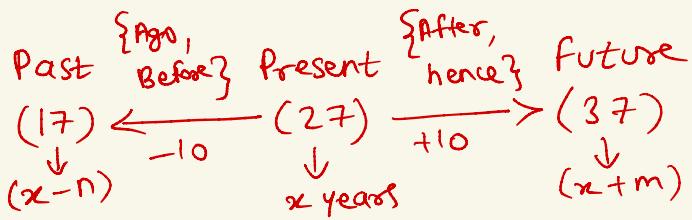


Aptitude Practice

Problems on Ages



① The present ages of A and B are in the ratio 4:5 and after 5 yrs they will be in the ratio 5:6. The present age of A is,

→ ~~Past~~ Present Future

$$4:5 \xrightarrow{+5} 5:6$$

$$\frac{4x:5x}{\cancel{x}} \xrightarrow{\quad} 4x+5:5x+5 = 5:6$$

$$\frac{4x+5}{5x+5} = \frac{5}{6}$$

$$24x+30 = 25x+25$$

$$x = 5$$

$$A = 20 \text{ yrs}$$

$$B = 25 \text{ yrs}$$

$$a:b = \frac{a}{b}$$

② The respective ratio of the present ages of Swati and Trupu is 4:5. Six years hence, the respective ratio of their ages will be 6:7. What is the difference b/w their ages?

→ ~~Past~~ Present Future

$$4:5 \xrightarrow{+6} 6:7$$

$$\frac{4x:5x}{\cancel{x}} \xrightarrow{\quad} 4x+6:5x+6 = 6:7$$

$$\frac{4x+6}{5x+6} = \frac{6}{7}$$

$$28x+42 = 30x+36$$

$$2x = 6$$

$$x = 3$$

③ The ratio of present ages of two brothers is 1:2 and 5yr back, the ratio was 1:3. what will be the ratio of their ages after 5yrs?
 (back) \rightarrow and $\rightarrow +5$ \rightarrow future

Their ages after 5 yrs?

\rightarrow Past $\xleftarrow{-5}$ Present $\xrightarrow{+5}$ Future

$1:3$ $1:2$ $\frac{15}{15} : \frac{25}{25} \Rightarrow 3:5$

$\frac{x-5}{2x-5} = \frac{1}{3}$

$x = 10$

$3x - 15 = 2x - 5$

$2x - 10 = 20$

④ The ratio of the present ages of Anju and Sandhya is $13:17$, respectively. Four years ago, the respective ratio of their ages was $11:15$. What will be the resp. ratio of their ages six years hence?

⑤ 4 yr ago, the ratio of the ages of A and B was 2:3 and after 4yr, it will become 5:7. find their present ages.

$$\begin{array}{c}
 \xrightarrow{\text{Part}} \xleftarrow{-4} \xrightarrow{\text{Present}} \xrightarrow{+4} \xrightarrow{\text{Future}} \\
 \begin{array}{c}
 2:3 \quad : \quad \frac{(2x+8)}{(3x+8)} \quad 5:7 \\
 \text{Diff} = 8 \text{ yrs} \\
 \frac{2x+8}{3x+8} = \frac{5}{7} \\
 14x + 56 = 15x + 40 \\
 x = 16
 \end{array}
 \begin{array}{c}
 3x+4 \\
 = 3 \times 16 + 4 \\
 = 48 + 4 \\
 = 52
 \end{array}
 \end{array}$$

⑥ Marsha is 40 yr old and Ruth is 60 yrs old. How many years ago was the ratio of their ages 3:5?

$$\begin{array}{c}
 \xrightarrow{\text{(Past)}} \xleftarrow{x} \xrightarrow{\text{(Present)}} \\
 3:5 \quad \xrightarrow{40:60} \\
 \frac{40-x}{60-x} = \frac{3}{5} \\
 200 - 5x = 180 - 3x \\
 2x = 20 \\
 x = 10 \quad \text{40 ago}
 \end{array}$$

⑦ The ratio of the ages of two persons is 4:7 and age of one of them is greater than that of the other by 30 yrs. The sum of their ages (in years) is,

$$\begin{array}{c}
 \xrightarrow{\text{A:B}} \xrightarrow{4:7} \\
 4x:7x \longrightarrow 7 \times 10 = 70 \\
 4x \sim 7x = 30 \text{ yrs} \\
 3x = 30 \\
 x = 10
 \end{array}
 \begin{array}{c}
 \therefore 40 + 70 \\
 (= 110)
 \end{array}$$

8) The resp. ratio betw the present ages of parag and sapna is 21:19. Six years ago, the resp. ratio betw their ages was 9:8. How old is Lina, if her present age is 12 yr less than sapna's present age?

$$\rightarrow \begin{array}{c} (\text{Past}) \\ 9:8 \xleftarrow{-6} (\text{Present}) \\ \text{Parag: Sapna} \\ 21 : 19 \\ 21x : 19x \\ \hline 168x - 48 = 171x - 54 \\ 3x = 6 \\ \boxed{x = 2} \end{array}$$

$$\begin{array}{c} 21x : 19x \\ \downarrow \\ 21 \times 2 : 19 \times 2 \\ = 42 : 38 \\ = \boxed{38} \end{array}$$

$$\begin{array}{c} \text{Sapna} \\ 38 \text{yr} - 12 \text{yr} = \boxed{26 \text{yr}} \\ \text{Lina} \end{array}$$

Twice / Thrice the Age

Note: ① father Age is three times the age of his Son.

Son	Father
$= x$	$= 3x$

② father is aged three times more than his son.

Son	Father
$= x$	$= x + 3x$
	$= 4x$

Ques) I am three times as old as my son. 15 yr hence I will be twice as old as my son. The sum of our ages is,

$3x$	Father	Son
$\leftarrow 3x$		$x \rightarrow x + 15$
$\boxed{= 45}$		

After 15 yr,

$$3x + 15$$

$$3x + 15 = 2(x + 15)$$

$$3x + 15 = 2x + 30$$

$$\boxed{x = 15}$$

$$\therefore 45 + 15$$

$$\boxed{= 60}$$

② The sum of present ages of A and B is 11 times the difference of their ages. 5 years hence, their total ages will be 13 times the difference of their ages. What is the present age of elder one?

$$\rightarrow (A+B) = 11(A-B)$$

$$A+B = 11A - 11B$$

$$11A - A = B + 11B$$

$$10A = 12B$$

$$\frac{10}{5}A = \frac{12}{6}B$$

$$\frac{A}{B} = \frac{6}{5}$$

$$\Rightarrow A:B = 6:5$$

$$\underline{6x+5} \quad \underline{5x+5}$$

$$6x+5 + 5x+5 = 13(6x-5x)$$

$$11x + 10 = 13x$$

$$2x = 10$$

$$x = 5$$

$$\begin{aligned} A &= 6x \\ &= 6 \times 5 \\ &= 30 \end{aligned}$$

③ The present age of a father is 3 yrs more than three times the age of his son. 3 yrs hence, father's age will be 10 yrs more than twice the age of the son. The father's present age is,

$$\rightarrow \begin{array}{lll} \text{Son} & \text{father} & \\ \text{Present Age} & x & (3+3x) \\ \text{Future Age} & \underline{x+3} & \underline{3x+6} \end{array} \quad \begin{aligned} 3x+6 &= 10 + 2(x+3) \\ 3x+6 &= 10 + 2x+6 \\ x &= 10 \\ 3+3x &= 3+3 \times 10 = 33 \end{aligned}$$

④ 10 years ago daughter's age was two-fifth of her mother's age that time. While 10 years hence her age will be three-fifth of her mother's age then. Find the difference in the ages of the two.

