Exam 1 Intermediate Work — Aman Patel

1i. $S = \{(1,2),(1,1),(1,5),(7,1),(2,2),(2,5),(5,1),(5,2),(5,5)\}$ $10 \cdot 10 = 100$ outcomes

1ii. 3 2 6 3 4 7 6 7 10

1b. $\frac{2}{} \cdot \frac{2}{} = 4$ outcomes / 100 total $= 0.04$

$\frac{3}{} \cdot \frac{2}{} + \frac{2}{} \cdot \frac{3}{} = 12$ outcomes / 100 total $= 0.12$

1c. $P(X \leq 4) = P(X=2) + P(X=3) + P(X=4) = 0.04 + 0.12 + 0.09 = 0.25$

$P(3 \leq X \leq 5) = P(X=3) + P(X=4) + P(X=5) = 0.12 + 0.09 + 0 = 0.21$

$P(3 < X < 5) = P(X=4) = 0.09$

1d ~~1 − pbinom(7, 20, 0.25) = 0.102~~ $P(X=10) = 0.25$ $P(Y \geq 8) = 1 - P(Y \leq 8) + P(Y=8)$

$= 1 - \text{pbinom}(8, 20, 0.25) + \text{dbinom}(8, 20, 0.25)$

$= 0.102$

2ai. If N wins, N must win last game

$\{\quad 5N, 3F \quad\}N$

$\binom{8}{5} = \frac{8!}{5!3!} = 56$ outcomes where N wins in 9 games

$\therefore$ 112 total 9-game outcomes

2aii.
$9: 2 \cdot \binom{8}{5} = 112$
$8: 2 \cdot \binom{7}{5} = 42$
$7: 2 \cdot \binom{6}{5} = 12$
$6: 2 \quad \binom{5}{5} = 2$
} 168 outcomes

2b. # of outcomes where F wins in 9 games: 56

Total outcomes $= 168 + 2 \cdot \binom{9}{5} + 2 \cdot \binom{10}{5} = 168 + 252 + 504 = 924$

$\frac{56}{924} = 0.061$

2c. N N F F F F F F = 1 Total: 6-game: 1

N N {__ 5F, 1N ____} F 7-game: 4

$1 + \binom{6}{5} = 1 + 6 = 7$ outcomes 8-game: $\binom{5}{3} = 10 + 1 = 11$

$\frac{7}{210} = 0.033$ 9-game: $\binom{6}{3} = 20 + 6 = 26$

10-game: $\binom{7}{3} + \binom{7}{5} = 35 + 21 = 56$

11-game: $\binom{8}{5} + \binom{8}{5} = 56 + 56 = 112$

Total = 210

2d. $0.61 \cdot 0.42 \cdot 0.61 \cdot 0.42 \cdot 0.61 \cdot 0.42 = 0.61^3 \cdot 0.42^3 = 0.017$

3a $\frac{b+4}{2} \cdot b = 1$

$5b = 1$

$b = 0.2$

3b $f(x) = \begin{cases} 0, & \text{when } -\infty < x < 4 \\ 0.8, & \text{when } 4 \le x < 8 \\ 0.2, & \text{when } 8 \le x < 10 \\ 0, & \text{when } 10 \le x < \infty \end{cases}$

$4 \cdot 0.2 = 0.8$

$2 \cdot 0.2/2 = 0.2$

3c Weighted average

$b \cdot 0.8 + \frac{2b}{3} \cdot 0.2$

$8 \text{ — } 10$

$\frac{1}{3} \cdot 2 = \frac{2}{3}$ $\quad 8 + \frac{2}{3} = \frac{26}{3}$

$E(X) = 6.53$

3d. $P(X = 4)$ undefined in pdf

$P(X \le 8) = P(X < 4) + P(4 \le X < 8) = 0 + 0.8 = 0.8$

$P(X > 9) = P(9 < X < 10) + P(10 \le X < \infty) = 1 \cdot 0.1/2 + 0 = 0.05$

$P(8 < X < 9) = 1 - \frac{0.1 + 0.2}{2} = 0.15$

4a. $X \sim N(85, 9)$

$P(X > 90) = 1 - P(X \le 90) = 1 - \text{pnorm}(90, 85, 3) = 0.048$

4b. $E(Z) = E(Y) \cdot 1.8 + 32 = 27.8 \cdot 1.8 + 32 = 82.04$

$\text{Var}(Z) = (\sigma(Y) \cdot 1.8)^2 = 3.24^2 = 10.50$

4c $E(V) = E(X) + E(Y) = 85 + 82.04 = 167.04$

$\text{Var}(V) = \text{Var}(X) + \text{Var}(Y) = 9 + 10.50 = 19.50$

$P(V > 180) = 1 - P(V \le 180) = 1 - \text{pnorm}(180, 167.04, \sqrt{19.50}) = 0.002$

4d $E(W) = E(Y) - E(X) = 82.04 - 85 = -2.96$

$\text{Var}(W) = \text{Var}(Y) + \text{Var}(X) = 10.50 + 9 = 19.50$

$P(W > 0) = 1 - P(W \le 0) = 1 - \text{pnorm}(0, -2.96, \sqrt{19.50}) = 0.251$