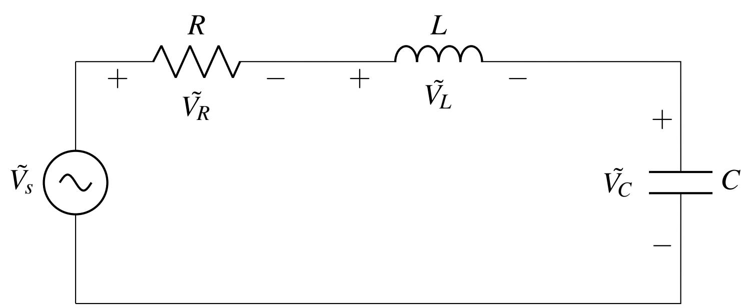
1. RLC Circuit

In this question, we will take a look at an electrical systems described by second-order differential equations and analyze it in the phasor domain. Consider the circuit below where $\tilde{V}_s$ is a sinusoidal signal, $L = 1\,\text{mH}$, and $C = 1\,\text{nF}$:



(a) **Transform the circuit into the phasor domain.**

(b) **Solve for the transfer function** $H_C(\omega) = \frac{\tilde{V}_C}{\tilde{V}_s}$ **in terms of** R**,** L**, and** C**.**

(c) **Solve for the transfer function** $H_L(\omega) = \frac{\tilde{V}_L}{\tilde{V}_s}$ **in terms of** R**,** L**, and** C**.**

(d) **Solve for the transfer function** $H_R(\omega) = \frac{\tilde{V}_R}{\tilde{V}_s}$ **in terms of** R**,** L**, and** C**.**

2. Bode Plots for Filters

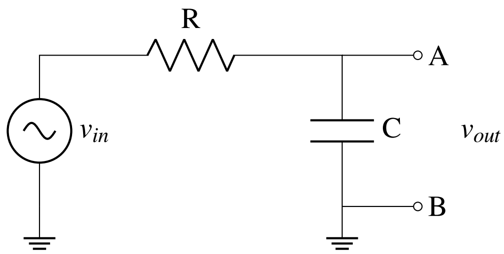


Figure 1: Low Pass Filter

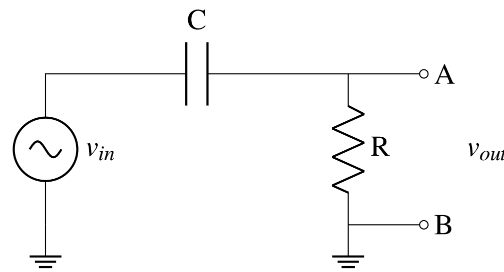
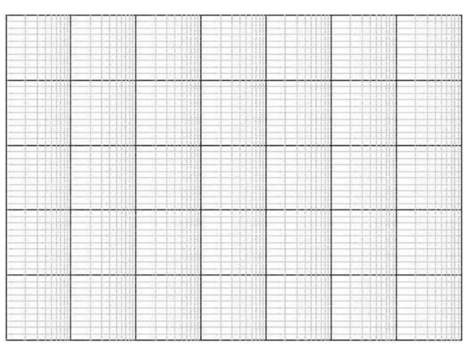


Figure 2: High Pass Filter

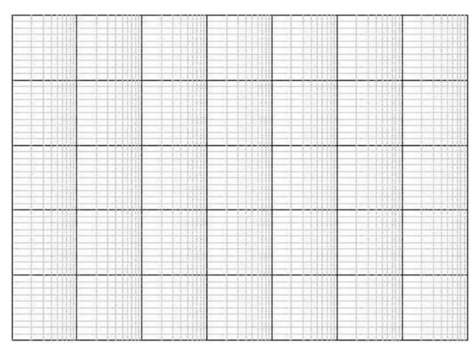
(a) First, consider the following transfer function of a Low-Pass Filter:
$H(\omega) = \frac{1}{j\omega C_1 R_1 + 1}$ where $R_1 = 100\Omega$ and $C_1 = 100pF$

i. What is its cutoff frequency?
Hint: Recall that the cutoff frequency is the frequency at which the magnitude of the transfer function is $\frac{1}{\sqrt{2}}$.

ii. Sketch its phase and magnitude.



