Trust Prediction using Temporal Dynamics

SMAI Project

Mentor: Dr. Naresh Manwani

Social trust Relation

- In open society based applications, we assume there are dependencies between users.
- Any user’s preference depends on the user’s trust on other users.
- Initial attempts assumes trust relations to be static.
- In real scenario, trust relations are dynamic and change with time.

Homophily Regularization:

The existence of homophily in trust relations and homophily effect indicates that users with higher similarity are more likely to establish trust relations. We define ζ(i, j) as the homophily coefficient between u_i and u_j , satisfying:

(1) $\zeta(i, j) \in [0, 1]$;

(2) $\zeta(i, j) = \zeta(j, i)$;

(3) the larger ζ(i, j) is, the more likely a trust relation is established between u_i and u_j . With homophily coefficient, homophily regularization is to minimize the following term as,

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} \zeta(i, j) \|\mathbf{U}(i, :) - \mathbf{U}(j, :)\|_2^2 ,$$

After some derivations, we can get the matrix form of homophily regularization,

$$
\begin{aligned}
&\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\zeta(i,j)\|\mathbf{U}(i,:)-\mathbf{U}(j,:)\|_2^2\\
&=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{d}\zeta(i,j)\left(\mathbf{U}(i,k)-\mathbf{U}(j,k)\right)^2\\
&=\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{d}\zeta(i,j)\mathbf{U}^2(i,k)\\
&\qquad-\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{d}\zeta(i,j)\mathbf{U}(i,k)\mathbf{U}(j,k)\\
&=\sum_{k=1}^{d}\mathbf{U}^\top(:,k)(\mathbf{D}-\mathcal{Z})\mathbf{U}(:,k)\\
&=Tr(\mathbf{U}^\top\mathcal{L}\mathbf{U}),
\end{aligned}
$$

where L = D - Z, is the Laplacian Matrix and D is the diagonal matrix with the i‘th diagonal element D(i,i)=$\sum_{j=1}^{n} Z(j,i)$ Z is the homophily coefficient matrix among n instances, defined as,

$$Z = \begin{pmatrix} \varsigma(1,1) & \varsigma(1,2) & \cdots & \varsigma(1,n) \\ \varsigma(2,1) & \varsigma(2,2) & \cdots & \varsigma(2,n) \\ \vdots & \vdots & \ddots & \vdots \\ \varsigma(n,1) & \varsigma(n,2) & \cdots & \varsigma(n,n) \end{pmatrix}$$

Social Trust Prediction Model

A trust prediction framework is proposed based on low rank matrix factorization as:

$$\min_{U,V} \| G - UVU^{\mathrm{T}} \|_F^2$$

where $G = \sum G_i$, $U \in R^{n*d}$ is the user preference matrix and d is the number of facets of user preference. $V \in R^{d*d}$ captures the more compact correlations among U, such as G(i,j) as $U(i,:)\ V\ U^T(j,:)$,

Updating Rule Derivation:

$$L = \|\mathbf{X} - \mathbf{U}\mathbf{V}^T\|_F^2 + \alpha\|\mathbf{U}\|_F^2 + \beta\|\mathbf{V}\|_F^2 - Tr(\Lambda_1\mathbf{U}^T) - Tr(\Lambda_2\mathbf{V}^T).$$

We have the following KKT condition,

$$\Lambda_1 \circ \mathbf{U} = \mathbf{0},$$
$$\Lambda_2 \circ \mathbf{V} = \mathbf{0},$$

