BCS054

Ans 1(a) (i) 0.00000123 5432

Chapping = 0.12354322 × 10^{-5}

= 0.1235 × 10^{-5}

Narmatised form

+	1	2	3	5	-	0	5

257890000012

Rounding = 0.257890000012 × 10^{12}

= 0.2579 × 10^{12}

Normalised from

+	2	5	7	9	+	1	2

(ii) 0 255640 × 10^{1}

We have to use chapping rouding = 0.2556 × 10^{1}

Absalute error = 0.255640 × 10^{1} − 0.2556 × 10^{1}

= 0.0004 × 10^{1} = 4 × 10^{-5} = 4 × 10^{-4}

2.55640

We have to use chapping bading

= 2.55600

Relative error = (2.55640 − 2.55600) / 2.55640

= 1.56 × 10^{-4}

(iii) x_1 = 0.1235 × 10^{-99}

x_2 = 0.2579 × 10^{-101}

To Compute the value of $x_2 - x_2$

result $x = x_2 - x_1$ = 0.13440 × 10^{-99} = 0.01344 × 10^{-101}

But the exponent -101 of the result is less than the smallest export -99 that can be stored in our standard form

+/-	D1	d2	d3	d4	+/-	e1	e0

The error, due to the fact the result cannot be stored

(iv) $a = 0.1235 \times 10^{-5}$ and $b = 0.2579 \times 10^{+2}$

$= 0.03185065 \times 10^{-1}$

into normalized form

+	0	3	1	8	6	-	1

(v) Overflow = Exponent is too type

$a = 0.00000000003234$

$= 0.3234 - 13$

$b = 0.2000 \times 10^{99}$

$c = a \times b = m \times 10^{c}$ where m is in normalized floting form, then if $0.1 \leq |m|$ then $m = m_1 \times m_2$

$e \leftarrow e, t e_2 = -13 + 99 = 86$

else $m = m_1 \times m_2 \times 10$ and

$e \leftarrow e_1 + e_2 - 1 = -13 + 99 - 1 = 85$

In both case a cannot be stoted in the 2 decimal digit space allotted to e or alternatively we can show the overflow throught the following argument $e - 13 + 99 = 86 \geq 99$ cannot be stored in 2 decimal

(vi) In learning how floating Point numbers are represented in Computers I have Come across the term " bias value " that I do not required understand.

Ans 1(b) Loss of Significant digit during floating Point Computative is known as Subtractive Concellation. floating Point Computation means representation of real number in such a way that trade off between range and Precision is maintained. These Significant figures are lost due to Subtraction of nearly equal floating - Point numbers.

Ans 1(c) => maclaurin's Theorm =)
Statement → if Closed interval $[0, x]$ define on function f then differential

(i) The order if $(n-1)$ in Continuous on $[0, x]$.
(ii) The exist in $f^n(x)$ interval $(0, x)$
(iii) if $P \in N$ then $\theta \in (0,1)$
because

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^{n-1}}{(n-1)} f^{n-1}(0) + R_n$$

Where, $R_n = \frac{x^n (1-\theta)^{n-p}}{p(n-1)!} f^n(\theta x)$

Now The given function =)

$f(x) = e^{2x}$ at $x=0$

$f'(x) = e^{2x} \cdot 2 = 2e^{2x}$

$f''(x) = 4e^{2x}$

$f'''(x) = 8e^{2x}$

we are find four term then =)

$f(x) = f(0) + x f'(0) + \frac{x^2}{2} f''(0) + \frac{x^3}{3!} f'''(0)$

$e^x \Rightarrow e^0 + x \cdot 2 \cdot e^0 + \frac{x^2}{2} 4 \cdot e^0 + \frac{x^3}{3} \cdot 8e^0$

$= 1 + 2x + 2x^2 + 4^2 x^3/3$ in terms $(0, x)$

(d) Truncation error made by truncation an infinite sum and approximating it by a finite sum. for instance, if we approximate the sine function by the first two non-zero term of its Taylor series as in $\sin(x) = x - \frac{1}{a} x^2$, for the resulting error is a truncating error. it is present even with infinite precision arithmetic, because its caused by trucation of the infinte Talyal series to form the algorithm.

