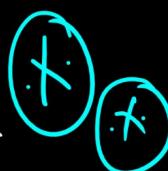


PYTHON PROGRAMMING

GATE DA/DSA

Agenda:

→ Recursive Function



↳ Definition, Examples

↳ Types of Recursion

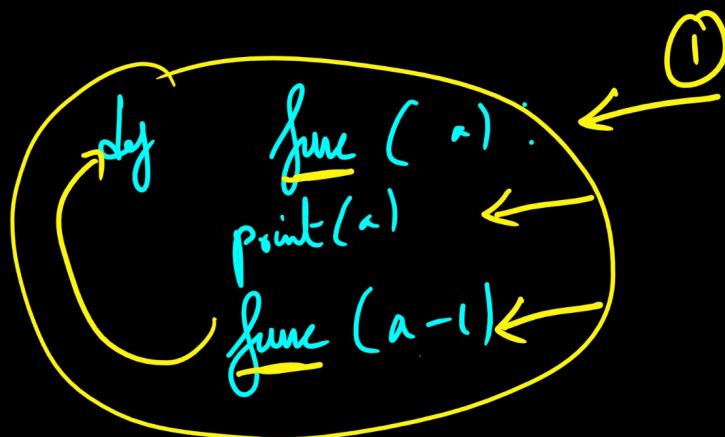
- Direct Recursion
- Indirect ..
- Head Recursion
- Tail Recursion
- Finite Recursion
- Infinite ..



↓
2 problems in
Python

just on
Recursion concept.

Recursive Functions: The process in which a function calls itself directly or indirectly is called recursion and the corresponding function is called a Recursive Function.



func(10)

10	4	-3
9	3	.
8	2	.
7	1	.
6	0	.
5	-1	.
4	-2	.

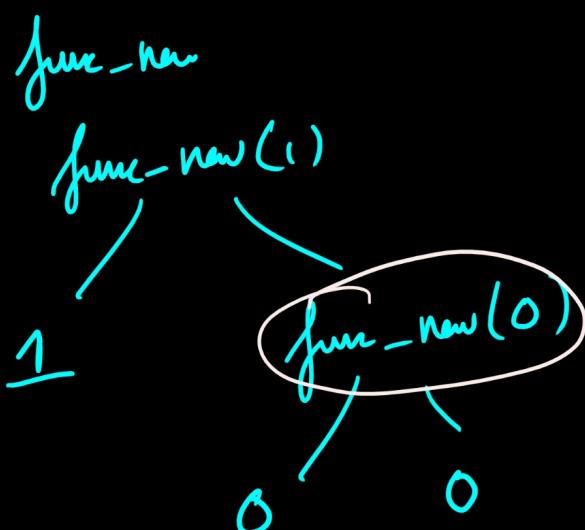
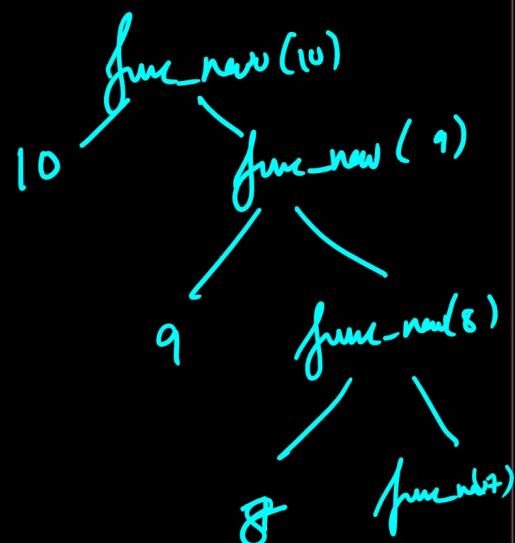
Every recursive function should have base condition
inorder to stop the infinite recursion calls.

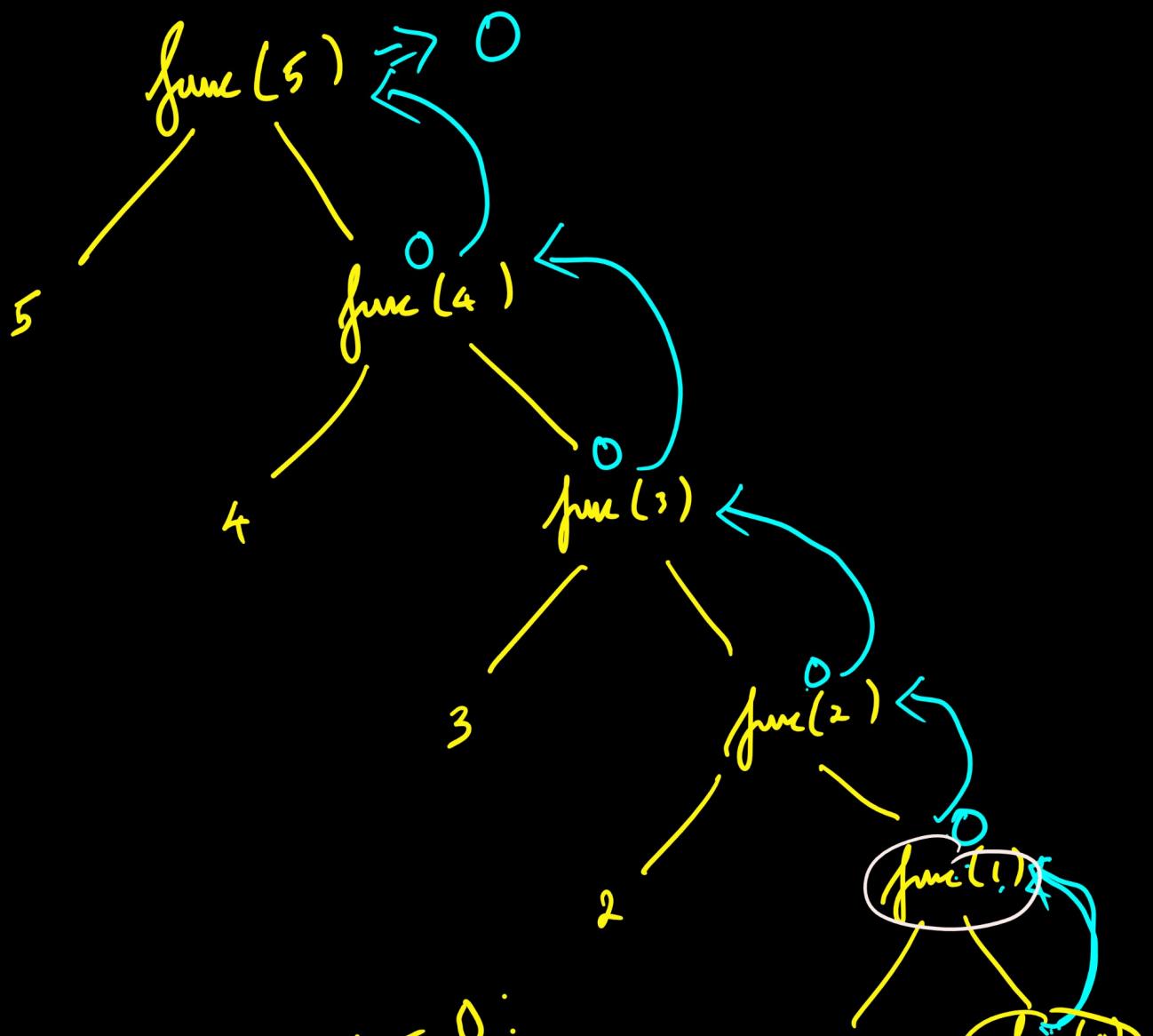
```
def func_new(a):  
    print("Value of a : -->",a)  
    if a <= 0:  
        return a  
    else:  
        return func_new(a-1)
```

