Trust Prediction using Temporal Dynamics

SMAI Project
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Social trust Relation

- In open society based applications, we assume there are dependencies between users.

- Any user's preference depends on the user's trust on other users.

- Initial attempts assumes trust relations to be static.

- In real scenario, trust relations are dynamic and change with time.

Homophily Regularization:

The existence of homophily in trust relations and homophily effect indicates that users with higher similarity are more likely to establish trust relations. We define $\zeta(i, j)$ as the homophily coefficient between u_i and u_j, satisfying:

(1) $\zeta(i, j) \in [0, 1]$;

(2) $\zeta(i, j) = \zeta(j, i)$;

(3) the larger $\zeta(i, j)$ is, the more likely a trust relation is established between u_i and u_j. With homophily coefficient, homophily regularization is to minimize the following term as,

$$\min \quad \sum_{i=1}^{n} \sum_{j=1}^{n} \zeta(i, j) \| \mathbf{U}(i, :) - \mathbf{U}(j, :) \|_2^2,$$

After some derivations, we can get the matrix form of homophily regularization,

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\zeta(i,j)\|\mathbf{U}(i,:)-\mathbf{U}(j,:)\|_2^2$$

$$=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{d}\zeta(i,j)(\mathbf{U}(i,k)-\mathbf{U}(j,k))^2$$

$$=\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{d}\zeta(i,j)\mathbf{U}^2(i,k)$$

$$-\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{d}\zeta(i,j)\mathbf{U}(i,k)\mathbf{U}(j,k)$$

$$=\sum_{k=1}^{d}\mathbf{U}^\top(:,k)(\mathbf{D}-\mathcal{Z})\mathbf{U}(:,k)$$

$$=Tr(\mathbf{U}^\top\mathcal{L}\mathbf{U}),$$

where L = D - Z, is the Laplacian Matrix and D is the diagonal matrix with the i'th diagonal element D(i,i)=$\sum_{j=1}^{\tilde{n}} \mathcal{Z}(j,i)$ Z is the homophily coefficient matrix among n instances, defined as,

$$\mathcal{Z} = \begin{pmatrix} \zeta(1,1) & \zeta(1,2) & \cdots & \zeta(1,n) \\ \zeta(2,1) & \zeta(2,2) & \cdots & \zeta(2,n) \\ \vdots & \vdots & \ddots & \vdots \\ \zeta(n,1) & \zeta(n,2) & \cdots & \zeta(n,n) \end{pmatrix}$$

Social Trust Prediction Model

A trust prediction framework is proposed based on low rank matrix factorization as:

$$\min_{U,V} \| G - UVU^{\mathrm{T}} \|_F^2$$

where $G = \sum G_i$, $U \in R^{n*d}$ is the user preference matrix and d is the number of facets of user preference. $V \in R^{d*d}$ captures the more compact correlations among U, such as G(i,j) as U(i,:) V U^{T}(j,:),

Updating Rule Derivation:

$$L = \|\mathbf{X} - \mathbf{U}\mathbf{V}^T\|_F^2 + \alpha\|\mathbf{U}\|_F^2 + \beta\|\mathbf{V}\|_F^2 - Tr(\Lambda_1\mathbf{U}^T) - Tr(\Lambda_2\mathbf{V}^T).$$

We have the following KKT condition,

$$\Lambda_1 \circ \mathbf{U} = \mathbf{0},$$
$$\Lambda_2 \circ \mathbf{V} = \mathbf{0},$$

