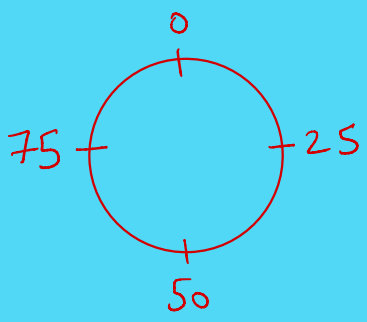
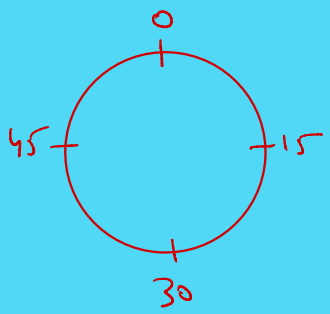
Aptitude Practice





① 7.5 hrs
→ 7 hrs 30 min

② 6.25 hrs
→ 6 hrs 15 min

③ 9.75
→ 9 hrs 45 min

Note : If the efficiency of outlet pipe is greater than inlet pipes than tank will not get filled.

Pipes and Cisterns (Note: 7.5 hrs → 7:30 hrs)

① Tap A can fill a tank in 12 hrs and Tap B can fill a tank in 36 hrs. If both taps are opened together, then how much time (in hrs) will be taken to fill the tank?

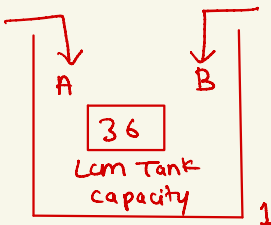
→ $\frac{1}{12} + \frac{1}{36}$

$= \frac{3+1}{36}$ ← Lcm

$= \frac{4}{36}$ 9

$= \frac{1}{9}$ ∴ 9 hours

A = 12 hr, B = 36 hr



A ⇒ 12 hr → 36 L
1 hr → 3 L
2 hr → 6 L
3 hr → 9 L
4 hr → 12 L
⋮
12 hr → 36 L

B ⇒ 36 hr → 36 L
1 hr → 1 L
⋮
36 hr → 36 L

A + B
1 hr ⇒ 3L + 1L ⇒ 4L
2 hr ⇒ 6L + 2L ⇒ 8L
3 hr ⇒ 9L + 3L ⇒ 12L
⋮
9 hr ⇒ 27L + 9L ⇒ 36L

Level ①

① Two pipes A and B can fill a tank in 20 and 30 min resp. If both the pipes are used together, then how long it will take to fill the tank?

→ A = 20 min, B = 30 min

$A+B = \frac{1}{20} + \frac{1}{30}$

$= \frac{3+2}{60} = \frac{5}{60} = \frac{1}{12}$ ∴ 12 minutes

Lcm → 60

② Pipes A and B can fill an empty tank in 8 hrs and 12 hrs resp, while a third pipe C can empty it in 6 hrs. If all pipes opened together, how much time will they take to fill it completely?

→ A → 8 hr
B → 12 hrs (inlet)
C → 6 hrs (outlet)

$A+B+C = \frac{1}{8} + \frac{1}{12} - \frac{1}{6}$ (− outlet)

$= \frac{3+2-4}{24}$

$= \frac{5-4}{24}$

$= \frac{1}{24}$ hrs ∴ 24 hrs

③ Two pipes A and B can fill a cistern in 24 min and 30 min, resp. There is also an outlet C. If all the three pipes are opened together, the cistern is filled in 20 min. How much time will be taken by C to empty the full cistern?

→ A = 24 min
B = 30 min
C = ? min

$A+B+C = 20\text{min}$

$\frac{1}{24}+\frac{1}{30}-\frac{1}{x}=\frac{1}{20}$ (outlet)

$\frac{1}{x}=\frac{1}{24}+\frac{1}{30}+\frac{1}{20}$

$=\frac{5+4-6}{120} \Rightarrow \frac{9-6}{120} \Rightarrow \frac{3}{120} \Rightarrow \frac{1}{40}$

∴ 40 minutes

④ Tap A can fill a water tank in 25 min, tap B can fill the same tank in 40 min and tap C can empty the tank in 30 min. If all the three taps are opened together, in how many min will the tank be completed filled up or emptied?

