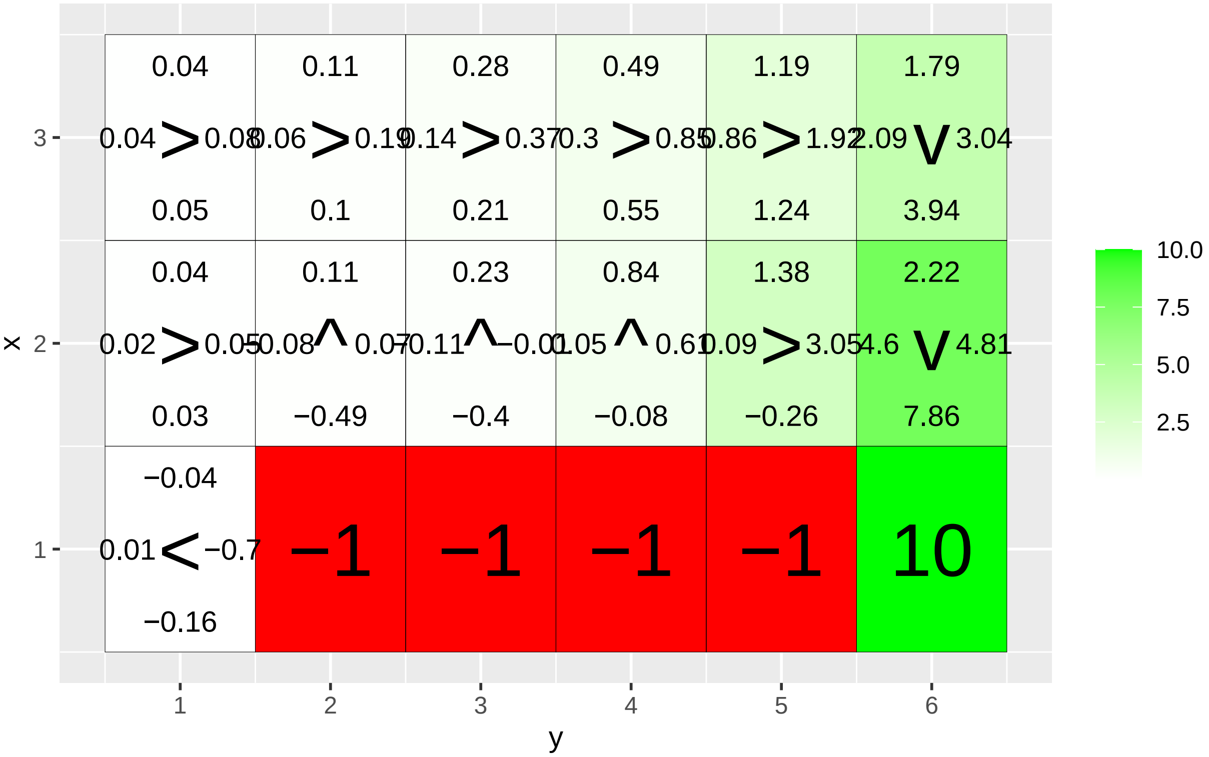


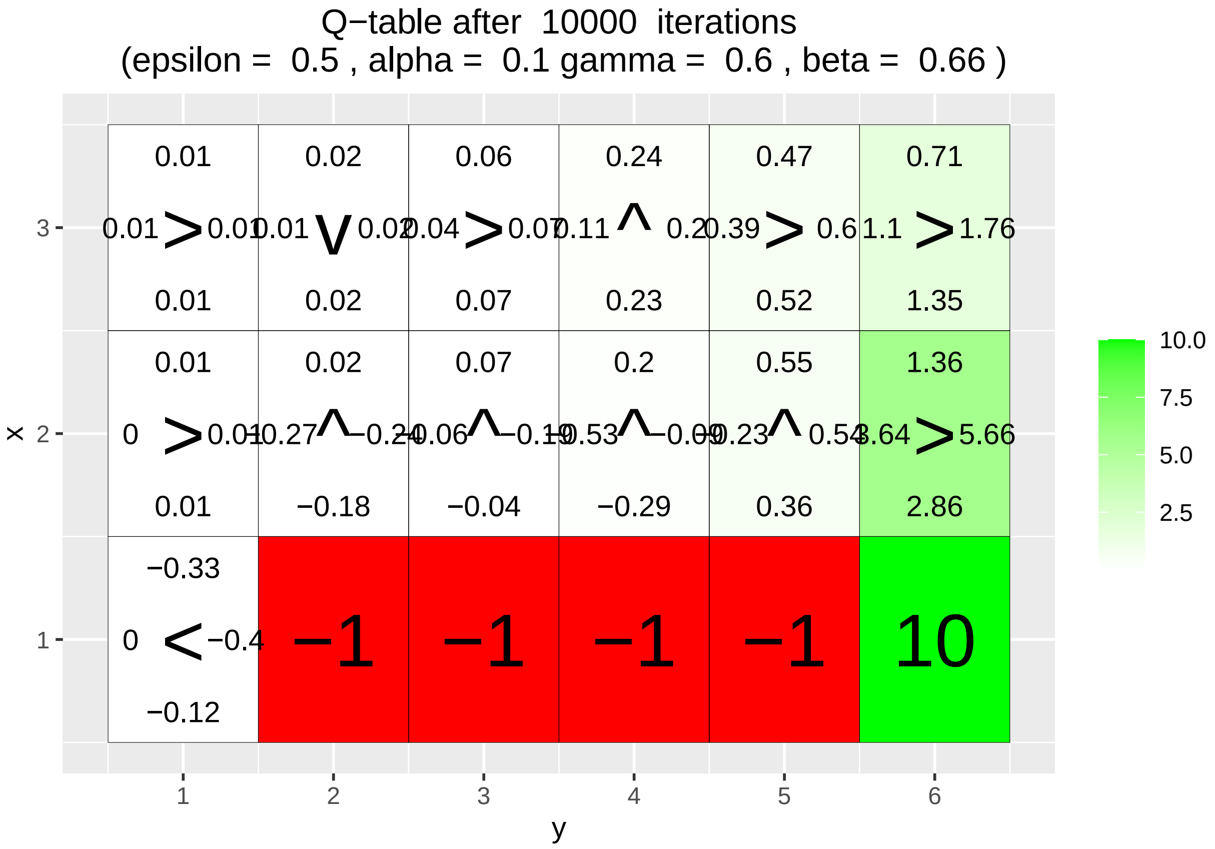


Q−table after 10000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.6 , beta = 0)


Q−table after 10000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.6 , beta = 0.2)
0.17
0.26
0.47
0.91
1.81
2.87
0.14 > 0.23
0.15 > 0.45
0.27 > 0.87
0.62 > 1.52
1.67 > 2.79
2.51 V 3.39
0.09
0.18
0.66
1.27
2.48
4.75
0.14
0.27
0.54
1.05
1.7
3.28
0.07 > 0.14
0.02 ^ 0.11
0.05 > 1.05
0.52 > 2.34
1.23 > 4.53
4.21 V 5.21
0.04
−0.98
−0.87
−0.94
0.25
9.69
0.05
0.03 ^ −0.89
−0.12
−1
−1
−1
−1
10
x
3
2
1
y
1
2
3
4
5
6
10.0
7.5
5.0
2.5

Q−table after 10000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.6 , beta = 0.4)





As we are increasing β, we can see that model will not move in chosen direction and thus its movement is getting more and more random and with β at 0.6 (as movement in chosen direction have probability of (1-β)), it will mostly move towards right or left of chosen direction thus it might get harder for agent to reach optimal state in grid.

Thus β value can be used to cover uncertainty about action taken by agent in any given environment.

To elaborate, since β is the probability of slipping, when we increase it we are increasing the uncertainty of movement in the optimal direction. When β is increased from 0 to o.66 we are most likely going to move to the left or right of the optimal direction (as the probability of moving towards optimal direction is $1 - \beta = 0.36$ now).

It can be seen that with a β value of 0.66, agent instead of trying to increase its rewards, tries to reduce the gain of additional negative reward.

This could be understood from the example of a finance point of view. In the general case, we try to maximize reward (gain) for investment but when uncertainty because of events like Covid-19 is so high, the system is trying to manage net value such that its focus is now to avoid losses(negative reward). This can be understood by the robot's decision to collide with the wall in grid position 1,1 instead of taking top or bottom direction as there was a possibility of slipping to a negative -1 reward state at 1,2.

Question 2

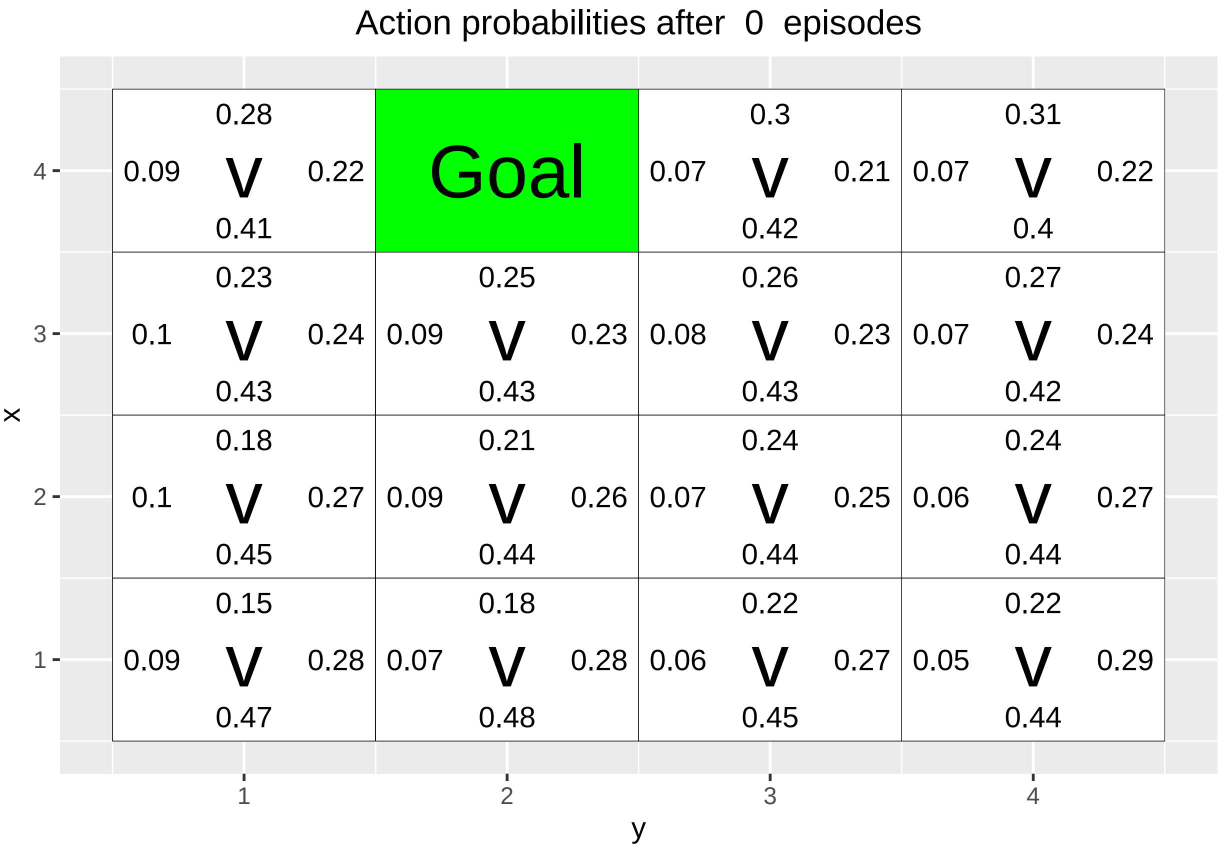
Reinforcement Learning

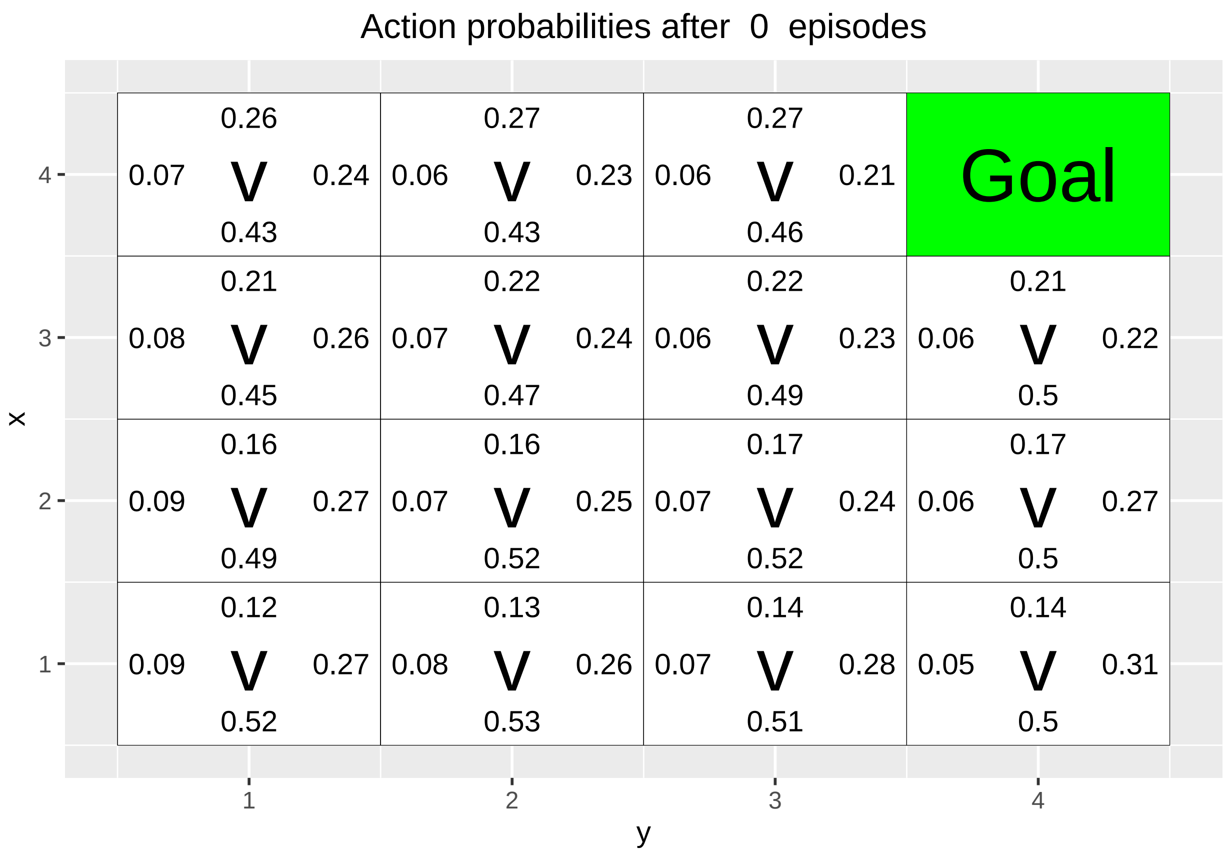
Task Setup

We will work with a 4 × 4 grid. We want the agent to learn to navigate to a random goal position in the grid. The agent will start in a random position and it will be told the goal position. The agent receives a reward of 5 when it reaches the goal. Since the goal position can be any position, we need a way to tell the agent where the goal is. Since our agent does not have any memory mechanism, we provide the goal coordinates as part of the state at every time step, i.e. a state consists now of four coordinates: Two for the position of the agent, and two for the goal position. The actions of the agent can however only impact its own position, i.e. the actions do not modify the goal position. Note that the agent initially does not know that the last two coordinates of a state indicate the position with maximal reward, i.e. the goal position. It has to learn it. It also has to learn a policy to reach the goal position from the initial position. Moreover, the policy has to depend on the goal position, because it is chosen at random in each episode. Since we only have a single non-zero reward, we do not specify a reward map. Instead, the goal coordinates are passed to the functions that need to access the reward function.

