Math Reference Sheet for Physics

Amanuel Anteneh

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This PDF contains mathematical formulas that I've used at least somewhat frequently while doing my undergraduate physics degree. Still a work in progress. You are to free to use/edit it as you see fit.

1 Trigonometry

Half Angle:
$\sin^2\theta = \frac{1-\cos 2\theta}{2}, \quad \sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}, \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

Double Angle:

2 Linear Algebra

Inverse: $A^{-1} = \frac{1}{det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ **Transpose**: $(AB)^T = B^T A^T$

Diagonalizable matrix:

A $n \times n$ matrix A for which there exists a diagonal matrix D and an invertable matrix P such that: $A = PDP^{-1}$.

$$D = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix} \qquad P = \begin{bmatrix} \vec{\lambda_1} & \vec{\lambda_2} & \cdots & \vec{\lambda_n} \end{bmatrix}$$

Where λ_n is an eigenvalue of A and $\vec{\lambda_n}$ is it's corresponding eigenvector.

3 Complex Analysis

Modulus: $|z|^2 = z\bar{z}, \qquad |z_1 z_2| = |z_1||z_2|, \qquad |\frac{z_1}{z_2}| = \frac{|z_1|}{|z_2|}, \qquad |z^n| = |z|^n$

Conjugate: $\overline{x \pm y} = \bar{x} \pm \bar{y}, \quad \overline{xy} = \bar{x}\bar{y}, \quad \overline{(\frac{x}{y})} = \frac{\bar{x}}{\bar{y}}$

Euler's Identity: $e^{i\theta} = \cos\theta + i\sin\theta$

4 Probability

Average value (mean) of random variable X:

Discrete: $\mu_x = \sum_{n=1}^{N} x_n P(x_n)$

Continuous: $\mu_x = \int_{-\infty}^{\infty} xP(x)dx$

5 Quantum Mechanics

Completeness Relation: $\mathbb{1} = \sum_{j=1} |\phi_j\rangle \langle\phi_j|$

Change of basis for QM state vector for spin-$\frac{1}{2}$ particle to S_x basis:
$|\psi\rangle \xrightarrow{S_x} \begin{bmatrix} \langle +x|\psi\rangle \\ \langle -x|\psi\rangle \end{bmatrix}$

Unitary Matrix:
$AA^\dagger = \mathbb{1}$

Hermitian Matrix:
$A = A^\dagger$

Orthonormal Matrix:
$A^T = A^{-1}$

Orthogonal basis states:
Inner product of basis vectors is 0.
$\langle\psi_1|\psi_2\rangle = 0$

Normalized basis states:
$|\langle\psi_1|\psi_1\rangle|^2 = 1$

Gaussian Integrals:
$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}}$
$\int_{-\infty}^{\infty} e^{-ax^2+bx}\,dx = e^{\frac{b^2}{4a}}\sqrt{\frac{\pi}{a}}$