$$\rightarrow \begin{array}{ll} \text{Daughter} = x, \text{Mother} = y & \\ (x-10) & (y-10) \end{array} \quad \begin{aligned} 5x-50 &= 2y-20 \\ 5x-2y &= 30 \quad \text{--- (3)} \end{aligned}$$

$$(x-10) = \frac{2}{5}(y-10) \quad \text{--- (1)}$$

$$(x+10) = \frac{3}{5}(y+10) \quad \text{--- (2)}$$

$$5x+50 = 3y+30$$

$$5x-3y = 20 \quad \text{--- (4)}$$

put $y=50$ in eqn (3)

$$5x-2 \times 50 = 30$$

$$5x-100 = 30$$

$$5x = 130$$

$$x = 26$$

$$\text{Mother} = y = 50 \text{ yrs}$$

$$\text{Daughter} = x = 26 \text{ yrs}$$

$$\text{--- (3)} - \text{--- (4)}$$

$$\begin{aligned} 5x-2y &= 30 \\ 5x-3y &= 20 \\ \hline y &= 50 \end{aligned}$$

$$\therefore \text{Diff} = M-D$$

$$= 50-26$$

$$= 24 \text{ years}$$

⑤ Father is aged three times more than his son Arun. After 8 years, he would be two and a half times of Arun's age. After further 8 years, how many times would he be of Arun's age?

$$\begin{array}{lll}
 \rightarrow & \text{son} & \text{father} \\
 \begin{array}{l} \text{Present age} \\ +8 \\ \text{Future age} \\ +8 \end{array} & x & x+3x = 4x \quad \therefore \text{Son} = \underline{\underline{8 \text{ yr}}}, \text{father} = \underline{\underline{4 \times 8}} \\
 & x+8 & 4x+8 \quad = \underline{\underline{32 \text{ yr}}} \\
 & 4x+8 = 2\frac{1}{2}(x+8) & \downarrow +8 \\
 & 4x+8 = \frac{5}{2}(x+8) & 16 \text{ yr} \\
 & 8x+16 = 5x+40 & \downarrow +8 \\
 & 3x = 40 - 16 & 24 \\
 & 3x = 24 & \downarrow +8 \\
 & \boxed{x=8} & 40 \\
 & & \downarrow +8 \\
 & & 48 \\
 & & \text{2 times}
 \end{array}$$

⑥ If the ages of A and C are added to twice the age of B, the total becomes 59. If the ages of B and C are added to thrice the age of A, the total becomes 68 and if the age of A is added to thrice the age of B and thrice the age of C, the total becomes 108. What is the age of A?

$$\rightarrow A + C + 2B = 59 \quad \text{--- } ①$$

$$B + C + 3A = 68 \quad \text{--- } ②$$

$$A + 3B + 3C = 108 \quad \text{--- } ③$$

$$② \times 3 \Rightarrow 3B + 3C + 9A = 204$$

$$③ \Rightarrow \overline{3B + 3C + A} = \overline{108}$$

$$8A = 96$$

$$\boxed{A = 12}$$

7) Meena married 10 yrs ago. Today her age is $\frac{7}{5}$ times her age at the time of her marriage. Her daughter's age is $\frac{1}{5}$ of her age. What is the ratio of Meena's age to her daughter's age after 5 years?

$$\rightarrow \text{Meena} = x$$

$$(x - 10)$$

$$x = \frac{7}{5}(x - 10)$$

$$5x = 7x - 70$$

$$2x = 70$$

$$\underline{\text{Meena age}} \quad \boxed{x = 35 \text{ years}}$$

$$\text{Daughter} = \frac{1}{5} \times 35$$

$$\boxed{= 7 \text{ years}}$$

After 5 yrs

$$m = 35 + 5 = 40 \text{ yrs}$$

$$D = 7 + 5 = 12 \text{ yrs}$$

$$\therefore 40 : 12 \Rightarrow \boxed{10 : 3}$$

8) The sum of the ages of father and son is 45 yrs. Five yrs ago the product of their ages was 4 times the father's age at that time. The present ages of father & son are?

$$\rightarrow \begin{array}{cc} (\text{Son}) & (\text{father}) \\ x & y \end{array}$$

Present

$$\boxed{x + y = 45}$$

$$x - 5 + y - 5 = 45$$

Past

$$\boxed{x + y = 35}$$

$$x \times y = 4y$$

$$\boxed{xy = 9}$$

Put $x = 9$ in eqn Past

$$x + y = 35$$

$$9 + y = 35$$

$$\boxed{y = 31}$$

(Past age father)

$$\boxed{x = 9}$$

(Past age son)

$$+5 \quad +5$$

$$\boxed{9 \text{ yrs}} \quad (\text{son})$$

$$\boxed{36 \text{ yrs}} \quad (\text{father})$$

⑨ In a family, a couple has a son and daughter. The age of the father is four times that of his daughter and the age of the son is half of his mother. The wife is ten years younger to her husband and the brother is six years older than his sister. What is the age of the mother?

→	father	mother	son	daughter
	$4x$	$4x - 10$	$x + 6$	x

$$x + 6 = \frac{1}{2}(4x - 10)$$

$$2x + 12 = 4x - 10$$

$$2x = 22$$

$$\boxed{x = 11}$$

$$\text{mother} = 4x - 10$$

$$= 4 \times 11 - 10$$

$$= 44 - 10$$

$$\boxed{= 34 \text{ years}}$$

⑩ 6 yrs ago, three times the age of B was 2 more than the age of A at that time. 6 yrs hence, twice age of B will be equal to A's age that time. Find the total of their ages.

→	B	A
	x yrs	y yrs
	$(x - 6)$	$(y - 6)$
	$3(x - 6) = 2 + (y - 6)$	
	$3x - 18 = y - 4$	
	$\boxed{3x - y = 14} \quad \textcircled{1}$	
	$2(x + 6) = (y + 6)$	
	$2x + 12 = y + 6$	
	$\boxed{2x - y = -6} \quad \textcircled{2}$	

$$\begin{array}{r} -3x - y = 14 \\ 2x - y = -6 \\ \hline x = 20 \end{array}$$

$$\text{put } x = 20 \text{ in eqn } \textcircled{1}$$

$$3x - y = 14$$

$$-y = 14 - 60$$

$$-y = -46$$

$$\boxed{y = 46}$$

$$A + B$$

$$46 + 20$$

$$\boxed{= 66} \text{ yrs}$$

⑪ Sneha's mother's age is five yrs more than twice the age of Sneha. When Sneha was born, her brother Rahul was four years old and her father two years older than her mother. If the average age of her mother and father is 46 yrs. Find the ratio of age of Rahul to that of Sneha.

Average on Ages

Note ① Average Age of father and Mother is 34

$$\frac{F+m}{2} = 34 \quad \Rightarrow \quad F+m = 68$$

② Average age of father, mother and son is 52
 $\frac{f+m+s}{3} = 52 \Rightarrow f+m+s = 156$

$$\underline{f+m+s} = 52 \Rightarrow f+m+s = 156$$

(Q1) In a family, the average age of father and mother is 35 yrs. The avg. age of the father, mother and their only son is 27 yrs. What is the age of the son?

$$\rightarrow \frac{f+m}{2} = 35 \Rightarrow f+m=70$$

$$\frac{F+m+s}{2} = 27 \Rightarrow F+m+s=81$$

$$70 + 5 = 81$$

S = 11

years

② 5 yrs ago, the avg. age of A, B, C and D was 45 yrs. With E joining them now, the avg. age of all the five is 49 yrs. How old is E?

$$\rightarrow \text{Past} \xleftarrow{-5\text{yr}} \text{Present}$$

A, B, C, D $\frac{A+B+C+D}{4} = 45$	A, B, C, D, E $\frac{A+B+C+D+E}{5} = 49$
--	---

$A+B+C+D = 180$
↳ Past

$A+B+C+D+E = 245$
↳ Present

$A+B+C+D = 200$
↳ Present

∴ to bring Past to present

$$A+B+C+D = 180$$

$$+5 +5 +5 +5$$

$$A+B+C+D = 180 + 20$$

$$= \underline{\underline{200}}$$

∴ ① - ②

$E = 45 \text{ year}$

③ The avg. age of a husband and wife, who were married 4 yrs ago, was 25 yrs at the time of their marriage. The avg. age of the family consisting of husband, wife and child, born during the interval is 20 yrs today. The age of the child is,

$$\rightarrow \text{Past} \xleftarrow{-4\text{yr}} \text{Present}$$

$\frac{H+W}{2} = 25$ $H+W = 50$ ↳ Past	$\frac{H+W+C}{3} = 20$ $H+W+C = 60$ ↳ Present
--	---