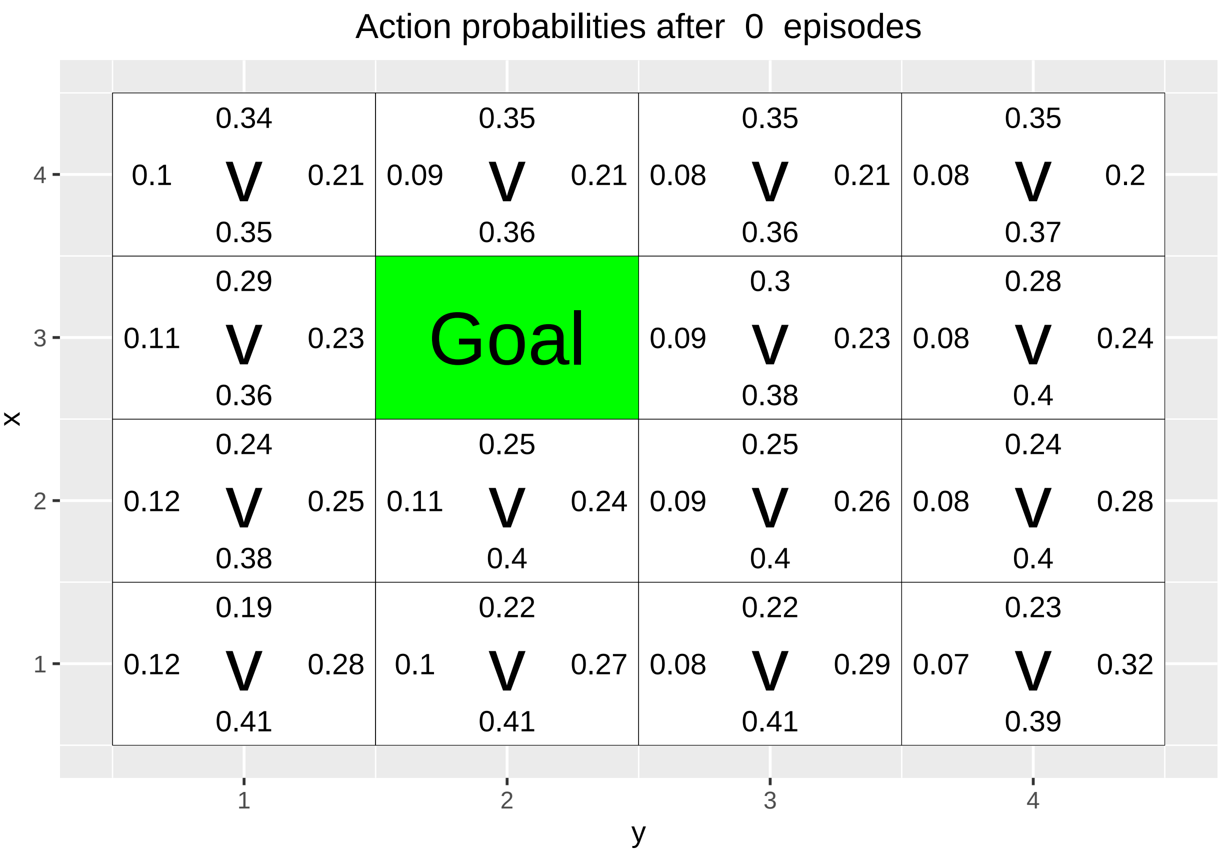
Environment D (training with random goal positions)

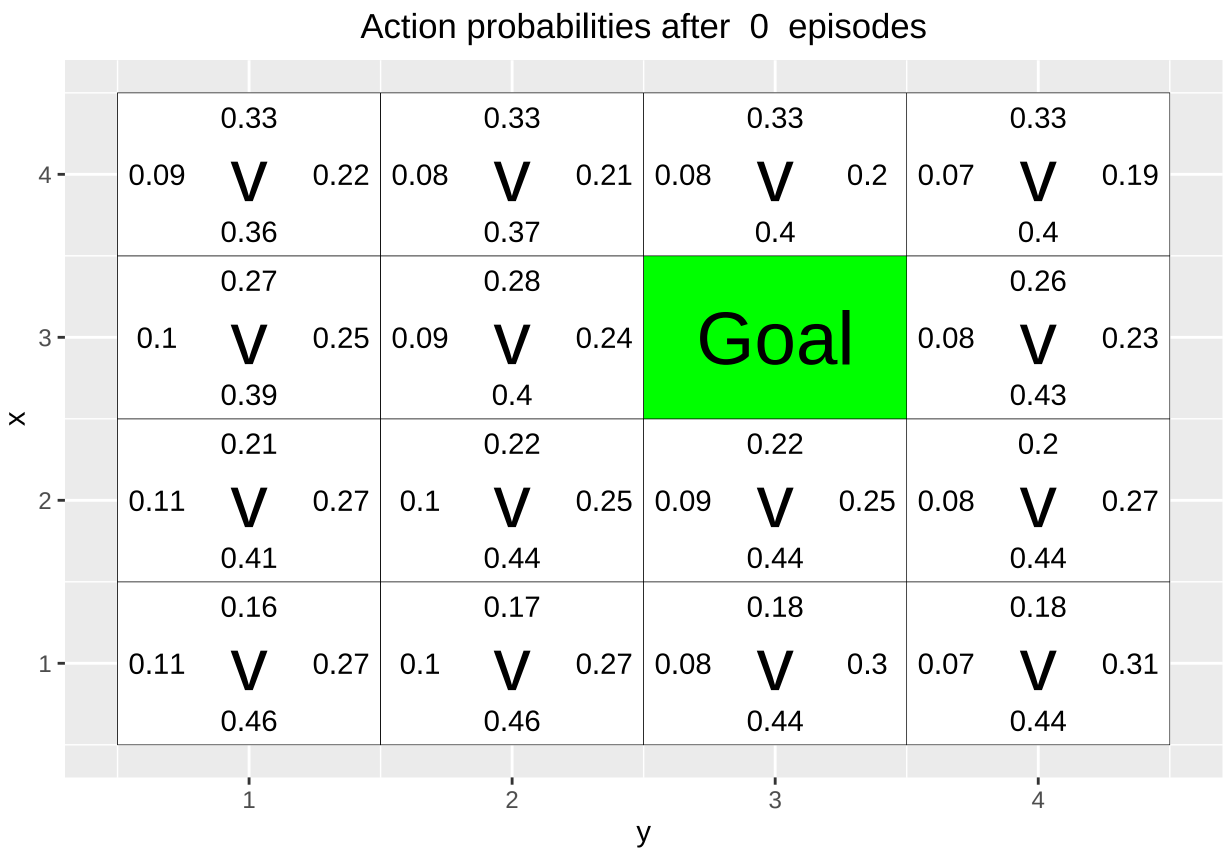
Task: In this task, we will use eight goal positions for training and, then, validate the learned policy on the remaining eight possible goal positions. The training and validation goal positions are stored in the lists train goals and val goals.