where $\circ$ denotes the Hadamard product. We then have

$$\begin{aligned}
\frac{\partial L}{\partial \mathbf{U}} &= \frac{\partial Tr(\mathbf{V}\mathbf{U}^T\mathbf{U}\mathbf{V}^T - 2\mathbf{X}^T\mathbf{U}\mathbf{V}^T) + \alpha Tr(\mathbf{U}^T\mathbf{U}) - Tr(\Lambda_1\mathbf{U}^T)}{\partial \mathbf{U}} \\
&= 2(\mathbf{U}\mathbf{V}^T\mathbf{V} - \mathbf{X}\mathbf{V} + \alpha\mathbf{U}) - \Lambda_1, \\
\frac{\partial L}{\partial \mathbf{V}} &= \frac{\partial Tr(\mathbf{V}\mathbf{U}^T\mathbf{U}\mathbf{V}^T - 2\mathbf{X}^T\mathbf{U}\mathbf{V}^T) + \beta Tr(\mathbf{V}^T\mathbf{V}) - Tr(\Lambda_2\mathbf{V}^T)}{\partial \mathbf{V}} \\
&= 2(\mathbf{V}\mathbf{U}^T\mathbf{U} - \mathbf{X}^T\mathbf{U} + \beta\mathbf{V}) - \Lambda_2.
\end{aligned}$$

Let $\frac{\partial L}{\partial \mathbf{U}} = 0$ and $\frac{\partial L}{\partial \mathbf{V}} = 0$ as another KKT condition, we have

$$\Lambda_1 = 2(\mathbf{U}\mathbf{V}^T\mathbf{V} - \mathbf{X}\mathbf{V} + \alpha\mathbf{U}),$$
$$\Lambda_2 = 2(\mathbf{V}\mathbf{U}^T\mathbf{U} - \mathbf{X}^T\mathbf{U} + \beta\mathbf{V}).$$

Now we combine Eq. 8 and Eq. 10, we have

Continuation:

An **auxiliary function** $G(\mathbf{U}, \mathbf{U}^t)$ of function $L(\mathbf{U})$ is a function that satisfies

$$G(\mathbf{U}, \mathbf{U}) = L(\mathbf{U}), \;\; G(\mathbf{U}, \mathbf{U}^t) \geq L(\mathbf{U}). \tag{13}$$

Then, if we take $\mathbf{U}^{t+1}$ such that

$$\mathbf{U}^{t+1} = \arg\min_{U} G(\mathbf{U}, \mathbf{U}^t), \tag{14}$$

we have

$$L(\mathbf{U}^{t+1}) \leq G(\mathbf{U}^{t+1}, \mathbf{U}^t) \leq G(\mathbf{U}^t, \mathbf{U}^t \leq L(\mathbf{U}^t)). \tag{15}$$

This proves that $L(U)$ is monotonically decreasing.

Step 1 - Finding an appropriate auxiliary function needs to take advantage of two inequalities,

$$z \geq 1 + logz, \forall z > 0, \tag{16}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{k} \frac{(\mathbf{AS}'\mathbf{B})(i,j)\mathbf{S}(i,j)^2}{\mathbf{S}'(i,j)} \geq Tr(\mathbf{S}^T\mathbf{ASB}),$$

$$\forall \mathbf{A} \in \mathbb{R}_+^{m\times m}, \mathbf{B} \in \mathbb{R}_+^{k\times k}, \mathbf{S}' \in \mathbb{R}_+^{m\times k}, \mathbf{S} \in \mathbb{R}_+^{m\times k}. \tag{17}$$

The proof for Eq. 17 can be found in [5] (Proposition 6).

After removing irrelevant terms, the objective function Eq. 6 in terms of $\mathbf{U}$ can be written as

$$\begin{aligned} &Tr(\mathbf{VU}^T\mathbf{UV}^T - 2\mathbf{X}^T\mathbf{UV}^T) + \alpha Tr(\mathbf{U}^T\mathbf{U}) \\ =&Tr(\mathbf{U}^T\mathbf{UV}^T\mathbf{V} - 2\mathbf{U}^T\mathbf{XV}) + \alpha Tr(\mathbf{U}^T\mathbf{U}) \end{aligned} \tag{18}$$

$$\sum_{i}\sum_{j} e^{-n_i(m-g_{ij}^+)} \| G(i,j) - U(i,:)VU^T(j,:)\|^2$$

$$+ \alpha\sum_i \|n_i\|^2 + \beta\|U\|^2 + \gamma\|V\|^2$$

$$U, V \geq 0 \quad ; n_i \geq 0.$$

$$= \|G(i,j) - U(i,:)VU^T(j,:)\|_2^2$$

$$= -\sum_j c_{ij}(m-g_{ij}^+)\, e^{-n_i(m-g_{ij}^+)} + 2\alpha n_i$$

$$2\alpha n_i + \sum c_{ij}\, g_{ij}^+\, e^{-n_i(m-g_{ij}^+)}$$

$$- \sum_j c_{ij}\, m\, e^{-n_i(m-g_{ij}^+)}$$

$$L(U) = Tr(G^TG + (UV^TU^T)(UVU^T) - 2G^TUVU^T) + \beta\, Tr(U^TU)$$

$$\frac{\partial L}{\partial U} = UV^TU^TU(V+V^T) + UVU^TU(V^T+V) + 2\beta U - \frac{\partial [G^TUVU^T]}{\partial U}$$

