Fall 2020 CS4641 Homework 1

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Deadline: September 9, Wednesday, 11:59 pm

- No extension of the deadline is allowed. Late submission will lead to 0 credit.
- Discussion is encouraged on Piazza as part of the Q/A. However, all assignments should be done individually.

Structure

Homework 1 will have two components to it: the theory questions in this file along with a programming portion in a Jupyter notebook. The homework is worth a total of 110 points, where 10 of these are bonus points. The grading breakdown is as follows:

1. **Theory** ($85 + 10$ **bonus**): problems 1-4 are worth 85 points and problem 5 is worth 10 bonus points.
2. **Programming** (15): there are four subproblems in this part of the assignment (in the .ipynb file)

Instructions

- We will be using Gradescope this semester for submission and grading of assignments.
- Your write up must be submitted in PDF form, you may use either Latex or markdown, whichever you prefer. **We will not accept handwritten work.**
- Please make sure to start answering each question on a new page. It makes it more organized to map your answers on GradeScope. When submitting your assignment, you must correctly map pages of your PDF to each question/subquestion to reflect where they appear. Improperly mapped questions may not be graded correctly. For further submission instructions, you may find this video helpful.

- **Early Bird Special:** If you can submit 2 out of the first 4 questions by September 2nd, you get a bonus of 5 points for this assignment.
- Make your submission as follows:
  - Questions 1-4: Submit it in A1 Written assignment of gradescope in .pdf format
  - Question 5: Submit it in A1 Written: Bonus Questions assignment of gradescope in .pdf format
  - Submit 2 questions from Q 1-4 in A1 Written: Early Bird assignment of gradescope for early bird bonus
  - Submit the programming solutions to A1 Programming assignment of gradescope as per the instructions in .ipynb file

1 Linear Algebra [20 points]

1.1 Determinant and Inverse of Matrix [10pts]

Given a matrix M:

$$M = \begin{bmatrix} 2 & -4 & 1 \\ 4 & 1 & \text{x} \\ 2 & 1 & 1 \end{bmatrix}$$

(a) Calculate the determinant of M for x = -2. [2pts] (Calculation process required.)

$$\begin{aligned} \begin{vmatrix} 2 & -4 & 1 \\ 4 & 1 & \text{x} \\ 2 & 1 & 1 \end{vmatrix} &= 2\begin{vmatrix} 1 & 1 & x \\ 1 & 1 & \end{vmatrix} + 4\begin{vmatrix} 4 & x \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix} && (1) \\ &= 2(1-x) + 4(4-2x) + 2 && (2) \\ &= 20 - 10x && (3) \end{aligned}$$

(b) Calculate M^{-1} for x = -2.. [4pts] (Calculation process required)
(**Hint:** Please double check your answer and make sure $MM^{-1} = I$)

$$
\left(\begin{array}{ccc|ccc} 2 & -4 & 1 & 1 & 0 & 0 \\ 4 & 1 & -2 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & -4 & 1 & 1 & 0 & 0 \\ 0 & 9 & -4 & -2 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array}\right)
$$

$$
\sim \left(\begin{array}{ccc|ccc} 2 & -4 & 1 & 1 & 0 & 0 \\ 0 & 9 & -4 & -2 & 1 & 0 \\ 0 & 5 & 0 & -1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & -4 & 1 & 1 & 0 & 0 \\ 0 & 5 & 0 & -1 & 0 & 1 \\ 0 & 9 & -4 & -2 & 1 & 0 \end{array}\right)
$$

$$
\sim \left(\begin{array}{ccc|ccc} 2 & -4 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 9 & -4 & -2 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & -4 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & -4 & -\frac{1}{5} & 1 & -\frac{9}{5} \end{array}\right)
$$

$$
\sim \left(\begin{array}{ccc|ccc} 2 & -4 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & 1 & \frac{1}{20} & -\frac{1}{4} & \frac{9}{20} \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & -4 & 0 & \frac{19}{20} & \frac{1}{4} & -\frac{9}{20} \\ 0 & 1 & 0 & -\frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & 1 & \frac{1}{20} & -\frac{1}{4} & \frac{9}{20} \end{array}\right)
$$

$$
\sim \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & \frac{3}{20} & \frac{1}{4} & -\frac{7}{20} \\ 0 & 1 & 0 & -\frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & 1 & \frac{1}{20} & -\frac{1}{4} & \frac{9}{20} \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{40} & \frac{1}{8} & -\frac{7}{40} \\ 0 & 1 & 0 & -\frac{1}{5} & 0 & \frac{1}{5} \\ 0 & 0 & 1 & \frac{1}{20} & -\frac{1}{4} & \frac{9}{20} \end{array}\right)
$$

$$
M^{-1} = \begin{pmatrix} \frac{3}{40} & \frac{1}{8} & -\frac{7}{40} \\ -\frac{1}{5} & 0 & \frac{1}{5} \\ \frac{1}{20} & -\frac{1}{4} & \frac{9}{20} \end{pmatrix} \tag{4}
$$