where $\circ$ denotes the Hadamard product. We then have

$$\frac{\partial L}{\partial \mathbf{U}} = \frac{\partial Tr(\mathbf{V}\mathbf{U}^T\mathbf{U}\mathbf{V}^T - 2\mathbf{X}^T\mathbf{U}\mathbf{V}^T) + \alpha Tr(\mathbf{U}^T\mathbf{U}) - Tr(\Lambda_1\mathbf{U}^T)}{\partial \mathbf{U}}$$
$$= 2(\mathbf{U}\mathbf{V}^T\mathbf{V} - \mathbf{X}\mathbf{V} + \alpha\mathbf{U}) - \Lambda_1,$$
$$\frac{\partial L}{\partial \mathbf{V}} = \frac{\partial Tr(\mathbf{V}\mathbf{U}^T\mathbf{U}\mathbf{V}^T - 2\mathbf{X}^T\mathbf{U}\mathbf{V}^T) + \beta Tr(\mathbf{V}^T\mathbf{V}) - Tr(\Lambda_2\mathbf{V}^T)}{\partial \mathbf{V}}$$
$$= 2(\mathbf{V}\mathbf{U}^T\mathbf{U} - \mathbf{X}^T\mathbf{U} + \beta\mathbf{V}) - \Lambda_2.$$

Let $\frac{\partial L}{\partial \mathbf{U}} = 0$ and $\frac{\partial L}{\partial \mathbf{V}} = 0$ as another KKT condition, we have

$$\Lambda_1 = 2(\mathbf{U}\mathbf{V}^T\mathbf{V} - \mathbf{X}\mathbf{V} + \alpha\mathbf{U}),$$
$$\Lambda_2 = 2(\mathbf{V}\mathbf{U}^T\mathbf{U} - \mathbf{X}^T\mathbf{U} + \beta\mathbf{V}).$$

Now we combine Eq. 8 and Eq. 10, we have

Continuation:

An **auxiliary function** $G(\mathbf{U}, \mathbf{U}^t)$ of function $L(\mathbf{U})$ is a function that satisfies

$$G(\mathbf{U}, \mathbf{U}) = L(\mathbf{U}), \ G(\mathbf{U}, \mathbf{U}^t) \geq L(\mathbf{U}). \tag{13}$$

Then, if we take $\mathbf{U}^{t+1}$ such that

$$\mathbf{U}^{t+1} = \arg \min_U G(\mathbf{U}, \mathbf{U}^t), \tag{14}$$

we have

$$L(\mathbf{U}^{t+1}) \leq G(\mathbf{U}^{t+1}, \mathbf{U}^t) \leq G(\mathbf{U}^t, \mathbf{U}^t \leq L(\mathbf{U}^t)). \tag{15}$$

This proves that $L(U)$ is monotonically decreasing.

Step 1 - Finding an appropriate auxiliary function needs to take advantage of two inequalities,

$$z \geq 1 + log z, \ \forall z > 0, \tag{16}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{k} \frac{(\mathbf{A S' B})(i,j) \mathbf{S}(i,j)^2}{\mathbf{S'}(i,j)} \geq Tr(\mathbf{S}^T \mathbf{A S B}),$$

$$\forall \mathbf{A} \in \mathbb{R}_+^{m \times m}, \mathbf{B} \in \mathbb{R}_+^{k \times k}, \mathbf{S'} \in \mathbb{R}_+^{m \times k}, \mathbf{S} \in \mathbb{R}_+^{m \times k}. \tag{17}$$

The proof for Eq. 17 can be found in [5] (Proposition 6).

After removing irrelevant terms, the objective function Eq. 6 in terms of $\mathbf{U}$ can be written as

$$Tr(\mathbf{V U}^T \mathbf{U V}^T - 2\mathbf{X}^T \mathbf{U V}^T) + \alpha Tr(\mathbf{U}^T \mathbf{U})$$

$$= Tr(\mathbf{U}^T \mathbf{U V}^T \mathbf{V} - 2\mathbf{U}^T \mathbf{X V}) + \alpha Tr(\mathbf{U}^T \mathbf{U}) \tag{18}$$

