

GEORGIA INSTITUTE OF TECHNOLOGY
ISYE 6644 – SIMULATION

PANDEMIC FLU SPREAD
PROJECT REPORT

Group-12

Ayush Gordhan Agarwal

Ajay Krishna Manoj

Harshal Mittal

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1. ABSTRACT

The project is about modeling of pandemic spread in a classroom of 21 elementary school kids with one carrier named Tommy. The probability of any classmate being infected by carrier is 0.02 and once infected, the carrier is infectious for 3 days. We have a base assumption that all kids and days are independent to get an i.i.d. Bern(p) trial. To model the infection spread, we surveyed our problem using binomial and chain binomial models like SIR (Susceptible, Infected and Recovered) model to predict how long this epidemic will last. Later, we made more realistic assumptions on exposure, immunity, and initial vaccinated populations to model the spread in varied settings. For our simulation, we used the SIR model without any other parameters. Through our simulations (1000 simulations), we found that the average infection lasts in the class lasts between 4-19 days with an average of 9 days.

2. MOTIVATION

Our motivation behind choosing this problem was that we wanted to simulate a small part of the pandemic. Though this does not come close to simulating the actual scenario, we can always build upon this and expand our project once we are able to simulate the situation keeping the assumptions in mind. We have understood the problem and researched into the methodology which we will adopt for our simulation. The current COVID times have taught us how important it is to be prepared with not only the prediction models, but also the simulation models to determine the spread of infectious diseases over a period. This can help authorities to be well-prepared in advance in case a breakout occurs. Although we are working with a very limited scope, our model will help with basic understanding of the statistics behind the rate of spread of infectious diseases and provide valuable insights for a small sample of the population.

3. BACKGROUND AND DESCRIPTION OF PROBLEM

We have identified several approaches to model pandemic spread. The most popular one being the SIR model (Susceptible, Infectious, and/or Recovered model). SIR model is found to be reasonably good for predicting spread of diseases that are transmitted from human to human, and where recovery confers lasting resistance. Along with SIR, we have also modelled different adaptations of SIR models to 4 different scenarios.

3.1 SIR MODEL

The SIR model is an epidemiological and population-based model. It can compute the (theoretical) number of people that would be affected by the pandemic flu in an enclosed place in a short period of time. The model gets its name from the fact that there are several equations relating to the number of people that are susceptible (S) to the flu, infected (I) by the flu and recovered (R) from it. Each variable changes over time t and is very dynamic in nature. The models that we would talk about after this are derivatives of this model. This is the

most basic model and very important to understand fully before moving ahead. It consists of 3 components:

- S is number of susceptible people, who can be potentially infected
- I is number of infected people. These people could affect others as well
- R is the number of recovered people

Assumptions:

- The people are in an enclosed space
- All of them are at the same risk of being affected
- People who have recovered are immune from the disease

Model Parameters:

S = number of susceptible people
 I = number of Infected people
 R = number of recovered people
 N = total number of people
 T = time (number of days)
 β = infection rate (per person/no of days)
 γ = recovery rate (/no of days)



$$\frac{dS}{dt} = -(\beta IS)$$

$$\frac{dI}{dt} = (\beta IS) - (\gamma I)$$

$$\frac{dR}{dt} = (\gamma I)$$

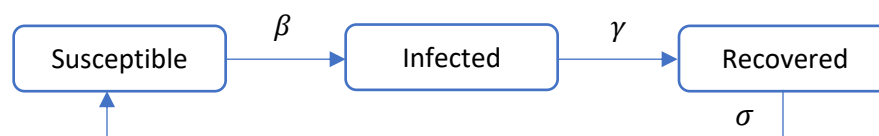
3.2 SIRS MODEL

This model is very similar to the SIR model. There is just one change. It may happen that the people that are immune after recovering from the disease may not remain immune forever and may get the flu after some period.

So, those individuals become susceptible again. This is a more realistic approach as compared to the SIR model.

