$$T(n) = T(n/3) + T(2n/3) + n \quad ; \ n>1$$

$$= 1 \quad \text{if } n=1$$

$n/3$ $-n$ $-n$ $-n$ $n/3^l$ 1 1 l

$$T(n) = n \rightarrow n$$

$$T(n/3) = n/3 \qquad T(2n/3) = \frac{2n}{3} \rightarrow \frac{n}{3} + \frac{2n}{3} = n$$

$$T(n/3^2) \qquad T\left(\frac{2n}{3^2}\right) \qquad T\left(\frac{2n}{3^2}\right) \qquad T\left(\frac{2^2 n}{3^2}\right) \rightarrow O(n)$$

$$T(n/3^l) \qquad T\left(\frac{2^k n}{3^k}\right)$$

$$T(1) \qquad T(1)$$

$$\frac{n}{3^l} = 1$$

$$n = 3^l$$

$$l = \log_3 n$$

$$\frac{n}{\left(\frac{3}{2}\right)^k} = 1$$

$$\left(\frac{3}{2}\right)^k = n$$

$$k = \log_{3/2} n$$

T_1

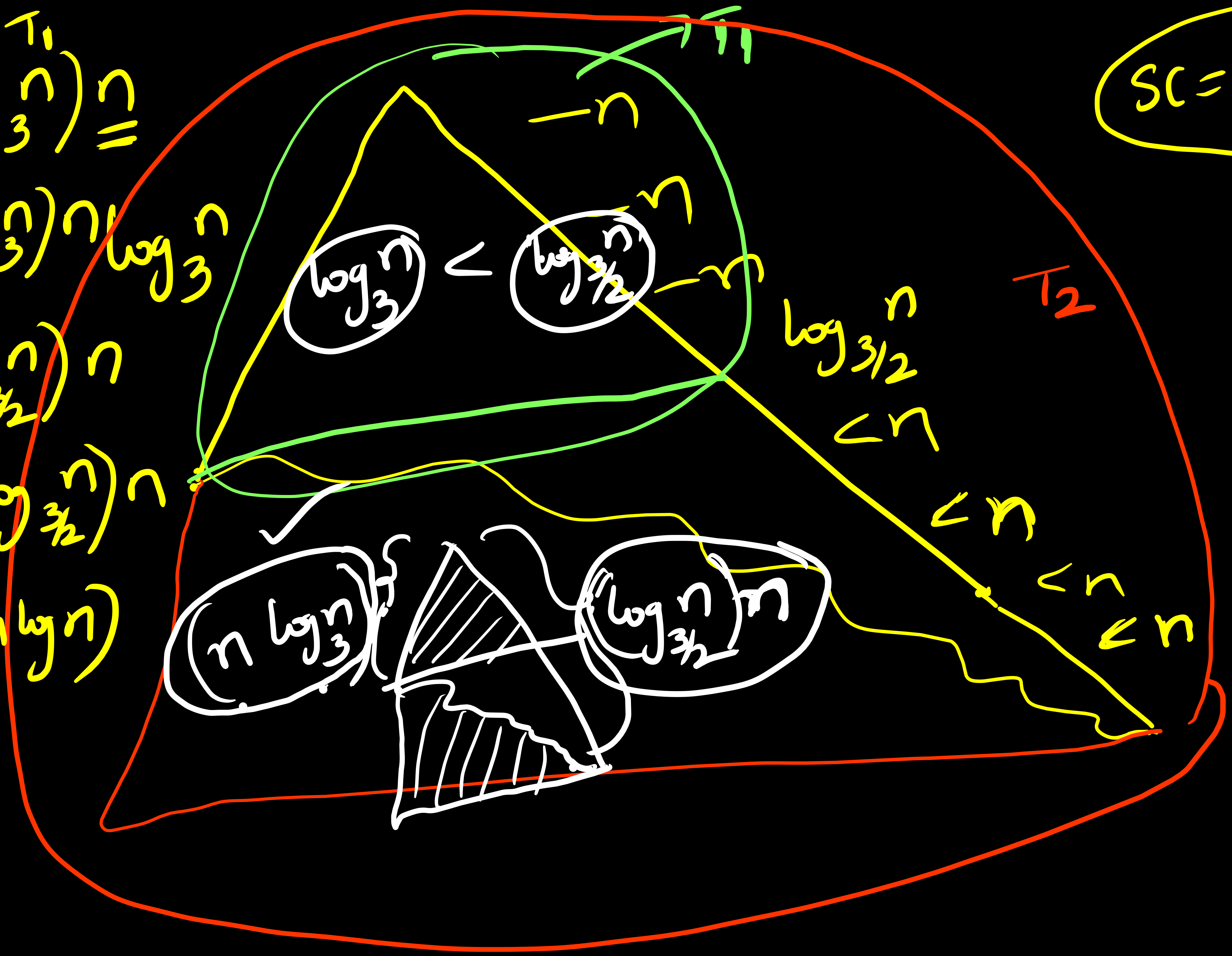
$T(n) \geq (\log_3 n)\, n$

$\Omega((\log_3 n)\, n)$

$T(n) \leq (\log_{3/2} n)\, n$

$T(n) = O((\log_{3/2} n)\, n)$

$T(n) = \Theta(n \log n)$



$SC = O(\log_{3/2} n) = O(\log n)$

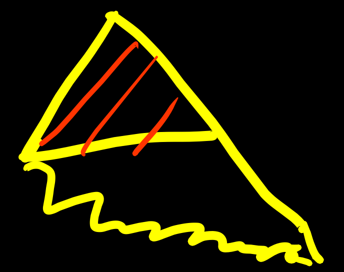
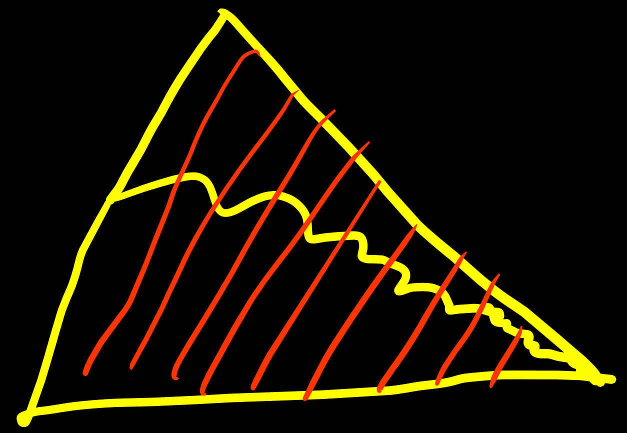
$\log_{3/2} n > \log_3 n$

