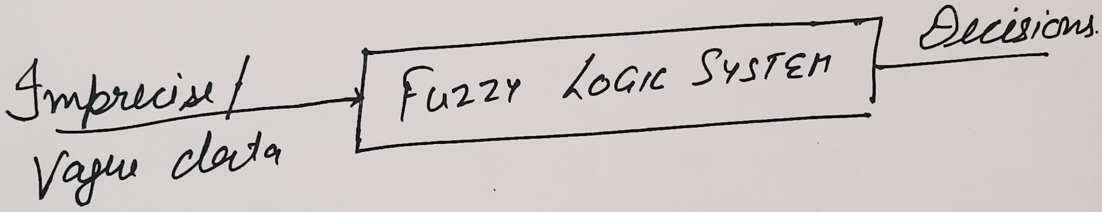
APPLICATION OF SOFT COMPUTING

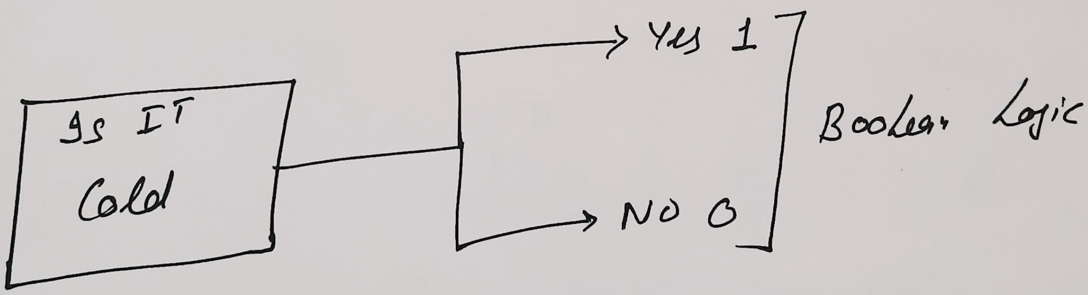
Unit-3

FUZZY LOGIC-I

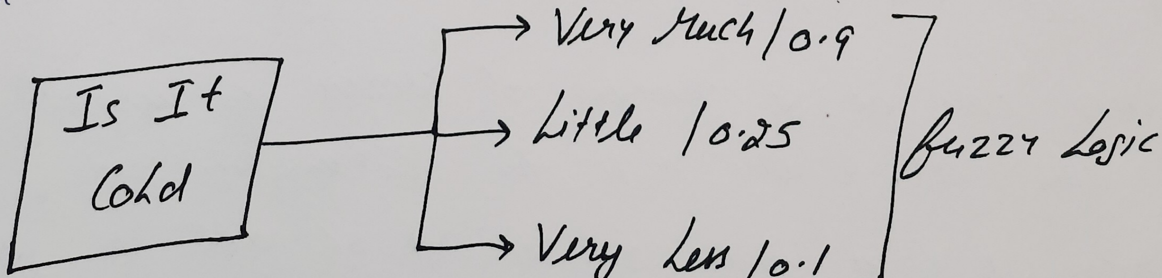
INTRODUCTION: The term fuzzy refers to things that are not clear or vague.

* In Real world many time we encounter a situation when we can't determine wheather the state is true or false. Their fuzzy logic provide very valuable flexibility for reasoning.





In fuzzy logic, An Intermediate value too present which is partially true or partially false.



CRISP SETS : There are three basic Method for representing crisp sets.

1 List Method : A set is defined by naming all its Members. This is suitable for finite set only

Set A having Members 0, 1, 2, 3, 4, 5 is written as

$$A = \{0, 1, 2, 3, 4, 5\}.$$

2 The Rule Method : A set is defined by a property satisfied by its Members

$$A = \{x \mid x \text{ is a positive Integer less than } 6\}$$

Here symbol '|' read as "such that"

3 CHARACTERISTIC FUNCTION : A set is represented by a function called characteristic function.

$$F_A(x) = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{for } x \notin A \end{cases}$$

SOME FACTS ABOUT CRISP SETS:

- The No of element in a particular crisp set give Cardinality of that set
- A set with Zero Cardinality → Empty set / $\{\phi\}$
- There can be 2^n numbers of subset Corresponding to a given set where n is the Cardinality of set
- Power set of a given set is set of all the possible

DEFINATION of CRISP SET:

* A Crisp set / Classical set is a well defined Collection of objects (say U) having identical properties such as Countability & finiteness.
* An object either belong to the set or not.
* Set → denoted by upper case letter & their Members by Lower Case Letter

  For eg. $x \in A$, $x \notin A$, $B \subseteq A$
* A set B is said to be the subset of A If all element of set B are also Contained in set A.

Crisp Logic: Crisp Logic are based on the first order Logic. It Can not provide an appropriate way to interpret the Imprecise and non-Categorical data.

Example: We are supposed to find the answer to the Question. Does she have a pen? If yes assign Value 1 otherwise 0.