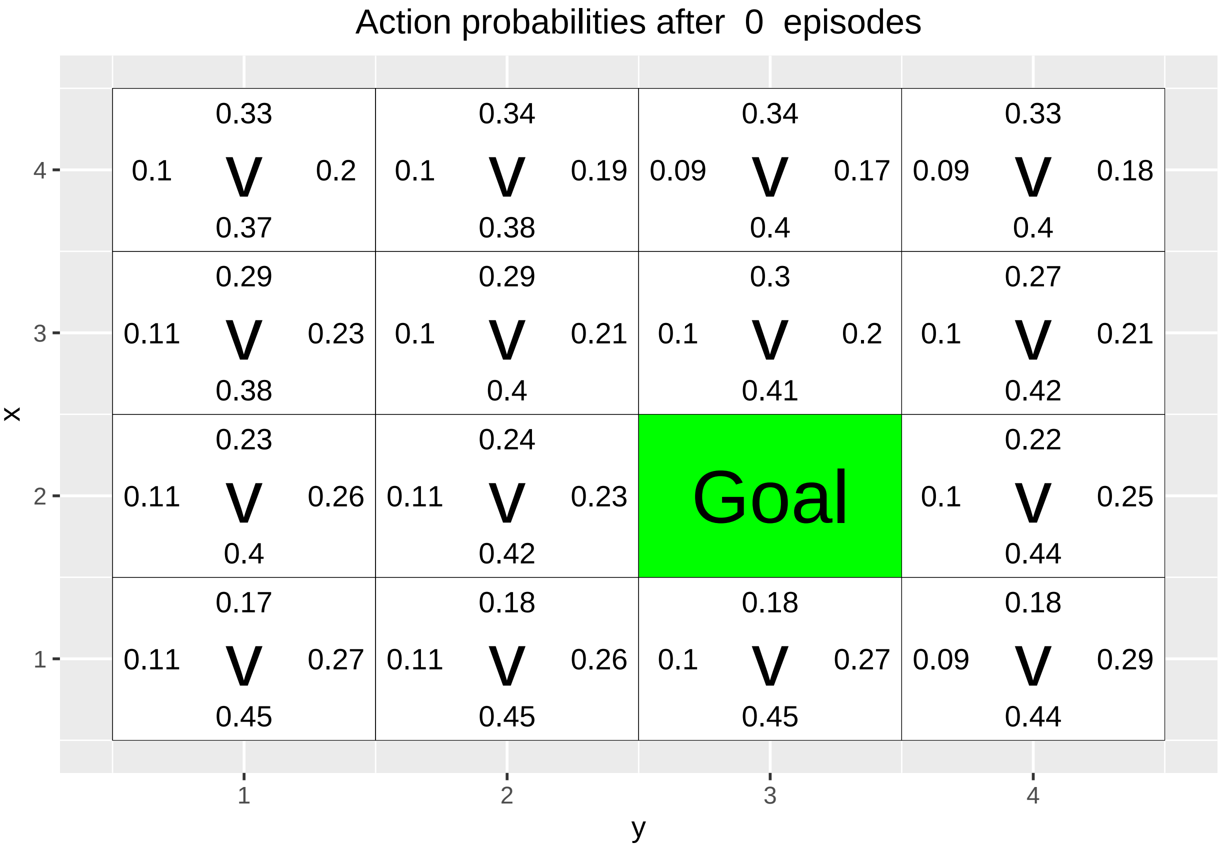


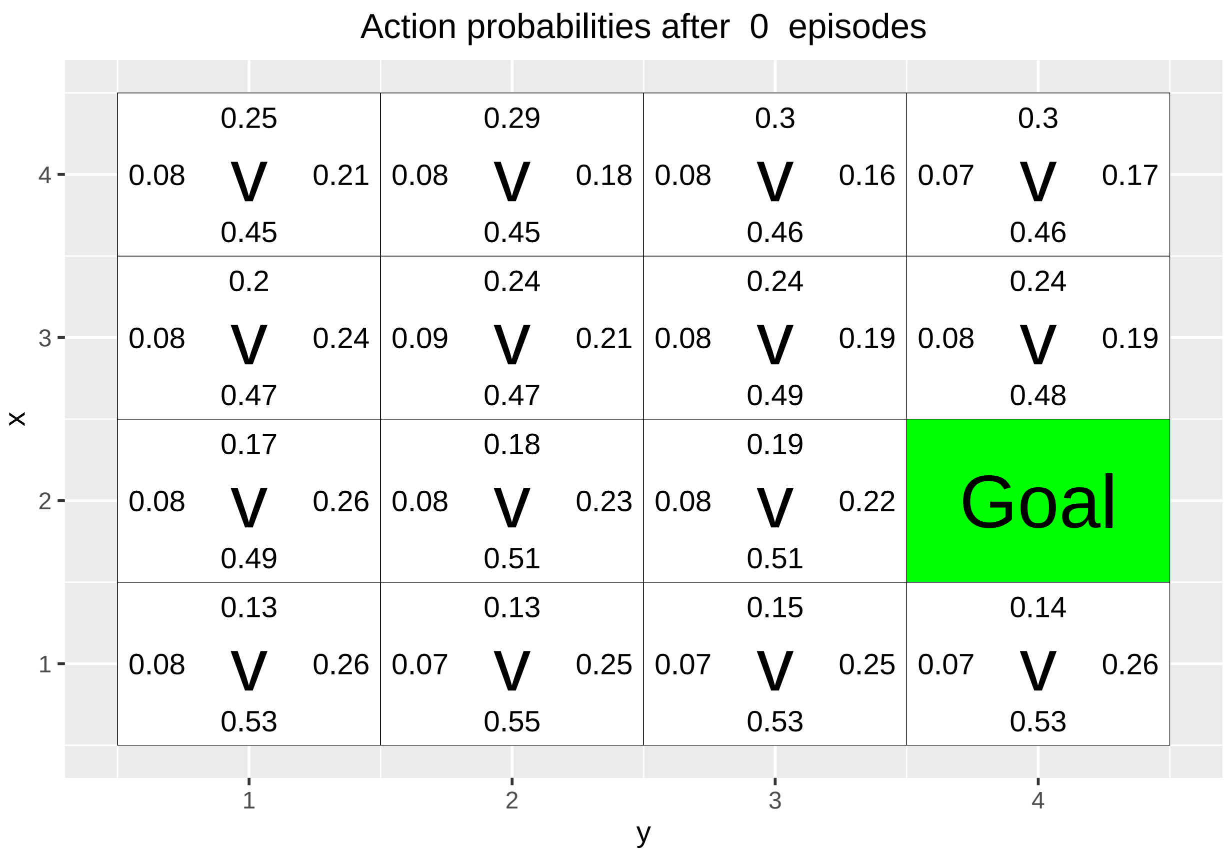


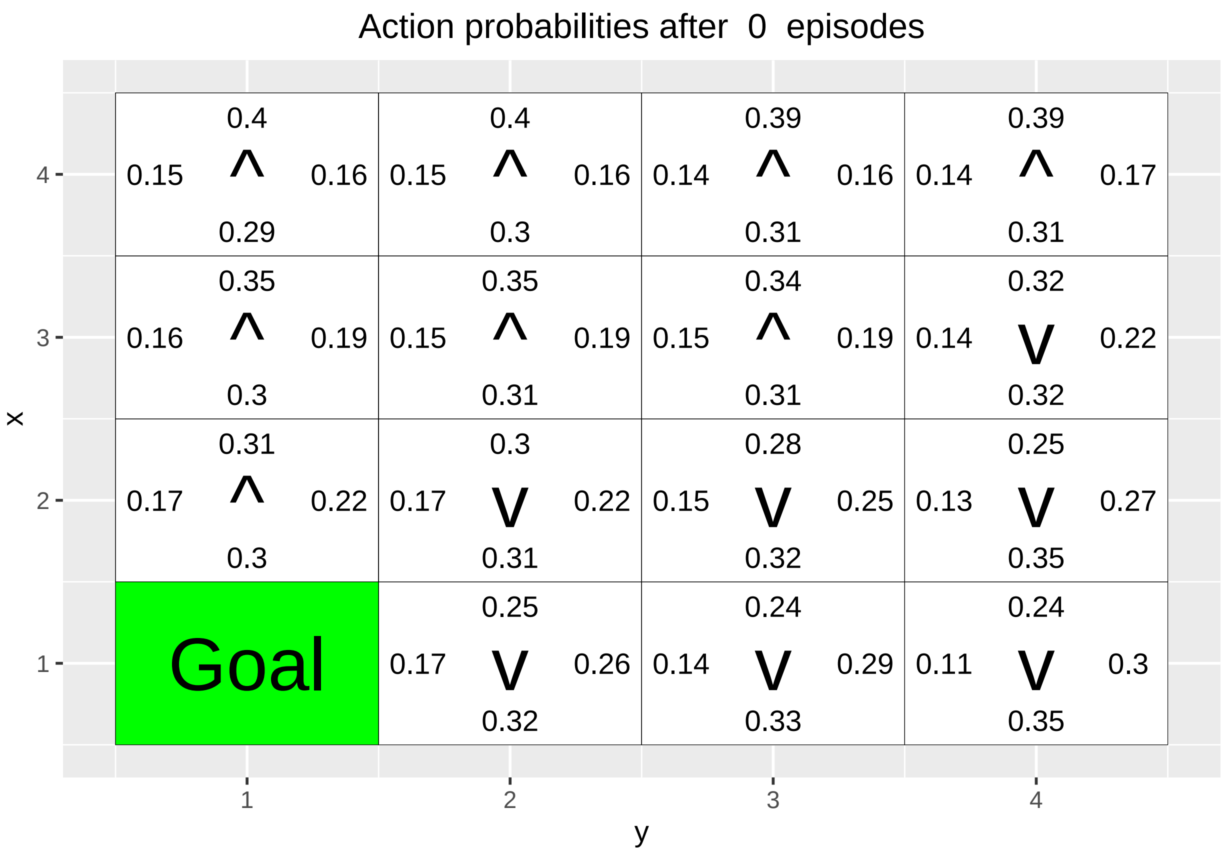
Action probabilities after 0 episodes

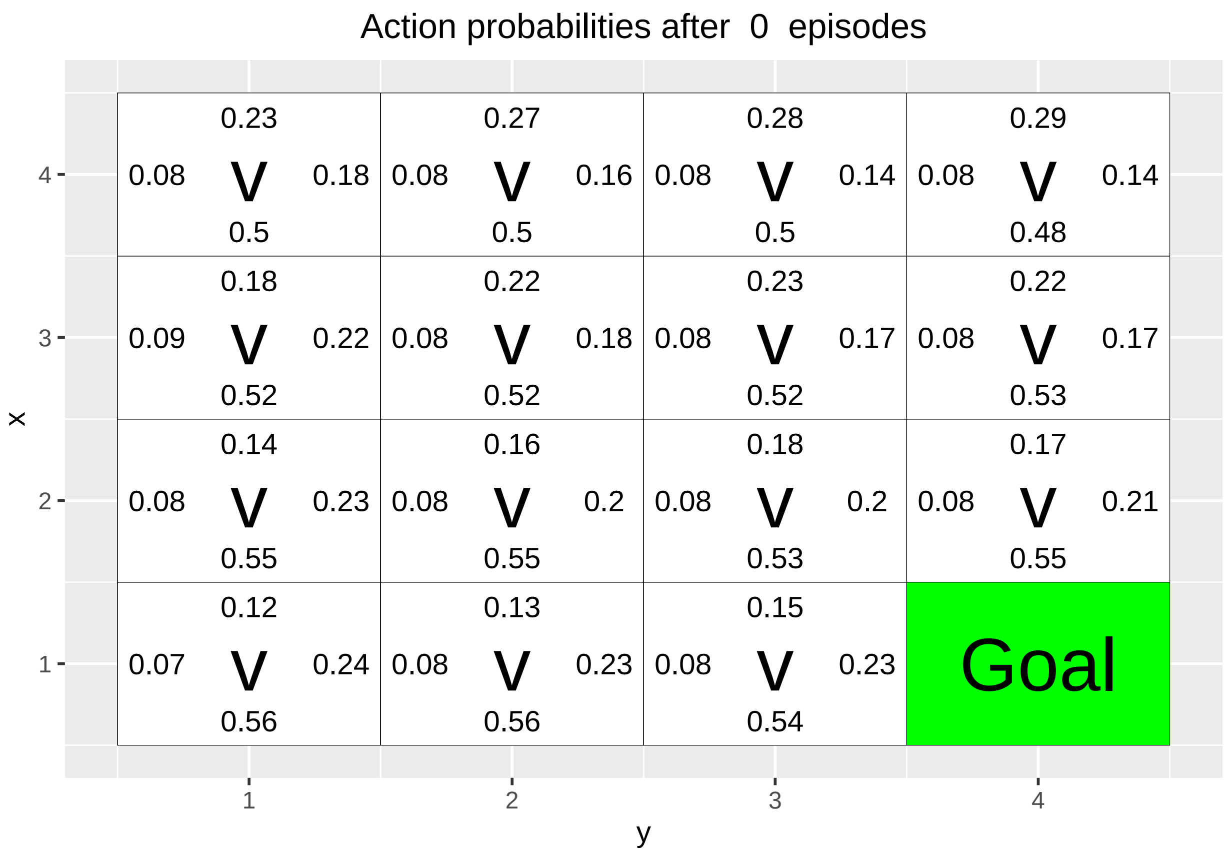

Action probabilities after 0 episodes


Action probabilities after 0 episodes
0.33
0.09 V 0.22
0.36
0.33
0.08 V 0.21
0.37
0.33
0.08 V 0.2
0.4
0.33
0.07 V 0.19
0.4
0.27
0.1 V 0.25
0.39
0.28
0.09 V 0.24
0.4
Goal
0.26
0.08 V 0.23
0.43
0.21
0.11 V 0.27
0.41
0.22
0.1 V 0.25
0.44
0.22
0.09 V 0.25
0.44
0.2
0.08 V 0.27
0.44
0.16
0.11 V 0.27
0.46
0.17
0.1 V 0.27
0.46
0.18
0.08 V 0.3
0.44
0.18
0.07 V 0.31
0.44
4
3
2
1
x
1
2
3
4
y


Action probabilities after 0 episodes
0.33 0.34 0.34 0.33
0.1 V 0.2 0.1 V 0.19 0.09 V 0.17 0.09 V 0.18
0.37 0.38 0.4 0.4
0.29 0.29 0.3 0.27
0.11 V 0.23 0.1 V 0.21 0.1 V 0.2 0.1 V 0.21
0.38 0.4 0.41 0.42
0.23 0.24 0.22
0.11 V 0.26 0.11 V 0.23 Goal 0.1 V 0.25
0.4 0.42 0.44
0.17 0.18 0.18 0.18
0.11 V 0.27 0.11 V 0.26 0.1 V 0.27 0.09 V 0.29
0.45 0.45 0.45 0.44
x
4
3
2
1
1
2
3
4
y


Action probabilities after 0 episodes
0.25
0.08 V 0.21
0.45
0.29
0.08 V 0.18
0.45
0.3
0.08 V 0.16
0.46
0.3
0.07 V 0.17
0.46
0.2
0.08 V 0.24
0.47
0.24
0.09 V 0.21
0.47
0.24
0.08 V 0.19
0.49
0.24
0.08 V 0.19
0.48
0.17
0.08 V 0.26
0.49
0.18
0.08 V 0.23
0.51
0.19
0.08 V 0.22
0.51
Goal
0.13
0.08 V 0.26
0.53
0.13
0.07 V 0.25
0.55
0.15
0.07 V 0.25
0.53
0.14
0.07 V 0.26
0.53
x
4
3
2
1
1
2
3
4
y


