Coordinates (and other things to calculate in geometry) (Black)

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1 Examples

1. [IMO'98/5] Let I be the incenter of triangle ABC. Let the incircle of ABC touch the sides BC, CA, and AB at K, L, and M, respectively. The line through B parallel to MK meets the lines LM and LK at R and S, respectively. Prove that angle RIS is acute.

2. [IMO'99/5] Two circles G_1 and G_2 are contained inside the circle G, and are tangent to G at the distinct points M and N, respectively. G_1 passes through the center of G_2. The line passing through the two points of intersection of G_1 and G_2 meets G at A and B. The lines MA and MB meet G_1 at C and D, respectively.

   Prove that CD is tangent to G_2.

3. [IMO'00/6] $A_1A_2A_3$ is an acute-angled triangle. The foot of the altitude from A_i is K_i and the incircle touches the side opposite A_i at L_i. The line K_1K_2 is reflected in the line L_1L_2. Similarly, the line K_2K_3 is reflected in L_2L_3 and K_3K_1 is reflected in L_3L_1. Show that the three new lines form a triangle with vertices on the incircle.

2 Problems

1. [IMO'01/5] In a triangle ABC, let AP bisect $\angle BAC$, with P on BC, and let BQ bisect $\angle ABC$, with Q on CA. It is known that $\angle BAC = 60^\circ$ and that $AB + BP = AQ + QB$. What are the possible angles of triangle ABC?

2. [IMO'02/2] BC is a diameter of a circle center O. A is any point on the circle with $\angle AOC > 60^\circ$. EF is the chord which is the perpendicular bisector of AO. D is the midpoint of the minor arc AB. The line through O parallel to AD meets AC at J. Show that J is the incenter of triangle CEF.

3. [IMO'03/3] A convex hexagon has the property that for any pair of opposite sides the distance between their midpoints is $\sqrt{3}/2$ times the sum of their lengths. Show that all the hexagon's angles are equal.

4. [IMO'04/5] In a convex quadrilateral $ABCD$ the diagonal BD does not bisect the angles ABC and CDA. The point P lies inside $ABCD$ and satisfies $\angle PBC = \angle DBA$ and $\angle PDC = \angle BDA$. Prove that $ABCD$ is a cyclic quadrilateral if and only if $AP = CP$.

5. [IMO'05/5] Let $ABCD$ be a fixed convex quadrilateral with $BC = DA$ and BC not parallel with DA. Let two variable points E and F lie of the sides BC and DA, respectively and satisfy $BE = DF$. The lines AC and BD meet at P, the lines BD and EF meet at Q, the lines EF and AC meet at R. Prove that the circumcircles of the triangles PQR, as E and F vary, have a common point other than P.