UNIT-4

Propositional logic

Lecture No-32

Statement A stmt is a declarative sentence which is true or false, but not both, In other words, a stmt is a declarative sentence which has a defined truth value.

Example: following are statements (or proposition)

		Truth Value
(i)	$\{x : x^2 = 36\} = \{6, -6\}$	True
(2)	Blood is red	True
(3)	$5+4 = 10$	false.

following are not statements –

(i) How are you (Interrogative)
(2) Please go from here (Request)
(3) May God help you. (Wish)

Statement variables: The symbols which are used to represent statements, are called statement letters or sentence variables.

logical Connectives

Rank	Connective Word	Name of Connective Symbol	Symbol
1	Not	Denial or Negation	$\sim$ or $\neg$
2	And	Conjunction	$\wedge$
3	OR	Disjunction	$\vee$
4	If then	conditional	$\rightarrow$ or $\Rightarrow$
5.	iff or if & only if	Biconditional	$\leftrightarrow$ or $\Leftrightarrow$

a is equal to 4 or b is equal 4.
In symbolic language it is written as -
Let $p \equiv$ a is equal to 4 , $q \equiv$ b is equal 4.
so $\boxed{p \vee q.}$

Kind of Sentences :-

Usually the sentences are of two kinds -
(1) simple or Atomic (2) Compound or Molecular.

(1) Simple Sentence :-
A simple sentence has no connective.
(2) Compound sentence
It is composed of various connectives.
In conditional sentence $p \to q$, p is called 'premise', 'antecedent' or hythesis.
q is called 'consequent' or 'conclusion'.

Truth Value of a statement

Acc. to definition of statement, a statement has a definite truth value which is either true or false.
for example -
15 is an odd no. (Its truth value is true)
9 is a prime no (Its truth value is false.

Open statement A statement, which contains one or more variable such that when certain values are substituted for variables, becomes an statemen[t] is called open statement.
Consider $x+3=9$. If we put 6 in place of x, then this sentence becomes a truth stmt.

Proposition

If p, q, r --- are simple statement, then the compound statement $P(p, q, r ---)$ is called a proposition.
The truth value of proposition P depends on the truth value of variables p, q, r ---.

Ex Find the truth table for the proposition $(\neg P \wedge q)$

P	q	$\neg P$	$\neg P \wedge q$
F	F	T	F
F	T	T	T
T	F	F	F
T	T	F	F

Well formed Formula (WFF)

A statement formula is a string consisting of variables, parentheses, or connective symbols. A statement formula is called wff if -

(i) A statement variable p alone is a wff.

(ii) If p is wff, then $(\sim p)$ is well formed formula.

(iii) If p & q are wff, then $(p \wedge q)$, $(p \vee q)$, $(p \to q)$ and $(p \leftrightarrow q)$ are wff.

(iv) A string of symbols is a wff if & only if it is obtained by finitely many applications of the rule (i), (ii) & (iii).

Que Write the following in symbols:-

(i) What so ever, I say, he refuses from that.

p = I say, q = he refuses from that. $p \wedge q$.

(ii) I shall reach the station in time, otherwise I shall miss the train.

p = I shall reach the station in time

q = I shall miss the train.

statement: $p \vee q$

(iii) I shall go to Delhi, but I shall not see zoo.

$p \equiv$ I shall go to Delhi

$q \equiv$ I shall see zoo

statement: $p \Rightarrow \sim q$

(iv) In the position when he is ill, I shall care

$p \equiv$ he is ill

$q \equiv$ I shall care

statement: $p \Rightarrow q$.

(v) Whenever I take leave, I fall ill.

$p \equiv$ I take leave

$q \equiv$ I fall ill.

statement: ~~p⇒q~~ $p \wedge q$

(vi) Until I shall not be called, till then, I will remain here.

$p \equiv$ I shall be called

$q \equiv$ I shall remain here

statement $\neg p \Rightarrow q$

(vii) Not only men, but also women & children were killed.

p = men were killed, $q \equiv$ women were killed

r = children were killed

statement: $p \wedge (q \wedge r)$

(viii) If he will do labour, he will success. If he will success, he will get pleasure. Thus by doing Labour one gets pleasure.

