Practical No. 1

Topic · limits and continuity.

(1) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$ (2) $\lim_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

(3) $\lim_{x \to \pi/6} \left[\dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$ (4) $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

5) Examine the continuity of the following function at gives points.

i) $$F(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}} & \text{for } 0 < x \le \pi/2 \\ \dfrac{\cos x}{\pi - 2x}, & \text{for } \pi/2 < x < \pi \end{cases} \quad \text{at } x = \pi/2$$

ii) $$F(x) = \begin{cases} \dfrac{x^2-9}{x-3} & 0 < x < 3 \\ x+3 & 3 \le x < 6 \\ \dfrac{x^2-9}{x+3} & 6 \le x < 9 \end{cases} \quad \text{at } x = 3 \text{ and } x = 6$$

6) Find value of k, so that the function $f(x)$ is cts at the indicated point.

i) $$f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & x < 0 \\ k & x = 0 \end{cases} \quad \text{at } x = 0$$

(ii) $F(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$
$= K$ $\quad x = 0$ $\Big\}$ at $x = 0$

(iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$
$= K$ $\quad x = \pi/3$ $\Big\}$ at $x = \pi/3$

(7) Discuss the Continuity of the following functions which of these functions which of these functions have a removable discontinuity Redefine the function so as to remove the discontinuity

(i) $f(x) = \dfrac{1 - \cos 3x}{x \tan x}$ $\quad x \neq 0$
$= 9$ $\quad x = 0$ $\Big\}$ at $x = 0$

(ii) $f(x) = \dfrac{(e^{3x} - 1)\sin x^{\circ}}{x^2}$ $\quad x \neq 0$
$= \pi/6$ $\quad x = 0$ $\Big\}$ at $x = 0$

(8) If $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is continuous at $x = 0$ find $F(0)$.

(9) If $f(x) = \dfrac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x}$ for $x \neq \pi/2$ is continuous at $x = \pi/2$ find $f(\pi/2)$

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$=\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{a}}\right]$$

$$=\lim_{x\to a}\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$=\frac{1}{3}\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x})+(2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$=\frac{1}{3}\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$=\frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$=\frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{2\sqrt{3a}}$$

$$=\frac{1}{3}\times\frac{2\sqrt{a}+2\sqrt{a}}{2\sqrt{3a}}$$

$$=\frac{1}{3}\,\frac{\cancel{4}^{2}}{\cancel{2}}\,\frac{\sqrt{a}}{\sqrt{3}\times\sqrt{a}}$$

$$=\frac{1}{3}\times\frac{2}{\sqrt{3}}$$

$$=\frac{2}{3\sqrt{3}}$$

2. $$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$$

$$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]\times\left[\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$$

$$\lim_{y\to 0}\cdot\frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$$

$$\lim_{y\to 0}\cdot\frac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$$

$$\frac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$$

$$\frac{1}{\sqrt{a}\,(2\sqrt{a})}$$

$$\frac{1}{2a}$$

3. $$\lim_{x\to\pi/6}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$$

By substituting $x-\pi/6=h$

$x=h+\pi/6$

where $h\to 0$

$$\lim_{h\to 0}\frac{\cos(h+\pi/6)\cdot\sqrt{3}\sin(h+\pi/6)}{\pi-6(h+\pi/6)}$$

$$\lim_{h \to 0} \frac{\cos h \cos \pi/6 \cdot \sinh \cdot \sin \pi/6 - \sqrt{3}\,\sinh \cos \pi/6 + \cosh \sin \pi/6}{\pi - 6\left(\frac{6h + \pi}{6}\right)}$$

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$$\lim_{h \to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} \cdot \sinh \frac{1}{\sqrt{2}} - \sqrt{3}\left(\sin \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{\sqrt{2}}\right)}{\pi - 6h + \pi}$$

$$\lim_{h \to 0} \frac{\cos \frac{\sqrt{3}}{2} h - \sin h/2 - \sin 3h/2 - \cos \frac{\sqrt{3}}{2} h}{-6h}$$

$$\lim_{h \to 0} \frac{-\sin \frac{4h}{2}}{-6h}$$

$$\lim_{h \to 0} \frac{\sin 4h}{12h}$$

$$\frac{1}{3} \lim_{h \to 0} \frac{\sinh}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$$

4. $\lim_{x\to\infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

Sol:

By rationalising Numerator & Denominator both

$$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right] \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}$$

$$\lim_{x\to\infty} \left[\frac{(x^2+5-x^2+3)\left(\sqrt{x^2+3} + \sqrt{x^2+1}\right)}{(x^2+3-x^2-1)\left(\sqrt{x^2+5}\right) + \sqrt{x^2-3})} \right]$$

$$\lim_{x\to\infty} \frac{8\left(\sqrt{x^2+3} + \sqrt{x^2+1}\right)}{2\left(\sqrt{x^2+5} + \sqrt{x^2-3}\right)}$$

$$4 \lim_{x\to\infty} \frac{\sqrt{x^2\left(1+3/x^2\right)} + \sqrt{x^2\left(1+1/x^2\right)}}{\sqrt{x^2\left(1+5/x^2\right)} + \sqrt{x^2\left(1-3/x^2\right)}}$$

After applying limit

we get,

$= 4$

$$f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}, \text{ for } 0<x\le \pi/2$$

$$= \frac{\cos x}{\pi - 2x}, \text{ for } \pi/2 < x < \pi$$

$$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$$

$\therefore f(\pi/2) = 0$

f at $x = \pi/2$ define.

(ii) $\lim_{x\to\pi/2^+} f(x) = \lim_{x\to\pi/2} \frac{\cos x}{\pi - 2x}$

By substituting method

$x - \pi/2 = h$

$x = h + \pi/2$

where $h \to 0$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)}$$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{-2h}$$

using $\cos(A+B) = \cos A \cos B - \sin A \sin B$

i.e

$$= \lim_{h\to 0} \frac{\cos h \cdot \cos \pi/2 - \sinh \sin\pi/2}{-2h}$$

$$= \lim_{h\to 0} \frac{\cosh \cdot 0 - \sinh}{-2h}$$

$$\lim_{h\to 0} \frac{-\sin h}{-2h}$$

$$\cancel{\lim} \ 1/2 \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= 1/2$$

b) $\lim_{x\to \pi/2^-} f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$

$$\lim_{x\to\pi/2^-} \cdot \frac{2\sin x \cdot \cos x}{\sqrt{2}\ \sin^2 x}$$

$$\lim_{x\to\pi/2^-} \frac{2\sin x \cdot \cos x}{\sqrt{2}}$$

$$\lim_{x\to\pi/2^-} \frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x\to\pi/2^-} \cos x$$

$\therefore$ LHL $\neq$ RHL

$\therefore$ f is not continous at $x=\pi/2$

2. $\lim_{x \to 6^+} \frac{x^2 - 9}{x + 3}$

$\lim_{x \to 6^+} \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x \to 6^+} (x-3) = 6 - 3 = 3$

