TJUSAMO Contest # 3

3 problems - 2 hours

December 14th, 2006

On your answer sheet, remember to write your ID number, problem number, current page number, and total page number in the upper right hand corner. Also, leave sufficient margins, write clearly and rigorously, and do not attempt to fool or confuse the graders. Good luck!

1. Let O be the circumcenter of a convex quadrilateral $ABCD$. Let W, X, Y, Z be the foot of the perpendicular from O to the segments AB, BC, CD, DA, respectively. If brackets denote area, prove that $[ABCD] = [ABZ] + [BCW] + [CDX] + [DAY]$.

2. Prove that for every natural a, there exist infinitely many naturals n such that $10^n a - 1$ is composite.

3. Find all finite sequences of nonnegative integers $z_0, z_1, \ldots, z_{n-1}$, such that n is a natural, and for any integer i such that $0 \leq i < n$, z_i represents the number of integers j such that $0 \leq j < n$ and $z_j = i$. One such sequence is $3, 2, 1, 1, 0, 0, 0$.