New Zealand Mathematical Olympiad Committee

July Problems

These problems are intended for students who might already have taken part in the September problems, or who are thinking of taking part in 2009. The difficulty will gradually increase over the course of the year, building up to problems comparable to those you will be asked to solve in the September problems for selection to the Christchurch camp in January.

As we're now coming closer to the time when the September problems are actually released, this month's problems come from the 2006 September problems (among the medium difficulty problems in that set.)

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1. For a real number x, let $\lfloor x \rfloor$ denote the largest integer less than or equal to x. Find all real numbers x such that:
$$x\lfloor x\lfloor x\lfloor x\rfloor\rfloor\rfloor = 88.$$

2. Find all values of m, n and p with $p^n + 144 = m^2$ where m and n are positive integers and p is a prime.

3. Let ABC be an isosceles triangle with $AB = AC$, and let D be the midpoint of BC. Let E be the foot of the perpendicular from D to AB and F the midpoint of DE. Prove that AF is perpendicular to CE.

4. Six points A,B,C,D,E and F are given in the plane, no three of which are collinear. Each segment XY where X and Y are any two of the six points is drawn using either red or blue ink. Show that there must be four distinct points Q, R, S and T from among these six such that the segments QR, RS, ST and TQ are all the same colour.