Additional Model Parameter:

σ = re-infection rate (/no of days)



$$\frac{dS}{dt} = -(\beta IS) + (\sigma R)$$

$$\frac{dI}{dt} = (\beta IS) - (\gamma I)$$

$$\frac{dR}{dt} = (\gamma I) - (\sigma R)$$

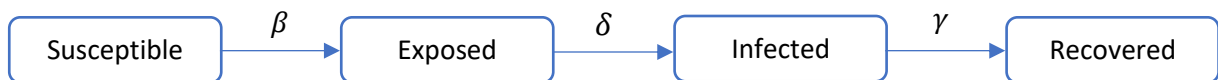
3.3 SEIR MODEL

This model works in a similar fashion as the above two models. However, difference is that of "E" i.e., the exposed individuals. These are the individuals that have been infected but are not yet infectious.

This makes the model much more realistic as compared to the previous models. As you can see in the equations, there's an extra differential equation for the exposed component. All the other equations remain the same as seen in the SIR model.

Additional Model Parameter:

δ = exposure rate (/no of days)



$$\frac{dS}{dt} = -(\beta IS)$$

$$\frac{dE}{dt} = (\beta IS) - (\delta E)$$

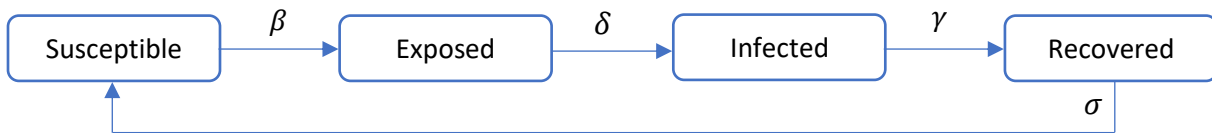
$$\frac{dI}{dt} = (\delta E) - (\gamma I)$$

$$\frac{dR}{dt} = (\gamma I)$$

3.4 SEIRS MODEL

This is a combination of the SEIR and the SIRS model. In this model, we consider the exposed individuals i.e. the individuals that have been infected but are not yet infectious. Also, just like the SIRS model, the people that are immune after recovering from the disease may not remain immune forever and may get the flu after some period. So, those individuals become susceptible again.

This is an even more realistic approach as compared to the previous models.



$$\frac{dS}{dt} = -(\beta IS) + (\sigma R)$$

$$\frac{dE}{dt} = (\beta IS) - (\delta E)$$

$$\frac{dI}{dt} = (\delta E) - (\gamma I)$$

$$\frac{dR}{dt} = (\gamma I) - (\sigma R)$$

4. EXPLORATION OF DIFFERENT MODELS

To start with simulating the model, we would need to state the basic assumptions and parameters from the given problem.

Model Assumptions:

- As we are modelling for a school, there are going to be 5 working days and 2 weekend days in a week. We are going to assume that all the days of the simulation model fall on weekdays (working days)
- All the students come to school, i.e., no one is absent. Also, there are no additional students added into the system or removed.
- At any given point of time, there are always going to be 21 kids in the model. (*Constant N= 21*). This means that there is no student that is removed from the setting for the duration of the simulation/experiment. We will also not consider extreme cases such as deaths and removal from the system.
- During the period of infection, the infected person has an equal chance of spreading the disease irrespective of whether the severity increases or decreases. (*Constant p=0.02*)
- We also assume Tommy and his classmates are close friends and always hang out together. Therefore, we consider that all his classmates meet one another every day, i.e., each infected person interacts with all 20 other students every day.

These assumptions help us to set the infection spread each day as an i.i.d Bern(p) trials. The spread across multiple days, hence, becomes a chain-binomial model. Here, we are using a fixed β to be 0.02. All the scenarios were modelled using R statical programming language.

4.1 SIR MODEL

To model the infection spread using SIR Model, we are making one more assumption to satisfy the Recovery aspect of the SIR Model i.e., once a person recovers from the disease, he/she is no longer susceptible to getting it again.