$p \equiv$ he will do Labour

$q \equiv$ he will success

$r \equiv$ he will get pleasure

Statement: $(p \Rightarrow q \wedge q \Rightarrow r) \Rightarrow (p \Rightarrow r)$

(ix) The necessary & sufficient condition for raining is that the sky should be cloudy.

$p \equiv$ raining $\quad q \equiv$ cloudy sky.

Statement: $p \Leftrightarrow q$

(x) If teams do not arrive or weather is bad, then there will be no match.

$p \equiv$ teams do not arrive

$q \equiv$ weather is bad

$r \equiv$ there will be no match.

$(p \vee q) \Rightarrow r$

Qu Whenever Ram & Shyam are present in the party then there is some trouble in the party. Today there is no trouble in the party. Hence Ram & shyam are not present in the party.

Write the above statment in symbolic notation.

$p \equiv$ Ram & shyam are present in party

$q \equiv$ To be trouble in party

statement is: $p \Rightarrow q, \sim q \Rightarrow \sim p$

$\{(p \Rightarrow q) \wedge (\sim q)\} \Rightarrow \sim p$

Que $p \equiv$ Romesh is player
$q \equiv$ Mohan is wise boy.
then write the following symbols in sentences.

(i) $\sim p \vee \sim q$ (ii) $\sim p \Leftrightarrow q$ (iii) $p \wedge \sim q$
(iv) $p \wedge q$ (v) $\sim(p \wedge q)$

(i) Neither Romesh is player nor Mohan is wise boy.
(ii) Romesh is not a player iff Mohan is a wise boy.
(iii) Romesh is player & mohan is not wise boy.
(iv) Romesh is player & Mohan is wise boy.
(v) It is not true that Romesh is player & Mohan is a wise boy.

Que $p \equiv$ it is cold $q \equiv$ It is raining.

(i) $\sim\sim p$
It is not true that it is cold.

(ii) $(p \wedge \sim q) \Rightarrow q$
If it is cold & ~~it is~~ not raining, then it is raining

(iii) $q \vee \sim p$
It is raining or it is not cold.

Que

Which of the following are statements? Also state their truth values -

(i) Is 3 a prime No? Not the statement [Interrogative]

(ii) $x^2 - 5x + 6 = 0$ Not the statement since truth value depends on value of x.

(iii) There will be snow in December. statement [since it can be judged to be true or false]

(iv) Give me 10 Rs. [Not statement]

(v) Ramesh is poor but honest. (statement)

It can be judged to be true or false.

It is true if Ramesh is poor & honest.

& false if Ramesh is poor but not honest.

Truth Tables

1. Conjunction ($\wedge$)

Suppose p & q are two statements. When two stmt are combined by using word "and" then $p \wedge q$ is conjunction.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(ii) Disjunction

suppose p & q are two statements. When they are joined by using the word "or" then we get a new statement represented by $p \vee q$.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(iii) Negation ($\sim$) or ($\neg$) Not

p	$\sim p$
T	F
F	T

(iv) Conditional statement

Let p & q be any two statements. The statement of the type "if p then q" is called conditional stmt & is represented by "$p \to q$".

It is read as $p \to q$
p is called premise/antecedent
q is called consequent.

We observe that the conditional statement is false only in condition 'when antecedent is true & consequent is false'

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$\boxed{\text{NOTE } p \to q \text{ is logically equivalent to } \neg p \vee q}$

(v) Bi-conditional ($\leftrightarrow$)

Let p & q be two statements.
The statement of type $p \to q$ and $q \to p$

i.e. $(p \to q) \wedge (q \to p)$ is called biconditional statement, also written as $p \leftrightarrow q$ or $p \Leftrightarrow q$

Biconditional stmt is also read as
"p if & only if q" or "p iff q"

Biconditional statement is true only in those cases when its both components have same truth value.

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Kinds of Conditional

Conditional	Name of kind
$p \to q$	Direct Implication
$q \to p$	Converse Implication
$\sim p \to \sim q$	Inverse Implication
$\sim q \to \sim p$	Contra-positive Implication.

Que Consider the conditional statement p.

If the floods destroy my house or fires destroy my house, then my insurance company will pay me.

