Tutorial - 1

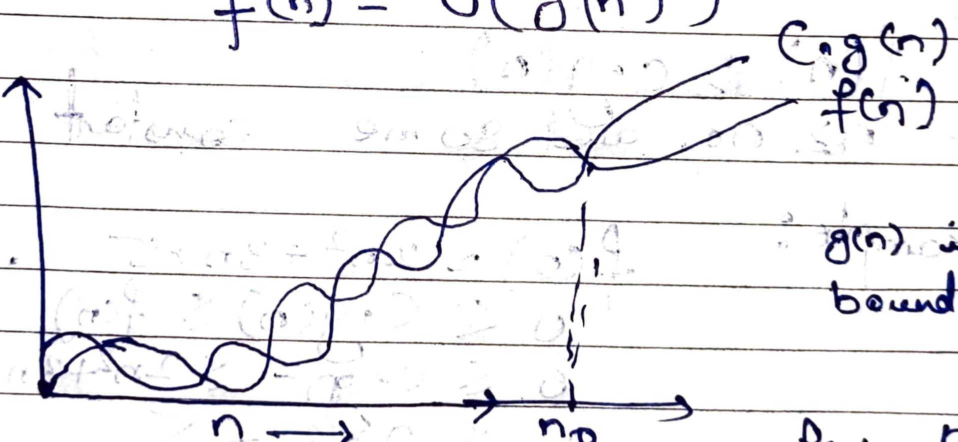
Ques 1: What do you mean by Asymptotic Notations. Define different type of notations along with example.

Ans :- Asymptotic Notations: Means tending to infinity. They are used to used to tell the complexity when input is very large.

→ Different types of Notations :

1. **Big oh (O) Notation :**

$$f(n) = O(g(n))$$



g(n) is "tight" upper bound of f(n)

$f(n) = O(g(n))$
if $f(n) \le c.g(n)$
$\forall\ n \ge n_0$ and some constant

Example :

```
for (i=1; i<=n; i++)
{   printf("*");  ——  O(1)
}
```

$\Rightarrow T(n) = O(n)$

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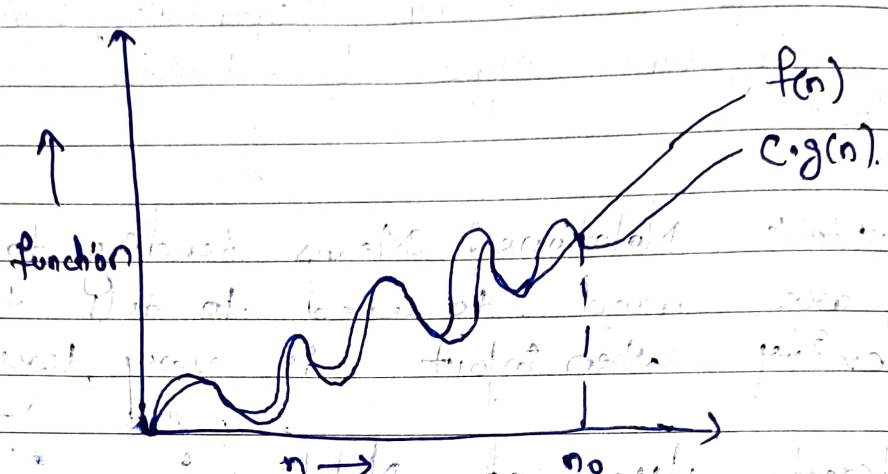
2. Big Omega (Ω):

$$f(n) = \Omega(g(n))$$



g(n) is "tight" lower bound of f(n)

$$f(n) = \Omega(g(n))$$

iff

$$f(n) \geq c \cdot g(n)$$

$\forall$ $n \geq n_0$, and some constant $c > 0$

Example:

$f(n) = 2n^2 + 3n + 5$, $g(n) = n^2$

$0 \leq c \cdot g(n) \leq f(n)$

$0 \leq c \cdot n^2 \leq 2n^2 + 3n + 5$

$c \leq 2 + \frac{3}{n} + \frac{5}{n}$

on putting $n = \infty$, $\frac{3}{n} \to \infty$, $\frac{5}{n^2} \to \infty$

$\Rightarrow c = 2$

$\Rightarrow 2n^2 \leq 2n^2 + 3n + 5$

on putting $n = 1$

$2 \leq 2 + 3 + 5$

$2 \leq 10$ True

$\Rightarrow c = 2$, $n = n_0 = 1$

$0 \leq 2n^2 \leq 2n^2 + 3n + 5$

$\therefore f(n) = \Omega(n^2)$

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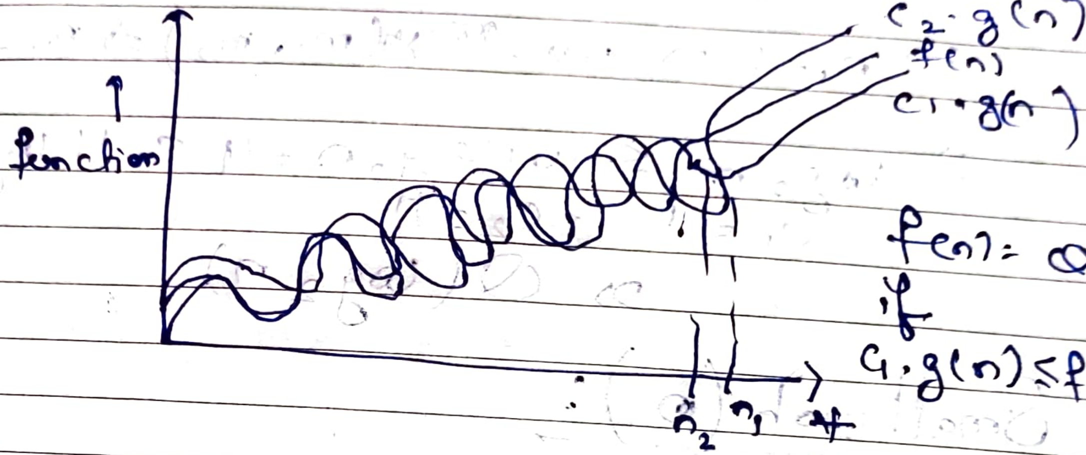
3. Big Theta (Θ):

$$f(n) = \Theta(g(n))$$



$f(n) = \Theta(g(n))$

if

$c_1 \cdot g(n) \leq f(n) \leq c_2 g$

$\forall$

$n \geq \max(n_1, n_2)$

and some constant

$c_1 > 0$ and $c_2 > 0$

Example: $f(n) = 10\log_2 n + 4$, $g(n) = \log_2 n$

$f(n) \leq c_2 \cdot g(n)$

$\Rightarrow 10\log_2 n + 4 \leq 10\log_2 n + \log_2 n$

$10\log_2 n + 4 \leq 11\log_2 n$

$c_2 = 11$

$\Rightarrow 4 \leq 11\log_2 n - 10\log_2 n$

$4 \leq \log_2 n$

$16 \leq n$

Here

$\forall n \geq 16$

$n_2 = 16$

$\angle \cdot c_2 = 1$

2

$f(n) \geq c_1 \cdot g(n)$

$10 \log_2 n + 4 \geq \cdot \log_2 n$

$c_1 = 1, n > 0$

$\Rightarrow n_1 = 1 \Rightarrow n_0 = max(m_1, m_2) \Rightarrow n_0 = 16$

$\Rightarrow \log_2 n \leq 10 \log_2 n + 4 \leq 11 \log_2 n$

$c_1 = 1, c_2 = 11$

$\Rightarrow \Theta(\log_2 n)$

4. **Small oh (o):**

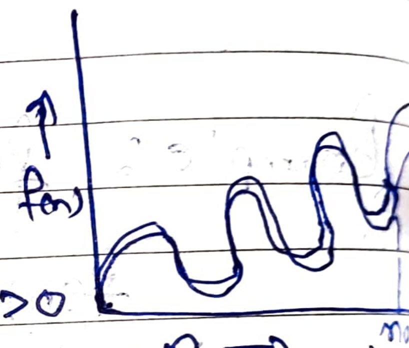
$f(n) = o(g(n))$

g(n) is upper bound of f(n)

$f(n) = o(g(n))$

if $f(n) < c \cdot g(n)$

∀ $n > n_0$ and ∀ constant, $c > 0$



5. **Small Omega (ω):**

$f(n) = \omega(g(n))$

g(n) is lower bound of f(n)

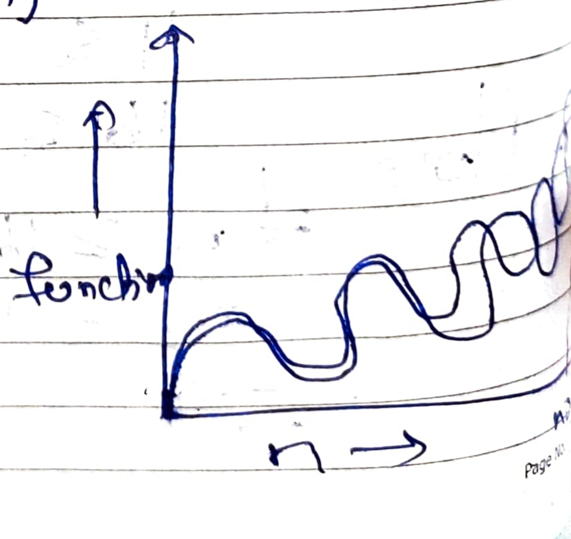
$f(n) = \omega(g(n))$

when

$f(n) > c \cdot g(n)$

∀ $n > n_0$

and ∀ $c > 0$



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Ques 2: What should be the time complexity of
for (i = 1