func_new(10) ←

if $a \leq 0$:
 return a

10
9
8
7
6
5
4
3
2
1



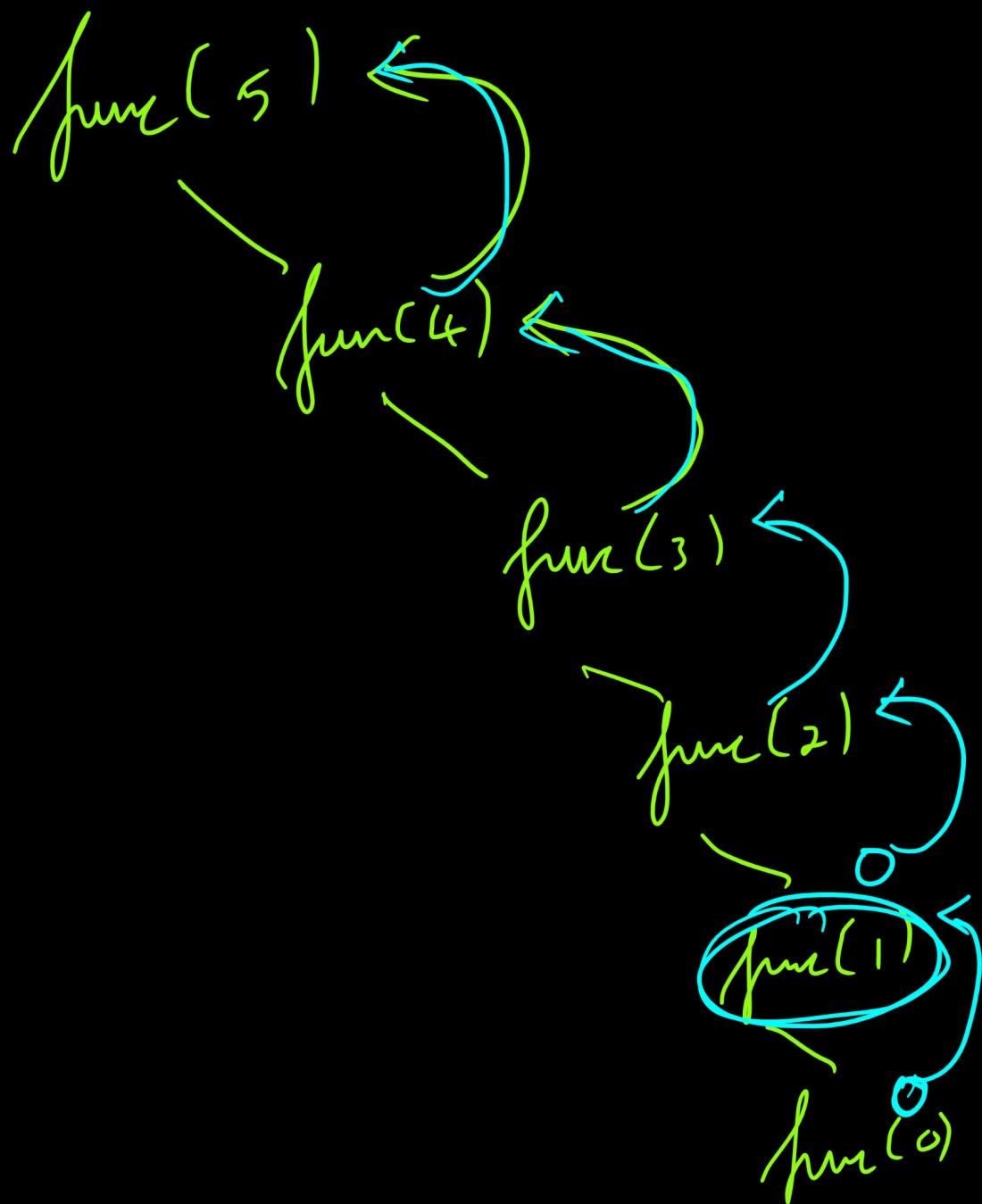


if $a \leq 0$:
 return a

else : $f(a-1)$: $f(a-1) \rightarrow f(a)$
 ~~$f(a-1)$~~
 return a

$a = f(a-1)$

$f(a)$
 $f(a-1)$



factorial
of a number :

$$n!$$

$$n! = n \times (n-1)!$$

$$5! = 5 \times 4!$$

$$4! = 4 \times 3!$$

$$3! = 3 \times 2!$$

$$3! = 3 \times 2 = 3 \times 2 \times 1 = 6$$

$$0! = 1$$

small subproblem
that you
know.

