New Zealand
Maths Olympiad Committee
February Problems

These problems are intended for students who might already have taken part in the September problems, or who are thinking of taking part in 2008. They will appear on an irregular basis, as and when I get around to preparing them. There are no prizes or competition involved, just the opportunity to improve your problem solving, and perhaps learn some new maths.

I welcome you to try them, and to send me any solutions you find. I'll try to acknowledge these, and might include (with credit!) any particularly clever or nice solutions from you in the "official solutions". These will appear on the web in about two months time, or can be obtained from me by email earlier if you provide evidence that you've tried the problems seriously. The solutions will contain hyperlinks where you can learn about some of the concepts involved and pursue them further.

This set of problems are modifications of ones appearing on Croatian competitions in 2005.

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1. Three positive real numbers a, b and c satisfy the condition:

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1.$$

   Prove that $(a-1)(b-1)(c-1) \geq 8$.

2. Points E and F on the sides AB and AD of parallelogram $ABCD$ have the property that $EF \parallel BD$. Prove that triangles BCE and CDF have the same area.

3. Let P be a polynomial with integer coefficients. If $P(8) = 2008$, is it possible that $P(2008)$ could be a perfect square?

4. Let a_1, a_2, $\ldots$, a_{99} be positive integers, all less than 100 (but not necessarily distinct). Supposing that the sum of any two or more of them is never a multiple of 100, show that they must all be equal.