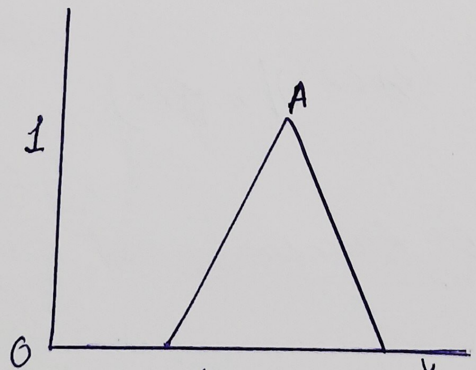
Unit-4 (APPLICATION OF SOFT COMPUTING)

Fuzzy Logic-II (FUZZY MEMBERSHIP, RULES)

1. Membership FUNCTIONS: INTRODuction:

* We know that fuzzy logic is not logic that is fuzzy but logic that is used to describe the fuzziness

* The fuzziness is best characterized by a membership function. In other words, membership function represent the degree of truth in fuzzy logic.



Membership function of fuzzy set A

* Membership function characterize fuzziness (ie all the Information in fuzzy set) wheather the element in fuzzy set are discrete or Continous

* Membership function can be defined as a technique to solve practical problem by experience rather than knowledge

* Membership function are represented by graphical form.

* Rules for defining fuzzy set are fuzzy too.

Mathematical Notation:

SET A in the universe of Information U can be defined as a set of ordered pairs and can be represented as

$$A = \{(x, \mu_A(x)) | x \in U)$$

Here $\mu(A)(\cdot)$ membership function

$$\mu A(\cdot) \in [0,1]$$

$\mu A(\cdot)$ maps U to the Membership space M.

dot $(\cdot)$ in the Membership function represent the element in a fuzzy set (wheather it is discrete or continous)

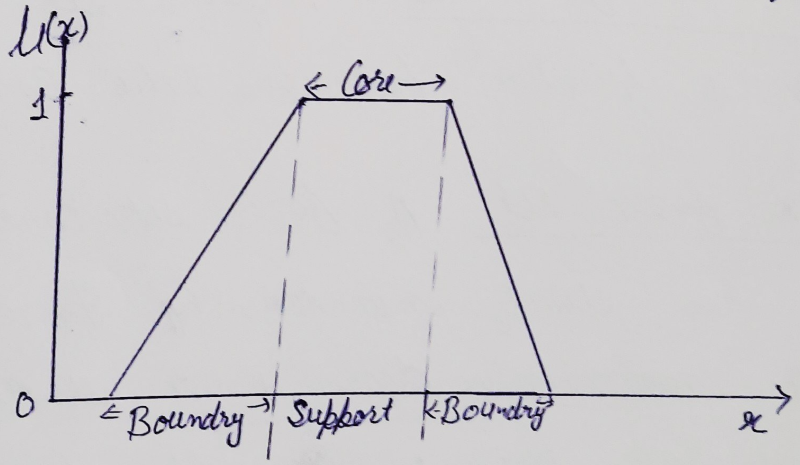
FEATURES OF MEMBERSHIP FUNCTION:

The Three Main basic features Involved in membership function are

1 **Core**: For some fuzzy set $\underset{\sim}{A}$, It is defined as that region of universe that is characterized by complete membership in the set $\underset{\sim}{A}$. The Core has elements x of the universe such that

$$\mu_{\underset{\sim}{A}}(x) = 1$$

The Core of a fuzzy set may be empty set



2 **Support**: It is defined as that region of universe that is characterized by a non-zero membership in the set $\underset{\sim}{A}$.

$$\mu_{\underset{\sim}{A}}(x) > 0$$

fuzzy singleton: A fuzzy set whose support is a single element in X with $\mu_A(x) = 1$

3 **BOUNDRY**: for any fuzzy set $\underset{\sim}{A}$, the boundry of a membership function is the region of univers that is characteried by a non-zero but incomplete membership in the set.

$$0 < \mu_A(x) < 1$$

TYPES of fuzzy set:

1) **Normal fuzzy set**: A fuzzy set whose membership function has at least one element x in the Universe whose Membership value is unit

2) **Subnormal fuzzy set**: A fuzzy set where no membership function has its value equal to 1.

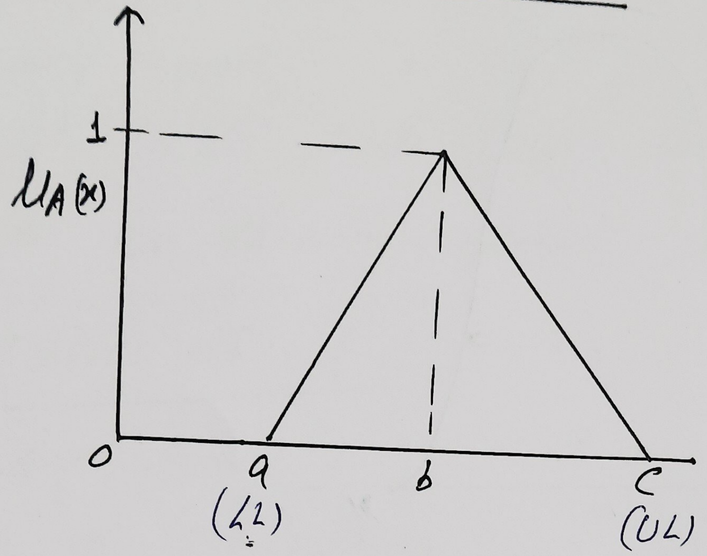
3) **Convex fuzzy set**: A fuzzy set whose membership value are strictly monotonically Increasing or strictly monotonically decreasing or strictly Monotonically Increasing than strictly monotonically decreasing with Increasing value for element in the univers.

$$\mu_A(x_2) \geq \min[\mu_A(x_1), \mu_A(x_3)]$$

4) **Non-Convex fuzzy set**: The Membership value of the membership function are not strictly monotonically Increasing or decreasing or strictly Monotonically Increasing than decreasing.

TYPES OF MEMBERSHIP FUNCTION

1 Triangular Membership function:



LL - Lower Limit

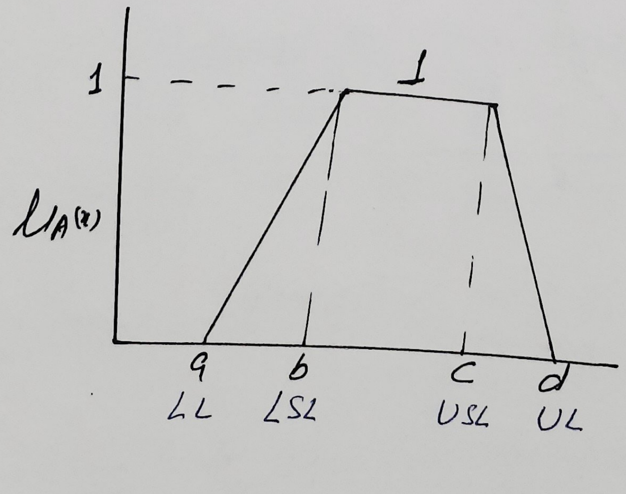
UL - Upper Limit

LSL - Lower support Limit

USL - Upper support Limit

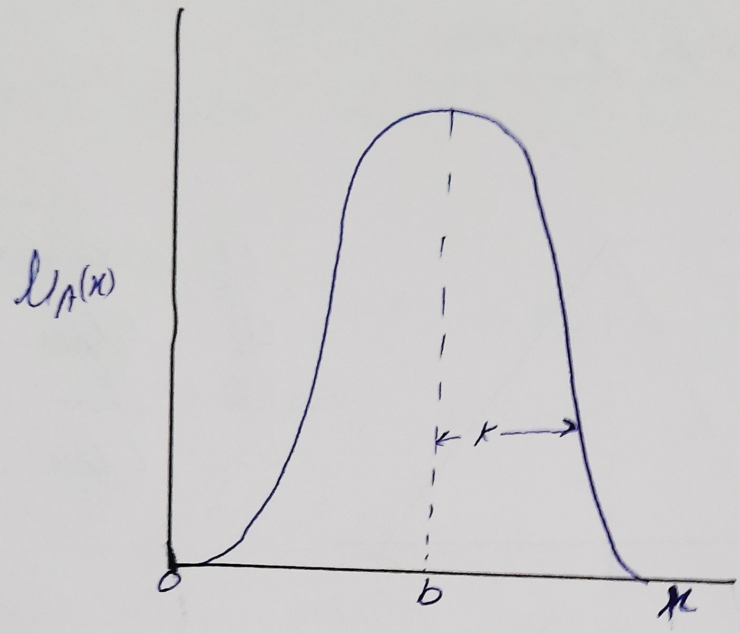
$$\mu_A(x) = \begin{bmatrix} 0 & , \text{ if } x < a \\ \frac{x-a}{b-a} & , \text{ if } a \le x \le b \\ \frac{c-x}{c-b} & \text{ if } b \le x \le c \\ 0 & \text{ if } x > c \end{bmatrix}$$