(c) What is the relationship between the determinant of M and the determinant of M^{-1} from parts (a) and (b)? [2pts]
The determinants of M and M^{-1} are the same.

(d) What happens to the inverse of the matrix if x =2 ? Why? [2pts]
M becomes a singular matrix, as the determinant becomes 0, which means an inverse does not exist.

1.2 Singular Value Decomposition [10pts]

Given a matrix A:

$$
A = \begin{bmatrix} 2 & 6 & 0 \\ -9 & 3 & 0 \end{bmatrix}
$$

Compute the Singular Value Decomposition (SVD) by following the steps below. Your full calculation process is required.

(a) Calculate all eigenvalues of AA^T and A^TA. [3pts]

$$
\begin{aligned}
AA^T &= \begin{pmatrix} 40 & 0 \\ 0 & 90 \end{pmatrix} \\
AA^T - \lambda I &= \begin{pmatrix} 40-\lambda & 0 \\ 0 & 90-\lambda \end{pmatrix} \\
det(AA^T - \lambda I) &= 0 \\
&= (40-\lambda)(90-\lambda) \\
\lambda &= 40, 90 \\
A^TA &= \begin{pmatrix} 85 & -15 & 0 \\ -15 & 45 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
A^TA - \lambda I &= \begin{pmatrix} 85-\lambda & -15 & 0 \\ -15 & 45-\lambda & 0 \\ 0 & 0 & -\lambda \end{pmatrix} \\
det(A^TA - \lambda I) &= 0 \\
&= -\lambda \begin{pmatrix} 85-\lambda & -15 \\ -15 & 45-\lambda \end{pmatrix} \\
&= -\lambda((85-\lambda)(45-\lambda) - 225) \\
&= -\lambda(3600 - 130\lambda + \lambda^2) \\
\lambda &= 0, 40, 90
\end{aligned}
$$

(b) Calculate all eigenvectors of AA^T normalized to unit length. [3pts]

$$
\begin{aligned}
(AA^T - \lambda I)v &= 0 \\
\lambda &= 40 \\
\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 50 & 0 \end{array}\right) \\
v_1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
\lambda &= 90 \\
\left(\begin{array}{cc|c} -50 & 0 & 0 \\ 0 & 50 & 0 \end{array}\right) \\
v_2 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\end{aligned}
$$

(c) Calculate all eigenvectors of A^TA normalized to unit length. [3pts]

$$(A^TA - \lambda I)v = 0$$

$$\lambda = 0$$

$$\left(\begin{array}{ccc|c} 85 & -15 & 0 & 0 \\ -15 & 45 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda = 40$$

$$\left(\begin{array}{ccc|c} 45 & -15 & 0 & 0 \\ -15 & 5 & 0 & 0 \\ 0 & 0 & -40 & 0 \end{array}\right)$$

$$v_2 = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \lambda \qquad = 90$$

$$\left(\begin{array}{ccc|c} -5 & -15 & 0 & 0 \\ -15 & -45 & 0 & 0 \\ 0 & 0 & -49 & 0 \end{array}\right)$$

$$v_3 = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$$

(d) Write out the SVD of matrix A in the following form [1pts]:

$$A = U\Sigma V^T$$

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{90} & 0 & 0 \\ 0 & \sqrt{40} & 0 \end{pmatrix} \begin{pmatrix} -\frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 \\ \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Hints:

- The square roots of the positive eigenvalues make up the singular values, the diagonal entries in Σ. They will be arranged in descending order, all other values in Σ are 0
- eigenvectors of A^TA make up the rows of V^T
- eigenvectors of AA^T make up the rows of U
- Reconstruct matrix A from the SVD to check your answer

2 Expectation, Co-variance and Independence [25pts]

Suppose X, Y and Z are distinct random variables. Let X follow a distribution with probability mass function

$$p_X(x) = \begin{cases} q & x = c \\ 1 - q & x = -c \end{cases}$$

where $c > 0$ is a positive constant and $q \in [0, 1]$. Let Y be independent from X and follow the standard normal (Gaussian) distribution, i.e., $Y \sim N(0, 1)$, and let us define Z by $Z = \frac{Y}{X}$. (Note: the computations below may be functions of c and q)

