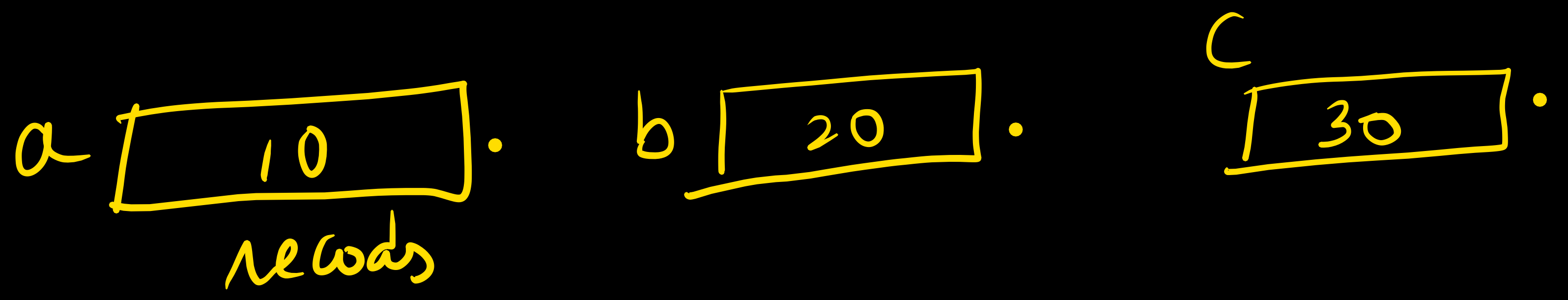
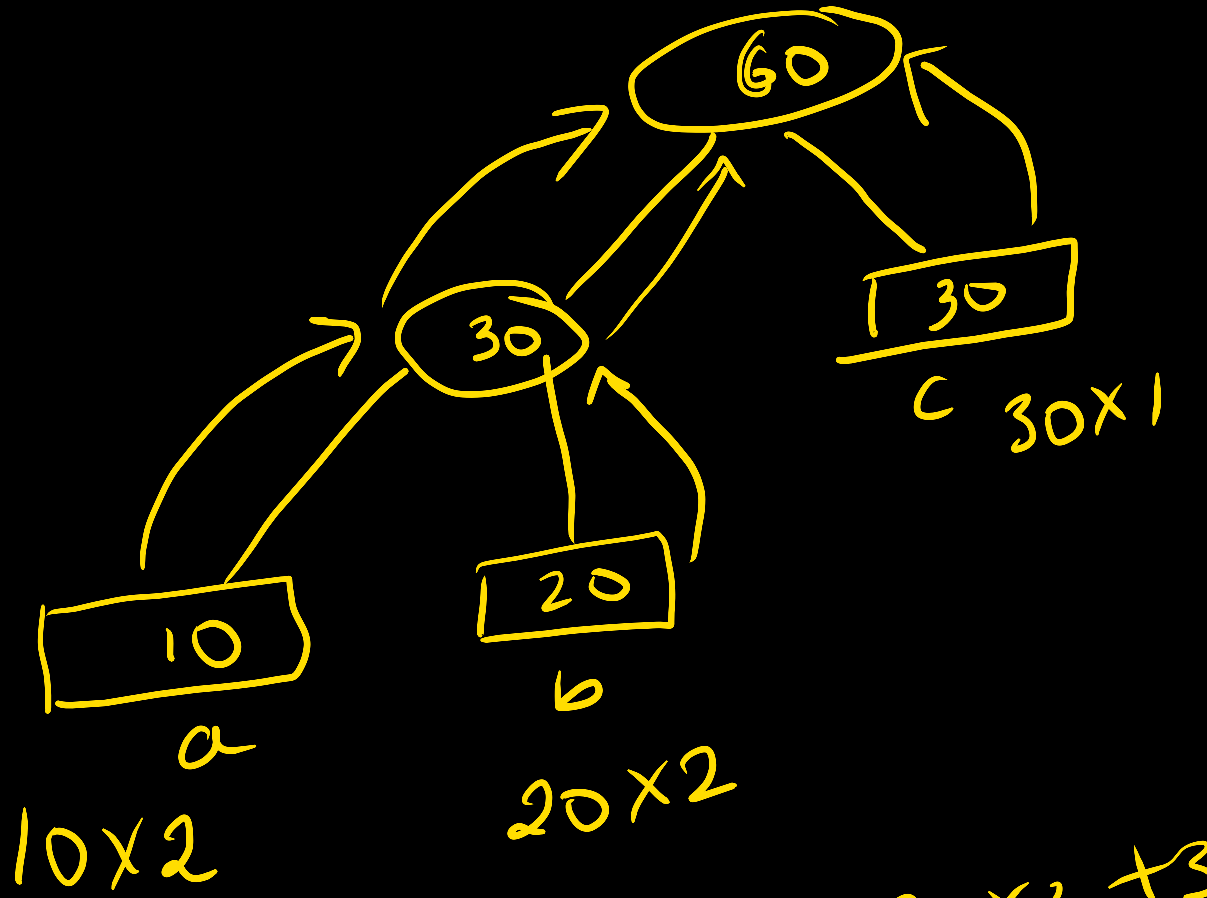


huffman coding.

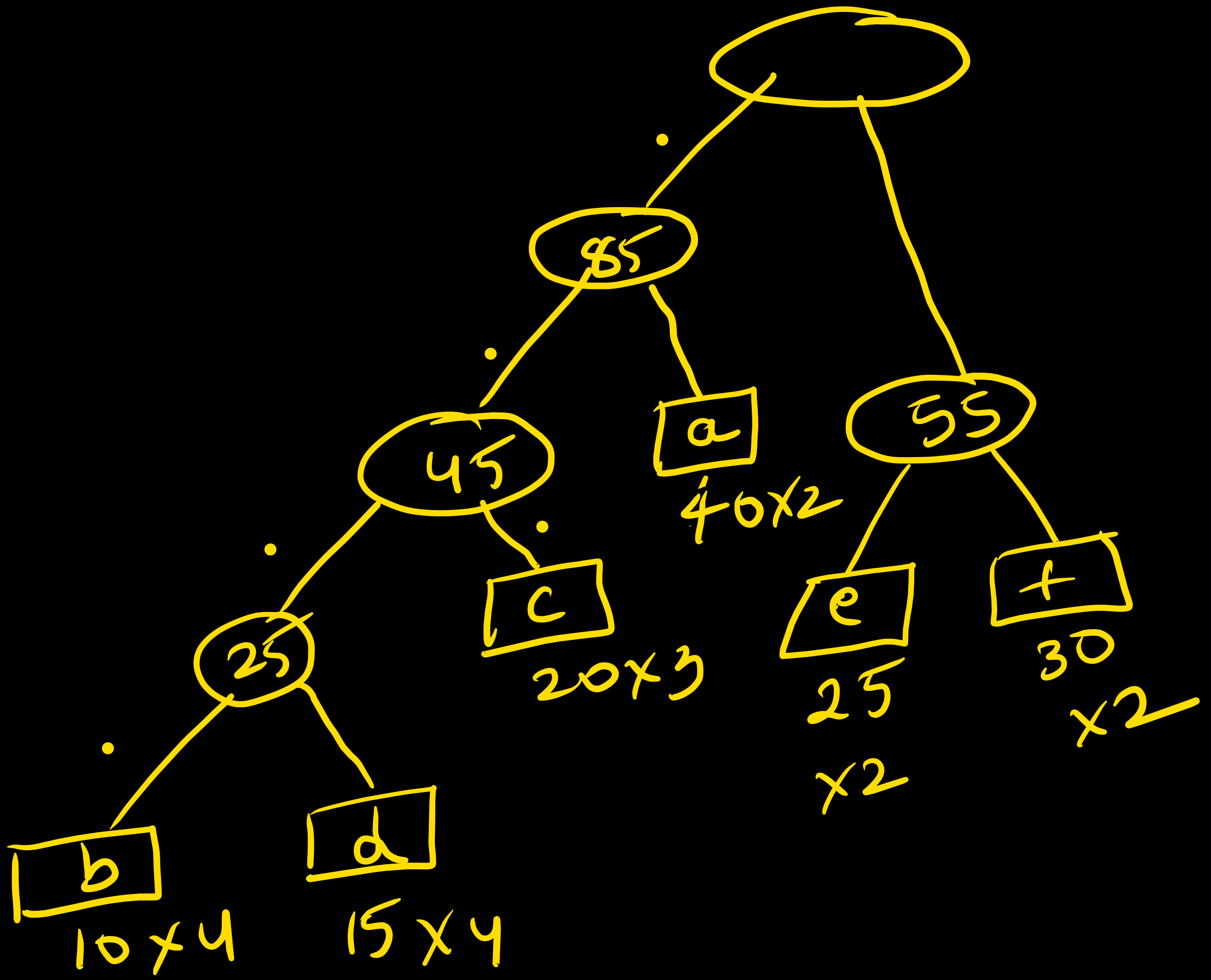
weighted external path length.



