

**COMPUTER SCIENCE**  
SEGEWICK / WAYNE  
PART II: ALGORITHMS, THEORY, AND MACHINES

# 12. Stacks and Queues

Section 4.3

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**COMPUTER SCIENCE**  
SEGEWICK / WAYNE  
PART II: ALGORITHMS, THEORY, AND MACHINES

## 12. Stacks and Queues

- APIs
- Clients
- Strawman implementation
- Linked lists
- Implementations

CS.12.A.StacksQueues.APIs

### Data types and data structures

**Data types**

- Set of values.
- Set of operations on those values.
- Some are built in to Java: int, double, String, ...
- Most are not: Complex, Picture, Charge, ...

**Data structures**

- Represent data.
- Represent relationships among data.
- Some are built in to Java: 1D arrays, 2D arrays, ...
- Most are not: linked list, circular list, tree, ...

**Design challenge for every data type: Which data structure to use?**

- Resource 1: How much memory is needed?
- Resource 2: How much time do data-type methods use?

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### Stack and Queue APIs

A **collection** is an ADT whose values are a multiset of items, all of the same type.

Two fundamental collection **ADTs** differ in just a detail of the specification of their operations.

Stack operations	Queue operations
<ul style="list-style-type: none"> <li>• Add an item to the collection.</li> <li>• Remove and return the item <i>most</i> recently added (LIFO).</li> <li>• Test if the collection is empty.</li> <li>• Return the size of the collection.</li> </ul>	<ul style="list-style-type: none"> <li>• Add an item to the collection.</li> <li>• Remove and return the item <i>least</i> recently added (FIFO).</li> <li>• Test if the collection is empty.</li> <li>• Return the size of the collection.</li> </ul>

Stacks and queues both arise naturally in countless applications.

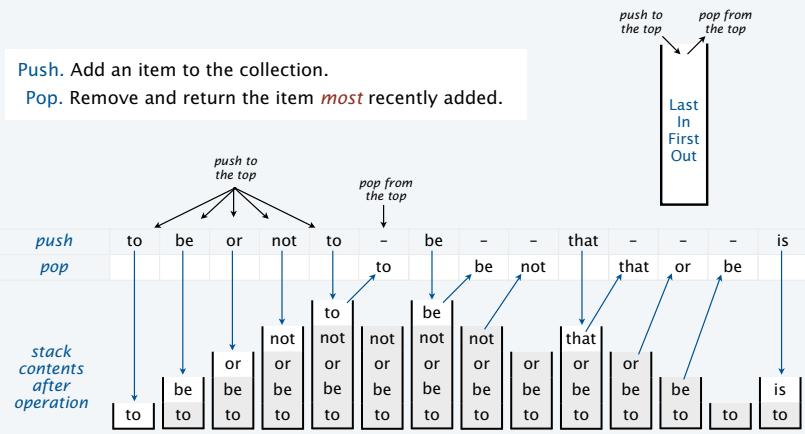
A key characteristic. **No limit** on the size of the collection.

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## Example of stack operations

**Push.** Add an item to the collection.

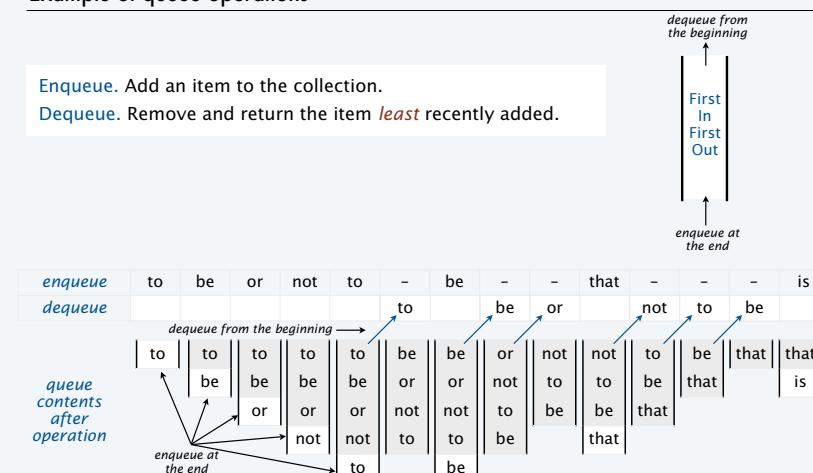
**Pop.** Remove and return the item *most* recently added.