$\lim_{x \to 6^-} (x+3) = 3+6 = 9$

$\therefore$ LHL $\neq$ RHL

function is not Continous

(i) $f(x) = \frac{1 - \cos 4x}{x^2} \quad x < 0$

$= k \quad x = 0$

at $x = 0$

Sol^n: f is continuous at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{1 - \cos 4x}{x^2} = k$

$\lim_{x \to 0} \frac{2 \sin^2 2x}{x^2} = k$

$2 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$

$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$\boxed{k = 8}$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$
$\qquad = k \qquad x = 0$ $\Big\}$ at $x = 0$

Soln: $f(x) = (\sec^2 x)^{\cot^2 x}$

$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$

Using $\tan^2 x - \sec^2 x = 1$
$\sec^2 x = \tan^2 x + 1$
$\cot^2 x = \frac{1}{\tan^2 x}$

$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$

we know, that

$$\lim_{x \to 0} (1 + px)^{1/px} = e$$

$\therefore = e$

$\therefore k = e$

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$
$\qquad = k \qquad x = \pi/3$ $\Big\}$ at $x = \pi/3$

$= x - \pi/3 = h$

$x = h + \pi/3$

where $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3} \tan\frac{\pi}{3} - \tan h}{1 - \tan\pi/3 \cdot \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}\left(1 - \tan\pi/3 \cdot \tan h\right) - \left(\tan\pi/3 + \tan h\right)}{1 - \tan\pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\left(\sqrt{3} - \sqrt{3}\times\sqrt{3} \tan h\right) - \left(\sqrt{3} + \tan h\right)}{1 - \tan\pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\left(\sqrt{3} - 3\tan h - \sqrt{3} - \tan h\right)}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{4 \tan h}{3h(1-\sqrt{3}\tan h)}$$

$$4/3 \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{(1-\sqrt{3}\tan h)}$$

$$= 4/3 \frac{1}{1 - \sqrt{3}(0)}$$

$$- \frac{1}{3}(1)$$

$$= \frac{4}{3}$$

i) $f(x) = \begin{cases} \frac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = 9 & x = 0 \end{cases}$ at $x = 0$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$\lim_{x\to 0} = \frac{2\sin^2 \frac{3}{2} x}{x \tan x}$$

$$\lim_{x\to 0} \frac{\frac{2\sin^2 \frac{3x}{2}}{x^2} \cdot x^2}{\frac{x\tan x}{x^2} \cdot x^2}$$

$$= 2\lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1} = \frac{9}{2}$$

$$\lim_{x\to 0} f(x) = \frac{9}{2} \quad 9 = f(0)$$

$\therefore$ f is not continuous at $x = 0$

Redefine function.

$$f(x) = \begin{cases} \frac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \frac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x\to 0} f(x) = f(0)$

7) (ii) $f(x) = (e^{3x} - 1)\sin x^\circ \quad x \neq 0$
$= \pi/6 \quad x = 0$ } at $x = 0$

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x\to 0} \frac{3\cdot e^{\alpha}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3\lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$\log e \cdot \pi/180 = \pi/60 \therefore = f(0)$

f is continuous at $x = 0$

8. $f(x) = \dfrac{e^{x^2} - \cos x}{x^2} \quad x = 0$ is continuous at $x = 0$

$\therefore$ Given, f is continuous at $x = 0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$= \lim_{x\to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

9)

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$$= \lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$$

multiply with 2 on num & Denomenator

$$= 1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$$

$$f(x) = \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \qquad x \neq \pi/2$$

$f(0)$ is continuous at $x = \pi/2$

$$\lim_{x\to\pi/2} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$\lim_{x\to\pi/2} \frac{2 - 1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$\lim_{x\to\pi/2} \frac{1+\sin x}{1-\cancel{\sin^2 x}(\sqrt{2}+\sqrt{1+\sin x})}$$

$$\lim_{x\to\pi/2} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

$$= \frac{-|x| + \tan^2 a}{\tan^2 a} = \frac{-\sec^2 a}{\tan^2 a} = \frac{-\frac{1}{\cos^2 a}}{} $$

$\therefore$ [f is differtiable $\forall a \in R$] $= -\operatorname{cosec}^2 a$

(iii) Cosec$x$

$$f(x) = \operatorname{cosec} x$$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x - a = h$
$x = a + h$, as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)} = \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

(iii) sec x

$f(x) = \sec x$

$$DF(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$DF(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a + a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \sin -h/2}{\cos a \cos(a+h) \times -h/2}$$

$$= -\frac{1}{2} \times \frac{2 \sin a}{\cos a \times \cos a}$$

$$= \tan a \sec a.$$

8.2

If $f(x) = 4x+1 \quad x \leq 2$ at $x=2$, then
$= x^2+5 \quad x>0$
Find function is differentiable or not

LHD:

$$DF(2-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$DF(2-) = 4$

RHD: $DF(2+) = \lim_{x \to 2+} \frac{x^2+5-9}{x-2}$

$$= \lim_{x \to 2+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$$

$DF(2^+) = 4$

RHD = LHD

F is differentiable at $x=2$

Q.3) If $f(x) = 4x + 7 \quad x < 3$
$= x^2 + 3x + 1 \quad x \geq 3$ at $x = 3$

Find F is differentiable or not

Ans: RHD : $DF(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$

$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3 + 1)}{x - 3}$

$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - 19}{x - 3}$

$= \lim_{x \to 3^+} \frac{x^2 + 3x - 18}{x - 3}$

$= \lim_{x \to 3^+} \frac{(x + 6)(x - 3)}{x - 3} = 3 + 6 = 9$

$DF(3^+) = 9$

LHD $= DF(-3)$

$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3}$

$= \lim_{x \to 3^-} \frac{4x + 7 - 19}{x - 3}$

$= \lim_{x \to 3} \frac{4(x - 3)}{(x - 3)}$

$DF(3^+) = 4$

RHD $\neq$ LHD.

$\therefore$ f is not differentiable at $x = 3$

Q.4) If $f(x) = 8x-5 \quad x \le 2$
$= 3x^2 - 4x + 7, \quad x > 2$ at $x = 2$

Find f is differentiable or not

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD $= DF(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$

$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$

$= \lim_{x \to 2} \frac{(3x+2) + 2(x-2)}{x-2}$

$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$

$= 3 \times 2 + 2$

$DF(2^+) = 8$

$= 8$

LHD : $DF(2^-)$

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$= \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$

$= \lim_{x \to 2^-} \frac{(x-2)}{(x-2)}$

$DF(2^-) = 8$

$\therefore$ LHD

Practical No. 3

TITLE : Application of derivatives.

