GROUP NUMBER: 18

FIN F313: SECURITIES ANALYSIS AND PORTFOLIO MANAGEMENT

ASSIGNMENT-1 OPTIMAL PORTFOLIO

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1. INTRODUCTION

Risk management is an important factor any investor takes into account when looking into the investment process for him/her to achieve optimal returns. This balance between risk and return can be achieved through an investment strategy that involves asset allocation- adjusting the proportion of investments in different asset classes and furthermore securities depending on the risk tolerance and goals of the investor. The investment process involves two steps of asset allocation and security selection and this report tries to reach the investment decisions through a top-down approach. The proportion of investments between risky and risk-free assets is done taking into account the age of the investor and the process of allocation between different assets is achieved making use of Markowitz Portfolio Theory and Single Index Model.

2. AIM AND SCOPE

The objective is to find the optimal portfolio allocation between 7 risky assets and risk-free assets taking into account the risk aversion index(A) of the investor. This report is limited to allotting between a risk-free asset (government securities) and stock equity. The investment step of allotment between different asset classes like allotment between debt, equity, and other real assets is not considered. The second step of the investment process which includes securities selection is done by choosing 7 equity securities trading on NSE, each from different industries to take advantage of the diversification benefit. The report focuses more on how the allocation between the chosen securities should be and how it changes for investors with various risk profiles and not on the security and asset class selection itself.

3. LIST OF FIRMS

Before proceeding with the Calculations, Client's profile and fund manager's information are used to select industries from different sectors for investments. For these purposes we have selected 7 different industries from various sectors.

i. Apollo HOSPITALS

Industry: HealthCare

It is the biggest private healthcare organization in India and has a hearty presence over the medical services environment, including Hospitals, Pharmacies, Primary Care and Diagnostic Clinics and a few retail wellbeing models. With stunning growth, it now has more than 71 hospitals, 3400+ Pharmacies and 10000+ beds all across India. It has a well-established track record of growth of CAGR 23% from FY14-20.

ii. Godrej PROPERTIES

Industry: Real-Estate

Godrej Properties was set up in 1989 as a subsidiary of Godrej Group, which is one of the largest conglomerates in India. Today, it is one of the leading players of the Indian real estate sector and has presence in over 12 urban communities. It ranks amongst India's most trusted brands as per the brand trust report of 2019 and its revenue and profit has grown by nearly 100x in the past 12 years.

iii. SIEMENS Limited

Industry: Automation

Siemens is one of the world's biggest producers of energy-efficient, resource-saving technologies and pioneers in infrastructure and energy solutions, automation and software for industry and is a leader in medical diagnosis. It also provides business-to-business financial solutions, rail automation and wind power solutions. Today it has about 22 factories across the country and 11 R&D Centers.

iv. Colgate Palmolive

Industry: Consumer goods

Colgate Palmolive is an American multinational company based in the production, distribution and provision of household, health care, personal care and veterinary products. It is a successor company to The Mennen company and is responsible for the top 300 companies for the production of plastic waste. So, as a result, the company is investing in recycling technology to reduce the waste produced and hence saving the environment for future growth.

v. Bharat Forge Ltd.

Industry: Forging metals and machinery

Bharat Forge Ltd. is a Pune based MNC involved in power, oil & gas, automotive, construction & mining, marine, aerospace. It has a joint venture with a France based company named Alstom and which is mostly involved in the production of turbines and generators. Alstom holds 51% of shares and Bharat forge holds 49% in the combined venture. Bharat forge is expanding its business in the defense sector as well in the coming years.

vi. FEDERAL Bank Limited

Industry: Banking

Federal Bank Limited is an Indian private sector, a scheduled business bank settled in Aluva, Kochi. The Bank additionally has its Representative Offices abroad at Abu Dhabi and Dubai.

With a client base of 10 million, including 1.5 million NRI clients and an enormous organization of settlement accomplices over the world, Federal Bank professes to deal with over 15% of India's internal settlements. During the year 2019-20, it has delivered robust growth in all the business segments and the net worth of the bank grew by 90 %.

vii. WELSPUN India Ltd.

Industry: Textile

Welspun India is a textile manufacturer based in Mumbai. It is the world's leading manufacturer in the production of terry towels. They export more than 90% of production to more than 50 countries in almost all the continents all over the world. The share prices have been stable for the last 2 years and there is an expectation for steady growth in the coming years.

4. SINGLE INDEX MODEL

i. Introduction

It is an asset pricing model proposed by William Sharpe in 1963 and is used to trade off risk and returns of stocks. It is based on some assumptions.

- Same set of common factors affects all securities.
- Return and risk of a security is due to common factors and due to firm specific factors.
- Covariance and correlation in security returns is due to common factors only.