Action probabilities after 0 episodes
4
0.4
0.15 ^ 0.16
0.29
0.4
0.15 ^ 0.16
0.3
0.39
0.14 ^ 0.16
0.31
0.39
0.14 ^ 0.17
0.31
3
0.35
0.16 ^ 0.19
0.3
0.35
0.15 ^ 0.19
0.31
0.34
0.15 ^ 0.19
0.31
0.32
0.14 V 0.22
0.32
2
0.31
0.17 ^ 0.22
0.3
0.3
0.17 V 0.22
0.31
0.28
0.15 V 0.25
0.32
0.25
0.13 V 0.27
0.35
1
Goal
0.25
0.17 V 0.26
0.32
0.24
0.14 V 0.29
0.33
0.24
0.11 V 0.3
0.35
x
1
2
3
4
y



```
## episode 10
## episode 20
## episode 30
## episode 40
## episode 50
## episode 60
## episode 70
## episode 80
## episode 90
## episode 100
## episode 110
## episode 120
## episode 130
## episode 140
## episode 150
## episode 160
## episode 170
## episode 180
## episode 190
## episode 200
## episode 210
## episode 220
## episode 230
## episode 240
## episode 250
## episode 260
## episode 270
## episode 280
## episode 290
## episode 300
## episode 310
## episode 320
## episode 330
## episode 340
## episode 350
## episode 360
## episode 370
## episode 380
## episode 390
## episode 400
## episode 410
## episode 420
## episode 430
## episode 440
## episode 450
## episode 460
## episode 470
## episode 480
## episode 490
## episode 500
## episode 510
## episode 520
## episode 530
## episode 540
## episode 550
## episode 560
## episode 570
## episode 580
## episode 590
## episode 600
## episode 610
## episode 620
## episode 630
## episode 640
## episode 650
## episode 660
## episode 670
## episode 680
## episode 690
## episode 700
## episode 710
## episode 720
## episode 730
## episode 740
## episode 750
## episode 760
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## episode 780
## episode 790
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## episode 820
## episode 830
## episode 840
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## episode 860
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## episode 1070
## episode 1080
## episode 1090
## episode 1100
## episode 1110
## episode 1120
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## episode 1170
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## episode 1190
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## episode 1360
## episode 1370
## episode 1380
## episode 1390
## episode 1400
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## episode 1450
## episode 1460
## episode 1470
## episode 1480
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## episode 1500
## episode 1510
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## episode 1570
## episode 1580
## episode 1590
## episode 1600
## episode 1610
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## episode 1880
## episode 1890
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## episode 2350
## episode 2360
## episode 2370
## episode 2380
## episode 2390
## episode 2400
## episode 2410
## episode 2420
## episode 2430
## episode 2440
## episode 2450
## episode 2460
## episode 2470
## episode 2480
## episode 2490
## episode 2500
## episode 2510
## episode 2520
## episode 2530
## episode 2540
## episode 2550
## episode 2560
## episode 2570
## episode 2580
## episode 2590
## episode 2600
## episode 2610
## episode 2620
## episode 2630
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## episode 2660
## episode 2670
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## episode 2690
## episode 2700
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## episode 2720
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## episode 2770
## episode 2780
## episode 2790
## episode 2800
## episode 2810
## episode 2820
## episode 2830
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## episode 2850
## episode 2860
## episode 2870
## episode 2880
## episode 2890
## episode 2900
## episode 2910
## episode 2920
## episode 2930
## episode 2940
## episode 2950
## episode 2960
## episode 2970
## episode 2980
## episode 2990
## episode 3000
## episode 3010
## episode 3020
## episode 3030
## episode 3040
## episode 3050
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## episode 3100
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## episode 3340
## episode 3350
## episode 3360
## episode 3370
## episode 3380
## episode 3390
## episode 3400
## episode 3410
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## episode 3430
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## episode 3580
## episode 3590
## episode 3600
## episode 3610
## episode 3620
## episode 3630
## episode 3640
## episode 3650
## episode 3660
## episode 3670
## episode 3680
## episode 3690
## episode 3700
## episode 3710
## episode 3720
## episode 3730
## episode 3740
## episode 3750
## episode 3760
## episode 3770
## episode 3780
## episode 3790
## episode 3800
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## episode 3950
## episode 3960
## episode 3970
## episode 3980
## episode 3990
## episode 4000
## episode 4010
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## episode 4890
## episode 4900
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## episode 4920
## episode 4930
## episode 4940
## episode 4950
## episode 4960
## episode 4970
## episode 4980
## episode 4990
## episode 5000
```
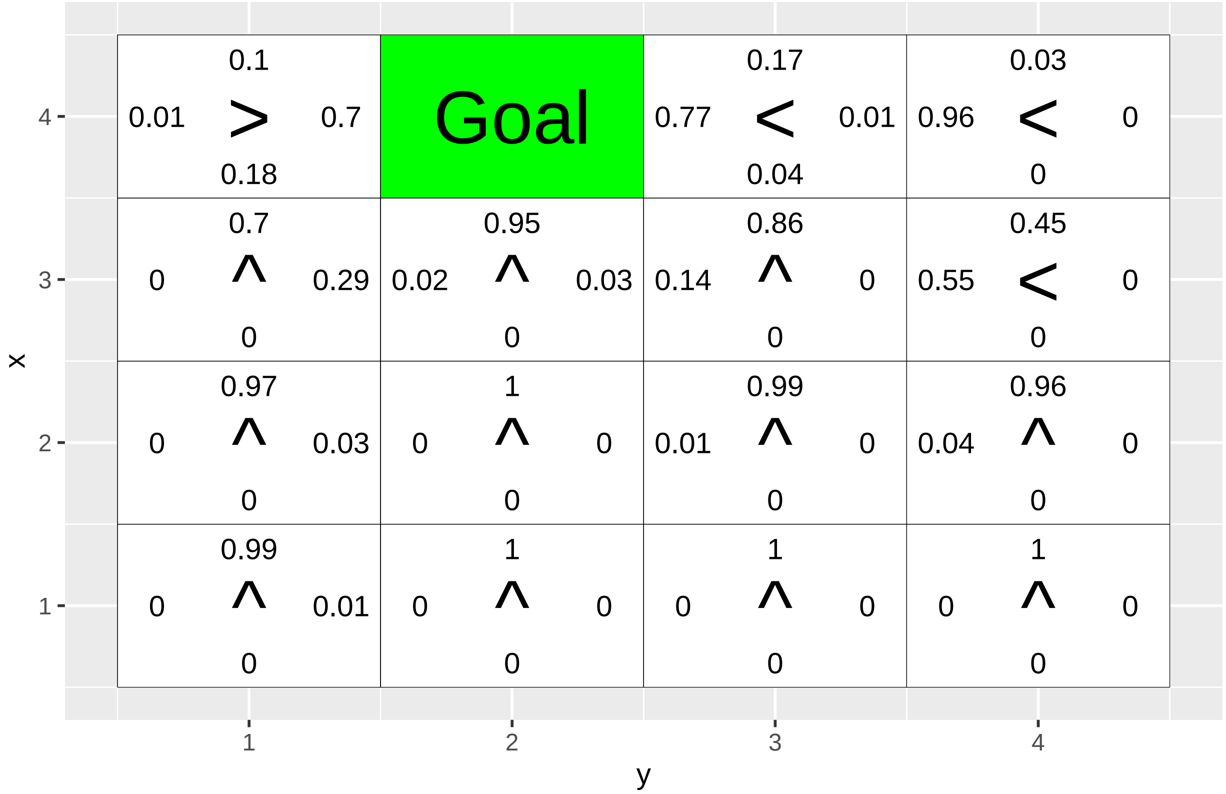
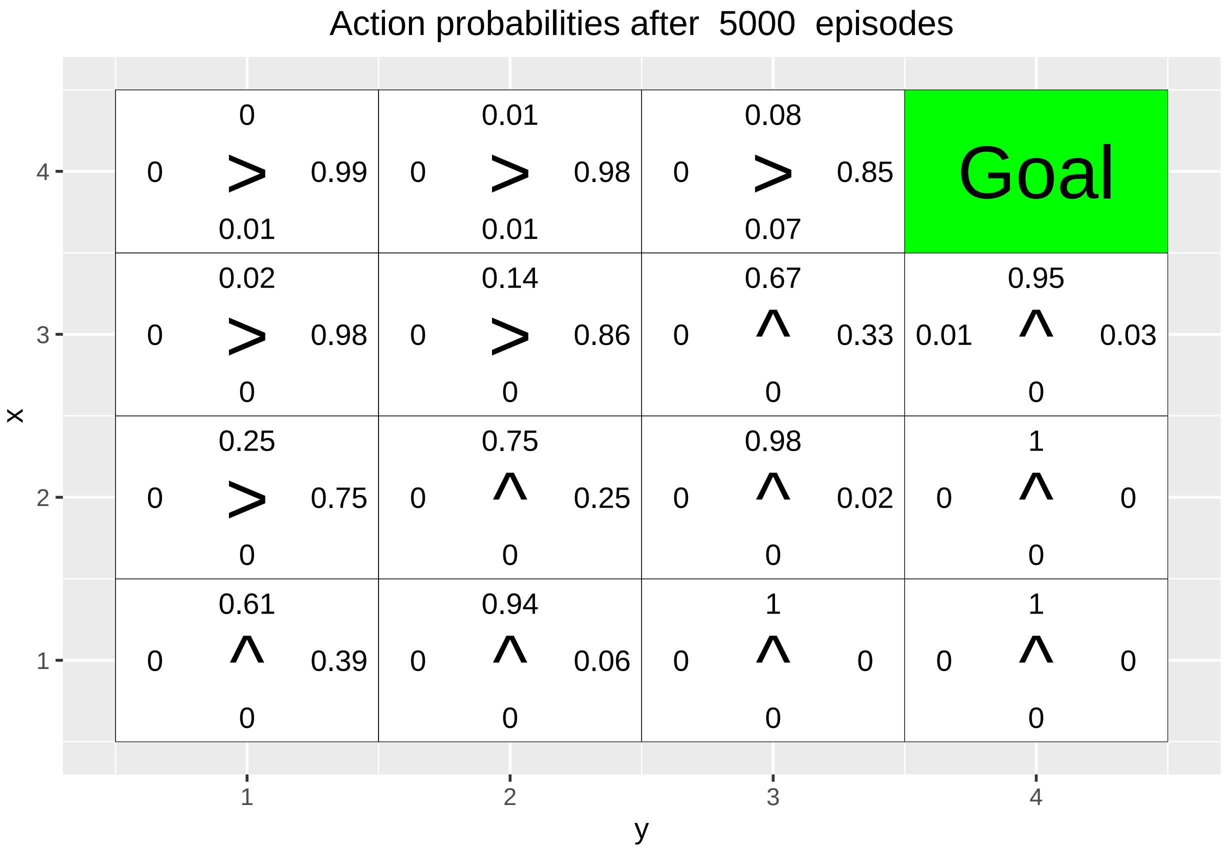
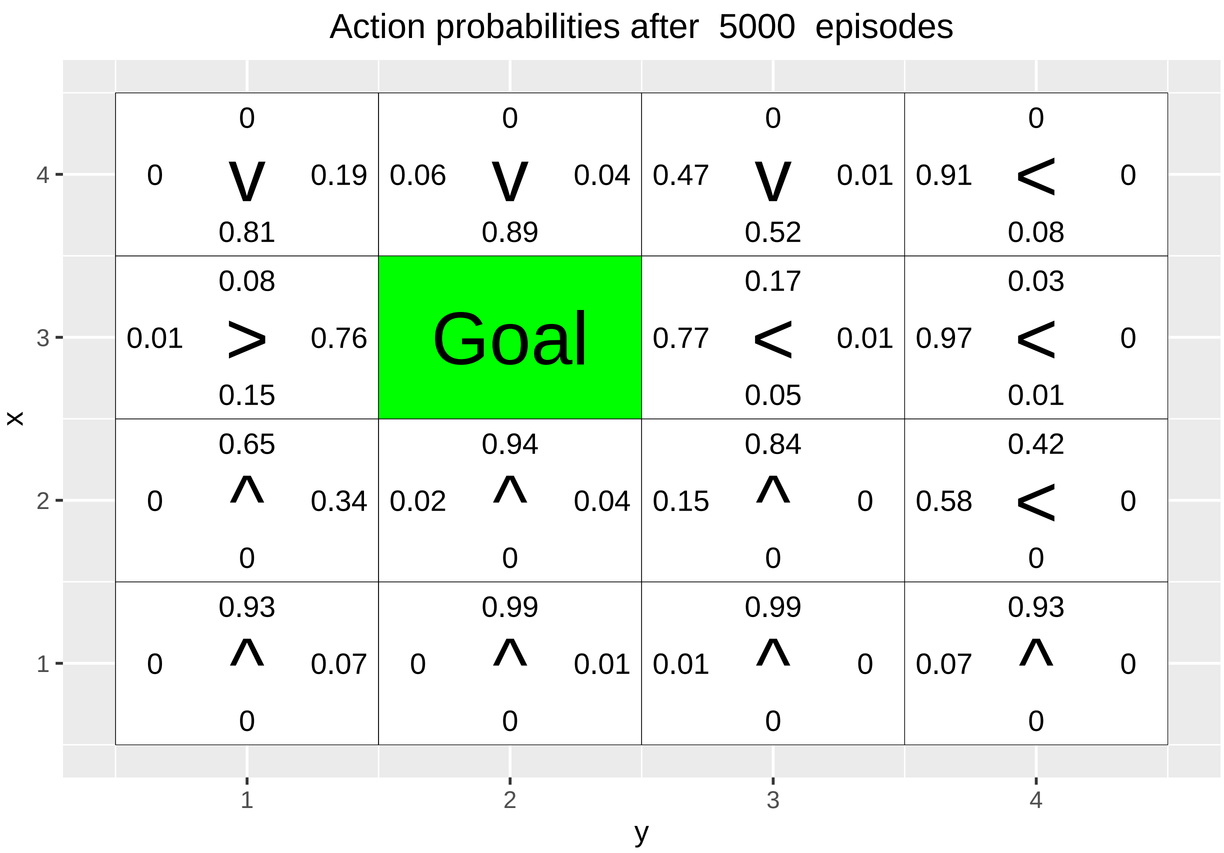
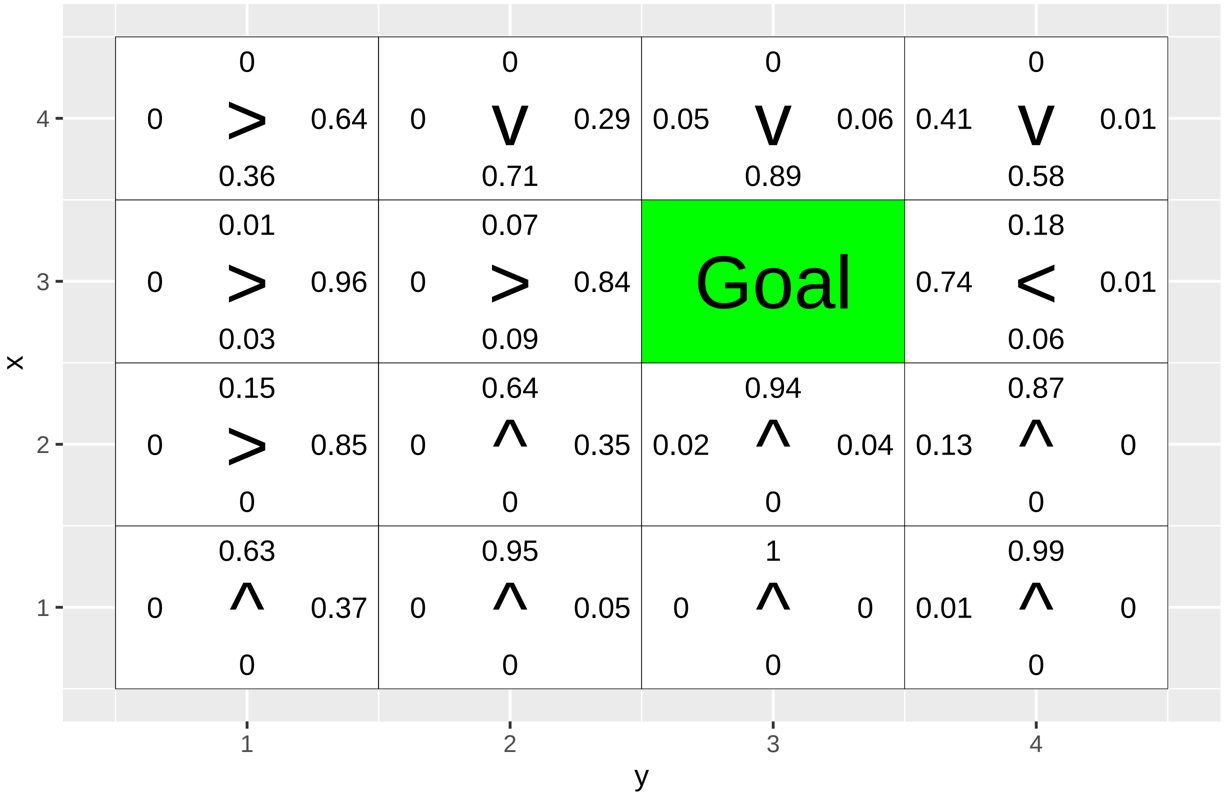



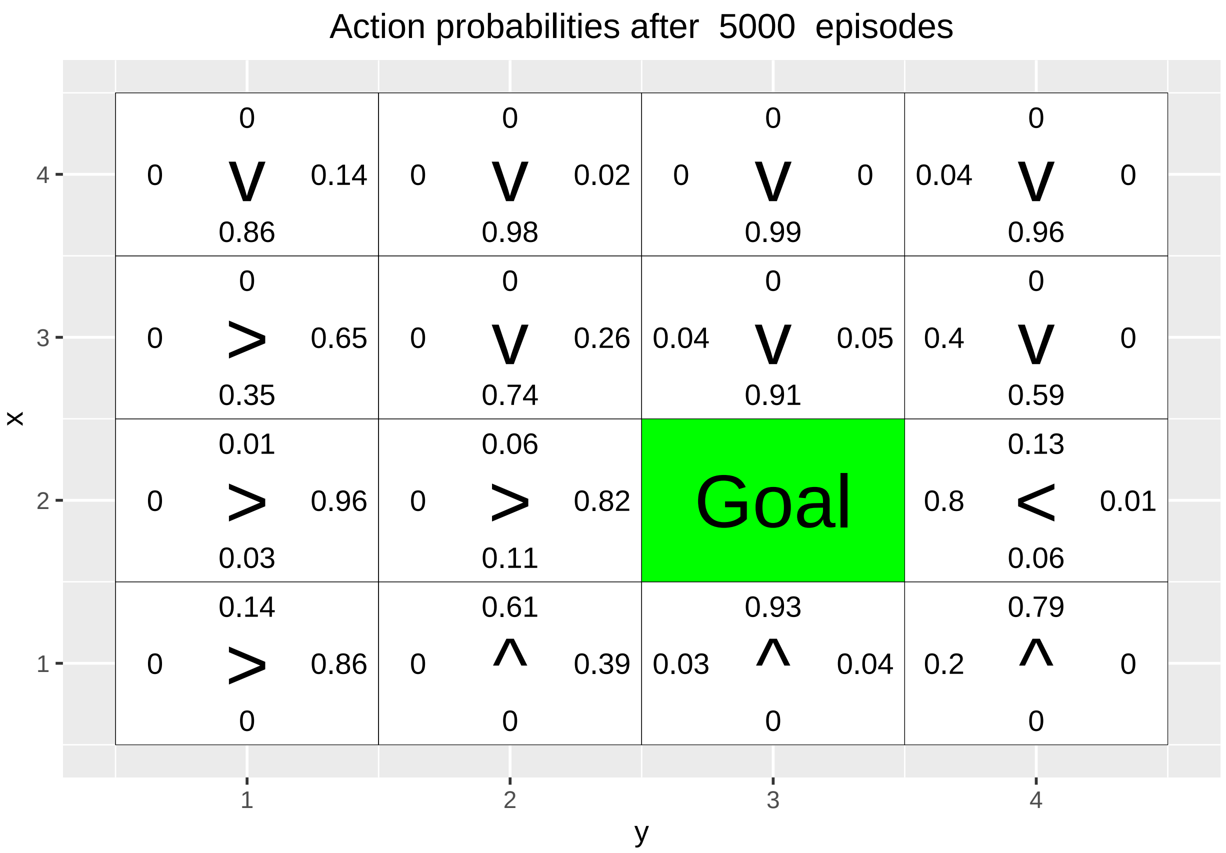
Action probabilities after 5000 episodes

