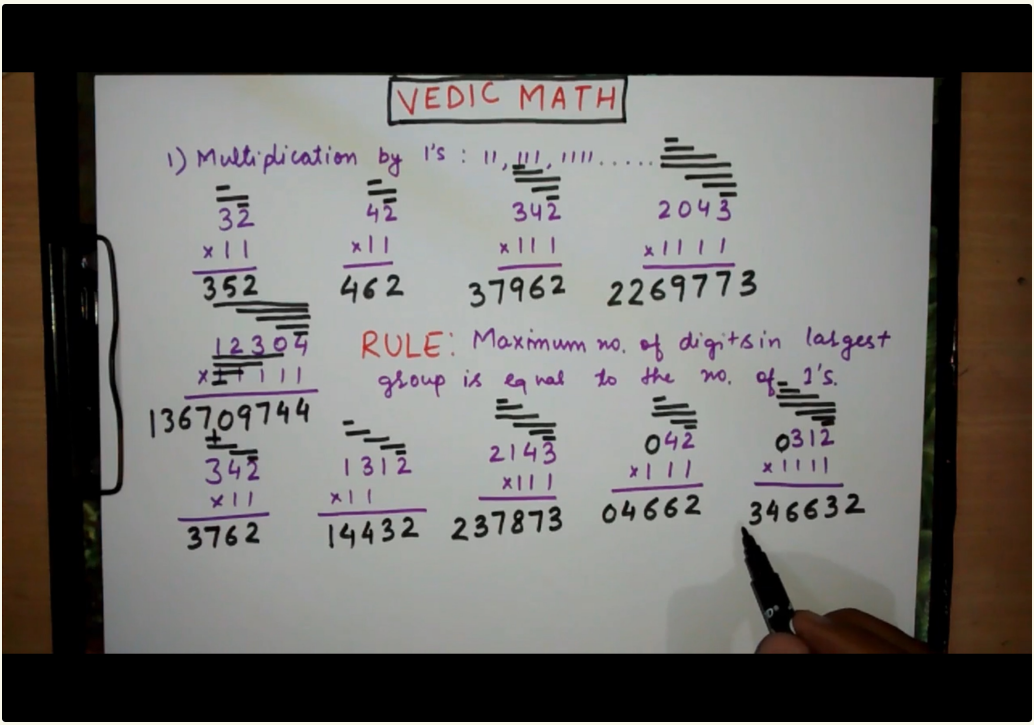
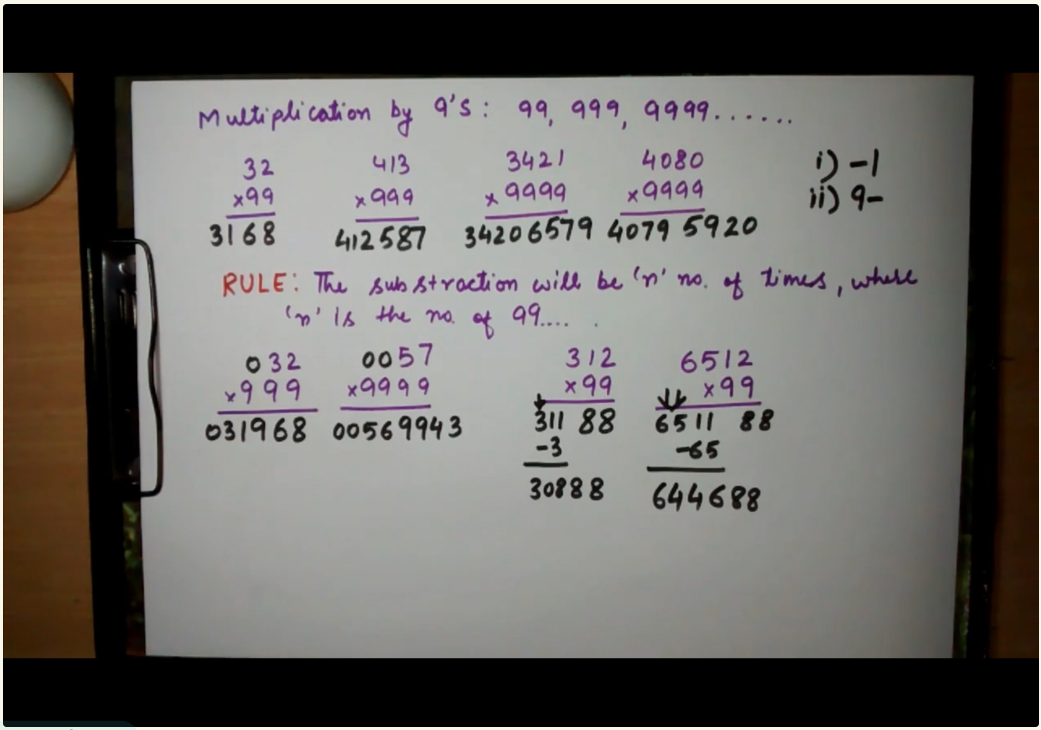
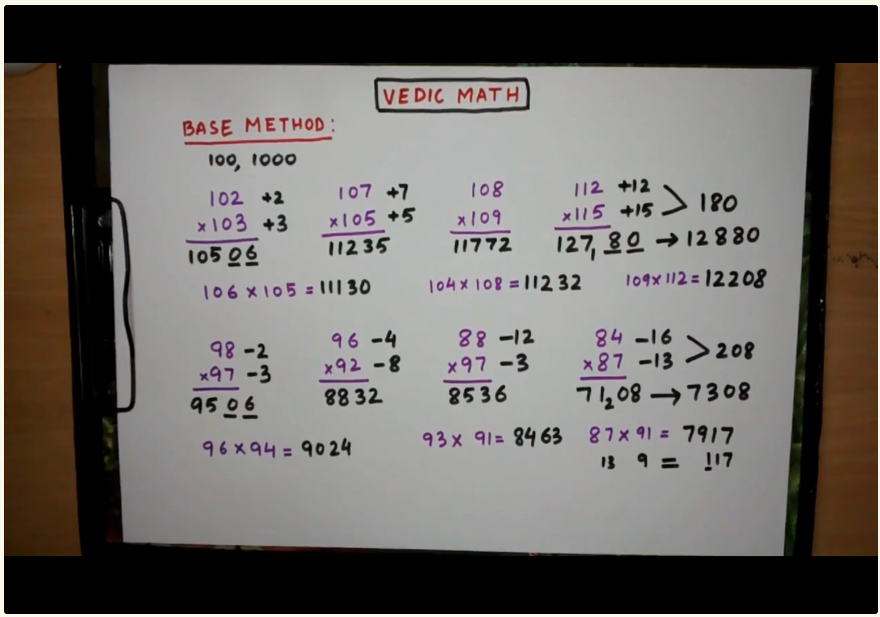
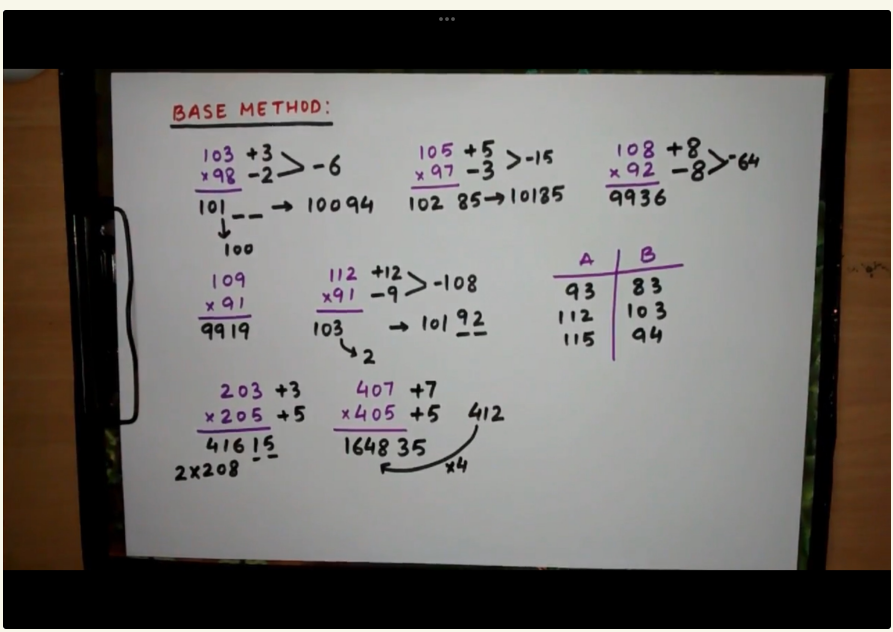
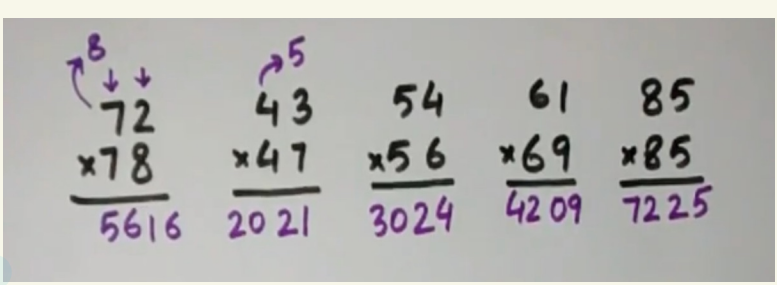
Aptitude Practice