Any broad market index like NIFTY50 can be considered as common factors for affecting all securities returns. For any firm, Returns can be broadly classified into the expected excess return of the individual stock due to firm-specific factors represented as alpha coefficient (α), the return due to macroeconomic events that affect the market, and the unexpected microeconomic events that affect only the firm.

$$R_i(t) = \alpha_i + \beta_i R_m(t) + e_i(t)$$

$R_i = r_i - r_f$: Security's excess return

α_i: Security's excess return when market's excess return is zero

β_i: sensitivity of security's return to market return

$R_m = r_M - r_f$: Excess return/ risk-premium of the market

e_i: Unexpected event relevant only to this security and has zero mean. Return of portfolio due to portfolio specific factors (firm specific factors).

ii. Expected return and Variance

$$E(R_i) = \alpha_i + B_i E(R_m)$$

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{e_i}^2$$

iii. Covariance and Correlation

With this equation, just the betas of the individual protections and the market difference should be assessed to ascertain covariance. Consequently, the index model enormously decreases the quantity of estimations that would somehow must be made to show a huge arrangement of thousands of securities.

$$\sigma_{ij} = \beta_i \beta_j \sigma_m^2$$

Correlation:
It lies between -1 and 1 and represents the amount of relation between two securities. Positive value signifies both securities moves in the same direction and negative value signifies the opposite movement as the value of any one changes.

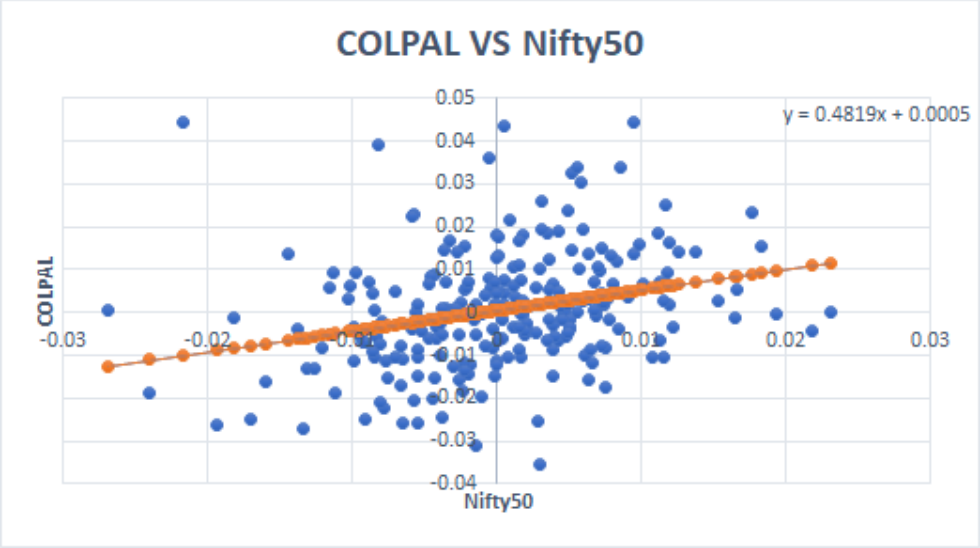
iv. Methodology

We used the Data analysis tool of MS Excel to regress daily excess returns of individual securities on daily excess returns of the market (here, Nifty50) and thus also obtained the best fit plot for the returns along with the values of alpha and betas for the securities.

v. Expected Returns and Variance of selected securities using Single Index Model

The expected returns and variances are taken using historical adjusted closed equity prices. The problem with the model lies with finding the covariance of the different securities with each other, considering each of the securities are from different industries or sectors. The Single Index Model (SIM) aims to arrive at this solution to find the covariances statistically.

