Report Reinforcement Learning

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Question 1

Q-Learning

Q-Learning is an off-policy Reinforcement Learning algorithm. It uses table to store Q-Values of all possible state-action possible pairs. Table is updated using Bellman equation while action selection can be done using $\epsilon - greedy$ policy or some other policy.

We will work with a grid world environment consisting of H × W tiles laid out in a 2-dimensional grid. An agent acts by moving up, down, left or right in the grid-world. This corresponds to the following Markov decision process:

State Space : S = {(x,y|x ϵ {1 , …. , H} , y ϵ {1,…..,H}} Action Space : A = {up,down,left,right}

Additionally, we assume state space to be fully observable. The reward function is a deterministic function of the state and does not depend on the actions taken by the agent. We assume the agent gets the reward as soon as it moves to a state. The transition model is defined by the agent moving in the direction chosen with probability (1$-\beta$). The agent might also slip and end up moving in the direction to the left or right of its chosen action, each with probability β\$/2. The transition model is unknown to the agent, forcing us to resort to model-free solutions. The environment is episodic and all states with a non-zero reward are terminal. Throughout this lab we use integer representations of the different actions: Up=1, right=2, down=3 and left=4

Environment A

For our first environment, we will use H = 5 and W = 7. This environment includes a reward of 10 in state (3,6) and a reward of -1 in states (2,3), (3,3) and (4,3). We specify the rewards using a reward map in the form of a matrix with one entry for each state. States with no reward will simply have a matrix entry of 0. The agent starts each episode in the state (3,1). The function vis_environment is used to visualize the environment and learned action values and policy.

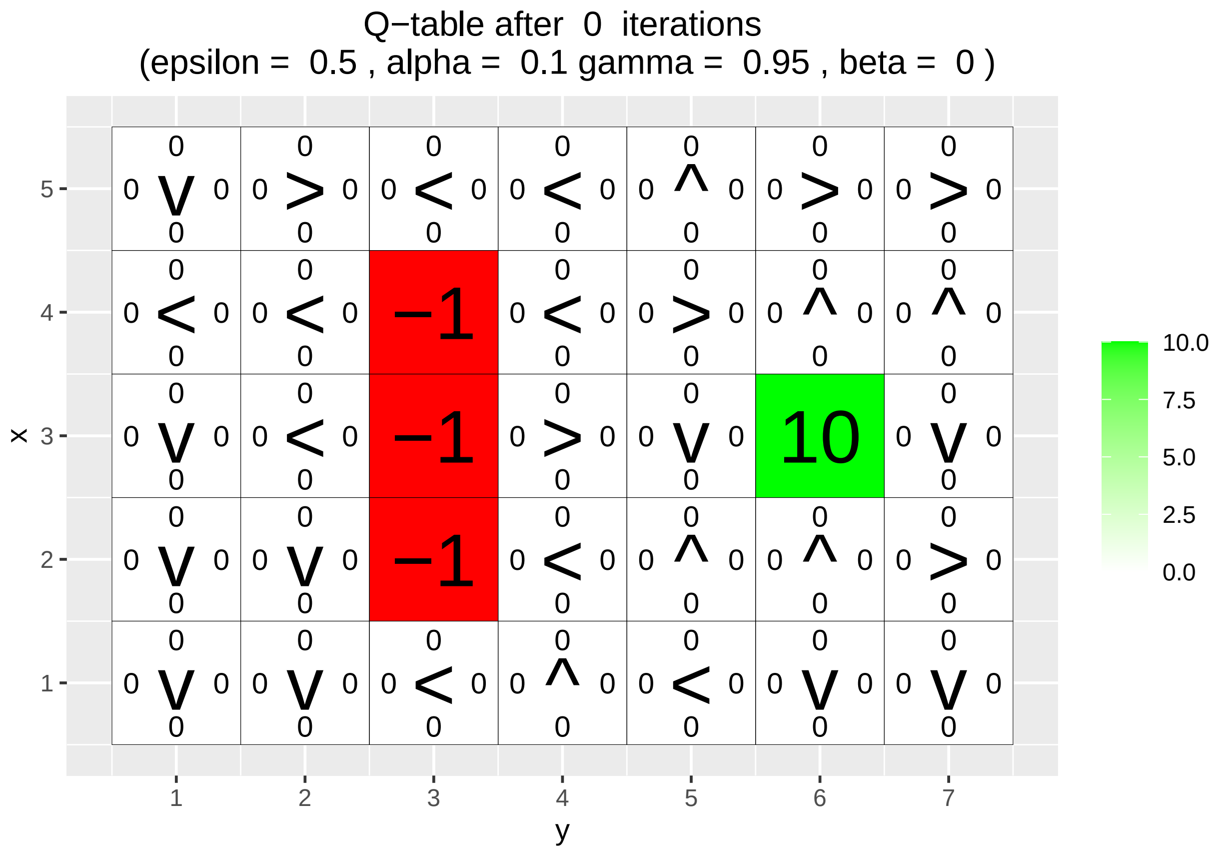
Q-Learning is off-policy, it evaluates one policy (target policy) while observing behavior policy. Finding optimal action value function Q_* under arbitrary behavior policy is achieved by policy iteration. Q converges to optimal action value function $Q_*(S, A)$ and its greedy converges to optimal policy under appropriate choice of learning rate (α) over time.

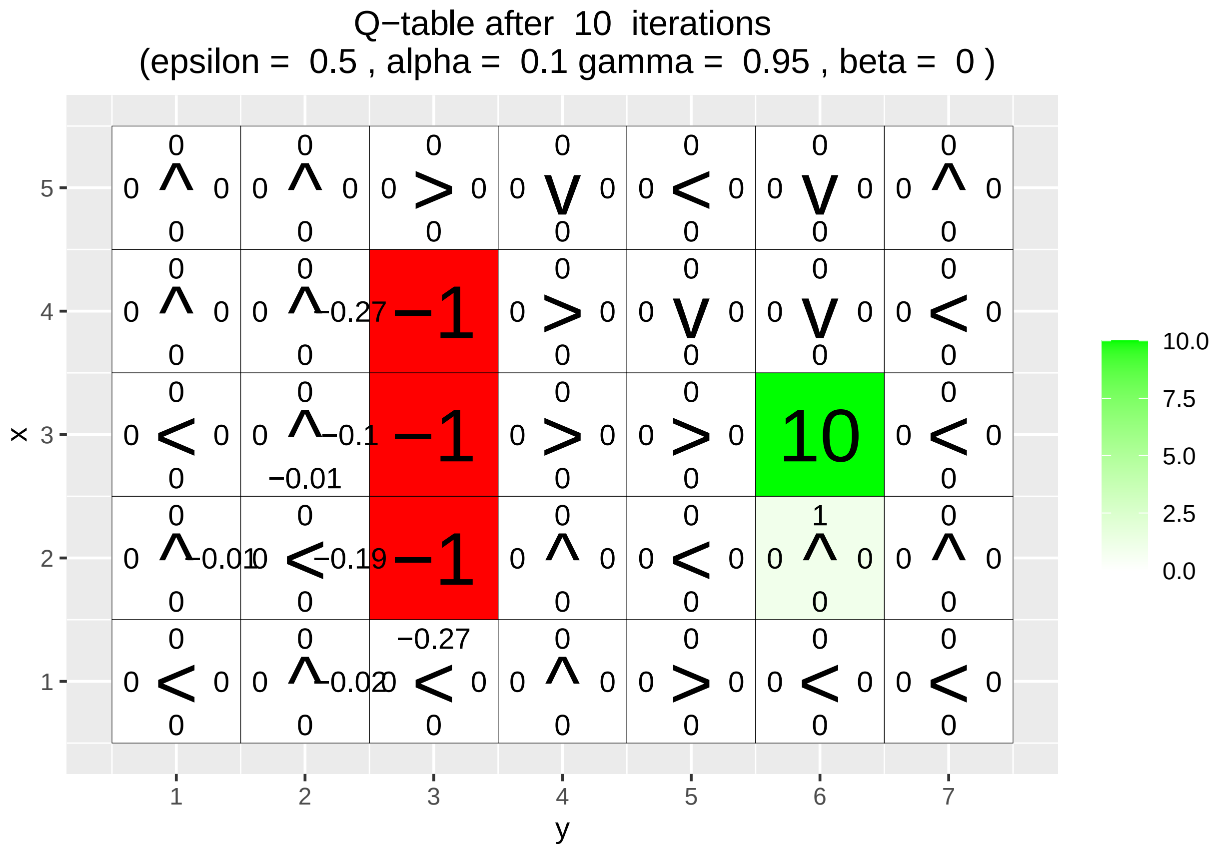
$$Q_*(S, A) = Q(S, A) + \alpha(R + \gamma(max_a Q(S', a)) - Q(S, A))$$

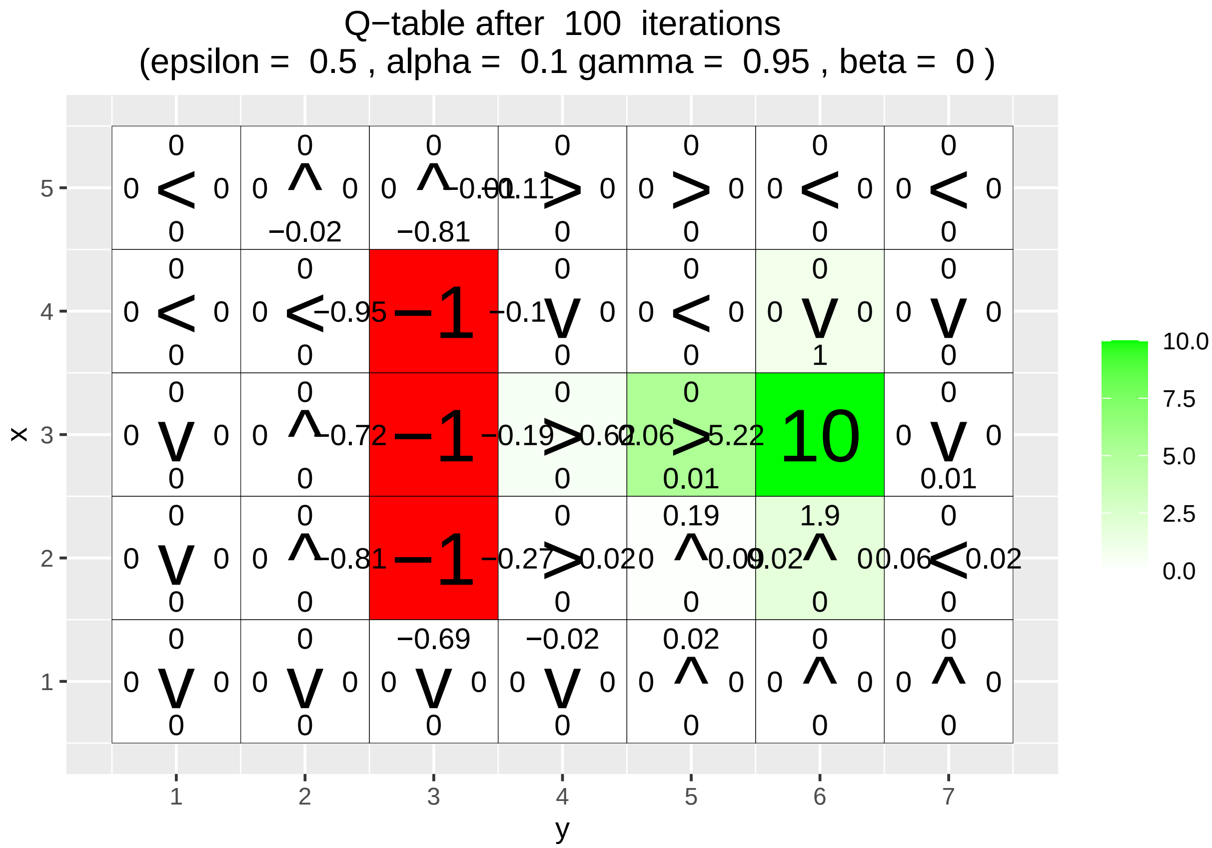
Here :

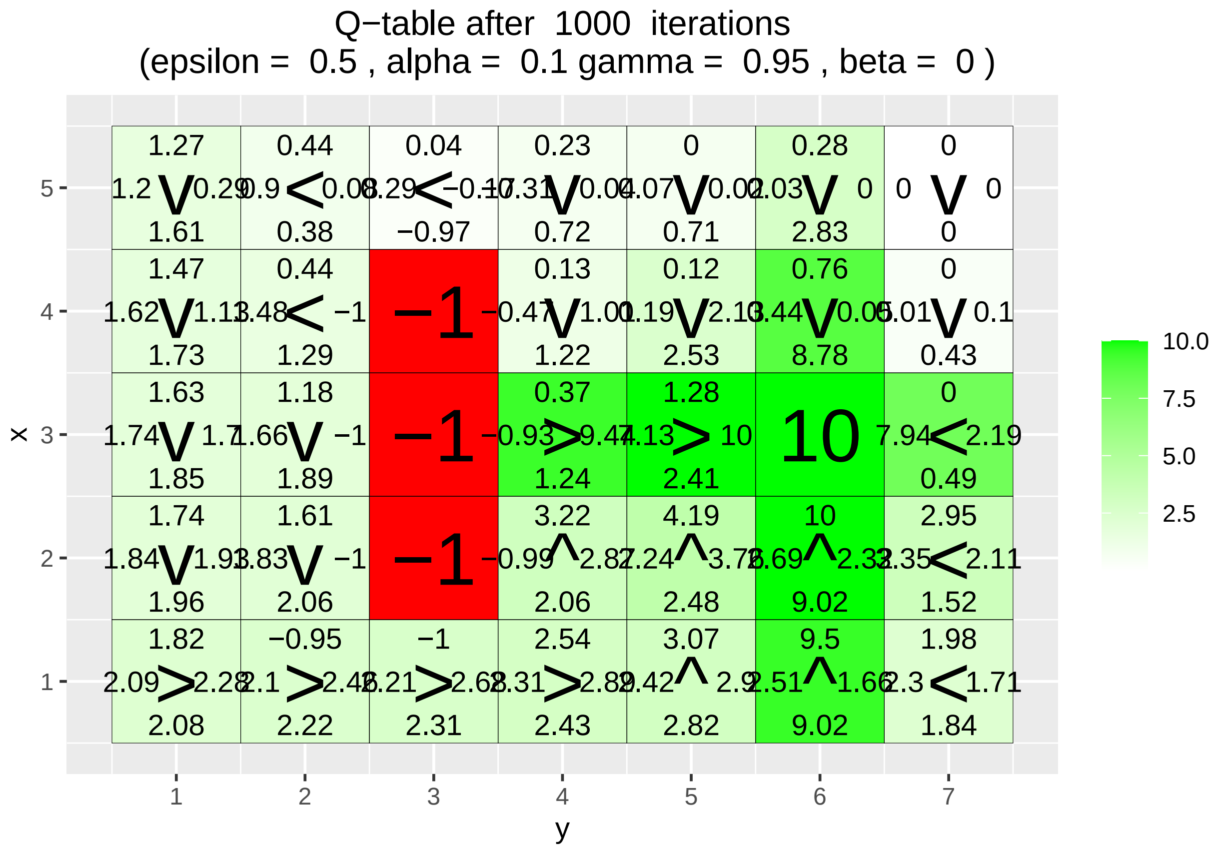
S : is current state A : Action chosen by ϵ-Greedy policy S': Transition state based on current state and chosen action A R : is Reward for moving to transition state S' a : Action chosen by Greedy Policy α :

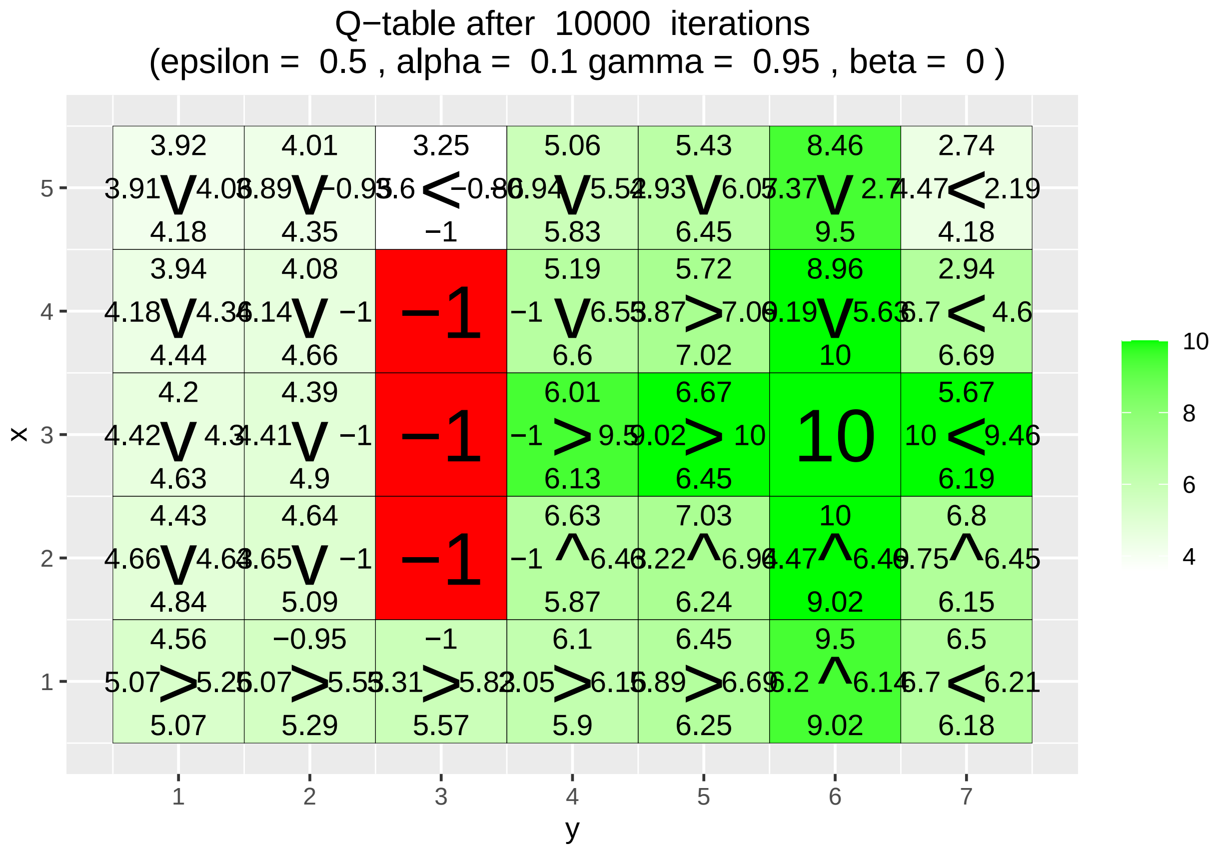
Learning Rate γ : Discount factor, ranges between (0 , 1) which control importance of future rewards in comparison to current one.




Q−table after 10 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)
x
y
10.0
7.5
5.0
2.5
0.0
−1
10
−0.27
−0.1
−0.01
−0.19
−0.02


Q−table after 100 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)
x
y
10.0
7.5
5.0
2.5
0.0
−1
10


Q−table after 1000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)
x
y
10.0
7.5
5.0
2.5



What has the agent learned after the first 10 episodes ?

It can be seen from above Q-Table named "Q-Table after 10 Iterations". With very few iterations, agent movement is quite random and yet there is no significant learning.

Is the final greedy policy (after 10000 episodes) optimal? Why / Why not ?

If can be seen that model has learned path towards maximum reward position state and thus when left in any grid, it will converge to global maxima position. Thus final greedy postion is optimal.

Does the agent learn that there are multiple paths to get to the positive reward ? If not, what could be done to make the agent learn this ?

In Q-table for 10000 iterations, looking at maximum rewards for each grid, it can be seen that agent did learn multiple path to obtain maximum reward based on where it start but it will follow same path in test state to reach for maximum reward.

Environment B

Investigate how the ϵ and γ parameters affect the learned policy by running 30000 episodes of Q-learning with $\epsilon = 0.1, 0.5$, $\gamma = 0.5, 0.75, 0.95$, $\beta = 0$ and $\alpha = 0.1$

Exploration vs Exploitation

Exploration allows an agent to improve its current knowledge about each action, hopefully leading to long-term benefit. Improving the accuracy of the estimated action-values, enables an agent to make more informed decisions in the future.

Exploitation on the other hand, chooses the greedy action to get the most reward by exploiting the agent's current action-value estimates. But by being greedy with respect to action-value estimates, may not actually get the most reward and lead to sub-optimal behavior.

When an agent explores, it gets more accurate estimates of action-values. And when it exploits, it might get more reward. It cannot, however, choose to do both simultaneously, which is also called the exploration-exploitation dilemma.

Use of ϵ parameter:

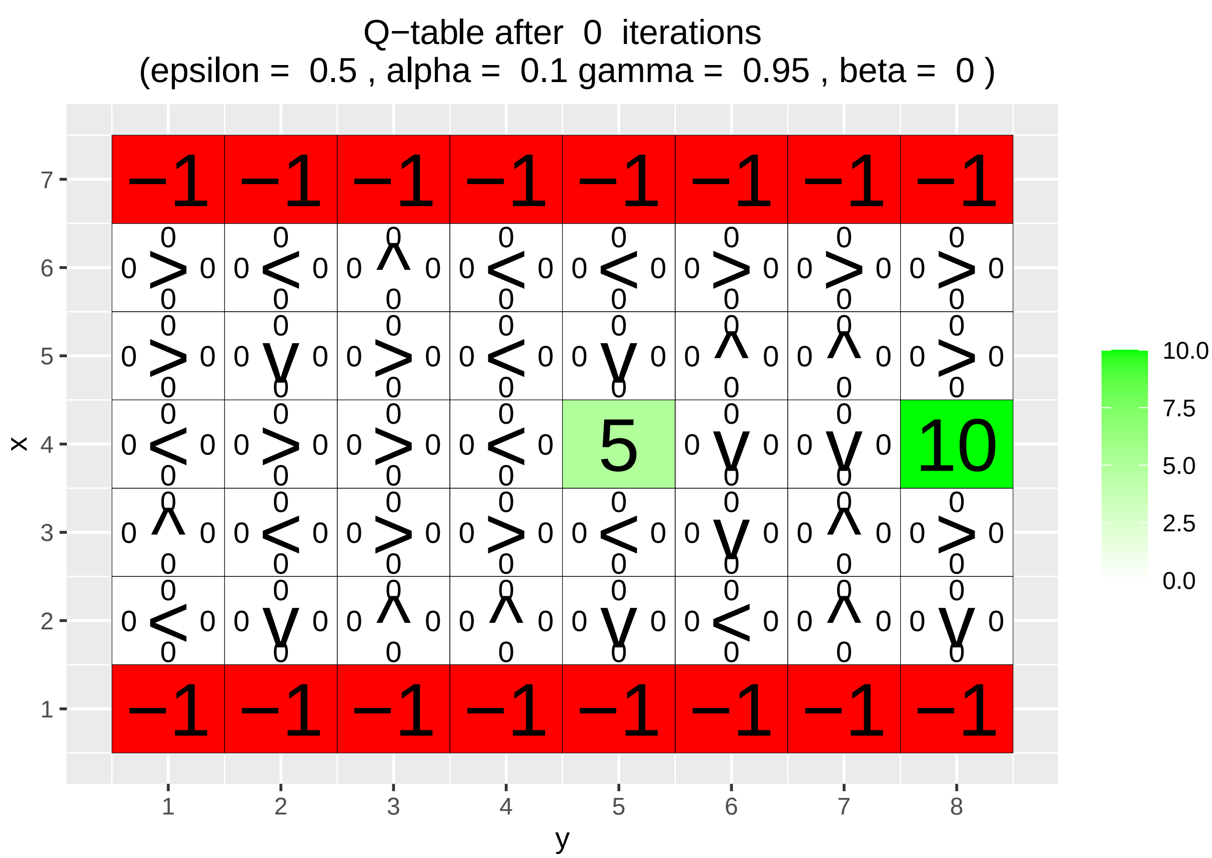
ϵ-Greedy is a simple method to balance exploration and exploitation by choosing between exploration and exploitation randomly.

