

Number Theory (contd.)

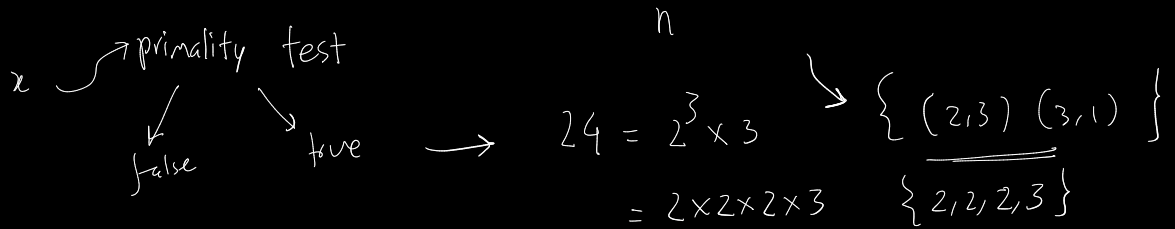
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GCD LCM

bitwise sieve

Prime Numbers



$$\sqrt{10^9} = \frac{3 \times 10^4}{2}$$

$$\frac{10^9}{2^{36}}$$

$$\{ (p_i, a_i) \}$$

9

$$\log_2 10^9 \approx 30$$

integer

$$p^2 = 10^9$$

$$\log p^2 = \log 10^9$$

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$$\log p^2 = \log 10^9$$

factorize (n)

- output a list of prime factors

$$2 \dots n$$

$$24$$

$$24 \rightarrow 12 \rightarrow 6 \rightarrow 3$$

$$(2,3)$$

$$3 \rightarrow 3/3 \rightarrow 1$$

$$(3,1)$$

$$18 \rightarrow 9 \rightarrow 3 \rightarrow 1$$

$$9 \rightarrow 3 \rightarrow 1$$

$$(2,1)$$

$$(3,2)$$

$$4 \times 5$$

$$n = 20$$

$$20 \rightarrow 10 \rightarrow 5$$

$$(2,2)$$

$$5 \rightarrow 5 \rightarrow 1$$

Prime Factorization

$$2,2$$

$$4,1$$

$$i=4 \rightarrow 1$$

$$i=4 \rightarrow$$

$$i=5 \rightarrow 5 \rightarrow 1$$

$$(5,1)$$

15! ~~काइने~~ ~~अथवा~~ ~~अथवा~~

Trailing Zeros

$$\boxed{n \leq 20}$$

$$\underline{n \leq 10^5}$$

$$\underline{1080} \rightarrow$$

$$5! = 120$$

~~720~~

Given a number n .

$$\underline{6! = 720}$$

$n!$ — Trailing Zeros.

$$4! = 24$$

$$\underline{n!}$$

$$5! = 120$$

$$\underline{20!}$$

$$n! \quad \underline{108800} \xrightarrow{\uparrow \uparrow} 10 - 10880$$

$$1088$$

$$\underline{5!} = 1 \times 2 \times 3 \times 4 \times 5$$

$$\underline{2 \times 5}$$

$$10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$$

$$3 \times 3$$

$$1$$

$$\underline{2 \times 2}$$

$$\underline{2 \times 3}$$

$$\underline{2 \times 2 \times 2}$$

$$\underline{2 \times 5}$$

$$\underline{2 \times 5}$$

$$\underline{n_2 \geq n_5}$$

$$\begin{matrix} 2 - 8 \\ 5 - 2 \end{matrix}$$

$$2 \times 5$$

for $i = 1 \dots n$

cnt += no. times 5 (i)

cnt

$$\frac{n}{2}$$

$$\frac{n}{4}$$

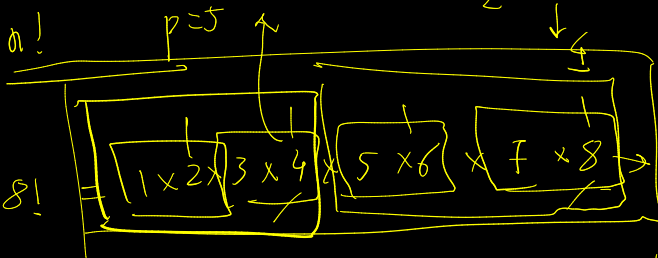
$$\frac{n}{8}$$

$$\underline{10^5} - 10^5$$

$$\underline{10^{10}}$$

$$p=2 \quad 4=2 \times 2 \rightarrow 2$$

$$\frac{9}{2} = 4.5 \rightarrow 4$$



काइने 2 थप 2 थप

$$2: 4 \rightarrow \left\lfloor \frac{n}{2} \right\rfloor$$

$$4: 2 \rightarrow \left\lfloor \frac{n}{4} \right\rfloor$$

$$8: 1 \rightarrow \left\lfloor \frac{n}{8} \right\rfloor$$

$$\underline{4+2+1=6}$$

$$\underline{n!} \quad \underline{p}$$

$$\left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \dots + \left\lfloor \frac{n}{p^k} \right\rfloor$$

Co-primes

संयोगात्मिक

12 7

6 4
~~1, 2, 3, 6~~ ~~1, 2, 4~~

Co-prime if
gcd = 1

14
 1, 2, 7, 14

9
 1, 3, 9

6
 1, 2, 3, 6

~~15~~
 1, 3, 5, 15

cop (i, j)

gcd(i, j) > 1

Euler's Totient Function

$\phi(n)$

(n) 1 ... n

1 ... n-1

phi(n)

phi(5) = 4
 1, 2, 3, 4, (5)

phi(6) = 1 + 1 = 2
 1, 2, 3, 4, 5
~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~

Coprime (4, 6)
 2

$n \log n$

$n(1 - \frac{1}{p_1})$

$12(1 - \frac{1}{2})$ phi(12) = (p_i, a_i)

$(12 - \frac{12}{2}) = 6$ $(1 - \frac{1}{3})$
12 = 2, 3 = $(6 - \frac{6}{3})$

$\phi(12) = 12 \times (1 - \frac{1}{2}) \times (1 - \frac{1}{3})$
= $6 \times \frac{2}{3} = 4$

$p_0, p_1, p_2, \dots, p_k$

$\phi(n) = n \times (1 - \frac{1}{p_0}) \times (1 - \frac{1}{p_1}) \dots \times (1 - \frac{1}{p_k})$

$6 \times (1 - \frac{1}{2}) \times (1 - \frac{1}{3})$

1, 2, 3, 4, 5, 6
7, 8, 9, 10, 11, 12 = $3 \times \frac{2}{3} = 2$

