

```

1 void f4(int n) {
2     int count = 0;
3     for(int i = 1; i ≤ n; i++) {
4         for(int j = 1; j ≤ n; j++) {
5             count += 1;
6         }
7     }
8 }

```

nested

~~$3nc$~~

$i=1 \rightarrow \text{loop } j \rightarrow [1, n] \rightarrow 3nc$   
 $i=2 \rightarrow \text{loop } j \rightarrow [1, n] \rightarrow 3nc$   
 $i=3 \rightarrow \text{loop } j \rightarrow [1, n] \rightarrow 3nc$   
 $\vdots$   
 $i=n \rightarrow \text{loop } j \rightarrow [1, n] \rightarrow 3nc$   
 Total  $\rightarrow 3nc + 3nc + 3nc \dots 3nc$  ( $n$  times)  
 is  $\rightarrow 3n^2c \rightarrow \underline{O(n^2)}$

For every iteration of  $i$ , multiple instructions of loop of  $j$  will be executed.

If we sum up no. of instructions executed by  $j$ , for every value of  $i$  then we can get total instructions

```

1 void f5(int n) {
2     int count = 0;
3     for(int i = 1; i ≤ n; i++) {
4         for(int j = i; j ≤ n; j++) {
5             count += 1;
6         }
7     }
8 }

```

$N=3$

$O(N^2)$

*repeated*

$i=1$  loop  $j \rightarrow [1, N] \rightarrow N$   
 $i=2$  loop  $j \rightarrow [2, N] \rightarrow N-1$   
 $i=3$  loop  $j \rightarrow [3, N] \rightarrow N-2$   
 $\vdots$   
 $i=N$  loop  $j \rightarrow [N, N] \rightarrow 1$

Total ins  $\rightarrow N + (N-1) + (N-2) + (N-3) \dots \dots \dots 3 + 2 + 1$

$$\rightarrow \frac{N(N+1)}{2} \Rightarrow \frac{N^2}{2} + \frac{N}{2}$$

$O(N^2)$

$N=3$

$$\begin{array}{r}
 1 + 2 + 3 \\
 \hline
 3(1) \rightarrow 6
 \end{array}$$

max  
3 place

[3, 1, 2, 4, 9, 8]

```
1
2 longestConsecutiveSequence(arr) {
3   let mp = {};
4   for(let i = 0; i < arr.length; i++) {
5     mp[arr[i]] = true;
6   }
7   let ans = -1;
8   for(let i = 0; i < arr.length; i++) {
9     if(mp[arr[i] - 1]) {
10      continue;
11    } else {
12      let len = 0;
13      let x = arr[i];
14      while(mp[x]) {
15        len++;
16        x++;
17      }
18      ans = Math.max(ans, len);
19    }
20  }
21  return ans;
22 }
```

3N

O(N)

i = 3

i = 0

i = 2

key value

3 - true

1 - true

2 - true

4 - true

9 - true

8 - true

~~i = 0 / 2 / 3 / 4~~

S

x = 1 2 3 4 5

O(N)

4 iterations [x = 1 → 2 → 3 → 4]

x = 8 9 10

2 iterations [x = 8 → 9]



```
1 void f6(int n) {  
2     int count = 0;  
3     while(n > 0) {  
4         count++;  
5         n /= 2;  
6     }  
7 }
```

$O(\log n)$

total instructions

In every iteration of while loop, we exec

3 ins

Assume total  $\rightarrow$   $K$  iterations

1<sup>st</sup> iteration

$n/1$

↓

$n/2^0$

2<sup>nd</sup>

↓

$n/2$

↓

$n/2^1$

3<sup>rd</sup>

↓

$n/4$

↓

$n/2^2$

4<sup>th</sup>

↓

$n/8$

↓

$n/2^3$

.....  $K^{\text{th}}$

↓

$n/2^{K-1}$

→ 1

$$\frac{n}{2^{k-1}} = 1$$

$$n = 2^{k-1}$$

taking  $\log_2$  both sides

$$\log_2 n = \log_2 2^{(k-1)}$$

$$\log_2 n = (k-1) \log_2 2$$

$$\log_2 n = k-1$$

$$k = \log_2 n + 1$$

$$\log_3 n \rightarrow \frac{\log_2 n}{\log_2 3}$$

total  $\rightarrow 3(\log_2 n + 1)$   
ins

$\downarrow$   
 $O(\log n)$



```
1 void f7(int n) {  
2     int count = 0;  
3     for(int i = 1; i ≤ n; i *= 2) {  
4         count++;  
5     }  
6 }
```