Write converse, inverse & contrapositive of statement

Sol^n $p \equiv$ The flood destroy my house

$q \equiv$ The fires destroy my house.

$r \equiv$ My insurance company will pay me.

$(p \vee q) \Rightarrow r$

Its inverse is

$\sim(p \vee q) \Rightarrow \sim r$ can be written $\sim p \wedge \sim q \Rightarrow \sim r$

If the flood doesnot destroy my house and fire doesn't destroy my house, then my insurance company will not pay me.

Converse is : $r \rightarrow p \vee q$

If my Insurance company pay me then the flood will destroy my house or fire will destroy my house.

Contrapositive is : $\sim r \rightarrow \sim(p \vee q)$

$\sim r \rightarrow \sim p \wedge \sim q$

If my insurance company doesn't pay me, then the flood will not destroy my house and fire will not destay my house.

Que The converse of a statement is given. Write inverse & contrapositive statements

"If I come early, then I can get car".

Solⁿ "If I cannot get the car, then I shall not come early". (Inverse)

"If I don't come early, then I can not get the car". (Contrapositive)

Tautology

A tautology is a proposition which is true for all truth values of its sub propositions or components.

"A tautology is also called logically valid or logically true."

Ex $T_{p(n)} = \{a \in U : p(a) \text{ is true}\}$

then prove that $T_{p \leftrightarrow q} = (T_p \wedge T_q) \vee (T_p^c \wedge T_q^c)$

and $T_{p \to q} = T_p^c \vee T_q$

Soln (i) $T_{p \leftrightarrow q} = (T_p \wedge T_q) \vee (T_p^c \wedge T_q^c)$

T_p	T_q	$T_{p \leftrightarrow q}$	$T_p \wedge T_q$	T_p^c	T_q^c	$T_p^c \wedge T_p^c$	$(T_p \wedge T_q) \vee (T_p^c \wedge T_q^c)$
T	T	T	T	F	F	F	T
T	F	F	F	F	T	F	F
F	T	F	F	T	F	F	F
F	F	T	F	T	T	T	T

from above table we find that

$$T_{p \leftrightarrow q} = (T_p \wedge T_q) \vee (T_p^c \wedge T_q^c)$$

(ii) $T_{p \to q} = T_p^c \vee T_q$

T_p	T_q	$T_{p \to q}$	T_p^c	$T_p^c \vee T_q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

from table. $T_{p \to q} = T_p^c \vee T_q$ Proved

Contrapositive

Ques Prove that $(p \leftrightarrow q) \wedge (q \leftrightarrow r) \rightarrow (p \leftrightarrow r)$ is a tautology.

Solⁿ For convenience, let $p \leftrightarrow r = A$ and $(p \leftrightarrow q) \wedge (q \leftrightarrow r) = B$.

p	q	r	$p \leftrightarrow q$	$q \leftrightarrow r$	$p \leftrightarrow r = A$	$(p \leftrightarrow q) \wedge (q \leftrightarrow r) = B$	$B \rightarrow A$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	T	F	T
T	F	F	F	T	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	F	T	F	T
F	F	T	T	F	F	F	T
F	F	F	T	T	T	T	T

Que Show that the truth value of the following formulas are independent of their components.

(i) $(p \wedge (p \to q)) \to q$

p	q	$p \to q$	$p \wedge (p \to q) = A$	$A \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(ii) $(p \to q) \Leftrightarrow (\sim p \vee q)$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$	$(p \to q) \Leftrightarrow (\sim p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Contradiction :-

A contradiction is a proposition which is always false for all truth value of its propositions or components.

Ex Prove that $p \wedge (\sim p)$ is a contradiction.

p	$\sim p$	$p \wedge (\sim p)$
T	F	F
F	T	F

Since all entries in the column of $p \wedge (\sim p)$ are of F only, its a contradiction.

Que Prove that $(p \vee q) \wedge (\sim p) \wedge (\sim q)$ is a contradiction.

Soln

p	q	$p \vee q$	$\sim p$	$\sim q$	$(p \vee q) \wedge (\sim p)$	$(p \vee q) \wedge (\sim p) \wedge (\sim q)$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	T	F
F	F	F	T	T	F	F

Since, all entries in the last column are of 'F' & so it is a contradiction.

