A Little Linear Algebra

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Overview

1. What's a matrix?
2. A Few Examples
3. Who Cares?

What's a matrix?

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A *matrix* is a *representation* of a linear transformation $T : V \to W$ given bases of V and W.

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A complicated example: Let $V = W = P(x)$ where $P(x)$ is the vector space of all polynomials (with real numbers as coefficients). Define T by

$$T(p)(x) = \int_{-\infty}^{\infty} p(y) e^{-\frac{(x-y)^2}{2}} dy$$

Some Intuition

Let's look at the matrix

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$$

What happens to a circle?

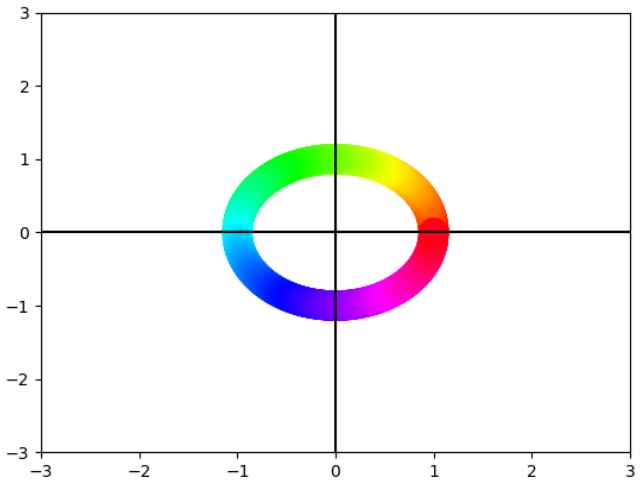

Figure: A unit circle

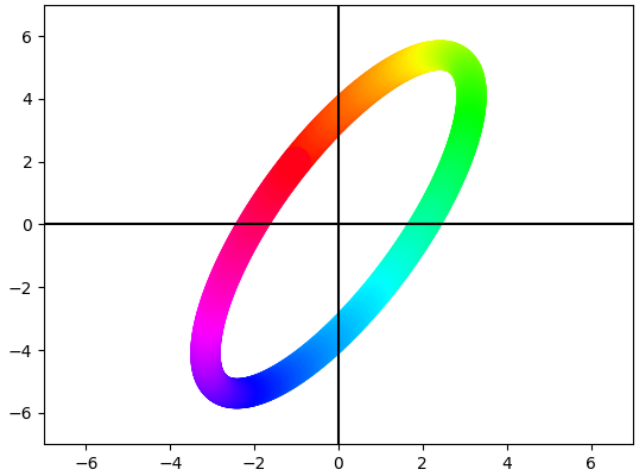