lower the base
higher the num

$T_1 \leq T \leq T_2$

$SC =$ maximum depth $= \log_{3/2} n$

$$n\log_3 n \le T(n) \le n\log_{3/2} n$$

$$T(n) = O(n\log_{3/2} n)$$

$$= \Omega(n\log_3 n)$$

$$= \Theta(n\log n)$$

Tree - $T(n) = \underbrace{T(n/2) + T\left(\frac{3n}{2}\right)}$ → Tree

Tree

$T(n) = \underbrace{T(n-1)}_{\text{one term}} + n$ → Substitution

$T(n) = aT(n/b) + n$ → master.

$$T(n) = \begin{cases} 1 \\ T(n/5) + T(4n/5) + n \end{cases}$$

$\frac{n}{5^k} = 1$

$\Rightarrow k = \log_5 n$

k

$T(n) \longrightarrow n$

$T(n/5) - n/5 \quad T(4n/5) - \frac{4n}{5} \longrightarrow \frac{n}{5} + \frac{4n}{5} = n$

$T(n/5^2) \quad T\left(\frac{4n}{5^2}\right) \quad T\left(\frac{4n}{5^2}\right) \quad T\left(\frac{4^2 n}{5^2}\right) \longrightarrow n.$

$T(n/5^k)$

$T(1)$

$T\left(\frac{n}{(5/4)^l}\right)$

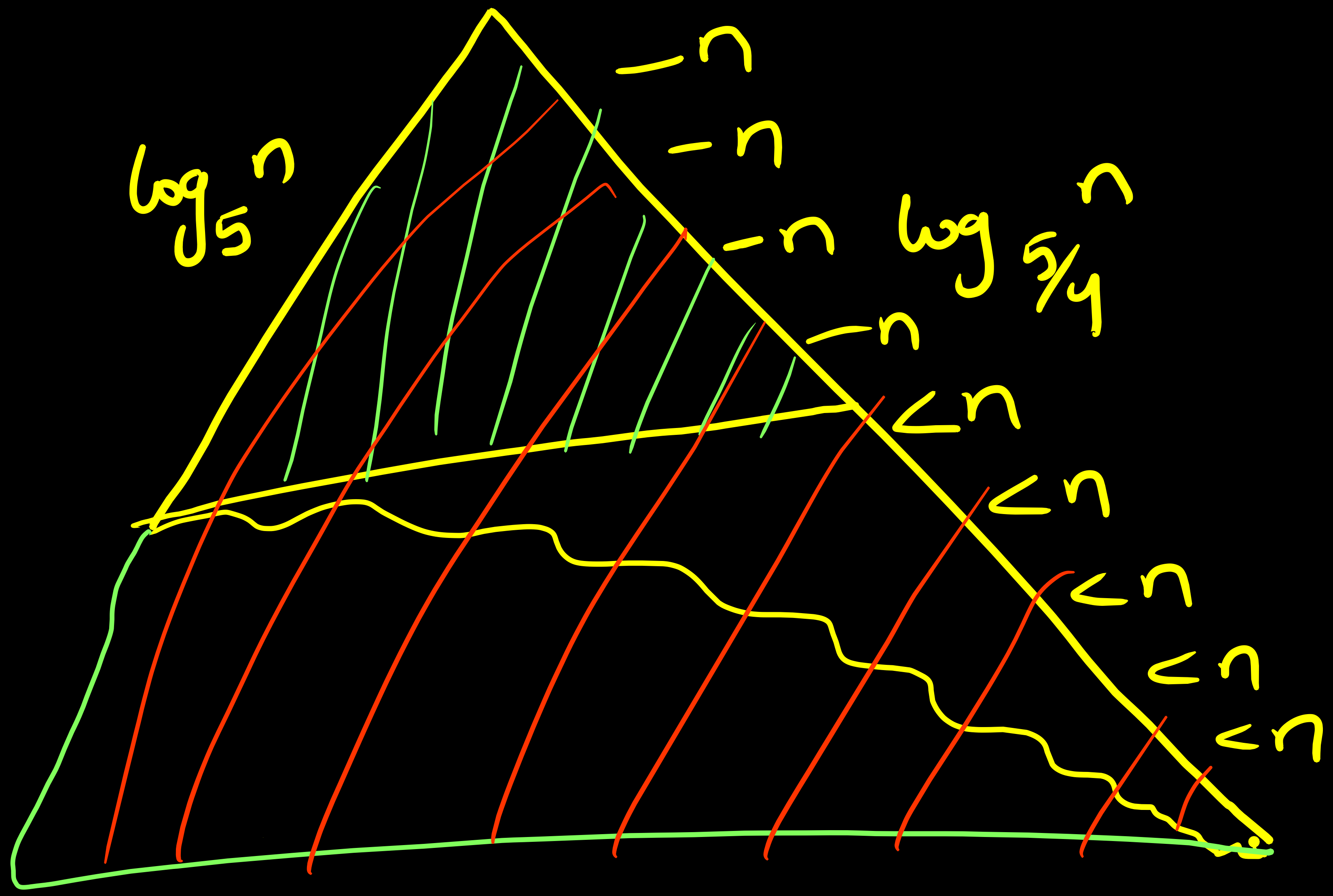
$T(1)$

l levels

$\frac{n}{(5/4)^l} = 1$

$\Rightarrow n = (5/4)^l$

$\Rightarrow l = \log_{5/4} n$



$$T(n) = O(n \log_{5/4} n)$$

$$T(n) = \Omega(n \log_5 n)$$

$$T(n) = \Theta(n \log n)$$

$$SC = O(\log_{5/4} n) = O(\log n)$$

$$n \log_5 n \le T(n) \le n \log_{5/4} n$$

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(n/10) + T(9n/10) + n \end{cases}$$

