New Zealand Mathematical Olympiad Committee

April Problems

These problems are intended for students who might already have taken part in the September problems, or who are thinking of taking part in 2009. The difficulty will gradually increase over the course of the year, building up to problems comparable to those you will be asked to solve in the September problems for selection to the Christchurch camp in January.

I welcome you to try them, and to send me any solutions you find. I'll try to acknowledge these, and might include (with credit!) any particularly clever or nice solutions from you in the "official solutions". These will appear on the web in about two months time, or can be obtained from me by email earlier if you provide evidence that you've tried the problems seriously.

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1. Call a number *mystical* if it is possible to begin with the sum of its digits, and then by a sequence of operations where we replace $n = a+b$, by $m = a \times b$ (where a and b are positive integers) eventually obtain the number again. For example, 35 is mystical because, starting from $8 = 3 + 5$ we can write $8 = 6 + 2$, getting $12 = 6 \times 2$, and then $12 = 7 + 5$ getting $35 = 7 \times 5$. Is 2009 mystical?

2. There are five paper triangles on a table – each is equilateral of side length 10 cm. Show that, no matter how they are oriented, it is possible to cover any one of them up using the other four, simply by sliding them (i.e. without any rotation). Does this remain true if the triangles are all congruent, but not necessarily equilateral?

3. A 20 kg block of cheese was on display at the local A and P show. Towards the end of the day, they started selling off bits of it. After each of the first ten customers bought a chunk the exhibitor announced "if everyone buys an amount equal to the average amount sold to each customer so far, there's just enough left for 10 more pieces". Could that announcement be correct (in all ten cases)? If so, how much cheese was left after the first 10 purchases?

4. Can the product of two consecutive positive integers equal the product of two consecutive even positive integers?

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http://www.mathsolympiad.org.nz