

GREEDY ALGORITHMS

→ optimizing problems

general structure

getOptimal (Item arr[], int n) {

① initialize res = 0

② while (All items are not considered)

{

i = selectAnItem();

if (feasible(i))

res = res + i;

}

③ Return Res

}

* Greedy algorithms may not work all the time

Activity Selection problem

I/P → {(2,3), (1,4), (5,8), (6,10)}

O/P → 2

machine can do only 1 activity at a time and we have to count max. number of activities happen on single machine.

I/P → {(1,3), (2,4), (3,8), (10,11)}

O/P → 3

Greedy

- sort according to finish-time
- initialize the solution with
 max first activity (i.e min finish-time)
- do the following for remaining
 - ① if current activity overlaps with
 the last picked activity, ignore
 the current activity
 - ② Else add the current activity

I/P → $\{(3,8), (2,4), (1,3), (10,11)\}$

sorted → $\{(1,3), (2,4), (3,8), (10,11)\}$

ans $\{(1,3), (3,8), (10,11)\}$

Implementation

```
int maxActivities (pair<int, int> arr[], int n) {  
    sort(arr, arr+n, mycmp);  
    int prev = 0;  
    int res = 1;  
    for (int curr = 1; curr < n; curr++) {  
        if (arr[curr].first >= arr[prev].second)  
        {  
            res++;  
            prev = curr;  
        }  
    }  
    return res;  
}
```

```
bool mycmp (pair<int, int> a, pair<int, int> b)  
    return a.second < b.second;
```

FRACTIONAL KNAPSACK

- collect maximum value in knapsack.

I/P

weight	50	20	30
values	600	500	400

knapsack capacity = 70

$$O/P \rightarrow I_2 + I_3 + \frac{20}{50} \times 600$$

$$= 500 + 400 + 20 \times 12$$

$$= 1140$$

Algorithm

- ① calculate ratio (value/weight) for every item.
- ② sort all items in decreasing order.
- ③ initialise res = 0, cur-cap = given-cap.
- ④ Do the following for every item I in sorted order

if (I .weight \leq cur-cap) {
 cur-cap -= I .weight
 res += I .value;

}
 else {

$$\text{res} += \frac{\text{cur-cap}}{I.\text{weight}} \times I.\text{value};$$

return res.

}

return res.

④

Job Sequencing

I/P \rightarrow deadline
 profit

J_0	J_1	J_2	J_3
4	1	1	1
70	80	30	100

O/P \rightarrow 170

I/P \rightarrow deadl.
 profit

J_0	J_1	J_2	J_3	J_4
2	1	2	1	3
100	50	10	20	30

\rightarrow 180

- \rightarrow one unit by every job
- \rightarrow only one job can be assigned at a time
- \rightarrow time start with 0
- \rightarrow maximize the profit

Algorithm

- ① sort jobs in decreasing order of profit
- ② initialize the result as first job in the sorted list
- ③ Do following for the remaining $(n-1)$ jobs.
 - (a) if this job can not be added, ignore it
 - (b) else add it to the latest possible slot.

4	1	1	5	5
50	5	20	10	80

 \Rightarrow

J_4	J_0	J_2	J_3	J_1
5	4	1	5	1
80	50	20	10	5

\downarrow

J_2		J_3	J_0	J_4
	0	1	2	3
				4
				5

$$80 + 50 + 20 + 10$$

Huffman Coding

- used for lossless compression
- variable length coding

Example problem → "abaabaca...."

100 chars

Frequency	
a → 70	00
b → 20	01
c → 10	10

fixed length encoding → $2 \times 100 = 200$

variable length encoding

different characters might have different length codes

greedy idea

most frequent char has smallest code

prefix Requirement for decompression

no code should be prefix of any other

a → 70	0	1	0	1
b → 20	10	01	0	0
c → 10	11	00	01	11

prefix free not prefix free

111
↓
aaa
or
caa
or
aac

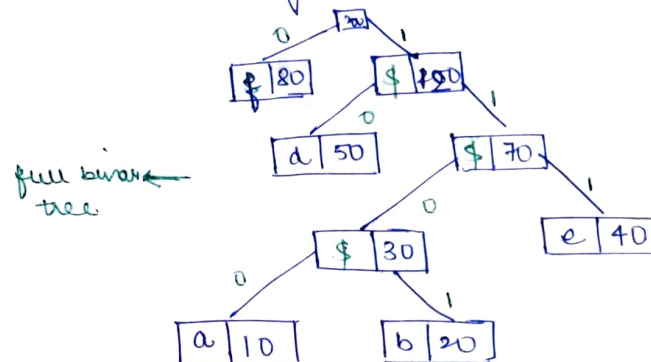
space → $70 \times 1 + 20 \times 2 + 10 \times 2$

