Set Theory (Unit-I)

Lecture -1

Set:- • set is an ordered collection of objects.
• The object in a set is called an element or member.

• Set denoted by capital letters. eg A, B, C
• Elements denoted by small letter a, b --
• set are describe by 2 methods-

(I) **Roster or list Method** - elements are listed in braces.

eg $A = \{2, 3, 5, 7, 11, 13\}$.

$B = \{2, 4, 6 ---\}$

(II) **Set Builder Method** - elements are describe by the property they satisfy.

eg $A = \{x : x$ is a prime no less then $15\}$

$B = \{x : x = 2n, n \in N\}$.

Empty set :- A set containg no element.
eg - set of even prime nos greater than 10

Empty set denoted by $\{\}$ or ϕ.

Subset :- A set A is said to be a subset of set B if every element of A is also an element of B. It is denoted by '$\subseteq$'
$A \subseteq B$ eg $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, 4, 7, 8\}$ then
$A \subseteq B$.

- **Super Set :-** A set A is said to be a superset of set B, if B is a subset of A. It is denoted by $\underset{\text{Super Set}}{A} \supseteq \underset{\text{sub set}}{B}$

- **Proper Subset :-** A set ~~is~~ A is said to be a proper subset of B if A is a subset of B and there is at least one element in B, which is not an element of A.

- **Universal Set :-** A set which contains all objects under consideration is called as Universal set and is denoted by U.

- Two sets are said to be equal iff they have same elements. eg $A = \{2, 5, 7, 9\}$ and $B = \{5, 2, 7, 9\}$ then set A and B are equal.

- **Operations —**
  - **Union (∪) :-** set of all elements in A or in B or in both. $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
    eg $A = \{1, 2, 3, 5, 7\}$, $B = \{2, 5, 10, 11\}$ then
    $A \cup B = \{1, 2, 3, 5, 7, 10, 11\}$
  - **Intersection (∩) :-** set of all elements, that are common in A as well as B.
    eg $A = \{1, 2, 3, 4, 5, 7\}$, $B = \{2, 5, 7, 10, 11\}$
    $A \cap B = \{2, 5\}$. $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
  - **Difference (-) :-** set of all elements that are in A, but not in B, is called difference between A and B, and denoted by $A - B$
    eg $A = \{2, 1, 4, 5\}$, $B = \{4, 9, 11\}$; $A - B = \{2, 1, 5\}$

Cardinality :- The total no. of elements in a set is called cardinality of a set.

eg $A = \{2, 3, 5, 6\}$ then cardinality of A is 4 and denoted by $|A|$.

Complement of a set :- If U is a universal set and A is its subset, then complement of A denoted by A' is all elements of U that are not in A.

eg $U = \{x : x \in N, x \leq 15\}$ and if

$A = \{x : x \in U \text{ and } 3 | x\}$ then

$A^c = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14\}$.

Power Set :- A power set of a set A, denoted by P(A), is set of all subsets of A.

eg $A = \{1, 2, 3\}$ then

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1, 2, 3\}\}$

- If Number of elements in A is n, then the number of elements in the power set of A is 2^n.

Cartesian Product :- Let A and B be two sets. The product set of A and B denoted by $A \times B$, is set of all ordered pairs from A and B then

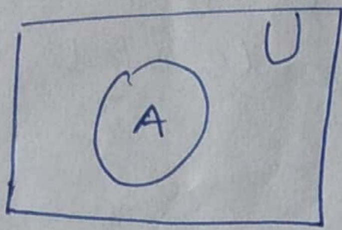
$A \times B = \{(a, b) : a \in A, b \in B\}$

eg $A = \{1, 2, 3\}$, $B = \{4, 5\}$ then

$A \times B = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$.

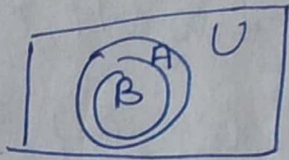
Venn Diagram:- A venn diagram is a pictorial representation by enclosed area in the plane

- Universal set is represented by rectangle
- Other sets are represented by circle within it.

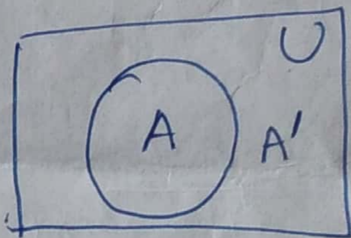


- Venn Diagrams can sometimes be used to determine whether or not an argument is valid.

