Number Theory (contd.)

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GCD LCM

bitwise sieve

- Prime Numbers

x → primality test → false / true → $24 = 2^3 \times 3 = 2 \times 2 \times 2 \times 3$ → { (2,3) (3,1) }

{2, 2, 2, 3}

n

$\sqrt{10^9} = \frac{3 \times 10^4}{2}$ $\quad 10^9 =$ $\quad \{ (p_i, a_i) \}$

9

2^{30} $\quad p^{0}$ $\quad = 2$

$p^x = 10^9$

$\log p^x = \log 10^9$

$x \log p = \log 10^9$

$\log_2 10^9 \approx 30$

integer

factorize (n)

- output a list of prime factors

2 . . . . . n

n %

24

2 − $24/2 \rightarrow 12/2 \rightarrow 6/2 \rightarrow 3$

(2,3)

3 − $3/3 \rightarrow 1$

(3,1)

18

i=2 → 18 → 9
(2,1)

i=3 → 9 → 3 → 1
(3,2)

~~i=4~~ → 1

(2,1)
(3,2)

4x5

n: 20

→ i=2 → 20 → 10 → 5
(2,2)

i=3 → 5 ~~(3,0)~~

→ i=4 →

i=5 → 5 → 1

(5,1)

Prime Factorization

n=5

2,2

4,1

Array : static

int arr[10];

arr[i]=5

STL

vector

vector v = {1,2,3}

$3\times3 \nmid 3$

$i * i \le n$

v.push_back(x)

{1, 2, 3, x}

24

v.pop_back()

{1,2,3}

i=2

$12 \to 6 \to 3$

(2,3)

v.size()

v[i]

X i=3

v[i]

n=1

{1,7,3}

v[i]= 7

$\le \sqrt{n}$

$\boxed{n>1} \to$ n always prime

$p < \sqrt{N}$

$\pi(x) \approx \frac{x}{\lg x}$

$\pi(\sqrt{N})$

$\boxed{O(\pi(\sqrt{N}))}$

$\pi(x)$

$< x$

$\pi(10) = 4$

$\pi(15) = 6$

2, 3, 5, 7, 11, 13

Number of divisors

12 : $\boxed{1, 2, 3, 4, 6, 12}$

NoD (n)

$O(\sqrt{n})$

NoD(12)

= 6

$12 = 2^2 \times 3^1$

NoD(4)

3

1,2,4

$1 \to 2^0 \times 3^0$

$2 \to \boxed{2^1 \times 3^0}$

$3 \to 2^0 \times 3^1$

$4 \to 2^2 \times 3^0$

$6 \to 2^1 \times 3^1$

$12 \to 2^2 \times 3^1$

$2^0 \to 3^0, 3^1$

2^1

2^2

12

$66 = 2^2 \times 3^1 \times 5^1$

$2^0 \to 3^0 \to 5^0 \to 1$

$3^0 \to 5^1 \to 5$

$3^1 \to 5^0 \to 3$

$3^1 \to 5^1 \to 15$

2^1

2^2

$2^0 \to 2 * 2 = 4$

$2^0, 2^1, 2^2$

$3 \times 2 \times 2$

= 12

Prime factorization of n:

$(p_0, a_0) \quad (p_1, a_1) \; - - - \quad (p_k, a_k)$ $\qquad \Sigma$

prime $\qquad$ count

$$\text{NoD} = \boxed{(a_0+1)(a_1+1) \cdots (a_k+1)} = \prod_i (a_i+1)$$

$12 \to 2^2 \times 3^1$

$a_0 = 2$

$a_1 = 1$

$$\text{NoD}(12) = (2+1)(1+1) = 6$$

Sum of divisors

$60 = 2^2 \times 3^1 \times 5^1$

~~$2^0 + 2^1 + 2^2$~~

$$(2^0 + 2^1 + 2^2) \times (3^0 + 3^1) \times (5^0 + 5^1)$$

$$= \underset{1}{2^0 3^0} + \underset{3}{2^0 3^1} + \underset{2}{2^1 3^0} + \underset{6}{2^1 3^1} + \underset{4}{2^2 3^0} + \underset{12}{2^2 3^1}$$

multiplication of (summation of all powers of that prime)

nod / sod

15! কতগুলো শূন্য থাকবে

Trailing Zeroes

$\boxed{n \le 20}$ $\quad n \le 10^5 \quad$ 1080

~~7k~~

$5! = 120$

$4! = 24$

$6! = 720$

Given a number n.

n! — Trailing Zeroes.

n!

