ABESIT Ghaziabad (290)

Course: Engg. Physics (KAS 101T)

Unit - IV

Wave optics

Faculty Name: Dr. Anchala

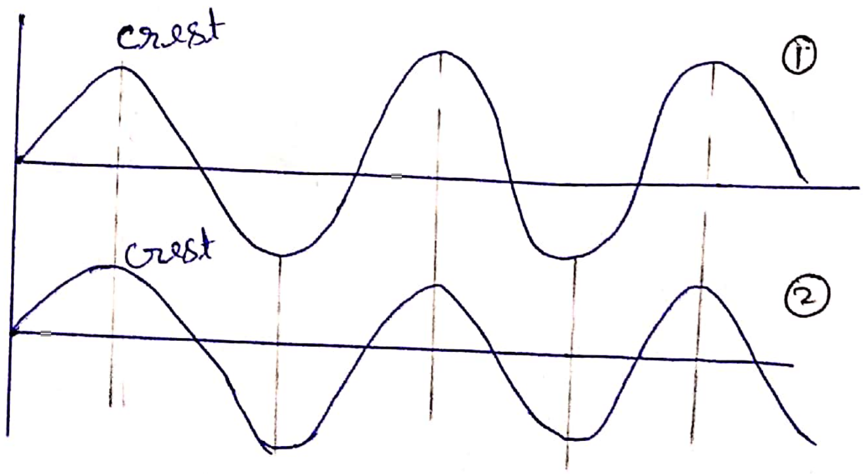
Lecture 36: Coherent sources, Interference in uniform thin film (Parallel)

Outcome: • Recall concept of coherent sources, Production of Coherent sources

• Explain Interference, Describe Interference due to thin films

Coherent Sources:- Two sources are said to be coherent if they emit light which have always a constant phase difference between them. It means that the sources must emit radiations of the same wavelength.

The light originating from a source consists of wave trains. When the light waves maintain crest to crest and trough to trough then they are said to be coheren.



Production :- If two sources are derived from a single sources by some device, then any phase-change in one is simultaneously accompained by the same phase-change in the other. Thus the phase difference between the two sources remains constant.

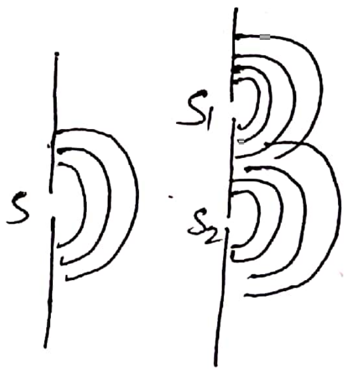
Methods of obtaining coherent sources :- In actual praactice it is not possible to have two independent coherent sources of light, but for experimental purpose two virtual coherent sources are derived from a single sources by some devices.

If two sources are derived from a single sources by some devices, then any phase-change in one is simultaneously accompanied by the same phase change in the other. So that the phase difference between the two sources remains constant.

ex.

(i) Young's double slit Experiment

In the device, two narrow slit S_1 and S_2 receive light from the same narrow slit S. Hence S_1 and S_2 act as a coherent sources.



Interference:- When the two light waves of the same frequency and having a constant phase difference travel simultaneously in the same region, then there is a modification in the intensity of light in the region of superposition. This modification in the intensity of light resulting from the superposition of waves is called interference.

Constructive and destructive interference:- At certain points the waves superimpose in such a way that the resultant intensity is the sum of the intensities due to individual waves. The interference produced at these points is called constructive interference. When the resultant amplitude is equal to the difference of individual amplitude, the interference is known as destructive interference.

The phenomenon of interference may be grouped into two categories depending up to the formation of two coherent sources in practice.

(1) Division of Wave front:- The incident wavefront is divided into two parts by utilising the phenomenoa of reflection, refraction or diffraction. These two parts of the same wavefront travel unequal distances and reunit at some angle to produce interference pattern.

Ex. Young's double slit experiment
Fresnel's biprism

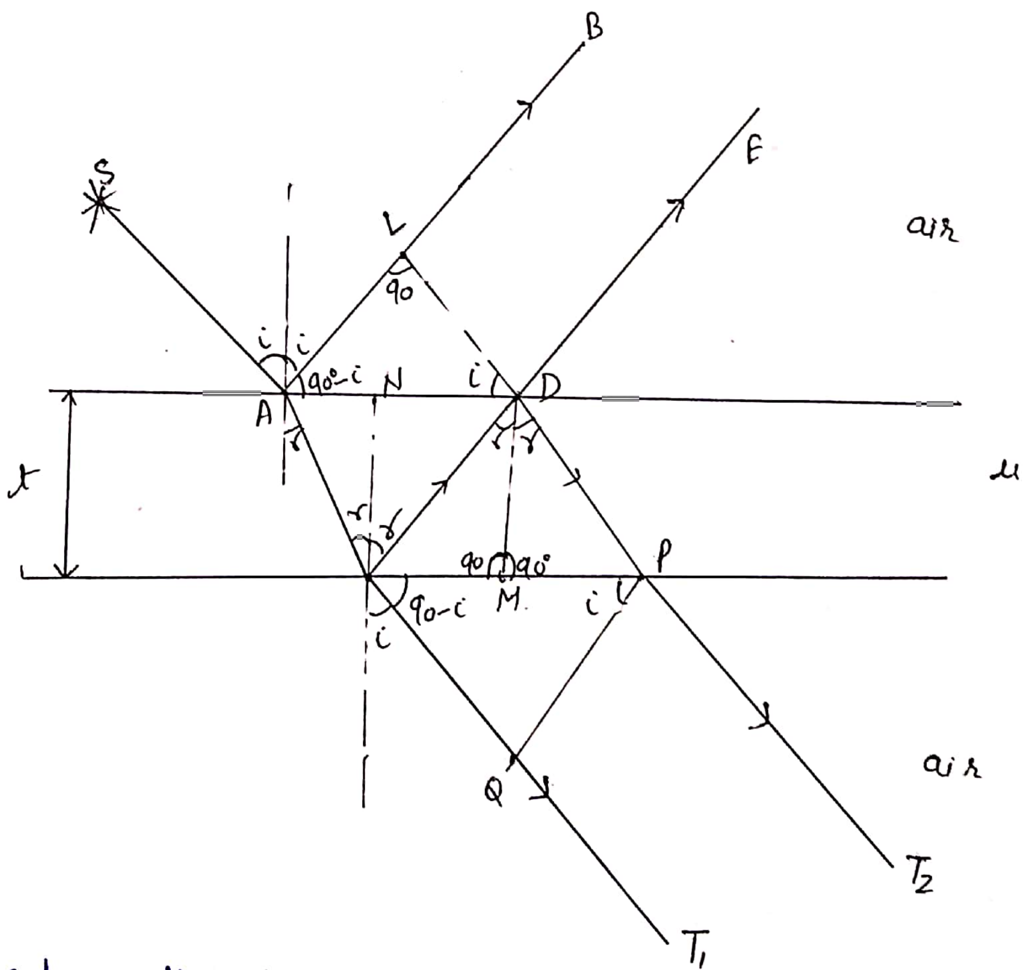
Division of amplitude :- The amplitude of the incoming beam is divided into two parts either by parallel reflection or refraction. These divided parts reunit after traversing different path and produce interference

Ex. Newton's Ring.

Interference in Thin film of uniform Thickness :-

A film is said to be the thin film if its thickness is about the order of wavelength of visible light (approx 5500Å). A thin film may be an air film enclosed between two transparent sheet or a soap bubble.

ex. When a thin film of oil spreads on the surface of water and is exposed to white light beautiful colours are seen.



Consider a thin transparent film of thickness t and refractive index μ. When a light ray SA is incident at angle i is partly reflected along AB and partly refracted along AC at angle r.

The refracted part AC is reflected from the point C on the lower surface of the film along CD and finally emerges out along DE. As the ray AB and DE are derived from the same incident ray, they are coherent.

To calculate the path difference between AB and DE, the perpendiculars DL and CN are drawn on AB and AD respectively.

Path difference Δ = path (AC+CD) in film − path AL in air

$$\Delta = \mu(AC + CD) - AL \quad \text{——— (1)}$$

In the right angle Δ ACN and CND, where $\angle ACN = r$, $CN = t$

$$\cos r = \frac{CN}{AC}, \quad AC = \frac{CN}{\cos r} = \frac{t}{\cos r}$$

then $$AC = CD = \frac{t}{\cos r} \quad \text{——— (2)}$$

In the right angle Δ ADL, where $\angle ADL = i$

$$\sin i = \frac{AL}{AD}, \quad AL = AD \sin i$$

$$AL = (AN + ND) \sin i \quad \text{——— (3)}$$

Now in Δ ACN and CND

$$\tan r = \frac{AN}{NC} \text{ and } \frac{ND}{NC} = \tan r$$

$$AN = NC \tan r, = t \tan r$$

$$ND = t \tan r$$

$$AN = ND = t \tan r \quad \text{——— (4)}$$

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using equation (4) in equ. (3)
then

$$AL = (t \tan r + t \tan r) \sin i$$

$$AL = 2t \tan r (\sin i)$$

using snell's law

$$\mu = \frac{\sin i}{\sin r} \quad \text{or} \quad \sin i = \mu \sin r$$

Therefore

$$AL = 2t \tan r (\mu \sin r)$$

$$AL = 2\mu t \frac{\sin^2 r}{\cos r} \quad \text{——} (5) \qquad \left(\because \tan r = \frac{\sin r}{\cos r}\right)$$

substituting the values of AC, CD and AL in equation (1)

$$\Delta = \mu (AC + CD) - AL$$

$$\Delta = \mu \left(\frac{2t}{\cos r}\right) - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$\Delta = \mu \left[\frac{2t}{\cos r} - \frac{2t \sin^2 r}{\cos r}\right]$$

$$\Delta = \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right] = \frac{2\mu t}{\cos r} \cdot \cos^2 r$$

$$\Delta = 2\mu t \cos r \quad \text{——} (6)$$

As the ray AB is reflected ray from denser medium, therefore there occurs an additional path difference $\frac{\lambda}{2}$.

Hence

optical path difference $\Delta = 2\mu t \cos r + \frac{\lambda}{2}$ ——(7)

Condition of Maxima and Minima in Reflected system

(i) for Constructive interference, path difference should be an even multiple of $\frac{\lambda}{2}$

$$2\mu t \cos r + \frac{\lambda}{2} = 2n\frac{\lambda}{2}$$

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = n\lambda - \frac{\lambda}{2}$$

$$\boxed{2\mu t \cos r = (2n-1)\frac{\lambda}{2}}$$ where $n = 1, 2, 3 \ldots$

(ii) for destructive interference, path difference should be an odd multiple of $\frac{\lambda}{2}$. Hence

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2} - \frac{\lambda}{2}$$

$$\boxed{2\mu t \cos r = n\lambda}$$

Transmitted System:-

Due to simultaneous reflection and refraction we obtain two transmitted rays CT_1 and RT_2. These rays have originated from the same point source, hence they have a constant phase difference and to produce interference. In order to calculate the path difference between CT_1 and PT_2 the ⊥rs PQ and DM are drawn on CT_1 and CP.

Path difference Δ = Path $(CD + DP)$ in film − Path CQ in air

$$\Delta = \mu(CD + DP) - CQ$$

$$\Delta = 2\mu t \cos r$$

In this case, there is no additional phase change. Hence in this case effective path difference between these rays is $2\mu t \cos r$.

Conditions for Maxima and Minima in Transmitted system (light):-

(i) for Constructive interference or Maxima

$$2\mu t \cos r = 2n\frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda$$

(ii) for destructive interference or Minima

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

Thus the conditions of maxima and minima in the transmitted light are just opposite to those for reflected light. Hence, the point of the film which appears bright in reflected light, appears dark in transmitted light.

Hence, the interference pattern of reflected and transmitted monochromatic light are complementary.

ABESIT Ghaziabad (290)

Course: Engg. Physics (KAS 101T)

Unit IV

Wave optics

Factulty Name: Dr. Anchala

Lecture 37: Interference in wedge shaped film

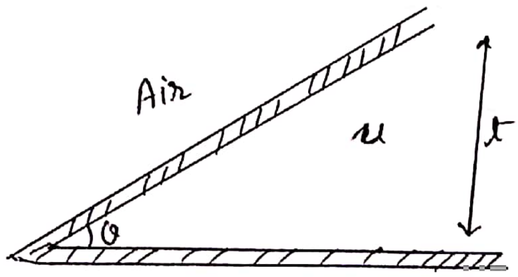
Necessity of extended sources

Outcome: Explain interference due to wedge shaped film

Deduce the spacing between consecutive bright/dark bands.

Wedge Shape film:- A thin film having zero thickness at one end progressively increasing to a particular thickness at the other end is called wedge shape film.

It can be formed by two glass slides over each other at one edge and seprated by a thin space at the opposite edge.