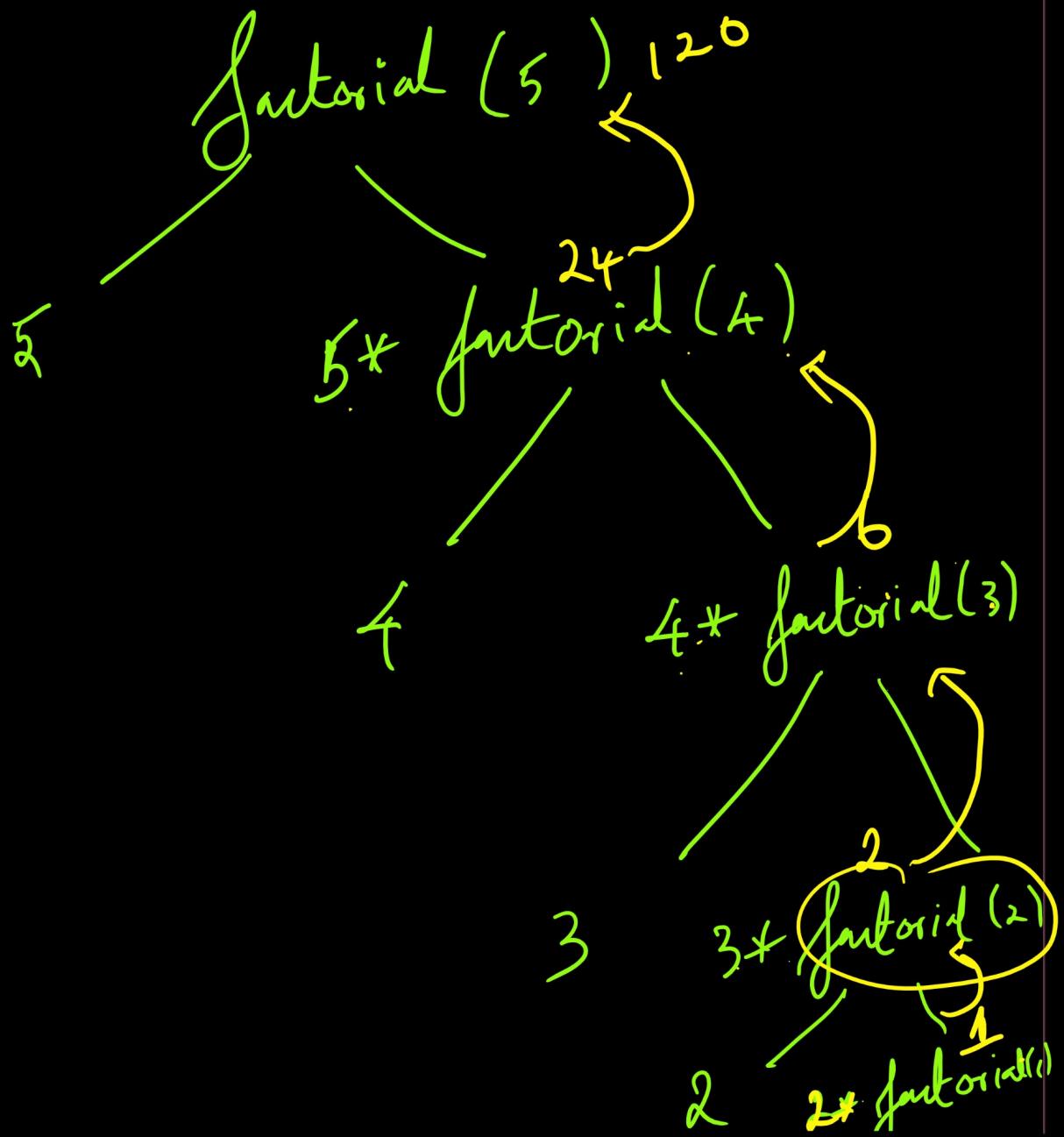
$$2! = 2 \times 1! = 2$$

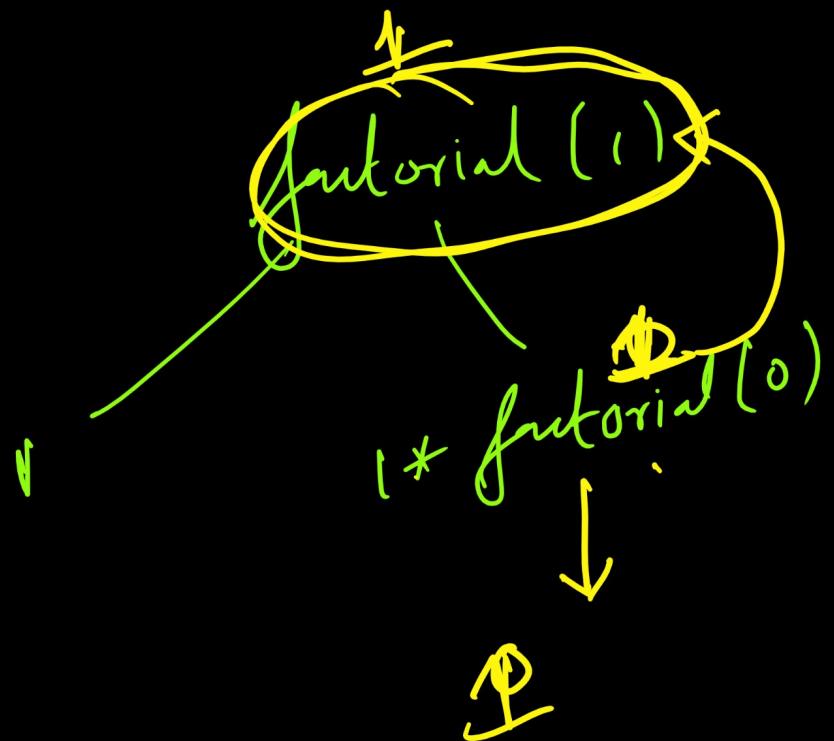
$$1! = 1 \times 0! \rightarrow 1 \times 1 = 1$$

$$0! = 1$$

```
def factorial(n):  
    print("value of n: -->",n)  
    if n==0:  
        return 1  
    return n*factorial(n-1)
```

```
ret_val = factorial(5)  
print("ret val: -->", ret_val)
```





Sum of first n numbers
starting from 1.

Sum of first 5.

$$5 + \text{sum}(n-1)$$

$$5 + \text{sum}(4)$$

$$4 + \text{sum}(3)$$

$3 + \text{num}(2)$
 \downarrow \uparrow
 $2 + \text{num}(1)$
 \downarrow
 $1 + \text{num}(0)$
 \downarrow
 if $\text{num} = 0$
 return 0

