SESSIONAL PAPERS

ARANJAY STATIONARY

Assignment :- 1

Question 1 :- Find a root of the eqn $x^3 - x = 0$ b/w 1 and 2 to four place of decimal by bisection method.

Sol. let $f(x) = x^3 - x - 4 = 0$

Initialisation :- At $x = 1$, $f(1) = 1^3 - 1 - 4 = -4$ and at $x = 2$, $f(2) = 2^3 - 2 - 4 = 2$. Clearly $f(1) \cdot f(2) = -8 < 0$. Therefore, the root of $f(x) = 0$ lies b/w 1 and 2.

first iteration :

$$x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$$

$f(1) = -4$, $f(2) = 2$, $f(x_1) = (1.5)^3 - 1.5 - 4 = -2.125$

Root lies b/w (1.5) & 2

Second iteration

$$x_2 = \frac{1.5+2}{2} = 1.75$$

$f(1.75) = (1.75)^3 - 1.75 - 4 = -0.39062$

Root lies b/w 1.75 & 2.

Third iteration

$$x_3 = \frac{1.75+2}{2} = 1.875$$

0.71679

$f(1.875) = (1.875)^3 - 1.875 - 4 =$ ~~0.14184~~

roots lies b/w (1.75, 1.875)

fourth iteration :- $x_4 = \frac{1.75 + 1.875}{2} = 1.8125$

$f(1.8125) = (1.8125)^3 - 1.8125 - 4 = 0.14184$

root is lies b/w (1.75, 1.8125)

fifth iteration :

$x_5 = \frac{1.75 + 1.8125}{2} = 1.78125$

$f(1.78125) = (1.78125)^3 - 1.78125 - 4 = 0.12960$

Root is lies b/w (1.78125, 1.8125)

Sixth itreation :

$x_6 = \frac{1.7128 + 1.8125}{2} = 1.79687$

$f(1.79687) = (1.79687)^3 - 1.79687 - 4 = 0.00477$

Root is lies b/w (1.78125, 1.79687).

Seventh iteration

$x_7 = \frac{1.78125 + 1.79687}{2} = 1.78906$

$f(1.78906) = (1.78906)^3 - 1.78906 - 4 = 0.0672.$

Root is lies b/w (1.78906, ~~1.7996~~ 1.79687

Eighth iteration :- $x_8 = \frac{1.78906 + 1.079296}{2} = 1.79296$

$f(1.79296) = (1.79296)^3 - 1.79296 - 4 = -0.02913$

Root's lies b/w (1.79296 & 1.79687)

Ninth iteration :- $x_9 = \frac{1.79296 + 1.79687}{2} = 1.79491$

$f(1.79491) = (1.79491)^3 - 1.79491 - 4 = -0.0251$

Root's lies b/w (1.79491, 1.79687)

Tenth iteration :- $x_{10} = \frac{1.79491 + 1.79491}{2} = 1.79589$

$f(1.79589) = (1.79589)^3 - 1.79589 - 4 = -0.00375$

Root's lies b/w (1.79589 & 1.79687)

Eleventh iteration :-

$x_{11} = \cancel{1.79638} \; \frac{1.79589 + 1.79687}{2} = 1.79638$

$f(1.79638) = (1.79638)^3 - 1.79638 - 4 = \cancel{1.0795}$ 0.0005

The root's b/w (1.79589, 1.79638)

Here, we see that the digit in the first three places of decimal are the same in the interval (1.79589, 1.79638) Therefore, the value of the root to three place of decimal is 1.796.

Ques-2. Find a real of the equation $x^3 - 9x + 1 = 0$ by the method of false position.

Solve :-

Initialisation: let = $f(x) = x^3 - 9x + 1$

$f(2) = -9, \quad f(3) = 1 \Rightarrow f(2) \cdot f(3) < 0$

root lies b/w 2 and 3.