Algebra of Proposition :-

1. Idempotent law

$p \vee p \leftrightarrow p$ (i) $p \vee p = p$

$p \wedge p \leftrightarrow p$ (ii) $p \wedge p = p$

2. Commutative law

(i) $p \vee q = q \vee p$ (ii) $p \wedge q = q \wedge p$

3. Associative law

(i) $(p \vee q) \vee r \leftrightarrow p \vee (q \vee r)$ or $(p \vee q) \vee r = p \vee (q \vee r)$

(ii) $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

4. Distributive law

(i) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

(ii) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

5. De-Morgan's law

(i) $\sim(p \vee q) = (\sim p) \wedge (\sim q)$

(ii) $\sim(p \wedge q) = (\sim p) \vee (\sim q)$

6. Identity law

(i) $p \wedge t = t \wedge p = p$

(ii) $p \vee f = f \vee p = p$

7. Complement law

(i) $p \vee (\sim p) = t$ (ii) $p \wedge (\sim p) = f$

8. Absorption law

(i) $p \vee (p \wedge q) = p$ (ii) $p \wedge (p \vee q) = p$

9. Null or Dominance law

(i) $p \wedge F = F = F \wedge p$

(ii) $p \vee T = T = T \vee p$

10. <u>Involution laws</u>

$$\sim(\sim p) = p$$

11. <u>Conditional Rules</u>

$$p \to q = (\sim p \vee q)$$

12. <u>Contrapositive Law</u>

$$(p \to q) = (\sim q) \to (\sim p)$$

13. <u>Biconditional or Equivalence Rules</u>

(i) $(p \to q) \wedge (q \to p) = p \leftrightarrow q$

(ii) $p \leftrightarrow q = (p \wedge q) \vee [(\sim p) \wedge (\sim q)]$

14. <u>Exponential Law</u>

$$(p \wedge q) \to r = p \to (q \to r)$$

15. <u>Absurdity law</u>

$$(p \to q) \wedge (p \to \sim q) = (\sim p)$$

Que UPTU Prepare the truth table of the statement -

$$A = (p \to q \wedge r) \vee (\sim p \wedge q)$$

or

Find a formula A, that uses the variable p, q & r such that A is contradiction.

Solⁿ $(p \to q \wedge r) \vee (\sim p \wedge q)$

p	q	r	$q \wedge r$	$p \to (q \wedge r)$	$\sim p$	$\sim p \wedge q$	$(p \to q \wedge r) \vee (\sim p \wedge q) = A$
T	T	T	T	T	F	F	T
T	T	F	F	F	F	F	F
T	F	T	F	F	F	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

Que show that $((p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))) \vee (\sim p \wedge \sim q) \vee (\sim p \vee r)$ is a tautology without using truth table.

Soln

$$\equiv ((p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))) \vee (\sim p \wedge \sim q) \vee (\sim(p \vee r)$$

$$\equiv ((p \vee q) \wedge \sim(\sim p \wedge \sim(q \wedge r)) \vee (\sim(p \vee q) \vee \sim(p \vee r))$$

$$\equiv ((p \vee q) \wedge (p \vee (q \wedge r))) \vee \sim((p \vee q) \wedge (p \vee r))$$

$$((p \vee q) \wedge ((p \vee q) \wedge (p \vee r))) \vee \sim((p \vee q) \wedge (p \vee r))$$

$$\equiv (p \vee q) \wedge (p \vee q) \wedge (p \vee r) \vee \sim((p \vee q) \wedge (p \vee r))$$

$$\equiv \underbrace{[(p \vee q) \wedge (p \vee r)]}_{x} \vee \underbrace{\sim((p \vee q) \wedge (p \vee r))}_{\sim x}$$

$$\equiv$$

$$\equiv x \vee \sim x$$

$$\equiv T$$

Hence it is tautology

Argument :- An argument is a process which yields a conclusion, from a given set of propositions called premises.

Let premises be $P_1, P_2 -- P_n$ & let argument yield the conclusion q, then such an argument is denoted by -

$$P_1, P_2 --- P_n \vdash q$$

Valid Argument

An argument $P_1, P_2 -- P_n \vdash q$ is called valid if q is true whenever all its premises $P_1, P_2 --- P_n$ are true.