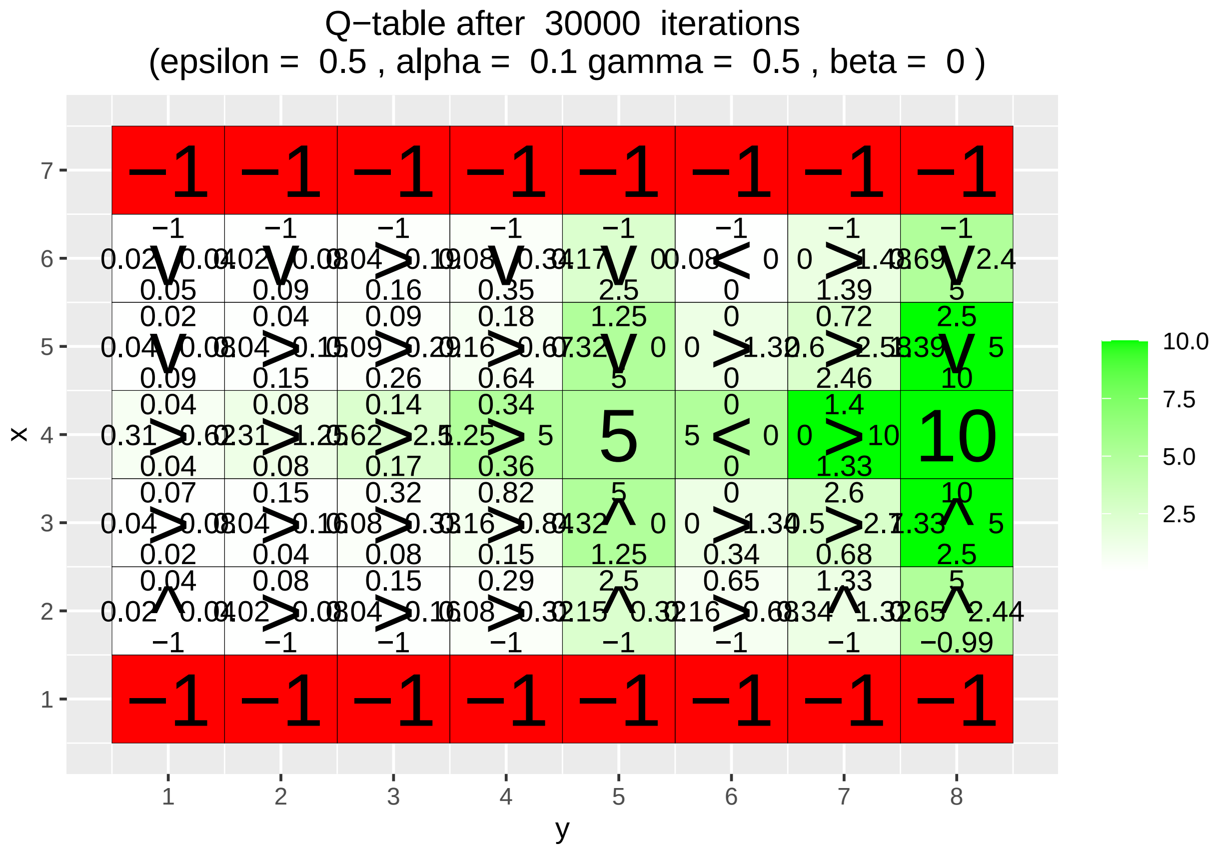
Now with probability of 1-ϵ we decide to exploit and with probability of ϵ we decide to explore.

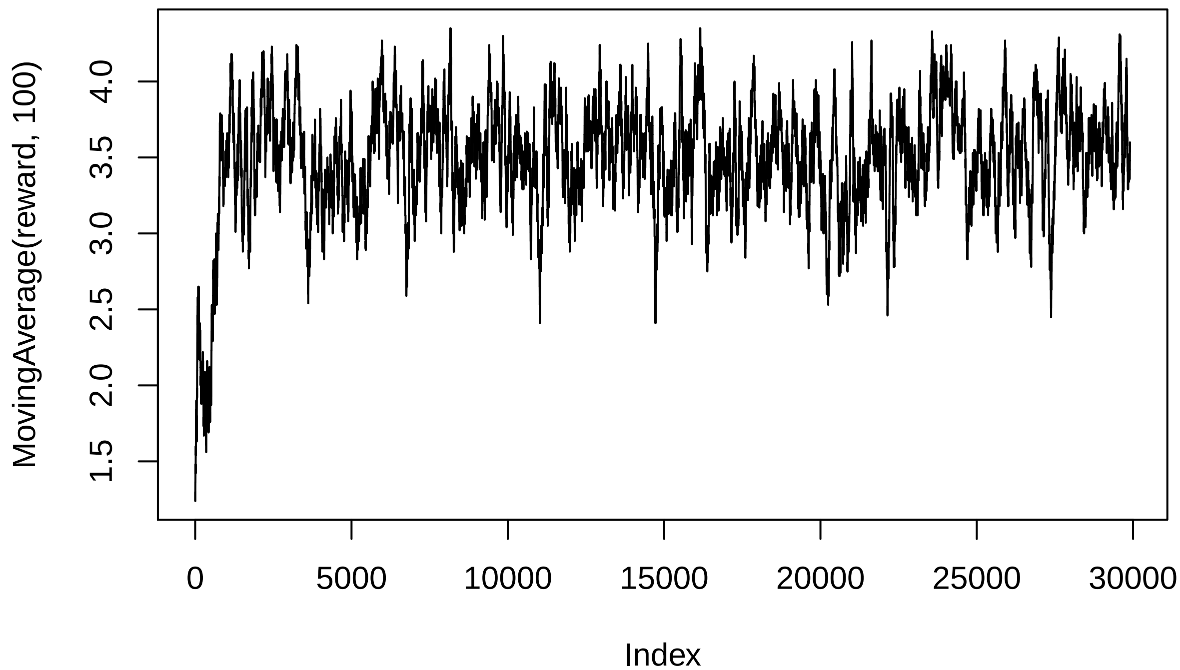
Use of γ (discount factor) parameter:

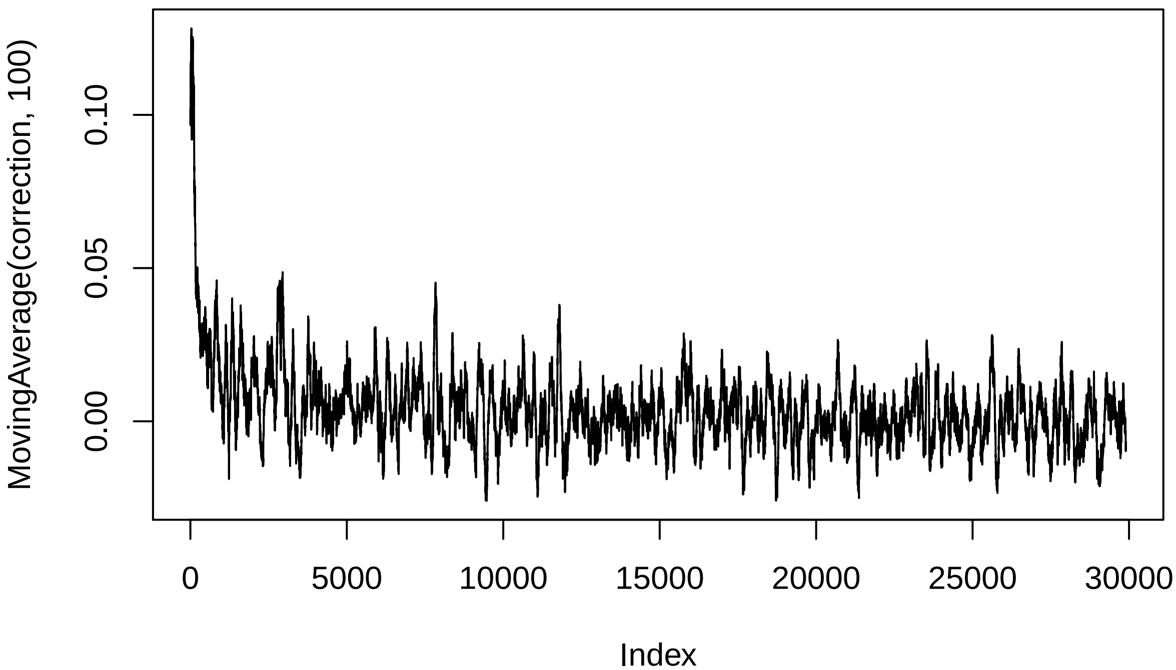
The discount factor essentially determines how much the reinforcement learning agents cares about rewards in the distant future relative to those in the immediate future. If $\gamma = 0$, the agent will be completely shortsighted and only learn about actions that produce an immediate reward. If $\gamma = 1$, the agent will evaluate each of its actions based on the sum total of all of its future rewards.

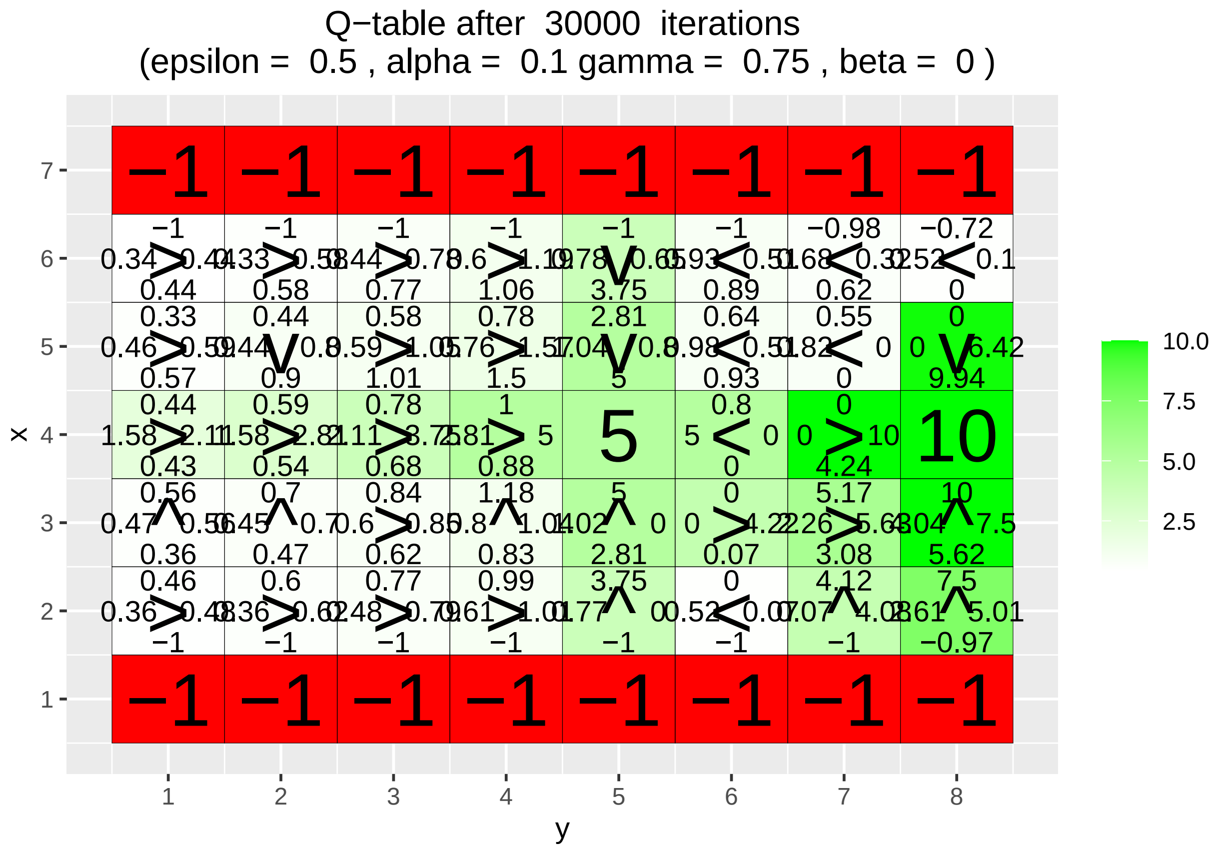
```
## Epsilon set as 0.5
```

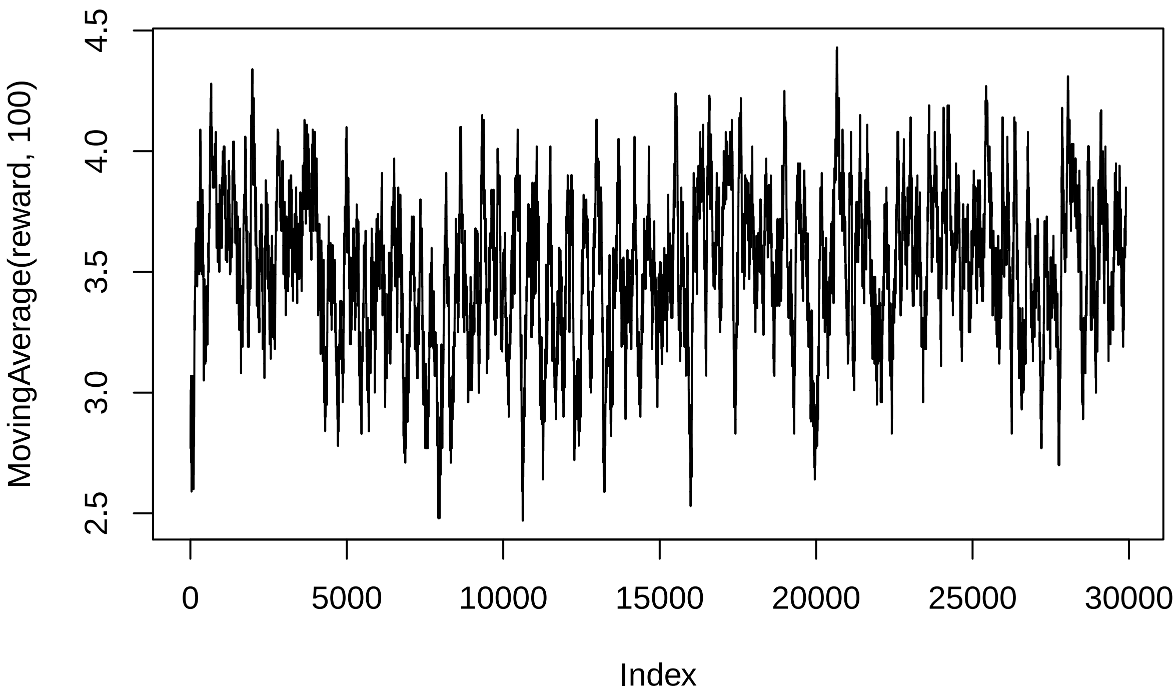



Q−table after 30000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.5 , beta = 0)
x
y
10.0
7.5
5.0
2.5

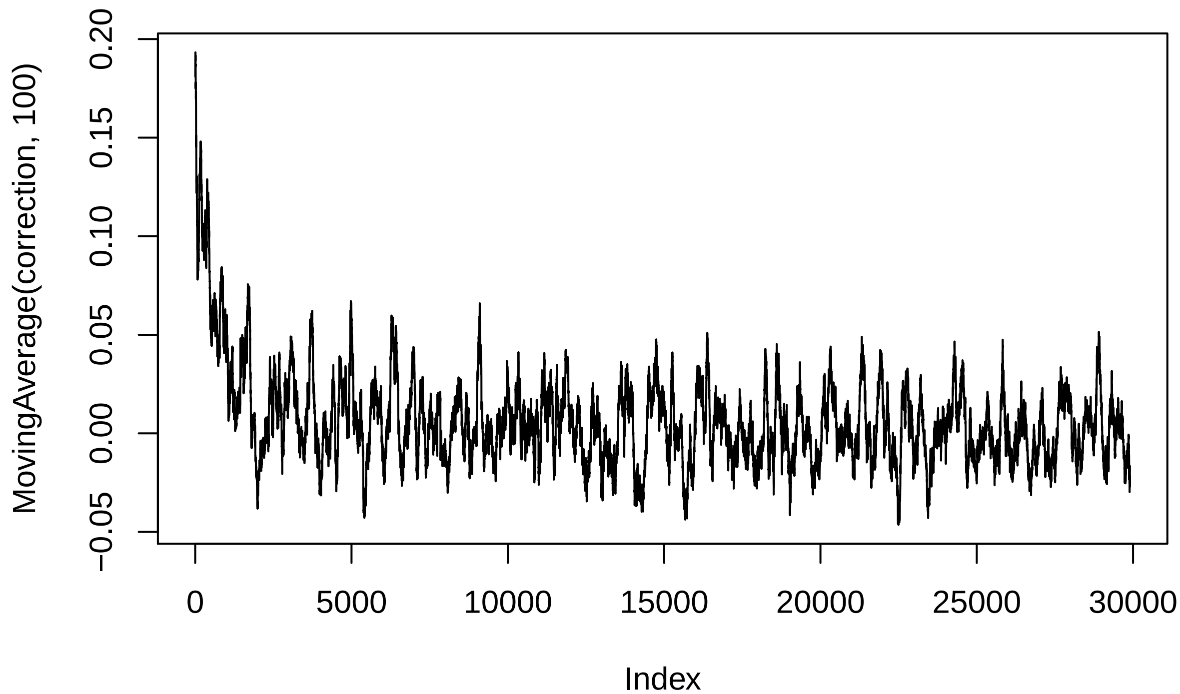

MovingAverage(reward, 100)
1.5
2.0
2.5
3.0
3.5
4.0
0
5000
10000
15000
20000
25000
30000
Index

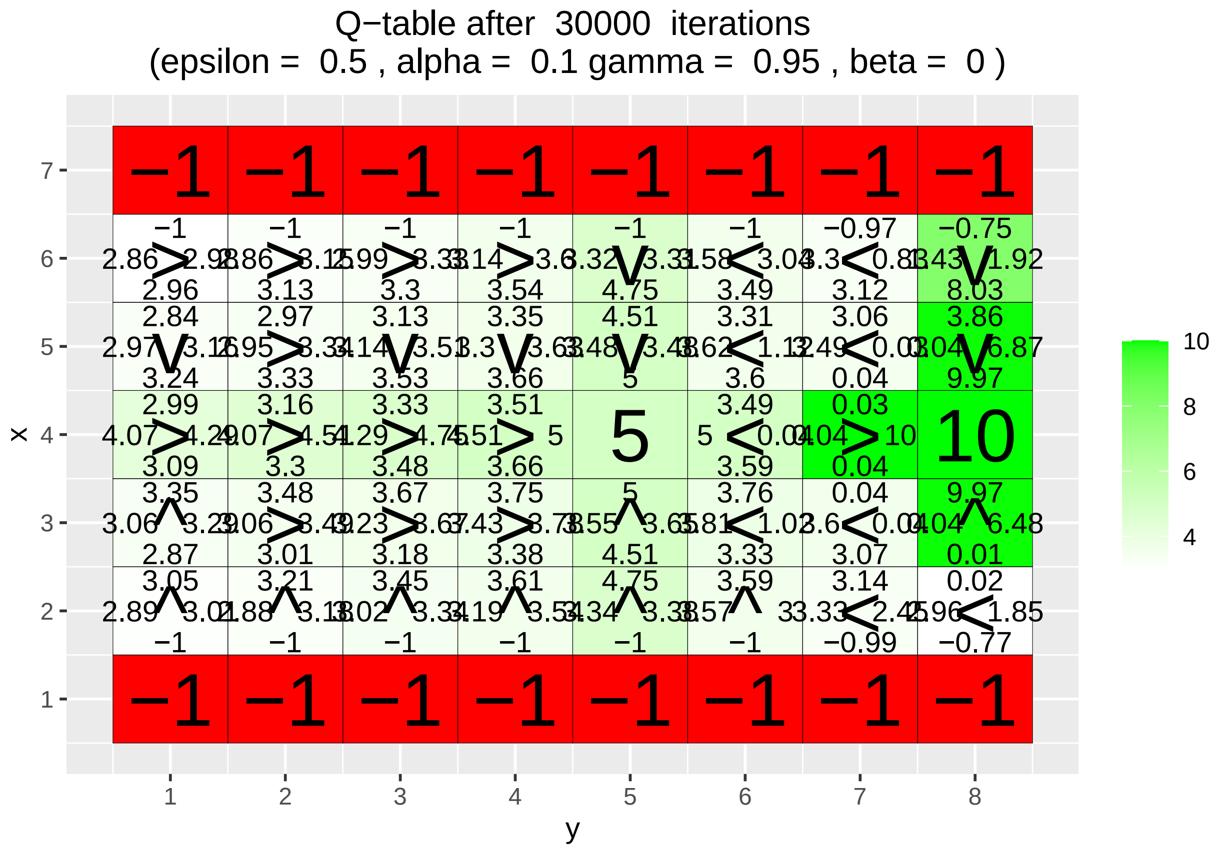

MovingAverage(correction, 100)
0.00
0.05
0.10
0
5000
10000
15000
20000
25000
30000
Index

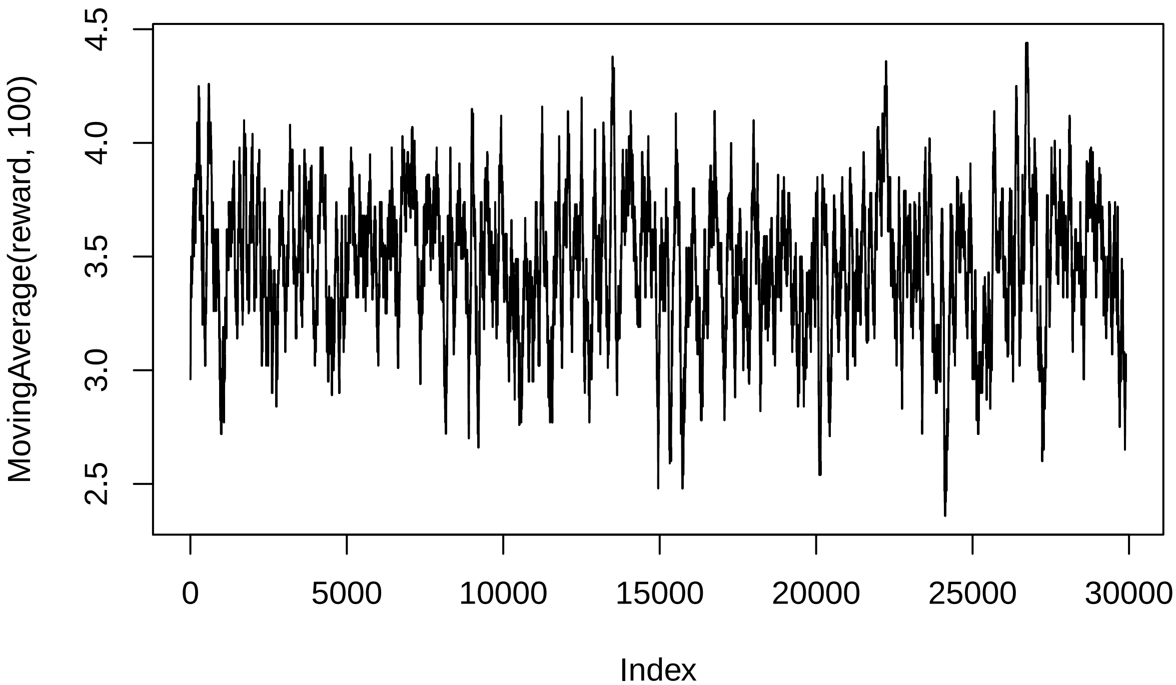

Q−table after 30000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.75 , beta = 0)
x
y
10.0
7.5
5.0
2.5

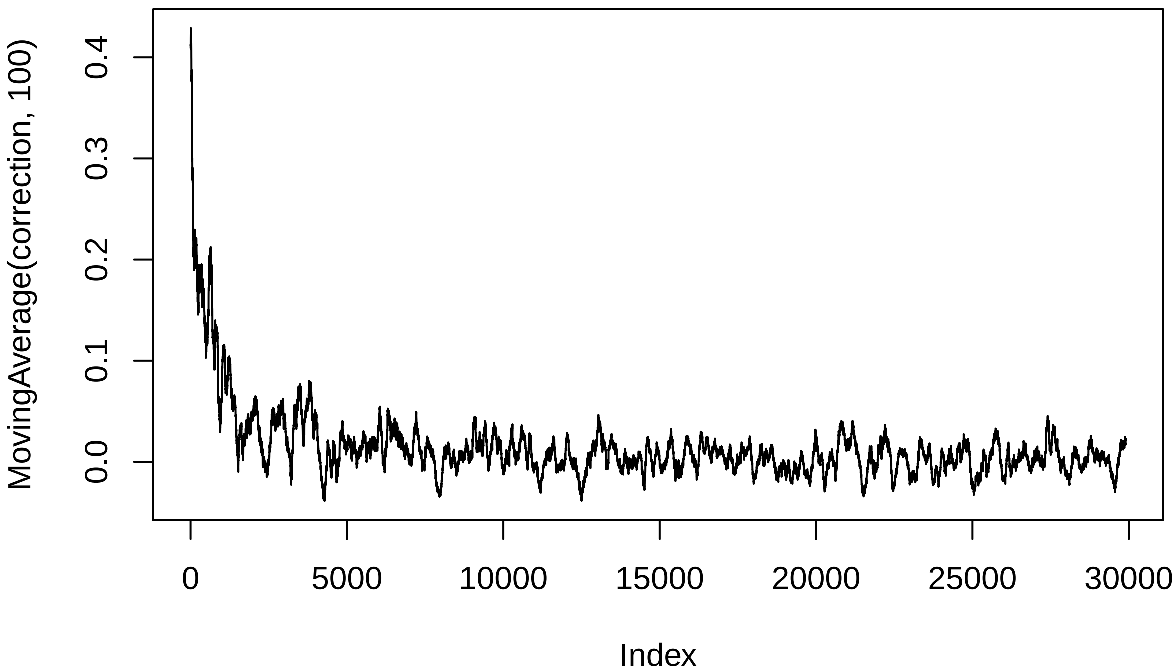


MovingAverage(reward, 100)
2.5
3.0
3.5
4.0
4.5
0
5000
10000
15000
20000
25000
30000
Index


MovingAverage(correction, 100)
−0.05
0.00
0.05
0.10
0.15
0.20
0
5000
10000
15000
20000
25000
30000
Index


