

# Written Assignment

ECO 4000 Statistical Analysis for Economics and Finance

02 April, 2022

**Maximum Points : 25**

**Due Date : 13 April, 2022 (Before Class)**

**1.  $Y = \hat{\beta}_0 + \hat{\beta}_1 X + e$ . where,  $Y$  are the observed values,  $\hat{\beta}_0 + \hat{\beta}_1 X = \hat{Y}$  are the fitted values, and  $e$  are the residuals generated from the OLS regression. Prove the following algebraic properties of the OLS (5 points)**

- a.  $\bar{e} = 0$  (1 point)
- b.  $\bar{\hat{Y}} = \bar{Y}$  (1 point)
- c.  $cov(X, e) = 0$  (1 point)
- d.  $cov(\hat{Y}, e) = 0$  (2 points)

**2. Determine whether following models are linear regression models or not.(2 points)**

- 1.  $Y_i = \beta_0 + \beta_1 \left( \frac{1}{X_i} \right) + u_i$  (0.5 points)
- 2.  $Y_i = \beta_0 + \beta_1 \ln X_i + u_i$  (0.5 points)
- 3.  $\ln Y_i = \beta_0 + \beta_1^2 \ln X_i + u_i$  (0.5 points)
- 4.  $Y_i = \beta_0 + (0.75 - \beta_0) e^{-\beta_2 (X_i - 2)} + u_i$  (0.5 points)

**3. Following are three data points on dependent (Y) and one explanatory variable(X). Fit a regression model by calculating  $\hat{\beta}_0$  and  $\hat{\beta}_1$ .(3 points)**

Y	X
8	10
15	7
24	3

4. The following model is a simplified version of the multiple regression model used by Biddle and Hamermesh (1990) to study the tradeoff between time spent sleeping and working and to look at other factors affecting sleep. where sleep and totwrk (total work) are measured in minutes per week and educ and age are measured in years.(6 points)

$$\text{sleep} = \beta_0 + \beta_1 \text{totwrk} + \beta_2 \text{educ} + \beta_3 \text{age} + u$$

- If adults trade off sleep for work, what is the sign of  $\beta_1$ ?(1 point)
- What signs do you think  $\beta_2$  and  $\beta_3$  will have?(1 point)
- Using the data, the estimated equation is

$$\widehat{\text{sleep}} = 3,638.25 - 0.148 \times \text{totwrk} - 11.13 \times \text{educ} + 2.20 \times \text{age}$$

(112.28)
(0.017)
(5.88)
(1.45)

$$n = 706, R^2 = 0.113$$

If someone works five more hours per week, by how many minutes is sleep predicted to fall?(1 point)

- Discuss the sign and magnitude of the estimated coefficient on *educ*.(1 point)
- Would you say *totwrk*, *educ*, and *age* explain much of the variation in sleep? What other factors might affect the time spent sleeping? Are those likely to be correlated with *totwrk*? If we do not include those factors in our model, what kind of bias the coefficient on *totwrk* will have?(2 points)

5. Once again using the same study as above for our purposes, the estimated equation is as follows:(6 points)

$$\widehat{\text{sleep}} = 3,638.25 - 0.148 \times \text{totwrk} - 11.13 \times \text{educ} + 2.20 \times \text{age}$$

(112.28)
(0.017)
(5.88)
(1.45)

$$n = 706, R^2 = 0.113$$

- Is either *educ* or *age* **individually** significant at the 5% level against a two-sided alternative? Show your work.(1 point)
- Construct a 95% confidence interval for a coefficient on *total work*. Show your work.(1 point)
- Dropping *educ* and *age* from the equation gives

$$\widehat{\text{sleep}} = 3,586.38 - 0.151 \times \text{totwrk}$$

(38.91)
(0.017)

$$n = 706, R^2 = 0.103$$

Are *educ* and *age* **jointly** significant in the original equation at the 5% level? Justify your answer.(2 points)

- d. Does including *educ* and *age* in the model greatly affect the estimated trade off between sleeping and working?(1 point)
- e. Suppose that the sleep equation contains heteroskedasticity. What does this mean about the tests computed in parts **a** and **c**?(1 point)

**6. Continuing with the same study as above, now consider the estimated equation as follows:(3 points)**

$$\widehat{sleep} = 3,840.83 - 0.163 \times totwrk - 11.71 \times educ - 8.70 \times age + 0.128 \times age^2 + 87.75 \times male$$

(235.11)
(0.018)
(5.86)
(11.21)
(0.134)
(34.33)

$$n = 706, R^2 = 0.123, \overline{R^2} = 0.117$$

The variable *sleep* is total minutes per week spent sleeping at night, *totwrk* is total weekly minutes spent working, *educ* and *age* are measured in years, and *male* is a gender dummy.

- a. All other factors being equal, is there evidence that men sleep more than women? How strong is the evidence?(1 point)
- b. Is there a statistically significant tradeoff between working and sleeping? What is the estimated trade off?(1 point)
- c. What other regression do you need to run to test the null hypothesis that, holding other factors fixed, age has no effect on sleeping?(1 point)