

Arrays, Strings & Linked Lists Lecture 8

Wednesday, 31 July 2024

6:05 AM

Binary Search

Sorted array:-

Find 14



$$T(n) = T(n/2) + O(1)$$

↓

$$T(n) = O(\log n)$$

Linear Search:- $O(n)$

$$T(n/4) + 1 + 1 = T(n/4) + 2$$

$$T(n/8) + 1 + 1 + 1 = T(n/8) + 3$$

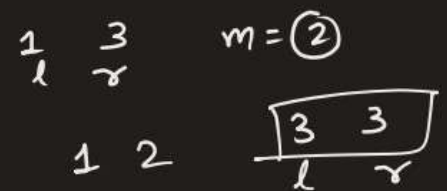
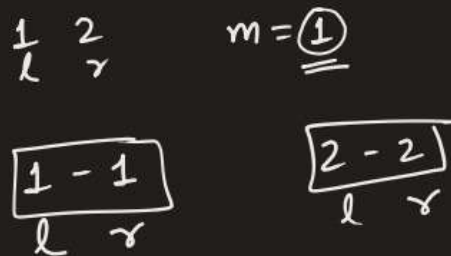
$$= T(n/2^k) + k$$

$$= T(n/2^{\log n}) + \log n$$

$$= T(n/n) + \log n$$

$$= \underline{\log n}$$

<https://leetcode.com/problems/binary-search/description/>



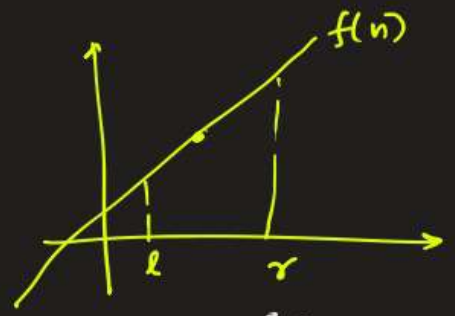
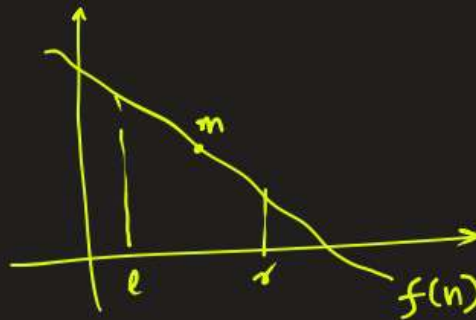
```
def bs(nums, l, r, target):
    if l == r:
        if nums[l] == target:
            return l
        else:
            return -1
    m = (l+r)//2
    if nums[m] >= target:
        return bs(nums, l, m, target)
    else:
        return bs(nums, m+1, r, target)

class Solution:
    def search(self, nums: List[int], target: int) -> int:
        return bs(nums, 0, len(nums)-1, target)
```

How to identify binary search problems?

- ① Input is sorted
- ② function (Input) is monotonous.

$f(n)$



Sorted array:-

Find (14)

all elem
 ≥ 14 T
 < 14 F

0	1	2	3	4	5	6	7	8	9	10
1	3	4	5	8	9	12	13	<u>14</u>	16	20
F	F	F	F	F	F	F	F	<u>T</u>	T	T

First occurrence of
True

<https://leetcode.com/problems/sqrtx>

$$\underline{\underline{n = 132}}$$

$$\sqrt{132} \approx 11.489$$

$$\lfloor \sqrt{132} \rfloor = \underline{\underline{11}}$$

[1]	2	3	4	5	6	7	8	9	10	11	12	13	...	132
F	F	F	F	F	F	F	F	F	F	T	T	T	T	T

$$n \rightarrow \text{long long}$$

$$n \sim 10^{18}$$

$$\log n$$

$$18 \times \log 10 \\ = 18 \times 3.1 \\ \sim \underline{\underline{50-60}}$$

$$\textcircled{11} \rightarrow \textcircled{12}$$

$$\frac{132}{2} = \underline{\underline{66}}$$

$$1 - \underline{\underline{66}}$$

$$1 - 33$$

$$1 - 17$$

$$8 - 17$$

$$8 - 12$$

$$10 - 12$$

$$\boxed{11 - 12}$$

$$\textcircled{2}$$

$$\textcircled{3}$$

$$\rightarrow \textcircled{11}$$

$$66^2 > 132$$

$$33^2 > 132$$

$$17^2 > 132$$

$$8^2 < 132$$

$$12^2 > 132$$

$$10^2 < 132$$

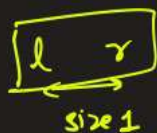
$$11^2 < 132$$

$$\boxed{\log n} \checkmark$$

$$i^2 < n$$

$$\textcircled{\sqrt{n}} \checkmark$$

$$\left. \begin{array}{l} bs(l, m) \\ bs(m+1, r) \end{array} \right\}$$



$$\begin{array}{cc} 1 & 4 \quad m=2 \\ 1 & 2 \quad \leftarrow m=1 \\ 3 & 4 \end{array} \rightarrow \begin{array}{cc} 1 & 1 \checkmark \\ 2 & 2 \checkmark \end{array}$$

$$\begin{array}{cc} 1 & 3 \quad m=2 \\ 1 & 2 \rightarrow \begin{array}{cc} 1 & 1 \\ 2 & 2 \end{array} \\ 3 & 3 \end{array}$$