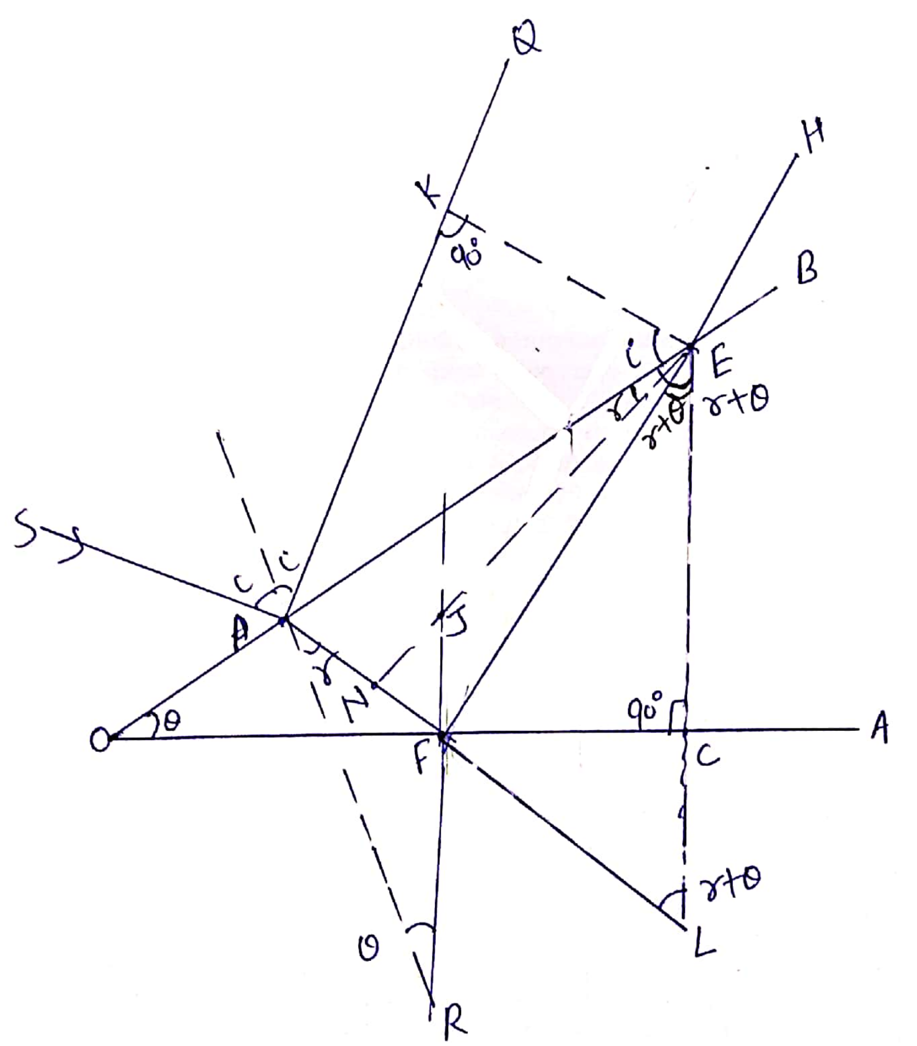
wedge shaped film

Interference in wedge shaped film:-

Let us consider wedge shaped film of refractive index μ enclosed by two surface at angle θ. When a light ray SP is incident on the film, it is partly reflected along PQ and

Partly refracted along PF. At point F of lower surface of thin film, the light ray PF is reflected along FE. finally it emerges along EH. Since the reflected rays PQ and EH are derived from the same incident ray SP they act as coherent light rays and produce an interference pattern.

To find out the path difference between the reflected rays PQ and EH, Let us draw perpendiculars EN on PF and EK on PQ.



The optical path difference between Rays PQ and EH is

$\Delta =$ Path $(PF + FE)$ in film $-$ PK in air

$$\Delta = \mu(PF+FE) - PK$$

$$\Delta = \mu(PN+NF+FE) - PK \quad \text{—— (1)}$$

Here $\angle PEK = i$ and $\angle PEN = r$

In the right angle triangle ΔPEK

$$\sin i = \frac{PK}{PE}$$

and in right angle ΔPEN

$$\sin r = \frac{PN}{PE}$$

from Snell's law

$$\mu = \frac{\sin i}{\sin r} = \frac{PK/PE}{PN/PE} = \frac{PK}{PN}$$

$$PK = \mu PN \quad \text{—— (2)}$$

using in equ. (1)

Path difference $\Delta = \mu(PN+NF+FE) - \mu PN$

$$\Delta = \mu(NF+FE) \quad \text{—— (3)}$$

Now draw a normal EC on OA. Extending EC and PF, they meet at point L.

In right angle triangle ECF and FCL, we have

$EC = CL = t$

$FE = FL$

and $\angle FCL = \angle FCE = 90°$

Hence from geometry $\angle FLE = \angle FEC = r+\theta$

Then equation (3) becomes

$$\Delta = \mu(NF+FL)$$

$$\Delta = \mu NL \quad \text{—— (4)}$$

Also, in right angle Δ ENL

$$\cos(r+\theta) = \frac{NL}{EL}$$

$$NL = EL\cos(r+\theta)$$

$$NL = 2t\cos(r+\theta) \quad \text{——— (5)}$$

Path difference $\Delta = 2\mu t\cos(r+\theta)$

Since the ray SP is reflected from denser medium, an additional path difference of $\frac{\lambda}{2}$ is produced between the reflected ray PQ and EH, then

Path difference $\Delta = 2\mu t\cos(r+\theta) + \frac{\lambda}{2}$

Condition of Maxima (Constructive interference)

$$2\mu t\cos(r+\theta) + \frac{\lambda}{2} = 2n\frac{\lambda}{2}$$

$$2\mu t\cos(r+\theta) = n\lambda - \frac{\lambda}{2}$$

$$\boxed{2\mu t\cos(r+\theta) = (2n-1)\frac{\lambda}{2}}$$

In this case, the film will apper bright.

Condition of Minima (destructive interference)

$$2\mu t\cos(r+\theta) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

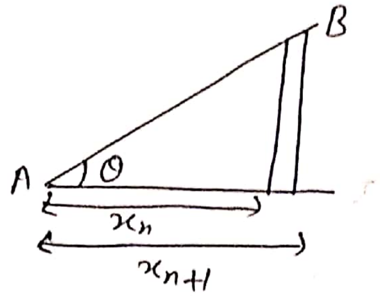
$$2\mu t\cos(r+\theta) = \frac{2n\lambda}{2} + \frac{\lambda}{2} - \frac{\lambda}{2}$$

$$\boxed{2\mu t\cos(r+\theta) = n\lambda}$$

Spacing between two Consecutive bright / dark fringe (Fringe width)

Fringe width is the distance between two consecutive bright / dark fringe.

Using wedge shaped film formula for n^{th} maxima



$$2\mu t\cos(r+\theta) = n\lambda \quad \text{——— (1)}$$

for normal incidence $r=0$

So this equation will be

$$2\mu t\cos\theta = n\lambda \quad \text{——— (2)}$$

Let x_n be the distance of n^{th} dark fringe from the point A of the film then

$$\tan\theta = \frac{t}{x_n}$$

$$t = x_n\tan\theta \quad \text{——— (3)}$$

Subsituting this value in equ. (2)

$$2\mu x_n\tan\theta\cos\theta = n\lambda \qquad \left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$$

$$2\mu x_n\sin\theta = n\lambda \quad \text{——— (4)}$$

Similarly if $(n+1)^{th}$ dark fringe is obtained at x_{n+1}, then

$$2\mu x_{(n+1)}\sin\theta = (n+1)\lambda \quad \text{——— (5)}$$

equation (5) - equ. (4)

$$2\mu x_{n+1}\sin\theta - 2\mu x_n\sin\theta = (n+1)\lambda - n\lambda$$

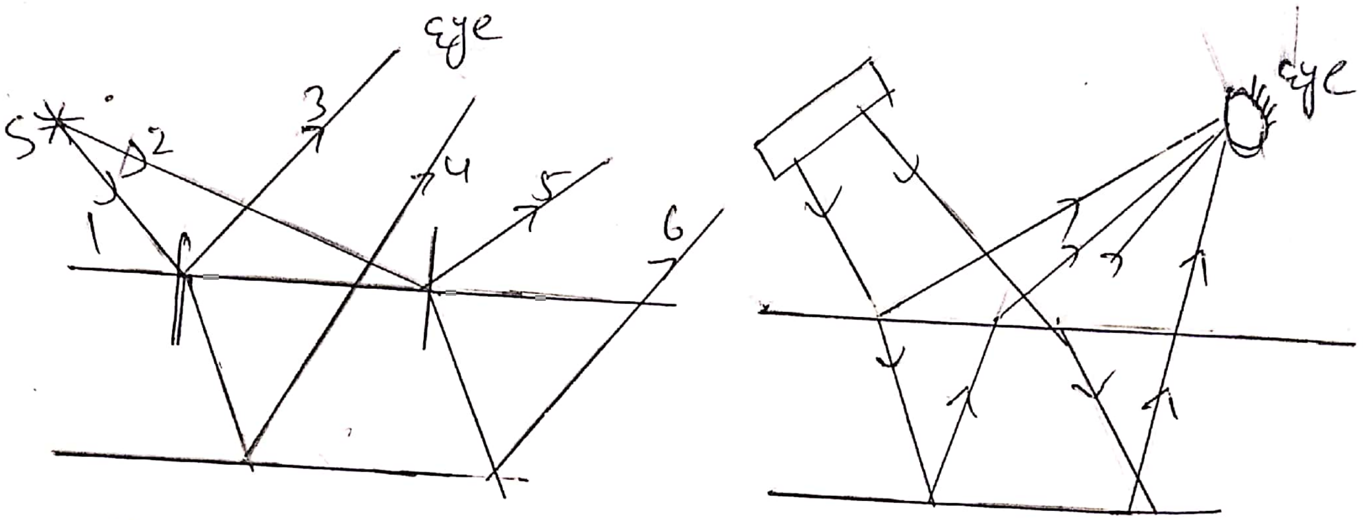
$$2\mu\sin\theta\,(x_{n+1} - x_n) = \lambda$$

fringe width $w = (x_{n+1} - x_n)$

Therefore $w = \dfrac{\lambda}{2\mu\sin\theta}$ for small angle $\sin\theta \simeq \theta$

$$\boxed{w = \frac{\lambda}{2\mu\theta}}$$ where θ is measure in radian.

Necessity of Extended Source



When a thin film is illuminated by a point source then for different incident rays, different pairs of interfering rays are obtained along different directions. All these pairs can not be seen by the eye simultaneously. Hence only a limited portion of the thin film is visible. On the other hand, if we use an extended source instead of the point source, then the different pairs of interfering reflected rays will reach the eye from a large portion of the thin film. Hence an extended source of light is necessary to view a film simultaneously.

ABES I T Ghaziabad (290)

Course: Engg. Physics (KAS 101T)

Unit-IV

wave optics

Faculty Name: Dr. Anchala

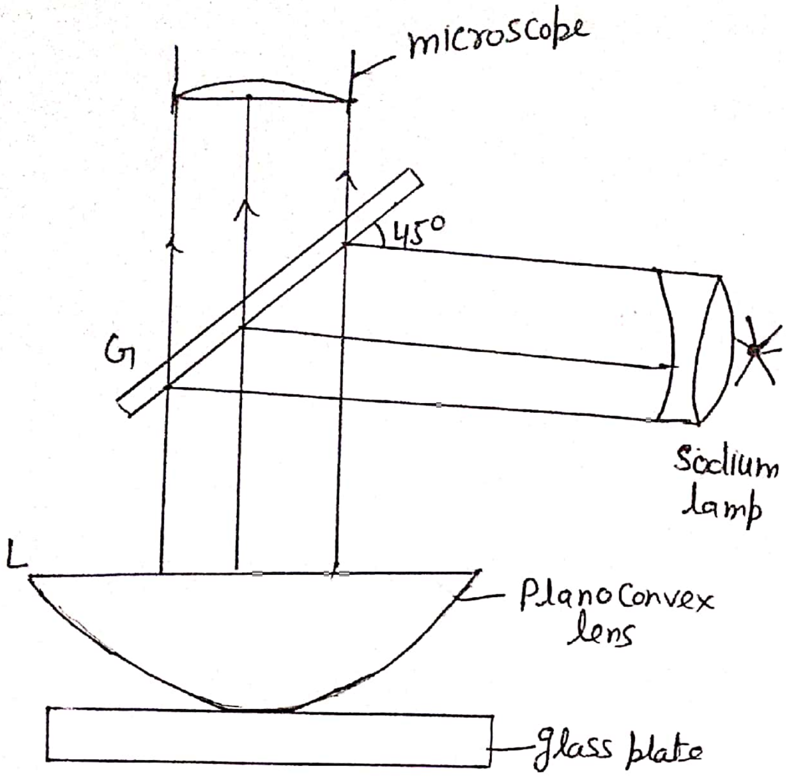
Lecture 38: Newton's Rings, Application of Newton's ring

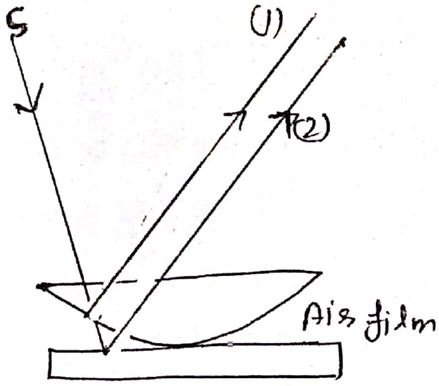
Outcome 1- Demonstrate Newton's rings experiment
2-Deduce diameter of dark and bright rings
3- Determine the wavelength of monochromatic source using Newton's ring
4- Determine the refractive index of liquid using Newton's ring

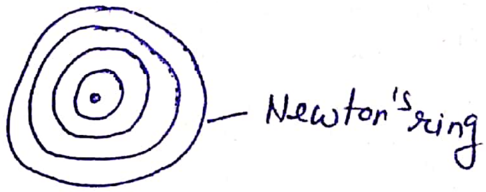
Newton's Rings :-

formation of Newton's rings:- When a plano-convex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate, an air film of gradually increasing thickness from the point of contact is formed. If monochromatic light is allowed to fall normally on this film, then alternate bright and dark concentric rings with their centre dark are formed. These rings are known as Newton's rings.

Newton's rings are formed because of the interference between the waves reflected from the top and bottom surface of an air film formed between the lens and plate.