## Example of queue operations

**Enqueue.** Add an item to the collection.

**Dequeue.** Remove and return the item *least* recently added.



## Parameterized data types

**Goal.** Simple, safe, and clear client code for collections of any type of data.

### Java approach: Parameterized data types (generics)

- Use placeholder type name in definition.
- Substitute concrete type for placeholder in clients. ← stay tuned for examples

<b>Stack API</b>	<pre>public class Stack&lt;Item&gt; {     Stack&lt;Item&gt;()      create a stack of items, all of type Item     void push(Item item) add item to stack     Item pop()          remove and return the item most recently pushed     boolean isEmpty()   is the stack empty?     int size()          # of objects on the stack }</pre>
<b>Queue API</b>	<pre>public class Queue&lt;Item&gt; {     Queue&lt;Item&gt;()      create a queue of items, all of type Item     void enqueue(Item item) add item to queue     Item dequeue()        remove and return the item least recently enqueued     boolean isEmpty()    is the queue empty?     int size()           # of objects on the queue }</pre>

## Performance specifications

**Challenge.** Provide guarantees on performance.

**Goal.** Simple, safe, clear, and *efficient* client code.

### Performance specifications

- All operations are constant-time.
- Memory use is linear in the size of the collection, when it is nonempty.
- No limits within the code on the collection size.

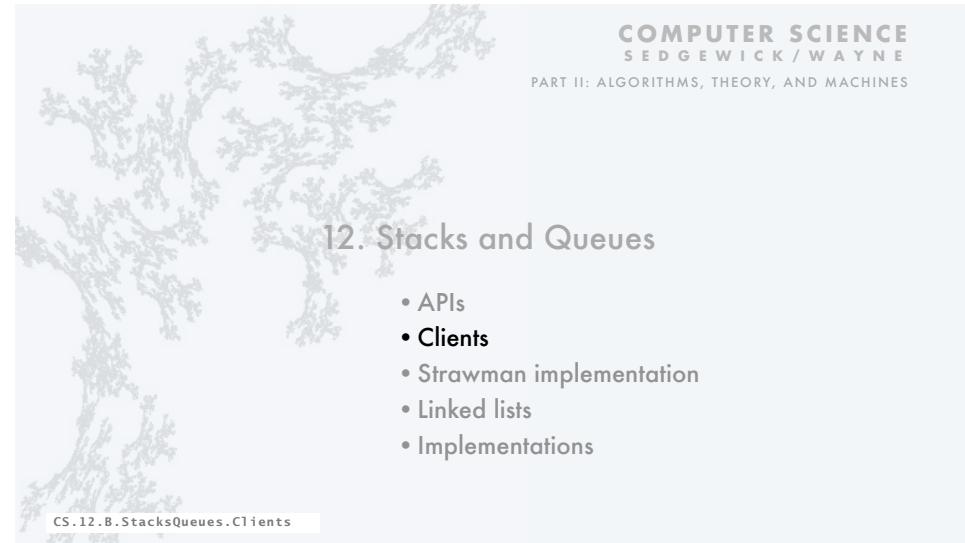
Typically required for client code to be *scalable*

**Java.** Any implementation of the API implements the stack/queue abstractions.

**RS+KW.** Implementations that do not meet performance specs *do not* implement the abstractions.



CS.12.A.StacksQueues.APIs



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CS.12.B.StacksQueues.Clients

### Stack and queue applications

#### Queues

- First-come-first-served resource allocation.
- Asynchronous data transfer (StdIn, StdOut).
- Dispensing requests on a shared resource.
- Simulations of the real world.



#### Stacks

- Last-come-first-served resource allocation.
- Function calls in programming languages.
- Basic mechanism in interpreters, compilers.
- Fundamental abstraction in computing.



### Queue client example: Read all strings from StdIn into an array

#### Challenge

- Can't store strings in array before creating the array.
- Can't create the array without knowing how many strings are in the input stream.
- Can't know how many strings are in the input stream without reading them all.