$\frac{n}{5} +$

$$\boxed{\log_2 n = O(\log_3 n)}$$

$\log_{10} n$ ✓

$$O(n \log_{10/9} n)$$

$$= O(n \log n) \checkmark$$

$$\Omega(n \log_{10} n)$$

$$= O(n \log n)$$

$$SC = O(\log_{10/9} n) \checkmark$$

$$= O(\log n)$$

$$T(n) = O(n \log_{10/9} n)$$

$$= \Theta(n \log n)$$

$$T(n) = \Omega(n \log_{10} n)$$

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(n/10) + T(7n/10) + n \end{cases}$$

$$T(n) = T(n/2) + T(n/2) + n$$

$$TC = \Theta(n\log n)$$

$$SC = \Theta(\log n)$$

$$T(n) = T(n/5) + T(4n/5) + n$$

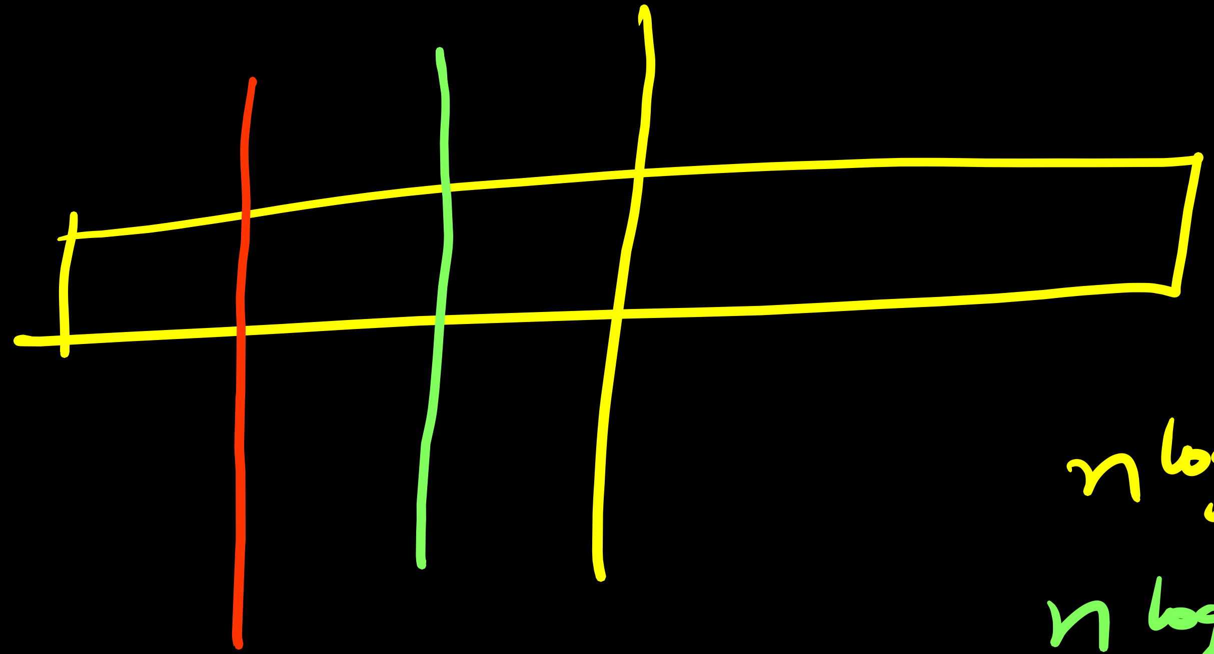
$$TC = \Theta(n\log n)$$

$$SC = \Theta(\log n)$$

$$TC = \Theta(n\log n)$$

$$SC = \Theta(\log n)$$

$$T(n) = T(n/10) + T\left(\frac{9n}{10}\right) + \cancel{?}$$

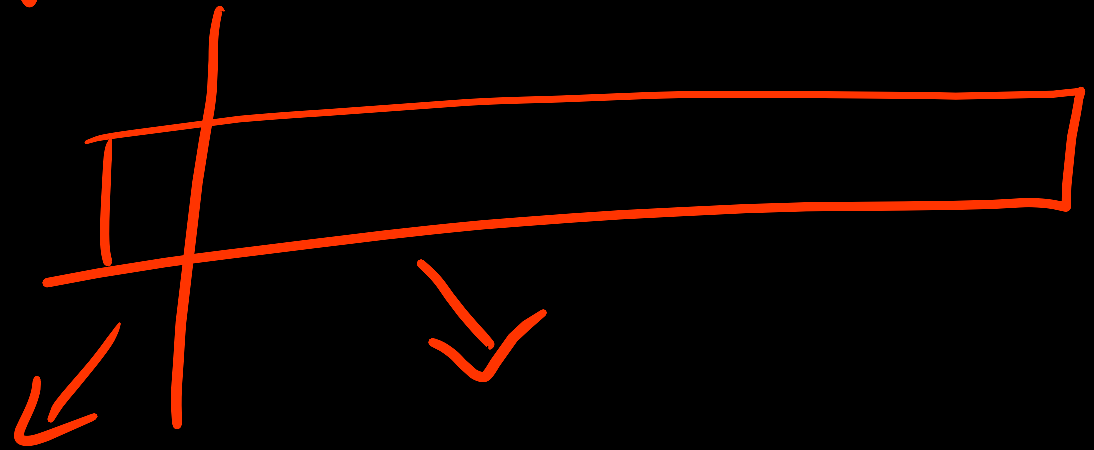


$n \log n$

$n \log n$

$n \log n$

quick sort.