Q−table after 30000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)
x
y


MovingAverage(reward, 100)
2.5
3.0
3.5
4.0
4.5
0
5000
10000
15000
20000
25000
30000
Index




```
## Epsilon set as 0.1
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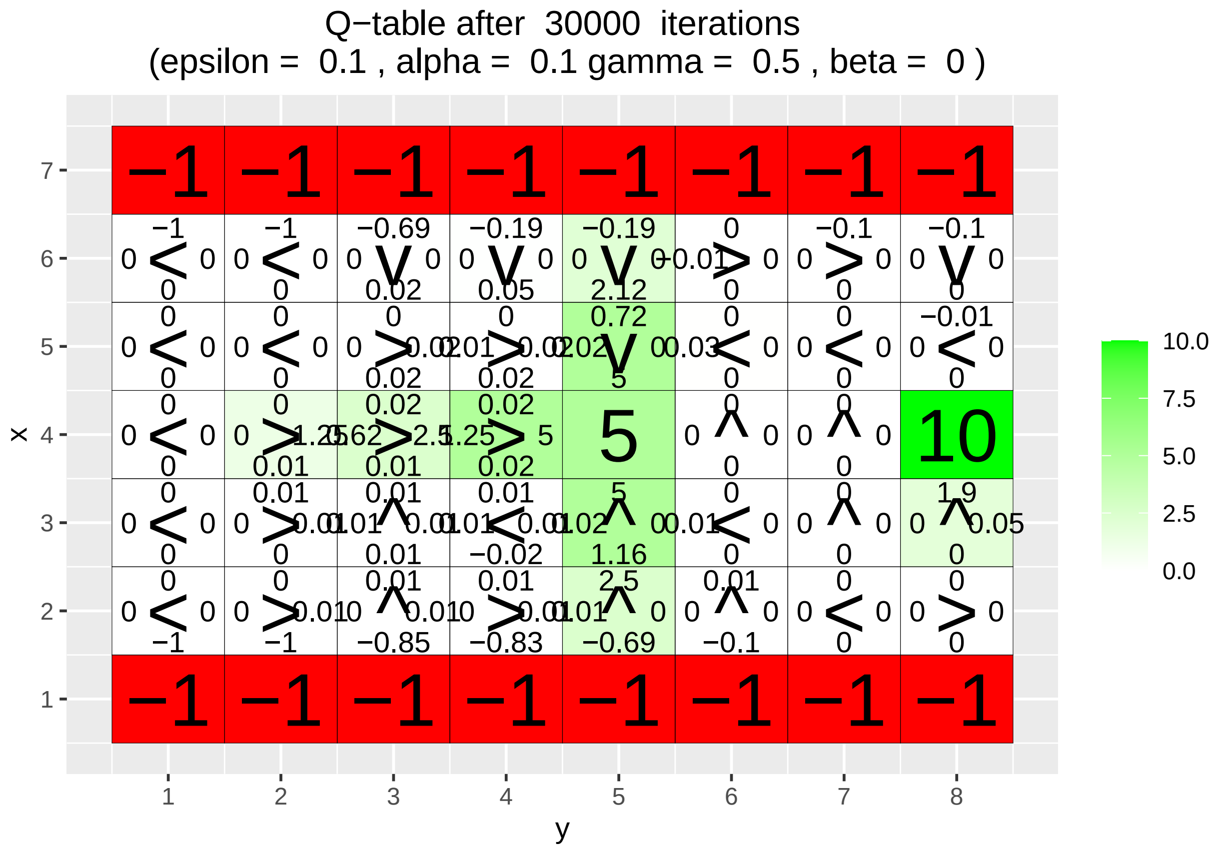
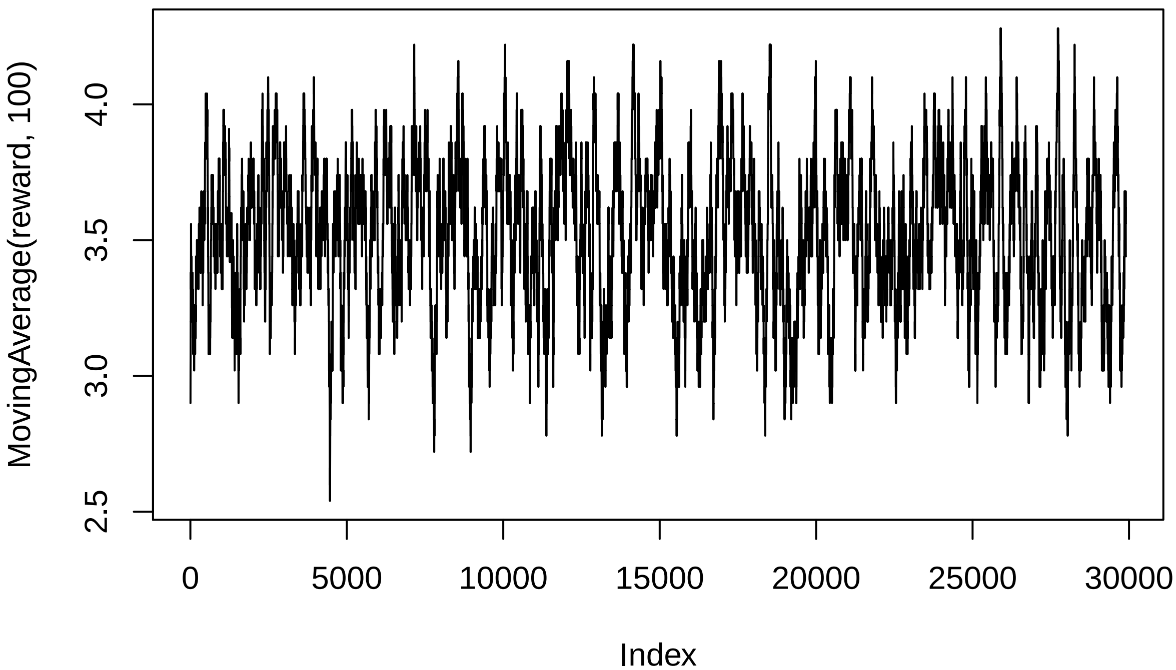
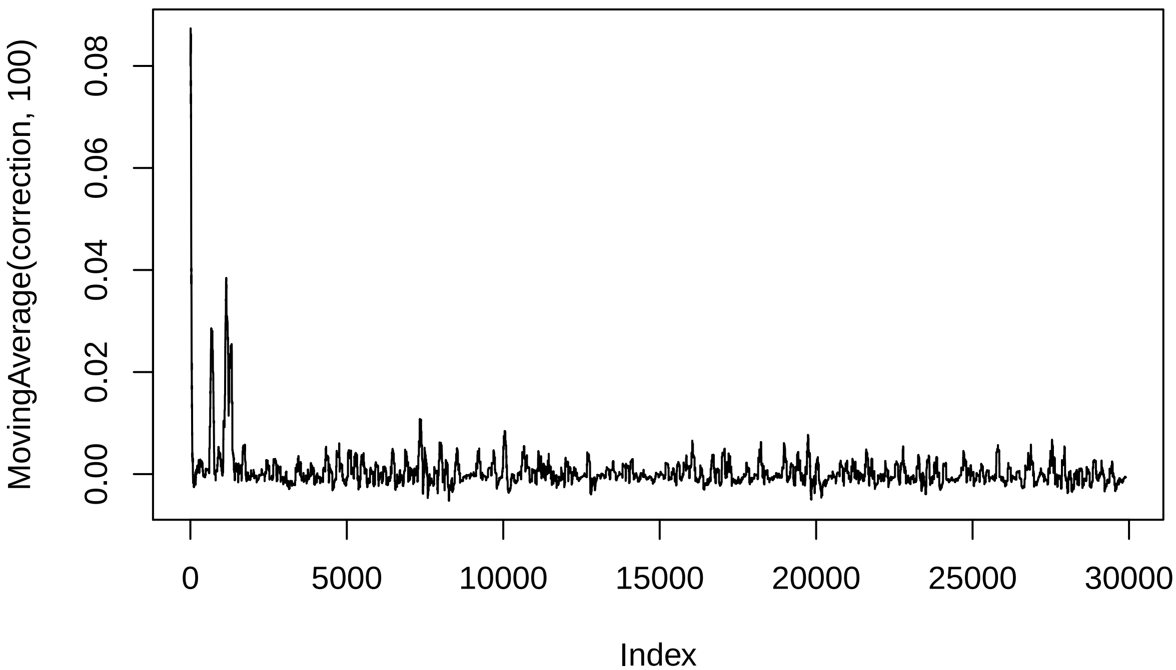
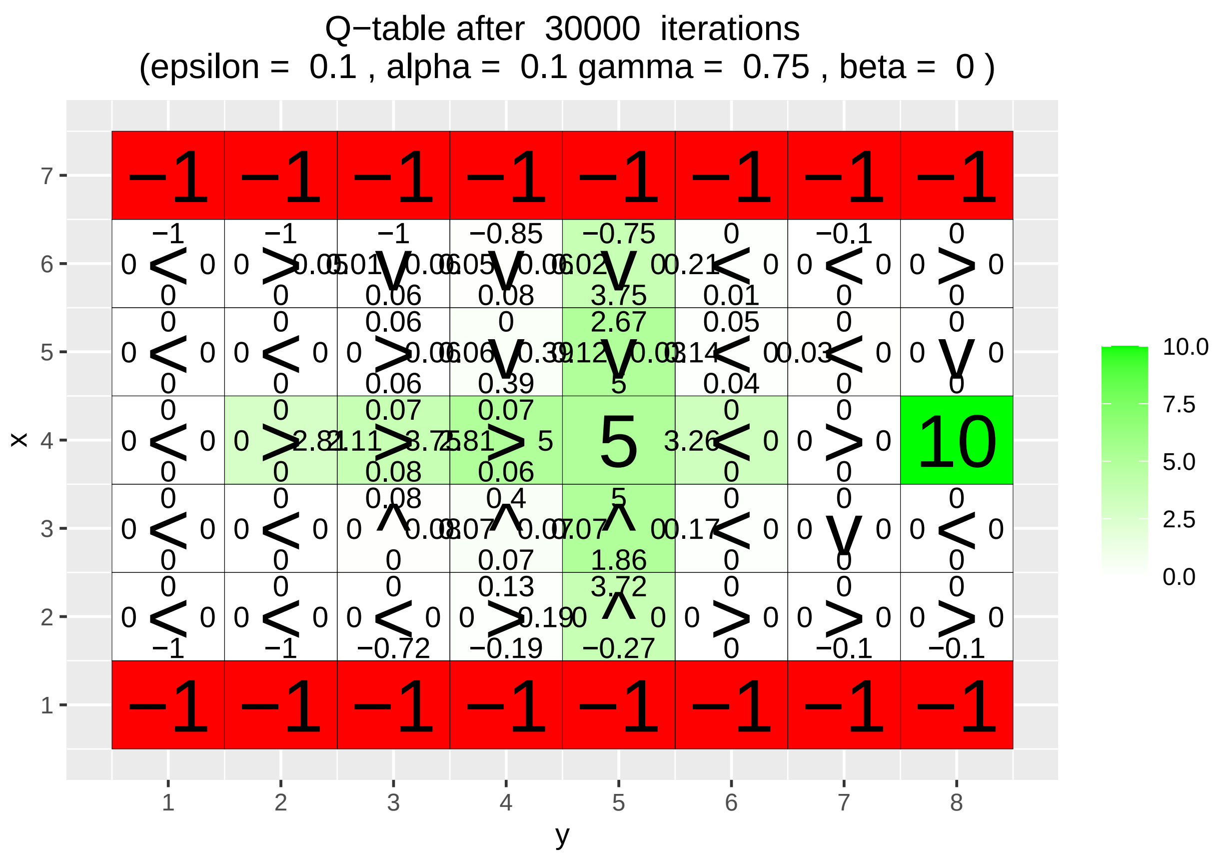


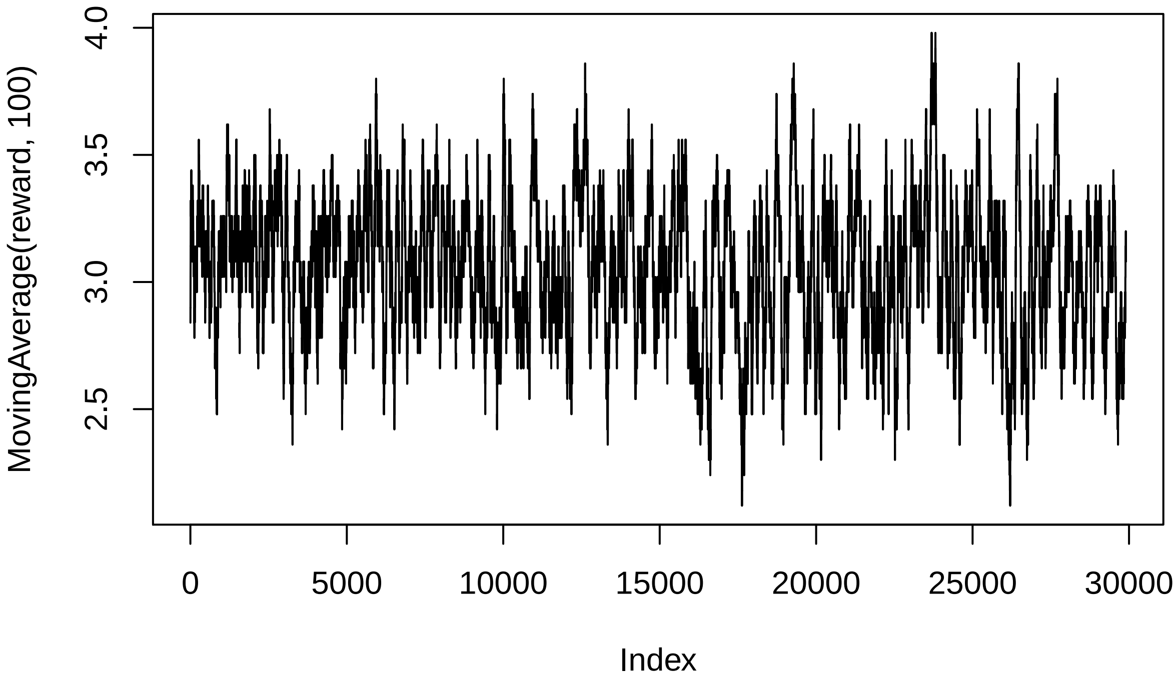
Q−table after 30000 iterations
(epsilon = 0.1 , alpha = 0.1 gamma = 0.5 , beta = 0)
x
y
10.0
7.5
5.0
2.5
0.0

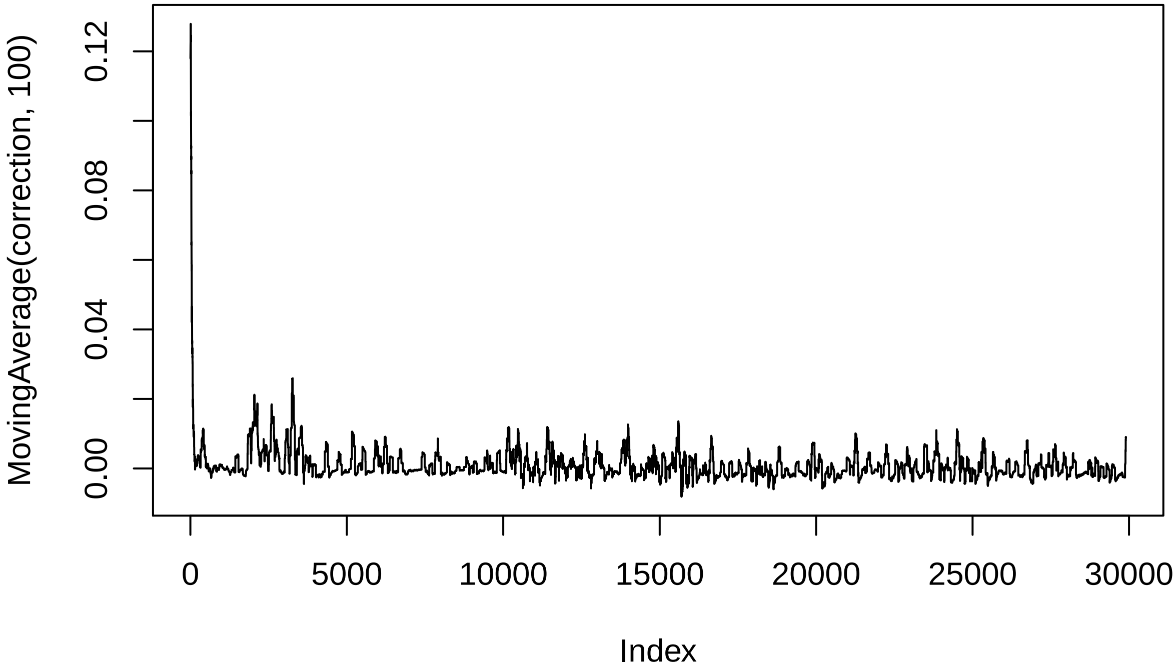

MovingAverage(reward, 100)
2.5
3.0
3.5
4.0
0
5000
10000
15000
20000
25000
30000
Index


MovingAverage(correction, 100)
0.00
0.02
0.04
0.06
0.08
0
5000
10000
15000
20000
25000
30000
Index

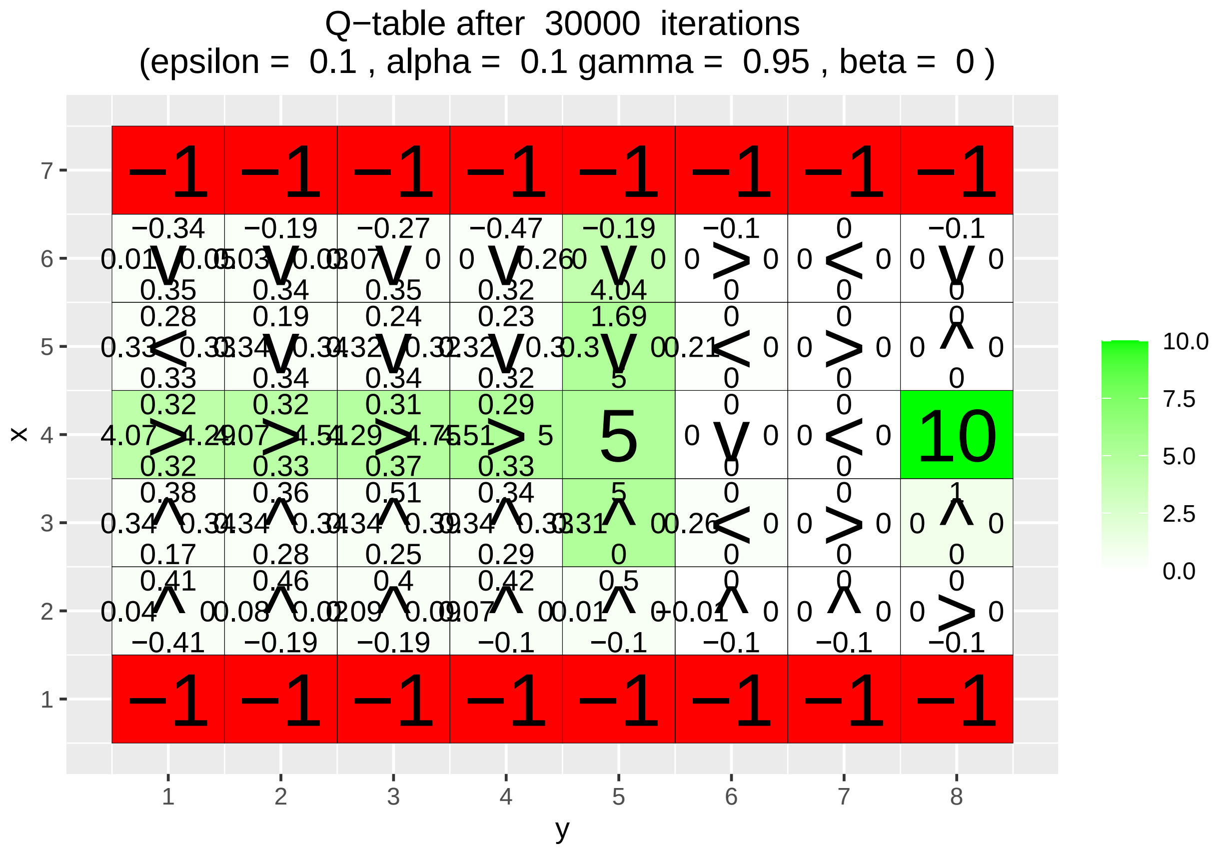


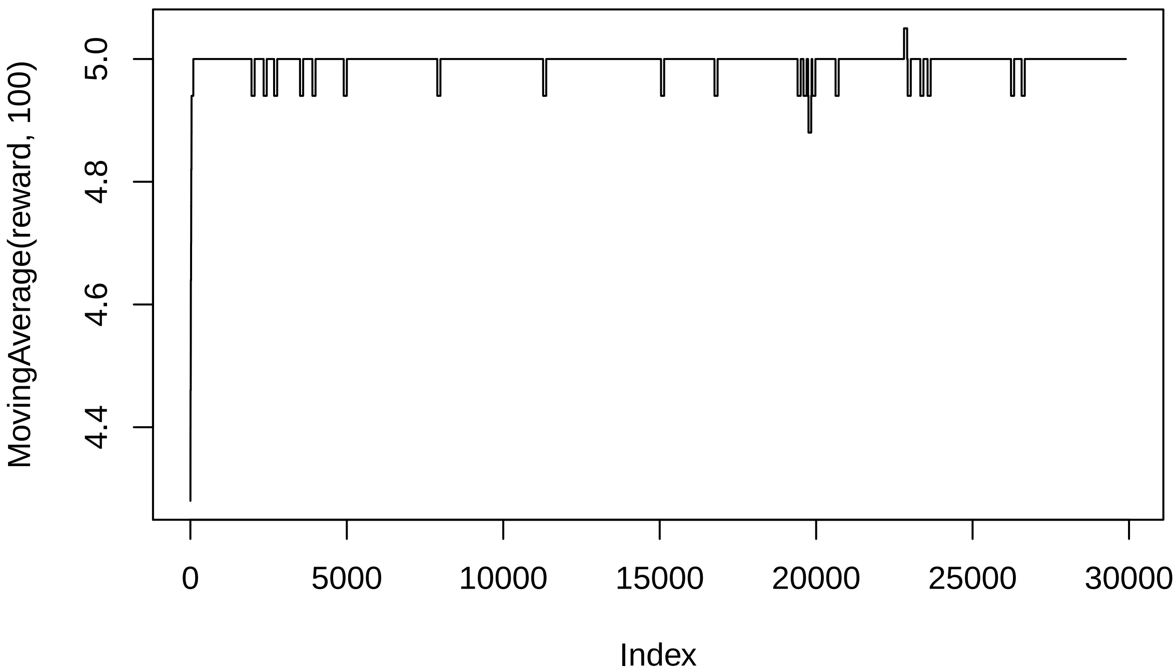
Q−table after 30000 iterations
(epsilon = 0.1 , alpha = 0.1 gamma = 0.75 , beta = 0)
x
y
10.0
7.5
5.0
2.5
0.0

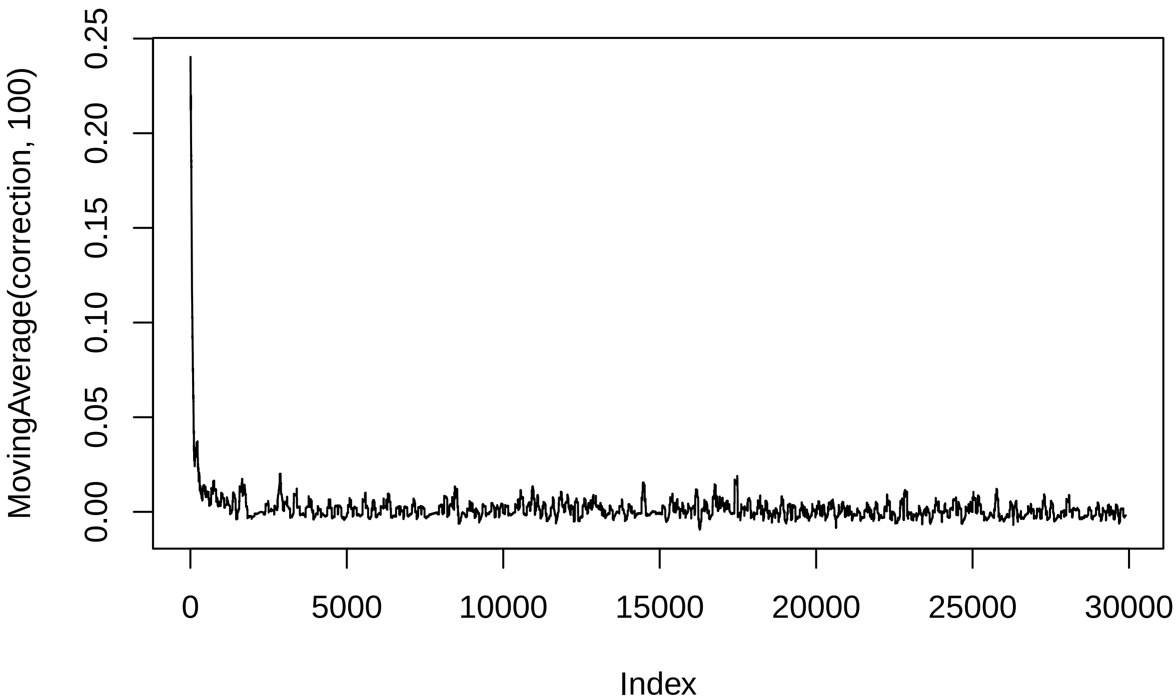

MovingAverage(reward, 100)
4.0
3.5
3.0
2.5
0
5000
10000
15000
20000
25000
30000
Index



MovingAverage(correction, 100)
0.00
0.04
0.08
0.12
0
5000
10000
15000
20000
25000
30000
Index




MovingAverage(reward, 100)
5.0
4.8
4.6
4.4
0
5000
10000
15000
20000
25000
30000
Index



Epsilon Impact

When $\epsilon = 0.5$, we give equal probability to exploit and explore thus it can be seen that we are able to find multiple directions towards high reward stages while wen we set $\epsilon = 0.1$, system is more exploiting in nature and select path with only high rewards and thus we are not able to find multiple directions towards high reward stages which can be seen by less number of green grids when $\epsilon = 0.1$.

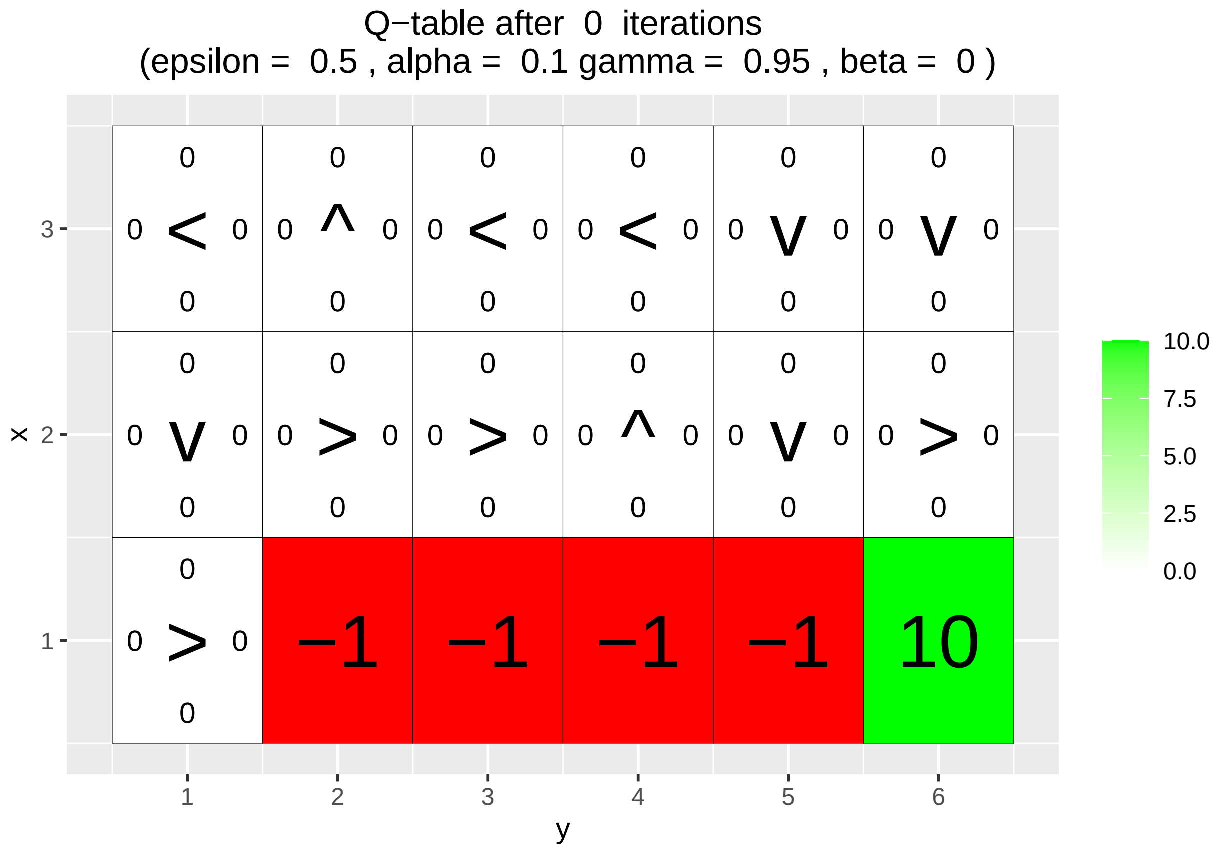
Gamma Impact

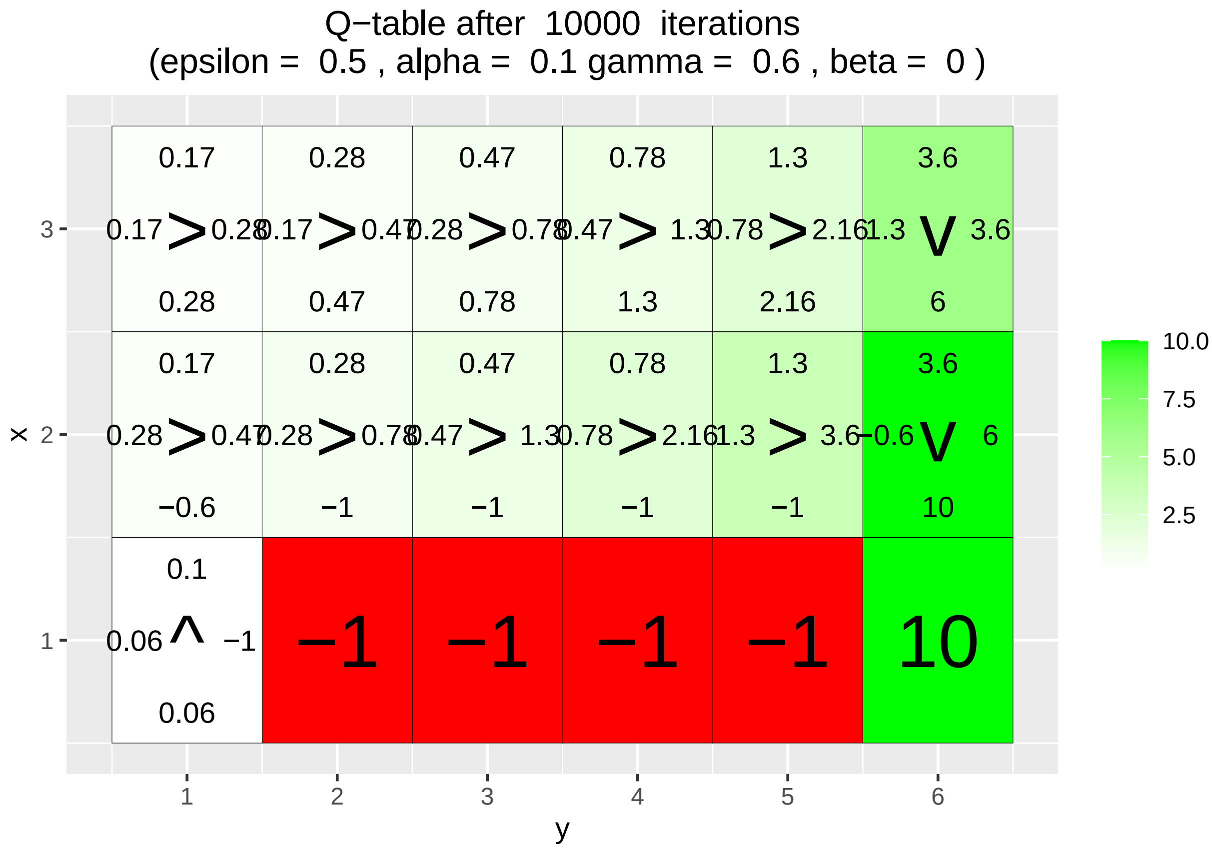
With different values of γ (discount factor) it can be seen that correction is low for $\gamma = 0.5$ while since it is looking for high reward in near future only and when we increase this to $\gamma = 0.75$ we can see higher variance in correction as now we are more interested in long term rewards than short term one and when we increase it $\gamma = 0.95$, we are giving very very high value to future rewards and thus system is more tuned for future rewards.

Environment C (the effect of beta)

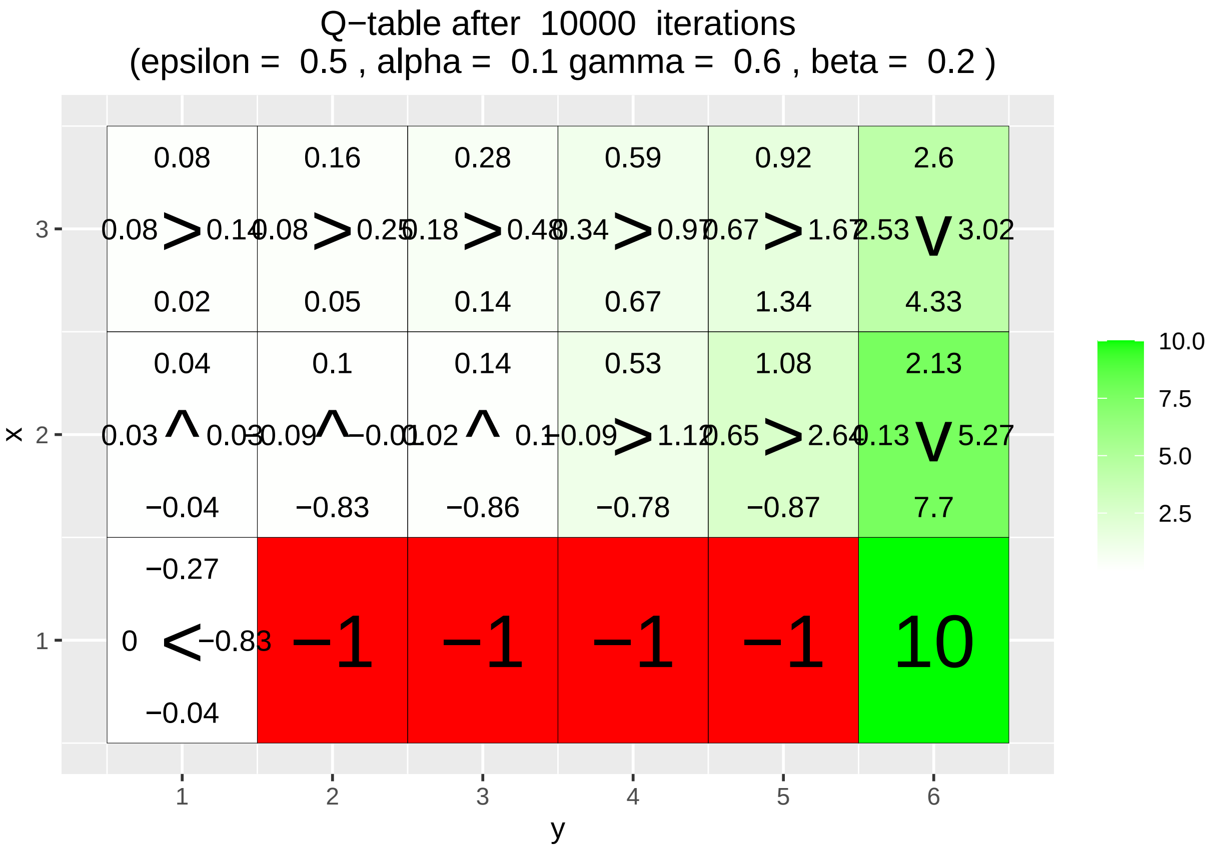
Your task is to investigate how the β parameter affects the learned policy by running 10000 episodes of Q-learning with $\beta = 0$, 0.2, 0.4, 0.66, $\epsilon = 0.5$, $\gamma = 0.6$ and $\alpha = 0.1$

Since the transition model is defined by the agent moving in the direction chosen with probability $(1-\beta)$. The agent might also slip and end up moving in the direction to the left or right of its chosen action, each with probability $\beta/2$

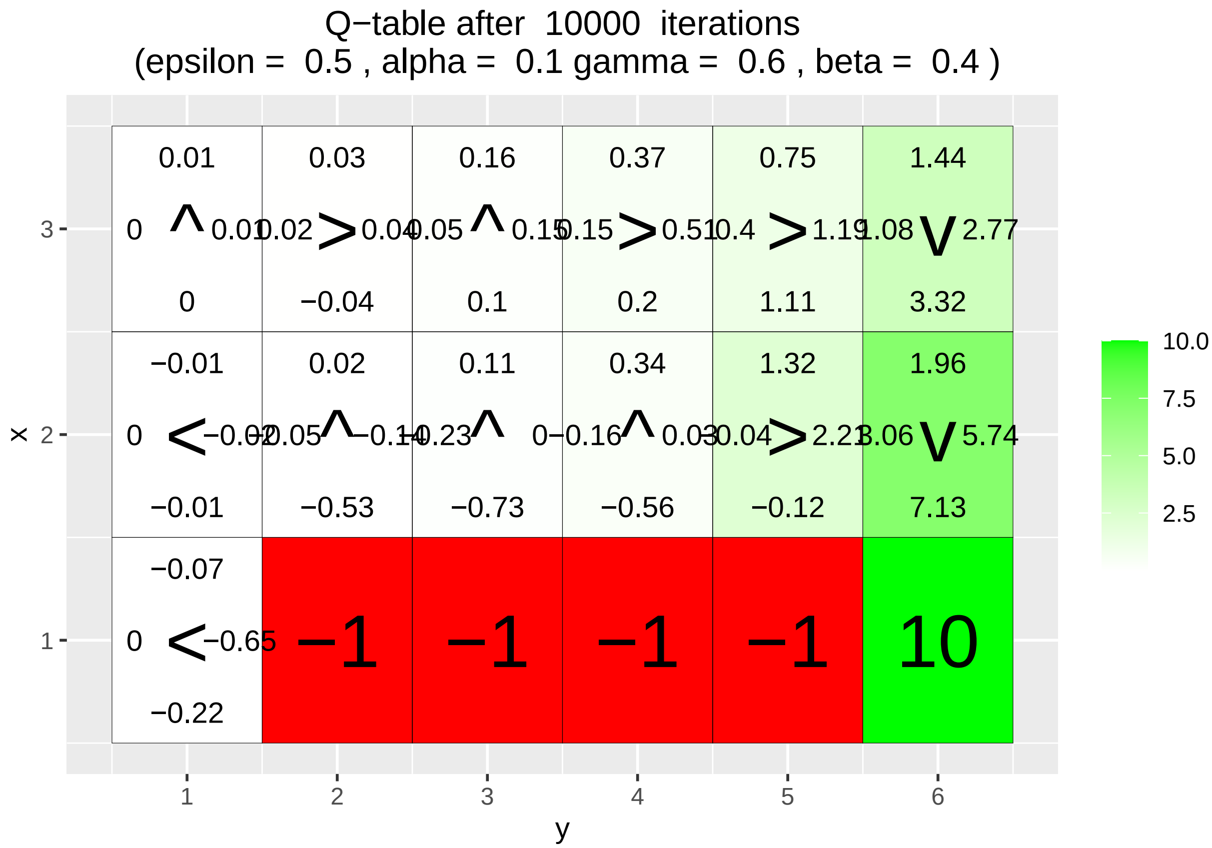

Q−table after 0 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.95 , beta = 0)
x
y
−1
−1
−1
−1
10
10.0
7.5
5.0
2.5
0.0

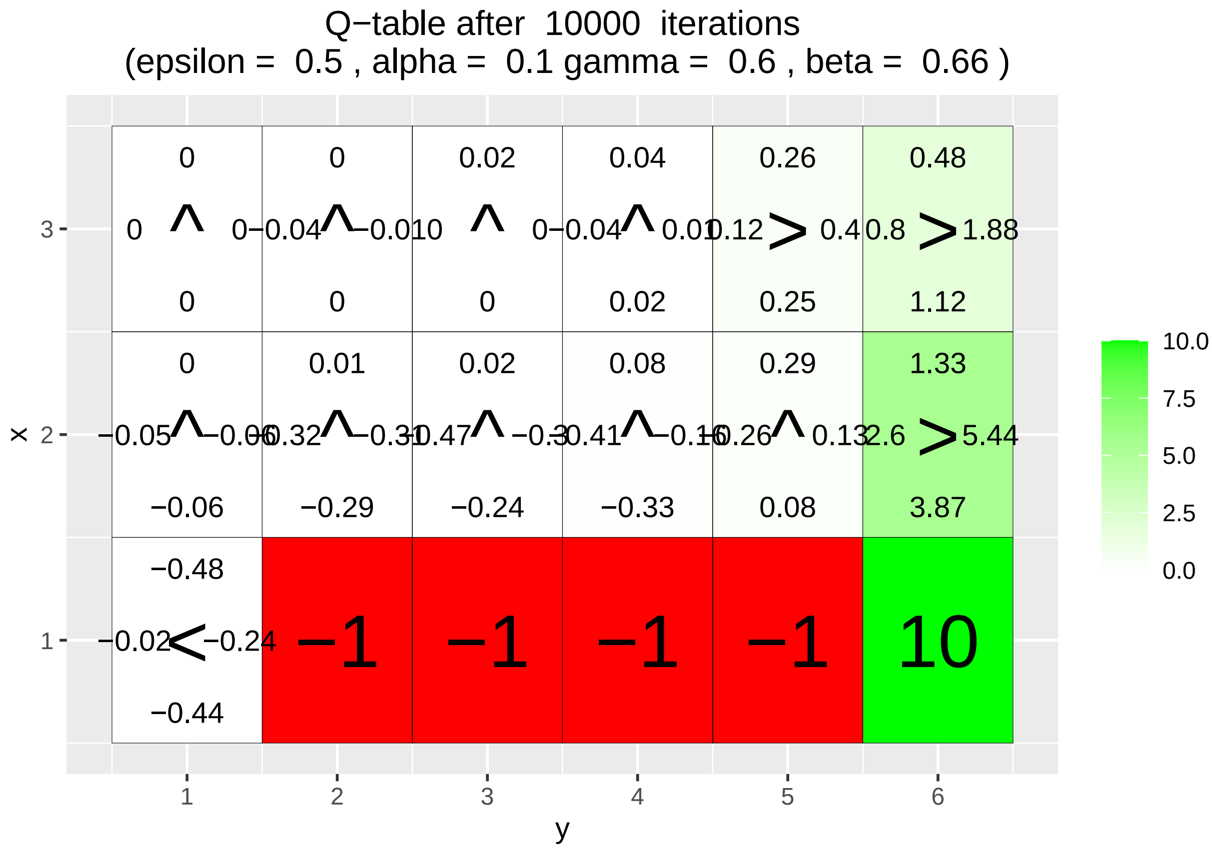


Q−table after 10000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.6 , beta = 0)
x
y
10.0
7.5
5.0
2.5



Q−table after 10000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.6 , beta = 0.2)
0.08
0.08 > 0.14
0.02
0.16
0.08 > 0.25
0.05
0.28
0.18 > 0.48
0.14
0.59
0.34 > 0.97