The parameters (β , γ) for our problem are:

- $\beta = 0.02$
- $\gamma = 1/3$

The initial state of the system are:

- $S=20$, the rest of the class is susceptible
- $I=1$, Tommy is the carrier of the disease
- $R=0$, no one has immunity/recovered from the disease

4.1.1 OBSERVATIONS

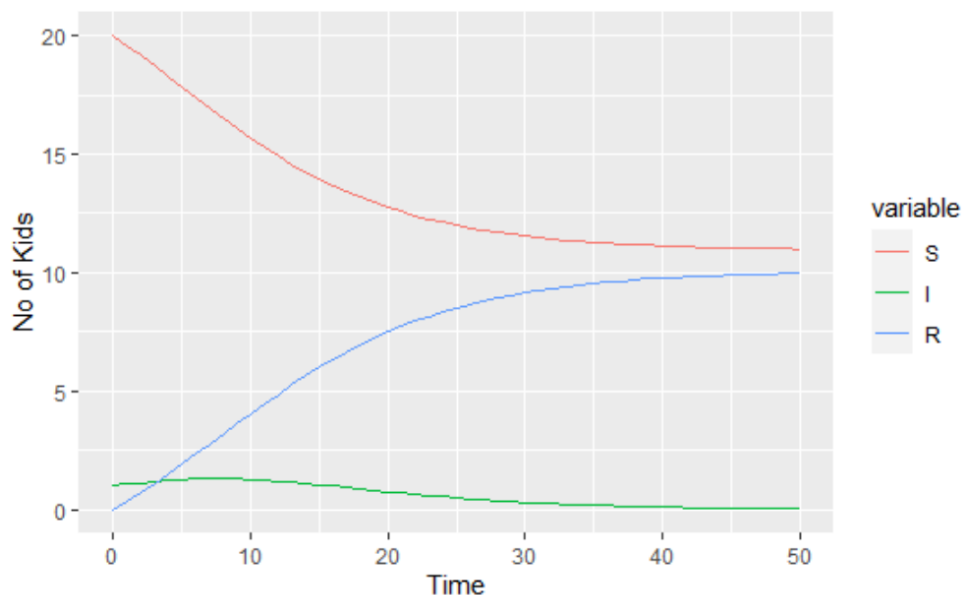


FIGURE A: SIR OBSERVATION

4.1.2 FINDINGS

We see that kids stop getting infected by Day 25 (threshold taken as 0.5) while the recovered population and susceptible population reach an equilibrium to about 10 kids per group. This means that about 10-11 people get infected in total in a span of 50 days. The maximum number of active infections in class at a point happens on Day 9.

4.2 SIRS MODEL

To model the infection spread using SIRS Model, we are assuming once a kid recovers from the disease, he/she will become susceptible again after 20 days. The immunity from recovering from the disease lasts only 20 days until the kid becomes infectious again.

The parameters (β , γ) for our problem are:

- $\beta = 0.02$
- $\gamma = 1/3$
- $\sigma = 1/20$

The initial state of the system are:

- $S=20$, the rest of the class is susceptible
- $I=1$, Tommy is the carrier of the disease
- $R=0$, no one has immunity/recovered from the disease