**Solution:** Use a Queue<String>.

```
public class QEx
{
    public static String[] readAllStrings()
    { /* See next slide. */ }

    public static void main(String[] args)
    {
        String[] words = readAllStrings();
        for (int i = 0; i < words.length; i++)
            StdOut.println(words[i]);
    }
}
```

Note: StdIn has this functionality

```
% more moby.txt
moby
dick
herman
melville
call me ishmael some years ago never
mind how long precisely having
little or no money
...
...
```

```
% java QEx < moby.txt
moby
dick
herman
melville
call me
ishmael
some
years
...
```

## Queue client example: Read all strings from StdIn into an array

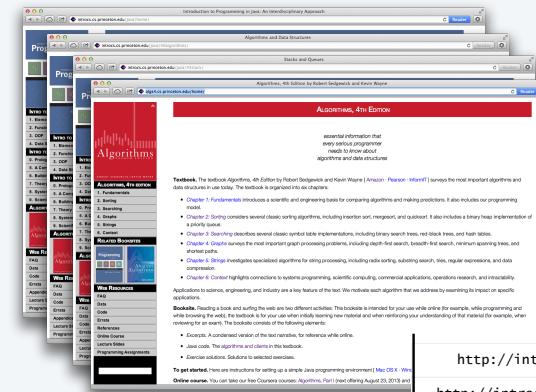
**Solution:** Use a Queue<String>.

- Store strings in the queue.
- Get the size when all have been read from StdIn.
- Create an array of that size.
- Copy the strings into the array.

```
public class QEx
{
    public static String[] readAllStrings()
    {
        Queue<String> q = new Queue<String>();
        while (!StdIn.isEmpty())
            q.enqueue(StdIn.readString());
        int N = q.size();
        String[] words = new String[N];
        for (int i = 0; i < N; i++)
            words[i] = q.dequeue();
        return words;
    }

    public static void main(String[] args)
    {
        String[] words = readAllStrings();
        for (int i = 0; i < words.length; i++)
            StdOut.println(words[i]);
    }
}
```

## Stack example: "Back" button in a browser



### Typical scenario

- Visit a page.
- Click a link to another page.
- Click a link to another page.
- Click a link to another page.
- Click "back" button.
- Click "back" button.
- Click "back" button.

<http://introcs.cs.princeton.edu/java/43stack/>

<http://introcs.cs.princeton.edu/java/40algorithms/>

<http://introcs.cs.princeton.edu/java/home/>

## Autoboxing

**Challenge.** Use a *primitive type* in a parameterized ADT.

### Wrapper types

- Each primitive type has a wrapper reference type.
- Wrapper type has larger set of operations than primitive type.
- Example: `Integer.parseInt()`.
- Instances of wrapper types are objects.
- Wrapper type can be used in a parameterized ADT.

primitive type	wrapper type
int	Integer
char	Character
double	Double
boolean	Boolean

**Autoboxing.** Automatic cast from primitive type to wrapper type.

**Auto-unboxing.** Automatic cast from wrapper type to primitive type.

```
Simple client code →
  Stack<Integer> stack = new Stack<Integer>();
  stack.push(17);           // Autobox  (int -> Integer)
  int a = stack.pop();     // Auto-unbox (Integer -> int)
```

## Stack client example: Postfix expression evaluation

**Infix.** Standard way of writing arithmetic expressions, using parentheses for precedence.

**Example.**  $(1 + ((2 + 3) * (4 * 5))) = (1 + (5 * 20)) = 101$

**Postfix.** Write operator *after* operands (instead of in between them).

**Example.** 1 2 3 + 4 5 \* \* + ← also called "reverse Polish" notation (RPN)



Jan Łukasiewicz  
1878–1956

**Remarkable fact.** No parentheses are needed!

```
1 2 3 + 4 5 * * +
1 (2 + 3) 4 5 * * + ← find first operator, convert
                        to infix, enclose in 0
1 ((2 + 3) * (4 * 5)) + ← iterate, treating subexpressions
                           in parentheses as atomic
                           (1 + ((2 + 3) * (4 * 5)))
```

**Next.** With a stack, postfix expressions are easy to evaluate.



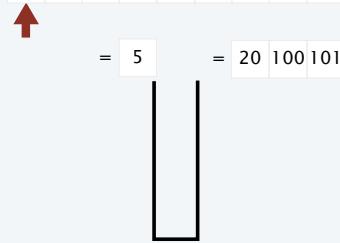
Made slide rules obsolete ()

## Postfix arithmetic expression evaluation

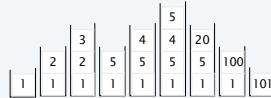
### Algorithm

- While input stream is nonempty, read a token.
- Value: Push onto the stack.
- Operator: Pop operand(s), apply operator, push the result.

