UNIT-1 MECHANICS of SOLID & BEAM

LECTURE: 01

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Body:

Particle: A particle is an object that has infinite small volume (occupies negligible space) but has a mass which can be considered to be concentrated at a point.

Body: A body has a definite shape and consists of number of particles.

Rigid body: A body which does not undergo any change in its dimensions under the action of external load is called a rigid body.

Practically, no body is perfectly rigid but assumed to be rigid for engineering application.

Elastic body: A body regains its original dimensions when the external load acting on it is completly removed is called as elastic body.

Ex- Mild steel, copper, aluminium, brass etc.

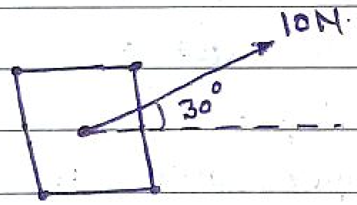
Plastic body: A body which gets permanent deformed under the action of load and not coming back to its original dimensions on removal of load is called as plastic body.

Types of body

- Rigid body
- Deformable body
  - Elastic body
  - Plastic body

FORCE: It is define as an external agency which tends to change the speed or direction of a system.
Force is a vector quantity and S.I unit of force is Newton (N).

Characteristics of force:
Force is characterized by following properties
(I) Magnitude: This represent the value of force ie 10 N, 20N.
(ii) Direction: It is represented by line of action and the angle it forms with some fixed axis.
(iii) Nature: Nature of force is represented by arrowhead. Generally it termed as push or pull.
(iv) Point of application: The point at which the force acts is known as point of application.



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Magnitude: 10 N.
Direction. 30° from x axis
Nature: Pull (Tensile) outwards to the centre (+ve)
Point of application: Centre of body.

Types of Load

- Direct Load or Axial load
  - Tensile load
  - Compressive load
- Ecentric load or Non-axial load
- shear load

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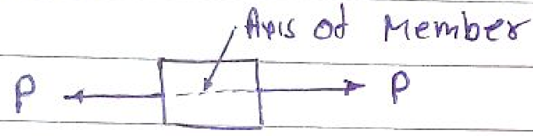
1- Direct load or axial load:

When a load whose line of axis coincides with the axis of a body, then it is called as direct load or axial load.

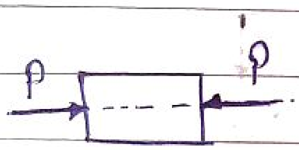
Direct load always acts at the centroid of a member.

It is of two types.

(i) Tensile force: When two equal and opposite pull is axially applied on a body then it is called as tensile force.

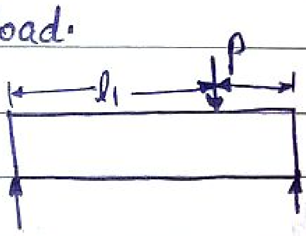


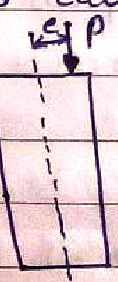
(ii) Compressive force: When two equal and opposite push is axially applied on a body, then it is called as compressive force.



2- Ecentric load:

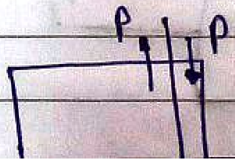
When a load whose line of action do not coincide with the axis of the body then it is called as an ecentric load.





3- Shear force: A force which acts tangential to the plane under consideration, is called shear force.

When a force acts along the plane or area of a body.

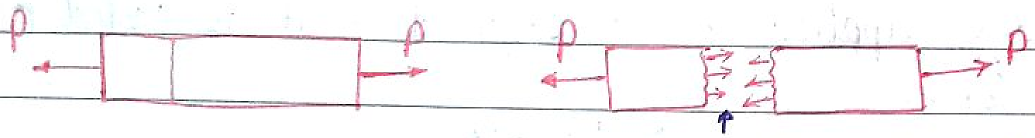


STRESS:

The internal resisting force developed per unit cross section area is defined as stress. It is also called as normal stress.