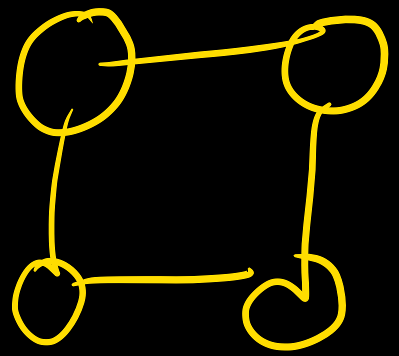
$10 \times 2 + 20 \times 2 + 30 \times 1$

Gate:

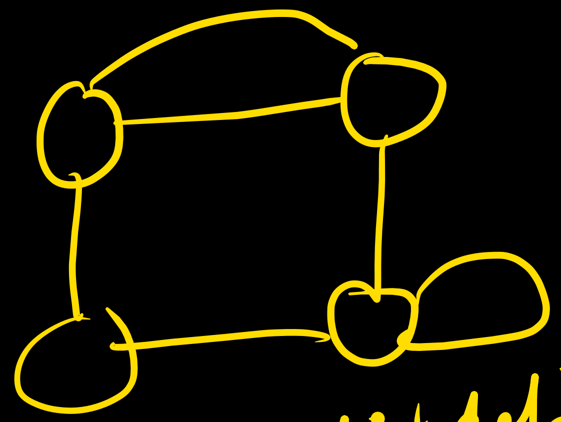
a	b	c	d	e	f				
40	10	20	15	25	30	25	45	55	85



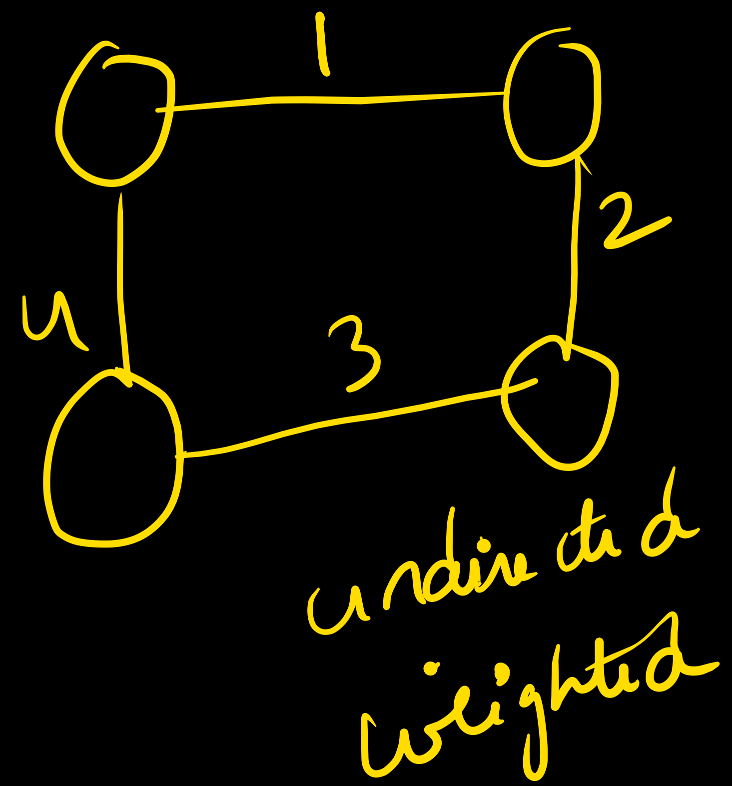
Graphs:

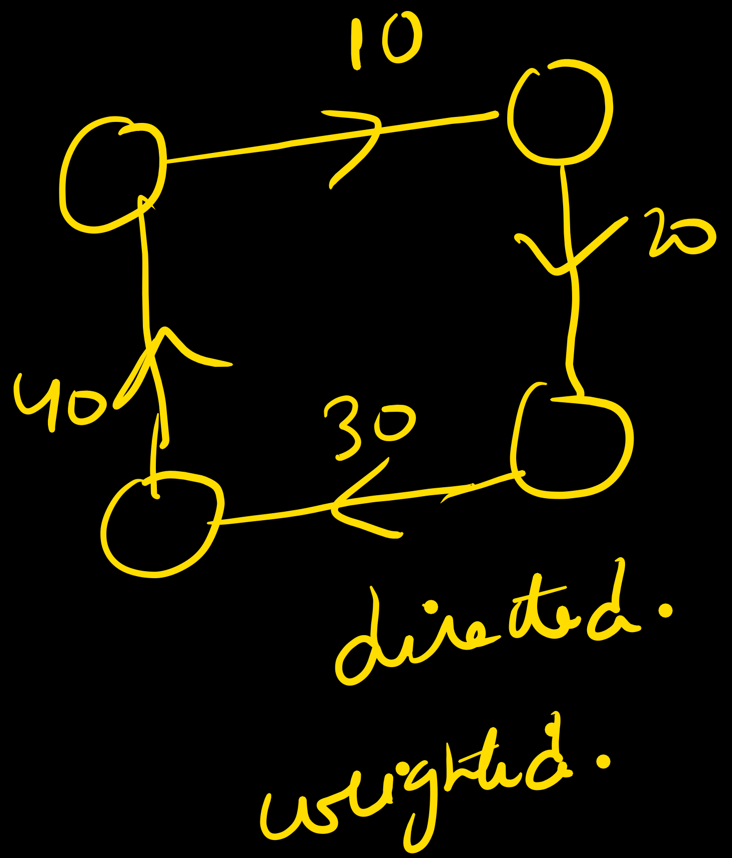


simple



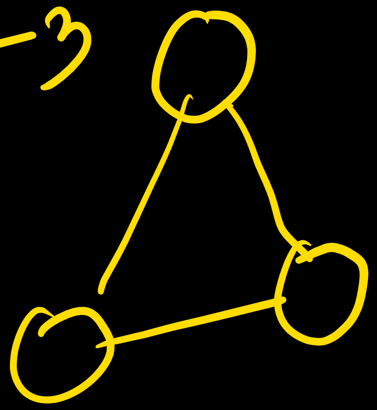
multigraphs

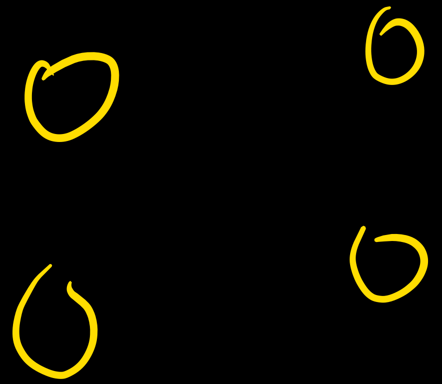

1
2
3
4
undirected
weighted


10
20
30
40
directed.
weighted.

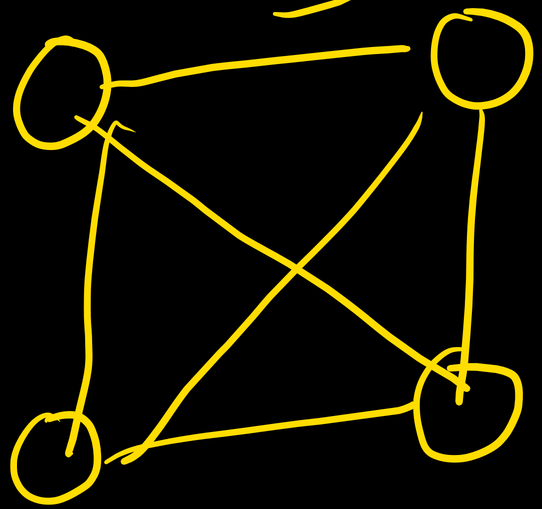
maximum no of edges

K_3





K_4



complete graph.