1 2 3 + 4 5 \* \* +



1 2 3 + 4 5 \* \* +



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## Stack client example: Postfix expression evaluation

```
public class Postfix
{
    public static void main(String[] args)
    {
        Stack<Double> stack = new Stack<Double>();
        while (!StdIn.isEmpty())
        {
            String token = StdIn.readString();
            if (token.equals("*"))
                stack.push(stack.pop() * stack.pop());
            else if (token.equals("+"))
                stack.push(stack.pop() + stack.pop());
            else if (token.equals("-"))
                stack.push(-stack.pop() + stack.pop());
            else if (token.equals("/"))
                stack.push((1.0 / stack.pop()) * stack.pop());
            else if (token.equals("sqrt"))
                stack.push(Math.sqrt(stack.pop()));
            else
                stack.push(Double.parseDouble(token));
        }
        StdOut.println(stack.pop());
    }
}
```

```
% java Postfix
1 2 3 + 4 5 * * +
101.0
```

```
% java Postfix
1 5 sqrt + 2 /
1.618033988749895
```

$$\frac{1 + \sqrt{5}}{2}$$

### Perspective

- Easy to add operators of all sorts.
- Can do infix with two stacks (see text).
- Could output machine language code.
- Indicative of how Java compiler works.

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## Real-world stack application: PostScript

**PostScript (Warnock-Geschke, 1980s): A turtle with a stack.**

- Postfix program code (push literals; functions pop arguments).
- Add commands to drive virtual graphics machine.
- Add loops, conditionals, functions, types, fonts, strings....

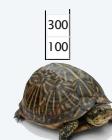
**PostScript code**

```
100 100 moveto
100 300 lineto+
300 300 lineto+
300 100 lineto+
stroke
```

push(100) → call "moveto" (takes args from stack)

define a path

draw the path

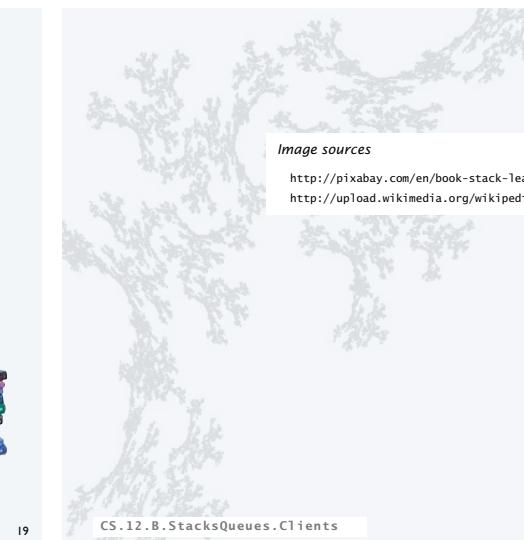


**A simple virtual machine, but not a toy**

- Easy to specify published page.
- Easy to implement on various specific printers.
- Revolutionized world of publishing.



**Another stack machine: The JVM (Java Virtual Machine)!**



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### Image sources

<http://pixabay.com/en/book-stack-learn-knowledge-library-168824/>  
[http://upload.wikimedia.org/wikipedia/commons/2/20/Cars\\_in\\_queue\\_to\\_enter\\_Gibraltar\\_from\\_Spain.jpg](http://upload.wikimedia.org/wikipedia/commons/2/20/Cars_in_queue_to_enter_Gibraltar_from_Spain.jpg)

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## 12. Stacks and Queues

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CS.12.C.StacksQueues.Strawman

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### Strawman ADT for pushdown stacks

#### Warmup: simplify the ADT

- Implement only for items of type String.
- Have client provide a stack *capacity* in the constructor.



Strawman API	
StrawStack(int max)	create a stack of capacity max
void push(String item)	add item to stack
String pop()	return the string most recently pushed
boolean isEmpty()	is the stack empty?
int size()	number of strings on the stack

Rationale. Allows us to represent the collection with an array of strings.

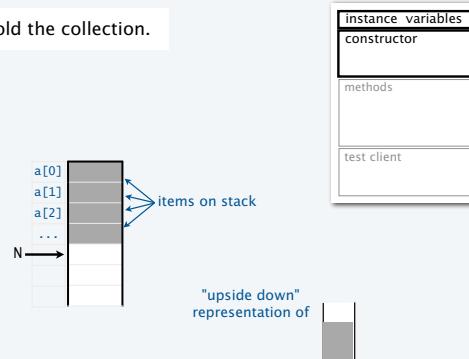
### Strawman implementation: Instance variables and constructor

Data structure choice. Use an array to hold the collection.

```
public class StrawStack
{
    private String[] a;
    private int N = 0;

    public StrawStack(int max)
    { a = new String[max]; }

    ...
}
```



### Strawman stack implementation: Test client

```
public static void main(String[] args)
{
    int max = Integer.parseInt(args[0]);
    StrawStack stack = new StrawStack(max);
    while (!StdIn.isEmpty())
    {
        String item = StdIn.readString();
        if (item.equals("-"))
            StdOut.print(stack.pop());
        else
            stack.push(item);
    }
    StdOut.println();
}
```



% more tobe.txt  
to be or not to - be -- that - - - is  
% java StrawStack 20 < tobe.txt  
to be not that or be

What we expect, once the implementation is done.

