

Derivative of Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

we want to calculate this

$$\sigma(x) = \frac{d}{dx} \left(\frac{1}{1+e^{-x}} \right)$$

For calculating this we have to apply Reciprocal Rule

$$\frac{d}{dx} \left(\frac{1}{x} \right) = \frac{d}{dx} (x)^{-1} = -\frac{1}{x^2}$$

$$\frac{d}{dx} \left(\frac{1}{1+e^{-x}} \right) = \frac{d}{dx} \left[(1+e^{-x})^{-1} \right]$$

$$= -\frac{1}{(1+e^{-x})^2} \cdot \frac{d}{dx} (1+e^{-x})$$

$$= -\frac{1}{(1+e^{-x})^2} \cdot \frac{d}{dx} (e^{-x})$$

$$= -\frac{e^{-x}}{(1+e^{-x})^2} \cdot \frac{d}{dx} (-x)$$

$$= -\frac{e^{-x}}{(1+e^{-x})^2} \cdot (-1)$$

$$= \frac{1 \cdot e^{-x}}{(1+e^{-x})(1+e^{-x})} = \frac{1}{1+e^x} \cdot \frac{e^{-x}}{1+e^{-x}}$$

$$\sigma(x) = \left[\frac{e^{-x}}{1+e^{-x}} \right]$$

$$\begin{aligned} e^{-x} &= e^{-x} \\ -x &= -x \end{aligned}$$

$$= \sigma(x) \left[\frac{1 + e^{-x}}{1 + e^{-x}} - 1 \right]$$

$$= \sigma(x) \left[\frac{1 + e^{-x}}{1 + e^{-x}} - \underbrace{\frac{1}{1 + e^{-x}}}_{\text{Sigmoid}} \right]$$

$$= \sigma(x) [1 - \sigma(x)]$$

So derivative of sigmoid is

$$\sigma'(x) = \sigma(x) [1 - \sigma(x)]$$

Sigmoid derivative is

$$= \text{Sigmoid} \times (1 - \text{Sigmoid})$$