

Aptitude Practice

VEDIC MATH

1) Multiplication by 1's : 11, 111, 1111.....

$$\begin{array}{r} \overline{\overline{3}}\overline{\overline{2}} \\ \times 11 \\ \hline \overline{\overline{3}}\overline{\overline{5}}\overline{\overline{2}} \end{array}$$

$$\begin{array}{r} \overline{\overline{4}}\overline{\overline{2}} \\ \times 11 \\ \hline \overline{\overline{4}}\overline{\overline{6}}\overline{\overline{2}} \end{array}$$

$$\begin{array}{r} \overline{\overline{3}}\overline{\overline{4}}\overline{\overline{2}} \\ \times 111 \\ \hline \overline{\overline{3}}\overline{\overline{7}}\overline{\overline{9}}\overline{\overline{6}}\overline{\overline{2}} \end{array}$$

$$\begin{array}{r} \overline{\overline{2}}\overline{\overline{0}}\overline{\overline{4}}\overline{\overline{3}} \\ \times 1111 \\ \hline \overline{\overline{2}}\overline{\overline{2}}\overline{\overline{6}}\overline{\overline{9}}\overline{\overline{7}}\overline{\overline{3}} \end{array}$$

$$\begin{array}{r} \overline{\overline{1}}\overline{\overline{2}}\overline{\overline{3}}\overline{\overline{0}}\overline{\overline{4}} \\ \times \overline{\overline{1}}\overline{\overline{1}}\overline{\overline{1}}\overline{\overline{1}} \\ \hline \overline{\overline{1}}\overline{\overline{3}}\overline{\overline{6}}\overline{\overline{7}}\overline{\overline{0}}\overline{\overline{9}}\overline{\overline{7}}\overline{\overline{4}} \end{array}$$

RULE: Maximum no. of digits in largest group is equal to the no. of 1's.

$$\begin{array}{r} \overline{\overline{1}}\overline{\overline{3}}\overline{\overline{1}}\overline{\overline{2}} \\ \times 11 \\ \hline \overline{\overline{1}}\overline{\overline{4}}\overline{\overline{3}}\overline{\overline{2}} \end{array}$$

$$\begin{array}{r} \overline{\overline{2}}\overline{\overline{1}}\overline{\overline{4}}\overline{\overline{3}} \\ \times 111 \\ \hline \overline{\overline{2}}\overline{\overline{3}}\overline{\overline{7}}\overline{\overline{8}}\overline{\overline{7}}\overline{\overline{3}} \end{array}$$

$$\begin{array}{r} \overline{\overline{0}}\overline{\overline{4}}\overline{\overline{2}} \\ \times 111 \\ \hline \overline{\overline{0}}\overline{\overline{4}}\overline{\overline{6}}\overline{\overline{6}} \end{array}$$

$$\begin{array}{r} \overline{\overline{0}}\overline{\overline{3}}\overline{\overline{1}}\overline{\overline{2}} \\ \times 1111 \\ \hline \overline{\overline{3}}\overline{\overline{4}}\overline{\overline{6}}\overline{\overline{6}}\overline{\overline{3}}\overline{\overline{2}} \end{array}$$

Multiplication by 9's : 99, 999, 9999.....

$$\begin{array}{r} 32 \\ \times 99 \\ \hline 3168 \end{array}$$

$$\begin{array}{r} 413 \\ \times 999 \\ \hline 412587 \end{array}$$

$$\begin{array}{r} 3421 \\ \times 9999 \\ \hline 34206579 \end{array}$$

$$\begin{array}{r} 4080 \\ \times 9999 \\ \hline 40795920 \end{array}$$

i) -1
ii) 9-

RULE: The subtraction will be 'n' no. of times, where 'n' is the no. of 99....

$$\begin{array}{r} 032 \\ \times 999 \\ \hline 031968 \end{array}$$

$$\begin{array}{r} 0057 \\ \times 9999 \\ \hline 00569943 \end{array}$$

$$\begin{array}{r} 312 \\ \times 99 \\ \hline 31188 \\ -3 \\ \hline 30888 \end{array}$$

$$\begin{array}{r} 6512 \\ \times 99 \\ \hline 651188 \\ -65 \\ \hline 644688 \end{array}$$

VEDIC MATH

BASE METHOD:

100, 1000

$$\begin{array}{r} 102 +2 \\ \times 103 +3 \\ \hline 105 \underline{06} \end{array} \quad \begin{array}{r} 107 +7 \\ \times 105 +5 \\ \hline 112 \underline{35} \end{array} \quad \begin{array}{r} 108 \\ \times 109 \\ \hline 117 \underline{72} \end{array} \quad \begin{array}{r} 112 +12 \\ \times 115 +15 \\ \hline 127 \underline{80} \end{array} > 180$$

$$106 \times 105 = 11130 \quad 104 \times 108 = 11232 \quad 109 \times 112 = 12208$$

$$\begin{array}{r} 98 -2 \\ \times 97 -3 \\ \hline 95 \underline{06} \end{array} \quad \begin{array}{r} 96 -4 \\ \times 92 -8 \\ \hline 88 \underline{32} \end{array} \quad \begin{array}{r} 88 -12 \\ \times 97 -3 \\ \hline 85 \underline{36} \end{array} \quad \begin{array}{r} 84 -16 \\ \times 87 -13 \\ \hline 71 \underline{08} \end{array} > 208$$

$$96 \times 94 = 9024 \quad 93 \times 91 = 8463 \quad 87 \times 91 = 7917$$

$$18 - 9 = 117$$

BASE METHOD:

$$\begin{array}{r} 103 +3 > -6 \\ \times 98 -2 \\ \hline 101 \end{array} \quad \begin{array}{r} 105 +5 > -15 \\ \times 97 -3 \\ \hline 102 \end{array} \quad \begin{array}{r} 108 +8 > -64 \\ \times 92 -8 \\ \hline 99 \end{array}$$

$$\downarrow 100$$

$$109$$

$$\times 91$$

$$\hline 9919$$

$$112 +12 > -108$$

$$\times 91 -9$$

$$\hline 103 \end{array}$$

$$\downarrow 2 \rightarrow 101 \underline{92}$$

$$\begin{array}{c|c} A & B \\ \hline 93 & 83 \\ 112 & 103 \\ 115 & 94 \end{array}$$

$$\begin{array}{r} 203 +3 \\ \times 205 +5 \\ \hline 416 \underline{15} \end{array} \quad \begin{array}{r} 407 +7 \\ \times 405 +5 \\ \hline 1648 \underline{35} \end{array} \quad 412$$

$$\begin{array}{r} 8 \\ \downarrow \uparrow \\ 72 \end{array}$$

$$\times 78$$

$$\hline 5616$$

$$\begin{array}{r} 5 \\ \uparrow \downarrow \\ 43 \end{array}$$

$$\times 47$$

$$\hline 2021$$

$$\begin{array}{r} 54 \\ \times 56 \end{array}$$

$$\hline 3024$$

$$\begin{array}{r} 61 \\ \times 69 \end{array}$$

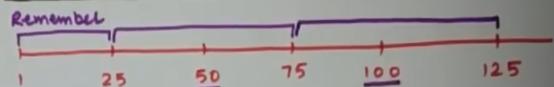
$$\hline 4209$$

$$\begin{array}{r} 85 \\ \times 85 \end{array}$$

$$\hline 7225$$

VEDIC MATH

$$\begin{array}{ll}
 1^2 = 1 & 18^2 = 324 \\
 2^2 = 4 & 19^2 = 361 \\
 3^2 = 9 & 20^2 = 400 \\
 4^2 = 16 & 21^2 = 441 \\
 5^2 = 25 & 22^2 = 484 \\
 6^2 = 36 & 23^2 = 529 \\
 7^2 = 49 & 24^2 = 576 \\
 8^2 = 64 & 25^2 = 625 \\
 9^2 = 81 & \\
 10^2 = 100 & \\
 11^2 = 121 & \\
 12^2 = 144 & \\
 13^2 = 169 & \\
 14^2 = 196 & \\
 15^2 = 225 & \\
 16^2 = 256 & \\
 17^2 = 289 &
 \end{array}$$



$$\begin{array}{ccc}
 53^2 = 28\underset{-}{0}9 & & 56^2 = 3136 \\
 +3 \nearrow & & 25+6 \\
 25 & & \\
 \end{array}$$

$$\begin{array}{ccc}
 61^2 = 37\underset{-}{2}1 & 64^2 = 4096 & 72^2 = 51\underset{-}{8}4 \\
 25+11 \nearrow & 25+14 & 25+22 \\
 & & \\
 \end{array}$$

$$\begin{array}{ccc}
 48^2 = 23\underset{-}{0}4 & & 42^2 = 1764 \\
 -2 & & 25-8 \\
 & & \\
 \end{array}$$

$$\begin{array}{ccc}
 38^2 = 1444 & & 28^2 = 7\underset{-}{8}4 \\
 25-12 & & 25-22 \\
 & & \\
 \end{array}$$