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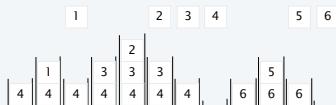
## Pop quiz 1 on stacks

Q. Can we always insert pop() commands to make items come out in sorted order?

Example 1. 6 5 4 3 2 1 - - - - - -

Example 2. 1 - 2 - 3 - 4 - 5 - 6 -

Example 3. 4 1 - 3 2 - - 6 5 - -



## Strawman implementation: Methods

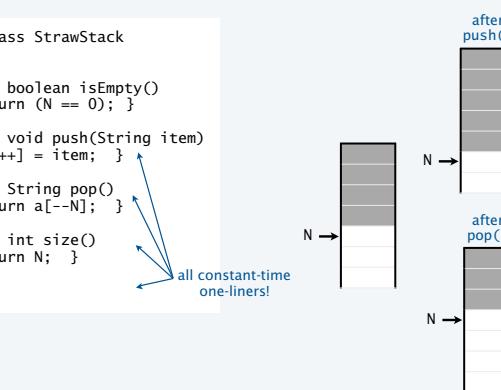
Methods define data-type operations (implement APIs).

```
public class StrawStack
{
    ...
    public boolean isEmpty()
    { return (N == 0); }

    public void push(String item)
    { a[N++] = item; }

    public String pop()
    { return a[--N]; }

    public int size()
    { return N; }
}
```



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## Strawman pushdown stack implementation

```
public class StrawStack
{
    private String[] a;
    private int N = 0;

    public StrawStack(int max)
    { a = new String[max]; }

    public boolean isEmpty()
    { return (N == 0); }

    public void push(String item)
    { a[N++] = item; }

    public String pop()
    { return a[--N]; }

    public int size()
    { return N; }

    public static void main(String[] args)
    {
        int max = Integer.parseInt(args[0]);
        StrawStack stack = new StrawStack(max);
        while (!StdIn.isEmpty())
        {
            String item = StdIn.readString();
            if (item.equals("-"))
                StdOut.print(stack.pop() + " ");
            else
                stack.push(item);
        }
        StdOut.println();
    }
}
```

Annotations:

- instance variables: points to `a` and `N`.
- constructor: points to the constructor `public StrawStack(int max)`.
- methods: points to the methods `push()`, `pop()`, and `size()`.
- test client: points to the `main()` method.
- % more tobe.txt  
to be or not to - be - - that - - is  
% java StrawStack 20 < tobe.txt  
to be not that or be

## Trace of strawman stack implementation (array representation)

	push	to	be	or	not	to	-	be	-	-	that	-	-	-	is
	pop														
stack contents after operation		a[0]	to	to	to	to	to	to	to	to	to	to	to	to	to
		a[1]	be	be	be	be	be	be	be	be	be	be	be	be	be
		a[2]	or	or	or	or	or	or	or	or	or	or	or	or	or
		a[3]													
		a[4]													
		a[5]													
		a[6]													
		a[7]													
		a[8]													
		a[9]													
		a[10]													
		a[11]													
		a[12]													
		a[13]													
		a[14]													
		a[15]													
		a[16]													
		a[17]													
		a[18]													
		a[19]													

Annotations:

- Significant wasted space when stack size is not near the capacity (typical). Arrows point to the empty slots in the array.

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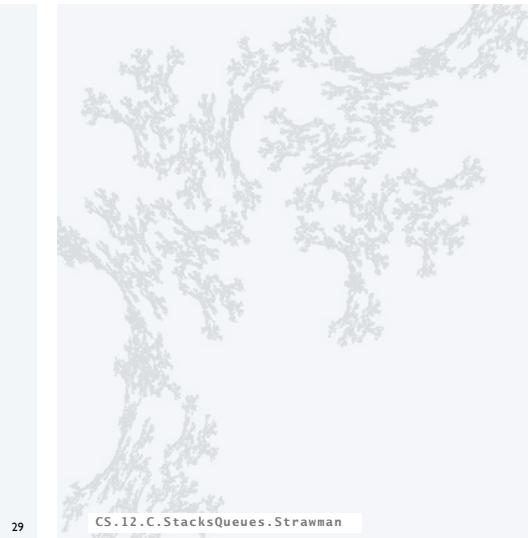
## Benchmarking the strawman stack implementation

StrawStack implements a *fixed-capacity collection that behaves like a stack* if the data fits.