→ A → 25 min
B → 40 min
C → 30 min

$A+B+C=\frac{1}{25}+\frac{1}{40}-\frac{1}{30}$ (outlet)

$=\frac{24+15-20}{600} \Rightarrow \frac{19}{600} \Rightarrow \frac{600}{19}\text{ min}$

$\Rightarrow 31\frac{11}{19}\text{ min}$

Efficiency

Efficiency $\propto \frac{1}{\text{Time}}$

X times faster

① One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36 min, then the slower pipe alone will be able to fill the tank in,

→ A = 10 hrs, B = 30 hrs

∴ A = x, B = 3x

$\frac{1}{x}+\frac{1}{3x}=\frac{1}{36}$

$\frac{3x+x}{3x^2}=\frac{1}{36}$

$4x \times 36^{12} = 3x^2$

$x = 48\text{min}$

$B = 3x$
$= 3 \times 48$
$= 144\text{ min}$
$= 2\text{ hr } 24\text{ min}$

② one fill pipe A is 3 times faster than second fill pipe B and takes 32 min less than the fill pipe B. When will the cistern be full if both pipes are opened together?

→ $A = x$, $B = 3x$

$A \sim B = 32$ min

$x \sim 3x = 32$ min

$2x = 32$ min

$\boxed{x = 16 \text{ min}}$

$A = x = 16$ min, $B = 3x = 3 \times 16 = 48$ min

$\frac{1}{16} + \frac{1}{48} \Rightarrow \frac{3+1}{48} \Rightarrow \frac{4}{48} \Rightarrow \frac{1}{12} \Rightarrow$ 12 min

③ Inlet A is four times faster than inlet B to fill a tank. If A alone can fill it in 15 min, how long will it take if both the pipes are opened together?

→ $A = x$, $B = 4x$

$\frac{1}{x} + \frac{1}{4x} = ?$

$\frac{1}{15} + \frac{1}{60} = ?$

LCM 60

$\frac{4+1}{60} = ?$

$\frac{5}{60} \Rightarrow \frac{1}{12}$

∴ 12 min

X hours faster

① If two pipes function simultaneously, a reservoir will be filled in 12 hrs. One pipe fills the reservoir 10 hrs faster than the other. Hrs taken by second pipe take to fill the reservoir - hrs

→ $A = 10$ hrs, $B = 20$ hrs.

$A = x$, $B = x + 10$

$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$

$\frac{x+10+x}{x(x+10)} = \frac{1}{12}$

$\frac{2x+10}{x^2+10x} = \frac{1}{12}$

$24x + 120 = x^2 + 10x$

$x^2 - 14x - 120 = 0$

$x^2 - 14x - 120 = 0$

$x^2 - 20x + 6x - 120 = 0$

$x(x-20) + 6(x-20) = 0$

$(x-20)(x+6) = 0$

$\boxed{x = 20}$, $\boxed{x = -6}$ (cancelled)

∴ $x = 20$ hrs

∴ 2nd pipe $= x + 10$

$= 20 + 10$

$= 30$ hrs

-120 → -20, $+6$

② A reservoir is fitted with two pipes A and B. pipe A can fill the reservoir 5 hrs faster than pipe B. If both pipe together fill the reservoir in 6 hrs, the reservoir will be filled by A alone in,

→ $A = x$, $B = x + 5$

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$$

$$\frac{x+5+x}{x(x+5)} = \frac{1}{6}$$

$$12x + 30 = x^2 + 5x$$

$$x^2 - 7x - 30$$

$x^2 - 7x - 30$

$x^2 - 10x + 3x - 30$

$x(x-10) + 3(x-10)$

$x = 10$, $x = -3$

$x = 10$ hrs

$A = 10$ hrs

-30 : -10, $+3$

Percentage

① Capacity of tap B is 80% more than that of A. If both the taps are opened simultaneously, they take 45 hrs to fill the tank. How long will B take to fill the tank alone?

→ $A = x + 80\%$ of x , $B = x$

$A = 180\%\ x$, $B = x$

$A = \frac{180}{100} \times x$, $B = x$

$A = \frac{9x}{5}$, $B = x$

$$\frac{1}{\frac{9x}{5}} + \frac{1}{x} = \frac{1}{45}$$

$$\frac{5}{9x} + \frac{1}{x} = \frac{1}{45}$$

$$\frac{5x + 9x}{9x^2} = \frac{1}{45}$$

$$\frac{14x}{9x^2} = \frac{1}{5}$$

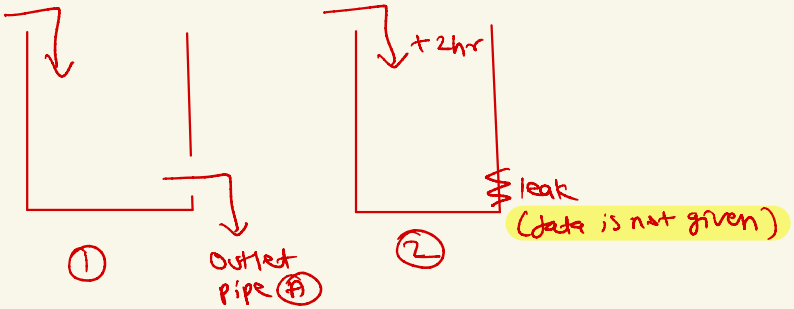
$x = 70$ hrs

$B = x$

$= 70$ hrs

Cistern Leak Concept

Note: In outlet pipe the data is given but in leak data is not given (eg) efficiency of pipe etc.