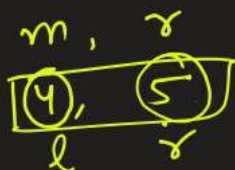
$$\left. \begin{array}{l} bs(l, \underline{m}) \\ bs(\underline{m}, r) \end{array} \right\}$$

$$\begin{array}{cc} 1 & 4 \quad m=2 \\ \boxed{1 & 2} \\ 2 & 4 \end{array}$$

$$\begin{array}{cc} 1 & 3 \quad m=2 \\ \boxed{1 & 2} \leftarrow \\ \boxed{2 & 3} \leftarrow \\ l & r \end{array}$$

$$\begin{array}{cc} 3 & 5 \\ m = \underline{4} \end{array} \quad (18)$$

$$\underline{m * m} \leq \underline{r}$$



$$3 \quad 5 \quad m=4 \quad (16)$$

```
class Solution {
public:
    int bs(long l, long r, long x) {
        if(l == r-1)
            return l;
        long m = (l+r)/2;
        if(m*m > x)
            return bs(l, m, x);
        else
            return bs(m, r, x);
    }
    int mySqrt(int x) {
        return bs(0, (long)x+1, x);
    }
};
```

<https://www.geeksforgeeks.org/problems/koko-eating-bananas/1>

1, 2
F T

$n = 4$

piles = [3, 6, 7, 11]

$H = 8$

$S = 1$

$O(n)$

$3 + 6 + 7 + 11 = 27$ hours ✓

[0, 0, 0, 0]
[0, 0, 0, 1]
[0, 0, 0, 3]
[0, 0, 0, 5]
[0, 0, 0, 7]
[0, 0, 0, 9]
[0, 0, 0, 11]

$S = 2$

$O(n)$

[3, 6, 7, 11]
↓ ✓
[1, 6, 7, 11]
↓ ✓
[0, 6, 7, 11] →

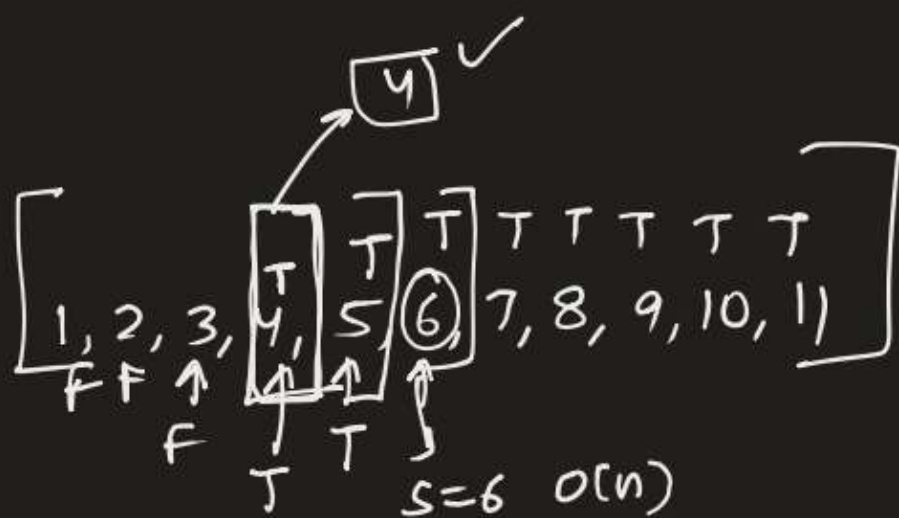
[0, 0, 7, 11] ✓
↑ ✓
[0, 2, 7, 11]
↑ ✓
[0, 4, 7, 11]

[0, 0, 5, 11]
↓ ✓
[0, 0, 3, 11]
↓ ✓
[0, 0, 1, 11] →

[0, 0, 0, 11]

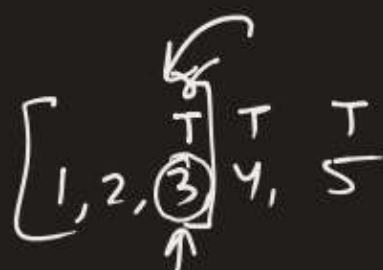
15 hours

$$\lceil \frac{3}{2} \rceil + \lceil \frac{6}{2} \rceil + \lceil \frac{7}{2} \rceil + \lceil \frac{11}{2} \rceil = 2 + 3 + 4 + 6 = 15$$



$s=6 \quad O(n)$

$$\begin{aligned} & \left\lceil \frac{3}{6} \right\rceil + \left\lceil \frac{6}{6} \right\rceil + \left\lceil \frac{7}{6} \right\rceil + \left\lceil \frac{11}{6} \right\rceil \\ &= 1 + 1 + 2 + 2 \\ &= 6 \end{aligned}$$



$$\begin{aligned} & \text{FF F} \\ & 1, 2, 3, \left[\begin{array}{c} 4 \\ m+1 \end{array}, \begin{array}{c} 5 \\ r \end{array} \right] \end{aligned}$$

$$O(n \cdot \log m) = O(10^4 \cdot \log 10^9)$$

$$\sim 9 \times 10^4 \times 3$$

$$\sim \underline{\underline{10^6}}$$

$F F F F F F$
 $1, 2, 3, 4, 5, 6, \underbrace{7, 8, 9, 10, 11}_{r}$
 \uparrow
 (m)
 $\underbrace{\hspace{1.5cm}}_{m+1}$

$\underline{T}: \underline{h} \leq H \quad \checkmark$
 $\underline{F}: \underline{h} > \underline{H} \quad \checkmark$

if $h[m] \leq H$:
 $bs(l, m)$

if $h[m] > H$:
 $bs(m+1, r)$

```
def bs(piles, H, l, r):  
    if l==r:  
        return l  
    m = (l+r)//2  
    h = 0  
    for p in piles:  
        h += p//m  
        if p%m != 0:  
            h += 1  
    if h <= H:  
        return bs(piles, H, l, m)  
    else:  
        return bs(piles, H, m+1, r)
```

```
class Solution:  
    def Solve(self, N, piles, H):  
        return bs(piles, H, 1, max(piles))
```