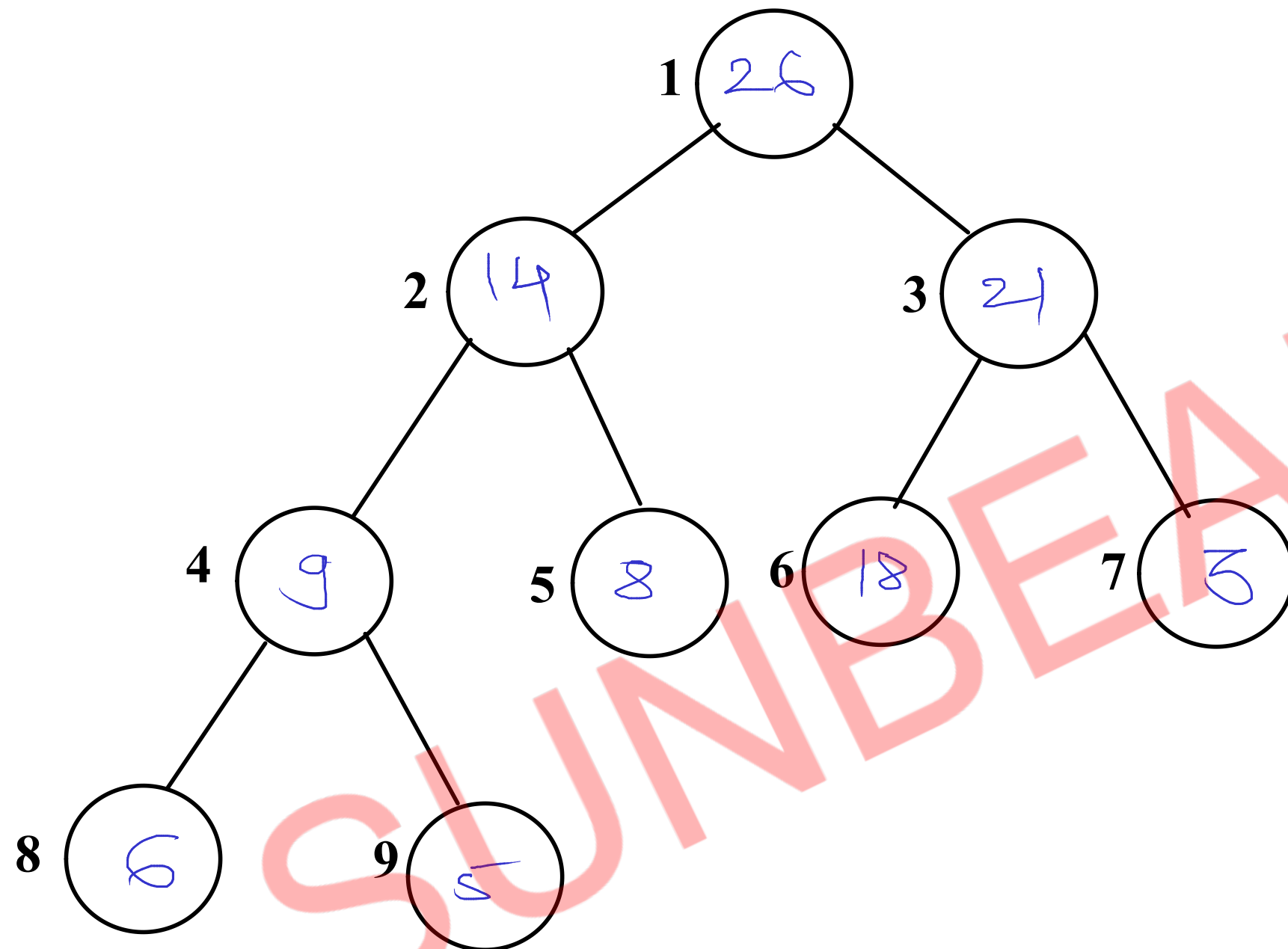


# Heapify



|   |    |   |    |   |    |    |   |   |
|---|----|---|----|---|----|----|---|---|
| 6 | 14 | 3 | 26 | 8 | 18 | 21 | 9 | 5 |
| 1 | 2  | 3 | 4  | 5 | 6  | 7  | 8 | 9 |

$$9/2 = 4$$

$$T(n) = O(n)$$

for( $i = \text{size}/2$ ;