VEDIC MATH

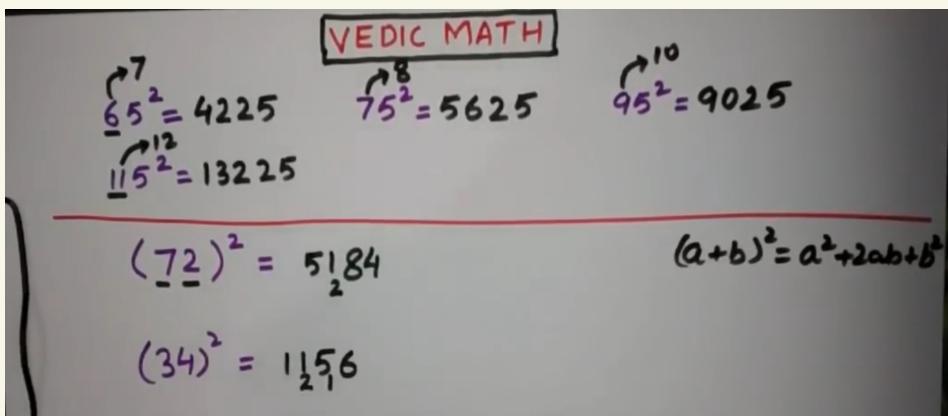
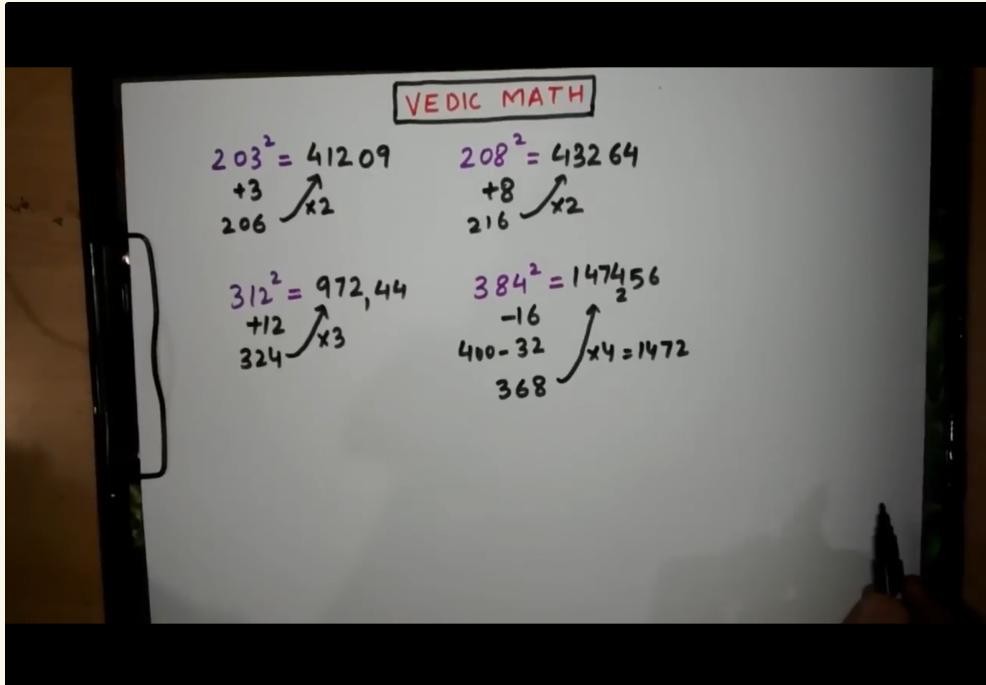
$$\begin{array}{ccc}
 103^2 = 106\underset{-}{0}9 & 107^2 = 11449 & 113^2 = 127\underset{-}{6}9 \\
 +3 & +7 & +13 \\
 & & \\
 \end{array}$$

$$\begin{array}{ccc}
 119^2 = 141\underset{-}{3}61 & & 106, 108, 112, 117, 122 \\
 +19 & & \\
 & & \\
 \end{array}$$

$$\begin{array}{ccc}
 98^2 = 96\underset{-}{0}4 & 96^2 = 9216 & 92^2 = 8464 \\
 -2 & -4 & -8 \\
 & & \\
 \end{array}$$

$$\begin{array}{ccc}
 88^2 = 77,44 & & 83^2 = 68\underset{-}{2}89 \\
 -12 & & -17 \\
 100-24 & & \\
 & & \\
 \end{array}$$

Squares



Square Root

N	N^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81

$$\begin{array}{r} \overline{42} \\ 7 \times \boxed{39} \quad 6 \\ \hline 3 \quad \text{unit} \\ \end{array}$$
$$\begin{array}{r} \overline{72} \\ 9 \times \boxed{73} \quad 6 \\ \hline 86 \end{array}$$

N	N^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81

$$\begin{array}{r} \overline{7744} <^2 \textcircled{88} \\ 4489 <^3 \textcircled{67} \\ \times 12 \quad 132 \\ \hline 13456 <^4 \textcircled{116} \\ 6241 <^1 \textcircled{79} \\ \hline 7056 <^4 \textcircled{84} \end{array}$$

Cube Root

N	N^3
1	1 ✓
2	8
3	27
4	64 ✓
5	125 ✓
6	216 ✓
7	343
8	512
9	729 ✓

32768
54872
571787
205379
79507

tables from 1 to 100

$\frac{120+12}{44} \times 3$	$\frac{200+16}{54} \times 4$	$\frac{300+15}{63} \times 5$	$\frac{540+12}{92} \times 6$
$\underline{132}$	$\underline{216}$	$\frac{315}{\cancel{10}} \times \cancel{7}$	$\frac{552}{60} \times 72$
$\frac{280+16}{74} \times 4$	$\frac{720+54}{86} \times 9$	$\frac{990+63}{79} \times 7$	$\frac{640}{72} \times 8$
$\underline{296}$	$\underline{774}$	$\underline{553}$	$\underline{712}$

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SSC CGL, Bank PO, CDS,

$9+4$

$$\begin{array}{r} 31 \\ \times 43 \\ \hline 1333 \end{array}$$

 $6+20$

$$\begin{array}{r} 34 \\ \times 52 \\ \hline 1768 \end{array}$$

$$\begin{array}{r} 43 \\ \times 32 \\ \hline 1376 \end{array}$$

$$\begin{array}{r} 54 \\ \times 48 \\ \hline 2592 \end{array}$$

$$49 + \overset{1}{\cancel{3}}2 = 50 + 31$$

$$\begin{array}{r} 83 \\ \times 47 \\ \hline 3901 \end{array}$$

$$\begin{array}{r} 78 \\ \times 47 \\ \hline 3666 \end{array}$$

$$\begin{array}{r} 67 \\ \times 83 \\ \hline 5561 \end{array}$$

$$\begin{array}{r} 86 \\ \times 78 \\ \hline 6708 \end{array}$$

Number System

$$\sqrt{9} = 3$$

$$\sqrt{144} = 12$$

Numbers

1

$$(4+3i)$$

real

img. no.

Real Number

Imaginary Number

it has
end

Rational Number
 $\sqrt{9}, \sqrt{144}$

Irrational Number
 $\sqrt{2}, \sqrt{3}$

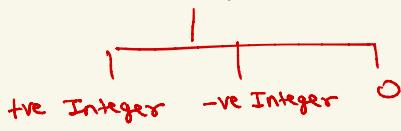
$$\sqrt{2} = 1.41421356\dots$$

$$\sqrt{3} = 1.7320508\dots$$

no end

Integers

Fraction $\left\{ \frac{1}{3}, \frac{3}{7} \right\}$



$$+ve \rightarrow 1, 2, 3, 4 \dots n$$

$$-ve \rightarrow -1, -2, -3, -4 \dots n$$

Natural
(1, 2, 3, ..., n)

Whole
(0, 1, 2, 3, ..., n)

Even Odd Prime

Composite Number
 {if a number which have
 more than two factors is
 called Composite number}

$$\begin{array}{r}
 15 \\
 \downarrow \\
 1 \times 15 \\
 15 \times 1 \\
 3 \times 5 \\
 5 \times 3
 \end{array}$$

Divisibility Rules

5 → If last digit '5' or '0'. eg. 105, 100

10 → If last digit '0'. eg. 1050, 1000

2 → If last digit even or zero. eg. 1658, 1650

3 → Add sum of digits and then divide by 3. eg: $342 \rightarrow 3+4+2 = \frac{9}{3} = 3$

9 → Add sum of digits and then divide by 9. eg: $243243 \rightarrow 2+4+3+2+4+3 = \frac{18}{9} = 2$

