Aptitude Practice

Permutation

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

0! = 1
1! = 1
2! = 2
3! = 6
4! = 24
5! = 120
6! = 720
7! = 5040
8! = 40320

Permutation → all possible
Combination → required possible

① ② ③

Combination

1-2
2-3
1-3
} ③

Permutation

1-2
2-1
2-3
3-2
1-3
3-1
} ⑥

Total ways the word Arranged

(i) Non-Repeated letters

CAT (1 2 3) ⇒ 3! ⇒ 6 ways

(ii) Repeated letters

SISTER (1 2 3 4 5 6) ⇒ $\frac{6!}{2!} = \frac{720}{2}$ ⇒ 360 ways

LETTER (1 2 3 4 5 6) = $\frac{6!}{2! \times 2!} = \frac{\cancel{720}^{\cancel{360}\ 180}}{\cancel{2 \times 2}}$ ⇒ 180 ways

Vowels always comes together (i) Non Repeated letter

JUDGE ⇒ _J_D_G_(UE) ⇒ 4! × 2! ⇒ 24 × 2 = 48
UE (or) EU ⇒ 2

MACHINE ⇒ _M_C_H_N_(AIE) ⇒ 5! × 3! ⇒ 120 × 6 = 720
AIE (or) EIA ⇒ 3

EDUCATION ⇒ _D_C_T_N_(AIOUE) ⇒ 5! × 5! ⇒ 120 × 120 = 14400
AIOUE ⇒ 5!

Vowels always comes together (ii) Repeated letter

SISTER ⇒ _S_S_T_R_(I E)

two 'S' Repeated → $\frac{5! \times 2!}{2!} = \frac{120 \times 2}{2} = 120$

ANIMATION ⇒ _N_M_T_N_ (A I A I O)

$\frac{5! \times 5!}{2! \times 2! \times 2!} = \frac{120 \times 120}{2 \times 2 \times 2} = 1800$

CORPORATION ⇒ _C_R_P_R_T_N_(O O A I O)

⇒ $\frac{7! \times 5!}{2! \times 3!}$ ⇒ $\frac{5040 \times 120}{2 \times 6}$ ⇒ 5040

Consonants always comes together

JUDGE ⇒ _ U _ E _ (J D G)

⇒ 3! × 3! ⇒ 6 × 6 ⇒ 36

BANKER ⇒ _ A _ E _ (B N K R)

3! × 4! ⇒ 6 × 24 ⇒ 144

SISTER ⇒ _ I _ E _ (S S T R)

$\frac{3! \times 4!}{2!}$ ⇒ $\frac{6 \times 24}{2} = 72$

Some letters Comes together

① CRYSTAL

'AY' Comes together?

_C_R_S_T_L_(A Y)

6! × 2! ⇒ 720 × 2

⇒ 1440

② EDUCATION

'UET' Comes together

_D_C_A_I_O_N(U E T)

7! × 3! ⇒ 5040 × 6

⇒ 30240

Vowels Never Comes together

① MACHINE

7! − MACHINE

Vowels comes together

MACHINE

M_C_H_N_(AIE)

5! × 3! ⇒ 120 × 6 = 720

5040 − 720 ⇒ Ans 4320

② SISTER

$\frac{6!}{2!}$ 360

Vowels comes together

SISTER

S_S_T_R_(IE)

$\frac{5! \times 2!}{2!} \Rightarrow \frac{120 \times 2}{2} \Rightarrow 120$

360 − 120 = 240

③ CORPORATION

$\frac{11!}{3! \times 2!}$

Vowels comes together

CORPORATION

C_R_P_R_T_N_(OOAIO)

$\frac{7! \times 5!}{2! \times 3!}$

$\frac{11!}{3! \times 2!} - \frac{7! \times 5!}{2! \times 3!}$

No two Vowels Comes together

formula $n p_r = \frac{n!}{(n-r)!}$ $[n \geq r]$

A B C D

_ _

4 letter in 2 place

$4 p_2 = \frac{4!}{(4-2)!}$

① BANKER

_B_N_K_R_ (AE)

$5 p_2 \times 4!$

② SISTER

_S_S_T_R (IE)

$\frac{5 p_2 \times 4!}{2!}$

ODD position / Even position

① How many ways the word 'DETAIL' be arranged in such a way that vowels occupy only ODD position?