It does *not* implement the stack API or meet the performance specifications.

Stack API	StrawStack requires client to provide capacity
Stack<Item>	public class Stack<Item>
	Stack<Item> X create a stack of items, all of type Item
	void push(Item item) add item to stack
StrawStack works only for strings →	X Item pop() remove and return the item most recently pushed
	boolean isEmpty() is the stack empty?
Performance specifications	int size() # of items on the stack
	<ul style="list-style-type: none"><li>All operations are constant-time. ✓</li><li>Memory use is linear in the size of the collection, X when it is nonempty.</li><li>No limits within the code on the collection size. X</li></ul>

Nice try, but need a new *data structure*.



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## Data structures: sequential vs. linked

### Sequential data structure

- Put objects next to one another.
- Machine: consecutive memory cells.
- Java: array of objects.
- Fixed size, arbitrary access. ← *i*th element

Array at C0	Linked list at C4
addr	addr
C0	C0 "Carol"
C1	C1 null
C2	C2 "Bob"
C3	C3 "Alice"
C4	C4 CA
C5	C5 CA
C6	C6 CA
C7	C7 CA
C8	C8 CA
C9	C9 CA
CA	CA Bob
CB	CB Alice

### Linked data structure

- Associate with each object a *link* to another one.
- Machine: link is memory address of next object.
- Java: link is reference to next object.
- Variable size, sequential access. ← *next* element
- Overlooked by novice programmers.
- Flexible, widely used method for organizing data.

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## Simplest singly-linked data structure: linked list

### Linked list

- A recursive data structure.
- Def. A *linked list* is null or a reference to a *node*.
- Def. A *node* is a data type that contains a reference to a node.
- Unwind recursion: A linked list is a sequence of nodes.

```
private class Node
{
    private String item;
    private Node next;
}
```

### Representation

- Use a private *nested class* Node to implement the node abstraction.
- For simplicity, start with nodes having two values: a String and a Node.

### A linked list



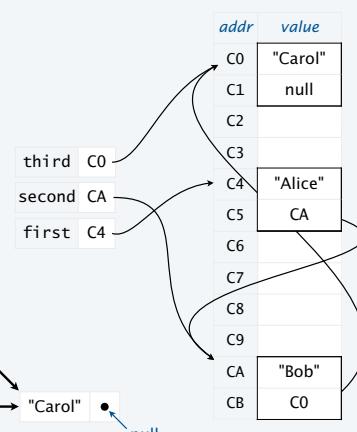
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## Building a linked list

```
Node third = new Node();
third.item = "Carol";
third.next = null;

Node second = new Node();
second.item = "Bob";
second.next = third;

Node first = new Node();
first.item = "Alice";
first.next = second;
```

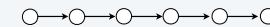


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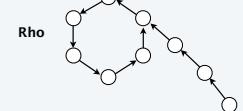
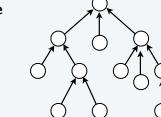
## Singly-linked data structures

Even with just one link ( $\circlearrowright$ ) a wide variety of data structures are possible.

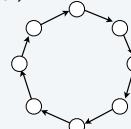
### Linked list (this lecture)



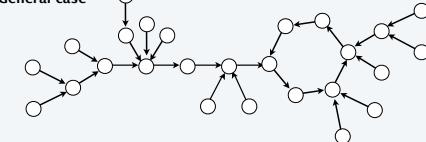
### Tree



### Circular list (TSP)



### General case



From the point of view of a particular object, all of these structures look the same.

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Multiply linked structures: many more possibilities!

## List processing code

### Standard operations for processing data structured as a singly-linked list

- Add a node at the beginning.
- Remove and return the node at the beginning.
- Add a node at the end (requires a reference to the last node).
- Traverse the list (visit every node, in sequence).

### An operation that calls for a *doubly-linked list* (slightly beyond our scope)

- Remove and return the node at the end.