So a Logic which demand a binary (0/1) type of handling is known as Crisp Logic

Operation on Classical / Crisp set:

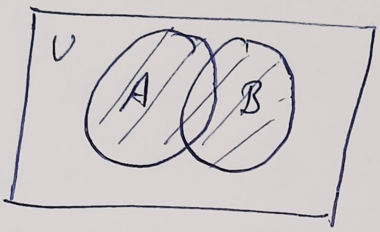
1 Union: The Union of a set is denoted by $(A \cup B)$

$$A \cup B = \{x / x \in A \text{ or } x \in B\}$$

Example: $A = \{10, 11, 12, 13\}$ $B = \{13, 14, 15\}$

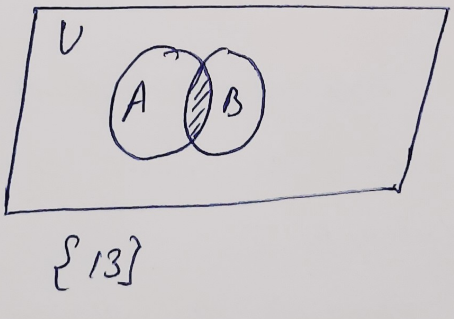
$$A \cup B = \{10, 11, 12, 13, 14, 15\}$$

The Common element occur only once.



2 Intersection: $A \cap B$, is a set of element which are in both A & B.

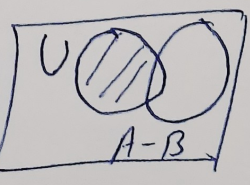
$$A \cap B = \{x / x \in A \text{ AND } x \in B\}$$

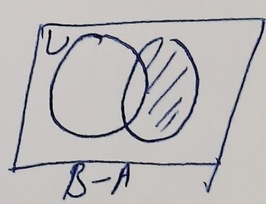


$$A \cap B = \{13\}$$

3 Difference / Relative Complement: denoted by A-B.

* Set of element which are only in A But Not in B.





$$A - B = \{x / x \in A \text{ AND } x \notin B\}$$

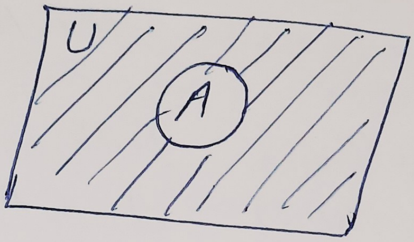
$(A - B) = \{10, 11, 12\}$ $B - A = \{14, 15\}$

$$(A-B) \neq (B-A)$$

4 **Complement of a set**: denoted by A'.

→ It is the set of element which are ~~only~~ not in A

$$A' = \{x \mid x \notin A\}$$



5 **Cartesian product / Cross product**:

The Cartesian product of a number of set $A_1, A_2 -$

→ denoted by $A_1 \times A_2 - A_n$ Can be defined as all possible ordered pairs $(x_1, x_2 - x_n)$

where $x_1 \in A_1$, $x_2 \in A_2 -$ $x_n \in A_n$

$A = \{a, b\}$ $B = \{1, 2\}$

$A \times B = \{(a,1)\ (a,2)\ (b,1)\ (b,2)\}$

$B \times A = \{(1,a)\ (1,b)\ (2,a)\ (2,b)\}$

PROPERTIES OF CRISP SETS

1 **Commutative Law:** $A \cup B = B \cup A$,
$A \cap B = B \cap A$

2 **Associative Law:** $A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

3 **Distributive Law:** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4 **IDEMPOTENCY/Law of Tautology:** $A \cup A = A$

5 **Identity:** $A \cap A = A$, $A \cup \phi = A$, $A \cap X = A$,
$A \cap \phi = \phi$ $A \cup X = X$ $X \rightarrow$ Universal set

6 **Transitivity:** If $A \subseteq B \subseteq C$ then $A \subseteq C$, $\subseteq \rightarrow$ subset

7 **Absorption Law:** $A \cap (A \cup C) = A$
$A \cup (A \cap C) = A$

8 **Involution or Law of double:** $\bar{\bar{A}} = A$

9 **Excluded Middle Law:** $A \cup \bar{A} = X$

10 **Law of Contradiction:** $A \cap \bar{A} = \phi$

11 **De-Morgan's Law:** $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

Fuzzy Sets

* A fuzzy set is a set having degree of Membership b/w 1 or 0.
* Fuzzy set may be viewed as an extention of generalization of the basic concept of the Crisp set.

1. The Membership of an element to a set become "fuzzy" or "Vague" Concep. If some elemens have issue of wheather they belong to a set or not may not be Clear
2. The Membership of an element to a set ~~become~~ may be measured by a degree, Commonly known as "Membership degree b/w 0 & 1.

__def. fuzzy Set:__ A fuzzy set is a Combination of an element having a changing degree of Membership in the set.

* fuzzy set A over a universe of discourse U (is called a finite or Infinite Interval within which the fuzzy set Can take a value) is a set of pairs

$A = \{\mu_A(x)/x \mid x \in X/U \cdot \mu_A(x) \in [0,1] \in R\}$

$\mu_A(x) \rightarrow$ Membership degree of the element x to the fuzzy set A {0 to 1}