→ D E T A I L (EAI)

1 2 3 4 5 6

$3 p_3 \times 3! \Rightarrow 3! \times 3! \Rightarrow 36$

② How many ways the word 'MACHINE' be arranged in such a way that vowels occupy only ODD position?

→ M A C H I N E (AIE)

1 2 3 4 5 6 7

$4 p_3 \times 4! \Rightarrow \frac{4!}{1!} \times 4! \Rightarrow 24 \times 24 \Rightarrow 576$

③ How many ways the word 'POUNDING' be arranged in such a way that vowels occupy only ODD position?

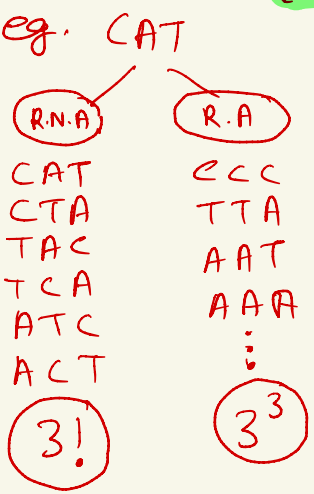
→ P O U N D I N G (OUI)

1 2 3 4 5 6 7 8

$\frac{4 p_3 \times 5!}{2!}$

(i) Repetition Allowed (R.A) (ii) Repetition not allowed (R.N.A)

eg. CAT



(a) CRYSTAL

(b) PIRATES

(c) BOOK

(D) OFFICERS

R.N.A	R.A
$7!$	7^7
$7!$	7^7
$\frac{4!}{2!}$	$\frac{4^4}{2!}$
$\frac{8!}{2!}$	$\frac{8^8}{2!}$

Miscellaneous

How many ways the word 'PLAYGROUND' can be arranged such that,

AOU ← 3 vowels

→ (i) If the word starts with letter 'y'?

Y _ _ _ _ _ _ _ _ _

1 2 3 4 5 6 7 8 9 10

$1 \times 9! = \boxed{9!}$

(ii) If the word start with Letter 'y' and ends at 'G'?

Y _ _ _ _ _ _ _ _ G

1 2 3 4 5 6 7 8 9 10

$1 \times 8! \times 1 = \boxed{8!}$

(iii) The word should ends with a vowel?

_ _ _ _ _ _ _ _ _ (A O U)

1 2 3 4 5 6 7 8 9 10

$\boxed{3 \times 9!}$

(iv) The word start with a vowel and ends with a consonent?

(A O U) _ _ _ _ _ _ _ _ _ (PLGRND)

1 2 3 4 5 6 7 8 9 10

(3) (7)

$\boxed{3 \times 8! \times 7}$

(v) No two vowels comes together?

PLAYGROUND

_ P _ L _ Y _ G _ R _ N _ D _ (OAU)

$\boxed{{}^{8}P_{3} \times 7!}$

(vi) Vowels occupy odd positions? (AOU)

1 2 3 4 5 6 7 8 9 10

PLYGRND

7!

$\boxed{{}^{5}P_{3} \times 7!}$

Problems Based on Numbers

How many ways 3 digit / 4 digit / 5 digit ... can be formed?

(a) Non-zero Numbers.

(i) 1, 2, 8, 7

_ _ _ _ $= 4!$

(ii) 2, 1, 4, 6, 9

_ _ _ _ _ $= 5!$

(iii) 2, 3, 4, 2

_ _ _ _ $= \frac{4!}{2!}$

(iv) 1, 3, 2, 4, 9, 1

_ _ _ _ _ _ $= \frac{6!}{2!}$

(v) 9, 3, 7, 4, 1

_ _ _ $= {}^5P_3$

4, 2, 9, 6 $= 4! = 24$

How many ④ digit numbers less than 4K ⇒ 6 numbers

$\underline{2}\ \underline{3!}\ \underline{\ }\ \underline{\ } = 6$

How many ④ digit numbers greater than 4K ⇒ 18 numbers

$\underline{4}\ \underline{3!}\ \underline{\ }\ \underline{\ } = 6$

$\underline{6}\ \underline{3!}\ \underline{\ }\ \underline{\ } = 6$

$\underline{9}\ \underline{3!}\ \underline{\ }\ \underline{\ } = 6$

24

How many ways 3 digit / 4 digit / 5 digit ... can be formed?

