

## Arrays & Strings Lecture 1

Saturday, 20 July 2024

2:56 PM

① Find the missing number in an array

<https://leetcode.com/problems/missing-number/>

$0 \rightarrow n$

$n+1$

$a = \{0, 3, 4, 2\}$

①

$|a| = 4$

$[0-4]$

$|a| = \underline{n}$

$[0, n]$

$a = \{2, 0, 1, 5, 3\}$

④

$|a| = 5$

$[0, 5]$

(i) Brute force strategy.

```
for (i: 0 → n) {  
  flag = false;  
  for (a in arr) {  
    if (a == i) {  
      flag = true;  
      break;  
    }  
  }  
  if (flag == false)  
    return i;  
}
```

Time complexity:-  $O(n^2)$

Space :-  $O(1)$

(ii) Better approach.

Sort the array.

0, 1, 2, 4, 5, ...

Find Missing Element (arr, n) {

sort(arr);  $\rightarrow n \log n$

{ for (i: 0  $\rightarrow$  n) {  
if (arr[i]  $\neq$  i) return i;  
}

}

$T = O(n \log n)$   
 $S = O(1)$

(iii) Better approach. (Hashing).

$0 \rightarrow n$

$\begin{cases} \text{map} \rightarrow (K, V) \\ \text{un-map} \rightarrow \\ \text{dict}() \end{cases}$

vector < bool > hash (n+1, false);

$0 \rightarrow T/F$

$i \rightarrow T/F$

$\{0, 2, 4, 1, 5\}$   
 $\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \\ \text{True} & \text{True} & \text{False} & \text{True} & \text{True} & \text{True} \end{matrix}$  missing

$\text{for } (\underline{a} : \underline{arr}) \{$   
 $\quad \underline{\text{hash}}[\underline{a}] = \underline{\text{True}}; \}$   $O(n)$   
 $\}$

$T = O(n)$   
 $S = O(n)$

$\text{for } (\underline{i} : 0 \rightarrow n) \{$   
 $\quad \text{if } (\underline{\text{hash}}[\underline{i}] == \text{False}) \quad \underline{\text{return } i}; \}$   $O(n)$

$\}$

(iv) optimal

Sum of the numbers:

$$[0, n] \rightarrow \boxed{\frac{n(n+1)}{2}} \leftarrow O(1)$$

$$\{0, 1, 2, 3, 4\} \rightarrow \textcircled{10}$$

$$\{\underline{3}, \underline{4}, \underline{1}, 0\} \rightarrow \frac{\textcircled{8}}{\boxed{2}}$$

sum  $\leftarrow 0$

for (a: arr) {  
    sum += a;  
}

}  $O(n)$

return  $\frac{(n * (n+1))}{2} - \text{sum};$

$T = O(n)$

$S = O(1)$

(v) Best Solution:

- $a \oplus a = 0$

- $a \oplus 0 = a$

$$a \oplus a$$

$$\begin{array}{r} 11011 \\ 11011 \\ \hline 00000 \end{array}$$

XOR	a	b	$a \wedge b$
	T	T	F
	T	F	T
	F	T	T
	F	F	F

$$\oplus \begin{array}{r} 11011 \\ 00000 \\ \hline 11011 \end{array}$$

{ 3, 2, 0, 5, 1 }    |a| = 5,    [0, 5]

$\cancel{3} \wedge \cancel{2} \wedge \cancel{0} \wedge \cancel{5} \wedge \cancel{1} \wedge \cancel{0} \wedge \cancel{1} \wedge \cancel{3} \wedge \cancel{2} \wedge \cancel{0} \wedge \cancel{5} \wedge \cancel{1} \wedge \boxed{4}$

```

ans = 0
for (a : arr) {
    ans = ans ^ a;
}
for (i : 0 → n) {
    ans = ans ^ i;
}
return ans;
    
```

T =  $O(n)$   
S =  $O(1)$

<https://leetcode.com/problems/missing-number/>

```
class Solution {
public:
    int missingNumber(vector<int>& nums) {
        int ans = 0;
        for(auto n: nums) ans ^= n;
        for(int i=0; i<=nums.size(); i++) ans ^= i;
        return ans;
    }
};
```

XOR of all elements in array  
XOR of all elements in  $[0, n]$

$=X=$

Subarrays & Subsequences

2, 3, 4, 1, 5, 6, 7, 8  
✓ ✓ ✓ ✓ ✓ ✓

2, 1, 7, 8 ✓  
1, 3, 2 X

contiguous

- ✗ Every subsequence is a subarray? NO
- Every subarray is a subsequence? Yes



$arr = [1, 2, 3, 4]$      $|arr| = 4$      $4, 4!, 2^4 = 16,$

# Subarrays of  $arr = 10$  (if we don't add empty subarray)

$arr = [1, 2, 3, 4]$

$\begin{matrix} 0, 1, 2, 3 \\ \vdots \quad \vdots \end{matrix}$

$$4 + 3 + 2 + 1 = 10$$

$$\boxed{\frac{n(n+1)}{2}}$$

$i=0, j=0$   $\boxed{1}$

$i=0, j=1$   $\boxed{1, 2}$

$i=0, j=2$   $\boxed{1, 2, 3}$

$i=0, j=3$   $\boxed{1, 2, 3, 4}$

# Subarrays =  $O(n^2)$

# Subsequences for  $arr$ ?

$arr = [1, 2, 3, 4]$

$2^4$  (if you are counting empty)



