

$$T(n) = \begin{cases} 1 & ; n=1 \\ 8T(n/2) + n^2 & \text{if } n>1 \end{cases}$$

Do it in master theorem method in exam.

Here we do substitution method for practice

$$T(n) = 8 T(n/2) + n^2 \rightarrow ①$$

$$T(n/2) = 8 T(n/4) + \left(\frac{n}{2}\right)^2 \rightarrow ②$$

$$T(n/4) = 8 T(n/8) + \left(\frac{n}{4}\right)^2 \rightarrow ③$$

Subs ② in ①

$$\begin{aligned} T(n) &= 8 \left(8 T(n/4) + \left(\frac{n}{2}\right)^2 \right) + n^2 \\ &= 8^2 T(n/2) + 2n^2 + n^2 \rightarrow ④ \end{aligned}$$

Subs ③ in ④

$$\begin{aligned}
 & 8^2 T\left(\frac{n}{2}\right) + 2n^2 + n^2 \\
 = & 8^2 \left(8 T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^2 \right) + 2n^2 + n^2 \\
 = & 8^3 T\left(\frac{n}{2^3}\right) + 2^2 n^2 + 2n^2 + n^2 \\
 = & 8^K T\left(\frac{n}{2^K}\right) + 2^{K-1} n^2 + 2^{K-2} n^2 + \dots + 2^2 n^2 + 2n^2 + n^2 \\
 = & 8^K T\left(\frac{n}{2^K}\right) + n^2 \left(2^{K-1} + 2^{K-2} + \dots + 2^2 + 2^1 + 2^0 \right)
 \end{aligned}$$

$$8^k T\left(\frac{n}{2^k}\right) + n^2 \left(2^{k-1} + 2^{k-2} + \dots + 2^2 + 2^1 + 2^0\right)$$

Let $\frac{n}{2^k} = 1 \Rightarrow n = 2^k \Rightarrow k = \log n$, $T\left(\frac{n}{2^k}\right) = 1 + \frac{n}{2^k} = 1$