$n \log n$

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(n/10) + T(7n/10) + n \end{cases}$$

$$\frac{n}{10} + \frac{7n}{10} = \frac{8n}{10} \neq n$$

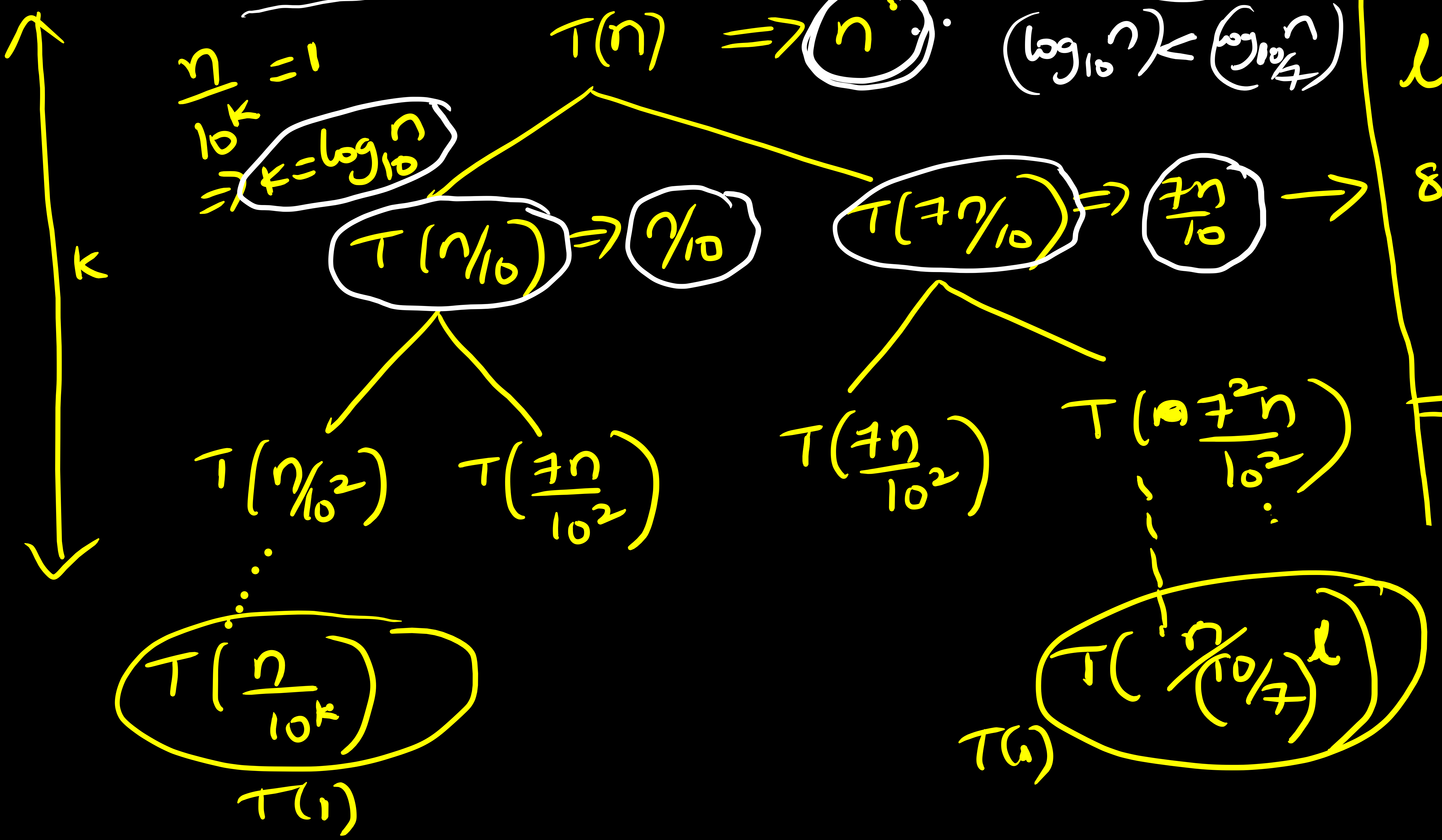
$$\frac{n}{5} + \frac{4n}{5} = n$$

$$\frac{n}{10} + \frac{9n}{10} = n.$$

5min break

5min break.

