

Math Reference Sheet for Physics

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This PDF contains mathematical formulas that I've used at least somewhat frequently while doing my undergraduate physics degree. Still a work in progress. You are free to use/edit it as you see fit.

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1 Trigonometry

Half Angle:

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}, \quad \sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}, \quad \cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

Double Angle:

2 Linear Algebra

Inverse: $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ **Transpose:** $(AB)^T = B^T A^T$

Diagonalizable matrix:

A $n \times n$ matrix A for which there exists a diagonal matrix D and an invertible matrix P such that:
 $A = PDP^{-1}$.

$$D = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix} \quad P = \begin{bmatrix} \vec{\lambda}_1 & \vec{\lambda}_2 & \cdots & \vec{\lambda}_n \end{bmatrix}$$

Where λ_n is an eigenvalue of A and $\vec{\lambda}_n$ is its corresponding eigenvector.

3 Complex Analysis

Modulus: $|z|^2 = z\bar{z}$, $|z_1 z_2| = |z_1| |z_2|$, $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$, $|z^n| = |z|^n$

Conjugate: $\overline{x \pm y} = \bar{x} \pm \bar{y}$, $\overline{xy} = \bar{x}\bar{y}$, $\overline{\left(\frac{x}{y}\right)} = \frac{\bar{x}}{\bar{y}}$

Euler's Identity: $e^{i\theta} = \cos \theta + i \sin \theta$

4 Probability

Average value (mean) of random variable X :

Discrete: $\mu_x = \sum_{n=1}^N x_n P(x_n)$

Continuous: $\mu_x = \int_{-\infty}^{\infty} xP(x)dx$

5 Quantum Mechanics

Completeness Relation: $\mathbb{1} = \sum_{j=1} |\phi_j\rangle \langle \phi_j|$

Change of basis for QM state vector for spin- $\frac{1}{2}$ particle to S_x basis:

$$|\psi\rangle \xrightarrow{S_x} \begin{bmatrix} \langle +x|\psi\rangle \\ \langle -x|\psi\rangle \end{bmatrix}$$

Unitary Matrix:

$$AA^\dagger = \mathbb{1}$$

Hermitian Matrix:

$$A = A^\dagger$$

Orthonormal Matrix:

$$A^T = A^{-1}$$

Orthogonal basis states:

Inner product of basis vectors is 0.

$$\langle \psi_1 | \psi_2 \rangle = 0$$

Normalized basis states:

$$|\langle \psi_1 | \psi_1 \rangle|^2 = 1$$

Gaussian Integrals:

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} e^{-ax^2+bx} dx = e^{\frac{b^2}{4a}} \sqrt{\frac{\pi}{a}}$$