$$= 2U[V^TU^TUV + V^TU^TUV^T + 2\beta I - G^TUV - G^TUV]$$

$$\Rightarrow U(i,k) \leftarrow U(i,k)\sqrt{\frac{a_i^T(k)}{[U(i,:)A_i](k)}}$$

$$a_i = \sum_{j=1}^{n} b_{ij}^+\, G(i,j)\, VU^T(j,:) + \sum_{j=1}^{n} b_{ji}^+\, G(j,i)\, V^TU^T(j,:)$$

$$A_i = \sum_j b_{ij}\, VU^T(j,:)U(j,:)V^T + \sum_j b_{ji}^+\, V^TU^T(j,:)U(j,:)V + \beta I$$

$$\frac{\partial L}{\partial V} = U^TUVU^TU + U^TUV^TU^T \ldots - 2GU^TU$$

$$\Rightarrow V(i,k) \leftarrow V(i,k)\sqrt{\frac{B(i,k)}{C(i,k)}}$$

$$B = \sum_i\sum_j b_{ij}\, G(i,j)\, U^T(i,:)$$

$$C = \sum_i\sum_j b_{ij}\, U^T(i,:)U(j,:) \ldots U^T($$

Temporal Weight Matrix Factorization in trust prediction

- Social trust relation depend on time distance between the current time and the time on which trust relation was established.
- These trust relations decay with time.
- The rates of decay may be different for different for different users.
- Prediction error is related to the time distance.
- In prediction error we want to incorporate the effect of time on trust relations.

Continued...

- Let $g_{i,j}^{t}$ be the first timestamp when u_i trusted u_j .
- Earlier trust relations reflect users' previous preferences and should have less influence on the current trust prediction .
- Exponential time function $e^{-\eta_i (m - g_{ij}^{t})}$ is added. 'm' is current time.

In the error term, we multiply the time decay factor, the error in prediction of a trust relation from user u_i to u_j be.

$$e^{-\eta_i (m - g_{ij}^{t})} \| G(i,j) - U(i,:) V U^{T}(j,:) \|_2^2$$

Minimizing the Error in prediction

Given the error function mentioned before, we write its lagrange to minimize the error as follows.

$$\min_{U,V,\eta_i} \sum_{i=1}^{n} \sum_{j=1}^{n} e^{-\eta_i} (m - g_{ij}^{t}) \| G(i,j) - U(i,:)VU^{T}(j,:) \|_2^2 + \alpha \sum_{i=1}^{n} \| \eta_i \|_2^2$$
$$+ \beta \| U \|_F^2 + \gamma \| V \|_F^2 \qquad s.t.\ U, V \geq 0, \quad \eta_i \geq 0, \ \forall i \in [1, n]$$

Algorithm:

Algorithm 1 TWMF for Trust Prediction

Input: $\{G_1, G_2, \cdots, G_m\}$, α, β and γ.

Output: $\hat{G}$

1: $G = \sum_{i=1}^{m} G_i$

2: Initialize V randomly;

3: Initialize η_i randomly;

4: Initialize $U(i,:)$ randomly

5: **while** not reach convergence or the maximal iteration

6: Update $\eta_i \leftarrow \eta_i \sqrt{\frac{\sum_{j=1}^{n} c_{ij} e^{-\eta_i (m - g_{ij}^t)}}{2\alpha\eta_i + \sum_{j=1}^{n} c_{ij} e^{-\eta_i (m - g_{ij}^t)}}}$;

7: Update $U(i,k) \leftarrow U(i,k) \sqrt{\frac{a_i^{\mathrm{T}}(k)}{[U(i,:) A_i](k)}}$