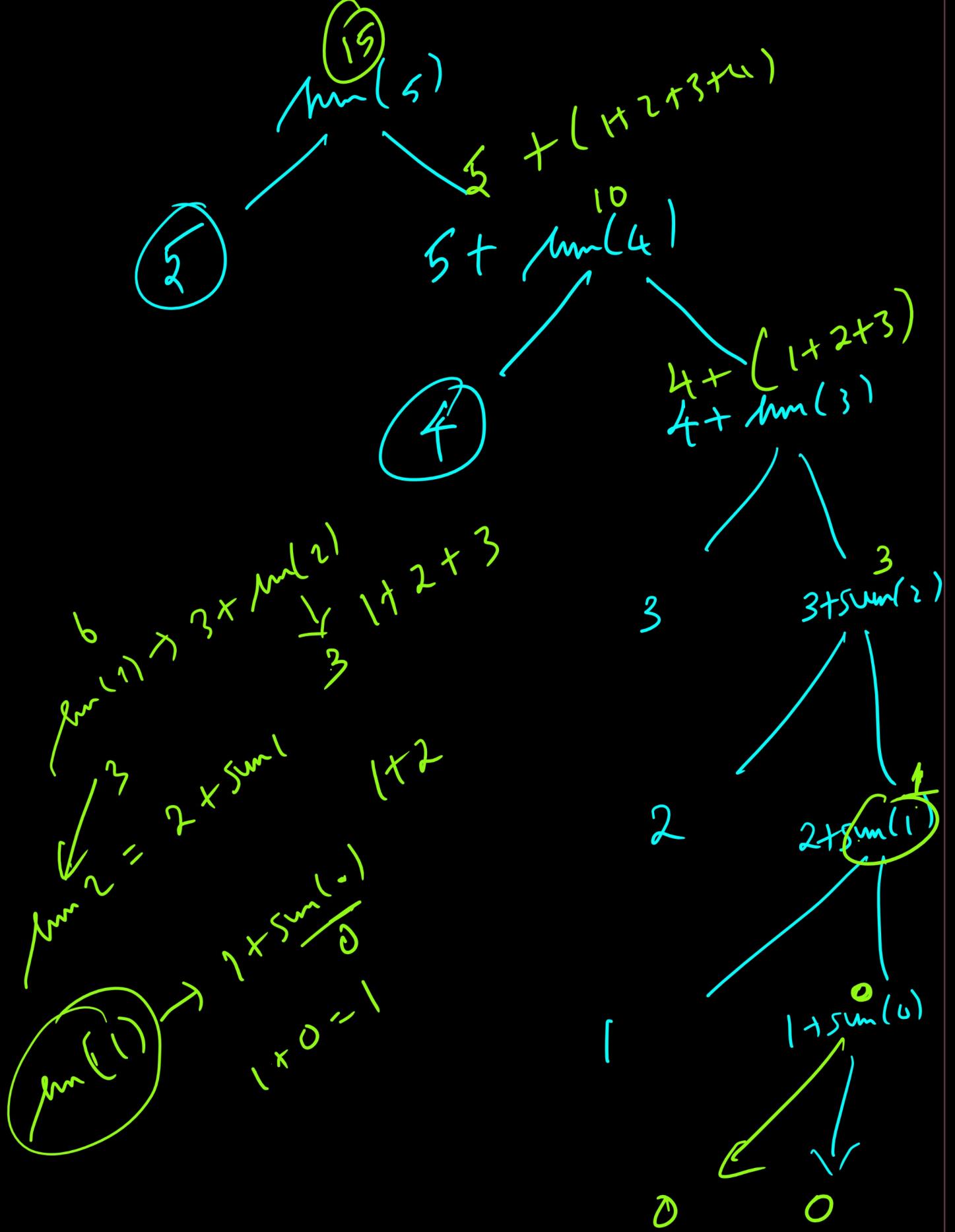
```

def sum_n_numbers(n):
  print("n val : -->",n)
  if n==0: ←
    return 0
  return n+sum_n_numbers(n-1)
  
```

```

ret_val = sum_n_numbers(5) ←
print(ret_val)
  
```

$\text{num}(5)$



Head Recursion :

In head recursion, the recursive call is made at the beginning of the function body.

Tail Recursion :

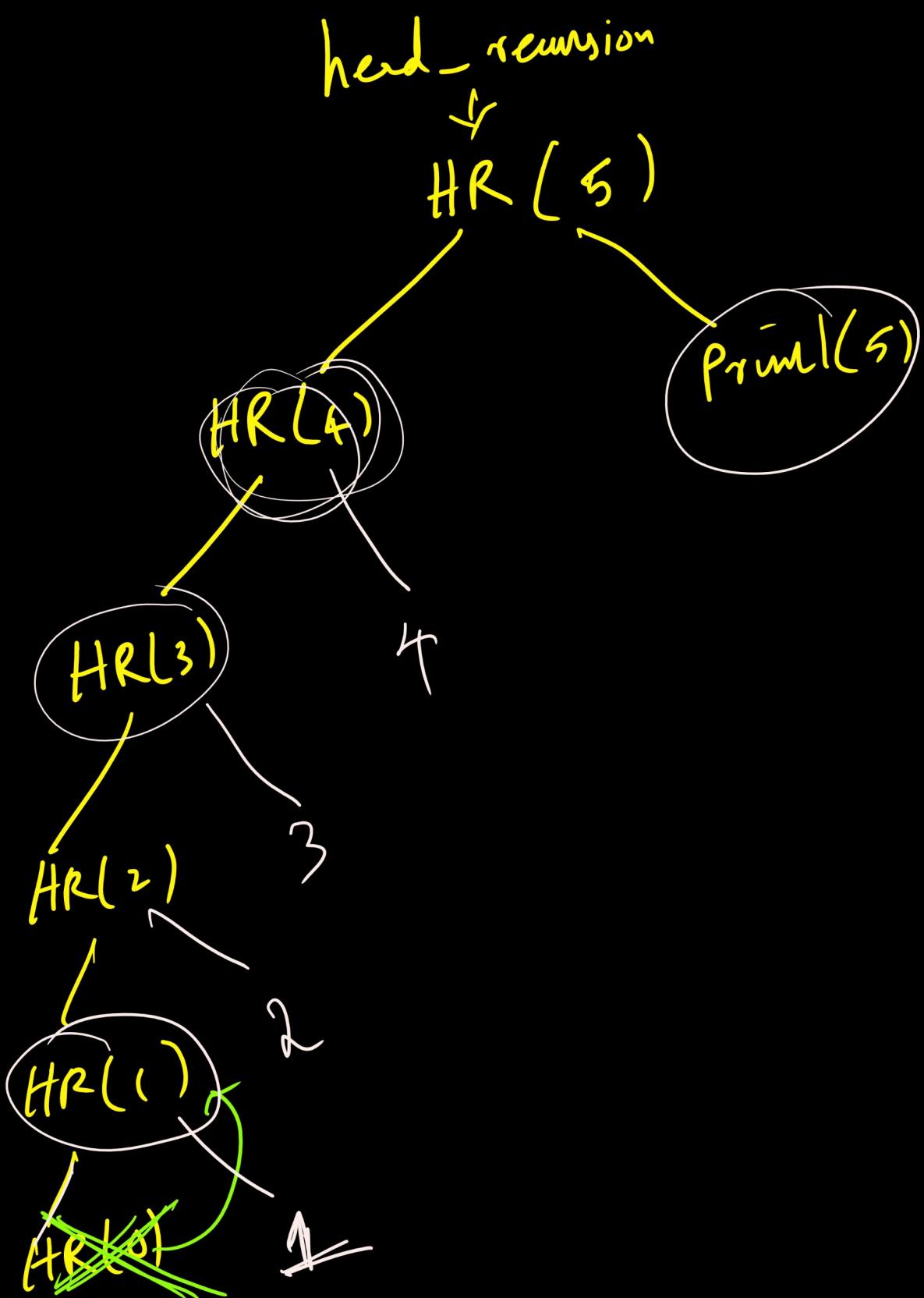
Tail recursion occurs when the recursive call is last thing executed by the function.