① A cistern is normally filled in 8 hrs, but it takes 10 hrs to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in,

→ $\frac{1}{8} - \frac{1}{10}$ ← lcm(8,10) = 40

$= \frac{5-4}{40} \Rightarrow \frac{1}{40} \Rightarrow$ 40 hrs

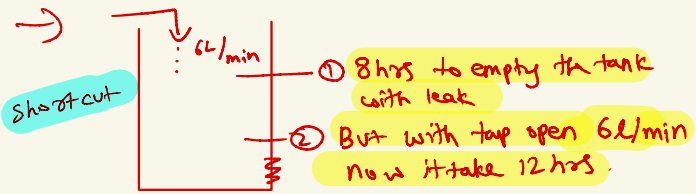
② There is a leak in the bottom of a cistern. When the cistern is thoroughly repaired, it would be filled in 12 min It now takes 18 min longer. If the cistern is full, how long would the leak take to empty the cistern?

→ Without leak: 12 min; With leak: 30 min (18+12 min)

$\frac{1}{12} - \frac{1}{30} \Rightarrow \frac{5-2}{60} \Rightarrow \frac{3}{60} \Rightarrow \frac{1}{20}$ ∴ 20 min

③ A cistern has a leak which would empty it in 8 hours. A tap is turned on, which admits 6 litres a minutes into the cistern and it is now emptied in 12 hours, How many litres can the cistern hold?

→



Short cut

1 min = 6L
60 min = 6 × 60
1 hr = 360L

$\frac{1}{8} \sim \frac{1}{12} = \frac{360}{x}$

LCM(8,12) = 24

$\frac{3-2}{24} = \frac{360}{x}$

$\frac{1}{24} = \frac{360}{x}$

$x = 360 \times 24$

$x = 8640$

Traditional

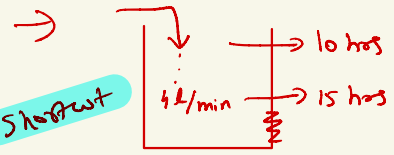
$\frac{1}{8} - \frac{1}{12} = \frac{1}{24}$ LCM(8,12) = 24

$\frac{1}{24}$ hrs $\Rightarrow \frac{1}{24 \times 60 \text{ min}} \Rightarrow \frac{1}{1440 \text{ min}}$ ∴ 1 min = $\frac{1}{1440}$ part filled

1 part = 1440 min
= 1440 × 6L
= 8640 Litre

④ A tank has a leak which would empty the completely filled tank in 10 hrs. If the tank is full of water and a tap is opened which admits 4 litres of water per minute in the tank, the leak takes 15 hrs to empty the tank. How many litres of water does the tank hold?

→



Shortcut

$\frac{1}{10} \sim \frac{1}{15} = \frac{240}{x}$

LCM(10,15) = 30

1 min = 4L
60 min = 4 × 60
1 hr = 240L

$\frac{3-2}{30} = \frac{240}{x}$

$\frac{1}{30} = \frac{240}{x}$ ∴ $x = 240 \times 30$

= 7200 litre

Traditional

$\frac{1}{10} - \frac{1}{15} = \frac{3-2}{30}$

$= \frac{1}{30}$ hrs

$\frac{1}{30 \times 60 \text{ min}} = \frac{1}{1800}$

1 min = $\frac{1}{1800}$ part filled

1 part = 1800 min
= 1800 × 4 ⇒ 7200 litre

Alternate Taps

Two taps → 1 cycle = A+B

Three taps → 1 cycle = A+B+C

① Two pipe A and B can fill the tank in 20 min and 30 min resp. If they are opened alternately for 1 min each, beginning with pipe A. In how many hrs the tank is fill,

→ A B A B . . .

1min 1min

$\frac{1}{20} + \frac{1}{30}$

LCM(20,30) = 60

2min = $\frac{5}{60} = \frac{1}{12}$

∴ 2 min = $\frac{1}{12}$

4 min = $\frac{2}{12}$

8 min = $\frac{4}{12}$

10 min = $\frac{5}{12}$

12 min = $\frac{6}{12}$

14 min = $\frac{7}{12}$

16 min = $\frac{8}{12}$

18 min = $\frac{9}{12}$

20 min = $\frac{10}{12}$

22 min = $\frac{11}{12}$

24 min = $\frac{12}{12}$

Trick LCM(20,30) = 60 L tank capacity

A = 20 min = 1 min = 3 litre

B = 30 min = 1 min = 2 litre

2 min = 5 litre fill

×12 ×12

24 min 60 litre

② Two pipe P and Q can fill the tank in 12 hrs and 4 hrs resp. If they are opened on alternate hrs each, beginning with pipe P. In how many hrs the tank is fill.