4 → if last two digit divisible by 4. eg. $\frac{1624}{4} \checkmark$

8 → if last three digit divisible by 8. eg. $\frac{15120}{8} \checkmark$

6 → if number is divisible by 2 & 3 eg. $\frac{126}{2}, \frac{126}{3}$

12 → if number is divisible by 3 & 4

14 → if number is divisible by 7 & 2

15 → if number is divisible by 5 & 3

18 → if number is divisible by 9 & 2

7 → $34(\textcircled{3})^{x^2} \rightarrow 34 - 6 = 28/7 \checkmark$

17 → $190(\textcircled{4})^{x^2} \rightarrow 190 - 20 = 170/17 \checkmark$

19 → $36(\textcircled{1})^{x^2} \rightarrow 36 + 2 = 38/19 \checkmark$

13 → $32(\textcircled{5})^{x^4} \rightarrow 32 + 20 = 52/13 \checkmark$

11 → $\frac{\overbrace{9 \ 1}^{\text{even}} \ \overbrace{7 \ 4}^{\text{even}}}{\overbrace{1 \ 0}^{\text{odd}} \ \overbrace{0 \ 9}^{\text{odd}}} \rightarrow (\text{sum of odd digits}) - (\text{sum of even digits}) = 16 - 5 = 11/11 \checkmark$

$\underbrace{6 \ 3 \ 9 \ 1}_{20-9=11} \ \underbrace{5 \ 5}_{11/11 \checkmark}$

Unit Digit (last digit)

$$\textcircled{1} \quad \underline{936} + \underline{972} + \underline{221} = \underline{\underline{9}}$$

$$\textcircled{2} \quad \underline{2369} + \underline{2434} = 9 + 4 = \underline{13}$$

$$\textcircled{3} \quad \underline{93} \times \underline{46} = \underline{18}$$

$$\textcircled{4} \quad 24 \times 98 \times 236 = 4 \times 8 \times 6 = \underline{32} \times 6 = \underline{12}$$

$$\textcircled{5} \quad \underline{1936} - \underline{243} = \underline{\underline{3}}$$

$$\textcircled{6} \quad 1934 - 239 = \cancel{1}^4 - 9 = 5$$

$$\textcircled{7} \quad 7^3 = 7 \times 7 \times 7 = \underline{49} \times \underline{7} = \underline{63}$$

$$\textcircled{8} \quad 7^4 = 7^2 \times 7^2 = \underline{49} \times \underline{49} = \underline{81}$$

$$\textcircled{9} \quad 81^3 = \underline{81} \times \underline{81} \times \underline{81} = 1$$

model ①

$$0^n = 0, 1^n = 1, 5^n = 5, 6^n = 6$$

$$\text{eg. } 26^2 = \underline{26} \times \underline{26} = \underline{36}$$

$$26^{142} = 26 \times 26 \times \dots = \underline{\underline{6}}$$

$$\textcircled{1} \quad \underline{266}^3 + \underline{395}^5 = ? \quad \begin{cases} 6^n = \text{last digit 6} \\ 5^n = \text{last digit 5} \end{cases}$$

$$\textcircled{2} \quad \underline{2311}^{124} \times \underline{646}^{94} = ? \quad \begin{cases} 1^n = \text{last digit 1} \\ 6^n = \text{last digit 6} \end{cases}$$

$$\underline{\underline{1}} \times \underline{\underline{6}} = \underline{\underline{6}}$$

Model ②

$$\left| \begin{array}{l} g^{\text{odd}} = 4 \\ g^{\text{even}} = 6 \end{array} \right| \left| \begin{array}{l} g^{\text{odd}} = 9 \\ g^{\text{even}} = 1 \end{array} \right.$$

$$\textcircled{1} \quad \underline{24g^{33}} + \underline{25g^{36}} + \underline{256^{133}} = ? \quad \left\{ \begin{array}{l} g^{\text{odd}} = 9 \\ g^{\text{even}} = 6 \\ 6^n = 6 \end{array} \right.$$

$\underline{-} - 9 + \underline{-} - 6 + \underline{-} - 6$
 $= 2\underline{1} \leftarrow \text{last digit ans is } '1'$

Model ③

(2, 3, 7, 8)

Note: Divide the power by 4 then put remainder in original

$$\textcircled{1} \quad \underline{212^{79}} = \underline{2^3} = \underline{8} \leftarrow \text{last digit} \quad 4 \overline{) 79} \begin{matrix} 19 \\ 4 \\ \hline 39 \\ 36 \\ \hline 3 \end{matrix}$$

$$\textcircled{2} \quad \underline{73^{54}} = \underline{3^2} = \underline{9} \leftarrow \text{last digit} \quad 4 \overline{) 59} \begin{matrix} 13 \\ 4 \\ \hline 14 \\ 12 \\ \hline 2 \end{matrix} \quad \frac{3}{3} \leftarrow \text{remainder}$$

$$\textcircled{3} \quad \underline{378^{41925}} = \underline{8^1} = \underline{8} \leftarrow \text{last digit} \quad 4 \overline{) 25} \begin{matrix} 6 \\ 24 \\ \hline 1 \end{matrix}$$

$$\textcircled{4} \quad \underline{214^{2164}} = \underline{4^{\circ} \times} \\ = \underline{4^4} \\ = \underline{\frac{4 \times 4 \times 4 \times 4}{16 \times 16}} \quad \leftarrow \text{last digit} \\ = \underline{36}$$

$$4 \overline{) 64} \begin{matrix} 16 \\ 4 \\ \hline 24 \\ 24 \\ \hline 0 \end{matrix}$$

Example Question (unit digit @ last digit)

$$\textcircled{1} \quad \underline{124}^{376} + \underline{124}^{375} = 6 + 4 \quad \left\{ \begin{array}{l} 4^{\text{even}} = 6 \\ 4^{\text{odd}} = 4 \end{array} \right.$$

$$\textcircled{2} \quad \underline{25}^{6527} + \underline{36}^{526} + \underline{73}^{54} = 5 + 6 + 3^2 \quad \left\{ \begin{array}{l} 5^n = 5 \\ 6^n = 6 \\ 3^n = 4 \sqrt[4]{54} \\ \frac{4}{14} \\ \frac{12}{2} \\ 3^2 = ? \end{array} \right.$$

$$\begin{aligned} &= 5 + 6 + 9 \\ &= 20 \end{aligned}$$

$$\textcircled{3} \quad 7^{295} \times 3^{158} \times 341^{476} = ?$$

$$\begin{aligned} &\quad -3 \times -9 \times -1 \\ &= 27 \leftarrow \text{last digit} \end{aligned}$$

$$7^n = \sqrt[4]{95} \quad \left| \begin{array}{l} 7^3 = 343 \\ 3^2 = 9 \end{array} \right. \quad \left| \begin{array}{l} 3^n = \sqrt[4]{58} \\ 3^2 = 9 \end{array} \right. \quad \left| \begin{array}{l} 1^n = 1 \\ 1^2 = 1 \end{array} \right.$$

$$\textcircled{4} \quad [(251)^{98} + (21)^{29} - (106)^{100} + (705)^{36} - 16^4 + 259] = ?$$

$$\begin{aligned} &1 + 1 - 6 + 5 - 6 + 9 \\ &= 16 - 12 = 4 \leftarrow \begin{array}{l} \text{unit digit} \\ \text{last digit} \end{array} \end{aligned}$$

$$1^n = 1, 6^n = 6, 5^n = 5$$

Last two Digits
By Mohit Goyal

Rules

① Ending in 1

$$(\dots a1)^{b} = \dots \underline{axb} \ 1$$

Eg. $(134\overset{5}{1})^{2342\overset{3}{1}} = \dots \frac{5}{1}$ last two digit

Eg. $(217\overset{3}{1})^{217\overset{3}{1}} = \dots \frac{9}{1}$ last two digit

② Even No.

$$(24)^{\text{odd}} = 24$$

last two digit

$$(24)^{\text{even}} = 76 \rightarrow \text{eg. } 2^{20} = 10485\cancel{7}6$$

$$(76)^N = 76 \text{ last two digits} \rightarrow \text{eg. } 76^2 = 57\cancel{7}6$$

$$76 \times 2^n = \text{last two digits of } 2^n \rightarrow \text{eg. } 76 \times 2^5 \rightarrow 24\cancel{3}2$$

$2^5 \rightarrow 32$

③ Ending in 5

If second last digit & power odd it will end in 75 otherwise 25

$$(-\overset{\text{odd}}{5})^{\text{odd}} \rightarrow 75, \text{ otherwise } \rightarrow 25$$

$$\begin{matrix} \overset{\text{odd}}{7}5 \\ \text{odd} \end{matrix} \xrightarrow{65} 75$$

$$19\overset{\text{odd}}{9}5 \xrightarrow{2014} 25$$

$$\overset{\text{odd}}{2}5 \xrightarrow{75} 25$$

$$19\overset{\text{odd}}{9}5 \xrightarrow{2015} 75$$

④ Odd No. (3, 7, 9)