$i > 1$ ;

$i--$ )

linear search

:-

$$T(n) = O(n)$$

Binary search

:-

$$T(n) = O(\log n)$$

Linked List search

:-

$$T(n) = O(n)$$

Binary Tree search

:-

$$T(n) = O(n)$$

BSTree search

:-

$$T(n) = O(\log n)$$

Hash Table

:-

$$T(n) = O(1)$$

# Hashing

- hashing is a technique in which data can be inserted, removed and searched in constant average time ( $O(1)$ )
- implementation of this technique is known hash table
- hash table is nothing but fixed size array in which elements are stored in key-value pair

Array - hash table

index - slot

- keys are always unique but values can be duplicates
- every key is mapped with one slot of the hash table.
- this mapping is done by a mathematical function known as "hash function"

# Hashing

$$h(k) = k \% \text{size}$$

size = 10

|     |       |  |  |
|-----|-------|--|--|
| Key | value |  |  |
| 8   | v1    |  |  |
| 3   | v2    |  |  |
| 10  | v3    |  |  |
| 4   | v4    |  |  |
| 6   | v5    |  |  |
| 13  | v6    |  |  |

|        |   |
|--------|---|
| 10, v3 | 0 |
|        | 1 |
|        | 2 |
| 3, v2  | 3 |
| 4, v4  | 4 |
|        | 5 |
| 6, v5  | 6 |
|        | 7 |
| 8, v1  | 8 |
|        | 9 |

collision

Hash Table

Collision:

- if multiple distinct keys yield same slot

collision handling techniques:

- 1) closed Addressing
- 2) open Addressing

$$h(8) = 8 \% 10 = 8$$

$$h(3) = 3 \% 10 = 3$$

$$h(10) = 10 \% 10 = 0$$

$$h(4) = 4 \% 10 = 4$$

$$h(6) = 6 \% 10 = 6$$

$$h(13) = 13 \% 10 = 3$$

Add:  $\rightarrow O(1)$

1) find slot

2)  $arr[slot] = (key, value)$

Search:  $\rightarrow O(1)$

1) find slot

2) return  $arr[slot](value)$

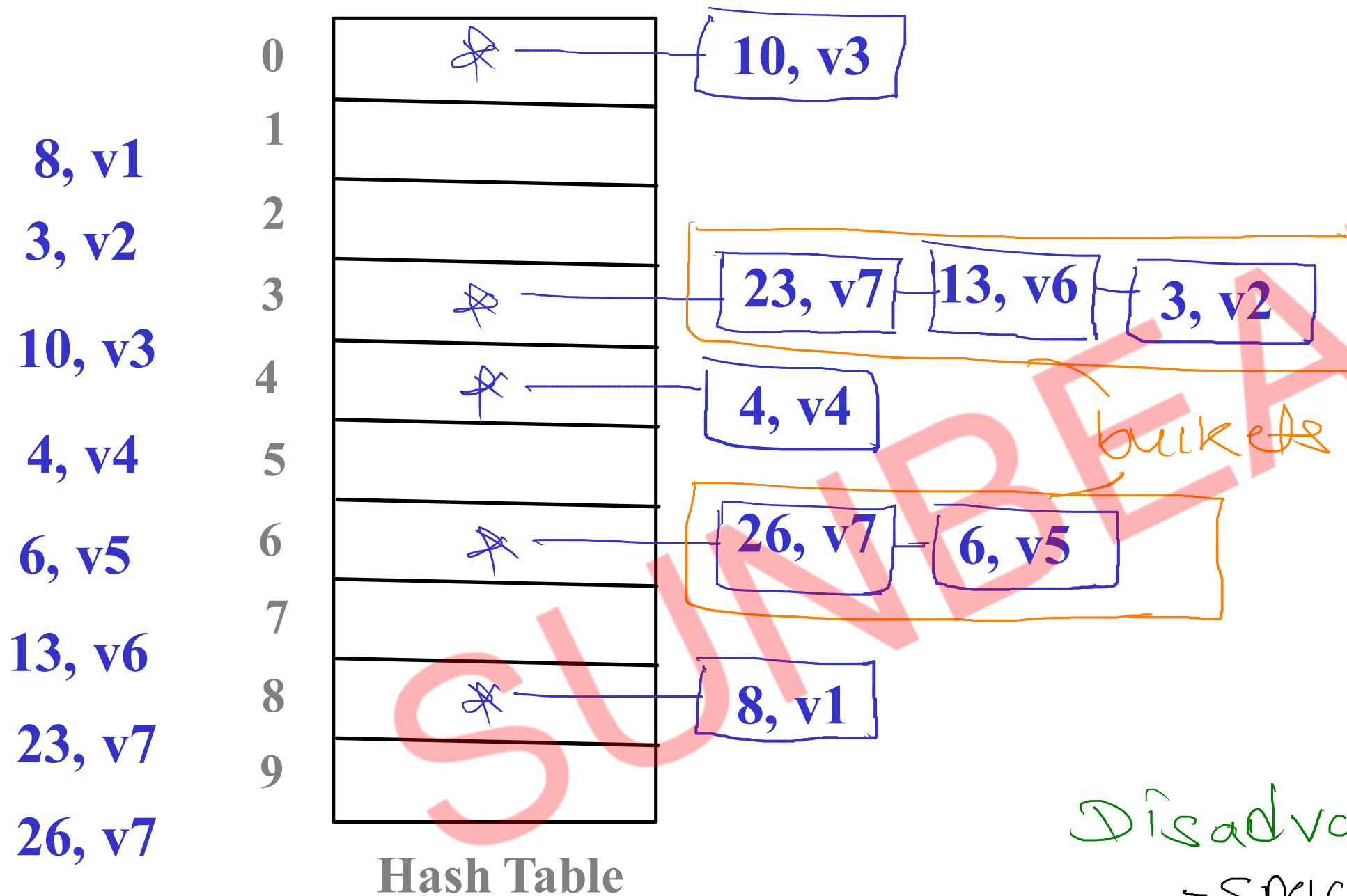
Delete:  $\rightarrow O(1)$

1) find slot

2)  $arr[slot] = null$

## Closed Addressing/ Separate Chaining / Chaining

size = 10



$$h(k) = k \% \text{size}$$

$$h(8) = 8 \% 10 = 8$$

$$h(3) = 3 \% 10 = 3$$

$$h(10) = 10 \% 10 = 0$$

$$h(4) = 4 \% 10 = 4$$

$$h(6) = 6 \% 10 = 6$$

$$h(13) = 13 \% 10 = 3$$

$$h(23) = 23 \% 10 = 3$$

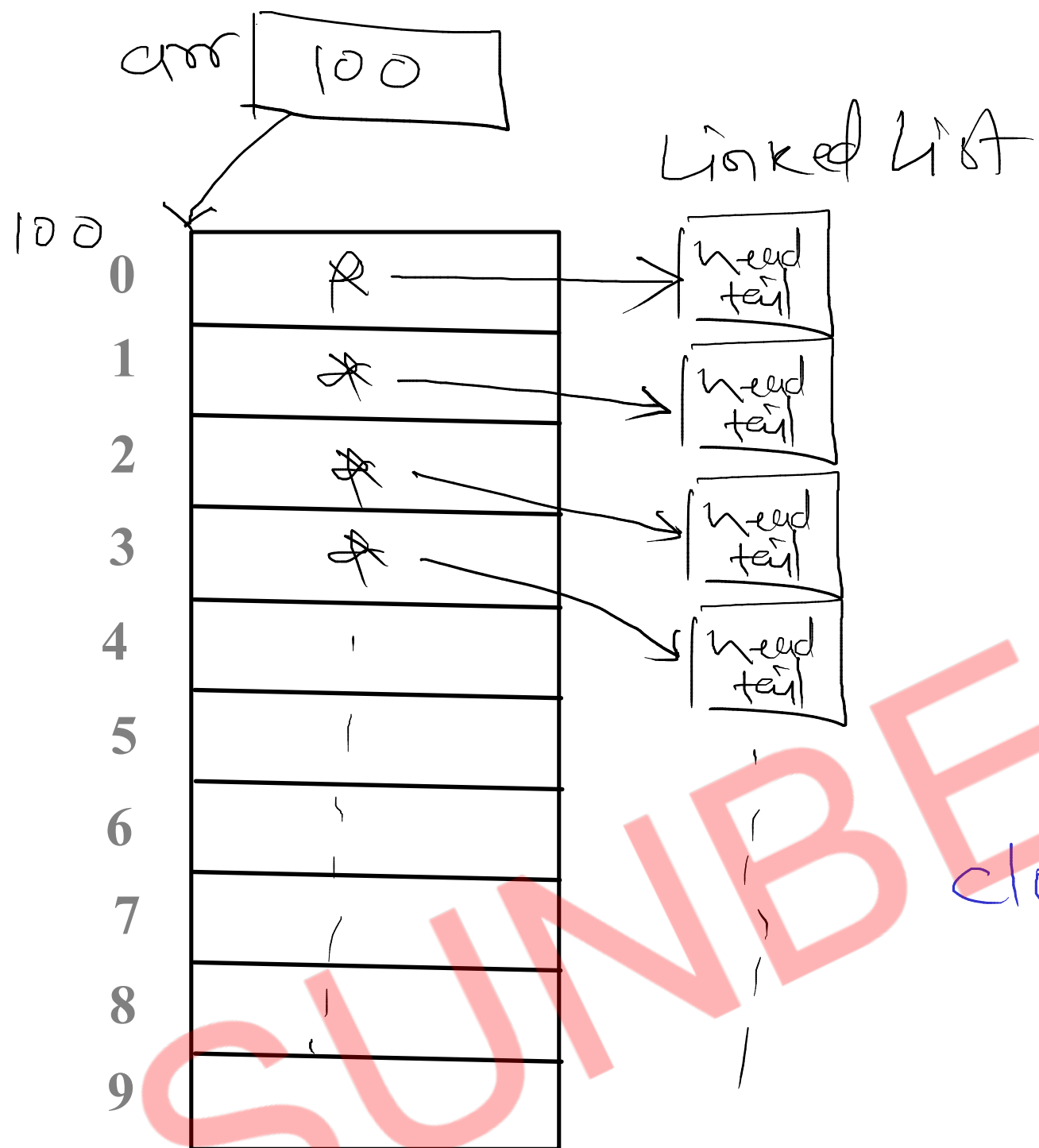
$$h(26) = 26 \% 10 = 6$$

Disadvantage:

- space inside memory is max
- data(key, value) is stored outside the table area (space)
- worst case  $T(n) = O(n)$

Advantage:

- multiple key value pairs can be stored



closed Addressing  
↳ open probing

open Addressing  
↳ closed probing

# Open Addressing - Linear Probing

size = 10

|        |        |   |