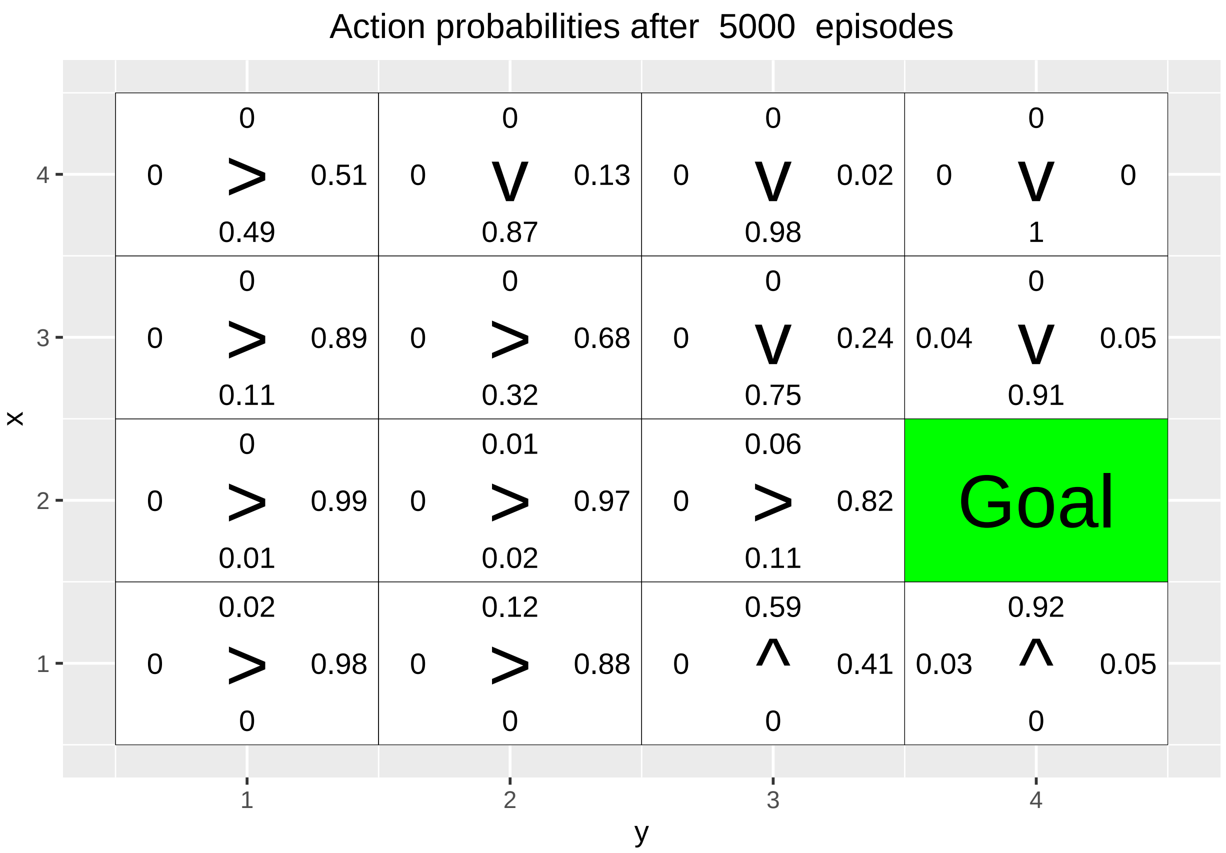

Action probabilities after 5000 episodes
x
y
1
2
3
4
1
2
3
4
0
0 > 0.99
0.01
0.01
0 > 0.98
0.01
0.08
0 > 0.85
0.07
Goal
0.02
0 > 0.98
0
0.14
0 > 0.86
0
0.67
0 ^ 0.33
0
0.95
0.01 ^ 0.03
0
0.25
0 > 0.75
0
0.75
0 ^ 0.25
0
0.98
0 ^ 0.02
0
1
0 ^ 0
0
0.61
0 ^ 0.39
0
0.94
0 ^ 0.06
0
1
0 ^ 0
0
1
0 ^ 0
0

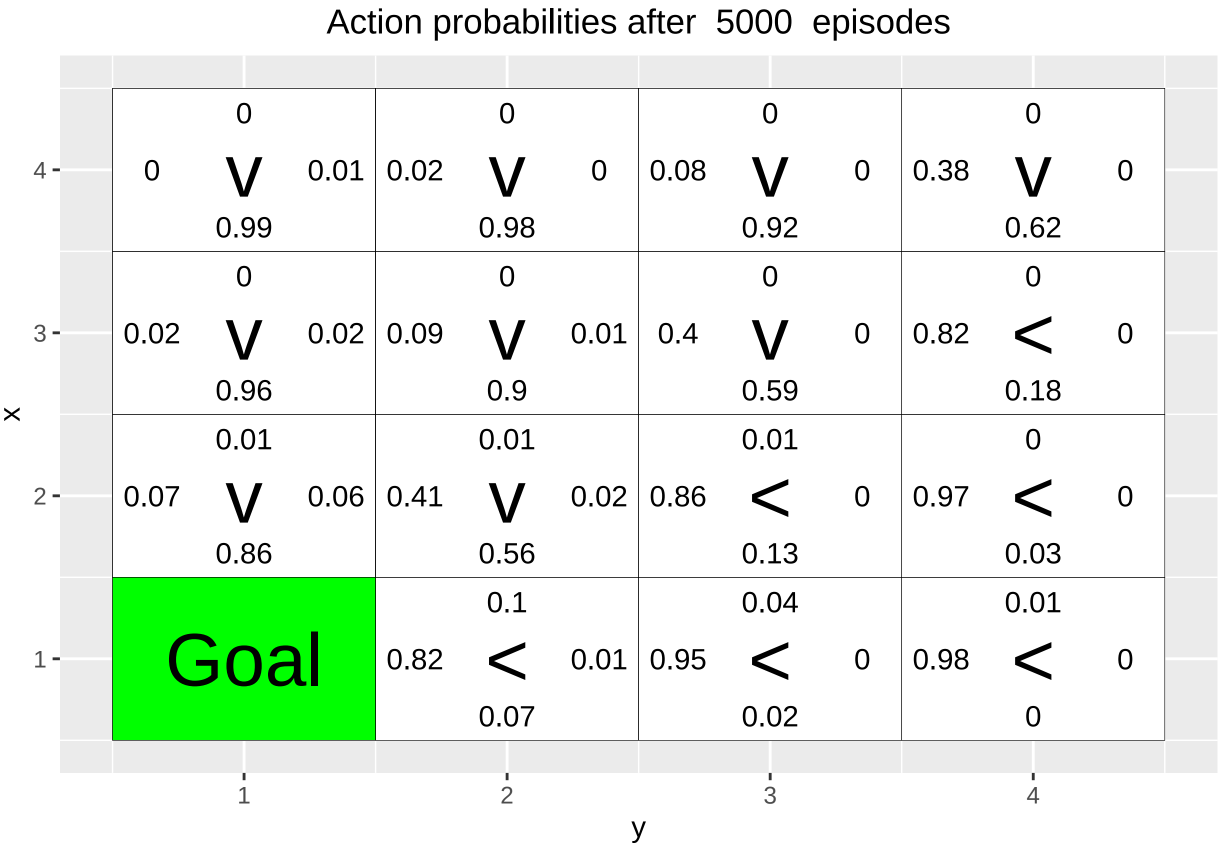

Action probabilities after 5000 episodes
0
0 V 0.19
0.81
0
0.06 V 0.04
0.89
0
0.47 V 0.01
0.52
0
0.91 < 0
0.08
0.08
0.01 > 0.76
0.15
Goal
0.17
0.77 < 0.01
0.05
0.03
0.97 < 0
0.01
0.65
0 ^ 0.34
0
0.94
0.02 ^ 0.04
0
0.84
0.15 ^ 0
0
0.42
0.58 < 0
0
0.93
0 ^ 0.07
0
0.99
0 ^ 0.01
0
0.99
0.01 ^ 0
0
0.93
0.07 ^ 0
0
x
4
3
2
1
1
2
3
4
y



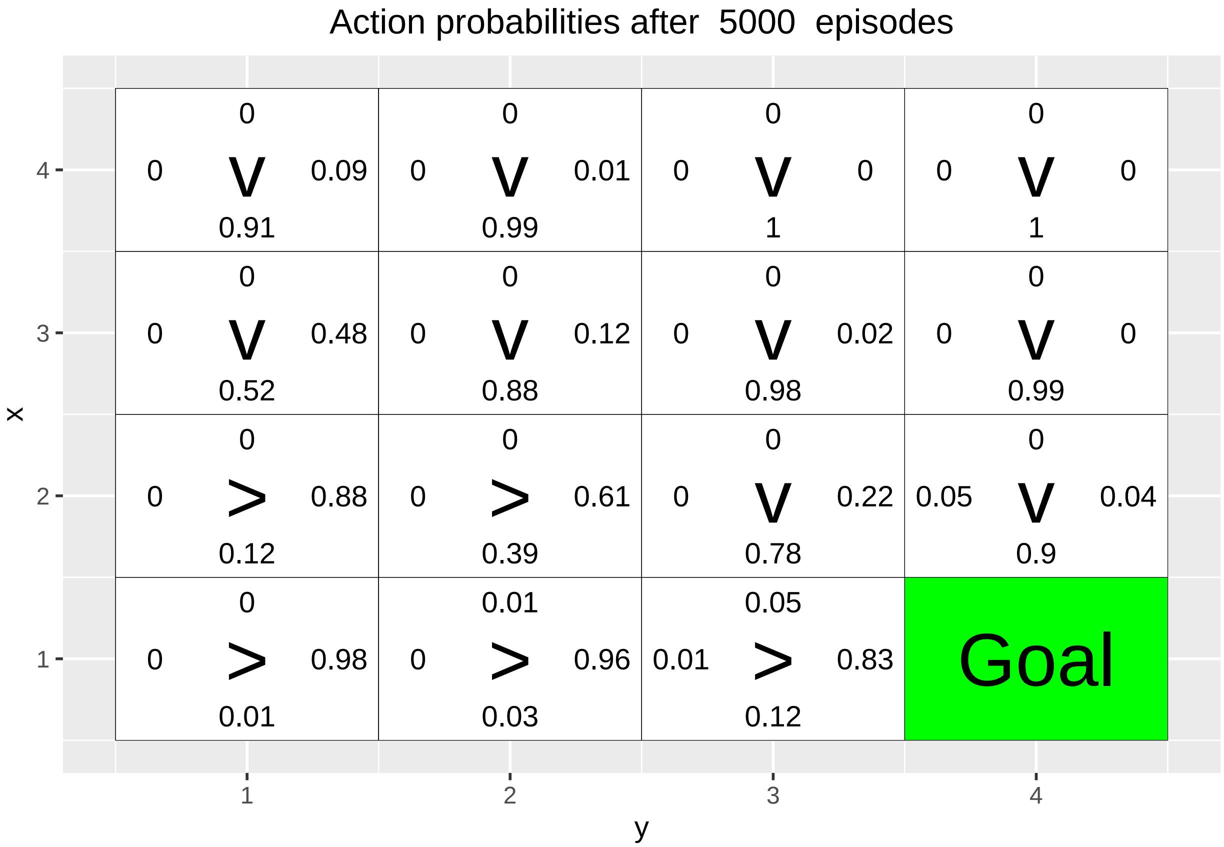
Action probabilities after 5000 episodes


Action probabilities after 5000 episodes
0
0 V 0.14
0.86
0
0 V 0.02
0.98
0
0 V 0
0.99
0
0.04 V 0
0.96
0
0 > 0.65
0.35
0
0 V 0.26
0.74
0
0.04 V 0.05
0.91
0
0.4 V 0
0.59
0.01
0 > 0.96
0.03
0.06
0 > 0.82
0.11
Goal
0.13
0.8 < 0.01
0.06
0.14
0 > 0.86
0
0.61
0 ^ 0.39
0
0.93
0.03 ^ 0.04
0
0.79
0.2 ^ 0
0
x
4
3
2
1
y
1
2
3
4


Action probabilities after 5000 episodes
x
4
3
2
1
y
1
2
3
4
0
0 > 0.51
0.49
0
0 V 0.13
0.87
0
0 V 0.02
0.98
0
0 V 0
1
0
0 > 0.89
0.11
0
0 > 0.68
0.32
0
0 V 0.24
0.75
0
0.04 V 0.05
0.91
0
0 > 0.99
0.01
0.01
0 > 0.97
0.02
0.06
0 > 0.82
0.11
Goal
0.02
0 > 0.98
0
0.12
0 > 0.88
0
0.59
0 ^ 0.41
0
0.92
0.03 ^ 0.05
0



Action probabilities after 5000 episodes



Has the agent learned a good policy? Why / Why not ?

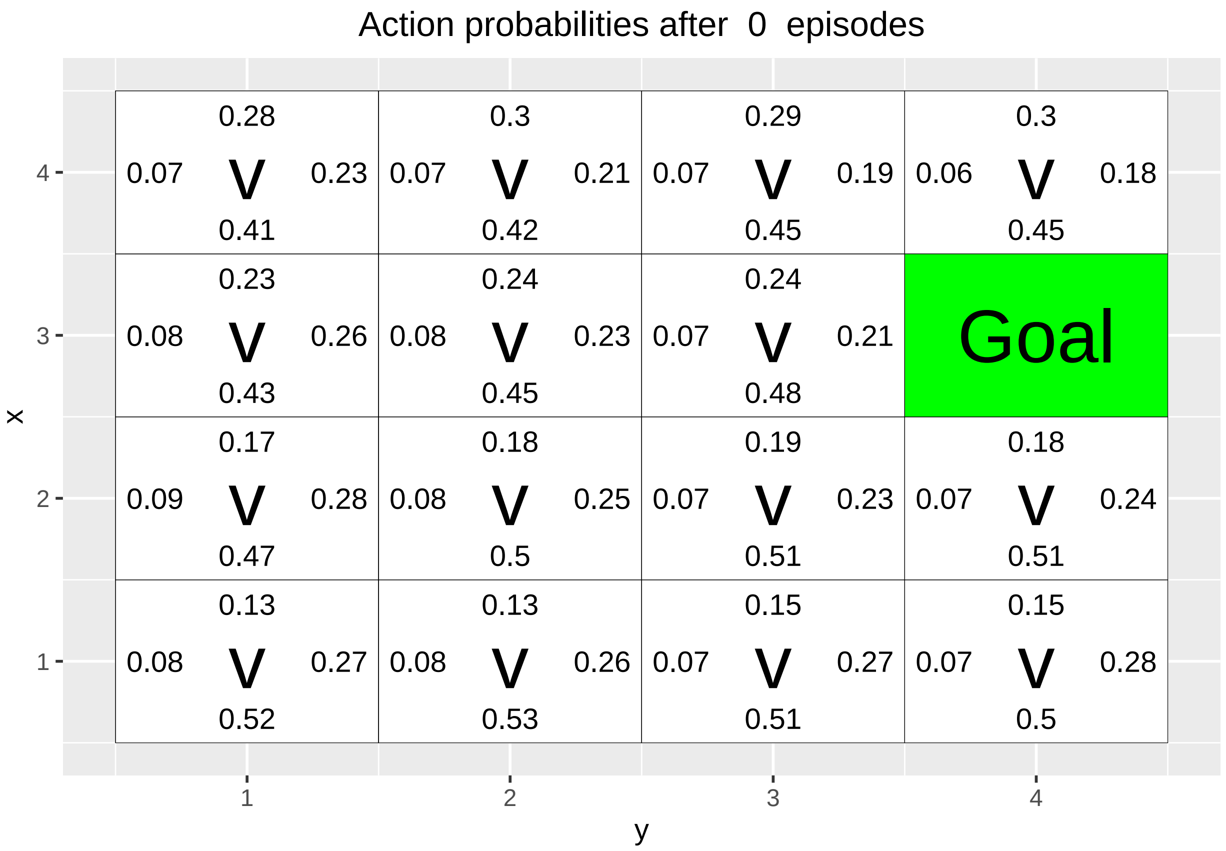
It can be seen that the agent is able to learn the good policy as it had access to all grid positions (as training data) and thus it is able to generalize and learn good policy.