Ans 2 (a) The Given equation ⇒

$2x + y + 5z = 18$ — (1)

$5x + 3y - 2z = 2$ — (2)

$x - 6y + 2z = 1$ — (3)

first of all we multiply by eq (3) from (2) and Subtract from eq (1) →

$2x - 12y + 4z = 2$

$\underline{-\ 2x \pm y \pm 5z = 18}$

$-13y - z = 16$

$13y + z = 16$ — (4)

Now multiply by 4 of equ (4) and Subtrating from eq (3)

$52y + 4z = 64$

$\underline{-\ 11y + 4z = 1}$

$63y = 63$

$y = \frac{63}{63} = 1$

$y = 1$

Put the value of y in eq (3) then

$-11 \times 1 + 4z = 1$

$-11 + 4z = 1$

$4z = 1 + 11$

$4z = 12$

$z = \cancel{12}^{3} / \cancel{4}$

$z = 3$

Now the value of y and z in eq (1) and find the value of x =)

$2x + y + 5z = 18$

$2x + 1 + 5 \times 3 = 18$

$2x + 1 + 15 = 18$

$2x + 16 = 18$

$2x = 18 + 6$

$2x = 2$

$x = \cancel{2}/\cancel{2}\ 1$

$x = 1$

Thus theing elimination method. the value of $x = 1,\ y = 1,\ z = 3$

Ans 2 (b) (i) Jacobi method -

$6x + 4y - z = 25$

$4x - 8y + 3z = -1$

$-3x + 2y + 5z = 0$

first of all, we find the value of x, y, z for four iteration method ⇒

$$x = \frac{1}{6}[25 - 4y + z]$$

$$y = \frac{1}{8}[-1 - 4x - 3z] = \frac{1}{8}[1 + 4x + 3z]$$

$$z = \frac{1}{5}[0 + 3x - 2y] = \frac{1}{5}[3x - 2y]$$

Ist iteration ⇒ $x = 0 = y = 0, z = 0$

$$x' \Rightarrow \frac{1}{6}[25 - 4 \times 0 + 0]$$

$$= \frac{1}{6}[25] = \frac{25}{6} \Rightarrow \boxed{4.16}$$

$$y' = \frac{1}{8}[1 + 4 \times 0 + 3 \times 0] = \frac{1}{8} = 0.125$$

$$z' \Rightarrow \frac{1}{5}[3 \times 0 - 2 \times 0] = 0$$

Now for IInd iteration =

$$x' = \frac{25}{6} \quad y = \frac{1}{8}, z = 0$$

$$x'' \Rightarrow \frac{1}{6}\left[25 - 4 \times \frac{1}{8} + 0\right] = \frac{1}{6}\left[25 - \frac{1}{2}\right]$$

$$= \frac{1}{6} \times \frac{49}{2} = \frac{49}{12} = 4.08$$

$$y'' = \frac{1}{8}\left[1 + 4 \times \frac{25}{6} + 3 \times 0\right]$$

$$= \frac{1}{8}\left[1 + \frac{100}{6}\right] \Rightarrow \frac{1}{8} \times \frac{106}{3} = \frac{53}{24} \Rightarrow [2.29]$$

$$z'' = \frac{1}{5}\left[3 \times \frac{25}{2} - 2 \times \frac{1}{8}\right]$$

$$= \frac{1}{5}\left[\frac{25}{2} - \frac{1}{4}\right] \Rightarrow \frac{1}{5}\left[\frac{50 - 1}{4}\right] = \frac{49}{20} = 2.45$$

Now for IIIrd iteration $\Rightarrow$

$$x'' = \frac{49}{12} \quad y'' = \frac{53}{24} \quad z'' = \frac{49}{20}$$

$$x^{III} = \frac{1}{6}\left[25 - 4 \times \frac{25}{24} + \frac{49}{20}\right]$$

$$= \frac{1}{6}\left[\frac{25}{1} - \frac{53}{6} + \frac{49}{20}\right]$$

$$= \frac{1}{6}\left[\frac{1500 - 530 + 147}{60}\right]$$

$$= \frac{1}{6}\left[\frac{1647 - 530}{60}\right] \Rightarrow \frac{1}{6}\left[\frac{1117}{60}\right] \Rightarrow \frac{1117}{360} =$$

$[3.102]$

$$y''' = \frac{1}{8}\left[1 + 4 \times \frac{49}{12} + 3 \times \frac{49}{20}\right]$$

$$= \frac{1}{8}\left[\frac{1}{1} + \frac{49}{3} + \frac{147}{20}\right]$$

$$= \frac{1}{8}\left[\frac{60 + 980 + 441}{60}\right] = \frac{1}{8}\left[\frac{1481}{60}\right] = \frac{1481}{480} = 3.085$$

$$z''' = \frac{1}{5}\left[3 \times \frac{49}{12} - 2 \times \frac{53}{24}\right] = \frac{1}{5}\left[\frac{49}{4} - \frac{53}{12}\right] =$$