Convert making last digit 1
and apply rule no. ①

Eg. 19^{256}
 $(19^2)^{128}$
 $(3\overset{\text{odd}}{6}1)^{128} \leftarrow \text{now apply rule ①}$
 $\frac{axb}{6 \times 8 = 48} \frac{1}{1}$
last two digit (8)

⑤ $x^2, (50-x)^2, (50+x)^2, (100-x)^2, (100+x)^2 \dots$
Right & last two digits same \Rightarrow $x^2 \equiv 1$

$$\begin{aligned} \rightarrow x = 23 &\rightarrow 23^2 \rightarrow \underline{\underline{\underline{29}}} \\ (50-x) &\rightarrow 27^2 \rightarrow \underline{\underline{\underline{29}}} \\ (50+x) &\rightarrow 73^2 \rightarrow \underline{\underline{\underline{29}}} \\ (100-x) &\rightarrow 77^2 \rightarrow \underline{\underline{\underline{29}}} \\ (100+x) &\rightarrow 123^2 \rightarrow \underline{\underline{\underline{29}}} \end{aligned}$$

Even No. Rule ② Example

$$\begin{aligned} 2^{105} \\ (2^{20})^5 \times 2^5 &\quad (\because 2^{20} \rightarrow \underline{\underline{\underline{76}}}) \\ (-\underline{\underline{76}})^5 \times 2^5 &\quad (\because 76^5 \rightarrow \underline{\underline{\underline{76}}}) \end{aligned}$$

$$76 \times 2^5 \quad (\because 76 \times 2^5 \rightarrow \underline{\underline{\underline{\underline{}}}})$$

$$2^5 \rightarrow \text{last two digit} \quad 2^n$$

$$\underline{\text{ans}} \quad \underline{\underline{\underline{32}}}$$

Remainder Theorem

Normal Number
 $\frac{23}{5}$

Polynomial Number
 $(x^2 + 7x + 2) \div (x^2 + 2)$

$2^1 = 2$	$3^1 = 3$
$2^2 = 4$	$3^2 = 9$
$2^3 = 8$	$3^3 = 27$
$2^4 = 16$	$3^4 = 81$
$2^5 = 32$	$3^5 = 243$

Note: Remainder value always less than Denominator

$$\textcircled{1} \quad \frac{23}{5} = \begin{matrix} \text{remainder} \\ 3 \end{matrix}$$

\curvearrowleft Denominator

$$\textcircled{2} \quad \frac{17}{6} = \begin{matrix} \text{remainder} \\ 5 \end{matrix}$$

$$\textcircled{3} \quad \frac{86 \times 23}{11} = \frac{86}{11} \times \frac{23}{11} = \begin{matrix} \text{remainder} \\ 9 \end{matrix}$$

$$\textcircled{4} \quad \frac{38 + 71 + 86}{16} = \frac{38}{16} + \frac{71}{16} + \frac{86}{16} = \begin{matrix} \text{remainders} \\ 6+7+6 \end{matrix} = \begin{matrix} \text{Remainder} \\ 19 \end{matrix} = \frac{19}{16} = \begin{matrix} \text{Remainder} \\ 3 \end{matrix}$$

acc. to note

$$\textcircled{5} \quad \frac{123 \times 124 \times 125}{9} = \frac{123}{9} \times \frac{124}{9} \times \frac{125}{9} = \begin{matrix} \text{remainders} \\ (6)(7)(8) \end{matrix}$$

$$= \frac{6 \times 7 \times 8}{9}$$

$$= \frac{42 \times 8}{9} = \begin{matrix} \text{remainder} \\ (6) \end{matrix}$$

$$= \frac{6 \times 8}{9} = \frac{48}{9} = \begin{matrix} \text{remainder} \\ (3) \end{matrix} = \textcircled{3}$$

$$\textcircled{6} \quad \frac{7^5}{9} = \frac{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7}}{9} = \begin{matrix} \text{remainders} \\ (3)(3)(3)(3)(3) \end{matrix}$$

$$= \frac{3 \times 3 \times 3 \times 3 \times 3}{9} = \frac{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}}{9} = \begin{matrix} \text{remainders} \\ (1)(1)(1) \end{matrix} = 1 \times 1 \times 3 = \begin{matrix} \text{remainder} \\ 3 \end{matrix}$$

$$\textcircled{7} \quad \frac{2^{75}}{5} = \frac{2^{75/4}}{5} \quad \therefore 18 \times 4 = 72 + \frac{3}{75}$$

$$= \frac{(2^4)^{18} \times 2^3}{5}$$

$$= \frac{(16)^{18} \times 8}{5}$$

$$= \frac{(1)^{18} \times 8}{5} \quad \begin{array}{c} 3 \\ 5 \overline{) 16} \\ \downarrow 15 \\ \downarrow 1 \end{array} \leftarrow \text{remainder}$$

$$= \frac{1 \times 8}{5} = \frac{8}{5} \stackrel{(3)}{=} 3 \text{ remainder}$$

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16 \quad \boxed{2^4 = 16} \leftarrow \text{bcz near to } \boxed{15} \text{ & In Que Denominator is 5}$$

$$2^5 = 32$$

$$\textcircled{8} \quad \frac{7^{129}}{5} = \frac{2^{129}}{5} \quad \therefore \frac{7}{5} = 2 \text{ remainder}$$

$$= \frac{2^{129/4}}{5} \quad \begin{array}{c} 32 \times 4 = 128 \\ \downarrow 1 \\ \downarrow 129 \end{array}$$

$$= \frac{(2^4)^{32} \times 2^1}{5} \quad \begin{array}{c} 2^1 = 2 \\ 2^2 = 4 \\ 2^3 = 8 \\ 2^4 = 16 \quad \boxed{2^4 = 16} \leftarrow \text{bcz near to } \boxed{15} \text{ & In Que Denominator is 5} \\ 2^5 = 32 \end{array}$$

$$= \frac{(16)^{32} \times 2}{5} \quad \therefore \frac{+ (1) \text{ remainder}}{5} \quad \therefore 1^n = 1$$

$$= \frac{1 \times 2}{5} = 2 \text{ remainder}$$

$$\textcircled{9} \quad \frac{517^{517}}{5} = \frac{(2)^{517}}{5} \quad \therefore \frac{\sqrt[10]{517}}{\sqrt[5]{515}}$$

$$= \frac{(2^4)^{129} \times 2^1}{5} \quad \begin{array}{c} 129 \times 4 = 516 \\ \downarrow 1 \\ \downarrow 517 \end{array} \quad \begin{array}{c} 103 \\ \sqrt[5]{515} \\ \downarrow 2 \end{array} \leftarrow \text{remainder}$$

$$= \frac{(16)^{125} \times 2^1}{5}$$

$$= \frac{1 \times 2^1}{5} = \frac{1 \times 2}{5} = 2 \text{ remainder}$$

Q) What is the remainder when 2243^{165} divided by 5?

$$\rightarrow \frac{2243^{165}}{5} = \frac{(3)^{165}}{5} \therefore \frac{2243}{5} = 3 \text{ remainder}$$

$$\begin{aligned}3^1 &= 3 \\3^2 &= 9 \\3^3 &= 27\end{aligned}$$

$$= \frac{(3)^{165}}{5}$$

$$\begin{aligned}\frac{81}{5} &= 1 \text{ remainder} \\&= (1)^{41} \\&= 1^n \\&= 1\end{aligned} \quad \begin{aligned}= (3^4)^{41} \times 3^1 \\= (81)^{41} \times 3\end{aligned}$$

nearby
multiple of 5
bcz in Ques
denominator is 5

$$\begin{aligned}3^4 &= 81 \\3^5 &= 243\end{aligned}$$

$$= \frac{1 \times 3}{5} = 3 \text{ remainder}$$

Sum of Series

Model ①

Natural Numbers

=

$$(1+2+3+4+\dots+n)$$

$$\left[n \frac{(n+1)}{2} \right]$$

$$1^2 + 2^2 + 3^2 + \dots + n^2$$

$$\left[n(n+1) \frac{(2n+1)}{6} \right]$$

$$1^3 + 2^3 + 3^3 + \dots + n^3$$

$$\left[n \frac{(n+1)}{2} \right]^2$$

Model ②

Natural Numbers

=

$$1+3+5+7+\dots+33$$

odd series

$$(x^2)$$

$$\downarrow x = \frac{n+1}{2}$$

$$x = \frac{33+1}{2}$$

$$= \frac{34}{2}^{17}$$

$$(17)^2 = \underline{\underline{289}} \text{ ans}$$

$$2+4+6+8+\dots+56$$

even series

$$(x(x+1))$$

$$\downarrow x = \frac{n}{2}$$

$$= \frac{56}{2}$$

$$= 28$$

$$x(x+1)$$

$$25(25+1)$$

$$= 625 + 25$$

$$= \underline{\underline{650}} \text{ ans}$$

$$51+53+55+\dots+99$$

odd series

$$(1+3+5+\dots+99) - (1+3+5+\dots+49)$$

Total - First

$$x^2$$

$$\downarrow x = \frac{n+1}{2}$$

$$= \frac{99+1}{2}$$

$$= 50$$

$$x^2 = 50$$

$$= 2500$$

$$x^2$$

$$\downarrow x = \frac{n+1}{2}$$

$$= \frac{49+1}{2}$$

$$= 25$$

$$x^2 = 25$$

$$= 625$$

$$2500 - 625$$

$$= 1875$$

$$=$$

Example 8

$$\textcircled{1} \quad 1+2+3+\dots+30 = ? \quad n \frac{(n+1)}{2} = \frac{30 \times 31}{2} = 15 \times 31 = \underline{\underline{465}}$$

