Stats_790_Assignment_1

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Question 1: Write a short description of any opinions, thoughts, or questions you are left with after reading Breiman, 2001, and one of the responses to it

Question 2: Pick a figure from ESL Chapter 2 and write, R, Python, or Julia code to replicate it

Question 3: ADA Problem 1.2

Question 4: ADA Problem 1.7

Suppose that the global mean is our linear smoother in this case. Recall that our influence matrix, w is a nxn matrix with the weight, w_{ij} saying how much each observation y_{ij} contributes to the fitted values.

Observing the classic mean formula of

$$\mu = (\frac{1}{n}) * (\Sigma_i^n y_i)$$

We observe the weight is essentially $\frac{1}{n}$ for every entry of the influence matrix w.

To observe this property visually, we construct an influence matrix as so,

$$w = \begin{pmatrix} 1/n & 1/n & \cdots & 1/n \\ 1/n & 1/n & \cdots & 1/n \\ \vdots & \ddots & \vdots & \\ 1/n & 1/n & \cdots & 1/n \end{pmatrix}$$

By Equation (1.70) of the textbook that states

$$df(\hat{\mu}) = tr(w)$$

we observe that the trace of w is essentially $tr(w) = \frac{1}{n} + \frac{1}{n} + ... + \frac{1}{n} = 1$.

Thus we conclude that the degrees of freedom for when the global mean is the linear smoother is $df = 1$.

Question 5: ADA Problem 1.8

Let us consider the case when k-nearest neighbors regression acts as our linear smoother. Recall that our influence matrix, w is a nxn matrix with the weight, w_{ij} saying how much each observation y_{ij} contributes to the fitted values.

We observe in (1.55) of the textbook that $\hat(w(x_i, x))$ is equal to $1/k$ when x_i is one of the k nearest neighbors of x and 0 otherwise.

As a result, we obtain a similar matrix to the one in Question (4), where the diagonal entries are strictly $1/k$, which makes sense since the distance would be 0 about the entries of x if they are not considered a k nearest neighbor.

Thus an approximate matrix w for this question can be constructed as so,

$$w = \begin{pmatrix} 1/k & \cdots & \\ & 1/k & \cdots \\ \vdots & \ddots & \vdots \\ & \cdots & 1/k \end{pmatrix}$$

Since the matrix is once again nxn, we can take the trace of the matrix by Equation (1.70) to obtain the following,

$$tr(w) = \frac{1}{k} + \frac{1}{k} + ... + \frac{1}{k} = \frac{n}{k}$$

As a result, we see that the degrees of freedom for when k-nearest neighbor regression is a linear smoother is $df = \frac{n}{k}$.

Question 6: ESL Problem 2.8