= 130 < 200

Huffman algorithm

I/P → ['a', 'd', 'b', 'e', 'f']
[10, 50, 20, 40, 80]

1) Build a binary tree



full binary tree

- Every input character is a leaf
- Every left child edge is labelled as 0 and right is labelled as 1
- Every root to leaf path represent Huffman code

a → 0 0 1 1 d → 0 1
b → 1 0 1 1 f → 0
e → 1 1 1

Implementation

- ① we will use min heap data structure here
- ② create the leaf nodes for every element of an array and put them in min heap
- ③ now ex while (h.size() > 1)

* left = h.extractMin();

* right = h.extractMin();

* create a new node with

→ character 'f'

→ frequency as left.freq + right.freq

→ left and right children as left & right

* insert new node into h

* The only node left in h is required binary tree

```

void printCodes (root, str="") {
    if (root == null)
        return;
    if (root->data != '$')
        print (root->ch + " " + str);
    return;
    printCodes (root->left, str+"0");
    printCodes (root->right, str+"1");
}

```

Implementation

```

struct Node {
    int freq;
    char ch;
    Node *left, *right;
    Node (int f, char c, Node *L = NULL,
          Node *R = NULL)
    {
        freq = f;
        ch = c;
        left = L;
        right = R;
    }
};

```

```

void createTree (int arr[], int freq[], int n) {
    priority_queue < Node*, vector < Node* >,
                  compare > > h;

    for (int i = 0; i < n; i++)
        h.push (new Node (freq[i], arr[i]));

    while (h.size() > 1) {
        Node *l = h.top(); h.pop();
        Node *r = h.top(); h.pop();
    }
}

```

```

Node *node = new Node ('$', l->freq + r->freq, l, r);
h.push (node);
}
printCodes (h.top(), "");
}

```

void printCodes() ← already written

```

struct compare {
    bool operator () (Node *l, Node *r) {
        return l->freq < r->freq;
    }
}

```

time comp → $O(N \log N) +$
 $O(N \log N) +$
 $O(\log N)$.

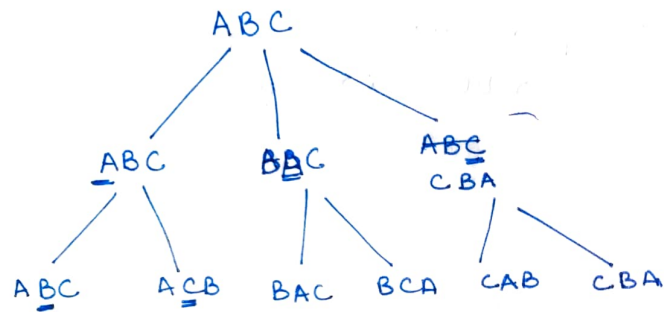
Backtracking

Given a string, print all the permutation which do not contain "AB" as a substring

string \rightarrow "ABC"

Naive solution

generate all permutations and before printing we check if "ab" is a substring



Fix a character and swap the next one with the remaining of string.

void permute (string str, int l , int r) {

if ($l == r$) {

print(str);
return;

}

else {

for (int $i = l$; $i \leq r$; $i++$) {

swap(str[l], str[i]);

permute(str, $l+1$, r);

swap(str[l], str[i]);

}

}

We can not call recursion after the currently generated string contains 'AB' not in it.
It can reduce a lot of recursive calls.
Hence before calling swap & permute we can just include bool is-safe

```
bool is-safe (string str, int l, int i, int r) {  
    if ( $l == 0$  && str[l-1] == 'A' && str[i] == 'B')  
        return false;  
    if ( $r = l+1$  && str[i] == 'A' && str[i] == 'B')  
        return false;
```

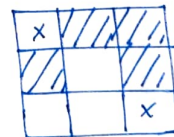
return true;

}

\Rightarrow Cut down
Recursive
calls

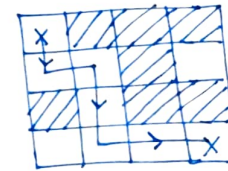
Rat in a Maze

{ {1, 0, 0, 0},
I/P \rightarrow {0, 1, 0, 0},
{1, 1, 1, 1}}

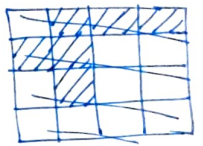


(NO)

