New Zealand Mathematical Olympiad Committee

2011 May Problems

These problems are intended to help students prepare for the 2011 camp selection problems, used to choose students to attend our week-long residential training camp in January.

The solutions will be posted in about one month's time, but can be obtained before then by email if you write to one of us with evidence that you've tried the problems seriously.

Good luck!

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1. Let x_1 and x_2 be the distinct roots of the equation $2x^2 - 3x + 4 = 0$. Compute the value of
$$\frac{1}{x_1^3} + \frac{1}{x_2^3}.$$

2. Six points are given inside a square of side-length 10, such that the distance between any two of them is an integer. Prove that at least two of these distances are the same.

3. Find all possible ways of expressing 2010 as a sum of (one or more) consecutive positive integers.

4. In a right triangle the median and bisector of the right angle divide the hypotenuse in three parts. The lengths of these parts, in a certain order, from an arithmetic sequence. Find all possible ratios of the lengths of the legs of the triangle (i.e., of the sides adjacent to the right angle).

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