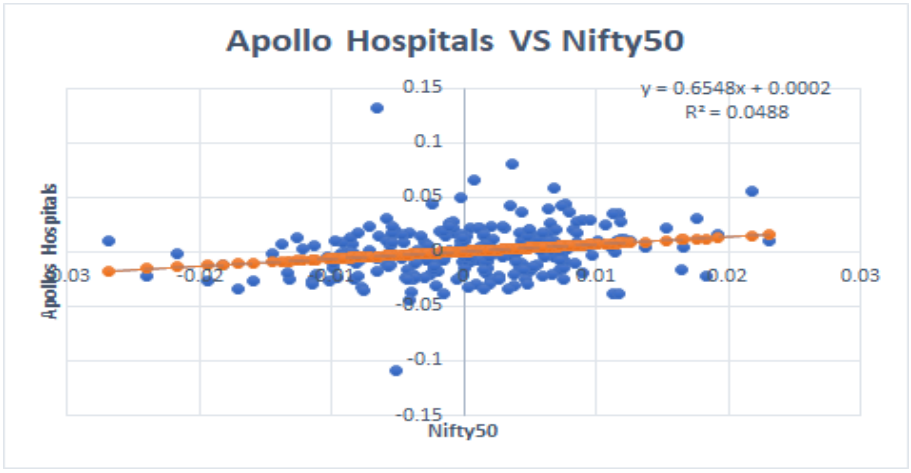
COLPAL



Linear regression of COLPAL excess returns v/s market return

α = 0.000491
β = 0.481942
$E(R)$ = 0.046
σ^2 = 0.045

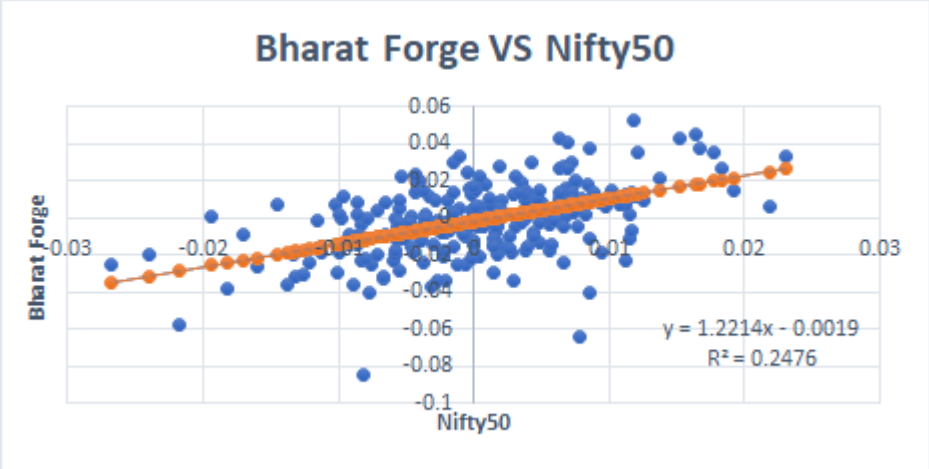
Apollo Hospitals



Linear regression of Apollo Hospitals excess returns v/s market return

α = 0.000173
β = 0.654774
$E(R)$ = 0.062
σ^2 = 0.136

Bharat Forge



Linear regression of Bharat Forge excess returns v/s market return

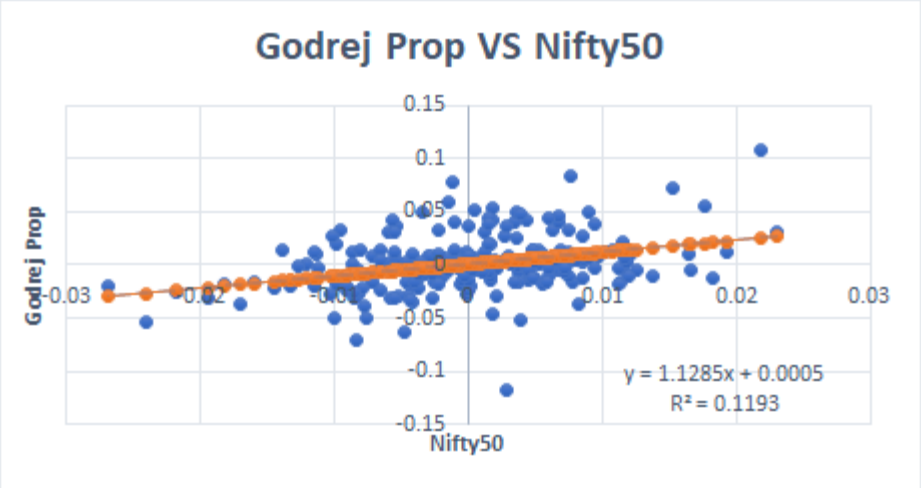
$\alpha = -0.00192$

$\beta = 1.221399$

$E(R) = 0.114$

$\sigma^2 = 0.093$

Godrej Properties



Linear regression of Godrej Prop. excess returns v/s market return

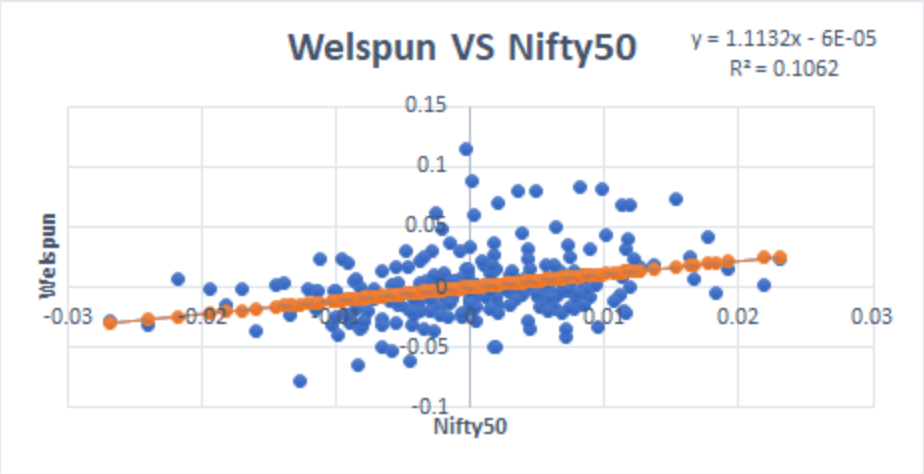
$\alpha = 0.000458$

$\beta = 1.12852$

$E(R) = 0.108$

$\sigma^2 = 0.165$

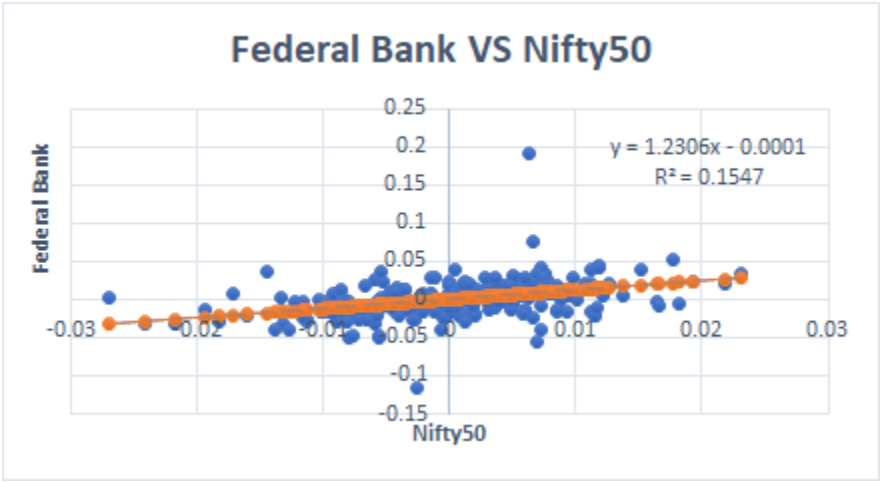
Welspun India Ltd.



Linear regression of Welspun India excess returns v/s market return

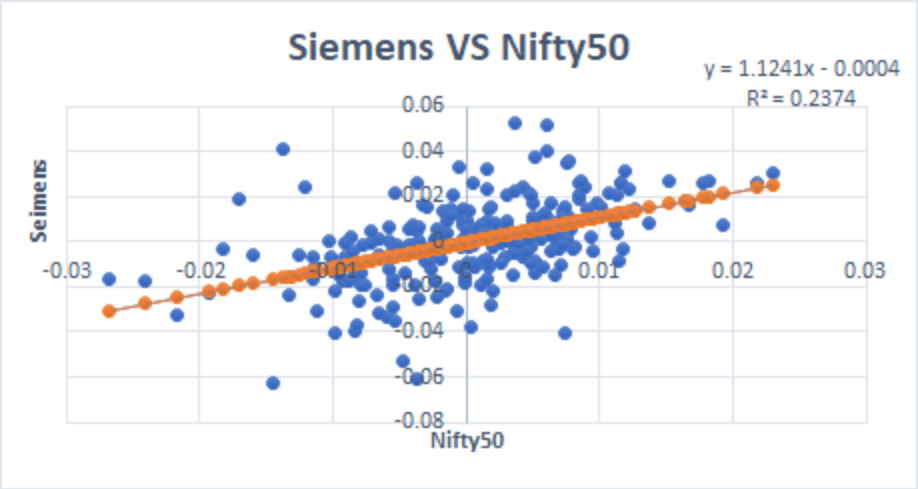
α = -5.78E-05
β = 1.11323472
$E(R)$ = 0.095
σ^2 = 0.181

Federal Bank



Linear regression of Federal Bank excess returns v/s market return

α = -0.00013
β = 1.230581