Q.1] Find the intervals in which function is increasing or decreasing.

a] $f(x) = x^3 - 5x - 11$

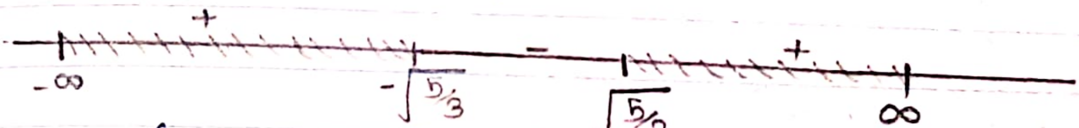
Solution: f is increasing if & only if $f'(x) > 0$

$\therefore f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\therefore 3x^2 - 5 > 0$

$\therefore x = \pm\sqrt{\frac{5}{3}}$

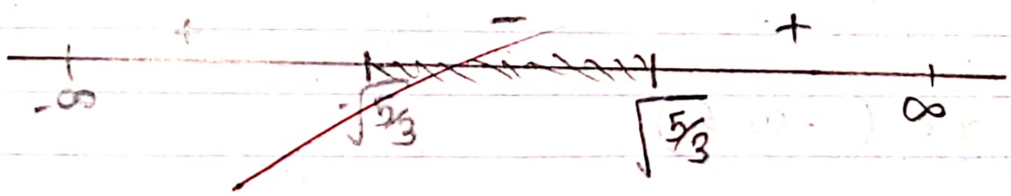


$\therefore x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

Now f is decreasing if & only if $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore x = \pm\sqrt{5/3}$



$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

b] $f(x) = x^2 - 4x$

solution: f is increasing if & only if $f'(x) > 0$

$\therefore f(x) = x^2 - 4x$

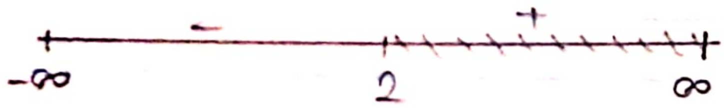
$\therefore f'(x) = 2x - 4$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$\therefore x - 2 > 0$

$\therefore x = 2$



$\therefore x \in (2, \infty)$

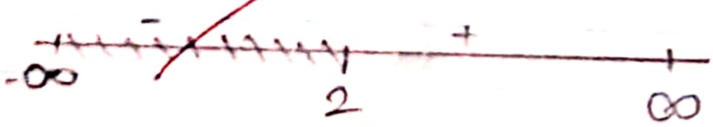
Now f is decreasing if & only if $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$\therefore x = 2$



$\therefore x \in (-\infty, 2)$

$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

b] $f(x) = x^2 - 4x$

solution: f is increasing if & only if $f'(x) > 0$

$\therefore f(x) = x^2 - 4x$

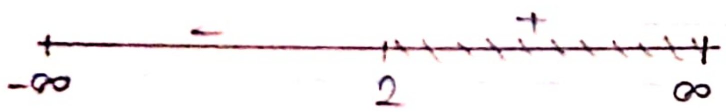
$\therefore f'(x) = 2x - 4$

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$\therefore x - 2 > 0$

$\therefore x = 2$



$\therefore x \in (2, \infty)$

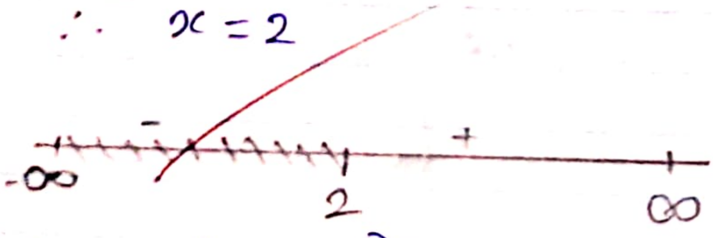
Now f is decreasing if & only if $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$\therefore x = 2$



$\therefore x \in (-\infty, 2)$

d] $f(x) = x^3 - 27x + 5$

Solution : f is increasing if & only if

$f'(x) > 0$

$\therefore f(x) = x^3 - 27x + 5$

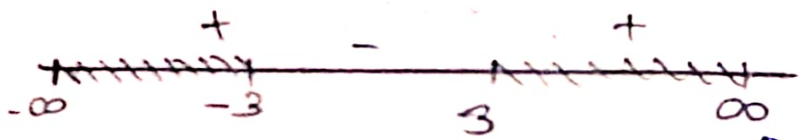
$\therefore f'(x) = 3x^2 - 27$

$\therefore 3x^2 - 27 > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore x^2 - 9 > 0$

$\therefore x = 3, -3$



Now f is decreasing if & only if

$\therefore f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore x^2 - 9 < 0$

$\therefore x = 3, -3$

$\therefore x \in (-3, 3)$

Q.2) find the intervals in which function is concave upwards & Concave downwards

a) $y = 3x^2 - 2x^3$

<u>Solution</u> : $\therefore \quad y = f(x)$

$\therefore \quad f(x) = 3x^2 - 2x^3$

$\therefore \quad f'(x) = 6x - 6x^2$

$\therefore \quad f''(x) = 6 - 12x$

$\therefore$ f is concave upward if & only if

$f''(x) > 0$

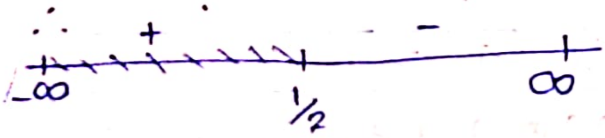
$\therefore \quad 6 - 12x > 0$

$\therefore \quad 6(1 - 2x) > 0$

$\therefore \quad 6(1 - 2x) > 0$

$\therefore \quad (1 - 2x) > 0$

$\therefore \quad -(2x - 1) > 0$



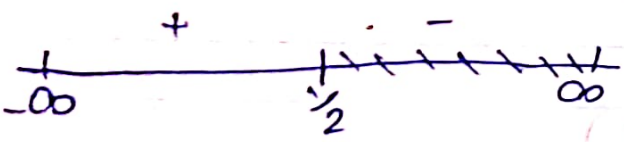
$x \in \left(-\infty, \tfrac{1}{2}\right)$

$\therefore$ f is concave downward if & only if

$f''(x) < 0$

$\therefore \quad 6(1 - 2x) < 0$

$\therefore \quad -(2x - 1) < 0$



$x \in \left(\tfrac{1}{2}, \infty\right)$

6] $y = x^4 - 6x^3 + 12x^2 + 5x + 4$

solution:

$\therefore y = f(x)$

$\therefore f(x) = x^4 - 6x^3 + 12x^2 + 5x$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$

$\therefore f$ is concave upward if & only if

$f''(x) > 0$

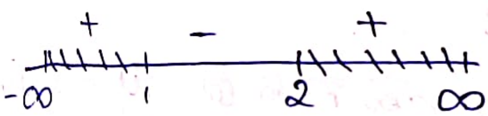
$12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore (x-2)(x-1) > 0$

$\therefore x = 2, 1$



$x \in (-\infty, 1) \cup (2, \infty)$

$\therefore f$ is concave downward if and only if

$f''(x) < 0$

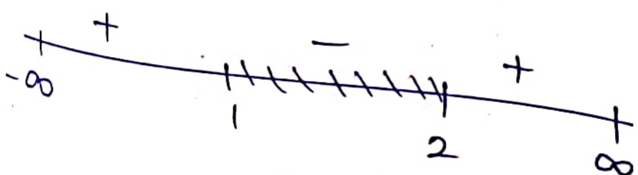
$\therefore 12x^2 - 36x + 24 < 0$

$\therefore 12(x^2 - 3x + 2) < 0$

$\therefore x^2 - 3x + 2 < 0$

$\therefore (x-2)(x-1) < 0$

$\therefore x = 2, 1$



$x \in (1, 2)$

$\cdot\, x + 5$

Solution :