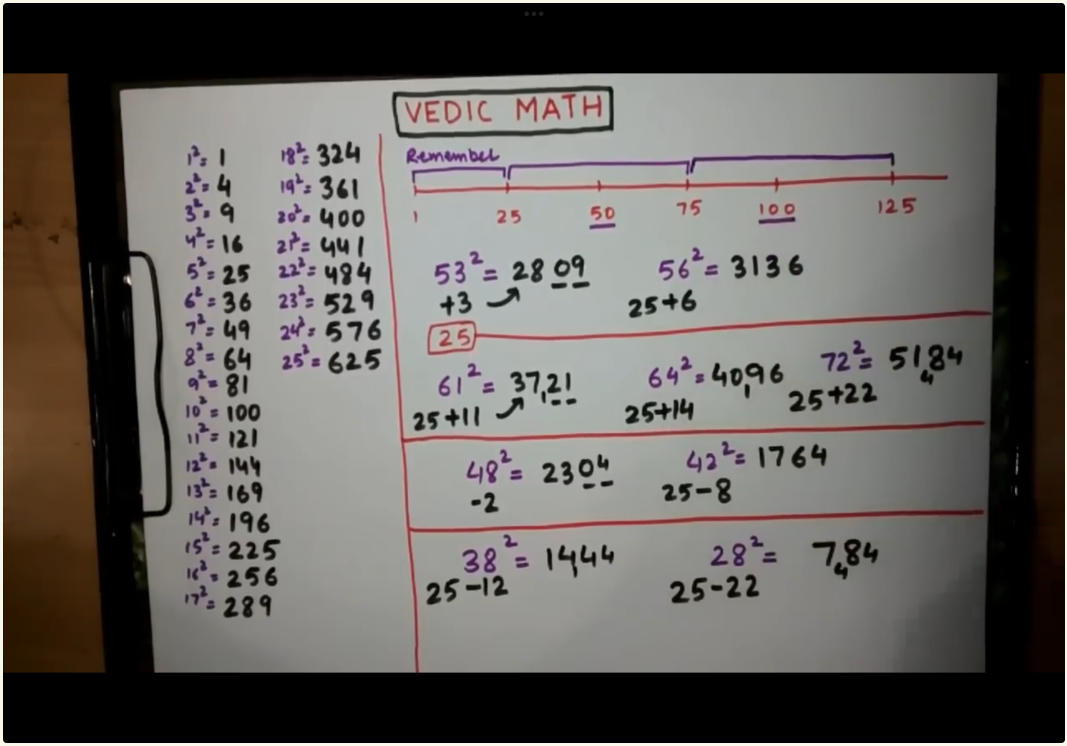


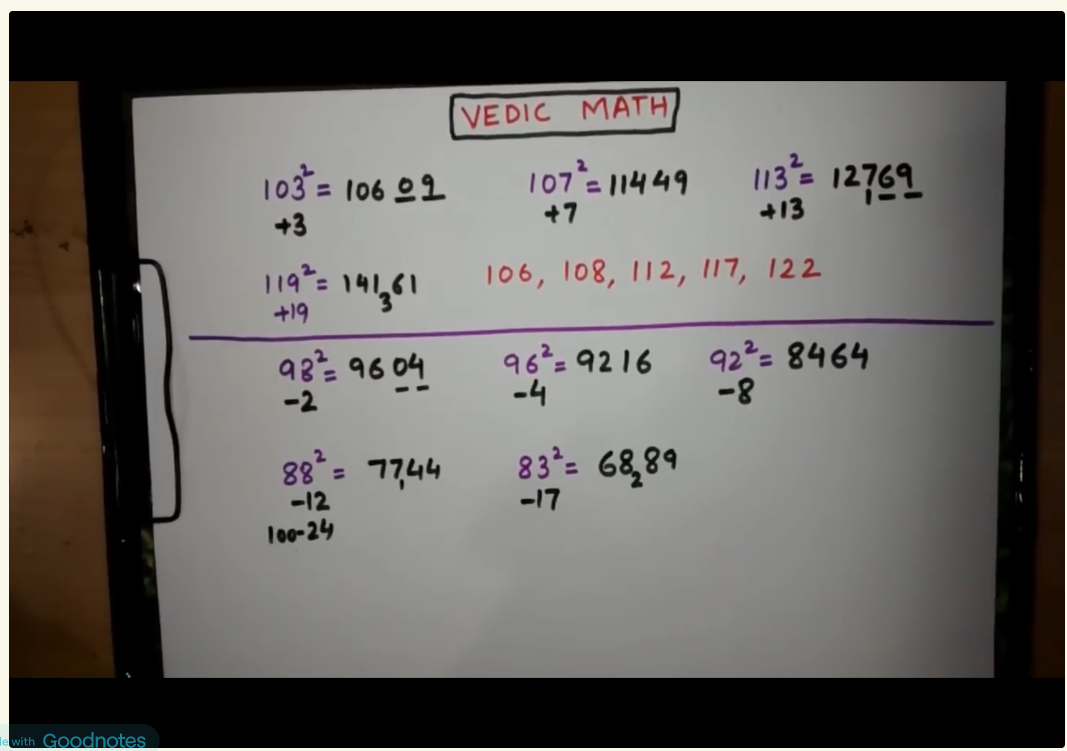




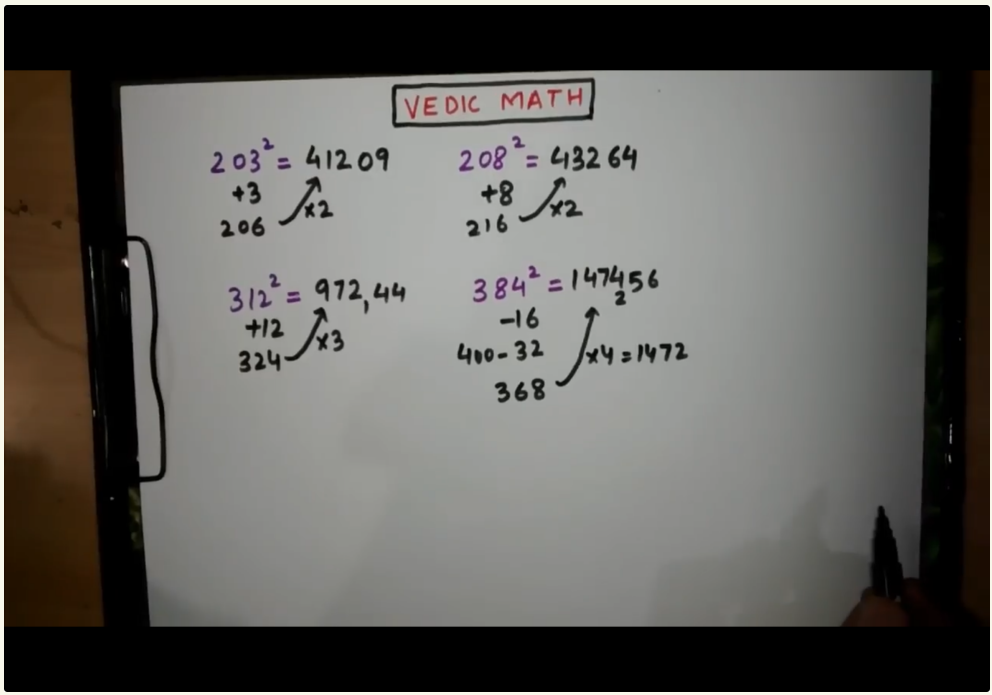


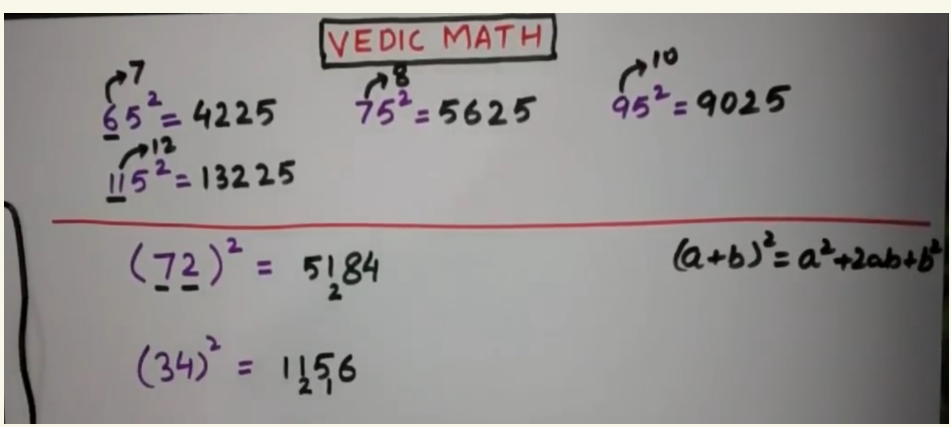




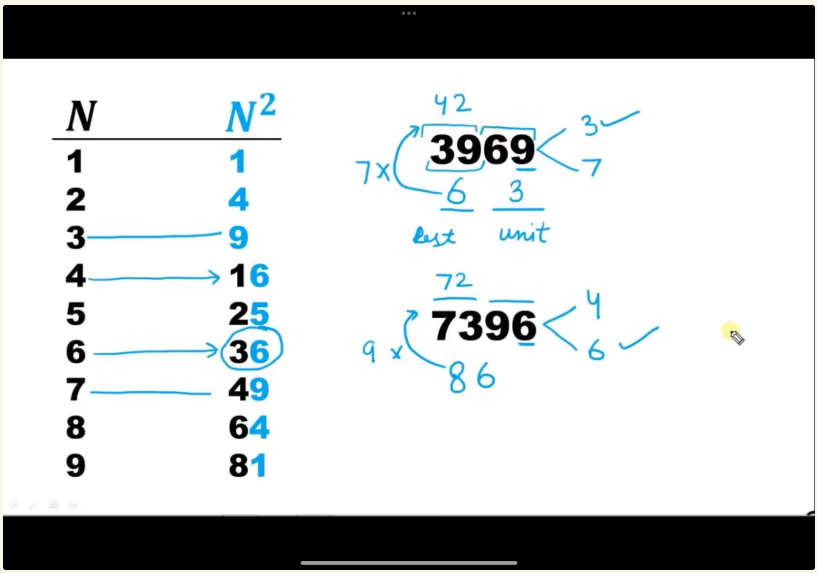


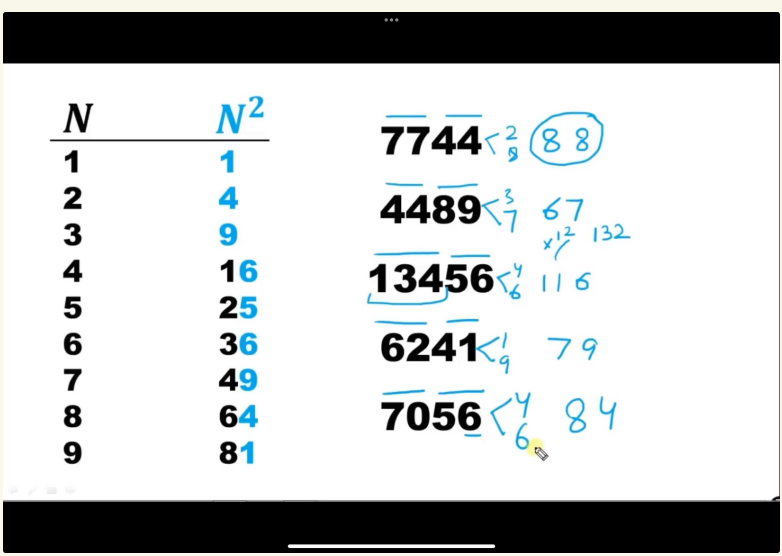
Squares



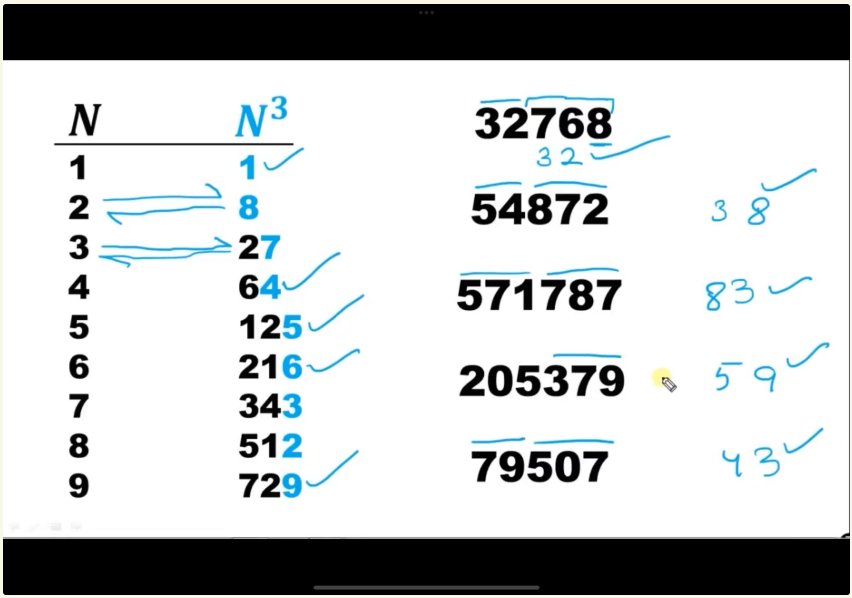


Square Root

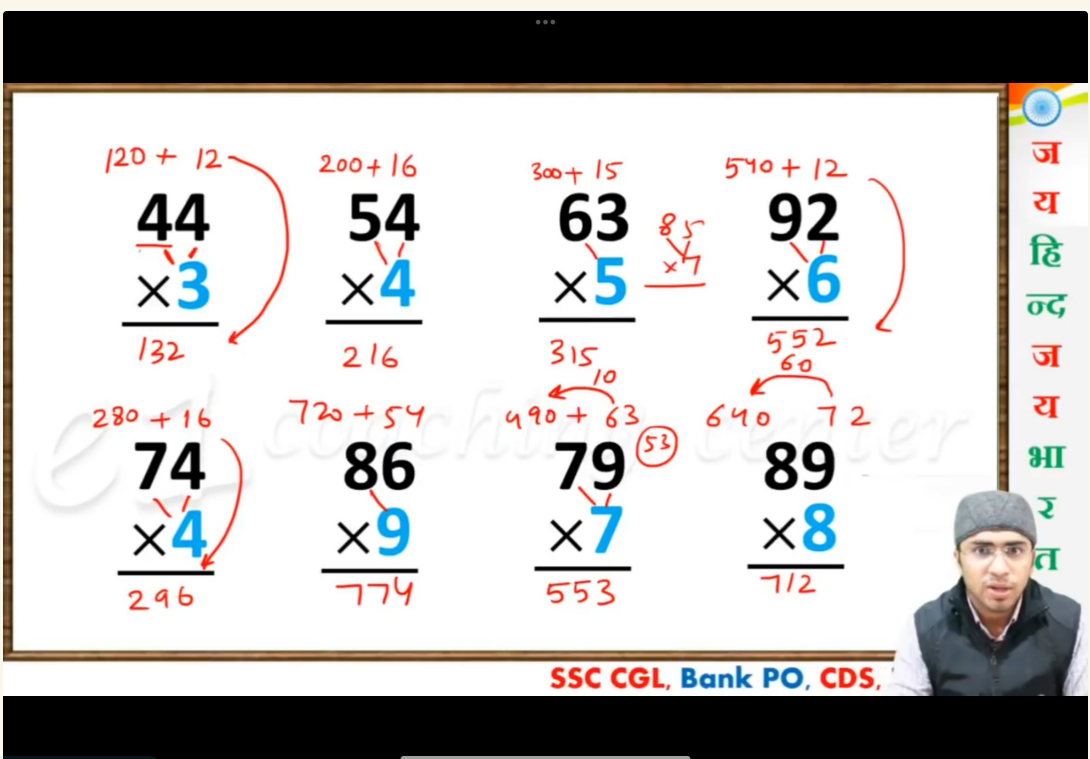


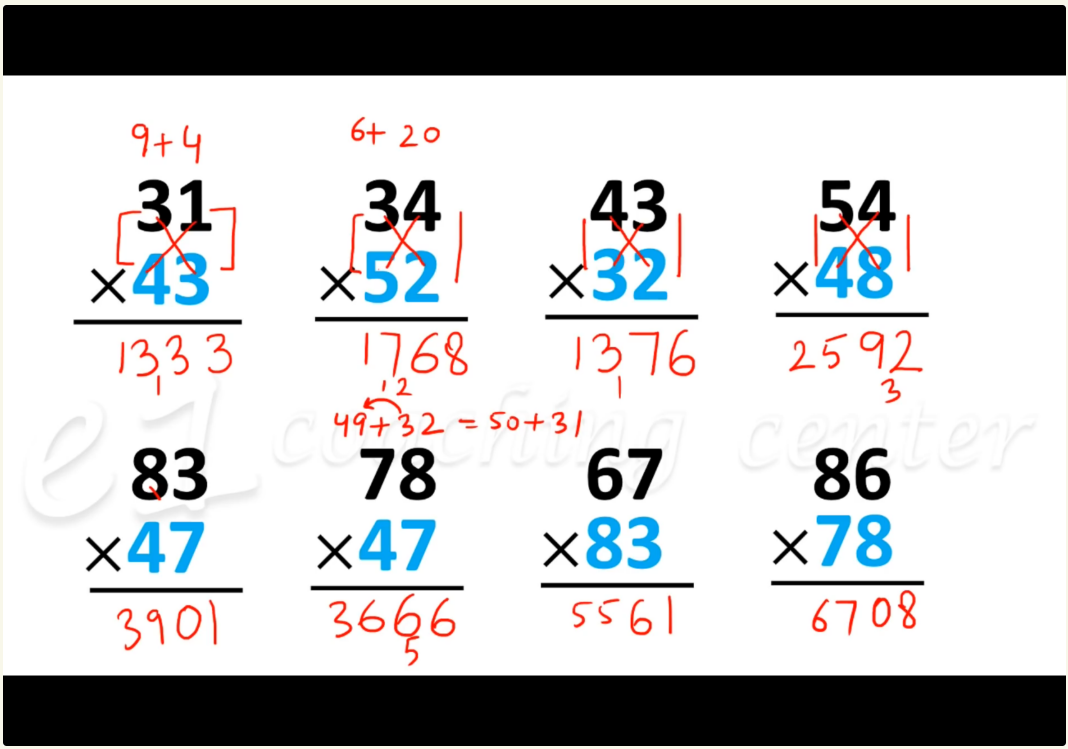


Cube Root



tables from 1 to 100





Feel Free To Learn

Number System

$\sqrt{i} = -1$

* A number contains 'i' then it is clas imaginary number

Numbers

$(4+3i)$ — 4: real, 3: img. no.

$\sqrt{9} = 3$

$\sqrt{144} = 12$

- Real Number
  - Rational Number — $\sqrt{9}$, $\sqrt{144}$ (it has end)
    - Integer
      - tve Integer — Natural (1, 2, 3 ... n)
      - -ve Integer
      - 0 — Whole (0, 1, 2, 3 ... n)
    - fraction $\{\frac{1}{3}, \frac{3}{7}\}$
  - Irrational Number — $\sqrt{2}$, $\sqrt{3}$ (no end)
    - $\sqrt{2} = 1.41421356\ldots$
    - $\sqrt{3} = 1.7320508\ldots$
- Imaginary Number

+ve → 1, 2, 3, 4 ... n

-ve → -1, -2, -3, -4 ... n

Natural (1, 2, 3 ... n)

- Even
- odd
- Prime
- Composite Number
  * if a number which have more than two factor clas Composite number

15

↓

1 × 15

15 × 1

3 × 5

5 × 3

Divisibility Rules

5 → If last digit '5' or '0'. eg. 105, 100

10 → If last digit '0'. eg. 1050, 1000

2 → If last digit Even or zero. eg. 1658, 1650

3 → Add sum of digits and then divide by 3. eg: 342 → $3+4+2 = \frac{9}{3} = 3$

9 → Add sum of digits and then divide by 9. eg: 243243 → $2+4+3+2+4+3 = \frac{18}{9} = 2$

4 → if last two digit divisible by 4. eg. $\frac{1624}{4}$ ✓

