<u>SOP (CS536)</u>
<u>Assignment 1</u>

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Assignment 1
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ASSINGNMENT - 1

Q1, Let e_1 and e_2 be two expressions

a, In general does $e_1 = e_2$ logically implies $e_1 \equiv e_2$? If yes, briefly justify it (in a sentence or two); if no, give a counter example.

→ No, $e_1 = e_2$ doesn't logically implies $e_1 \equiv e_2$. As, $e_1 = e_2$ implies that the two expressions e_1 and e_2 have the same value which is semantically equivalent. On, the other hand, $e_1 \equiv e_2$ means that the expressions e_1 and e_2 ~~and~~ are syntactically equivalent that is it means something about grammar. These two expressions are textually identical.

For ex.. let's take two expressions,

$e_1 = 7+5$

$e_2 = 9+3$

The two expressions are semantically equal "$e_1 = e_2$" as they give value which is 12. However, their "Syntactic expression" ~~as~~ that is textual ~~means~~ structure is different.

b, In general, does $e_1 \neq e_2$ logically implies $e_1 \not\equiv e_2$? Again, give a brief justification or counter example.

→ Yes, if $e_1 \neq e_2$ it logically implies that $e_1 \not\equiv e_2$. As, $e_1 \neq e_2$ means that the expressions e_1 and e_2 do not have the same value.
Furthermore, syntactic equality logically implies semantic equality ie... $A \equiv B \Rightarrow A = B$.

Q2, Write each of the these statements in the form, "if p, then q" in English:

a, It is necessary to wash the boss's car to get promoted.

→ If you want to get promoted, then it is necessary to wash the boss's car.

b, Winds from the South imply a spring thaw

→ If there are winds from the South, then it is a spring thaw.

c, Willy gets caught whenever he cheats.

→ If Willy cheats, then he gets caught.

d, You will reach the summit unless you begin your climb too late.

→ If you begin your climb too late, then you will not reach summit.

e, You can access the website only if you pay a subscription fee.

→ If you pay a subscription fee, then you can access the website.

Q3, Create truth table for proposition $p \wedge \neg(q \vee r) \rightarrow q \vee r \rightarrow \neg p$. Is it tautology, contingency or contradiction and why.

						let e_1	let e_2	
$\neg p$	p	q	r	$q \vee r$	$\neg(q \vee r)$	$p \wedge \neg(q \vee r)$	$q \vee r \rightarrow \neg p$	$e_1 \rightarrow e_2$
F	T	T	T	T	F	F	F	
F	T	T	F	T	F	F	F	
F	T	F	T	T	F	F	F	
F	T	F	F	F	T	T	T	
T	F	T	T	T	F	F	T	
T	F	T	F	T	F	F	T	
T	F	F	T	T	F	F	T	
T	F	F	F	F	T	F	T	

$q \vee r \rightarrow \neg p$ has the same truth value as $\neg(q \vee r) \vee \neg p$

$e_1 \rightarrow e_2$ has the same truth value as $\neg e_1 \vee e_2$.

~~eese~~ $\neg e_1$	e_2	$\neg e_1 \vee e_2$ (or $e_1 \rightarrow e_2$)
F	F	
F	F	
F	F	
T	T	
F	T	
F	T	
F	T	
F	T	

$\neg p$	p	q	r	$q \vee r$	$\neg(q \vee r)$	$p \wedge \neg(q \vee r)$ (let e_1)	$q \vee r \rightarrow \neg p$ or $\neg(q \vee r) \vee \neg p$ (let e_2)	$\neg e_1$	$e_1 \rightarrow e_2$ $(\neg e_1 \vee e_2)$
F	T	T	T	T	F	F	F	T	T
F	T	T	F	T	F	F	F	T	T
F	T	F	T	T	F	F	F	T	T
F	T	F	F	F	T	T	T	F	T
T	F	T	T	T	F	F	T	T	T
T	F	T	F	T	F	F	T	T	T
T	F	F	T	T	F	F	T	T	T
T	F	F	F	F	T	F	T	T	T

remember, $q \vee r \rightarrow \neg p$ is $\neg(q \vee r) \vee \neg p$ is e_2

$p \wedge \neg(q \vee r)$ is e_1

This, proposition is a tautology from $e_1 \rightarrow e_2$ column.