$\because \quad y = f(x)$

$\therefore \quad f(x) = x^3 - 27x + 5$

$\therefore \quad y'(x) = 3x^2 - 27$

$\therefore \quad f''(x) = 6x$

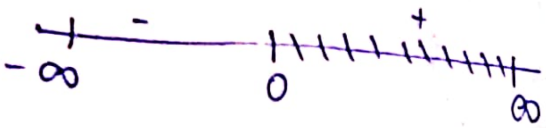
f is concave upward if & only if

$F''(x) > 0$

$\therefore \; 6x > 0$

$\therefore \; x > 0$

$\therefore \quad x = 0$



$\therefore x \in (0, \infty)$

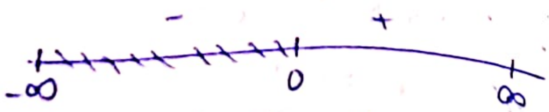
f is concave downward if & only if

$\therefore \quad f''(x) < 0$

$\therefore \quad 6x < 0$

$\therefore \; x < 0$

$\therefore \; x = 0$



$\therefore x \in (-\infty, 0)$

d] $y = 69 - 24x - 9x^2 + 2x^3$

Solution :

$\therefore \quad y = f(x)$

$\therefore \quad f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore \quad f'(x) = -24 - 18x + 16x^2$

$\therefore \quad f''(x) = -18 + 12x$

$\therefore$ f is concave upward if & only if

$\therefore \quad F''(x) > 0$

$\therefore \quad -18 + 12x > 0$

$\therefore \quad 6(2x - 3) > 0$

$\therefore \quad 2x - 3 > 0$

$\therefore \quad x = 3/2$

$-$ $\qquad$ $+$

$-\infty \qquad 3/2 \qquad \infty$

$\therefore \quad x \in (+3/2, \infty)$

$\therefore$ f is concave downward if & only if

$\cancel{f}$ $f''(x) < 0$

$\therefore$ $\cancel{f}$ $-18 + 12x < 0$

$\therefore \quad 6(2x - 3) < 0$

$\therefore \quad 2x - 3 < 0$

$\therefore \quad x = 3/2$

$-$ $\qquad$ $+$

$-\infty \qquad 3/2 \qquad \infty$

$x \in \left(-\infty, \frac{3}{2}\right)$

e) $y = 2x^3 + x^2 - 20x + 4$

Solution: $\therefore y = f(x)$

$\therefore f(x) = 2x^3 + 2x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f''(x) = 12x + 2$

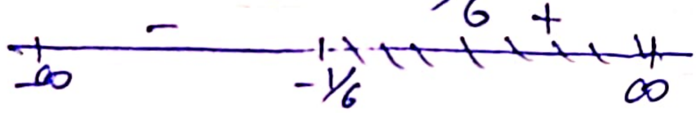
$\therefore f$ is concave upwards if & only if $f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 2(6x + 1) > 0$

$\therefore 6x + 1 > 0$

$\therefore x = -1/6$



$\therefore x \in (-1/6, \infty)$

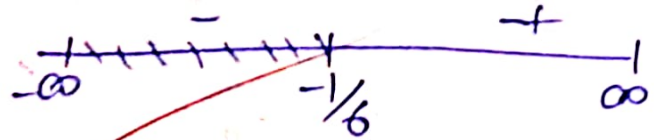
$\therefore f$ is concave downward if & only if $f''(x) < 0$

$\therefore 12x + 2 < 0$

$\therefore 2(6x + 1) < 0$

$\therefore 6x + 1 < 0$

$\therefore x < -1/6$



$\therefore \cancel{x \in (-\infty, -1/6)}$

$\therefore \cancel{x \in (-1/6, \infty)}$

$\therefore \cancel{x \in (-\infty, -1/6)}$

$\cancel{x \in (-1/6, +\infty)}$

$\therefore x \in (-\infty, -1/6)$

Q-1)

Practical No. 4

i) $F(x) = x^2 + \frac{16}{x^2}$

Solution: $f'(x) = 2x - \frac{32}{x^3}$

The maximum / minimum

$\therefore f'(x) = 0$

$\therefore 2x - \frac{32}{x^3} = 0$

$\therefore 2x = \frac{32}{x^3}$

$\therefore x^4 = 16$

$\therefore \boxed{x = \pm 2}$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = f''(-2)$

$2 + \frac{96}{(\pm 2)^4} = 2 + \frac{96}{16} = 8 > 0$

$\therefore f(x)$ is minimum at $x = \pm 2$

$\therefore f(2)$ ~~is~~ 8 is the minimum value.

2) $f(x) = 3 - 5x^3 + 3x^5$

Solution:

$f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^4 - 15x^2$

for maxima / minima

$f'(x) = 15x^4 - 15x^2 = 0$

$\therefore x^4 - x^2 = 0$

$\therefore x^2(x^2 - 1) = 0$

$\therefore x = 0, -1, 1$

$f''(x) = 60x^3 - 30x$

$f''(0) = 0$

$f''(-1) = -60 + 30 = -30 < 0$

$f''(1) = 60 - 30 = 30 > 0$

$f(x)$ is maximum at -1 & minimum at 1

$f(-1) = 3 + 5 - 3 = 5$

$f(1) = 3 - \cancel{5} + 3 = 1$

(iii) $f(x) = x^3 - 3x^2 + 1$

Solution : $f'(x) = 3x^2 - 6x$

for maxima / minima

$f'(x) = 0$

$3x^2 - 6x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$f''(x) = 6x - 6$

$f''(0) = -6 < 0$

$f''(2) = 6 > 0$

$f(x)$ is maxima at $x = 0$, minimum at $x = 2$

$f(0) = 1$

$f(2) = -3$

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

<u>Solution:</u> $f'(x) = 6x^2 - 6x - 12$

for maxima/minima

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$(x+1)(x-2) = 0$

$x = -1, 2$

$f''(x) = 12x - 6$

$f''(-1) = -12 - 6 = -18 < 0$

$f'(2) = 24 - 6 = 18 > 0$

$\therefore$ $f(x)$ is maximum at $x = -1$ and minimum at $x = 2$

$f(-1) = 8$

$\therefore$ $f(2) = -19$

Q.2)

i) $f(x) = 3x^2 - 6x - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

$x_1 = 0$

$f(x_0) = 9.5$

$f'(x_0) = 55$

$$x_1 = x_0 = \frac{f(x_0)}{f'(x_1)}$$

$$= \frac{0 - 9.5}{-55}$$

$x = 0.1727$

$f(x_1) = -0.0828$

$f(x_2) = -55.9467$

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$f(x_1)$: $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.1727 - \frac{0.0828}{-55.9467}$