$|H(\omega)|$

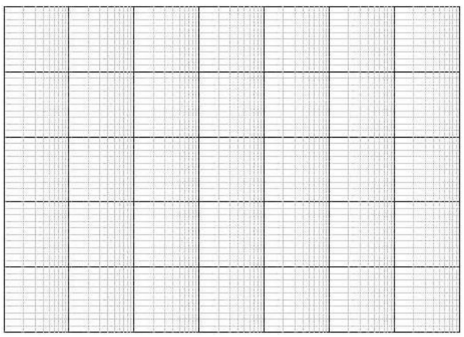


$\angle H(\omega)$

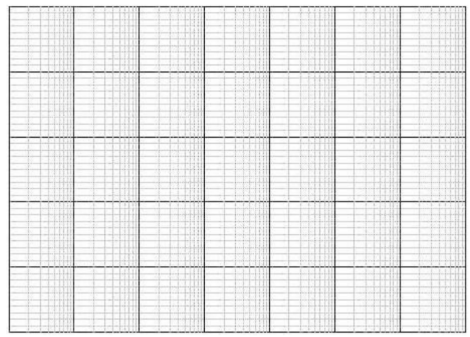
(b) Now consider the transfer function of a High-Pass Filter: $H(\omega) = \frac{j\omega C_2 R_2}{j\omega C_2 R_2 + 1}$ where $R_2 = 1k\Omega$ and $C_1 = 10nF$

i. What is its cutoff frequency?

ii. Sketch its phase and magnitude.



$|H(\omega)|$

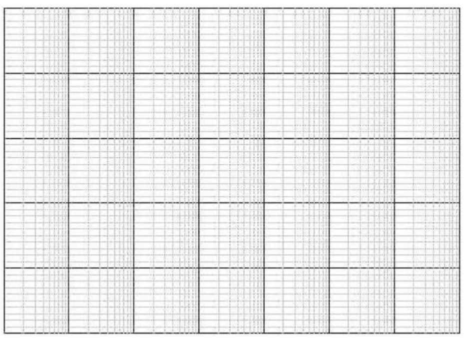


$\angle H(\omega)$

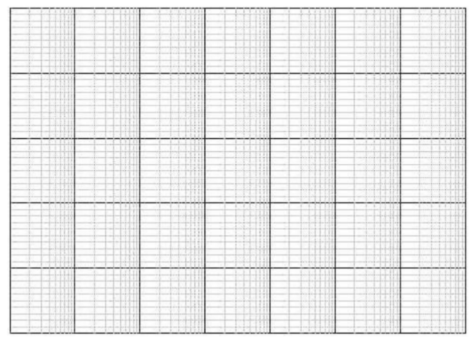
(c) What happens if we cascade these two filters together with a unity-gain buffer in between them? Consider the resulting transfer function: $H(\omega) = \frac{1}{j\omega C_1 R_1 + 1} \cdot \frac{j\omega C_2 R_2}{j\omega C_2 R_2 + 1}$

i. What are its cutoff frequencies?

ii. Sketch its phase and magnitude. *Hint: How can we combine the plots of the individual filters together?*



$|H(\omega)|$



$\angle H(\omega)$

3. RLC Bandstop Filter

One way to compose a bandpass filter is by combining a low pass and high pass filter via a buffer. Another way to compose a bandpass is to use a RLC circuit of the following form:

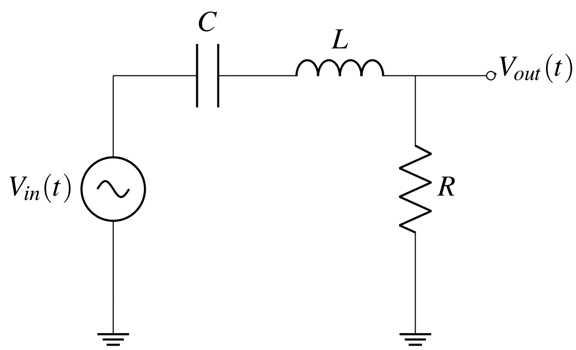


Figure 3: RLC Bandpass Filter

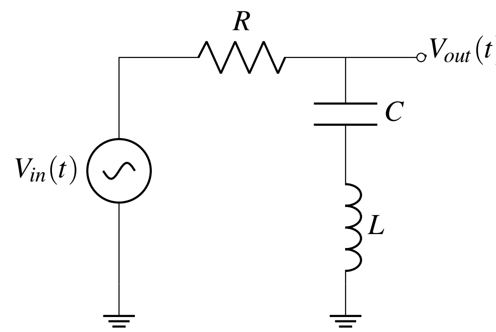


Figure 4: Unknown Behavior

Let's explore what happens when swap the location of the resistor with the location of the capacitor and inductor.

(a) Write out the transfer function of the circuit of unknown behavior.

(b) What is the magnitude of the transfer function?

(c) Sketch the magnitude of this transfer function.
HINT: The name of this question is bandstop filter. What do you think that means?

(d) When is the magnitude of the transfer function zero? When is it one?

(e) How would you describe the behavior of this circuit? Why do you think circuits with this type of circuit behavior are classified as notch/bandstop filters? Why is this specific filter called a *resonsant* bandstop filter?

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