$$= \frac{1}{5}\left[\frac{147 - 53}{12}\right] = \frac{94}{60} = [1.566]$$

Now, IVth iteration

$$x''' = \frac{1117}{360}, \; y''' = \frac{1481}{480} = z''' = \frac{94}{60}$$

$$x^{IV} = \frac{1}{6}\left[25 - 4 \times \frac{1481}{480} + \frac{94}{60}\right]$$

$$= \frac{1}{6}\left[\frac{3000 - 1481 + 188}{120}\right]$$

$$= \frac{1}{6}\left[\frac{3188 - 1481}{120}\right] = \frac{1707}{720} = \frac{569}{240} = 2.370$$

$$y^{IV} = \frac{1}{8}\left[1 + 4 \times \frac{1117}{360} + 3 \times \frac{94}{60}\right]$$

$$= \frac{1}{8}\left[\frac{1}{1} + \frac{1117}{90} + \frac{94}{20}\right]$$

$$= \frac{1}{8}\left[\frac{180 + 2234 + 846}{180}\right]$$

$$= \frac{1}{8}\left[\frac{3260}{180}\right] = \frac{3260}{1440} = \frac{326}{144} = \frac{163}{72} = 2.263$$

$$z^{IV} = \frac{1}{5}\left(3 \times \frac{1117}{360} - 2 \times \frac{1481}{480}\right)$$

$$= \frac{1}{5}\left[\frac{1117}{120} - \frac{1481}{240}\right] = \frac{1}{5} \times \frac{2234 - 1481}{240} = \frac{753}{1200}$$

$$= 0.6275$$

Ans2(b) (iii) Gauss - Seidel method:

$$\begin{bmatrix} 6 & 4 & -1 \\ 4 & -8 & 3 \\ -3 & 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2.5 \\ -1 \\ 0 \end{bmatrix}$$

Gauss Siedel method are the same type of Jacobi's method:-

$$6x + 4y - z = 25 \quad \text{—} (1)$$

$$4x - 8y + 3z = -1 \quad \text{—} (2)$$

$$-3x + 2y + 5z = 0 \quad \text{—} (3)$$

from eq (1), (2) and (3) →

IInd intertuin :

$$x'' = \frac{25 - 4y + z}{6}$$

Put the value of x, y and z from Ist interation.

$$x'' = \frac{25 - 4 \times 2.208 + 1.6164}{6}$$

$$= \frac{25 - 8.832 + 1.6164}{6}$$

$$= \frac{26.6164 - 8.822}{6} = \frac{17.7844}{6} = 2.9640$$

$$y'' = \frac{1 + 4x + 3z}{8}$$

where the value of x put from x'' iteration and the value of z put $z' = 1.6164$

$$y'' = \frac{1 + 4 \times 2.9640 + 3 \times 1.6164}{8}$$

$$= \frac{1 + 11.8560 + 4.8492}{8}$$

$$= \frac{12.8560 + 4.8492}{8} = \frac{17.7052}{8} = 2.21315$$

$$z'' = \frac{3 \times 2.9640 - 2 \times 2.21315}{5}$$

$$= \frac{8.8920 - 4.42630}{5} \quad \frac{4.46570}{5} = 0.89314$$

IIIrd iteration:

$$x''' = \frac{25 - 4y + z}{6}$$

$$x''' = \frac{25 - 4 \times 2.21315 + 0.89314}{6}$$

$$= \frac{25 - 8.85260 + 0.89314}{6}$$

$$= \frac{17.04054}{6} = 2.84009$$

$$y''' = \frac{1 + 4x + 3z}{8}$$

$$y''' \Rightarrow \frac{1 + 4 \times 2.84009 + 3 \times 0.89314}{8}$$

$$\Rightarrow \frac{1 + 11.36036 + 2.67942}{8}$$

$$= \frac{12.36036 + 2.67942}{8} = \frac{15.03978}{8}$$

$$= 1.87997$$

$$z''' = \frac{3x - 2y}{5}$$

$$z''' = \frac{3 \times 2.84009 - 2 \times 1.87997}{5}$$

$$= \frac{8.52027 - 3.75994}{5} = \frac{4.76033}{5}$$

$$= 0.95206$$

Date : / /
Page No. 13

IV iteration =

$$x^{IV} = \frac{25-4y+z}{6}$$

$$= \frac{25-4\times1.87997+0.95206}{6}$$

$$= \frac{25-7.51988+0.95206}{6}$$

$$= \frac{25.95206-7.51988}{6}$$

$$= \frac{18.43218}{6} = 3.07203$$

$$y^{IV} = \frac{1+4x+3z}{8}$$

$$y^{IV} = \frac{1+4\times3.0720+3\times0.95206}{8}$$

$$= \frac{1+12.28817+2.85618}{8}$$

$$= \frac{16.14430}{8} = 2.18037$$

$$z^{IV} = \frac{3x+2y}{5}$$

$$z^{IV} = \frac{3\times3.07203-2\times2.18037}{5}$$

$$= \frac{9.21609-4.36074}{5}$$

$= \frac{4.85535}{5} = 0.97107$

Gauss's Sielded method gives better approximation

Ans3. $f(x) = 3x^4 - 5x^2 - 11x - 13 = 0$

(a) By Regula Falsi method -

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \times f(x_n)$$

$f(0) = -13$ $f(1) = -26$ $f(2) = -7$ $f(3) = 152$

$f(-2) = 37$ $f(-1) = \cancel{2}\ -4$

Interval $[-2, -1]$

Take $x_0 = -2$ $x_1 = -1$

$$f(x_2) = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_1)$$

$$= -1 = \frac{-1+2}{-4-37} \times -4$$

$$= -1 - \frac{1}{+41} \times / 4$$

$$= \frac{-1}{1} - \frac{4}{41} = \frac{-41-4}{41} \Rightarrow \frac{-45}{41}$$

(b) Newton Raphson method :-

Take $x_0 = 1$

$$x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$$

$$= -1 - \frac{37}{-13} \Rightarrow \frac{-1}{1} + \frac{37}{13}$$

$$= \frac{37-13}{13} = \frac{24}{13} = [1.84]$$

$$x_2 = x_1 - \left(\frac{f(x_1)}{f'(x_1)}\right)$$

$$= 1.84 - \left[\frac{-15.78}{67.35}\right]$$

$$= 1.84 + \frac{15.78}{67.35} = 2.07$$

(c) Bisection Method:

$f(a)\ f(b) < 0$

$f(-2) = 37 \quad f(-1) = -4$

$37 \times -4 < 0$

$$\frac{a+b}{2} = \frac{-2-1}{2} = \frac{-3}{2} = -1.5$$

Second method

$x_0 = -2 \quad x_1 = 1$

$$x_2 = x_1 - \frac{(x_1 - x_0) f(x_1)}{f(x_1) - f(x_0)}$$

$$= -1 - \frac{(-1+2)-4}{-4-37}$$

$$= -1 + \frac{4}{-41} = \frac{-1}{1} - \frac{4}{41} = \frac{-41-4}{41} = \frac{-45}{41}$$

$$= -1.09$$

Ans 4: (i) Give data interpolating:

(a) $x = 1 \quad 3 \quad 6 \quad 10$

$f(x) = 1 \quad 5 \quad 26 \quad 82$

Here $x_0 = 1 \quad x_1 = 3 \quad x_2 = 6 \quad x_3 = 10$

$y_0 = 1 \quad y_1 = 5 \quad y_2 = 26 \quad y_3 = 82$

$x = 4$ (Given)

$y = ?$

By Lagrange's interpolating formula:

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} (y_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

$$y(4) = \frac{(4-3)(4-6)(4-10)}{(1-3)(1-6)(1-10)} \times 1 + \frac{(4-1)(4-6)(4-10)}{(3-1)(3-6)(3-10)} \times 5$$

$$+ \frac{(4-1)(4-3)(4-10)}{(6-1)(6-3)(6-10)} \times 26 + \frac{(4-1)(4-3)(4-6)}{(10-1)(10-3)(10-6)} \times 82$$

$$y(4) = \frac{1 \times -2 \times -6}{-2 \times 5 \times -9} \times 1 + \frac{3 \times -2 \times -6}{2 \times -3 \times -7} \times 5 + \frac{3 \times 1 \times -6}{5 \times 3 \times -4} \times 26$$

$$+ \frac{3 \times 1 \times -2}{9 \times 7 \times 4} = \frac{-12}{90} + \frac{36}{42} \times 5 + \frac{18}{60} \times 26$$

$$= \frac{6}{252} \times 82$$

$$\Rightarrow \frac{-12}{90} + \frac{180}{42} + \frac{39}{5} - \frac{41}{21}$$

$$= \frac{-2}{15} + \frac{180}{42} + \frac{39}{5} - \frac{41}{21}$$

$$= \frac{-84 + 27000 + 4914}{630} = 1230$$

$$= \frac{7614 - 1314}{630} = \frac{6300}{630} = 10$$

$$y(4) = 10$$

(b) Give that

$x = 6 \quad 20 \quad 42 \quad 90$

$y = (f(x)) = 1 \quad 3 \quad 5 \quad 8$

$x_0 = 6 \quad x_1 = 20 \quad x_2 = 42 \quad x_3 = 90$

$y_0 = 1 \quad y_2 = 3 \quad y_2 = 5 \quad y_3 = 8$

Using the Lagrange's inverse interpalation formula:-

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} \times x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} \times x_1$$