$x_2 = 0.1712$

$f(x_1) = 0.0011$

$f'(x_2) = -55.9393$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.1712 - \frac{0.0011}{-55.9393}$

$\therefore x_3 = 0.1717$

$\therefore x = 0.1712$ in the root of the equation.

ii) $f(x) = x^3 - 4x - 4$

$f'(x) = 3x^2 - 4$

$f(2) = -9$

$f(3) = -9$

$f(3) = 6$

6 is closes to 0 in number line

$x_0 = 3$

$f(x_0) = 23$ $\quad \therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_1)}$

$x_1 = 2.7391$

$f(x_1) = 0.5942$ $\quad f'(x_1) = 18.508$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{0.5942}{18.508} = \underline{2.707}$

$f(x_2) = 0.0085$, $f'(x_2) = 17.9835$

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$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_3 = 2.7065$

$f(x_3) = -0.0085$

$f'(x_3) = 17.9757$

$$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.7065 - \frac{0.0005}{17.9757}$$

$\therefore x_4 = 2.7065$

$\therefore$ 2.7065 is the root of the given function.

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = 1 - 1.8 - 10 + 6.2$

$f(2) = -2.2$

$\therefore$ -2.2 is closer to 0 on the number line.

$x = 2$

$f(x_0) = -2.2$

$f'(x_0) = -5.2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{-2.2}{-5.2}$$

$\boxed{x_1 = 1.5769}$

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$+ 0(1.5769) + 17$

$= 0.6762$

$f'(x_1) = -8.217$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.5769 - \frac{0.6162}{-8.217}$

$x_2 = 1.6592$

$f(x_2) = 0.0204$

$f'(x_2) = -7.7143$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.6592 - \frac{0.0204}{-7.7144}$

$x_3 = 1.6618$

$f(x_3) = 0$

$\therefore$ 1.6618 is the root of the function

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Practical No. 5

Topic : Integration.

i) $I = \int \frac{1}{\sqrt{x^2+2x-3}}\, dx$

$= \int \frac{1}{\sqrt{x^2+2x+4}}\, dx$

$= \int \frac{1}{\sqrt{x \mp 2^2}}\, dx$

ii) $I = |x| \quad x+1 + \sqrt{x^2+2x-3} \; | + c$

iii) $I = \int (4e^{3x} + \int 1 \, dx$

$= \frac{4e^2}{3} + x + c$

iii) $I = \int (\cancel{2} e 2x^2 - 3 \sin x + 5\sqrt{x}) \, dx$

$= 2 \int x^2 d \, dx - \int \sin x \, dx + 5 \int x^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{5x^2}{3} \; x^{3/2} + c$

$I = \frac{2x^3}{3} + 3\cos x + 10/3 \; x\sqrt{x} + c$

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9v) $I = \int \frac{x^3 + 3x + 4}{\sqrt{x}} \, dx$

subs $\sqrt{x} = t$

$\therefore \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$, $\frac{dx}{\sqrt{x}} = 2dt$

$I = \int \frac{(\sqrt{x})^6 + 3(\sqrt{x})^2 + 4}{\sqrt{x}} \, dx$

$= 2\int \frac{t^6 + 3t^2 - 4dt}{\sqrt{x}}$

$= 2\int \frac{t^7}{7} + \frac{3t^3}{3} + \cancel{d}t \Big] + c$

$= 2\left[\frac{x^{3/2}}{7} + t^{3/2} + 4x^{1/2} \right] + c$

v) $I = \cancel{\int t^2 \sin (2t)^2 \, dt}$

$= \int t^4 \sin 2t^4 \, dt$

subs $t^4 = x$

$4t^3 = \frac{dx}{dt}$

$t^3 dt = \frac{1}{4} dx$

$I = \frac{1}{4} \int x \sin (2x) dx$

$\int \frac{d}{dx} \left[\int \sin 2x \, dx \right] dx \Big]$

$= \frac{1}{4}\left[\frac{-x\cos 2x}{2} + \frac{1}{2} \int \cos 2x \, dx \right]$

$= \frac{1}{10} \sin 2x - \frac{x\cos 2x}{8} + c$

$I = \frac{1}{16} \sin 2t^4 - t^4 \cdot \frac{\cos 2t^4}{8} + c$

vi) $I = \int \sqrt{x}\,(x^2 - 1)\,dx$

$I = \int x^2 \sqrt{x}\,dx - \int \sqrt{x}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= \frac{2}{7} x^3 \sqrt{x} - \frac{2}{3} x\sqrt{x} + c$

vii) $I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$\frac{1}{x^2} = t$

$\frac{-2}{x^3} = \frac{-dt}{x^2}$

$I = -\frac{1}{2} \int \sin t \, dt$

$= \frac{\cos t}{t} + c$

$$I = \frac{\cos(1/x^2)}{2} + c$$

8

viii) $I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\, dx$

$\therefore \cos x\, dx = dt$

subs $\sin x = t$

$$I = \int \frac{1}{t^{2/3}}\, dt$$

$$= \frac{t^{-2/3+1}}{-2/3+1} + c$$

$$= 3t^{1/3} + c$$

$$= 3\sqrt[3]{\sin x} + c$$

ix) $I = \int e^{\cos^2 x} \cdot \sin 2x\, dx$

subs $\cos^2 x = t$

$\therefore -2\cos x \cdot \sin x = \frac{dt}{dx}$

$\therefore \sin 2x\, dx = -dt$

$\therefore I = -\int e^t\, dt$

$= -e^t + c$

$= -e^{\cos^2 x} + c$

$I = \int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

$x^3 - 3x^2 + 1 = t$

3

$\Rightarrow x^3 - 3x^2 + 1 = t$

$\Rightarrow 3x^2 - 6x = \frac{dt}{dx}$

$\therefore (x^2 - 2x)\, dx = dt/3$

$I = 1/3 \int 1/t \, dt$

$= 1/3 \log|t| + c$

$= 1/3 \log|x^3 - 3x^2 + 1| + c.$

P.2

Practical No 6

Topic: Application of Integration & numerical Integration.