It is represented by "σ". and unit of stress is N/mm^2

$$\boxed{\text{stress} = \frac{\text{Internal resisting force}}{\text{Cross section area}} = \frac{P}{A}}$$



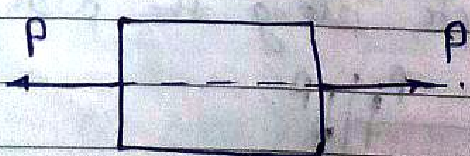
Internal resisting force.

$$\boxed{1\,MPa = 1\,N/mm^2}$$

Types of stress

- Direct stress or Normal stress
  - Tensile stress
  - Compressive stress
- Shear stress
- Bending stress

1- Direct Stress: stress which acts normal to the plane on which the force acts axially are called as direct stress or normal stress (σ).



It is of two types

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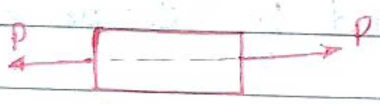
(a) Tensile stress: When two equal and opposite pull is applied on the member, then member is subjected to tension and stress is so developed into the body is called as tensile stress.



Positive

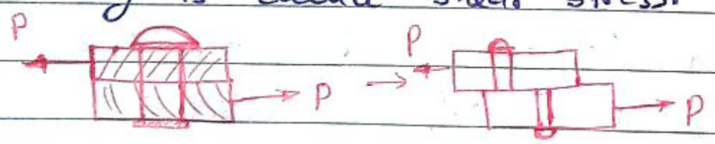
$$\boxed{\sigma_t = \frac{P}{A}} \quad N/mm^2$$

(b) Compressive Stress: When two equal and opposite push is applied on the body then body is subjected to compression & stress is so developed into the body is called as compressive stress.

Negative

$$\boxed{\sigma_c = \frac{P}{A}} \quad N/mm^2$$

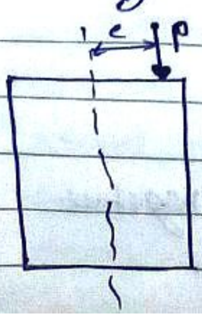
2- Shear stress: When equal and opposite force acts tengentially on a body across the resisting section then the body tends to shear off. In such case the stress is so developed in the body is called shear stress.



$$\boxed{\tau = \frac{P}{A}} \quad N/mm^2$$

3- Bending stress: When an ecentric load acts on a body then stress is so developed is called as bending stress.

In case of beam, when the load acts on transverse section, then resistance offered by the internal stress to bending is called as bending stress.



$$\boxed{\sigma_b = \frac{P}{A}} \quad N/mm^2$$

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STRAIN:

When a body of elastic material is acted upon by an axial force, it undergoes change in dimensions. This change in dimension per original dimension is called as strain. It is denoted by "e" or "ε".

It has no units.

$$\boxed{\text{strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}}$$

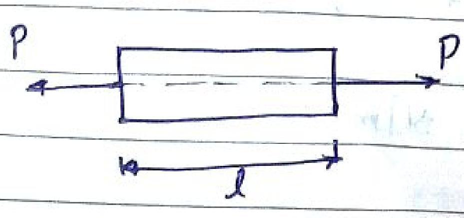
Types of strain

- Linear strain or Longitudinal strain
  - Tensile strain
  - Compressive strain
- Lateral strain
- Volumetric strain
- Shear strain

1- Linear Strain: Change in length in the direction of applied load per original length is called as linear strain. It is denoted by 'e' or 'ε'.

If the applied load is tensile then strain is called tensile strain.

If the applied load is compressive then strain is called compressive strain.



$$\boxed{e = \frac{\delta L}{L}}$$

Tensile is positive and compressive is negative.

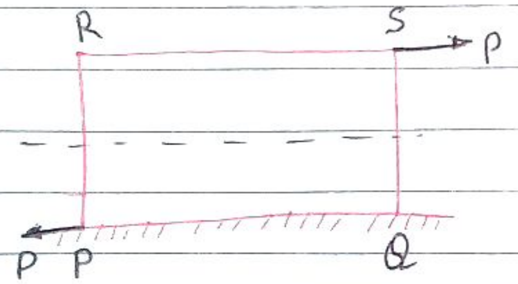
2- Lateral strain: The change in lateral dimensions (width, thickness) (perpendicular to the applied force) to the original lateral dimensions is called lateral strain or secondary strain.