Sol^n Values of $i = \underbrace{1, 2, 4, 6, 16 \ldots n}_{K \text{ terms}}$

As this is a G.P with $a = 1, r = 2$

Now, K^{th} term :- $t_K = ar^{K-1}$

$n = 1 \cdot 2^{K-1}$

$n = 2^{K-1}$

taking $\log_2$ on to the both sides.

$\Rightarrow \log_2 n = \log_2 2^{K-1}$

$\log_2 n = (K-1) \log_2 2$

$\log_2 n = K - 1 \Rightarrow K = 1 + \log_2 n$ $[\because \log_2 2 = 1]$

$\therefore$ Time Complexity $T(n) = O(K)$

$= O(1 + \log_2 n)$

$= O(\log_2 n)$

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Ques 3: $T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

Soln

$T(n) = 3T(n-1)$ .... (1)

put $n = n-1$ in eqn 1

$T(n-1) = 3T(n-1-1)$ .... (2)

$T(n-1) = 3T(n-2)$

put value of $T(n-1)$ from eqn (2) in eqn (1)

$T(n) = 3[3T(n-2)]$ (3)

$T(n) = 9T(n-2)$ ....

put $n = n-2$ in eqn (1)

$T(n-2) = 3T(n-3)$ .... (4)

put value of $T(n-2)$ in eqn (3)

$T(n) = 3[9T(n-3)]$

$T(n) = 27T(n-3)$ .... (5)

On generalisation eqn (5)

$T(n) = 3^K T(n-K)$

Put $n-K = 0$

$\Rightarrow T(n) = 3^K T(0)$

$= 3^K$ $\quad (\because T(0) = 1)$

$\therefore T(n) = O(3^n)$

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Ques4: $T(n) = \{2T(n-1) - 1$ if $n>0$, otherwise $1\}$

Soln

$$T(n) = 2T(n-1) - 1 \quad \dots (1)$$

put $n = n-1$ in eqn (1)

$$T(n-1) = 2T(n-1-1) - 1$$
$$T(n-1) = 2T(n-2) - 1 \quad \dots 2$$

put value of $T(n-1)$ from (2) in (1)

$$T(n) = 2[2T(n-2) - 1] - 1$$
$$T(n) = 4T(n-2) - 2 - 1 \quad \dots (3)$$

Put $n = n-2$ in eqn (1)

$$T(n-2) = 2T(n-3) - 1$$

put value of $T(n-2)$ in eqn (3)

$$T(n) = 4[2T(n-3) - 1] - 2 - 1$$
$$T(n) = 8T(n-3) - 4 - 2 - 1$$

on generalising

$$T(n) = 2^K T(n-K) - 2^{K-1} - 2^{K-2} \dots (1)$$

Put $n-K=0 \Rightarrow n=K$, $T(0)=1$ (Given)

$$T(n) = 2^n T(0) - 2^{n-1} - 2^{n-2} - \dots \quad (1)$$
$$= 2^n - \underbrace{[2^{n-1} + 2^{n-2} \dots + 1]}_{K \text{ terms}},$$

$\Rightarrow a = 2^{n-1}$, $r = 1/2$

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Sum of GP $= \dfrac{2^{n-1}\left[1-\left(\frac{1}{2}\right)^{n-1}\right]}{1-\frac{1}{2}}$

$= 2^n - 2$

$\Rightarrow T(n) = 2^n - [2^n - 2] = 2$
$= O(2)$

$\boxed{T(n) = O(1)}$

Ques 5 What should be the time complexity of

```
int i=1, s=1;
while (s<=n) {
    i++; s = s+i;
    printf("#"); }
```

Soln

i	s
1	1
2	3
3	6
⋮	10
⋮	15
⋮	⋮
n	n

k times

$S = \underbrace{1, 3, 6, 10, 15, \ldots n}_{k \text{ terms}}$

k^{th} term, $t_k = t_{k-1} + k$

$k = t_k - t_{k-1}$ ---- (1)