Theory:- As the rings are observed in reflected light the effective path difference is given by,

$$\Delta = 2\mu t \cos r + \frac{\lambda}{2}$$

where μ is the refractive index of the film and t is the thickness of the film, $r \rightarrow$ the angle of refraction

for normal incidence $r = 0$, $\cos r = 1$

for air film $\mu = 1$

Hence $\Delta = 2t + \frac{\lambda}{2}$

Condition for Maxima (Bright rings)

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n-1)\frac{\lambda}{2} \quad \text{where } n = 1, 2, 3 \ldots$$

Condition for Minima (dark rings)

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

Hence $$2t = n\lambda$$

Centre of Newton's rings:- At the point of contact, the thickness of the film $t = 0$

So the path difference $\Delta = \frac{\lambda}{2}$

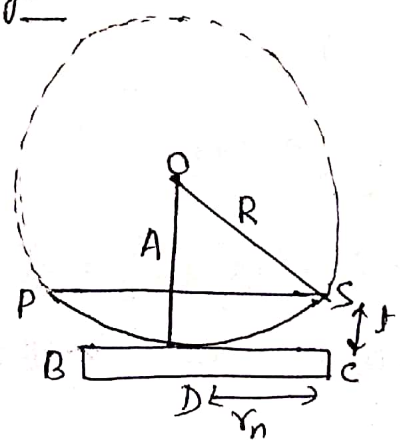
This is the condition of minimum intensity.

Hence the centre of Newton's rings appears dark.

Diameters of Bright and Dark rings:

To evaluate the diameters of bright and dark rings

Consider a Plano-Convex lens of radius of curvature R placed on glass plate.



$OD = OS = R$ (Radius of curvature)

$AD = t$ (thickness of air film)

$AS = r_n$ (Radius of ring)

and
$$OA = OD - AD$$
$$OA = (R - t)$$

So $\Delta\, OAS$

$$(OS)^2 = (OA)^2 + (AS)^2$$
$$R^2 = (R-t)^2 + r_n^2$$

$$R^2 = R^2 + t^2 - 2Rt + r_n^2$$

$$r_n^2 = 2Rt \qquad (\because t^2 \text{ is negligibly small})$$

$$t = \frac{r_n^2}{2R} \quad \text{——(1)}$$

Diameter of Bright Rings:-

for bright rings, the condition for constructive interface is

$$2t = (2n-1)\frac{\lambda}{2}$$

using equ. (1)

$$2 \cdot \frac{r_n^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$r_n^2 = R(2n-1)\frac{\lambda}{2}$$

$$r_n = \sqrt{\frac{(2n-1)\lambda R}{2}}$$

The diameter of bright ring

$$D_n = 2r_n = 2\sqrt{\frac{(2n-1)\lambda R}{2}}$$

$$D_n = \sqrt{2(2n-1)\lambda R}$$

$D_n \propto \sqrt{(2n-1)}$, So the diameter of bright rings is proportional to square root of odd natural nubers.

Diameters of Dark rings: condition for destructive interference is satisfied for dark ring is

$$2t = n\lambda$$

$$2 \cdot \frac{r_n^2}{2R} = n\lambda$$

$$r_n^2 = Rn\lambda$$

$$R^2 = R^2 + t^2 - 2Rt + r_n^2$$

$$r_n^2 = 2Rt \qquad (\because t^2 \text{ is negligibly small})$$

$$t = \frac{r_n^2}{2R} \quad \text{——— (1)}$$

Diameter of Bright Rings:-

for bright rings, the condition for constructive interface is

$$2t = (2n-1)\frac{\lambda}{2}$$

Using equ. (1)

$$2 \cdot \frac{r_n^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$r_n^2 = R(2n-1)\frac{\lambda}{2}$$

$$r_n = \sqrt{\frac{(2n-1)\lambda R}{2}}$$

The diameter of bright ring

$$D_n = 2r_n = 2\sqrt{\frac{(2n-1)\lambda R}{2}}$$

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Diameters of Dark rings: condition for destructive interference is satisfied for dark ring is

$$2t = n\lambda$$

$$2 \cdot \frac{r_n^2}{2R} = n\lambda$$

$$r_n^2 = Rn\lambda$$

$r_n = \sqrt{Rn\lambda}$

Diameter of n^{th} ring

$D_n = 2r_n$

$D_n = 2\sqrt{Rn\lambda}$

$D_n = \sqrt{4n\lambda R}$

$D_n \propto \sqrt{n}$

The diameter of dark rings are proportional to the square roots of natural numbers.

Applications of Newton's ring experiment

(1) **Determination of wavelength of Sodium Light:**

We know that the diameter of nth dark ring

$D_n^2 = 4n\lambda R$ ——— (1)

Diameter of $(n+p)^{th}$ dark ring

$D_{n+p}^2 = 4(n+p)\lambda R$ ——— (2)

equation (2) − equ. (1)

$D_{n+p}^2 - D_n^2 = 4(n+p)\lambda R - 4n\lambda R$

$D_{n+p}^2 - D_n^2 = 4p\lambda R$

Then

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

In Newton's rings experiment, when there is an air film between glass plate and lens, then

The diameter of n^{th} dark ring

$$D_n^2 = 4n\lambda R$$

and $$D_{n+p}^2 = 4(n+p)\lambda R$$

Therefore $$(D_{n+p}^2 - D_n^2)_{air} = 4p\lambda R \text{ ——— } (1)$$

The liquid whose refractive index is to be determined is placed between the lens and the glass plate of the Newton's ring experiment setup.

For liquid film, diameter of n^{th} dark ring

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

and for $(n+p)^{th}$ dark ring

$$D_{n+p}^2 = \frac{4(n+p)\lambda R}{\mu}$$

$$\therefore (D_{n+p}^2 - D_n^2)_{liquid} = \frac{4(n+p)\lambda R}{\mu} - \frac{4n\lambda R}{\mu}$$

$$(D_{n+p}^2 - D_n^2)_{liquid} = \frac{4p\lambda R}{\mu} \text{ ——— } (2)$$

dividing equ (1) by (2)

$$\boxed{\frac{(D_{n+p}^2 - D_n^2)_{air}}{(D_{n+p}^2 - D_n^2)_{liquid}} = \mu}$$

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Effect of introduction of liquid between the plates and lens in Newton's rings :-

When a liquid of refractive index μ is introduced between plate and lens, then the diameter of n^{th} dark ring is

$$(D_n^2)_{liquid} = \frac{4n\lambda R}{\mu} \quad \text{—— (1)}$$

and in case of air

$$(D_n^2)_{air} = 4n\lambda R \quad \text{—— (2)}$$

Dividing equ (1) by equ. (2)

$$\frac{(D_n^2)_{liquid}}{(D_n^2)_{air}} = \frac{1}{\mu}$$

$$(D_n)_{liquid} = \frac{(D_n)_{air}}{\sqrt{\mu}}$$

As $\mu > 1$ and hence $(D_n)_{liquid} < (D_n)_{air}$

This shows that when a liquid of refractive index μ is introduced between glass plate and lens, the diameter of the ring decreases.

Ques. Describe and explain the formation of Newton's rings in reflected monochromatic light, Prove that in reflected light (i) diameters of bright rings are proportional to the square roots of odd natural numbers and (ii) the diameters of dark rings are proportional to the square roots of natural numbers.

ABESIT Ghaziabad (290)

Course: Engg. Physics (KAS101T)

Unit – IV

Wave optics

Faculty Name: Dr. Anchala

Lecture 39: Numerical Problems

Outcome: To understand how to solve numerical based on interference

Q.1 - A parallel beam of sodium light of wavelength 5880 A° is incident on a thin glass plate of refractive index 1.5 such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of the plate which will make it appear dark by reflection.

Ans. Condition for dark fringe in the reflected system

$$2\mu t \cos r = n\lambda$$

Given $\mu = 1.5$, $\lambda = 5880\,A° = 5880 \times 10^{-8}\,cm$, $r = 60°$

for smallest thickness $n = 1$

$$t = \frac{n\lambda}{2\mu \cos r} = \frac{1 \times 5880 \times 10^{-8}}{2 \times 1.5 \cos 60}$$

$$t = 3920 \times 10^{-8}\,cm = 3920\,A°$$

Q.2 Calculate the thickness of the thinnest film ($\mu = 1.4$) in which interference of violet component ($\lambda = 4000\,A°$) of incident light can take place by reflection.

Ans Condition for Bright fringes in the reflected system is

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2}$$

for thinnest film $n=1$, $\lambda = 4000\,A^{\circ} = 4000\times10^{-8}\,cm$

and for normal incidence $r=0$

$$2\mu t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{4\mu} = \frac{4000\times10^{-8}}{4\times1.4}$$

$$t = 714.3\,A^{\circ}$$

Q.3 A parallel beam of sodium light ($\lambda = 5890\,A^{\circ}$) strikes a film of oil floating on water. when viewed at an angle 30° from the normal, 8^{th} dark band is seen. Determine the thickness of the film. (refractive index of oil $=1.5$)

Ans. Condition for dark fringe

$$2\mu t\cos r = n\lambda$$

given $\lambda = 5890\,A^{\circ} = 5890\times10^{-8}\,cm$, $n=8$, $\mu=1.5$

angle of incidence $i = 30$

from Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 30}{1.5}$$

$$\sin r = \frac{1}{2\times1.5} = 0.33$$

$$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-(0.33)^2} = 0.943$$

Hence $2\mu t\cos r = n\lambda$

$$t = \frac{n\lambda}{2\mu\cos r} = \frac{8\times5890\times10^{-8}}{2\times1.5\times0.943}$$

$$t = 1.665\times10^{-4}\,cm$$

(3)

Q.4 White light falls normally on a film of a soapy water whose thickness is 1.5×10^{-5} cm and refractive index 1.33. Which wavelength in the visible region will be reflected strongly.

Ans. Condition for Bright fringe

$$2\mu t\cos r = (2n-1)\frac{\lambda}{2}$$

for normal incidence $r=0$, then

$$2\mu t = (2n-1)\frac{\lambda}{2},$$

given $t = 1.5\times10^{-5}$ cm, $\mu = 1.33$

$$\lambda = \frac{4\mu t}{2n-1}$$

for $n=1$,

$$\lambda = \frac{4\times1.33\times1.5\times10^{-5}}{(2n-1)} = \frac{7.98\times10^{-5}}{2n-1}$$

for $n=1$, $\lambda = \frac{7.98\times10^{-5}}{1} = 7.98\times10^{-5}$ cm

$n=2$, $\lambda = \frac{7.98\times10^{-5}}{3}$, $\lambda = 2.66\times10^{-5}$ cm

$n=3$, $\lambda = \frac{7.98\times10^{-5}}{3}$, $\lambda = 1.596\times10^{-5}$ cm

$n=4$, $\lambda = \frac{7.98\times10^{-5}}{5}$, $\lambda = 1.14\times10^{-5}$ cm

Out of above wavelengths only 7.98×10^{-5} cm lies near the upper limit of visible region. Hence, wavelength 7980 A° is most strongly reflected.

Wedge shaped film related Numericals

Q.1 Light of wavelength 6000Å falls normally on a thin wedge-shaped film of refractive index 1.4 forming fringes that are 2 mm apart. Find the angle of wedge in seconds.

Ans Fringe width

$$w = \frac{\lambda}{2\mu\theta}$$

$$\theta = \frac{\lambda}{2\mu w}$$

Given $\lambda = 6000\,Å = 6000\times10^{-8}\,cm$, $\mu = 1.4$, $w = 2.00\,mm = 0.20\,cm$

$$\theta = \frac{6000\times10^{-8}}{2\times1.4\times0.20} = 10.71\times10^{-5}\,rad$$

$$\theta = 10.71\times10^{-5}\times\frac{180}{\pi} \qquad \left(\because rad = \frac{180}{\pi}\right)$$

$$\theta = 0.0061^\circ$$

$$\theta = 0.0061\times60\times60\ second = 21.96\ second$$

Q.2 A wedge-shaped air-film having an angle of 40'' is illuminated by monochromatic light and fringes are observed vertically through a microscope. The distance between two consecutive bright fringe is 0.12 cm. Calculate the wave length of light used.

Ans The fringe width,

$$w = \frac{\lambda}{2\mu\theta}$$

$$\lambda = 2\mu w\theta, \qquad \mu = 1\ (\text{air film})$$

$$w = 0.12\,cm,\ \theta = 40'' = \left(\frac{40}{3600}\right)^\circ = \left(\frac{1}{90}\right)^\circ\times\frac{\pi}{180}$$

$$\theta = 1.94\times10^{-4}\ radian.$$

$$\lambda = 2\times1\,(1.94\times10^{-4}\,rad)\times .12\,cm$$

$$\lambda = 0.4656\times10^{-4}\,cm.$$

Q.3 Using Sodium light $\lambda = 5893 A^\circ$ interference fringes are formed by reflection from a thin air wedge. When viewed ~~fr~~ 10 fringes are observed in a distance of 1cm. Calculate the angle of the wedge.

Ans. Fringe width $\omega = \frac{\lambda}{2\mu\theta}$ $(\mu = 1 \text{ for air})$

$\lambda = 5893 A^\circ, = 5893 \times 10^{-8} cm$

$\omega = \frac{1cm}{10}$

$\theta = \frac{5893 \times 10^{-8} cm}{2 \times \frac{1}{10}} = 2946.5 \times 10^{-7}$ radian.

Newton's rings related Numerical Problems

Ques.1 Newton's rings are observed normally in reflected light of wavelength 6000 A°. The diameter of the 10th dark ring is 0.50 cm. Find the radius of curvature of the lens and the thickness of the film.