Q4, Give full parenthesization for each of the following prepositions / predecates

a, $p \wedge \neg r \vee s \Rightarrow \neg q \wedge r \Rightarrow \neg p \Leftrightarrow \neg s \rightarrow t$.

Solution:- $p \wedge (\neg r) \vee s \rightarrow (\neg q) \wedge r \rightarrow (\neg p) \leftrightarrow (\neg s) \rightarrow t$

$(p \wedge (\neg r)) \vee s \rightarrow ((\neg q) \wedge r) \rightarrow (\neg p) \leftrightarrow (\neg s) \rightarrow t$.

$(((p \wedge (\neg r)) \vee s) \rightarrow (((\neg q) \wedge r) \rightarrow (((\neg p) \leftrightarrow (\neg s)) \rightarrow t)))$

b, $\forall m \cdot 0 < m < n \wedge \exists 0 \leq j \leq m \cdot b[0] \leq b[j] \wedge b[j] \leq b[m]$.

Solution:- $(\forall m \cdot ((0 < m < n) \wedge \exists (0 \leq j < m)) \cdot ((b[0] \leq b[j]) \wedge (b[j] \leq b[m])))$

Q5, Use only logic rules to prove the following logic equalities. In each step, clarify which rules you use :-

a, $\neg p \to q \to r \iff q \to (p \vee r)$

$LHS = \neg p \to q \to r$

$\iff \neg p \to (q \to r)$ { implication is right associative }

$\iff \neg p \to (\neg q \vee r)$ { definition of implication }

$\iff p \vee (\neg q \vee r)$ { definition of implication }

$\iff \neg q \vee (p \vee r)$ { Associativity }

$\iff q \to (p \vee r)$ { definition of Implication }

$\iff RHS$ proved

b, $(p \to q) \wedge (p \to r) \Leftrightarrow p \to q \wedge r$

RHS = $p \to q \wedge r$.

$\Leftrightarrow p \to (q \wedge r)$

$\Leftrightarrow \neg p \vee (q \wedge r)$ {definition of implication}

$\Leftrightarrow (\neg p \vee q) \wedge (\neg p \vee r)$ { definition of "Distributivity"
~~De Morgan's Laws~~ }

$\Leftrightarrow (p \to q) \wedge (p \to r)$ { definition of Implication }

$\Leftrightarrow$ L.H.S proved

Q6, Use Quantified prediates to express the following Sentences :-

a, The sum of two negative integers will always be negative.

$\rightarrow$ $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z} \cdot (((x<0) \wedge (y<0)) \longrightarrow (x+y<0))$

b, For each positive integer, there doesn't exist any negative integer that has the same value as it.

$\rightarrow$ $\forall x \in \mathbb{Z} \cdot ((x>0) \longrightarrow \neg \exists y \in \mathbb{Z} \cdot ((y<0) \wedge (x=y)))$

c, For any two integers, there is always another integer equals to their (the first two integer's) product.

$\rightarrow$ $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, \exists p \in \mathbb{Z} \cdot (p = x * y).$

Q7, Negate each prediate you created in question 6, then simplify them to remove the negation operators. While simplifying, clarify which logic rule(s) you use in each step.

a, $\neg [\forall x \in Z, \forall y \in Z \cdot (((x<0) \wedge (y<0)) \rightarrow ((x+y)<0))]$

$\Rightarrow \exists x \cdot \exists y \cdot \neg [((x<0) \wedge (y<0)) \rightarrow ((x+y)<0))]$

{in this step we applied De Morgan's law in prediate logic}.

$\exists x \cdot \exists y \cdot [((x<0) \wedge (y<0)) \wedge \neg ((x+y)<0))]$

{in this step we applied negation of implication}

$\exists x \cdot \exists y \cdot [((x<0) \wedge (y<0)) \wedge ((x+y) \geq 0)]$

{in this step we applied De Morgan's law in propositional logic}

Q7, b, $\neg[\forall x \in Z \{((x>0) \rightarrow \neg \exists y \in Z \cdot (y<0 \wedge y=x)]$

$\exists x \in Z \ \neg[((x>0) \rightarrow \neg \exists y \in Z \cdot (y<0 \wedge y=x)]$

{De Morgan's law in predicate logic}

$\exists x \in Z \cdot ((x>0) \wedge \neg (\neg \exists y \in Z \cdot (y<0 \wedge y=x))$

{Negation of implication}.