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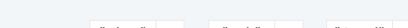
## List processing code: Remove and return the first item

Goal. Remove and return the first item in a linked list `first`.



```
item = first.item;
```

item  
"Alice"



```
first = first.next;
```

item  
"Alice"



```
return item;
```

item  
"Alice"



## List processing code: Add a new node at the beginning

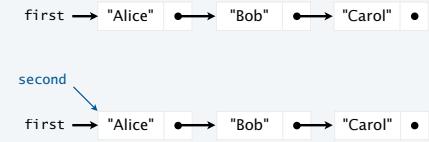
Goal. Add `item` to a linked list `first`.

item  
"Dave"



```
Node second = first;
```

second



```
first = new Node();
```

first



```
first.item = item;  
first.next = second;
```

second



## List processing code: Traverse a list

Goal. Visit every node on a linked list `first`.

```
Node x = first;  
while (x != null)  
{  
    StdOut.println(x.item);  
    x = x.next;  
}
```



StdOut  
Alice  
Bob  
Carol

## Pop quiz 1 on linked lists

Q. What is the effect of the following code (not-so-easy question)?

```
...  
Node list = null;  
while (!StdIn.isEmpty())  
{  
    Node old = list;  
    list = new Node();  
    list.item = StdIn.readString();  
    list.next = old;  
}  
for (Node t = list; t != null; t = t.next)  
    StdOut.println(t.item);  
...
```

## Pop quiz 2 on stacks

Q. Give code that uses a stack to print the strings from StdIn on StdOut, in reverse order.

## Pop quiz 2 on linked lists

Q. What is the effect of the following code (not-so-easy question)?

```
...
Node list = new Node();
list.item = StdIn.readString();
Node last = list;
while (!StdIn.isEmpty())
{
    last.next = new Node();
    last = last.next;
    last.item = StdIn.readString();
}
...
```

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PART II: ALGORITHMS, THEORY, AND MACHINES

## 14. Stacks and Queues

- APIs
- Clients
- Strawman implementation
- Linked lists
- Implementations

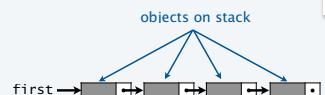
CS.12.E.StacksQueues.Implementations

## Pushdown stack implementation: Instance variables and constructor

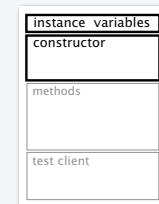
Data structure choice. Use a [linked list](#) to hold the collection.

```
public class Stack<Item>
{
    private Node first = null;
    private int N = 0;

    private class Node
    {
        private Item item;
        private Node next;
    }
    ...
}
```

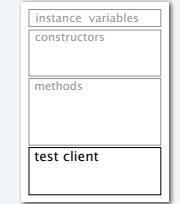


Annoying exception (not a problem here).  
Can't declare an array of Item objects (don't ask why).  
Need cast: Item[] a = (Item[]) new Object[N]



## Stack implementation: Test client

```
public static void main(String[] args)
{
    Stack<String> stack = new Stack<String>();
    while (!StdIn.isEmpty())
    {
        String item = StdIn.readString();
        if (item.equals("-"))
            System.out.print(stack.pop() + " ");
        else
            stack.push(item);
    }
    StdOut.println();
}
```



% more tobe.txt  
to be or not to - be -- that --- is  
% java Stack < tobe.txt  
to be not that or be

What we expect, once the implementation is done.

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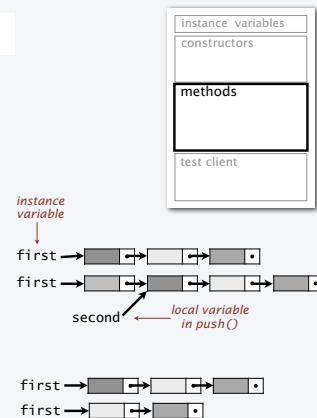
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## Stack implementation: Methods

Methods define data-type operations (implement the API).

```
public class Stack<Item>
{
    ...
    public boolean isEmpty()
    { return first == null; }
    public void push(Item item)
    {
        Node second = first;
        first = new Node();
        first.item = item;
        first.next = second;
        N++;
    }
    public Item pop()
    {
        Item item = first.item;
        first = first.next;
        N--;
        return item;
    }
    public int size()
    { return N; }
    ...
}
```

might also use N == 0  
add a new node to the beginning of the list  
remove and return first item on list