2 TRAPEZOIDAL Membership function:



$$\mu_A(x) = \begin{bmatrix} 0, \text{ if } x \le a, \text{ OR } x \ge d \\ \frac{x-a}{b-a}, \quad a \le x \le b \\ 1, \quad b \le x \le c \\ \frac{d-x}{d-c}, \quad c \le x \le d \end{bmatrix}$$

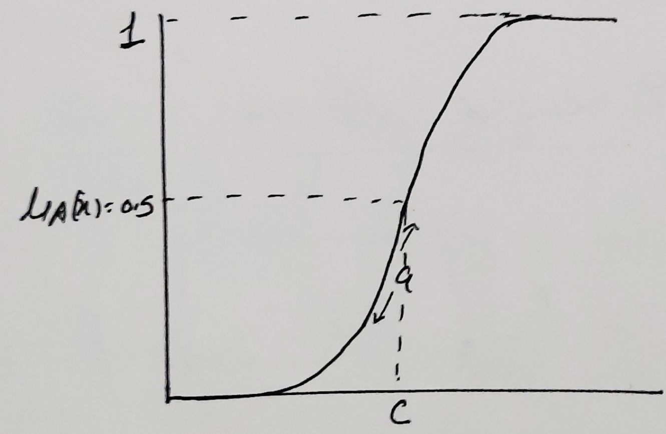
3 GAUSSIAN MEMBERSHIP FUNCTION:



$$\mu_A(x) = e^{-\frac{(x-b)^2}{2k^2}}$$

k = standard deviation > 0

4 SIGMOIDAL MEMBERSHIP FUNCTION



$a \rightarrow$ slope

$$\mu_A(x) = \frac{1}{1+e^{-a(x-c)}}$$

FUZZY INFERENCE SYSTEM (FIS)

* Fuzzy Inference system is the key unit of fuzzy logic system having decision making as it's primary work.
* It uses 'IF. THEN' rule along with connectors 'OR' and 'AND' for drawing essential decision rule.

CHARACTERISTICS OF FIS:

* The output from FIS is always a fuzzy set irrespective of it's Input which can be fuzzy or crisp.
* It is necessary to have fuzzy output when it is used as a controller
* A difuzzification unit would be there with FIS to convert fuzzy variables into crisp variable.

FUNCTIONAL BLOCK OF FIS:

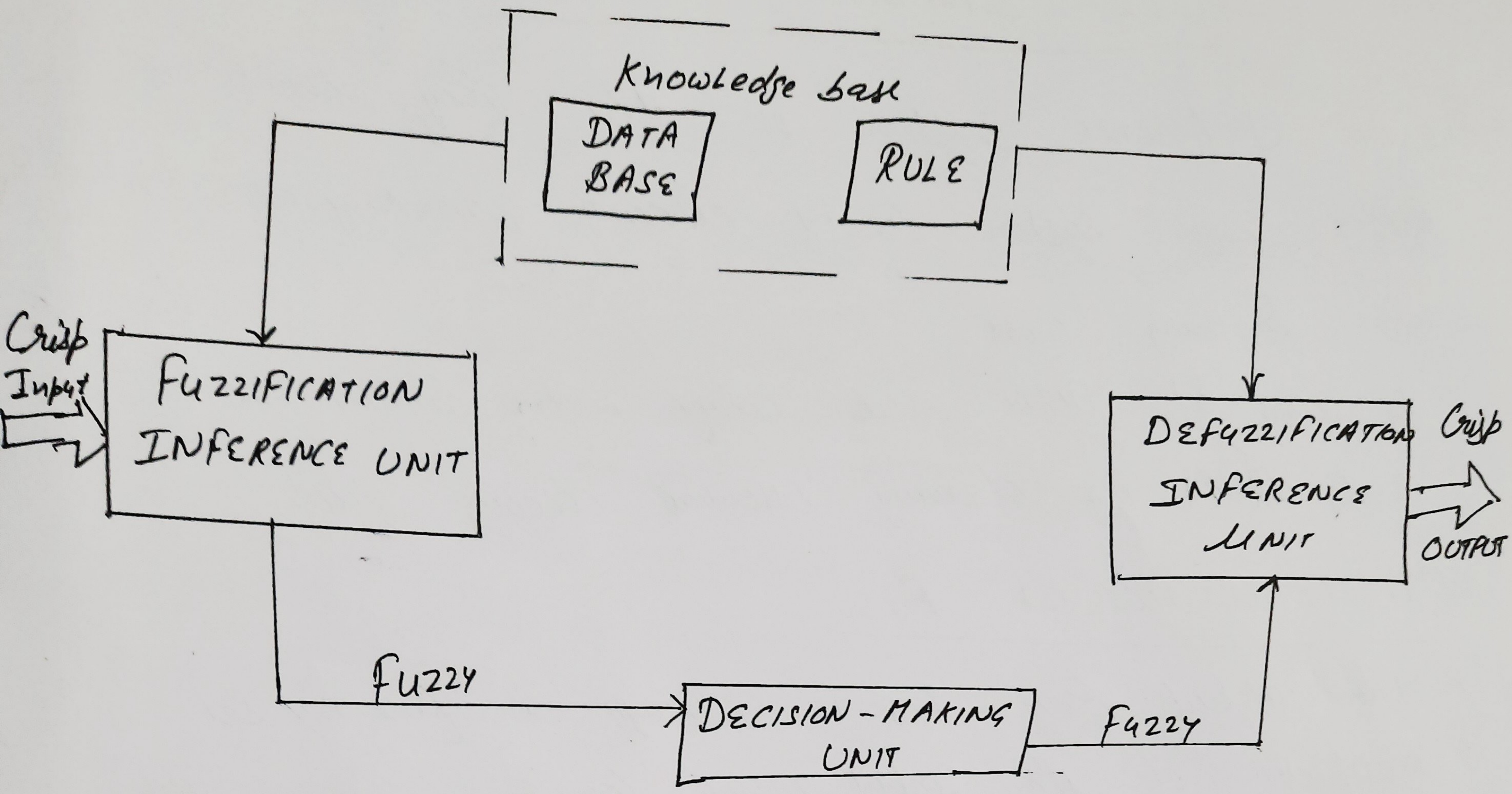
RULE BASE: It contain fuzzy IF-THEN rule

Database: It defines the membership function of fuzzy sets used in fuzzy rule

Decision-making unit: It perform operation on Rule

FUZZIFICATION INTERFACE UNIT → It convert the crisp quantities ~~quantifies~~ into fuzzy quantities

DEFUZZIFICATION Interface Unit: It convert the fuzzy ~~quantifies~~ quatities into crisp quantities



BLOCK DIAGRAM OF FIS

WORKING OF FIS :

* FUZZIFICATION Unit support the application of numerous fuzzification Methods. and Convert the Crisp Input into fuzzy Input

* Knowledge base - Collection of rule base & database is formed upon the Conversion of Crisp Input into fuzzy Input

* Defuzzification: fuzzy Input is finally Converted into Crisp output

METHOD OF FIS:

* Mamdani fuzzy INFERENCE SYSTEM
* Takagi - Sugeno fuzzy Model (TS Method)

Mamdani fuzzy INFERENCE SYSTEM:

This sys was proposed in 1975 by Ebhasim Mamdani. It was anticipated to Control a steam engine and boiler Combination by Synthesizing a set of fuzzy rule obtained from people working on the system.