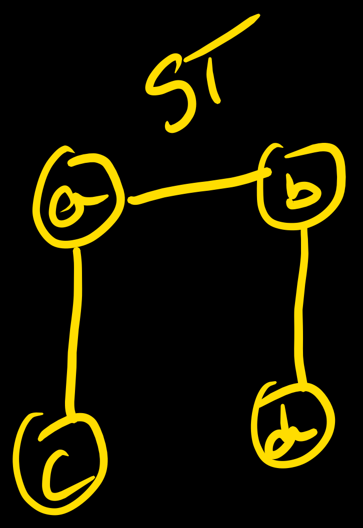
n nodes.

$^nC_2 = \frac{n(n-1)}{2} = O(n^2)$

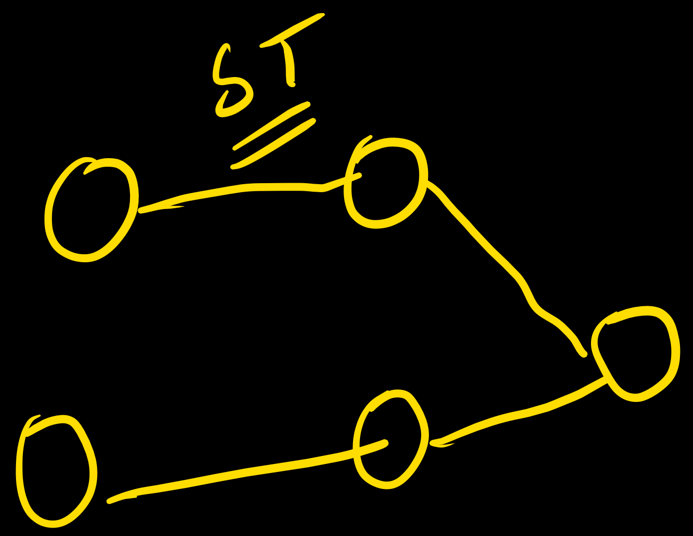
4 - 6.

& $V = O(\cancel{\log}\sqrt{E})$

$E = O(V^2)$



How many edges are required to connect them



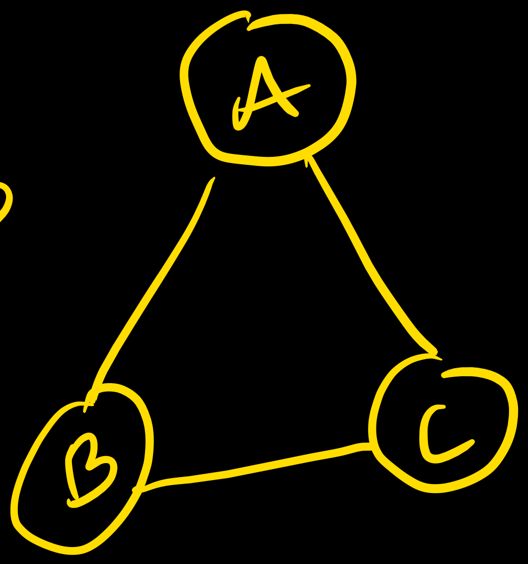
$n \rightarrow$ nodes

$(n-1)$ minimum to connect it.

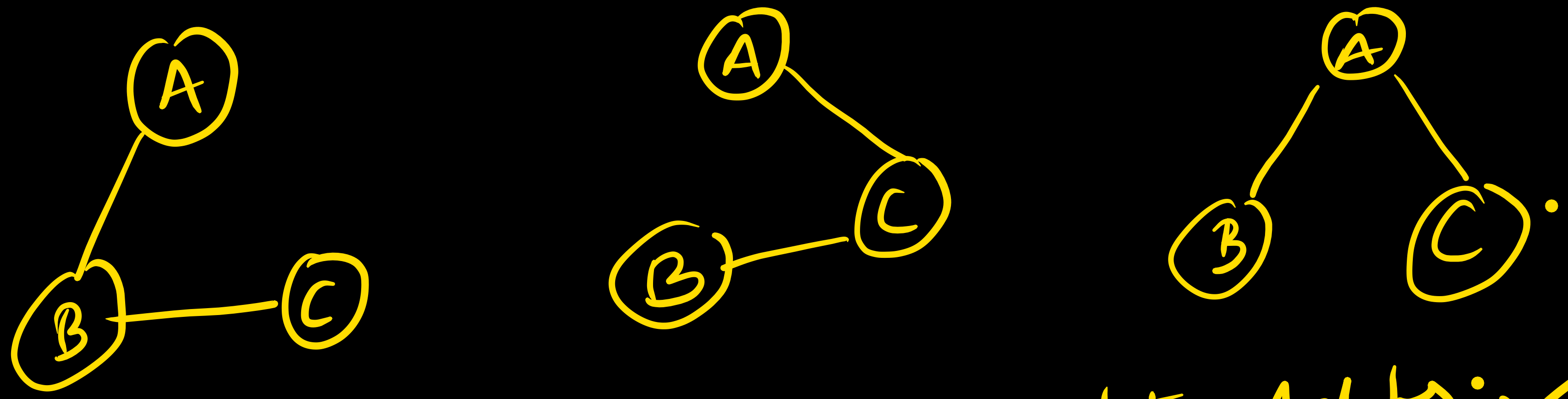
If we connect a 'n' vertex graph with 'n-1' edges

the 'it' is called Spanning tree.

K_3 $=$



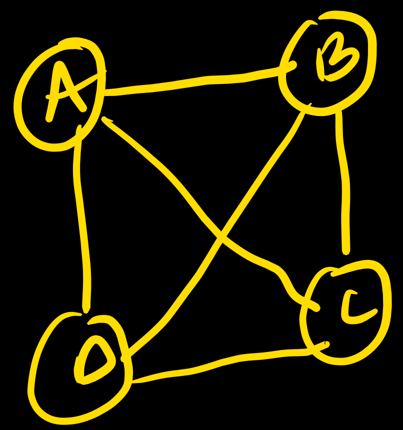
$\Rightarrow$



n^{n-2} $\rightarrow$ complete graphs. ✓

ST

K_4 $=$



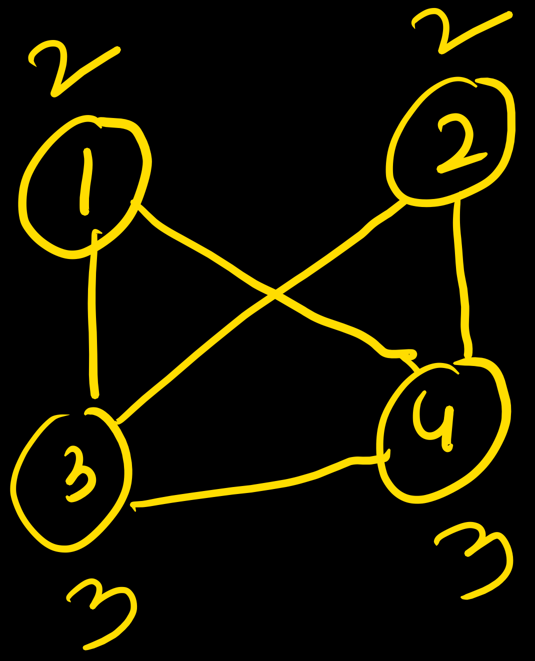
$\Rightarrow$

K_n

K_4 $\quad 4^{4-2} = 16$

K_5 $\quad 5^{5-2} = 125.$

Kirchoff theorem:



Procedure:

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 1 & 1 & 1 & 0 \end{array}$$

(i) Diagonal 0's → degree

(ii) Non diagonal 1's → -1

(iii) Non diagonal 0's → 0's

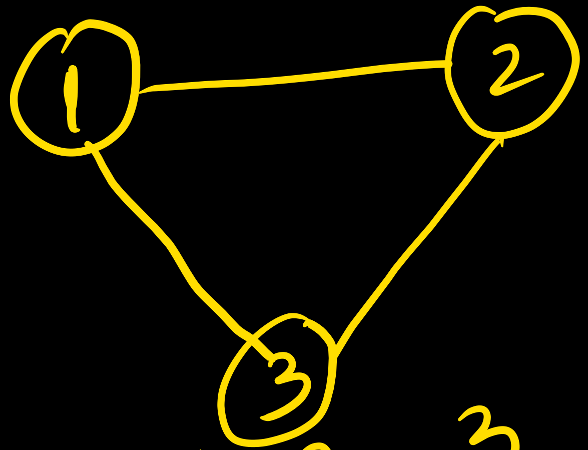
linear algebra

No of ST = Cofactor of any element.

