Notes on Probability and Statistics

- **Experiment** is any procedure that can be infinitely repeated and has a well-defined set of possible outcomes, known as the sample space (S).

- **Event (E)** is a subset of the sample space of an experiment. i.e., $E \subseteq S$

- **Probability (P)** is the likelihood of an event occurring and is given as:.

$$P(A) = \frac{no.\ of\ favourable\ outcomes\ to\ A}{Total\ no.\ of\ possible\ outcomes}$$

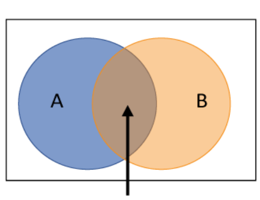
- **Mutually Exclusive Events :**
  If two events are mutually exclusive then the probability of both the events occurring at the same time is equal to zero. i.e. $P(A \cap B) = 0$.
  For example, while tossing of a coin, coming up heads or tails are two mutually Exclusive Events.

- **Mutually Exhaustive Events :** When two events 'A' and 'B' are exhaustive, it means that one of them must occur. i.e. $P(A \cup B) = 1$.
  For example, while tossing of a coin, coming up heads or tails are two mutually Exhaustive Events.

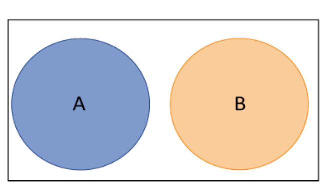
- **Independent Events :** Two events are independent if the occurrence of one does not change the probability of the other occurring.
  If two events 'A' and 'B' are independent, then : $P(A \cap B) = P(A).P(B)$

- **Conditional Probability :** is defined as the likelihood of an event or outcome occurring, based on the occurrence of a previous event or outcome.
  P(Event A will occur given that Event B has already occurred) = $P(A|B) = \frac{P(A \cap B)}{P(B)}$
  Two events A and B are mutually independent if $P(A|B) = P(A)$.
  also, $P(A|B) \neq P(B|A)$.

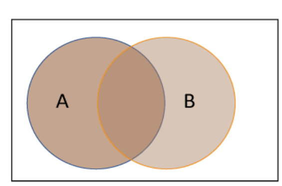
- **Basic rules of probability :**
  Addition rule : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  Multiplication rule : $P(A \cap B) = P(A|B).P(B) = P(B|A).P(A)$
  Complement rule : $P(A^c) = 1 - P(A)$



$A \cap B$
A and B



$A \cap B = \varnothing$
A and B is the empty set



$A \cup B$
A or B

- **Bayes Theorem :** Conditional probability of each of a set of possible causes (B), given an observed outcome (A) is given as:

$$P(B|A) = \frac{P(A|B).P(B)}{P(A \cap B)}$$

E.g. A man speaks truth ¾ times. He draws a card and reports 'King'. What is the probability of it actually being a king?

P(Man speak truth)= P(T) =¾, P(drawing a king)=P(K)= 4/52 = ¼

P(drawing a king given that the man speaks truth) = P(K|T)

$$P(K|T) = \frac{P(T|K).p(K)}{P(T)} = \frac{(3/4).(1/13)}{(1/13).(3/4) + (12/13).(1/4)} = 0.2$$

- **Types of a Random variable :**

  Discrete random variable: A random variable with a finite or countable number of possible values. e.g. random variable of the outcome when a dice is thrown. It can take integer values in range 1 to 6.

  Continuous random variable: A random variable that can take an infinite number of possible values. E.g. a random variable representing the height of students in a class.

- **Probability Distribution Function (PDF)** is a function that is used to give the probability of all the possible values that a random variable can take.

  If the random variable is discrete, then it is called **Probability Mass Function**.

  For the continuous random variable, it is called **Probability Density Function**.

- **Cumulative Distribution Function (CDF):** returns the probability that a random variable will take a value less than or equal to x.

- **Expectation** of a discrete random variable(X) having a Probability mass function P(x) is the weighted average of possible values that X can take.

  i.e. $$E(X) = \sum_{1}^{n} x_i.P(x_i)$$

  Note :For a uniformly distributed random variable, Expectation is equal to the mean.