Steps for Computing the Output:

Step1: Set of fuzzy rules needs to be determined

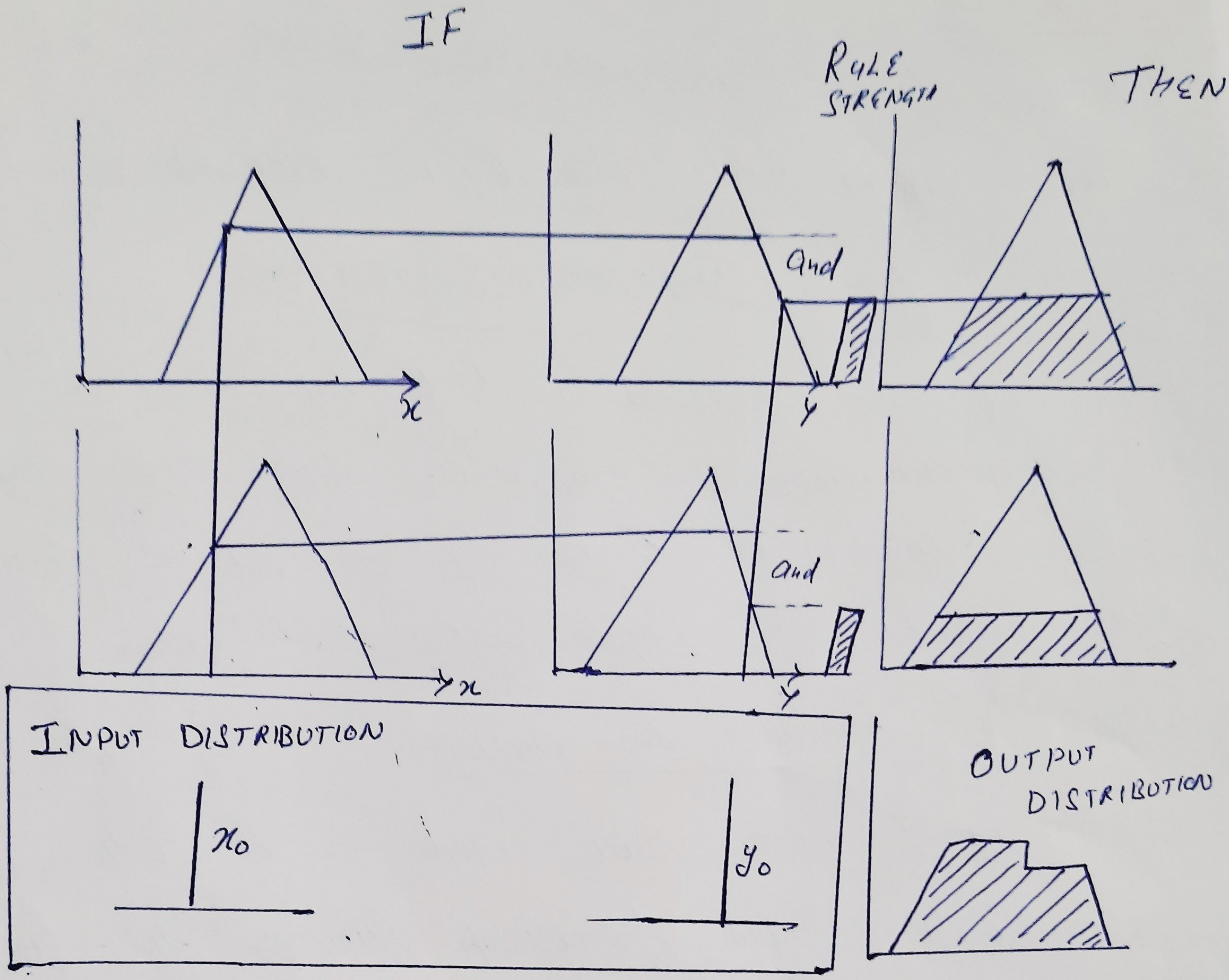
Step2: By Using Input Membership function, the Input would be made fuzzy

Step3: Now establish the Rule Strength by Combining the fuzzified Input According to fuzzy Rule

Step 4: Determine the Consequent of Rule by Combining the rule Strength and the output Membership function

Step 5: For getting output distribution Combine all the Consequents

Step5: Finally, a defuzzified output distribution is obtained



Takagi - Sugeno Fuzzy Model (TS Method)

This model was proposed by Takagi, sugeno & Kang in 1985. Format of this rule is given by

IF x is A and y is B THEN $Z = f(x,y)$

Here AB are fuzzy sets in Antecedents and $Z = f(x,y)$ is a Crisp function in the Consequent.

FUZZY INFERENCE PROCESS:

Step 1: FUZZIFYING THE INPUTS: Here the Input of the System are made fuzzy.

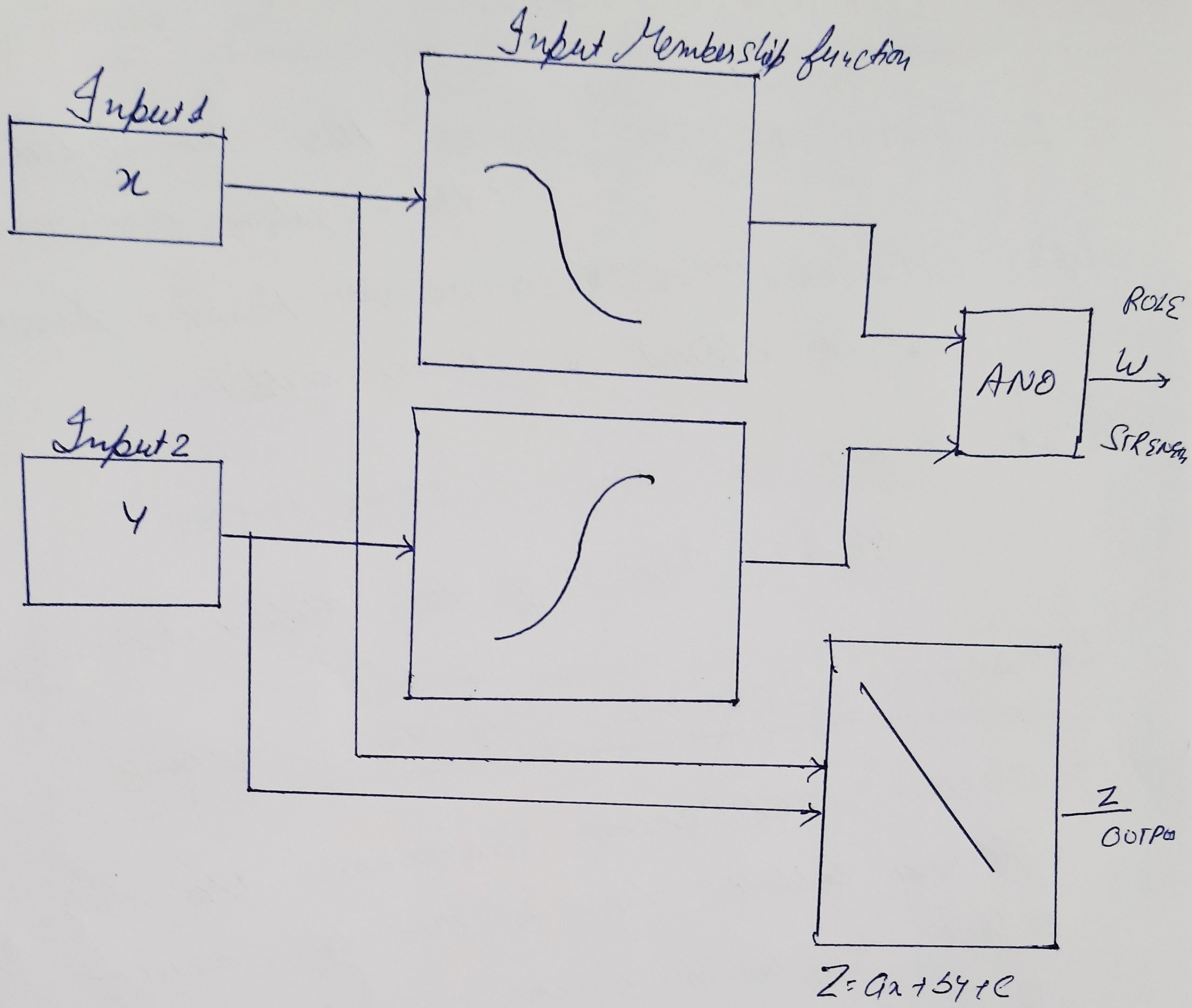
Step 2: APPLYING THE FUZZY operator: Fuzzy operators Must be applied to get the output

ROLE FORMAT OF THE SUGENO FORM

If $x=7$ and $a=y$ then output is $Z=ax+by+c$

Comparison BETWEEN THE TWO METHOD

- OUTPUT MEMBERSHIP FUNCTION - The Main difference between them is on the basis of output membership function. The sugeno output Membership functions are either Linear or Constant
- AGGREGATION & DEFUZZIFICATION PROCEDURE → The difference between them also Lies in the Consequence of fuzzy rule & due to the same their aggregation & defuzzification procedure also differ
- Mathematical Rule: More Mathematical rule exist for the sugeno rule than the Mamdani rule
- Adjustable Parameter: The sugeno Controller has More adjustable Parameter than the Mamdani controller