$$+ \frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} + x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)} \times x_3$$

$$x = \frac{(7-3)(7-5)(7-8)}{(1-3)(1-5)(1-8)} \times 6 + \frac{(7-1)(7-5)(7-8)}{(3-1)(3-5)(3-8)} \times 20$$

$$+ \frac{(7-1)(7-3)(7-8)}{(5-1)(5-3)(5-8)} \times 42 + \frac{(7-1)(7-3)(7-5)}{(8-1)(8-3)(8-5)} \times 90$$

$$x = \frac{4\times 2\times 1}{-2\times -4\times -7} \times 6 + \frac{6\times 2\times -1}{2\times -2\times -5} \times 20 + \frac{6\times 4\times -1}{4\times 2\times -3} \times 42$$

$$+ \frac{6\times 4\times 2}{7\times 5\times 3} \times 90$$

$$= \frac{-6}{7} - 12 + 48 + \frac{288}{7}$$

$$= \frac{288}{7} - \frac{6}{7} + \frac{36}{1}$$

$$\Rightarrow \frac{288-6+252}{7} = \frac{534}{7} = 76.28$$

Ans 5 (a) (i) Given that

$x = 2011 \quad 2013 \quad 2015 \quad 2017 \quad 2019$

$y = 5 \quad 15 \quad 45 \quad 102 \quad 193$

Using Stirlling difference formula:-

$$y_p = y_0 + P\mu\delta y_0 + \frac{P^2}{2!}\delta^2 y_0 + \frac{P(P^2-1)}{3!}\mu\delta^3 y_0$$

$$+ \frac{P^2(P^2-1)}{4!}\delta^4 y_0$$

$x = 2014$
$h = 2$

$x_p = x_0 + hp$

$2014 = 2013 + 2p$

$p = \frac{1}{2} = 0.5$

$\boxed{p = 0.5}$

$\boxed{y_0 = 15}$

Δ	Δ^1	Δ^2	Δ^2
5		20	
15	30	27	7
45			7
102	54	34	
193			

28

Now, $\mu \delta y_0 = \frac{30+57}{2} = \frac{87}{2} = 43.5$

Put all the value in stirling formula:-

$$y_A = 15 + 0.5 \times 43.5 + \frac{(2.5)^2}{2!}(43.5)^2 + \frac{0.5(0.25-1)(43.5)^3}{3!}$$

$$+ \frac{(0.5)^2(0.25-1)(43.5)^4}{4!}$$

$$15 + 21.75 + \frac{0.25}{2} \times 1892.25 + \frac{0.5 \times -0.75}{3 \times 2} \times 82302.875$$

$$+ \frac{(0.25) \times -0.75}{4 \times 3 \times 2} \times 3580610.0625$$

$15 + 21.75 + 236.53 - 5144.55 - 27973.51$

$= -32844.78$

Ans 5 (ii) Given that

x :	2011	2013	2015	2017	2019
y :	5	15	45	102	193

using Newton's forward formula estimate the deposits for the year 2012.

$$y = f(x) + P\,\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0$$

$$+ \frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 y_0 + \ldots$$

h = equal interval b/w x

$h = 2$

$x = 2012$

$x_0 = 2011$

$$P = \frac{x - x_0}{h} = \frac{2012 - 2011}{2} = \frac{1}{2} = 0.5$$

x	$f(x)$	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
2011	5				
		10			
2013	15		20		
		30		7	
2015	45		27		0
		57		7	
2017	102		34		
		91			
2019	113				

Put these value in formula:-

$$y = 5+0.5(10) + \frac{0.5(0.5-1)}{2} \times 20 + \frac{(0.5)(0.5-1)(0.5-2)}{3\times 2} \times 7$$

$$= 5+5+\frac{(0.5)(-0.5)}{2} \times 20 + \frac{(0.5)(-0.5)(-1.5)}{6} \times 7$$

$$= 10+(-2.5) + \frac{0.25 \times 1.5}{6} \times 7$$

$$= 10-2.5 + \frac{875}{1000 \times 2}$$

$$= 10-2.5 + \frac{437.5}{1000}$$

$$= 7.5 + 0.4375 = 7.9375$$

(iii) Give that

x:	2011	2013	2015	2017	2019
y:	5	15	45	102	193

Using Newton's backward difference formula:-

$$y(x_n + rh) = y_n + r\nabla y_n + \frac{r(r+1)}{2!} \nabla^2 y_n + \frac{r(r+1)(r+2)}{3!} \nabla^3 y_n + \ldots$$

x $f(x)$ Δy $\nabla^2 y$ $\nabla^3 y$ $\Delta^4 y$

~~2011~~

2011	5			
		10		
2013	15		20	
		30		7
2015	45		27	0
		57		7
2017	102		34	
		91		
2019	192			