0.67
0.92
0.67 > 1.67
1.34
2.6
2.53 V 3.02
4.33
0.04
0.03 ^ 0.03
−0.04
0.1
−0.09 ^ −0.01
−0.83
0.14
0.02 ^ 0.1
−0.86
0.53
−0.09 > 1.12
−0.78
1.08
0.65 > 2.64
−0.87
2.13
0.13 V 5.27
7.7
−0.27
0 < −0.83
−0.04
−1
−1
−1
−1
10
x
3
2
1
y
1
2
3
4
5
6
10.0
7.5
5.0
2.5


Q−table after 10000 iterations
(epsilon = 0.5 , alpha = 0.1 gamma = 0.6 , beta = 0.4)
x
y
−1
10
10.0
7.5
5.0
2.5



As we are increasing β, we can see that model will not move in chosen direction and thus its movement is getting more and more random and with β at 0.6, it will mostly move towards right or left of chosen direction thus it might get harder for agent to reach optimal state in grid.

Thus β value can be used to cover uncertainty about action taken by agent in any given environment.

Question 2

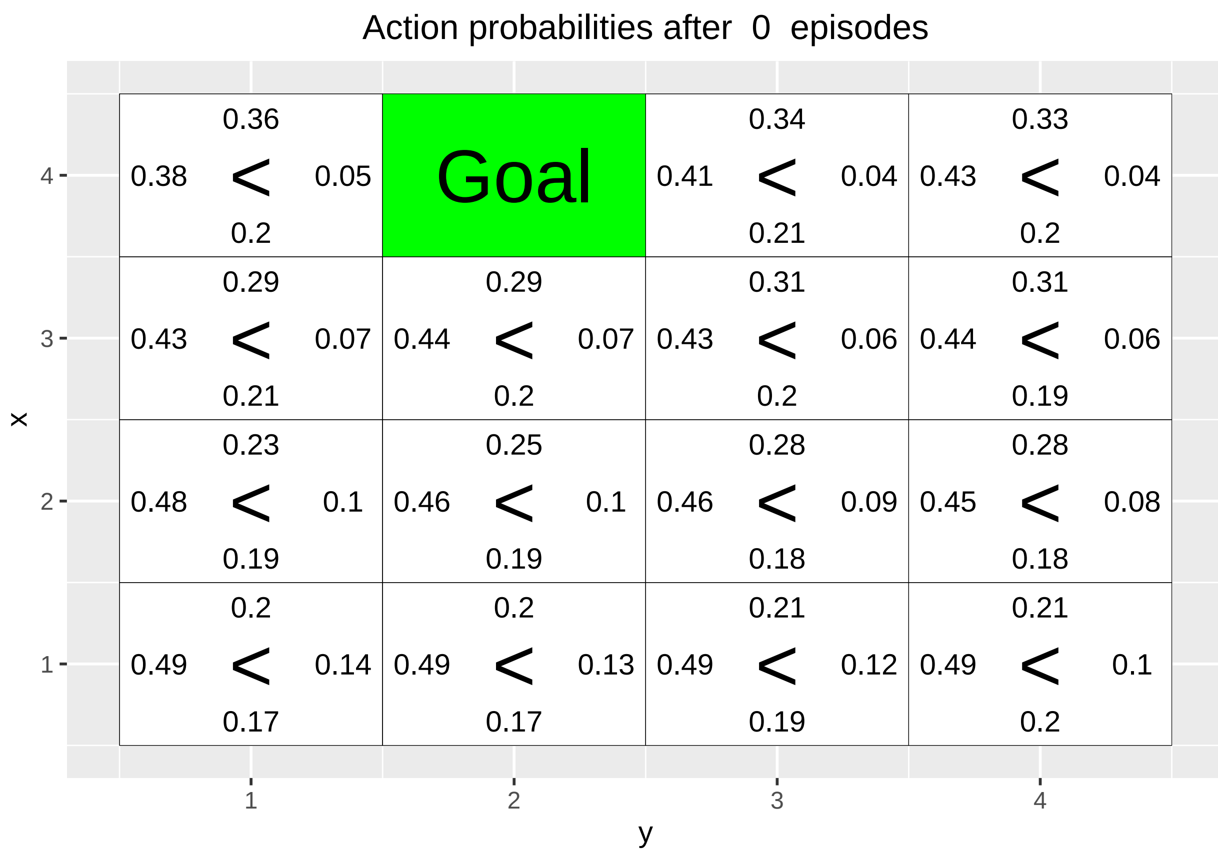
Reinforcement Learning

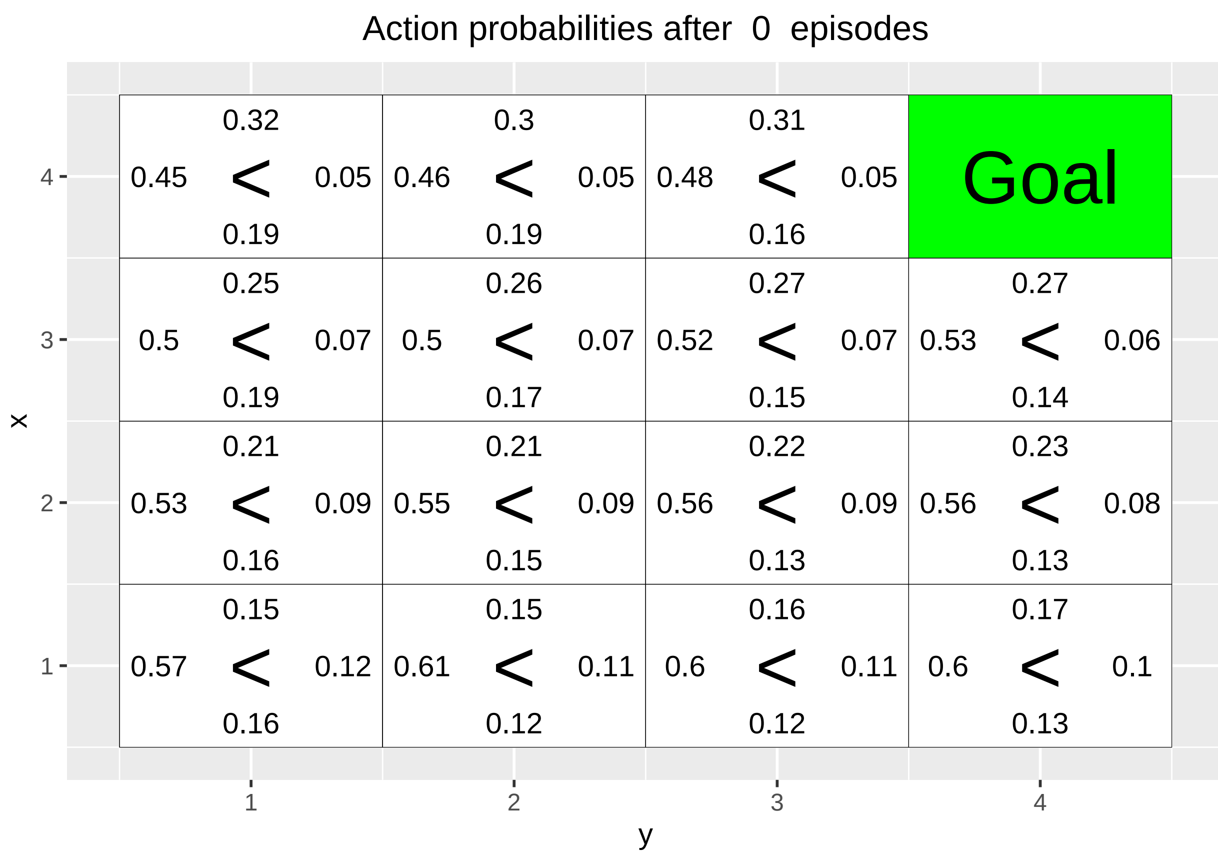
Task Setup

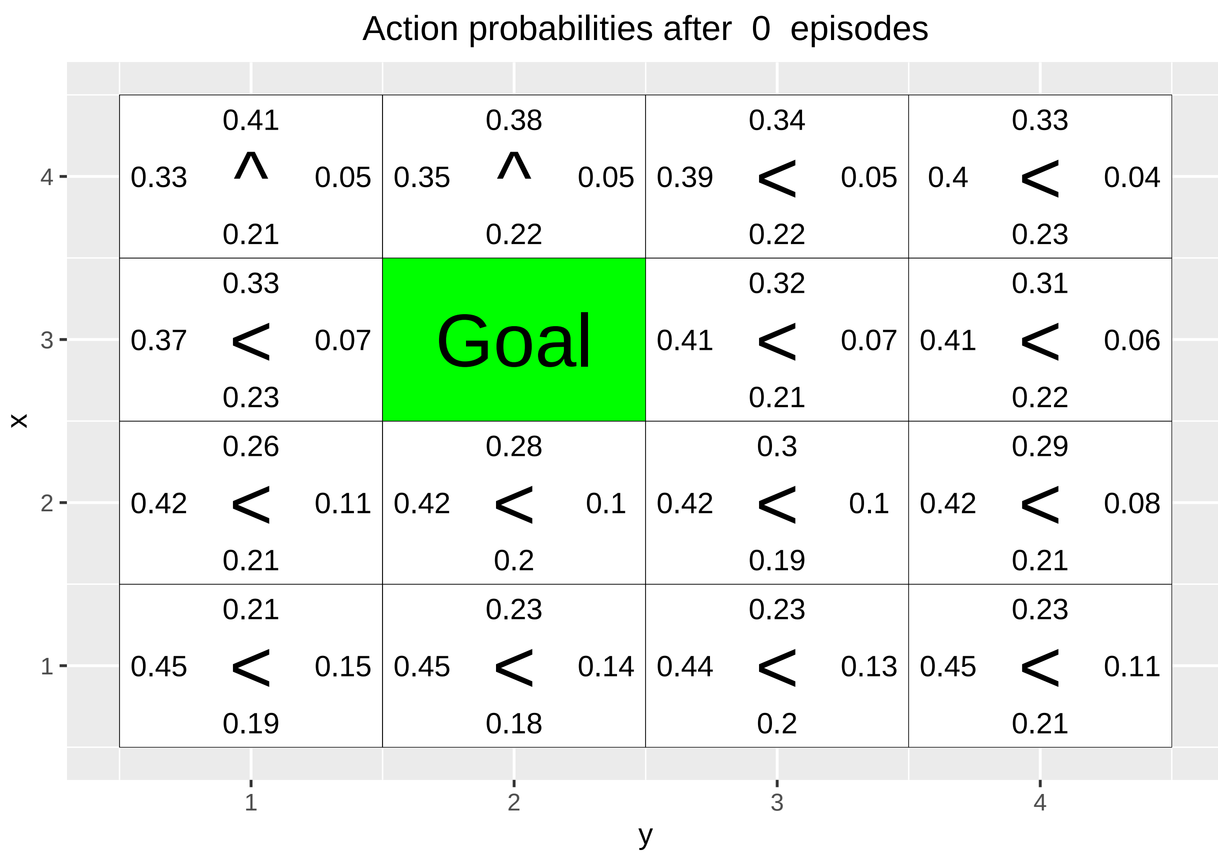
We will work with a 4×4 grid. We want the agent to learn to navigate to a random goal position in the grid. The agent will start in a random position and it will be told the goal position. The agent receives a reward of 5 when it reaches the goal. Since the goal position can be any position, we need a way to tell the agent where the goal is. Since our agent does not have any memory mechanism, we provide the goal coordinates as part of the state at every time step, i.e. a state consists now of four coordinates: Two for the position of the agent, and two for the goal position. The actions of the agent can however only impact its own position, i.e. the actions do not modify the goal position. Note that the agent initially does not know that the last two coordinates of a state indicate the position with maximal reward, i.e. the goal position. It has to learn it. It also has to learn a policy to reach the goal position from the initial position. Moreover, the policy has to depend on the goal position, because it is chosen at random in each episode. Since we only have a single non-zero reward, we do not specify a reward map. Instead, the goal coordinates are passed to the functions that need to access the reward function.

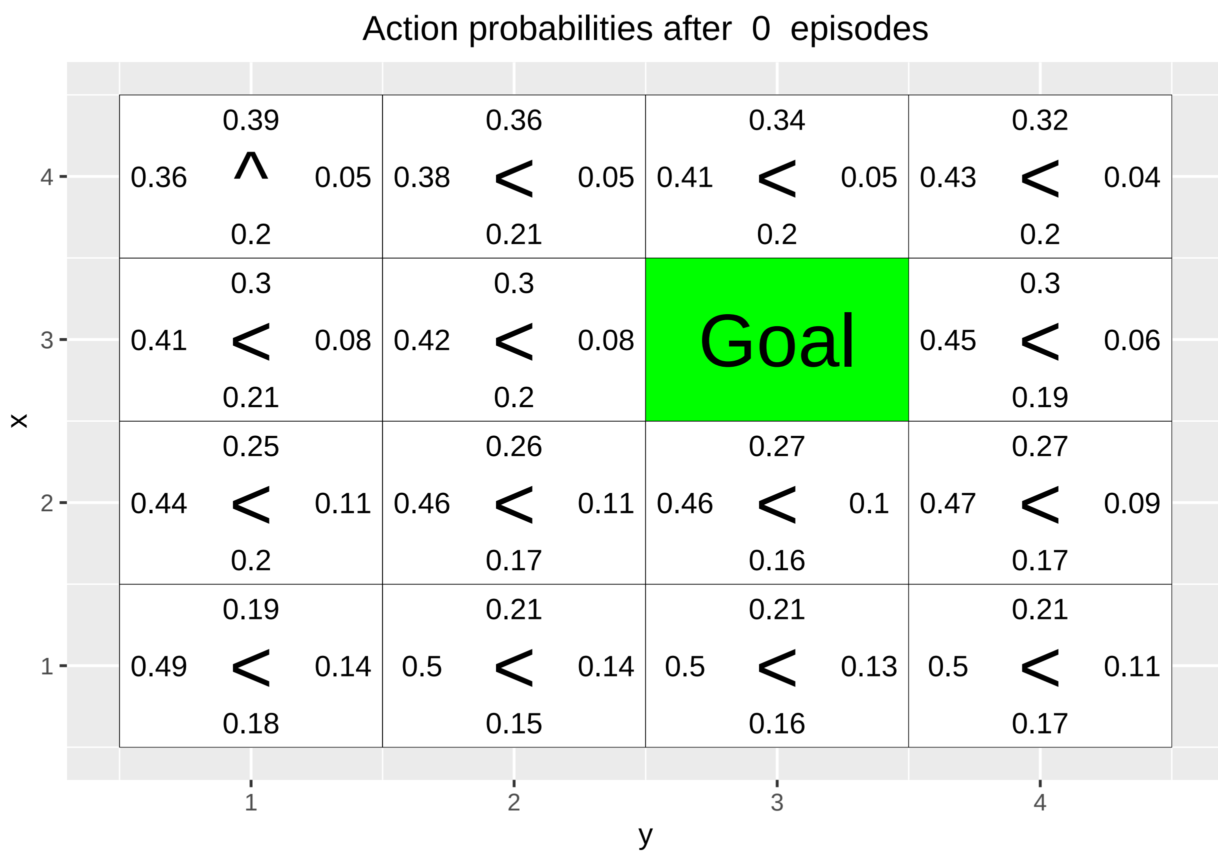
Environment D (training with random goal positions)

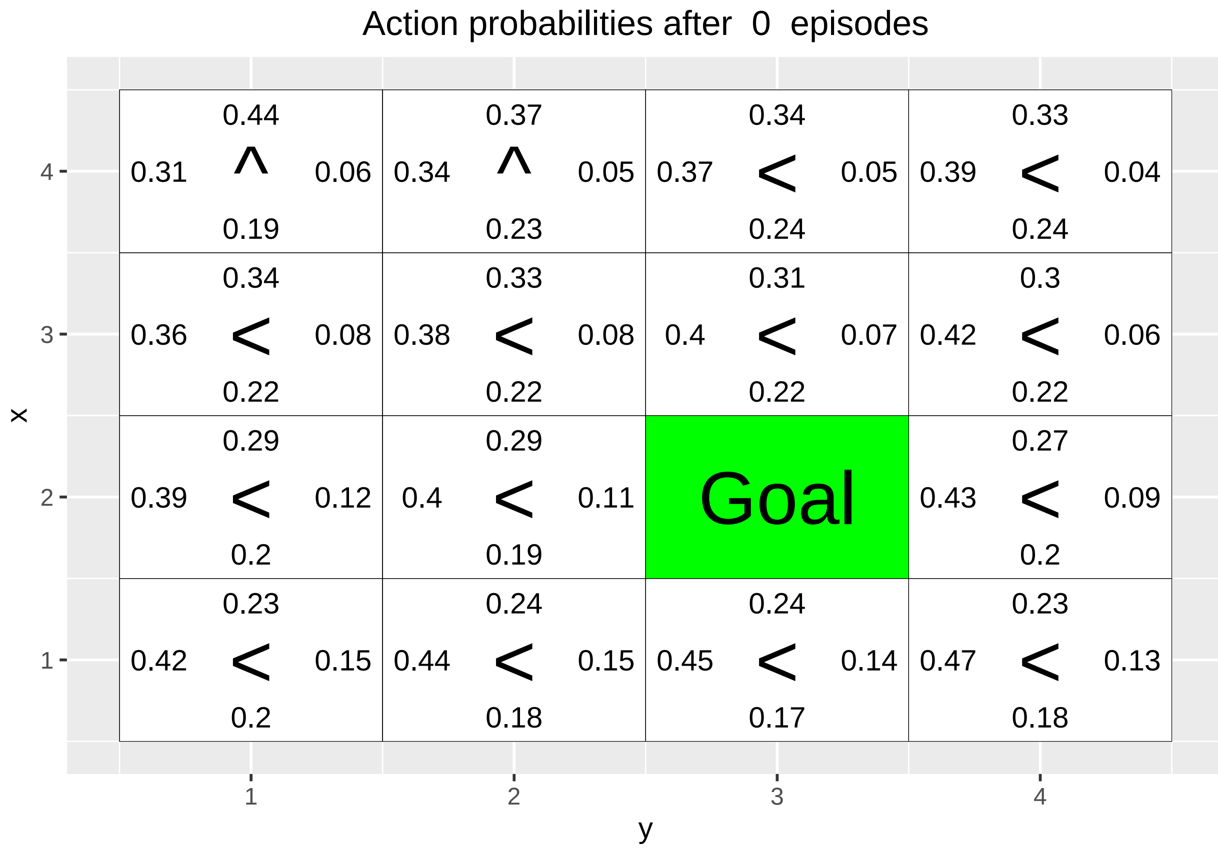
Task: In this task, we will use eight goal positions for training and, then, validate the learned policy on the remaining eight possible goal positions. The training and validation goal positions are stored in the lists train goals and val goals.

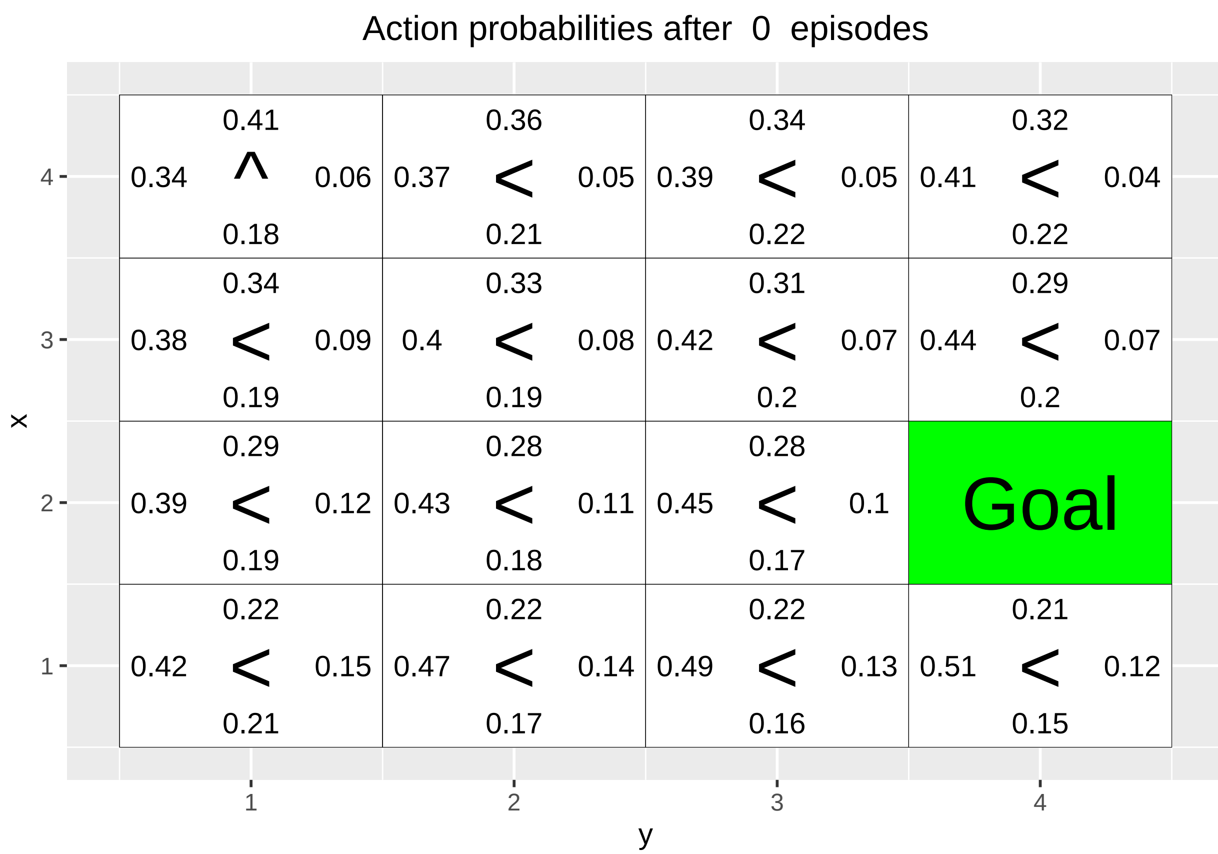



Action probabilities after 0 episodes
0.32
0.45 0.05
0.19
0.3
0.46 0.05
0.19
0.31
0.48 0.05
0.16
Goal
0.25
0.5 0.07
0.19
0.26
0.5 0.07
0.17
0.27
0.52 0.07
0.15
0.27
0.53 0.06
0.14
0.21
0.53 0.09
0.16
0.21
0.55 0.09
0.15
0.22
0.56 0.09
0.13
0.23
0.56 0.08
0.13
0.15
0.57 0.12
0.16
0.15
0.61 0.11
0.12
0.16
0.6 0.11
0.12
0.17
0.6 0.1
0.13
x
y
1
2
3
4

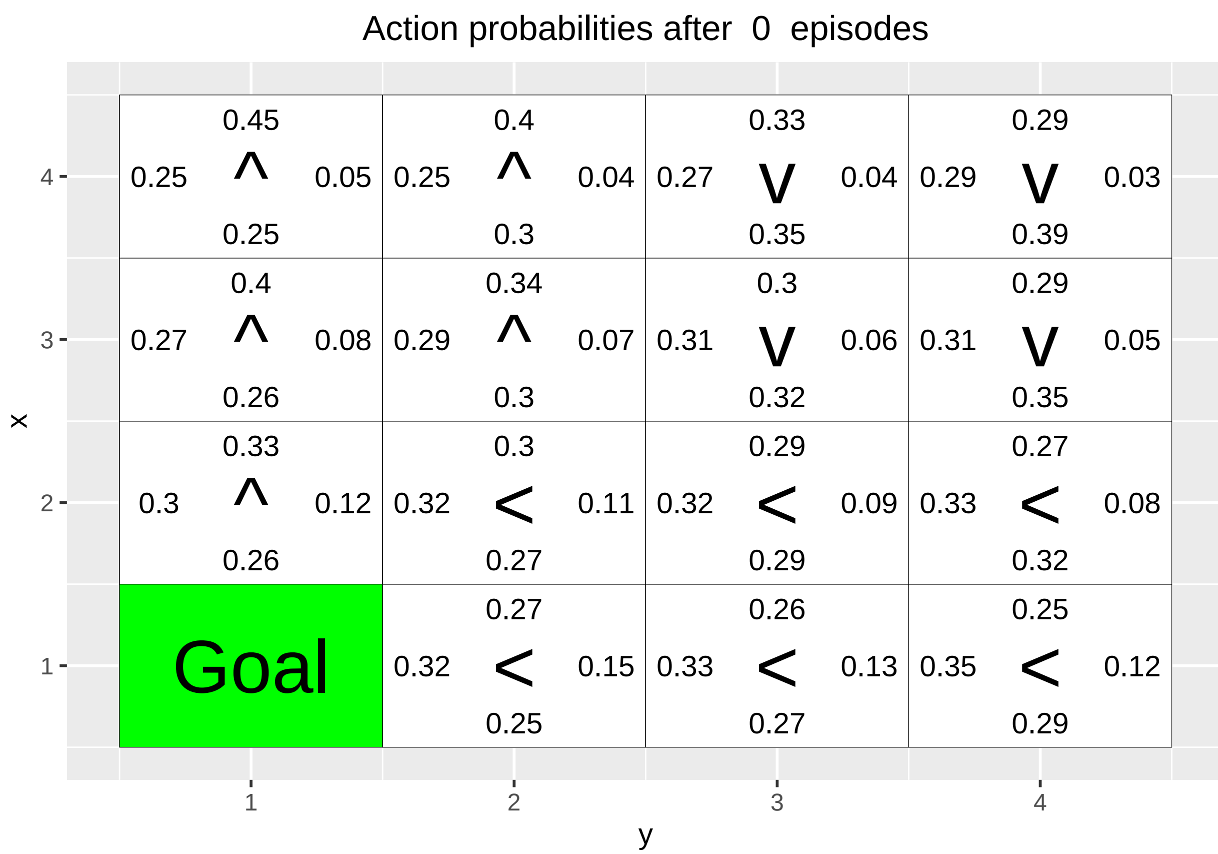

Action probabilities after 0 episodes
0.41
0.33 ^ 0.05
0.21
0.38
0.35 ^ 0.05
0.22
0.34
0.39 < 0.05
0.22
0.33
0.4 < 0.04
0.23
0.33
0.37 < 0.07
0.23
Goal
0.32
0.41 < 0.07
0.21
0.31
0.41 < 0.06
0.22
0.26
0.42 < 0.11
0.21
0.28
0.42 < 0.1
0.2
0.3
0.42 < 0.1
0.19
0.29
0.42 < 0.08
0.21
0.21
0.45 < 0.15
0.19
0.23
0.45 < 0.14
0.18
0.23
0.44 < 0.13
0.2
0.23