Input Membership function
Inputs
x
Input2
Y
ROLE
AND
W
STRENGTH
Z
OUTP
Z=ax+by+c

FUZZY IF-then RULE

Fuzzy set and fuzzy set operations are the subject & verb of fuzzy logic. If Then rule statement are used to formulate the conditional statement that comprise fuzzy logic

IF Premise (antecedent), THEN Conclusion (Consequence)

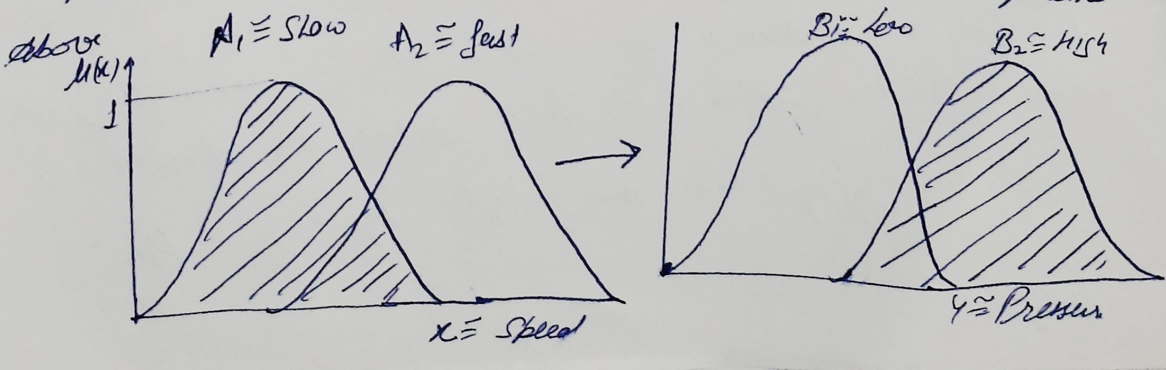
It typically express an Inference such that if we know a fact (Premise, hypothesis, antecedent) then we can infer, drive another fact called a Consequence or Conclusion

$$\text{If } \underbrace{x \text{ is } A_1}_{\text{antecedent/Premise}}, \text{ THEN } \underbrace{y \text{ is } B_2}_{\text{Consequent}}$$

In other words, the conditional statement can be expressed in Mathematical form

$$\text{If } A_1 \text{ then } B_2 \quad \text{or} \quad A_1 \to B_2$$

Example: If speed is slow Then pressure should be high. speed & pressure of a steam Engine can be expressed as above



RULE FORM: In general Three form of rule exist for any Linguistic Variable.

1. Assignment statement
   eg. x is not Large AND Not Very Small
2. Conditional statement
   eg. IF x is Very big THEN y is Medium
3. Unconditional statement
   eg. set pressure high

* **Compound Rule**: A Linguistic statement expressed by a human might involve compound rule structure. By using basic properties and operations defined for fuzzy set, any compound rule structure may be decomposed and reduced to a number of simple canonical rule.

1) **Conjunctive Antecedent**:

IF x is A AND x is A^2 .. AND x is A^n THEN y is B^s

Can be written as

$$\boxed{\text{IF } x \text{ is } A^S \text{ THEN } y \text{ IS } B^S}$$

where $A^S = A^1 \cap A^2 \cap A^3 \ldots \cap A^n$

A^S can be expressed by means of membership function based on the definition of fuzzy Intersection operation as

$$\mu_{A^S}(x) = \min[\mu_{A^1}(x), \mu_{A^2}(x) \ldots \mu_{A^n}(x)]$$

2) **DISJUNCTIVE ANTECEDENT:** For a Multiple disjunctive antecedent of the form

IF x is A^1 OR x is A^2 -- OR x is A^n THEN y is B^s

Can be written as

$$\boxed{\text{IF } x \text{ is } A^s \text{ THEN } y \text{ is } B^s}$$

Where $A^s = A^1 \cup A^2 \cup A^3 .. \cup A^n$

A^s Can be expressed by means of membership function based on the defination of fuzzy union operation

$$\mu_{A^s}(x) = \max\left[\mu_{A^1}(x), \mu_{A^2}(x) \ldots \mu_{A^s}(x)\right]$$

* **AGGREGATION OF RULE:**

* Most rule based system have more than one rule

> The Process of obtaining the overall of the Consequent from the Individual Consequent Contributed by each rule in the rule base is the aggregation of rules

* In Case of a System of rule that must be jointly satisfied. The rules are Connected by AND Connective OR The Aggregated O/P Can be found by fuzzy Intersection of the entire individual rule Consequent y

$$y = y^1 \text{ AND } y^2 \text{ AND } y^3 . - \text{ AND } y^n$$

or

$$y = y^1 \cap y^2 \cap y^3 \text{ --- } \cap y^n$$

Which is defined by the Membership function

$$\mu_y(y) = \min[\mu_{y^1}(y)\ \mu_{y^2}(y) \ldots \mu_{y^r}(y)] \text{ for } y \in Y$$

For the Case of dis junctive system of rule where at least one rule must be satisfied.

$$Y = Y^1 \text{ or } Y^2 \text{ OR} \ldots \text{OR } Y^r$$

$$Y = Y^1 \cup Y^2 \ldots \cup Y^r$$

WHICH is defined by the Membership function

$$\mu_y(y) = \max[\mu_{y^1}(y)\ \mu_{y^2}(y) \ldots \mu_{y^r}(y)] \text{ for } y \in Y$$

FUZZY IMPLICATIONS

- A Fuzzy Rule generally assume the form

$\boxed{R: \text{IF } x \text{ is } A, \text{ THEN } y \text{ IS } B.}$

Where A & B are Linguistic Value defined by fuzzy set on universe of Discourse X and Y resp.

- The Rule is also Called a "fuzzy IMPLICATION" Or fuzzy Conditional statement.

- Fuzzy IMPLICATION is an Important Connective in Fuzzy Control sys. because the Control Strategies are embodied by set of IF-THEN rule

- $R \; A \to B$ or $A \to B$

- In essence, The expression describes a relation between two Variable x & y

- This suggest that a fuzzy rule can be defined as a binary relation R $R: X \times Y$

DIFFERENT INTERPRETATIONS

1 'A is coupled with B'

$$R = A \to B = A \times B = \int_{X \times Y} \frac{\mu_A(x) \otimes \mu_B(y)}{(x,y)}$$

$\otimes$ is T norm operator

2 "A entails B": four different formulae

a) Material Implication

$$R = A \to B = \neg A \cup B$$

b) Propositional Calculus

$$R = A \to B = \neg A \cup (A \cup B)$$

c) Extended Propositional Calculus

$$R = A \to B = (\neg A \cap \neg B) \cup B$$

d) Generalized Modus Ponens (GMP)

$$\mu_R(x,y) = \sup\{c \mid \mu_A(x) \otimes c \leq \mu_B(y), 0 \leq c \leq 1\}$$

All the other above formulae reduce to the familiar identity $A \to B = \neg A \cup B$

DIFFERENT IMPLICATIONS

1 KLeen Diene's Implication:

$$R_K = C_Y(A') \cup C_Y(B)$$

Where C is the Cylindrical entention

$$\mu_{R_K} = \max[1-\mu_A(x), \mu_B(y)]$$

2 Lukasiewicz's Implication:

$$R_L = C_Y(A') \oplus C_Y(B)$$

$\oplus$ is bounded seem

$$\mu_{R_L}(x,y) = \min\{1, 1-\mu_A(x) + \mu_B(y)\}$$

3 Zadeh's Implication:

$$R_Z = [C_Y(A) \cap C_Y(B)] \cup C_Y(A')$$

$$\mu_{R_Z}(x,y) = \max[\min(\mu_A(x), \mu_B(y)), (1-\mu_A(x))]$$

4 STOCHASTIC IMPLICATION

It is based on the following equality

$$P\left(\frac{B}{A}\right) = [(1-P(A)] + P(A).P(B)$$

defined as $R_{St} = C_Y(A') \cup [C_Y(A) \cap C_Y(B)]$

$$\mu_{R_{St}}(x,y) = \min\{1, 1-\mu_A(x) + \mu_A(x).\mu_B(y)\}$$

* Based on these two Interpretation & Various T-norm / S-norm operators, a no. of qualified method can be formulated to calculate the fuzzy Relation $R: A \to B$

* Relation R can be viewed as fuzzy set with 2D

$$\mu_R(x,y) = f(\mu_A(x), \mu_B(y)) = f(a,b)$$

* f is called fuzzy Implication function & perform the task of transforming the membership grades of x in A & y in B into those of (x,y) in $A \to B$

'For first Interpretation'

→ "A is coupled with B" - $A \to B$ result from employing the most commonly used T-norm operations (min, algebric, bounded, drastic product)

