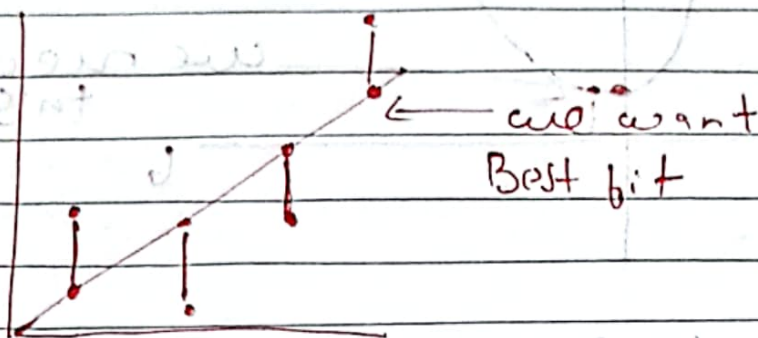


Gradient Descent

Intuition



Loss function
$$L = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

we know that $\hat{y}_i = mx_i + b$

$$L = \sum_{i=1}^n (y_i - (mx_i + b))^2$$

So Loss function is dependent on $L(m, b)$

Slope

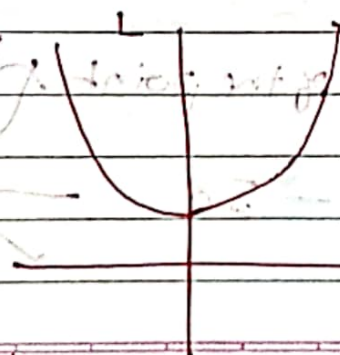
intercept

G_{pg}	L_{pg}
-	-
-	-
-	-
-	-

For understanding let's say $m = 78.35$

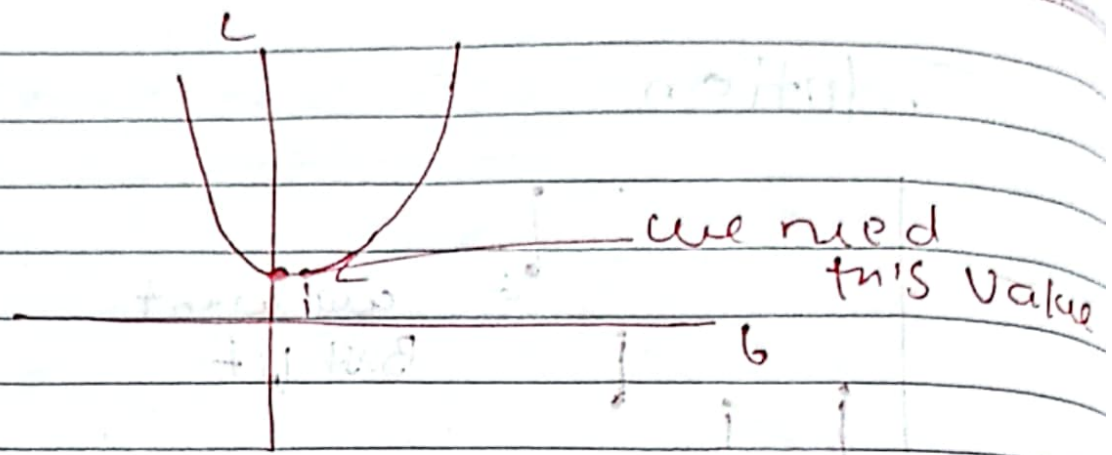
$$L = \sum_{i=1}^n (y_i - 78.35 * x_i - b)^2$$

we need the value of b so that L is minimum



because $L \propto b^2$

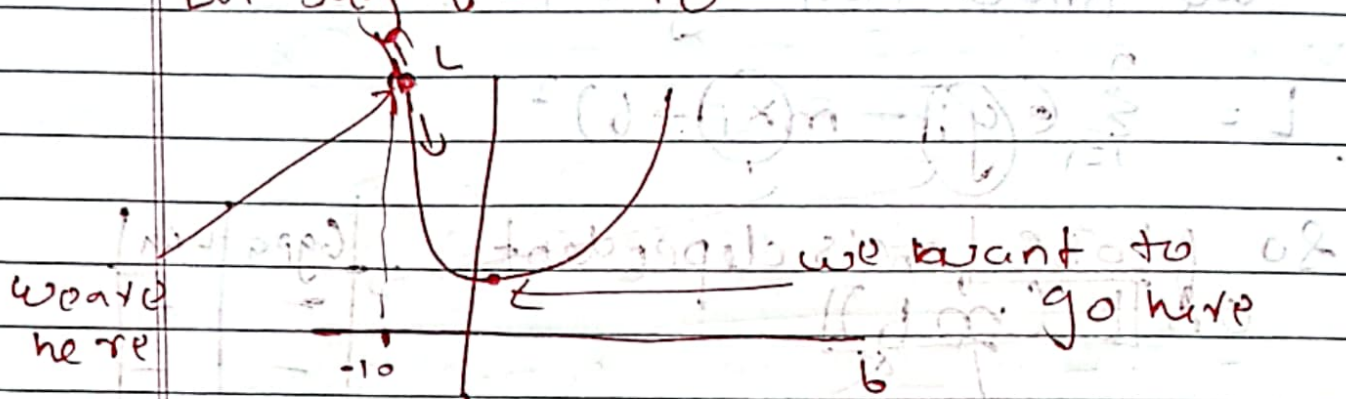
we need a value of b which makes L minimum



In Gradient Descent

Step (1): Select a Random b

Let say $b = -10$



So How would a Machine will know if we have to move forward or backward.