0.45 < 0.11
0.21
x
y
1
2
3
4

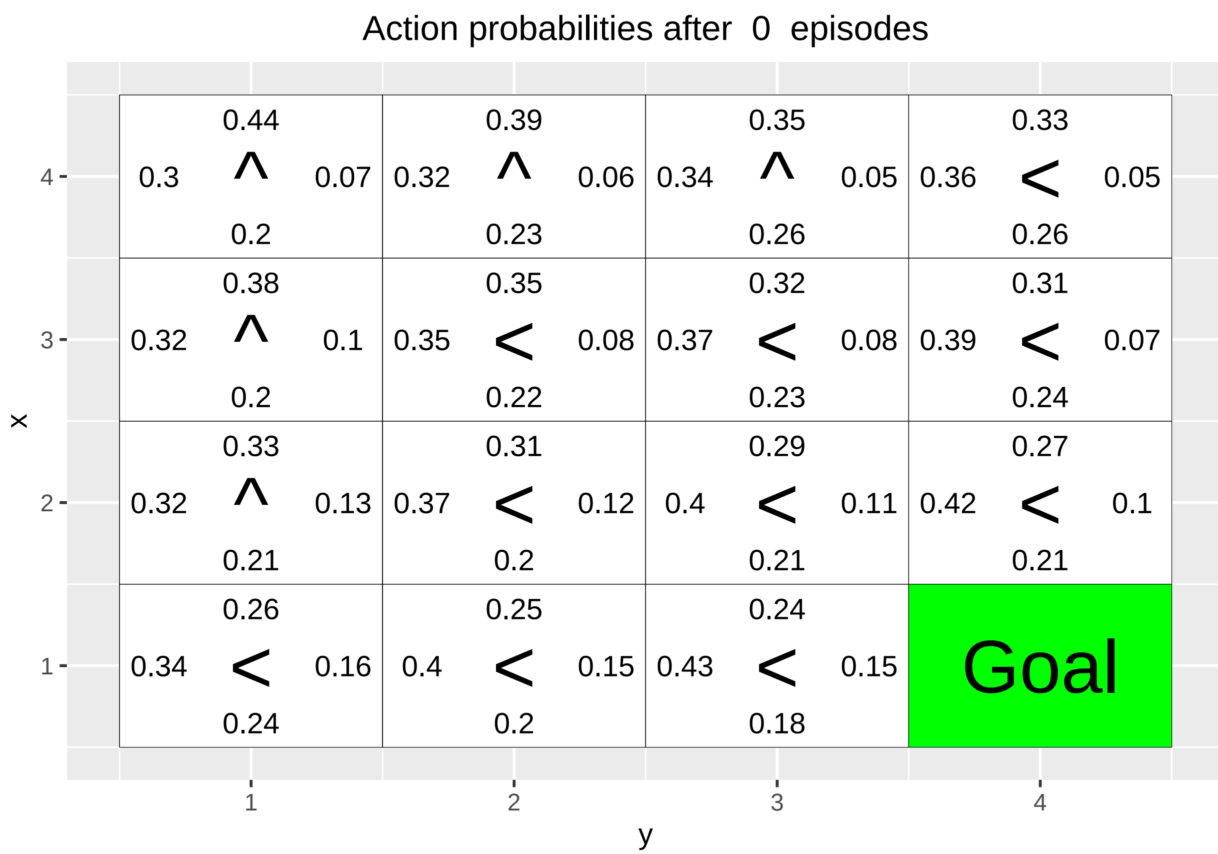

Action probabilities after 0 episodes
x
y
Goal


Action probabilities after 0 episodes
0.44
0.31 ^ 0.06
0.19
0.37
0.34 ^ 0.05
0.23
0.34
0.37 < 0.05
0.24
0.33
0.39 < 0.04
0.24
0.34
0.36 < 0.08
0.22
0.33
0.38 < 0.08
0.22
0.31
0.4 < 0.07
0.22
0.3
0.42 < 0.06
0.22
0.29
0.39 < 0.12
0.2
0.29
0.4 < 0.11
0.19
Goal
0.27
0.43 < 0.09
0.2
0.23
0.42 < 0.15
0.2
0.24
0.44 < 0.15
0.18
0.24
0.45 < 0.14
0.17
0.23
0.47 < 0.13
0.18
x
1
2
3
4
y



Action probabilities after 0 episodes
x
y
Goal


Action probabilities after 0 episodes
x
y
1
2
3
4
0.45
0.25 ^ 0.05
0.25
0.4
0.25 ^ 0.04
0.3
0.33
0.27 V 0.04
0.35
0.29
0.29 V 0.03
0.39
0.4
0.27 ^ 0.08
0.26
0.34
0.29 ^ 0.07
0.3
0.3
0.31 V 0.06
0.32
0.29
0.31 V 0.05
0.35
0.33
0.3 ^ 0.12
0.26
0.3
0.32 < 0.11
0.27
0.29
0.32 < 0.09
0.29
0.27
0.33 < 0.08
0.32
Goal
0.27
0.32 < 0.15
0.25
0.26
0.33 < 0.13
0.27
0.25
0.35 < 0.12
0.29

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## episode 4970
## episode 4980
## episode 4990
## episode 5000
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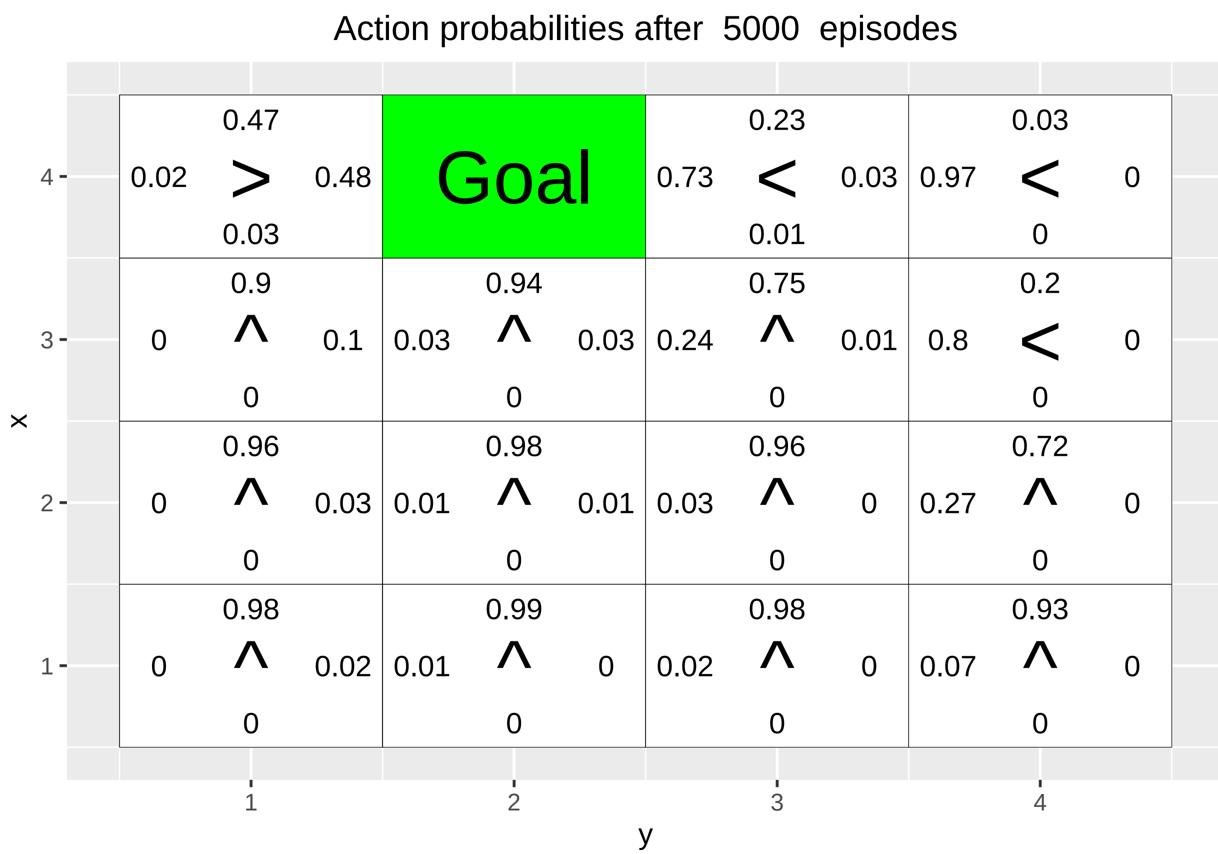
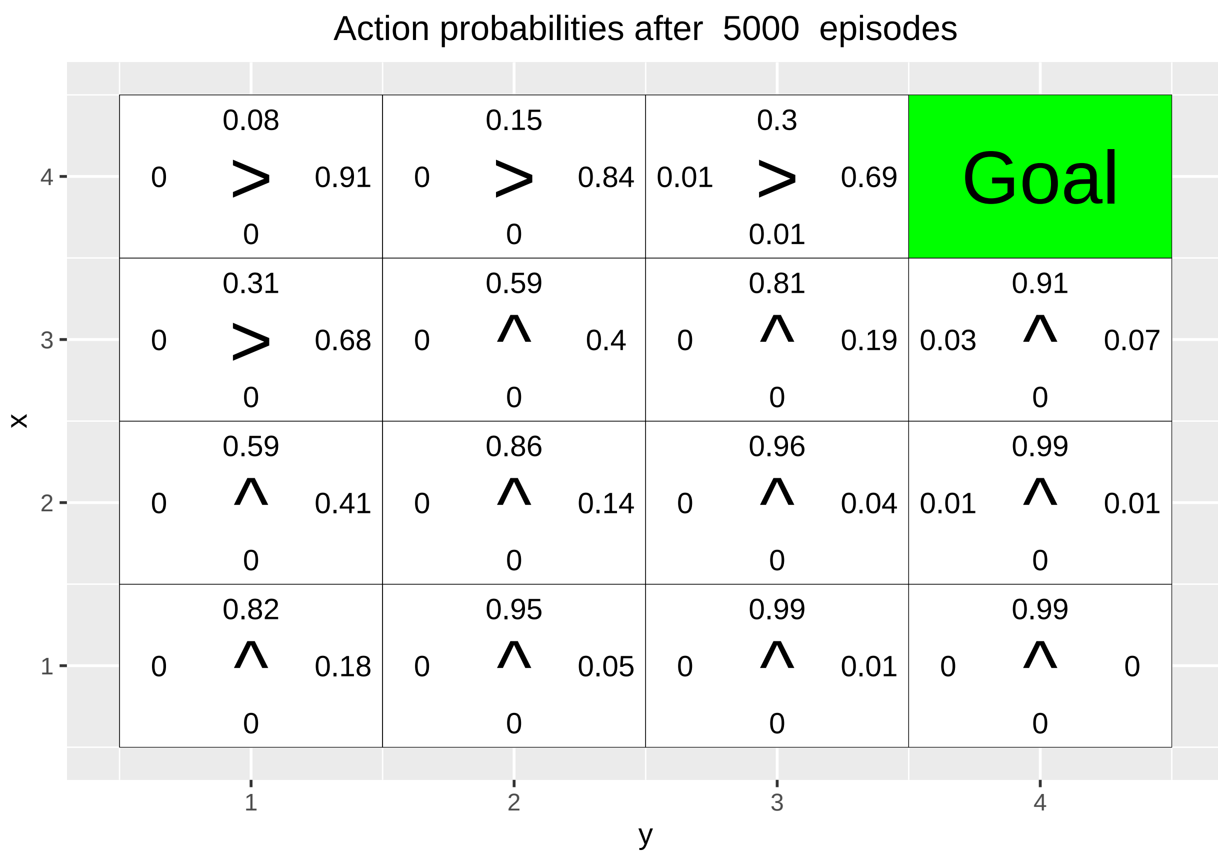
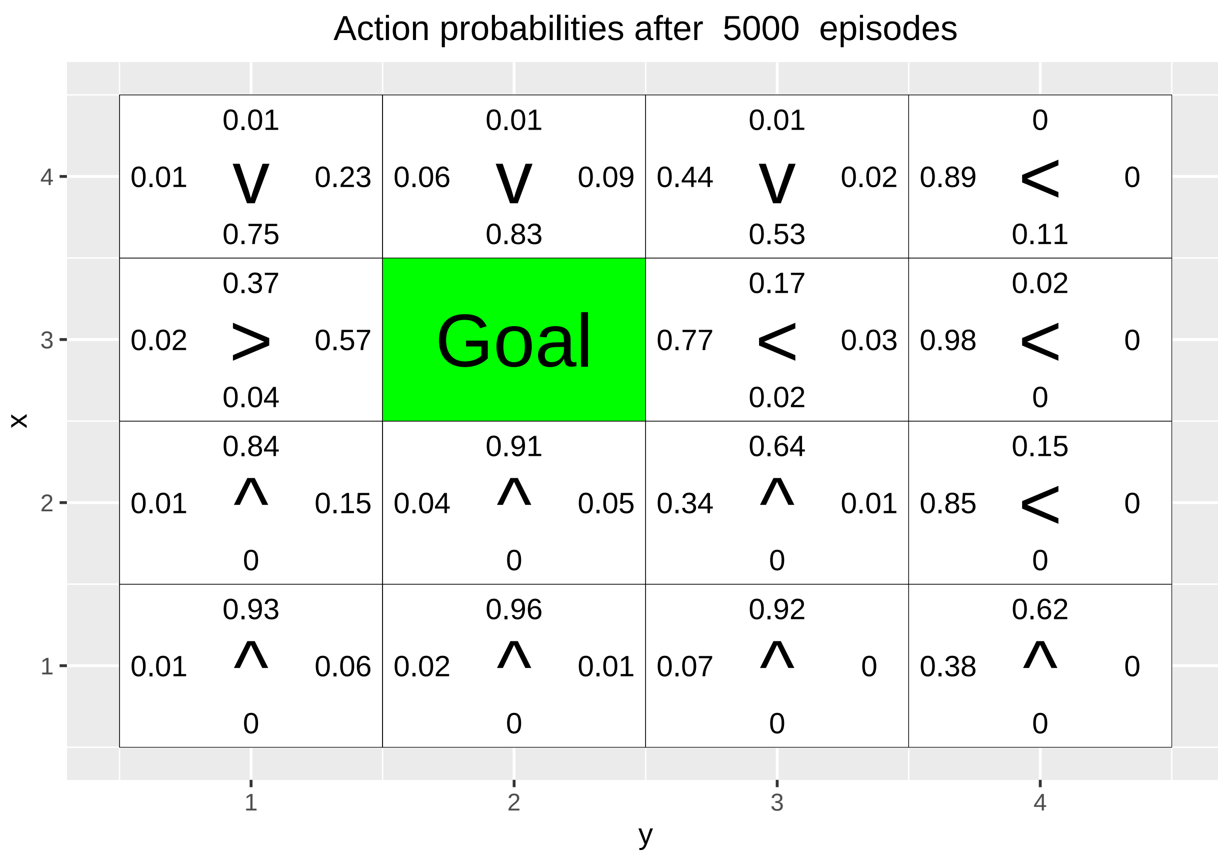
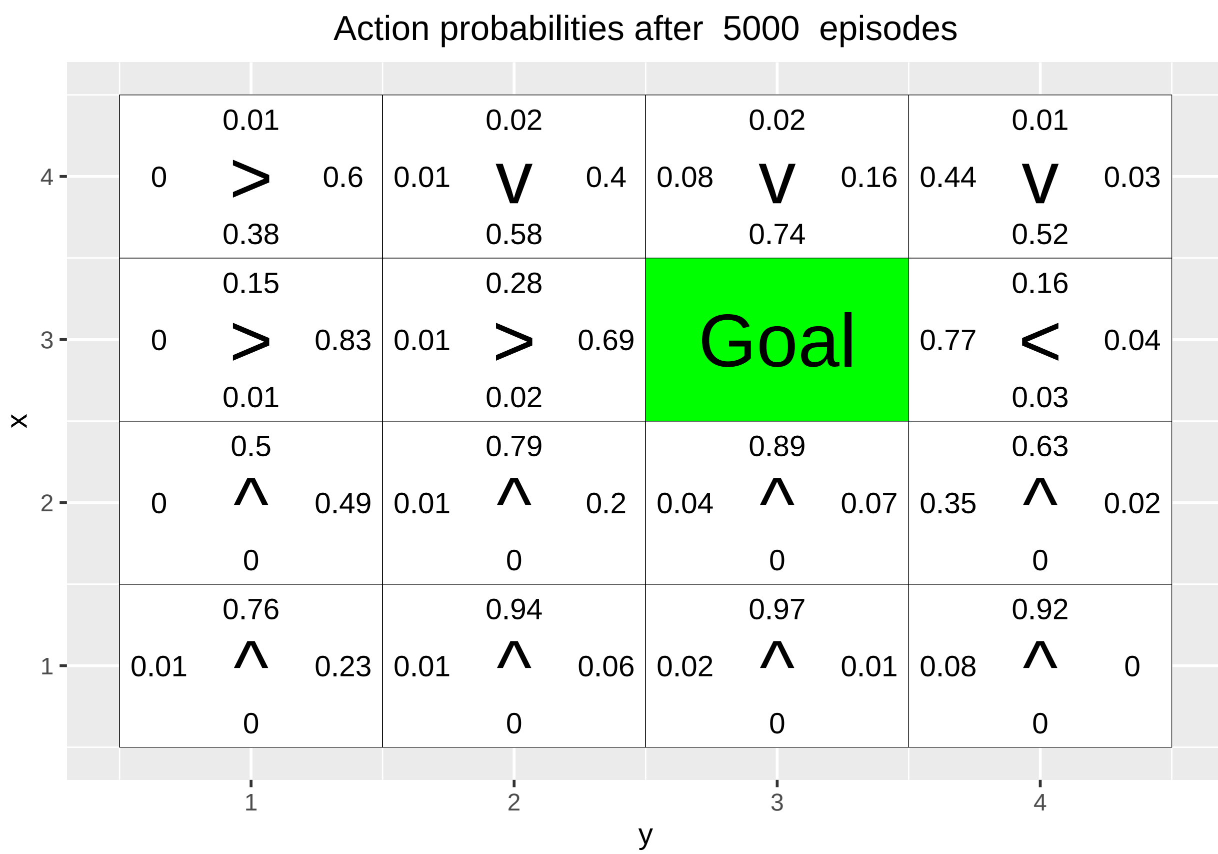
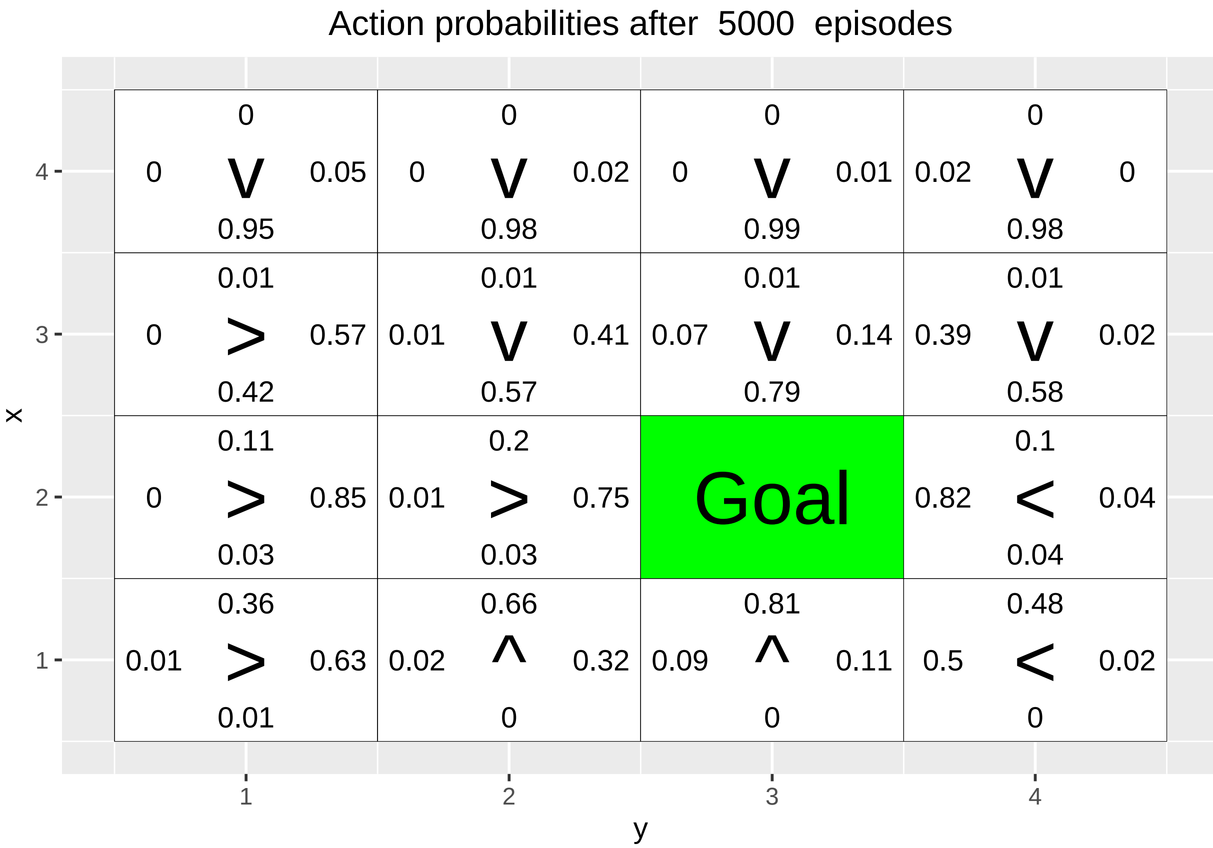

Action probabilities after 5000 episodes
0.47
0.02
>
0.48
0.03
Goal
0.23
0.73
<
0.03
0.01
0.03
0.97
<
0
0
0.9
0
^
0.1
0
0.94
0.03
^
0.03
0
0.75
0.24
^
0.01
0
0.2
0.8
<
0
0
0.96
0
^
0.03
0
0.98
0.01
^
0.01
0
0.96
0.03
^
0
0
0.72
0.27
^
0
0
0.98
0
^
0.02
0
0.99
0.01
^
0
0
0.98
0.02
^
0
0
0.93
0.07
^
0
0
x
y
1
2
3
4

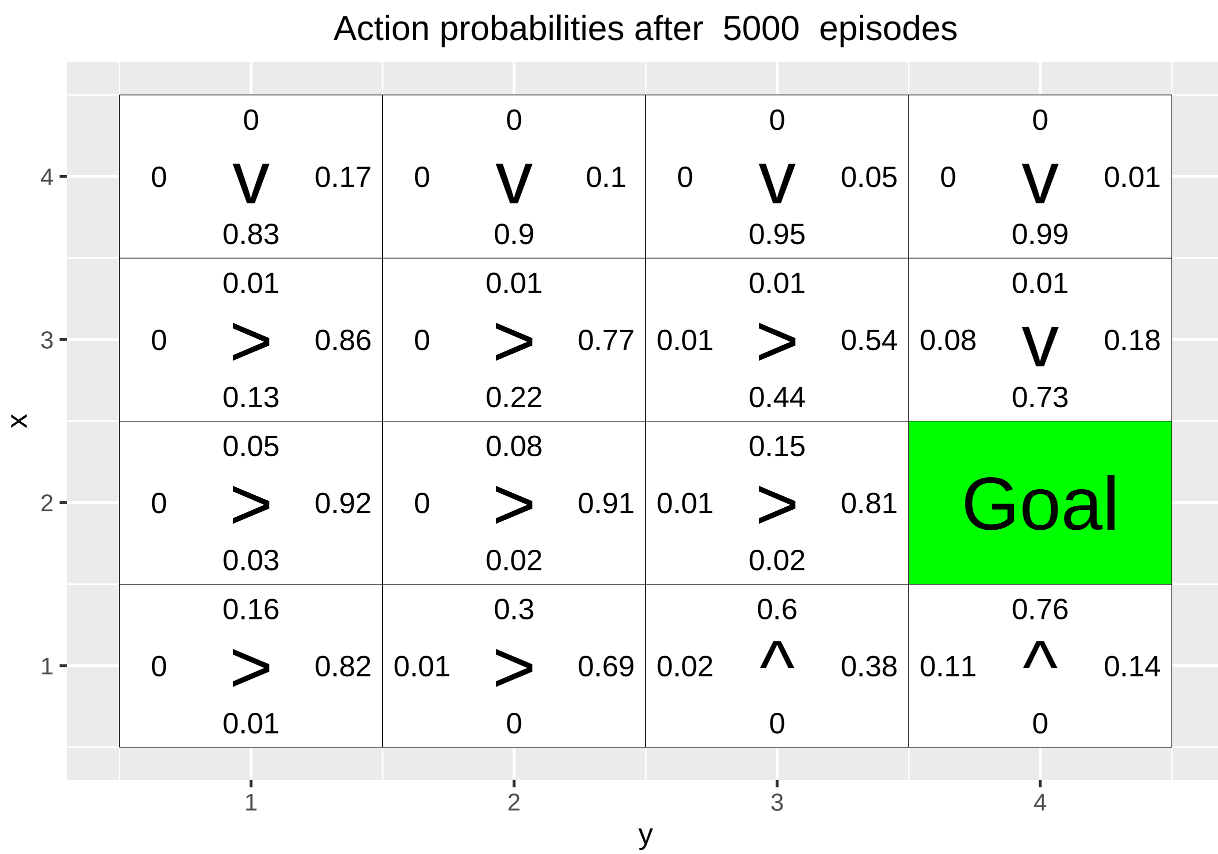


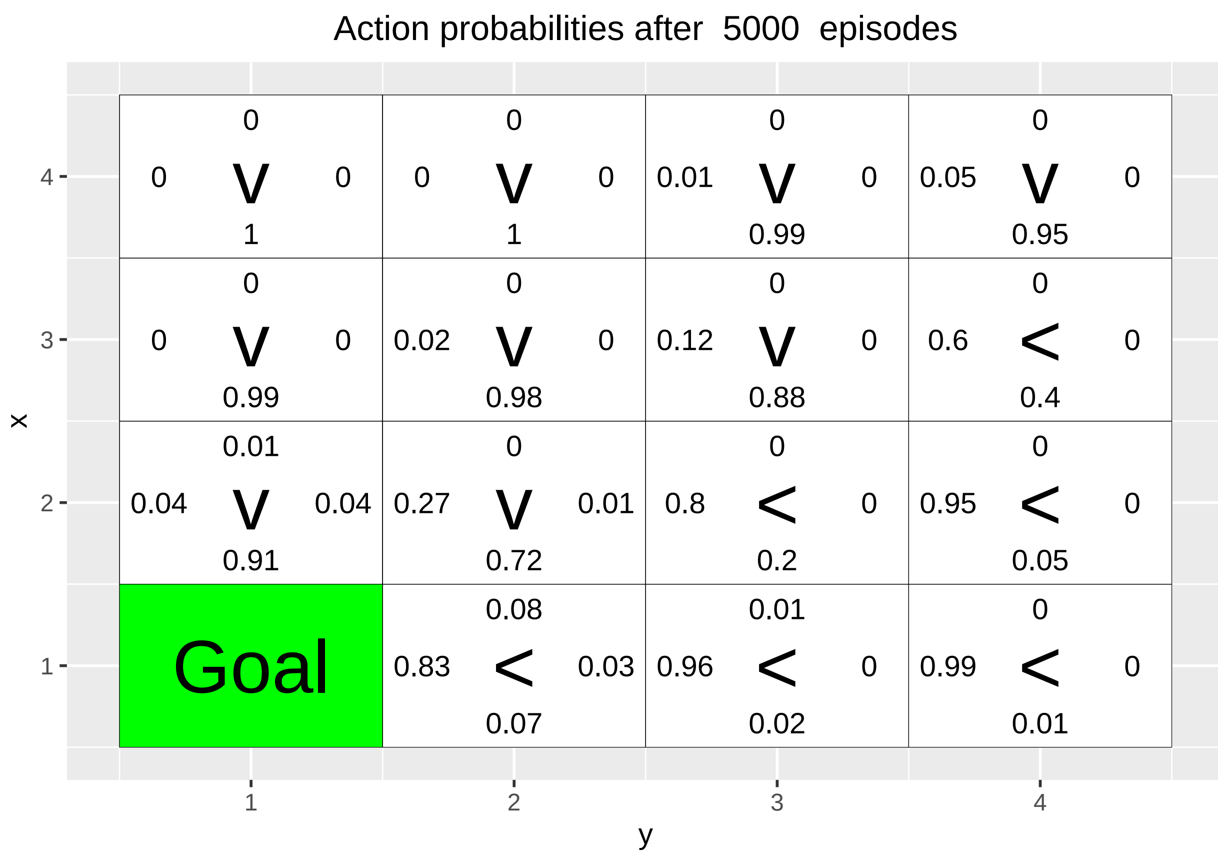
Action probabilities after 5000 episodes
Goal
0.08
0
0.91
0
0.15
0
0.84
0
0.3
0.01
0.69
0.01
0.31
0
0.68
0
0.59
0
0.4
0
0.81
0
0.19
0
0.91
0.03
0.07
0
0.59
0
0.41
0
0.86
0
0.14
0
0.96
0
0.04
0
0.99
0.01
0.01
0
0.82
0
0.18
0
0.95
0
0.05
0
0.99
0
0.01
0
0.99
0
0
0
x
y
1
2
3
4

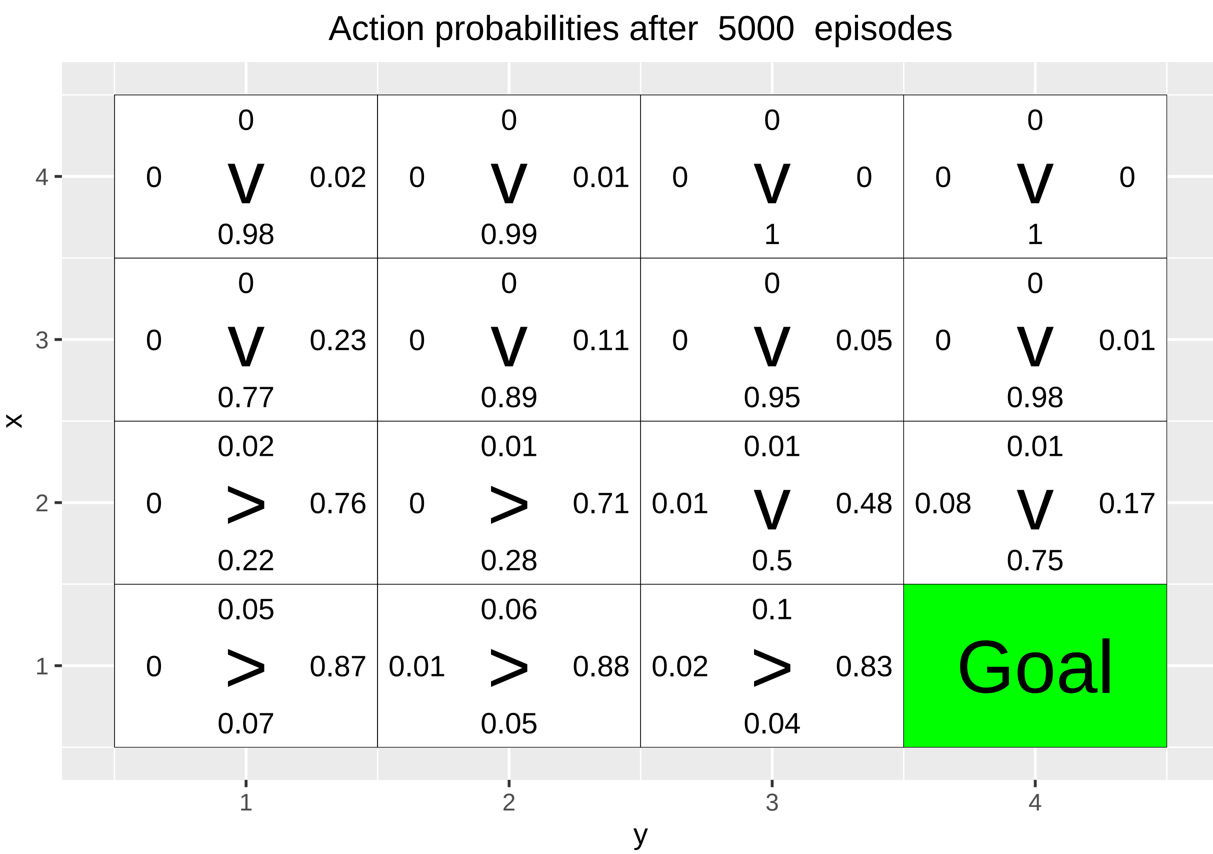

Action probabilities after 5000 episodes
Goal
x
y
0.01 V 0.01 0.23 0.75
0.01 V 0.06 0.09 0.83
0.01 V 0.44 0.02 0.53
0 < 0.89 0 0.11
0.37 > 0.02 0.57 0.04
0.17 < 0.77 0.03 0.02
0.02 < 0.98 0 0
0.84 ^ 0.01 0.15 0
0.91 ^ 0.04 0.05 0
0.64 ^ 0.34 0.01 0
0.15 < 0.85 0 0
0.93 ^ 0.01 0.06 0
0.96 ^ 0.02 0.01 0
0.92 ^ 0.07 0 0
0.62 ^ 0.38 0 0


Action probabilities after 5000 episodes
0.01
0 > 0.6
0.38
0.02
0.01 V 0.4
0.58
0.02
0.08 V 0.16
0.74
0.01
0.44 V 0.03
0.52
0.15
0 > 0.83
0.01
0.28
0.01 > 0.69
0.02
Goal
0.16
0.77 < 0.04
0.03
0.5
0 ^ 0.49
0
0.79
0.01 ^ 0.2
0
0.89
0.04 ^ 0.07
0
0.63
0.35 ^ 0.02
0
0.76
0.01 ^ 0.23
0
0.94
0.01 ^ 0.06
0
0.97
0.02 ^ 0.01
0
0.92
0.08 ^ 0
0
x
4
3
2
1
1
2
3
4
y


Action probabilities after 5000 episodes
0
0
0
0
0
0.05
0
0.02
0
0.01
0.02
0
0.95
0.98
0.99
0.98
0.01
0.01
0.01
0.01
0
0.57
0.01
0.41
0.07
0.14
0.39
0.02
0.42
0.57
0.79
0.58
0.11
0.2
0.1
0
0.85
0.01
0.75
Goal
0.82
0.04
0.03
0.03
0.04
0.36
0.66
0.81
0.48
0.01
0.63
0.02
0.32
0.09
0.11
0.5
0.02
0.01
0
0
0
x
y
1
2
3
4


Action probabilities after 5000 episodes
0
0 V 0.17
0.83
0
0 V 0.1
0.9
0
0 V 0.05
0.95
0
0 V 0.01
0.99
0.01
0 > 0.86
0.13
0.01
0 > 0.77
0.22
0.01
0.01 > 0.54
0.44
0.01
0.08 V 0.18
0.73
0.05
0 > 0.92
0.03
0.08
0 > 0.91
0.02
0.15
0.01 > 0.81
0.02
Goal
0.16
0 > 0.82
0.01
0.3
0.01 > 0.69
0
0.6
0.02 ^ 0.38