→ for second Interpretation

"A entails B" again 4 different fuzzy relation $A \to B$ have been reported in literature (Zadeh's arithmetic rule, Zadeh's max-min rule, Boolean, Goguen's)

5. GOGUEN's Implication:

One of the Requirement in Multi-Valued Logic is that $A \rightarrow B$ should satisfy

$$\mu_A(x) \cdot \mu_{A \rightarrow B}(x, y) \leq \mu_B(y)$$

The Goal is achieved when we use the defination

$$R_{GN} = \min\left[1, C_Y(A) \div C_Y(B)\right]$$

$$\mu_{R_{GN}} = \min\left[1, \frac{\mu_A(x)}{\mu_B(y)}\right]$$

6 Godel's Implication: It is one of the best known Implication formulae in Multi-valued Logic.

$$\mu_{R_g} = \begin{cases} 1 & \mu_A(x) \leq \mu_B(y) \\ \mu_B(y) & \text{OTHERWISE} \end{cases}$$

eg.

$$\mu_{A \xrightarrow{g} B}(0.5, 0.7) = 1$$

$$\mu_{A \xrightarrow{g} B}(0.8, 0.6) = 0.6$$

Above defination results in the following fuzzy relation that is frequently used in fuzzy Logic

$$R_g = \left[C_Y(A) \xrightarrow[g]{} C_Y(B)\right.$$

$$\mu_{R_g}(x, y) = \mu_A(x) \xrightarrow[g]{} \mu_B(y)$$

I <u>MAMDANI's Implication</u> w.r.t fuzzy Control this is the Most Important. It's definition is based on the Intersection operation

$$A \rightarrow B = A \cap B$$

$$R_M = [C_Y(A) \cap C_Y(B)]$$

$$\mu_{R_M}(x,y) = \min[\mu_A(x), \mu_B(y)]$$

- Also known as Control Implication, It is better than the Conventional PI Controller.
- Majority of application are through Mamdani Implication.
- Sometime subscript M is also written

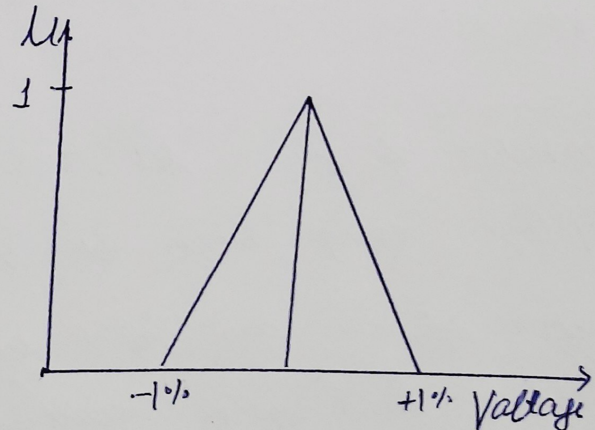
FUZZIFICATION

* Fuzzification is the process of Making a crisp Quantity fuzzy. We do this by simply recognizing that many of the Quantity that we Consider to be crisp and deterministic are actually not deterministic at All.

* They Carry uncertainity. If the form of uncertainity happen to arise because of imprecision, ambiguity or Vagueness. Then the Variable is probably fuzzy. & Can be represented by a membership function

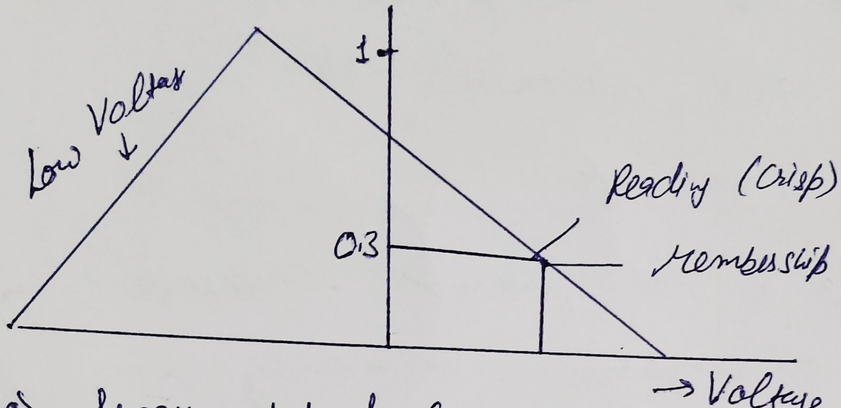
* In real world, Hardware such as a digital Voltmeter generates crisp data. But these data are subject to experimental error.

→ The Info. shown in fig. show one Possible range of errors for a typical reading but the associated membership function that might represent such Imprecision

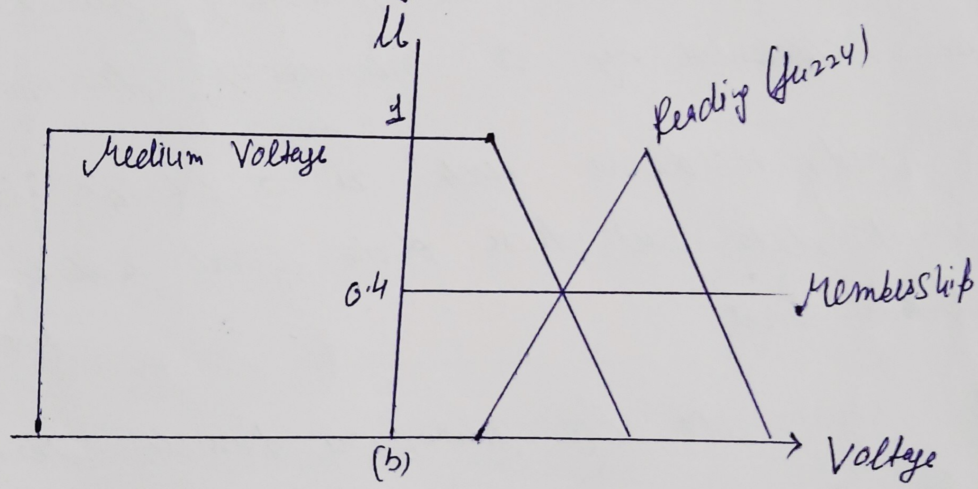


Membership function represented Imprecision in "Crisp Voltage reading"

The representation of Imprecise data as fuzzy set is a useful but not mandatory step when those data are used in fuzzy system.



a) fuzzy set & Crisp reading

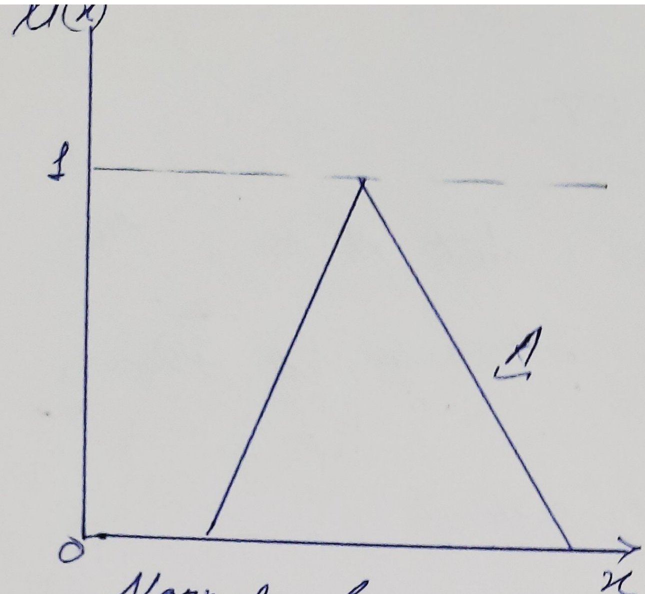


b) fuzzy set & fuzzy reading

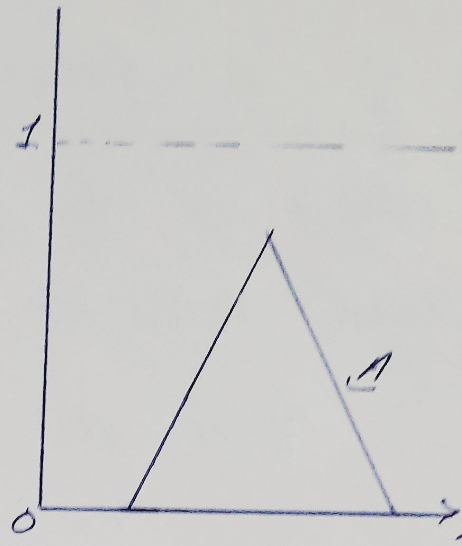
In fig (a) Crisp reading Intersect the fuzzy set Low Voltage" at a membership of 0.3