|--------|--------|---|
| 8, v1  | 10, v3 | 0 |
| 3, v2  |        | 1 |
| 10, v3 |        | 2 |
| 4, v4  | 3, v2  | 3 |
| 6, v5  | 4, v4  | 4 |
| 13, v6 | 13, v6 | 5 |
|        | 6, v5  | 6 |
|        |        | 7 |
|        | 8, v1  | 8 |
|        |        | 9 |

collision

$$h(k) = \text{key} \% \text{size}$$

$$h(k, i) = [h(k) + f(i)] \% \text{size}$$

$$f(i) = i$$

where  $i = 1, 2, 3, \dots$

↳ probe number

$$h(13) = 13 \% 10 = 3 \textcircled{c}$$

$$\begin{aligned} h(13, 1) &= [h(13) + f(1)] \% 10 \\ &= [3 + 1] \% 10 \\ &= 4 \text{ (1st probe)} \textcircled{c} \end{aligned}$$

$$\begin{aligned} h(13, 2) &= [3 + 2] \% 10 \\ &= 5 \text{ (2nd probe)} \end{aligned}$$

Probing:

- finding next free Hash Table slot whenever collision occurs

Primary clustering:

- need long runs of filled slots

"near" key position to find empty slot



# Open Addressing - Quadratic Probing

size = 10

8, v1

3, v2

10, v3

4, v4

6, v5

13, v6

collision

|        |   |
|--------|---|
| 10, v3 | 0 |
|        | 1 |
|        | 2 |
| 3, v2  | 3 |
| 4, v4  | 4 |
|        | 5 |
| 6, v5  | 6 |
| 13, v6 | 7 |
| 8, v1  | 8 |
|        | 9 |

