Geometry Problems, with emphasis on Pithot

1. Let ABC be an acute triangle with centroid G. If $BG + AC = CG + AB$ then ABC is isosceles. (Hint: calculate, calculate, calculate)

2. Let ABC be an acute triangle with centroid G. If $BG + AC = CG + AB$ then ABC is isosceles. (Hint: Take M the midpoint of BC and N the symmetrical of G across M. Then restate the problem using N instead of G. What is the locus of points B, C with that property? Now, go for areas.)

3. Let $ABCD$ be a convex quadrilateral. Prove that $ABCD$ is circumscribed if and only if $AB + CD = AD + BC$. (Pithot's Theorem)

4. Let ABC be an acute angled triangle and $M \in (AC), N \in (AB)$. Let $\{P\} = BM \cap CN$. Prove that $AMPN$ is circumscribed if and only if $BP + AC = CP + AB$. (a theorem similar to Pithot's)

5. Let ABC be an acute triangle. Let M be the midpoint of side BC and $P \in (AM)$. Prove that if $BP + AC = CP + AB$ then ABC is isosceles. (Hint: reread the previous problem and think of homotheties)

6. Let ABC be an acute triangle with centroid G. If $BG + AC = CG + AB$ then ABC is isosceles. (Hint: reread the previous problem. Arad county olympiad 2000, proposed by me)

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