8 → if last three digit divisible by 8. eg. $\frac{15120}{8}$ ✓

6 → if number is divisible by 2 & 3 eg. $\frac{126}{2}$, $\frac{126}{3}$

12 → if number is divisible by 3 & 4

14 → if number is divisible by 7 & 2

15 → if number is divisible by 5 & 3

18 → if number is divisible by 9 & 2

7 → $34\textcircled{3}^{\times 2}$ → $34 - 6 = 28/7$ ✓

17 → $190\textcircled{4}^{\times 5}$ → $190 - 20 = 170/17$ ✓

19 → $36\textcircled{1}^{\times 2}$ → $36 + 2 = 38/19$ ✓

13 → $32\textcircled{5}^{\times 4}$ → $32 + 20 = 52/13$ ✓

11 → $\overset{\text{odd}}{9}\overset{\text{even}}{1}\overset{\text{odd}}{7}\overset{\text{even}}{4}$ → (odd digit) − (even digit)

16 − 5

$= 11/11$ ✓

6 3 9 1 5 5

20 − 9 = $11/11$ ✓

Unit Digit (last digit)

① $936 + 972 + 221 = \_\_9$

② $2369 + 2434 = 9 + 4 = 13$

③ $93 \times 46 = 18$

④ $24 \times 98 \times 236 = 4 \times 8 \times 6 = 32 \times 6 = 12$

⑤ $1936 - 243 = \_\_\_3$

⑥ $1934 - 239 = \cancel{4}^{14} - 9 = 5$

⑦ $7^3 = 7 \times 7 \times 7 = 49 \times 7 = 63$

⑧ $7^4 = 7^2 \times 7^2 = 49 \times 49 = 81$

⑨ $81^3 = 81 \times 81 \times 81 = 1$

Model ①

$0^n = 0$, $1^n = 1$, $5^n = 5$, $6^n = 6$

eg. $26^2 = 26 \times 26 = 36$

$26^{142} = 26 \times 26 \times \ldots = \_\_6$

① $266^{13} + 395^{45} = ?$ $\begin{cases} 6^n = \text{last digit } 6 \\ 5^n = \text{last digit } 5 \end{cases}$

$6 + 5 = 11$

② $2311^{124} \times 646^{94} = ?$ $\begin{cases} 1^n = \text{last digit } 1 \\ 6^n = \text{last digit } 6 \end{cases}$

$\_\_1 \times \_\_6 = \_\_\_6$

model ②

$4^{odd} = 4$	$9^{odd} = 9$
$4^{even} = 6$	$9^{even} = 1$

① $249^{33} + 254^{36} + 256^{133} = ?$