$$\textcircled{2} \quad 1^2+2^2+3^2+\dots+12^2 = ? \quad n \frac{(n+1)(2n+1)}{6} = \cancel{12^2} \cancel{13 \times 25} = 50 \times 13 = \underline{\underline{650}}$$

$$\textcircled{3} \quad 1^3+2^3+3^3+\dots+9^3 = ? \quad \left[n \frac{(n+1)}{2} \right]^2 = \left(\frac{9 \times 10}{2} \right)^2 = (45)^2 = \underline{\underline{2025}}$$

$$\textcircled{4} \quad 1+3+5+\dots+49 = ? \quad (x^2) \rightarrow x = \frac{n+1}{2} = \frac{49+1}{2} = \frac{50}{2} = 25 \\ x^2 = (25)^2 = \underline{\underline{625}}$$

$$\textcircled{5} \quad 2+4+6+\dots+58 = ?$$

$$x(x+1) \rightarrow x = \frac{n}{2} = \frac{58}{2} = 29$$

$$x(x+1) \rightarrow 29(29+1) = 29 \times 30 = \underline{\underline{870}}$$

$$\textcircled{6} \quad 10^2+11^2+12^2+\dots+20^2 = ?$$

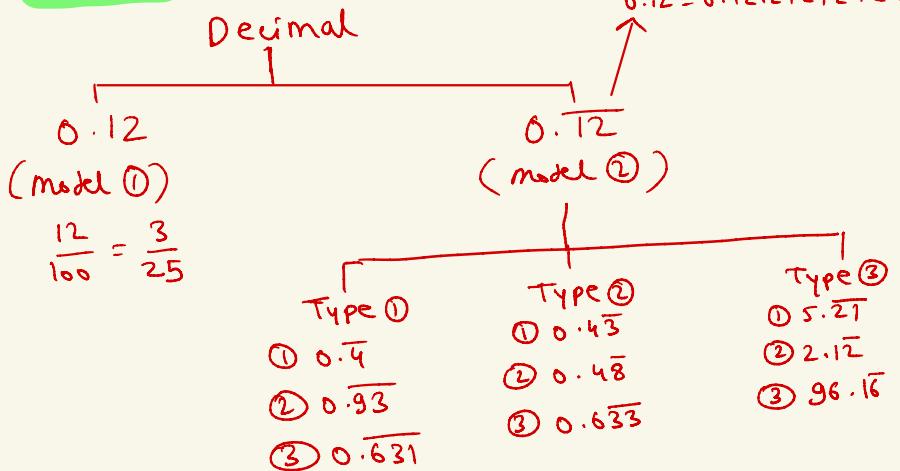
$$(1^2+2^2+\dots+20^2) - (1^2+2^2+\dots+9^2)$$

Total - First

$$n \frac{(n+1)(2n+1)}{6} - n \frac{(n+1)(2n+1)}{6}$$
$$\cancel{20 \times 21 \times 41} - \cancel{\frac{9 \times 10 \times 19}{6}}$$

$$2870 - 15 \times 19 = \underline{\underline{2585}}$$

Decimal to fraction



Type ①

$$\begin{array}{l} \textcircled{1} \\ \hookrightarrow \boxed{\frac{4}{9}} \end{array}$$

$$\begin{array}{l} \textcircled{2} \\ \hookrightarrow \boxed{\frac{93}{99}} \end{array}$$

$$\begin{array}{l} \textcircled{3} \\ \hookrightarrow \boxed{\frac{631}{999}} \end{array}$$

Type ②

$$\begin{array}{l} \textcircled{1} \\ 43 - 4 = \boxed{\frac{39}{90}} \end{array}$$

$$\begin{array}{l} \textcircled{2} \\ 48 - 4 = \boxed{\frac{44}{90}} \end{array}$$

$$\begin{array}{l} \textcircled{3} \\ 633 - 6 = \boxed{\frac{627}{990}} \end{array}$$

Type ③

$$\begin{array}{l} \textcircled{1} \\ 5 + 0.\overline{21} \end{array}$$

$$5 + \frac{21}{99}$$

$$= \boxed{\frac{516}{99}}$$

$$\begin{array}{l} \textcircled{2} \\ 2 + 0.\overline{12} \\ 2 + \frac{12-1}{90} \end{array}$$

$$2 + \frac{11}{90}$$

$$= \boxed{\frac{191}{90}}$$

$$\begin{array}{l} \textcircled{3} \\ 96 + 0.\overline{16} \\ 96 + \frac{16-1}{90} \\ 96 + \frac{15}{90} \\ = 8640 + 15 = \boxed{\frac{8655}{90}} \end{array}$$

Consecutive Numbers

1, 2, 3 ... \rightarrow cons. natural numbers ($x^{\textcircled{1}}, x+1, x+2, \dots, n$)

2, 4, 8 ... \rightarrow cons. Even numbers ($x^{\textcircled{2}}, x+2, x+4, x+6, \dots, n$)

1, 3, 5 ... \rightarrow cons. Odd numbers ($x^{\textcircled{3}}, x+2, x+4, x+6, \dots, n$)

Q. The sum of three consecutive numbers is 87. The middle number is.

$$\rightarrow x + x+1 + x+2 = 87$$

$$3x + 3 = 87$$

$$3x = 87 - 3$$

$$x = 28$$

$$\begin{matrix} x, & x+1, & x+2 \\ 28 & 29 & 30 \end{matrix}$$

Ans 29

Q. The sum of three consecutive odd numbers is 147.
Then, the middle number is.

$$\rightarrow x + x+2 + x+4 = 147$$

$$3x + 6 = 147$$

$$3x = 147 - 6$$

$$x = 47$$

$$\begin{matrix} x, & x+2, & x+4 \\ 47 & 49 & 51 \end{matrix}$$

Ans 49

Q. The sum of four consecutive even numbers is 748.
The smallest among them.

$$\rightarrow \underline{x} + \underline{x+2} + \underline{x+4} + \underline{x+6} = 748$$

$$4x + 12 = 748$$

$$4x = 748 - 12$$

$$x = 184$$

Ans 184

$$\begin{matrix} x, & x+2, & x+4, & x+6 \\ 184 & 186 & 188 & 190 \end{matrix}$$

Q = Out of six consecutive natural numbers, if the sum of first three is 27, what is the sum of the other three.

$$\rightarrow \frac{x}{\text{first three cons. natural numbers}} + \frac{x+1}{\text{}} + \frac{x+2}{\text{}} \stackrel{(8)}{=} x+3 \stackrel{(8)}{=} x+4 \stackrel{(8)}{=} x+5$$

$$3x+3 = 27 \quad \downarrow \quad \downarrow \quad \downarrow$$

$$3x = 24 \quad \text{Ans} \Rightarrow \boxed{36}$$

$$\boxed{x = 8}$$

finding a Number

Note $17 - 10 = 7$

- * Difference = Subtract (-)
- * More = Subtract (-)
- * Exceeds = Subtract (-)

Q = What is two-third of half of 369?

$$\rightarrow \frac{2}{3} \text{ of } \frac{1}{2} \text{ of } 369 = ? \quad \frac{2}{3} \times \frac{1}{2} \times \frac{369}{123} = ?$$

$$\boxed{\text{Ans} = 123}$$

Q = If one-third of one-fourth of a number is 15, then three-tenth of the number is

$$\rightarrow \frac{1}{3} \times \frac{1}{4} \times x = 15 \quad x = 15 \times 4 \times 3$$

$$= 180$$

$$\frac{3}{180} \times 180 \Rightarrow \boxed{\text{Ans} 54}$$

Q = $\frac{1}{2}$ of $\frac{3}{4}$ of a number is $2(\frac{1}{2})$ of 10. What is the number?

$$\rightarrow \frac{1}{2} \times \frac{3}{4} \times x = \frac{5}{2} \times 10^5$$

$$x = \frac{200}{3}$$

$$\boxed{\text{Ans } 66\frac{2}{3}}$$

Q If the sum of two numbers, one of which is $\frac{2}{5}$ times the other is 50, then the numbers are,

