New Zealand Mathematical Olympiad Committee

May Problems

These problems are intended for students who might already have taken part in the September problems, or who are thinking of taking part in 2009. The difficulty will gradually increase over the course of the year, building up to problems comparable to those you will be asked to solve in the September problems for selection to the Christchurch camp in January.

I welcome you to try them, and to send me any solutions you find. I'll try to acknowledge these, and might include (with credit!) any particularly clever or nice solutions from you in the "official solutions". These will appear on the web in about two months time, or can be obtained from me by email earlier if you provide evidence that you've tried the problems seriously.

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1. Alice and Bob each chose a positive integer, and told it to Eve. Eve wrote the sum of these numbers on one card, the product on another, hid one card and showed the other to Alice and Bob. Alice looked at the number on the card, which was 2010, and declared that she was unable to determine Bob's number. Knowing this, Bob said he was also unable to determine Alice's number.

   What was Bob's number? Can you determine Alice's number?

2. The increasing sequence 1, 3, 4, 9, 10, 12, 13,... consists of all those positive integers which are powers of 3 or sums of distinct powers of 3. Of the first googol (10^{100}) terms in the sequence, how many are powers of 3?

3. Quadrilateral $ABCD$ is circumscribed about a circle, which is tangent to the sides AB, BC, CD and DA at K, L, M, N respectively. Let S be the point of intersection of the line segments KM and LN, and suppose that the quadrilateral $SKBL$ is cyclic. Prove that the quadrilateral $SNDM$ must be too.

4. Fifteen rooks are placed on a 15×15 chess board in such a way that no rook is attacking any other. Each rook then makes a single knight's move. Prove that after this has taken place there must be a pair of rooks that are attacking each other.

May 8, 2009

http://www.mathsolympiad.org.nz