$H+W = 50$
 $+4 +4$
 $H+W = 58$
↳ Present

$H+W+C = 60$
 $58 + C = 60$
↳ Present

$C = 2 \text{ yr}$

④ Average age of 6 sons of a family is 8 yrs. Avg. age of son together with their parents is 22 yrs. If the father is older than the mother by 8 yrs, then the age of mother (in years) is,

$$\rightarrow \begin{array}{cc} \text{mother} & \text{father} \\ x \text{ yrs} & (x+8) \text{ yrs} \end{array}$$

$$\frac{6 \text{ son}}{6} = 8 \text{ yrs}$$

$$6 \text{ son} (\text{total Age}) = 48 \text{ yrs}$$

$$\frac{8 \text{ person}}{8} = 22$$

$$8 \text{ person } \{6 \text{ son} + m + f\} = 176 \text{ yrs}$$

$$48 + m + f = 176 \text{ yrs}$$

$$m + f = 128 \text{ yrs}$$

$$x + (x+8) = 128$$

$$2x = 120$$

$$\boxed{x = 60}$$

$$\text{Mother} = 60 \text{ yrs}$$

$$\text{Father} = 60 + 8$$

$$= 68 \text{ yrs}$$

⑤ When the avg. age of husband and wife and their son was 42 yrs, the son got married and a child was born just one year after the marriage. When child turned to be 5 yrs, then the avg. age of family became 36 yrs. What was the age of daughter-in-law at the time of marriage?

$$\rightarrow \boxed{M + F + S} \rightarrow M + F + S + W \xrightarrow[14 \text{ yrs}]{\text{child born}} \xrightarrow[5 \text{ yrs}]{5 \text{ yrs}} M + F + S + W + C \rightarrow 36 \text{ yrs}$$

$$\frac{M + F + S}{3} = 42$$

$$M + F + S = 126$$

$$\frac{(M + 6) + (F + 6) + (S + 6) + (W + 6) + 5}{5} = 36$$

$$M + F + S + W = (36 \times 5) - 29$$

$$\underbrace{M + F + S}_{126} + W = 180 - 29$$

$$W = 180 - 29 - 126$$

$$\boxed{W = 25} \text{ years}$$

daughter in law

Advanced

① The ratio of the age of a couple (man and his wife) is 5:3 when they get married. After 8 yrs their age's ratio becomes 7:5. When they purchase a new car their age's ratio is 3:2. Then after how much years they brought a new car?

$$\rightarrow \begin{array}{l} M:W \\ 5:3 \end{array} \quad \begin{array}{l} M:W \\ 7:5 \end{array}$$

$$\frac{5x : 3x}{5x+8} = \frac{7}{5} \quad \boxed{\begin{array}{l} 5x = 20 \\ 3x = 12 \end{array}}$$

$$25x + 40 = 21x + 56$$

$$\frac{4x = 16}{x = 4}$$

$$\frac{20+x}{12+x} = \frac{3}{2}$$

$$40 + 2x = 36 + 3x$$

$$\boxed{x = 4} \quad \text{Ans}$$

② In a family the father's age is $\frac{1}{3}$ of the grandfather's age. The mother's age is $\frac{7}{9}$ of the father's age and child age is $\frac{1}{7}$ of the mother's age. Find the difference b/w grandfather's age and child's age, if the mother's age is 21.

$$\rightarrow \text{mom} = 21, \quad C = \frac{1}{7} \times 21 = \boxed{\text{child} = 3 \text{ yrs}}$$

$$21 = \frac{7}{9} \times F$$

$$F = 21 \times \frac{9}{7}$$

$$\boxed{F = 27}$$

$$\text{G.F.} = 81$$

$$\text{Child} = 3$$

$$\boxed{78}$$

$$27 = \frac{1}{3} \times \text{G.F.}$$

$$\text{G.F.} = 3 \times 27$$

$$\boxed{= 81}$$

③ Preethi father's age was 2.5 times of Preethi's age, the ratio of the present age of Preethi's mother to Preethi's grandmother age is 5:9. 3 yrs hence Preethi's mother age was 43 and ratio of age of her grandfather, grandmother and Preethi is 13:12:3. What is Preethi's father age after 7 years?

$$\rightarrow \boxed{P.F = 2.5 \times P} \quad \underline{P.M. : P.G.M} = 5:9$$

$$5x : 9x$$

$$\checkmark \boxed{[50 : 72]}$$

$$\textcircled{8} \quad \underline{P.G.F : P.G.M : P} = \frac{13:12:3}{= 13x : 12x : 3x}$$

$$\downarrow$$

$$\boxed{\text{Preethi Age} = 18}$$

$$\boxed{P.F. = 2.5 \times 18}$$

$$\boxed{P.F = 45} + 7$$

$$\boxed{12x = 72}$$

$$\boxed{1x = 6}$$

$$\therefore P.F = 52 \text{ years}$$

④ The present age of Romila is one fourth of that of her father. After 6 yrs the father's age will be twice the age of Kapil. If Kapil celebrated fifth birthday 8 yrs ago, what is Romila's present age?

$$\rightarrow \begin{matrix} \text{Present} & \xrightarrow{+6} \\ R = \frac{1}{4} \times F & F = 2k \end{matrix}$$

$$F = 2(19)$$

$$5+8 \quad \boxed{k = 13} + 6 \rightarrow = 38 \text{ yrs}$$

$$R = \frac{1}{4} \times 38$$

$$\boxed{R = 8 \text{ yrs}}$$

⑤ X's age 3 yrs ago was three times the present age of Y. At present, Z's age is twice the age of Y. Also Z is 12 years younger than X. What is the present age of Z?

$$\rightarrow \text{Y's age} = 'a'$$

$$\text{'X' present age} = (3a + 3)$$

$$Z = 2a$$

$$x \sim z = 12$$

$$3a + 3 - 2a = 12$$

$$\boxed{a = 9}$$

$$z = 2 \times 9$$

$$\boxed{z = 18}$$

⑥ The resp. ratio b/w the present age of Ram and Selva is 3:X. Selva is 24 yrs old than prem. Prem's age after 7 years will be 23 yrs. Ram's age is 75% of Selva's age. What is the difference b/w the value of X and the age of Ram.

$$\rightarrow \text{Ram : Selva}$$

$$\boxed{3 : x}$$

$$3x : \text{?}$$

$$y \sim 30$$

$$= 26$$

$$\text{Selva} = 24 + \text{Prem}(16)$$

$$\boxed{\text{Selva} = 40}$$

$$\boxed{\text{Prem} = 16}$$

$$\text{Ram} = \frac{75}{100} \times 40$$

$$\boxed{\text{Ram} = 30}$$

$$\begin{matrix} +7 \\ (23) \end{matrix}$$

7) 4 years ago, the respective ratio between $\frac{1}{2}$ of A's age at that time and four times of B's age at that time was 5:12. Eight years hence $\frac{1}{2}$ of A's age at that time will be less than B's age at that time by 2 years. What is B's present age?

$$\begin{array}{c}
 \text{Past} & \xleftarrow{-4} \text{Pre} & \xrightarrow{8\text{ yrs}} \text{future} \\
 5:12 & & \\
 \frac{1}{2}x:A = 5 & 10x+4 & A = 10x+12 \\
 \boxed{A = 10x} & 3x+4 & B = 3x+12 \\
 4B = 12 & 3(2)+4 & \\
 \boxed{B = 3x} & = 6+4 & \\
 & \boxed{= 10} & \\
 & & \frac{10x+12}{2} = (3x+12)-2 \\
 & & 10x+12 = 3x+10 \\
 & & 10x+12 = 6x+20 \\
 & & 4x = 8 \\
 & & \boxed{x = 2}
 \end{array}$$