Ans. The diameter of nth dark ring is given by

$D_n^2 = 4n\lambda R$ or $R = \frac{D_n^2}{4n\lambda}$

Given $D_n = 0.50 cm$, $\lambda = 6000 A^\circ = 6000 \times 10^{-8} cm$, $n = 10$

$R = \frac{0.50 \times 0.50}{4 \times 10 \times 6000 \times 10^{-8}} = 104 cm$

$d \rightarrow$ Is the thickness of the film

$d = \frac{r^2}{2R} = \frac{\left(\frac{D}{2}\right)^2}{2R} = \frac{D^2}{8R}$

$d = \frac{0.50 \times 0.50}{8 \times 104}$

$d = 3 \times 10^{-4} cm$

Q.2 Newton's rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. If the diameter of the 15th bright ring is 0.590 cm and the diameter of 5th ring is 0.336 cm, what is the wavelength of light used?

Ans. If D_{n+p} and D_n be the diameters of $(n+p)^{th}$ and n^{th} bright rings then,

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}, \quad \text{Given } D_{15} = 0.590\,cm,\ D_5 = 0.336\,cm$$

$p = 10$, and $R = 100\,cm$

$$\therefore \lambda = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100} = 5.88 \times 10^{-5}\,cm$$

$$\lambda = 5880\,A^\circ$$

Q.3 In Newton's ring experiment the diameter of 4th and 12th dark rings are 0.400 cm and 0.700 cm respectively. Deduce the diameter of 20th dark ring.

Ans. If D_{n+p} and D_n be the diameters of $(n+p)^{th}$ and n^{th} dark rings respectively, then

$$D_{n+p}^2 - D_n^2 = 4p\lambda R \quad \text{———(1)}$$

Given, $n = 4$, $n+p = 12$, $p = 8$, $D_4 = 0.400\,cm$, $D_{12} = 0.700\,cm$

$$\therefore D_{12}^2 - D_4^2 = 4 \times 8 \times \lambda \times R \quad \text{———(2)}$$

Suppose the diameter of 20th dark ring is D_{20}, then

$$D_{20}^2 - D_4^2 = 4 \times 16 \times \lambda \times R \quad \text{———(3)}$$

Dividing equ.(2) by (3), we get

$$\frac{D_{12}^2 - D_4^2}{D_{20}^2 - D_4^2} = \frac{4\times 8}{4\times 16} = \frac{1}{2}$$

$$2(D_{12}^2 - D_4^2) = (D_{20}^2 - D_4^2)$$

$$D_{20}^2 = 2D_{12}^2 - D_4^2$$

$$D_{20}^2 = 2(.700)^2 - (.400)^2$$

$$D_{20}^2 = 0.98 - 0.16 = 0.82$$

So the diameter of 20^{th} dark ring $= \sqrt{0.82} = 0.906\,cm$

Q.4 Light containing two wavelengths λ_1 and λ_2 fall normally on a plano-convex lens of radius of curvature R resting on a glass plate. If the n^{th} dark ring due to λ_1 coincides with the $(n+1)^{th}$ dark ring due to λ_2, prove that the radius of the n^{th} dark ring of λ_1 is $\sqrt{\dfrac{\lambda_1 \lambda_2 R}{(\lambda_1 - \lambda_2)}}$

Ans. The diameter of n^{th} dark ring due to λ_1 is

$$D_n^2 = 4n\lambda_1 R \quad \text{---} (1)$$

The diameter of $(n+1)^{th}$ dark ring due to λ_2 is

$$D_{n+1}^2 = 4(n+1)\lambda_2 R \quad \text{---} (2)$$

Given that n^{th} dark ring due to λ_1 coincides with the $(n+1)^{th}$ dark ring due to λ_2, then

$$D_n^2 = D_{n+1}^2$$

$$4n\lambda_1 R = 4(n+1)\lambda_2 R$$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

Substituting the value of n in equation (1)

$$D_n^2 = \frac{4R\lambda_1\lambda_2}{\lambda_1-\lambda_2}$$

$$\left(\frac{D_n}{2}\right)^2 = \frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}R$$

then

$$r_n^2 = \frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}R$$

$$\boxed{r_n = \sqrt{\frac{\lambda_1\lambda_2 R}{\lambda_1-\lambda_2}}}$$

Q.5 The lower surface of a lens resting on a plane glass plate has a radius of curvature of 400 cm. When illuminated by monochromatic light, the arrangement produces Newton's rings and the 15^{th} bright ring has a diameter of 1.16 cm. Calculate the wavelength of monochromatic light.

Sol. The diameter of n^{th} bright rings is

$$D_n^2 = 2(2n-1)\lambda R \quad , n = 1,2,3\ldots$$

Given, $n = 15$, $D_{15} = 1.16$ cm, $R = 400$ cm,

$$\lambda = \frac{D_n^2}{2(2n-1)R}$$

$$\lambda = \frac{1.16\times1.16}{2\times29\times400} = 5429 \text{Å}$$

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ABESIT Ghaziabad (290)

Course: Engg. Physics (290)

Faculty Name: Dr. Anchala

Unit IV

Wave optics

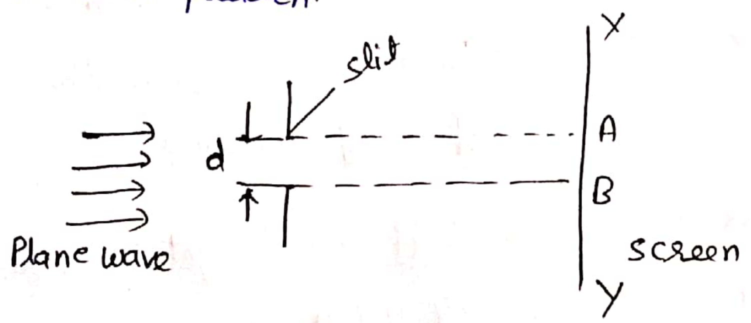
Lecture 40: Fraunhofer diffraction at single slit

Outcome: Recall diffraction of light

Explain single slit diffraction

Interpret the condition of maxima and minima

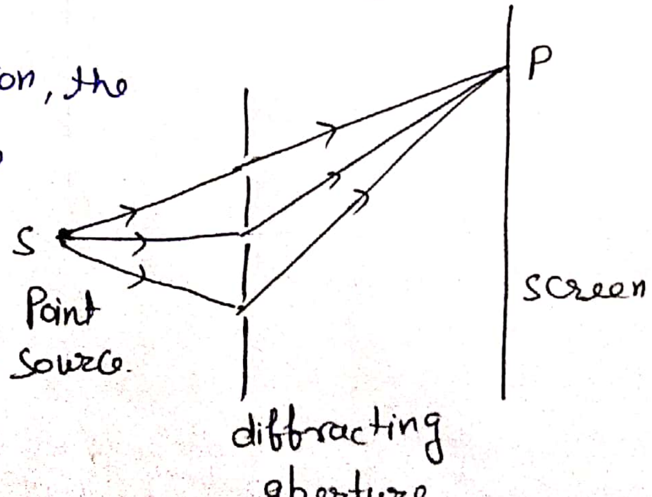
Diffraction:- The phenomenon of bending of light round the corners of an obstacle and their spreading into the geometrical shadow is called diffraction and the distribution of light intensity resulting in dark and bright fringes is called a diffraction pattern.



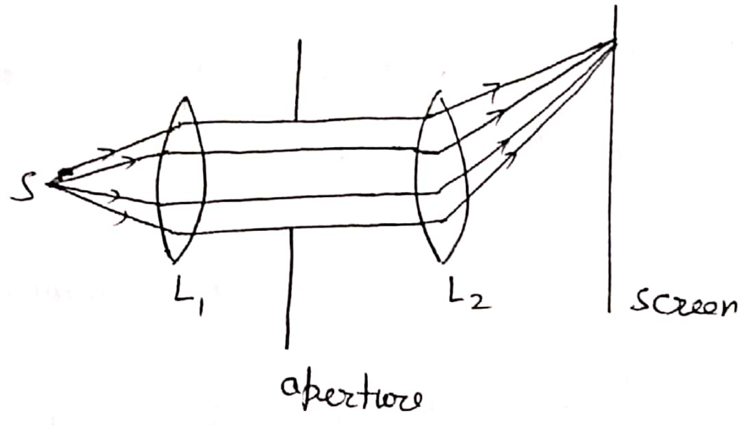
The diffraction phenomenon is usually divided into two categories

(1) Fresnel diffraction:-

In the fresnel diffraction, the source of light and the screen are generally at a finite distance from the diffracting aperture.



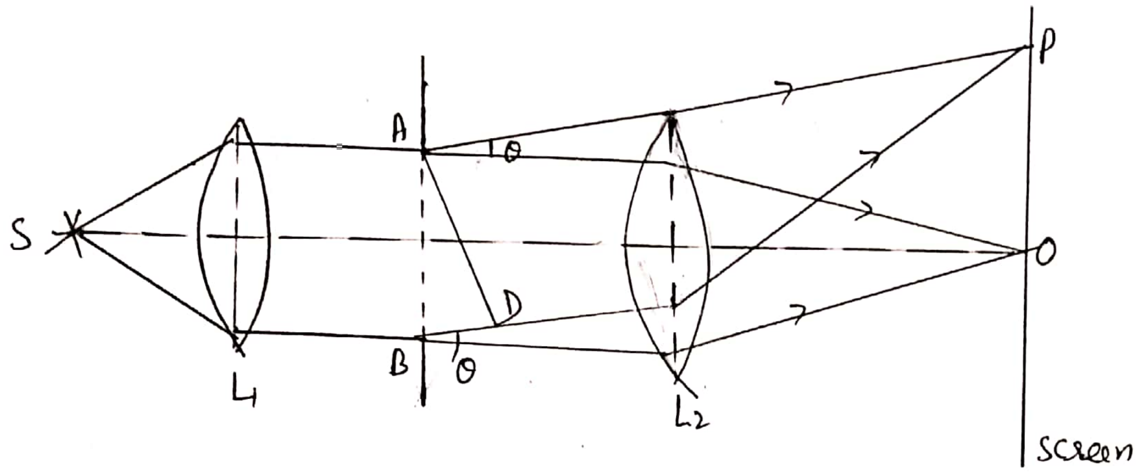
(2) Fraunhofer diffraction :- In the fraunhofer class of diffraction the source of light and screen are generally at infinite distance from the diffracting aperture.



Difference between Fresnel diffraction and Fraunhofer diffraction :-

Fresnel diffraction	Fraunhofer diffraction
(1) The source and the screen are at finite distance from the diffracting aperture	1) The source and screen are at infinite distance from the diffracting aperture.
(2) No mirrors or lenses are used for observation.	(3) Diffracted light is collected by using lenses.
(3) wavefront is either spherical or cylindrical	(3) The wave front is plane.

Fraunhofer Diffraction at a Single slit :-



Fraunhofer diffraction at single slit:- A slit is a rectangular aperture whose length is large compared to its breadth. A parallel beam of monochromatic light is made incident on a single slit AB of width a.

According to Huygen's theory, every point within the slit becomes the source of secondary wavelets, which spread out in all directions. The rays proceeding in the same direction as the incident rays are focussed at O, while those diffracted through an angle θ are focussed at P.

To find the resultant intensity at P. draw a perpendicular AD. The path difference between the secondary waves from A and B.

Path difference $\Delta = BD$

In ΔABD

$$\sin\theta = \frac{BD}{AB}$$

$$BD = AB\sin\theta \qquad (\because AB = a \text{ slit width})$$

$$BD = a\sin\theta$$

$$\text{Phase difference} = \frac{2\pi}{\lambda}(\text{Path difference})$$

$$\text{Phase difference} = \frac{2\pi}{\lambda}(a\sin\theta) \quad \text{---(1)}$$

Let AB is divided into n no. of small equal parts

Thus the phase difference between the waves from any two successive parts of the slit AB would be

$$\frac{1}{n}\left(\frac{2\pi}{\lambda}a\sin\theta\right)$$

Let $\frac{1}{n}\left(\frac{2\pi}{d} a \sin\theta\right) = \delta$ —— (2)

According to the theory of composition of n simple harmonic motion of equal amplitude (a) and common phase difference between successive vibrations

Resultant amplitudes of the rays diffracted at $\angle$ gle θ is given by

$$R = \frac{a \sin \frac{n\delta}{2}}{\sin \frac{\delta}{2}} = \frac{a \sin \left\{\frac{\pi a \sin\theta}{d}\right\}}{\sin\left\{\frac{\pi a \sin\theta}{nd}\right\}}$$

$$R = \frac{a \sin \alpha}{\left(\frac{\alpha}{n}\right)} \text{ —— (3)} \quad \text{where } \alpha = \frac{\pi a \sin\theta}{d}$$

and for large value of n, $\sin\left(\frac{\alpha}{n}\right) = \frac{\alpha}{n}$

So $R = \frac{na \sin\alpha}{\alpha} = \frac{A \sin\alpha}{\alpha}$, where $A = na$

Hence $R = \frac{A \sin\alpha}{\alpha}$ —— (4)

Resultant intensity at P

$I \propto (\text{Amplitude})^2$

$I \propto R^2$

$I = A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$ —— (5) (for simplicity the constant of proportionality being taken of unity)

Position of Maxima and Minima

(i) Principal Maxima;

Resultant Amplitude

$$R = \frac{A \sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{\lfloor 3} + \frac{\alpha^5}{\lfloor 5} - \frac{\alpha^7}{\lfloor 7} + \cdots\right]$$

$$R = A\left[1 - \frac{\alpha^2}{\lfloor 3} + \frac{\alpha^4}{\lfloor 5} - \frac{\alpha^6}{\lfloor 7} + \cdots\right]$$