## Stack implementation

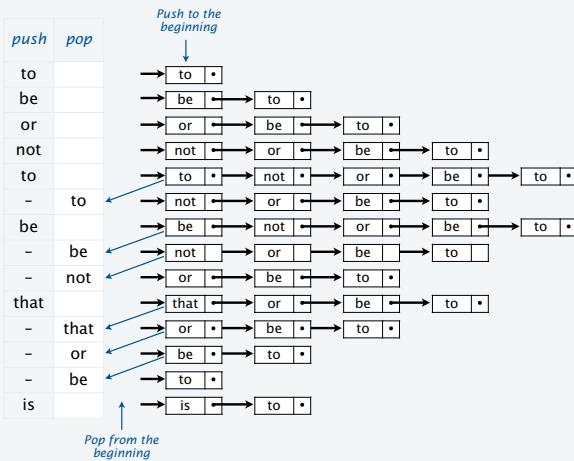
```
public class Stack<Item>
{
    private Node first = null;
    private int N = 0;
    private class Node
    {
        private Item item;
        private Node next;
    }
    public boolean isEmpty()
    { return first == null; }
    public void push(Item item)
    {
        Node second = first;
        first = new Node();
        first.item = item;
        first.next = second;
        N++;
    }
    public Item pop()
    {
        Item item = first.item;
        first = first.next;
        N--;
        return item;
    }
    public int size()
    { return N; }
    public static void main(String[] args)
    { // See earlier slide }
}
```

% more tobe.txt  
to be or not to - be -- that --- is  
% java Stack < tobe.txt  
to be not that or be

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## Trace of stack implementation (linked list representation)



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## Benchmarking the stack implementation

Stack implements the stack abstraction.

It *does* implement the API and meet the performance specifications.

### Stack API

<code>public class Stack&lt;Item&gt;</code>	
<code>Stack&lt;Item&gt;()</code>	<i>create a stack of items, all of type Item</i>
<code>void push(Item item)</code>	<i>add item to stack</i>
<code>Item pop()</code>	<i>remove and return the item most recently pushed</i>
<code>boolean isEmpty()</code>	<i>is the stack empty?</i>
<code>int size()</code>	<i># of items on the stack</i>

### Performance specifications

- All operations are constant-time. ✓
- Memory use is linear in the size of the collection, when it is nonempty. ✓
- No limits within the code on the collection size. ✓

Made possible by [linked data structure](#).

`dequeue()`: same code as `pop()`  
`enqueue()`: slightly more complicated

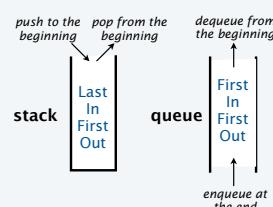
Also possible to implement the *queue* abstraction with a singly-linked list (see text).

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## Summary

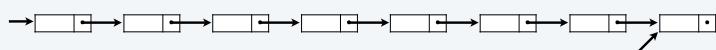
### Stacks and queues

- Fundamental collection abstractions.
- Differ only in order in which items are removed.
- Performance specifications: Constant-time for all operations and space linear in the number of objects.



### Linked structures

- Fundamental alternative to arrays.
- Enable implementations of the stack/queue abstractions *that meet performance specifications*.

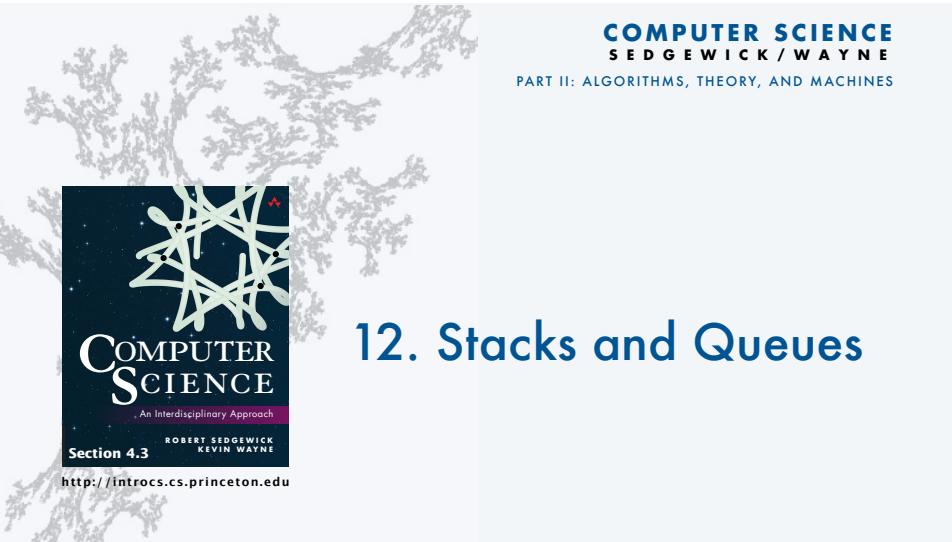


Next: [Symbol tables](#)

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## 12. Stacks and Queues