An argument is called valid if & only if the premises implies the conclusion.

Thus the argument $P_1, P, P_3 --- P_n \vdash q$ is said to be valid iff the statement

$$(P_1 \wedge P_2 \wedge P_3 ---- \wedge P_n) \vdash q \text{ is tautology.}$$

Representation of an Argument

An argument $P_1, P_2, -- P_n \vdash q$ is written as

$$\left.\begin{array}{l} P_1 \\ P_2 \\ \vdots \\ P_n \end{array}\right\} \text{Premises}$$
$$\overline{}$$
q (conclusion)

Que Show that the following argument is valid.

$$\begin{array}{c} p \vee q \\ \sim p \\ \hline q \end{array}$$

Sol^n We need to prove that $[(p \vee q) \wedge \sim p] \to q$ is tautology.

p	q	$\sim p$	$p \vee q$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Que Test the validity of following argument :-

If a man is bachelor, he is worried (premise)

If a man is worried, he dies young. (premise)

Bachelor dies young. (conclusion)

Sol^n

$$\begin{array}{c} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

The given argument is true by Law of syllogism

Que Test the validity of following argument

"If is rains then it will be cold. If it is cold then I shall stay at home. Since it rains, therefore I shall stay at home".

Soln

$p \to q$

$q \to r$

p

---

r

p = it rains, q = it will be cold.

r = I shall stay at home.

statement is given as -

$((p \to q) \wedge (q \to r) \wedge p) \to r$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$(p \to q) \wedge (q \to r) \wedge p = A$	$A \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

Since last column contains only T, hence the given argument is valid.

Que Test the validity of argument :-

If two sides of a triangle are equal, then opposite angles are equal.

Two sides of a triangle are not equal

∴ The opposite ~~side~~ angles are not equal

Soln

$p \to q$ (a premise)

$\sim p$ (a premise)

$\sim q$ (conclusion

$([p \to q] \wedge \sim p) \to \sim q$

p	q	$p \to q$	$\sim p$	$\sim q$	$(p \to q) \wedge \sim p$	$((p \to q) \wedge \sim p) \to \sim q$
T	T	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	T	F	T	F
F	F	T	T	T	T	T

The last column is not tautology. Hence the given argument is not valid.

Rules of Inferences :-

Rules of Inference	Tautology	Name
1. $\dfrac{P}{\therefore P \vee Q}$	$P \to P \vee Q$	Addition
2. $\dfrac{P \wedge Q}{\therefore P}$	$P \wedge Q \to P$	Simplification.
3. $\dfrac{\begin{matrix} P \\ Q \end{matrix}}{\therefore P \wedge Q}$	$(P) \wedge (Q) \to P \wedge Q$	Conjunction
4. $\dfrac{\begin{matrix} P \\ P \to Q \end{matrix}}{Q}$	$[P \wedge (P \to Q)] \to Q$	Modus Ponens. (law of detachment)
5. $\dfrac{\begin{matrix} \neg Q \\ P \to Q \end{matrix}}{\neg P}$	$[(\neg Q) \wedge (P \to Q)] \to \neg P$	Modus Tollen - (law of contraposition)
6. $\dfrac{\begin{matrix} P \to Q \\ Q \to R \end{matrix}}{P \to R}$	$[(P \to Q) \wedge (Q \to R)] \to (P \to R)$	Hypothetical syllogism.
7. $\dfrac{\begin{matrix} P \vee Q \\ \neg P \end{matrix}}{Q}$	$[(P \vee Q) \wedge (\neg P)] \to Q$	Disjunctive syllogism.
8. $\dfrac{\begin{matrix} P \vee Q \\ \neg P \vee R \end{matrix}}{Q \vee R}$	$[(P \vee Q) \wedge (\neg P \vee R)] \to (Q \vee R)$	Resolution
9. $\dfrac{\begin{matrix} (p \to q) \wedge (r \to s) \\ p \vee r \end{matrix}}{\therefore q \vee s}$	$(p \vee q) \wedge (r \to s)$	Constructive Dilemma.

