Communication Systems: Homework #3

Due on Aban 14, 1396 at 7:30am

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Problem 1

Determine wether these signals are energy-type or power-type. In each case, find the energy spectral density and the energy content or the power spectral density and the power content of the signal.

1. $e^{-\alpha|t|} sin(\beta t)\ \alpha, \beta > 0$
2. $sinc^2(3t)$
3. $\sum_{n=-\infty}^{+\infty} \Lambda(t-3n)$
4. $2u(-t)$

Problem 2

Find the energy spectral density and the energy content, or power-spectral density and the power content of the output of the following LTI system when driven by the signals of the previous problem.

$$h(t) = sinc(2t) \tag{1}$$

Problem 3

The voltage $f(t) = 10te^{-2t}u(t)$ is developed across a 50 ohm resistor.

- Calculate the total energy dissipated in the resistor.
- What fraction of the energy is contained within a low pass bandwidth of 1 rad/sec?
- What fraction of the energy is contained within a bandwidth of 2 rad/sec with a center frequency of 4 rad/sec?

Problem 4

Determine the autocorrelation function of each of the signals

1. $e^{-\alpha^2 t}u(t)$
2. $rect(\frac{t}{T})$

Problem 5

The lowpass signal $x(t)$ with a bandwidth of W is sampled with a sampling interval of T_s and the signal

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT_s)p(t - nT_s) \tag{2}$$

is generated, where p(t) is an arbitrary shaped pulse (not necessarily time-limited to the interval $[0, T_s]$).

1. Find the Fourier transform of $x_p(t)$.
2. Find the conditions for perfect reconstruction of $x(t)$ from $x_p(t)$.
3. Determine the required reconstruction filter.

Problem 6

Let $X_1, X_2, \ldots, X_n$ denote i.i.d. random variables, each with PDF $f_{\mathcal{X}}(x)$,

- If $Y = min\{X_1, X_2, \ldots, X_n\}$, find the PDF of Y.
- If $Z = max\{X_1, X_2, \ldots, X_n\}$, find the PDF of Z.

Problem 7

Show that for a Poisson random variable defined by the PMF $P(X = k) = \frac{\lambda^k}{k!}e^{-\lambda},\ k = 0, 1, 2, \ldots$, the characteristic function is given by $\psi_{\mathcal{X}}(\nu) = e^{\lambda(e^{j\nu}-1)}$. Use this result to show that $E[X] = \lambda$ and $VAR(X) = \lambda$.

Problem 8

Two random variables X and Y are distributed according to

$$f_{X,Y}(x, y) = \begin{cases} Ke^{-x-y}, & x \geq y \geq 0 \\ 0, & otherwise \end{cases} \tag{3}$$

1. Find K.
2. Find the marginal density functions of X and Y.
3. Are X and Y independent?

Problem 8 continued on next page...

4. Find $f_{X|Y}(x|y)$.

5. Find $E[X|Y = y]$.

6. Find $COV(X, Y)$ and $\rho_{X,Y}$.

Problem 9

Plot the time-frequency diagram of the sounds uploaded in CW using MATLAB spectrogram function and answer the following questions:

1. Explain the difference between a male sound and a female sound.

2. Describe the application of spectrogram in sound processing with the aid of the test.wav file.

3. Describe the relation between the time diagram, frequency diagram and spectrogram of a signal.

Try to adjust the parameters of spectrogram function to take an appropriate diagram.