$9^{odd} = 9$, $4^{even} = 6$, $6^{\wedge} = 6$

$\_\_9 + \_\_6 + \_\_6$

$= 21$ ← last digit ans is '1'

model ③

(2, 3, 7, 8)

Note: Divide the power by 4 then put remainder in original

① $212^{79} = 2^{3} = 8$ ← ans last digit

$4\overline{)79}$ → 19, 4, 39, 36, 3 ← remainder

② $73^{54} = 3^{2} = 9$ ← ans last digit

$4\overline{)54}$ → 13, 4, 14, 12, 2

③ $378^{41925} = 8^{1} = 8$ ← ans last digit

$4\overline{)25}$ → 6, 24, 1

④ $214^{2164} = 4^{\circ}\alpha$

$= 4^{4}$

$= 4 \times 4 \times 4 \times 4$

$= 16 \times 16 = 36$ ← ans last digit

$4\overline{)64}$ → 16, 4, 24, 24, 0

Example Question (unit digit ⇔ last digit)

① $124^{376} + 124^{375} = 6 + 4$

$= 10$

$4^{even} = 6$

$4^{odd} = 4$

② $25^{6527} + 36^{526} + 73^{54} = 5 + 6 + 3^2$

$= 5 + 6 + 9$

$= 20$

$5^n = 5$

$6^n = 6$

$3^n = 4\overline{)54}$ R 2 (13, 4, 14, 12)

$3^2 = ?$ ← 2

③ $7^{295} \times 3^{158} \times 341^{476} = ?$

$--3 \times --9 \times --1$

$= 27$ ← last digit

$7^n = 4\overline{)95}$ R 3 (23, 8, 15, 12)

$7^3 = 343$

$3^n = 4\overline{)58}$ R 2 (14, 4, 18, 16)

$3^2 = 9$

$1^n = 1$

④ $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{36} - 16^4 + 259] = ?$

$1 + 1 - 6 + 5 - 6 + 9$

$= 16 - 12 = 4$ ← unit digit ⇔ last digit

$1^n = 1$, $6^n = 6$, $5^n = 5$

Last two Digits

By mohit Goyal

Rules

eg. $(134\underset{a}{5}1)^{2342\overset{b}{3}} = \_\_\_\underset{\text{last two digit}}{\overset{a\times b}{5}\ 1}$

eg. $(217\underset{a}{3}1)^{217\overset{b}{3}} = \_\_\_\underset{\text{last two digit}}{\overset{a\times b}{9}\ 1}$

① Ending in 1

$(\_\_\_a1)^{(\_\_\_b)} = \_\_\_\overset{a\times b}{\ }\ 1$

② Even No.