$$\sum_{n_i}\sum_i\sum_j e^{-n_i(m-g_{ij})}\,\| G(i,j)-U(i,:)VU^T(j,:)\|^2$$

$$+\,\alpha\sum_i\|n_i\|^2+\beta\|U\|^2+\gamma\|V\|^2$$

$$U,V\ge 0\;;\; n_i\ge 0.$$

$$j=\| G(i,j)-U(i,:)VU^T(j,:)\|_2^2$$

$$=-\sum_j c_{ij}(m-g_{ij})\,e^{-n_i(m-g_{ij})+2\alpha n_i}$$

$$2\alpha n_i+\sum c_{ij}\,g_{ij}^+\,e^{-n_i(m-g_{ij})^+}$$

$$-\sum c_{ij}\,m\,e^{-n_i(m-g_{ij})}$$

$$H(u)=Tr\big(G^TG+(UV^TU^T)(UVU^T)-2G^TUVU^T\big)$$

$$+\beta\,Tr(U^TU)$$

$$\frac{\partial L}{\partial U}=UV^TU^TU(V+V^T)+UVU^TU(V^T+V)+2\beta U$$

$$-2G^TUVU^T$$

$$=2U\Big[V^TU^TUV+V^TU^TUV^T+2\beta$$

$$-\,G^TUV$$

$$-\,G^TUV\Big]$$

$$\Rightarrow\ U(i,k)\Longleftarrow U(i,k)\sqrt{\frac{a_i^T(k)}{[(U,:)A_i](k)}}$$

$$a_i=\sum_{j=1}^n c_{ij}\,G(i,j)\,VU^T(j,:)$$

$$+\sum_{j=1} c_{ji}\,G(j,i)\,V^TU^T(j,:)$$

$$A_i=\sum_j c_{ij}\,VU^T(j,:)U(j,:)V^T$$

$$+\sum_j c_{ji}\,V^TU(j,:)U(j,:)V+\beta I$$

$$\frac{\partial L}{\partial V}=U^TUV^TU^TU+U^TUV^TU^T$$

$$-\,2\,G_iU^TU$$

$$\Rightarrow\ V(i,k)\Longleftarrow V(i,k)\sqrt{\frac{B(i,k)}{C(i,k)}}$$

$$B=\sum_i\sum_j c_{ij}\,G(i,j)\,U^T(i,:)$$

$$C=\sum_i\sum_j c_{ij}\,U^T(i,:)U(j,:)$$

Temporal Weight Matrix Factorization in trust prediction

- Social trust relation depend on time distance between the current time and the time on which trust relation was established.
- These trust relations decay with time.
- The rates of decay may be different for different for different users.
- Prediction error is related to the time distance.
- In prediction error we want to incorporate the effect of time on trust relations.

Continued...

- Let $g_{i,j}^t$ be the first timestamp when u_i trusted u_j.
- Earlier trust relations reflect users' previous preferences and should have less influence on the current trust prediction.
- Exponential time function $e^{-\eta_i(m-g_{ij}^t)}$ is added. 'm' is current time.

In the error term, we multiply the time decay factor, the error in prediction of a trust relation from user u_i to u_j be.

$$e^{-\eta_i(m-g_{ij}^t)} \| G(i,j) - U(i,:)VU^{\mathsf{T}}(j,:) \|_2^2$$

Minimizing the Error in prediction

Given the error function mentioned before, we write its lagrange to minimize the error as follows.

$$\min_{U,V,\eta_i} \sum_{i=1}^{n} \sum_{j=1}^{n} e^{-\eta_i} (m - g_{ij}^{t}) \| G(i,j) - U(i,:)VU^{T}(j,:) \|_2^2 + \alpha \sum_{i=1}^{n} \| \eta_i \|_2^2$$

$$+ \beta \| U \|_F^2 + \gamma \| V \|_F^2 \qquad s.t. \ U, V \geq 0, \quad \eta_i \geq 0, \ \forall i \in [1, n]$$

Algorithm:

Algorithm 1 TWMF for Trust Prediction

Input: $\{G_1, G_2, \cdots, G_m\}$, α, β and γ.