Taking $x_0 = 2$, $x_1 = 3$ so that $f(x_0) = -9$ & $f(x_1) = 1$

By method of false position, first approximation:

$$x_2 = x_0 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} f(x_0)$$

$$x_2 = 2 - \frac{(3-2)}{1+9}(-9) = 2 + \frac{9}{10} = 2.9$$

Now

I iteration $f(x_2) = f(2.9) = (2.9)^3 - 9(2.9) + 1 = -0.711$

root lies b/w 2.9 and 3

~~$x_0 = 2.9$ $x_1 = 3$ $f(x_0) = -0.711$ $f(x_1) = 1$~~

$$x_3 = 2.9 = \frac{3 - 2.9}{1 + 0.711}(-0.711) = 2.9416$$

~~$f(x_3) = f(2.9416) = -0.0207$~~

Root lies b/w 2.9416 & 3

II Iteration $x_0 = 2.9416$, $x_1 = 3$ $f(x_0) = -0.0207$,

$f(x_1) = 1$

$$x_4 = 2.9416 - \frac{0.0584}{1.0207}(-0.0207) = 2.9428$$

$f(x_4) = -0.0003$

Root's lies b/w 2.9428 and 3.

Taking $x_0 = 2.9428$, $x_1 = 3$

$f(x_0)$

$$x_5 = 2.9428 - \frac{0.0572}{1.0003}(-0.0003) = 2.9428$$

Clearly $x_4 = x_5 = 2.9428$

Hence the root $x_4 = x_5$ 2.9428

Hence the real root is 2.942 correct to three decimal places.

Question-3 Find the real of $x^4 - x - 10 = 0$ by Newton-Raphson method.

Solution:-

$f(x) = x^4 - x - 10 = 0$

$f(0) = -10 = (-ve)$

$f(1) = 1 - 1 - 10 = -10 (-ve)$

$f(2) = 16 - 2 - 10 = 4 (+ve)$

$f(1)$ and $f(2)$ are opposite sign

root's of eq^n (1) lies b/w 1 and 2

Taking initial approximation $x_0 = 1.5$

By Newton-Raphson method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots$$

$$f(x) = x^4 - x - 10$$

$$f'(x) = 4x^3 - 1.$$

Hence ② becomes

$$x_{n+1} = x_n - \frac{(x_n^4 - x_n - 10)}{4x_n^3 - 1} = \frac{4x_n^4 - x_n - x_n^4 + x_n + 10}{4x_n^3 - 1}$$

$$x_{n+1} = \frac{3x_n^4 + 10}{4x_n^3 - 1} \quad \text{--- ③}$$

$n = 0$ in ③,

I approximation,

$$x_1 = \frac{3x_0^4 + 10}{4x_0^3 - 1} = \frac{3(1.5)^4 + 10}{4(1.5)^3 - 1} = \frac{25.1875}{12.5} = 2.015$$

$n = 1$

II approximation, $x_2 = \frac{3x_1^4 + 10}{4x_1^3 - 1} = \frac{3(2.015)^4 + 10}{4(2.015)^3 - 1} = 1.874$

$n = 2$

III approximation, $x_3 = \frac{3x_2^4 + 10}{4x_2^3 - 1} = \frac{3(1.874)^4 + 10}{4(1.874)^3 - 1} = 1.856$

$n = 3$

IV approximation,

$$x_4 = \frac{3(x_3)^4 + 10}{4(x_3)^3 - 1} = \frac{3(1.874)^4 + 10}{4(1.874)^3 - 1} = 1.856$$

Since $x_3 = x_4 = 1.856$ correct to 3 decimal places.

Question 4 :- Evaluate $\Delta^2 x^3$, when $h=1$

Solve :-
$$\Delta^2 x^3 = \Delta[(x+1)^3 - x^3] = \Delta[x^3 + 3x^2 + 3x + 1 - x^3]$$
$$= \Delta(3x^2 + 3x + 1$$
$$= 3\Delta x^2 + 3\Delta x + \Delta 1$$
$$[3(x+1)^2 - 3x^2] + [3(x+1) - 3x] + 0$$
$$3x^2 + 6x + 3 - 3x^2 + 3x + 3 - 3x$$
$$6x + 6$$
$$6(x+1)$$ Ans

Question-5 Given $u_2 = 13$, $u_3 = 28$, $u_4 = 49$, $u_5 = 76$
find $\Delta^2 u_2$ and $\Delta^3 u_2$.