$(24)^{odd} = 24$ ← last two digit

$(24)^{even} = 76 \rightarrow$ eg. $2^{20} = 1048576$

$(76)^N = 76$ last two digits → eg. $76^2 = 5776$

76×2^n = last two digits of 2^n → eg. $76 * 2^5 \rightarrow 2432$

$2^5 \rightarrow 32$

③ Ending in 5

If second last digit & power odd it will end in 75 otherwise 25

$(\_\underset{5\,odd}{5})^{odd} \rightarrow 75$, otherwise → 25

$75^{65} \rightarrow 75$ (odd, odd)

$19\boxed{9}5^{2014} \rightarrow 25$

$19\boxed{9}5^{2015} \rightarrow 75$ (odd, odd)

$\boxed{6}5^{75} \rightarrow 25$

④ odd No. (3, 7, 9)

Convert making last digit ①

and apply rule no. ①

eg 19^{256} (odd)

$(19^2)^{128}$

$(3\underset{b}{6}1)^{\frac{128}{2}}$ ← now apply rule ①

$\_\_\overset{a\times b}{8}\ 1$

last two digit 81

$6\times 8 = 48$

(5) $x^2, (50-x)^2, (50+x)^2, (100-x)^2, (100+x)^2 \ldots$

सभी के last two digits same होते हैं।

→ $x = 23 \rightarrow 23^2 \rightarrow \_\_\_29$

$(50-x) \rightarrow 27^2 \rightarrow \_\_\_29$

$(50+x) \rightarrow 73^2 \rightarrow \_\_\_29$

$(100-x) \rightarrow 77^2 \rightarrow \_\_\_29$

$(100+x) \rightarrow 123^2 \rightarrow \_\_\_29$

Even No. Rule (2) Example

2^{105}

$(2^{20})^5 \times 2^5$ ($\because 2^{20} \rightarrow$ last two digit 76 $\rightarrow \_\_\underline{7}\underline{6}\_$)

$(\_\_\underline{7}\underline{6})^5 \times 2^5$ ($\because 76^n \rightarrow \_\_\underline{7}\underline{6}$)

76×2^5 ($\because 76 \times 2^5 \rightarrow \_\_\_\_$ ↓ 2^n)

$2^5 \rightarrow$ last two digit

ans $\_\_\_\underline{3}\underline{2}$

Remainder Theorem

Normal Number: $\frac{23}{5}$

Polynomial Number: $(x^2+7x+2) \div (x^2+2)$

$2^1 = 2$	$3^1 = 3$
$2^2 = 4$	$3^2 = 9$
$2^3 = 8$	$3^3 = 27$
$2^4 = 16$	$3^4 = 81$
$2^5 = 32$	$3^5 = 243$

Note: Remainder value always low than Denominator

① $\frac{23}{5} = 3$ (remainder) — 5 is Denominator

② $\frac{17}{6} = 5$ (remainder)

③ $\frac{86 \times 23}{11} = \frac{\cancel{86}^{(9)}}{\cancel{11}} \times \frac{\cancel{23}^{(1)}}{\cancel{11}} = 9$ (Remainder, Remainder)

④ $\frac{38+71+86}{16} = \frac{\cancel{38}^{6}}{\cancel{16}} + \frac{\cancel{71}^{7}}{\cancel{16}} + \frac{\cancel{86}^{6}}{\cancel{16}}$ (Remainders)

Remainder $\propto$ 19 — acc. to note

$= \frac{\cancel{19}}{\cancel{16}} = 3$ (Remainder)

⑤ $\frac{123 \times 124 \times 125}{9} = \frac{\cancel{123}^{(6)}}{\cancel{9}} \times \frac{\cancel{124}^{(7)}}{\cancel{9}} \times \frac{\cancel{125}^{(8)}}{\cancel{9}}$ (Remainders)

$= \frac{6 \times 7 \times 8}{9}$

$= \frac{\cancel{42}^{(6)} \times 8}{\cancel{9}}$ ← remainder

$= \frac{6 \times 8}{9} = \frac{\cancel{48}^{(3)}}{\cancel{9}} = 3$ ← remainder

⑥ $\frac{7^5}{4} = \frac{\cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7} \times \cancel{7}}{\cancel{4}}$ (3) (3) (3) (3) (3) — remainder

$= \frac{3 \times 3 \times 3 \times 3 \times 3}{4}$

$= \frac{\cancel{9}^{(1)} \times \cancel{9}^{(1)} \times 3}{\cancel{4}}$ — remainder

$= 1 \times 1 \times 3 = 3$ — remainder

(7) $\frac{2^{75}}{5} = \frac{2^{75/4}}{5}$ $\quad \because 18 \times 4 = 72 + 3 = 75$

$= \frac{(2^4)^{18} \times 2^3}{5}$

$= \frac{(16)^{18} \times 8}{5}$

$= \frac{(1)^{18} \times 8}{5}$

$= \frac{1 \times 8}{5} = \frac{8}{5} = 3$ remainder

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$ ← bcz near to (15) & In Que Denominator is 5
$2^5 = 32$

$5\overline{)16}\ 3$, 15, 1 ← remainder

$\therefore 1^n = 1$

(8) $\frac{7^{129}}{5} = \frac{2^{129}}{5}$ $\quad \because \frac{7}{5} = 2$ remainder

$= \frac{2^{129/4}}{5}$

$32 \times 4 = 128 + 1 = 129$

$= \frac{(2^4)^{32} \times 2^1}{5}$

$= \frac{(16)^{32} \times 2}{5}$

$= \frac{1 \times 2}{5} = 2$ remainder

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$ ← bcz near to (15) & In Que Denominator is 5
$2^5 = 32$

$\therefore \frac{16}{5}$ (1) remainder $\quad \therefore 1^n = 1$

(9) $\frac{517^{517}}{5} = \frac{(2)^{517}}{5}$ $\quad \because 5\overline{)517}\ 103$, 515, 2 ← remainder

$129 \times 4 = 516 + 1 = 517$

$= \frac{(2^4)^{129} \times 2^1}{5}$

$= \frac{(16)^{129} \times 2^1}{5}$

$= \frac{(1)^{129} \times 2^1}{5} = \frac{1 \times 2}{5} = 2$ remainder

(10) What is the remainder when 2243^{165} divided by 5?

→ $\frac{2243^{165}}{5} = \frac{(3)^{165}}{5}$ ∴ $\frac{2243}{5}$ = 3 remainder

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$

nearby multiple of 5 bcz in Qve denominator is 5

$= \frac{(3)^{165/4}}{5}$

$= \frac{(3^4)^{41} \times 3^1}{5}$

$\frac{81}{5}$ = 1 remainder
$= (1)^{41}$
$= 1^n$
$= 1$

$= \frac{(81)^{41} \times 3}{5}$ → $= \frac{1 \times 3}{5} = 3$ remainder

Sum of Series

Model ①

Natural Numbers

$1+2+3+4+\ldots+n$ $\left[\frac{n(n+1)}{2}\right]$

$1^2+2^2+3^2+\ldots+n^2$ $\left[\frac{n(n+1)(2n+1)}{6}\right]$

$1^3+2^3+3^3+\ldots+n^3$ $\left[\frac{n(n+1)}{2}\right]^2$

Model ②

Natural Numbers

$1+3+5+7+\ldots+33$

odd series

(x^2)

$x = \frac{n+1}{2}$

$x = \frac{33+1}{2}$

$= \frac{34}{2}$ 17

$(17)^2 = 289$ ans

$2+4+6+8+\ldots+50$

even series

$x(x+1)$

$x = \frac{n}{2}$

$= \frac{50}{2}$

$= 25$

$x(x+1)$

$25(25+1)$

$= 625+25$

$= 650$ ans

$51+53+55+\ldots+99$

odd series

$(1+3+5+\ldots+99) - (1+3+5+\ldots+49)$

Total − first

x^2 ↳ $x = \frac{n+1}{2}$

$= \frac{99+1}{2}$

$= 50$

$x^2 = 50$

$= 2500$

x^2 ↳ $x = \frac{n+1}{2}$

$= 49+1$

$= \frac{50}{2} = 25$

$x^2 = 25$

$= 625$

$2500 - 625$

$= 1875$

=

Examples

① $1+2+3+\ldots+30 = ?$ $\quad \frac{n(n+1)}{2} = \frac{30\times31}{2} = 15\times31 = 465$

② $1^2+2^2+3^2+\ldots+12^2 = ?$ $\quad \frac{n(n+1)(2n+1)}{6} = \frac{12\times13\times25}{6} = 50\times13 = 650$

③ $1^3+2^3+3^3+\ldots+9^3 = ?$ $\quad \left[\frac{n(n+1)}{2}\right]^2 = \left(\frac{9\times10}{2}\right)^2 = (45)^2 = 2025$