(a) What is the expectation and variance of X? [6pts]

$$\begin{aligned} E(X) &= \sum_x x p_X(x) \\ &= cq + (-c)(1 - q) \\ &= cq - c + cq \\ E(X) &= c(2q - 1) \\ E(X^2) &= \sum_x x^2 p_X(x) = c^2 q + c^2(1 - q) \\ &= c^2 \\ Var(X) &= E(X^2) - (E(X))^2 \\ &= c^2 - (c(2q - 1))^2 \\ &= c^2 - (c^2(4q^2 - 4q + 1)) \\ &= c^2(4q - 4q^2) \\ Var(X) &= 4c^2 q(1 - q) \end{aligned}$$

(b) Show that Z also follows a normal distribution and compute its expected value and variance. [10pts]
Since X and Y are independent of each other, and $Z = \frac{X}{Y}$, their conditional and unconditional distributions are the same, and Z must have a normal distribution.

$$
\begin{aligned}
E(Z) &= E(\frac{Y}{X}) \\
&= E(Y) \times E(\frac{1}{X}) \\
E(Y) &= 0 \\
E(Z) &= 0 \\
Var(Z) &= E(Z^2) - (E(Z))^2 \\
&= E(\frac{Y^2}{X^2}) = E(Y^2) \times E(\frac{1}{X^2}) \\
E(Y^2) &= 1 \\
E(\frac{1}{X^2}) &= \sum_x \frac{p_X(x)}{x^2} \\
&= \frac{q}{c^2} + \frac{1-q}{c^2} \\
&= \frac{1}{c^2} \\
Var(Z) &= \frac{1}{c^2}
\end{aligned}
$$

(c) Compute $Cov(Y, Z)$. [5pts]

$$
\begin{aligned}
Cov(Y, Z) &= Cov(Y, \frac{Y}{X}) = E(\frac{Y^2}{X}) - E(Y)E(\frac{Y}{x}) \\
&= E(Y^2)E(\frac{1}{X}) \\
E(\frac{1}{X}) &= \sum_x \frac{p_X(x)}{x} = \frac{2q-1}{c} \\
Cov(Y, \frac{Y}{X}) &= 1 \times \frac{2q-1}{c} \\
Cov(Y, Z) &= \frac{2q-1}{c}
\end{aligned}
$$

(d) Are Y and Z independent? Explain. [4pts]
No, because $Cov(Y, Z) \neq 0$ and Z depends on Y in the equation $Z = \frac{Y}{X}$

3 Maximum Likelihood [20 pts]

3.1 Discrete Example [10 pts]

Suppose we have a 4-sided die and let X denote the random face that comes up on a throw. Its pmf is given by Table 1, where $\theta, p \in [0, 1]$. Suppose we throw

x	1	2	3	4
$p_X(x)$	θp	$(1-\theta)p$	$\theta(1-p)$	$(1-\theta)(1-p)$

Table 1: Pmf of X

the die a certain number of times and observe x_i i's, for $i = 1, \ldots, 4$ (i.e., face i comes up x_i times).

(a) What is the likelihood of this experiment given θ? (You should treat p as a constant) [4pts]

$$\begin{aligned} L(\mathcal{D}, \theta) &= (\theta p)^1((1-\theta)p)^2(\theta(1-p))^3((1-\theta)(1-p))^4 \\ &= p^3(1-p)^7\theta^4(1-\theta)^6 \end{aligned}$$

(b) What is the maximum likelihood estimate of θ? [6pts]

$$\begin{aligned} L(\mathcal{D}, \theta) &= p^3(1-p)^7\theta^4(1-\theta)^6 \\ \log(L(\mathcal{D}, \theta)) &= 3\log p + 7\log(1-p) + 4\log\theta + 6\log(1-\theta) \\ \frac{\partial}{\partial\theta}\log(L(\mathcal{D}, \theta)) &= \frac{4}{\theta} - \frac{6}{1-\theta} = 0 \\ &= 4(1-\theta) - 6\theta \\ &= 4 - 10\theta \\ \theta &= \frac{2}{5} \end{aligned}$$

3.2 Normal Distribution [10 pts]

Suppose we sample n i.i.d. points from a Gaussian distribution with mean μ and variance σ^2. Recall that the Gaussian pdf is given by

$$f(x; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Compute the maximum likelihood estimate of parameters μ and σ^2.