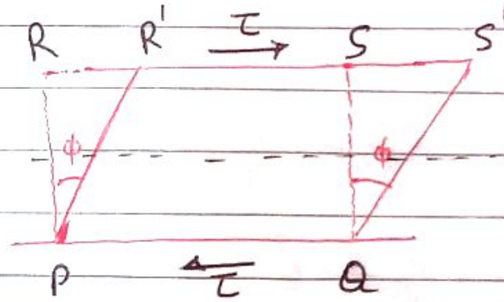
$$\boxed{e = \frac{\delta d}{d} = \frac{\delta t}{t}}$$

3- Volumetric strain: The change in volume due to external force per original volume is called as volumetric strain. It is denoted by e_v.

$$\boxed{e_v = \frac{\delta V}{V}}$$

4- Shear strain: It is defined as the change in right angle. It is denoted by ϕ.





$$\boxed{\phi = \frac{SS'}{SQ}}$$

NOTE: (i) Strain is the ratio of two similar quantities, hence it has no unit.

(ii) Longitudional strain is also called linear strain, Primary strain or simple strain.

(iii) Lateral strain is called secondary strain.

(iv) Volumetric strain is the algebraic sum of all linear strain.

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Poisson's ratio:

It is define as the ratio of lateral strain to longitudinal strain. It is denoted by μ or $\frac{1}{m}$.

$$\boxed{\mu = \frac{\text{Lateral strain}}{\text{linear strain}} = \frac{\delta d/d}{\delta l/l}}$$

Value of poisson's ratio generally varies from 0.25 to 0.5 for different material.

HOOKE'S LAW:

It state that stress is directly proportional to strain withen elastic limit for homogenius and isotropic material for one dimensional loading.

Mathematically,

$$\boxed{\sigma \propto \varepsilon}$$

$\sigma = E\,\varepsilon$ where E is elastic constant.

$\because \sigma = \frac{P}{A}$ & $\varepsilon = \frac{\delta L}{L}$

so $\frac{P}{A} = E \cdot \frac{\delta L}{L} \Rightarrow \boxed{\delta L = \frac{PL}{AE}}$

Elastic constant: There are three elastic constant namely modulus of elasticity or Young's modulus (E), modulus of rigidity or shear modulus (G or C) and bulk modulus (K).

~~These~~

Relation between E, G, K

$$\boxed{E = \frac{9KG}{3K+G}}$$

1- Young's Modulus: It is defined as the ratio of normal stress to the normal strain.

It is denoted by 'E'. & its unit is N/mm².

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$

2- Modulus of rigidity: It is defined as the ratio of shear stress to shear strain.

It is denoted by 'G' or 'C'. & its unit is N/mm².

$$\boxed{G = \frac{\tau}{\phi}}$$

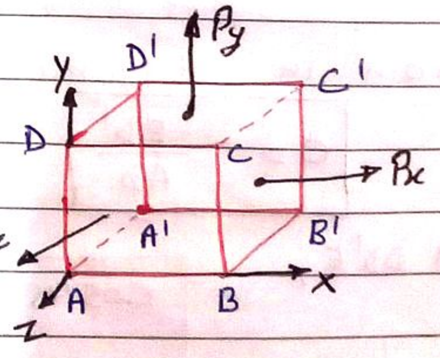
3- Bulk modulus: It is defined as the ratio of direct stress to the volumetric strain.

It is denoted by 'k'. & its unit is N/mm².

$$\boxed{K = \frac{\sigma}{e_v}}$$

Relation between E and K:

When three forces are applied on mutually perpendicular planes then stresses induced in the system is called as tri-axial stress system.



Strain in x- direction $e_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$

Strain in y- direction $e_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$

Strain in z- direction $e_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$

Total volumetric strain $e_v = \frac{\delta V}{V} = e_x + e_y + e_z$

$$e_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

If tri axial stress system consists of equal like stress

i.e. $\sigma_x = \sigma_y = \sigma_z = \sigma$

then

$$e_v = \frac{3\sigma}{E}(1-2\mu)$$

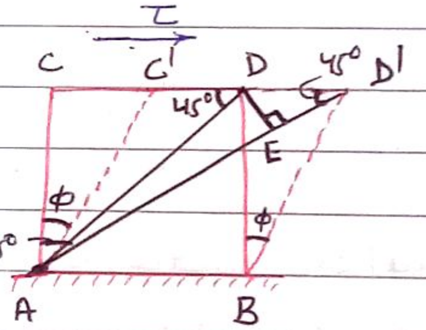
But,

Bulk modulus $K = \frac{\sigma}{e_v}$

$$K = \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)} \Rightarrow K = \frac{E}{3(1-2\mu)}$$

$$\boxed{E = 3K(1-2\mu)}$$

Relation between E and G:



Consider a square block ABCD and apply a shear stress τ.

strain in diagonal AD

$$\boxed{e = \frac{ED'}{AD}} \quad -(i) \qquad \because AE \simeq AD$$

Δ DD'E,

$$\cos 45 = \frac{ED'}{DD'} \Rightarrow \boxed{ED' = \frac{DD'}{\sqrt{2}}} \quad -(ii)$$

Δ ADC,

$$\cos 45 = \frac{CD}{AD} \Rightarrow \boxed{AD = \sqrt{2}\,CD} \quad -(iii)$$

Now from eq. (1)

$$e = \frac{DD'}{\sqrt{2}\cdot\sqrt{2}\,CD} \Rightarrow e = \frac{DD'}{2CD}$$

$$\boxed{e = \frac{DD'}{2BD}} \quad -(iv) \qquad \because CD = BD$$

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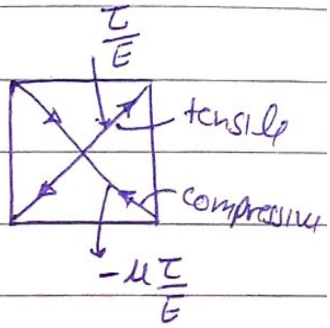
But from the geometry $\frac{DD'}{BD}$ = shear strain (ϕ)

So $\frac{DD'}{BD} = \phi$

Now from eq. (IV)

$$\boxed{e = \frac{\phi}{2}} \quad - (V)$$

Shear strain along the diagonal AD



$$e = \frac{\tau}{E} - \left(-\mu\frac{\tau}{E}\right) = \frac{\tau}{E}(1+\mu)$$

$$\boxed{e = \frac{\tau}{E}(1+\mu)} \quad - (VI)$$

Now from eq. (V) & (VI)

$$\frac{\phi}{2} = \frac{\tau}{E}(1+\mu)$$

$$\boxed{E = 2\left(\frac{\tau}{\phi}\right)(1+\mu)}$$

But modulus of rigidity $G = \frac{\tau}{\phi}$

So

$$\boxed{E = 2G(1+\mu)}$$

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Relation between E, G and K:

from the relation E and K.

$$E = 3K(1-2\mu) \Rightarrow E = 3K - 6K\mu$$

$$\boxed{\mu = \frac{3K-E}{6K}} \quad -(i)$$

from the relation E and G.

$$E = 2G(1+\mu) \Rightarrow E = 2G + 2G\mu$$

$$\boxed{\mu = \frac{E-2G}{2G}} \quad -(ii)$$

Compar eq. (i) & eq. (ii)

