

# Econometric Theory I

# Computer Lab Class III

Juergen Amann juergen.amann@nottingham.ac.uk Wednesday 12:00 - 13:00, C42 SCGB

#### Last week

#### Two exercises:

- Exercise 1: Examine the evidence of a gender pay gap.
  - Dummy variables.
  - What does it mean when we 'control for (a) variable(s)'?
- Exercise 2: Examine the determinants of educational attainment.
  - Joint parameter hypothesis testing of coefficients; e.g.  $H_0: \beta_2 = \beta_3$ .
  - Restricted Least Squares.

#### Model the birth weights of infants:

- Identify factors that may determine a child's birth weight.
- Interpret Stata output.
- One- vs. two-sided hypothesis testing.
- Over- and under-specification of an econometric model.
- Homoskedasticity vs. heteroskedasticity.

• What are the factors that determine a child's birth weight?

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- Our data set bwght.xls contains n = 1,388 observations on the following variables:
  - bwght;: birth weight in ounces
  - faminc<sub>i</sub>: family income in USD 1,000
  - order<sub>i</sub>: birth order of child
  - cigs; cigarettes smoked per day by mother during pregnancy
  - mothereduc<sub>i</sub>: number of years mother spent in full-time education

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- What is the expected sign for the variables? Which variables do you expect to be significant?

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- Estimate a regression of birth weight on a constant, family income and birth order.

```
. regress bught faminc order
    Source | SS df MS Number of obs
                                                = 1,388
                            ----- F(2, 1385)
                                                = 11.15
    Model | 9103.2286 2 4551.6143 Prob > F = 0.0000
  Residual | 565508.491 1,385 408.30938 R-squared = 0.0158
                       ----- Adj R-squared = 0.0144
    Total | 574611.72 1,387 414.283864 Root MSE
                                                    20, 207
          Coef. Std. Err. t P>|t| [95% Conf. Interval]
    bwght |
    faminc | .1235904 .0290392 4.26 0.000 .0666249 .1805559
    order | 1.439648 .6086762 2.37 0.018 .2456214 2.633675
    cons | 112.7618 1.45622 77.43 0.000
                                          109 9052
                                                  115,6185
```

• Test if variables are significant, interpret signs and magnitudes.

#### Results of regression 1:

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- order: For the same family income, the next-born child is expected to be approx. 1.14 ounces ( $\approx 32.4g$ ) heavier than his/her sibling born right before him/her.

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- order: For the same family income, the next-born child is expected to be approx. 1.14 ounces ( $\approx 32.4g$ ) heavier than his/her sibling born right before him/her.
- All coefficients are found to be highly significant.

# Birth weight: regression 1 • Look at the test for overall relation.

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```
Source I
                                             Number of obs
                                                                1,388
                                             F(2, 1385)
  Model |
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                             2 4551.6143
                                            Prob > F
                                                                 0.0000
Residual |
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                      1,385
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                                             R-squared
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                                                                 0.0144
            574611.72 1.387 414.283864
                                             Root MSE
  Total |
                                                                 20,207
```

• Remember [Slides 4, p.10]: 'One useful special case of the F-test is in testing whether all the explanatory variables taken together are significant in the regression model.'

Look at the test for overall relation.

Source	SS	df	MS	Number of obs		
Model	9103.2286	2	4551.6143	Prob > F R-squared	=	0.0000
				Adj R-squared		
Total	574611.72	1,387	414.283864	Root MSE	=	20.207

• Remember [Slides 4, p.10]: 'One useful special case of the F-test is in testing whether all the explanatory variables taken together are significant in the regression model.'

For our multivariate regression

$$\mathtt{bwght_i} = \beta_1 + \beta_2 \mathtt{faminc_i} + \beta_3 \mathtt{order_i} + u_i$$
 this means  $H_0: \beta_2 = \beta_3 = 0$ .

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• This test statistic follows an  $F_{(q,n-k)}$  distribution under  $H_0$  where q=2 (no. of tested parameters) and (n-k)=1,385.

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				Adj R-squared Root MSE	=	0.0144

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this means  $H_0$ :  $\beta_2 = \beta_3 = 0$ .

- This test statistic follows an  $F_{(q,n-k)}$  distribution under  $H_0$  where q=2 (no. of tested parameters) and (n-k)=1,385.
- Compare test statistic against critical value (and/or use p-value): We reject H<sub>0</sub> here; variables are 'jointly significant'.

• Interpret the goodness-of-fit of the regression, i.e. the  $R^2$ .