$5! = 120$ $\quad$ 20!

n! $\quad$ 108800 $\quad /10$ — 10880

1088

$5! = 1 \times 2 \times 3 \times 4 \times 5$

2×5

~~7k~~

$10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

$2\times2 \quad 2\times3 \quad 2\times2\times2 \quad 3\times3 \quad 2\times5$

2×5

2×5 $\quad n_2 \ge n_5$ $\quad$ 2 — 8 $\quad$ 5 — 2

for i = 1 ... n
  cnt += notimes 5 (i)

2 $\quad \frac{n}{x}$ $\quad O(n)$

$10^5 - 10^5$

10^{10}

cnt $\quad \frac{n}{2} \quad \frac{n}{4} \quad \frac{n}{8}$

$p = 2 \quad 4 = 2 \times 2$ → 2

$9/2 = 4.5$ → 4

n! $\quad p = 5$

এখানে 2 আছে 2 বারের

~~7k~~ 8! $\quad 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$

2 : 4 → $\lfloor \frac{n}{2} \rfloor$

4 : 2 → $\lfloor \frac{n}{4} \rfloor$

8 : 1 → $\lfloor \frac{n}{8} \rfloor$

8

~~16~~

n! $\quad$ p $\quad \lfloor \frac{n}{p} \rfloor + \lfloor \frac{n}{p^2} \rfloor + \lfloor \frac{n}{p^3} \rfloor + \dots + \lfloor \frac{n}{p^k} \rfloor$

Co-primes সহমৌলিক

12 7

6 4

~~1~~, 2, 3, 6 ~~1~~, 2, 4

co-prime if gcd = 1

14 9

1, 2, 7, 14 1, 3, 9

6 ~~15~~

1, 2, 3, 6 ~~1, 3, 5, 15~~

cop(i, j) gcd(i, j) > 1

Euler's Totient Function $\sigma(n)$

n 1 ... n

1 ... n-1 phi(n)

phi(5) = 4

1, 2, 3, 4, 5

phi(6) = 1 + 1 = 2

1, 2, 3, 4, 5

Coprime(4, 6)

2

n lg n

```
for i = 1 .... n-1 →
    if gcd(i, n) == 1 → lg n
        cnt++
```

$\sigma(n) = n \prod_i \left(1 - \frac{1}{p_i}\right)$

$n(1 - 1/2$

$12(1 - 1/2)$ phi(12) = (pi, ai

$p_0, p_1, p_2, \ldots p_k$

$\left(12 - \frac{12}{2}\right) = 6\left(1 - \frac{1}{3}\right)$

$= (6 - 6/3)$

$12 = 2, 3$

$\sigma(n) = n \times \left(1 - \frac{1}{p_0}\right) \times \left(1 - \frac{1}{p_1}\right) \ldots \times \left(1 - \frac{1}{p_k}\right)$

$\sigma(12) = 12 \times (1 - 1/2) \times (1 - 1/3)$

$= 6 \times 2/3 = 4$

$6 \times (1 - 1/2) \times (1 - 1/3)$

1, 2, 3, 4, 5, 6

7, 8, 9, 10, 11

$= 3 \times 2/3 = 2$

Factorization

NoD

SoD

Trailing Zeroes

Sieve

Euler's Totient Function

$\sigma(n)$

$[1 \ldots n)$

phi[10^5]

H.W

Given a bunch of numbers $n \leq 10^5$.

For each of them output phi(n)

restricted:

precompute the phi values

phi[n]