$$
\begin{aligned}
L(x, \mu, \sigma^2) &= \prod_{i=1}^{n} f(x_i, \mu, \sigma^2) \\
&= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2} \\
&= (2\pi)^{-n/2}(\sigma^2)^{-n/2} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2} \\
\log(L(x, \mu, \sigma^2)) &= -\frac{n}{2}\log 2\pi - \frac{n}{2}\log\sigma^2 - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2 \\
\frac{\partial}{\partial\mu}\log(L(x, \mu, \sigma^2)) &= \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{\sigma^2} = 0 \\
&= \sum_{i=1}^{n}(x_i-\mu)^2 = 0 \\
&= \sum_{i=1}^{n} x_i = n\mu \\
\hat{\mu} &= \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x} \\
\frac{\partial^2}{\partial\mu^2}\log(L(x, \mu, \sigma^2)) &= -\frac{n}{\sigma^2} \\
\frac{\partial^2}{\partial\mu^2}\log L_{\hat{\mu}=\bar{x}} &= -\frac{n}{\hat{x}} < 0 \\
\frac{\partial}{\partial\sigma^2}\log(L(x, \mu, \sigma^2)) &= \frac{-n}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{i=1}^{n}(x_i-\mu)^2 = 0 \\
\frac{1}{2(\sigma^2)^2}\sum_{i=1}^{n}(x_i-\mu)^2 &= \frac{n}{2\sigma^2} \\
\hat{\sigma^2} &= \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n} \\
\frac{\partial^2}{\partial\mu^2}\log(L(x, \mu, \sigma^2)) &= \frac{n}{2(\sigma^2)^2} - \frac{1}{(\sigma^2)^3}\sum_{i=1}^{n}(x_i-\mu)^2 \\
\frac{\partial^2}{\partial\mu^2}\log L_{\hat{\sigma}} &= \frac{n \times n^2}{2(\sum(x_i-\bar{x})^2)^2} - \frac{\sum(x_i-\bar{x})^2)^2 \times n^3}{(\sum(x_i-\bar{x})^2)^3} \\
&= \frac{-n^3}{2(\sum(x_i-\bar{x})^2)^2} < 0
\end{aligned}
$$

MLE of $\hat{\mu}$ is $\bar{x}$, MLE of $\hat{\sigma^2}$ is $\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$

4 Information Theory [20 points]

4.1 Marginal Distribution [7pts]

Suppose the joint probability distribution of two binary random variables X and Y are given as follows.

$X \backslash Y$	1	2
0	$\frac{1}{4}$	$\frac{1}{2}$
1	$\frac{1}{4}$	0

(a) Show the marginal distribution of X and Y, respectively. [4pts]
$P(X=0)=\frac{3}{4}$, $P(X=1)=\frac{1}{4}$, $P(Y=1)=\frac{1}{2}$, $P(Y=2)=\frac{1}{2}$

$$P_X(x) = \begin{cases} \frac{3}{4} & \text{if } x=0 \\ \frac{1}{4} & \text{if } x=1 \\ 0 & \text{otherwise} \end{cases}$$

$$P_Y(y) = \begin{cases} \frac{1}{2} & \text{if } y=1,2 \\ 0 & \text{otherwise} \end{cases}$$

(b) Find mutual information for the joint probability distribution in the previous question [3pts]

$$\begin{aligned} I(X,Y) &= P_{X,Y}(0.1)\log\frac{P_{X,Y}(0.1)}{P_X(0)P_Y(1)} + P_{X,Y}(0,2)\log\frac{P_{X,Y}(0,2)}{P_X(0)P_Y(2)} \\ &+ P_{X,Y}(1,1)\log\frac{P_{X,Y}(1,1)}{P_X(1)P_Y(1)} + P_{X,Y}(1,2)\log\frac{P_{X,Y}(1,2)}{P_X(1)P_Y(2)} \\ &= \frac{1}{4}\log\frac{\frac{1}{4}}{\frac{3}{4}\times\frac{1}{2}} + \frac{1}{2}\log\frac{\frac{1}{2}}{\frac{3}{4}\times\frac{1}{2}} + \frac{1}{4}\log\frac{\frac{1}{4}}{\frac{1}{4}\times\frac{1}{2}} + 0 \\ &= 0.2158 \end{aligned}$$

4.2 Mutual Information and Entropy [13pts]

Given a dataset as below.