Answer is: By using Slope

If Slope is (+ve) we have to decrease

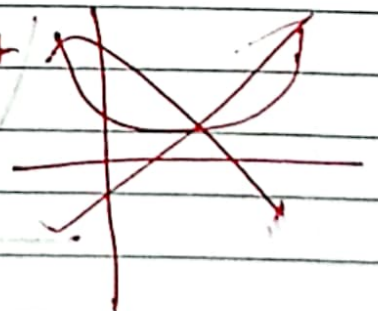
If Slope is (-ve) we have to increase b

$b_{new} = b_{old} - \text{slope of the point}$

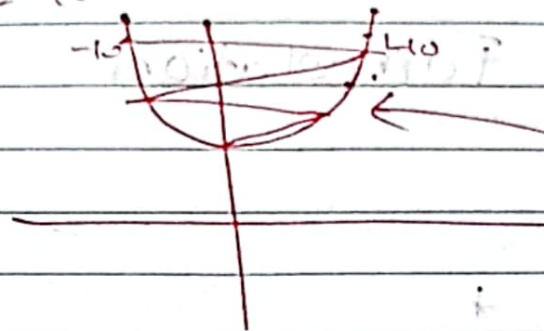
Let say slope = -50

$b_{new} = -10 - 80$

$b_{new} = 40$



$$b_{new} = 40$$



we use O.S.P

Learning rate

because we don't want this to happen

So

$$b_{new} = b_{old} - \alpha * \text{slope (partial derivative)}$$

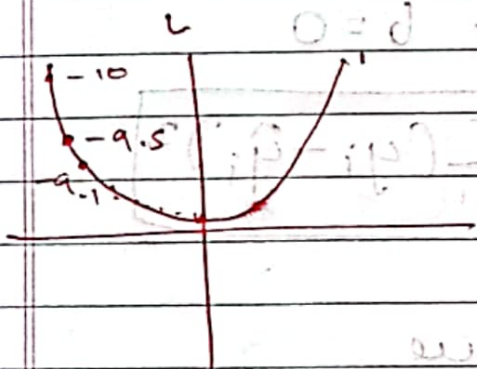
$$\alpha = 0.01$$

Learning rate

$$b_{new} = -10 - (0.01 \times -50)$$

$$= -10 + (0.01 \times 50)$$

$$= \underline{-9.5}$$



we use O.S.P

$$b_{new} = (-9.5) - (0.01 \times -40)$$

$$= -9.5 + (0.01 \times 40)$$

$$= -9.5 + 0.4$$

$$= \underline{-9.1}$$

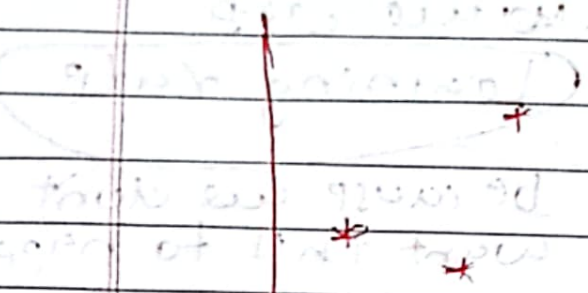
* When to stop because it is a iterative process

① Approach ① diff b_{new} b_{old} and b_{new} should be $\Rightarrow 0.0001$

② Iteration Limit

Epochs

* Mathematical Formulation



Lets say,

$$m = 28.35$$

Step (i) Start with Random Value

Lets say $b = 0$

How to find slope at $b = 0$

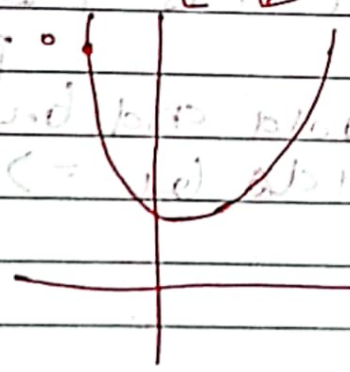
as we know

$$Loss = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

when we plot this

Slope eq. is

$$= -2 \sum_{i=1}^n (y_i - mx_i + b)$$



$$b = 0$$

(Partial derivative)

$$\text{Slope} = -2 \sum_{i=1}^n (y_i - mx_i - b)$$

$$= -2 \sum_{i=1}^n (y_i - mx_i - 0)$$

$$\text{Slope} = -2 \sum_{i=1}^n (y_i - 78.35x_i - 0)$$

Energy
target
Value

Energy
independent
Value

Then we will find
 b_{new} using

$$b_{\text{new}} = b_{\text{old}} - \alpha (-\text{slope})$$

$$= 0 - 0.01 \text{ Output}$$

$$b_{\text{new}} = \text{Value}$$

from this Value we will
find ~~new~~ b_{new} slope
then again b_{new} until
the given (epochs) ←
iteration