\rightarrow first no. = x , second no. = $\frac{2}{5}x$

$$x + \frac{2}{5}x = 50$$

$$\frac{5x + 2x}{5} = 50$$

$$7x = 250$$

$$x = \frac{250}{7}$$

$$\boxed{\text{first no.} = \frac{250}{7}}$$

$$\begin{aligned}\text{Second no.} &= \frac{2}{5} \times \frac{250}{7} \\ &= \frac{100}{7}\end{aligned}$$

Q: If $\frac{3}{4}$ of a number is 7 more than $\frac{1}{6}$ of the number, then $\frac{5}{3}$ of the number is,

\rightarrow first - second = 7

(more = subtract) $\quad (-)$

$$\frac{3}{4}x - \frac{1}{6}x = 7$$

$$\frac{3x}{4} - \frac{x}{6} = 7$$

$$\frac{9x - 2x}{12} = 7$$

$$7x = 7 \times 12$$

$$\boxed{x = 12}$$

$$\frac{5}{3}x \rightarrow \frac{5}{3}x + 2^4$$

$$\boxed{\text{Ans } 20}$$

Q If $\frac{1}{2}$ is added to a number and the sum is multiplied by 3, the result is 21. Then the number is,

$$\rightarrow \left(\frac{1}{2} + x \right) \times 3 = 21$$

$$\frac{3}{2} + 3x = 21$$

$$3x = 21 - \frac{3}{2}$$

$$3x = \frac{42 - 3}{2}$$

$$3x = \frac{39}{2}$$

$$x = \frac{13}{2}$$

Q If $\frac{4}{5}$ th of a number exceeds its $\frac{3}{4}$ th by 8, then the number is,

$$\rightarrow \frac{4}{5}x - \frac{3}{4}x = 8$$

$$\frac{16x - 15x}{20} = 8 \quad \boxed{x = 160}$$

Q The sum of three numbers is 2, the 1st number is $\frac{1}{2}$ times the second number and 3rd number is $\frac{1}{4}$ times the second number. Then the second number is,

$$\rightarrow \frac{x}{2} + \frac{x}{1} + \frac{x}{4} = 2$$

$\frac{1}{2}$ $\frac{1}{4}$

$$\frac{x}{2} + x + \frac{x}{4} = 2$$

$$\frac{2x + 4x + x}{4} = 2$$

$$7x = 2 \times 4$$

$$x = \frac{8}{7}$$

Miscellaneous

Q A person gives $\frac{1}{4}$ of his property to his daughter, $\frac{1}{2}$ to his son and $\frac{1}{5}$ for charity. How much has he given away.

$$\rightarrow \frac{1}{4} + \frac{1}{2} + \frac{1}{5} \Rightarrow \frac{5+10+4}{20} = \boxed{\frac{19}{20}}$$

Q In an office, there are 108 tables and 132 chairs. If $\frac{1}{6}$ of tables and $\frac{1}{4}$ of chairs are broken. How many people can work in the office if each person requires one table and one chair?

$$\rightarrow \begin{array}{c} \text{Tables} \\ \hline \hline 108 \end{array}$$

$$108 \times \frac{1}{6} = 18 \text{ broken}$$

$$\text{Good tables} = 108 - 18 = \underline{\underline{90}}$$

office = 90 people

$$\begin{array}{c} \text{Chairs} \\ \hline \hline 132 \end{array}$$

$$132 \times \frac{1}{4} = 33 \text{ broken}$$

$$\text{Good chairs} = 132 - 33 = \underline{\underline{99}}$$

Q. In a school $\frac{1}{10}$ of the boys are same in number as $\frac{1}{4}$ of the girls. The ratio of the boys to the girls in that school is,

$$\rightarrow \text{Boys} = x, \text{ Girls} = y$$

$$\frac{1}{10} \times x = \frac{1}{4} \times y$$

$$\frac{x}{10} = \frac{y}{4}$$

$$\frac{x}{y} = \frac{10}{4}$$

$$\boxed{\frac{x}{y} = \frac{5}{2}}$$

Q Two basket together have 640 oranges. If $\frac{1}{5}$ of the oranges in the first basket be taken out to the second basket. The number of oranges on the first basket is,



$$B_1 = x, B_2 = 640 - x$$

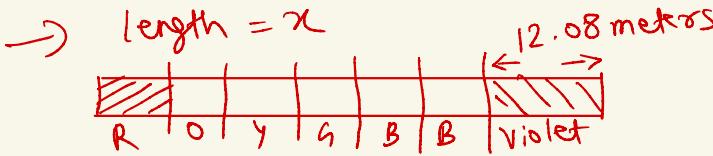
$$x - \frac{1}{5}x = 640 - x + \frac{x}{5}$$

$$\frac{4x}{5} = \frac{640 \times 5 - 4x}{5}$$

$$8x = 640 \times 5$$

$$| x = 400$$

Q $\frac{1}{10}$ of a rod is coloured red, $\frac{1}{20}$ orange, $\frac{1}{30}$ yellow, $\frac{1}{40}$ green, $\frac{1}{50}$ blue, $\frac{1}{60}$ black and the rest is violet. If the length of the violet portion of the rod is 12.08 meters then the length of the rod is,



$$x - \left[\frac{x}{10} + \frac{x}{20} + \frac{x}{30} + \frac{x}{40} + \frac{x}{50} + \frac{x}{60} \right] = 12.08$$

$$x - \left[1 - \frac{147}{600} \right] = 12.08$$

$$x \left[\frac{453}{600} \right] = 12.08$$

$$x = 12.08 \times \frac{600}{453}$$

$$x = \frac{12.08 \times 600}{453}$$

$$| x = 16 \text{ meters}$$

Q A man read $\frac{2}{5}$ th of the book on the first day. He read $\frac{1}{3}$ rd more on the second day than he read on the first day. 15 pages are left for the third day. The number of pages in the book is,

$$\rightarrow \text{Book} = x$$

$$\begin{array}{c} \text{1st day} \quad \text{2nd day} \quad \text{3rd day} \\ \uparrow \qquad \qquad \qquad \uparrow \\ \frac{2x}{5} + \frac{2x}{5} + \left(\frac{2x}{5} \times \frac{1}{3} \right) + 15 = x \end{array}$$

$$\frac{2x}{5} + \frac{2x}{5} + \frac{2x}{15} + 15 = x$$

$$\frac{4x}{5} + \frac{2x}{15} + 15 = x$$

$$12x + 2x + 225 = 15x$$

$$\boxed{x = 225}$$

Q. A man spends $\frac{1}{3}$ of his income on food, $\frac{2}{5}$ on his income on house rent and $\frac{1}{5}$ of his income on cloths. If he still has Rs 400 left with him. Then his income is,

$$\rightarrow \text{Income} = x$$

$$x - \left[\frac{1}{3}x + \frac{2}{5}x + \frac{1}{5}x \right] = 400$$

$$x \left[1 - \frac{5+6+3}{15} \right] = 400$$

$$x \left[\frac{15-14}{15} \right] = 400$$

$$x \left[\frac{1}{15} \right] = 400$$

$$x = 400 \times 15$$

$$\boxed{x = 6000}$$

Q In a class $\frac{3}{5}$ of the students are girls and rest are boys. If $\frac{2}{3}$ of the girls and $\frac{1}{4}$ of the boys are absent. What part of the total number of students are present?

$$\rightarrow \text{Total} = x$$

$$\text{Girls} = \boxed{\frac{3}{5}x}, \text{Boys} = x - \frac{3x}{5} = \boxed{\frac{2x}{5}}$$

$$= \frac{3x}{5} \times \frac{2}{3}$$

$$= \frac{2x}{5} \times \frac{1}{4}$$

$$\text{Girls} = \boxed{\frac{2x}{15}} \text{ absent}, \text{Boys} = \boxed{\frac{x}{10}} \text{ absent}$$

$$x - \frac{2x}{15} - \frac{x}{10}$$

$$\frac{30x - 4x - 3x}{30} = \frac{27x}{30}$$

$$\boxed{x = \frac{27}{30}}$$

Place + face value

place value

The value represented by a "digit" according to its "Position" in the number

hundred tens unit
5 7 2

place value of 5,

$$5 \times 100 = 500$$

$$7 \rightarrow 7 \times 10 = 70$$

$$2 \rightarrow 2 \times 1 = 2$$

face value

The face value of each "digit" in a number is "same" as the digit itself.

5 7 2

face value of 5

is 5.

7 → 7

2 → 2

Q What is the "place value" + "face value" of Boxed digit?