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 2 & 0 & -1 & -1 \\ 2 & 0 & 2 & -1 & -1 \\ 3 & -1 & -1 & 3 & \\ 4 & -1 & -1 & -1 & 3 \end{array}$$

$2(9-1)-(-1)(-3-1)+(-1)(1+3)$

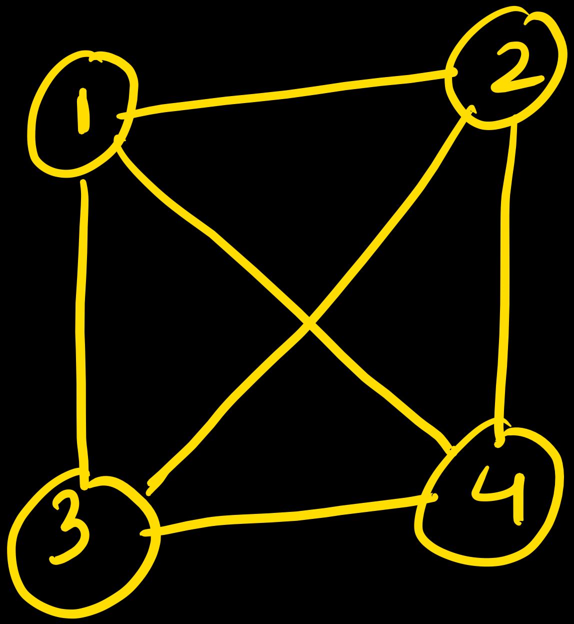
$= 8$ ST



$$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} \cancel{2} & \cancel{-1} & \cancel{-1} \\ \cancel{-1} & 2 & -1 \\ \cancel{-1} & -1 & 2 \end{bmatrix}$$

$4 - 1 = 3$ ✓



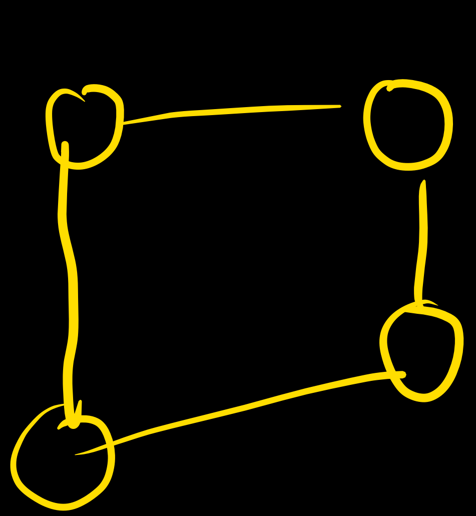
4^{4-2}

$$3(9-1) - (-1)(-3-1) + (-1)(1+3)$$

$$= 16$$

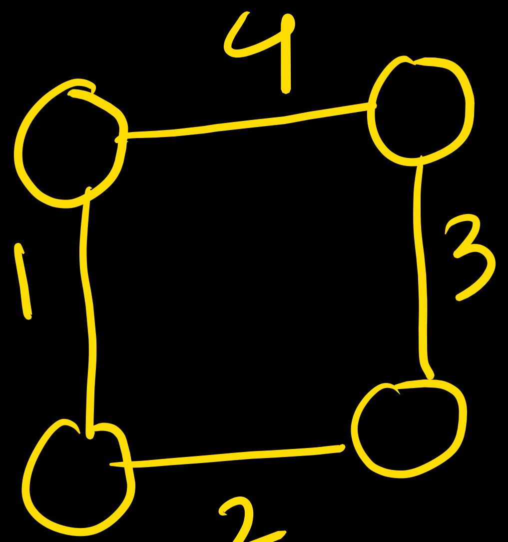
$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 1 & 1 & 1 & 0 \end{array}$$

$$\begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$$

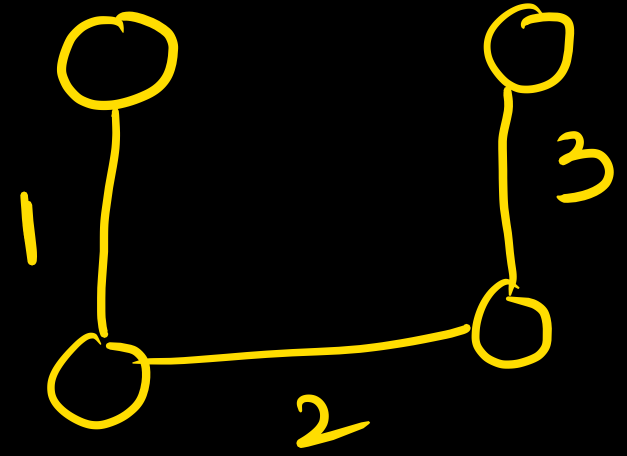


⇒ Kirchoff theorem → STs

min cost spanning tree.



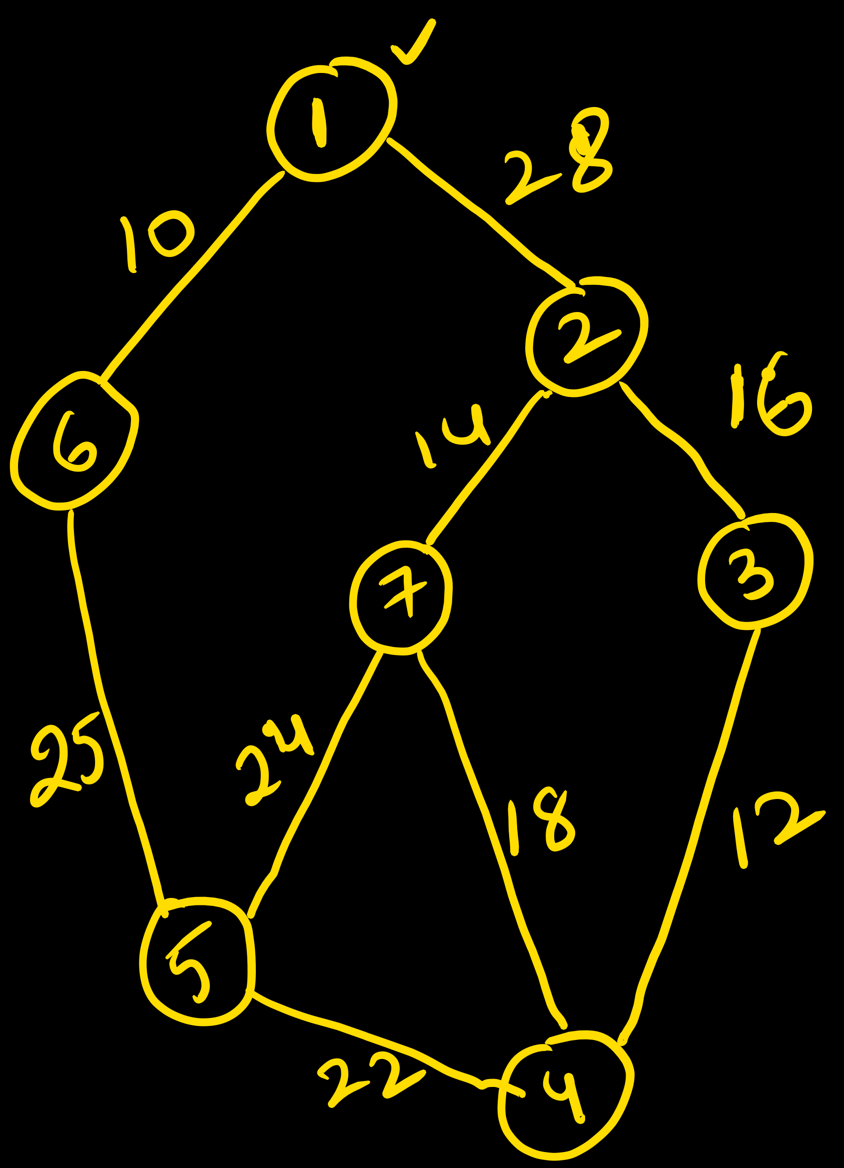
weighted



ST → minimum cost.

→ Prims ✓
and Kruskals } greedy.