In fig(b) The Intersection of fuzzy set "Medium Voltage" and a fuzzified Voltage reading occur at a Membership of 0.4

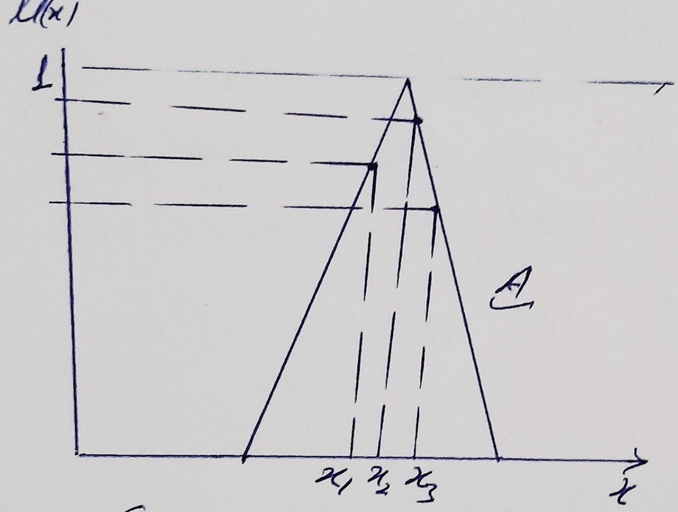
The set Intersection of the two fuzzy set is a small triangle, whose Largest membership occurs at the membership Value of 0.4



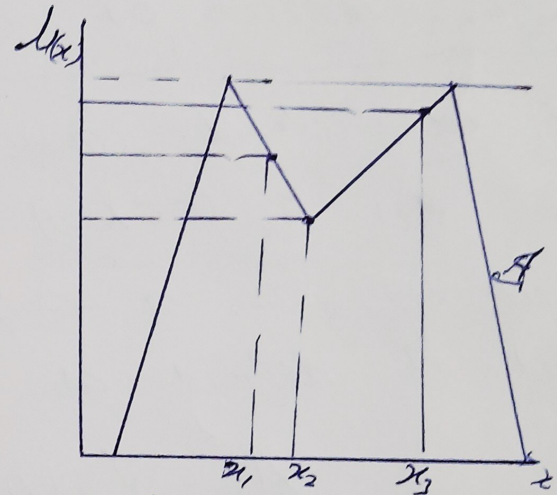
Normal fuzzy set



SubNormal fuzzy set

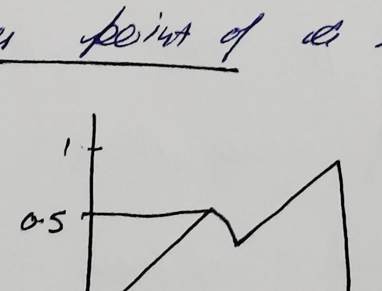


Convex Normal fuzzy set



Non-Convex Normal fuzzy set

* The Intersection of two Convex fuzzy set is also Convex fuzzy set.
* The element in the universe for which a Particular fuzzy set $\underset{\sim}{A}$ has its Value equal to 0.5 is called <u>Cross over point of a</u> Membership function



Height of a fuzzy SET:

The Height of a fuzzy set A is the Maximum value of the Membership function

$$\text{hight}(\underset{\sim}{A}) = \max[\mu_{\underset{\sim}{A}}(x)]$$

Defuzzification of CRISP SET

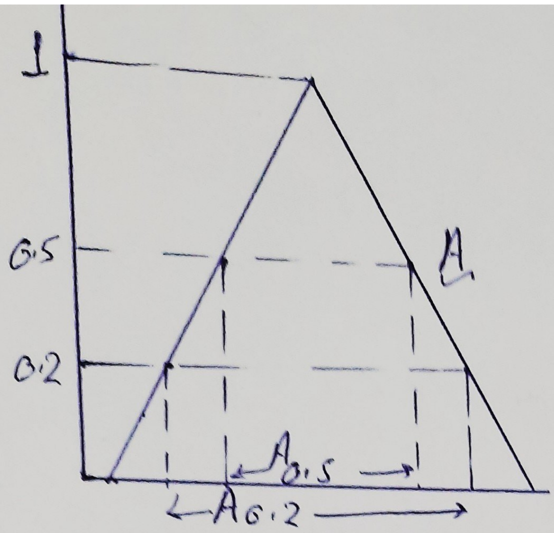
1 λ-Cut Method: Consider a fuzzy set $\underset{\sim}{A}$. The set A_λ $(0<\lambda<1)$ called the Lambda (λ)-Cut set, is a crisp set of the fuzzy set is defined as

$$A_\lambda = \{x \mid \mu_A(x) \geq \lambda\}; \quad \lambda = \in 0, 1)$$

The Properties of λ-Cut sets

1. $(\underset{\sim}{A} \cup \underset{\sim}{B})_\lambda = \underset{\sim}{A}_\lambda \cup \underset{\sim}{B}_\lambda$
2. $(\underset{\sim}{A} \cap \underset{\sim}{B})_\lambda = \underset{\sim}{A}_\lambda \cap \underset{\sim}{B}_\lambda$
3. $(\bar{\underset{\sim}{A}})_\lambda \neq (\bar{A_\lambda})$ except $\lambda = 0.5$
4. ~~eq~~: For any $\lambda \leq \beta$, where $0 \leq \beta \leq 1$, Is it true that $A_\beta \subseteq A_\lambda$ where $A^0 = X$

$$A_0^+ = \{x \mid \mu_{\underset{\sim}{A}}(x) > 0\}$$



The fourth property of fuzzy set shows a Continous - Valued fuzzy set with two λ- Cut Value.

$\lambda = 0.2$ $\beta = 0.5$ $A_{0.2}$ has greater domain than $A_{0.5}$.

ie $\lambda \leq \beta \, (0.2 \leq 0.5)$ $A_{0.5} \subseteq A_{0.2}$

→ LAMBDA- CUT FOR FUZZY RELATIONS:

The λ - Cut for fuzzy Relation is similar to that for fuzzy set.

Let $\underset{\sim}{R}$ be a fuzzy Relation where each row of the Relational Matrix is Considered as fuzzy set. The fuzzy relation can be Converted into a Crisp relation in the following Manner

$$R_\lambda = \{(x,y) \mid \mu_{\underset{\sim}{R}}(x,y) \geq \lambda\}$$

Where R_λ is a λ- Cut relation of the fuzzy Relation $\underset{\sim}{R}$. Here $\underset{\sim}{R}$ is 2-D Array

Similar to the properties of λ-cut fuzzy set, the λ-cut on fuzzy relation also obey certain properties

1> $(\underset{\sim}{R} \cup \underset{\sim}{S})_\lambda = R_\lambda \cup S_\lambda$

2> $(\underset{\sim}{R} \cap \underset{\sim}{S})_\lambda = R_\lambda \cap S_\lambda$

3> $(\bar{\underset{\sim}{R}})_\lambda \neq (\bar{R}_\lambda)$ except when $\lambda = 0.5$

4> For any $\lambda \leq \beta$ where $0 \leq \beta \leq 1$. It is true $R_\beta \subseteq R_\lambda$

Q Consider the discrete fuzzy set defined on the universe $X = \{a, b, c, d, e\}$

$$\underset{\sim}{A} = \left\{ \frac{1}{a} + \frac{0.9}{b} + \frac{0.6}{c} + \frac{0.3}{d} + \frac{0}{e} \right\}$$

$\lambda = 1, 0.9.$ $\qquad A_\lambda = \{x \mid \mu_A(x) \geq \lambda\}$

$\lambda = 1 \qquad A_1 = \left\{ \frac{1}{a} + \frac{0}{b} + \frac{0}{c} + \frac{0}{d} + \frac{0}{e} \right\}$

$\lambda = 0.9 \qquad A_{0.9} = \left\{ \frac{1}{a} + \frac{1}{b} + \frac{0}{c} + \frac{0}{d} + \frac{0}{e} \right\}$

$\lambda = 0.6 \qquad A_{0.6} = \left\{ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{0}{d} + \frac{0}{e} \right\}$

$\lambda = 0.3 \qquad A_{0.3} = \left\{ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{0}{e} \right\}$

$\lambda = 0 \qquad A_0 = \left\{ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e} \right\}$

$\lambda = 0^+ \qquad A_{0^+} = \left\{ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right\}$

Q Determine the crisp λ-cut relation when $\lambda = 0.1, 0^+, 0.3$ & 0.9 for the following relation $\underset{\sim}{R}$

$$\underset{\sim}{R} = \begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.3 & 0.7 & 0.1 \\ 0.8 & 0.9 & 1.0 \end{bmatrix}$$