$$T(n) = T(n/10) + T(7n/10) + n$$



SC

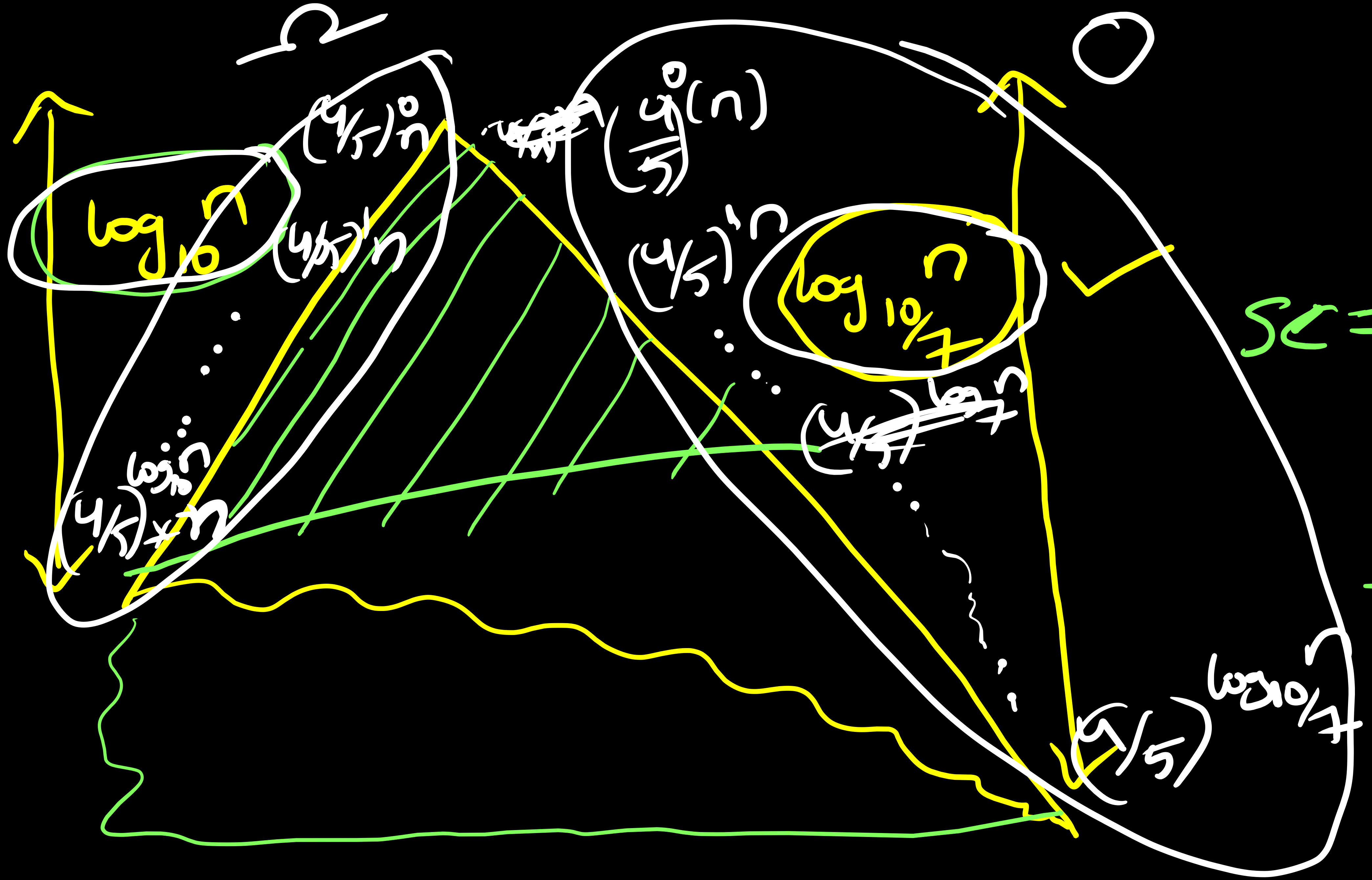
$(\log_{10} n) < (\log_{10/7} n)$

$\frac{n}{(10/7)^l} = 1$

$\Rightarrow l = \log_{10/7} n$

$\rightarrow 8n/10 = \frac{4n}{5}$

$\Rightarrow (4/5)^2 n$



SC = longest depth

$= O(\log_{10/7} n)$

$= O(\log n)$.

$$T(n) \le \left(\frac{4}{5}\right)^0 n + \left(\frac{4}{5}\right)^1 n + \left(\frac{4}{5}\right)^2 n + \cdots\cdots \left(\frac{4}{5}\right)^{\log_{10/7} n} n$$

$$n\left[(4/5)^0 + (4/5)^1 + (4/5)^2 + \cdots\cdots + (4/5)^{\log_{10/7} n}\right]$$

$$= n\left[\frac{a(r^n-1)}{r-1}\right] \quad r>1$$

initial term

$$r = 4/5 < 1 \quad \left[\frac{a(1-r^n)}{1-r}\right]$$

$$= n\left[\frac{1\left(1-(4/5)^{\log_{10/7} n + 1}\right)}{1-4/5}\right]$$

$\Rightarrow 0$ as $n \to \infty$

$$= O(n)$$

$$T(n) \geq \left(\tfrac{4}{5}\right)^0 n + \left(\tfrac{4}{5}\right)^1 n + \left(\tfrac{4}{5}\right)^2 n + \cdots\cdots + \left(\tfrac{4}{5}\right)^{\log_{10} n} n$$

$$T(n) \geq n\left(\left(\tfrac{4}{5}\right)^0 + \left(\tfrac{4}{5}\right)^1 + \left(\tfrac{4}{5}\right)^2 + \cdots \left(\tfrac{4}{5}\right)^{\log_{10} n}\right)$$

$$T(n) \geq n\left\{\frac{1\left(1 - \left(\tfrac{4}{5}\right)^{\log_{10} n + 1}\right)}{1 - 4/5}\right\}$$

$\left(\tfrac{4}{5}\right)^{\log_{10} n + 1} \to 0$ as $n \to \infty$

~~$T(\cdot) = 0$~~

$T(n) = O(n)$

$= \Omega(n)$

$= \Theta(n)$

$SC = O(\log n)$

$$T(n) \geq n(c)$$

$$T(n) = \Omega(n)$$

$$\lim_{n \to \infty} (r)^n = 0$$

$r < 1$

$r^k \to 0 \cdot -$

if $r < 1$ and

$k \to \infty$,

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(n/3) + T(2n/3) + n^2 & \text{if } n > 1 \end{cases}$$

$T(n) \Longrightarrow n^2$

$T(n/3) - (n/3)^2 \; T(2n/3) \rightarrow (2n/3)^2$

$\frac{n^2}{3^2} + \frac{4n^2}{3^2} = \left(\frac{5}{9}\right)^2 n^2$

$T(n/3^2) \quad T\left(\frac{2n}{3^2}\right) \quad T\left(\frac{2n}{3^2}\right) \quad T\left(\frac{2^2 n}{3^2}\right) \quad \left(\frac{5}{9}\right)^3 n^2$