→ P Q P Q . . .

1hr 1hr

$2\text{hr} = \frac{1}{12} + \frac{1}{4}$

$= \frac{1+3}{12} \Rightarrow \frac{4}{12} \Rightarrow \frac{1}{3}$

2 hr = $\frac{1}{3}$

4 hr = $\frac{2}{3}$

6 hr = $\frac{3}{3} = 1$

Trick LCM(12,4) = 12 L tank capacity

P = 12 hr = 1 hr = 1 litre

Q = 4 hr = 1 hr = 3 litre

2 hr = 4 litre

×3 ×3

6 hr 12 litre

③ Two pipe A and B can fill the tank in 4hrs and 5hrs resp. If they are opened on alternate hrs and if pipe B is opened first, in how many hrs the tank is fill,

→ B A B A ...

1 hr 1 hr

1/5 1/4

Lcm(5,4) ⇒ 20L capacity tank

B ⇒ 5hr = 1hr = 4L

A ⇒ 4hr = 1hr = 5L

(B+A) 2hr = 9L

1 cycle → (B+A) 2hr = 9/20

remaining 1/10

2 cycle → (B+A) 4hr = 18/20 = 9/10 ⇒ 1/10

3 cycle → only (B) bcz only 1/10 to be filled

B = 1/10 × 5 = 1/2 hr (∵ 1/5 = 5)

(B)

4hr 30min

④ Two pipe A and B can fill the tank in 4 min and 6min resp. If they are opened on alternately for 1 min each, in how many hrs the tank is fill

→ A B A B ...

↓ ↓

4min 6min

Lcm(4,6) = 12 litre capacity of tank

A = 4min = 1min = 3 Litre

B = 6min = 1min = 2 Litre

2min = 5 litre

$4\frac{2}{3}$ min

$\frac{2}{3} \times 60 = 40$ sec

∴ 4hr 40sec

1 cycle → 2min = 5/12

2 cycle → 4min = 10/12 ⇒ remaining $\frac{2}{12} \Rightarrow \frac{1}{6}$

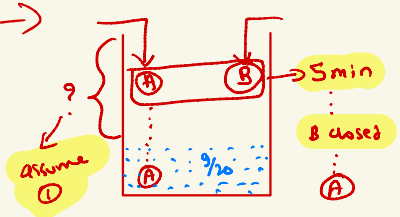
3 cycle → only (A) A = $\frac{1}{6_3} \times 4^2$

$= \frac{2}{3}$ min

on and off

① A tank be filled by one tap in 20 min. and by another in 25 min. Both the taps are kept open for 5 min. and then the second is turned off. In how many min more is the tank completely filled?

→



Normal

$A = \frac{1}{20}$, $B = \frac{1}{25}$

$(A+B)\times 5 = \left(\frac{1}{20}+\frac{1}{25}\right)\times 5$

$= \frac{5+4}{100}\times 5$

$= \frac{9}{20}$

$1 - \frac{9}{20} = \frac{11}{20}$

$\frac{11}{20}\times 20 = 11$ min more

Trick — equation method

(because B is closed; 1 if filled full or 0 if empty)

$\frac{5+x}{20} + \frac{5}{25} = 1$

$\frac{25+5x+20}{100} = 1$ (LCM(20,25) = 100)

$45+5x = 100$

$5x = 55$

$x = 11$

② A tank can be filled by a tap in 20 min and by another tap in 60 min. Both the taps are kept open for 10 min and then the first tap is shut off. After this, the tank will be completely filled in,

→ Normal

$(A+B)\times 10$

$\left(\frac{1}{20}+\frac{1}{60}\right)\times 10$

$\frac{4}{60}\times 10 = \frac{2}{3}$

$1 - \frac{2}{3} = \frac{1}{3}$

$\frac{1}{3}\times 60 = 20$ min

Trick — equation method

$\frac{10}{20} + \frac{10+x}{60} = 1$ (LCM(20,60) = 60)

$\frac{30+10+x}{60} = 1$

$40 + x = 60$

$x = 20$ min

③ Two pipes can fill a tank in 6 and 8 hrs. Both opened together, but after 1 and 1/2 hrs the pipe A turned off. How much time will it take to fill the tank?