4.2.1 OBSERVATIONS

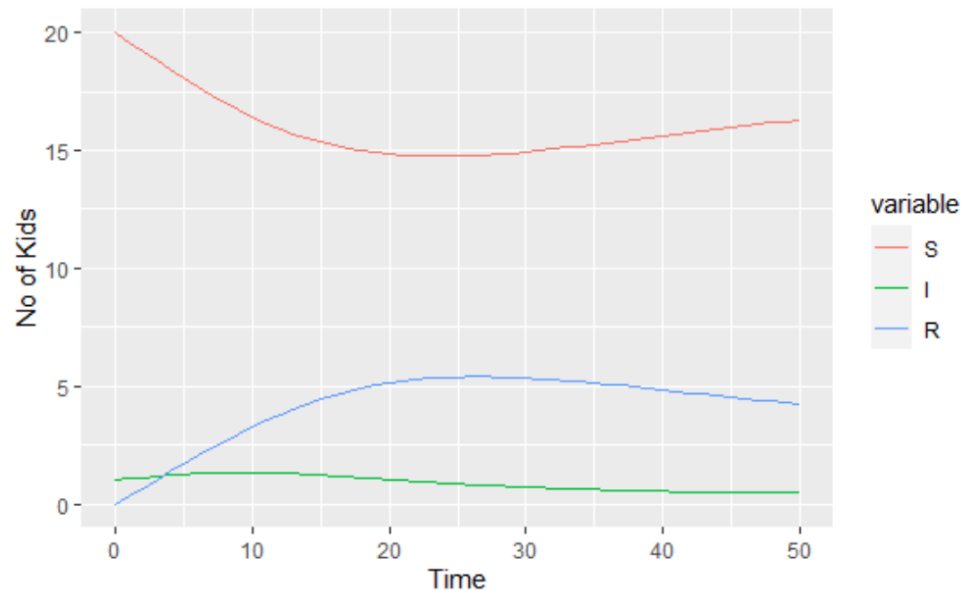


FIGURE B: SIRS OBSERVATION

4.2.2 FINDINGS

We see that kids stop getting infected by Day 45 (threshold taken as 0.5) while the recovered population and susceptible population reach to about 4 and 16 respectively. The total recovered group first increases and then drops post the 29th day. This is in line with our understanding that the recovered population becomes susceptible again.

4.3 SEIR MODEL

To model the infection spread using SEIR Model, we are assuming once a kid recovers from the disease, he/she will be immune to the disease. Also, it takes a specific period to be exposed to the disease for the child to be infectious.

The parameters (β , γ) for our problem are:

- $\beta = 0.02$
- $\gamma = 1/3$
- $\delta = 1/2$

The initial state of the system are:

- $S=20$, the rest of the class is susceptible
- $E=0$, exposed individuals are initially 0
- $I=1$, Tommy is the carrier of the disease
- $R=0$, no one has immunity/recovered from the disease

4.3.1 OBSERVATIONS

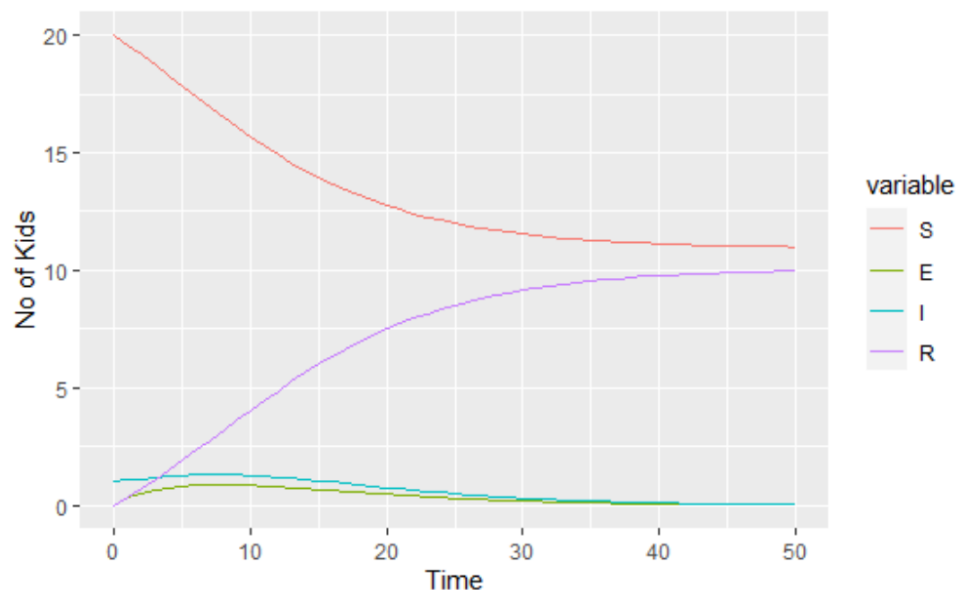


FIGURE C: SEIR OBSERVATION

4.3.2 FINDINGS

We see that kids stop getting exposed to the disease and becoming infectious by Day 25 (threshold taken as 0.5). The recovered population and susceptible population reach an equilibrium to about 10 kids per group. This means that about 10-11 people get infected in total in a span of 50 days. The maximum number of active infections in class at a point happens on Day 7.

4.4 SEIRS MODEL

To model the infection spread using SEIRS Model, we are assumption once a kid recovers from the disease, he/she will become susceptible again after 20 days. The immunity from recovering from the disease lasts only 20 days until the kid becomes infectious again. Also, it takes a specific period to be exposed to the disease for the child to be infectious.