Output: $\hat{G}$

1: $G = \sum_{i=1}^{m} G_i$

2: Initialize V randomly;

3: Initialize η_i randomly;

4: Initialize $U(i,:)$ randomly

5: **while** not reach convergence or the maximal iteration

6: Update $\quad \eta_i \leftarrow \eta_i \sqrt{\dfrac{\sum_{j=1}^{n} c_{ij} e^{-\eta_i (m - g_{ij}^t)}}{2\alpha\eta_i + \sum_{j=1}^{n} c_{ij} e^{-\eta_i (m - g_{ij}^t)}}}$;

7: Update $\quad U(i,k) \leftarrow U(i,k) \sqrt{\dfrac{a_i^{\mathrm{T}}(k)}{[U(i,:)A_i](k)}}$

8: $V(i,k) \leftarrow V(i,k) \sqrt{\dfrac{B(i,k)}{C(i,k)}}$

9: end **while**

10: $\hat{G} = UVU^{\mathrm{T}}$

where a_i and A_i are defined as follows:

$$a_i = \sum_{j=1}^{n} b_{ij}^t G(i,j) V U^{\mathrm{T}}(j,:) + \sum_{j=1}^{n} b_{ji}^t G(j,i) V^{\mathrm{T}} U^{\mathrm{T}}(j,:)$$

$$A_i = \sum_{j=1}^{n} b_{ij}^t V U^{\mathrm{T}}(j,:) U(j,:) V^{\mathrm{T}}]$$

$$+ \sum_{j=1}^{n} b_{ji}^t V^{\mathrm{T}} U(j,:)^{\mathrm{T}} U(j,:) V + \beta I$$

where B and C are defined as

$$B = \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij}^t G(i,j) U^{\mathrm{T}}(i,:) U(j,:)$$

$$C = \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij}^t U^{\mathrm{T}}(i,:) U(j,:) V^{\mathrm{T}} U^{\mathrm{T}}(i,:) U(j,:) + \gamma V$$

where $c_{ij} = \| G(i,j) - U(i,:) V U^{\mathrm{T}}(j,:) \|_2^2$

$$b_{ij}^t = e^{-\eta_i (m - g_{ij}^t)} \quad \text{and} \quad b_{ji}^t = e^{-\eta_j (m - g_{ji}^t)},$$

Experiment Settings

- Used dataset generated by *www.epinions.com* .
- We choose trust relations from t_i to t_{l+i-1} t as old trust relations O with the time window size
- N is the new trust relations established at t_{l+i}.
- A is the trust relation predicted excluding O.
- We sort values in A and take top N pairs as C.

Prediction Accuracy (PA) = |N∩C|/|N|

Results

- The following is the graph of The obtained results (accuracy) with varying values of m (time till which the trust relation is known) and gamma (varied from 0.1 to 1).
- The bars of same colors are shows the results obtained with same number of epochs.
- The average time taken for one epoch is about 210 seconds.
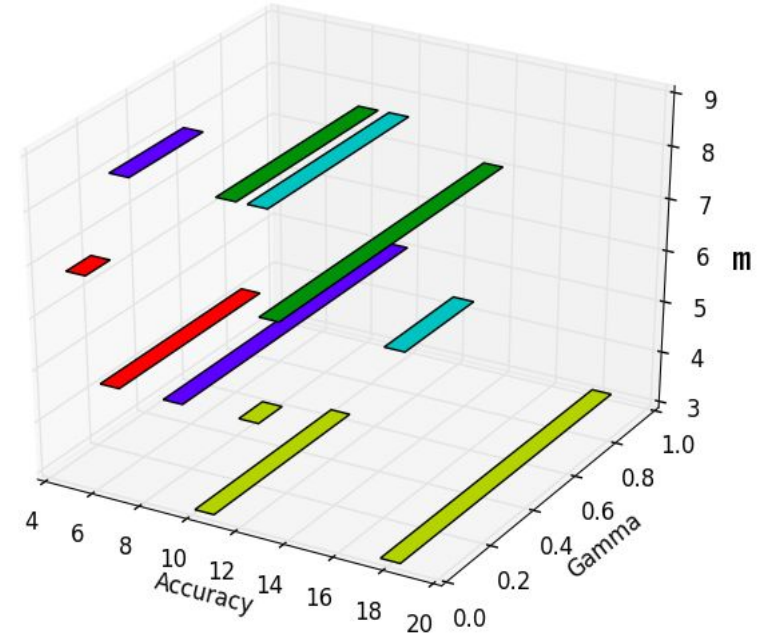
(Epocs,colour) - (10 - Red), (5,blue), (6,green), (4,Yellow), (3,Cyan)