$x_n = 2019$

$x = ?\quad , h = 2$

$x_n + xh = 2018$

$2019 + x \times 2 = 2018$

$2x = -1$

$\boxed{x = -0.5}$

$y_n = 193$

Put these value in formula:-

$$y(x) = 193 + (-0.5)91 + \frac{-(0.5)(-0.1+1)}{2!}34$$

$$+ \frac{(-0.5)(-0.5+1)(0.5)+2}{3!} \times 7$$

$$= 193 + (-45.5) + \frac{(-0.5)(0.5)}{2} \times 34 + \frac{(0.5)(0.5)(1.5)}{6} \times 7$$

$= 193 - 45.5 - 0.25 \times 17 - \frac{0.5 \times 0.5 \times 15 \times 7}{6 \times 20}$

$= 193 - 45.5 - 4.25 - \frac{0.25 \times 7}{4}$

$= 193 - 45.5 - 4.25 - \frac{1.75}{4}$

$= 193 - 45.5 - 4.25 - 0.4625$

$= 193 - 50.21 = 142.79$

(b) An expression of forward differrence operator in term of δ.

Shift operator $= E$ forward operator $= \delta$

$E = 1 + \delta$

$\delta = E - 1$

Ans 6 (a) Give data

x :	1	1.5	2	2.5
y :	3	7	12	18

$y = 2x^2 + 3x - 2$ for $x = 1.25$

The value of first and second derivatives by using forward difference method :-

$x_0 = 1 \quad x_1 = 1.5 \quad x_2 = 2 \quad x_3 = 2.5$

$h = x_1 - x_0 \Rightarrow 1.5 - 1 = 0.5$

$p = \frac{x - x_0}{h} = \frac{1.25 - 1}{0.5} = \frac{0.25}{0.5} \Rightarrow 0.5$

Date:

Page No.: 24

x	$f(x)$	Δy	Δy_2	Δy_3
1	3			
		4		
1.5	7		1	
		5		0
2	12			
		6		
2.5	18			

Forword interpolation formula:-

$y(x) = y_0 + P\Delta y_1 + \frac{P(P-1)}{2!}\Delta^2 y$

$= 3 + (0.5) \times 4 + \frac{(0.5)(0.5-1) \times (1)^2}{2!}$

$= 3+2 + \frac{0.5 \times -0.5}{2} = \frac{5-0.25}{2}$

$= 5 - 0.125$

$= 4.875$

$y = 2x^2 + 3x - 2$

Put $x = 1.25$

$y = 2(1.25)^2 + 3 \times 1.25 - 2$

$= 2 \times 1.5625 + 3.75 - 2$

$= 3.125 + 3.75 - 2$

$= 6.875 - 2 \Rightarrow 4.875$

Truncation Error $\Rightarrow 4.875 - 4.875 = 0$

(b) Given data

x :	1	1.5	2	2.5
y :	3	7	12	18

$y = 2x^2 + 3x - 2$ for $x = 1.25$

using Lagrange's interpolation formula:-

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

Put these Value in formula:-

$$y = \frac{(1.25-1.5)(1.25-2)(1.25-2.5)}{(1-1.5)(1-2)(1-2.5)} \times 3 + \frac{(1.25-1)(1.25-2)(1.25-2.5)}{(1.5-1)(1.5-2)(1.5-2.5)} \times 7$$

$$+ \frac{(1.25-1)(1.25-1.5)(1.25-2.5)}{(2-1)(2-1.5)(2.2.5)} \times 12$$

$$+ \frac{(1.25-1)(1.25-1.5)(1.25-2)}{(2.5-1)(2.5-1.5)(2.5-2)} \times 18$$

$$y = \frac{-0.25 \times -0.75 \times -1.25}{-0.5 \times -1 \times -1.5} \times 3 + \frac{0.25 \times -0.75 \times -1.25}{0.5 \times -0.5 \times -1}$$

$$+ \frac{0.25 \times -0.25 \times -1.25}{1 \times 0.5 \times -0.5} \times 12 + \frac{0.25 \times -0.25 \times -0.75}{1.5 \times 1 \times 0.5} \times 18$$

$$y = \frac{25 \times 75 \times 125 \times 10 \times 10}{5 \times 1 \times 15 \times 100 \times 100 \times 100} \times 3 + \frac{25 \times 75 \times 125 \times 10 \times 10}{5 \times 5 \times 1 \times 100 \times 100 \times 100} \times 7$$