$E(R)$ = 0.095
σ^2 = 0.152



Linear regression of Siemens excess returns v/s market return

α = -0.00045
β = 1.124123
$E(R)$ = 0.106
σ^2 = 0.082

5. VARIANCE-COVARIANCE MATRIX

This is obtained by using the following formula as given below.

$$\sigma_{ij} = \beta_i \beta_j \sigma_M^2$$

σij = Covariance between two securities

β = Slope of linear regression plot of security excess returns against market excess returns

σ_M = Variance due to market

Covariance between the seven securities are calculated using the Data-analysis function of excel and also this matrix will be used in Markowitz theory.

	COLPAL	APOLLOHOSP	BHARATFORG	GODREJPROP	WELSPUNIND	FEDERALBNK	SIEMENS
COLPAL	0.0452	0.0049	0.0091	0.0084	0.0083	0.0092	0.0084
APOLLOHOSP	0.0049	0.1362	0.0124	0.0115	0.0113	0.0125	0.0114
BHARATFORG	0.0091	0.0124	0.0934	0.0214	0.0211	0.0233	0.0213
GODREJPROP	0.0084	0.0115	0.0214	0.1654	0.0195	0.0215	0.0197
WELSPUNIND	0.0083	0.0113	0.0211	0.0195	0.1809	0.0212	0.0194
FEDERALBNK	0.0092	0.0125	0.0233	0.0215	0.0212	0.1517	0.0214
SIEMENS	0.0084	0.0114	0.0213	0.0197	0.0194	0.0214	0.0825

