

## 5 行列式の性質 (その 1) の解答例

演習 5.1 (1) 
$$\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} = \left| t \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \right| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 2.$$

(2) 
$$\begin{vmatrix} -a_{11} & -a_{12} & -a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 3a_{31} & 3a_{32} & 3a_{33} \end{vmatrix} = (-1) \cdot 2 \cdot 3 \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -12.$$

(3) 
$$\begin{vmatrix} a_{13} & a_{12} & a_{11} \\ a_{23} & a_{22} & a_{21} \\ a_{33} & a_{32} & a_{31} \end{vmatrix} = - \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -2.$$

(4) 
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} - 2a_{21} & a_{32} - 2a_{22} & a_{33} - 2a_{23} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 2.$$

### 演習 5.2

$$\begin{vmatrix} \sqrt{5} & -\sqrt{2} & -\sqrt{5} & \sqrt{2} \\ \sqrt{2} & \sqrt{5} & -\sqrt{2} & -\sqrt{5} \\ \sqrt{7} & -\sqrt{3} & \sqrt{7} & -\sqrt{3} \\ \sqrt{3} & \sqrt{7} & \sqrt{3} & \sqrt{7} \end{vmatrix} = \begin{vmatrix} \sqrt{5} & -\sqrt{2} & 0 & 0 \\ \sqrt{2} & \sqrt{5} & 0 & 0 \\ \sqrt{7} & -\sqrt{3} & 2\sqrt{7} & -2\sqrt{3} \\ \sqrt{3} & \sqrt{7} & 2\sqrt{3} & 2\sqrt{7} \end{vmatrix} \\ = \begin{vmatrix} \sqrt{5} & -\sqrt{2} \\ \sqrt{2} & \sqrt{5} \end{vmatrix} \begin{vmatrix} 2\sqrt{7} & -2\sqrt{3} \\ 2\sqrt{3} & 2\sqrt{7} \end{vmatrix} = 7 \cdot 40 = 280.$$

### 演習 5.3

$$\begin{vmatrix} A & B \\ B & A \end{vmatrix} = \begin{vmatrix} A+B & B+A \\ B & A \end{vmatrix} = \begin{vmatrix} A+B & (B+A) - (A+B) \\ B & A-B \end{vmatrix} \\ = \begin{vmatrix} A+B & O \\ B & A-B \end{vmatrix} = |A+B| \cdot |A-B|.$$