0
0.76
0.11 ^ 0.14
0
x
4
3
2
1
1
2
3
4
y


Action probabilities after 5000 episodes
Goal
x
y
1
2
3
4
0
V
1
0.99
0.95
0.01
0.05
0.02
0.12
0.98
0.88
0.6
0.4
0.04
0.91
0.27
0.72
0.8
0.2
0.95
0.83
0.08
0.07
0.03
0.96
0.02
0.99
<



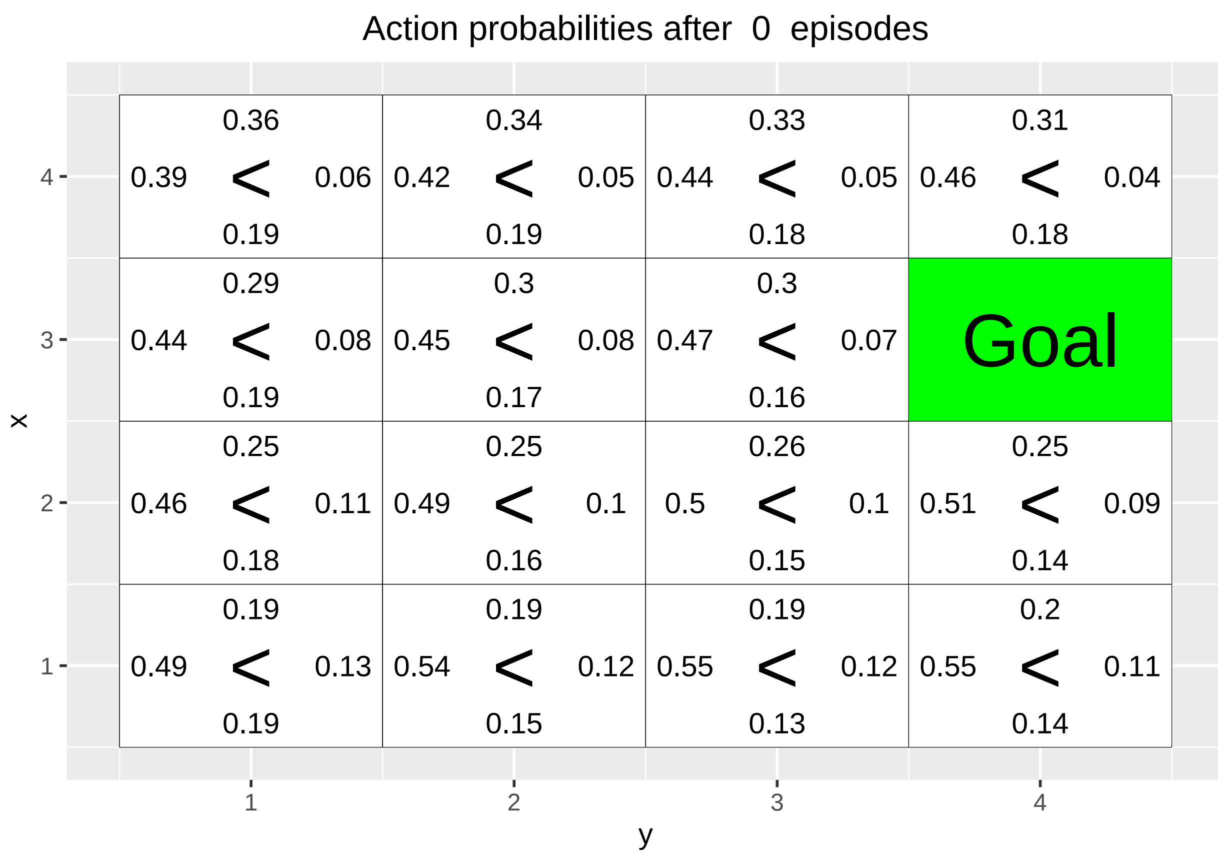
Has the agent learned a good policy? Why / Why not ? It can be seen that agent is able to learn the good policy as it have access to all data points and thus it is able to generalize and learn good policy.

Could you have used the Q-learning algorithm to solve this task ?

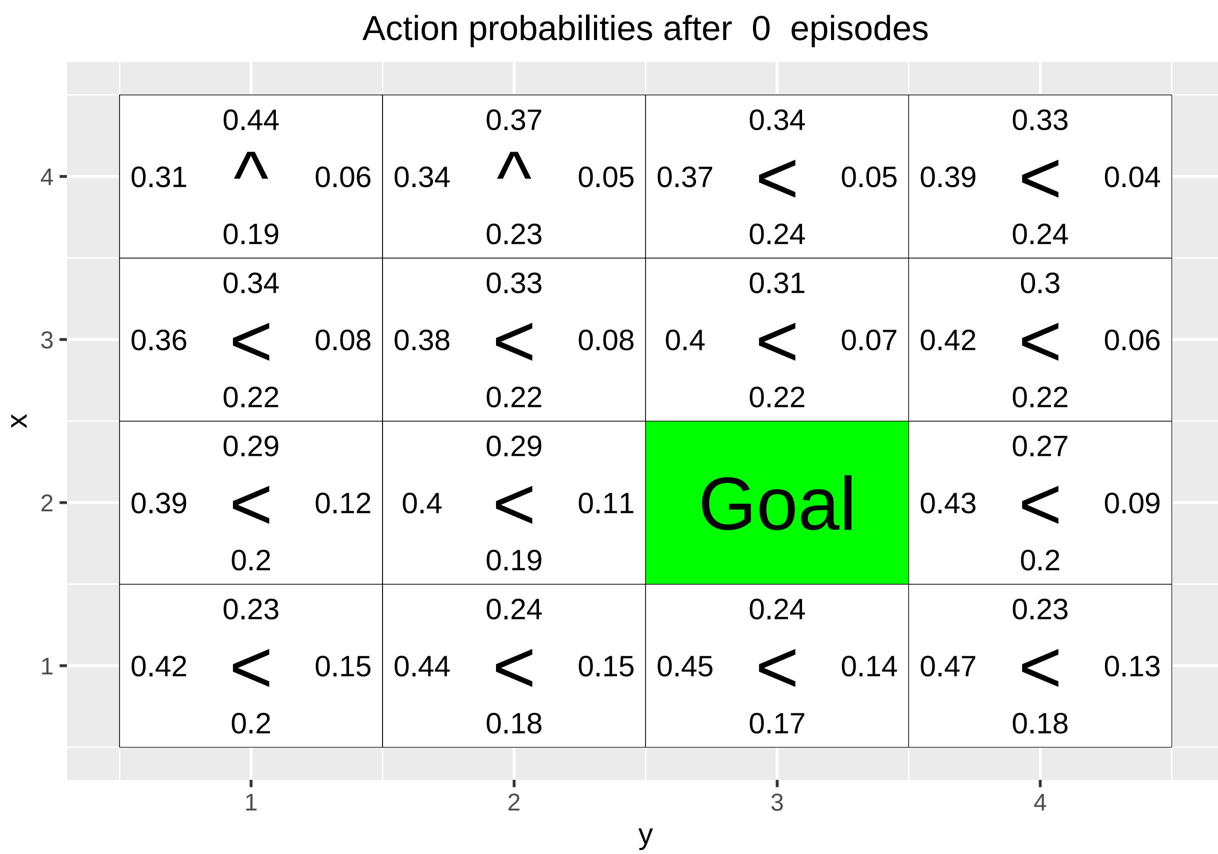
In Q-Learning we create q-matrix which contain states and rewards for state, now when we are not able to obtain state and optimal location is keep on changing like for above scenario, we cannot implement Q-Learning.

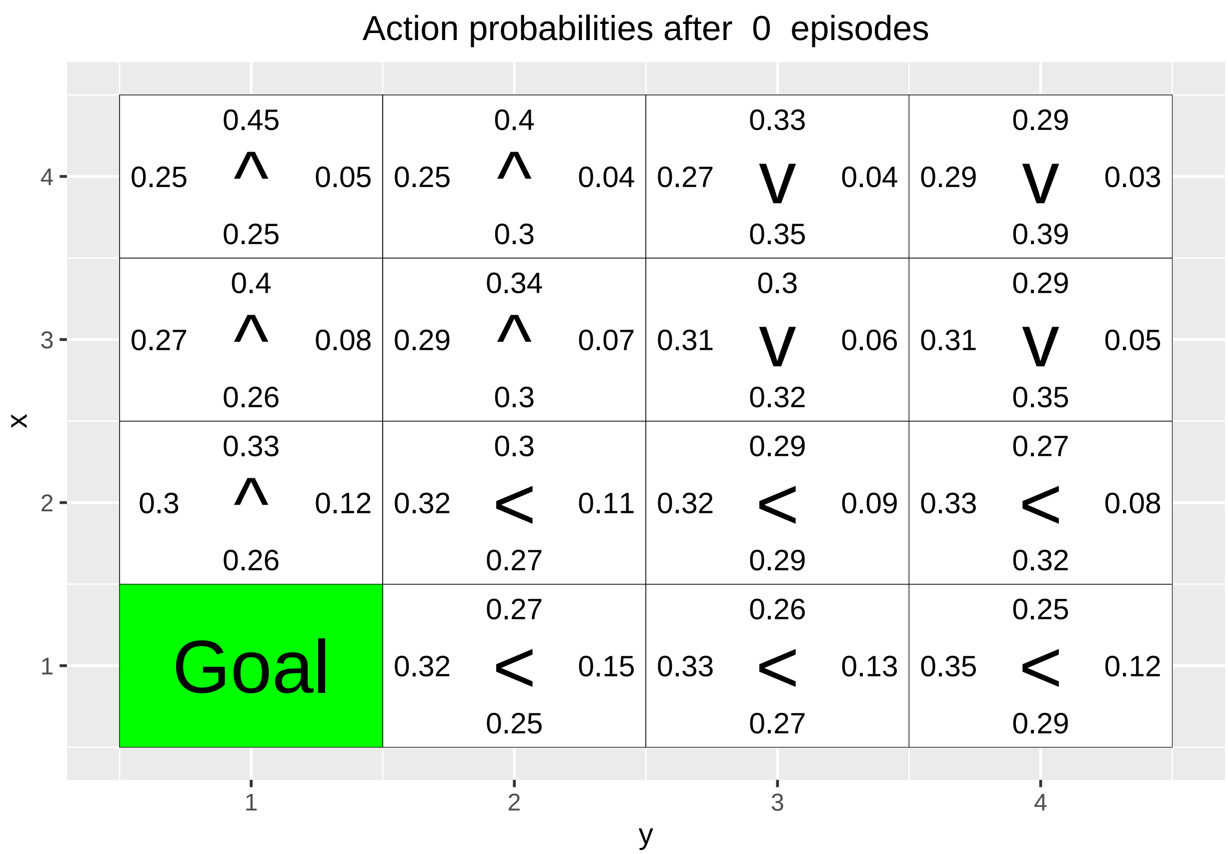
Environment E (training with top row goal positions)

Task : To repeat the previous experiments but this time the goals for training are all from the top row of the grid. The validation goals are three positions from the rows below.



Action probabilities after 0 episodes
0.36
0.39 < 0.06
0.19
0.34
0.42 < 0.05
0.19
0.33
0.44 < 0.05
0.18
0.31
0.46 < 0.04
0.18
0.29
0.44 < 0.08
0.19
0.3
0.45 < 0.08
0.17
0.3
0.47 < 0.07
0.16
Goal
0.25
0.46 < 0.11
0.18
0.25
0.49 < 0.1
0.16
0.26
0.5 < 0.1
0.15
0.25
0.51 < 0.09
0.14
0.19
0.49 < 0.13
0.19
0.19
0.54 < 0.12
0.15
0.19
0.55 < 0.12
0.13
0.2
0.55 < 0.11
0.14
x
y
1
2
3
4


Action probabilities after 0 episodes
x
y
Goal

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## episode 1780
## episode 1790
## episode 1800
## episode 1810
## episode 1820
## episode 1830
## episode 1840
## episode 1850
## episode 1860
## episode 1870
## episode 1880
```

```
## episode 1890
## episode 1900
## episode 1910
## episode 1920
## episode 1930
## episode 1940
## episode 1950
## episode 1960
## episode 1970
## episode 1980
## episode 1990
## episode 2000
## episode 2010
## episode 2020
## episode 2030
## episode 2040
## episode 2050
## episode 2060
## episode 2070
## episode 2080
## episode 2090
## episode 2100
## episode 2110
## episode 2120
## episode 2130
## episode 2140
## episode 2150
## episode 2160
## episode 2170
## episode 2180
## episode 2190
## episode 2200
## episode 2210
## episode 2220
## episode 2230
## episode 2240
## episode 2250
## episode 2260
## episode 2270
## episode 2280
## episode 2290
## episode 2300
## episode 2310
## episode 2320
## episode 2330
## episode 2340
## episode 2350
## episode 2360
## episode 2370
## episode 2380
## episode 2390
## episode 2400
## episode 2410
## episode 2420
```

```
## episode 2430
## episode 2440
## episode 2450
## episode 2460
## episode 2470
## episode 2480
## episode 2490
## episode 2500
## episode 2510
## episode 2520
## episode 2530
## episode 2540
## episode 2550
## episode 2560
## episode 2570
## episode 2580
## episode 2590
## episode 2600
## episode 2610
## episode 2620
## episode 2630
## episode 2640
## episode 2650
## episode 2660
## episode 2670
## episode 2680
## episode 2690
## episode 2700
## episode 2710
## episode 2720
## episode 2730
## episode 2740
## episode 2750
## episode 2760
## episode 2770
## episode 2780
## episode 2790
## episode 2800
## episode 2810
## episode 2820
## episode 2830
## episode 2840
## episode 2850
## episode 2860
## episode 2870
## episode 2880
## episode 2890
## episode 2900
## episode 2910
## episode 2920
## episode 2930
## episode 2940
## episode 2950
## episode 2960
```