Q.1) find the length of the following:

① $x = t\sin t \; ; \; y = 1 - \cos t \in [0, 2\pi]$

Solution:

$$\text{arclength} = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^3}$$

$$dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t} + dt = \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} 2\sin \tfrac{1}{2}\, dt$$

$$= \left[-9 \cos \tfrac{1}{2} \right]_0^{2\pi}$$

$$= (-4\cos\pi) + 4\cos 0$$

$$= \underline{8 \text{ units}}$$

$y = \sqrt{4 - x^2} \qquad x \in [-2, 2]$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$$L = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$$

$$= \int_{-2}^{2} \frac{1}{\sqrt{2^2 - x^2}}\, dx$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\pi/2 + \pi/2\right] = 2\pi$$

3] $y = x^{3/2}$ in $[0,4]$

na $\rightarrow \dfrac{dy}{dx} = \cancel{d}\, \tfrac{3}{2} x^{1/2}$

$$L = \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^4 \sqrt{1 + 9/4}\, dx$$

$$= \frac{1}{2} \int_0^4 \sqrt{4 + 9x}\, dx$$

$$= \frac{1}{2} \left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9} \right]_0^4$$

$$= \frac{1}{27} \left[(4+9x)^{3/2} \right]_0^4$$

$$= -\frac{1}{27} \left[(4+0)^{3/2} - (4+36)^{3/2} \right]$$

$$L = \frac{1}{27} \left(40^{3/2} - 8 \right) \text{ units.}$$

$x = 3\sin t$, $y = 3\cos t$ 81

$\frac{dx}{dt} = 3\cos t$; $\frac{dy}{dt} = -3\sin t$ $t \in (0, 2\pi)$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\ dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\ dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\ dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\ dt$$

$$= 3\int_0^{2\pi} 3 \cdot dt$$

$= 6\pi$ units.

(5) $x = \frac{1}{6} y^3 + \frac{1}{2} y$ on $y \in (1, 2)$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$$

$$\frac{dx}{dy} = \frac{y^2 - 1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$= \int_1^2 \sqrt{1 + \frac{(y^4 - 1)^2}{4y^4}}\ dy$$

→3

$$= \int_1^2 \sqrt{\frac{(y^4-1) + 4 \times y^4}{4y^4}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1/2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$$= \frac{17}{12} \text{ units.}$$

Q.II) i) $\int_0^4 x^2\, dx$ $x = 4$

$$h = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int_0^4 x^2\,dx = \frac{h}{3}\left[(y_0 + y_x) + 4(y_1 + y_3) + 2(y_2)\right]$$

$$= \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= \frac{64}{3}$$

$$\int_0^4 x^2\,dx = 21.533$$

iii) $\int_0^{\pi/3} \sqrt{\sin x}\,dx$ with $n = 6$

$$h = \frac{\pi/3 - 0}{6} = \pi/18$$

x	0	$\pi/18$	$\frac{2\pi}{18}$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$8\pi/18$
y	0	0.4166	0.58	0.70	0.80087		0.43

$$\int_0^{\pi/3} \sqrt{\sin x} = \frac{h}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049$$

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Practical No. 7

Solve the following differential equation.

1) $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $Q(x) = \frac{e^x}{2}$

$IF = e^{\int P(x)dx}$

$= e^{\ln x}$

$IF = x$

$y(IF) = \int Q(x)(IF)\,dx + c$

$yx = \int \frac{e^x}{x} x \, dx + c$

$= \int \frac{e^x}{x} x \cdot dx + c$

$= \int e^x \, dx + c$

$\therefore xy = e^x + c$

Ex!

2) Solution:

$$e^{x}\frac{dy}{dx} + 2e^{x}y = 1$$

$$\frac{dy}{dx} + 2y = e^{-x}$$

$P(x) = 2 \quad q(x) = e^{-x}$

$$I.F = e^{\int P(x)dx}$$
$$= e^{\int 2dx}$$
$$= e^{2x}$$

$$y(I.F) = \int q(x)\,(I.F)\,dx + C$$

$$y \cdot e^{2x} = \int e^{-x} e^{2x}\,dx + C$$
$$= \int e^{x}\,dx + C$$

$$ye^{2x} = e^{x} + C$$

3) Solution:

$$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$$

$$\therefore \frac{dy}{dx} + \frac{2}{x}y = \frac{\cos x}{x^2}$$

$P(x) = 2/x \quad Q(x) = \cos x/x^2$

$$I.F = e^{\int P(x)dx}$$
$$= e^{\int 2/x\,dx}$$
$$= e^{2\ln x}$$
$$= e^{\ln x^2}$$

$$IF = x^2$$

$$y(IF) = \int Q(x)\,(IF)\,dx + C$$

$$x^2 y = \int \frac{\cos x}{x^2} \times x^2\, dx + c$$

$$= \int \cos x + c$$

$$\boxed{x^2 y = \sin x + c}$$

Solution:

i)

$$x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$$

$$\therefore \frac{dy}{dx} + \frac{3}{x} y = \frac{\sin x}{x^3}$$

$P(x) = 3/x \qquad Q(x) = \sin x / x^2$

$$I.F = e^{\int P(x) dx}$$

$$= e^{\int 3/x\, dx}$$

$$= e^{3 \ln x}$$

$$= e^{\ln x}$$

$$\boxed{I.F = x^3}$$

$$y(I.F) = \int Q(x)\,(I.F)\, dx + c$$

$$x^3 y = \int \frac{\sin x}{x^3} \times x^3\, dx + c$$

$$= \int \sin x\, dx + c$$

$$x^3 y = -\cos x + C$$

v) Solution:

$$\text{i.e. } e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$$p(x) = 2 \quad Q(x) = 2x\,e^{-2x}$$

$$IF = e^{\int p(x)dx}$$
$$= e^{\int 2dx}$$
$$= e^{2x}$$

$$y(I.F) = \int Q(x)^{(I.F)} dx + c$$

$$ye^{2x} = \int 2x\,e^{-2x} \times e^{2x}\,dx + c$$

$$\therefore ye^{2x} = \int 2x\,dx + c$$

$$\therefore ye^{2x} = x^2 + c.$$

vi) Solution: $\frac{dy}{dx} = \sin^2(x - y + 1)$

put ~~x~~ $-y+1 = x$

Differentiating both sides

$$x - y + 1 = v$$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore 1 - \frac{dv}{dx} = \frac{dy}{dx}$$

$$1 - \frac{dv}{dx} = \sin^2 v$$

$$\frac{dv}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\therefore \int \sec^2 v\, dv = \int dx$$

$$\therefore \tan v = x + c$$

$$\therefore \underline{\tan(x + y - 1) = x + c}$$

vii) Solution: $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$$\sec^2 x \cdot \tan y\, dx = -\sec^2 y \cdot \tan y\, dy$$

$$\frac{\sec^2 x\, dx}{\tan x} = -\frac{\sec^2 y\, dy}{\tan y}$$

$$\therefore \log|\tan x| = -\log|\tan y| + c$$

$$\therefore \log(\tan x \cdot \tan y) = c$$

$$\therefore \underline{\tan x \cdot \tan y = e^c}$$

(VT 11)

$$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

put $2x+3y = v$

$$\therefore 2 + \frac{3dy}{dx} = \frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\therefore \frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= 3\left(\frac{v+1}{v+2}\right)$$

$$\int\left(\frac{v+2}{v+1}\right) dv = 3\int dx$$

$$\int\left(\frac{v+1}{v+1}\right) dv = \int \frac{1}{v+1} dv = 3x + c$$

$$v + \log|v+1| = 3x + c$$

$$2x + 3y + \log|2x + 3x + 1| = 3x + c$$

$$3y = x - \log|2x + 3y + 1| + c.$$

TOPIC : Euler's

1) $\frac{dy}{dx} = y + e^{x} - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ?$