The parameters (β , γ) for our problem are:

- $\beta = 0.02$
- $\gamma = 1/3$
- $\delta = 1/2$
- $\sigma = 1/20$

The initial state of the system are:

- $S=20$, the rest of the class is susceptible
- $E=0$, exposed individuals are initially 0
- $I=1$, Tommy is the carrier of the disease
- $R=0$, no one has immunity/recovered from the disease

4.4.1 OBSERVATIONS

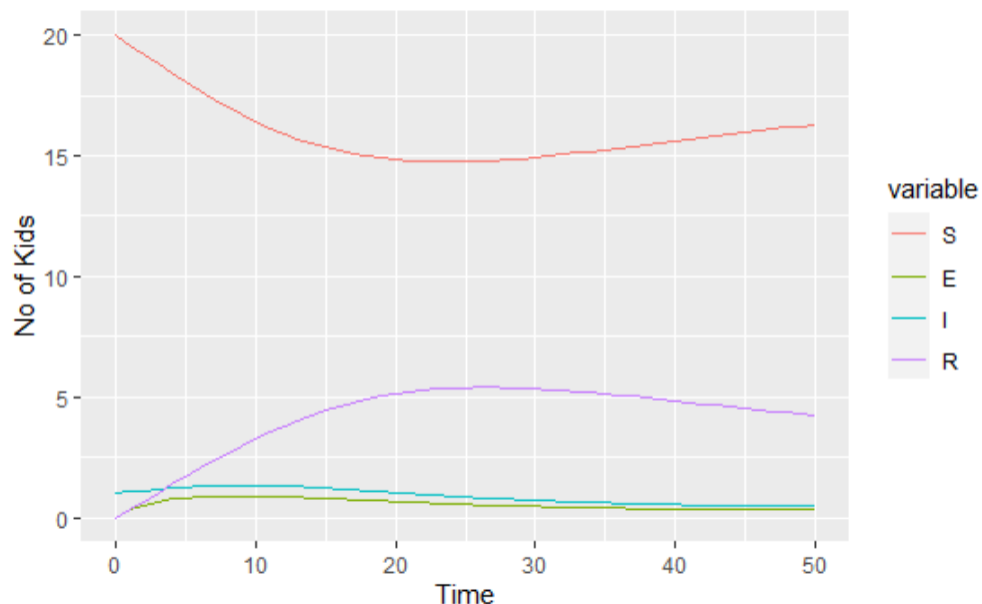


FIGURE D: SEIRS OBSERVATION

4.4.2 FINDINGS

We see that kids stop getting exposed to the disease and becoming infectious by Day 45 (threshold taken as 0.5). The recovered population and susceptible population reach to about 4 and 16 at then of 50 days, respectively. The total recovered group first increases and then drops post the 27th day. These results are very similar to the SIRS model and is in line with our understanding that the recovered population becomes susceptible again.

4.5 SIR WITH VACCINATED POPULATION

To model the infection spread using SIR Model, we are making one more assumption to satisfy the Recovery aspect of the SIR Model i.e., once a person recovers from the disease, he/she is no longer susceptible to getting it again.

Furthermore, we are assuming that 40% are vaccinated and, only 60% are susceptible to the disease.

Hence, the initial values of S and R, are multiplied by 0.6 and 0.4 respectively.