$\exists x \in Z \cdot ((x>0) \wedge \neg (\forall y \in Z \cdot (y<0 \wedge y=x)).$

{De Morgan's law in predicate logic}

$\exists x \in Z \ ((x>0) \wedge \exists y \in Z \cdot \neg(y<0 \wedge y=x))$

{De Morgan's law in predicate logic as
$\neg \forall y \Rightarrow \exists y$}.

$\exists x \in Z \ ((x>0) \wedge \exists y \in Z \cdot (y \geq 0 \vee y \neq x))$

{De Morgan's law in propositional logic}.

Q7, c, $\neg [\forall x \in Z, \forall y \in Z, \exists P \in Z \cdot (P = x*y)]$

$\exists x, \exists y, \forall P \in Z \cdot \neg [(P = x*y)]$
{ applying De Morgan's law in predicate logic }

$\exists x, \exists y, \forall P \in Z \cdot (P \neq x*y)$
{ De Morgan's law in Propositional logic }.

Q8, For each sub-question, please answer the following questions: Is state σ proper for proposition / predicate e? If no explain why; if yes, does σ satisfy e?

a, $\sigma = \{p = T\}$, $e \equiv (r \to p) \wedge (\neg r \to p)$

→ Yes, state σ is proper for e as with $p = T$ (True) the expression 'e' becomes,

$(r \to T) \wedge (\neg r \to T)$

using the definition of implication

$(\neg r \vee T) \wedge (r \vee T)$

So, if r = True or False, the expression is True, and σ satisfies e.

b, $\tau = \{p = F, q = T\}$, $\sigma = \tau \cup \{r = F\}$, $e \equiv p \wedge q$

→ State σ is proper for e.
Yes, σ satisfies e.

Q8, c, $\sigma = \{b=5, i=0, x=6\}$, $e \equiv x > b[i]$

→ Yes, σ is proper for proposition e.

→ No, σ doesn't satisfy e as when we put the values in the expression e it becomes
$6 > 5[0]$ which is not meaningful as b is not an array.

Q8 d, $\sigma = \{x=5, b=(5,8)\}$, $e \equiv x+1 = b[0]$.

→ Yes, state σ is proper for e as it contain all the values mentioned in proposition e.

→ ~~Yes~~ No, state σ satisfies e, as substituting the values we get,

$x+1 = 5+1 = 6$

and $b[0] = 5$

Thus, $x+1 \neq b[0]$, so it isn't satisfying

Q9, Find all states σ (containing only bindings for p, q and r) such that $\sigma \models p \leftrightarrow q \leftrightarrow r$.
Briefly explain each state.

Solution:- Let's look at all the states of σ, ~~containing~~ considering all possible combinations of variables p, q and r.
There would be, $2^3 = 8$ combinations possible
Note, $\sigma \models p \leftrightarrow q \leftrightarrow r$
as bi-conditional statements are right associative
$\Rightarrow p \leftrightarrow (q \leftrightarrow r)$
let's say $q \leftrightarrow r \Rightarrow e_1$
$\Rightarrow p \leftrightarrow e_1$
Truth table:-

p	q	r	$(q \leftrightarrow r)$ let's e_1	$p \leftrightarrow (q \leftrightarrow r)$ $(p \leftrightarrow e_1)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	T	T
F	T	T	T	F
F	T	F	F	T
F	F	T	F	T
F	F	F	T	F

Note

In the truth table above, first we captured the truth value of $(q \longleftrightarrow r)$ and then it's result that is e_1 is calculated with p as $p \longleftrightarrow (e_1)$ where, $e_1 = (q \longleftrightarrow r)$.

