New Zealand Mathematical Olympiad Committee

June Problems

These problems are intended for students who might already have taken part in the September problems, or who are thinking of taking part in 2009. The difficulty will gradually increase over the course of the year, building up to problems comparable to those you will be asked to solve in the September problems for selection to the Christchurch camp in January.

As we're now coming closer to the time when the September problems are actually released, this month's problems come from the 2006 September problems (among the easiest of the problems in that set.)

Michael Albert, 2009 NZ IMO team leader malbert@cs.otago.ac.nz

1. A square is inscribed in a regular dodecagon (12 sided polygon) by connecting every third vertex. If the length of each side of the square is 1, what is the length of each side of the dodecagon?

2. Prove that for every positive integer n:

$$\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{3n+1} > 1.$$

3. Let A be a positive integer with an even number of digits and suppose that there is an integer B whose digits are a rearrangement of those in A such that the sum of A and B is a power of 10. Show that A is divisible by 10.

4. Prove that for any positive integer n, $n(n+1)(n+2)$ is not a perfect square.

June 11, 2009

http://www.mathsolympiad.org.nz