Variance-Covariance Matrix

6. MARKOWITZ PORTFOLIO THEORY

i. Introduction

The allocation of weights between the different assets is done using the Markowitz portfolio theory. The theory aims to maximize the expected return of the portfolio of the investor at a given market risk or construct a portfolio that minimizes the risk for a given expected return. An important assumption of the theory is that the investor is risk-averse, the investor would prefer an investment that has a lower risk at a given return or higher expected return for a given risk.

ii. Methodology

The methodology includes finding the various risk-return opportunities available for different risky assets. Then we identify the optimal portfolio of risky assets by finding the weights which is obtained by maximizing the Sharpe Ratio of our portfolio. Finally, we obtain an appropriate portfolio by mixing the risk-free asset with the optimal risky portfolio by maximizing the utility function.

The significance of the theory is that given the expected returns, variances, and covariances of the securities, it is possible to construct the minimum variance portfolio and further the efficient frontier for any target return(expected).

	COLPAL	APOLLOHOSP	BHARATFORG	GODREJPROP	WELSPUNIND	FEDERALBNK	SIEMENS	NIFTY 50
Mean	0.001	0.000	-0.001	0.001	0.000	0.000	0.000	0.000
Var	0.013	0.023	0.019	0.026	0.027	0.025	0.018	0.008
Stdev	0.000	0.001	0.000	0.001	0.001	0.001	0.000	0.000
Ann. Mean	0.182	0.108	-0.312	0.243	0.090	0.082	-0.010	0.095
Ann. Stdev	0.213	0.369	0.306	0.407	0.425	0.390	0.287	0.124
Ann. Variance	0.045	0.136	0.093	0.165	0.181	0.152	0.082	0.015

Various Parameters of all the 7 Securities

iii. Minimum Variance Portfolio (M)

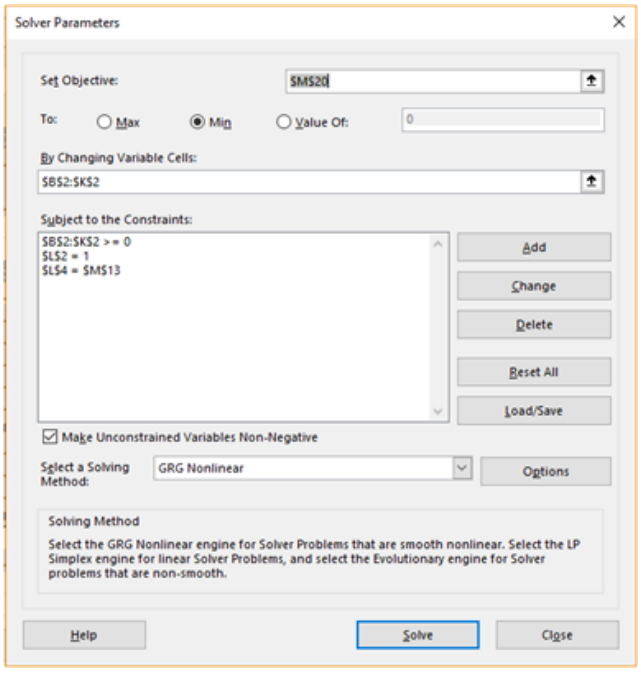
The combination of bond and equity that yields lowest risk is known as minimum variance portfolio. It is calculated in excel using Solver Function by minimizing the portfolio variance subjected to the following conditions.