Solution: $f(x) = y + e^{x} - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	2.1487	2.5
1	0.5	2.5	4.2925	3.5743
2	1	3.5743	8.2021	5.7205
3	1.5	5.7205		9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215.$

Q.2) $\frac{dy}{dx} = (1 + y^2)$ $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$y_0 = 0$ $y_0 = 0$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1			1

$y(1) = 1.2939$

Practical

Q.3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$x_0 = 0$ $y(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0.4472	1.0894
1	0.2	1.0894	0.6069	1.2105
2	0.4	1.2105	0.7040	1.3513
3	0.6	1.3513	0.7696	1.5051
4	0.8	1.5051		
5	1			

$\therefore y(1) = 1.5051$

Q.4) $\frac{dy}{dx} = 3x^2+1$ $y(1) = 2$ find $y(2)$ $h = 0.5$

~~y_0~~ $h = 0.25$

$y_0 = 2$ $x_0 = 1$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(\cancel{x}\,2) = 7.875$

5) $y_0 = 2 \quad x_0 = 1 \quad h = 0.25$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1722.6426	299.9960
4	2	299.9960		

$y(2) = 299.9960$

5) $\frac{dy}{dx} = \sqrt{xy+2}$; $y(1) = 1 \quad h = 0.2$

$x_0 = 1 \quad y_0 = 1 \quad h = 0.2$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

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Practical No. 9

TOPIC : Limits & partial order derivatives

Q.1 Evaluate the following limits

1) $$\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3x + y^2 - 1}{xy + 5}$$

$$= \lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3x + y^2 - 1}{xy + 5}$$

Apply limit,

$$= \frac{(-4)^3 - 3(-4) + (-1)^2 - 1}{(-4)(-1) + 5}$$

$$= \frac{-64 + 12 + 1 - 1}{4 + 5} = -\frac{52}{9}$$

2) $$\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$$

$$= \lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$$

$$= \cancel{\lim_{(x,y)\to(2,0)}} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$$ # Apply limit

$$= \frac{1(4 + 0 - 8)}{2} = \boxed{-2}$$

3) $\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2y^2}$

$= \lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2y^2}$

Apply limit

$= \dfrac{(1)^2-(1)^2-(1)^2}{(1)^3-(1)^2-(1)(1)} = \dfrac{1-1}{1-1} = \dfrac{0}{0}$

$\therefore$ Limit doesnot exist.

Q.2) find f_x, f_y for each of the following

1) $f(x,y) = xye^{x^2+y^2}$

$f(x) = y(1\, e^{x^2+y^2}) + xy(e^{x^2+y^2}\, 2x)$

$= ye^{x^2+y^2} + 2xye^{x^2+y^2}$

$f_y = x \cdot e^{x^2+y^2} + 2xy^2 e^{x^2+y^2}$

$f_x = ye^{x^2+y^2} + 2x^2y\, e^{x^2+y^2}$

$f_y = xe^{x}+y^2 + 2xy^2\, e^{x^2+y^2}$

ii) $f(x,y) = e^x \cos y$

$f_x = \cos y\, e^x$

$f_y = e^x - \sin y$

$f_y = -\sin y\, e^x$

$f(x, y) = x^3y^2 - 3x^2y + y^3 + 1$

Sol:

$f_x = y^2\, 3x^2 - 3y2x + 0 + 0$

$= 3x^2y^2 - 6xy$

$f_y = x^3 2y - 3x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

2) Using definition find values of f_x, f_y at $(0, 0)$ for $f(x, y) = \frac{2x}{1+y^2}$

$f_x(a, b) = \lim_{h \to 0} \frac{f(a+h, b) - f(a, b)}{h}$

$f_y(a, b) = \lim_{h \to 0} \frac{f(a+h, b) - f(a, b)}{h}$

According to given $(a, b) = (0, 0)$

$f_x(0, 0) = \lim_{h \to 0} \frac{f(h, 0) - f(0, 0)}{h}$

$= \lim_{h \to 0} \frac{2h - 0}{h} = 2$

$f_y(0, 0) = \lim_{h \to 0} \frac{f(0, h) - f(0, 0)}{h}$

$f(x) = 2$, $f_y = 0$

$f(x, y) = (0, 0) = \lim_{h \to 0} \frac{f(0, h) - f(0, 0)}{h}$

$= \lim_{h \to 0} \frac{0 - 0}{h} = 0$

$\therefore f(x) = 2 \quad fy = 0$

2) find all second order partial partial derivation of F. Also verify whether $fxy = fyx$

1) $f(x,y) = \frac{y^2 - xy}{x^2}$

$\therefore fxx = \frac{d^2 f}{dx^2} \quad fyy = \frac{d^2 f}{dy^2}$

Applying ouv rule

$f x = \frac{x^2(0-y) - (y^2 - x^4)2x}{x^4}$

$= \frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4}$

$\therefore fx = \frac{x^2 y - 2xy^2}{x^4}$

$\therefore fxx = \frac{x^4(2xy - 2y^2) - (x^2 y - 2xy^2)(4x^3)}{x^8}$

$= \frac{x^5 - 2x^4 y^2 - (4x^5 y - 8x^4 y^2)}{x^8}$

P.a

$$= \frac{-2x^5 y - 2x^4 y^2 - 4x^5 y + 8x^4 y^2}{x^8}$$

$$= \frac{6x^4 y^2 - 2x^5 y}{x^8}$$

$$= \frac{6y^2 - 2xy}{x^4}$$

$$Fy = \frac{1}{x^2}(2y - x)$$

$$\therefore Fy = \frac{2y - x}{x^2}$$

$$Fyy = \frac{1}{x^2} \quad 2 \quad = \frac{2}{x^2}$$

$$Fxy = \frac{2y - x}{x^2} = \frac{x^2(-1) - (2y - x)(2x)}{x^4}$$

$$Fxy = \frac{x - 4y}{x^3}$$

$$fyx = \frac{x^2 y - 2xy^2}{x^4}$$

$$= \frac{x - 4y}{x^3}$$

$\therefore \boxed{Fyx = Fxy}$ HENCE verified.

