TJUSAMO Contest 2: 11/02/2006

1. To *tile* a region R means to find a set S of non-overlapping regions such that their union is R. Suppose R is a square with side length 1. (i) Is it possible to tile R with finitely many circles? (ii) Is it possible to tile R with finitely many $30-60-90$ triangles such that their longest side has a rational length?

2. Consider an n row by m column grid G such that each element of the grid is a non-negative integer, the sum of the elements in the grid is n, each element in the grid is no less than the element below it or to its right, and no two elements in a single row differ by more than 1. Prove that the number of distinct grids G is independent of m and that it is equal to the number of ways of writing n as a sum of non-decreasing positive integers. (For example, when $n = 4$, there are 5 ways to do this: $1+1+1+1$, $1+1+2$, $1+3$, $2+2$, and 4).

3. To a polynomial $P(x) = ax^3+bx^2+cx+d$, of degree at most 3, one can apply two operations:

   (a) $ax^3 + bx^2 + cx + d$ becomes $dx^3 + cx^2 + bx + a$;
   (b) $ax^3 + bx^2 + cx + d$ becomes $a(x + t)^3 + b(x + t)^2 + c(x + t) + d$.

   Determine if one can transform, by applications of these operations, the polynomial $P_1(x) = x^3 + x^2 - 2x$ into $P_2(x) = x^3 - 3x - 2$.