$\mu_A(0) \rightarrow$ x is no way belong to the fuzzy set A

$\mu_A(1) \rightarrow$ x Completely belongs to fuzzy set A

$\mu_A(0.5)$ - greatest uncertainity being

Fuzzy Set / Universe of discourse X → TYPES

1 FINITE / discrete universe of discourse X

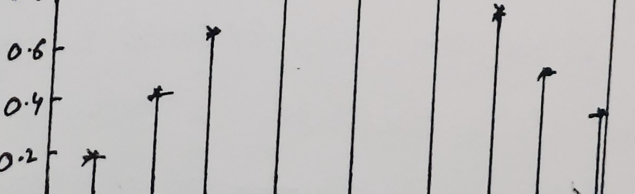
$X = \{x_1, x_2 -- x_n\}$

A - fuzzy set

$A = \mu_1(x_1)/x_1 + \mu_2(x_2)/x_2 + \mu_3(x_3)/x_3 ---- \mu_n/x_n$

μ_i with $i = 1, 2 -- n$ represents the Membership degree of the element x_i

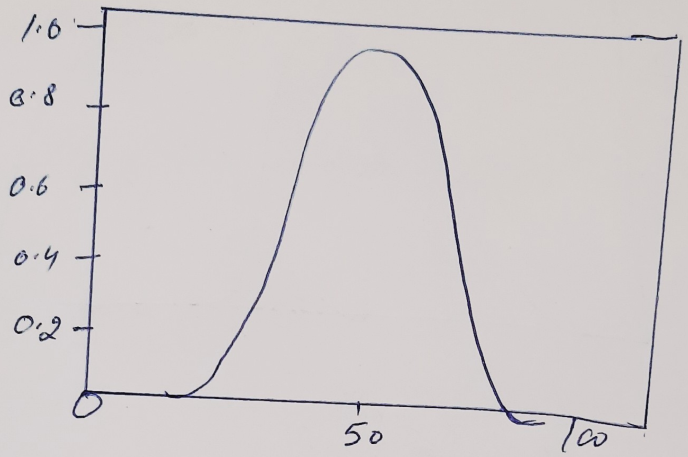
$A = \sum_{i=1}^{n} \mu_i(x_i)/x_i$



2 Infinite / Continous fuzzy SET.

→ A fuzzy set A over X can be represented by

$$\underset{\sim}{A} = \int \mu_A(x)/x$$



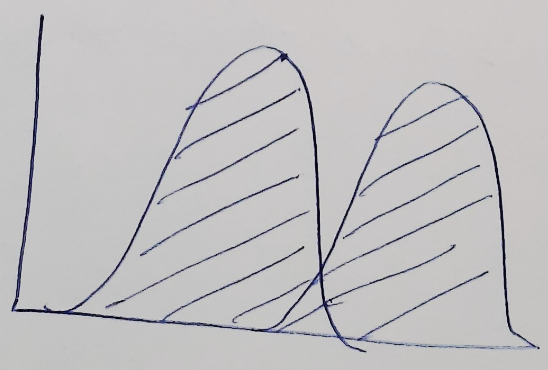
Fuzzy set operations

1 Union:

$$\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \max[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)]$$

$$= \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x) \text{ for all } x \in U$$

Where $\vee$ indicates max operation



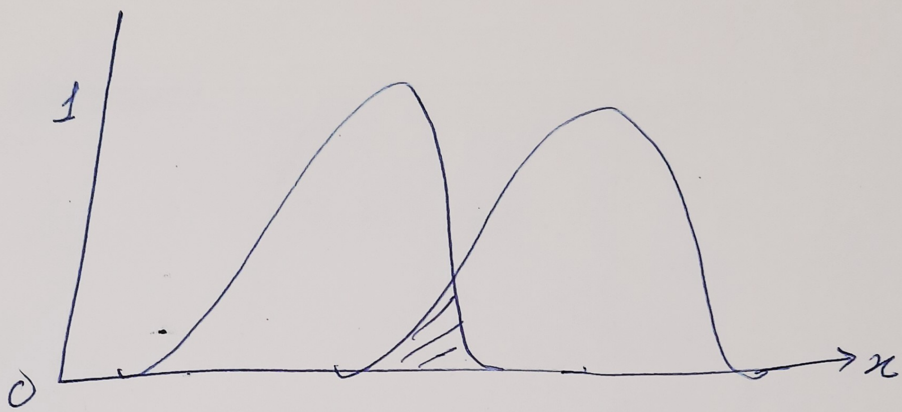
2 Intersection:

: denoted by $\underset{\sim}{A} \cap \underset{\sim}{B}$

$$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \min\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\right]$$

$$= \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x) \quad \text{for all } x \in U$$

Where $\wedge$ Indicates min operation

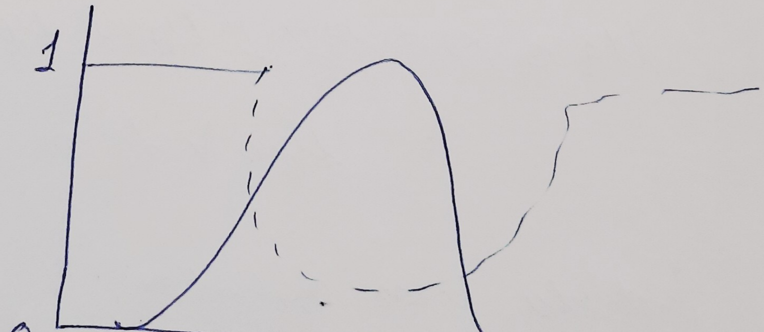


3 Complement:

$\mu_{\underset{\sim}{A}}(x) \in [0,1]$

The Complement of A, denoted by $\bar{A}$

$$\mu_{A}(x) = 1 - \mu_{A}(x) \quad \text{for all } x \in U$$



Complement of fuzzy Set A

More operation of fuzzy set

1 **Algeberic sum**: The Algebric sum $(A+B)$ of fuzzy set. fuzzy set $\underset{\sim}{A}$ & $\underset{\sim}{B}$ is defined as

$$\mu_{\underset{\sim}{A}+\underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x) - \mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x)$$

