

Econometric Theory I

Computer Lab Class III

Juergen Amann juergen.amann@nottingham.ac.uk Wednesday 12:00 - 13:00, C42 SCGB

Last week

Two exercises:

- Exercise 1: Examine the evidence of a gender pay gap.
 - Dummy variables.
 - What does it mean when we 'control for (a) variable(s)'?
- Exercise 2: Examine the determinants of educational attainment.
 - Joint parameter hypothesis testing of coefficients; e.g. $H_0: \beta_2 = \beta_3$.
 - Restricted Least Squares.

Modelling the birth weights of infants

• Identify factors that may determine a child's birth weight.

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- Homoskedasticity vs. heteroskedasticity.

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- Our data set bwght.xls contains N = 1,388 observations on the following variables:
 - bwght;: birth weight in ounces,
 - faminc_i: family income in USD 1,000,
 - order; birth order of child,
 - cigs_i: cigarettes smoked per day by mother during pregnancy,
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- What is the expected sign for the variables? Which variables do you expect to be significant?

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```
. regress bught faminc order
    Source | SS df MS Number of obs
                                                = 1,388
                            ----- F(2, 1385)
                                                = 11.15
    Model | 9103.2286 2 4551.6143 Prob > F = 0.0000
  Residual | 565508.491 1,385 408.30938 R-squared = 0.0158
                       ----- Adj R-squared = 0.0144
    Total | 574611.72 1,387 414.283864 Root MSE
                                                    20, 207
          Coef. Std. Err. t P>|t| [95% Conf. Interval]
    bwght |
    faminc | .1235904 .0290392 4.26 0.000 .0666249 .1805559
    order | 1.439648 .6086762 2.37 0.018 .2456214 2.633675
    cons | 112.7618 1.45622 77.43
                                  0.000
                                          109 9052
                                                   115 6185
```

• Test if variables are significant, interpret signs and magnitudes.

Results

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- order: For the same family income, the next-born child is expected to be approx. 1.14 ounces ($\approx 32.4g$) heavier than his/her sibling born right before him/her.

Results

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- order: For the same family income, the next-born child is expected to be approx. 1.14 ounces (\approx 32.4g) heavier than his/her sibling born right before him/her.
- All coefficients are found to be highly significant.

Regression 1
• Look at the test for overall relation.

Look at the test for overall relation.

Source	SS	df	MS	Number of obs		
Model	9103.2286	2	4551.6143	Prob > F R-squared	=	0.0000
+-				Adj R-squared	=	0.0144
Total	574611.72	1,387	414.283864	Root MSE	=	20.207

• Remember [Slides 4, p.10]: 'One useful special case of the F-test is in testing whether all the explanatory variables taken together are significant in the regression model.'

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- Remember [Slides 4, p.10]: 'One useful special case of the F-test is in testing whether all the explanatory variables taken together are significant in the regression model.'
- For our multivariate regression

$$\texttt{bwght}_{\texttt{i}} = \beta_1 + \beta_2 \texttt{faminc}_{\texttt{i}} + \beta_3 \texttt{order}_{\texttt{i}} + u_i$$

this means H_0 : $\beta_2 = \beta_3 = 0$.

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• This test statistic follows an $F_{(q,n-k)}$ distribution under H_0 where q=2 and (n-k)=1,385.

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Source	SS	df	MS	Number of obs	=	1,388
				F(2, 1385)	=	11.15
Model	9103.2286	2	4551.6143	Prob > F	=	0.0000
Residual	565508.491	1,385	408.30938	R-squared	=	0.0158
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this means H_0 : $\beta_2 = \beta_3 = 0$.

- This test statistic follows an $F_{(q,n-k)}$ distribution under H_0 where q=2 and (n-k)=1,385.
- Compare test statistic against critical value (and/or use p-value): We reject H₀ here; variables are 'jointly significant'.

• Interpret the goodness-of-fit of the regression, i.e. the R^2 .

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• See Slides 4, p.19-21 for more details.

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	9103.2286 565508.491	2		F(2, 1385) Prob > F R-squared	=	
+ Total	574611.72			Adj R-squared Root MSE		

- See Slides 4, p.19-21 for more details.
- Main message: Our model fits the data only very poorly (very low R^2)!

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- Main message: Our model fits the data only very poorly (very low R^2)!
- Only about 1.58% of the variation of the dependent variable about its mean is explained by regression 1.
- As before, we can calculate the R² using other parts of the regression output.

 * See how it's done!

 Estimate a regression of birth weight on a constant, family income, birth order and cigs.

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                                 MS
                                       Number of obs
                                                   = 1,388
    Source |
                                       F(3, 1384)
                                                   = 16.63
    Model | 19996.5211 3 6665.50703 Prob > F = 0.0000
  Residual | 554615.199 1,384 400.733525 R-squared = 0.0348
                                      Adj R-squared = 0.0327
    Total | 574611.72 1.387 414.283864 Root MSE
                                                      20.018
     bwght |
             Coef.
                     Std. Err. t P>|t|
                                             [95% Conf. Interval]
    famine | .0979201 .0291868 3.35 0.001 .040665 .1551752
     order | 1.616372 .603955 2.68 0.008 .4316058 2.801138
     cigs |
           -.4771537 .091518 -5.21 0.000 -.6566827 -.2976247
     _cons | 114.2143 1.4693
                              77.73 0.000
                                          111.3321 117.0966
```

Observe how the regression output has changed!

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- Observe how the regression output has changed!
- Conduct a 1-sided test on the cigs coefficient using statistical tables!

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 - Testing for the possibility of the relationship in both directions.