Figure: The unit circle after a linear transformation

Let's Break It Down

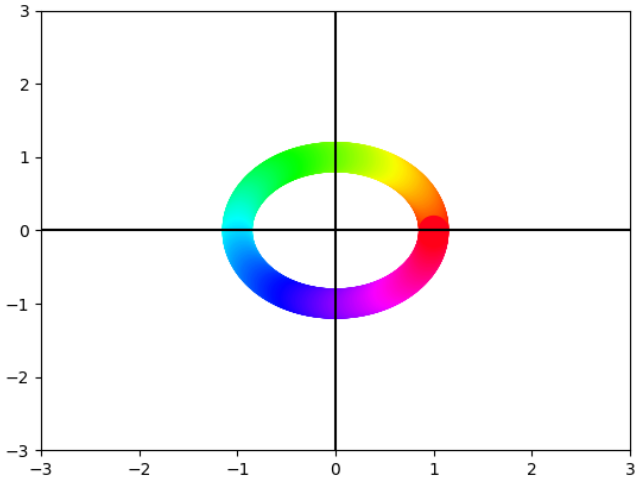

Figure: A unit circle

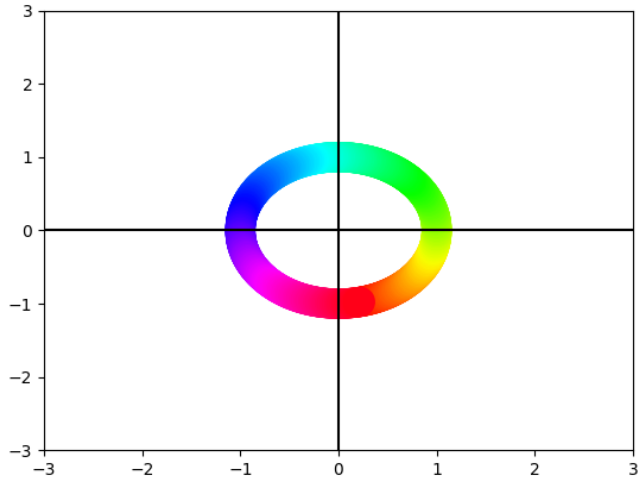

Figure: The unit circle after a rotation

Next Step

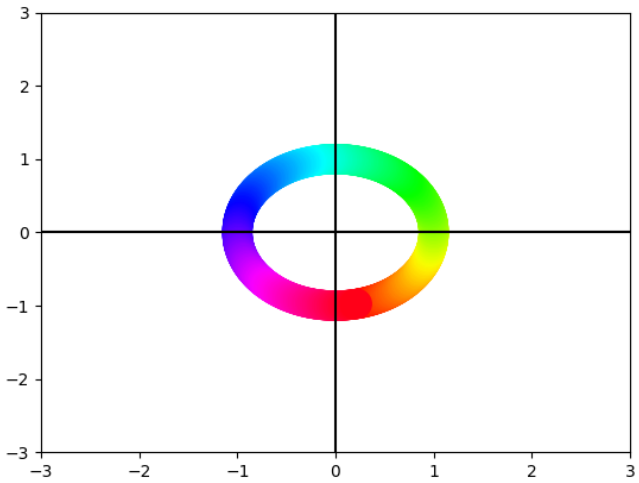

Figure: A unit circle after a rotation

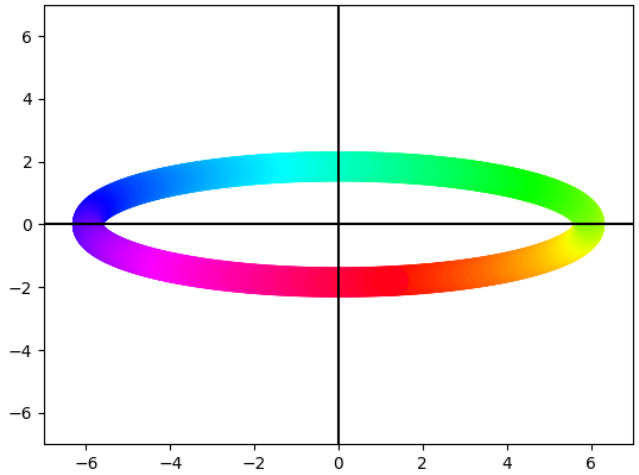

Figure: The unit circle after a rotation and stretch

Finally

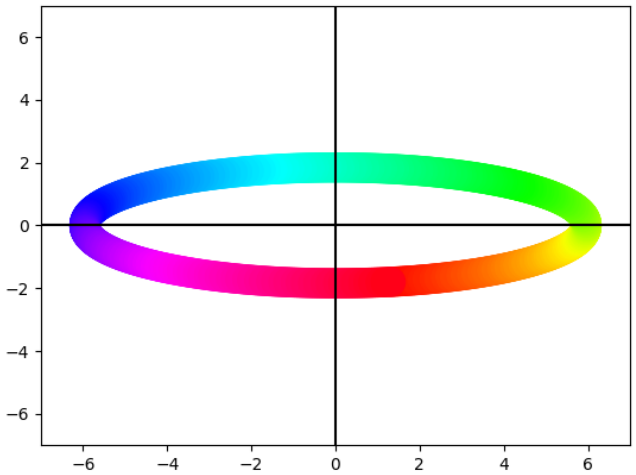

Figure: A unit circle after a rotation and stretch

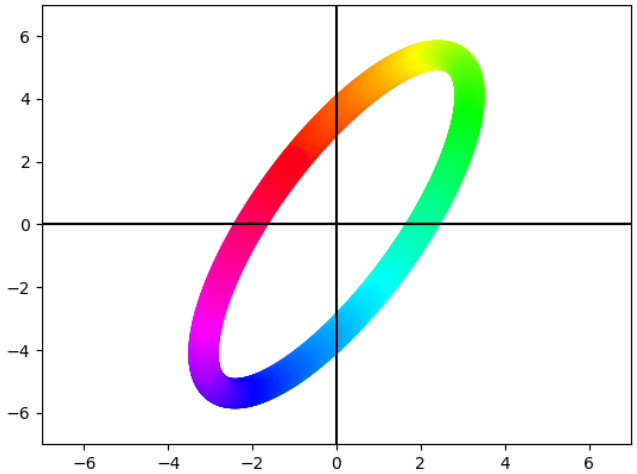

Figure: The unit circle after a rotation and stretch and another rotation

Some Intuition

Let's look a singular matrix

$$A = \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix}$$

What happens to a circle?