(b) Number Zero

How many ways 4 digit no. is formed by using the number

2, 0, 4, 9

$\underline{2}\ \underline{3!}\ \underline{\ }\ \underline{\ } = 6$

$\underline{4}\ \underline{3!}\ \underline{\ }\ \underline{\ } = 6$

$\underline{9}\ \underline{3!}\ \underline{\ }\ \underline{\ } = 6$

$\underline{0}\ \underline{\ }\ \underline{\ }\ \underline{\ }$ ← this will not form 4 digit numbers

18 = '4' digit number can form

Combination ① ② ③

Combination
1-2
2-3
1-3
} ③

Permutation
1-2
2-1
2-3
3-2
1-3
3-1
} ⑥

Permutation → all possible
Combination → required possible

Extra note ${}^{15}C_{11} \Rightarrow {}^{15}C_{4} \Rightarrow \frac{15\times14\times13\times12}{4\times3\times2\times1}$

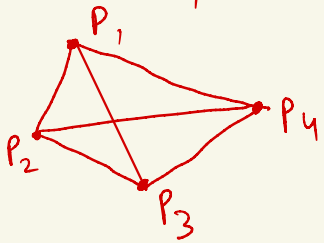
$${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

n → total no. of elements
r → target element to pick

① ② ③ Balls

$${}^{3}C_{2} = \frac{3\times2}{1\times2}$$

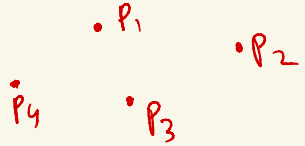
* How many △ can form by using 4 points ?



Note △ form with 3 points

${}^{4}C_{3} \Rightarrow \frac{4\times3\times2}{1\times2\times3} \Rightarrow 4$

* using 4 points how many straight line can be drawn ?



Note • —— • two points req. to draw straight line.

${}^{4}C_{2} \Rightarrow \frac{4\times3}{2\times1} \Rightarrow 6$

* ${}^{9}C_{4}$ — 9 → Total, 4 → Selection

Basic Que

$6! = 6\times5\times4\times3\times2\times1$

$(n-2)! = (n-2)(n-3)(n-4)\ldots$

$(n-8)! = (n-8)(n-9)(n-10)\ldots$

$^{14}C_2 \Leftrightarrow C(14,2)$

① If $C(n,7) = C(n,5)$, find n?

→ Tra. Method

$$^nC_7 = {}^nC_5$$

$$\frac{n!}{7!(n-7)!} = \frac{n!}{5!(n-5)!}$$

$$5!(n-5)! = 7!(n-7)!$$

$$5!(n-5)(n-6)(n-7)! = 7\times6\times5!(n-7)!$$

$$(n-5)(n-6) = 42$$

$$n^2 - 6n - 5n + 30 = 42$$

$$n^2 - 11n - 12 = 0$$

$$n^2 - 12n + n - 12 = 0$$

$$n(n-12) + 1(n-12) = 0$$

-12 : $-12, +1$

$$n = 12, \quad n \neq -1$$

Short cut

$$^nC_7 = {}^nC_5$$

Note ① Base should be same (or) add base value no. resultant should be top number

$$x = y$$

$$7 + 5 = n$$

$$(x) + (y) = n$$

$$n = 12$$

② If $C(n,8) = C(n,6)$, find $C(n,2)$

→ ${}^{n}C_{8} = {}^{n}C_{6}$ (x) (y)

$x + y = n$

$n = 8 + 6$

$\boxed{n = 14}$

$C(n,2) \Rightarrow {}^{14}C_{2} \Rightarrow \frac{14 \times 13}{1 \times 2}$

$\Rightarrow 91$

③ If ${}^{18}C_{r} = {}^{18}C_{r+2}$, find ${}^{r}C_{5}$

→ $x + y = n$

$r + r + 2 = 18$

$2r + 2 = 18$

$2r = 16$

$\boxed{r = 8}$

find ${}^{r}C_{5}$

${}^{8}C_{5} \Rightarrow \frac{8 \times 7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4 \times 5}$