Hash Table

$$h(k) = \text{key} \% \text{size}$$

$$h(k, i) = [h(k) + f(i)] \% \text{size}$$

$$f(i) = i^2$$

where  $i = 1, 2, 3, \dots$

$$h(13) = 13 \% 10 = 3 \text{ (C)}$$

$$\begin{aligned} h(13, 1) &= [h(13) + f(1)] \% 10 \\ &= [3 + 1] \% 10 \\ &= 4 \text{ (1st probe) (C)} \end{aligned}$$

$$\begin{aligned} h(13, 2) &= [3 + 4] \% 10 \\ &= 7 \text{ (2nd probe)} \end{aligned}$$

- no guarantee that key will get free slot

- primary clustering is removed

Secondary clustering

- need long runs of filled slots

"away" key position to find empty slot



## Open Addressing - Quadratic Probing

size = 10

23, v7

33, v8

|        |   |
|--------|---|
| 10, v3 | 0 |
|        | 1 |
| 23, v7 | 2 |
| 3, v2  | 3 |
| 4, v4  | 4 |
|        | 5 |
| 6, v5  | 6 |
| 13, v6 | 7 |
| 8, v1  | 8 |
| 33, v8 | 9 |

Hash Table

$$h(k) = \text{key \% size}$$

$$h(k, i) = [h(k) + f(i)] \% \text{size}$$

$$f(i) = i^2$$

where  $i = 1, 2, 3, \dots$

$$h(23) = 23 \% 10 = 3 \text{ (C)}$$

$$h(23, 1) = [3 + 1] \% 10 = 4 \text{ (1<sup>st</sup>) (C)}$$

$$h(23, 2) = [3 + 4] \% 10 = 7 \text{ (2<sup>nd</sup>) (C)}$$

$$h(23, 3) = [3 + 9] \% 10 = 2 \text{ (3<sup>rd</sup>) (C)}$$

$$h(33) = 33 \% 10 = 3 \text{ (C)}$$

$$h(33, 1) = [3 + 1] \% 10 = 4 \text{ (1<sup>st</sup>) (C)}$$

$$h(33, 2) = [3 + 4] \% 10 = 7 \text{ (2<sup>nd</sup>) (C)}$$

$$h(33, 3) = [3 + 9] \% 10 = 2 \text{ (3<sup>rd</sup>) (C)}$$

$$h(33, 4) = [3 + 16] \% 10 = 9$$

# Hashing - Double Hashing

size = 11

$$h1(k) = \text{key} \% \text{size}$$

$$h2(k) = 7 - (\text{key} \% 7)$$

$$h(k, i) = [h1(k) + i * h2(k)] \% \text{size}$$

$$h1(8) = 8 \% 11 = 8$$

$$h1(3) = 3 \% 11 = 3$$

$$h1(10) = 10 \% 11 = 10$$

$$h1(25) = 25 \% 11 = 3 \text{ (C)}$$

$$h2(25) = 7 - (25 \% 7) = 3$$

$$h(25, 1) = [3 + 1 * 3] \% 11 \\ = 6 \% 11 = 6 \text{ (1st)}$$

8, v1

3, v2

10, v3

25, v6

3, v2

25, v6

8, v1

10, v3

Hash Table

## Rehashing

$$\text{Load Factor} = \frac{n}{N}$$

$(\lambda)$

$$\frac{6}{10} = \underline{0.6} \rightarrow 60\% \text{ full}$$

**n** - Number of elements (key value pairs) in hash table

**N** - Number of slots in hash table

|            |                   |                            |
|------------|-------------------|----------------------------|
| if $n < N$ | Load factor $< 1$ | - free slots are available |
| if $n = N$ | Load factor $= 1$ | - no free slots            |
| if $n > N$ | Load factor $> 1$ | - can not insert at all    |

- Rehashing is make the hash table size twice of existing size if hash table is 70 or 75 % full
- In rehashing existing key value pairs are again mapped according to new hash table size