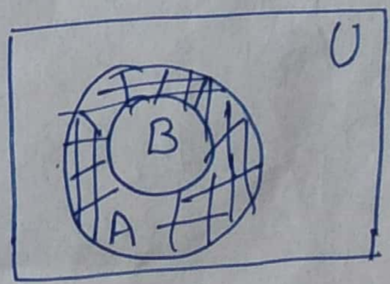
(I) Set B is a proper subset of A
ie $B \subset A$



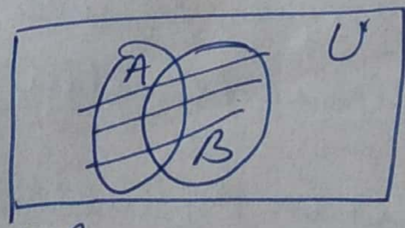
(II) The complement of set A
ie A' or A^c



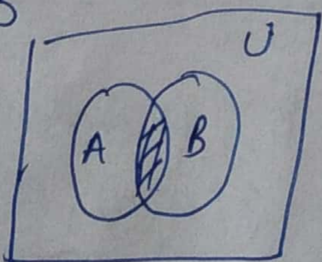
(III) The difference of set A and B
ie $A - B$



(IV) The union of set A and B
ie $A \cup B$



(V) The intersection of set A and B
ie $A \cap B$



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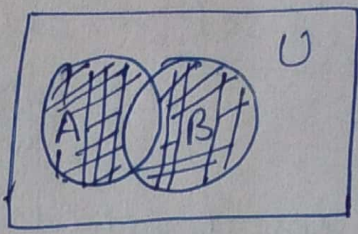
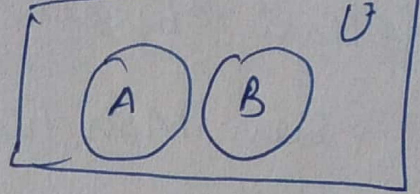
(VI) The symmetrical difference of set A and B ie $A \Delta B$



(VII) The set A & B are Disjoint ie $A \cap B = \phi$



Symmetric Difference:-

The symmetric difference of set A and B, denoted by $A \oplus B$, consists of those elements which belongs to A or B but not both ie

~~eg~~ $A \oplus B = (A \cup B) \setminus (A \cap B)$

OR $A \oplus B = (A/B) \cup (B/A)$

eg $A = \{1,2,3,4,5,6\}$, $B = \{4,5,6,7,8,9\}$ then

$A/B = \{1,2,3\}$, $B/A = \{7,8,9\}$

$A \oplus B = \{1,2,3,7,8,9\}$.

Duality:-

The dual E^* of E is the equation obtained by replacing each occurrence of $\cup, \cap, U$ and ϕ in E by $\cap, \cup, \phi$ and U respectively.

- If E is an identity, then its dual E^* is also an identity and this is known as "principle of duality".

eg $(U \cap A) \cup (B \cap A) = A$

$E^* = (\phi \cup A) \cap (B \cup A) = A$

Ordered Set :- set is an unordered collection of different objects.

The ordered set is defined as ordered collection of different objects. eg A = {3, 6, 7, 8, 9}.

B = {Sun, Mon, Tue, Wed, Thurs, Fri, Sat}.

Ordered Pairs :- An ordered pair of objects is a pair of objects arranged in some order.

- Thus in the set {a, b}, of two objects a is first and b is second object of pair. Thus (a, b) and (b, a) are two different ordered pair.
- An ordered triple is ordered triple of objects (a, b, c) where a is 1^{st}, b is 2^{nd} and c is 3^{rd} element of triple. It can also be written as {(a, b), c}.
- An ordered n-tuple is an ordered pair where the first component is an ordered (n-1) tuple and n^{th} element is second component.
- ~~And Ordered~~ eg. an ~~o~~ ordered set of 5 elements {a, b, c, d, e} can be written as {(((a, b), c), d), e}.

Lecture - 2

Identities of Sets and Multiset

- There are 2 methods of proving equations involving set operations -
  - (I) use what it means for an object x to be an element of each side ie use of identities
  - (II) Use of Venn Diagrams.

- **The Algebra of Sets** :- Identities / Law of the algebra of Sets.

(I) Idempoten law :-

$A \cup A = A$; $A \cap A = A$

(II) Associative law :-

$(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$

(III) Commutative law :-

$A \cap B = B \cap A$; $A \cup B = B \cup A$

(IV) Distributive law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ - union distribution over intersection

$A \cap B(\cup C) = (A \cap B) \cup (A \cap C)$ - intersection distributive over union