```
## episode 2970
## episode 2980
## episode 2990
## episode 3000
## episode 3010
## episode 3020
## episode 3030
## episode 3040
## episode 3050
## episode 3060
## episode 3070
## episode 3080
## episode 3090
## episode 3100
## episode 3110
## episode 3120
## episode 3130
## episode 3140
## episode 3150
## episode 3160
## episode 3170
## episode 3180
## episode 3190
## episode 3200
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## episode 3230
## episode 3240
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## episode 3270
## episode 3280
## episode 3290
## episode 3300
## episode 3310
## episode 3320
## episode 3330
## episode 3340
## episode 3350
## episode 3360
## episode 3370
## episode 3380
## episode 3390
## episode 3400
## episode 3410
## episode 3420
## episode 3430
## episode 3440
## episode 3450
## episode 3460
## episode 3470
## episode 3480
## episode 3490
## episode 3500
```

```
## episode 3510
## episode 3520
## episode 3530
## episode 3540
## episode 3550
## episode 3560
## episode 3570
## episode 3580
## episode 3590
## episode 3600
## episode 3610
## episode 3620
## episode 3630
## episode 3640
## episode 3650
## episode 3660
## episode 3670
## episode 3680
## episode 3690
## episode 3700
## episode 3710
## episode 3720
## episode 3730
## episode 3740
## episode 3750
## episode 3760
## episode 3770
## episode 3780
## episode 3790
## episode 3800
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## episode 3870
## episode 3880
## episode 3890
## episode 3900
## episode 3910
## episode 3920
## episode 3930
## episode 3940
## episode 3950
## episode 3960
## episode 3970
## episode 3980
## episode 3990
## episode 4000
## episode 4010
## episode 4020
## episode 4030
## episode 4040
```

```
## episode 4050
## episode 4060
## episode 4070
## episode 4080
## episode 4090
## episode 4100
## episode 4110
## episode 4120
## episode 4130
## episode 4140
## episode 4150
## episode 4160
## episode 4170
## episode 4180
## episode 4190
## episode 4200
## episode 4210
## episode 4220
## episode 4230
## episode 4240
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## episode 4260
## episode 4270
## episode 4280
## episode 4290
## episode 4300
## episode 4310
## episode 4320
## episode 4330
## episode 4340
## episode 4350
## episode 4360
## episode 4370
## episode 4380
## episode 4390
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## episode 4520
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## episode 4560
## episode 4570
## episode 4580
```

```
## episode 4590
## episode 4600
## episode 4610
## episode 4620
## episode 4630
## episode 4640
## episode 4650
## episode 4660
## episode 4670
## episode 4680
## episode 4690
## episode 4700
## episode 4710
## episode 4720
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## episode 4850
## episode 4860
## episode 4870
## episode 4880
## episode 4890
## episode 4900
## episode 4910
## episode 4920
## episode 4930
## episode 4940
## episode 4950
## episode 4960
## episode 4970
## episode 4980
## episode 4990
## episode 5000
```
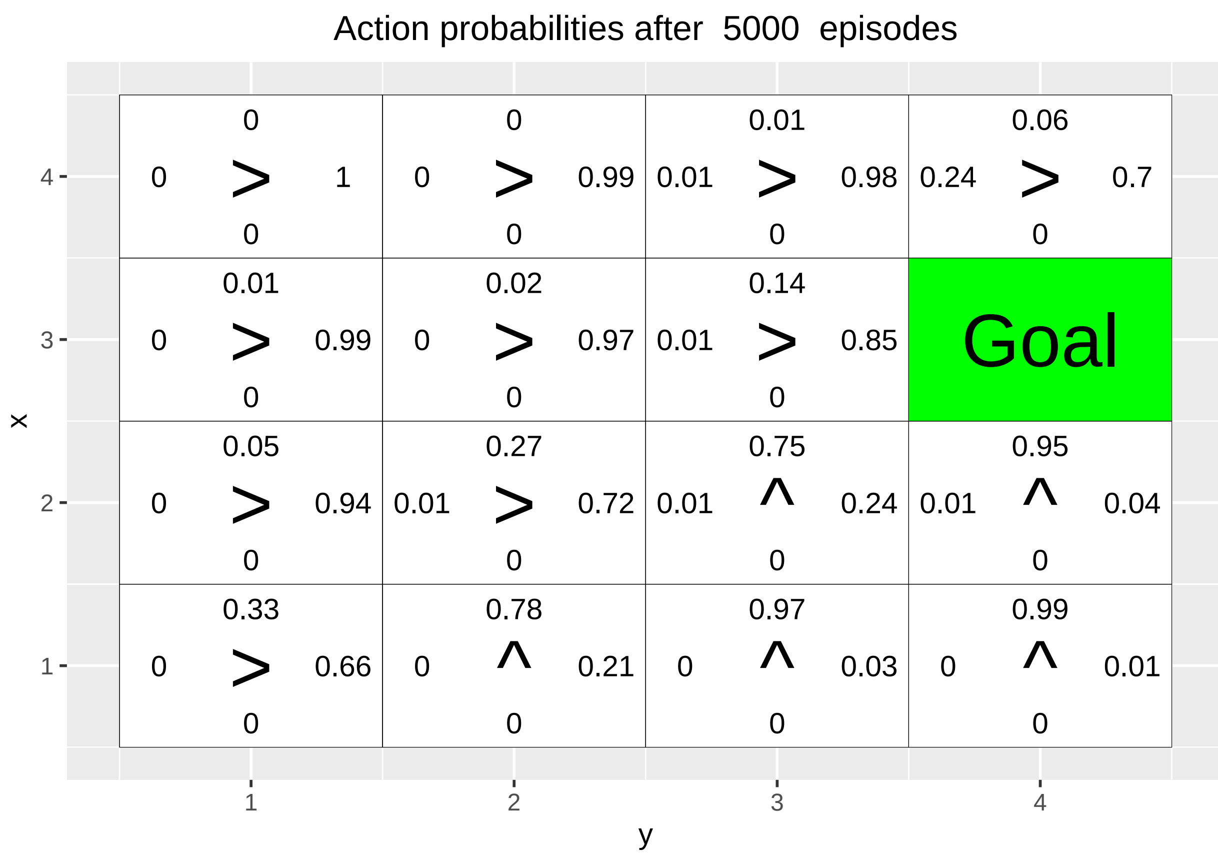
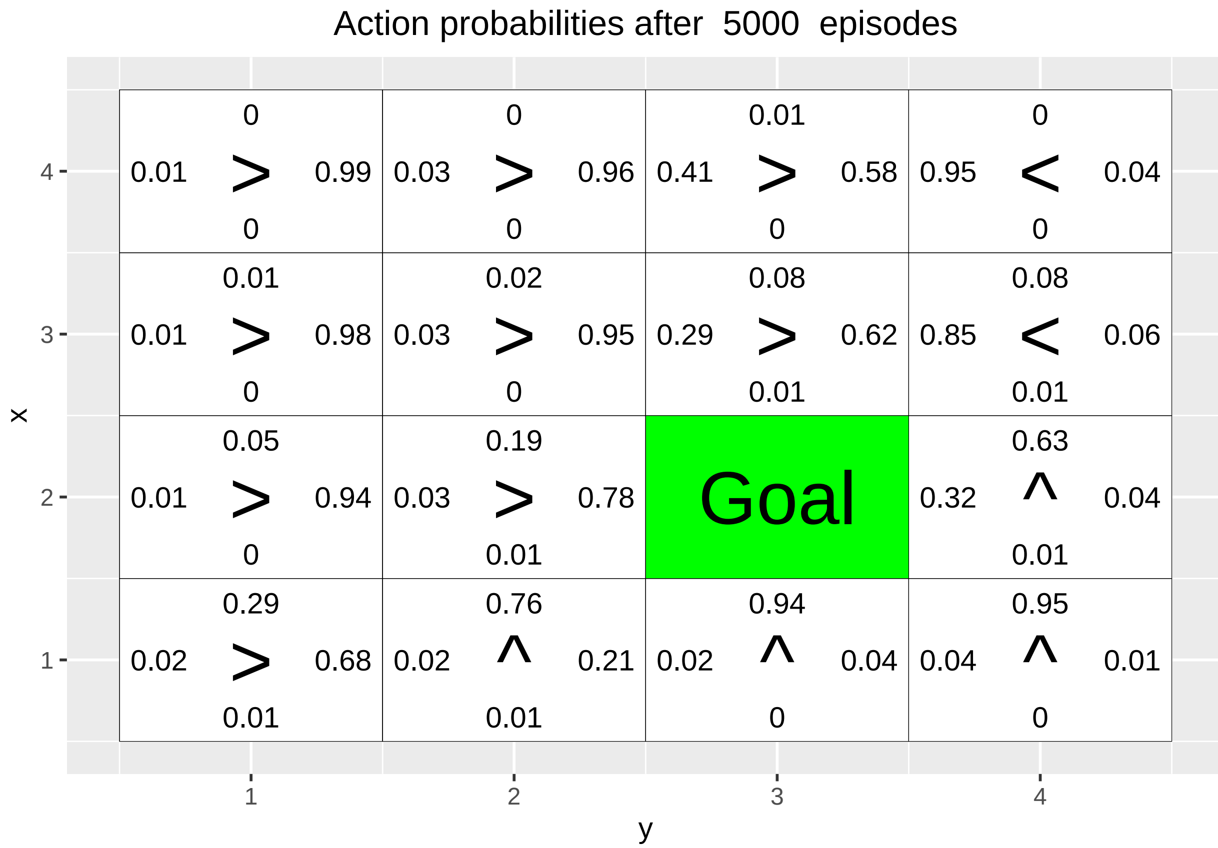
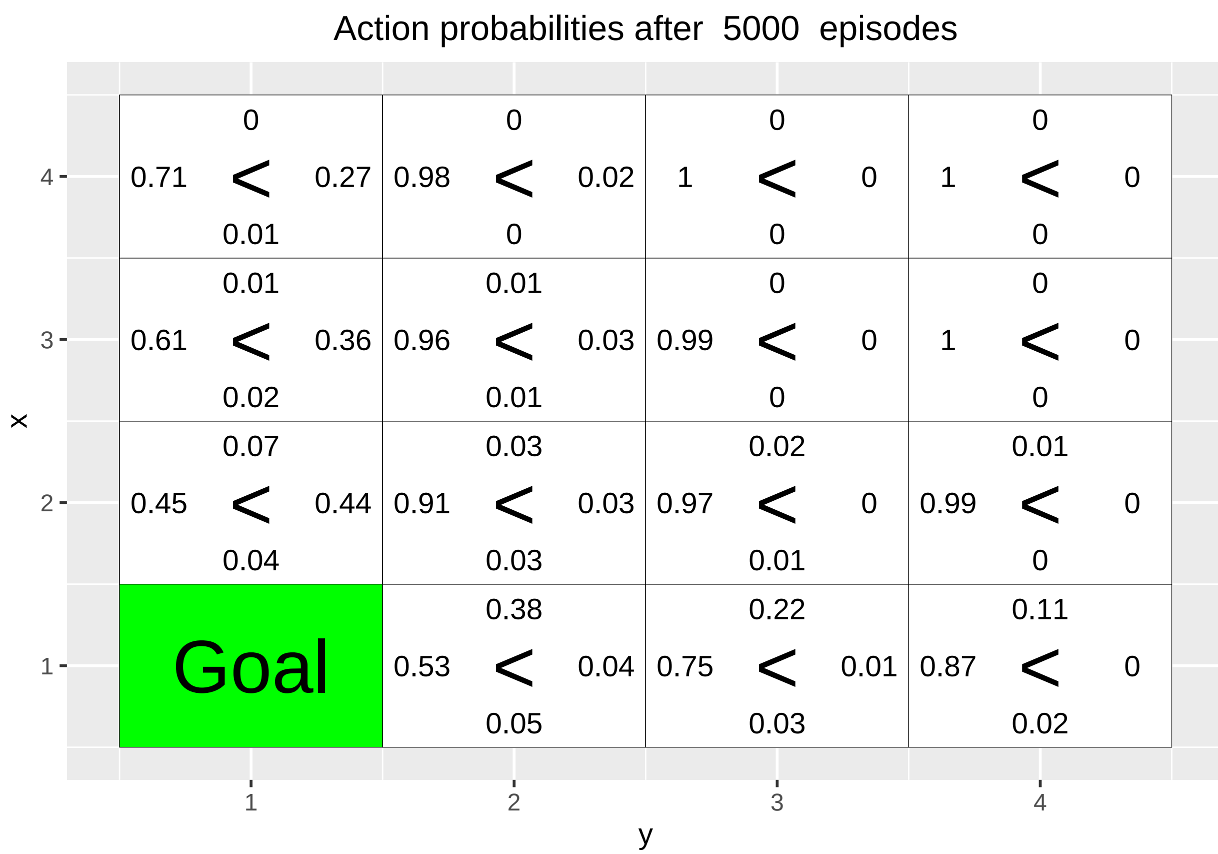