Prims: Gate



14
2 - 7

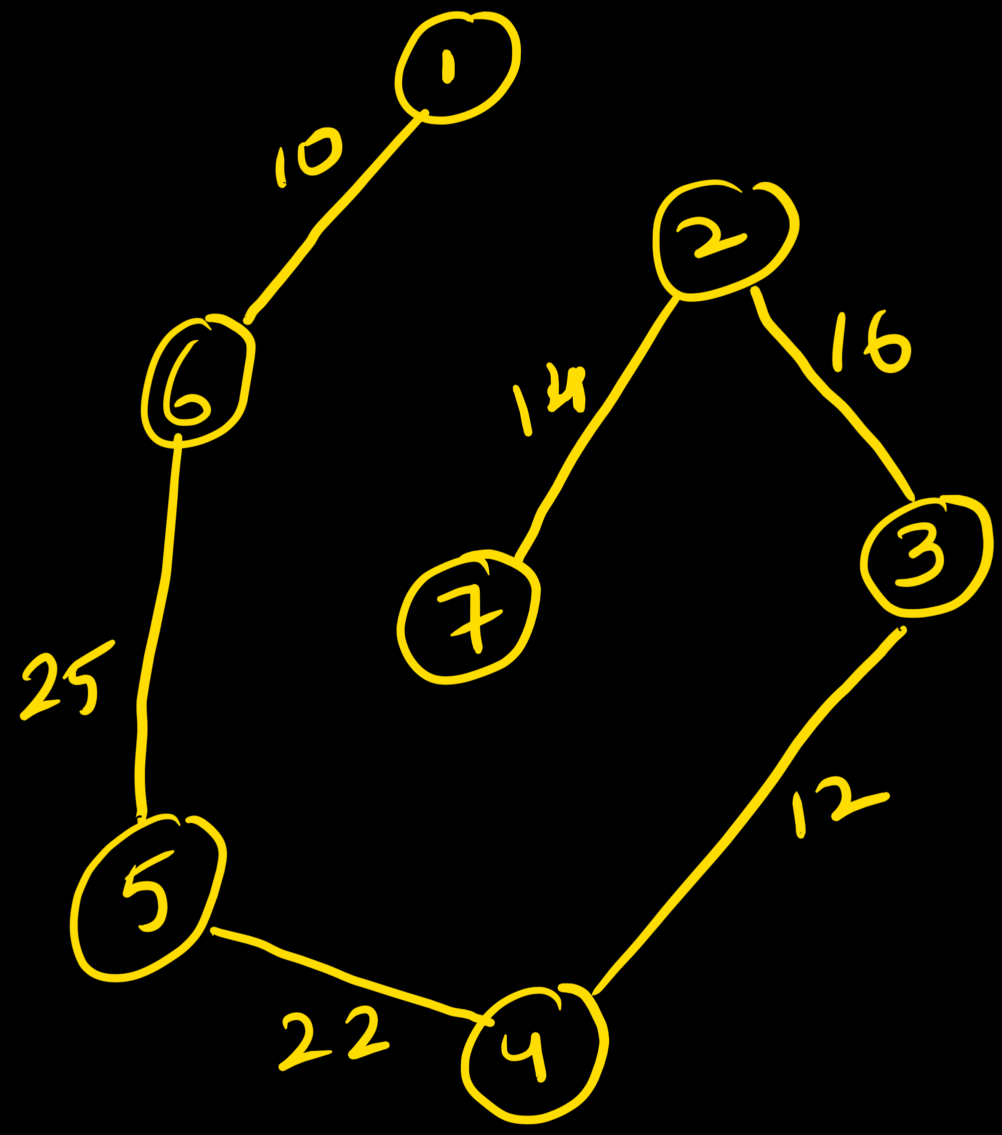
16
3 - 2

18 ~~24~~ 12
4 - 7 3

24 22
5 - 7 4

10 28.
1 - 6 2

25
6 - 5

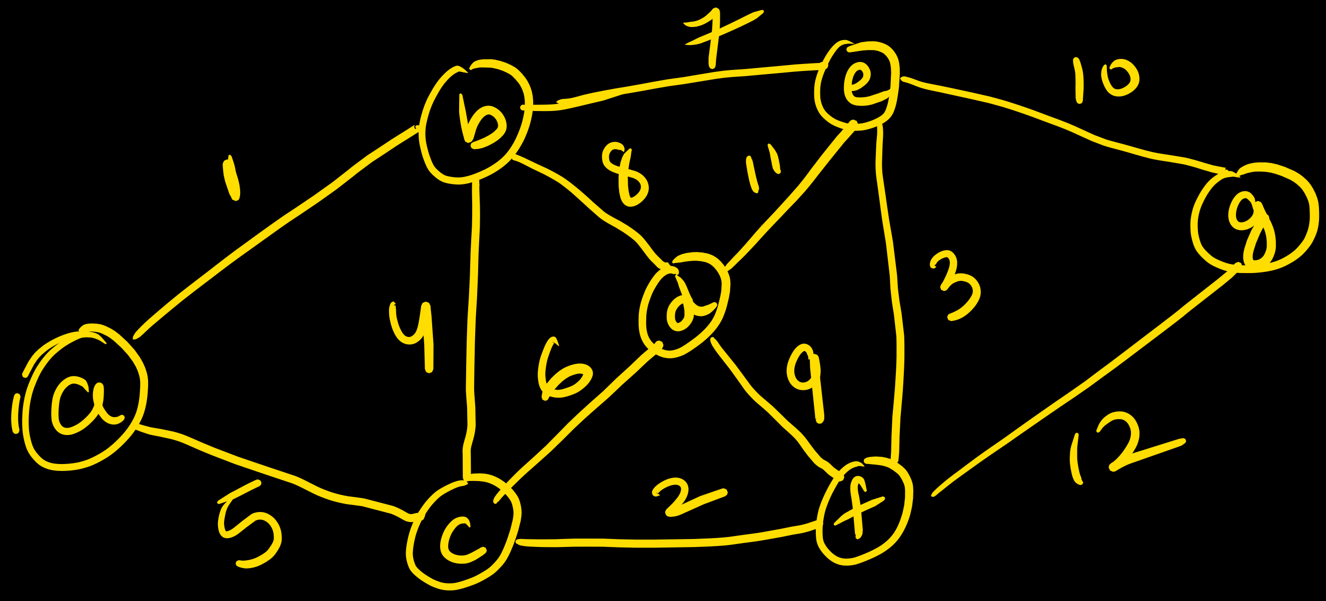


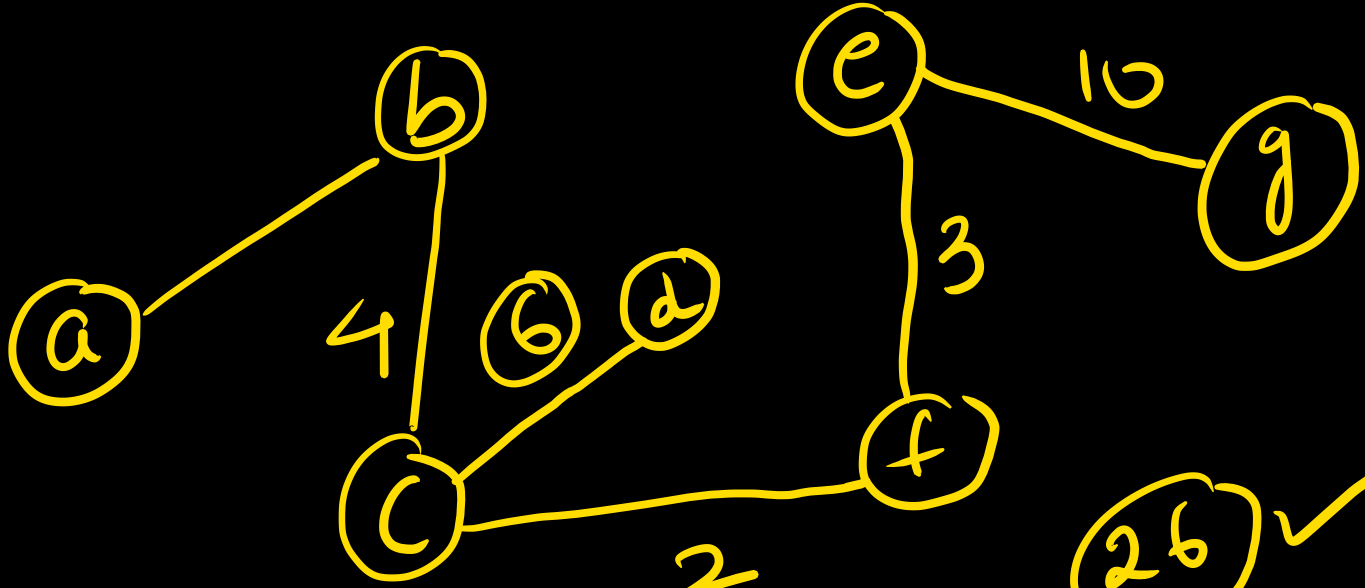
MCST

→ Prims.

