NT Practice

I guess the number 720 is just awesome.

1. (JSteinhardt) Let $\diamond(k)$ be the sum of k's digits in base $720+1$. How many positive integers m exist such that $\diamond(k) \equiv k \pmod{m}$ for all k?
2. (JSteinhardt) Find the number of divisors of the sum of the squares of the divisors of 720.
3. (JSteinhardt) Find the sum of the square divisors of 720.

Actually, 210 is pretty cool as well.

4. (Traditional) Find the right-most non-zero digit of 210! in base 210.
5. (Traditional) Find the length of the repeating part of $\frac{1}{17}$ in base 210.

But 47 is the coolest of all.

6. (Traditional) If $\displaystyle\sum_{i=1}^{46} \frac{1}{i} = \frac{p}{q}$, and r exists such that $47|p - qr$, then find $r - 47\lfloor \frac{r}{47} \rfloor$.
7. (Traditional) Find the smallest positive integer n such that $1^n + 2^n + \ldots + 46^n$ is not divisible by 47.

And finally, Haitao has asked permission to give you all a problem, and I said yes.

8. Let $p > 2$ be a prime and let $P(x) = a_0x^{p-1} + \ldots + a_{p-1}$ be a polynomial with integer coefficients. If for all integral x,y such that if p does not divide $x - y$, then p also does not divide $P(x) - P(y)$. Prove that $p|a_0$.