$$= \frac{25 \times 25 \times 125 \times 10 \times 10}{1 \times 5 \times 5 \times 100 \times 100 \times 100} \times 12 + \frac{25 \times 25 \times 75 \times 10 \times 10}{15 \times 1 \times 5 \times 100 \times 100 \times 100} \times 18$$

$$y = \frac{15}{16} + \frac{105}{16} - \frac{15}{4} + \frac{9}{2}$$

$$= \frac{15 + 105 - 60 + 18}{16}$$

$$= \frac{138 - 60}{16} \Rightarrow \frac{78}{16} \Rightarrow 4.875$$

Longrange's method is more accurate

36

Ans 7 (a) Given integral

$$y = \int_0^6 (2x^4 + 3x^3 + 11x^2) dx$$

$$\frac{dy}{dx} = \left[\frac{2x^5}{5} + \frac{3x^4}{4} - \frac{11x^3}{3} \right]_0^6$$

$$\Rightarrow \frac{2(6)^5}{5} + \frac{3(6)^4}{4} - \frac{11(6)^3}{3} - 0$$

$$\Rightarrow \frac{2 \times 7776}{5} + \frac{3 \times 1296}{4} - \frac{11 \times 216}{3}$$

$$\Rightarrow 3110.4 + 972 - 792$$

$$\Rightarrow 3110.4 + 180 \Rightarrow 3290.4$$

Acutual value = 3290.4

(a) Trapezoidal Rule:-

$$\int_0^6 (2x^4 + 3x^3 - 11x^2) dx$$

$$\Delta x = \frac{6 - a}{a}$$

Formula:-

$$\int_a^b (f(x)) dx = \frac{\Delta x}{2} \left[y_0 + 2y_1 + 2y_2 + \dots 2y_{n-1} + y_n \right]$$

Divide the interval [0,6] into six parts, each of width $h = 1$

x : 0 1 2 3 4 5 6

$y = (f)(x)$: 0 -6 12 144 528 1350 2844

Now put these value in formula :-

$$\int_0^6 (2x^4 + 3x^3 - 11x^2)\,dx = \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$\Rightarrow \frac{1}{2}\left[(0 + 2844) + 2(-6 + 12 + 144 + 528 + 1350)\right]$$

$$= 0.5\left[2844 + 2028 \times 2\right]$$

$$= 0.5\left[2844 + 4056\right] \Rightarrow 0.5 \times 6900$$

$$= 3450$$

(b) Simpson's $\frac{1}{3}$ Rule:-

$$\int_0^6 (2x^4 + 3x^3 - 11x^2)\,dx = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$\Rightarrow \frac{1}{3}\left[(0 + 2844) + 4(-6 + 144 + 1350) + 2(12 + 528)\right]$$