99 ✓

Gate:





26 ✓

~~a – b c~~

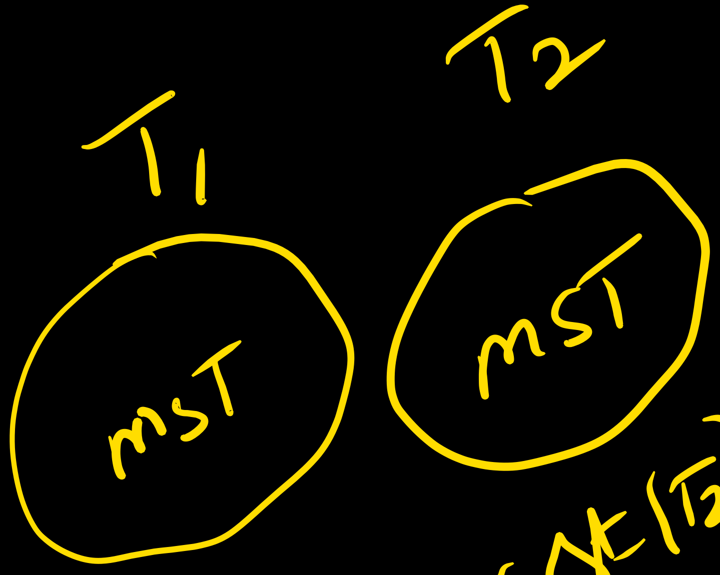
b – e d c

c – d f

f – d e g

e – g d

d – –

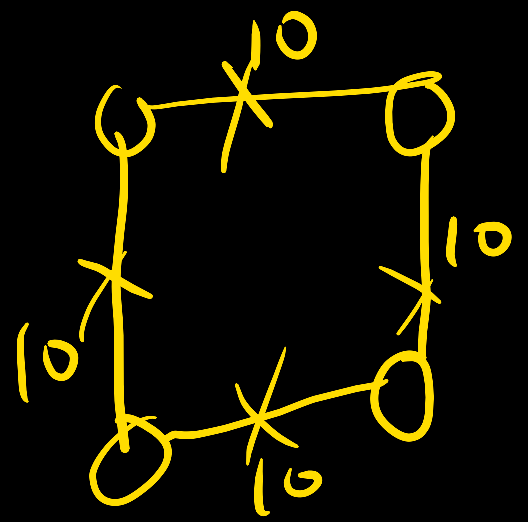


Is MST alway unique:

if the edge weights are unique, the MST is unique

if the edge weights are repeated then MST may not be unique

cost(T1) = cost(T2)



many MST, Sometimes when edge weights are repeating.

But cost of MST is unique

Algo till prims:

no standard algo.

$O(E \log V)$

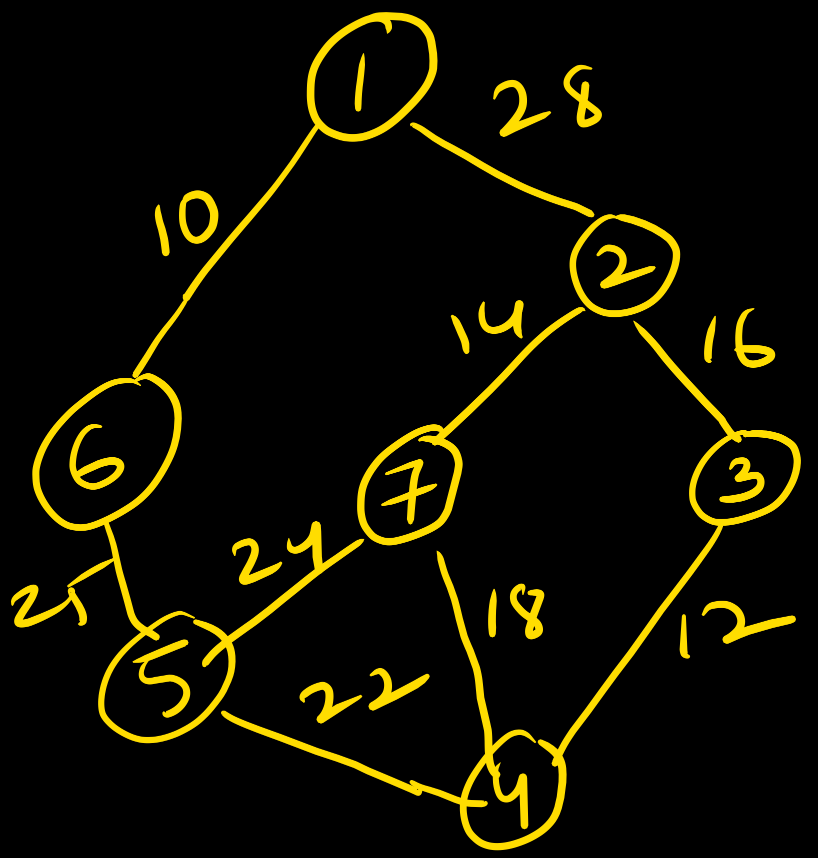
$O(V^2)$

Best implementation → fibonacci heaps.

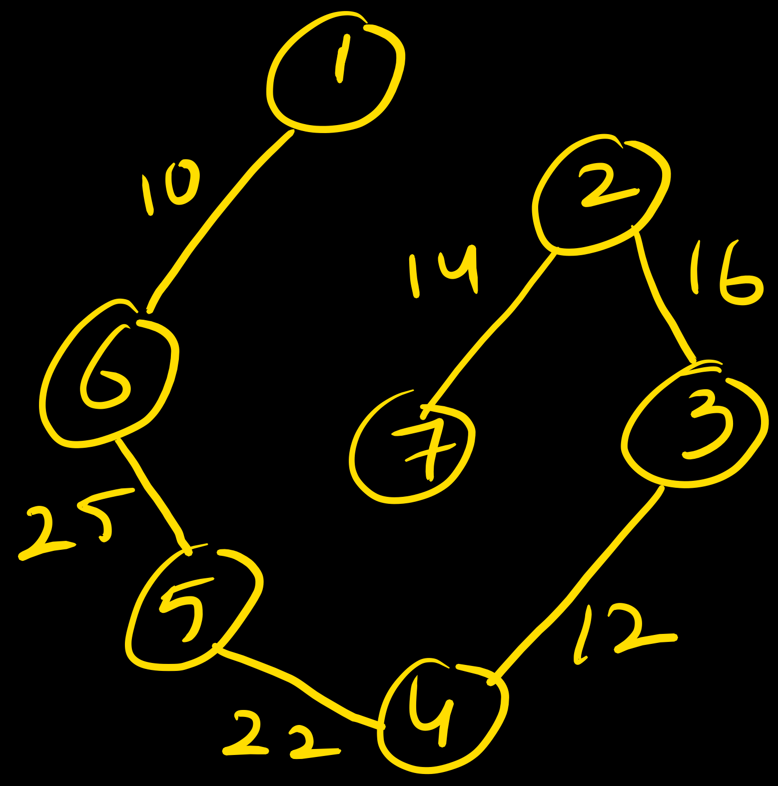
not in syllabus

min wt → Cycle

Kruskal's:-



MC → Same



MC = 0.

if all the edge wt are unique → Both prims, Krusk → Same MST

Algo:

Kruskal → disjoint set Data structure

↓

Not in syllabus.

Gate:

Let G be an undirected connected graph with distinct edge weights.
Let max_e be the edge with maximum weight and min_e be the edge with min weight. Which of the following is false

a) T Every minimum spanning tree of 'G' must contain min_e

only one min edge

b) T If max_e is in a minimum ST, then its removal must disconnect 'G'

F c) NO MST contains max_e

d) T 'G' has a unique MST

Gate:

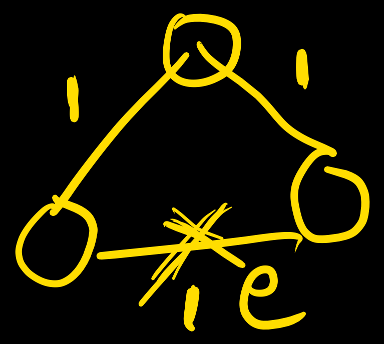
Let $\underline{w}$ be the minimum weight among all the weights in an undirected connected graph. Let 'e' be a specific edge of weight 'w'. Which of the following is False?