$$8^{\log_2 n} + n^2 \left(2^0 + 2^1 + 2^2 + \dots + 2^{\log n - 1}\right)$$

$GP = a \left(\frac{r^{n-1}}{r-1}\right) \quad r > 1$

$$n^{\log_2 8} + n^2 \left(\frac{2^{\log_2 n} - 1}{2 - 1}\right)$$

$$n^3 + n^2 \left(\frac{n-1}{2-1}\right) = n^3 + n^3 = O(n^3)$$

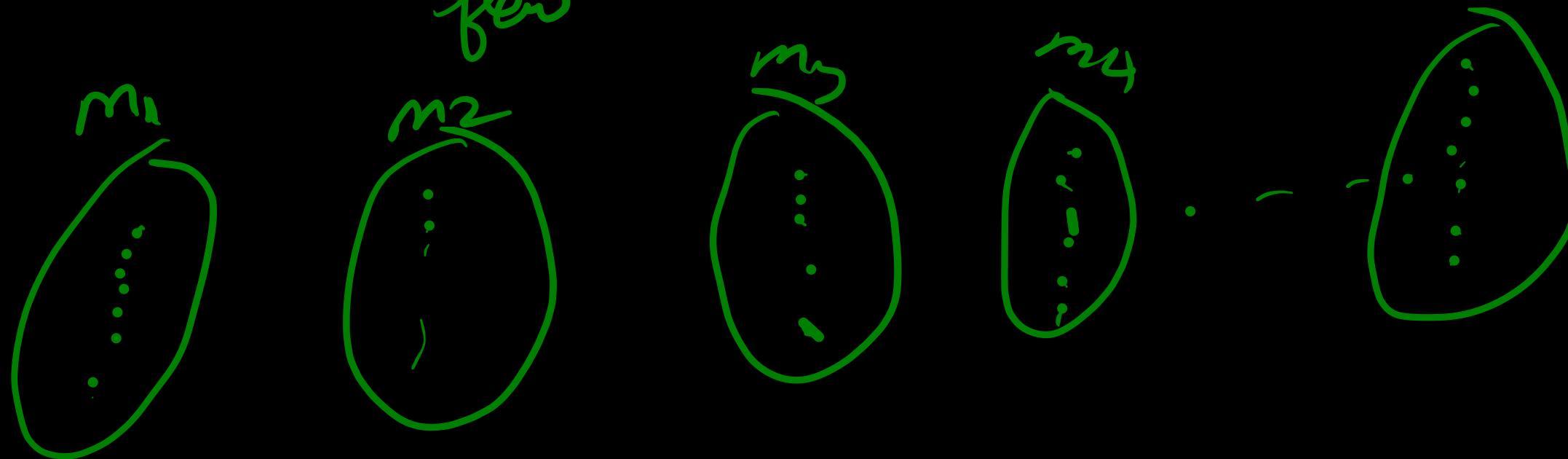
$$= \Theta(n^3)$$

→ Internet → models

Text books → model

AFS ① question.

few exercise quest.



$$T(n) = \begin{cases} 2 & \text{if } n=2 \\ 7T(n/2) + n^2, & n>2 \end{cases}$$

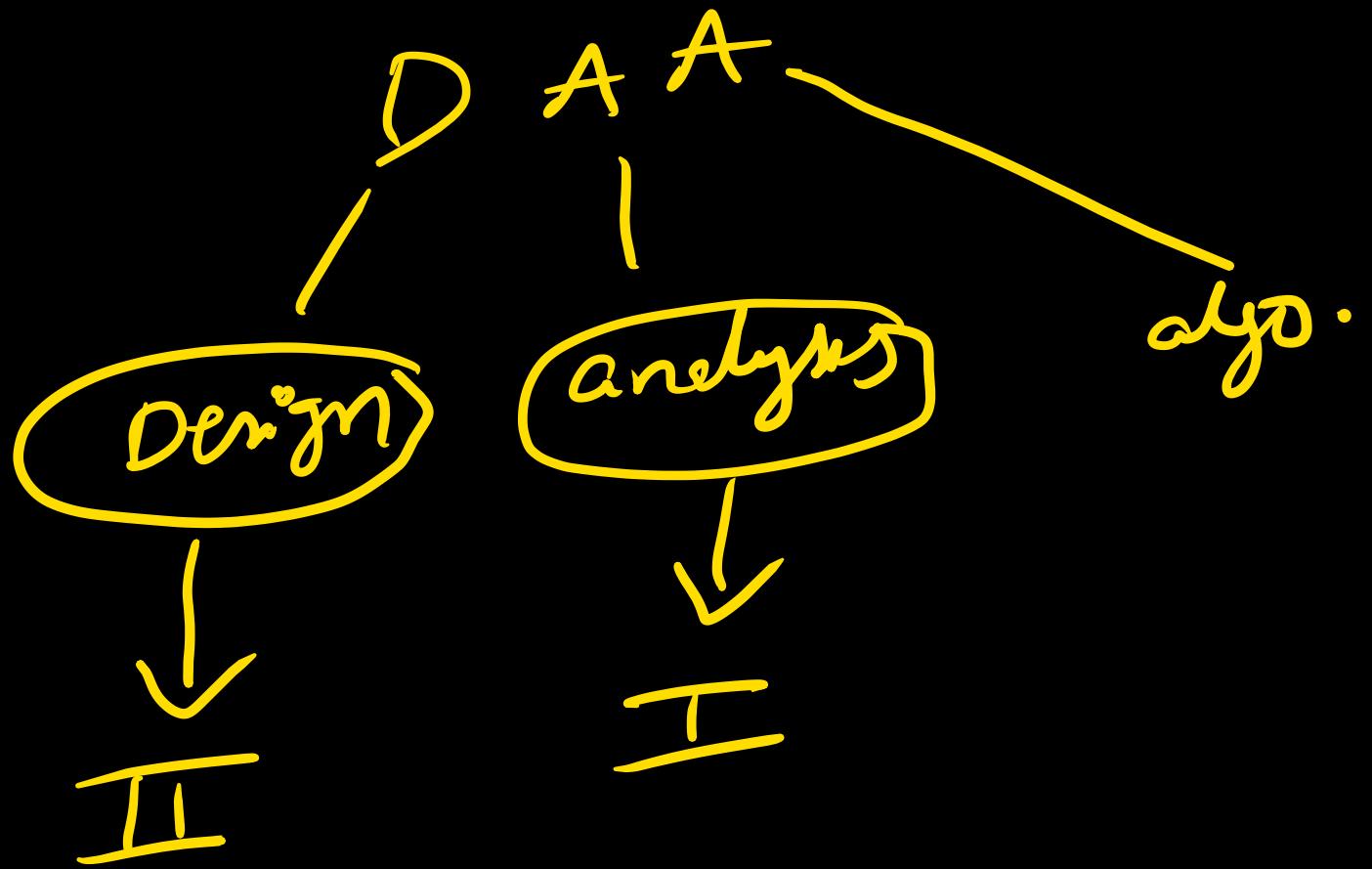
$$\begin{aligned} T(n) &= 7T(n/2) + n^2 \\ &= 7(7T(n/2^2) + (n/2)^2) + n^2 \\ &= 7^2 T(n/2^2) + 7/4 n^2 + n^2 \\ &= 7^2 [7T(n/2) + (n/2^2)^2] + 7/4 n^2 + n^2 \\ &= 7^3 T(n/2^3) + (7/4)^2 n^2 + (7/4)n^2 + n^2 \end{aligned}$$