Continued ..

Terminal — guest_aashay@stormrage: ~/Anuj/smai/trust_prediction-master

```
Time taken for c =  0.631978034973
Started a

Time taken for a =  1.47279405594
Started A

Time taken for A =  3.6650428772
Started B

Time taken for B =  0.40845990181
Started C

Time taken for C =  1.18050789833
Updating eta
8519/8519 : 1.099882615331
Time taken for eta =  190.221890926
Updating U

Time taken for U =  0.0736908912659
Updating V

Time taken for V =  0.000121831893921
275727
10530
0.120417853751
guest_aashay@stormrage:~/Anuj/smai/trust_prediction-master$
```

Continued ..

Implementation (Matrix optimizations)

Matrix Optimizations:-

① $$c = (G_m - UVU^T)^2$$

where $G_m = \sum_{i=1}^{m} g_i$ and g_i is the social trust matrix at time i.

U, V are variable matrices defined later.

② $$b_{ij} = e^{-\eta_i [m - g_{ij}^t]}$$

where η_i = decay rate of ith user.

g_{ij}^t = first time instance when user 'i' trusts 'j'.

③ $$a = \left[(b \cdot G_m) \times (V \times U^T)^T + (b \cdot G_m)^T \times (V^T \times U^T)^T \right]$$

$[\cdot]$ = dot product of matrices.

$[\times]$ = Matrix multiplication.

④ $$B = U^T \times (b \cdot G_m) \times U$$

Continuation ..

⑤ $tmp1[i] = U[i]^T \times U[i]$

$tmp2 = $ reshaped $tmp1 [(n) \times (d) \times (d) \longrightarrow (n) \times (d \times d)]$

$tmp3[i] = [(b[i])^* \times tmp2]$, reshape $([d, d])$

where $d = $ no of faceits features

$\boxed{C += (V_x^T U[i]^T \times U[i])^T \times tmp3[i]} \quad \forall i$

$\boxed{C += \gamma V}$

$\gamma \Rightarrow$ a variable that is varied in our model

$\beta \Rightarrow$ fixed variable

⑥ $\boxed{A = \beta I}$

$tmp1[i] = U[i]^T \times U[i]$

$tmp2 = $ reshaped $tmp1 [(n) \times (d) \times (d) \longrightarrow (n) \times (d \times d)]$

$tmp3 = (b \times tmp2)$ reshaped $([d, d])$

$tmp4 = (b^T \times tmp2)$ reshaped $([d, d])$

$\boxed{A[i] += V \times tmp3[i] \times V^T + V^T \times tmp4[i] \times V} \quad \forall i$

Dimensions of matrices :-

$c \Rightarrow n \times n$

$b \Rightarrow n \times n$

$a(i) \Rightarrow d \times 1$

$A(i) \Rightarrow d \times d$

$B \Rightarrow d \times d$

$c \Rightarrow d \times d$

$g \Rightarrow n \times n$

$G \Rightarrow n \times n$

$\forall i = [1, n]$
$\forall i = [1, n]$

$n = $ # of users.

$m = $ time till which trust matrices are known.

$d = $ # of faceit features

$U \Rightarrow n \times d$

$V \Rightarrow d \times d$

Predicted trust matrix :-

$G' = U V U^T$

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Github : https://github.com/amanshahi/trust_prediction

Thank You!