→ equation method

$$\frac{\frac{3}{2}}{6} + \frac{\frac{3}{2}+x}{8} = 1$$

lcm(6,8) = 24

$$\frac{\frac{3}{2}\times 4^{2} + \frac{3}{2}\times 3 + 3x}{24}$$

$$6 + \frac{4.5+3x}{24} = 1$$

$$6.5 + 3x = 24$$

$$3x = 13.5$$

$$x = 4.5$$

Total = 4.5 + 1.5

= 6 hrs

④ Two pipes A and B can fill a tank in 36 min and 45 min resp. Pipe C can empty it in 30 min, A and B are opened and after 7 min, C is also opened. In how many min will the tank be full?

→ equation method

$$\frac{7+x}{36} + \frac{7+x}{45} - \frac{x}{30} = 1$$

$$\frac{35+5x+28+4x-6x}{180} = 1$$

$$3x + 63 = 180$$

$$3x = 117$$

$$x = 39 \text{ min}$$

Total = 39 + 7

= 46 min

Normal

$$\left(\frac{1}{30}+\frac{1}{45}\right)\times 7$$

$$\frac{5+4}{180}\times 7 = \frac{7}{20}$$ ← total two tap

$$1 - \frac{7}{20} = \frac{13}{20}$$ ← remaining water

$$\frac{1}{36}+\frac{1}{45}-\frac{1}{30}$$

$$\frac{5+4-6}{180} = \frac{3}{180} = \frac{1}{60}$$

$$\frac{13}{20}\times 60 = 39 \text{ min}$$

7 + 39 min

46 min

(5) A pipe can fill a cistern in 12 min while a second pipe fills it in 15 min. But third pipe can empty that completely filled cistern in 6 min. The first two pipes are kept open for 5 min initially and then the third one is also opened. Then the further time (in min) taken to empty that cistern is,

→ Eq method

bcz empty

filled = 1
empty = 0

$$\frac{5+x}{12} + \frac{5+x}{15} - \frac{x}{6} = 0$$

$$\frac{25 + 5x + 20 + 4x - 10x}{60} = 0$$

$$-x + 45 = 0$$

$$x = 45 \text{ min}$$

Boats and Streams

Five formula

① Upstream (U) (Speed) = $B - S$

② Downstream (Speed) = $B + S$

③ Boat speed (B) = $\frac{U+V}{2}$

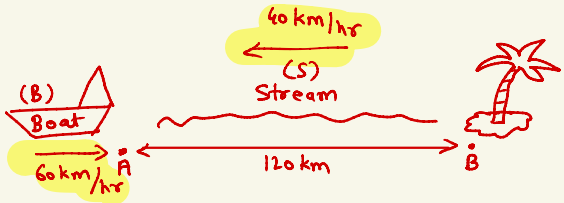
④ Stream Speed (S) = $\frac{U-V}{2}$

⑤ EQ. Distance

$$T_1(B-S) = T_2(B+S)$$

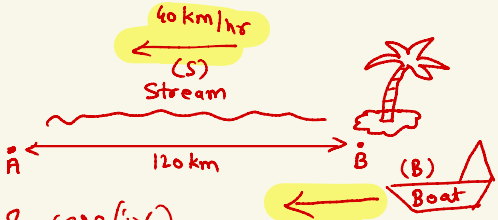
Case (i) Time taken = ?

Upstream (or) Against the stream



Case (ii)

Downstream (or) Along the stream



Case (iii) & Case (iv)

Given Que

$U = 20$ km/hr $\quad D = 100$ km/hr

$U = B - S$ — ① $\quad D = B + S$ — ②

① + ② $\quad U = B - S$

$+ \quad D = B + S$

$2B = U + D$

$$B = \frac{U+D}{2} \quad \text{(or)} \quad B = \frac{U+V}{2}$$

Case (v)

Case (i) Speed / Distance / Time

Case (ii) Speed / Distance / Time

Total Distance $_{\text{upstream}}$ = Total Distance $_{\text{downstream}}$

$D_U = D_D$

Distance = Speed × Time

$D = S \times T$

Distance = Speed × Time

$D = S \times T$

$$T_1(B-S) = T_2(B+S)$$

↳ upstream ↳ downstream

① A man can swim 3 km/hr in still water. If the velocity of the stream be 2 km/hr, the time taken by him to swim to a place 10 km upstream and back is,

⇒ $m = 3\text{km/hr}$, $s = 2\text{km/hr}$

Upstream $= m - s = 3 - 2 = 1\text{ km/hr}$

Downstream $= m + s = 3 + 2 = 5\text{ km/hr}$

Distance = 10 km

Time take T = Upstream + downstream

$$T = \frac{D}{S} + \frac{D}{S}$$

$$= \frac{10}{1} + \frac{10}{5} \Rightarrow 10 + 2 \Rightarrow 12\text{ hrs}$$

② A boat travels upstream from B to A and downstream from A to B in 3 hrs. If the speed of the boat in still water is 9 km/hr and the speed of the current is 3 km/hr, the distance between A and B is,

→ Total time taken (T) $= 3\text{hrs}$, $B = 9\text{ km/hr}$, Stream speed $= 3\text{km/hr}$

Distance = ?