For the given fuzzy set relation, the λ-cut relation is given by

$$R_\lambda = \{ (x,y) \mid \mu_R(x,y) \geq \lambda \}$$

$$\{ 1 \mid \mu_R(x,y) \geq \lambda ; 0 \mid \mu_R(x,y) < \lambda \}$$

$h = 0.1$

$$R_{0.1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$h = 0.9$

$$R_{0.9} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$h = 0^+$

$$R_{0^+} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$h = 0.3$

$$R_{0.3} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

DEFUZZIFICATION METHODS

Defuzzification to scalar → Defuzzification is the Conversion of a fuzzy quantity to a precise quantity

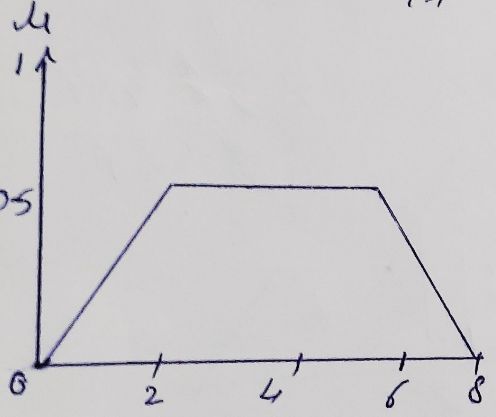
For example: suppose a fuzzy output is Comprised of two part

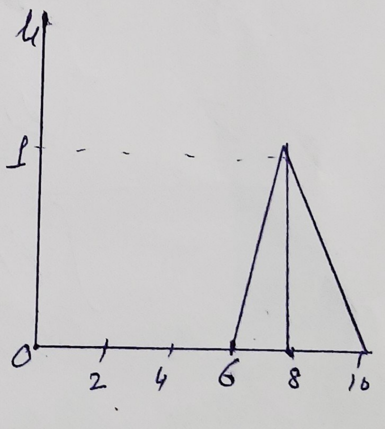
① $\underset{\sim}{C_1}$ a trapezodial shape

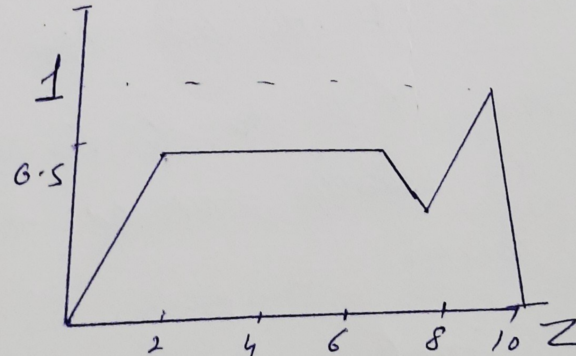
② $\underset{\sim}{C_2}$ a triangular membership shape

$\underset{\sim}{C} = \underset{\sim}{C_1} \cup \underset{\sim}{C_2}$ [max operator].

$$C_k = \bigcup_{i=1}^{k} \underset{\sim}{C_i} = C$$







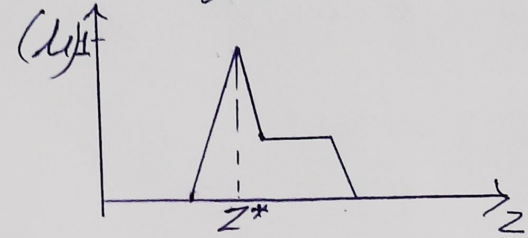
Method 1 for Defuzzification

1) Max-Membership Principle: (Also known as height Method)

This Method is limited for peaked output function.

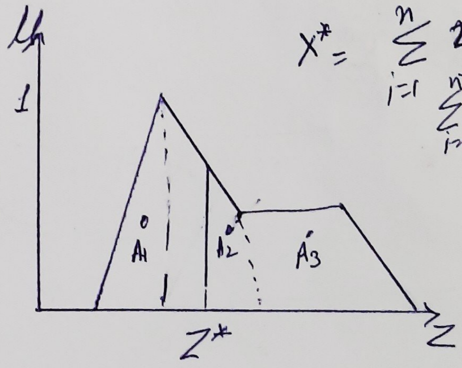
$$\mu_C(z^*) \geq \mu_C(z) \quad \text{for all } z \in Z$$

Where Z^* is the defuzzified Value



2 Centeroid Method: (Also known as center of area, center of gravity)

$$Z^* = \frac{\int \mu_C(z) \cdot z \, dz}{\int \mu_C(z) \cdot dz}$$



$$X^* = \frac{\sum_{i=1}^{n} x_i \cdot A_i}{\sum_{i=1}^{n} A_i}$$

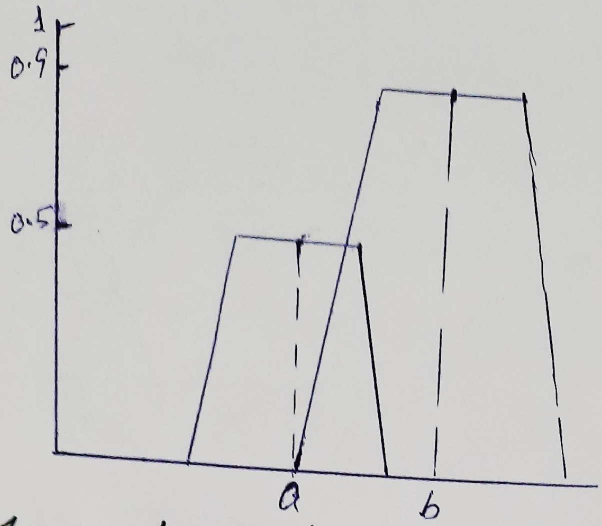
$\int$ denote Algebric Integration.

3 Weighted-Average Method:

It is the most frequently used in fuzzy application. It is one of the most computationally efficient Method. It is usually restricted to symmetrical output membership function

$$Z^* = \frac{\sum \mu_C(\bar{z}) \cdot \bar{z}}{\sum \mu_C(\bar{z})}$$

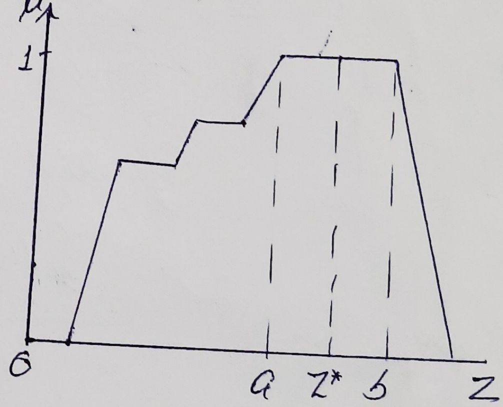
$\bar{z}$ is the Centeroid of each symmetric membership function



$$Z^* = \frac{a(0.5) + b(0.9)}{0.5 + 0.9}$$

4 **Mean- Max Membership**: also Called (Middle-of-Maxima). This Method is Closly related to the first method, except that the location of the Maximum Membership Can be non unique (ie Max Membership Can be a plateu rather than a single point)

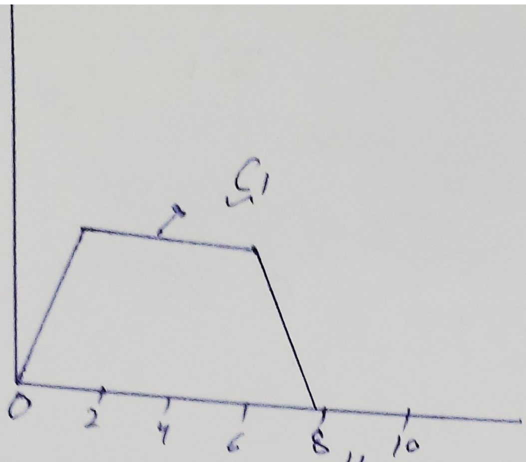
$$Z^* = \frac{a+b}{2}$$

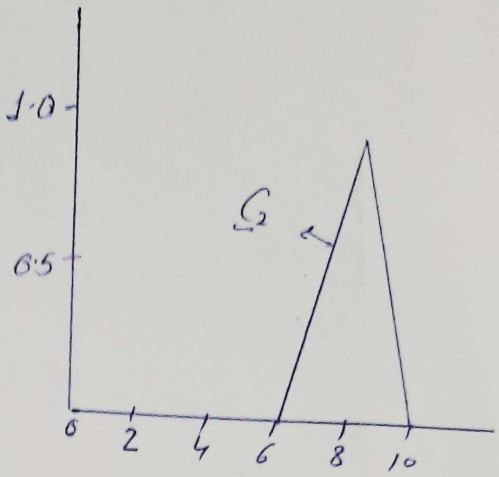


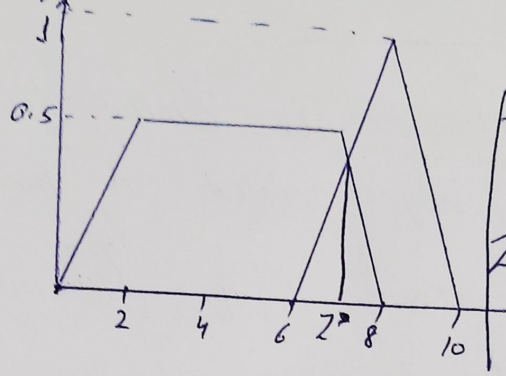
5 Centers of Sums: This is faster than many defuzzification Methods. The Method is not restricted to symmetric Membership function