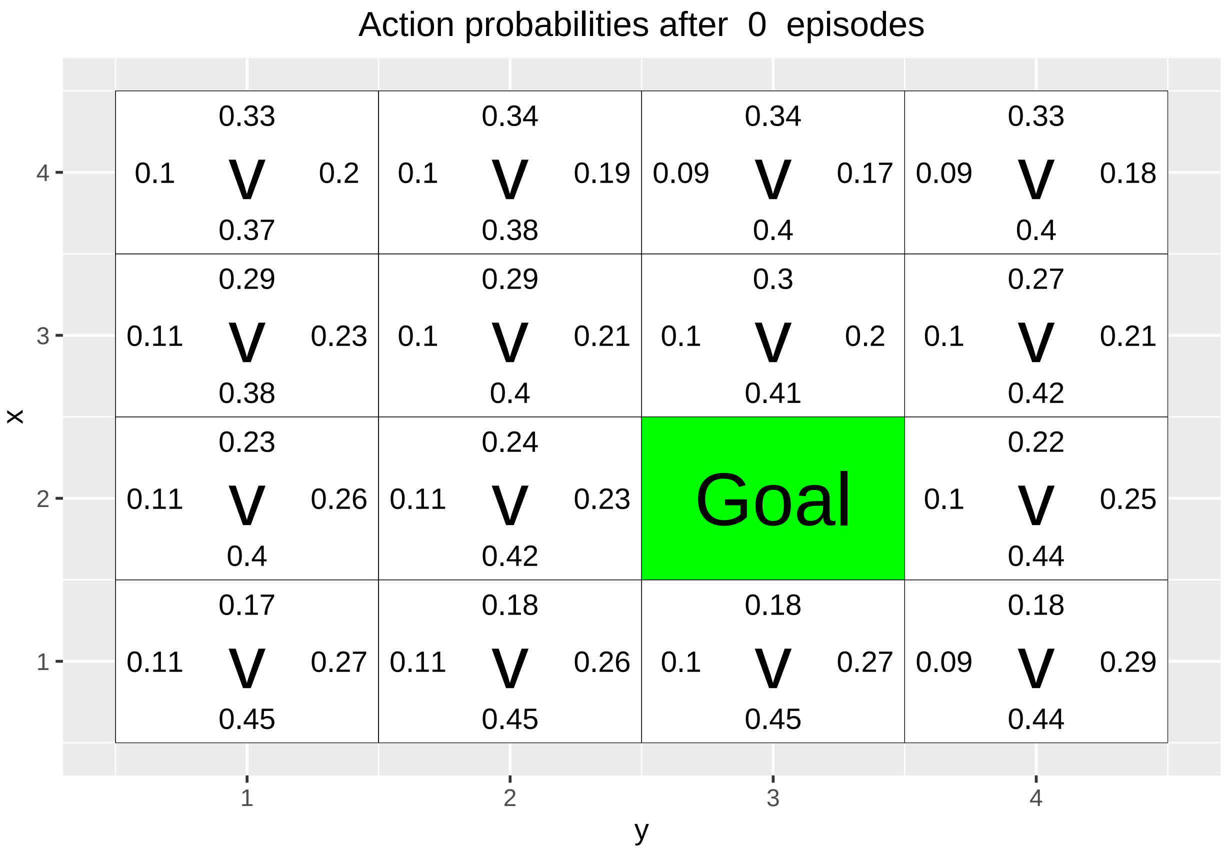
Could you have used the Q-learning algorithm to solve this task ?

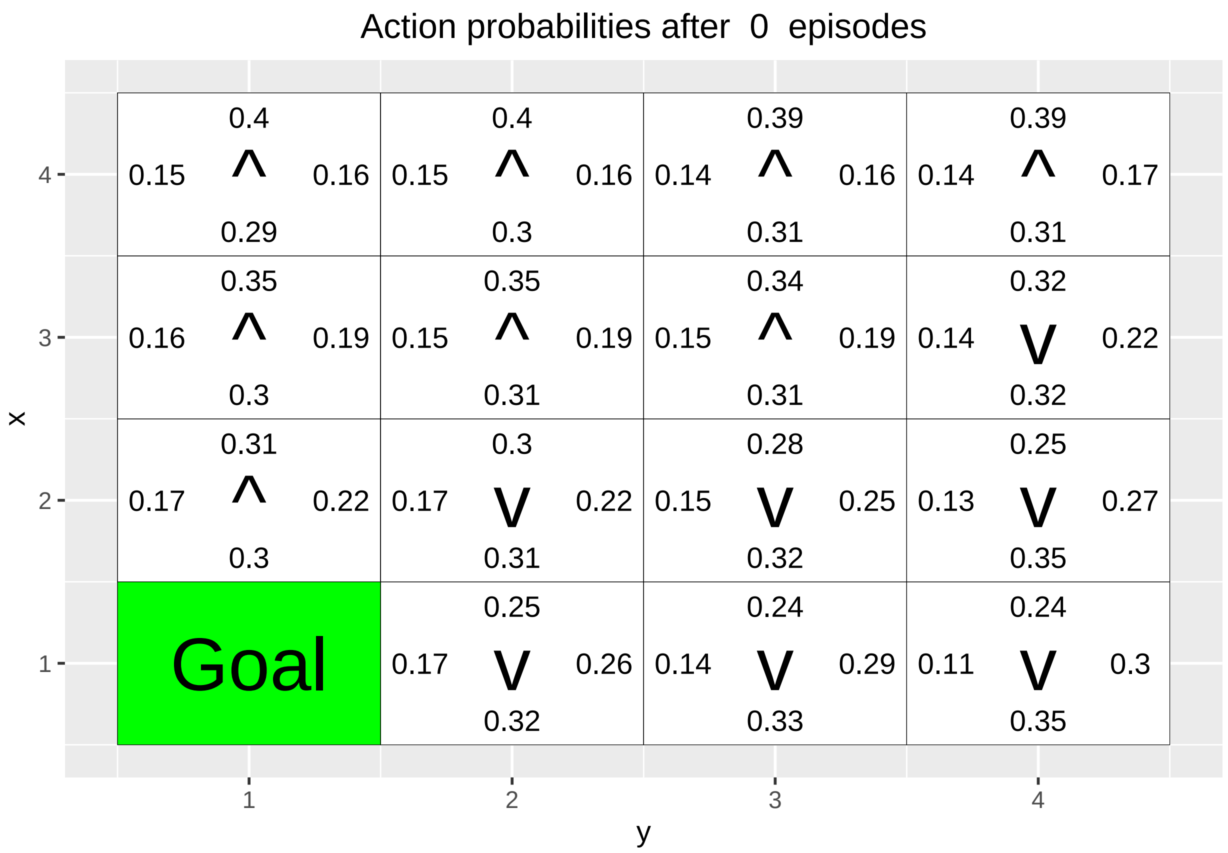
In Q-Learning we create a q-matrix which contain states and rewards for the state, now when we grid where the optimal location is kept on changing like for the above scenario, we cannot implement Q-Learning since the path to the optimal location will keep on changing and in order to do so, we have to train the model with different high reward positions which won't give us single generalized model. Thus applying Q-Learning for a similar problem is not the correct solution.

Environment E (training with top row goal positions)

Task : To repeat the previous experiments but this time the goals for training are all from the top row of the grid. The validation goals are three positions from the rows below.


Action probabilities after 0 episodes
0.28
0.07 V 0.23
0.41
0.3
0.07 V 0.21
0.42
0.29
0.07 V 0.19
0.45
0.3
0.06 V 0.18
0.45
0.23
0.08 V 0.26
0.43
0.24
0.08 V 0.23
0.45
0.24
0.07 V 0.21
0.48
Goal
0.17
0.09 V 0.28
0.47
0.18
0.08 V 0.25
0.5
0.19
0.07 V 0.23
0.51
0.18
0.07 V 0.24
0.51
0.13
0.08 V 0.27
0.52
0.13
0.08 V 0.26
0.53
0.15
0.07 V 0.27
0.51
0.15
0.07 V 0.28
0.5
x
4
3
2
1
1
2
3
4
y