(T) a) There is a minimum spanning tree containing 'e'

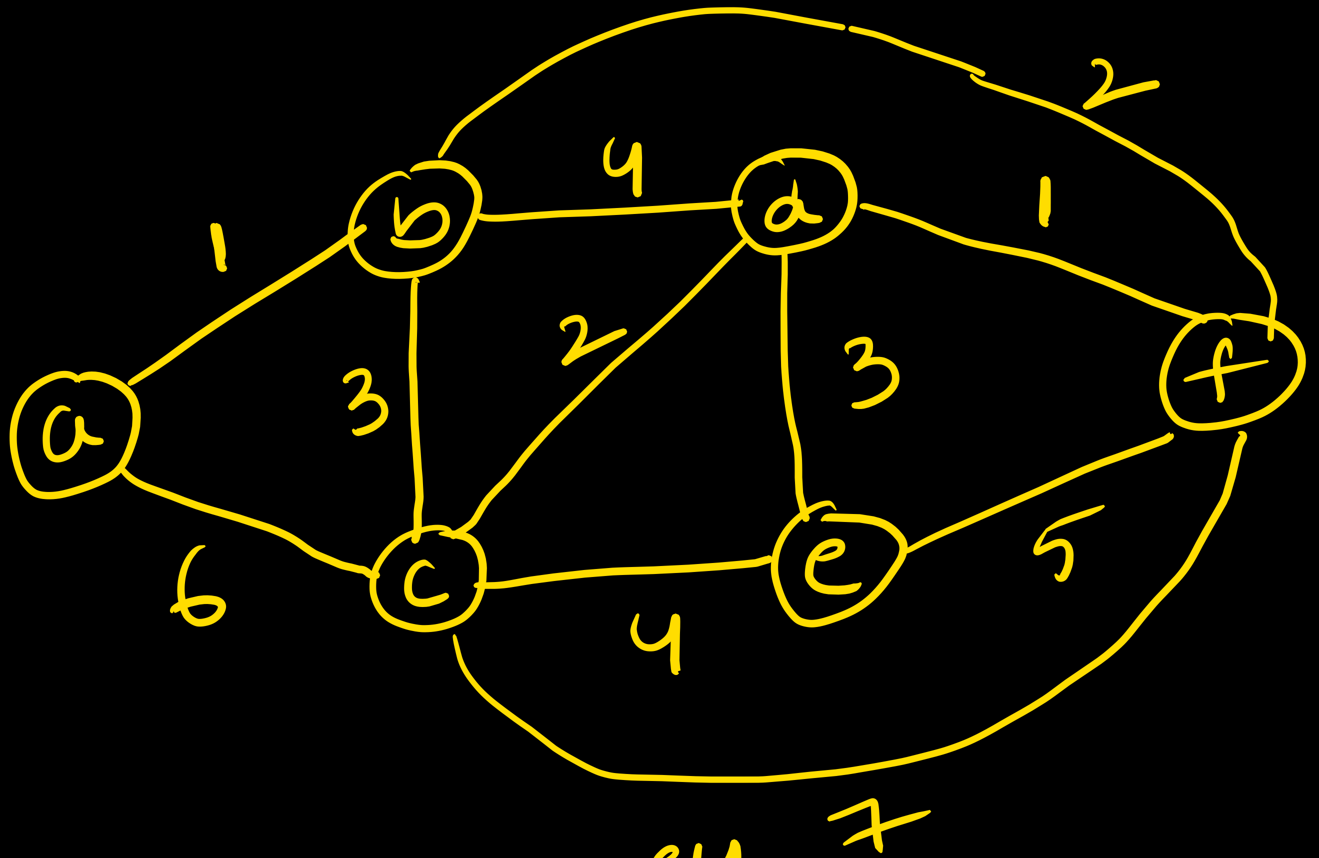
(T) b) If e is not present in MST 'T', then in a cycle formed by adding e to 'T', all edges have same weight

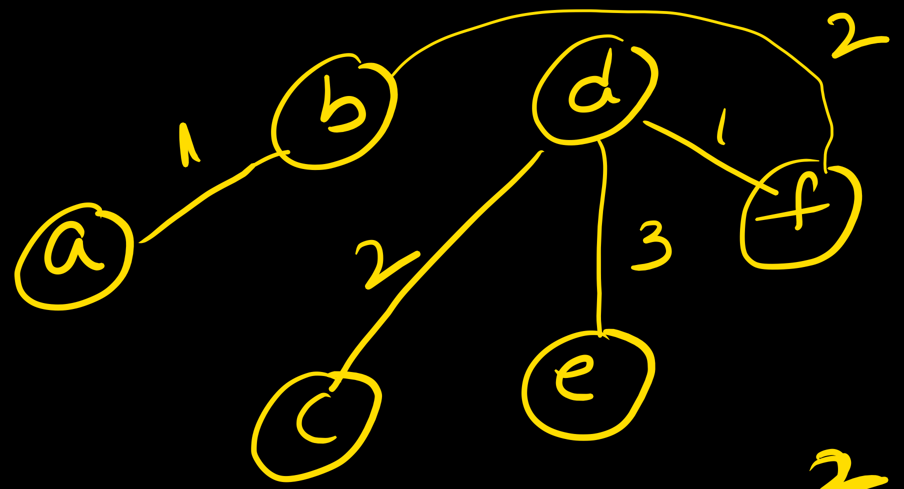
(T) c) Every MST has an edge of weight 'w' ✓ → K → min edge.

(F) d) 'e' is present in every MST



Gate:





Kruskal → not possible

✓ a) (a-b) (d-f) (b-f) (d-c) (d-e)
  1 1 2 2 3

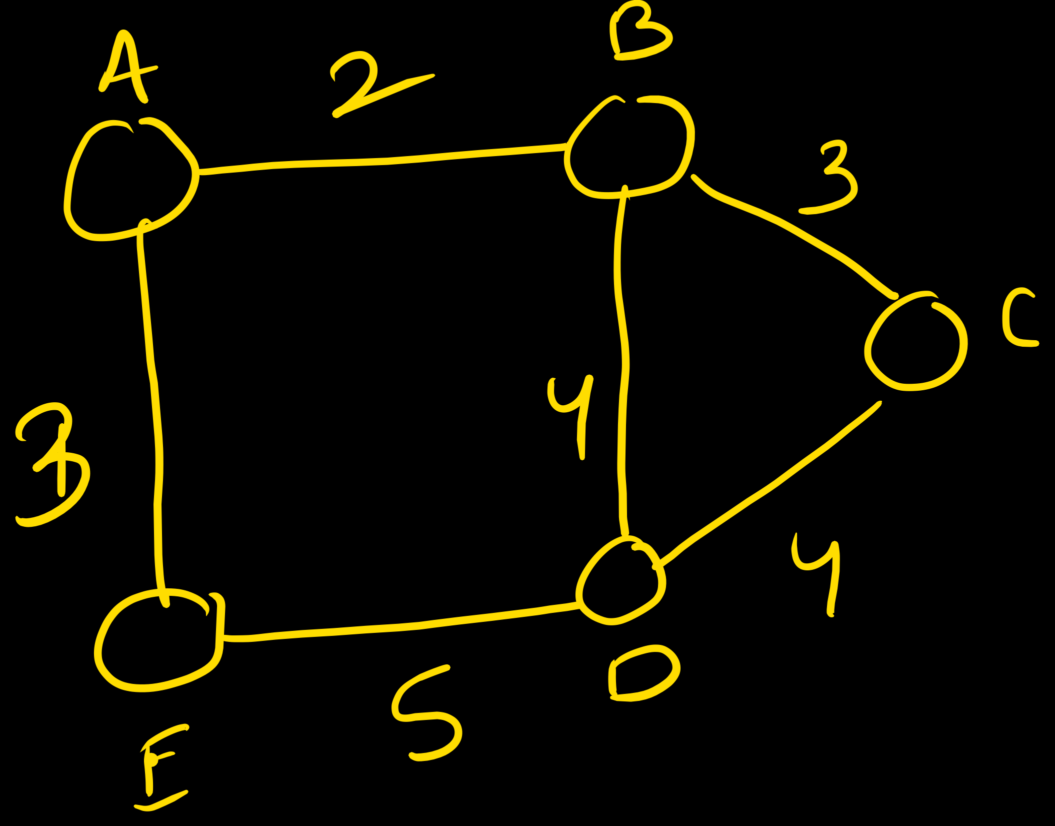
✓ b) (a-b) (d-f) (d-c) (b-f) (d-e)