$$\frac{3K-E}{6K} = \frac{E-2G}{2G}$$

$$3KG - EG = 3KE - 6KG$$

$$3KE + GE = 9KG$$

$$E[3K+G] = 9KG$$

$$\boxed{E = \frac{9KG}{3K+G}}$$

And

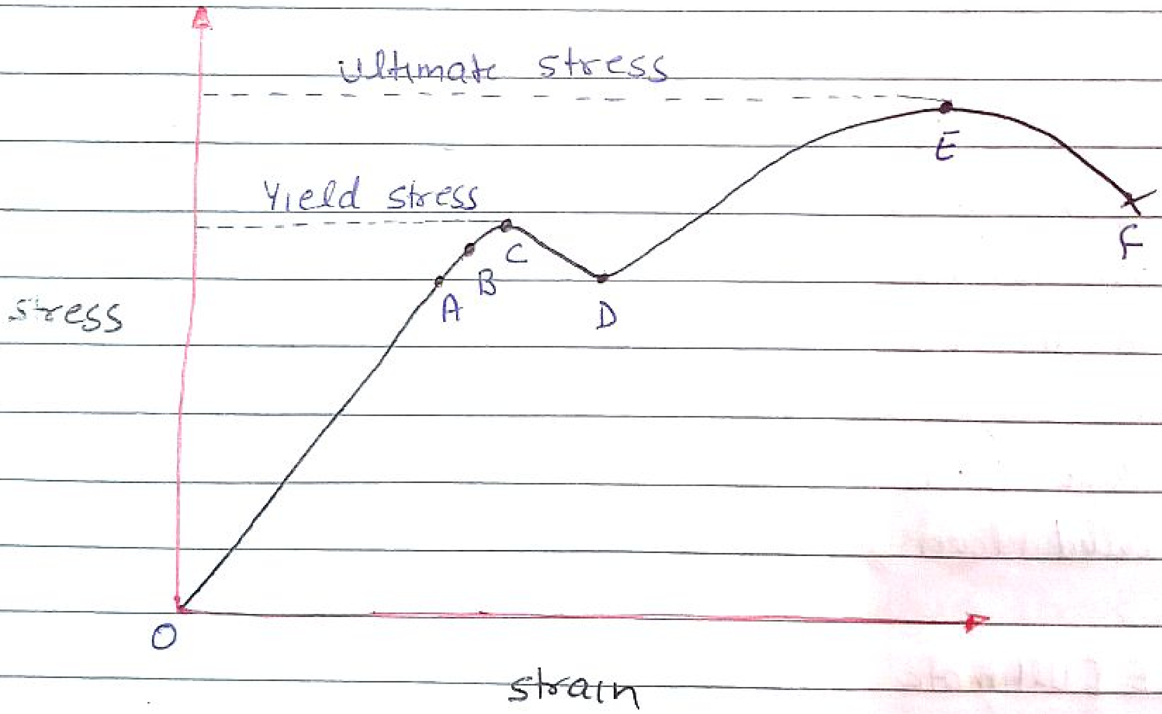
$$\boxed{G = \frac{3EK}{9K-E}}$$

$$\boxed{K = \frac{EG}{3(3G-E)}}$$

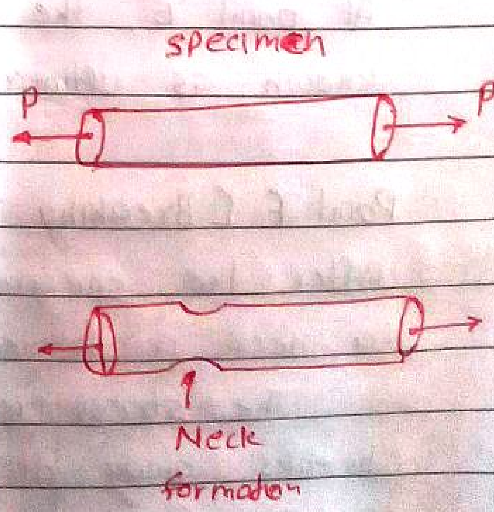
Tensile test diagram for ductile material OR stress- strain diagram for ductile material (Mild steel):

This test consist of gradually applied tensile force on a ductile material (Ex. Mild steel) speciman and noting the corresponding values of load and elongation until specimen fractures. This test is generally conducted on universal testing machine (UTM).

Stress - strain diagram is graphical representation of stress and strain due to tensile load.



A → Proportional limit
B → Elastic limit
C → Upper yield point
D → Lower yield point
E → Ultimate load point
F → Breaking point



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Point - A (Proportional limit):

From point O to A is a straight line, which represent that the stress is directly proportional to strain.

Beyond point A the curve slightly deviates from the straight line.

Point B [Elastic limit]:

Material has elastic properties upto point B. This point is known as elastic limit.

Thus it is defined as the stress developed in the material without any permanent deformation.

Point C and Point D [Yield point]:

The point at which strain goes on increasing rapidly with out further increase of stress is called as yield points.

If the material is stressed beyond point 'B', the plastic stage will reach i.e. permanent deformation.

Point 'C' is called upper yield point and point 'D' is called lower yield point.

Point E [Ultimate stress]:

At point 'E', the stress, attains its maximum value known as ultimate stress. After 'E' stress is decreses.

Point F [Breaking stress]:

After the specimen has reached the ultimate stress a neck is formed which decrease the cross section area.

The stress is therefore reduced until the specimen breaks away at point F. The stress at F is known as breaking stress.