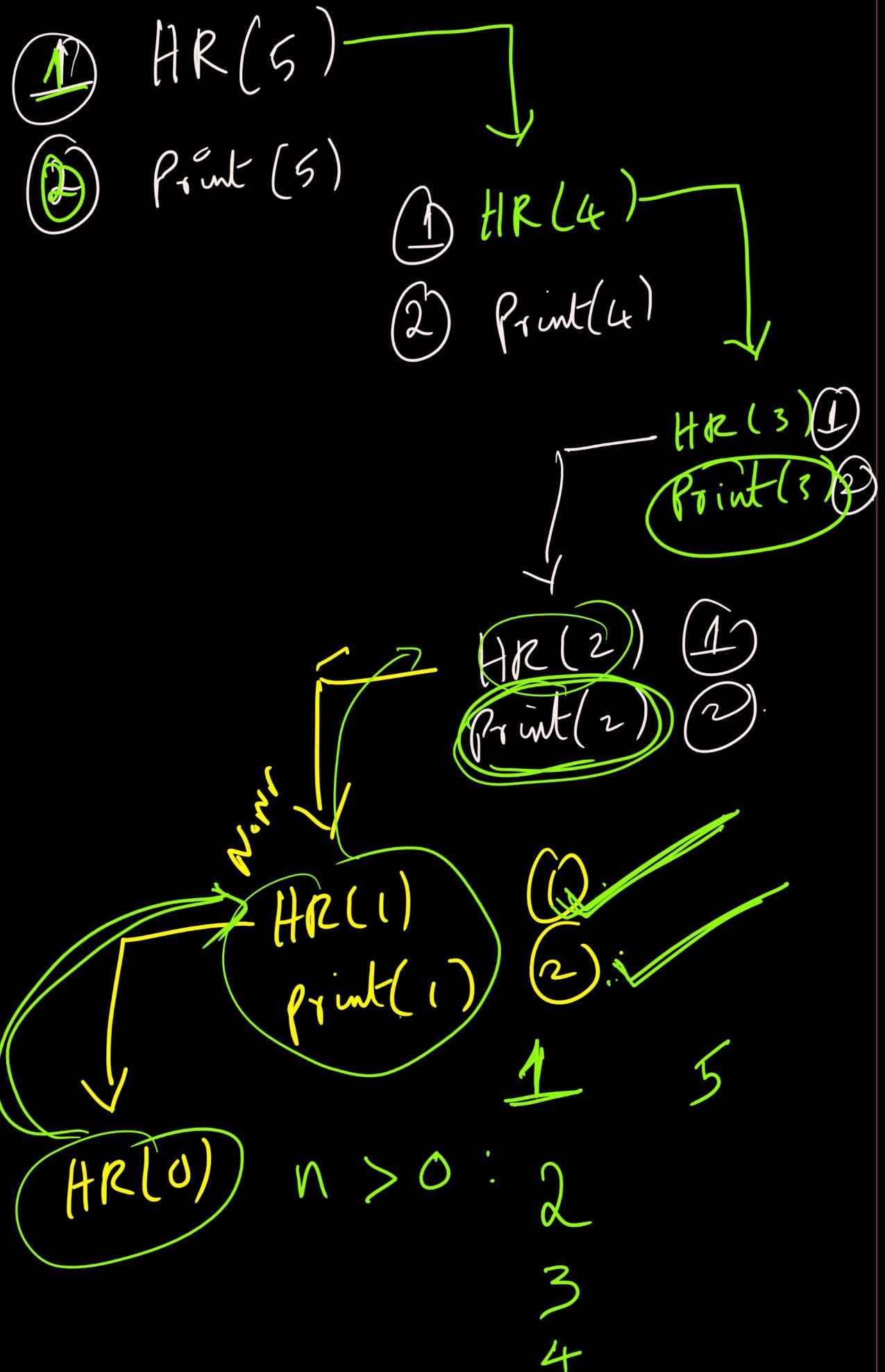
```
def head_recursion(n):
```

```
    if n>0:
```

```
        head_recursion(n-1)
```

```
        print("n val: -->", n)
```