$$\begin{aligned}
 & 7^3 T\left(\frac{n}{2^3}\right) + \left(\frac{7}{4}\right)^2 n^2 + \left(\frac{7}{4}\right)n^2 + n^2 \\
 & \xrightarrow{\text{3 times}} 7^k T\left(\frac{n}{2^k}\right) + \left(\frac{7}{4}\right)^{k-1} n^2 + \left(\frac{7}{4}\right)^{k-2} n^2 + \dots + \left(\frac{7}{4}\right)^0 n^2 \\
 & \quad \text{where } \frac{n}{2^k} = 2 \Rightarrow n = 2^{k+1} \Rightarrow \log n = k+1 \Rightarrow k = \log n - 1 \\
 & = 7^{\log n - 1} (2) + n^2 \left(\left(\frac{7}{4}\right)^0 + \left(\frac{7}{4}\right)^1 + \left(\frac{7}{4}\right)^2 + \dots + \left(\frac{7}{4}\right)^{\log n - 2} \right) \\
 & = \frac{7^{\log n}}{7} \cdot 2 + n^2 \left(\frac{\left(\frac{7}{4}\right)^{\log n - 1} - 1}{\frac{7}{4} - 1} \right)
 \end{aligned}$$

$$\begin{aligned}
 T(n) &= \frac{7}{7} \cdot n^2 + n^2 \left(\frac{(7/4)^{\log_2 7} - 1}{7/4 - 1} \right) \\
 &= n^{\log_2 7} + n^2 (7/4)^{\log_2 7} \\
 &= n^{\log_2 7} + n^2 \frac{(7/4)^{\log_2 7}}{7/4 - 1} \\
 &= n^{\log_2 7} + n^2 \frac{n^{\log_2 7}}{n^{\log_2 7}} = n^{\log_2 7} \\
 &= O(n^{2+81})
 \end{aligned}$$

Every question we are solving here is some algorithm.
~~we~~ Some of them are in syllabus.
 we will see them when the time comes.

Analysing algo.



$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ 2T(n/2) + n & \text{if } n>1 \end{cases}$$

$$\begin{aligned} T(n) &= 2T(n/2) + n \\ &= 2(2T(n/4) + n/2) + n \\ &= 2^2 T(n/2^2) + n + n \\ &= 2^2 (T(n/2^3) + n/4) + n + n \\ &= 2^3 T(n/2^3) + 3n \end{aligned}$$

$$\begin{aligned}
 &= 2^3 T\left(\frac{n}{2^3}\right) + 3n \\
 &\quad \text{\{ b time}} \\
 &= 2^k T\left(\frac{n}{2^k}\right) + kn \\
 &\quad \text{b} \leftarrow \frac{n}{2^k} = 1 \Rightarrow n = 2^k \Rightarrow k = \log_2 n \\
 &= 2^{\log_2 n} (1) + (\log_2 n) n \\
 &= n^{\log_2 2} + (\log_2 n) n = O(n \log n) \\
 &\qquad\qquad\qquad \Omega(n \log n) \\
 &\qquad\qquad\qquad \Theta(n \log n)
 \end{aligned}$$