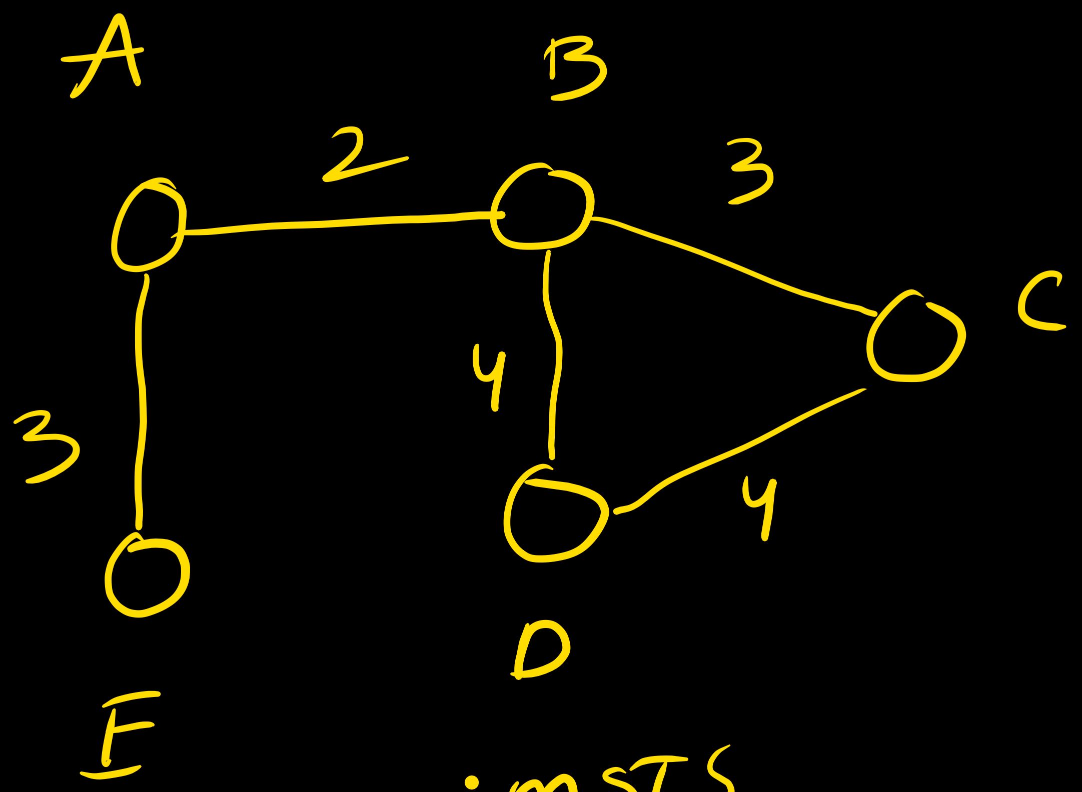
c) (d-f) (a-b) (d-c) (b-f) (d-e)

d) (d-f) (a-b) (b-f)
  1 1 2
(d-e)
3 .

Gate:

How many MSTs ?



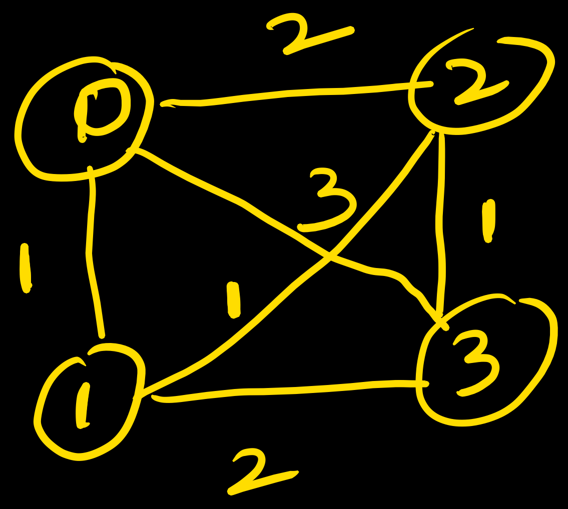


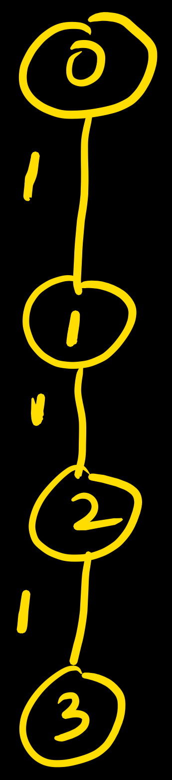
2 MSTs

Gate:

A Complete, undirected weighted graph 'G' is given on the vertices $\{0, 1, 2, \ldots\ldots n-1\}$ for any fixed 'n'. Draw MST of G if

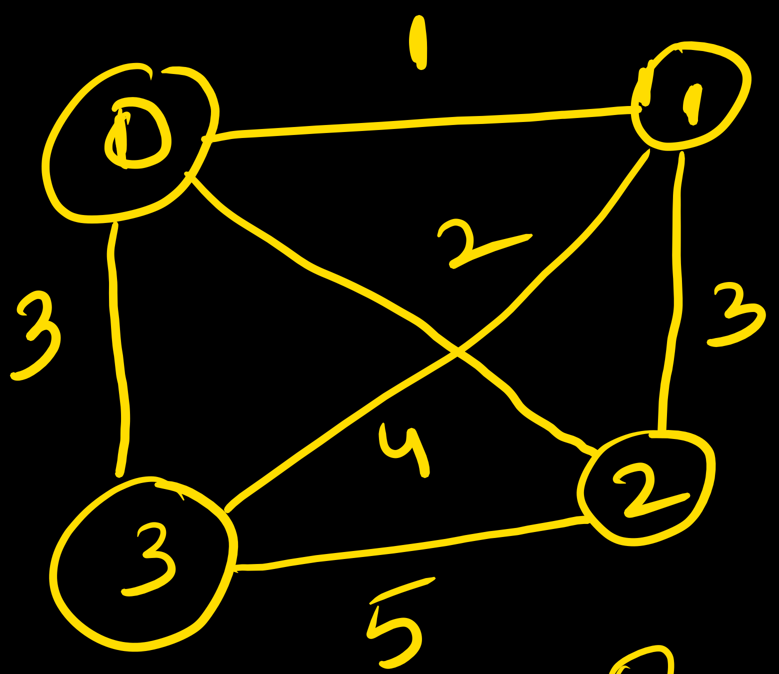
a) The weight of the edge is ~~(u-v)~~ (u,v) is $|u-v|$

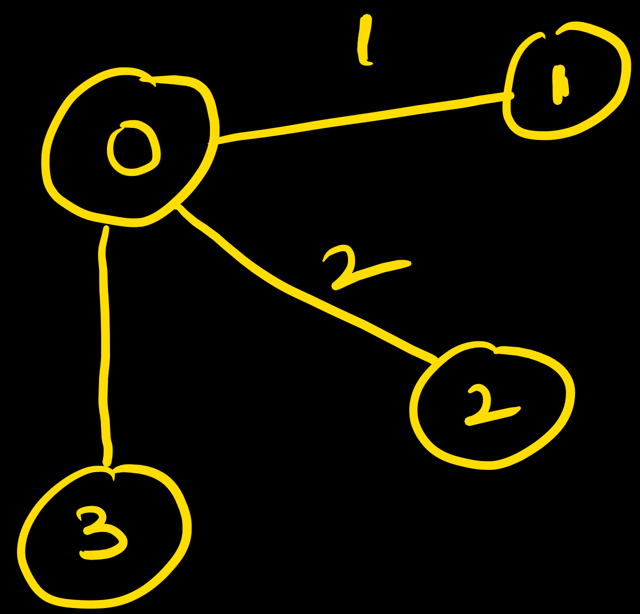


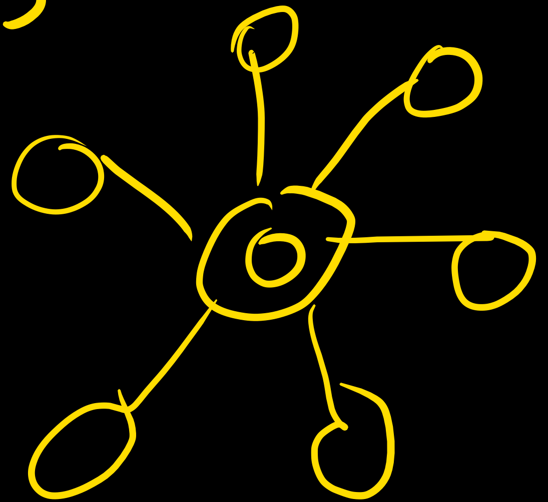


line graph

b) The weight of edge (u,v) is $(u+v)$







Star :