

# Stats\_790\_Assignment\_1

Amandeep Sidhu

2023-01-17

**Question 1:** Write a short description of any opinions, thoughts, or questions you are left with after reading Breiman, 2001, and one of the responses to it

**Question 2:** Pick a figure from ESL Chapter 2 and write, R, Python, or Julia code to replicate it

**Question 3:** ADA Problem 1.2

**Question 4:** ADA Problem 1.7

Suppose that the global mean is our linear smoother in this case. Recall that our influence matrix,  $w$  is a  $n \times n$  matrix with the weight,  $w_{ij}$  saying how much each observation  $y_{ij}$  contributes to the fitted values.

Observing the classic mean formula of

$$\mu = \left(\frac{1}{n}\right) * (\sum_i^n y_i)$$

We observe the weight is essentially  $\frac{1}{n}$  for every entry of the influence matrix  $w$ .

To observe this property visually, we construct an influence matrix as so,

$$w = \begin{pmatrix} 1/n & 1/n & \cdots & 1/n \\ 1/n & 1/n & \cdots & 1/n \\ \vdots & \vdots & \ddots & \vdots \\ 1/n & 1/n & \cdots & 1/n \end{pmatrix}$$

By Equation (1.70) of the textbook that states

$$df(\hat{\mu}) = tr(w)$$

we observe that the trace of  $w$  is essentially  $tr(w) = \frac{1}{n} + \frac{1}{n} + \dots + \frac{1}{n} = 1$ .

Thus we conclude that the degrees of freedom for when the global mean is the linear smoother is  $df = 1$ .

**Question 5:** ADA Problem 1.8

Let us consider the case when k-nearest neighbors regression acts as our linear smoother. Recall that our influence matrix,  $w$  is a  $n \times n$  matrix with the weight,  $w_{ij}$  saying how much each observation  $y_{ij}$  contributes to the fitted values.

We observe in (1.55) of the textbook that  $\hat{w}(x_i, x)$  is equal to  $1/k$  when  $x_i$  is one of the  $k$  nearest neighbors of  $x$  and 0 otherwise.

As a result, we obtain a similar matrix to the one in Question (4), where the diagonal entries are strictly  $1/k$ , which makes sense since the distance would be 0 about the entries of  $x$  if they are not considered a  $k$  nearest neighbor.

Thus an approximate matrix  $w$  for this question can be constructed as so,

$$w = \begin{pmatrix} 1/k & \cdots & \cdots & 0 \\ & 1/k & \cdots & \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & 1/k \end{pmatrix}$$

Since the matrix is once again  $n \times n$ , we can take the trace of the matrix by Equation (1.70) to obtain the following,

$$\text{tr}(w) = \frac{1}{k} + \frac{1}{k} + \cdots + \frac{1}{k} = \frac{n}{k}$$

As a result, we see that the degrees of freedom for when  $k$ -nearest neighbor regression is a linear smoother is  $df = \frac{n}{k}$ .

## Question 6: ESL Problem 2.8