8) 8 years ago, Jyoti's age was equal to Swati's present age. If the sum of Jyoti's age 10 yrs from now and Swati's age 6 yrs ago is 88 yrs. what was Jyoti's age 14 yrs if Jyoti is 8 yrs younger to Swati? (in yrs)

$$\begin{array}{ccc}
 \rightarrow & \text{Past} & \text{Present} \\
 \text{Swathi} & x-6 & x \\
 & & x+8 \\
 \text{Jyoti} & & \\
 & x+8 + 10 + x-6 = 88 & \\
 & 2x + 12 = 88 & \\
 & 2x = 88 - 12 & \\
 & 2x = 76 & \\
 \underline{\text{Swathi}} & \boxed{x = 38} &
 \end{array}$$

$$\begin{array}{c}
 \text{Swathi} \\
 \frac{38}{-8} \text{ Younger to Swathi} \\
 \underline{30} \leftarrow \text{Jyoti} \\
 -14 \text{ ago} \\
 \underline{16} \leftarrow \text{Jyoti age}
 \end{array}$$

⑨ Three yrs ago, the resp. ratio b/w A's age at that time and B's age at that time was 9:5. If A's age two yrs hence will be 17 yrs more than B's age five years hence, what is B's present age?

→ $\frac{\text{Past}}{9:5} \xleftarrow{-3} \frac{\text{Present}}{9x+3+2}$

$\frac{5x+3}{5x+5+3} = 5$

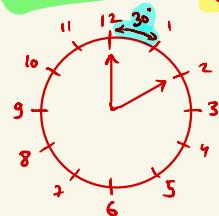
$= 28$ B's present age

$9x+3+2 - 5x-3-5 = 17$

$4x = 20$

$x = 5$

Clock



(H-H) → Hour hand → small
(M-H) → minute hand → large

Total angle = 360°

$$\textcircled{1} \text{ gap} = \frac{360^\circ}{12} = 30^\circ$$

$$\textcircled{2} 1\text{ hr} \Rightarrow 60\text{ min} = 30^\circ$$

$$\begin{array}{|c|c|} \hline 1\text{ min} & = \frac{1}{2}^\circ \\ \hline (\text{m-H}) & (\text{H-H}) \\ \hline \end{array}$$

$$\text{eg. } 20\text{ min} = 10^\circ$$

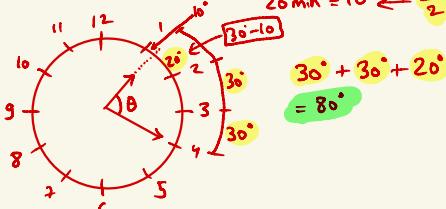
(m-H) (H-H)

$$45\text{ min} = 22.5^\circ$$

(m-H) (H-H)

① What is the angle betn min hand and hour hand at 1:20?

→



$$20\text{ min} = 10^\circ \leftarrow \frac{20}{2}$$

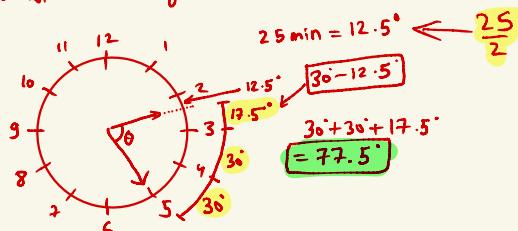
$$\begin{array}{|c|c|} \hline 20-10 & \\ \hline \end{array}$$

$$30^\circ + 30^\circ + 20^\circ$$

$$= 80^\circ$$

② What is the angle betn min hand and hour hand at 2:25?

→



$$25\text{ min} = 12.5^\circ \leftarrow \frac{25}{2}$$

$$\begin{array}{|c|c|} \hline 30-12.5 & \\ \hline \end{array}$$

$$30^\circ - 12.5^\circ$$

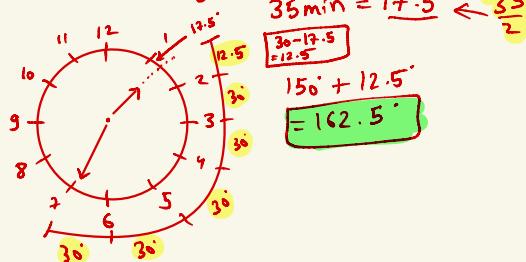
$$= 17.5^\circ$$

$$30^\circ + 30^\circ + 17.5^\circ$$

$$= 77.5^\circ$$

③ What is the angle betn minute hand and hour hand at 1:35?

→



$$35\text{ min} = 17.5^\circ \leftarrow \frac{35}{2}$$

$$\begin{array}{|c|c|} \hline 30-17.5 & \\ \hline \end{array}$$

$$30^\circ - 17.5^\circ$$

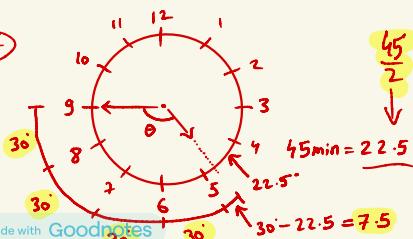
$$= 12.5^\circ$$

$$150^\circ + 12.5^\circ$$

$$= 162.5^\circ$$

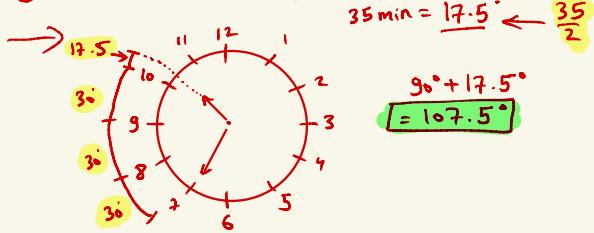
④ What is the angle betn min hand and hour hand at 4:45?

→

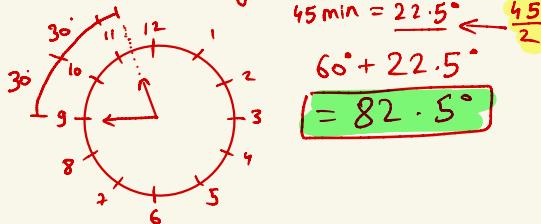


$$120^\circ + 7.5^\circ = 127.5^\circ$$

⑤ What is the angle b/w minute hand and hour hand at 10:35?



⑥ What is the angle b/w min hand and hour hand at 11:45?



formula method

$$= |30H - \frac{11}{2}(M)|$$

① 1:20 ?

$$= |30H - \frac{11}{2}(M)|$$

$$= |30(1) - \frac{11}{2}(\frac{10}{60})|$$

$$= |30 - 110|$$

$$= |-80| \Rightarrow 80^\circ$$

② 2:25 ?

$$= |30H - \frac{11}{2}(M)|$$

$$= |30(2) - \frac{11}{2}(25)|$$

$$= |60 - \frac{275}{2}|$$

$$= |60 - 137.5|$$

$$= |-77.5|$$

$$\Rightarrow 77.5$$

③ 11:45 ?

$$= |30H - \frac{11}{2}(M)|$$

$$= |30(11) - \frac{11}{2}(45)|$$

$$= |330 - \frac{495}{2}|$$

$$= |330 - 247.5|$$

$$= 82.5^\circ$$

Note: more than $180^\circ \Rightarrow$ Reflex angle

④ 12:30 ?

$$= |30H - \frac{11}{2}(M)|$$

$$= |30 \times 12 - \frac{11}{2}(30)|$$

$$= |360 - 165|$$

$$= 195^\circ \leftarrow \text{Reflex angle} \Rightarrow = 245^\circ$$

$$\therefore 360 - 195^\circ$$

$$= 165^\circ$$

⑤ 10:10 ?

$$= |30H - \frac{11}{2}(M)|$$

$$= |30 \times 10 - \frac{11}{2}(10)|$$

$$= |300 - 55|$$

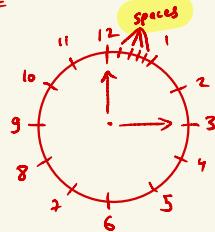
$$= 245^\circ$$

$$\therefore 360 - 245^\circ$$

$$= 115^\circ$$

Hands Together

Note ① "55 min spaces" are gained by minute hand in 60 minutes"

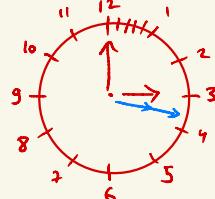


$$\begin{array}{l}
 \text{3:00 to 4:00} \\
 60 \text{ spaces} \quad 5 \text{ spaces} \\
 (\text{m.h}) \quad (\text{h.h}) \\
 \text{in 1 hr} \qquad \qquad \qquad \Rightarrow 60 \sim 5 \\
 \qquad \qquad \qquad = 55 \text{ spaces}
 \end{array}$$

m.h → minute hand
h.h → hour hand

Q1 At what time betn 3 o'clock and 4 o'clock, will the hands of a clock be together?