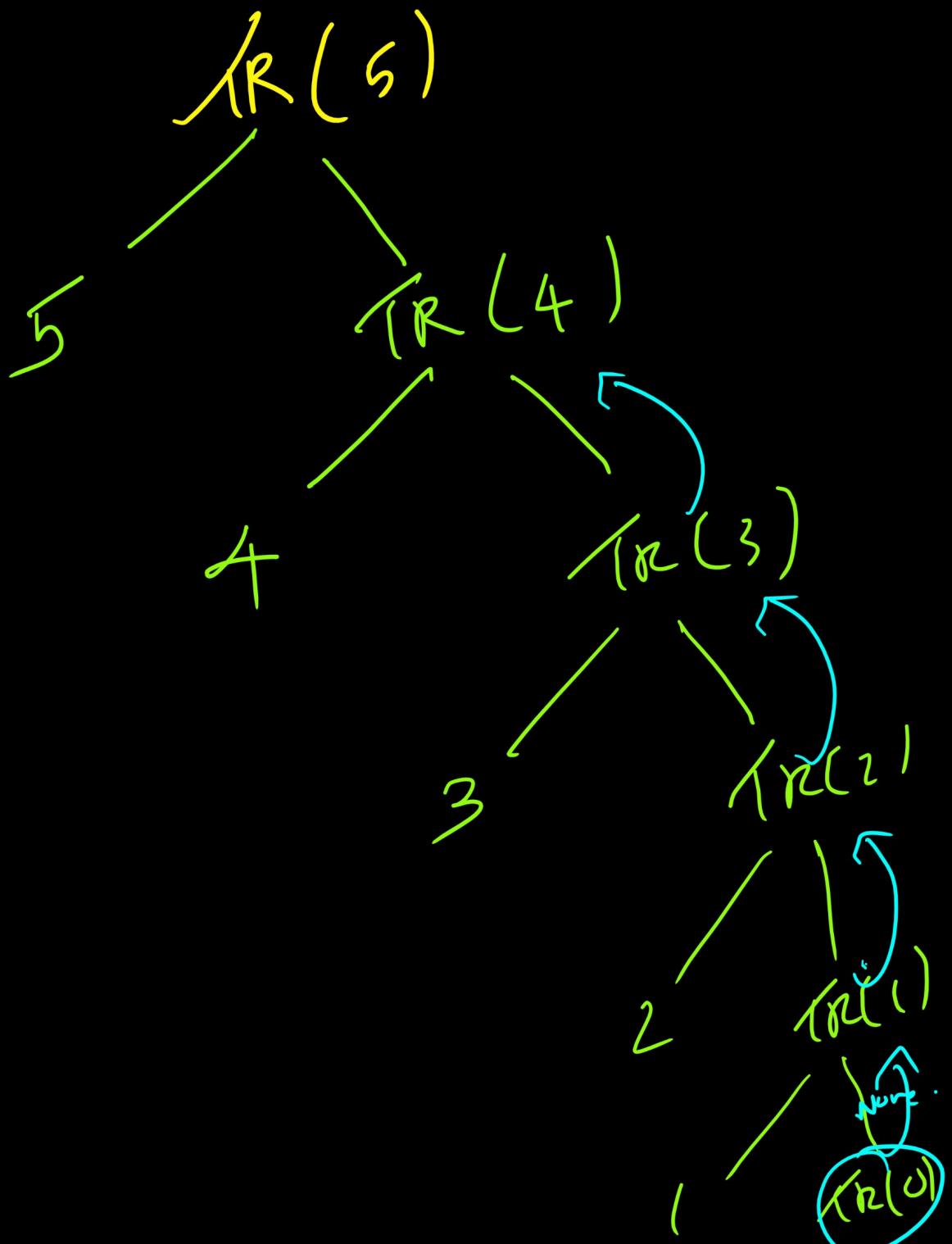
```
def tail_recursion(n):
```

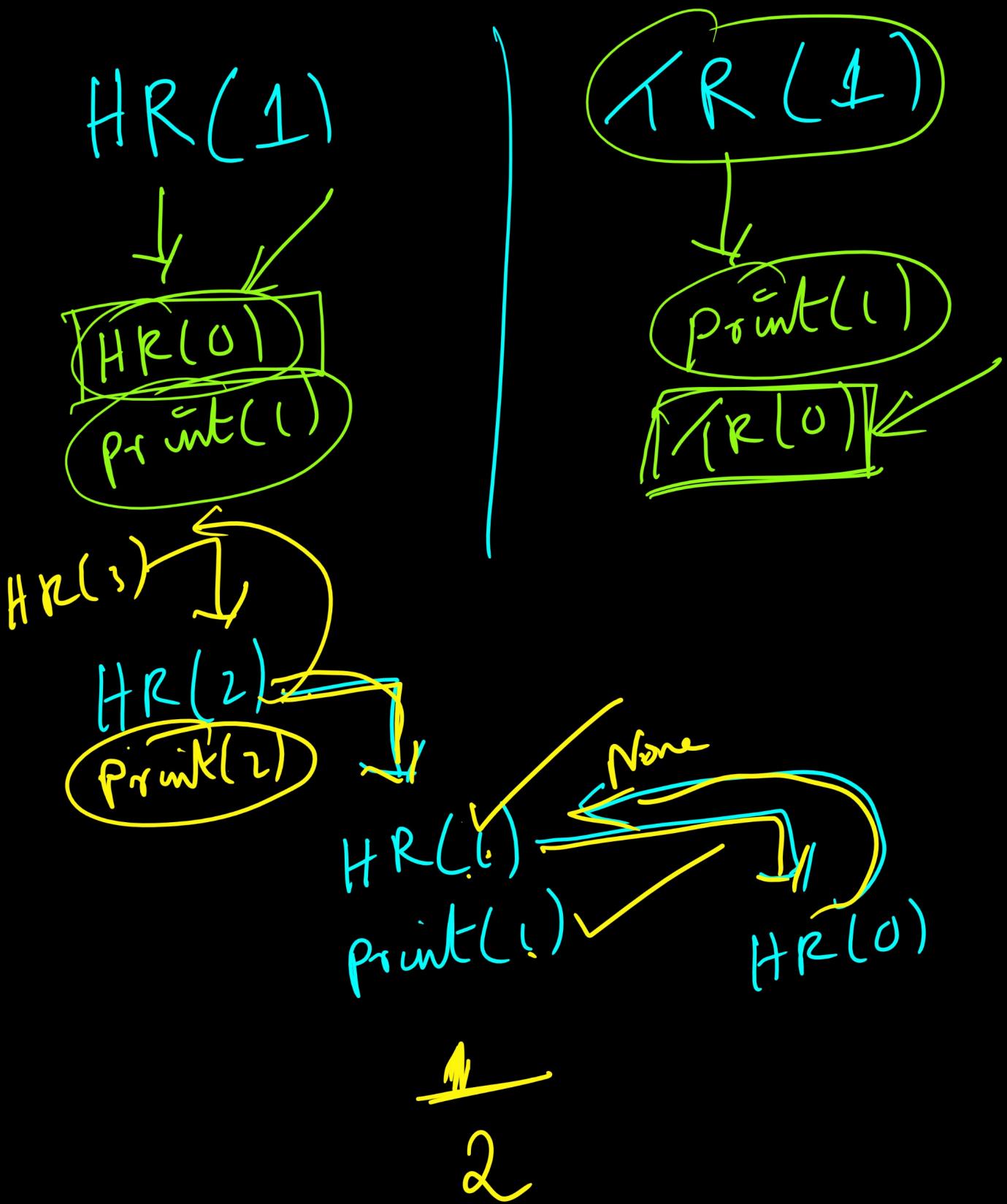
```
    if n>0:
```

```
        print("n val: -->", n)
```

```
        tail_recursion(n-1)
```

tail Recursion





Finite Revision:

Finite Revision terminates after a finite number of recursive calls.