	Place	face
① 6 [5] 2 →	50	5
② 7 1 [6] →	6	6
③ [3] 5 1 7 → 3000		3
④ [7] 8 6 → 700		7

'n' Terms

① find the sum

$$1 + 3 + 5 + 7 + 9 + \dots + 15$$

$$\Rightarrow x^2$$

$$x = \frac{n+1}{2}$$

$$= \frac{15+1}{2} = \frac{16}{2} = 8$$

$$(8)^2 = 64$$

Note:

used A.P. when 'to' is

given $\textcircled{15}$ terms used

② Find the sum

$$1 + 3 + 5 + 7 + 9 \dots \textcircled{to} 15 \text{ terms}$$

$$\text{A.P.} \rightarrow S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\rightarrow S_{15} = \frac{15}{2} [2(1) + (15-1)(2)]$$

Start

difference

start $\overbrace{(1+3+5+\dots \text{to } 15 \text{ terms})}^{2+2+2}$

$$= \frac{15}{2} [2 + 28]$$

$$= \frac{15}{2} \times 30 = \boxed{225}$$

③ Find the sum of first 19 terms of the sequence

→ ②, 7, 12, 17 ... to ⑯ terms
 $a = 2, n = 19, d = 5$

$$S_{19} = \frac{19}{2} [2(2) + (19-1)5]$$

$$= \frac{19}{2} [4 + 90]$$

$$= \frac{19}{2} \times 94 \Rightarrow \boxed{893}$$

④ find the sum of first 30 terms of the sequence

→ 17 + 19 + 21 + ... 30 terms

$a = 17, n = 30, d = 2$

$$S_{30} = \frac{30}{2} [2(17) + (30-1)(2)]$$
$$= 15 [34 + (60-2)]$$
$$= 15 [34 + 58]$$
$$= 15 [92]$$

$$\begin{array}{r} 15 \\ \times 92 \\ \hline 1380 \end{array}$$

$$= \boxed{1380}$$

Cyclicity

$$0 \rightarrow 0$$

$$1 \rightarrow 1$$

$$2 \rightarrow (2, 4, 8, 6)$$

$$3 \rightarrow (3, 9, 7, 1)$$

$$4 \rightarrow (4, 6)$$

$$5 \rightarrow (5)$$

$$6 \rightarrow (6)$$

$$7 \rightarrow (7, 9, 3, 1)$$

$$8 \rightarrow (8, 4, 2, 6)$$

$$9 \rightarrow (9^{\text{odd}}, 1^{\text{even}})$$

$$\ast 9^9 \rightarrow 9$$

$$\ast 9^{10} \rightarrow 1$$

$$\ast 7^{63}$$

$$7^{63}/4 (\because 63 \rightarrow \text{remainder } 3)$$

$$\begin{array}{ccccccc} \text{remainders} & 1 & 2 & 3 & 0 \\ (7, 9, 3, 1) & & & & & & \end{array}$$

Ans ③

$$2^1 = 2$$

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

$$2^8 = 256$$

$$2^9 = 512$$

$$3^1 = 3$$

$$3^2 = 9$$

$$3^3 = 27$$

$$3^4 = 81$$

$$3^5 = 243$$

$$3^6 = 729$$

$$3^7 = 2187$$

$$3^8 = 6561$$

$$3^9 = 19683$$

* find last digit of 2^{17}

$$2^{17}/4 (\because 17 \rightarrow \text{remainder } 1)$$

$$\begin{array}{ccccccc} \text{remainders} & 1 & 2 & 3 & 0 \\ (2, 4, 8, 6) & & & & & & \end{array}$$

Ans ②

* 3^{21}

$$3^{21}/4 (\because 21 \rightarrow \text{remainder } 1)$$

$$\begin{array}{ccccccc} \text{remainders} & 1 & 2 & 3 & 0 \\ (3, 9, 7, 1) & & & & & & \end{array}$$

Ans ③

Note $12^6/9$ always ①

Lcm

① Traditional method

find LCM for 8, 12, 6, 4 ?

low	16	24	12	8
↓	24	36	18	12
high	32	48	24	16
			20	
			24	

Lcm = 24
common in all given numbers

② Factorization method

find LCM 8, 12, 6, 4

$$\begin{array}{|c|c|}\hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \\ \hline \end{array} \Rightarrow 2^3$$

$$\begin{array}{|c|c|}\hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \\ \hline \end{array} \Rightarrow 2^2 \times 3^1$$

$$\begin{array}{|c|c|}\hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \\ \hline \end{array} \Rightarrow 2^1 \times 3^1$$

$$\begin{array}{|c|c|}\hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \\ \hline \end{array} \Rightarrow 2^2$$

Highest power
 $\downarrow \downarrow$
 $2^3 \times 3^1 = 24$

③ Long Division method

find Lcm 8, 12, 6, 4 ?

$$\begin{array}{|c|c|}\hline 2 & 8, 12, 6, 4 \\ \hline 2 & 4, 6, 3, 2 \\ \hline 2 & 2, 3, 3, 1 \\ \hline 3 & 1, 3, 3, 1 \\ \hline & 1, 1, 1, 1 \\ \hline \end{array} \Rightarrow 2^3 \times 3^1 = 24$$

④ Short Cut Method

Note check any prime no. is there or not

find Lcm 8, 12, 6, 4

Step ① take highest first 12

Step ② check the highest no. is divisible by other ~~or~~ NOT

$$\frac{12}{8} \checkmark \quad \frac{12}{6} \checkmark \quad \frac{12}{12} \checkmark \quad \frac{12}{4} \checkmark$$

Step ③ double that highest no. $12 \times 2 = 24$

then again check

$$\frac{24}{8} \checkmark \quad \frac{24}{6} \checkmark \quad \frac{24}{12} \checkmark \quad \frac{24}{4} \checkmark \quad \therefore \text{Lcm} = 24$$

Find Lcm 3, 11, 12 ($\because 11$ is prime no.)

if prime no. in que separate that no. ($11 \times \dots$)

now, Lcm (3, 12)

Biggest number 12

$$\frac{12}{3} \checkmark \quad \frac{12}{12} \checkmark \quad \text{lcm} = 12$$

Now multiply this Lcm with 11

$12 \times 11 = 132$ is the final Lcm

HCF

① Traditional Method

find HCF	27	18	36	$\frac{27}{9}, \frac{27}{3}, \frac{27}{1}$
high	9	9	18	$\frac{18}{9}, \frac{18}{6}, \frac{18}{3}, \frac{18}{2}, \frac{18}{1}$
↓	3	6	12	
low	1	3	9	
common	2	6	36	$\frac{36}{18}, \frac{36}{12}, \frac{36}{9}, \frac{36}{6}, \frac{36}{4}, \frac{36}{2}, \frac{36}{1}$
MCF = 9	1	4	5	
		2	4	
		1	2	

② Factorization Method

HCF 27, 18, 36

$$\begin{array}{c|cc}
 3 & 27 & 2 & 18 \\
 \hline
 3 & 9 & 3 & 9 \\
 3 & 3 & 3 & 3 \\
 \hline
 1 & & 1 &
 \end{array}
 \quad
 \begin{array}{c|cc}
 2 & 36 \\
 \hline
 2 & 18 \\
 3 & 9 \\
 3 & 3 \\
 \hline
 1 &
 \end{array}
 \quad
 \begin{array}{c|cc}
 2^2 & 3^2 \\
 \hline
 &
 \end{array}$$

$\Rightarrow 3^3$ $\Rightarrow 2 \times 3^2$ $\Rightarrow 2^2 \times 3^2$

common lowest powers

$$3^2 = 9$$

$$\text{HCF} = 9$$

③ Division Method

Note Divide the numbers until we get '0' remainder

find HCF of 70 and 90?

Note Always take small value as divisor

The diagram illustrates the division algorithm for finding the HCF of 70 and 90. It shows two columns of numbers: 90 and 70 above the first bracket, and 70 and 20 above the second bracket. Blue arrows point from the first column to the second, labeled "Divisor". Another blue arrow points from the first column to the second, labeled "remainders". A green circle highlights the number 10 at the bottom right, with an arrow pointing to it labeled "HCF = 10".

find HCF of 27, 18, 36

① HCF of 27, 18

$$\begin{array}{r} 1 \\ 18 \overline{)27} \\ 18 \\ \hline 2 \\ 9 \overline{)18} \\ 18 \\ \hline 0 \end{array}$$

HCF(27, 18)

$$\begin{array}{r} 4 \\ 9 \overline{)36} \\ 36 \\ \hline 0 \end{array}$$

final HCF = 9

④ Short - (ut Method)

Note Below 100 → use shortcut

Above 100 → use Division method

HCF (27, 18, 36)

Step ① Take smallest no. and get factors of it

Step ② factor of 18 → 9

Step ③ now check 9 is divisible by other two numbers (27, 36)

if yes then HCF = 9

eg HCF (24, 42, 30)