→



$$55 \text{ spaces} = 60 \text{ min}$$

$$15 \text{ min} = x$$

$$55 \times x = 60 \times 15$$

$$x = \frac{60 \times 15}{55}$$

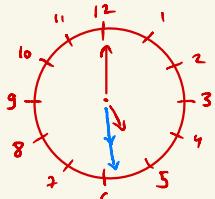
$$x = \frac{180}{11}$$

$$\begin{array}{r}
 16 \\
 11) 180 \\
 -17 \\
 \hline
 10 \\
 -11 \\
 \hline
 4
 \end{array}$$

$$x = \frac{180}{11} \Rightarrow 16 \frac{4}{11} \text{ past 3}$$

Q2 At what time betn 5 o'clock and 6 o'clock, will the hands of a clock be together?

→



$$55 \text{ spaces} = 60 \text{ min}$$

$$25 \text{ min} = x$$

$$55 \times x = 60 \times 25$$

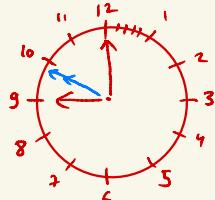
$$x = \frac{60 \times 25}{55}$$

$$x = \frac{300}{11}$$

$$x = \frac{300}{11} \Rightarrow 27 \frac{3}{11} \text{ past 5}$$

Q3 At what time betn 9 o'clock and 10 o'clock, will the hands of a clock be together?

→



(1 hr)

$$55 \text{ min spaces} = 60 \text{ min}$$

$$45 \text{ min spaces} = x$$

$$x = \frac{60 \times 45}{55}$$

$$x = \frac{540}{11}$$

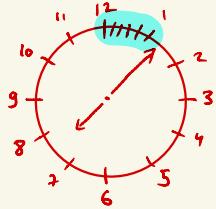
$$x = \frac{540}{11} \Rightarrow 49 \frac{1}{11} \text{ past 9}$$

Hands of the clock opposite

"55 min spaces are gained by minute hand in 60 min"

① At what time betw 7 o'clock ant 8 o'clock the hands of the clock be in opposite direction?

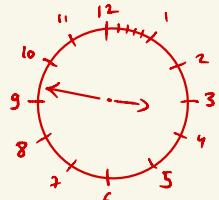
→



$$\begin{aligned} 55 &= 60 \\ 5 &= x \\ x &= \frac{60 \times 5}{55} \\ &= \frac{60}{11} \Rightarrow 5 \frac{5}{11} \text{ past 7} \end{aligned}$$

② At what time betw 3 o'clock and 4 o'clock the hands of the clock be in opposite direction?

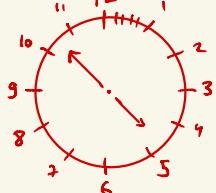
→



$$\begin{aligned} 55 &= 60 \\ 45 &= x \\ x &= \frac{60 \times 45}{55} \\ &= \frac{540}{11} \Rightarrow 49 \frac{1}{11} \text{ past 3} \end{aligned}$$

③ At what time betw 4 o'clock and 5 o'clock the hands of the clock be in opposite direction?

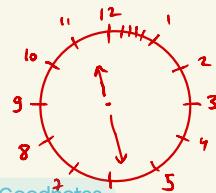
→



$$\begin{aligned} 55 &= 60 \\ 50 &= x \\ x &= \frac{60 \times 50}{55} \\ &= \frac{600}{11} \Rightarrow 54 \frac{6}{11} \text{ past 4} \end{aligned}$$

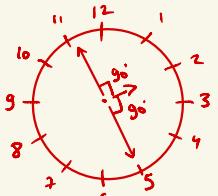
④ At what time betw 11 o'clock and 12 o'clock the hands of the clock be in opposite direction?

→



$$\begin{aligned} 55 &= 60 \\ 25 &= x \\ x &= \frac{60 \times 25}{55} \\ &= \frac{300}{11} \Rightarrow 27 \frac{3}{11} \text{ past 11} \end{aligned}$$

Right Angle

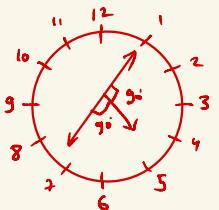


Note we get two ans
check and mark
which is in option

- Steps
- ① Draw hrs hand
- ② Draw perpendicular line
- ③ Find answer ②

① At what time b/w 4 o'clock and 5 o'clock, will the both hands of a clock be at right angle?

→



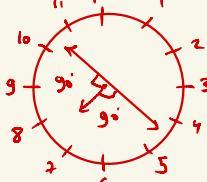
$$\begin{aligned} \textcircled{1} \\ 55 &= 60 \\ 5 &= x \\ x &= \frac{60 \times 5}{55} \\ &= \frac{60}{11} \end{aligned}$$

$$\Rightarrow 5\frac{5}{11} \text{ past 4}$$

$$\begin{aligned} \textcircled{2} \\ 55 &= 60 \\ 35 &= x \\ x &= \frac{60 \times 35}{55} \\ &= \frac{420}{11} \\ &= 38\frac{2}{11} \text{ past 4} \end{aligned}$$

② At what time b/w 7 o'clock and 8 o'clock, will the both hands of a clock be at right angle?

→

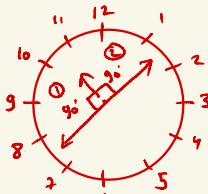


$$\begin{aligned} \textcircled{1} \\ 55 &= 60 \\ 20 &= x \\ x &= \frac{60 \times 20}{55} \\ &= \frac{240}{11} \\ &= 21\frac{9}{11} \text{ past 7} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \\ 55 &= 60 \\ 50 &= x \\ x &= \frac{60 \times 50}{55} \\ &= \frac{600}{11} \\ &= 54\frac{6}{11} \text{ past 7} \end{aligned}$$

③ At what time b/w 10 o'clock and 11 o'clock, will the both hands of a clock be at right angle?

