AM-GM Inequality: For positive reals $\omega_1, \ldots, \omega_n$ and $a_1, \ldots, a_n$ the following holds true:

$$\frac{\sum_{i=1}^{n} \omega_i a_i}{\sum_{i=1}^{n} \omega_i} \geq \sqrt[\sum_{i=1}^{n} \omega_i]{\prod_{i=1}^{n} a_i^{\omega_i}}$$

Yay that was pretty scary looking. Here's a slightly less intimidating form (though sometimes you will want to use the above form):

$$\frac{a_1 + a_2 + \ldots a_n}{n} \geq \sqrt[n]{a_1 a_2 \ldots a_n}$$

Surprisingly, this is all you will need to know for this lecture other than basic algebra and the fact that $x^2 \geq 0$. The rest will be problems, because inequalities are a difficult concept to master and the best way to master anything is to do a lot of problems!

1. Verify that $a^2 + b^2 \geq 2ab$, first with AM-GM and then with the fact that $x^2 \geq 0$.
2. If $abc = 1$, minimize $a^3 + b^3 + c^3$ for positive reals a,b,c.
3. For $x \geq 0$, minimize $x^2 + \frac{2}{x}$.
4. For $a, b \geq 0$, maximize $\frac{2ab}{a^2+b^2}$.
5. For $a, b \geq 0$, maximize $\frac{6ab}{a^2+9b^2}$.
6. For $x \geq 0$, minimize $x + \frac{1}{x+1}$ (no Calculus!).
7. For $x \geq 0$, minimize $\frac{x^3+2}{x+3}$.
8. Which is bigger, 2007! or $(1003)^{2007}$?
9. If you can have 1 of one coin, 5 of another coin, and 10 of a third coin, and can choose pennies, nickels, and dimes, in what way should you choose the coins to maximize your net gain?
10. This intuitive result can in fact be generalized: $x^2 + y^2 + z^2 \geq xy + yz + zx$. Show that this is always true.