New Zealand Mathematical Olympiad Commitee

January Problems

These problems are intended for students who might already have taken part in the September problems, or who are thinking of taking part in 2009. The difficulty will gradually increase over the course of the year, building up to problems comparable to those you will be asked to solve in the September problems for selection to the Christchurch camp in January.

I welcome you to try them, and to send me any solutions you find. I'll try to acknowledge these, and might include (with credit!) any particularly clever or nice solutions from you in the "official solutions". These will appear on the web in about two months time, or can be obtained from me by email earlier if you provide evidence that you've tried the problems seriously.

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1. Rex the dog is attached by an 8 m long chain to the south-west corner of his 1m square doghouse (whose sides run in the four primary compass directions.) A mischievous cat leads him, at full stretch of the chain at all times, for two circuits around the doghouse, beginning from a point directly south of the south-west corner. How much farther does he travel in the first lap than in the second?

2. In a 3×3 magic square, the three row sums, the three column sums, and the two diagonal sums must be the same. Given the partially completed square below, what is the value of D?

A	B	6
D	E	F
G	9	2

3. How many pairs (a, b) are there where a and b are integers in the range from 1 to 100 inclusive, and a^b is a perfect square.

4. Is it possible to partition the set $\{1, 2, 3, \ldots, 45\}$ into nine 5-element subsets (this means that every number from the set belongs to exactly one of the subsets) in such a way that in each of the subsets we can find three elements whose sum is equal to the sum of the other two?

January 19, 2009

http://www.mathsolympiad.org.nz