Important Points in Geometry

1. G the centroid ($\bigcap$ medians).
2. I the incenter. ($\bigcap$ bisectors)
3. O the circumcenter. ($\bigcap$ perpendicular bisectors)
4. H the orthocenter. ($\bigcap$ altitudes)
5. I_a, I_b, I_c the excenters. ($\bigcap$ two exterior bisectors with a third interior one).
6. N Nagel's point. ($\bigcap$ of cevians joining vertices with the point of tangency between the opposite side and excircle).
7. R Gergonne's point. ($\bigcap$ of cevians joining vertices with the point of tangency between the opposite side and incircle).
8. W Euler's point. (center of Euler's circle)
9. Brocard's point. The Brocard's point. (Unique point P inside ΔABC so that $\angle PBC = \angle PCA = \angle PAB$).
10. ω center of circle around the podar triangles of two isogonic points M, N.
11. K Lemoine's point. ($\bigcap$ of symmedians)
12. N' Nagel's second point. ($\bigcap$ of perpendiculars from excenters to sides).
13. S Spiecker's point. (incenter of the median triangle).
14. T Toricelli's point. (unique point T inside ΔABC so that $\angle BPC = \angle CPA = \angle APB = 120$).
15. M Miquel's point. ($\bigcap$ of circumcircles of big triangles in the complete quadrilateral).
16. Brianchon's point. ($\bigcap$ of main diagonals of a circumscribed hexagon).
17. M Mathot's point. ($\bigcap$ of the perpendiculars from the midpoints of sides of cyclic quadrilaterals to the opposite sides).

Properties:

1. $2\overrightarrow{OG} = \overrightarrow{GH}$.
2. $\overrightarrow{OG} = \overrightarrow{GM}$ in a cyclic quadrilateral where M is Mathot's point.
3. $\overrightarrow{OW} = \overrightarrow{WH}$ where W is Euler's point.

4. $\overrightarrow{IG} = 2\overrightarrow{GS} = \overrightarrow{SN}$, where I is the incenter, G the centroid, S Spiecker's point and N Nagel's point.

5. The angle PBC is at most 30, where P is Brocard's point.

6. Prove that T minimizes the expression $XA + XB + XC$ for X inside triangle ABC. T is Toricelli's point.

7. If M and N are isogonic then $M\omega = \omega N$.

8. Miquel's point lies on the circumcircles of the small triangles in the complete quadrilateral.

9. Miquel's point lies on the outer diagonal of the quadrilateral if and only if this is cyclic.

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