(V) identity law :-

$A \cup \phi = A$; $A \cap U = A$

$A \cup U = U$; $A \cap \phi = \phi$

(vi) complement law :-

$A \cup A^c = U$; $A \cap A^c = \phi$

$U^c = \phi$; $\phi^c = U$

(vii) Involution law :-

$(A^c)^c = A$

(viii) DeMorgan's law :-

$(A \cup B)^c = A^c \cap B^c$; $(A \cap B)^c = A^c \cup B^c$

(ix) $A - B = A \cap B^c$

(x) $(A-B)-C = A-(B \cup C)$

(xi) A if $A \cap B = \phi$ then $(A \cup B) - B = A$

(x) $A - (B \cap C) = (A-B) \cap (A-C)$

=

eg -

The inclusion-Exclusion Principle:-

If A and B are finite sets then

$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$ or

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

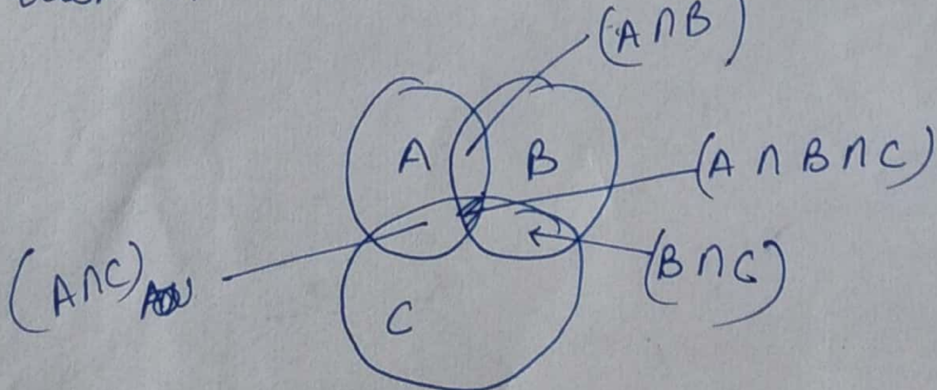
ie include $n(A)$ and $n(B)$ and exclude $n(A \cap B)$

Now let A, B and C are sets then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C) - n(B \cap C)$$

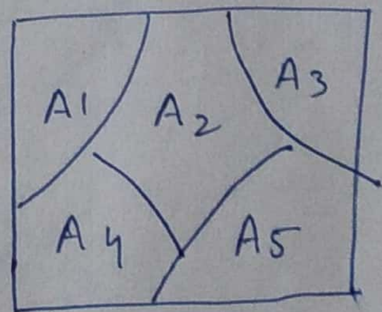
The Addition Principle for disjoint sets:-

If a task T_1 can be performed in exactly n ways and a task T_2 can be performed in exactly m ways, then the nos of ways of performing task T_1 & T_2 is $n+m$



Partitions:-

Let S be a non-empty set. A partition of S is subdivision of S into non overlapping, nonempty subsets. ie



(i) Each a in S $\in A_i$ (one of A_i)

(ii) $A_i \neq A_j$ then $A_i \cap A_j = \phi$.

Generalized set Operations

The set operations of union and intersection can be extended to any nos of sets, finite or infinite. Consider a finite no. of sets $A_1, A_2 - A_m$, then the union and intersection of these sets are denoted

by $A_1 \cup A_2 \cup \ldots \cup A_m = \bigcup_{i=1}^{m} A_i$

$= \{x : x \in A_i \text{ for some } A_i\}$

and

$A_1 \cap A_2 \cap \ldots \cap A_m = \bigcap_{i=1}^{m} A_i$.

$= \{x : x \in A_i \text{ for every } A_i\}$.

Multisets :-

- Multisets is an unordered collection of elements where an elements can occurs as a member more than once.
- Multiset is a set in which elements are not necessarily different.

  eg = {1, 1, 2, 2, 3, 4, 4}
- The notations used to represent multiset is as $S = \{n_1 \cdot a_1, n_2 \cdot a_2 \cdots n_i \cdot a_i\}$

ie a_1 occurs n_1 times, a_2 occurs n_2 times and the no. $n_i = 1, 2, 3 \cdots$ are called <u>multiplicies</u> of the elements a_i.

So $S = \{2 \cdot 1, 2 \cdot 2, 1 \cdot 3, 2 \cdot 4\}$

- The multiplicity of an element in a multiset is defined to be the no. of times the element appears in the multiset.
- The cardinality of a multiset is the no. of different no. of elements. eg $A = \{1, 1, 2, 3,\}$ then $|A| = 3$

- <u>Operations on Multiset</u> :-
- <u>Union $(A \cap B)$</u> :- The union of multisets A and B is the multiset where the multiplicity of an element is the max of its multiplicities of A and B.

eg $A = \{1, 1, 1, 2, 2, 3\}$ then $A \cup B = \{1, 1, 1, 4, 2, 2, 3, 3\}$.
$B = \{1, 1, 4, 3, 3\}$

- **intersection $(A \cap B)$:-** is the multiset where the multiplicity of an element is the minimum of its multiplicities in A and B.

  $A \cap B = \{1, 1, 3\}$.

- The difference of A and B is the multiset where the multiplicity of an element is the multiplicity of element in A less its multiplicity in ~~B~~ B unless this difference is -ve, in this case the multiplicity is zero.

  eg ~~$A = \{1,1,1, 2,2,3,4,4,5\}$~~

  eg $A = \{1, 1, 1, 2, 2, 3, 4, 4, 5\}$

  $B = \{1, 1, 2, 2, 2, 3, 3, 4, 4, 6\}$

  $A - B = \{1, 5\}$

- The sum of A and B is the multiset where the multiplicity of an element is sum of multiplicities in set A and set B, denoted by $A + B$

  eg $A = \{1, 1, 2, 3, 3\}$, $B = \{1, 2, 2, 4\}$

  $A + B = \{1, 1, 1, 2, 2, 2, 3, 3, 4\}$

81, 82, 100, 101, 102, 114 117, 122, 123

83, 88, 89, 100, 102, 111, 118.

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