New Zealand
Maths Olympiad Committee
May Problems

These problems are intended for students who might already have taken part in the September problems, or who are thinking of taking part in 2008. They will appear on an irregular basis, as and when I get around to preparing them. There are no prizes or competition involved, just the opportunity to improve your problem solving, and perhaps learn some new maths.

I welcome you to try them, and to send me any solutions you find. I'll try to acknowledge these, and might include (with credit!) any particularly clever or nice solutions from you in the "official solutions". These will appear on the web (somewhere, sometime!), or can be obtained from me by email if you provide evidence that you've tried the problems seriously. The solutions will contain hyperlinks where you can learn about some of the concepts involved and pursue them further.

This month's problems, perhaps a bit more difficult than recently, are modifications of ones that have appeared recently in competitions from Bulgaria, Greece and Moldova.

Michael Albert, 2008 NZ IMO team leader (malbert@cs.otago.ac.nz)

1. Let k be a positive integer such that $k(k+3)$ is a perfect square. Prove that k is not a multiple of 3.

2. Prove that, for every positive integer n:
$$\frac{1}{11}+\frac{2}{21}+\frac{3}{31}+\cdots+\frac{n}{10n+1}<\frac{n}{10}.$$

3. A social network has 2008 members. Any two of them who are not already friends have at least one friend in common. What is the minimum possible number of pairs of people in the network who are already friends?

4. A cyclic quadrilateral $ALNB$ is given. The point M is the midpoint of the arc not containing A and B determined by LN. Let D be the intersection of AM and BL, and E the intersection of AN and BM. Prove that $DE \parallel LN$.