Each states are as follows

i, ~~p~~ $\sigma = \{p = T, q = T, r = T\}$, then, $p \longleftrightarrow q \longleftrightarrow r \Rightarrow$ True.

ii, $\sigma = \{p = T, q = T, r = F\}$, then $p \longleftrightarrow q \longleftrightarrow r \Rightarrow$ False, as, $q \longleftrightarrow r \Rightarrow F$ and $p \longleftrightarrow e_1 \Rightarrow F$.

iii, $\sigma = \{p = T, q = F, r = T\}$, then $p \longleftrightarrow q \longleftrightarrow r \Rightarrow$ False, as, $q \longleftrightarrow r \Rightarrow F$ and $p \longleftrightarrow e_1 \Rightarrow F$.

iv, $\sigma = \{p = T, q = F, r = F\}$, then $p \longleftrightarrow q \longleftrightarrow r \Rightarrow$ True. as, $q \longleftrightarrow r \Rightarrow T$ and, $p \longleftrightarrow e_1 \Rightarrow T$.

v, $\sigma = \{p = F, q = T, r = T\}$ then $p \longleftrightarrow q \longleftrightarrow r \Rightarrow$ False. as, $q \longleftrightarrow r \Rightarrow T$ and $p \longleftrightarrow e_1 \Rightarrow F$

vi, $\sigma = \{p = F, q = T, r = F\}$, then, $p \longleftrightarrow q \longleftrightarrow r \Rightarrow$ True, as, $q \longleftrightarrow r \Rightarrow F$ and $p \longleftrightarrow e_1 \Rightarrow T$.

vii, $\sigma = \{p = F,\ q = F,\ r = T\}$, then $p \longleftrightarrow q \longleftrightarrow r = True$,
as $q \longleftrightarrow r \Rightarrow F$ and $p \longleftrightarrow e_1 \Rightarrow T$

viii, $\sigma = \{p = F,\ q = F,\ r = F\}$, then, $p \longleftrightarrow q \longleftrightarrow r = False$,
as $q \longleftrightarrow r \Rightarrow T$ and $p \longleftrightarrow e_1 \Rightarrow T$.

Q 10, Define predicate functions

a, Define predicate function isGreater(b, m, x) which returns True if and only if positive m is not larger than the length of array b, and integer x is greater than each of the first m numbers in b. For example, under state $\sigma = \{b = (2, 4, 1, 6)\}$, isGreater$(b, 3, 5)$ returns True, isGreater$(b, 3, 4)$ returns False. Remind that, you can use Size(b) to find the length of array b.

<u>Solution</u>:-

$$\text{isGreater}(b, m, x) \equiv (0 < m \le \text{Size}(b)) \wedge (\forall i \cdot 0 \le i < m \rightarrow x > b[i])$$

$$\cancel{\forall m \in \mathbb{Z}\ (m \le \text{Size}(b)) \wedge (\forall i \in \mathbb{Z} : 1 \le i \le m,\ b[i] < x)}$$

Another way, it can be written is,

This function returns only if m is not larger than the length of array 'b' and 'x' is greater than each of the first 'm' numbers in array 'b'.

Pseudo code for this function.

```
if m > size (b)  or  m < 1
        return False;
else
    for i in range (m)
        if b[i] >= x
            return False;
    return True;
    End for
fi
```

For, is Greater (b, 3, 5)
where, $b = [2, 4, 1, 6]$, $m = 3$, $x = 5$.
This function would return True because m is not greater than the size of b (that is 4) and $x = 5$ is greater than the first $m = 3$ numbers in array b.

For, is Greater (b, 3, 4)
This function would return False because although m is not greater than the size of array b, but $x^{=4}$ is not greater than the first $m = 3$ numbers, as $b[1] = 4$ is equal to $x = 4$.

Q10, b, Define predicate function has Greater (a, b) which returns True if and only if every integer in array a is greater than some integer in array b.
For example, has Greater ((4, 5, 2, 3), (8, 1, 5, 8)) returns True.

Solution:- ~~has Greater (a, b) $\equiv$ ($\forall x \in a, \exists y \in b \cdot x > y$~~)

has Greater $(a, b) \equiv \forall a \cdot 0 \le i < size(a) \cdot \exists b \cdot 0 \le j < size(b) \cdot a[i] > b[j]$

Pseudo code for this function is,

```
for i in a
    found-Greater := False;
  for j in b:
      if i > j
          found-Greater := True;
          break
  if not found-Greater
        return False;
return True;
```

has Greater ((4, 5, 2, 3), (8, 1, 5, 8)) would return True, as every element in array a is greater some of the elements in array b.
a[2] = 2, is greater than b[1] = 1

and has Greater ((4, 6, 8), (5, 7, 9)) would return False because, a[0]=4 is not greater than b[0] = 5.