8: $V(i,k) \leftarrow V(i,k) \sqrt{\frac{B(i,k)}{C(i,k)}}$

9: end **while**

10: $\hat{G} = UVU^{\mathrm{T}}$

where a_i and A_i are defined as follows:

$$a_i = \sum_{j=1}^{n} b_{ij}^t G(i,j) V U^{\mathrm{T}}(j,:) + \sum_{j=1}^{n} b_{ji}^t G(j,i) V^{\mathrm{T}} U^{\mathrm{T}}(j,:)$$

$$A_i = \sum_{j=1}^{n} b_{ij}^t V U^{\mathrm{T}}(j,:) U(j,:) V^{\mathrm{T}}]$$

$$+ \sum_{j=1}^{n} b_{ji}^t V^{\mathrm{T}} U(j,:)^{\mathrm{T}} U(j,:) V + \beta I$$

where B and C are defined as

$$B = \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij}^t G(i,j) U^{\mathrm{T}}(i,:) U(j,:)$$

$$C = \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij}^t U^{\mathrm{T}}(i,:) U(j,:) V^{\mathrm{T}} U^{\mathrm{T}}(i,:) U(j,:) + \gamma V$$

where $c_{ij} = \| G(i,j) - U(i,:) V U^{\mathrm{T}}(j,:) \|_2^2$

$$b_{ij}^t = e^{-\eta_i (m - g_{ij}^t)} \quad \text{and} \quad b_{ji}^t = e^{-\eta_j (m - g_{ji}^t)},$$

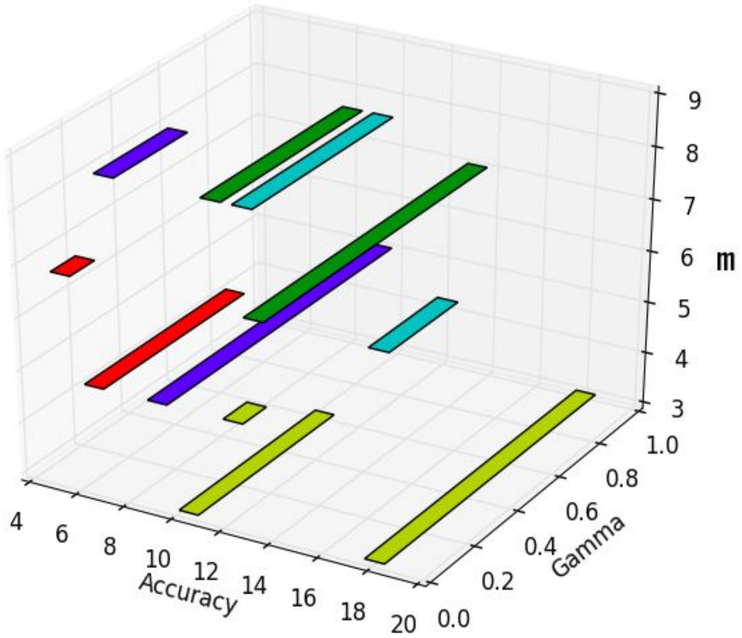
Experiment Settings

- Used dataset generated by www.epinions.com .
- We choose trust relations from t_i to t_{l+i-1} t as old trust relations O with the time window size
- N is the new trust relations established at t_{l+i}.
- A is the trust relation predicted excluding O.
- We sort values in A and take top N pairs as C.