$O(\log n)$

1<sup>st</sup> 2<sup>nd</sup> 3<sup>rd</sup> ..... K<sup>th</sup>  
↓ ↓ ↓ ↓  
3 3 3 3

lets assume we have K iterations

total ops → 3K

1<sup>st</sup> 2<sup>nd</sup> 3<sup>rd</sup> 4<sup>th</sup> 5<sup>th</sup> ..... K<sup>th</sup>  
↓ ↓ ↓ ↓ ↓  
i = 1 2 4 8 16 .....  $2^{k-1}$   
 $(2^0)$   $(2^1)$   $(2^2)$   $(2^3)$   $(2^4)$  .....  $(2^{k-1})$  → last value of i viz  $\leq n$

$$2^{k-1} \leq n$$

taking log both sides

$$\log_2 2^{k-1} \leq \log_2 n \rightarrow (k-1) \frac{\log_2 2}{1} \leq \log_2 n$$

$$k-1 \leq \log_2 n \rightarrow (k \leq \log_2 n + 1) \rightarrow O(\log n)$$



```
1 void f8(int n) {  
2     int count = 0;  
3     for(int i = 1; i ≤ n; i++) {  
4         for(int j = 1; j ≤ sqrt(n); j++) {  
5             count++;  
6         }  
7     }  
8 }
```

$i=1 \rightarrow j \Rightarrow [1, \sqrt{n}] \rightarrow \sqrt{n}$   
 $i=2 \rightarrow j \Rightarrow [1, \sqrt{n}] \rightarrow \sqrt{n}$   
 $i=3 \rightarrow j \Rightarrow [1, \sqrt{n}] \rightarrow \sqrt{n}$   
 $\vdots$   
 $i=N \rightarrow j \Rightarrow [1, \sqrt{N}] \rightarrow \sqrt{N}$

$O(N\sqrt{N})$

```

1 void f9(int n) {
2     int ans = 0;
3     for(int i = 0; i < n; i++) {
4         ans += i;
5     }
6     for(int i = 0; i < n; i++) {
7         for(int j = 0; j < n; j++) {
8             ans += (i+j);
9         }
10    }
11 }

```

$$O(n + n^2) \rightarrow \underline{\underline{O(n^2)}}$$

$$i=0 \quad j \rightarrow [0, n-1] \rightarrow n$$

$$i=1 \quad j \rightarrow [0, n-1] \rightarrow n$$

⋮

$$i=n-1 \quad j \rightarrow [0, n-1] \rightarrow n$$

$$\underline{\underline{N \times N}}$$





```
1 void f10(int n) {  
2     int count = 0;  
3     for(int i = n; i > 0; i /= 3) {  
4         count++;  
5     }  
6 }
```

$$\begin{array}{ccccccc} 1^3 & 2^3 & 3^3 & \dots & K \\ \downarrow & \downarrow & \downarrow & & \downarrow \\ n & n/3 & n/9 & & n/3^{K-1} \end{array}$$

$$\frac{n}{3^{K-1}} \approx 1$$

$$n \approx 3^{K-1}$$

$$\log_3 n \approx \log_3 3^{K-1}$$

$$\log_3 n \approx K-1$$

$$K \rightarrow \underline{O(\log n)}$$



```
1 void f11(int n) {  
2     int i = 1, s = 1;  
3     while(s ≤ n) {  
4         i++;  
5         s += i;  
6     }  
7 }  
8
```

$10(\sqrt{n})$

$$i = 1$$

$$s = 1$$

$$i = 2$$

$$s = (1 + 2)$$

$$i = 3$$

$$s = (1 + 2 + 3)$$

$$i = 4$$

$$s = (1 + 2 + 3 + 4)$$

⋮

$$i = k$$

$$s = (1 + 2 + 3 + 4 + \dots + k)$$

if loop terminate  
here

$$s \leq n \rightarrow (1 + 2 + 3 + 4 + \dots + k) \leq n$$

$$\frac{k(k+1)}{2} \leq n \rightarrow \begin{aligned} &\approx k^2 \leq n \\ &\approx k \leq \sqrt{n} \end{aligned}$$