$\Rightarrow k = n - t_{k-1}$

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loop runs K times

Time Complexity –

$O(1+1+1+n-t_{n-1})$

but $t_{n-1} = c$ (constant)

$\therefore$ Time Complexity $= O(3+n-1)$

$= O(n)$

Ques 6. Time Complexity of

```
Void function (int n) {
O(1) – int i, count = 0;
  for(i=1; i*i <= n; i++)
     Count ++;     O(1)
}
```

Soln

i*i

1^2

2^2

3^2

4^2

:

.

n

$i*i = \underbrace{1^2, 2^2, 3^2, 4^2 .. n}_{K \text{ terms}}$

$\Rightarrow K^{th}$ term $= T_K = K^2$

$K^2 = n$

$K = n^{1/2}$

Time Complexity

$= O(1+1+1+n^{1/2})$

$= O(n^{1/2})$

$= O(\sqrt{n})$

Ques 7. Time Complexity of

```
void function (int n)
{ int i, j, k, count = 0;
  for (i = n/2 ; i <= n ; i++)
    for (j = 1 ; j <= n ; j = j * 2)
      for (k = 1 , k <= n ; k = k * 2)
        count ++
}
```

Soln:- $i \rightarrow n/2 , \frac{n+2}{2} , \frac{n+4}{2} , \frac{n+6}{2} , \ldots$ upto n

$= \frac{n+0\times2}{2} + \frac{n+1\times2}{2} + \frac{n+2\times2}{2} , \ldots$ upto n

General form -

$$t_k = \frac{n + k * 2}{2}$$

total terms = k+1

$$t_{k+1} = n$$

$$\Rightarrow \frac{n + (k+1)*2}{2} = n$$

$$n + 2k + 2 = 2n$$

$$2k = n - 2$$

$$k = \frac{n}{2} - 1$$

i	j	k
$n/2$	$\log_2 n$ time	$(\log_2 n)^2$
$\frac{n+2}{2}$	$\log_2 n$ ti.	⋮
⋮	⋮	⋮
n	$\log_2 n$ time	$(\log_2 n)^2$
$\frac{n}{2} - 1$ time		

$$\Rightarrow \left(\frac{n}{2}-1\right)(\log_2 n)^2$$

$$= O\left(\frac{n}{2}\log_2^2 n - \log_2 n\right)$$

$$= O(n \log^2 n)$$

Ques 8 Time complexity of

```
function (int n) {
    if (n==1) return;        --- O(1)
for (i=1 to n) {             --- O(n)
        for (j=1 to n) {     --- O(n)
            printf("*");     --- O(1)
} }
```

Sol^n for function call

$\underbrace{n, n-3, n-6, n-9, \ldots\ldots 1}_{k \text{ terms}}$

AP with $d = -3$, $a = n$

$a_n = a + (n-1)d$

$1 = n + (k-1)(-3)$

$\frac{1-n}{(-3)} = k-1$

$k-1 = \frac{n-1}{3}$

$k = \frac{n-1+3}{3}$

$\boxed{k = \frac{n+2}{3}}$

Hence function has a recursive call $\frac{n+2}{3}$ times

$\Rightarrow$ Time complexity $=$ ~~O(n)~~ $\left(\frac{n+2}{3}\right)(n)(n)$

$O(n^3)$

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Ques 9 Time Complexity of

```
Void function (int n) {
for (i=1 to n) {
    for (j=1 ; j<=n ; j=j+i)
        printf("*");
} }
```

Soln:
for $i=1 \rightarrow j=1,2,3,4, \ldots n = n$
for $i=2 \rightarrow j=1,3,5,7, \ldots n = n/2$
for $i=3 \rightarrow j=1,4,7, \ldots n = n/3$

for $(i=n \Rightarrow j=1, \ldots \quad n=1$

$$\Rightarrow \sum_{j=n}^{n} n + n/2 + n/3 + n/4 + \ldots + 1$$

$$\sum_{j=n}^{1} n\left[1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\right]$$

$$\sum_{j=n}^{1} n\log n$$

$$T(n) = [n\log n]$$

$$T(n) = O(n\log n)$$

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Ques 10: For the functions, n^k and c^n, what is the asymptotic notation relationship between these functions. Assume that $k >= 1$ and $c > 1$ are constants. find out the value of c and no for which relation holds.

Soln As given n^k and c^n
relation b/w n^k and c^n is

$$\boxed{n^k = O(c^n)}$$

as $n^k \leq a c^n$ $\forall$ $n \geq n_0$ for a constant $a > 0$

for $n_0 = 1$
$c = 2$
$1^k \leq a 2^1$

$\therefore$

$$\boxed{n_0 = 1, \text{ and } c = 2}$$