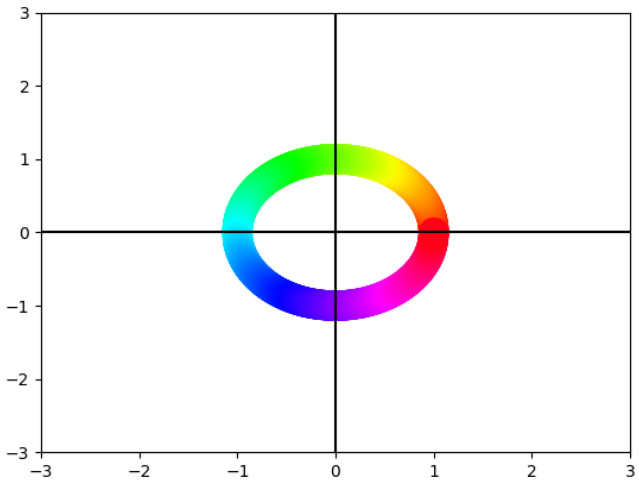

Figure: A unit circle

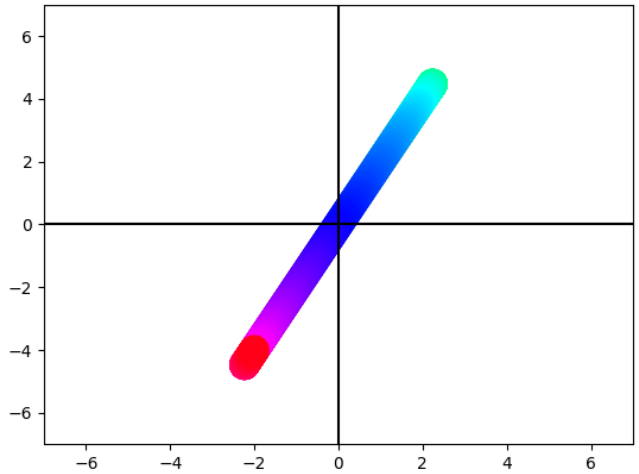

Figure: The unit circle after a singular linear transformation

Still Start With a Rotation

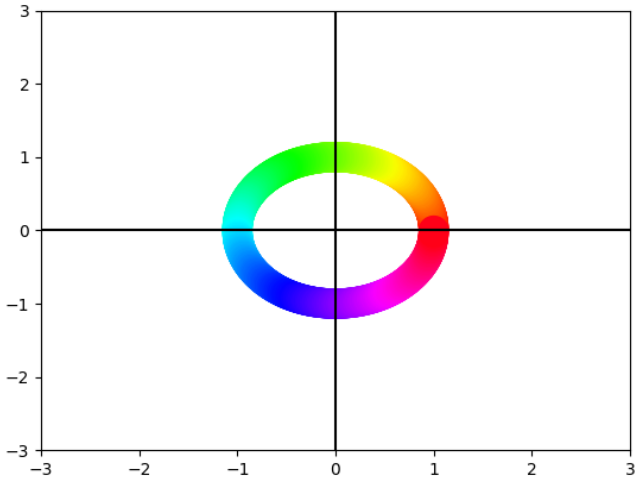

Figure: A unit circle

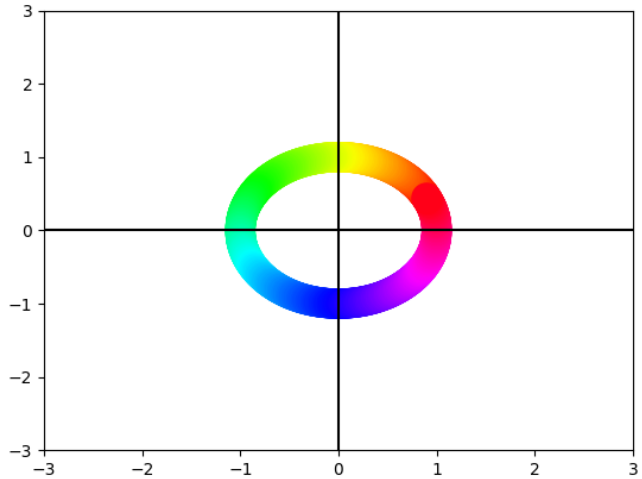

Figure: The unit circle after a rotation

"Stretch" is more of a "Smash"