Action probabilities after 0 episodes
0.33
0.1 V 0.2
0.37
0.34
0.1 V 0.19
0.38
0.34
0.09 V 0.17
0.4
0.33
0.09 V 0.18
0.4
0.29
0.11 V 0.23
0.38
0.29
0.1 V 0.21
0.4
0.3
0.1 V 0.2
0.41
0.27
0.1 V 0.21
0.42
0.23
0.11 V 0.26
0.4
0.24
0.11 V 0.23
0.42
Goal
0.22
0.1 V 0.25
0.44
0.17
0.11 V 0.27
0.45
0.18
0.11 V 0.26
0.45
0.18
0.1 V 0.27
0.45
0.18
0.09 V 0.29
0.44
x
4
3
2
1
1
2
3
4
y



```
## episode 10
## episode 20
## episode 30
## episode 40
## episode 50
## episode 60
## episode 70
## episode 80
## episode 90
## episode 100
## episode 110
## episode 120
## episode 130
## episode 140
## episode 150
## episode 160
## episode 170
## episode 180
## episode 190
## episode 200
## episode 210
## episode 220
## episode 230
## episode 240
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## episode 260
## episode 270
## episode 280
## episode 290
## episode 300
## episode 310
## episode 320
## episode 330
## episode 340
## episode 350
## episode 360
## episode 370
## episode 380
## episode 390
## episode 400
## episode 410
## episode 420
## episode 430
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## episode 450
## episode 460
## episode 470
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## episode 500
## episode 510
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## episode 610
## episode 620
## episode 630
## episode 640
## episode 650
## episode 660
## episode 670
## episode 680
## episode 690
## episode 700
## episode 710
## episode 720
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## episode 750
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## episode 780
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## episode 800
## episode 810
## episode 820
## episode 830
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## episode 850
## episode 860
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## episode 900
## episode 910
## episode 920
## episode 930
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## episode 950
## episode 960
## episode 970
## episode 980
## episode 990
## episode 1000
## episode 1010
## episode 1020
## episode 1030
## episode 1040
## episode 1050
## episode 1060
## episode 1070
## episode 1080
## episode 1090
## episode 1100
## episode 1110
## episode 1120
## episode 1130
## episode 1140
## episode 1150
## episode 1160
## episode 1170
## episode 1180
## episode 1190
## episode 1200
## episode 1210
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## episode 1350
## episode 1360
## episode 1370
## episode 1380
## episode 1390
## episode 1400
## episode 1410
## episode 1420
## episode 1430
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## episode 1450
## episode 1460
## episode 1470
## episode 1480
## episode 1490
## episode 1500
## episode 1510
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## episode 1570
## episode 1580
## episode 1590
## episode 1600
## episode 1610
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## episode 1680
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## episode 1890
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## episode 1950
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## episode 1980
## episode 1990
## episode 2000
## episode 2010
## episode 2020
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## episode 2270
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## episode 2340
## episode 2350
## episode 2360
## episode 2370
## episode 2380
## episode 2390
## episode 2400
## episode 2410
## episode 2420
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## episode 2440
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## episode 2480
## episode 2490
## episode 2500
## episode 2510
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## episode 2590
## episode 2600
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## episode 2670
## episode 2680
## episode 2690
## episode 2700
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## episode 2950
## episode 2960
## episode 2970
## episode 2980
## episode 2990
## episode 3000
## episode 3010
## episode 3020
## episode 3030
## episode 3040
## episode 3050
## episode 3060
## episode 3070
## episode 3080
## episode 3090
## episode 3100
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## episode 3340
## episode 3350
## episode 3360
## episode 3370
## episode 3380
## episode 3390
## episode 3400
## episode 3410
## episode 3420
## episode 3430
## episode 3440
## episode 3450
## episode 3460
## episode 3470
## episode 3480
## episode 3490
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## episode 3580
## episode 3590
## episode 3600
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## episode 3620
## episode 3630
## episode 3640
## episode 3650
## episode 3660
## episode 3670
## episode 3680
## episode 3690
## episode 3700
## episode 3710
## episode 3720
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## episode 3950
## episode 3960
## episode 3970
## episode 3980
## episode 3990
## episode 4000
## episode 4010
## episode 4020
## episode 4030
## episode 4040
## episode 4050
## episode 4060
## episode 4070
## episode 4080
## episode 4090
## episode 4100
## episode 4110
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## episode 4370
## episode 4380
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## episode 4470
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## episode 4570
## episode 4580
## episode 4590
## episode 4600
## episode 4610
## episode 4620
## episode 4630
## episode 4640
## episode 4650
## episode 4660
## episode 4670
## episode 4680
## episode 4690
## episode 4700
## episode 4710
## episode 4720
## episode 4730
## episode 4740
## episode 4750
## episode 4760
## episode 4770
## episode 4780
## episode 4790
## episode 4800
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## episode 4860
## episode 4870
## episode 4880
## episode 4890
## episode 4900
## episode 4910
## episode 4920
## episode 4930
## episode 4940
## episode 4950
## episode 4960
## episode 4970
## episode 4980
## episode 4990
## episode 5000
```
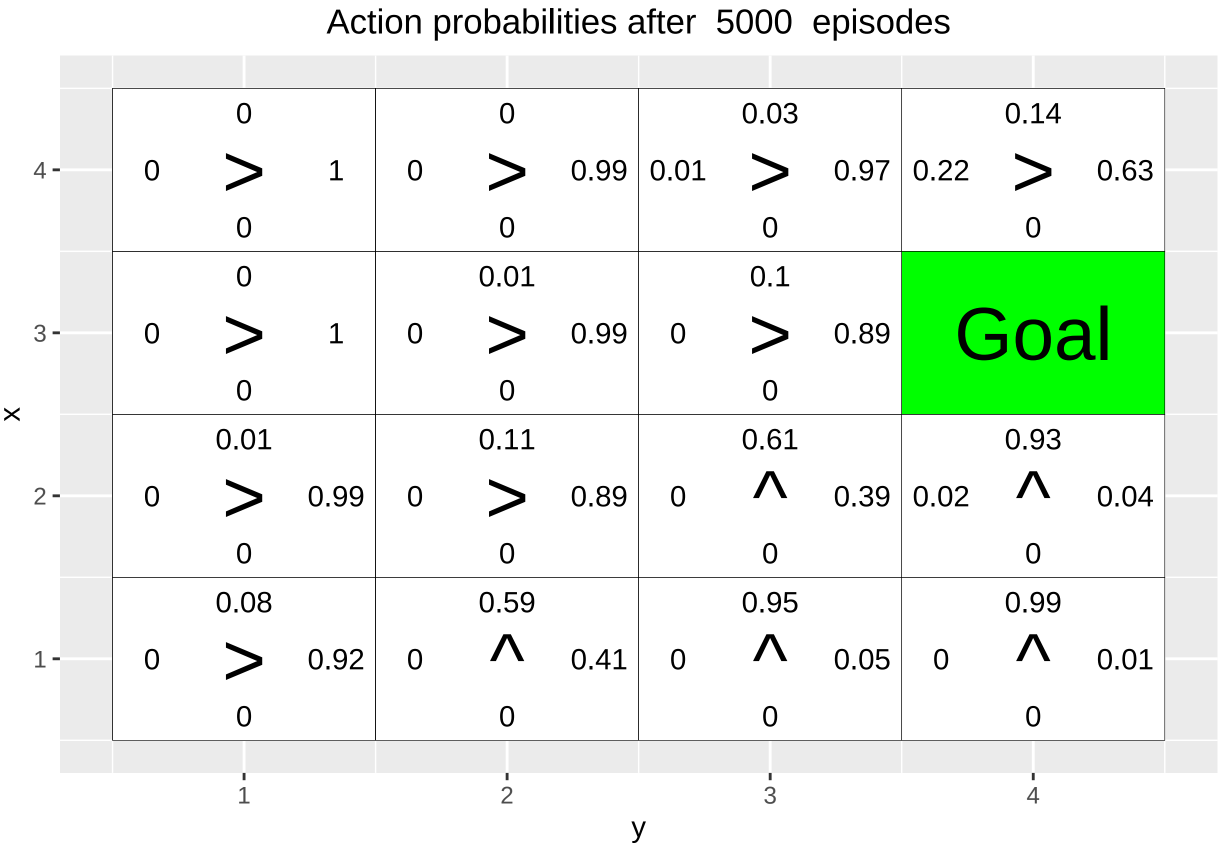
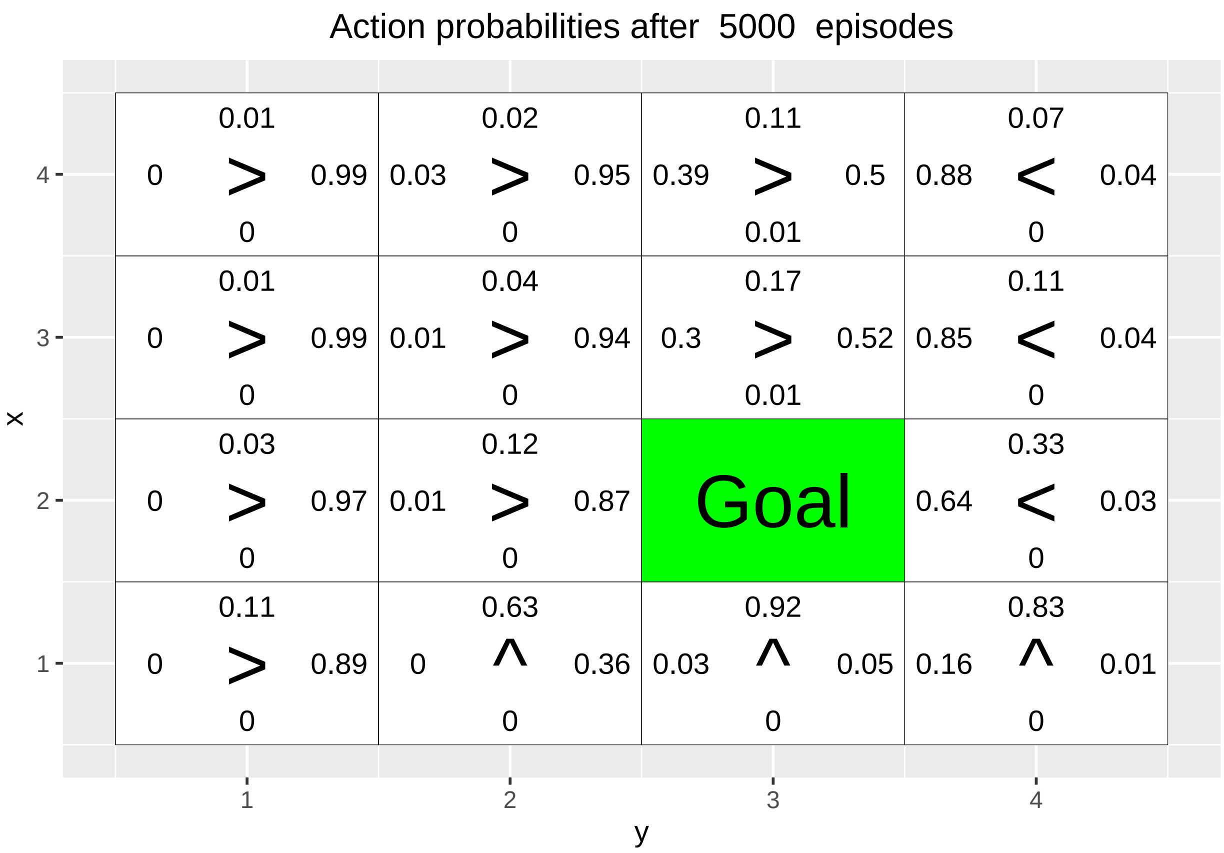
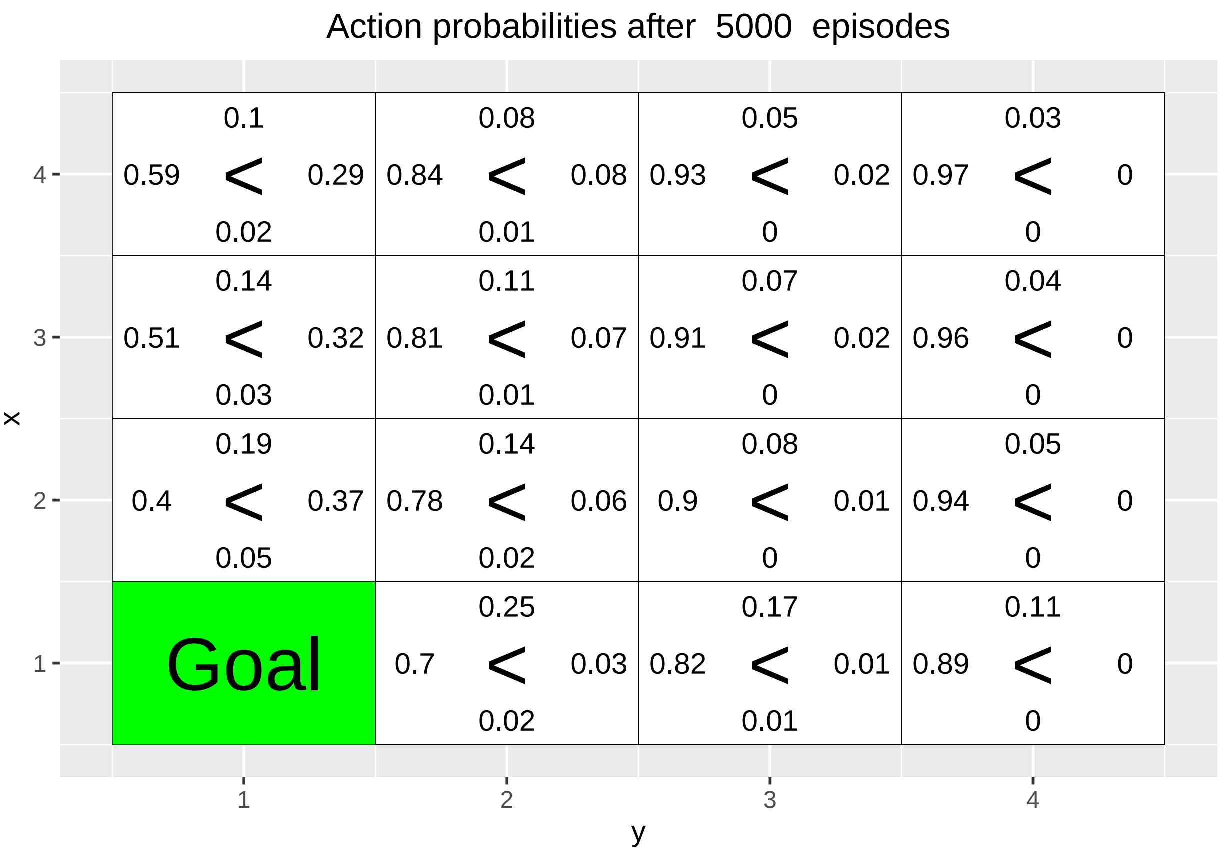


Action probabilities after 5000 episodes
0
0 > 1
0
0
0 > 0.99
0
0.03
0.01 > 0.97
0
0.14
0.22 > 0.63
0
0
0 > 1
0
0.01
0 > 0.99
0
0.1
0 > 0.89
0
Goal
0.01
0 > 0.99
0
0.11
0 > 0.89
0
0.61
0 ^ 0.39
0
0.93
0.02 ^ 0.04
0
0.08
0 > 0.92
0
0.59
0 ^ 0.41
0
0.95
0 ^ 0.05
0
0.99
0 ^ 0.01
0
x
4
3
2
1
1
2
3
4
y


Action probabilities after 5000 episodes
0.01
0
>
0.99
0
0.02
0.03
>
0.95
0
0.11
0.39
>
0.5
0.01
0.07
0.88
<
0.04
0
0.01
0
>
0.99
0
0.04
0.01
>
0.94
0
0.17
0.3
>
0.52
0.01
0.11
0.85
<
0.04
0
0.03
0
>
0.97
0
0.12
0.01
>
0.87
0
Goal
0.33
0.64
<
0.03
0
0.11
0
>
0.89
0
0.63
0
^
0.36
0
0.92
0.03
^
0.05
0
0.83
0.16
^
0.01
0
x
4
3
2
1
y
1
2
3
4



Has the agent learned a good policy? Why / Why not ?

Since the agent only had access to 1st row of the grid as data, the neural network was not able to learn all paths for the whole grid, so we are not able to create a generalized model because of which agent is not able to learn optimal policy.

If the results obtained for environments D and E differ, explain why

In D agent was able to generalize based on given training data as it had access to the whole grid while it lacks to do it for E as it only had access to 1st row of the grid thus better generalization was obtained in D while E we were not able to do so because of which we have obtained different results. This can be seen by the path selected in case of validation for different goal positions.