④ $1+3+5+\ldots+49 = ?$ $\quad (x^2) \rightarrow x = \frac{n+1}{2}$

$= \frac{49+1}{2} = \frac{50}{2} = 25$

$x^2 = (25)^2 = 625$

⑤ $2+4+6+\ldots+58 = ?$

$x(x+1) \rightarrow x = \frac{n}{2}$

$= \frac{58}{2} = 29$

$x(x+1) \rightarrow 29(29+1) = 29\times30 = 870$

⑥ $10^2+11^2+12^2+\ldots+20^2 = ?$

$(1^2+2^2+\ldots+20^2) \quad (1^2+2^2+\ldots+9^2)$

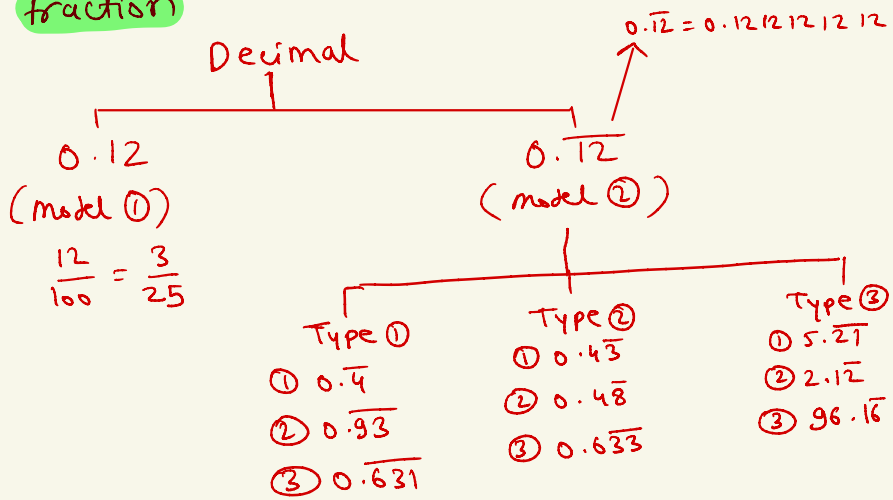
Total − First

$\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$

$\frac{10\times21\times41}{6} - \frac{3\times5\times19}{6}$

$2870 - 15\times19 = 2585$

Decimal to fraction



Type ①

① 0.$\overline{4}$ ↳ $\frac{4}{9}$

② 0.$\overline{93}$ ↳ $\frac{93}{99}$

③ 0.$\overline{631}$ ↳ $\frac{631}{999}$

Type ②

① 0.4$\overline{3}$

$\frac{43-4}{90} = \frac{39}{90}$

② 0.48$\overline{}$

$\frac{48-4}{90} = \frac{44}{90}$

③ 0.6$\overline{33}$

$\frac{633-6}{990} = \frac{627}{990}$

Type ③

① 5.$\overline{21}$

$5 + 0.\overline{21}$

$5 + \frac{21}{99}$

$= \frac{516}{99}$

② 2.1$\overline{2}$

$2 + 0.1\overline{2}$

$2 + \frac{12-1}{90}$

$2 + \frac{11}{90}$

$= \frac{191}{90}$

③ 96.1$\overline{6}$

$96 + 0.1\overline{6}$

$96 + \frac{16-1}{90}$

$96 + \frac{15}{90}$

$= \frac{8640+15}{90} = \frac{8655}{90}$

Consecutive Numbers

1, 2, 3 ... → cons. natural numbers ($x^{①}, x+1, x+2, \ldots, n$)

2, 4, 8 ... → cons. Even numbers ($x^{②}, x+2, x+4, x+6, \ldots, n$)

1, 3, 5 ... → cons. odd numbers ($x^{③}, x+2, x+4, x+6, \ldots, n$)

Q. The sum of three consecutive numbers is 87. The middle number is.

→ $x + x+1 + x+2 = 87$

$3x + 3 = 87$

$3x = 87 - 3$

$\boxed{x = 28}$

$x, x+1, x+2$

28, 29, 30

Ans 29

Q. The sum of three consecutive odd numbers is 147. Then, the middle number is.

→ $x + x+2 + x+4 = 147$

$3x + 6 = 147$

$3x = 147 - 6$

$\boxed{x = 47}$

$x, x+2, x+4$

47, 49, 51

Ans 49

Q. The sum of four consecutive even numbers is 748. The smallest among them.

→ $x + x+2 + x+4 + x+6 = 748$

$4x + 12 = 748$

$4x = 748 - 12$

$\boxed{x = 184}$

$x, x+2, x+4, x+6$

184, 186, 188, 190

Ans 184

Q out of six consecutive natural numbers, if the sum of first three is 27, what is the sum of the other three.

→ $x + x+1 + x+2$ (first three cons. natural number)

$3x + 3 = 27$

$3x = 24$

$\boxed{x = 8}$

$x+3$ (8) $x+4$ (8) $x+5$ (8)

$11 + 12 + 13$

Ans ⇒ $\boxed{36}$

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finding a Number

Note $17 - 10 = 7$

* Difference = Subtract (-)
* More = Subtract (-)
* Exceeds = Subtract (-)

Q what is two-third of half of 369?

→ $\frac{2}{3}$ of $\frac{1}{2}$ of 369 = ?

$\frac{2}{3} \times \frac{1}{2} \times 369 = ?$

123

ans = 123

Q If one-third of one-fourth of a number is 15, then three-tenth of the number is

→ $\frac{1}{3} \times \frac{1}{4} \times x = 15$

$x = 15 \times 4 \times 3$

$= 180$

$\frac{3}{10} \times 180 \Rightarrow$ ans 54

Q 1/2 of 3/4 of a number is 2(1/2) of 10. what is the number

→ $\frac{1}{2} \times \frac{3}{4} \times x = \frac{5}{2} \times 10^5$

$x = \frac{200}{3}$

ans $66\frac{2}{3}$

Q If the sum of two numbers, one of which is $\frac{2}{5}$ times the other is 50, then the numbers are,

→ first no. = x, second no. = $\frac{2}{5}x$

$$x + \frac{2}{5}x = 50$$

$$\frac{5x + 2x}{5} = 50$$

$$7x = 250$$

$$x = \frac{250}{7}$$

first no. = $\frac{250}{7}$

Second no. = $\frac{2}{5} \times \frac{250}{7}$

$= \frac{100}{7}$

Q. If $\frac{3}{4}$ of a number is 7 more than $\frac{1}{6}$ of the number, then $\frac{5}{3}$ of the number is,

→ first − second = 7

(more = (−) subtract)

$$\frac{3}{4} \times x - \frac{1}{6} \times x = 7$$

$$\frac{3x}{4} - \frac{x}{6} = 7$$

$$\frac{9x - 2x}{12} = 7$$

$$7x = 7 \times 12$$

$$x = 12$$

$\frac{5}{3} \times x \rightarrow \frac{5}{3} \times 12^{4}$

ans 20

Q If $\frac{1}{2}$ is added to a number and the sum is multiplied by 3, the result is 21. Then the number is,

$\rightarrow \left(\frac{1}{2}+x\right)\times 3 = 21$

$\frac{3}{2}+3x = 21$

$3x = 21 - \frac{3}{2}$

$3x = \frac{42-3}{2}$

$3x = \frac{39}{2}$ 13

$x = \frac{13}{2}$

Q If 4/5th of a number exceeds its 3/4th by 8, then the number is,

$\rightarrow \frac{4}{5}x - \frac{3}{4}x = 8$

$\frac{16x-15x}{20} = 8$

$x = 160$

Q The sum of three numbers is 2, the 1st number is 1/2 times the second number and 3rd number is 1/4 times the second number. Then the second number is,

$\rightarrow \frac{x}{2} + x + \frac{x}{4} = 2$

(1/2 and 1/4)

$\frac{x}{2} + x + \frac{x}{4} = 2$

$\frac{2x+4x+x}{4} = 2$

$7x = 2\times 4$

$x = \frac{8}{7}$

Miscellaneous

Q A person gives 1/4 of his property to his daughter. 1/2 to his son and 1/5 for charity. How much has he given away.

→ $\frac{1}{4} + \frac{1}{2} + \frac{1}{5} \Rightarrow \frac{5+10+4}{20} = \boxed{\frac{19}{20}}$

Q In an office, there are 108 tables and 132 chairs. If 1/6 of tables and 1/4 of chairs are broken. How many people can work in the office if each person requires one table and one chair?

→ Tables

108

$\cancel{108}^{18} \times \frac{1}{\cancel{6}} = 18$ broken

Good tables = 108 − 18 = 90

Chairs

132

$\cancel{132}_{33} \times \frac{1}{\cancel{4}} = 33$ broken

Good chairs = 132 − 33 = 99

Office = 90 people

Q. In a school 1/10 of the boys are same in number as 1/4 of the girls. The ratio of the boys to the girls in that school is,

→ Boys = x , Girl = y

$\frac{1}{10} \times x = \frac{1}{4} \times y$

$\frac{x}{10} = \frac{y}{4}$

$\frac{x}{y} = \frac{10}{4}$

$\boxed{\frac{x}{y} = \frac{5}{2}}$

Q Two basket together have 640 oranges. If $\frac{1}{5}$ of the oranges in the first basket be taken out to the second basket. The number of oranges on the first basket is,

→

640

Basket ① x | Basket ② $640 - x$

$B_1 = x$, $B_2 = 640 - x$

$$x - \frac{1}{5}x = 640 - x + \frac{x}{5}$$

$$\frac{4x}{5} = \frac{640 \times 5 - 4x}{5}$$

$$8x = 640 \times 5$$

$$\boxed{x = 400}$$

Q $\frac{1}{10}$ of a rod is colored red, $\frac{1}{20}$ orange, $\frac{1}{30}$ yellow, $\frac{1}{40}$ green, $\frac{1}{50}$ blue, $\frac{1}{60}$ black and the rest is violet. If the length of the violet portion of the rod is 12.08 meters then the length of the rod is,