2 **Algebric product:**

$$\mu_{\underset{\sim}{A} \cdot \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x)$$

3 **Bounded sum:**

$A \oplus B$ of two ~~f~~fuzzy set

$$\mu_{\underset{\sim}{A} \oplus \underset{\sim}{B}}(x) = \min\left[1, \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x)\right]$$

4 **Bounded difference**: $(A \odot B)$

$$\mu_{\underset{\sim}{A} \odot \underset{\sim}{B}}(x) = \max\left[0, \mu_{\underset{\sim}{A}}(x) - \mu_{\underset{\sim}{B}}(x)\right]$$

FUZZY RELATION

Fuzzy Relation relate element of one universe (say X) to those of another universe (say Y) through the Cartesian product.

* A fuzzy Relation is a fuzzy set defined on the Cartesian product of the Classical set $[X_1, X_2 - X_n]$ Where tuple $(x_1, x_2, x_3 - x_n)$ may have varying degree of membership $\mu_R(x_1, x_2 -- x_n)$ within the Relation

$$R(x_1, x_2 -- x_n) = \int_{X_1, X_2 -- X_n} \mu_R(x_1, x_2 -- x_n) / (x_1, x_2 -- x_n) \quad x_i \in X_i$$

A fuzzy Relation b/w two set X & Y is called <u>Binary fuzzy Relation</u>

→ A Binary Relation R (X, Y) is reffered to as bipartite graph when $X \neq Y$

& Directed graph when $X = Y$

$$X = \{x_1, x_2 -- x_n\} \quad Y = \{y_1, y_2 -- y_m\}$$

A Fuzzy Relation R (X, Y) Can be expressed by an $n \times m$ matrix as follow

FUZZY SET Relation $R(X,Y)$ Can be Represented by $n \times m$ matrix

$$R(x,y) = \begin{bmatrix} \mu_R(x_1, y_1) & \mu_R(x_1, y_2) & \cdots & \mu_R(x_1, y_m) \\ \mu_R(x_2 y_1) & \mu_R(x_2 y_2) & & \mu_R(x_2, y_m) \\ \vdots & \vdots & & \\ \mu_R(x_n y_1) & \mu_R(x_n y_2) & & \mu_R(x_n y_m) \end{bmatrix}$$

The domain of a binary fuzzy Relation $R(X,Y)$ is the fuzzy set, $dom(X,Y)$ having the membership function as

$$\mu_{domain\, R}(x) = \max_{y \in Y} \mu_R(x,y) \quad \forall x \in X$$

The Range of a binary fuzzy relation $R(X,Y)$ is the fuzzy set $ran\, R(X,Y)$ having the Membership function as

$$\mu_{range\, R}(y) = \max_{x \in X} \mu_R(x,y) \quad \forall y \in Y$$

Consider a universe $X = \{x_1, x_2, x_3, x_4\}$ & the binary fuzzy relation on X as

$$R(X,X) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix} 0.2 & 0 & 0.5 & 0 \\ 0 & 0.3 & 0.7 & 0.8 \\ 0.1 & 0 & 0.4 & 0 \\ 0 & 0.6 & 0 & 1 \end{bmatrix} \end{array}$$

IDENTITY RELATION:

The Cartesian product for set $A \times A = A^2$ is called Identity Relation

$A = \{2, 4, 6\}$

$U = \{(2,2)\ (2,4)\ (2,6)\ (4,2)\ (4,4)\ (4,6)\ (6,6)\ (6,2)\ (6,4)\}$

$I_A = \{(2,2), (4,4), (6,6)\}$

Cardinality of Classical Relation:

Consider n element of universe X being related to m element of universe Y.

$X = n_X$ (Cardinality of X)

$Y = n_Y$ (Cardinality of Y)

$n_{X \times Y} = n_X \times n_Y$

The Cardinality of the Power set $P(X \times Y)$

$n_P(X \times Y) = 2^{(n_X \times n_Y)}$

Operation on Classical Relations.

Let R and S be two seperate Relations on the Cartesian universe $X \times Y$.

1 Union $(R \cup S)$

$\chi_{R \cup S}(x,y) : \chi_{R \cup S}(x,y) = \max[\chi_R(x,y), \chi_S(x,y)]$

$\chi_{R \cap S}(x,y) = \min[\chi_R(x,y), \chi_S(x,y)]$

3 Complement

$$\bar{R} = \chi_{\bar{R}}(x,y) = (1 - \chi_R(x,y))$$

4 Identity $\quad \phi \rightarrow \phi_R \quad X \rightarrow E_R$

where

$$\phi_R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad E_R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\phi_R \rightarrow$ Null Relation $\qquad E_R \rightarrow$ Complete Relation.

PROPERTIES OF CLASSICAL SET.