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 - Given a fixed significance level of, say, $\alpha=0.05$, a two-sided test allots half of our α to testing the statistical significance in **one** direction and the other half of α to test in the **other** direction.
 - This means cigs is considered significantly different from 0 if the test statistic is in the top 2.5% or bottom 2.5% of its probability distribution, resulting in a p-value of less than 0.05.

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 - Tests either if the estimated parameter is significantly greater **or** significantly smaller than zero, but not both.

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 - A one-sided test allots **all** of our α to testing the statistical significance in **one** direction only.
 - This means that, depending on the chosen tail, cigs is considered significantly lower (higher) than 0 if the test statistic is in the bottom (top) 5% of its probability distribution, resulting in a p-value of less than 0.05.

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 - This means that, depending on the chosen tail, cigs is considered significantly lower (higher) than 0 if the test statistic is in the bottom (top) 5% of its probability distribution, resulting in a p-value of less than 0.05.
 - The one-tailed test provides more power to detect an effect in one direction by not testing the effect in the other direction.

One-sided test in our example

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                                 -5.21 0.000
     cons |
            114.2143
                                 77.73 0.000
                                                 111.3321
                       1.4693
                                                          117.0966
```

- Above $H_0: \beta_{cigs} = 0, H_a: \beta_{cigs} \neq 0$ (two-sided).
- We want to test for the one-sided alternative that $\beta_{cigs} < 0$. Note that the effect has the correct 'predicted sign'.
- To get the p-value, simply divide $p Val_{cigs}/2$ because the effect is going in the 'predicted' direction. This is P(|-5.21|).
- We reject H₀ in favour of the one-sided alternative indicating that smoking has a significantly negative impact on birth weight.

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				Number of ob			
				F(4, 1383) Prob > F			
	554580.264			R-squared	=	0.0349	
Total	574611.72	1,387	414.283864	Root MSE	=	20.025	
bwght	Coef.	Std. Err.	t F		Conf. In	nterval]	
faminc	.0937846	.0323843	2.90 0	.004 .0302	569	.1573122	
order	1.631028	.6061915	2.69	.007 .4418	743	2.820183	
cigs	4730827	.0925814	-5.11 C	.0006546	978 -	.2914676	
mothereduc	.076161	.2580337	0.30	.7684300	187 .	.5823408	
	112 217	2 276042	22 56 0	.000 106.6	005	110 0/15	

Interpret results!

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 If an important explanatory variable is missing from our model, the estimated coefficients are biased (omitted variable bias)!

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Which is the best specification for our problem?

- Regression 1: low R^2 , cigs missing; probably under-specified.
- Regression 3: mothereduc probably irrelevant; probably over-specified.
- Regression 2: note that adj. $R_{reg.3}^2 < adj. R_{reg.2}^2$; probably best fit.

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 However, OLS is no longer efficient (smallest variance) and estimated standard errors are biased.
 - ⇒ Hypothesis testing is invalid!

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White test for heteroskedasticity:

```
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White's test for Ho: homoskedasticity
    against Ha: unrestricted heteroskedasticity
    chi2(9) = 0.63
    Prob > chi2 = 0.9999
```

• We fail to reject H_0 of homoskedasticity.

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- No statistical evidence of heteroskedasticity.
- This is in line with our visual analysis.

Thank you and see you next time!

Juergen Amann

juergen.amann@nottingham.ac.uk Wednesday 12:00 - 13:00, C42 SCGB

Overall relation 'by hand' (Go back)
There are two ways how we can recover the previous statistic ourselves:

Joint parameter test (remember, we did this last week!)

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$$F^* = \frac{R^2/(q)}{(1-R^2)/(n-k)}.$$

```
. display 0.0158/(1 - 0.0158) * 1385/2
```

From the lecture slides [Slides 3, pp.20]:

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             9103.2286 2
                               4551.6143
   Residual | 565508.491 1,385 408.30938 R-squared = 0.0158
                                         Adj R-squared = 0.0144
     Total |
             574611.72 1.387 414.283864
                                         Root MSE
                                                        20.207
. display 9103.2286 / 574611.72
```

R^2 'by hand' \bullet Go back

From the lecture slides [Slides 3, pp.20]:

• The R^2 measure of goodness-of-fit is defined by $R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$

```
. regress bught faminc order
     Source |
                                     MS
                                            Number of obs
                                                            1,388
                             df
                                            F(2, 1385)
                                                          = 11.15
                                            Prob > F =
     Model |
              9103.2286
                                 4551.6143
                                                              0.0000
             565508.491
                       1,385 408.30938
                                            R-squared = 0.0158
   Residual |
                                            Adj R-squared = 0.0144
     Total |
                       1,387 414,283864
              574611.72
                                            Root MSE
                                                              20,207
. display 9103.2286 / 574611.72
```

• The adjusted R^2 incorporates a penalty for including additional variables: $adj.R^2=1-\left[\left(\frac{T-1}{T-k}\right)(1-R^2)\right]$

From the lecture slides [Slides 3, pp.20]:

• The R^2 measure of goodness-of-fit is defined by $R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$

```
. regress bught faminc order
    Source |
                           df
                                   MS
                                         Number of obs
                                                      = 1,388
                                         F(2, 1385)
                                                      = 11.15
                                         Prob > F = 0.0000
     Model |
             9103.2286 2
                               4551.6143
   Residual | 565508.491 1,385 408.30938 R-squared = 0.0158
                                         Adj R-squared = 0.0144
     Total |
             574611.72 1.387 414.283864
                                         Root MSE
                                                          20,207
. display 9103.2286 / 574611.72
```

• The adjusted R^2 incorporates a penalty for including additional variables: $adj.R^2=1-\left[\left(\frac{T-1}{T-k}\right)(1-R^2)\right]$

```
. display 1 - ((1388-1)/(1383)*(1-20031.4556/574611.72))
0.0144
```