Let's say epoch = 10

then we will do this 10 time

Full intuition

Now we bind m and b both

Step ①:- Initialize random value
for m and b
lets $m = 1$ and $b = 0$

Epochs = 100 and $\alpha = 0.01$

for i in epochs
 $b = b - \alpha \cdot \text{slope}$
 $m = m - \alpha \cdot \text{slope}$

So,

$$L = \sum (y_i - \hat{y}_i)^2$$

$$L = \sum (y_i - mx_i - b)^2$$

Now Loss function dependent on

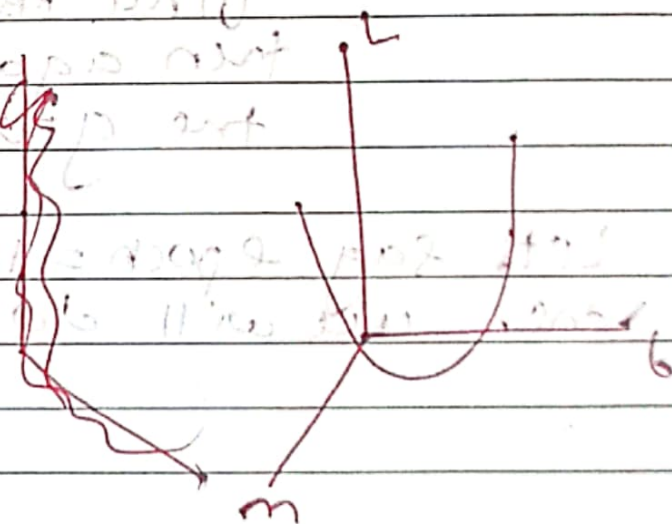
m and b

$L(m, b)$

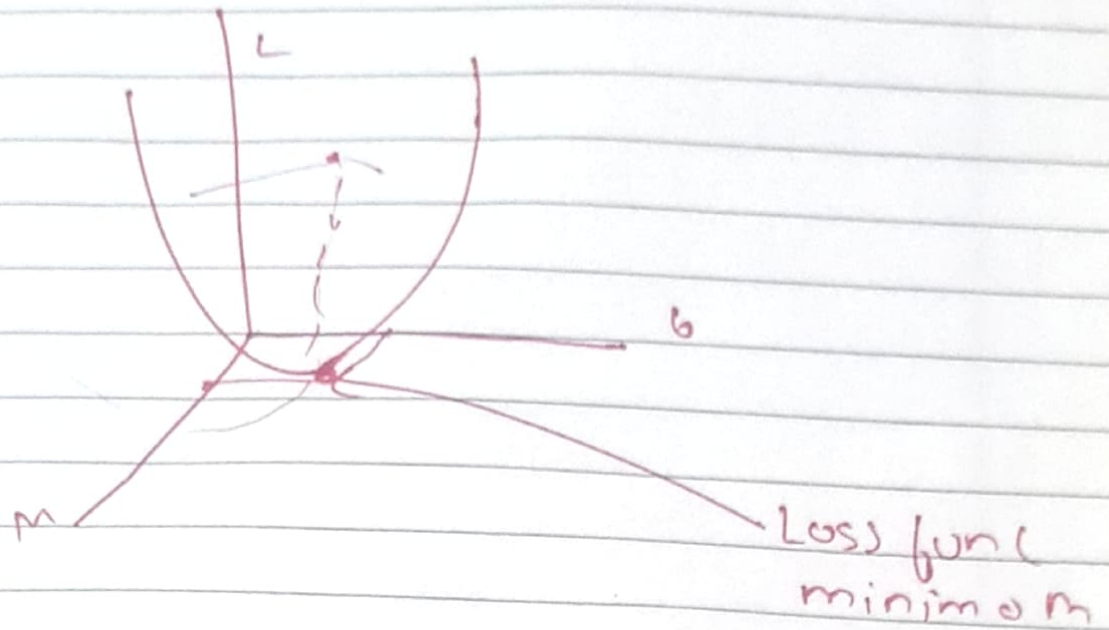
$L(m, b)$

$L(m, b)$

$L(m, b)$



See 39 graph in Github repo



Now slope formula will be

$$b\text{-slope} = \frac{\partial L}{\partial b}$$

$$m\text{-slope} = \frac{\partial L}{\partial m}$$

Partial
Derivative of the
Loss function at
Point m and b

$$\Sigma (y_i - mx_i - b)^2$$

Derivative wrt 'b'

$$\frac{\partial L}{\partial b} = \Sigma (y_i - mx_i - b)^2$$

$$\frac{\partial L}{\partial b} = -2 \Sigma (y_i - mx_i - b)$$

Derivative wrt 'm'

$$\frac{\partial L}{\partial m} = -2 \Sigma (y_i - mx_i - b) x_i$$

$$\frac{\partial L}{\partial m} = -2 \Sigma (y_i - mx_i - b) x_i$$