→ length $= x$

$$x - \left[\frac{x}{10} + \frac{x}{20} + \frac{x}{30} + \frac{x}{40} + \frac{x}{50} + \frac{x}{60}\right] = 12.08$$

$$x - \left[1 - \frac{147}{600}\right] = 12.08$$

$$x\left[\frac{453}{600}\right] = 12.08$$

$$x = \frac{12.08 \times 600}{453}$$

$$x = \frac{1208 \times 6}{453}$$

$$\boxed{x = 16 \text{ meters}}$$

Q A man read $\frac{2}{5}$th of the book on the first day. He read $\frac{1}{3}$rd more on the second day than he read on the first day. 15 pages are left for the third day. The number of pages in the book is,

→ Book = x

1st day, 2nd day, 3rd day

$$\frac{2x}{5} + \frac{2x}{5} + \left(\frac{2x}{5} \times \frac{1}{3}\right) + 15 = x$$

$$\frac{2x}{5} + \frac{2x}{5} + \frac{2x}{15} + 15 = x$$

$$\frac{4x}{5} + \frac{2x}{15} + 15 = x$$

$$12x + 2x + 225 = 15x$$

$$\boxed{x = 225}$$

Q. A man spends $1/3$ of his income on food, $\frac{2}{5}$ on his income on house rent and $\frac{1}{5}$ of his income on cloths. If he still has Rs 400 left with him. Then his income is,

→ Income = x

$$x - \left[\frac{1}{3}x + \frac{2}{5}x + \frac{1}{5}x\right] = 400$$

$$x\left[1 - \frac{5+6+3}{15}\right] = 400$$

$$x\left[\frac{15-14}{15}\right] = 400$$

$$x\left[\frac{1}{15}\right] = 400$$

$$x = 400 \times 15$$

$$\boxed{x = 6000}$$

Q In a class $\frac{3}{5}$ of the students are girls and rest are boys. If $\frac{2}{9}$ of the girls and $\frac{1}{4}$ of the boys are absent. What part of the total number of students are present?

→ Total $= x$

Girls $= \boxed{\frac{3}{5}x}$, Boys $= x - \frac{3x}{5} = \boxed{\frac{2x}{5}}$

$= \frac{3x}{5} \times \frac{2}{3}$ $= \frac{2x}{5} \times \frac{1}{4}$

Girls $= \boxed{\frac{2x}{15}}$ absent, Boys $= \boxed{\frac{x}{10}}$ absent

$$x - \frac{2x}{15} - \frac{x}{10}$$

$$\frac{30x - 4x - 3x}{30} = \frac{27x}{30}$$

$$\boxed{x = \frac{27}{30}}$$

Place + face value

Place value

The value represents by a "digit" according to "its Position" in the number

hundred ten unit
5 7 2

Place value of 5,

$5 \times 100 = 300$

$7 \rightarrow 7 \times 10 = 70$

$2 \rightarrow 2 \times 1 = 2$

face value

The face value of each "digit" in a number is "same" as the digit itself.

5 7 2

face value of 5 is (5).

$7 \rightarrow 7$

$2 \rightarrow 2$

Q What is the "place value" + "face value" of Boxed digit?

		Place	face
①	6[5]2 →	50	5
②	71[6] →	6	6
③	[3]517 →	3000	3
④	[7]86 →	700	7

'n' Terms

① find the sum

$1+3+5+7+9+\ldots+15$

$\Rightarrow x^2$

$\hookrightarrow x = \frac{n+1}{2}$

$= \frac{15+1}{2} = \frac{16}{2} = 8$

$(8)^2 = 64$

Note: used A.P. when 'to' is given (or) terms used

② find the sum

$1+3+5+7+9\ldots$ (to) 15 terms

$$A.P. \rightarrow S_n = \frac{n}{2}[2a+(n-1)d]$$

$\rightarrow S_{15} = \frac{15}{2}[2(1)+(15-1)(2)]$

(1 = Start, 2 = difference)

Start (1 + 3 + 5 + 7 ... to 15 terms)

(2, 2, 2)

$= \frac{15}{2}[2+28]$

$= \frac{15}{2} \times 30^{15} = 225$

③ Find the sum of first 19 terms of the sequence

→ ②, 7, 12, 17 ... to ⑲ terms

(12 − 7 = 5, 17 − 12 = 5)

$a = 2$, $n = 19$, $d = 5$

$$S_{19} = \frac{19}{2}[2(2) + (19-1)5]$$

$$= \frac{19}{2}[4 + 90]$$

$$= \frac{19}{2} \times 94^{47} \Rightarrow \boxed{893}$$

④ Find the sum of first 30 terms of the sequence

→ $17 + 19 + 21 + \ldots$ 30 terms

$a = 17$, $n = 30$, $d = 2$

$$S_{30} = \frac{30^{15}}{2}[2(17) + (30-1)(2)]$$

$$= 15[34 + (60-2)]$$

$$= 15[34 + 58]$$

$$= 15[92]$$

$$= \boxed{1380}$$

$$\begin{array}{r} 15 \\ \times 92 \\ \hline 10 \\ 47 \\ 9 \\ \hline 1380 \end{array}$$

Cyclicity

$0 \rightarrow 0$

$1 \rightarrow 1$

$2 \rightarrow (2, 4, 8, 6)$

$3 \rightarrow (3, 9, 7, 1)$

$4 \rightarrow (4, 6)$

$5 \rightarrow (5)$

$6 \rightarrow (6)$

$7 \rightarrow (7, 9, 3, 1)$

$8 \rightarrow (8, 4, 2, 6)$

$9 \rightarrow (9^{\text{odd}}, 1^{\text{even}})$

* $9^9 \rightarrow 9$
* $9^{10} \rightarrow 1$

$2^1 = 2$ $3^1 = 3$

$2^2 = 4$ $3^2 = 9$

$2^3 = 8$ $3^3 = 27$

$2^4 = 16$ $3^4 = 81$

$2^5 = 32$ $3^5 = 243$

$2^6 = 64$ $3^6 = 729$

$2^7 = 128$ $3^7 = 2187$

$2^8 = 256$ $3^8 = 6561$

$2^9 = 512$ $3^9 = 19683$

* Find last digit of 2^{17}

$2^{17/4}$ ($\because \frac{17}{4} \rightarrow$ remainder ①)

remainder	1	2	3	0
	(2,	4,	8,	6)

Ans ②

* 3^{21}

$3^{21/4}$ ($\because \frac{21}{4} \rightarrow$ remainder ①)

remainder	1	2	3	0
	(3,	9,	7	1)

Ans ③

* 7^{63}

$7^{63/4}$ ($\because \frac{63}{4} \rightarrow$ remainder ③)

remainder	1	2	3	0
	(7,	9,	3,	1)

Ans ③

Note $12^{6}/_{19}$ always ①

LCM

① Traditional Method

Find LCM for 8, 12, 6, 4 ?

low ↓ high

8	12	6	4
16	24	12	8
24	36	18	12
32	48	24	16
			20
			24

LCM = 24 (common in all given numbers)

② Factorization Method

Find LCM 8, 12, 6, 4

2 | 8
2 | 4
2 | 2
1

$\Rightarrow 2^3$

2 | 12
2 | 6
3 | 3
1

$\Rightarrow 2^2 \times 3^1$

2 | 6
3 | 3
1

$\Rightarrow 2^1 \times 3^1$

2 | 4
2 | 2
1

$\Rightarrow 2^2$

Highest power

$2^3 \times 3^1 = 24$

③ Long Division Method

Find LCM 8, 12, 6, 4 ?

2 | 8, 12, 6, 4
2 | 4, 6, 3, 2
2 | 2, 3, 3, 1
3 | 1, 3, 3, 1
1, 1, 1, 1

$\Rightarrow 2^3 \times 3^1 = 24$

(4) Short cut Method

Note check any prime no. is there or not

find Lcm 8, 12, 6, 4

Step (1) take highest first (12)

Step (2) check the highest no. is divisible by other (or) not

$\frac{12}{8}$ ✗ $\frac{12}{6}$ ✓ $\frac{12}{12}$ ✓ $\frac{12}{4}$ ✓

Step (3) double that highest no. $12 \times 2 = 24$

then again check

$\frac{24}{8}$ ✓ $\frac{24}{6}$ ✓ $\frac{24}{12}$ ✓ $\frac{24}{4}$ ✓ $\therefore$ LCM = 24

---

find Lcm 3, 11, 12 ($\because$ (11) is prime no.)

if prime no. in que separate that no. (11 × ....)

now, Lcm (3, 12)

Biggest number (12)

$\frac{12}{3}$ ✓ $\frac{12}{12}$ ✓ Lcm = (12)

Now multiply this Lcm with (11)

$12 \times 11 = 132$ is the final Lcm

HCF

① Traditional method

Find HCF 27, 18, 36

27	18	36
9	9	18
3	6	12
1	3	9
	2	6
	1	4
		2
		1

high ↓ low

common

HCF = 9

$\frac{27}{9}, \frac{27}{3}, \frac{27}{1}$

$\frac{18}{9}, \frac{18}{6}, \frac{18}{3}, \frac{18}{2}, \frac{18}{1}$

$\frac{36}{18}, \frac{36}{12}, \frac{36}{9}, \frac{36}{6}, \frac{36}{4}, \frac{36}{2}, \frac{36}{1}$

② Factorization method

HCF 27, 18, 36

3 | 27
3 | 9
3 | 3
1

$\Rightarrow 3^3$

2 | 18
3 | 9
3 | 3
1

$\Rightarrow 2 \times 3^2$

2 | 36
2 | 18
3 | 9
3 | 3
1

$\Rightarrow 2^2 \times 3^2$

Common lowest powers

$3^2 = 9$

HCF = 9

③ Division method

Note: Divide the number until we get '0' remainder

Find HCF of 70 and 90?

