TJUSAMO Contest # 1

3 problems - 2 hours

October 5th, 2006

1. Find all ordered triplets (x, y, z) of integers that satisfy the equation:

$$x^2 + y^2 + z^2 = 2xyz$$

2. [Putnam 1993] A deck containing $2n$ cards numbered from 1 to $2n$ is shuffled and n cards are dealt to each of two players, Alice and Bob. Starting with Alice, the two players take turns discarding one of their remaining cards. A player can win at anytime by discarding a card that causes the sum of the numbers of all the discarded cards to be divisible by $2n + 1$. If both Alice and Bob play flawlessly, what is the probability that Alice wins?

3. [IMO 1985] In a convex cyclic quadrilateral ABCD, a circle has its center on side AB and is tangent to BC, CD, and DA. Prove that AD + BC = AB.