Action probabilities after 5000 episodes
Goal
x
y
1
2
3
4
0 > 1
0 > 0.99
0.01 > 0.98
0.24 > 0.7
0.01 > 0.99
0.02 > 0.97
0.14 > 0.85
0.05 > 0.94
0.27 > 0.72
0.75 ^ 0.24
0.95 ^ 0.04
0.33 > 0.66
0.78 ^ 0.21
0.97 ^ 0.03
0.99 ^ 0.01



Action probabilities after 5000 episodes
0
0.01
>
0.99
0
0
0.03
>
0.96
0
0.01
0.41
>
0.58
0
0
0.95
<
0.04
0
0.01
0.01
>
0.98
0
0.02
0.03
>
0.95
0
0.08
0.29
>
0.62
0.01
0.08
0.85
<
0.06
0.01
0.05
0.01
>
0.94
0
0.19
0.03
>
0.78
0.01
Goal
0.63
0.32
^
0.04
0.01
0.29
0.02
>
0.68
0.01
0.76
0.02
^
0.21
0.01
0.94
0.02
^
0.04
0
0.95
0.04
^
0.01
0
x
y
1
2
3
4



Has the agent learned a good policy? Why / Why not ?

Since agent doen not have access to similar data, it was not able to generalize and thus obtained policy is not good.

If the results obtained for environments D and E differ, explain why

In D agent was able to generalize based on given training data which is access to whole grid while it lack to do it for E as it only had access to 1st row of grid.

References

- A Concise Introduction to Reinforcement Learning

*Epsilon-Greedy Algorithm in Reinforcement Learning

Appendix

```
knitr::opts_chunk$set(echo = TRUE)

library(ggplot2)

# If you do not see four arrows in line 16, then do the following:
# File/Reopen with Encoding/UTF-8

#arrows <- c("↑", "→", "↓", "←")
arrows <- c("^", ">", "v", "<")
action_deltas <- list(c(1,0), # up
```

```r
                    c(0,1), # right
                    c(-1,0), # down
                    c(0,-1)) # left

vis_environment <- function(iterations=0, epsilon = 0.5, alpha = 0.1, gamma = 0.95, beta = 0){

  # Visualize an environment with rewards.
  # Q-values for all actions are displayed on the edges of each tile.
  # The (greedy) policy for each state is also displayed.
  #
  # Args:
  #   iterations, epsilon, alpha, gamma, beta (optional): for the figure title.
  #   reward_map (global variable): a HxW array containing the reward given at each state.
  #   q_table (global variable): a HxWx4 array containing Q-values for each state-action pair.
  #   H, W (global variables): environment dimensions.

  df <- expand.grid(x=1:H,y=1:W)
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,1],NA),df$x,df$y)
  df$val1 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,2],NA),df$x,df$y)
  df$val2 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,3],NA),df$x,df$y)
  df$val3 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,q_table[x,y,4],NA),df$x,df$y)
  df$val4 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y)
    ifelse(reward_map[x,y] == 0,arrows[GreedyPolicy(x,y)],reward_map[x,y]),df$x,df$y)
  df$val5 <- as.vector(foo)
  foo <- mapply(function(x,y) ifelse(reward_map[x,y] == 0,max(q_table[x,y,]),
                                     ifelse(reward_map[x,y]<0,NA,reward_map[x,y])),df$x,df$y)
  df$val6 <- as.vector(foo)

  print(ggplot(df,aes(x = y,y = x)) +
          scale_fill_gradient(low = "white", high = "green", na.value = "red", name = "") +
          geom_tile(aes(fill=val6)) +
          geom_text(aes(label = val1),size = 4,nudge_y = .35,na.rm = TRUE) +
          geom_text(aes(label = val2),size = 4,nudge_x = .35,na.rm = TRUE) +
          geom_text(aes(label = val3),size = 4,nudge_y = -.35,na.rm = TRUE) +
          geom_text(aes(label = val4),size = 4,nudge_x = -.35,na.rm = TRUE) +
          geom_text(aes(label = val5),size = 10) +
          geom_tile(fill = 'transparent', colour = 'black') +
          ggtitle(paste("Q-table after ",iterations," iterations\n",
                        "(epsilon = ",epsilon,", alpha = ",alpha,"gamma = ",gamma,", beta = ",beta,")")
          theme(plot.title = element_text(hjust = 0.5)) +
          scale_x_continuous(breaks = c(1:W),labels = c(1:W)) +
          scale_y_continuous(breaks = c(1:H),labels = c(1:H)))

}

GreedyPolicy <- function(x, y){

  # Get a greedy action for state (x,y) from q_table.
  #
```

```r
  # Args:
  #   x, y: state coordinates.
  #   q_table (global variable): a HxWx4 array containing Q-values for each state-action pair.
  #
  # Returns:
  #   An action, i.e. integer in {1,2,3,4}.
  Rewards = q_table[x,y,]
  maxRewardIndx = which(Rewards == max(Rewards))
  #randomly selecting max value and setting it as action

  if (length(maxRewardIndx) == 1){
    InitSelected = maxRewardIndx
  }else{
  InitSelected = sample(maxRewardIndx , size = 1)
  }

  return(InitSelected)
  # Your code here.

}#GreedyPolicy

EpsilonGreedyPolicy <- function(x, y, epsilon){

  # Get an epsilon-greedy action for state (x,y) from q_table.
  #
  # Args:
  #   x, y: state coordinates.
  #   epsilon: probability of acting randomly.
  #
  # Returns:
  #   An action, i.e. integer in {1,2,3,4}.

  # Your code here.
  Rewards = q_table[x,y,]
  maxRewardIndx = which(Rewards == max(Rewards))
  #randomly selecting max value and setting it as action
  if (length(maxRewardIndx) == 1){
    InitSelected = maxRewardIndx
  }else{
  InitSelected = sample(maxRewardIndx , size = 1)
  }
  #unif
  p = runif(1)
  if(p < epsilon)
    {
      return(sample(setdiff(c(1,2,3,4) ,InitSelected) , size = 1)) #e
  }else{
      return(InitSelected) #1-e
    }
}#EpsilonGreedyPolicy


transition_model <- function(x, y, action, beta){
```

```r
  # Computes the new state after given action is taken. The agent will follow the action
  # with probability (1-beta) and slip to the right or left with probability beta/2 each.
  #
  # Args:
  #   x, y: state coordinates.
  #   action: which action the agent takes (in {1,2,3,4}).
  #   beta: probability of the agent slipping to the side when trying to move.
  #   H, W (global variables): environment dimensions.
  #
  # Returns:
  #   The new state after the action has been taken.

  delta <- sample(-1:1, size = 1, prob = c(0.5*beta,1-beta,0.5*beta))
  final_action <- ((action + delta + 3) %% 4) + 1
  foo <- c(x,y) + unlist(action_deltas[final_action])
  foo <- pmax(c(1,1),pmin(foo,c(H,W)))

  return (foo)
}

q_learning <- function(start_state, epsilon = 0.5, alpha = 0.1, gamma = 0.95,
                       beta = 0){

  # Perform one episode of Q-learning. The agent should move around in the
  # environment using the given transition model and update the Q-table.
  # The episode ends when the agent reaches a terminal state.
  #
  # Args:
  #   start_state: array with two entries, describing the starting position of the agent.
  #   epsilon (optional): probability of acting greedily.
  #   alpha (optional): learning rate.
  #   gamma (optional): discount factor.
  #   beta (optional): slipping factor.
  #   reward_map (global variable): a HxW array containing the reward given at each state.
  #   q_table (global variable): a HxWx4 array containing Q-values for each state-action pair.
  #
  # Returns:
  #   reward: reward received in the episode.
  #   correction: sum of the temporal difference correction terms over the episode.
  #   q_table (global variable): Recall that R passes arguments by value. So, q_table being
  #   a global variable can be modified with the superassigment operator <<-.

  # Your code here.
  episode_correction = 0

  repeat{
    # Follow policy, execute action, get reward.
    x = start_state[1]
    y = start_state[2]
    A = EpsilonGreedyPolicy(x,y,epsilon)
    a = GreedyPolicy(x,y)
    #calculate new state
    TransState = transition_model(x,y,A,beta)
```

```r
    reward = reward_map[TransState[1],TransState[2]]

    NewCorrection = alpha * (reward + gamma*(q_table[TransState[1],TransState[2],a]) - q_table[x,y,A])
    episode_correction = episode_correction + NewCorrection

    # Q-table update.
    q_table[x,y,A] <<- q_table[x,y,A] + NewCorrection

    start_state = TransState

    if(reward!=0)
      # End episode.
      return (c(reward,episode_correction))
  }

}


H <- 5
W <- 7

reward_map <- matrix(0, nrow = H, ncol = W)
reward_map[3,6] <- 10
reward_map[2:4,3] <- -1

q_table <- array(0,dim = c(H,W,4))

vis_environment()

for(i in 1:10000){
  foo <- q_learning(start_state = c(3,1))

  if(any(i==c(10,100,1000,10000)))
    vis_environment(i)
}

#test run for additional 1000 run

# for(i in 1:100000){
#   foo <- q_learning(start_state = c(3,1))
#
#   if(any(i==c(10,100,1000,10000 , 100000)))
#     vis_environment(i)
# }


# Environment B (the effect of epsilon and gamma)

cat("Epsilon set as 0.5")

H <- 7
W <- 8
```

```r
reward_map <- matrix(0, nrow = H, ncol = W)
reward_map[1,] <- -1
reward_map[7,] <- -1
reward_map[4,5] <- 5
reward_map[4,8] <- 10

q_table <- array(0,dim = c(H,W,4))

vis_environment()

MovingAverage <- function(x, n){

  cx <- c(0,cumsum(x))
  rsum <- (cx[(n+1):length(cx)] - cx[1:(length(cx) - n)]) / n

  return (rsum)
}

for(j in c(0.5,0.75,0.95)){
  q_table <- array(0,dim = c(H,W,4))
  reward <- NULL
  correction <- NULL

  for(i in 1:30000){
    foo <- q_learning(gamma = j, start_state = c(4,1))
    reward <- c(reward,foo[1])
    correction <- c(correction,foo[2])
  }

  vis_environment(i, gamma = j)
  plot(MovingAverage(reward,100),type = "l")
  plot(MovingAverage(correction,100),type = "l")
}



cat("Epsilon set as 0.1")

for(j in c(0.5,0.75,0.95)){
  q_table <- array(0,dim = c(H,W,4))
  reward <- NULL
  correction <- NULL

  for(i in 1:30000){
    foo <- q_learning(epsilon = 0.1, gamma = j, start_state = c(4,1))
    reward <- c(reward,foo[1])
    correction <- c(correction,foo[2])
  }

  vis_environment(i, epsilon = 0.1, gamma = j)
  plot(MovingAverage(reward,100),type = "l")
  plot(MovingAverage(correction,100),type = "l")
```

```
}



H <- 3
W <- 6

reward_map <- matrix(0, nrow = H, ncol = W)
reward_map[1,2:5] <- -1
reward_map[1,6] <- 10

q_table <- array(0,dim = c(H,W,4))

vis_environment()


for(j in c(0,0.2,0.4,0.66)){
  q_table <- array(0,dim = c(H,W,4))

  for(i in 1:10000)
    foo <- q_learning(gamma = 0.6, beta = j, start_state = c(1,1))

  vis_environment(i, gamma = 0.6, beta = j)
}

# install.packages("keras")
#install.packages("tensorflow")
library(tensorflow)
library(keras)

# install.packages("ggplot2")
# install.packages("vctrs")
library(ggplot2)

# If you do not see four arrows in line 19, then do the following:
# File/Reopen with Encoding/UTF-8

#arrows <- c("↑", "→", "↓", "←")
arrows <- c("^", ">", "v", "<")
action_deltas <- list(c(1,0), # up
                      c(0,1), # right
                      c(-1,0), # down
                      c(0,-1)) # left

vis_prob <- function(goal, episodes = 0){

  # Visualize an environment with rewards.
  # Probabilities for all actions are displayed on the edges of each tile.
  # The (greedy) policy for each state is also displayed.
  #
  # Args:
  #   goal: goal coordinates, array with 2 entries.
  #   episodes, epsilon, alpha, gamma, beta (optional): for the figure title.
```

```
  #   H, W (global variables): environment dimensions.

  df <- expand.grid(x=1:H,y=1:W)
  dist <- array(data = NA, dim = c(H,W,4))
  class <- array(data = NA, dim = c(H,W))
  for(i in 1:H)
    for(j in 1:W){
      dist[i,j,] <- DeepPolicy_dist(i,j,goal[1],goal[2])
      foo <- which(dist[i,j,]==max(dist[i,j,]))
      class[i,j] <- ifelse(length(foo)>1,sample(foo, size = 1),foo)
    }

  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,1]),df$x,df$y)
  df$val1 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,2]),df$x,df$y)
  df$val2 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,3]),df$x,df$y)
  df$val3 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,dist[x,y,4]),df$x,df$y)
  df$val4 <- as.vector(round(foo, 2))
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),NA,class[x,y]),df$x,df$y)
  df$val5 <- as.vector(arrows[foo])
  foo <- mapply(function(x,y) ifelse(all(c(x,y) == goal),"Goal",NA),df$x,df$y)
  df$val6 <- as.vector(foo)

  print(ggplot(df,aes(x = y,y = x)) +
          geom_tile(fill = 'white', colour = 'black') +
          scale_fill_manual(values = c('green')) +
          geom_tile(aes(fill=val6), show.legend = FALSE, colour = 'black') +
          geom_text(aes(label = val1),size = 4,nudge_y = .35,na.rm = TRUE) +
          geom_text(aes(label = val2),size = 4,nudge_x = .35,na.rm = TRUE) +
          geom_text(aes(label = val3),size = 4,nudge_y = -.35,na.rm = TRUE) +
          geom_text(aes(label = val4),size = 4,nudge_x = -.35,na.rm = TRUE) +
          geom_text(aes(label = val5),size = 10,na.rm = TRUE) +
          geom_text(aes(label = val6),size = 10,na.rm = TRUE) +
          ggtitle(paste("Action probabilities after ",episodes," episodes")) +
          theme(plot.title = element_text(hjust = 0.5)) +
          scale_x_continuous(breaks = c(1:W),labels = c(1:W)) +
          scale_y_continuous(breaks = c(1:H),labels = c(1:H)))

}

transition_model <- function(x, y, action, beta){

  # Computes the new state after given action is taken. The agent will follow the action
  # with probability (1-beta) and slip to the right or left with probability beta/2 each.
  #
  # Args:
  #   x, y: state coordinates.
  #   action: which action the agent takes (in {1,2,3,4}).
  #   beta: probability of the agent slipping to the side when trying to move.
  #   H, W (global variables): environment dimensions.
  #
```

```r
  # Returns:
  #   The new state after the action has been taken.

  delta <- sample(-1:1, size = 1, prob = c(0.5*beta,1-beta,0.5*beta))
  final_action <- ((action + delta + 3) %% 4) + 1
  foo <- c(x,y) + unlist(action_deltas[final_action])
  foo <- pmax(c(1,1),pmin(foo,c(H,W)))

  return (foo)
}

DeepPolicy_dist <- function(x, y, goal_x, goal_y){

  # Get distribution over actions for state (x,y) and goal (goal_x,goal_y) from the deep policy.
  #
  # Args:
  #   x, y: state coordinates.
  #   goal_x, goal_y: goal coordinates.
  #   model (global variable): NN encoding the policy.
  #
  # Returns:
  #   A distribution over actions.

  foo <- matrix(data = c(x,y,goal_x,goal_y), nrow = 1)

  # return (predict_proba(model, x = foo))
  return (predict_on_batch(model, x = foo)) # Faster.

}

DeepPolicy <- function(x, y, goal_x, goal_y){

  # Get an action for state (x,y) and goal (goal_x,goal_y) from the deep policy.
  #
  # Args:
  #   x, y: state coordinates.
  #   goal_x, goal_y: goal coordinates.
  #   model (global variable): NN encoding the policy.
  #
  # Returns:
  #   An action, i.e. integer in {1,2,3,4}.

  foo <- DeepPolicy_dist(x,y,goal_x,goal_y)

  return (sample(1:4, size = 1, prob = foo))

}

DeepPolicy_train <- function(states, actions, goal, gamma){

  # Train the policy network on a rolled out trajectory.
  #
  # Args:
```

```r
  #   states: array of states visited throughout the trajectory.
  #   actions: array of actions taken throughout the trajectory.
  #   goal: goal coordinates, array with 2 entries.
  #   gamma: discount factor.

  # Construct batch for training.
  inputs <- matrix(data = states, ncol = 2, byrow = TRUE)
  inputs <- cbind(inputs,rep(goal[1],nrow(inputs)))
  inputs <- cbind(inputs,rep(goal[2],nrow(inputs)))

  targets <- array(data = actions, dim = nrow(inputs))
  targets <- to_categorical(targets-1, num_classes = 4)

  # Sample weights. Reward of 5 for reaching the goal.
  weights <- array(data = 5*(gamma^(nrow(inputs)-1)), dim = nrow(inputs))

  # Train on batch. Note that this runs a SINGLE gradient update.
  train_on_batch(model, x = inputs, y = targets, sample_weight = weights)

}

reinforce_episode <- function(goal, gamma = 0.95, beta = 0){

  # Rolls out a trajectory in the environment until the goal is reached.
  # Then trains the policy using the collected states, actions and rewards.
  #
  # Args:
  #   goal: goal coordinates, array with 2 entries.
  #   gamma (optional): discount factor.
  #   beta (optional): probability of slipping in the transition model.

  # Randomize starting position.
  cur_pos <- goal
  while(all(cur_pos == goal))
    cur_pos <- c(sample(1:H, size = 1),sample(1:W, size = 1))

  states <- NULL
  actions <- NULL

  steps <- 0 # To avoid getting stuck and/or training on unnecessarily long episodes.
  while(steps < 20){
    steps <- steps+1

    # Follow policy and execute action.
    action <- DeepPolicy(cur_pos[1], cur_pos[2], goal[1], goal[2])
    new_pos <- transition_model(cur_pos[1], cur_pos[2], action, beta)

    # Store states and actions.
    states <- c(states,cur_pos)
    actions <- c(actions,action)
    cur_pos <- new_pos

    if(all(new_pos == goal)){
```

```r
      # Train network.
      DeepPolicy_train(states,actions,goal,gamma)
      break
    }
  }

}




# Environment D (training with random goal positions)

H <- 4
W <- 4

# Define the neural network (two hidden layers of 32 units each).
model <- keras_model_sequential()
model %>%
  layer_dense(units = 32, input_shape = c(4), activation = 'relu') %>%
  layer_dense(units = 32, activation = 'relu') %>%
  layer_dense(units = 4, activation = 'softmax')

compile(model, loss = "categorical_crossentropy", optimizer = optimizer_sgd(lr=0.001))

initial_weights <- get_weights(model)

train_goals <- list(c(4,1), c(4,3), c(3,1), c(3,4), c(2,1), c(2,2), c(1,2), c(1,3))
val_goals <- list(c(4,2), c(4,4), c(3,2), c(3,3), c(2,3), c(2,4), c(1,1), c(1,4))

show_validation <- function(episodes){

  for(goal in val_goals)
    vis_prob(goal, episodes)

}

set_weights(model,initial_weights)

show_validation(0)

for(i in 1:5000){
  if(i%%10==0) cat("episode",i,"\n")
  goal <- sample(train_goals, size = 1)
  reinforce_episode(unlist(goal))
}

show_validation(5000)


# Environment E (training with top row goal positions)

train_goals <- list(c(4,1), c(4,2), c(4,3), c(4,4))
```

```r
val_goals <- list(c(3,4), c(2,3), c(1,1))

set_weights(model,initial_weights)

show_validation(0)

for(i in 1:5000){
  if(i%%10==0) cat("episode", i,"\n")
  goal <- sample(train_goals, size = 1)
  reinforce_episode(unlist(goal))
}

show_validation(5000)
```