Commutativity, Associativity, distributivity, Involution & Idempotency hold good for Classical Relation. Similarly De-Morgan's Law, Excluded Middle Law hold also good for Crisp Relation

Composition of classical Relation

Let R be a Relation that map element from Universe X to universe Y.

$$R \subseteq X \times Y \qquad S \subseteq Y \times Z$$

$\Rightarrow$ Second set of R must be same as first set of S

A Relation T is formed that relate the same element of Universe X Contained in R with the same element of Universe Z Contained in S

$X = \{a_1, a_2, a_2\}$ $\quad Y = \{b_1, b_2, b_3\}$

$Z = \{c_1, c_2, c_3\}$

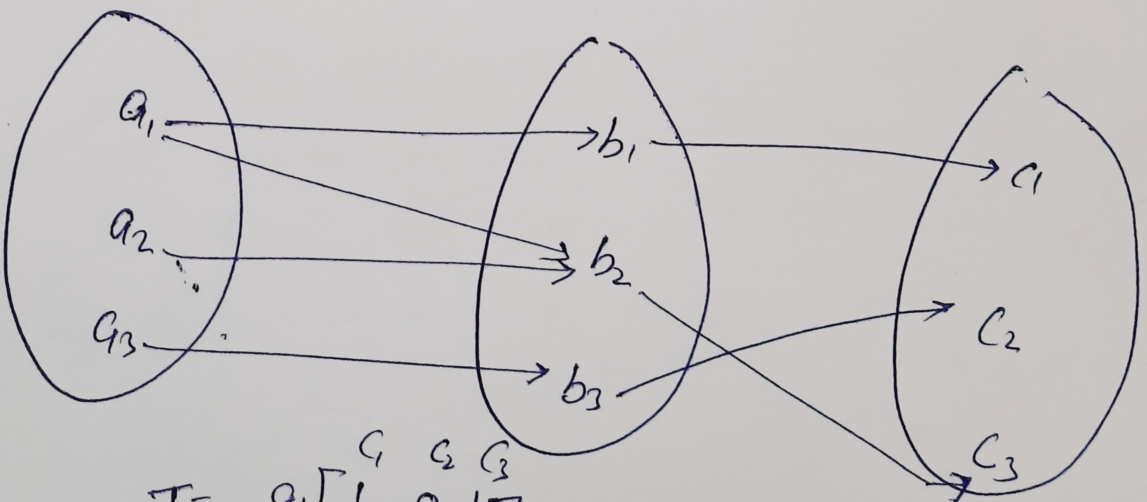
$R = X \times Y = \{(a_1,b_1)\ (a_1,b_2)\ (a_2,b_2)\ (a_3,b_3)\}$

$S = Y \times Z = \{(b_1,c_1)\ (b_2,c_3)\ (b_3,c_2)\}$

$T = R \circ S = \{(a_1,c_1)\ (a_2,c_3)\ (a_3,c_2)\ (a_1,c_3)\}$

Matrix Representation of R & S

$$R = \begin{array}{c|ccc} & b_1 & b_2 & b_3 \\ \hline a_1 & 1 & 1 & 0 \\ a_2 & 0 & 1 & 0 \\ a_3 & 0 & 0 & 1 \end{array} \qquad S = \begin{array}{c|ccc} & c_1 & c_2 & c_3 \\ \hline b_1 & 1 & 0 & 0 \\ b_2 & 0 & 0 & 1 \\ b_3 & 0 & 1 & 0 \end{array}$$



$$T = \begin{array}{c|ccc} & c_1 & c_2 & c_3 \\ \hline a_1 & 1 & 0 & 1 \\ a_2 & 0 & 0 & 1 \\ a_3 & 0 & 1 & 0 \end{array}$$

$T = R \circ S = \{(a_1 c_1)\ (a_2 c_3)\ (a_3 c_2)\ (a_1 c_3)\}$

The Composition operation are of two types

1) Max-Min Composition

2) Max-PRODUCT Composition

1. $T = R \circ S$

$$\chi_T(x,z) = \bigvee_{y \in Y} [\chi_R(x,y) \wedge \chi_S(y,z)]$$

2. <u>MAX PRODUCT COMPOSITION</u>:

$T = R \circ S$

$$\chi_T(x,z) = \bigvee_{y \in Y} [\chi_R(x,y) \cdot \chi_S(y,z)]$$

Fuzzy & CRISP RELATIONS

1 CRISP RELATIONS:

* Cartesian product of crisp set of two set A & B denoted by $A \times B$

$$A \times B = \{(a,b) \mid a \in A,\ b \in B\}$$

For eg. $A = \{a, b, c\}$ $B = \{1, 2\}$

$A \times B = \{(a,1)\ (a,2)\ (b,1)\ (b,2)\ (c,1)\ (c,2)\}$

$B \times A = \{(1,a)\ (2,a)\ (1,b)\ (2,b)\ (1,c)\ (2,c)\}$

⇒ Consider the element defined in the universes X & Y

$X = \{2, 4, 6\}$

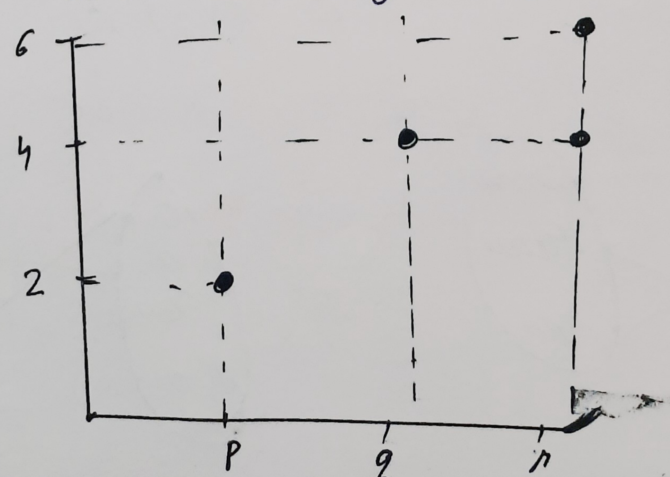
$Y = \{p, q, r\}$

$\cancel{X} \times \cancel{X} = \{(p,2)\ (p,4)\ (p,6)\ (q,2)\ (q,4)\ (q,6)\ (r,2)\ (r,4)\ (r,6)\}$