{ {1, 0, 0, 0},
I/P \rightarrow {1, 1, 0, 1},
{0, 1, 0, 0},
{1, 1, 1, 1}}



(Yes)



only two
moves req.
 $\rightarrow (i+1)(j)$
 $\rightarrow (i)(j+1)$

right &
left
down

⇒ boolean solveMaze() {

if (solveMazeRec(0,0) == false)

return false;

else {

print(sol);

return true;

}

⇒ bool isSafe(int i, int j) {

return (i < n && j < n && m[i][j] == 1)

}

⇒ boolean solveMazeRec(int i, int j) {

if (i == N-1 && j == N-1) { sol[i][j] = 1;
return true; }

if (isSafe(i,j) == true) {

sol[i][j] = 1;

if (solveMazeRec(i+1, j) == true)

return true;

if (solveMazeRec(i, j+1) == true)

return true;

sol[i][j] = 0;

}

return false

}

N Queen Problem

- placing n queens so that no two can attack each other.
- A queen can attack horizontally, vertically and diagonally



N = 4

O/P → Yes

		1	
1			
			1
	1		

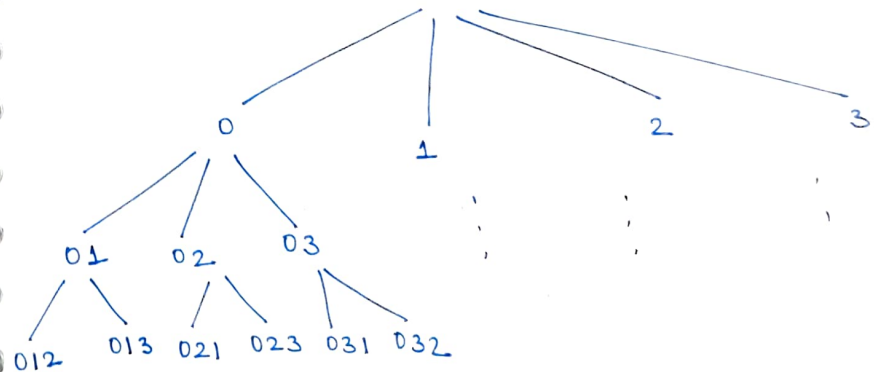
N = 3 O/P → NO

Super Naive Solution

generate all possible permutations i.e. $n^2 C_n$.

Naive Solution

* placing queens in different columns. hence generate permutations of in their columns itself.



```

⇒ bool solve() {
    if (solveRec(0) == false)
        return false;
    else {
        printMatrix(board);
        return true;
    }
}

```

```

⇒ bool solveRec(int col) {
    if (col == N) return true;
    for (int i = 0; i < N; i++) {
        if (isSafe(i, col)) {
            board[i][col] = 1;
            if (solveRec(col + 1))
                return true;
            board[i][col] = 0;
        }
    }
    return false;
}

```

```

⇒ bool isSafe(int row, int col) {
    // horizontal check
    for (i = 0; i < N; i++)
        if (board[row][i]) return false;
}

```

```

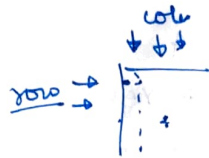
    // upper diagonal
    for (i = row, j = col;
         i >= 0 && j >= 0; i--, j--)
        if (board[i][j]) return false;
}

```

```

    // lower diagonal
    for (i = row, j = col; j >= 0 && i < N; i++, j--)
        if (board[i][j]) return false;
    return true;
}

```



SUDUKU PROBLEMS

The size of sudoku
is always a square
4x4, 9x9, 16x16

Numbers we are gonna
fill is always from
1 → N

- Every number in every row is different
- Every number in every column is different.
- Every number in size subgrid 2x2 must also be different.

```

bool isSafe(int i, int j, int n) {
    for (int k = 0; k < n; k++) {
        if (grid[k][i] == n || grid[i][k] == n)
            return false;
    }
}

```

```

int s = sqrt(n);
int rs = i - i / s;
int cs = j - j / s;

for (int i = 0; i < s; i++) {
    for (int j = 0; j < s; j++) {
        if (grid[i + rs][j + cs] == n)
            return false;
    }
}
}

```

```

bool solve() {
    int i, j;
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            if (grid[i][j] == 0) break;
}

```



```
if (i == n && j == n)
    return true;
```

```
for (int num = 1; num ≤ n; num++) {
    if (isSafe(i, j, num)) {
        grid[i][j] = num;
        if (solve()) return true;
        grid[i][j] = 0;
    }
}
return false;
```

}