PH107-Quiz 1 (2019) Solutions

Solution to Q1

(a) Photon energy = $\frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{550\times10^{-9}} J = 3.616\times10^{-19} J$ 0.5

Intensity of light = 10 $\mu W/cm^2 = 10^{-5} J/s/cm^2 = \frac{10^{-5}}{3.616\times10^{-19}} photon/s/cm^2$ 0.5

Photoelectrons produced per second = $\frac{10^{-5}}{3.616\times10^{-19}}\times0.5\times0.05 = 6.9\times10^{11}$ 0.5

Maximum current (A)= $\frac{6.9\times10^{11}}{6.22\times10^{18}} = 1.11\times10^{-7} A$ 0.5

(b) $K_{\max} = \frac{1}{2}mV_{\max}^2 = \frac{1}{2}(mc^2)(V_{\max}/c)^2 = 0.248eV$

$V_s = 0.248V$ 0.5

(c) $\phi = \frac{hc}{\lambda} - eV_S = \frac{1239.8}{500} - 0.248 = 2.254 - 0.248 = 2 \text{ eV}$ 0.5

(d) $K_{\max} = \frac{1}{2}mV_{\max}^2 = \frac{1}{2}\times511\times10^3\times\left(\frac{6.22\times10^5}{3\times10^8}\right)^2 = 1.098\,\text{eV} = eV_s$ 0.5

$\frac{hc}{\lambda} = eV_S + \phi = 3.098$

$\lambda = \frac{1239.8}{3.098} = 400 \text{ nm}$ 0.5

Solution to Q2

(a) Energy of the nth state of a positronium atom

$$E_n = \frac{-13 \cdot 6}{n^2} \times \frac{\mu_{(p)}}{m_e}\ eV = \frac{-13.6}{n^2} \times \frac{m_e/2}{m_e}\ eV = -\frac{6.8}{n^2}\ eV$$

0.5

Energy of emitted photon

$$\varepsilon_{\text{ph}} = E_3 - E_1 = \frac{8}{9} \times 6.8\ eV = 6.04\ eV$$

0.5

Maximum kinetic energy of photoelectron

$$\text{K.E} = \varepsilon_{\text{ph}} - \phi = 3.04\ eV$$

0.5

$$V_{max} = \sqrt{\frac{2 \times K.E}{m_e}} = \left[\frac{2 \times 3.04 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}\right]^{\frac{1}{2}} = \sqrt{1.068 \times 10^{12}} = 1.033 \times 10^6\ m/s$$

0.5

(b) Ignoring the term due to kinetic energy of the recoiling positronium atom in the energy balance, the momentum of the emitted photon

$$\mathrm{p}_{ph} = \frac{\varepsilon_{ph}}{c}$$

Therefore, momentum of the recoiling positronium atom $= \frac{\varepsilon_{ph}}{c}$

$$(2m_e)v = \frac{\varepsilon_{ph}}{c}$$

$$v = \frac{\varepsilon_{ph}}{2m_e c}$$

0.5

$$\frac{\varepsilon_{ph}}{2m_e c} = \frac{\varepsilon_{ph}}{2m_e c^2} \times c = \frac{6.04}{2 \times 0.511 \times 10^6} \times 3 \times 10^8$$

$$\therefore v = 1773\ m/s$$

0.5

Solution to Q3

(a) $T_1 = 173 + 273\,\mathrm{K}$, $\lambda_{1\max} = \dfrac{2.897\times10^{-3}\,m.K}{446\,K} =$ 6497 nm 0.5

(b) $T_2 = \dfrac{2.897\times10^{-3}\,m.K}{650\times10^{-9}\,m} = 4457$ K 0.5

(c) Ratio of intensities: $\dfrac{I_2}{I_1} = \left(\dfrac{T_2}{T_1}\right)^4 = 9973 = 10^4$

OR $\dfrac{I_1}{I_2} = \left(\dfrac{T_1}{T_2}\right)^4 = 10^{-4}$ 0.5

(d)