Prediction Accuracy (PA) = $|N \cap C| / |N|$

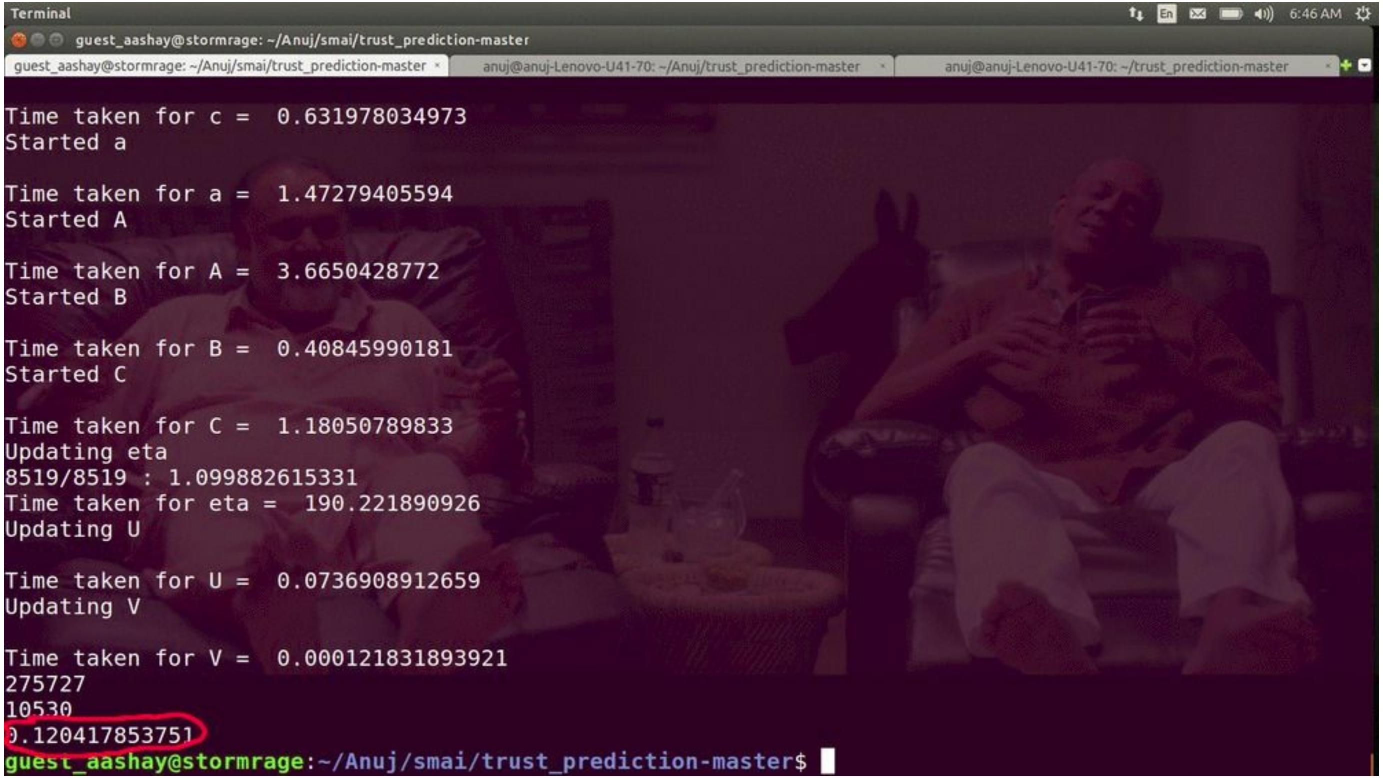
Results

- The following is the graph of The obtained results (accuracy) with varying values of m (time till which the trust relation is known) and gamma (varied from 0.1 to 1).
- The bars of same colors are shows the results obtained with same number of epochs.
- The average time taken for one epoch is about 210 seconds.



(Epocs,colour) - (10 - Red), (5,blue), (6,green), (4,Yellow), (3,Cyan)