l

k

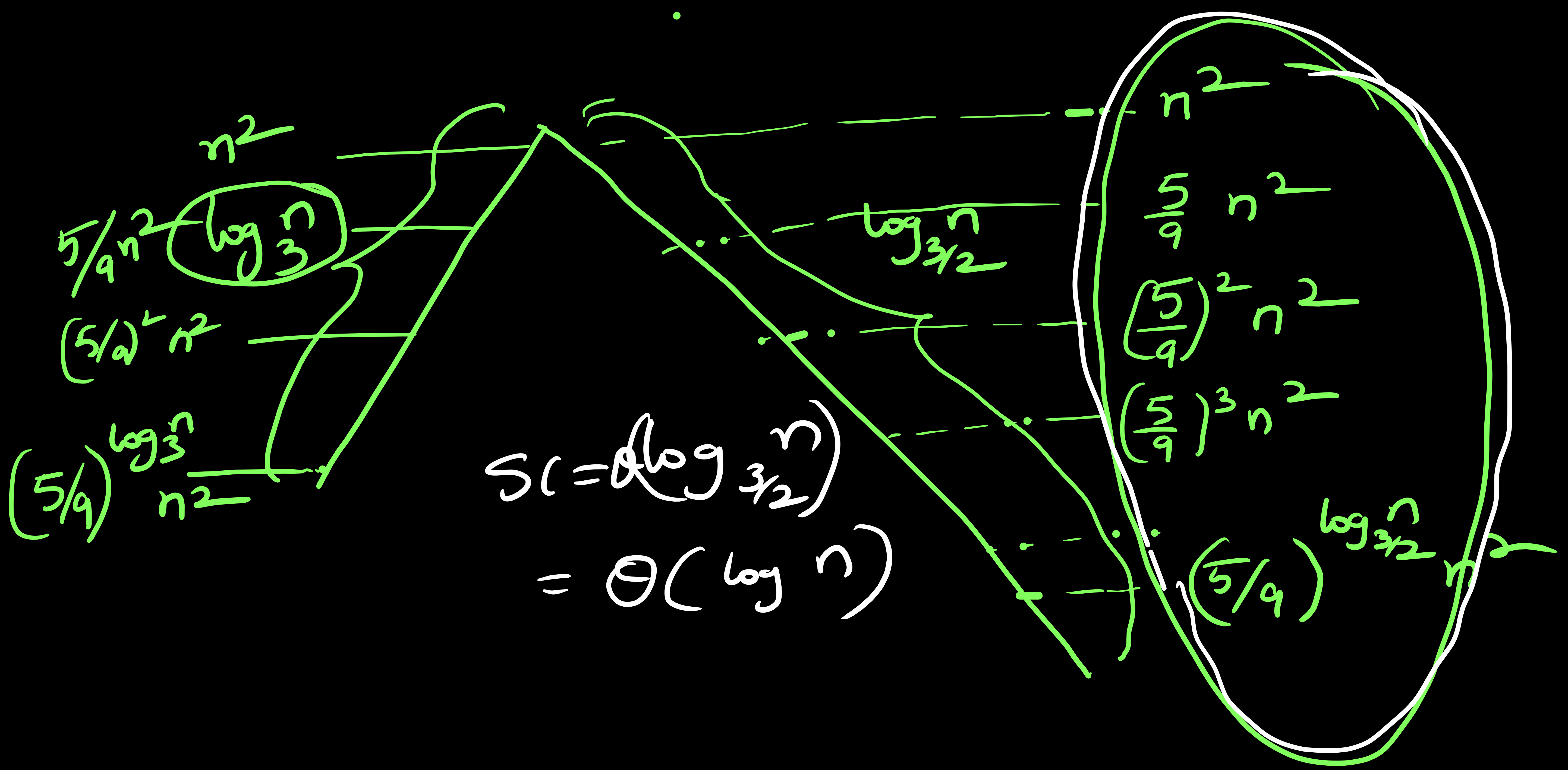
$\frac{n}{(3/2)^k} = 1$

$\Rightarrow k = \log_{3/2} n$

$T(n/3^l) \qquad T\left(\frac{n}{(3/2)^k}\right)$

$n/2^l = 1 \Rightarrow l = \log_3 n$

$T(1)$


n^2
$5/9\, n^2$
$\log_3 n$
$(5/9)^2 n^2$
$(5/9)^{\log_3 n}\, n^2$
$\log_{3/2} n$
n^2
$\frac{5}{9} n^2$
$\left(\frac{5}{9}\right)^2 n^2$
$\left(\frac{5}{9}\right)^3 n^2$
$(5/9)^{\log_{3/2} n}\, n^2$
$SC = O(\log_{3/2} n)$
$= \Theta(\log n)$

$$T(n) \le n^2\left((5/9)^0 + (5/9)^1 + \left(\frac{5}{9}\right)^2 + \cdots \left(\frac{5}{9}\right)^{\log_{3/2} n} \right)$$