The amplitude is maximum when the negative terms vanish i.e α should be zero

$$R = A$$

$$\Rightarrow \frac{\pi a \sin\theta}{\lambda} = 0$$

$$a\sin\theta = 0, \quad \sin\theta = 0$$

$$\theta = 0$$

Hence principal maxima in the diffraction pattern at single slit is obtained at $\theta = 0$ i.e

The maximum amplitude at P will be

$$R = A$$

$\therefore$ Maximum intensity, $I_{max} = R^2_{max}$

$$I_{max} = A^2$$

or $\boxed{I_0 = A^2}$

Position of Minima:- $R = \dfrac{A\sin\alpha}{\alpha}$

R will be minimum when $\sin\alpha = 0$ but $\alpha \neq 0$

$$\text{or } \sin\alpha = \sin m\pi$$

$$\alpha = \pm m\pi \quad , m = 1, 2, 3 \ldots \quad m \neq 0$$

Now $\alpha = \dfrac{\pi a \sin\theta}{\lambda}$

then $\dfrac{\pi a \sin\theta}{\lambda} = \pm m\pi$

$$a\sin\theta = \pm m\lambda$$

where $m = 1, 2, 3 \ldots$ gives the directions of first, second and third .. minima (Here $m \neq 0$ because $m = 0$ or $\theta = 0$ corresponds to principal maxima)

Position of Secondary Maxima:- In addition, to principal maxima, there are less intense secondary maxima

The secondary maxima condition will be

$$\frac{dI}{d\alpha} = 0$$

$$\frac{d}{d\alpha}\left[\frac{A^2 \sin^2\alpha}{\alpha^2}\right] = 0$$

$$\frac{d}{d\alpha}\left(\frac{\sin^2\alpha}{\alpha^2}\right) = 0$$

$$\frac{\alpha^2 \cdot 2\sin\alpha \cdot \cos\alpha - \sin^2\alpha \cdot 2\alpha}{(\alpha^2)^2} = 0$$

$$\frac{2\alpha\sin\alpha\,(\alpha\cos\alpha - \alpha\sin\alpha)}{\alpha^4} = 0$$

$$\alpha\cos\alpha - \alpha\sin\alpha = 0$$

$$\alpha = \tan\alpha$$

This equation can be solved by plotting the graph between $y = \alpha$ and $y = \tan\alpha$ on the same graph.

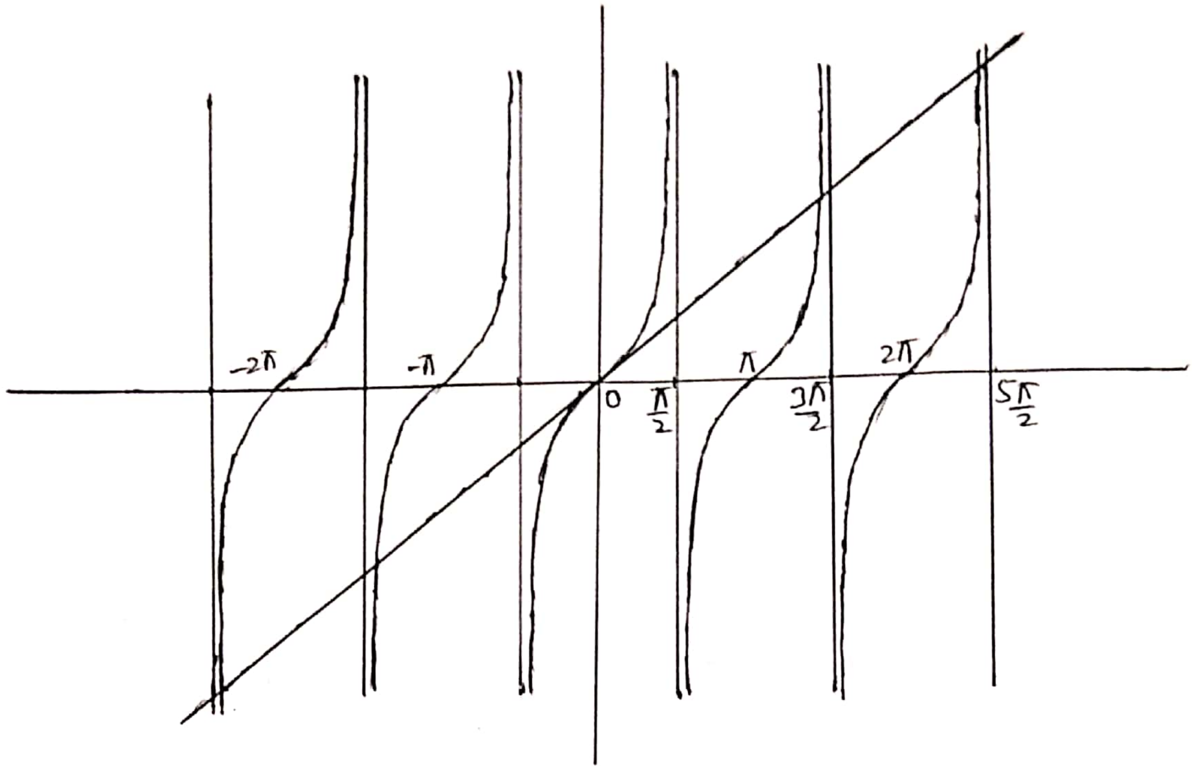
The points of intersection of two curves give the position of secondary maxima.

The first value of $\alpha = 0$ gives the principal maxima. The remaining values of α which gives the secondary maxima are

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots$$

The more exact values of α are

$$\alpha = 1.430\pi, 2.46\pi, 3.47\pi \ldots$$



Relative intensities of principal maxima and Secondary Maxima

(i) for principal Maximum (Central maximum) at $\alpha = 0$

$$I = I_0$$

(ii) Intensity of 1st secondary maximum at $\alpha = \pm \frac{3\pi}{2}$

$$I_1 = \frac{A^2\left(\sin\frac{3\pi}{2}\right)^2}{\left(\frac{3\pi}{2}\right)^2} = \frac{4}{9\pi^2}A^2 = \frac{A^2}{22} = \frac{I_0}{22}$$

(iii) The intensity of 2nd secondary maximum (at $\alpha = \pm\frac{5\pi}{2}$)

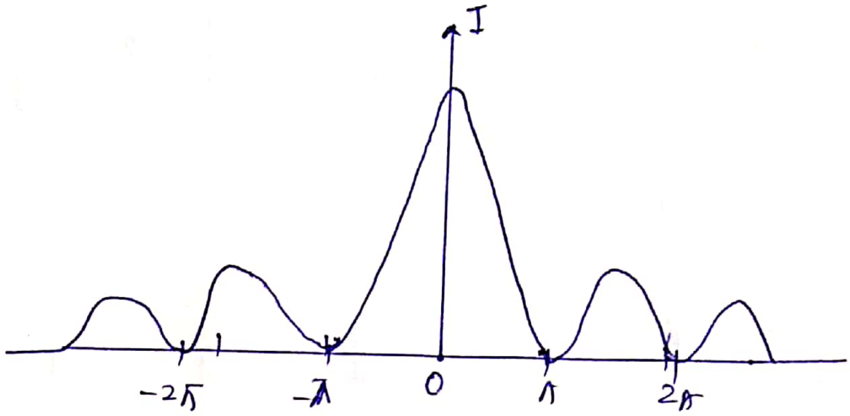
$$I_2 = \frac{A^2 \sin\left(\frac{5\pi}{2}\right)^2}{\left(\frac{5\pi}{2}\right)^2} = \frac{4}{25\pi^2}A^2 = \frac{A^2}{62} = \frac{I_0}{62}$$

Hence the relative intensities

$$I_0 : I_1 : I_2 :$$

$$I_0 : \frac{4}{9\pi^2} I_0 : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} :$$

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} :$$



Intensity in the diffraction pattern of a single slit

ABESIT Ghaziabad (290)

Course: Engg. Physics (KAS 101T) Unit - IV

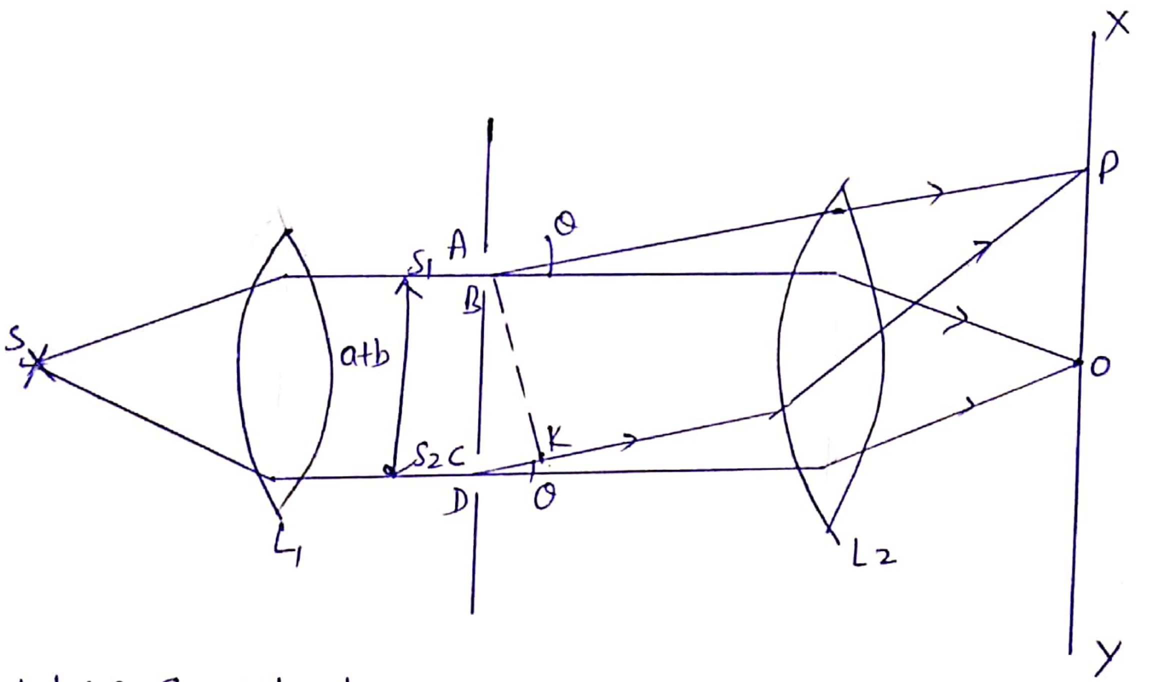
Faculty Name: Dr. Archala Wave optics

Lecture: 41 - Fraunhofer diffraction at double slit

Outcome: Explain double slit diffraction

Explain - Intensity distribution in double slit

Fraunhofer diffraction at double slit:-



Let us consider two parallel slits AB and CD. Let a monochromatic source of light having wavelength λ be incident normally upon two slits AB and CD, each of width 'a' seprated by opaque space of width 'b'. The distance between the corresponding points of the two slits is $(a+b)$. Suppose the light diffracted by the slits be focused on the screen XY.

Explanation: According to the principle of Huygen theory

(1) All points in slits S_1 and S_2 will send secondary wavelets

(2) All secondary waves moving along the incident wave will be focused at O and the diffracted wave be focused at P.

(3) From the theory of diffraction at a single slit the resultant amplitude

$$R = \frac{A \sin\alpha}{\alpha} \quad \text{——— (1)}$$

where $$\alpha = \frac{\pi a \sin\theta}{\lambda} \quad \text{——— (2)}$$

Now, consider the two slits AB and CD as equivalent to two coherent sources placed at the middle of S_1 and S_2 of the slits, each sending a wavelet of amplitude $\frac{A\sin\alpha}{\alpha}$ in direction θ. The the resultant amplitude at P on the screen will be result of interference between two waves of same amplitude.

Let us drop a perpendicular S_1K on S_2K

So the path difference between the wavelets from S_1 and S_2 in the direction θ.

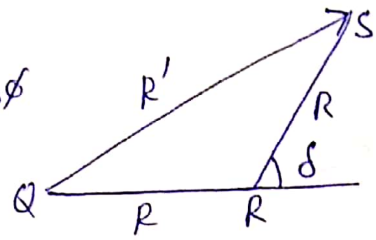
$$\text{Path difference } \Delta = S_2K = S_1S_2 \sin\theta$$

$$S_2K = (a+b)\sin\theta \quad \text{——— (3)}$$

$$\text{Phase difference } \phi = \frac{2\pi}{\lambda}(a+b)\sin\theta \quad \text{——— (4)}$$

To find the resultant of two waves emerging from slit S_1 and S_2 we apply vector addition method,

$$(QS)^2 = (QR)^2 + (RS)^2 + 2(QR)(RS)\cos\phi$$



$$R'^2 = R^2 + R^2 + 2R.R\cos\phi$$

$$R'^2 = 2R^2(1+\cos\phi)$$

$$R'^2 = 4R^2\cos^2\frac{\phi}{2} \quad \text{—— (5)}$$

Substituting the values of R and ϕ from equ (1) and (4) in equ (5)

$$R'^2 = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cdot\cos^2\beta \quad \text{—— (6)}$$

where $\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$ —— (7)

Then resultant intensity at P

$$I = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cdot\cos^2\beta \quad \text{—— (8)}$$

This is the required expression for the intensity distribution due to a fraunhofer diffraction at a double slit.

Hence the resultant intensity depends upon two factors,

(1) $\frac{\sin^2\alpha}{\alpha^2}$ → which gives diffraction pattern due to each single slit

2) $\cos^2\beta$ → which gives the interference pattern.

