New Zealand Mathematical Olympiad Committee

2010 June Problems

These problems are intended to help students prepare for the 2010 camp selection problems, used to choose students to attend our week-long residential training camp in Christchurch in January.

In recent years the camp selection problems have been known as the "September Problems", as they were made available in September. This year we're going to trial moving the selection problems earlier in the year, releasing them in July and moving the due date to August. This will allow more time for pre-camp training, building up to Round One of the British Mathematical Olympiad in December.

The solutions will be posted in about two month's time, but can be obtained before then by email if you write to me with evidence that you've tried the problems seriously.

Good luck!

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1. A circle O_1 of radius 2 and a circle O_2 of radius 4 are externally tangent at the point P. Points A and B, both distinct from P, are chosen on O_1 and O_2 respectively so that A, P and B are collinear. Determine the length of the line segment PB if the length of AB is 4.

2. Prove that the number $9^n + 8^n + 7^n + 6^n - 4^n - 3^n - 2^n - 1^n$ is divisible by 10 for all non-negative integers n.

3. Suppose that each point in the plane is coloured either black or white. Show that there is a set of three points of the same colour which form the vertices of an equilateral triangle.

4. If $1 = d_1 < d_2 < \cdots < d_k = n$ are all the positive divisors of a positive integer $n > 1$, prove that

$$d_1 + d_2 + \cdots + d_k > k\sqrt{n}.$$

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