Upstream $= B - S = 9 - 3 = 6\text{ km/hr}$

Downstream $= B + S = 9 + 3 = 12\text{ km/hr}$

$U = 6\text{ km/hr}$

$V = 12\text{ km/hr}$

Total time taken T = Upstream $\frac{D}{S}$ + Downstream $\frac{D}{S}$

$$3 = \frac{x}{6} + \frac{x}{12}$$

LCM(6, 12) = 12

$$3 = \frac{2x + x}{12}$$

$$3 = \frac{3x}{12}$$

$$x = 12\text{ km}$$

③ In a stream running at 2km/hr, a motor boat goes 10km upstream and returns to the starting point in 55 min find out the speed of the motorboat in still water.

→ $B = x$, $S = 2$ km/hr, $D = 10$ km, $T = 55$ min

Upstream $= (x - 2)$

downstream $= (x + 2)$

$U = (x-2)$

$V = (x+2)$

Time taken upstream + downstream

$$T = \frac{D}{S} + \frac{D}{S}$$

(55 min → convert to hrs ∴ $\frac{55}{60}$)

$$\frac{11}{12} = \frac{10}{x-2} + \frac{10}{x+2}$$

$$\frac{11}{12} = \frac{10(x+2) + 10(x-2)}{(x-2)(x+2)}$$

$$\frac{11}{12} = \frac{10x + 20 + 10x - 20}{x^2 - 2^2}$$

$$11(x^2 - 4) = 20x \times 12$$

$$11x^2 - 44 = 240x$$

$$11x^2 - 240x - 44 = 0$$

$11x^2 - 240x - 44 = 0$

-44×11

242×2

$11 \quad 11$

22 , $\frac{2}{11}$

∴ Ans $x = 22$ km/hr

④ A motor-boat can travel at 10 km/hr n still water. It travelled 91 km downstream in a river and then returned to the same place, taking altogether 20 hrs. find the rate of the flow of river.

→ $m = 10$ km/hr, $D = 91$ km, $T = 20$ hr

Speed of the stream $(S) = ?$

Upstream $(U) = m - S = 10 - x$

Downstream $(V) = m + S = 10 + x$

$$\text{Time} = \frac{D}{S} + \frac{D}{S}$$

$$20 = \frac{91}{10-x} + \frac{91}{10+x}$$

$$20 = \frac{91(10+x) + 91(10-x)}{10^2 - x^2}$$

$$20(10^2 - x^2) = 910 + 91x + 910 - 91x$$

$$20(10^2 - x^2) = 1820$$

$$2000 - 20x^2 = 1820$$

$$180 = 20x^2$$

$$x^2 = 9$$

∴ $x = 3$ km/hr

⑤ The speed of the current is 5 km/hr. A motorboat goes 10 km upstream and back again to starting point in 50 min. The speed (in km/hr) of the motorboat in still water is,

→ $U = x - 5$

$V = x + 5$

$T = \frac{D}{S} + \frac{D}{S}$ (50 min → hr → $\frac{50}{60}$ hr)

$\frac{5x}{6x} = \frac{10}{x-5} + \frac{10}{x+5}$

$\frac{5}{6} = \frac{10(x+5) + 10(x-5)}{x^2 - 25}$

$\frac{5}{6} = \frac{10x + 10}{x^2 - 25}$

$5x^2 - 125 = 120x$

$5x^2 - 120x - 125 = 0$

∴ Ans ⇒ $x = 25$ km/hr

Option method

(c) 25

$\frac{50}{60} = \frac{10}{25-5} + \frac{10}{25+5}$

$= \frac{10}{20} + \frac{10}{30}$

$\frac{50}{60} = \frac{30+20}{60}$

LHS = RHS ∴ ans = 25

⑥ A man can row 30 km downstream and return in a total of 8 hrs. If the speed of the boat in still water is four times the speed of the current, then the speed of the current is, (y)

→ $D = 30$ km, $T = 8$ hrs

Boat	stream speed
x	y
40	10
$(4y)$	(y)

$\therefore x = 4y$

$T = \frac{D}{S} + \frac{D}{S}$

$8 = \frac{30}{x-y} + \frac{30}{x+y}$

$8 = \frac{30}{3y} + \frac{30}{5y}$

$8 = \frac{10}{y} + \frac{6}{y}$

$8 = \frac{16}{y}$

$y = 2$ km/hr

⑦ A boat takes 26 hrs for travelling downstream from point A to point B and coming back to point C midway betn A and B. If the velocity of the stream is 4 km/hr and the speed of the boat in still water is 10 km/hr, what is the distance betn A and B?