4.5.1 OBSERVATIONS

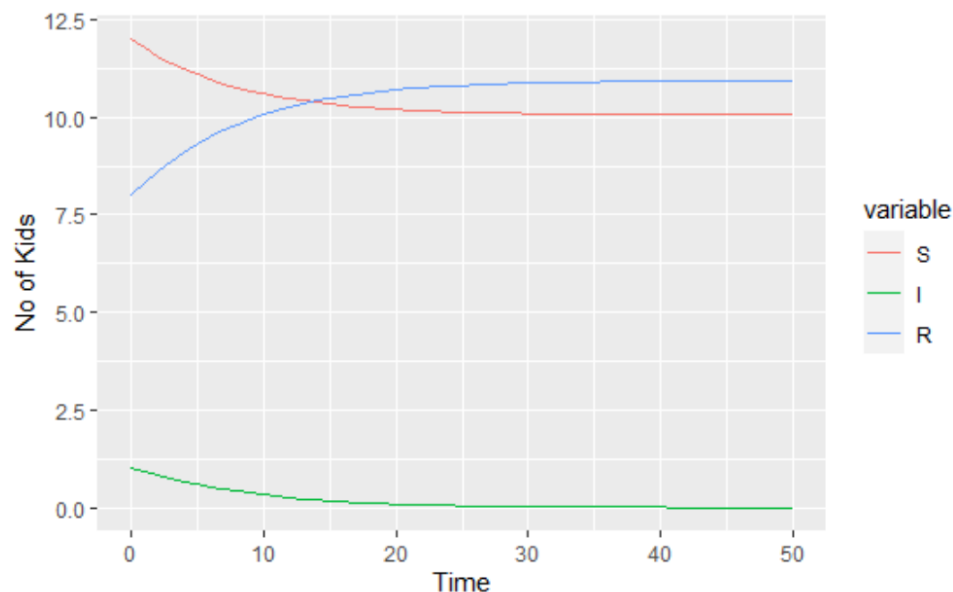


FIGURE E: SIR WITH VACCINATIONS OBSERVATION

4.5.2 FINDINGS

The class has 8 kids already immune/recovered. Thus, we see that kids stop getting infected by Day 7 (threshold taken as 0.5), which is much earlier than the SIR Model. The recovered population and susceptible population reach an equilibrium to about 11 and 10 kids, respectively. This means that from the initial susceptible population of 12 only 3 get infected by the end of 50 days. The maximum number of active infections in class at a point happens on Day 9. This tells us the importance of getting vaccinated!

5. SIMULATING THE PANDEMIC FLU SPREAD

From the previous section, we have seen different variations of SIR models with β to be taken as $0.4/20 = 0.02$ (expectation value). Here, in this section, we are simulating the pandemic spread using the basic SIR model 1000 times with β taken as a Binomial distribution (20, 0.02). The simulations were carried out in R programming language. Below, we are providing solutions to the given problem statement.

(a) What is the distribution of the number of kids that Tommy infects on Day 1?

The distribution of the number of kids that Tommy affects on Day 1 is a Binomial Distribution with $n = 20$ and $p = 0.02$.

$$I \sim \text{Bin}(20, 0.02)$$

From our 1000 simulations, the distribution of the number of infected is shown below:

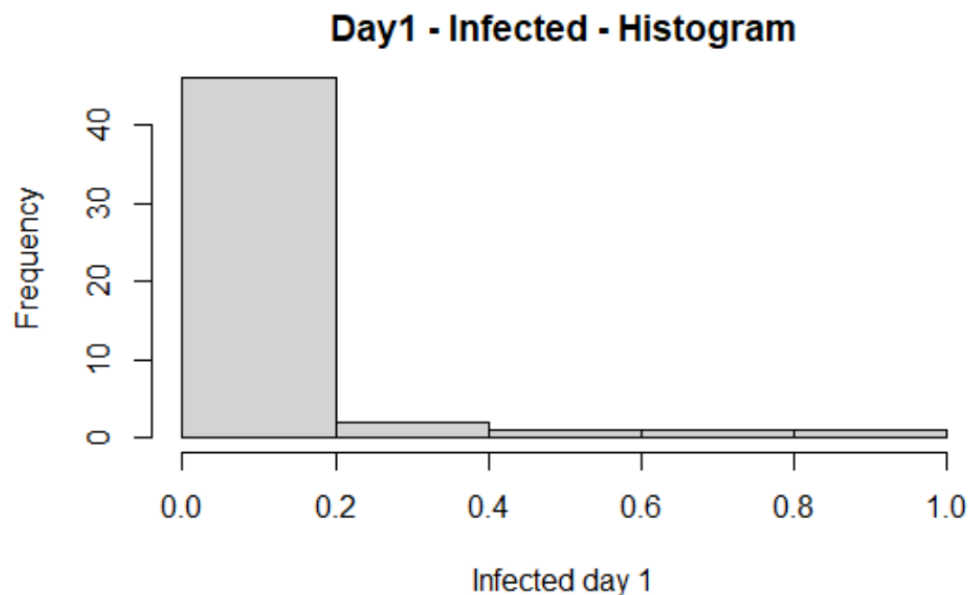


FIGURE F: DAY1 INFECTED HISTOGRAM

The mean infection on Day 1 = 0.355 from above.

(b) What is the expected number of kids that Tommy infects on Day 1?

$$E[I \text{ on day 1}] = np = 20 * 0.02 = 0.4$$

The expected number of kids Tommy infects on Day 1 is 0.4 approximately.