from this set one may select a subset such that

$$R = \{(p,2)\ (q,4)\ (r,4)\ (r,6)\}$$

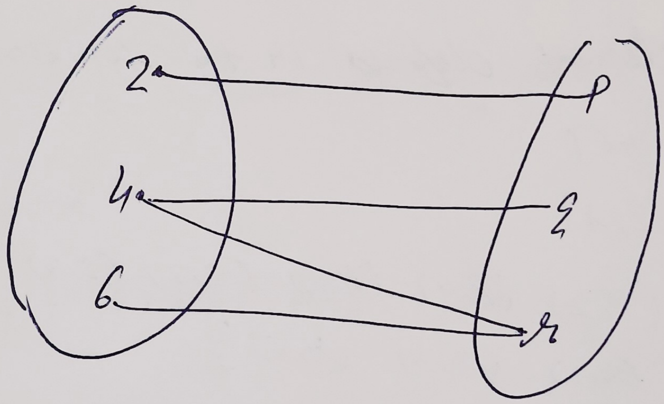
Subset R can be represented using a Co-ordinate diagram as shown in fig 8-1



* The Relation could be represented using a matrix

R	p	q	r
2	1	0	0
4	0	1	1
6	0	0	1

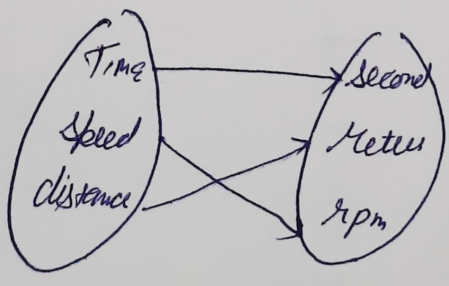
* The Relation could be Expressed by Mapping Representation.

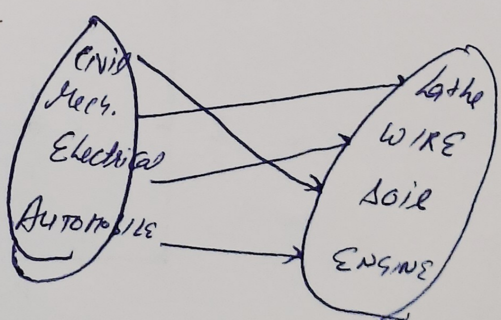


It can be Represented as

$R : Y \times X$

$$\chi_R(x,y) = \begin{cases} 1, & (x,y) \in R \\ 0 & (x,y) \notin R \end{cases}$$





Operations on FUZZY RELATION:

Let R & S be fuzzy Relation on the Cartesian space X × Y.

1 **Union:** $\mu_{R\cup S}(x,y) = \max[\mu_R(x,y), \mu_S(x,y)]$

2 **Intersection:** $\mu_{R\cap S}(x,y) = \min[\mu_R(x,y), \mu_S(x,y)]$

3 **Complement:** $\mu_{\bar{R}}(x,y) = 1 - \mu_R(x,y)$

4 **Containment:** $R \subset S \Rightarrow \mu_R(x,y) \leq \mu_S(x,y)$

5 **Inverse:** The Inverse of a fuzzy Relation R on X × Y

denoted by R^{-1}

$$R^{-1}(y,x) = R(x,y) \text{ for all pairs } (y,x) \in Y \times X$$

6 **PROJECTION** $(R \downarrow Y)$ denote the projection of R onto Y.

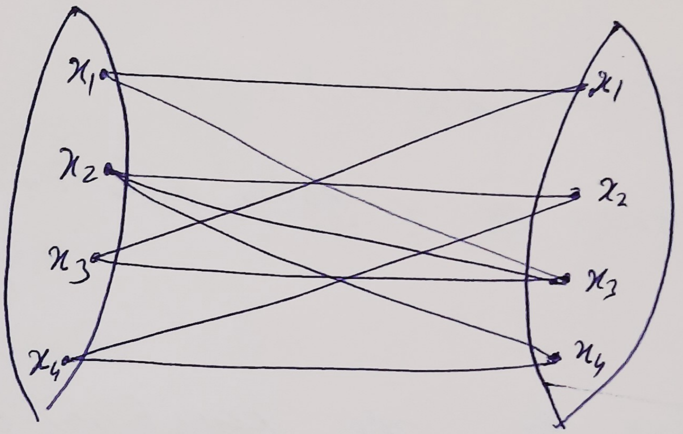
$$\mu_{[R\downarrow Y]}(x,y) = \max_x \mu_R(x,y)$$