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solution :

v) $f(x,y) = x^3 + 3x^2 y^2 = \log(x^2+1)$

$f(x) = \frac{d}{dx} x^3 + 3x^2 y^2 - \log(x^2+1)$

$fx = 3x^2 + 6xy^2 - \frac{2x}{x^2+2}$

$fy = \frac{d}{dy} x^3 + 3x^2 y^2 - \log(x^2+1)$

$= 0 + 6yx^2 - 0$

$fy = \frac{d}{dy} fx$

$= \frac{d}{dy} y \cdot \cos(xy) + e^x \cdot ey$

$fyy = -xy^2 \cdot in(xy) + \cos(xy) + e^x \cdot e^y$

$fyx = \frac{d}{dx} fy$

$= \frac{d}{dx} x \cdot \cos(xy) e^x \cdot e^y$

$Fyx = -xy \sin(x,y) y \cdot \cos(xy) + e^x - e^4$

$\therefore fxy = fy - x$

Q-8) i) $f(x,y) = \sqrt{x^2+y^2}$

$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \times 2x$

$fx(a,b) = 1/\sqrt{2}$

$$= \sqrt{2}$$

$$f_y = \frac{2y}{2\sqrt{x^2+y^2}}$$

$$f_y = \frac{y}{\sqrt{x^2+y^2}}$$

$$\therefore f_y(1,4) = \frac{1}{\sqrt{2}}$$

Q.4) 3)

Soln.

$$f(x,y) = \sin(xy) + e^{x+y}$$

$$f(x,y) = \sin(xy) + e^x \cdot e^y$$

$$f_x = \frac{d}{dx}(\sin(xy) + e^x \cdot e^y)$$

$$f_x = y\cos(xy) + e^x \cdot e^y$$

$$= x\cos(xy) + e^x \cdot e^y$$

$$f_{xx} = \frac{d}{dx} f_x$$

$$= \frac{d}{dx} y\cos(xy) + e^x \cdot e^y$$

$$f_{yy} = \frac{d}{dy} f_y$$

$$= \frac{d}{dy}(x\cos(xy) + e^x \cdot e^y)$$

$$f_{yy} = -x^2 \sin^2(xy) + e^x \cdot e^y$$

5) (iii) $f(x, y) = 1 - x + y \sin x$ at $\pi/2$

$f(\pi/2 ; 0) = 1 - \pi/2 \; y \; 0 + \sin \pi/2$

$f(\pi/2, 0) = 2 - \pi/2$

$\therefore f_x = -1 + y \cos x \qquad f_y = \sin x$

$f_x [\pi/2, 0] = -1 + 0 - \cos \pi/2$

$= -1$

$f_y (\pi/2, 0) = \sin \pi/2$

$= 1$

(iv) $f(x, y) = \log x + \log y$

$f(1, 1) = \log 1 + \log 1$

$= 0$

$f_x = 1/x \qquad \cancel{f_y} = 1/y$

$L(x, y) = F(1, 1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 0 + 1[x-1] + 1[y-1]$

$L(x, y) = x + y - 2$

Practical No: 10

1) find the directional derivative of the following function at given points in the direction of given vector.

i) $f(x,y) = x + 2y - 3 \quad a = (1,-1) \quad u = 3i - j$

Here $u = 3i - j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2 - (-1)^2} = \sqrt{9+1} = \sqrt{10}$

Unit vector $= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f = \left(1 + \frac{3}{\sqrt{10}}\right), \left(-1, -\frac{h}{\sqrt{10}}\right)$

$f(a+hv) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1, -\frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hv) = -4 + \frac{h}{\sqrt{10}}$

$f.(a) = \lim_{h \to 0} \frac{f(a+hv) - f(a)}{h}$

$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

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i) Here $u = i + 5j$ is not a unit vector. 72

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

unit along r is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3, 4) = (4)^2 - 4(3) + 1 = 5$

$f(a + hv) = f(3, 4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$f(a) = \frac{25}{26} + \frac{36}{\sqrt{26}}$ //

ii) $2x + 3y$ $a(1, 2)$ $u = (3i + 4i)$

Here $u = 3i + 4j$ is not a unit vector

$|\bar{u}| = \sqrt{25} = 5$

Unit vector $= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1, 2) = 2(1) + 3(2) = 8$

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$$f(a+hv) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$$

$$f(a+hv) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$$

$$= \frac{18h}{6} + 8$$

$$f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$$

$$= \frac{18}{5}$$

Q.2) find gradient vector for the following function at point :

i) $f(x,y) = x^y + x^a \quad a = (1,1)$

$f(x) = y - x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^x$

$f(x,y) = (f_x , f_y)$

$= \left(y x^{y-1} + y^2 \log y , x^y \log y + x y^{x-1}\right]$

$y(1,1) = (1+0, 1+0)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1}x) \cdot y^2$ a $(1,-1)$

$f(x) = \frac{1}{1+x^2} \quad -y^2$

$f_y = 2y \tan^{-1}x$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

Q.3) i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f(x) = \cos y \, 2x + e^{xy} y$

$f(y) = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1,0) = \therefore x_0 = 1 \; y_0 = 0$

eqn. tangent

$f_x(x - x_0) = (1)^2 \cancel{(\sin 0)} + e^0 \cdot 1$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 1.$

$2(x-1) + 1(y-0) = 0$

$2x + y - 2 = 0$ It is the required eqn of tangent.

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Eq^n of normal

$ax + by + c = 0$

$-bx + ay + d = 0$

$1(\cdot) + 2(y) + d = 0$

$x + 2y + d = 0$ at $(\cdot, 0)$

$\Rightarrow 1 + 2(0) + d = 0$

$d + 1 = 0$

$d = -1$

Q 2) find the equation of tangent & normal line to each of the following surface

1) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$f_x = 2x + 0 + 0 + z$

$= 2x + z$

$f_y = 2z + 3$

$f_z = -2y + x$

$f(x)(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f(y)(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f(z)(x_0, y_0, z_0) = 0$

Equation of tangent

$f(x)(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x - 2) + 3(y - 1) + 0(z - 0) = 0$

$= 4x + 3y - 11 = 0 \rightarrow$ reqd eq^n of tangent

Equation of normal at (...)

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x - 2}{4} = \frac{y - 1}{3} = \frac{z + 1}{0}$$

(6) find the local maxima & minima for the following

$$f(x, y) = 3x^2 + y^2 - 3xy + 6x - 4y$$

$$= 6x + 0 - 3y + 6 - 0$$

$$= 6x - 3y + 6$$

$$f_y = 0 + 2y - 3x + 0 - 4$$
$$= 2y - 3x - 4$$

$$f(x) = 0$$

$$6x - 3y + 6 = 0$$
$$3(2x - y + 2) = 0$$
$$2x - y + 2 = 0$$
$$2x - y = -2 \quad \text{—} (1)$$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ — (2)

multiply eq^n (1) with 2

$4x - 2y = -4$ — (1)

$2y - \cancel{3x}\ y = 2$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ — (2)

multiply eq^n.(1) with 2

$4x - 2y = -4$

$2y - 3x = 4$

$\boxed{x = 0}$

Substitute value of x in eq^n (1)

$2(0) - y = -2$

$-y = -2$

$\boxed{y = 2}$

$\therefore$ critical points are $(0, 2)$

Scanned with CamScanner

$x = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0,2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f(xy) = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$f(x,y) = x^2 - y^2 + 2x + 8y - 70$ 76

$f(x) = 2x + 2$

$f(y) = -2y + 8$

$f(x) = 0$ $\therefore 2x + 2 = 0$

$x = -1$

$f y = 0$ $-2y + 8 = 0$

$y = -8/-2 = 4$

$\therefore y = 4$

$\therefore$ Critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + (2)(-1) + 8(4) - 70$

$1 + 16 - 2 + 32 + 70$

$= 17 + 30 - 70$

35

$= 37 - 70$

$= 33$

$\boxed{f(x,y) = 33}$

AK

05/02/2024