12

8

6

$$\frac{42}{12} \cancel{\times} \quad \frac{30}{12} \cancel{\times}$$

$$\frac{42}{8} \cancel{\times} \quad \frac{30}{8} \cancel{\times}$$

$$\frac{42}{6} \checkmark \quad \frac{30}{6} \checkmark$$

∴ HCF = 6

Lcm and Hcf of given fraction

$$\frac{2}{5}, \frac{3}{15}, \frac{6}{25}$$

$$\text{Lcm} = \frac{\text{Lcm of Numerator}}{\text{Hcf of Denominator}} = \frac{\text{lcm}(2, 3, 6)}{\text{Hcf}(5, 15, 25)} = \frac{6}{5}$$

$$\text{Hcf} = \frac{\text{Hcf of Numerator}}{\text{lcm of Denominator}} = \frac{\text{hcf}(2, 3, 6)}{\text{lcm}(5, 15, 25)} = \frac{1}{75}$$

$$\text{lcm} = \frac{2}{3}, \frac{4}{9}, \frac{5}{6}, \frac{7}{6}$$

$$N \rightarrow \text{lcm}(2, 4, 8, 7) \quad \checkmark$$

prime

$$4 \times 5 \times 7$$
$$\text{lcm} = 140$$

$$D \rightarrow \text{hcf}(3, 9, 6, 12)$$

$$\text{hcf} = 3$$

$$\therefore \underline{\underline{\text{Ans Lcm } \frac{140}{3}}}$$

$$\text{Hcf} = \frac{4}{9}, \frac{10}{21} \text{ and } \frac{20}{63}$$

$$N \rightarrow \text{hcf}(4, 10, 20) \rightarrow \text{hcf} = 2$$

$$D \rightarrow \text{lcm}(9, 21, 63) \rightarrow \text{lcm} = 63$$

$$\underline{\underline{\text{Ans hcf } \frac{2}{63}}}$$

Lcm and Hcf for Decimal

Find Lcm for 1.2, 1.5, 2 and 5?

$$\rightarrow \begin{array}{cccc} 1.2 & 1.5 & 2.0 & 5.0 \\ \times 10 \hookrightarrow & 12 & 15 & 20 & 50 \end{array}$$

$$2 \times 2 \times 3 \times 5 \times 5 = \text{Lcm} \frac{300}{10}$$

$$\therefore \text{final Lcm} = 30$$

2	12, 15, 20, 50
2	6, 15, 10, 25
3	3, 15, 5, 25
5	1, 5, 5, 25
S	1, 1, 1, 5
	1, 1, 1, 1

Find Lcm 1.20, 0.24 and 6

$$\rightarrow \begin{array}{cccc} 1.20 & 0.24 & 6.00 \\ \times 100 \hookrightarrow & 120 & 24 & 600 \end{array}$$

$$\text{Lcm}(120, 24, 600) \rightarrow \frac{600}{100}$$

$$\therefore \text{final Lcm} = 6$$

2	120, 24, 600
2	60, 12, 300
2	30, 6, 150
3	15, 3, 75
S	5, 1, 25
S	1, 1, 5
	1, 1, 1

$$\text{Lcm} = 2^3 \times 3^1 \times 5^2 = 600$$

Find Hcf for 6.16 and 13?

$$\rightarrow \begin{array}{cc} 6.16 & 13.00 \\ \times 100 \hookrightarrow & 616 & 1300 \end{array}$$

$$\text{Hcf}(616, 1300) \rightarrow 4$$

$$\begin{aligned} \text{final Hcf} &= \frac{4}{100} \\ &= 0.04 \\ &\equiv \end{aligned}$$

$$\begin{array}{r} 616 \overline{)1300} \\ 1232 \overline{)8} \\ 68 \overline{)616} \\ 612 \overline{)4} \\ 17 \\ \hline \end{array}$$

Hcf \equiv 4

Product of Numbers

① $12, 9 \Rightarrow 108$

$$\text{Lcm} \Rightarrow 36 \quad \text{Lcm} \times \text{Hcf} = 36 \times 3$$
$$\text{Hcf} \Rightarrow 3 \quad = 108$$

$$\therefore \text{Product of two numbers} = \text{Lcm} \times \text{Hcf}$$

② Lcm and Hcf of two No. are 1260 and 63 resp. If one of the number is 315 find other number

$$\rightarrow N_1 \times N_2 = \text{Lcm} \times \text{Hcf}$$

$$315 \times N_2 = 1260 \times 63$$
$$N_2 = \frac{1260 \times 63}{315}$$

$$\therefore N_2 = 252$$

Lcm - same/Different Remainder

① find the least number which when divided by 4, 9 and 12 will leave in each case a remainder 3

$$\rightarrow \text{Lcm}(4, 9, 12) = 36 \quad \text{Same remainder Que}$$

$$\text{then add remainder to it} \Rightarrow 36 + 3 = \text{Ans } 39$$

Divisor $\rightarrow 9 \overline{) 29}^{\text{Quotient}} \begin{matrix} 3 \\ 27 \\ \hline 2 \end{matrix}$ remainder

② find the least number which when divided by 10, 9 and 8 leaves a remainders 9, 8 and 7 resp. in each case.

Different remainder Que

$$\rightarrow \begin{array}{c} 10 \\ | \\ 9 \end{array} \quad \begin{array}{c} 9 \\ | \\ 8 \end{array} \quad \begin{array}{c} 8 \\ | \\ 7 \end{array}$$

$$\text{Lcm}(10, 9, 8) \rightarrow 360$$

then subtract with difference

$$360 - 1 = \text{Ans } 359$$

Least 4 or 5 - digit Number

① Find the least no. of 4 digit which is divisible by 4, 6, 8 and 10

1000

- 100 → least 2 digit Number
- 999 → Highest 3 digit Number
- 1000 → Least 4 digit Number
- 9999 → Highest 4 digit Number
- 10000 → Least 5 digit Number
- 99999 → Highest 5 digit Number

→

Short Cut method

1000 ← 4 digit no.

$\text{Lcm}(4, 6, 8, 10) \rightarrow 120$

$120 \times 8 \rightarrow 960 \checkmark$

$120 \times 9 \rightarrow 1080 \checkmark$
Ans

Traditional method

1000 ← 4 digit no.

$(\text{Lcm}(4, 6, 8, 10) \rightarrow 120)$

$1000 + (\frac{\text{Lcm}}{4} - 40)$ remainder

$\text{Ans} \sqrt{1080}$

$$\begin{array}{r} 8 \\ 120 \overline{) 1000} \\ 960 \\ \hline 40 \end{array}$$

② Find the least number of five Digit which is divisible by 16, 18, 24 and 30

10000

→ SHORTCUT

$$\text{Lcm}(16, 18, 24, 30) \rightarrow 720$$

$$\begin{aligned} 720 \times 10 &\rightarrow 7200 \checkmark \\ 720 \times 13 &\rightarrow 9360 \checkmark \\ 720 \times 14 &\rightarrow 10080 \checkmark \\ &\text{Ans} \end{aligned}$$

Traditional

$$\text{Lcm}(16, 18, 24, 30) \rightarrow 720$$

$$\begin{array}{r} 13 \\ 720 \overline{) 10000} \\ 9360 \\ \hline 640 \end{array}$$

$$\begin{aligned} 10000 + 720 - 64 \\ \text{Ans } 10080 \end{aligned}$$

Note

LCM	HCF
1000 ⊕ 9999 ⊖	9999 ⊖
10000 ⊕ 99999 ⊖	99999 ⊖

③ find the smallest 4 digit number such that when divided by 12, 18, 21 and 28 it leaves remainder 3 in each case.

∴ smallest 4 digit number → 1000
or least

$$\text{Lcm}(12, 18, 21, 28) \rightarrow 252$$

④ $\rightarrow 252 \times 3 = 756$

Traditional method

$$252 \times 4 = 1008$$

$$\text{Lcm}(12, 18, 21, 28) \rightarrow 252$$

$$1008 + 3 = 1011 \text{ Ans}$$

$$\begin{array}{r} 3 \\ 252 \overline{) 1000} \\ 756 \\ \hline 244 \\ 1000 + 252 - 244 + 3 \\ \text{Ans } 1011 \end{array}$$

Greatest 4 or 5 digit number

① Find the greatest 4 digit number which when divided by 12, 15, 20 and 35 leaves no remainder

Greatest 4 digit number → 9999

Greatest 5 digit number → 99999

$$\rightarrow \text{Lcm}(12, 15, 20, 35) \Rightarrow 420$$

$$420 \times 20 \rightarrow 8400$$

$$420 \times 23 \rightarrow 9660 \text{ Ans}$$



random
to get near ans

Traditional method

$$\text{Lcm}(12, 15, 20, 25) \Rightarrow 420$$

$$\begin{array}{r} 23 \\ 420 \overline{) 9999} \\ 840 \\ \hline 1599 \\ 1260 \\ \hline 339 \end{array}$$

$$9999 - 339$$

$$\text{Ans } 9660$$

② Find the greatest 5 digit number which when divided by 8, 9 and 10 leaves 3 as a remainder in each case. **greatest 5 digit no. 99999**

$$\rightarrow \text{Lcm}(8, 9, 10) \rightarrow 360$$

$$\begin{array}{r} 277 \\ 360 \overline{)99999} \\ 720 \\ \hline 2799 \\ 2520 \\ \hline 279 \\ 2520 \\ \hline 279 \end{array}$$

$$99999 - 279 + 3$$

$$\text{Ans } 99723$$