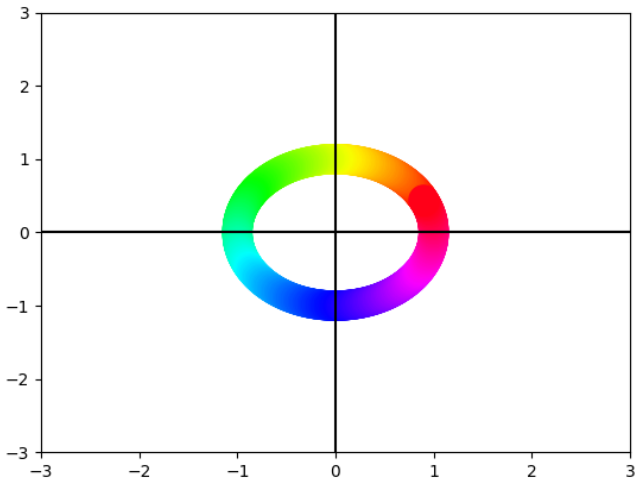

Figure: A rotated unit circle

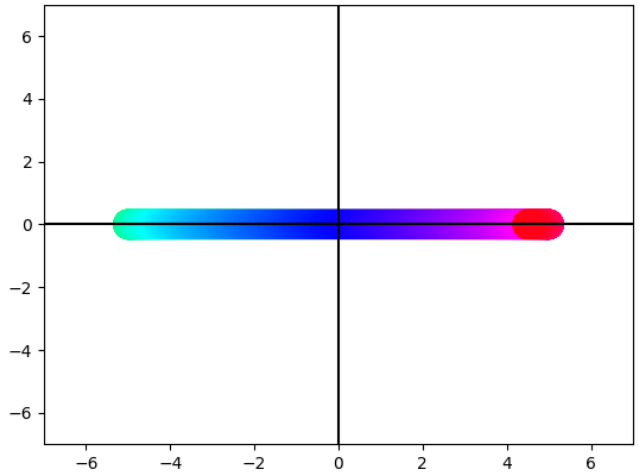

Figure: The unit circle after a rotation and a smash

Finally

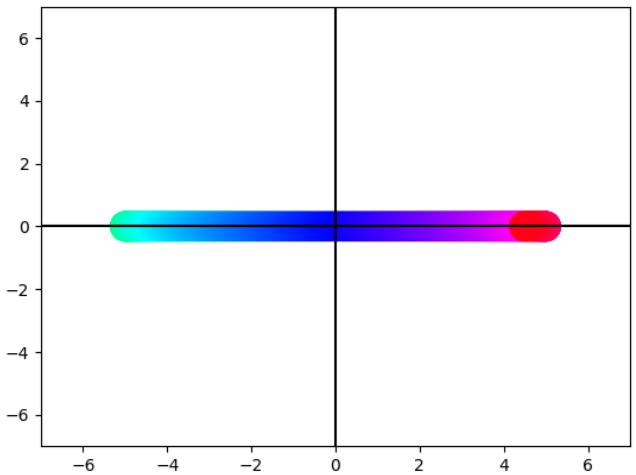

Figure: A rotated smashed unit circle

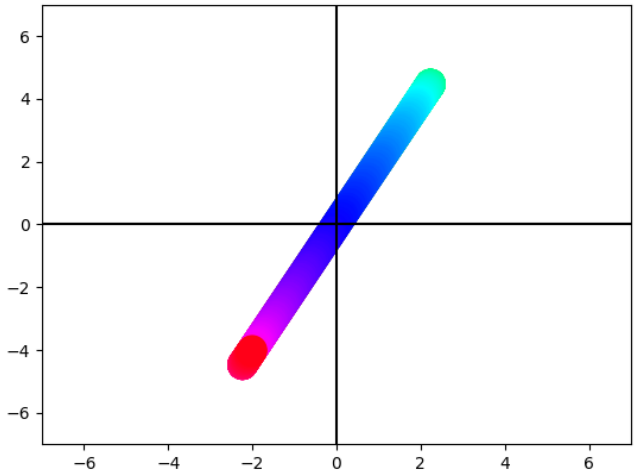

Figure: A final rotation

A system

Look at $Ax = b$ where

$$A = \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix}$$

and

$$b = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

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How many solutions does this have?

A Picture