References

- A Concise Introduction to Reinforcement Learning

*Epsilon-Greedy Algorithm in Reinforcement Learning

Appendix

```
knitr::opts_chunk$set(echo = TRUE)

library(ggplot2)

# If you do not see four arrows in line 16, then do the following:
# File/Reopen with Encoding/UTF-8
```

```r
#arrows <- c("↑", "→", "↓", "←")
arrows <- c("^", ">", "v", "<")
action_deltas <- list(c(1,0), # up
                      c(0,1), # right
                      c(-1,0), # down
                      c(0,-1)) # left

vis_environment <- function(iterations=0, epsilon = 0.5, alpha = 0.1, gamma = 0.95, beta = 0){

  # Visualize an environment with rewards.
  # Q-values for all actions are displayed on the edges of each tile.
  # The (greedy) policy for each state is also displayed.
  #
  # Args:
  #   iterations, epsilon, alpha, gamma, beta (optional): for the figure title.
  #   reward_map (global variable): a HxW array containing the reward given at each state.
  #   q_table (global variable): a HxWx4 array containing Q-values for each state-action pair.
  #   H, W (global variables): environment dimensions.

  df <- expand.grid(x=1:H,y=1:W)
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,1],NA),df$x,df$y)
  df$val1 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,2],NA),df$x,df$y)
  df$val2 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,3],NA),df$x,df$y)
  df$val3 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,4],NA),df$x,df$y)
  df$val4 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y)
    ifelse(reward_map[x,y] == 0,arrows[GreedyPolicy(x,y)],reward_map[x,y]),df$x,df$y)
  df$val5 <- as.vector(foo)
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,max(q_table[x,y,]),
                                     ifelse(reward_map[x,y]<0,NA,reward_map[x,y])),df$x,df$y)
  df$val6 <- as.vector(foo)

  print(ggplot(df,aes(x = y,y = x)) +
          scale_fill_gradient(low = "white", high = "green", na.value = "red", name = "") +
          geom_tile(aes(fill=val6)) +
          geom_text(aes(label = val1),size = 4,nudge_y = .35,na.rm = TRUE) +
          geom_text(aes(label = val2),size = 4,nudge_x = .35,na.rm = TRUE) +
          geom_text(aes(label = val3),size = 4,nudge_y = -.35,na.rm = TRUE) +
          geom_text(aes(label = val4),size = 4,nudge_x = -.35,na.rm = TRUE) +
          geom_text(aes(label = val5),size = 10) +
          geom_tile(fill = 'transparent', colour = 'black') +
          ggtitle(paste("Q-table after ",iterations," iterations\n",
                        "(epsilon = ",epsilon,", alpha = ",alpha,"gamma = ",gamma,", beta = ",beta,")")
          theme(plot.title = element_text(hjust = 0.5)) +
          scale_x_continuous(breaks = c(1:W),labels = c(1:W)) +
          scale_y_continuous(breaks = c(1:H),labels = c(1:H)))

}

GreedyPolicy <- function(x, y){
  # Get a greedy action for state (x,y) from q_table.
  #
  # Args:
  #   x, y: state coordinates.
  #   q_table (global variable): a HxWx4 array containing Q-values for each state-action pair.
  #
  # Returns:
  #   An action, i.e. integer in {1,2,3,4}.
  Rewards = q_table[x,y,]
  maxRewardIndx = which(Rewards == max(Rewards))
  #randomly selecting max value and setting it as action

  if (length(maxRewardIndx) == 1){
    InitSelected = maxRewardIndx
  }else{
  InitSelected = sample(maxRewardIndx , size = 1)
  }

  return(InitSelected)
  # Your code here.

}#GreedyPolicy

EpsilonGreedyPolicy <- function(x, y, epsilon){

  # Get an epsilon-greedy action for state (x,y) from q_table.
  #
  # Args:
  #   x, y: state coordinates.
  #   epsilon: probability of acting randomly.
  #
  # Returns:
  #   An action, i.e. integer in {1,2,3,4}.

  # Your code here.
  Rewards = q_table[x,y,]
  maxRewardIndx = which(Rewards == max(Rewards))

  #randomly selecting max value and setting it as action
  if (length(maxRewardIndx) == 1){
    InitSelected = maxRewardIndx
  }else{
  InitSelected = sample(maxRewardIndx , size = 1)
  }
  #unif
  p = runif(1)
  if(p < epsilon)
    {
      return(sample(c(1,2,3,4) , size = 1)) #e
  }else{
      return(InitSelected) #1-e
    }
}#EpsilonGreedyPolicy
```

```
transition_model <- function(x, y, action, beta){

  # Computes the new state after given action is taken. The agent will follow the action
  # with probability (1-beta) and slip to the right or left with probability beta/2 each.
  #
  # Args:
  #   x, y: state coordinates.
  #   action: which action the agent takes (in {1,2,3,4}).
  #   beta: probability of the agent slipping to the side when trying to move.
  #   H, W (global variables): environment dimensions.
  #
  # Returns:
  #   The new state after the action has been taken.

  delta <- sample(-1:1, size = 1, prob = c(0.5*beta,1-beta,0.5*beta))
  final_action <- ((action + delta + 3) %% 4) + 1
  foo <- c(x,y) + unlist(action_deltas[final_action])
  foo <- pmax(c(1,1),pmin(foo,c(H,W)))

  return (foo)
}

q_learning <- function(start_state, epsilon = 0.5, alpha = 0.1, gamma = 0.95,
                       beta = 0){

  # Perform one episode of Q-learning. The agent should move around in the
  # environment using the given transition model and update the Q-table.
  # The episode ends when the agent reaches a terminal state.
  #
  # Args:
  #   start_state: array with two entries, describing the starting position of the agent.
  #   epsilon (optional): probability of acting greedily.
  #   alpha (optional): learning rate.
  #   gamma (optional): discount factor.
  #   beta (optional): slipping factor.
  #   reward_map (global variable): a HxW array containing the reward given at each state.
  #   q_table (global variable): a HxWx4 array containing Q-values for each state-action pair.
  #
  # Returns:
  #   reward: reward received in the episode.
  #   correction: sum of the temporal difference correction terms over the episode.
  #   q_table (global variable): Recall that R passes arguments by value. So, q_table being
  #   a global variable can be modified with the superassigment operator <<-.

  # Your code here.
  episode_correction = 0

  repeat{
    # Follow policy, execute action, get reward.
    x = start_state[1]
    y = start_state[2]
    A = EpsilonGreedyPolicy(x,y,epsilon)
    #a = GreedyPolicy(x,y)
    #calculate new state
    TransState = transition_model(x,y,A,beta)

    reward = reward_map[TransState[1],TransState[2]]

    #NewCorrection = alpha * (reward + gamma*(q_table[TransState[1],TransState[2],a]) - q_table[x,y,A])
    NewCorrection = reward + gamma* max(q_table[TransState[1],TransState[2],]) - q_table[x,y,A]
    episode_correction = episode_correction + NewCorrection

    # Q-table update.
    #q_table[x,y,A] <<- q_table[x,y,A] + NewCorrection
    q_table[x,y,A] <<- q_table[x,y,A] + alpha * NewCorrection

    start_state = TransState

    if(reward!=0)
      # End episode.
      return (c(reward,episode_correction))
  }

}


H <- 5
W <- 7

reward_map <- matrix(0, nrow = H, ncol = W)
reward_map[3,6] <- 10
reward_map[2:4,3] <- -1

q_table <- array(0,dim = c(H,W,4))

vis_environment()

for(i in 1:10000){
  foo <- q_learning(start_state = c(3,1))

  if(any(i==c(10,100,1000,10000)))
    vis_environment(i)
}

#test run for additional 1000 run

# for(i in 1:100000){
#   foo <- q_learning(start_state = c(3,1))
#
#   if(any(i==c(10,100,1000,10000 , 100000)))
#     vis_environment(i)
# }


# Environment B (the effect of epsilon and gamma)
```

```
cat("Epsilon set as 0.5")

H <- 7
W <- 8

reward_map <- matrix(0, nrow = H, ncol = W)
reward_map[1,] <- -1
reward_map[7,] <- -1
reward_map[4,5] <- 5
reward_map[4,8] <- 10

q_table <- array(0,dim = c(H,W,4))

vis_environment()

MovingAverage <- function(x, n){

  cx <- c(0,cumsum(x))
  rsum <- (cx[(n+1):length(cx)] - cx[1:(length(cx) - n)]) / n

  return (rsum)
}

for(j in c(0.5,0.75,0.95)){
  q_table <- array(0,dim = c(H,W,4))
  reward <- NULL
  correction <- NULL

  for(i in 1:30000){
    foo <- q_learning(gamma = j, start_state = c(4,1))
    reward <- c(reward,foo[1])
    correction <- c(correction,foo[2])
  }

  vis_environment(i, gamma = j)
  plot(MovingAverage(reward,100),type = "l")
  plot(MovingAverage(correction,100),type = "l")
}




cat("Epsilon set as 0.1")

for(j in c(0.5,0.75,0.95)){
  q_table <- array(0,dim = c(H,W,4))
  reward <- NULL
  correction <- NULL

  for(i in 1:30000){
    foo <- q_learning(epsilon = 0.1, gamma = j, start_state = c(4,1))
    reward <- c(reward,foo[1])
    correction <- c(correction,foo[2])
  }

  vis_environment(i, epsilon = 0.1, gamma = j)
  plot(MovingAverage(reward,100),type = "l")
  plot(MovingAverage(correction,100),type = "l")
}



H <- 3
W <- 6

reward_map <- matrix(0, nrow = H, ncol = W)
reward_map[1,2:5] <- -1
reward_map[1,6] <- 10

q_table <- array(0,dim = c(H,W,4))

vis_environment()


for(j in c(0,0.2,0.4,0.66)){
  q_table <- array(0,dim = c(H,W,4))

  for(i in 1:10000)
    foo <- q_learning(gamma = 0.6, beta = j, start_state = c(1,1))

  vis_environment(i, gamma = 0.6, beta = j)
}

# install.packages("keras")
#install.packages("tensorflow")
library(tensorflow)
library(keras)

# install.packages("ggplot2")
# install.packages("vctrs")
library(ggplot2)

# If you do not see four arrows in line 19, then do the following:
# File/Reopen with Encoding/UTF-8

#arrows <- c("↑", "→", "↓", "←")
arrows <- c("^", ">", "v", "<")
action_deltas <- list(c(1,0), # up
                      c(0,1), # right
                      c(-1,0), # down
                      c(0,-1)) # left

vis_prob <- function(goal, episodes = 0){

  # Visualize an environment with rewards.
  # Probabilities for all actions are displayed on the edges of each tile.
```

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  # The (greedy) policy for each state is also displayed.
  #
  # Args:
  #   goal: goal coordinates, array with 2 entries.
  #   episodes, epsilon, alpha, gamma, beta (optional): for the figure title.
  #   H, W (global variables): environment dimensions.

  df <- expand.grid(x=1:H,y=1:W)
  dist <- array(data = NA, dim = c(H,W,4))
  class <- array(data = NA, dim = c(H,W))
  for(i in 1:H)
    for(j in 1:W){
      dist[i,j,] <- DeepPolicy_dist(i,j,goal[1],goal[2])
      foo <- which(dist[i,j,]==max(dist[i,j,]))
      class[i,j] <- ifelse(length(foo)>1,sample(foo, size = 1),foo)
    }

  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,1]),df$x,df$y)
  df$val1 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,2]),df$x,df$y)
  df$val2 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,3]),df$x,df$y)
  df$val3 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,4]),df$x,df$y)
  df$val4 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,class[x,y]),df$x,df$y)
  df$val5 <- as.vector(arrows[foo])
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),"Goal",NA),df$x,df$y)
  df$val6 <- as.vector(foo)

  print(ggplot(df,aes(x = y,y = x)) +
          geom_tile(fill = 'white', colour = 'black') +
          scale_fill_manual(values = c('green')) +
          geom_tile(aes(fill=val6), show.legend = FALSE, colour = 'black') +
          geom_text(aes(label = val1),size = 4,nudge_y = .35,na.rm = TRUE) +
          geom_text(aes(label = val2),size = 4,nudge_x = .35,na.rm = TRUE) +
          geom_text(aes(label = val3),size = 4,nudge_y = -.35,na.rm = TRUE) +
          geom_text(aes(label = val4),size = 4,nudge_x = -.35,na.rm = TRUE) +
          geom_text(aes(label = val5),size = 10,na.rm = TRUE) +
          geom_text(aes(label = val6),size = 10,na.rm = TRUE) +
          ggtitle(paste("Action probabilities after ",episodes," episodes")) +
          theme(plot.title = element_text(hjust = 0.5)) +
          scale_x_continuous(breaks = c(1:W),labels = c(1:W)) +
          scale_y_continuous(breaks = c(1:H),labels = c(1:H)))

}

transition_model <- function(x, y, action, beta){

  # Computes the new state after given action is taken. The agent will follow the action
  # with probability (1-beta) and slip to the right or left with probability beta/2 each.
  #
  # Args:
```

```r
  #   x, y: state coordinates.
  #   action: which action the agent takes (in {1,2,3,4}).
  #   beta: probability of the agent slipping to the side when trying to move.
  #   H, W (global variables): environment dimensions.
  #
  # Returns:
  #   The new state after the action has been taken.

  delta <- sample(-1:1, size = 1, prob = c(0.5*beta,1-beta,0.5*beta))
  final_action <- ((action + delta + 3) %% 4) + 1
  foo <- c(x,y) + unlist(action_deltas[final_action])
  foo <- pmax(c(1,1),pmin(foo,c(H,W)))

  return (foo)
}

DeepPolicy_dist <- function(x, y, goal_x, goal_y){

  # Get distribution over actions for state (x,y) and goal (goal_x,goal_y) from the deep policy.
  #
  # Args:
  #   x, y: state coordinates.
  #   goal_x, goal_y: goal coordinates.
  #   model (global variable): NN encoding the policy.
  #
  # Returns:
  #   A distribution over actions.

  foo <- matrix(data = c(x,y,goal_x,goal_y), nrow = 1)

  # return (predict_proba(model, x = foo))
  return (predict_on_batch(model, x = foo)) # Faster.

}

DeepPolicy <- function(x, y, goal_x, goal_y){

  # Get an action for state (x,y) and goal (goal_x,goal_y) from the deep policy.
  #
  # Args:
  #   x, y: state coordinates.
  #   goal_x, goal_y: goal coordinates.
  #   model (global variable): NN encoding the policy.
  #
  # Returns:
  #   An action, i.e. integer in {1,2,3,4}.

  foo <- DeepPolicy_dist(x,y,goal_x,goal_y)

  return (sample(1:4, size = 1, prob = foo))

}
```

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DeepPolicy_train <- function(states, actions, goal, gamma){

  # Train the policy network on a rolled out trajectory.
  #
  # Args:
  #   states: array of states visited throughout the trajectory.
  #   actions: array of actions taken throughout the trajectory.
  #   goal: goal coordinates, array with 2 entries.
  #   gamma: discount factor.

  # Construct batch for training.
  inputs <- matrix(data = states, ncol = 2, byrow = TRUE)
  inputs <- cbind(inputs,rep(goal[1],nrow(inputs)))
  inputs <- cbind(inputs,rep(goal[2],nrow(inputs)))

  targets <- array(data = actions, dim = nrow(inputs))
  targets <- to_categorical(targets-1, num_classes = 4)

  # Sample weights. Reward of 5 for reaching the goal.
  weights <- array(data = 5*(gamma^(nrow(inputs)-1)), dim = nrow(inputs))

  # Train on batch. Note that this runs a SINGLE gradient update.
  train_on_batch(model, x = inputs, y = targets, sample_weight = weights)

}

reinforce_episode <- function(goal, gamma = 0.95, beta = 0){

  # Rolls out a trajectory in the environment until the goal is reached.
  # Then trains the policy using the collected states, actions and rewards.
  #
  # Args:
  #   goal: goal coordinates, array with 2 entries.
  #   gamma (optional): discount factor.
  #   beta (optional): probability of slipping in the transition model.

  # Randomize starting position.
  cur_pos <- goal
  while(all(cur_pos == goal))
    cur_pos <- c(sample(1:H, size = 1),sample(1:W, size = 1))

  states <- NULL
  actions <- NULL

  steps <- 0 # To avoid getting stuck and/or training on unnecessarily long episodes.
  while(steps < 20){
    steps <- steps+1

    # Follow policy and execute action.
    action <- DeepPolicy(cur_pos[1], cur_pos[2], goal[1], goal[2])
    new_pos <- transition_model(cur_pos[1], cur_pos[2], action, beta)

    # Store states and actions.
    states <- c(states,cur_pos)
    actions <- c(actions,action)
    cur_pos <- new_pos

    if(all(new_pos == goal)){
      # Train network.
      DeepPolicy_train(states,actions,goal,gamma)
      break
    }
  }

}




# Environment D (training with random goal positions)

H <- 4
W <- 4

# Define the neural network (two hidden layers of 32 units each).
model <- keras_model_sequential()
model %>%
  layer_dense(units = 32, input_shape = c(4), activation = 'relu') %>%
  layer_dense(units = 32, activation = 'relu') %>%
  layer_dense(units = 4, activation = 'softmax')

compile(model, loss = "categorical_crossentropy", optimizer = optimizer_sgd(lr=0.001))

initial_weights <- get_weights(model)

train_goals <- list(c(4,1), c(4,3), c(3,1), c(3,4), c(2,1), c(2,2), c(1,2), c(1,3))
val_goals <- list(c(4,2), c(4,4), c(3,2), c(3,3), c(2,3), c(2,4), c(1,1), c(1,4))

show_validation <- function(episodes){

  for(goal in val_goals)
    vis_prob(goal, episodes)

}

set_weights(model,initial_weights)

show_validation(0)

for(i in 1:5000){
  if(i%%10==0) cat("episode",i,"\n")
  goal <- sample(train_goals, size = 1)
  reinforce_episode(unlist(goal))
}

show_validation(5000)
```

```
# Environment E (training with top row goal positions)

train_goals <- list(c(4,1), c(4,2), c(4,3), c(4,4))
val_goals <- list(c(3,4), c(2,3), c(1,1))

set_weights(model,initial_weights)

show_validation(0)

for(i in 1:5000){
  if(i%%10==0) cat("episode", i,"\n")
  goal <- sample(train_goals, size = 1)
  reinforce_episode(unlist(goal))
}

show_validation(5000)
```