→



$$\begin{aligned} \textcircled{1} \\ 55 &= 60 \\ 35 &= x \\ x &= \frac{60 \times 35}{55} \\ &= \frac{420}{11} \\ &= 38\frac{2}{11} \text{ past 10} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \\ 55 &= 60 \\ 5 &= x \\ x &= \frac{60 \times 5}{55} \\ &= \frac{60}{11} \\ &= 5\frac{5}{11} \text{ past 10} \end{aligned}$$

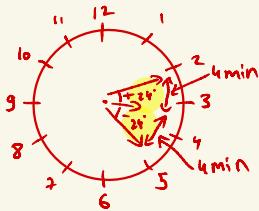
Apart

$$60 \text{ min} \rightarrow 360^\circ$$

$$1 \text{ min} \rightarrow 6^\circ$$

$$\theta = 30H - \frac{11}{2}M$$

- ① At what time betn 3 o'clock and 4 o'clock will both the hands of a clock be 4 min apart?
 $\rightarrow (3-4) 4 \text{ min apart?}$



① 1 min = 6°
 $\therefore 1 \text{ min} = 6^\circ \times 4 = 24^\circ$

② $\theta = 30H - \frac{11}{2}M$
 $+ 24 = 30(3) - \frac{11}{2}M$ ⑤ $- 24 = 30(3) - \frac{11}{2}M$

$$\frac{11}{2}M = 90 - 24$$

$$\frac{11}{2}M = 114$$

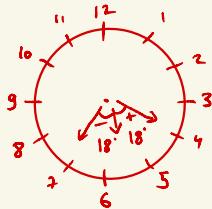
$$\frac{11}{2}M = 66$$

$$M = \frac{228}{11} = 20\frac{8}{11} \text{ past } 3$$

$$M = 12 \text{ min}$$

- ② At what time betn 5 o'clock and 6 o'clock will both the hands of a clock be 3 min apart?

\rightarrow



(5-6) 3 min Apart

$$6^\circ \times 3 = 18^\circ$$

$$+ 18$$

 $18 = 30(5) - \frac{11}{2}M$

$$\frac{11}{2}M = 150 - 18$$

 $= 132 \times 2$
 4

$$M = 24 \text{ min past } 5$$

$$- 18$$

 $- 18 = 30(5) - \frac{11}{2}M$

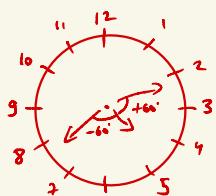
$$\frac{11}{2}M = 150 + 18$$

 $M = 168 \times 2$

$$M = \frac{336}{11} = 30\frac{6}{11} \text{ past } 5$$

- ③ At what time betn 4 o'clock and 5 o'clock will both the hands of a clock be 10 min apart?

\rightarrow



(4-5) 10 min apart.

$$1 \text{ min} \rightarrow 6^\circ \times 10 = 60^\circ$$

$$+ 60$$

 $60 = 30(4) - \frac{11}{2}M$

$$\frac{11}{2}M = 120 - 60$$

$$M = \frac{120}{11}$$

$$M = 10\frac{10}{11} \text{ past } 4$$

$$- 60$$

 $- 60 = 30(4) - \frac{11}{2}M$

$$\frac{11}{2}M = 120 + 60$$

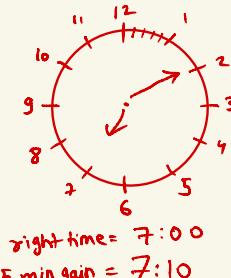
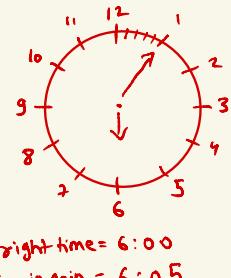
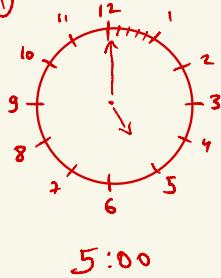
$$M = \frac{360}{11}$$

$$M = 32\frac{8}{11} \text{ past } 4$$

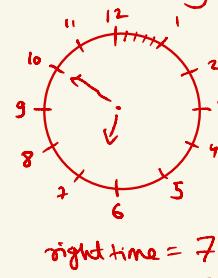
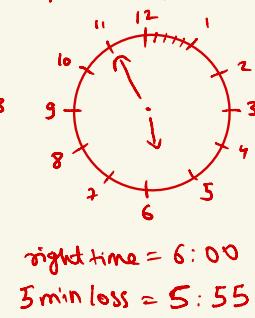
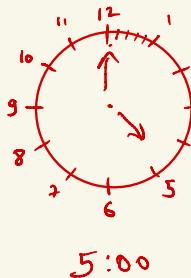
Loss @ Gain

* clock gains 5 min every hour.

Note ①



* clock loses 5 min every hour.



Note ②

$$45 \frac{4}{5} \text{ hours}$$

$$\Rightarrow 45 \text{ hours } \frac{4}{5} \times 60 \leftarrow$$

1hr = 60 min

45 hours 48 minutes

$$23 \frac{1}{3} \text{ hrs}$$

$$\Rightarrow 23 \frac{1}{3} \times 60 \leftarrow$$

1hr = 60 min

23 hrs 20 min

Q1) Akash saw the clock when it was set right at 8 am. The clock gains 5 min in an hour. What time will it show at 8 pm on the same day?

$$\rightarrow 8\text{am} - 8\text{pm} (12 \text{ hrs})$$

$$1 \text{ hr} \rightarrow 5 \text{ min}$$

$$12 \text{ hrs} \rightarrow 12 \times 5 \Rightarrow 60 \text{ min}$$

original
8pm
9pm

Q2) If a clock gains 5 min every hour and it is set correctly at 5 am, then at what time will it show at 10 am on the same day?

$$\rightarrow 5\text{am} - 10\text{am} (5 \text{ hrs})$$

$$1 \text{ hr} \rightarrow 5 \text{ min}$$

$$5 \text{ hrs} \rightarrow 5 \times 5 = 25 \text{ min}$$

original wrong
10:00 am 10:25 am

(Q3) Raju observed that a watch loses 5 sec in one hr which was set right at 7 am. What time will it show at 2 pm on the same day?

$$\rightarrow 7 \text{ am} \rightarrow 2 \text{ pm (7 hrs)}$$

$$1 \text{ hr} \rightarrow 5 \text{ sec (loss)}$$

$$7 \text{ hrs} \rightarrow 7 \times 5 = 35 \text{ sec (loss)}$$

original wrong

2:00 pm 1:59:25
hr min sec

(Q4) A clock is set correctly at 1 pm. If it loses 3 min every hour, what time will the clock show when the correct time is 10 am the next day?

$$\rightarrow 1 \text{ pm} \rightarrow 10 \text{ am}$$

$$(1 \text{ pm} - 1 \text{ am}) + 9 \text{ hrs}$$

$$12 \text{ hrs} + 9 \text{ hrs} = 21 \text{ hrs}$$

$$1 \text{ hrs} = 3 \text{ min (loss)}$$

$$21 \text{ hrs} = 21 \times 3 = 63 \text{ min}$$

original wrong

10:00 am 8:57 am

(Q5) A watch losses 5 sec every hr. The watch was set correctly on wednesday at 6:30 am. what time did it show at 6:30 am in the next wednesday?

$$\rightarrow 6:30 \text{ am} \longleftrightarrow 6:30 \text{ am} \quad 24 \text{ hrs} \rightarrow 24 \times 5 = 120 \text{ sec} \\ \text{Next wed (7 days)} \quad = 2 \text{ min (1 day loss)}$$

$$1 \text{ hr} \rightarrow 5 \text{ sec}$$

In 7 days,

$$2 \text{ min} \times 7 = 14 \text{ min (loss)}$$

original wrong

6:30 am 6:16 am

(Q6) A watch gains 5 sec in 3 min was set right at 6:00 am. What time will it shows at 8:00 pm on the same day?

$$\rightarrow 6:00 \text{ am} \longleftrightarrow 8:00 \text{ pm} \quad (\underbrace{6:00 \text{ am} \text{ to } 6:00 \text{ pm}}_{12 \text{ hrs}} + 2 \text{ hrs})$$

$$\begin{array}{r} 5 \text{ sec} \rightarrow 3 \text{ min} \\ \times 20 \qquad \qquad \times 20 \\ \hline 100 \text{ sec} \rightarrow 1 \text{ hr} \end{array}$$

$$\begin{array}{r} 12 \text{ hrs} \\ = 4 \text{ hrs} \\ \hline \end{array}$$

Convert to 1 hrs

$$\begin{array}{r} 23 \\ 6 \sqrt{140} \\ \underline{-12} \\ 18 \\ \underline{\times 2} \\ 23 \frac{2}{6} \text{ min} \end{array}$$

$$23 \text{ min } \frac{2}{6} \times 60$$

$$23 \text{ min } 2 \text{ sec}$$

total gain

original wrong

8:00 pm 8:23:20
hr min sec

CALENDAR

2019 → Ordinary year: (365 days)