→ $T = 26$ hrs, $B = 10$, $S = 4$ km/hr

B ← C → A

$U = 10 - 4 = 6$ km/hr

$V = 10 + 4 = 14$ km/hr

$T = \frac{D}{S} + \frac{D}{S}$

$26 = \frac{D/2}{6} + \frac{D}{14}$

$26 = \frac{D}{12} + \frac{D}{14}$

$26 = \frac{70 + 60}{84}$

$26 = \frac{150}{84}$

$D = 84 \times 2$

$= 168$ km

⑧ A boat has to travel upstream 20km distance from point X of a river to point Y. The total time taken by boat in travelling from Point X to Y and Y to X is 41 min. What is the speed of the boat.

→ D = 20km , T = 41min

(41 min → hr → $\frac{41}{60}$ hr)

$T = \frac{D}{S} + \frac{D}{S}$

$\frac{41}{60} = \frac{20}{x-y} + \frac{20}{x+y}$

Stream speed not given (x)
Speed of boat not given (y)

two unknown value → Any Data Inadequate

Based on two Equation

① A boat covers 12km upstream and 18km downstream in 3 hrs, while it covers 36 km upstream and 24 km downstream in 6($\frac{1}{2}$) hrs. What is the speed of the current?

→ Time taken upstream + downstream

$T = \frac{D}{S} + \frac{D}{S}$

$3 = \frac{12}{x-y} + \frac{18}{x+y}$ —①

Time taken upstream + downstream

$T = \frac{D}{S} + \frac{D}{S}$

$\frac{13}{2} = \frac{36}{x-y} + \frac{24}{x+y}$ —②

$3 = \frac{12}{x-y} + \frac{18}{x+y}$ —①

$\frac{13}{2} = \frac{36}{x-y} + \frac{24}{x+y}$ —②

①×3 ⇒ $9 = \frac{36}{x-y} + \frac{54}{x+y}$ (×3)

$\frac{13}{2} = \frac{36}{x-y} + \frac{24}{x+y}$

$9 - \frac{13}{2} = \frac{30}{x+y}$

$\frac{5}{2} = \frac{30}{x+y}$

$x + y = 12$ put in eqn ①

$3 = \frac{12}{x-y} + \frac{3}{2}$

$\frac{12}{x-y} = 3 - \frac{3}{2}$

$4\,\frac{12}{x-y} = \frac{3}{2}$

$x - y = 8$

$x + y = 12$

$x - y = 8$

$2y = 12 - 8$

$2y = 4$

$y = 2$

Speed of current 2 km/hr

② A motor-boat travelling at a same speed can cover 25 km upstream and 39 km downstream in 8 hrs. At the same speed, it can travel 35 km upstream and 52 km downstream in 11 hrs. The speed of the stream is,

→ (Time taken) (upstream) (downstream)

$$T = \frac{D}{S} + \frac{D}{S}$$

$$8 = \frac{25}{x-y} + \frac{39}{x+y} \quad -①_{\times 4}$$

$$11 = \frac{35}{x-y} + \frac{52}{x+y} \quad -②_{\times 3}$$

$$32 = \frac{100}{x-y} + \frac{156}{x+y}$$

$$-33 = \frac{105}{x-y} + \frac{156}{x+y}$$

$$-1 = \frac{-5}{x-y}$$

$$\boxed{x-y=5}$$

put $\boxed{x-y=5}$ in eqn ①

$$8 = \frac{25}{5} + \frac{39}{x+y}$$

$$3 = \frac{39}{x+y}$$

$$\boxed{x+y=13}$$

$$x - y = 5$$
$$-\quad x + y = 13$$
$$-2y = -8$$
$$\boxed{y=4}$$

Speed of stream

$$\boxed{y = 4\ km/hr}$$

③ A boat cover 24 km upstream and 36 km downstream in 6hr while it covers 36 km upstream and 24 km downstream in 6 $(\frac{1}{2})$ hrs. The speed of the current is,

→

$$6 = \frac{24}{x-y} + \frac{36}{x+y} \quad -①_{\times 2}$$

$$\frac{13}{2} = \frac{36}{x-y} + \frac{24}{x+y} \quad -②_{\times 3}$$

$$12 = \frac{48}{x-y} + \frac{72}{x+y}$$

$$\frac{39}{2} = \frac{108}{x-y} + \frac{72}{x+y}$$

$$12 - \frac{39}{2} = \frac{60}{x-y}$$

$$\frac{15}{2} = \frac{60}{x-y}$$

$$\boxed{x-y=8}$$

put $\boxed{x-y=8}$ in eqn ①

$$6 = \frac{24}{8} + \frac{36}{x+y}$$

$$3 = \frac{36}{x+y}$$

$$\boxed{x+y=12}$$

$$x - y = 8$$
$$-\quad x + y = 12$$
$$-2y = -4$$
$$\boxed{y = 2\ km/hr}$$

④ A boat goes 4km upstream and 4km downstream in 1hr. The same boat goes 5km downstream and 3km upstream in 55 min. What is the speed of the boat in still water?