<u>Condition of Maxima and Minima:</u>

The diffraction term $\frac{\sin^2\alpha}{\alpha^2}$ gives a central maxima at $\theta = 0$ with the altenately minima and secondary maxima of decreasing intensity on both sides.

The direction of the minima are given by

$$\sin\alpha = 0, \text{ but } \alpha \neq 0$$

$$\sin\alpha = \sin m\pi$$

$$\alpha = \pm m\pi$$

or $$\alpha = \frac{\pi a \sin\theta}{\lambda}$$

then $$\frac{\pi a \sin\theta}{\lambda} = \pm m\pi$$

$$\boxed{a\sin\theta = \pm m\lambda} \quad \text{where } m = 1, 2, 3 \cdots$$

The term $\cos^2\beta$ gives a set of bright and dark fringes of equal width.

The direction of the maxima (bright) are given by

$$\cos^2\beta = 1$$

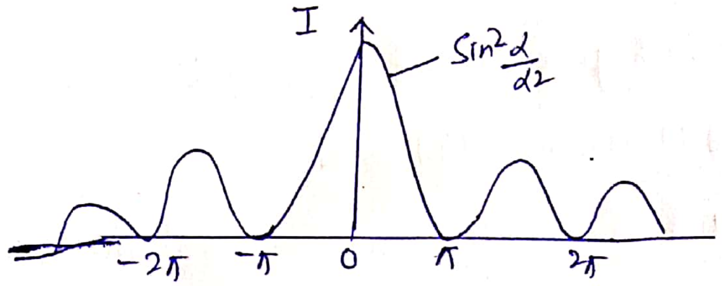
$$\Rightarrow \beta = \pm n\pi$$

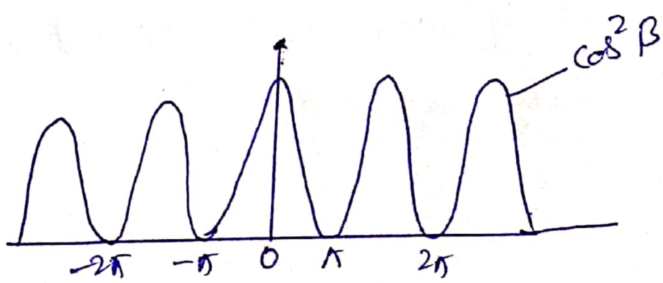
or $$\beta = \frac{\pi(a+b)\sin\theta}{\lambda}$$

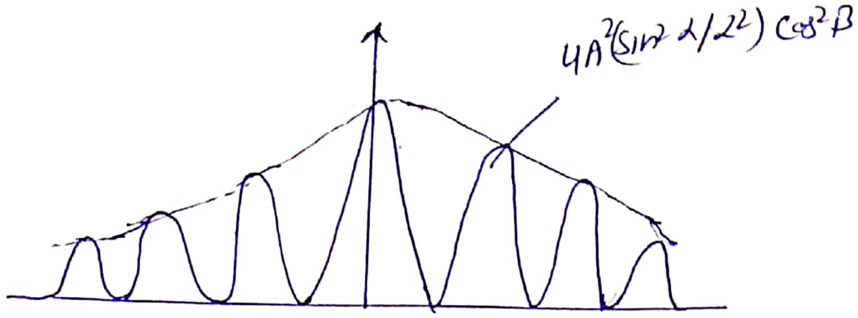
$$\therefore \frac{\pi(a+b)\sin\theta}{\lambda} = \pm n\pi$$

$$(a+b)\sin\theta = \pm n\lambda, \text{ where } n = 0, 1, 2 \cdots$$

corresponds to zero order, 1st, 2nd order--- maximas.







(The resultant intensity distribution pattern)

Important Points

(i) Effect of increasing the slit width :- on increasing the slit width 'a' the Central peak becomes sharper but the fringe spacing remains unchanged.

(ii) Effect of increasing the distance b/w the slits :
On increasing the sepration between the slit 'b' the fringes become closer together. Hence more interference maxima fall within the Central maximum.

(iii) Effect of increasing the wavelength λ :-
When the wavelength of the monochromatic light falling on the slits increases, the envelope becomes broader. and the fringes move farther apart.

ABESIT Ghaziabad (290)

Course: Engg. Physics

Unit IV- Wave optics

Faculty Name: Dr. Archala

Lecture 42: Absent Spectra and Diffraction grating

Outcome: compute missing orders in double slit

Define plane transmission grating

Absent Spectra or Missing order in double slit diffraction :-

If the slit width 'a' is kept constant and the opaque distance 'b' between the slits is varied, then it is observed that the spacing between two consecutive maxima changes. for a certain value of 'b' certain interference maxima become absent from the pattern

As the direction of interference maxima are given by

$$(a+b)\sin\theta = n\lambda \quad \text{——— (1)}$$

The direction of diffraction minima are given by

$$a\sin\theta = m\lambda \quad \text{——— (2)}$$

If the values of a and b are such that both the equations are satisfied for the same value of θ, then a certain interference maximum will overlap the minimum and hence the spectrum order will be missing

Dividing equ. (1) by equation (2)

$$\frac{a+b}{a} = \frac{n}{m}$$

(i) If $b=a$, i.e the width of transparencies a and opacities b are equal, then

$n = 2m$, i.e for $m = 1,2,3\ldots$

$n = 2, 4, 6$

Hence 2^{nd}, 4^{th}, 6^{th} order maxima will be absent

(2) If $b = 2a$

then $n = 3m$ i.e for $m = 1,2,3\ldots$

$n = 3, 6, 9$ will be absent

Hence 3^{rd}, 6^{th} and 9^{th} order of maxima will be absent.

Diffraction Grating:-

An arrangement of large number of parallel slits having same width and sebrated by equal opaque spaces is known as a diffraction grating

A grating is made by drawing a series of very fine equidistant and parallel lines on an optically plane glass plate by means of diamond pen. The light cannot pass through the lines while the space between the lines is transparent to the light and act as a slit

There are about 15,000 lines per Inch. Let a be the width of each slit and 'b' the width of each opaque space between the slits. Then $(a+b)$ is called the grating element.

Course : Engg. Physics (KAS 101T)

Unit - VI

wave optics

Faculty Name: Dr. Anchala

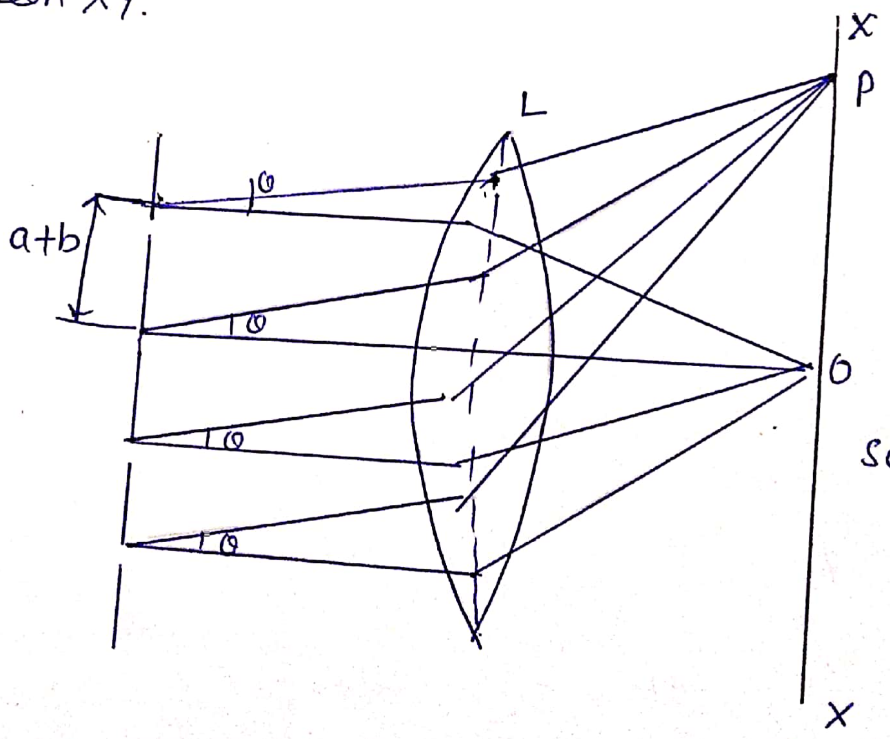
Lecture 43: Spectra with grating, Dispersive Power

Out Come: 1-Explain N slit diffraction, 2-Interpret the Condition of Maxima and Minima

3- Deduce maximum order in diffraction grating

4- Deduce the dispersive power of grating.

Spectra with Grating:- Let a parallel beam of monochromatic light of wavelength λ be incident normally on the grating. The light diffracted through N slits is focussed by a convex lens on the screen XY.



Theory:- In this Case the resultant amplitude in the direction θ is given by

$$R' = \frac{A \sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}$$

and Intensity is $I = R'^2 = \frac{A^2 \sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta}$ —— (1)

where $\beta = \frac{\pi(a+b)\sin\theta}{\lambda}$ and $\alpha = \frac{\pi a \sin\theta}{\lambda}$

Hence the intensity distribution is the product of two terms. The first term $\frac{A^2\sin^2\alpha}{\alpha^2}$ represents the diffraction pattern due to single slit. Second term $\frac{\sin^2 N\beta}{\sin^2\beta}$ represents the interference pattern due to N-slits.

(i) Position of Principal Maxima

Intensity will be maximum, if $\sin\beta = 0$

$\Rightarrow \beta = \pm n\pi$, when $n = 0, 1, 2, 3 \ldots$

Also, we have

$$\sin N\beta = 0$$

So $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$ i.e indeterminate

Applying 'L' Hospital rule i.e

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to \pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$\lim_{\beta \to \pm n\pi} \frac{N\cos N\beta}{\cos\beta} = \pm N$$

$\therefore$ ~~Sin~~

Therefore, the intensity of principal maxima is proportional to N

Substituting the value of $\frac{\sin N\beta}{\sin \beta} = N$ in equ (1)

$$I = \frac{A^2 \sin^2\alpha}{\alpha^2} N^2 \quad \text{—— (2)}$$

These maxima are most intense and are called 'Principal maxima'.

<u>The directions of principal Maxima are</u>

$$\sin\beta = 0, \text{ i.e } \beta = \pm n\pi \text{ , where } n = 0, 1, 2 \ldots$$

or $$\frac{\pi(a+b)\sin\theta}{\lambda} = \pm n\pi$$

or $$\boxed{(a+b)\sin\theta = \pm n\lambda} \quad \text{—— (3)}$$

This is known as grating equation.

· If we put $n=0$ in equ (3) we get the zero order maximum.
for $n = 1, 2, 3 \ldots$ we obtain the first, second, third —— order principal maxima respectively.

<u>Minima</u> :- The intensity is minimum when $\sin N\beta = 0$ but $\sin\beta \neq 0$

Therefore

$$N\beta = \pm m\pi$$

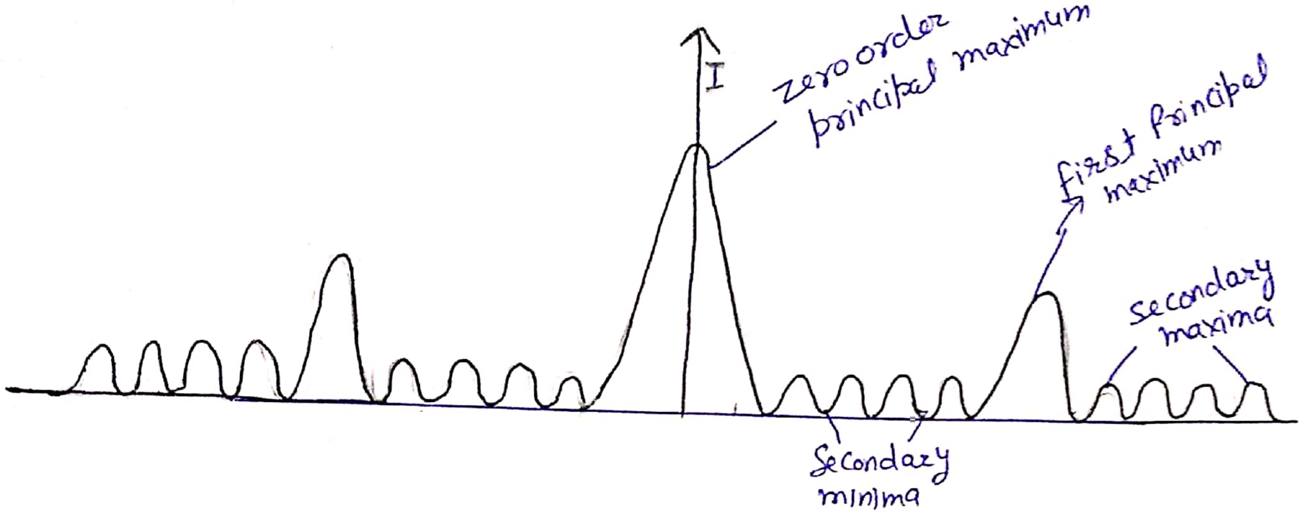
or putting the value of β

$$N \cdot \frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi$$

$$\boxed{N(a+b)\sin\theta = \pm m\lambda} \quad \text{—— (4)}$$

where $m \neq 0, N, 2N \ldots nN$ because these values of m make $\sin\beta = 0$ which gives principal maxima.

It is clear $m = 1, 2, 3 \cdots (N-1)$ give minima and then at $m = N$, we get principal maximum of first order. Thus there are $(N-1)$ minima between two successive principal maxima.



Condition for Missing order or Absent Spectra with a Diffraction grating:-

When condition for principal maximum of n^{th} order is simultaneously satisfied with the condition of m^{th} order minima, then n^{th} order of the principal maximum will be absent from the diffraction pattern. These are called known as absent spectra.