Solve :-

(i) $\Delta^2 u_2 = (E-1)^2 u_2$
$$= (E^2 - 2E + 1) u_2$$
$$E^2 u_2 - 2E u_2 + u_2$$
$$u_4 - 2u_3 + u_2$$
$$49 - 2(28) + 13$$
$$= 6$$

(ii) $\Delta^3 u_2 = (E-1)^3 u_2$
$$= (E^3 - 3E^2 + 3E - 1) u_2$$
$$E^3 u_2 - 3E^2 u_2 + 3E u_2 - u_2$$
$$u_5 - 3u_4 + 3u_3 - u_2$$
$$76 - 3(49) + 3(28) - 13$$
$$= 0$$ Ans

Question 6:- Represent the function:
$f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ and its successive differences in, factorial notation.

Solve:-

Let $f(x) = Ax^{[4]} + Bx^{[3]} + Cx^{[2]} + Dx^{[1]} + E$ in factorial notation. — (1)

$$x^4 - 12x^3 + 24x^2 - 30x + 9 = Ax(x-1)(x-2)(x-3) + Bx(x-1)(x-2) + Cx(x-1) + Dx + E \quad \text{— (2)}$$

Putting $x=0$ in eqn (2) $E = 9$

Again putting $x=1$ in eqn (2) $1 - 12 + 24 - 30 + 9 = D + E$

$D + E = -8$

$D = -17$

Putting $x = 2$ in eqn (2)

$16 - 96 + 96 - 60 + 9 = 2B + 2C + D$

$2C + 2E + E = 35$

$C = -5$

Again - - - $x = 3$ - - - (2)

$6B + 6C + 3D + E$

$= -108$

$\Rightarrow B = -6$

equating the coefficient of x^4, $A=1$
from eqⁿ

$f(x) = x^{[4]} - 6x^{[3]} - 5x^{[2]} - 17x^{[1]} + 9$

$\Delta f(x) = 4x^{[3]} - 6\times3x^{[2]} - 2\times5x^{[1]} - 17\times1 + 0$
$4x^{[3]} - 18x^{[2]} - 10x^{[1]} - 17$
$\Delta^2 f(x) = 12x^{[2]} - 36x^{[1]} - 10$
$\Delta^3 f(x) = 24x^{[1]} - 36$
$\Delta^4 f(x) = 24$
$\Delta^5 f(x) = 0$ Ans.

Question :- 7 Estimate the sale for 1966 using Newton forward interpolation formula :

Year :	1931	1941	1951	1961	1971	1981
Sale in thousand :	12	15	20	27	39	52.

Solution :- Given interval $h = 10$

Year (x)	Sale (y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1931	12					
		3				
1941	15		2			
		5		0		
1951	20		2		3	
		7		3		-10
1961	27		5		-7	
		12		-4		
1971	39		1			
		13				
1981	52					

we know that Newton forward interpolation is

$$y(x) = f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

Here $u = \frac{x - x_0}{h}$

$$u = \frac{1966 - 1931}{10} = 3.5$$

$$y_{1966} = f(1966) = 12 + (3.5)\times 3 + \frac{(3.5)(2.5)}{2!}\times 2 + \frac{(3.5)(2.5)(1.5)}{3!}\times 0$$

$$+ \frac{(3.5)(2.5)(1.5)(0.5)}{4!}\times 3 + \frac{(3.5)(2.5)(1.5)(0.5)(-0.5)}{5!}\times(-10)$$

$$12 + 10.5 + 8.75 + 0.8203 + 0.2734$$

$$= 32.34$$

Thus the sale for the year 1966 = 32.34 thousands.

Ans

Question:- 8. Given the values

x :	5	7	11	13	17
$f(x)$:	150	392	1452	2366	5202

Evaluate $f(9)$, using Newton's divided difference formula.

Solve :-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0 = 5$	150				
		121			
$x_1 = 7$	392		24		
		265		1	
$x_2 = 11$	1452		32		0
		457		1	
$x_3 = 13$	2366		42		
		709			
$x_4 = 17$	5206				

By Newton's divided interpolation formula :

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + \ldots \quad (1)$$

Here $x = 9$ and putting value from table in (1)

$$f(9) = 150 + (9-5)(121) + (9-5)(9-7)(24) + (9-5)(9-7)(9-11)(1) + 0$$

$$= 810$$ Ans