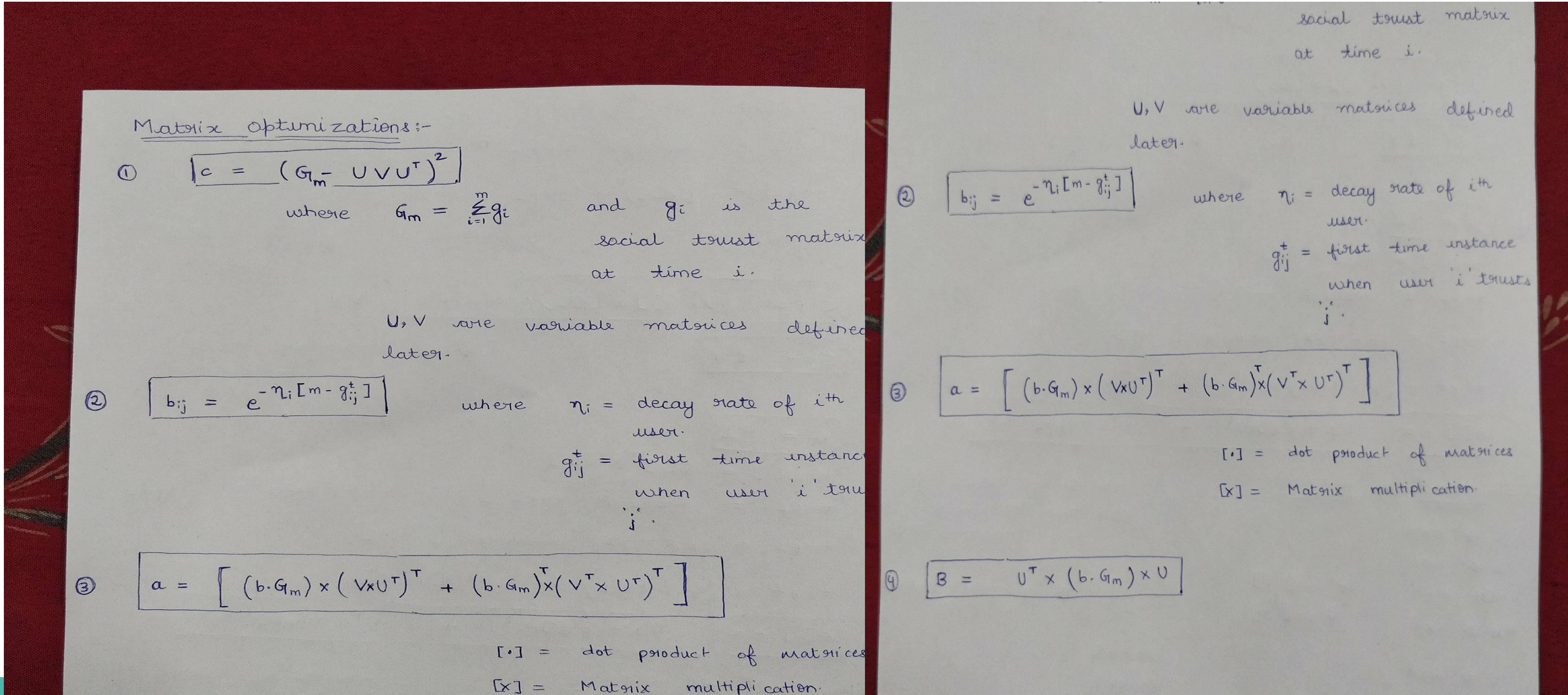
Continued ..


Terminal
6:46 AM
guest_aashay@stormrage: ~/Anuj/smai/trust_prediction-master
guest_aashay@stormrage: ~/Anuj/smai/trust_prediction-master
anuj@anuj-Lenovo-U41-70: ~/Anuj/trust_prediction-master
anuj@anuj-Lenovo-U41-70: ~/trust_prediction-master
Time taken for c = 0.631978034973
Started a
Time taken for a = 1.47279405594
Started A
Time taken for A = 3.6650428772
Started B
Time taken for B = 0.40845990181
Started C
Time taken for C = 1.18050789833
Updating eta
8519/8519 : 1.099882615331
Time taken for eta = 190.221890926
Updating U
Time taken for U = 0.0736908912659
Updating V
Time taken for V = 0.000121831893921
275727
10530
0.120417853751
guest_aashay@stormrage:~/Anuj/smai/trust_prediction-master$

Continued ..

```
Terminal
guest_aashay@stormrage: ~/Anuj/smai/trust_prediction-master
guest_aashay@stormrage: ~/Anuj/smai/trust_prediction-master    anuj@anuj-Lenovo-U41-70: ~/Anuj/trust_prediction-master    anuj@anuj-Lenovo-U41-70: ~/trust_prediction-master
Time taken for V =  0.000141143798828
epochs # 2
Started c

Time taken for c =  0.648800849915
Started a

Time taken for a =  1.4781460762
Started A

Time taken for A =  3.63032507896
Started B

Time taken for B =  0.429671049118
Started C

Time taken for C =  1.22097206116
Updating eta
8519/8519 : 1.099882615331
Time taken for eta =  196.742071867
Updating U

Time taken for U =  0.0756840705872
Updating V

Time taken for V =  0.000118017196655
epochs # 3
```

Implementation (Matrix optimizations)



Matrix optimizations:-

① $\boxed{c = (G_m - UVU^T)^2}$

where $G_m = \sum_{i=1}^{m} g_i$ and g_i is the social trust matrix at time i.