Note: Always take small value as divisor

1
70 | 90
70
20 | 70
60
10 | 20
20
0

Divisor → 70

remainder → 20

HCF = 10

Find HCF of 27, 18, 36

① HCF of 27, 18

$$18 \overline{)27}\ (1,\ 18,\ 9 \overline{)18}\ (2,\ 18,\ 0$$

HCF(27, 18)

$$9 \overline{)36}\ (4,\ 36,\ 0$$

find HCF = 9

④ Short-Cut Method

Note: Below 100 → use shortcut

Above 100 → use Division method

HCF (27, 18, 36)

Step ① Take smallest no. and get factor of it

Step ② factor of 18 → 9

Step ③ now check 9 is divisible by other two numbers (27, 36)

if yes then HCF = 9

---

eg HCF (24, 42, 30)

12

8

6

$\frac{42}{12}$ ✗ $\frac{30}{12}$ ✗

$\frac{42}{8}$ ✗ $\frac{30}{8}$ ✗

$\frac{42}{6}$ ✓ $\frac{30}{6}$ ✓

∴ HCF = 6

Lcm and Hcf of given fraction

$\frac{2}{5}, \frac{3}{15}, \frac{6}{25}$

$$\text{Lcm} = \frac{\text{Lcm of Numerator}}{\text{Hcf of Denominator}} = \frac{\text{Lcm}(2,3,6)}{\text{HCF}(5,15,25)} = \frac{6}{5}$$

$$\text{HCF} = \frac{\text{HCF of Numerator}}{\text{Lcm of Denominator}} = \frac{\text{HCF}(2,3,6)}{\text{Lcm}(5,15,25)} = \frac{1}{75}$$

$\text{Lcm} = \frac{2}{3}, \frac{4}{9}, \frac{5}{6}, \frac{7}{6}$

N → Lcm(2, 4, ~~5~~, ~~7~~)

prime

$4 \times 5 \times 7$

Lcm = 140

D → HCF(3, 9, 6, 12)

HCF = 3

∴ Ans Lcm $\frac{140}{3}$

HCF = $\frac{4}{9}, \frac{10}{21}$ and $\frac{20}{63}$

N → HCF(4, 10, 20) → HCF = 2

D → Lcm(9, 21, 63) → Lcm = 63

Ans HCF $\frac{2}{63}$

Lcm and Hcf for Decimal

Find Lcm for 1.2, 1.5, 2 and 5 ?

→ 1.2 1.5 2.0 5.0

×10 ↳ 12 15 20 50

$2 \times 2 \times 3 \times 5 \times 5 = \text{Lcm } \frac{300}{10}$

∴ final Lcm = 30

2	12, 15, 20, 50
2	6, 15, 10, 25
3	3, 15, 5, 25
5	1, 5, 5, 25
5	1, 1, 1, 5
	1, 1, 1, 1

Find Lcm 1.20, 0.24 and 6

→ 1.20 0.24 6.00

×100 ↳ 120 24 600

Lcm(120, 24, 600) → $\frac{600}{100}$

∴ final Lcm 6

2	120, 24, 600
2	60, 12, 300
2	30, 6, 150
3	15, 3, 75
5	5, 1, 25
5	1, 1, 5
	1, 1, 1

Lcm = $2^3 \times 3^1 \times 5^2 = 600$

Find HCF for 6.16 and 13 ?

→ 6.16 13.00

×100 ↳ 616 1300

HCF(616, 1300) → 4

final HCF = $\frac{4}{100}$

= 0.04

$$\begin{array}{r} 2 \\ 616 \overline{)1300} \\ 1232 \\ \hline 68 \overline{)616} \\ 612 \\ \hline 4 \overline{)68} \\ 68 \\ \hline 00 \end{array} \quad \begin{array}{r} 9 \\ \\ \\ 17 \\ \\ \\ \end{array}$$

HCF = 4

Product of Numbers

① 12, 9 ⇒ 108

Lcm ⇒ 36
HCF ⇒ 3

Lcm × HCF = 36 × 3
= 108

∴ Product of two Number = Lcm × HCF

② Lcm and HCF of two No. are 1260 and 63 resp. If one of the number is 315 find other number

→ $N_1 \times N_2 = Lcm \times HCF$

$315 \times N_2 = 1260 \times 63$

$N_2 = \frac{1260 \times 63}{315} = 252$

∴ $N_2 = 252$

Lcm – Same / Different Remainder

① Find the least number which when divided by 4, 9 and 12 will leave in each case a remainder 3

Divisor → 9) 29 ← Dividend
3 ← Quotient
27
2 ← Remainder

→ Lcm(4, 9, 12) = 36

Same remainder Qve

then add remainder to it ⇒ 36 + 3 = Ans 39

② Find the least number which when divided by 10, 9 and 8 leaves a remainders 9, 8 and 7 resp. in each case.

Different remainder Qve

→

10) 1 (9
9) 1 (8
8) 1 (7

Lcm (10, 9, 8) → 360

then subtract with difference

360 − 1 = Ans 359

Least 4 or 5 -digit Number

100 → least 3 digit Number
999 → Highest 3 digit Number
1000 → Least 4 digit Number
9999 → Highest 4 digit Number
10000 → Least 5 digit Number
99999 → Highest 5 digit Number

① Find the least no. of 4 digit which is divisible by 4, 6, 8 and 10

4 digit → 1000

→

Short cut method

1000 ← 4 digit no.

LCm(4,6,8,10) → 120

120 × 8 → 960 ✗

120 × 9 → 1080 ✓

Ans 1080

Traditional method

1000 ← 4 digit no.

Lcm (4,6,8,10) → 120

4 digit no. + (Lcm − remainder)

1000 + (120 − 40)

Ans 1080 ✓

$$\begin{array}{r|l} & 8 \\ 120 & 1000 \\ & 960 \\ \hline & 40 \end{array}$$

---

② Find the least number of five Digit which is divisible by 16, 18, 24 and 30

five Digit → 10000

→ SHORTCUT

LCm (16, 18, 24, 30) → 720

720 × 10 → 7200 ✗

720 × 13 → 9360 ✗

720 × 14 → 10080 ✓

Ans

Traditional

LCm (16, 18, 24, 30) → 720

$$\begin{array}{r|l} & 13 \\ 720 & 10000 \\ & 9360 \\ \hline & 640 \end{array}$$

10000 + 720 − 64

Ans 10080

Note

LCM	HCF
1000 (+)	9999 (−)
10000 (+)	99999 (−)

③ Find the smallest 4 digit number such that when divided by 12, 18, 21 and 28 it leaves remainder 3 in each case.

→ ∴ smallest (or least) 4 digit number → 1000

LCM(12, 18, 21, 28) → 252

→ 252 × 3 = 756

252 × 4 = 1008

1008 + 3 = 1011 Ans

(or)

Traditional method

LCM(12, 18, 21, 28) → 252

$$252\overline{)1000}\ 3$$

$$\begin{array}{r} 756 \\ \hline 244 \end{array}$$

1000 + 252 − 244 + 3

Ans 1011

Greatest 4 or 5 digit number

Greatest 4 digit number → 9999

Greatest 5 digit number → 99999

① Find the greatest 4 digit number which when divided by 12, 15, 20 and 35 leaves no remainder

→ LCM(12, 15, 20, 35) ⇒ 420

420 × 20 → 8400

420 × 23 → 9660 ans

↑ random to get near ans

Traditional method

LCM(12, 15, 20, 25) ⇒ 420

$$420\overline{)9999}\ 23$$

$$\begin{array}{r} 840 \\ \hline 1599 \\ 1260 \\ \hline 339 \end{array}$$

9999 − 339

Ans 9660

② Find the greatest 3 digit Number which when divided by 8, 9 and 10 leaves 3 as a remainder in each case.

greatest 5 digit no. 99999

→ LCM(8, 9, 10) → 360

$$360 \overline{)99999} \quad 277$$

720

2799

2520

279

279

99999 − 279 + 3

Ans 99723