$$Min\ \sigma_{P2} = w'\boldsymbol{\Sigma}w$$

Constraint: w'1= 1

w- vector of asset weights

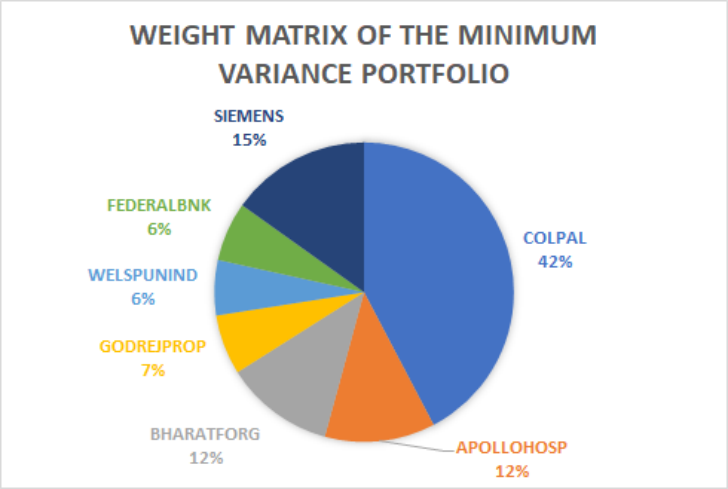
Σ - variance-covariance matrix



Solver Parameters

	Minimum Variance Portfolio [M]								
	Weight Matrix	Constraint	Mu	Unit	Portfoli_Ret.	Portfolio_Var	Portfolio_Risk	Risk free rate	Sharpe Ratio
COLPAL	0.4232	1.0000	0.0463	1.0000	0.0752	0.0237	0.1540	0.0681	0.0463
APOLLOHOSP	0.1192		0.0623	1.0000					
BHARATFORG	0.1179		0.1140	1.0000					
GODREJPROP	0.0649		0.1076	1.0000					
WELSPUNIND	0.0597		0.0950	1.0000					
FEDERALBNK	0.0637		0.0950	1.0000					
SIEMENS	0.1513		0.1060	1.0000					

Minimum Variance Portfolio of the Selected Securities



Data Analysis - The table and the pie chart above shows the weights of different risky securities for the minimum variance portfolio. The weights of all the securities are the same for both the cases i.e. Short Selling Banned and Short Selling Allowed since the weights of all the securities are positive. The weight for Colgate Palmolive is maximum as the risk adjusted expected return involved in the security is comparatively less. The weight for Federal Bank is the lowest because of the high risk involved in generating a high rate of returns.

iv. Efficient Portfolio (E)

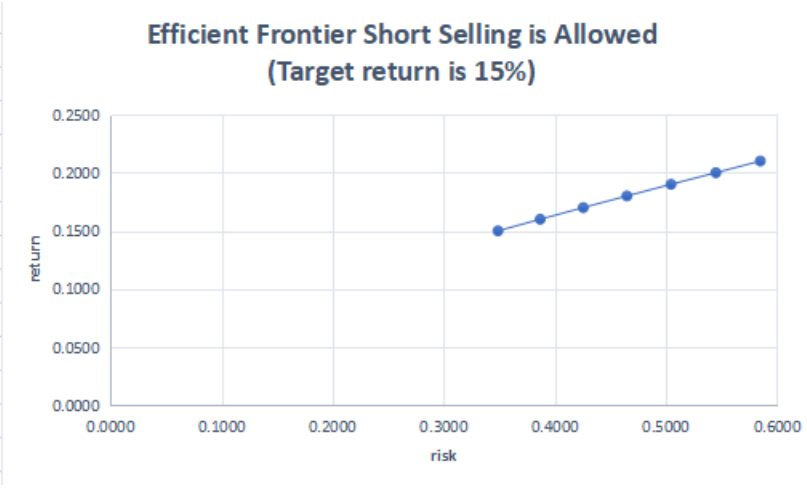
Efficient frontier is the upward sloping part of the investment opportunity set originating from the minimum variance portfolio. All portfolios that lie on an efficient frontier are called efficient portfolios. Efficient frontier was also calculated with a target return of 10% using the Solver function in Excel by minimizing the portfolio variance subject to the following conditions:

- When Short Selling is Allowed (Target return 15%)

$Min\ \sigma_{P2}$ = w'$\boldsymbol{\Sigma}$w
Constraint: w'1= 1
w- vector of asset weights
Σ - variance-covariance matrix

	Efficient Portfolio target Return 15% Short Selling allowed						
	Weight Matrix	Constraint	μ	Unit	Portfoli_Ret.	Portfolio_Var	Portfolio_Risk
COLPAL	-0.63820	1.00000	0.04625	1.00000	0.15000	0.12171	0.34887
APOLLO HOSP	-0.07630		0.06234	1.00000			
BHARAT FORGE	0.61840		0.11405	1.00000			
GODREJ PROP	0.25660		0.10761	1.00000			
WELSPUN IND	0.13060		0.09489	1.00000			
FEDERAL BANK	0.14040		0.09482	1.00000			
SIEMENS	0.56850		0.10629	1.00000			

Efficient Frontier from the selected securities when Short Selling is Allowed



Efficient Frontier when Short Selling is allowed

Data Analysis - From the above data, it can be concluded that when short selling is allowed, as we move towards a higher return the proportion of investment in the high-risk assets increases which is the case with the Efficient Portfolio. For higher returns high risk is involved. (Risk - Return tradeoff). Here Bharat Forge has the maximum weight followed by Siemens and Godrej as the risk adjusted returns are higher for these securities among others selected for the portfolio. Colgate Palmolive and Apollo Hospital has minimum risk adjusted returns and hence negative weight due to short position in the securities.