```
Source |
             SS
                       df
                              MS
                                     Number of obs
                                                    1,388
                                     F(2, 1385)
                                                 = 11.15
                                     Prob > F =
  Model I
          9103.2286
                     2
                           4551.6143
                                                     0.0000
Residual |
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                                                   20.207
```

• See Slides 4, p.19-21 for more details.

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- See Slides 4, p.19-21 for more details.
- Main takeaway here: Our model fits the data only very poorly (very low  $R^2$ ).

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Source |
              SS
                                        Number of obs
                                                          1.388
                          df
                                 MS
                                        F(2, 1385)
                                                          11.15
                                        Prob > F
  Model
           9103 2286
                                                           0.0000
                          2
                              4551 6143
Residual |
          565508.491
                    1,385 408,30938
                                        R-squared
                                                           0.0158
                                        Adj R-squared
                                                      = 0.0144
  Total | 574611.72 1,387 414.283864
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- Main takeaway here: Our model fits the data only very poorly (very low  $R^2$ ).
- Only about 1.58% of the variation of the dependent variable about its mean is explained by regression 1.

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- Main takeaway here: Our model fits the data only very poorly (very low  $R^2$ ).
- Only about 1.58% of the variation of the dependent variable about its mean is explained by regression 1.
- As before, we can calculate the R<sup>2</sup> using other parts of the regression output.

  \* See how it's done!

 Estimate a regression of birth weight on a constant, family income, birth order and cigs.

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```
. regress bught faminc order cigs
                                       Number of obs
                                                   = 1,388
    Source |
                                 MS
                                       F(3, 1384)
                                                   = 16.63
    Model | 19996.5211 3 6665.50703 Prob > F = 0.0000
  Residual | 554615.199   1,384   400.733525   R-squared = 0.0348
                        ----- Adj R-squared = 0.0327
    Total | 574611.72 1.387 414.283864 Root MSE
                                                   = 20.018
     bwght |
           Coef. Std. Err. t P>|t| [95% Conf. Interval]
    famine | .0979201 .0291868 3.35 0.001 .040665 .1551752
     order | 1.616372 .603955 2.68 0.008 .4316058 2.801138
           -.4771537 .091518 -5.21 0.000 -.6566827 -.2976247
     cigs |
     _cons | 114.2143 1.4693
                              77.73 0.000
                                         111.3321 117.0966
```

• Observe how the regression output has changed!

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- Observe how the regression output has changed!
- Conduct a 1-sided test on the cigs coefficient using statistical tables!

- Two-sided:
  - Testing for the possibility of the relationship in both directions.

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  - Given a fixed significance level of, say,  $\alpha=0.05$ , a two-sided test allots half of our  $\alpha$  to testing the statistical significance in one and the other half of  $\alpha$  to the other direction.
  - This means cigs is considered significantly different from 0 if the test statistic is in the top 2.5% or bottom 2.5% of its probability distribution, resulting in a p-value less than 0.05.

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  - Tests either if the estimated parameter is significantly greater **or** significantly smaller than zero, but not both.

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  - A one-sided test allots **all** of our  $\alpha$  to testing the statistical significance in *one* direction only.
  - This means that, depending on the chosen tail, cigs is considered significantly lower (higher) than 0 if the test statistic is in the bottom (top) 5% of its probability distribution, resulting in a p-value less than 0.05.

One- vs two-sided tests?