U, V are variable matrices defined later.

② $\boxed{b_{ij} = e^{-\eta_i [m - g^t_{ij}]}}$

where η_i = decay rate of i^{th} user.

g^t_{ij} = first time instance when user 'i' trusts 'j'.

③ $\boxed{a = \left[(b \cdot G_m) \times (V \times U^T)^T + (b \cdot G_m)^T \times (V^T \times U^T)^T \right]}$

[•] = dot product of matrices

[x] = Matrix multiplication.

social trust matrix at time i.

U, V are variable matrices defined later.

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[•] = dot product of matrices

[x] = Matrix multiplication.

④ $\boxed{B = U^T \times (b \cdot G_m) \times U}$

Continuation ..

⑤ $tmp1[i] = U[i]^T \times U[i]$

$tmp2 =$ reshaped $tmp1\ [\ (n) \times (d) \times (d) \longrightarrow (n) \times (d \times d)\ ]$

$tmp3[i] = [(b[i])^T \times tmp2]$. reshape $([d, d])$

where d = no of faceits features

$$\boxed{C \mathrel{+}= (V^T U[i]^T \times U[i])^T \times tmp3[i]} \quad \forall i$$

$$\boxed{C \mathrel{+}= \gamma V}$$

$\gamma \Rightarrow$ a variable that is varied in our model

⑥ $\boxed{A = \beta I}$

$\beta \Rightarrow$ fixed variable

$tmp1[i] = U[i]^T \times U[i]$

$tmp2 =$ reshaped $tmp1\ [\ (n) \times (d) \times (d) \longrightarrow (n) \times (d \times d)\ ]$

$tmp3 = (b \times tmp2)$ reshaped $([d, d])$

$tmp4 = (b^T \times tmp2)$ reshaped $([d, d])$

$$\boxed{A[i] \mathrel{+}= V \times tmp3[i] \times V^T + V^T \times tmp4[i] \times V} \quad \forall i$$

Dimensions of matrices :-

n = # of users

$$\boxed{C \mathrel{+}= \gamma V}$$

$\gamma \Rightarrow$ a variable that is varied in our model

⑥ $\boxed{A = \beta I}$

$\beta \Rightarrow$ fixed variable

$tmp1[i] = U[i]^T \times U[i]$

$tmp2 =$ reshaped $tmp1\ [\ (n) \times (d) \times (d) \longrightarrow (n) \times (d \times d)\ ]$

$tmp3 = (b \times tmp2)$ reshaped $([d, d])$

$tmp4 = (b^T \times tmp2)$ reshaped $([d, d])$

$$\boxed{A[i] \mathrel{+}= V \times tmp3[i] \times V^T + V^T \times tmp4[i] \times V} \quad \forall i$$

Dimensions of matrices :-

$c \Rightarrow n \times n$

$b \Rightarrow n \times n$

$a(i) \Rightarrow d \times 1$ $\quad \forall\ i = [1, n]$

$A(i) \Rightarrow d \times d$ $\quad \forall\ i = [1, n]$

$B \Rightarrow d \times d$

$C \Rightarrow d \times d$

$g \Rightarrow n \times n$

$G \Rightarrow n \times n$

$N \Rightarrow n \times d$

n = # of users.

m = time till which trust matrices are known.

d = # of faceit features

$U \Rightarrow n \times d$

$V \Rightarrow d \times d$

Predicted trust Matrix :-

$G' = U V U^T$

Presented By: Team 30

Amandeep Shahi (201501087)

Anuj Goyal (201501129)

Swapnil Gupta (201501175)

Aayush Tiwari (201531031)

Thank You!