- When Short Selling is not allowed (Target return=8%)

$$Min\ \sigma_{P2} = w'\Sigma w$$

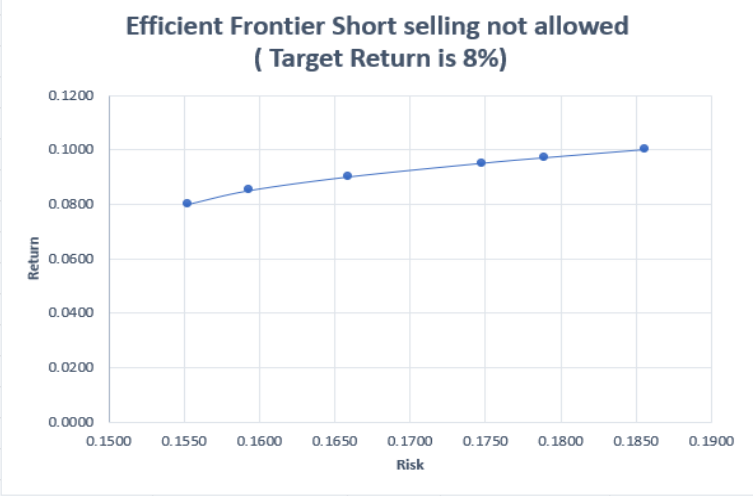
Constraint: w'1= 1, w>0

w- vector of asset weights

Σ - variance-covariance matrix

	Efficient Portfolio target Return 8% (Short selling not allowed)						
	Weight Matrix	Constraint	μ	Unit	Portfoli_Ret.	Portfolio_Var	Portfolio_Risk
COLPAL	0.35560	1.00000	0.04625	1.00000	0.08000	0.02410	0.15525
APOLLO HOSP	0.10670		0.06234	1.00000			
BHARAT FORGE	0.14990		0.11405	1.00000			
GODREJ PROP	0.07720		0.10761	1.00000			
WELSPUN IND	0.06420		0.09489	1.00000			
FEDERAL BANK	0.06860		0.09482	1.00000			
SIEMENS	0.17790		0.10629	1.00000			

Efficient Frontier from the selected securities when short selling is not allowed



Efficient Frontier when Short Selling is not Allowed

Data Analysis: From the above data, it can be concluded that when short selling is not allowed, as we move towards a higher return the proportion of investment in the high-risk assets increases which is the case with the Efficient Portfolio. For higher returns high risk is involved. (Risk - Return tradeoff). Here Colgate Palmolive has the maximum weight as the returns are higher for this security among others selected for the portfolio. Godrej Prop, Welspun Ind, Federal Bank and Siemens have comparatively lesser weights.

v. Optimal Allocation of Risky Assets

When a risk-free asset is introduced in a portfolio the optimal allocation of risky assets on the efficient frontier is obtained by the point of tangency of the frontier and the Capital Allocation Line. This point is obtained by maximizing the Sharpe ratio of the CAL.

- Optimal Portfolio when Short Selling is Allowed

Data Analysis - The below table shows that the portfolio returns are 26.41% at the risk of 80.71%. Colpal and Apollo Hospital have negative weights which shows they are shorted due to their low risk adjusted return. Bharat Forge and Siemens have positive weights which shows they have a higher risk adjusted return.

	Optimal Risky Portfolio with riskfree rate = 6.8067% [O](with short selling)								
	Weight Matrix	Constraint	μ	Unit	Portfoli_Ret.	Portfolio_Var	Portfolio_Risk	Risk free rate	Sharpe Ratio
COLPAL	-2.2646	1.0000	0.0463	1.0000	0.2641	0.6514	0.8071	0.0681	0.2429
APOLLOHOSP	-0.3753		0.0623	1.0000					
BHARATFORG	1.3862		0.1140	1.0000					
GODREJPROP	0.5516		0.1076	1.0000					
WELSPUNIND	0.2421		0.0950	1.0000					
FEDERALBNK	0.2641		0.0950	1.0000					
SIEMENS	1.1958		0.1060	1.0000					

- Optimal Portfolio when Short Selling is Not Allowed

Data Analysis - The below table shows that the portfolio returns are 10.82% at the risk of 20.92 %. Bharat Forge and Siemens have maximum weights due to their high risk adjusted return. Colpal and Apollo Hospitals have zero weights due to their low risk adjusted return and as shorting them is not allowed.

	Optimal Risky Portfolio with riskfree rate = 6.8067% [O](without short selling)								
	Weight Matrix	Constraint	μ	Unit	Portfoli_Ret.	Portfolio_Var	Portfolio_Risk	Risk free rate	Sharpe Ratio
COLPAL	0.0000	1.0000	0.0463	1.0000	0.1082	0.0438	0.2092	0.0681	0.1918
APOLLOHOSP	0.0000		0.0623	1.0000					
BHARATFORG	0.3981		0.1140	1.0000					
GODREJPROP	0.1557		0.1076	1.0000					
WELSPUNIND	0.0570		0.0950	1.0000					
FEDERALBNK	0.0552		0.0950	1.0000					
SIEMENS	0.3340		0.1060	1.0000					

7. OPTIMAL COMPLETE PORTFOLIO

Thumb rule- We assume that any investor would invest (100-age)% of his/her capital into risky assets and remaining into risk free assets.