- One-sided:
  - Tests either if the estimated parameter is significantly greater
     or significantly smaller than zero, but not both.
  - IOW it tests for the possibility of the relationship in one direction and completely disregarding the other direction.
  - A one-sided test allots **all** of our  $\alpha$  to testing the statistical significance in *one* direction only.
  - This means that, depending on the chosen tail, cigs is considered significantly lower (higher) than 0 if the test statistic is in the bottom (top) 5% of its probability distribution, resulting in a p-value less than 0.05.
  - The one-tailed test provides more power to detect an effect in one direction by not testing the effect in the other direction.

• One-sided test in our example

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```
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     bwght |
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                       .0291868
                                 3.35
                                        0.001
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                                                -.6566827 -.2976247
     cons |
            114.2143
                                 77.73 0.000
                                                 111.3321
                       1.4693
                                                          117.0966
```

- Above  $H_0: \beta_{cigs} = 0, H_a: \beta_{cigs} \neq 0$  (two-sided).
- We want to test for the one-sided alternative that  $\beta_{cigs} < 0$ . Note that the effect has the correct 'predicted sign'.
- To get the p-value, simply divide  $p Val_{cigs}/2$  because the effect is going in the 'predicted' direction. This is P(|-5.21|).
- We reject H<sub>0</sub> in favour of the one-sided alternative indicating that smoking has a significantly negative impact on birth weight.

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Source   SS							•
	20031.4556						
	554580.264				ared		
+				Adj F	R-squared	=	0.0321
Total	574611.72	1,387	414.283864	Root	MSE	=	20.025
bwght	Coef.	Std. Err.	t P	> t	[95% Con		
faminc	.0937846	.0323043		.004	.0002000		
	.0937846 1.631028						2.820183
order		.6061915	2.69 0	.007	.4418743	3	
order   cigs	1.631028	.6061915 .0925814	2.69 0 -5.11 0	.007	.4418743 6546978	} }	2914676

Interpret results!

From the lecture [slides 4, pp. 14]:

 If an important explanatory variable is missing from our model, the estimated coefficients are biased (omitted variable bias)!

#### From the lecture [slides 4, pp. 14]:

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- If an irrelevant explanatory variable is included in our model, the estimated coefficients are still unbiased but inefficient, i.e. its variance is greater when the irrelevant variable is included.

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• regression 1: low  $R^2$ , cigs missing; probably under-sepcified.

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#### Which is the best specification for our problem?

- regression 1: low  $R^2$ , cigs missing; probably under-sepcified.
- regression 3: mothereduc probably irrelevant; probably over-sepcified.

#### From the lecture [slides 4, pp. 14]:

- If an important explanatory variable is missing from our model, the estimated coefficients are biased (omitted variable bias)!
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#### Which is the best specification for our problem?

- regression 1: low  $R^2$ , cigs missing; probably under-sepcified.
- regression 3: mothereduc probably irrelevant; probably over-sepcified.
- regression 2: note that adj.  $R_{reg.3}^2 < adj$ .  $R_{reg.2}^2$ ; probably best fit.

See lecture [slides 4, pp. 37]:

• **Homoskedasticity**: One of the classical assumptions of the linear regression model [Slides 3, p.3].

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     However, OLS is no longer efficient (smallest variance) and estimated standard errors are biased.

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- Heteroskedasticity: Errors have non-constant variances.
  - Example: More variation in consumption behaviour for rich compared to poor families.
  - Causes: measurement error, sub-population differences, miss-specification (omitted variables) etc. (many causes possible).
  - Consequences: Coefficient estimates remain unbiased.
     However, OLS is no longer efficient (smallest variance) and estimated standard errors are biased.
    - ⇒ Hypothesis testing is invalid!

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```

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```
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```
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```

White test for heteroskedasticity:

- We fail to reject H<sub>0</sub> of homoskedasticity; no statistical evidence of heteroskedasticity.
- This is in line with our visual analysis.

# Thank you and see you in two weeks!

Juergen Amann

juergen.amann@nottingham.ac.uk Wednesday 12:00 - 13:00, C42 SCGB

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Joint parameter test (remember, we did this last week!)

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Joint parameter test (remember, we did this last week!)

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. test faminc = order = 0
(1) faminc - order = 0
(2) faminc = 0

F( 2, 1385) = 11.15
    Prob > F = 0.0000
```