From our 1000 simulations, the mean infection on Day 1 = 0.355 which is close to the expected value.

(c) What is the expected number of kids that are infected by Day 2 (you can count Tommy if you want)?

$$E[I \text{ on day 2}] = np = 19.6 * 0.028 = 0.5488$$

$$E[I_1 + I_2] = 0.4 + 0.5488 = 0.9488$$

Hence, expected number of kids Tommy infected BY Day 2 is 0.95 approximately.

From our 1000 simulations, the distribution of the number of infected for Day 2 is shown below:

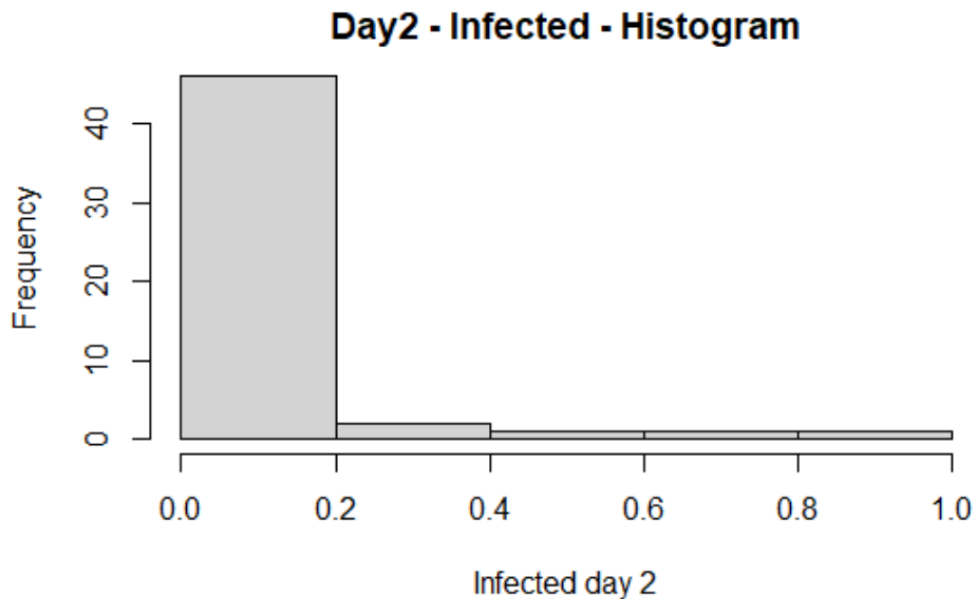


FIGURE G: DAY2 INFECTED HISTOGRAM

From our simulations, the mean infection on Day 2 = 1.991 (including Tommy). Subtracting Tommy, we get 0.991 which is close to the expected value.

(d) Simulate the number of kids that are infected on Days 1,2, Do these many times. What are the (estimated) expected numbers of kids that are infected by Day i, i = 1, 2, . . .? Produce a histogram detailing how long the “epidemic” will last.

We have done 1000 simulations and below is the number of kids actively infective from Day 1 to Day 50. From the plot and our analysis, we can see that after 15 days the number of infections tend to remain constant. The maximum number of students that are infected at a given time is 2.59.