* * * *

Q) $T(n) = \begin{cases} 1 & n=1 \\ 2T(n/2) + n \log n & \text{if } n>1 \end{cases}$

$$\begin{aligned} T(n) &= 2T(n/2) + n \log n \\ &= 2(2T(n/4) + n/2 \log n/2) + n \log n \\ &= 2^2 T(n/2^2) + 2n \log n/2 + n \log n \end{aligned}$$

$$= 2^2 \overbrace{\tau(\gamma_{2^2})}^1 + 2n \log \frac{n}{2} + n \log n$$

$$= 2^2 \left[\tau(\gamma_8) + \frac{n}{2^2} \log \frac{n}{2^2} \right] + n \log \frac{n}{2} + n \log n$$

$$= 2^3 \tau(\gamma_{2^3}) + n \log \frac{n}{2^2} + n \log \frac{n}{2} + n \log \frac{n}{2^3}$$

$\underbrace{\qquad\qquad}_{K \text{ times}}$

$$2^K \tau(\gamma_{2^3}) + n \log \frac{n}{2^{K-1}} + n \log \frac{n}{2^{K-2}} + \dots + n \log n$$

$$2^K \downarrow T\left(\frac{n}{2^K}\right) + n \log \frac{n}{2^{K-1}} + n \log \frac{n}{2^{K-2}} + \dots + n \log n$$

$$= n \quad \text{let } \frac{n}{2^K} = 1 \Rightarrow 2^K = n \Rightarrow K = \log n.$$

$$= n(1) + n \left(\log \frac{n}{2^0} + \log \frac{n}{2^1} + \dots + \log \frac{n}{2^{K-1}} \right) \xrightarrow{\log \frac{n}{2^{K-1}}} \log \frac{n}{2^{\log n - 1}}$$

$$= n + n(\log n - \log 2^0 + \log n - \log 2^1 + \dots + \log n - \log 2^{\log n - 1})$$

$$= n + n \left(\underbrace{\log n - \log 2^0}_{\text{cancel}} + \underbrace{\log n - \log 2^1}_{\text{cancel}} + \dots + \underbrace{\log n - \log 2^{\log n-1}}_{\text{cancel}} \right)$$

$$= n + n \left(\cancel{\log n} \cancel{\log n} - \log_2 2^0 - \log_2 2^1 + \dots + \log_2 2^{\log n-1} \right)$$

$$= n + n \left[(\log n)^2 - \cancel{(1+2+3+\dots+\log n-1)} \right]$$

Sum of $\log n-1$ terms

$$= n + n \left(\cancel{(\log n)^2} - \frac{(\log n-1)(\log n)}{2} \right) \quad \checkmark$$

$$\cancel{n(\log n)^2}$$

$$\cancel{n \log n} \times$$

$$= n + n \frac{(\log n)^2}{2}$$

all getting
Tree
matter

Substitution

$$T(n) = 2T(n-1) + K$$

$\underbrace{\qquad\qquad\qquad}_{\text{one term.}}$

$$T(n) = \underbrace{T(n-1) + T(n-2)}_{\text{more than one term}}$$
$$T(n) = aT(n/b) + K$$

$\underbrace{\qquad\qquad\qquad}_{\text{not for all get}}$

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ 2T(n-1) + n & \text{if } n>1 \end{cases}$$

$$\begin{aligned} T(n) &= 2 \cancel{T(n-1)} + n \\ &= 2(2T(n-2) + n-1) + n \\ &= 2^2 \cancel{T(n-2)} + 2(n-1) + n \\ &= \underline{2^2(2T(n-3) + (n-2)) + 2(n-1) + n} \\ &= \underline{2^3 T(n-3) + 2^2(n-2) + 2^1(n-1) + n} \end{aligned}$$

$$= 2^3 T(n-3) + 2^2(n-2) + 2^1(n-1) + n$$

$\frac{T(n)}{-2T(n)}$
 $-T(n)$

K times

$$2^k T(n-k) + 2^{k-1}(n-(\cancel{k-1})) + 2^{k-2}(n-(\cancel{k-2})) + \dots + 2^1(n-1) + 2^0(n-0)$$

$$n-k=1 \Rightarrow k=n-1$$

$$= 2^{n-1} T(1) + 2^{n-2}(n-(\cancel{n-1})) + 2^{n-3}(n-(\cancel{n-2})) + \dots + 2^1(n-1) + 2^0(n-0)$$

$$T(n) = (2^{n-1})_1 + (2^{n-2})_2 + (2^{n-3})_3 + \dots + (2^1)_{(n-1)} + (2^0)_{(n-0)} \quad \checkmark$$

