$X + Y$ International Mathematical Olympiad

Cambridge 2018

language: English

1. A $2n \times 2n$ board is divided into $4n^2$ small squares in the manner of a chessboard. Each small square is painted with one of four colours so that every 2×2 block of four small squares involves all four colours. Prove that the four corner squares of the board are painted with different colours.

2. Which positive integers n have the property that $\{1, 2, \ldots, n\}$ can be partitioned into two subsets A and B so that the sum of the squares of the elements of A is the sum of the squares of the elements of B?

3. This problem concerns polynomials in X with real coefficients. Let $f(X) = 2013X + 1$. Suppose that $g(X)$ and $h(X)$ are polynomials such that $f(g(X)) = g(f(X))$ and $f(h(X)) = h(f(X))$. Prove that $g(h(X)) = h(g(X))$.

Time allowed: 4 hours 30 minutes

Each problem is worth 7 points