

Binary Search



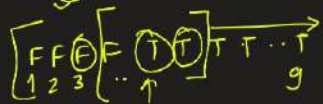
<https://leetcode.com/problems/find-the-smallest-divisor-given-a-threshold/>

1 1 1 1
1, 2, 5, 9

6

$O(n)$

$x=1 \rightarrow$	$1+2+5+9=17 > 6$	F
$x=2 \rightarrow$	$1+1+3+5=10 > 6$	F
$x=3 \rightarrow$	$1+1+2+3=7 > 6$	F
$x=4 \rightarrow$	$1+1+2+3=7 > 6$	F
$x=5 \rightarrow$	$1+1+1+2=5 \leq 6$	T

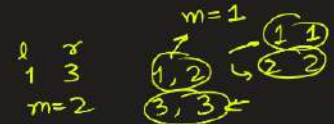


$$f(x) = \sum_{a \in arr} \lceil \frac{a}{x} \rceil$$

$f(x) \leq \text{Threshold} \rightarrow \text{True} \rightarrow [l, m]$
 $f(x) > \text{Threshold} \rightarrow \text{False} \rightarrow [m+1, r]$

l, m
 m, r
 base 2 elements

base 1 element



```
def bs(nums, t, l, r):
    if l == r:
        return l
    m = (l+r)//2
    s = 0
    for n in nums:
        s += n//m
        if n%m != 0:
            s += 1
    if s <= t:
        return bs(nums, t, l, m)
    else:
        return bs(nums, t, m+1, r)

class Solution:
    def smallestDivisor(self, nums: List[int], threshold: int) -> int:
        return bs(nums, threshold, 1, max(nums))
```

<https://www.geeksforgeeks.org/problems/minimum-days-to-make-m-bouquets/1>

$O(n)$

$m = 2$ (need m such boy)
 $k = 3$ (no of adjacent flowers needed to create 1 boy)

bloomDay = [5, 5, 5, 5, 10, 5, 5]

Day	Flowers	# boy	Result
day 1	x x x x x x x	0	F
day 2	x x x x x x x	0	F
day 5	✓ ✓ ✓ ✓ x ✓ ✓	1	F
day 6	✓ ✓ ✓ ✓ x ✓ ✓	1	F
day 10	✓ ✓ ✓ ✓ ✓ ✓ ✓	2	T
day 11			T

at least m boy

$\lfloor n/k \rfloor < m$ (1)

bog. is an increasing function on # days
 min value of # days such that # bog. $\geq m$

day(x) How many bog. can we create?
 $k=3$

days $\rightarrow x$
 min days 1
 max days $\max(\log arr) \sim 10^9$
 $\log(10^9) \times O(n) \sim 35 \times 10^5 \sim 3.5 \times 10^6$
 $\log(10^9) \sim 9 \times \log 10 \sim 9 \times 3.2 \sim 35$

$x \rightarrow f(x)$ # bog.
 $f(x) < m \quad F \rightarrow \underline{m+1, r}$
 $f(x) \geq m \quad T \rightarrow \underline{l, m}$
 1 2 3 4 max(arr)
 F F F T T T

base
 1

```

class Solution {
public:
    // At xth day returns the maximum no of
    // bouquets you can make
    int nb(vector<int> &bd, int k, int x) {
        int mc = 0, ans = 0;
        for(auto b: bd) {
            if(b<=x) mc++;
            else mc = 0;
            if(mc == k) {
                ans++;
                mc = 0;
            }
        }
        return ans;
    }
};

```

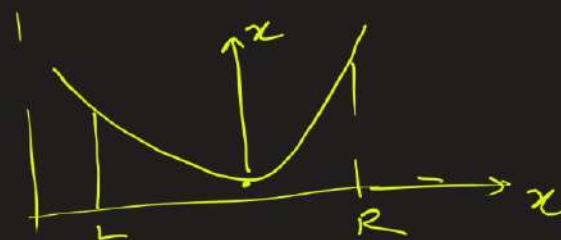
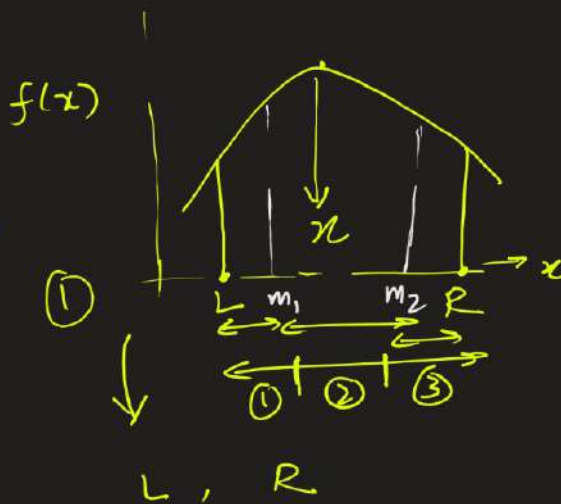
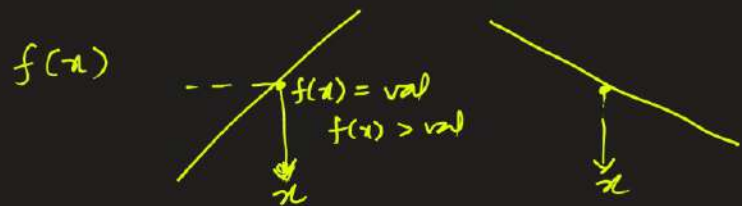
```

int bs(vector<int> &bd, int m, int k, int l, int r) {
    if(l == r)
        return l;
    int mid = (l+r)/2;
    int s = nb(bd, k, mid);
    if(s < m)
        return bs(bd, m, k, mid+1, r);
    else
        return bs(bd, m, k, l, mid);
}

int solve(int M, int K, vector<int> &bloomDay) {
    if(bloomDay.size()/K < M) return -1;
    return bs(
        bloomDay, M, K, 1,
        *max_element(
            bloomDay.begin(),
            bloomDay.end()
        )
    );
}