$$\{1, 2, 3, 4\} \rightarrow$$

$$\uparrow \uparrow \uparrow \uparrow$$

$$2 \times 2 \times 2 \times 2 = \underline{\underline{16}}$$

$$\binom{4}{0} = 1$$

$$\binom{4}{1} = 4$$

$$\binom{4}{2} = 6$$

$$\binom{4}{3} = 4$$

$$\binom{4}{4} = 1$$

$$\binom{4}{0} + \binom{4}{1} + \dots + \binom{4}{4}$$

$$= 2^4 = \underline{\underline{16}}$$

$$\# \text{ Subseq.} = 2^n$$

$$\# \text{ Subsequences} = \boxed{O(2^n)}$$

array of size  $n$  (arr) is given. How many pair of elements  $i, j$  exist such that  $i, j \in \text{arr}$  and  $i \neq j$

$$\binom{n}{2} \text{ pair of elements.} = O(n^2)$$

$$\# \text{ triplets} = \binom{n}{3} = O(n^3)$$

$\Rightarrow x =$

- ② { Longest Subarray ✓ with sum equals k  
# Subarrays ✓  
whether a subarray exists ✓ }

<https://leetcode.com/problems/subarray-sum-equals-k>

# Subarrays with sum = k.

arr = { 1, 1, 2, 1, 3, 2, 1, 1, 1, 1, 4, 2 }

k = 6

2, 1, 3  
1, 3, 2  
3, 2, 1  
2, 1, 1, 1, 1  
1, 1, 4  
4, 2

} ⑥

(i) Brute force :-

check every subarray.

Find the sum

$sum == k$      $ans++$ ;

```
ans = 0;
```

```
for (i : 0 → n-1) {
```

```
    for (j : i → n-1) {
```

```
        sum = 0;
```

```
        for (l : i → j) {
```

```
            sum += arr[l];
```

```
        }
```

```
        if (sum == k) ans++;
```

```
    }
```

```
}
```

```
return ans;
```

1, 1, 2, 1, 3, 2, 1

T =

$O(n^3)$

$S = O(1)$

$\boxed{1, 1, 2, 1}$  3, 2, 1  
 $i=0$   $j=3$   $j=4$   
 $\vdots$

$s = 5$      $5 + 3 = 8$

$\} \rightarrow \underline{\underline{O(n^2)}}$

<https://leetcode.com/problems/subarray-sum-equals-k/>

```

class Solution {
public:
    int subarraySum(vector<int>& nums, int k) {
        int ans = 0, n = nums.size();
        for(int i=0; i<n; i++) {
            int sum = 0;
            for(int j=i; j<n; j++) {
                sum += nums[j];
                if(sum == k) ans++;
            }
        }
        return ans;
    }
};

```

$T = O(n^2)$

$S = O(1)$

Optimal :-

Optimizing :

	0	1	2	3	4	5	6	7	8	9	10	11		
arr =	{	1	, 1	, 2	, 1	, 3	, 2	, 1	, 1	, 1	, 4	, 2	}	
preSum =	{	1	, 2	, <u>4</u>	, 5	, 8	, 10	, 11	, 12	, <u>13</u>	, 14	, 18	, 20	}
suffSum =	{	20	, 17	, 18	, <u>16</u>	, 15	, 12	, 10	, 9	, 8	, 7	, 6	, 2	}

$$\text{Sum}[3, 8] =$$

$$\begin{array}{ccc} \text{pre}[8] - \text{pre}[2] & = & 9 \\ \downarrow & & \downarrow \\ \text{Sum}(0 \rightarrow 8) & & \text{Sum}(0 \rightarrow 2) \end{array}$$

$$\begin{array}{ccc} \text{Sum}[3, 8] = \text{Suff}[3] - \text{Suff}[9] & = & 16 - 7 = 9 \\ \downarrow & & \downarrow \\ \text{Sum}(3 \rightarrow n-1) & & \text{Sum}(9 \rightarrow n-1) \end{array}$$

	0	1	2	3	4	5	6	7	8	9	10	11		
arr =	{	1	, 1	, 2	, 1	, 3	, 2	, 1	, 1	, 1	, 4	, 2	}	
pre =	{	1	, 2	, 4	, 5	, 8	, 10	, 11	, 12	, 13	, 14	, 18	, 20	}

$$\underline{k=6}$$

$$\text{Sum till } 9 = 14$$

A Subarray  
 $\left\{ \begin{array}{l} \text{prefix Sum} \\ \text{Suffix Sum} \end{array} \right\}$  XOR

$$\text{pre}[i] = \sum_{j=0}^i \text{arr}[j]$$

⑦

$$14 - 6 = 8$$

arr = [1, -1, 2, -1, 3, -2, -1, -1, 1, 1, 1, 4, -2]  
 pre = [1, 0, 2, 1, 4, 2, 1, 0, 1, 2, 3, 7, 5]

k = 1

Sum = ~~0~~ ~~1~~ ~~0~~ ~~2~~ ~~1~~ ~~4~~ ~~2~~  
Sum - k = ~~0~~ ~~3~~ ~~1~~

4:1  
2:2  
0:1  
1:2

hashmap

frequency

int → int  
 string → int  
 int → string  
 int → pair  
 pair → int

↳ dictionary

map <k, v>

log #k

unordered\_map

<k, v>  
 O(1)  
 O(#k) } O(1)

ans = ~~0~~ ~~1~~ ~~2~~ 4

ans = ans + map [Sum - k]

```
class Solution {
public:
    int subarraySum(vector<int>& nums, int k) {
        unordered_map<int, int> m;
        int s=0, ans=0;
        m[0] = 1;
        for(auto n: nums) {
            s += n;
            ans += m[s-k];
            m[s]++;
        }
        return ans;
    }
};
```

$T = O(n)$  ✓  
 $S = \underline{O(n)}$  ✓