$\Rightarrow 56$

④ If ${}^{2n}C_{r} = {}^{2n}C_{r+2}$, find r

→ $r + r + 2 = 2n$

$2r + 2 = 2n$

$2r = 2n - 2$

$\boxed{r = n - 1}$

⑤ find $\sum_{r=1}^{5} C(5,r)$

Σ → total (or) collection of terms

or → +, and → ×

→ $C(5,1)$ (or) $C(5,2)$ (or) $C(5,3)$ (or) $C(5,4)$ (or) $C(5,5)$

${}^{5}C_{1} + {}^{5}C_{2} + {}^{5}C_{3} + {}^{5}C_{4} + {}^{5}C_{5}$

$5 + \left[\frac{5 \times 4}{1 \times 2}\right] + \left[\frac{5 \times 4 \times 3}{1 \times 2 \times 3}\right] + 5 + 1$

$5 + 10 + 10 + 5 + 1 \Rightarrow 31$

Based on Committee

and → ×
or → +

Set ①

Different committees are to be made as per the requirement given in each question. In how many different ways can it be done?
8 Students out of which 5 are doctors and 3 are scientist.

Que① A committee of 4 in which 3 are doctors and 1 is scientist?

→ 3 doctor and 1 Scientist

$${}^5C_3 \times {}^3C_1 \Rightarrow \frac{5\times\cancel{4}^{2}\times\cancel{3}}{1\times\cancel{2}\times\cancel{3}} \times 3 \Rightarrow 30$$

Que② A committee of 5 in which 3 are doctors

→ $${}^5C_3 \times {}^3C_2 \Rightarrow \frac{5\times\cancel{4}^{2}\times\cancel{3}}{1\times\cancel{2}\times\cancel{3}} \times \frac{3\times\cancel{2}}{1\times\cancel{2}} \Rightarrow 30$$

Que③ A committee of 2 in which there is no Doctors.

→ $${}^3C_2 \Rightarrow \frac{3\times\cancel{2}}{1\times\cancel{2}} \Rightarrow 3$$

Que④ A committee of 3 in which there is no Scientist.

→ $${}^5C_3 \Rightarrow \frac{5\times\cancel{4}^{2}\times\cancel{3}}{1\times\cancel{2}\times\cancel{3}} \Rightarrow 10$$

Que⑤ A committee of 2 in which either both are Doctors or both are scientist.

→ $${}^5C_2 + {}^3C_2 \Rightarrow \frac{5\times\cancel{4}^{2}}{1\times\cancel{2}} + \frac{3\times\cancel{2}}{1\times\cancel{2}} \Rightarrow 10+3 \Rightarrow 13$$

Que⑥ A committee of 3 in which at least 1 doctors are there

note: Atleast → min(1) doctor (or) 2 doctor (or) 3 doctor
↳ min ①-max ③

→

$$\{{}^5C_1 \times {}^3C_2\} + \{{}^5C_2 \times {}^3C_1\} + \{{}^5C_3\}$$

$$\Rightarrow (5\times 3) + \left(\frac{5\times 4}{1\times 2}\times 3\right) + \left\{\frac{5\times 4\times 3}{1\times 2\times 3}\right\} \Rightarrow 55$$

Set 2

A committee 5 members is to be formed out of 3 Trainees 4 professors and 6 Research associates. In how many different ways can this be done if,

(Q1) The committee should have all 4 professors and 1 research associate or all 3 trainees and 2 professors?

→ Group (5) 3 Trainees 4 Prof. 6 Research

(4P and 1R) or (3T and 2 Prof)

$({}^4C_4 \times {}^6C_1) + ({}^3C_3 \times {}^4C_2)$

$(1 \times 6) + (1 \times 6) \Rightarrow$ 12

(Q2) The committee should have 2 trainees and 3 research associates?

→ 3T, 4P, 6R

${}^3C_2 \times {}^6C_3 \Rightarrow 3 \times \frac{6 \times 5 \times 4}{1 \times 2 \times 3} \Rightarrow$ 60

(Q3) The committee should have 2 trainees?

→ 3T, 4P, 6R

${}^3C_2 \times {}^{10}C_3 \Rightarrow 3 \times \frac{10 \times 9 \times 8}{1 \times 2 \times 3} \Rightarrow$ 360

(Q4) The committee should not contain any professors?