② Use one of the results from lecture [Slides 4, p.11], e.g.:

$$F^* = \frac{R^2/(q)}{(1-R^2)/(n-k)}$$

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$$F^* = \frac{R^2/(q)}{(1-R^2)/(n-k)}$$

```
. display 0.0158/(1 - 0.0158) * 1385/2
11.117151
```

## R<sup>2</sup> 'by hand' Go back

From the lecture slides [Slides 3, pp.20]:

• The  $R^2$  measure of goodness-of-fit is defined by  $R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$ .

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    Source |
                SS
                                  MS
                                        Number of obs = 1.388
                                        F(4, 1383) = 12.49
                                       Prob > F = 0.0000
     Model |
            20031.4556
                          4 5007.8639
   Residual |
           554580.264 1,383 400.998022
                                      R-squared =
                                                       0.0349
                    ----- Adj R-squared = 0.0321
     Total |
           574611.72 1.387 414.283864 Root MSE
                                                       20.025
```

#### R<sup>2</sup> 'by hand' Go back Go back

From the lecture slides [Slides 3, pp.20]:

• The  $R^2$  measure of goodness-of-fit is defined by  $R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$ 

```
. regress bught faminc order cigs mothereduc
    Source |
                SS
                                  MS
                                        Number of obs = 1.388
                                        F(4, 1383) = 12.49
                                       Prob > F = 0.0000
     Model |
            20031.4556
                          4 5007.8639
   Residual |
           554580.264
                     1,383 400.998022
                                      R-squared = 0.0349
        --+---- Adj R-squared = 0.0321
     Total |
           574611.72 1.387 414.283864 Root MSE
                                                       20.025
. display 20031.4556 / 574611.72
0.03486085
```

#### $R^2$ 'by hand' $\bigcirc$ Go back

From the lecture slides [Slides 3, pp.20]:

• The  $R^2$  measure of goodness-of-fit is defined by  $R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$ 

```
. regress bught faminc order cigs mothereduc
                  SS
                                     MS
                                            Number of obs = 1.388
     Source |
                                            F(4, 1383) = 12.49
     Model
              20031.4556
                             4
                                 5007.8639
                                           Prob > F = 0.0000
             554580.264
                       1,383 400,998022
                                          R-squared =
                                                            0.0349
   Residual |
                                           Adj R-squared = 0.0321
     Total |
              574611.72
                       1,387 414,283864
                                            Root MSE
                                                             20.025
. display 20031.4556 / 574611.72
0.03486085
```

• The adjusted  $R^2$  incorporates a penalty for including additional variables:  $adj.R^2=1-\left[\left(\frac{T-1}{T-k}\right)(1-R^2)\right]$ 

#### $R^2$ 'by hand' $\bullet$ Go back

From the lecture slides [Slides 3, pp.20]:

• The  $R^2$  measure of goodness-of-fit is defined by  $R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$ 

```
. regress bught faminc order cigs mothereduc
    Source |
                                  MS
                                         Number of obs = 1.388
                                         F(4, 1383) = 12.49
                                        Prob > F = 0.0000
     Model I
            20031.4556
                     4 5007.8639
   Residual | 554580.264 1,383 400.998022
                                       R-squared = 0.0349
                         ----- Adj R-squared = 0.0321
     Total |
             574611.72 1.387 414.283864
                                         Root MSE
                                                = 20.025
. display 20031.4556 / 574611.72
0.03486085
```

• The adjusted  $R^2$  incorporates a penalty for including additional variables:  $adj.R^2=1-\left[\left(\frac{T-1}{T-k}\right)(1-R^2)\right]$ 

```
. display 1 - ((1388-1)/(1383)*(1-20031.4556/574611.72))
0.03206942
```