The condition of principal maximum in the grating is

$$(a+b)\sin\theta = n\lambda \quad \text{——— (1)}$$

Condition of minima in single slit

$$a\sin\theta = m\lambda \quad \text{——— (2)}$$

Now dividing equ (1) by equ (2)

$$\frac{(a+b)\sin\theta}{a\sin\theta} = \frac{n}{m}$$

$$\frac{(a+b)}{a} = \frac{n}{m} \quad \text{---} (3)$$

This is the required condition of missing order spectra in the diffraction pattern.

(I) When $b = a$, then from equ. (3)

$$\frac{2a}{a} = \frac{n}{m}$$

$$n = 2m$$

for $m = 1, 2, 3 \ldots$

$$n = 2, 4, 6$$

Hence 2^{nd}, 4^{th}, 6^{th} ... orders are absent will be absent from diffraction pattern

(ii) If $b = 2a$, then

$$\frac{a+2a}{a} = \frac{n}{m}$$

$$\frac{3a}{a} = \frac{n}{m}, \Rightarrow n = 3m$$

When $m = 1, 2, 3 \ldots$

then $n = 3, 6, 9 \ldots$

Hence, when width of the opacities (b) of the grating is doubled to that of transparencies (a), then 3^{rd}, 6^{th}, 9^{th} order maxima will be absent.

Maximum numbers of orders with a diffraction Grating :-

The maximum number of spectra available with a diffraction grating in the visible region can be calculated by using the grating equation for normal incidence as

$$(a+b)\sin\theta = n\lambda$$

$$n_{max} = \frac{(a+b)}{\lambda} \quad (\sin\theta = 1)$$

The maximum possible value of angle of diffraction is 90°.
Thus, the grating element determines the maximum possible order

Ex (i) If the grating element $(a+b)$ lies between λ and 2λ i.e $(a+b) < 2\lambda$, then

$$n_{max} < \frac{2\lambda}{\lambda} < 2$$

i.e for normal incidence only first order will be obtained.
Hence, if the width of a grating element is less than twice the wavelength of light, then only first order is possible.

(ii) If $(a+b)$ is in between 2λ and 3λ i.e $(a+b) < 3\lambda$

$$n_{max} < \frac{3\lambda}{\lambda} < 3$$

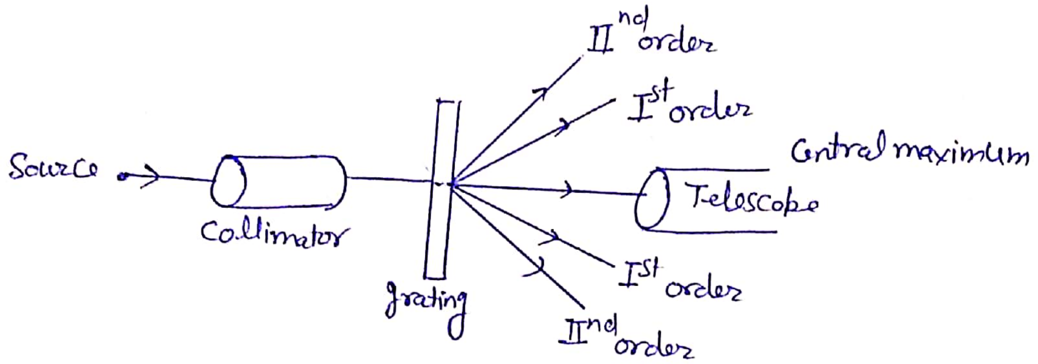
Hence, for normal incidence, when $(a+b) < 3\lambda$ only two orders are available.
Hence if the width of the grating element is less than that thrice the wavelength of light, then only two order are possible.

Ques. Show that only first order spectra is possible if the width of the grating element is less than twice the wavelength of light.

Determination of Wavelength using a plane transmission grating:-

One of main application of diffraction grating is the measurement of wavelength of an unknown source of light. The relation used in a diffraction grating is for the principal maxima obtained in the direction θ given by

$$(a+b)\sin\theta = n\lambda$$



Diffraction grating can be used in the laboratory to determine the wavelength of a monochromatic source.

The experimental procedure is as follows.

(1) The spectrometer (consisting of the collimator and telescope) is adjusted for parallel light which is the condition required for Fraunhofer diffraction

(2) The diffraction grating is mounted on a prism table between the collimator and the telescope and adjusted for normal incidence.

(3) The position of telescope is adjusted so that the cross wire of its eyepiece coincides with the central maximum and the corresponding reading on the vernier scale is noted.

(4) The telescope is moved to one side of the Central maximum and reading are recorded for the first and second order principal maxima.

(5) The telescope is now moved to the other side of the Central maximum and reading are recorded for the first and second order principal maxima.

(6) The angles of the first and second order maxima from the Central maxima are calculated.

(7) The grating element is calculated using the equation

$$(a+b) = \frac{1}{\text{No of lines per unit length}} = \frac{2.54}{N} \text{ cm}$$

(8) The wavelength is determined from first order using

$$(a+b)\sin\theta = \lambda$$

and from the second order using

$$(a+b)\sin\theta = 2\lambda$$

(9) The mean wavelength is then determined which is the wavelength of the unknown monochromatic sources.

Ques. What do you understand by missing order spectrum?

Ques. Give the theory of plane transmission grating and show how would you use it to determine the wavelength of light.

Ques What is diffraction grating? Derive ano expression for dispersive power of grating and explain it.

Dispersive Power:- The dispersive power of a diffraction grating is defined as the rate of change of the angle of diffraction with the change of the wavelength of light used.

If the wavelength of light is suppose changed from λ to $\lambda+d\lambda$ then if angle of diffraction is changed from θ to $\theta+d\theta$ then $\frac{d\theta}{d\lambda}$ represents the dispersive power.

The direction of principal maxima (grating equation) is given by

$$(a+b)\sin\theta = n\lambda \quad \text{——— (1)}$$

Differentiating equation (1)

$$(a+b)\cos\theta \, d\theta = n\, d\lambda$$

$$\boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}} \Rightarrow \text{dispersive power}$$

Now

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\sqrt{1-\sin^2\theta}}$$

from equation (1) $\sin\theta = \frac{n\lambda}{(a+b)}$

Then

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\sqrt{1-\left(\frac{n\lambda}{a+b}\right)^2}}$$

$$\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{\frac{(a+b)^2}{n^2} - \lambda^2}}$$

Conclusion:-

① The dispersive power is directly proportional to n i.e as the number of order increases, the dispersive power of grating increases.

(2) The dispersive power is inversely proportional to (a+b), i.e grating element, so, smaller is the grating element, higher the dispersive power.

(3) The dispersive power is inversely proportional to $\cos\theta$, i.e for large values of θ, the dispersive power increases.

Ques. What do you understand by missing orders spectrum? What particular spectra would be absent if the width of transparencies and opacities of the grating are equal.

Ques. Give the theory of plane transmission grating and show how would you use it to determine the wavelength of light.

Ques. Give the construction and theory of plane transmission grating and explain the formation of spectra by it. Explain what are absent spectra in the grating.

ABESIT Ghaziabad (290)

Course: Engg. Physics (KAS101T)

Unit IV

Wave optics

Faculty Name: Dr. Anchala

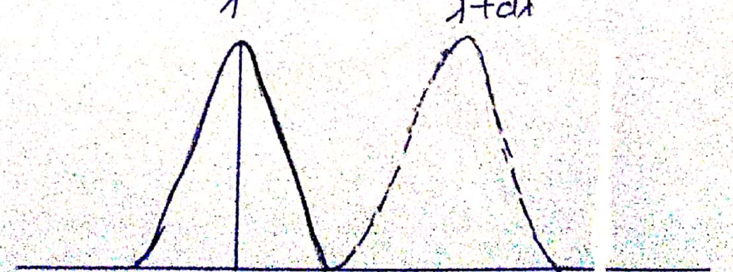
Lecture 44: Rayleigh's criterion of Resolution.
Resolving Power

outcome: Deduce Rayleigh criterion of resolution
Deduce resolving power of grating spectra

Rayleigh's Criterion of Resolution:- The ability of an optical instrument to just resolve the image of two closely space object is called resolving power

Two spectral lines of equal intensities are said to be just resolved if the principal maximum of the diffraction pattern due to one falls on the first minimum of the diffraction pattern due to other and vice versa.

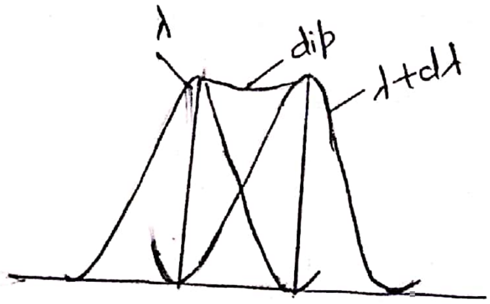
Consider two close wavelength λ and $\lambda+d\lambda$. The sepration between their central maximum will depend upon the wavelength difference $d\lambda$. If $d\lambda$ is sufficiently large so that the central maximum due to both wavelength are quite sepreated and two lines are distinctly resolved.



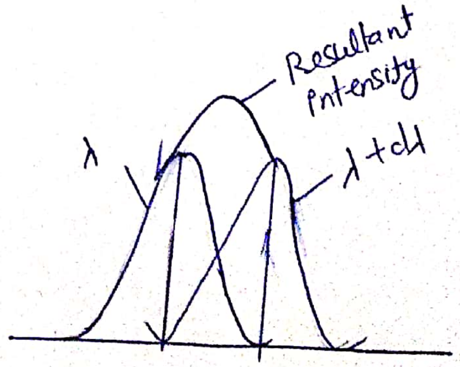
(Well resolved)

If the difference in wavelength is made smaller and have a limiting value for which the angular sepration between their principal maxima is such that the principal central maximum due to one source coincides with the first minimum of the other and vice-versa, then the curve shows a distinct dip in the middle, indicating the presence of two spectral lines corresponding to wavelength λ and $\lambda+d\lambda$.

Two spectral lines under this condition are said to be just resolved.



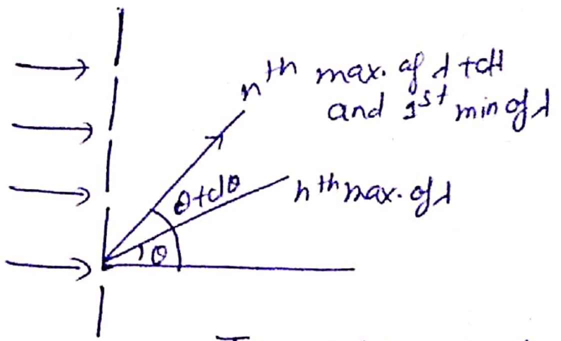
If the wavelength difference ($d\lambda$) is smaller than the limiting value, then the two principal maxima show considerable overlapping. The resultant intensity curve indicates no dip and appears as if it is a diffraction pattern of only one spectral line. Hence the two spectral lines under this condition are not seprated.



Resolving Power of a diffraction Grating:- The resolving power of a grating is defined as its ability to show the two neighbouring spectral lines in a spectrum as separate. It can also be defined as the ratio of wavelength (λ) of any spectral line to the wavelength difference ($d\lambda$) between neighbouring for which the spectral lines can be just resolved at wavelength λ.

It can be expressed as

$$\frac{\lambda}{d\lambda} = nN.$$



Transmisson Gration

Let a parallel beam of light of wavelength λ and $\lambda+d\lambda$ be incident normally on a diffraction grating.

Rayleigh Criterion is fulfilled if first minima of λ and principal maxima of $(\lambda+d\lambda)$ die in the direction of $(\theta+d\theta)$.

The principal maxima of λ in the direction θ, is given by

$$(a+b)\sin\theta = n\lambda \quad \text{——(1)}$$

The equation of minina is

$$N(a+b)\sin\theta = m\lambda$$

where N is the total number of lines on the grating and m is the order of minima, which takes all integer values except 0, N, 2N ... nN, since for these values, maxima are obtained.

Thus, first minimum adjacent to the nth principal maximum in the direction $(\theta + d\theta)$ can be obtained for

$$m = nN + 1, \text{ then}$$

$$N(a+b)\sin(\theta + d\theta) = (nN+1)\lambda$$

$$(a+b)\sin(\theta + d\theta) = \frac{(nN+1)\lambda}{N} \quad \text{——— (2)}$$

n^{th} maximum of $(\lambda + d\lambda)$ in direction $(\theta + d\theta)$ then

$$(a+b)\sin(\theta + d\theta) = n(\lambda + d\lambda) \quad \text{——— (3)}$$

Comparing equation (2) and (3)

$$\frac{(nN+1)\lambda}{N} = n(\lambda + d\lambda)$$

$$(nN+1)\lambda = nN(\lambda + d\lambda)$$

$$nN\lambda + \lambda = nN\lambda + nN\,d\lambda$$

$$\lambda = nN\,d\lambda$$

$$\boxed{\frac{\lambda}{d\lambda} = nN}$$ This is resolving power of the diffraction grating.

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ABESIT Ghaziabad (290)

Course: Engg. Physics (290)

Unit IV

Wave optics

Lecture 45: Numerical Problem related to Diffraction

Outcome: To understand how to solve numerical based on Diffraction.

Q.1 In a plane transmission grating the angle of diffraction for the second order principal maxima for $\lambda = 5\times10^{-5}$ cm is $30°$. Calculate the number of lines in one cm of the grating surface.

