New Zealand Mathematical Olympiad Committee

2010 March Problems

These problems are intended to help students prepare for the 2010 camp selection problems, used to choose students to attend our week-long residential training camp in Christchurch in January.

In recent years the camp selection problems have been known as the "September Problems", as they were made available in September. This year we're going to trial moving the selection problems earlier in the year, releasing them in July and moving the due date to August. This will allow more time for pre-camp training, building up to Round One of the British Mathematical Olympiad in December.

The solutions will be posted in about two month's time, but can be obtained before then by email if you write to me with evidence that you've tried the problems seriously.

Good luck!

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1. Two players, A and B, are playing the following game. They take turns writing down the digits of a six-digit number from left to right; A writes down the first digit, which must be nonzero, and repetition of digits is not permitted. Player A wins the game if the resulting six-digit number is divisible by 2, 3 or 5, and B wins otherwise.

   Prove that A has a winning strategy.

2. Prove that $n^n - n$ is divisible by 24 for all odd positive integers n.

3. Let a and b be real numbers. Prove that the inequality

   $$\frac{(a+b)^3}{a^2 b} \geq \frac{27}{4} \tag{1}$$

   holds.

   When does equality hold?

4. Let $ABCD$ be a quadrilateral. The circumcircle of the triangle ABC intersects the sides CD and DA in the points P and Q respectively, while the circumcircle of CDA intersects the sides AB and BC in the points R and S. The straight lines BP and BQ intersect the straight line RS in the points M and N respectively. Prove that the points M, N, P and Q lie on the same circle.

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