10. $$\frac{\begin{array}{c}(p \to q) \wedge (r \to s) \\ \sim q \wedge s\end{array}}{\sim p \wedge r}$$ Destructive Dilemma

11. $$\frac{p \to q}{\therefore\ p \to (p \wedge q)}$$ Absorption.

Que Show that argument

$p, p \to q, q \to r \vdash r$ is valid.

1. p (premise given)
2. $p \to q$ (premise given)
3. $q \to r$ (premise given)
4. $p \to r$ (using 2 & 3) Hypothetical syllogism
5. r (using 1 & 4) Modus Ponen.

step 5 (conclusion)

so it is valid.

Que Test the validity of argument.

If 8 is even then 2 does not divide 9. Either 7 is not prime or 2 divides 9. But 7 is prime, therefore 8 is odd.

Sol^n Let p: 8 is even
q 2 divies 9.
r 7 is prime

1. $p \to \neg q$ (Given premise)
2. $\neg r \vee q$ (Given premise)
3. r (Given premise)

4. $r \wedge (\neg r \vee q)$ Conjunction using (2 & 3)
$(r \wedge \neg r) \vee q$
$F \vee q \cong q$

5. $\neg p$ Modus Tollen using (1 & 4)

(Hence conclusion)

Que "It's not sunny this afternoon & it is colder than yestody We will go swimming only if it is sunny. If we donot go for swimming, than we will take a trip, and If we take a trip, then we will be at home by sunset' leads to conclusion 'We will be home by sunset.

Sol^n $p \equiv$ It is sunny this afternoon
$q \equiv$ it is colder than yesterday.
$r \equiv$ we will go for swimming
$s \equiv$ we will take a trip
$t \equiv$ We will be at home by sunset.

$\neg p \wedge q, \quad r \rightarrow p, \quad \neg r \rightarrow s, \quad s \rightarrow t \vdash t$

1. $\neg p \wedge q$ (Given Premise)
2. $\neg p$ (simplification using 1)
3. $r \rightarrow p$ (Given premise)
4. $\neg r$ Modus tollen using 2 & 3
5. $\neg r \rightarrow s$ (Given premise)
6. s Modus ponens using 4 & 5.
7. $s \rightarrow t$ (Given Premise)
8. t Modus ponens using step 6 & 7.

Predicate Logic

Predicate or Proposition funⁿ or open sentences

let A be a given set. An expression denoted by $P(x)$ is called propositional funⁿ or simply an open sentence on A if $P(a)$ is true or false for each $a \in A$ ie $P(a)$ has a truth value for each $a \in A$.

In other words, $P(x)$ is called propositional funⁿ or open sentence if $p(x)$ becomes a statement whenever any element $a \in A$ is substituted for the variable x.

Ex let $P(x)$ is $x+4<9$

then $P(x)$ is a propositional funⁿ on the set of natural numbers N.

Clearly $P(x)$ is true for $n = 1, 2, 3, 4$ & false for $x = 5, 6, 7$ --- Hence $P(x)$ becomes a statement whenever any element $a \in N$ is substitute for x.

Truth Set Definition

let $P(x)$ be a propositional funⁿ & D be its domain. The set of elements $d \in D$ with the property that $P(d)$ is true is called the Truth set $T(P)$ of $P(x)$.

symbolically :-

$$T(P) = \{x : x \in D,\ P(x) \text{ is true}\} \text{ or}$$

$$T(P) = \{x : P(x)\}$$

Ex If $x+3>6$ be a propositional funⁿ defined on N. then find truth set of $P(x)$.

We have $P(x) \equiv 'x+3>6'$, $D \equiv N$

$T(P) = \{x : x+3>6, x \in N\} = \{4, 5, 6 \ldots\}$

Ex If $P(x)$ be $x>2$ & $D \equiv \{3, 4, 5 \ldots\}$ then

$T(P) = \{3, 4, 5 \ldots\} = D$ ie

$P(x)$ is true for every $x \in D$.

Quantifiers

The restrictions namely 'for every' & 'for some' are called quantifiers.

There are two types of quantifiers.

1) Universal Quantifier 2) Existential Quantifiers.

1) Universal Quantifier :-

The symbol $\forall$ which is read as "for every" or 'for all' is called universal quantifier.