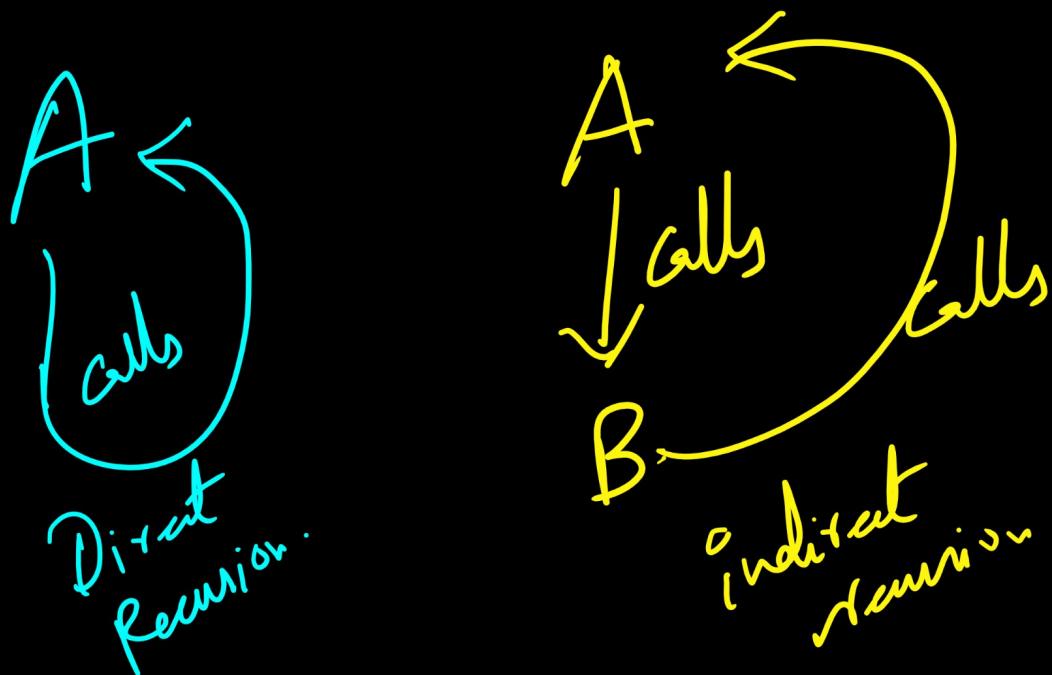
Infinite Revision: function calls itself indefinitely. base condition won't get satisfied

Direct Revision:

Direct Revision occurs when a function calls itself directly.

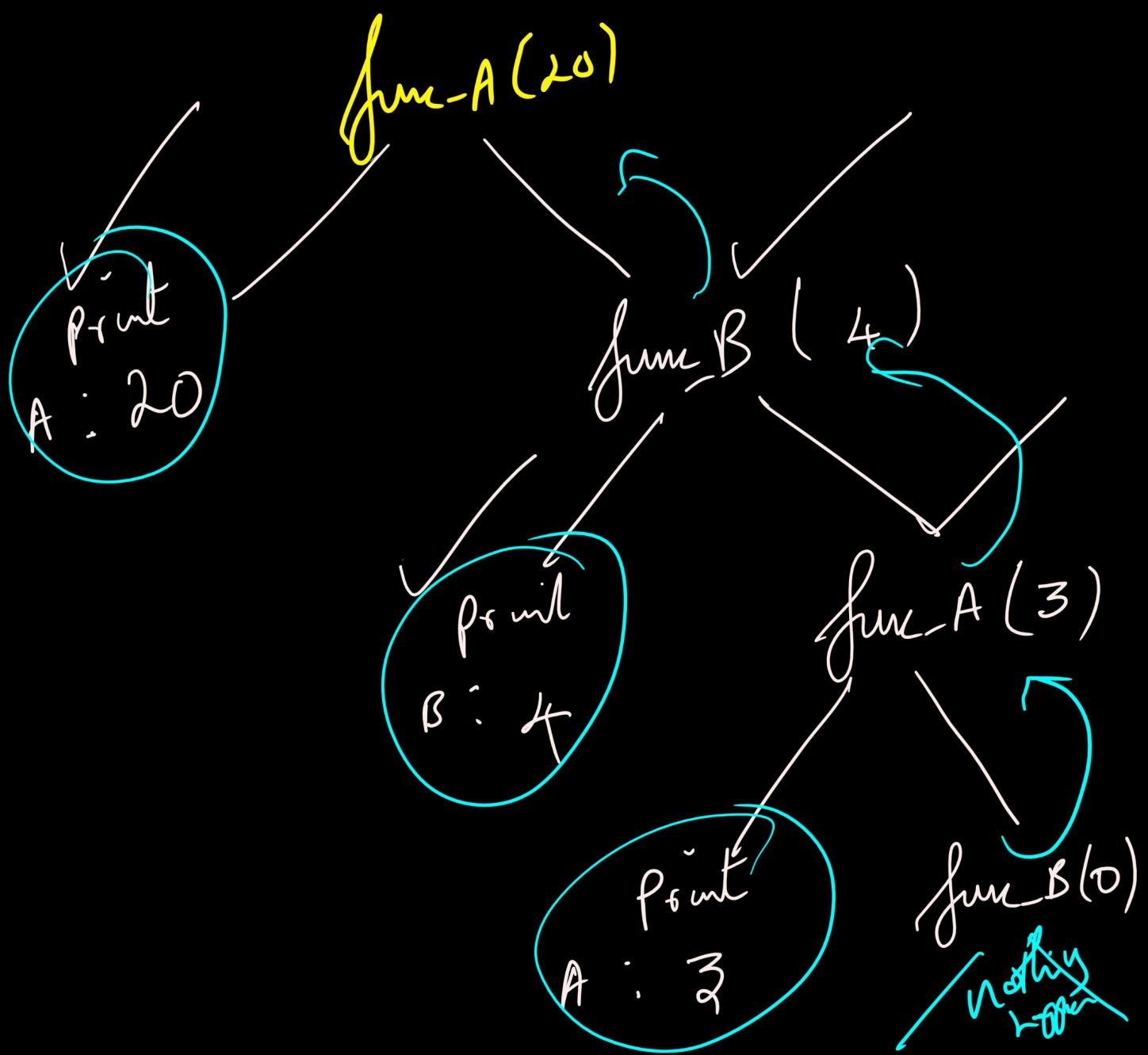
Indirect Recursion:

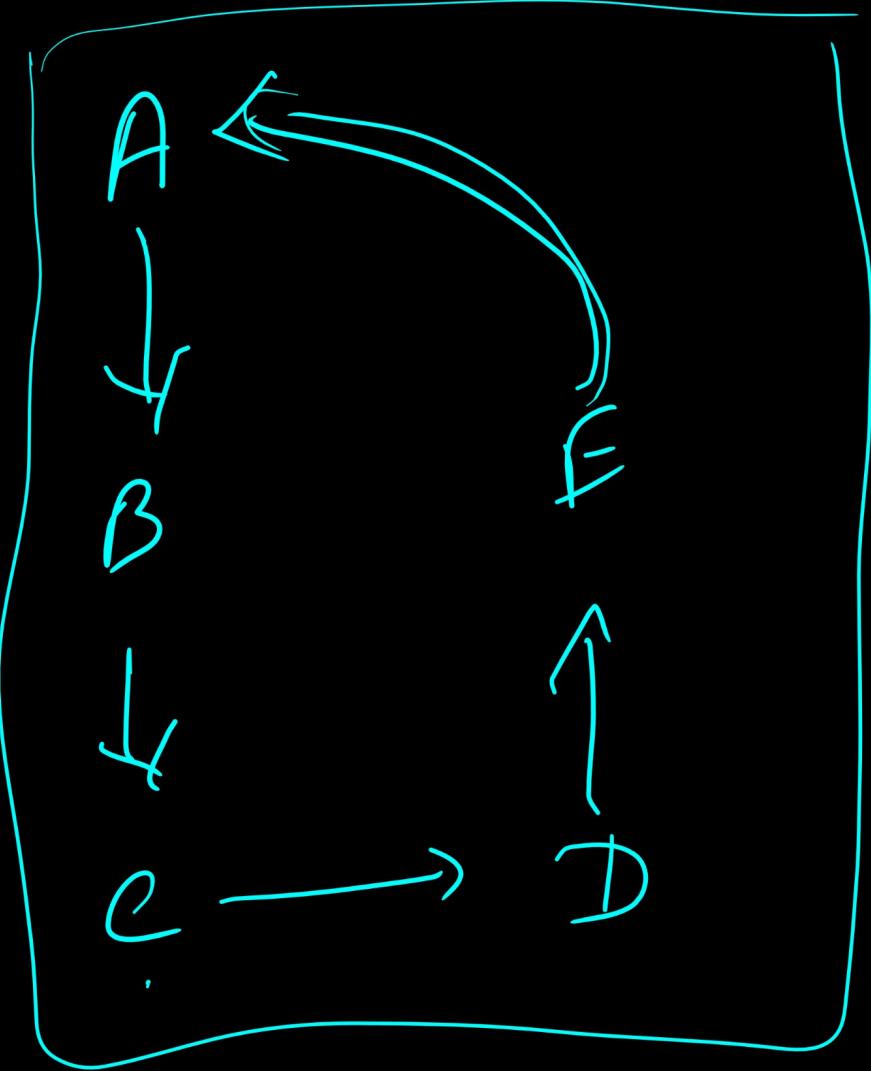
Indirect recursion occurs when a function calls another function that eventually calls first function again directly / indirectly.



```
def func_A(n):
    if n>0:
        print("In func A: -->", n)
        func_B(n//5)
```

```
def func_B(n):
    if n>0:
        print("In func B: -->", n)
        func_A(n-1)
```





Nested Recursion:

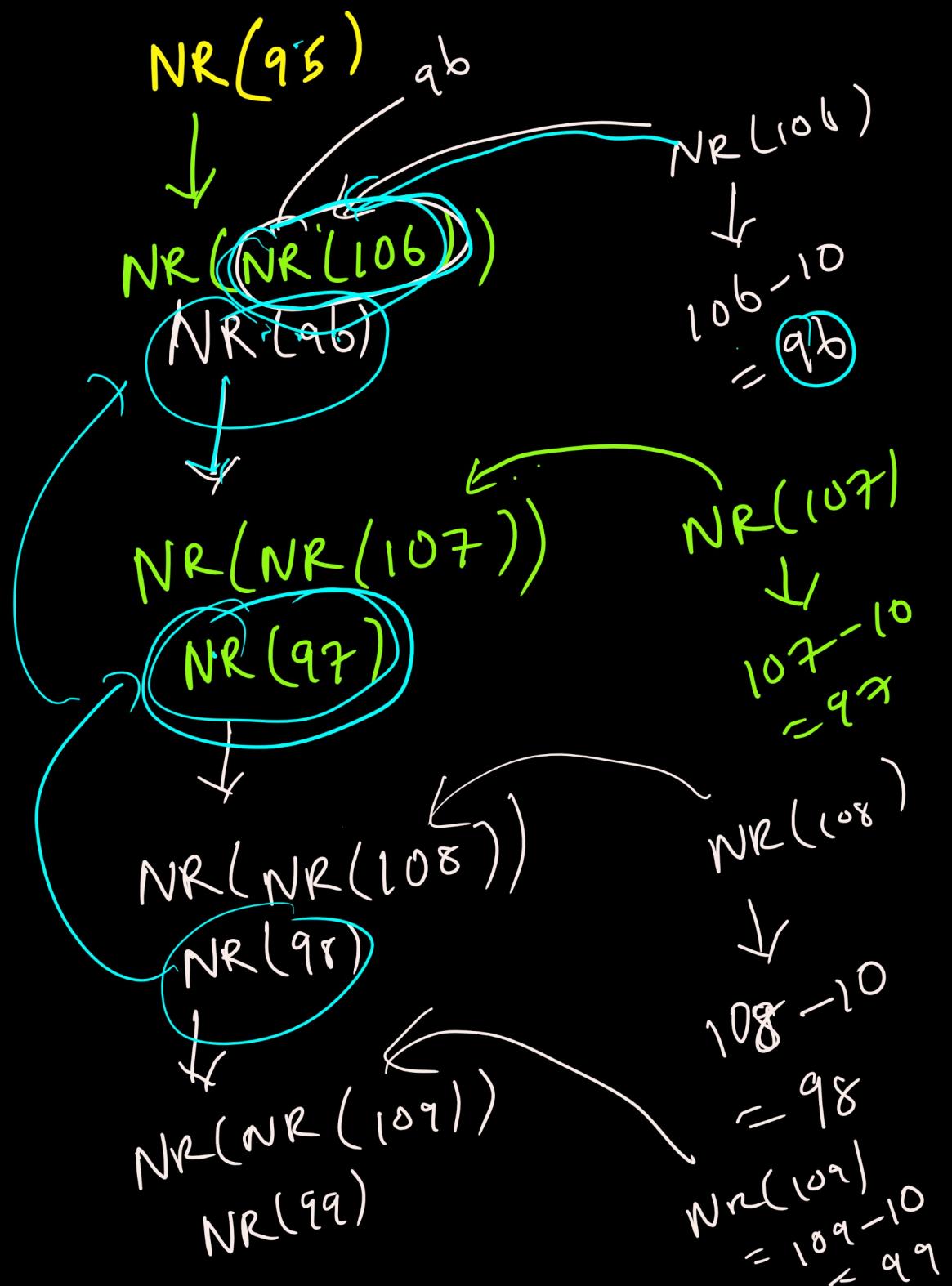
In this recursion, a recursive function will pass the parameter in a recursive call. That means recursion inside recursion.

```

def nested_recursion(n):
    if n>100:
        return n-10
    return nested_recursion(nested_recursion(n+11))

```

nested_recursion(95)



$NR(99)$
 \downarrow
 $NR(NR(110))$
 $NR(100)$
 \downarrow
 $NR(NR(111))$
 $NR(101)$
 \downarrow
 $101-10$
 $= 91$

$NR(110)$
 \downarrow
 $M^0 - 10$
 $= 100$

$NR(111)$
 \downarrow
 $111-10$
 $= 101$

$NR(95)$
 \downarrow
 $WR(NR(106))$
 $NR(106)$
 \downarrow
 $WR(NR(x))$

RF → Continued

↓
Anonymous function (Lambda function)
a Map filter
Saturday