\downarrow
 $T(n)$

Combination of AP and GP

$$T(n) = 2^0(n-0) + 2^1(n-1) + 2^2(n-2) + \dots + 2^{n-3}(3) + 2^{n-2}(2) + 2^n(1)$$

$$-2 \times T(n) = -2^0(n-0) - 2^1(n-1) - 2^2(n-2) - 2^{n-3}(4) - 2^{n-2}(3) - 2^{n-1}(2)$$

$$-T(n) = 2^0(n) - (2^1 + 2^2 + 2^3 + \dots + 2^{n-3} + n \sum_{i=2}^{n-2} 2^i + \dots + 2^n)$$

$$\frac{T(n)}{-2T(n)} = \frac{-T(n)}{-T(n)}$$

$$-T(n) = n - [2^1 + 2^2 + 2^3 + \dots + 2^n]$$

$$T(n) = (2^1 + 2^2 + 2^3 + \dots + 2^n) - n$$

$$= \frac{2(2^{n-1})}{2-1} - n = O(2^n) \Theta(2^n) \Omega(2^n)$$



$$\begin{aligned} T(n) &= \frac{n^{1/2} T(n^{1/2}) + n}{n^{1/2}} \\ &= n^{1/2} [n^{1/4} T(n^{1/4}) + n^{1/2}] + n \\ &= n^{3/4} [T(n^{1/4}) + n + n] \\ &= n^{3/4} (n^{1/2^3} T(n^{1/2^3}) + n^{1/4}) + n + n \\ &= n^{7/2^3} T(n^{1/2^3}) + 3n \\ &= \frac{n^{2^3 - 1}}{2^3} T(n^{1/2^3}) + 3n \end{aligned}$$

$$\begin{aligned}
 &= n^{\frac{2^3 - 1}{2^3}} T(n^{\frac{1}{2^3}}) + 3n \\
 &\quad \text{↓ K times} \\
 &= n^{\frac{2^K - 1}{2^K}} T(n^{\frac{1}{2^K}}) + Kn \\
 &= n^{1 - \frac{1}{2^K}} T(n^{\frac{1}{2^K}}) + Kn \\
 &\quad \text{let } n^{\frac{1}{2^K}} = 2 \rightarrow \text{termination condition} \\
 &\quad \Theta(n^{\log \log n}) \\
 &\quad \Theta(n^{\frac{1}{2^K} \log \log n}) \\
 &= \left(\frac{n}{2}\right)(1) + \log \log n(n) = n + n \log \log n \\
 &= O(n \log \log n)
 \end{aligned}$$

$$\begin{aligned}
 &T(n^{\frac{1}{2^K}}) \\
 &n^{\frac{1}{2^K}} = 2 \\
 &\log n^{\frac{1}{2^K}} = \log 2 \\
 &\frac{1}{2^K} \log n = 1 \Rightarrow 2^K = \log n \\
 &\Rightarrow K = \log \log n
 \end{aligned}$$

Now = 5 minutes to 31
↓
10 minutes to 1st.

$$\log \log n = \log 2^k$$
$$\Rightarrow \underline{\log \log n} = \underline{k}.$$

Revise 4 times ✓

$$\log n^{1/2^k} = \log 2$$
$$\frac{1}{2^k} \log n = 1 \Rightarrow \log n = 2^k \Rightarrow \cancel{\log \log n} = \log 2^k$$

Recursive tree method :

$$T(n) = T(n-1) + T(n-2) + \gamma.$$

max than two terms

$$T(n) =$$

$$T(n) = \begin{cases} 1 & n=1 \\ T(\gamma_2) + T(\gamma_2) + n ; & \text{if } n>1 \end{cases}$$

You can apply masters theorem

But we have practice RT method

$$\underline{\underline{T(n)}} = \underline{\underline{T(n/2)}} + \underline{\underline{T(n/2)}} + \underline{\underline{n}}.$$

$T(n)$

$$2T(n/2) + n$$

↓
Substitution.