$Patient$	$Temperature\,(x_1)$	$Cough\,(x_2)$	$Fatigue\,(x_3)$	$Nausea\,(x_4)$	$COVID?\,(Y)$
1	< 37	Yes	$Absent$	$Absent$	Low
2	$37-38$	Yes	$Present$	$Present$	$High$
3	< 37	No	$Absent$	$Present$	Low
4	$37-38$	No	$Absent$	$Present$	Low
5	< 37	Yes	$Present$	$Absent$	$High$
6	> 38	No	$Absent$	$Absent$	Low
7	$37-38$	No	$Absent$	$Present$	Low
8	> 38	Yes	$Present$	$Absent$	$High$
9	< 37	No	$Present$	$Present$	$High$
10	$37-38$	Yes	$Present$	$Absent$	$High$
11	$37-38$	No	$Absent$	$Absent$	Low
12	< 37	Yes	$Present$	$Present$	$High$
13	> 38	Yes	$Absent$	$Absent$	$High$
14	$37-38$	Yes	$Present$	$Absent$	$High$

You are analyzing the relationship between the signs and symptoms of COVID-19 for early detection and assessment to reduce the transmission rate of SARS-Cov-2. We want to determine what symptoms might affect the contraction of COVID-19. Each input has four features (x_1, x_2, x_3, x_4): Temperature (in degree Celsius), Cough, Fatigue, Nausea. The outcome is the probability to contract COVID (High vs Low), which is represented as Y.

(a) Find entropy $H(Y)$. [2pts]

$$H(Y) = -\frac{4}{7}\log_2\frac{4}{7} - \frac{3}{7}\log_2\frac{3}{7} = 0.9852$$

(b) Find conditional entropy $H(Y|x_1)$, $H(Y|x_4)$, respectively. [5pts]

$$\begin{aligned}
H(Y|x_1) &= \frac{5}{14}(-\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5}) + \frac{3}{7}(-\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2}) + \frac{3}{14}(-\frac{1}{3}\log_2\frac{1}{3} - \frac{2}{3}\log_2\frac{2}{3}) \\
&= 0.9721 \\
H(Y|x_4) &= \frac{4}{7}(-\frac{3}{8}\log_2\frac{3}{8} - \frac{5}{8}\log_2\frac{5}{8}) + \frac{3}{7}(-\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}log_2\frac{1}{2}) \\
&= 0.9740
\end{aligned}$$

(c) Find mutual information $I(x_1, Y)$ and $I(x_4, Y)$ and determine which one (x_1 or x_4) is more informative. [4pts]

$$
\begin{aligned}
I(x_1, Y) &= H(Y) - H(Y|x_1) \\
&= 0.9852 - 0.9721 = 0.0131 \\
I(x_4, Y) &= H(Y) - H(Y|x_4) \\
&= 0.9852 - 0.9740 = 0.0112
\end{aligned}
$$

Since $I(x_1, Y) > I(x_4, Y)$, x_1 is more informative.

(d) Find joint entropy $H(Y, x_3)$. [2pts]

$$
\begin{aligned}
H(Y, x_3) &= \frac{3}{7}\log_2\frac{7}{3} + \frac{1}{14}\log_2 14 + \frac{1}{2}\log_2 2 \\
&= 1.2958
\end{aligned}
$$

5 Bonus for All [10 pts]

5.1 Mutual Information [3 pts]

Prove that the mutual information is symmetric, i.e., $I(X, Y) = I(Y, X)$ and $x_i \in X, y_i \in Y$

5.2 Probabilities [7 pts]

Due to the recent social distancing requirement, Amazon is re-evaluating their delivery policies. In order to properly update their policy, Amazon is analyzing data from previous records. Delivery time can be classified as early, on time or late. Delivery distance can be classified as within 5 miles, between 5 and 10 miles and over 10 miles. From the previous records, 10% of deliveries arrive early, and 50% arrive on time. 60% of orders are within 5 miles and 25% of orders are between 5 and 10 miles. The probability for arriving on time if delivery distance is over 10 miles is 0. The probability of a shipment arriving on time and having a delivery distance between 5 and 10 miles is 15%. The probability for arriving early if delivery distance is within 5 miles is 20%.

(a) What is the probability that the delivery will arrive on time if the distance is between 5 and 10 miles? [2 pts]

(b) What is the probability that the delivery will arrive on time if the distance is within 5 miles? [3 pts]

(c) What is the probability that the delivery will arrive late if the distance is within 5 miles? [2 pts]