  Properties of Expectation :

$$E(aX) = a.E(X)$$
$$E(X + b) = E(X) + b$$
$$E(aX + b) = a.E(X) + b$$

- **Variance** is a statistical measurement that is used to determine the spread of numbers in a data set with respect to the average value or the mean. It is given as :

$$Var(X) = \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

  Where, σ is the **standard deviation**. Therefore,

$$Variance = (Standard\ Deviation)^2$$

For a random variable *X*, **Variance** can be given as :

$$Var(X) = E(X^2) - [E(X)]^2$$

Properties of variance:

1. $Var(k.X) = k^2.Var(X)$

2. Assuming that the samples were collected independently.

   $Var(X_1 + X_2 + X_3 + ...) = Var(X_1) + Var(X_2) + Var(X_3) + ...$

3. $Var(X + c) = Var(X)$

- **Covariance** is the variance of two quantities with respect to each other. It is a measure of how much two random variables vary together.

$$Cov(X, Y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})$$

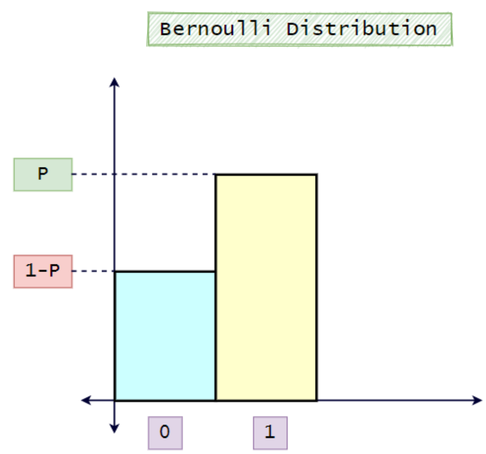
- **Discrete probability distributions:**

1. **Bernoulli distribution:**The distribution of a random variable which takes a binary, boolean output: 1 with probability p, and 0 with probability (1-p).
   Let X follows the Bernoulli distribution, i.e. $X \sim Bern(p)$
   then,

$$P(X = x) = \begin{cases} 1 - p & , \quad x = 0 \\ p & , \quad x = 1 \end{cases}$$

   and

$$E(X) = p$$
$$Var(X) = p(1 - p)$$



2. **Binomial distribution:** It is the probabiity distribution of getting *x* successes in *n* Bernoulli trials.
   Let X follows the Binomial distribution, i.e. $X \sim B(n, p)$

   where, n = number of trials , p = probability of success

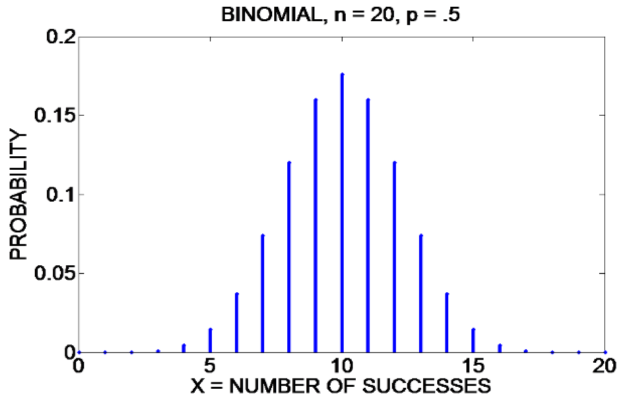
   then,

$$P(X = x) = {}^{n}C_{x}.p^{x}.(1 - p)^{n-x}$$

   and

$$E(X) = n.p$$
$$Var(X) = n.p.(1 - p)$$

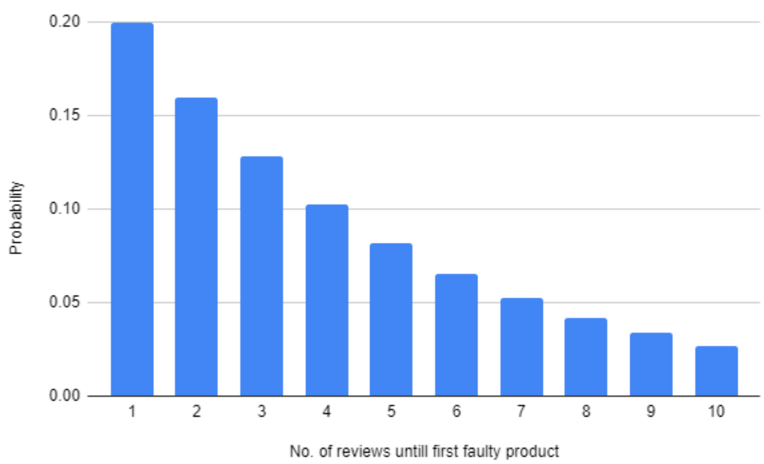
E.g. Probability Distribution of number of heads in 20 coin flips.



3. **Geometric distribution:** It is the probability distribution of number of Bernoulli trials needed to get one success.

   Its Probability mass function is given as : $P(k) = (1-p)^{k-1}.p$

   E.g. The probability of getting a faulty product after reviewing *k* non-faulty products.



4. **Poisson distribution:** It gives the probability of an event happening a certain number of times (x) within a given interval.

$$P(X = x) = \frac{\lambda^x.e^{-\lambda}}{x!}$$

Expectation, mean and variance are all equal to λ.