```

Ternary Search



$$m_1 = L + (R-L)/3 \quad \checkmark$$

$$m_2 = L + 2(R-L)/3 \equiv R - (R-L)/3 \quad \checkmark$$

$$L, \frac{f(m_1)}{O(n)}, \frac{f(m_2)}{O(n)}, R$$

$$\frac{O(n)}{L, R} \rightarrow \frac{f(m_1)}{O(n)} > \frac{f(m_2)}{O(n)}$$

$$(L, m_2) \quad \checkmark \quad (2/3)$$



$$\frac{O(n)}{L, R} \rightarrow \frac{f(m_1)}{O(n)} < \frac{f(m_2)}{O(n)}$$

$$(m_1, R) \quad \checkmark \quad (2/3)$$



$$\frac{O(n)}{L, R} \rightarrow \frac{f(m_1)}{O(n)} = \frac{f(m_2)}{O(n)}$$

$$(m_1, m_2) \quad \checkmark \quad (1/3)$$



$$T(n) = T(2n/3) + O(n)$$

$$\hookrightarrow T(n) = O(n \log n) = \checkmark \rightarrow BS$$

$$T(n) = T(2n/3) + O(1)$$

$$\hookrightarrow T(n) = O(\log n) \rightarrow BS$$

$$\frac{L}{n} \quad R$$



$$R-L < 3 \rightarrow$$

$$\left. \begin{array}{l} R-L=0 \quad (1) \\ R-L=1 \quad (2) \\ R-L=2 \quad (3) \end{array} \right\}$$

$$R-L < 2$$

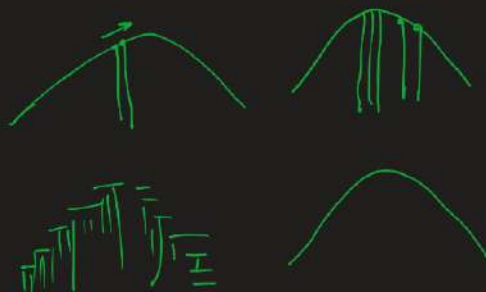
$$R-L=1 \quad (2)$$

$$R-L=0 \quad (1)$$

$$T(n) = T(n/2) + \text{---}$$

$$T(n) = T(2n/3) + \text{---}$$

<https://codeforces.com/problemset/problem/1978/B>



first k

$$\boxed{b, b-1, b-2, \dots, b-k+1}$$

remaining $n-k$

$$\boxed{a, a, a, \dots}$$

\downarrow

$$(n-k)a$$

max

$$\boxed{kb - \frac{k(k-1)}{2} + (n-k)a}$$

$n \checkmark$
 $a \checkmark$
 $b \checkmark$



$n=5$
 $a=5$
 $b=9$

$$\left\{ \begin{array}{ll} k=1 & 9 + 4(5) = 29 \\ k=2 & 9+8 + 3(5) = 32 \\ k=3 & 9+8+7 + 2(5) = 34 \\ k=4 & 9+8+7+6 + 1(5) = \boxed{35} \\ k=\boxed{5} & 9+8+7+6+5 = \boxed{35} \end{array} \right.$$

$$S(8) = 40$$

$n = 5$
 $a = 8$
 $b = 10$

$k=0 \quad 5(8) = 40$
 $k=1 \quad 10 + 4(8) = 42$
 $k=2 \quad 10 + 9 + 3(8) = 43$
 $k=3 \quad 10 + 9 + 8 + 2(8) = 43$
 $k=4 \quad 10 + 9 + 8 + 7 + 1(8) = 42$
 $k=5 \quad 10 + 9 + 8 + 7 + 6 + 0(8) = 40$



$\min(k) = 0$
 $\max(k) = 1$

```

ll pr(ll n, ll a, ll b, ll k) {
    return k*b - (k*(k-1))/2 + (n-k)*a;
}
ll ts(ll n, ll a, ll b, ll l, ll r) {
    if(r-l < 3) {
        ll ans = 0;
        FOR(i, l, r+1) ans = max(ans, pr(n, a, b, i));
        return ans;
    }
    ll m1 = l + (r-l)/3;
    ll m2 = r - (r-l)/3;
    ll p1 = pr(n, a, b, m1);
    ll p2 = pr(n, a, b, m2);
    if(p1 < p2) return ts(n, a, b, m1, r);
    if(p1 > p2) return ts(n, a, b, l, m2);
    return ts(n, a, b, m1, m2);
}
void solve() {
    ll n, a, b;
    cin >> n >> a >> b;
    cout << ts(n, a, b, 0, min(n, b)) << endl;
}

```

$l = 1 \quad r = 4$

$m_1 = r - l = 3$
 $(r-l)/3 = 1$

$m_1 = 2$

$m_2 = 3$

$1, 2, 3, 4$

$(2, 4) \quad (1, 3) \quad (2, 3)$

$2, 4$

$(r-l)/3 = 0$

$m_1 = 2$

$m_2 = 4$

$2, 2, 4, 4$

$(2, 4) \quad (2, 4) \quad (2, 4)$