Let $P(x)$ be a propositional funⁿ defined on set D then the if for every $x \in D$, $P(x)$ is true statement then the use of universal quantifier ($\forall$) is written as -

$$(\forall x \in D)\ P(x)$$

or

$$\forall x\ P(x) \quad \text{or} \quad \forall x,\ P(x)$$

Ex The statement $\forall n \in N \quad n+2>1$ is true since its Truth set

$$T(p) = \{n : n \in N,\ n+2>1\} = \{1, 2, 3 \ldots\} = N$$

2) Existential Quantifiers

The symbol $\exists$ which is read as "There Exists" or "for some" or "for at least one" is called the Existential Quantifier.

Let $P(x)$ be a propositional funⁿ defined on set D. If there exists an $x \in D$, such that $P(x)$ is true or for some $x \in D$, $P(x)$ is true statement.

$$(\exists x \in D)\, P(x) \quad \text{or} \quad \exists x\, P(x)$$

Hence, we find that truth set is not the empty set

$$T(P) = \{x : x \in D,\ P(x)\} \neq \phi$$

Negation of a Quantifier

$\forall$ let us consider proposition 'All Indians are honest'. If M denotes the set of Indians, then it can be written as -

$$\forall x \in M \ (x \text{ is honest})$$

This stmt will become false, if we say that, 'there is an Indian, who is not honest', or in symbolic form -

$$\exists x \in M \quad (x \text{ is not honest})$$

Therefore Negation of the given statement will be 'There exist an Indian, who is not honest'

Que Translate the following statements given in English into equivalent statement of Propositional / Predicate Calculas.

(1) Some pet dogs are dangerous.

$P(x)$: x is pet dog $D(x)$: x is dangerous.

$$\exists x \{P(x) \rightarrow D(x)\}$$

2) Some physicists are not good in chemistry

Let $P(x)$: x is physicist & $C(x)$: x is not good in chemistry

$$\exists x \{P(x) \rightarrow C(x)\}$$

3) Some patient like all doctors.

$P(x)$: x is patient $D(y)$: y is doctor

$L(x, y)$: x likes y.

$\therefore \exists x \; P(x) \cancel{L}(x, y)$ or $\forall y, \exists x \; (P(x) \rightarrow L(x, y))$

4) Some cats are black but all bufallos are black.

Let $C(x)$: x is cat $B(x)$: x is black $BF(x)$: x is bufallos

$$\exists x \in C(x) \rightarrow B(x)$$

and $\forall x \; BF(x) \rightarrow B(x)$

5) Sum of two positive Integers is greater than either of the integers.

Let $I(x)$: x is '+ve' integer $GT(x, y)$: x is greater than y

$Su(x, y)$: sum of x & y.

In Predicate calculas we have -

$$\forall x \forall y \; (I(x) \; I(y)) (GT(Su(x, y), y)$$

Que Write the following predicate into symbols & also write its negative in symbol.

"Every rational no. is a real no".

Sol^n Let $Q(x) \equiv x$ is rational no.

$R(x) \equiv x$ is real no.

$P(x) \equiv Q(x) \rightarrow R(x)$

Symbolic form of predicate is $\forall x \in P(x)$

Negative $\exists x \in [\neg P(x)]$

ie $\exists x [\neg Q(x) \rightarrow R(x)]$

Que Write the following predicates into symbolic form

(i) All ~~me~~ are not mortal

$\sim \forall x [M(x)]$

$M(x) \equiv x$ is mortal.

(ii) All are immortal.

$\forall x [\sim M(x)]$

(iii) Some people like to listen only instrumental music.

$P(x) \equiv x$ is people.

$I(x) \equiv x$ is instrumental music

$H(x, y) \equiv x$ likes to listen y.

$\exists x [P(x) \wedge H(x, y)] \rightarrow I(y)]$

(iv) All student are not wise.

$\sim \forall x [S(x) \wedge W(x)]$

$S(x) \equiv x$ is student, $W(x) \equiv x$ is wise.

Contingency: A proposition which is neither tautology nor contraction is called contingency. Here the last column of truth table contains both T & F.

Satisfiability :- A compount statement formula $A(P_1, P_2 .. P_n)$ is said to be satisfiable, if it has the truth value T for at least one combination of truth value of $P_1, P_2 ... P_n$.