$= \frac{1}{3}\left[2844 + 4 \times 1488 + 1080\right]$

$= \frac{1}{3}\left[2844 + 5952 + 1080\right]$

$= \frac{1}{3}\left[9876\right] = 3292$

Ans 8(a) Given that :-

$y' = 1 + 3xy$

$y(0) = 1$

$y(1.0) = ?$ when $= h = 0.25$, $h = 0.5$

$f(x, y) = 1 + 3xy$

The initial Condition $\Rightarrow$

$x_0 = 0$ $y_0 = 1$ $h = 0.25$

x iteration $= x_{n+1} = x_n + h$

Put $n = 0$

$x_1 = x_0 + h$

$x_1 = 0 + 0.25 = 0.25$

$x_1 = 0.25$

y iteration :- $y_{n+1} = y_n + h(f x_n, y_n)$

$n = 0$

$y_1 = y_0 + h(f(x_0, y_0)\}$

$y_1 = 1 + 0.25 \, (1 + 3xy)$

$= 1 + 0.25 \, (1 + 3 \, xy)$

$= 1 + 0.25 \, (1 + 3 \times 0 \times 1)$

$= 1 + 0.25 \Rightarrow 1.25$

$y_1 = 1.25$

$x_2 = x_1 + h$

$= 0.25 + 0.25 = 0.5$

$y_2 = y_1 + h \{f'(x_1 y_1)$

$= 1.25 + 0.25(1 + 3x_1 y_1)$

$= 1.25 + 0.25(1 + 3 \times 0.25 \times 1.25)$

$= 1.25 + 0.25(1 + 0.75 \times 1.25)$

$= 1.25 + 0.25(1 + 0.9)$

$= 1.25 + 0.25 \, (1 + 0.9)$

$= 1.25 + 0.25 \times 1.9$

$= 1.25 + 0.47 = 1.72$

$x_3 + x_2 + h$

$= 0.5 + 0.25 = 0.75$

$y_3 = y_2 + h f(x_2 \, y_2)$

$= 1.72 + 0.25 \, (1 + 3x_2 \, y_2)$

$= 1.72 + 0.25 \, (1 + 3 \times 0.5 \times 1.72)$

$= 1.72 + 0.25(1 + 1.5 \times 1.72)$

$= 1.72 + 0.25 \, (1 + 1.5 \times 1.72)$

$= 1.72 + 0.25 \, (1 + 1.5 \times 1.72)$

$= 1.72 + 0.25 (1 + 2.58)$
$= 1.72 + 0.25 (3.58)$
$= 1.72 + 0.89 = 2.61$

$x_4 = x_3 + h$
$= 0.75 + 0.25 = 1$

$y_4 = y_3 + h f(x_3, y_3)$
$= 2.61 + 0.25 (1 + 3x_3y_3)$
$= 2.61 + 0.25 (1 + 3 \times 0.75 \times 2.61)$
$= 2.61 + 0.25 (1 + 3 \times 0.75 \times 2.61)$
$= 2.61 + 0.25 (1 + 5.87)$
$= 2.61 + 0.25 (1 + 5.87)$
$= 2.61 + 0.25 \times 6.87$
$= 4.32$

$x_5 = x_4 + h \Rightarrow 1 + 0.25 = 1.25$
$y_5 = y_4 + h \times f(x_4, y_4)$
$= 1.25 + 0.25 (1 + 3x_4 y_4)$
$= 1.25 + 0.25 (1 + 3 \times 1 \times 4.32)$
$= 1.25 + 0.25 (1 + 12.96)$
$= 1.25 + 0.25 \times 13.96 = 4.74$

(ii) $h = 0.5$
$x_1 = x_0 + h$
$x_1 = 0 + 0.5 = 0.5$
$y_1 = y_0 + h f(x_0 + y_0)$
$= 1 + 0.5 (1 + 3 \times 0 \times 1$
$= 1 + 0.5 = 1.5$

Date :

Page No. : 27

$x_2 = x_1 + h$

$= 0.5 + 0.5 = 1.0$

$y = y_1 + h\, f(x_1, y_1)$

$= 1.5 + 0.5\,(1 + 3 \times 0.5 \times 1.5)$

$= 1.5 + 0.5\,(1 + 1.5 \times 1.5)$

$= 1.5 + 0.5\,(1 + 2.25)$

$= 1.5 + 0.5 \times (3.25)$

$= 1.5 + 1.625 = 3.125$

$x_3 = x_2 + h = 1 + 0.5 \Rightarrow 1.5$

$y_3 = 3.12 + 0.5\,(1 + 3 \times 1 \times 3.12)$

$= 3.12 + 0.5\,(1 + 3 \times 3.12)$

$= 3.12 + 0.5\,(1 + 9.36)$

$= 3.12 + 0.5\,(10.36)$

$= 3.12 + 5.180 = 8.30$

$x_4 = x_3 + h = 1.5 + 0.5 = 2$

$y_4 = 8.3 + 0.5\,(1 + 3 \times 1.5 \times 8.3)$

$= 8.3 + 0.5\,(1 + 37.35)$

$= 8.3 + 0.5\,(38.35)$

$= 27.4$

(b) Given differential equation $\Rightarrow$

$y' + x^2 y + x^2$ and $y(0) = 1$

formula =

$y_{n+1} = y_n + \frac{1}{2}(K_1 + K_2)$

where $K_1 = h\,(f(x_n, y_n))$

$K_2 = h\, f(x_n + h, y_n + K)$

$y(0.4) = ?$ $h = 0.2$

$x_{n+1} = x_0 + h$

$n = 0, 1, 2, \ldots$

$y(0) = 1 \quad x = 0, y = 1$

$f_1 = h f(x, y_0)$

$= 0.2 [0] \times 1 + (0)^2 = 0$

$f_2 = x + \frac{h}{2}$

$= 0 + \frac{0.2}{2} \Rightarrow 0.1$

$y + \frac{f_1}{2} \Rightarrow 0 + \frac{0.2}{2} = 0.1$

$f_2 = h f\left(x + \frac{h}{2}, y + \frac{f_1}{2}\right)$

$= 0.2 [(0.1), 0.1]$

$= 0.2$

$f_3 = h f\left(x_0 + \frac{h}{2}\right), y_0 + \frac{k_2}{2}\right)$

$f_4 = h (x_0 + h, y_0 + k_3)$