GP $\quad r < 1$

$$n^2\left(1 \frac{\left(1 - (5/9)^{\log_{3/2} n + 1}\right)}{1 - 5/9} \right)$$

$(5/9)^{\log_{3/2} n + 1} \to 0$ as $n \to \infty$

$$T(n) = O(n^2)$$

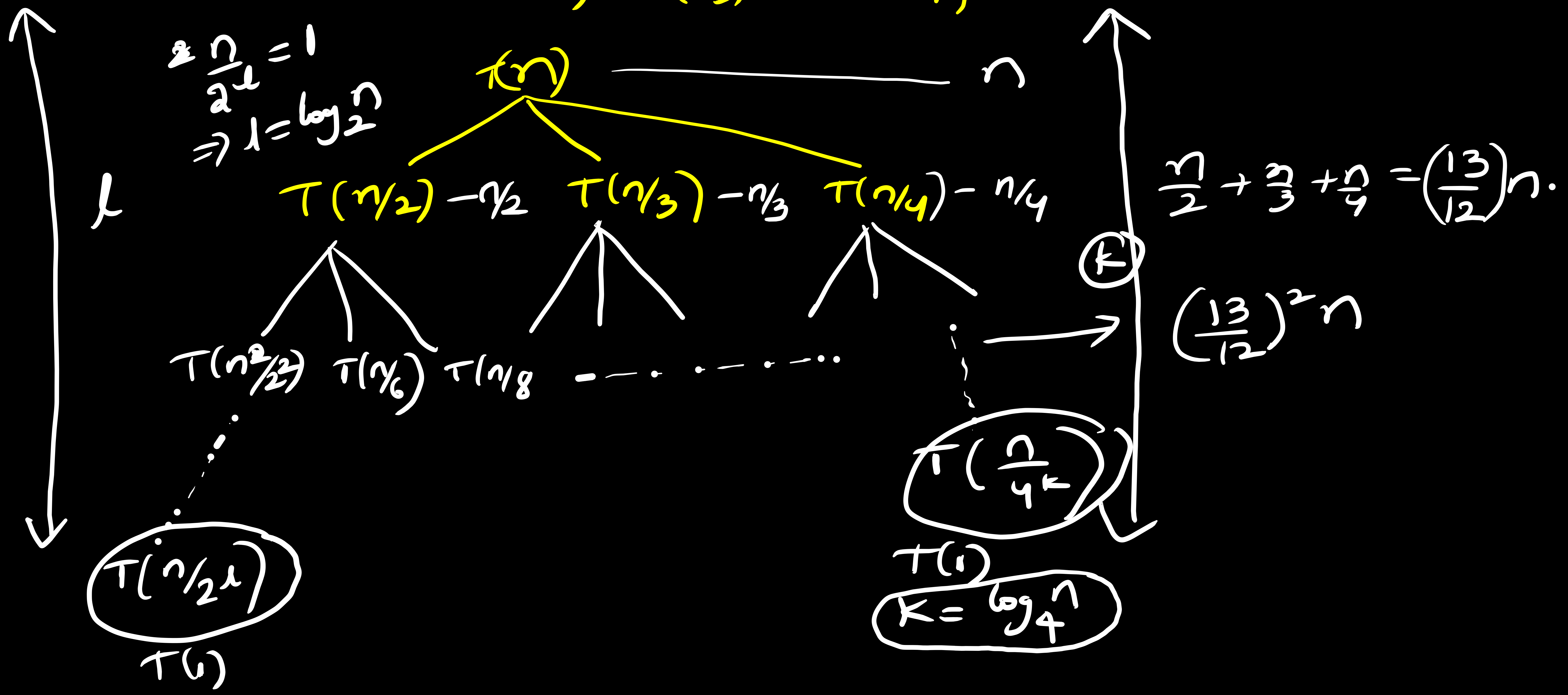
$$T(n) > n^2\left((5/9)^0 + (5/9)^1 + \cdots (5/9)^{\log_3 n} \right)$$

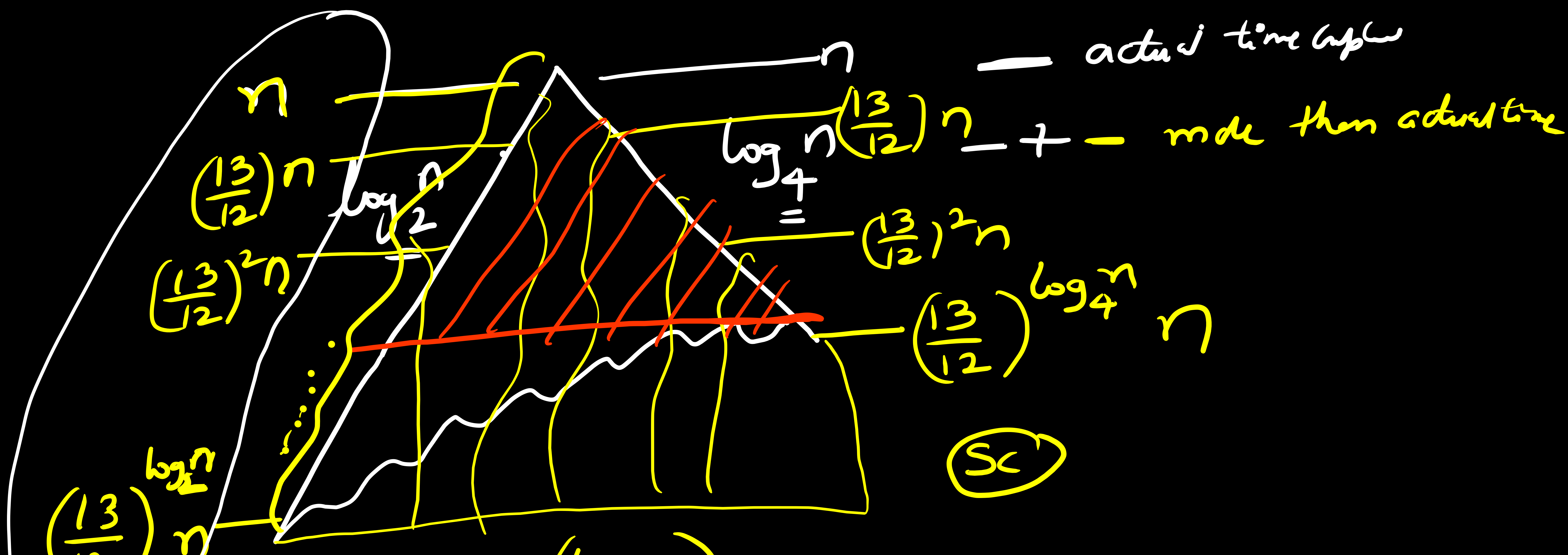
$$n^2\left(\frac{1\left(1 - (5/9)^{\log_3 n + 1}\right)}{1 - 5/9} \right)$$

$(5/9)^{\log_3 n + 1} \to 0$ as $n \to \infty$

$$\Omega(n^2)$$

$$T(n) = \begin{cases} 1 \\ T(n/2) + T(n/3) + T(n/4) + n & \text{if } n > 1 \end{cases}$$




n
$\left(\frac{13}{12}\right)n$
$\left(\frac{13}{12}\right)^2 n$
$\left(\frac{13}{12}\right)^{\log_2 n} n$
$\log_2 n$
n
$\left(\frac{13}{12}\right)n$
$\left(\frac{13}{12}\right)^2 n$
$\left(\frac{13}{12}\right)^{\log_4 n} n$
$\log_4 n$
— actual time complexity
— + — more than actual time
SC

$$SC = O(\log_2 n)$$

$$T(n) \leq n\left[\left(\frac{13}{12}\right)^0 + \left(\frac{13}{12}\right)^1 + \cdots\cdots + \left(\frac{13}{12}\right)^{\log_2 n}\right]$$

$$n\left[\frac{a(r^n-1)}{r-1}\right] = n\left(\frac{1\left(\frac{13}{12}^{\log_2 n+1} - 1\right)}{\frac{13}{12} - 1}\right)$$