The projection Concept can be extended to an n-ary relation $R(x_1, x_2 - x_n)$

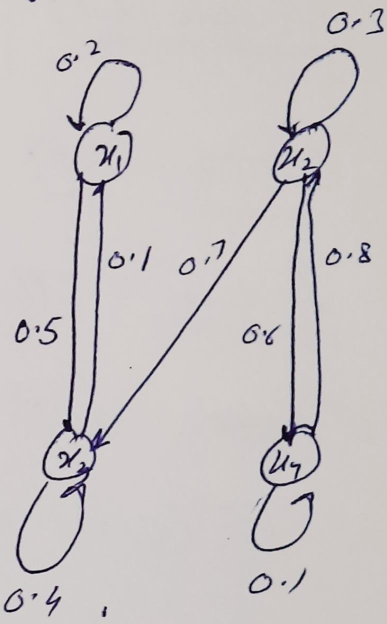
Consider a universe $X = \{x_1, x_2, x_3, x_4\}$ & the binary fuzzy relation on X as

$$R(X X) = \begin{array}{c c} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} 0.2 & 0 & 0.5 & 0 \\ 0 & 0.3 & 0.7 & 0.8 \\ 0.1 & 0 & 0.4 & 0 \\ 0 & 0.6 & 0 & 1 \end{bmatrix} \end{array}$$

The bipartite graph & simple fuzzy graph of R(X X)



Graphical Representation of fuzzy Relation i) Bipartite graph



ii) Simple fuzzy graph

Cardinality of fuzzy RELATION:

* Cardinality of fuzzy set on any universe is infinity. Hence the Cardinality of a fuzzy Relation between two or more universe is also Infinity

PROPETIES of FUZZY RELATION:

Commutativity, Associativity, distributivity, Idempotency & Identity hold good for fuzzy Relation. De. Morgan's Law hold Good for fuzzy Relation

* Excluded Middle Law & Law of Contradiction doesn't hold Good.

$$R \cap \bar{R} \neq \emptyset$$
$$R \cup \bar{R} \neq E \text{ (Whole set)}$$

FUZZY COMPOSITION: Let R be fuzzy Relation on Cartesian Space X × Y. S be a fuzzy Relation on Cartesian Space Y × Z.

Types of Composition

1 **Max-Min Composition:** $R(X,Y) \circ S(Y,Z) \Rightarrow T(XZ)$

$$\mu_T(x,z) = \mu_{R\circ S}(x,z) = \max_{y \in Y} \{\min[\mu_R(x,y), \mu_S(y,z)]\}$$

$$= \bigvee_{y \in Y} [\mu_R(x,y) \wedge \mu_S(y,z)] \quad \forall x \in X \quad z \in Z$$

2 **Min-Max Composition:**

$$\mu_T(x,z) = \mu_{R\circ S}(x,z) = \min_{y \in Y} [\max[\mu_R(x,y), \mu_S(y,z)]]$$

$$= \bigwedge_{y \in Y} [\mu_R(x,y) \vee \mu_S(y,z)] \quad \forall x \in X, z \in Z$$

3 MAX-PRODUCT COMPOSITION:

The Max-Product Composition of $R(X,Y)$ & $S(Y,Z)$ denoted by $R(X,Y) \cdot S(Y,Z) \Rightarrow T(X,Z)$

$$\mu_T(x,z) = \mu_{R \cdot S}(x,z) = \max_{y \in Y} [\mu_R(x,y) \cdot \mu_S(y,z)]$$

$$= \bigvee_{y \in Y} [\mu_R(x,y) \cdot \mu_S(y,z)]$$

The Properties of fuzzy Composition

$$R o S \neq S o R$$

$$(R o S)^{-1} = S^{-1} o R^{-1}$$

$$(R o S) o M = R o (S o M$$

fuzzy Cartesian Product:

$A \times B = R \qquad R \subset X \times Y$

The Membership function of fuzzy relation

$$\mu_R(x,y) = \mu_{A \times B}(x,y) = \min [\mu_A(x), \mu_B(y)]$$

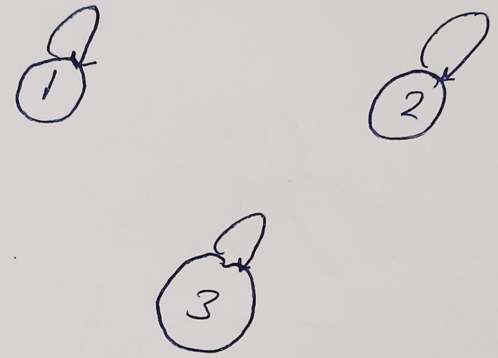
TOLERENCE & EQUIVALANCE RELATION

Consider the simple graph describe Three element in universe of discourse WHICH are labled as the vertices.

$$X = \{1, 2, 3\}$$

The Three characteristics properties of a Relation is discuss below.

1) Reflexivity



A Relation R on a universe X can also be thought of as a Relation X to X

The Relation is equivalance Relation if it hold the three property Reflexivity, Symmetry & Transitivity

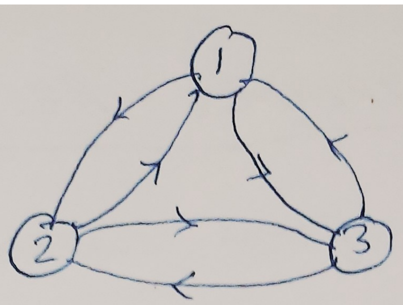
$$x_i x_i \in R \quad \text{or} \quad \chi_R(x_i, x_i) = 1$$

2 Symmetry:

$$\cancel{\chi_R(x_i, x_j) = \chi_R(x_j, x_i) \text{ or } (x_i, x_j) \in R}$$

or

$$\chi_R(x_i, x_j) = \chi_R(x_j, x_i) \text{ ie}$$
$$(x_i, x_j) \in R \Rightarrow (x_j, x_i) \in R$$