Stress - strain diagram for brittle material [Cast Iron]:

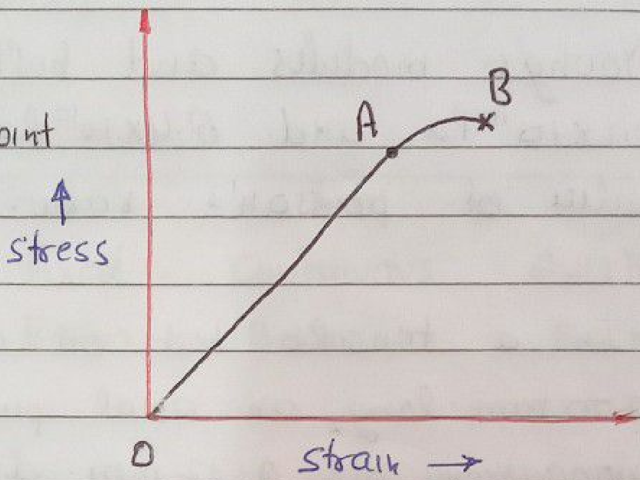
Material that fail in tension at relatively two value of strain are classified as brittle material.

Brittle materials are good for compressive stress.

Ex- Concrete, glass, ceramic material, cast iron etc.

A → Proportional limit

B → Breaking or fracture Point



Point A [Proportional limit]:

From point O to A is a straight line which represent that stress is directly proportional to strain.

Beyond point 'A', the curve slightly deviates from the straight line.

Point B [fracture stress]:

The material fails with only little alongation after the proportional limit is exceeded. The fracture stress is same as ultimate stress.

Factor of safety [FOS]:

It is defined as the ratio of maximum stress to the working stress. It is denoted by 'FOS' or 'N'. It has no unit.

$$\boxed{FOS = \frac{\text{maximum stress}}{\text{working stress}}}$$

$$\boxed{FOS = \frac{\text{Yield Point stress}}{\text{working stress}}}$$

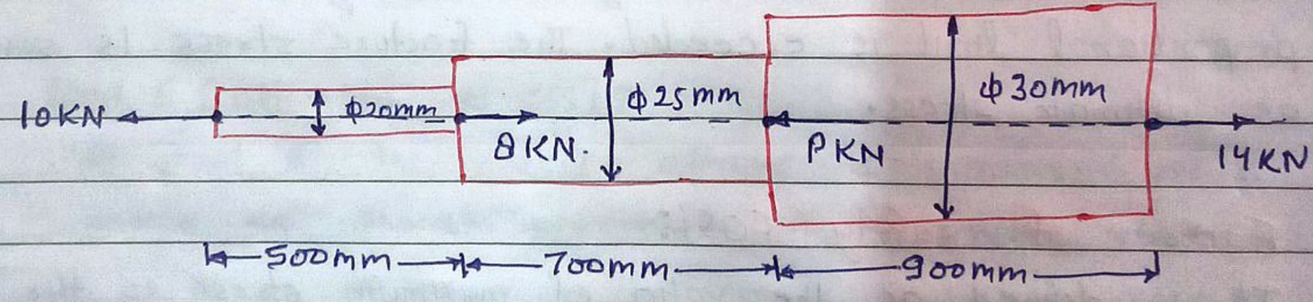
$$\boxed{FOS = \frac{\text{Ultimate stress}}{\text{working stress}}}$$

Ques.1: A rectangular block of size 200 mm x 300 mm x 500 mm is subjected to stresses of 500 N/mm² (Tensile), 600 N/mm² tensile and 700 N/mm² (compressive) in x, y, and z direction respectively. Calculate change in volume of the block.

Ques.2. Young's modulus and bulk modules of steel are 2.1×10^{11} Pa and 8.4×10^{10} Pa respectively. Determine the value of poisson's ratio.

Ques.3. In a tensile test on certain specimen 20 mm dia, 200 mm long, an axial pull of 100 KN produce an alongation of 0.32 mm and reduction in diameter is observed to be 0.0085 mm. Find the value of poisson's ratio and the three modulli.

Ques4. Determine the magnitude of 'P' for equilibrium and the total elongation of the bar. Also calculate min. stress induced. Take E = 210 GPa.



Ques.5 A 80 m long wire of 5 mm diameter is made of steel with E = 200 GPa and an ultimate tensile strength of 400 MPa. If factor of safety of 3.2 is desired determine:

(a) the largest allowable tension in the wire

(b) Corresponding elongation of the wire.