* last two digit Not divisible by 4

* feb 28 days

* 52 weeks + 1 odd day

2020 → Leap year: (366 days)

* last two digit divisible by 4

* feb 29 days

* 52 weeks + 2 odd days

* If century year divide by 400 e.g. 400, 1600, 2000

Odd days

0 - Sunday

1 - Mon

2 - Tue

3 - Wed

4 - Thurs

5 - Fri

6 - Sat

(e.g.) 10 days → 1 week + 3 odd days
(7 days)

16 days → 2 weeks + 2 odd days

21 days → 3 weeks + 0 odd day

(e.g.) $\frac{20}{4} = 5$

multiple of 4

$$\begin{array}{r} 0001 - 0020 \\ \hline 5 (\text{Leap Year}) + 15 (\text{ordinary years}) \end{array}$$

for century

0 - 100 leap yrs
 $\hookrightarrow (0-99) + (100)$

$\frac{100}{400} \Rightarrow 100 (0.4)$
24 (leap years) + 100 (ordinary year)

$\Rightarrow 24(L.Y) + 76(O.Y)$

$\Rightarrow 24 \times 2 \text{ odd day} + 76 \times 1 \text{ odd day}$

$\Rightarrow 48 + 76$

$\Rightarrow 124 \text{ odd day}$

$\hookrightarrow \div 7$

$\Rightarrow 17 \text{ weeks} + 5 \text{ odd day}$

Friday

$\therefore 31^{\text{st}} \text{ Dec } 2020$

Thursday

Note First 100 years \rightarrow 5 odd days
 200 years $\rightarrow 5 \times 2 \rightarrow$ 10 odd days
 $\rightarrow 1 \text{ week} + 3 \frac{\text{odd days}}{\text{wednesday}}$

300 years $\rightarrow 5 \times 3 \Rightarrow 15 \text{ odd days}$
 $\rightarrow 2 \text{ weeks} + 1 \frac{\text{odd day}}{\text{monday}}$

400 years $\rightarrow (5 \times 4 + 1) \Rightarrow 21 \text{ odd days}$
 bcz $\frac{400}{4} \rightarrow 3 \text{ weeks} + 0 \frac{\text{odd day}}{\text{sunday}}$
 \therefore we add 1 yr extra $\Rightarrow \frac{31^{\text{st}} \text{ Dec 400}}{\text{Sunday}}$

$$600 \text{ year} \rightarrow 400 \text{ yr} + 200 \text{ yr}$$

$$\downarrow$$

$$0(\text{odd day}) + 3(\text{odd day})$$

$$\Rightarrow 3 \frac{\text{odd day}}{\text{wed}}$$

$$700 \text{ year} \rightarrow 400 \text{ yr} + 300 \text{ yr}$$

$$\downarrow$$

$$0(\text{odd day}) + 1(\text{odd day})$$

$$\Rightarrow 1 \frac{\text{odd day}}{\text{monday}}$$

Important note

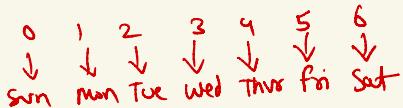
- ① $\frac{_}{4} \Rightarrow$ To find Leap year / ordinary year
- ② $\frac{_}{400} \Rightarrow$ Century year
- ③ $\frac{_}{7} \Rightarrow$ week + odd days
- ④ $0 - 6$ (odd days)
 \downarrow
Sunday

Day of the week (Non Leap year)

Jan	- 31	- 3 (odd day)
Feb	- 28	- 0
Mar	- 31	- 3
Apr	- 30	- 2
May	- 31	- 3
June	- 30	- 2
July	- 31	- 3
Aug	- 31	- 3
Sep	- 30	- 2
Oct	- 31	- 3
Nov	- 30	- 2
Dec	- 31	- 3

first
100 yr - 5 odd day
first
200 yr - 3 odd day
first
300 yr - 1 odd day
first
400 yr - 0 odd day

Odd day chart



31 days \rightarrow 3 odd day
30 days \rightarrow 2 odd day
28 days \rightarrow 0 odd day
29 days \rightarrow 1 odd day

① Which day of the week was 12 feb 1941?

\rightarrow 12 feb 1941
 $\frac{1941}{4} \rightarrow 485 \text{ d. } (\text{ordinary year})$

$$\begin{aligned}
 \frac{40}{4} &\Rightarrow 10 \text{ leap years} \\
 1000 + 300 + 40 + \text{Jan} + 12 \text{ feb} & \\
 \downarrow & \downarrow \quad \downarrow \quad \downarrow \\
 0 \text{ (odd day)} + 1 \text{ (odd day)} + & \quad + 3 \text{ (odd day)} + 12 \text{ days} \\
 (10 \text{ leap years} + 30 \text{ ordinary years}) & \\
 0 + 1 + (10 \times 2 \text{ (odd day)} + 30 \times 1 \text{ (odd day)}) + 3 + 12 & \quad (10 \text{ leap years} - 2 \text{ odd day}) \\
 1 + 50 + 3 + 12 & \quad (30 \text{ ordinary years} - 1 \text{ odd day}) \\
 50 + 16 & \\
 = 66 \text{ odd days} & \\
 \frac{66}{7} \rightarrow 9 \text{ weeks} + 3 \text{ odd days} & \quad \text{wednesday}
 \end{aligned}$$

② Which day of the week was 18 April 1963?

$$\rightarrow \frac{62}{4} = 15$$

(Leap year - 2 odd days)
(ordinary year - 1 odd day)

$$18 \text{ Apr } 1963 \quad \frac{\overline{2}}{4} \alpha \text{ (ordinary year)}$$

odd day $1600 + 300 + 62 \text{ yrs} + (\text{Jan - Mar}) + 18 \text{ days}$

$$0 + 1 + \left\lfloor \frac{62}{4} \right\rfloor + (3 + 0 + 3) + 18$$

$$(15 \times 1 \text{ y.} + 47 \times 0.5)$$

$$0 + 1 (30 + 47) + 6 + 18$$

$$77 + 6 + 18 + 1$$

$$\frac{102}{7} \rightarrow 14 \text{ week} + \frac{4 \text{ odd day}}{\hookrightarrow \text{ Thurs}}$$

③ Which day of the week was 21 July 1630?

$$\rightarrow 21 \text{ July } 1630$$

$$1600 + 29 \text{ yrs} + (\text{Jan - June}) + 21 \text{ days} \quad \frac{29}{4} = 7$$

odd days $0 + (7 \times 1 \text{ y.} + 22 \times 0.5) + (3 + 0 + 3 + 2 + 3 + 2) + 21 \text{ day}$

$$0 + (14 + 22) + 16 + 21$$

$$= 73 \text{ odd days}$$

$$\frac{\overline{7}}{7} \rightarrow 10 \text{ week} + \frac{3 \text{ odd days}}{\hookrightarrow \text{ wed}}$$

④ Which day of the week was 2 May 1921?

$$\rightarrow 2 \text{ May } 1921 \quad \frac{20}{4} = 5$$

$\frac{\overline{5}}{4} \alpha \text{ (ordinary years)}$

$$1600 + 300 + 20 \text{ yrs} + (\text{Jan - Apr}) + 2 \text{ days}$$

odd days $0 + 1 + (5 \times 1 \text{ y.} + 15 \times 0.5) + (3 + 0 + 3 + 2) + 2$

$$1 + 10 + 15 + 10$$

$$= 36 \text{ odd day}$$

$$\frac{\overline{7}}{7} \rightarrow 5 \text{ week} + \frac{1 \text{ odd day}}{\hookrightarrow \text{ mon}}$$

⑤ Which day of the week was 2 Dec 1818
 \rightarrow 2 Dec 1818 $\frac{17}{4} = 4 \text{ L.Y.}$ $\frac{1}{4} \text{ of (ordinary year)}$

odd days

$$1600 + 200 + 17 \text{ yrs} + (6 \times 3 + 4 \times 2) + 2$$

$$0 + 3 + (4 \times \text{L.Y.} + 13 \times \text{o.y.}) + 28$$

$$0 + 3 + (8 + 13) + 28$$

52 odd days

$\frac{52}{7} \rightarrow 7 \text{ weeks} + 3 \text{ odd days}$

wed

Day of the Week (Leap Year)

① What day of the week was 12 March 1940?