Start at

$$u(\nu)d\nu = \frac{8\pi h\nu^3 d\nu}{c^3}\left(e^{h\nu/k_BT} - 1\right)^{-1}$$

$$u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}d\lambda\left(e^{hc/\lambda k_BT} - 1\right)^{-1}$$

OR

$$u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}e^{-hc/\lambda k_BT}d\lambda$$

1

Convert to intensity

$$I(\lambda)d\lambda = \frac{c}{4}u(\lambda)d\lambda = \frac{2\pi hc^2}{\lambda^5}d\lambda\left(e^{hc/\lambda k_BT} - 1\right)^{-1} \approx \frac{2\pi hc^2}{\lambda^5}e^{-hc/\lambda k_BT}d\lambda$$

0.5

Calculate $\lambda_{\max}$ and identify $d\lambda$

$$\lambda_{\max} = \frac{2.897\times10^{-3}\,mK}{1278K} = 2267\text{ nm}$$

$$d\lambda = 1\text{ nm}$$

0.5

$$k_B T = 8.6 \times 10^{-5} \times 1278\, eV = 0.11 \text{ eV}$$

$$\frac{hc}{\lambda_{\max} k_B T} = \frac{4.135 \times 10^{-15}\, eV.s \times 3 \times 10^{8}\, m/s}{2267 \times 10^{-9}\, m \times 0.11\, eV} = 4.975$$

Final calculation

$$I(\lambda)d\lambda = \frac{2\pi \times (6.6 \times 10^{-34}\, J.s)(3 \times 10^{8}\, m/s)^2 \times 10^{-9}\, m}{(2267 \times 10^{-9}\, m)^5} \left(e^{4.975} - 1\right)^{-1} = 43.38 \text{ W/m}^2$$

0.5

Summary of the marking scheme for this problem,

If you	You get
Arrived at the expression of intensity, starting from energy density (as shown above) Copied the expression of intensity (given as a hint)	0.5+0.5+0.5=1.5 0.5
Identified dλ and calculated $\lambda_{max.}$	+0.5
Got the final answer correct	+0.5
Total	**2.5** 1.5
Attempted the problem (even using Wien's law, Stefan's law etc.)	1
Did not attempt the problem	0

Solution to Q4

(a)

1. Calculation of λ_2

$$\lambda_2 = \lambda_1 + \lambda_c(1 - \cos 90°)$$

$$= \lambda_1 + \lambda_c = 60 + 2.43 = 62.43 \text{ pm}$$

0.5

2. Calculation of $\Delta\lambda$ for Compton scattering by 2nd particle

$$\Delta\lambda = \lambda_3 - \lambda_2 = (\lambda_3 - \lambda_1) - (\lambda_2 - \lambda_1) = \delta\lambda - \lambda_c = 2.6686 - 2.43$$

$$= 0.2386\, pm$$

0.5

3. Calculation of α

$$\Delta\lambda = \frac{\lambda_c}{\alpha}$$

$$\alpha = \frac{\lambda_c}{\Delta\lambda} = \frac{2.43}{0.2386} = 10.18$$

1.0

(b) In the direction of the incoming photon (λ_2), the momentum balance reads

$$P_{||} = \frac{h}{\lambda_2} = \frac{6.6 \times 10^{-34}}{62.43 \times 10^{-1}} = 10.57 \times 10^{-24}\, Kg.m/s$$

2.0

Note: If you mistook the direction of the incoming photon to be the direction of the λ_1 –photon, you would have calculated the momentum of the particle in the direction perpendicular to the incoming photon (opposite to the outgoing photon λ_3)

$$P_{\perp} = \frac{h}{\lambda_3} = \frac{h}{\lambda_3} = \frac{6.6 \times 10^{-34}}{62.6686 \times 10^{-12}} = 10.53 \times 10^{-24}\, Kg.m/s$$

[you still get 0.5]