i. When the investor is 30 years old and short selling is allowed

COMPLETE PORTFOLIO (AGE= 30 yrs., Rf= 6.8067%)					
Types of Assets	Fraction				
Risky Assets	70.00%				
Risk Free Assets	30.00%				
			Complete Portfolio Risk		0.56497
COLPAL	-158.52%		**Complete Portfolio Return**		0.20529
APOLLOHOSP	-26.27%		**Complete Portfolio Variance**		0.3192
BHARATFORG	97.04%				
GODREJPROP	38.62%				
WELSPUNIND	16.95%				
FEDERALBNK	18.48%				
SIEMENS	83.71%				

Risk and Return of Complete Portfolio, Weights of each security incomplete portfolio

Data Analysis - A 30-year-old risk averse investor would realize a return of 20.529% with a portfolio risk of 56.49% when he invests 30% in risk free and 70% in risky assets considering short selling is allowed.

ii. When the investor is 30 years old and short selling is not allowed

COMPLETE PORTFOLIO (AGE= 30 yrs., Rf= 6.8067%)					
Types of Assets	Fraction				
Risky Assets	70.00%				
Risk Free Asset	30.00%				
			Complete Portfolio Risk		0.14646
COLPAL	0.00%		**Complete Portfolio Return**		0.09616
APOLLOHOSP	0.00%		**Complete Portfolio Variance**		0.02145
BHARATFORG	27.87%				
GODREJPROP	10.90%				
WELSPUNIND	3.99%				
FEDERALBNK	3.87%				
SIEMENS	23.38%				

Risk and Return of Complete Portfolio, Weights of each security in complete portfolio

Data Analysis - A 30-year-old risk-averse investor would realize a return of 96.16% with a portfolio risk of 14.64% when he invests 50% in risk-free and 50% in risky assets considering short selling is not allowed.

ii. When the investor is 50 years old and short selling is allowed

COMPLETE PORTFOLIO (AGE= 50 yrs., Rf= 6.8067%)					
Types of Assets	Fraction				
Risky Assets	50.00%				
Risk Free Assets	50.00%				
			Complete Portfolio Risk		0.40355
COLPAL	-113.23%		**Complete Portfolio Return**		0.16608
APOLLOHOSP	-18.76%		**Complete Portfolio Variance**		0.16286
BHARATFORG	69.31%				
GODREJPROP	27.58%				
WELSPUNIND	12.11%				
FEDERALBNK	13.20%				
SIEMENS	59.79%				

Risk and Return of Complete Portfolio, Weights of each security in complete portfolio

Data Analysis - A 50-year-old risk-averse investor would realize a return of 16.60% with a portfolio risk of 40.35% when he invests 50% in risk-free and 50% in risky assets considering short selling is allowed.

iv. When the investor is 50 years old and short selling is not allowed

COMPLETE PORTFOLIO (AGE= 50 yrs., Rf= 6.8067%)					
Types of Assets	Fraction				
Risky Assets	50.00%				
Risk Free Asset	50.00%				
			Complete Portfolio Risk		0.10461
COLPAL	0%		**Complete Portfolio Return**		0.08813
APOLLOHOSP	0%		**Complete Portfolio Variance**		0.01094
BHARATFORG	19.91%				
GODREJPROP	7.78%				
WELSPUNIND	2.85%				
FEDERALBNK	2.76%				
SIEMENS	16.70%				

Risk and Return of Complete Portfolio, Weights of each security in complete portfolio

Data Analysis: A 50-year-old risk-averse investor would realize a return of 8.813% with a portfolio risk of 10.461% when he invests 50% in risk-free and 50% in risky assets considering short selling is not allowed.

8. CONCLUSION

An investment in the bundle of the equity instruments asset class involves the requirement of a minimum risk of 15.4% for around 7.5% return. Considering COLPAL has been the least volatile security among the equity asset class, a higher weight to this security would ensure minimal risk.

Achieving a targeted rate of return in the cases when:
Short selling is involved would imply a negative weight to assets with lower risk adjusted return and a weight greater than 1 to securities with higher risk adjusted return. In this case, it is possible theoretically to achieve indefinite returns and the efficient frontier would hence be a positively-sloped ray. Since a higher expected return would require a higher risk factor, the proportion of more volatile securities would increase.

Short-selling is not allowed- A constraint that the weights of the assets should be positive would restrict the maximum return that can be expected on the portfolio. So the efficient frontier would end at the point with the highest return possible (expected return of the most risky asset).

Taking into account the different ages of the two investors (and hence different risk tolerance), optimal portfolio allocation would include a higher proportion of risky assets in the case of the younger investor due to availability of a longer investment horizon. The younger investor in this case gets a higher portfolio return compared to the 50 yr old investor in both the cases(without short selling and with short selling) but it was accompanied by higher variance of the portfolio. But the allocation of weights within the risky assets (among equity instruments) is the same for any risk profile- an advantage of the Modern Portfolio Theory.

9. REFERENCES

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