12 Mar 1940 $\frac{39}{4} = 9 \text{ Leap years}$

$\frac{1}{4} \checkmark$
(leap year)

$$1600 + 300 + 39 + (\text{Jan-Feb}) + 12$$

$$0 + 1 + (9 \times \text{L.Y.} + 3 \times \text{o.y.}) + (3 + 1) + 12$$

$$1 + (18 + 30) + 3 + 1 + 12$$

65 odd days

$\frac{65}{7} \rightarrow 9 \text{ weeks} + 2 \text{ odd days}$

Tue

odd day chart

0	1	2	3	4	5	6
sun	mon	tue	wed	thu	fri	sat

31 days $\rightarrow 3 \text{ odd day}$

30 days $\rightarrow 2 \text{ odd day}$

28 days $\rightarrow 0 \text{ odd day}$

29 days $\rightarrow 1 \text{ odd day}$

(leap year - 2 odd day)

(ordinary year - 1 odd day)

② What day of the week was 18 April 1964?

\rightarrow 18 Apr 1964 $\frac{53}{4} = 13 \text{ Leap years}$

$\frac{1}{4} \checkmark$
(leap years)

$$1600 + 300 + 63 \text{ yrs} + (\text{Jan-Mar}) + 18$$

$$0 + 1 + (15 \times \text{L.Y.} + 48 \times \text{o.y.}) + (3 + 1 + 3) + 18$$

$$1 + (30 + 48) + 7 + 18$$

104 odd days

$\frac{104}{7} \rightarrow 14 \text{ weeks} + 6 \text{ days}$

Sat

③ What day of the week was 21 July 1632?

$$\rightarrow 21 \text{ July } 1632 \quad \frac{31}{4} = 7 \text{ leap year} \quad \frac{\div 4}{\checkmark \text{ (leap year)}}$$

$$1600 + 31 \cancel{yrs} + (\text{Jan-June}) + 21$$

odd days

$$0 + [7 \times L.Y. + 24 \times O.Y.] + [3 + 1 + 3 + 2 + 3 + 2] + 21$$

$$14 + 24 + 12 + 5 + 21$$

$$\frac{76}{7} \rightarrow 10 \text{ week} + 6 \frac{\text{odd day}}{\checkmark \text{ Sat}}$$

④ What day of the week was 2 May 1920?

$$\rightarrow 2 \text{ May } 1920 \quad \frac{19}{4} = 4 \text{ leap year} \quad \frac{\div 4}{\checkmark \text{ Leap year}}$$

$$1600 + 300 + 19 \cancel{yrs} + [\text{Jan-Apr}] + 2 \text{ May}$$

$$0 + 1 + [4 \times L.Y. + 15 \times O.Y.] + [3 + 1 + 3 + 2] + 2$$

$$1 + 8 + 15 + 9 + 2$$

$$\frac{35}{7} \rightarrow 5 \text{ week } 0 \frac{\text{odd days}}{\checkmark \text{ Sun}}$$

⑤ What day of the week was 2 Dec 1816?

$$\rightarrow 2 \text{ Dec } 1816 \quad \frac{15}{4} = 3 \text{ leap year} \quad \frac{\div 4}{\checkmark \text{ Leap year}}$$

$$1600 + 200 + 15 \cancel{yrs} + [\text{Jan-Nov}] + 2 \text{ Dec}$$

$$0 + 3 + [3 \times L.Y. + 12 \times O.Y.] + [6 \times 3 + 4 \times 2 + 1] + 2$$

$$3 + 6 + 12 + 18 + 8 + 1 + 2$$

$$\frac{50}{7} \rightarrow 7 \text{ week} + 1 \frac{\text{odd day}}{\checkmark \text{ Mon}}$$

on what dates

① on what dates of April 2001 did Wednesday fall?

→ 1 Apr 2001
 $\frac{4}{4} \rightarrow$ of ordinary year

$$1600 + 400 + (\text{Jan - March}) + 1 \\ 0 + 0 + (3 + 0 + 3) + 1 \\ = 7 \\ \frac{7}{7} \rightarrow \text{week} + 0 \frac{\text{odd day}}{\text{sun}}$$

$$1 \text{ Apr } 2001 \rightarrow \text{Sun} \\ 4 \text{ Apr } 2001 \rightarrow \text{Wed} \\ \downarrow 11 \quad \downarrow 18 \quad \downarrow 25 \\ \} \text{ wed}$$

Ans 4, 11, 18, 25

② on what dates of May 1945 did Saturday fall?

→ 1 May 1945
 $\frac{44}{4} = 11 \text{ (L.Y.)} \rightarrow$ of ordinary year

$$1600 + 300 + 44 \text{ yrs} + (\text{Jan - Apr}) + 1 \\ 0 + 1 + [11 \times L.Y. + 33 \times O.Y.] + [3 + 0 + 3 + 2] + 1 \\ 1 + [22 + 33] + [9] \\ 10 + 55 \\ \frac{65}{7} \rightarrow 9 \text{ week} + 2 \frac{\text{odd day}}{\text{Tue}}$$

$$1 \text{ May } 1945 \rightarrow \text{Tue} \\ \vdots \\ 5 \text{ May } 1945 \rightarrow \text{Sat} \\ \downarrow 12 \quad \downarrow 19 \quad \downarrow 26 \\ \} \text{ Sat}$$

Ans 5, 12, 19, 26

③ on what dates of Feb 1842 did Thursday fall?

→ 1 Feb 1842
 $\frac{41}{4} = 10 \text{ (L.Y.)} \rightarrow$ of (ordinary) years

$$1600 + 200 + 41 \text{ yrs} + (\text{Jan}) + 1 \\ 0 + 3 + [10 \times L.Y. + 31 \times O.Y.] + 3 + 1 \\ 3 + 20 + 31 + 4 \\ = 58 \text{ days} \\ \frac{58}{7} \rightarrow 8 \text{ week} 2 \frac{\text{odd days}}{\text{Tue}}$$

$$1 \text{ Feb } 1842 \rightarrow \text{Tue} \\ \vdots \\ 3 \text{ Feb } 1842 \rightarrow \text{Thur} \\ + 7 \rightarrow 10 \\ + 7 \rightarrow 17 \\ + 7 \rightarrow 24$$

Ans 3, 10, 17, 24

④ On what date of June 1764 did Monday fall?

$$\rightarrow 1 \text{ June } 1764 \quad \frac{63}{4} = 15 \text{ leap years}$$

$\div 4 \hookrightarrow \checkmark (\text{Leap year})$

$$1600 + 100 + \frac{63}{4} \leftarrow 48 + (\text{Jan - May}) + 1 \text{ June}$$

$$0 + 5 + [15 \times \text{leap years} + 48 \times 0.4] + [3 + 1 + 3 + 2 + 3] + 1$$

$$5 + 30 + 48 + 12 + 1$$

$$78 + 18$$

96 days

$$\frac{\div 7}{\hookrightarrow} 13 \text{ weeks} + 5 \text{ extra days}$$

$\hookrightarrow \text{Fri}$

1 June 1764 \rightarrow Friday

$$\begin{array}{rcl} & & ! \\ & & 4 \text{ June } 1764 \rightarrow \text{mon} \\ +7 & \hookrightarrow & 11 \\ +7 & \hookrightarrow & 18 \\ +7 & \hookrightarrow & 25 \end{array}$$

Ans 4, 11, 18, 25