✓ $T\left(\frac{7n}{8}\right) + T\left(\frac{n}{8}\right) + n.$

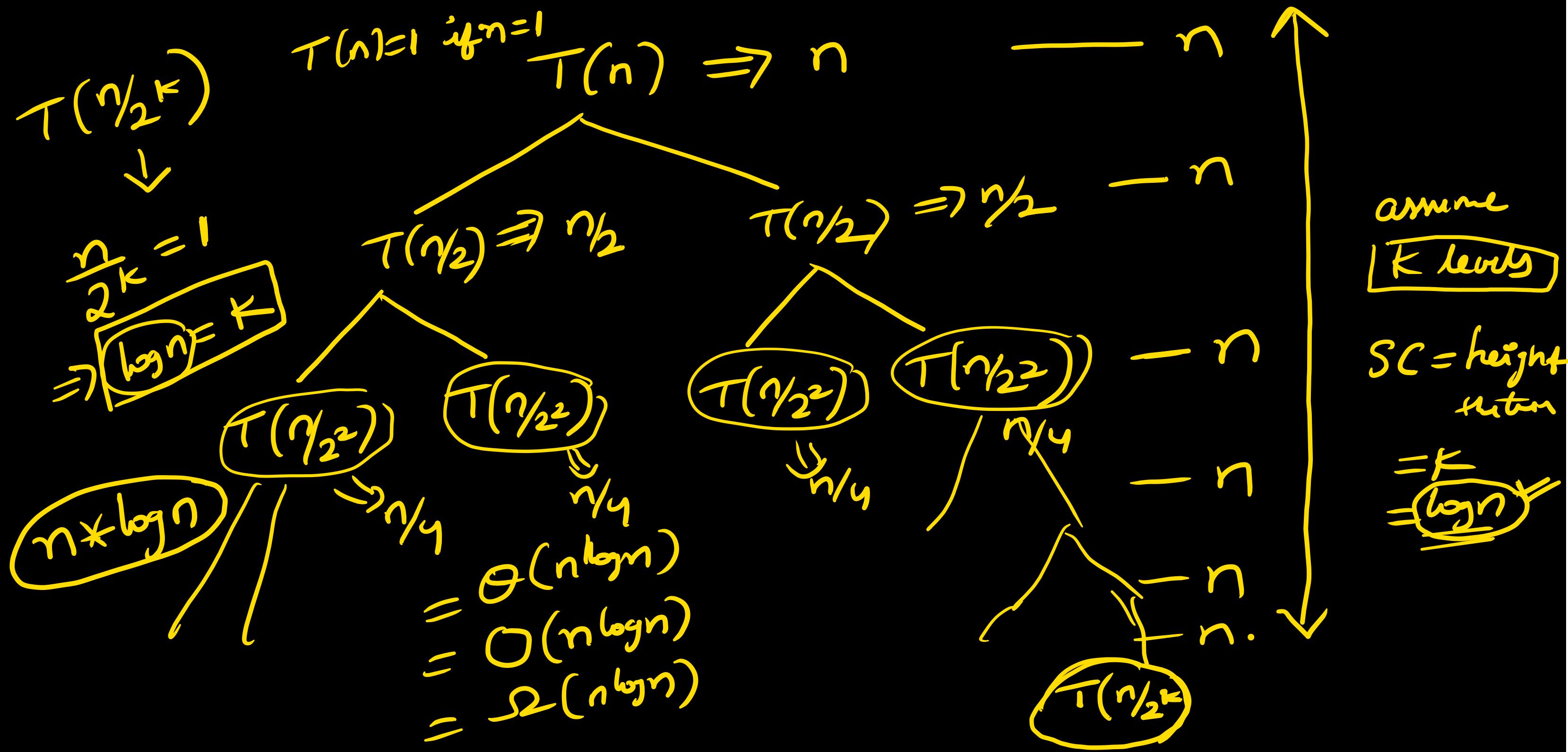
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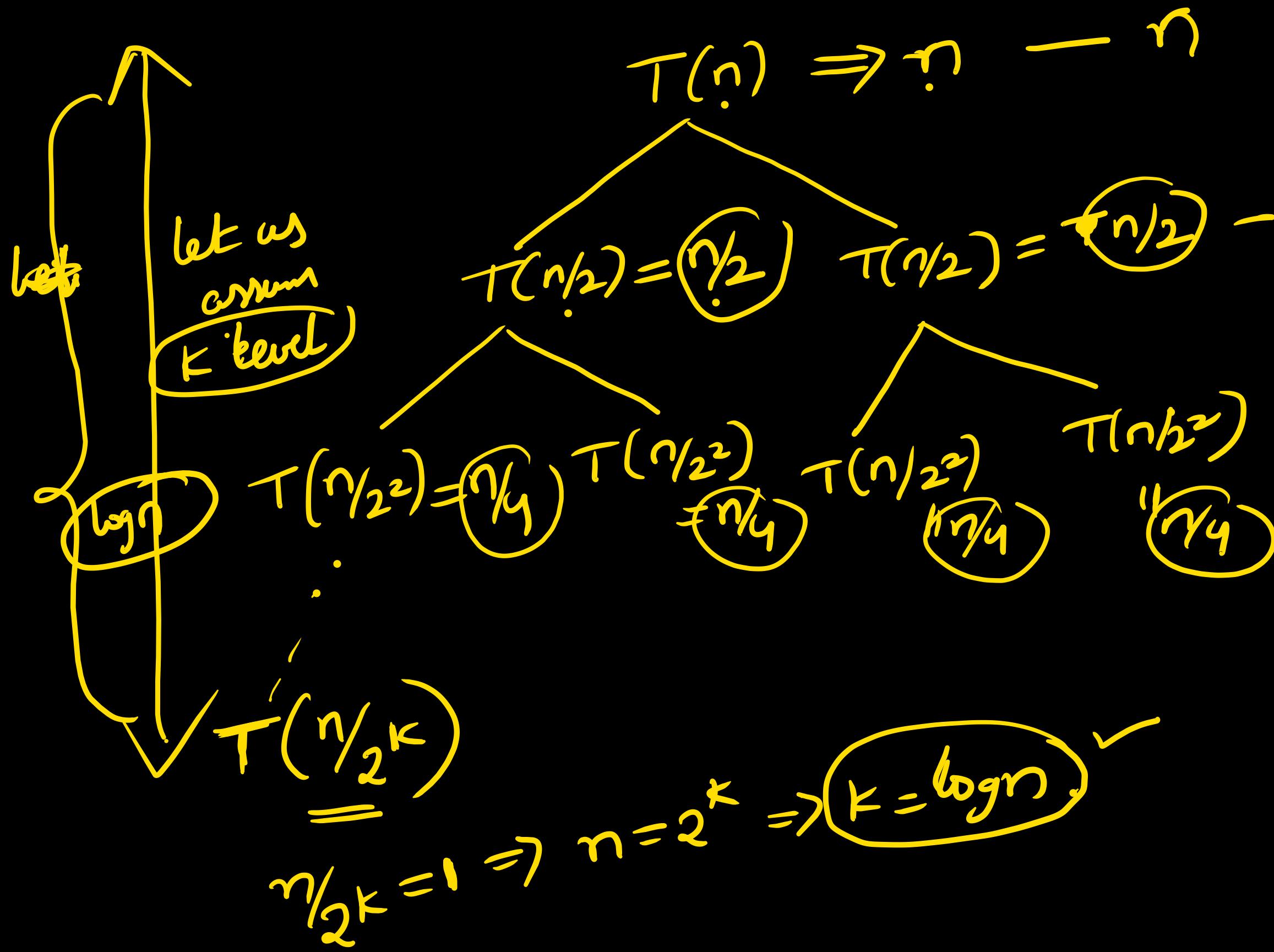
$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + \text{C}$$

Problem of size n

Problem of size $n/2$

Time taken for dividing problem into sub problems.





$$T(n) = T(n_2) + T(\gamma_2)_{+n}$$

$= 1$ if $n=1$

$$T(n) = T(n/2) + T(n/2) + \Theta(n)$$

$\Theta(n * \log n)$

$\Theta(n \log n)$

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$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(n/3) + T(2n/3) + n & \text{if } n>1 \end{cases}$$