→ ($\because 55\text{min} = \frac{55}{60}\text{ hr}$)

Time taken = upstream + downstream

$$T = \frac{D}{S} + \frac{D}{S}$$

$$1 = \frac{4}{x-y} + \frac{4}{x+y} \quad \text{—①} \times 3$$

$$\frac{11}{12} = \frac{3}{x-y} + \frac{5}{x+y} \quad \text{—②} \times 4$$

$\frac{11 \times 4}{12} = \frac{44}{12} = \frac{11}{3}$

$$3 = \frac{12}{x-y} + \frac{12}{x+y}$$

$$-\frac{11}{3} = \frac{12}{x-y} + \frac{20}{x+y}$$

$$3 - \frac{11}{3} = \frac{-8}{x+y}$$

$$-\frac{2}{3} = \frac{-8}{x+y}$$

$$x + y = 12$$

put $x+y=12$ in eqn ①

$$1 = \frac{4}{x-y} + \frac{4}{12}$$

$$1 = \frac{4}{x-y} + \frac{1}{3}$$

$$\frac{4}{x-y} = 1 - \frac{1}{3}$$

$$\frac{4}{x-y} = \frac{2}{3}$$

$$x - y = 6$$

$$x - y = 6$$
$$x + y = 12$$
$$2x = 18$$
$$x = 9$$

Speed of boat 9 km/hr

Twice / Thrice

① A person can row $7\left(\frac{1}{2}\right)$ km/hr in still water and he finds that it takes him twice as long as to row up as to row down the river. The speed of the stream is,

→ Speed = $7\frac{1}{2}$ km/hr = $\frac{15}{2}$ km/hr

row up → upstream (U)
row down → downstream (D)

Speed of stream (S) = y km/hr

$$U_T = 2 \times D_T$$

$$\frac{D}{S_U} = 2 \times \frac{D}{S_D}$$

$$S_D = 2 \times S_U$$

$$\left(\frac{15}{2} + y\right) = 2\left(\frac{15}{2} - y\right)$$

$$\frac{15+2y}{2} = 2\left(\frac{15-2y}{2}\right)$$

$$15 + 2y = 30 - 4y$$
$$6y = 30 - 15$$
$$6y = 15$$

$$y = \frac{15}{6} = 2.5$$

$y = 2.5$ km/hr

② A man can row at a speed of $4\left(\frac{1}{2}\right)$ km/hr in still water. If he takes 2 times as long to row a distance upstream as to row the same distance downstream, then the speed of the stream is,

→ Speed $= \frac{9}{2}$ km/hr

Speed of stream $(S_S) = y$ km/hr

$U_T = 2 \times D_T$

$\frac{D}{S_U} = 2 \times \frac{D}{S_D}$

$S_D = 2 \times S_U$

$\left(\frac{9}{2} + y\right) = 2\left(\frac{9}{2} - y\right)$

$\frac{9+2y}{2} = 2\left(\frac{9-2y}{2}\right)$

$9 + 2y = 18 - 4y$

$6y = 9$

$y = 9/6$

$y = 1.5$ km/hr

③ A boat goes 6km an hr in still water, it takes thrice as much time in going the same distance against the current comparison to direction of the current. The speed of the current is,

Speed of current $= y$ km/hr

→ $S_B = 6$ km/hr

* Against the current → Upstream
* Direction of the current → Downstream

$U_T = 3 \times D_T$

$\frac{D}{S_U} = 3 \times \frac{D}{S_D}$

$S_D = 3 \times S_U$

$(6+y) = 3 \times (6-y)$

$6 + y = 18 - 3y$

$4y = 18 - 6$

$4y = 12$

$y = 3$ km/hr

④ In a fixed time, a boy swims double the distance along the current that he swims against the current. If the speed of the current is 3km/hr, the speed of the boy in still water is,

→ $y = 3$km/hr

$x = ?$ km/hr

$D_D = 2 \times U_D$

$5x = 2 \times (5x)$

$(x+3) = 2(x-3)$

$(x+3) = 2x - 6$

$x = 9$ km/hr