→ 3T, 4P, 6R

${}^9C_5 \Rightarrow \frac{9 \times 8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4 \times 5} \Rightarrow 14 \times 9 \Rightarrow$ 126

Set 3

Different committees are to be made as per the requirement given in each question. In how many different ways can it be done? 10 men and 8 women out of which 5 men are teachers, 3 men are doctors and 2 are Scientists. Among women, 3 are Teachers, 2 are Doctors, 2 researchers and 1 Lawyer.

(Q1) A committee of 5 in which 3 men and 2 women are there

→ 10 men, 8 women

$^{10}C_3$ and $^{8}C_2 \Rightarrow \frac{10 \times \overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}}}{1 \times \cancel{2} \times \cancel{3}} \times \frac{\overset{4}{\cancel{8}} \times 7}{1 \times \cancel{2}} \Rightarrow 120 \times 28 \Rightarrow$ **3360**

(Q2) A committee of 4 in which at least 2 women are there?

→ 10 men, 8 women

Atleast → min-max 2/3/4 women

2W (or) 3W (or) 4W

$(^{8}C_2 \times {}^{10}C_2) + (^{8}C_3 \times {}^{10}C_1) + (^{8}C_4) \Rightarrow \left(\frac{\overset{2}{\cancel{8}} \times 7}{1 \times \cancel{2}} \times \frac{10 \times 9}{1 \times \cancel{2}}\right) + \left(\frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times 10\right) + \left(\frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4}\right)$

$\Rightarrow (14 \times 9 \times 10) + 560 + 70$

$\Rightarrow 1260 + 560 + 70 \Rightarrow$ **1890**

(Q3) A committee of 2?

→ 10 men 8 women

$^{18}C_2 \Rightarrow \frac{\overset{9}{\cancel{18}} \times 17}{1 \times \cancel{2}} \Rightarrow$ **153**

(Q4) A committee of 3 in which there is no teacher and no doctor?

→ 10 men | −8 people eliminated → 2 men

8 women | −5 women eliminated → 3 women

$^{5}C_3 \Rightarrow \frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3}}{1 \times \cancel{2} \times \cancel{3}} \Rightarrow$ **10**

Miscellaneous

① In how many ways can 5 cricketers be selected from a group of 10?

→ ${}^{n}C_r$ → n → total, r → select

${}^{n}C_r \Rightarrow {}^{10}C_5 \Rightarrow \frac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5} \Rightarrow$ **252**

② A question paper has 2 parts, Part A and Part B each containing 10 questions. If the student has to choose 8 from Part A and 5 from Part B in how many ways can he choose the question?

and → ×
or → +

→ P − A and P − B

$n = 10$ $n = 10$

${}^{10}C_8$ × ${}^{10}C_5 \Rightarrow {}^{10}C_2 \times {}^{10}C_5 \Rightarrow \frac{10 \times 9}{1 \times 2} \times \frac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5}$

$\Rightarrow 45 \times 252 \Rightarrow$ **11340**

③ Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour?

→ 6 Red, 5 White, 5 Blue → (9) balls

↓ 3 ↓ 3 ↓ 3

${}^{6}C_3 \times {}^{5}C_3 \times {}^{5}C_3 = \frac{6 \times 5 \times 4}{1 \times 2 \times 3} \times \frac{5 \times 4 \times 3}{1 \times 2 \times 3} \times \frac{5 \times 4 \times 3}{1 \times 2 \times 3}$

$= 20 \times 10 \times 10 \Rightarrow$ **2000**

④ In an examination, A has to select 4 questions from each part. There are 6, 7 and 8 questions in Part 1, Part 2, Part 3 resp. what is the number of possible combinations in which A can choose the questions?

→ 6 7 8

${}^{6}C_4 \times {}^{7}C_4 \times {}^{8}C_4 \Rightarrow \frac{6 \times 5}{1 \times 2} \times \frac{7 \times 6 \times 5}{1 \times 2 \times 3} \times \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} \Rightarrow 15 \times 35 \times 70$

$\Rightarrow$ **36750**

⑤ In how many ways can cricket team of 11 players be selected out of 16 players, if 2 particular player are always to be included?

