New Zealand
Maths Olympiad Committee
Camp 2009
Problem Set 1

1. A triangle ABC has incentre I, and AI intersects the circumcircle again at X. Show that X is the circumcentre of triangle BCI.

2. Find the smallest positive integer that can be written in the form $|36^m - 5^n|$ for some positive integers m and n.

3. Find all values of the real parameter m such that:

$$\left|\frac{x^2 - mx + 1}{x^2 + x + 1}\right| < 3$$

holds for all real numbers x.

4. Find the sum of all the five digit numbers formed by using the digits 1 through 5 exactly once each.

5. Determine all real solutions of the equation:

$$\left(16x^{200} + 1\right)\left(y^{200} + 1\right) = 16(xy)^{100}.$$