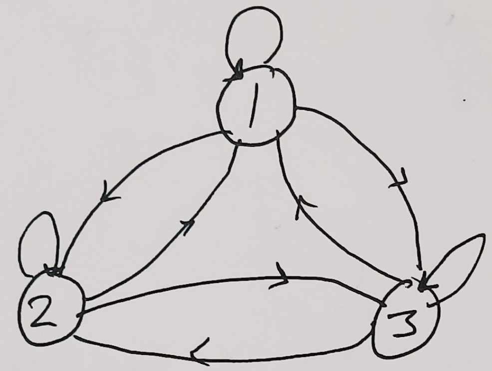


3 <u>Transitivity</u>:

$$\chi_R(x_i, x_j) \text{ and } \chi_R(x_j, x_k) = 1$$

so $\chi_R(x_i, x_k) = 1$

ie $(x_i, x_j) \in R$ $(x_j, x_k) \in R$ so $(x_i, x_k) \in R$



Classical Tolerance Relation:

A Tolerence relation R (also Called Proximity relation) on a universe X is a relation that exhibits only the properties of <u>reflexivity</u> and <u>symmetry</u>

A Relation R, Can be reformed into an equivalence Relation by at most (n-1) Compositions with itself. Where n is the Cardinal number of the set defining R

Q Suppose in an airline transportation system we have a universe composed of five elements: the cities Omaha, Chicago, Rome, London, Detroit. The airline is studying location of potential hubs in various countries.

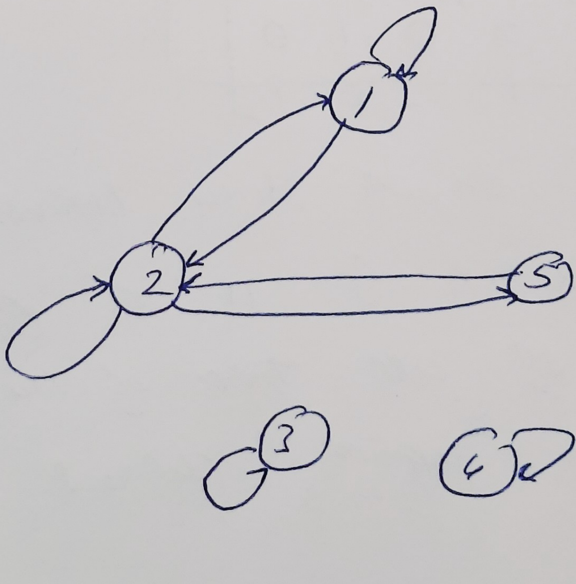
$$X = [x_1, x_2\ x_3\ x_4\ x_5] = [\text{Omaha, Chicago, Rome, London, Detroit}]$$

Suppose we have a Tolerance Relation R,

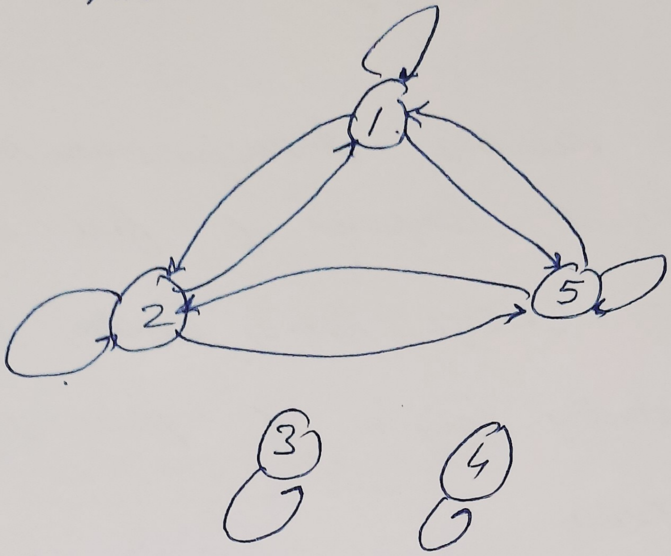
$$R_1 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

This Relation is reflexive → all diagonal are one

& symmetric → Transpose = Actual Relation



Five Vertex Equivalence Relation



$$R_1 \circ R_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Fuzzy TOLERANCE & EQUIVALENCE RELATIONS

$$R_1 \circ R_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} = R$$

A fuzzy Relation R, on a single universe X is also a Relation from X to X. It is a fuzzy equivalence relation if all three of the following properties for Matrix relation define it

1 Reflexivity $\mu_R(x_i, x_i) = 1$

2 Symmetry $\mu_{\underset{\sim}{R}}(x_i\ x_j) = \mu_{\underset{\sim}{R}}(x_j, x_i)$

3 Transitivity $\mu_R(x_i\ x_j) = \lambda_1 \quad \mu_R(x_j, x_k) = \lambda_2$

$$\mu_R(x_i, x_k) = \lambda$$

Where $\lambda \geq \min(\lambda_1, \lambda_2)$

FUZZY TOLERENCE RELATION: fuzzy tolerence Relation $\underset{\sim}{R_1}$ has the properties of reflexivity and Symmetry & Can be reformed into fuzzy equivalence relation by at most $(n-1)$ Composition

$$R_1^{(n-1)} = R_1 \circ R_1 \circ R_1 \cdots \circ R_1 = R$$