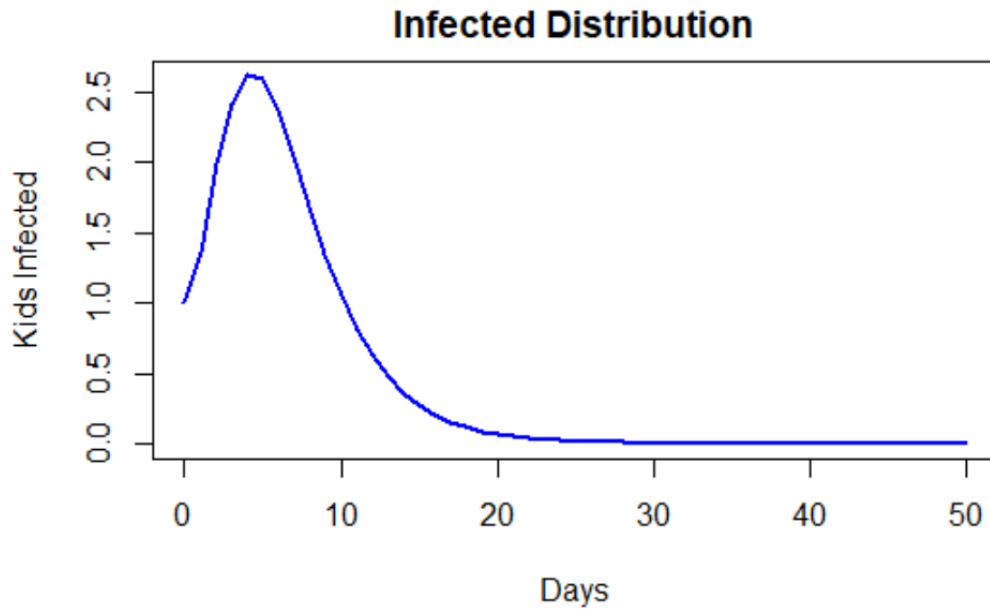


FIGURE H: DISTRIBUTION OF INFECTIONS OVER TIME

From our simulations, below is the histogram of the flu lengths with a threshold taken as 0.5. From the plot, we can see that 65% of times the pandemic lasts than 5 days but it is possible for the pandemic to last up to 19 days. There is a 29% chance that it can last up to 19 days. The mean duration of the pandemic is found to be 9.034 days.

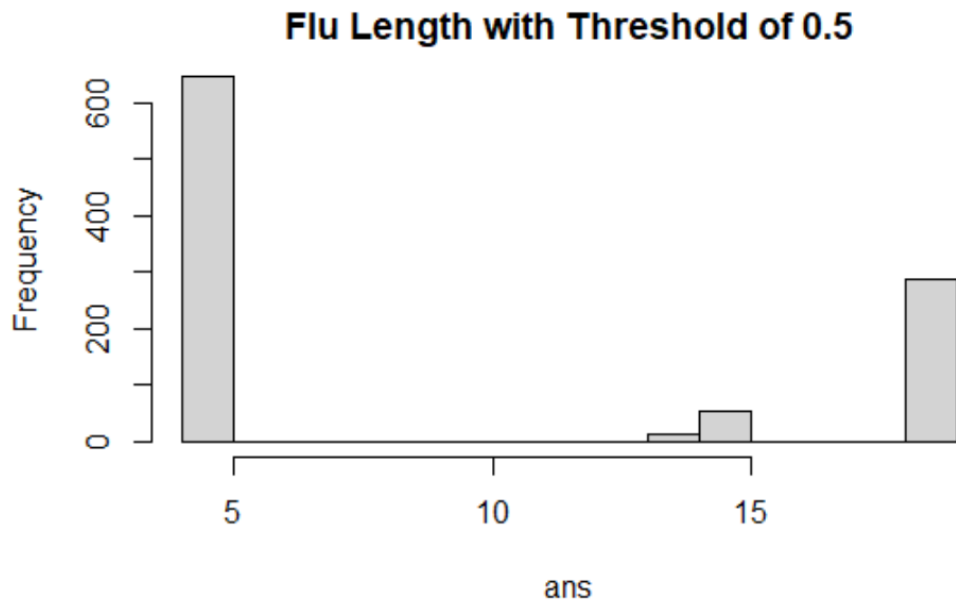


FIGURE I: HISTOGRAM OF LENGTH OF PANDEMIC

6. CONCLUSION

We explored 5 variations of SIR model namely SIR, SIRS, SIER, SIERS and SIR with vaccination using R programming language. The observations from these models displayed a similar trend and helped us understand who the disease spreads and how effective vaccination, and social distancing is in reducing the number of infected.

We also made 1000 simulations using the SIR model to understand the spread of the pandemic and answer the questions to the problem. From the simulations we learnt that the pandemic flu would not infect the whole class and a significant portion will remain unaffected. However, if we adopt a higher transmission rate, we will be seeing a different scenario.

We realize that modelling pandemic spread involves many assumptions and it is not easy to incorporate all factors into our analytical modelling. For future work, it would be interesting to model disease spread through social cliques and populations circles as part of a larger society. Herd immunity and natural immunity can also be considered here. This project made us exposed to this vast field and interesting aspect of simulations.

Moreover, we had FUN learning and working on the project!

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