→ Included / Excluded

16 players → 11 player (select) → 9 players (select)

↳ 2 player always included

∴ 14 players

$^{14}C_9 \Rightarrow {}^{14}C_5 \Rightarrow \frac{14\times13\times12\times11\times10}{1\times2\times3\times4\times5}$

$\Rightarrow 14\times13\times11$

$\Rightarrow 182\times11 \Rightarrow 2002$

⑥ In how many ways can a cricket team of 11 players be selected out of 16 players of 1 particular players is to be excluded?

→ Included / Excluded

16 player → 11 player

↳ 1 is coach

∴ 15 player

$^{15}C_{11} \Rightarrow {}^{15}C_4 \Rightarrow \frac{15\times14\times13\times12}{1\times2\times3\times4}$

$\Rightarrow 1365$

⑦ In how many ways can a cricket team of 11 players be selected out of 16 players if 2 particular players are to be included and 1 particular player is to be excluded?

→ Included / Excluded

16 player → 11 player → only 9 player

↳ 2 player already selected

∴ 14 player

↳ ① is eliminated

13 player

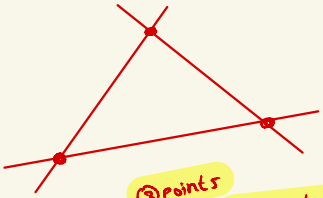
$^{13}C_9 \Rightarrow {}^{13}C_4$

$\Rightarrow \frac{13\times12\times11\times10}{1\times2\times3\times4}$

$\Rightarrow 143\times5$

$\Rightarrow 715$

Non-Collinear Points



③ points

② points to draw line

$${}^{n}C_{r} \Rightarrow {}^{3}C_{2} \Rightarrow \frac{3 \times 2}{1 \times 2} \Rightarrow 3$$

3 straight line we can draw

Collinear Points

(lies exactly on same line)

(or)

Same plane

only one straight line

total $\Rightarrow {}^{n}C_{r} - {}^{m}C_{r} + 1$ (m ← no. of collinear pt.)

$\Rightarrow {}^{3}C_{2} - {}^{3}C_{2} + 1$

$\Rightarrow 1$

① In a plane there are 16 non collinear points. find the number of straight lines formed?

$\Rightarrow$ 16 points $\quad {}^{n}C_{r} \Rightarrow {}^{16}C_{2} \Rightarrow \frac{16^{8} \times 15}{1 \times 2} \Rightarrow 120$

② There are 14 points in a plane out of which 4 points are collinear. find the number of straight line formed?

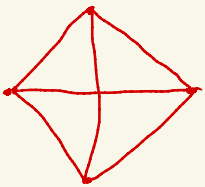
$\Rightarrow$ 14 points, 4 points collinear

$${}^{14}C_{2} - {}^{4}C_{2} + 1 \Rightarrow \frac{14^{7} \times 13}{1 \times 2} - \frac{4^{2} \times 3}{1 \times 2} + 1$$

$$\Rightarrow 91 - 6 + 1$$

$$\Rightarrow 86$$

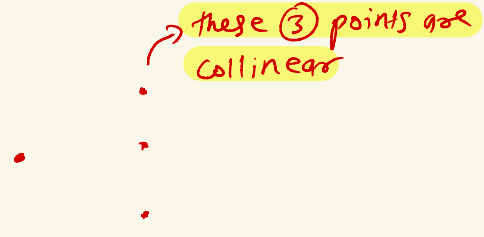
Triangle



④ points

③ req. to form triangle

$\Rightarrow {}^{n}C_{r} \Rightarrow {}^{4}C_{3} \Rightarrow$ (4)



$\Rightarrow {}^{n}C_{r} - {}^{m}C_{r}$

$\Rightarrow {}^{4}C_{3} - {}^{3}C_{3}$

① There are 14 points in a plane out of which 4 points are collinear. find the number of Triangle formed?

→ 14 points 4 points

${}^{n}C_{r} - {}^{m}C_{r}$

${}^{14}C_{3} - {}^{4}C_{3}$

$\frac{14 \times 13 \times \cancel{12}^{2}}{1 \times \cancel{2} \times \cancel{3}} - 4$

$\Rightarrow (14 \times 26) - 4$

$\Rightarrow 364 - 4$

$\Rightarrow$ **360**