Ans. The grating equation is given by

$$(a+b)\sin\theta = n\lambda$$

where $(a+b)$ is the grating element

$$\therefore \quad (a+b) = \frac{n\lambda}{\sin\theta}$$

The number of lines in one cm $= \frac{1}{a+b}$

$$\therefore \quad \frac{1}{a+b} = \frac{\sin\theta}{n\lambda}$$

Given $n = 2$, $\theta = 30°$ and $\lambda = 5\times10^{-5}$ cm

$$\therefore \quad \frac{1}{a+b} = \frac{\sin 30°}{2\times5\times10^{-5}} = \frac{10^{5}}{2\times2\times5} = 5000 \text{ lines/cm}$$

Q.2 Find the angular separation of 5048 Å and 5016 Å wavelength in second order spectrum obtained by a plane diffraction grating having 15000 lines per inch.

Ans We know that in n^{th} order spectrum obtained by plane diffraction grating the angular separation $d\theta$ is

$$d\theta = \frac{d\lambda}{\sqrt{\left(\frac{a+b}{n}\right)^2 - \lambda^2}}$$

or $$d\theta = \frac{n\,d\lambda}{\sqrt{(a+b)^2 - (n\lambda)^2}}$$

or $$d\theta = \frac{n\,d\lambda}{a+b} \quad \text{(approximately)}$$

Given $d\lambda = 5048 - 5016 = 32 \times 10^{-8}$ cm

$$a+b = \frac{2.54}{15000} = 1.69 \times 10^{-4}\ \text{cm}$$

$$d\theta = \frac{2 \times 32 \times 10^{-8}}{1.69 \times 10^{-4}} = 3.787 \times 10^{-4}\ \text{radian.}$$

Q.3. In a grating spectrum, which spectral line in 4th order will overlap with 3rd order line of 5461 A°?

Ans. The grating equation is $(a+b)\sin\theta = n\lambda$

If the n^{th} order of wavelength λ_1 coincides with the $(n+1)^{th}$ order of λ_2, then

$$(a+b)\sin\theta = n\lambda_1 = (n+1)\lambda_2$$

Given $n = 3$, $\lambda_1 = 5461$ A° $= 5461 \times 10^{-8}$ cm, $(n+1) = 4$, $\lambda_2 = ?$

$$\therefore \lambda_2 = \frac{n\lambda_1}{(n+1)} = \frac{3 \times 5461 \times 10^{-8}}{4} = 4096 \times 10^{-8}\ \text{cm}$$

$$\lambda_2 = 4096\ \text{A°}$$

Q.4 A diffraction grating has 40,000 lines. Find its resolving power in the second order for wavelength 5000 A°.

Ans. The resolving power

$$\left(\frac{\lambda}{d\lambda}\right) = nN$$

Given $N = 40,000$ lines

$n = 2$

$\lambda = 5000 A^\circ$

$\therefore$ R.P (resolving power) $= nN$

$R.P = 2 \times 40,000$

$\boxed{R.P = 80,000}$

Q.5 find the minimum number of lines in a plane diffraction grating required to just resolve the sodium doublet ($5890 A^\circ$ and $5896 A^\circ$) in the (i) first order (ii) Second order.

Ans. Resolving power $\frac{\lambda}{d\lambda} = nN.$

where n, is the order, and N the total number of lines on the grating

$$N = \frac{1}{n}\left(\frac{\lambda}{d\lambda}\right)$$

(i) for the first order, $n = 1$

Mean wavelength $\lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{5890 + 5896}{2} = 5893 A^\circ$

and $d\lambda = \lambda_2 - \lambda_1 = 5896 - 5890 = 6 A^\circ$

$$\therefore N = \frac{5893}{1 \times 6} = 982$$

(ii) for $n = 2$,

$$N = \frac{5893}{2 \times 6} = 491$$

Q.6 What must be minimum number of lines per cm. in a half inch width grating to resolve the wavelength 5890 and $5896 A^\circ$

Ans The resolving power of a grating is given by $\frac{\lambda}{d\lambda} = nN$

where N is the number of lines per inch in the grating

or

Here $\lambda = \frac{\lambda_1+\lambda_2}{2} = \frac{5890+5896}{2} = 5893 A^\circ$

and $d\lambda = \lambda_2 - \lambda_1 = 5896 - 5890 = 6 A^\circ$

In this given problem, order is not given so $n = 1$. Thus

$$N = \frac{1}{n}\left(\frac{\lambda}{d\lambda}\right) = \frac{1}{1}\left(\frac{5893}{6}\right) = 982$$

Since the grating is half inch wide, therefore the number of lines per inch $= 982 \times 2$

Hence, the minimum number of lines per cm $= \frac{982 \times 2}{2.54} = 773$

Q.7 Calculate the least width of a plane diffraction grating having 500 lines per cm to resolve the two sodium lines D_1 and D_2 ($5896 A^\circ$ and $5890 A^\circ$) in the second order.

Ans. The resolving power of a grating is given by $\frac{\lambda}{d\lambda} = nN$

for the second order, $n = 2$

Here $\lambda = \frac{\lambda_1+\lambda_2}{\lambda_2} = \frac{5890+5896}{2} = 5893 A^\circ$

$d\lambda = \lambda_2 - \lambda_1 = 5896 - 5890 = 6 A^\circ$

$\therefore$ The total no. of lines on the grating

$$N = \frac{1}{n}\left(\frac{\lambda}{d\lambda}\right) = \frac{1}{2}\left(\frac{5893}{2}\right) = 491$$

width of grating $N(a+b) = \frac{\text{Total lines on grating}}{\text{Lines per cm on grating}}$

$$= \frac{491}{500 \text{ lines/cm}} = .982 \text{ cm}$$

Q.(8) A plane transmission grating has 15,000 lines per inch. Find the resolving power of the grating and the smallest wavelength difference that can be resolved with a light of wavelength 6000A° in the second order. 5

Sol. The resolving power of a grating is $\frac{\lambda}{d\lambda} = nN$

Here $N = 15{,}000$ and $n = 2$

$$\therefore \frac{\lambda}{d\lambda} = 2 \times 15000 = 30{,}000$$

Given $\lambda = 6000\,A^\circ$

$\therefore$ The smallest wavelength difference

$$d\lambda = \frac{\lambda}{nN} = \frac{6000\,A^\circ}{30{,}000} = 0.2\,A^\circ$$

ABES IT Ghaziabad (290)

Course: Engg. Physics (KAS 101T) Unit IV

Faculty Name: Dr Anchala Wave optics

Lecture: Some questions related to wave optics

Q.1 Why do you need coherent sources for observing interference?
Or What is the condition of sustained interference?

Ans Coherent sources are required for observing interference pattern so that path difference between the interfering waves does not change with time

Q Why two independent light sources can not produce interference?

Ans. Two independent light sources like two bulbs or two candles cannot produce interference as they are incoherent sources. Such sources emit light waves whose phase changes with time. Hence the intensity at a point will change with time and there will be no stationary interference pattern.

Q. Define Incoherent sources.

Ans. Two sources of light are said to be incoherent if they emit light waves whose phases changes with time, the phase difference at a point at which they arrive also vary with time in a random way.

Ques. Why does a very thick film appear white in reflected system

or A thick film shows no colours in reflected white light. explain

Ans. The bright and dark appearance of the reflected light depends upon the values of μ, t and r. In case of white light even if t and r made constant μ varies with wavelength. Due to large thickness (t) a large number of wavelength will satisfy the condition of constructive interference and on the other hand at the same point the condition of destructive interference is also satisfied for another set of wavelength. Thus the resultant effect at any point will be a general white illumination. Hence a thick film shows no colour but appears white in reflected system.

Q. Why does an excessively thin film appear dark in reflected system?

Ans The effective path difference in reflected light for a film is $2\mu t \cos r \pm \frac{\lambda}{2}$.

If thickness 't' is very small, the path difference is

$$\therefore \Delta = \pm \frac{\lambda}{2}$$

This is a condition for minimum intensity.

$\therefore$ An excessively thin film appears dark in reflected light.

Q Why Newton's rings are circular?

Ans The fringes are circular because the air film is symmetrical about the point of contact of the lens with the glass plate. Every fringe is the locus of point of equal thickness of the film. And the locus of points of equal thickness of air film lie on a circle with the point of contact of lens and the glass plate as centre. Hence the fringes are circular.

Ques What is the effect of using a white light source in Newton's rings experiment instead of a monochromatic source?

Ans. When monochromatic light is used, then Newton's rings are alterntly dark and bright. The diameter of the ring depends upon the wavelength of light used. In case of white light, the diameter of rings of different colours will be different. There will be an overlapping of the rings of different colours over each other, the rings cannot be viewed.

Ques. What will be the effect on the intensity of principal maximum of diffraction pattern when single slit is replaced by double slit

Ans When single slit is replaced by double slit, the intensity of principal maximum becomes four times.

Q: Explain why diffraction is not observed for light passing through a window in a room.

Ans. for very large slit widths, like a window in a room, the minimum and also the secondary maxima will come very close to each other and merge due to which the diffraction pattern will not be observed.

Ques Explain the diffraction pattern due to a single slit when the slit width is smaller than the wavelength of light used.

Ans. The first minima on either side of the minima occurs in the direction θ is given by

$$a \sin\theta = \pm \lambda$$

When the slit is narrowed by reducing a, the angle of diffraction θ increases, which means that the central maximum becomes wider. When the width of slit is equal to the wavelength of light used, for first minimum

$$a \sin\theta = n\lambda \; ; \; n=1 \text{ and } a=\lambda$$

$$\therefore \sin\theta = 1$$

$$\theta = 90^\circ$$

i.e, the first minimum will be formed at 90° from the incident light. Hence the central maximum will occupy the

the enitre space on the screen. for all slits width less than the wavelength, only the Central maximum will be formed on the screen.

Q What is the effect of increasing the wavelength on single slit fraunhofer diffraction pattern?

Ans. As wavelength is increased, the angle at which n^{th} order minimum is formed increases, Hence the spectrum becomes wider.

Ques. Give the characteristics of grating spectra.

Ans (i) According to grating equ. $(a+b)\sin\theta = \pm n\lambda$ $(n=0,1,2 \dots)$ for a given order n, the angle of diffraction θ varies with λ. for longer wavelength λ, θ would be greater.

(2) If incident light is white, then each order will contain principal maxima of different wavelength in different directions.

(3) The principal maxima of all wavelength corresponding to $n=0$ will be in the direction $\theta=0$. Hence, zero order maxima will be absent.

(4) The grating spectral lines are straight.

Ques White light is incident on a soap film at an angle $\sin^{-1}\frac{4}{3}$ and reflected light is observed with a spectroscope. It is found that two consecutive dark bands corresponds to wavelength 6.1×10^{-5} and 6.0×10^{-5} cm. If the refractive index of the film be (4/3), calculate the thickness.

Ans. We know that the condition for dark band or fringe in the reflected light is

$$2\mu t\cos r = n\lambda$$

If n and $(n+1)$ are the orders of consecutive dark bands for wavelength λ_1 and λ_2 respectively, then

$$2\mu t\cos r = n\lambda_1 \quad \text{and} \quad 2\mu t\cos r = (n+1)\lambda_2$$

or

$$2\mu t\cos r = n\lambda_1 = (n+1)\lambda_2 \qquad \text{———} (1)$$

$$n\lambda_1 = (n+1)\lambda_2$$

$$n\lambda_1 = n\lambda_2 + \lambda_2$$

$$n(\lambda_1 - \lambda_2) = \lambda_2$$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

substituting this value of n in equ. (1) then

$$2\mu t\cos r = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

$$d = \frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_2)}\cdot\frac{1}{2\mu\cos r} \qquad \text{———} (2)$$

$$\cos r = \sqrt{1-\sin^2 r}$$

(from Snell's law $\mu = \frac{\sin i}{\sin r}$)

$$\cos r = \sqrt{1-\left(\frac{\sin i}{\mu}\right)^2}$$

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Given $\mu = \frac{4}{3}$, and $\sin i = \frac{4}{5}$

$$\therefore \quad \cos r = \sqrt{1 - \frac{(4/5)^2}{(4/3)^2}} = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

Substituting this value of $\cos r$ in equ(2), and also

$\lambda_1 = 6.1 \times 10^{-5}$ cm, $\lambda_2 = 6.0 \times 10^{-5}$ cm, $\mu = \frac{4}{3}$ .. then

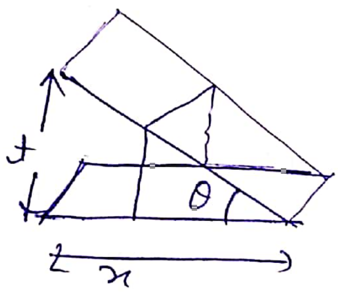
$$t = \frac{6.1 \times 10^{-5} \times 6.0 \times 10^{-5}}{(6.1 \times 10^{-5} - 6.0 \times 10^{-5}) \times 2 \times \frac{4}{3} \times \frac{4}{5}}$$

$$t = 0.0017 \text{ cm}$$

Q Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 interference fringes are observed between these edges in sodium light of $\lambda = 5890 A^\circ$ of normal incidence, find the diameter of the wire

Ans The fringe width in air-wedge for normal incidence is

$$w = \frac{\lambda}{2\theta} \quad \text{——(1)}$$



Let t be the thickness of the wire and x be the length of the glass surface from the point of contact then wedge angle $\theta = \frac{t}{x}$ ——(2)

therefore equ.(1)

$w = \frac{\lambda x}{2t}$, If n fringes are seen in the entire film

then $x = nw$

Therefore $w = \frac{\lambda n w}{2t}$ or $t = \frac{n\lambda}{2}$

Given $n = 20$, $\lambda = 5890 A^\circ = 5890 \times 10^{-8}$ cm,

$$t = \frac{20 \times 5890 \times 10^{-8}}{2} = 5.89 \times 10^{-4} \text{ cm}$$