$\log_3 n = \log_4 n$
$= \log n$

dominating terms

$a^{\log_c b} = b^{\log_c a}$

$\cancel{n \frac{13}{12}^{\log_2 n}}$

$$n\left(\frac{13}{12}\right)^{\log_2 n} = \left(n\, n^{\log_2 (13/12)}\right)$$

$$T(n) = O\left(n\, n^{\log_2 13/12}\right) \checkmark$$

$$T(n) \geq n \left\{ \left(\tfrac{13}{12}\right)^0 + \left(\tfrac{13}{12}\right)^1 + \cdots\cdots + \left(\tfrac{13}{12}\right)^{\log_4 n} \right)$$

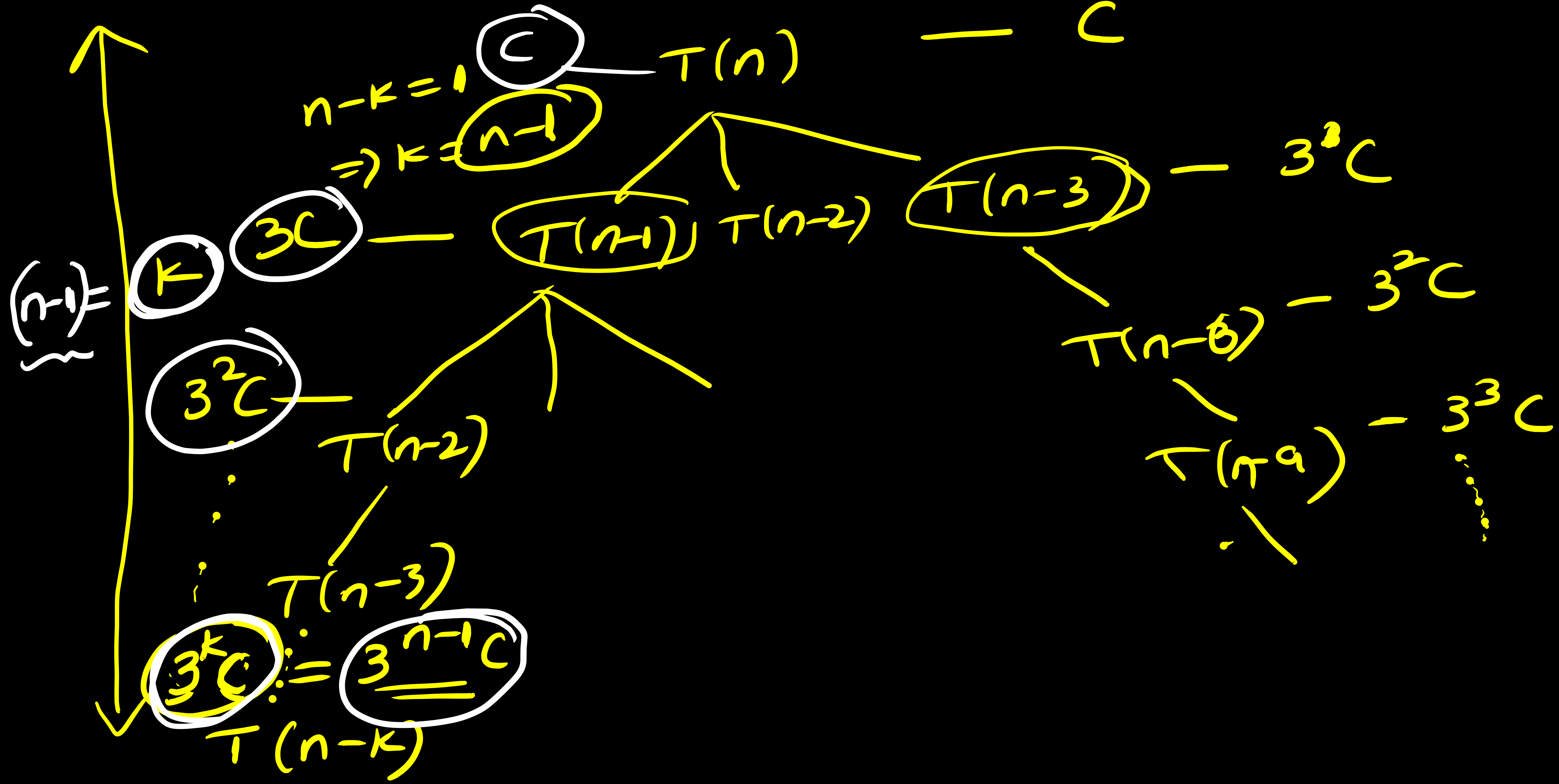
$$\geq n \left(1 \, \frac{\left(\left(\frac{13}{12}\right)^{\log_4 n + 1} - 1 \right.}{\frac{13}{12} - 1} \right)$$

$$\geq n \left(\frac{13}{12}\right)^{\log_4 n}$$

$$\geq n \cdot n^{\log_4 13/12}$$

$$= \Omega\left(n \cdot n^{\log_4 13/12}\right)$$

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(n-1)+T(n-2)+T(n-3)+c & \text{if } n>1 \end{cases}$$



$SC = O(n)$

$$T(n) \leq 3^0 c + 3^1 c + 3^2 c + \ldots\ldots + 3^{n-1}(c)$$

$$c\left(3^0 + 3^1 + 3^2 + \ldots\ldots + 3^{n-1}\right)$$

$$c\left(\frac{1(3^n - 1)}{3-1}\right) = c\,3^n$$

$$SC = O(n)$$

$$T(n) = O(3^n)$$

$$= \Omega(3^n)$$

$$= \Theta(3^n).$$

$$T(n) = 2 * T(n-2) + n.$$

$T(n)$ — n.

$T(n-2)\ (n-2)\ \ T(n-2)\ (n-2)$ $\qquad 2n-4.$

$T(n-4) \qquad T(n-4).$

$$T(n) = T(n-1) + T(n-2) + c \quad n > 1$$
$$= 1 \quad ; n = 1.$$