→ This process Involve the algebric sum of Individual output fuzzy set, Instead of their union

Drawback: ① The Intersecting areas are added twice.
② This Method Involves finding the Centroids of the Individual Membership function







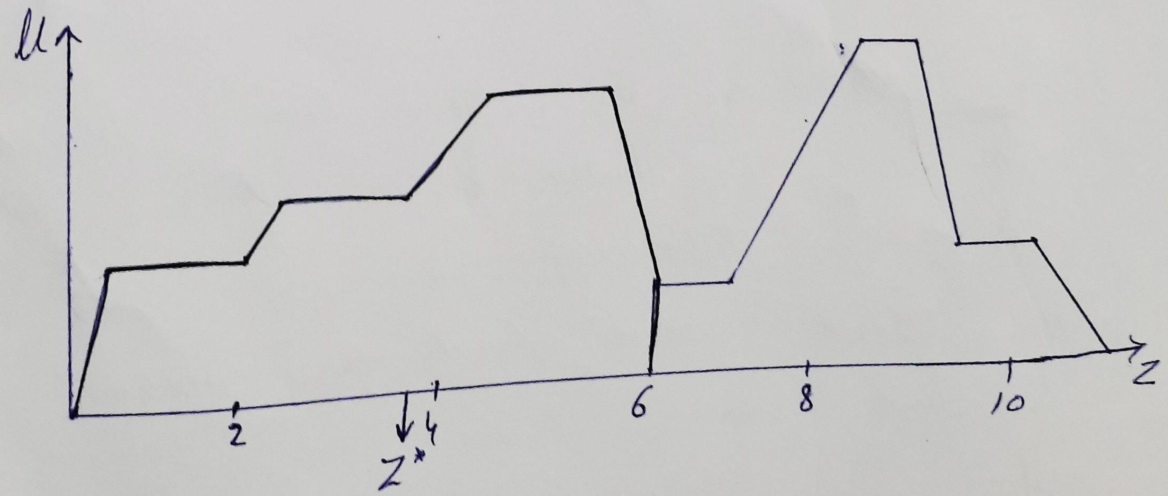
$$Z^* = \frac{\int_z \bar{z} \sum_{k=1}^{n} \mu_{C_k}(z)\, dz}{\int_z \sum_{k=1}^{n} \mu_{C_k}(z)\, dz}$$

6 <u>Center of Largest area</u>: If the output fuzzy set has at least two Convex regions, Then the Center of gravity (z^*) is Calculated using the area Centroid Method of the Convex fuzzy subregion with the Largest is used to obtain z^*

$$Z^* = \frac{\int \mu_{C_m}(z) \cdot z\, dz}{\int \mu_{C_m}(z)\, dz}$$

where C_m is the Convex subregion that has the Largest area making up C_k.

The Convex fuzzy subregion with the Largest area is used to obtain the defuzzified Value z^* of the o/p



7 FIRST (or Last) of Maxima: This Method uses the overall output or union of all individual o/p fuzzy set C_k to determine the smallest value of the domain with maximized membership degree in C_k

- First, the largest height in the union (denoted $hgt(C_k)$) is determined

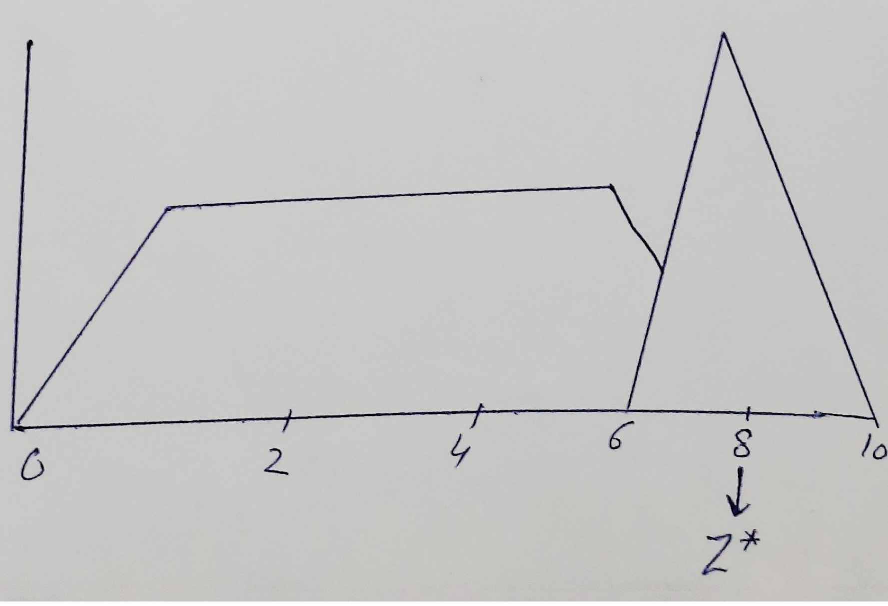
$$hgt(C_k) = \sup_{z \in Z} \mu_{C_k}(z)$$ [Supremum is the least upper bound]

Then the first of maxima is found

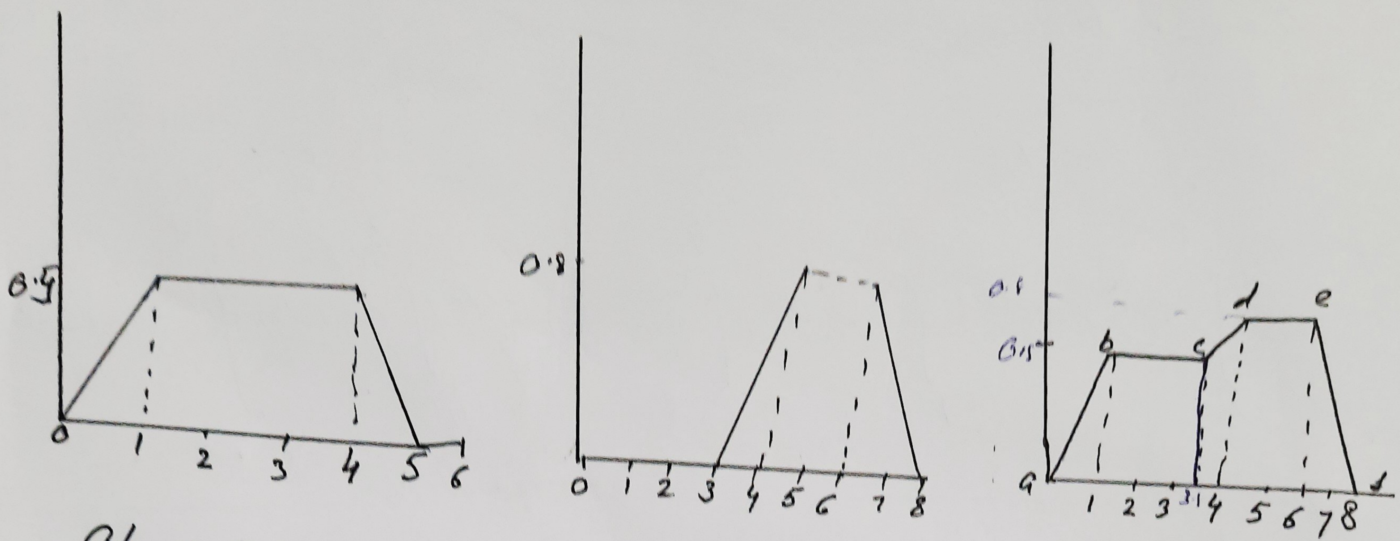
$$z^* = \inf_{z \in Z} \{z \in Z \mid \mu_{C_k}(z) = hgt(C_k)\}$$ [Infimum (inf) is the Greatest Lower bound]

An Alternative of this Method is called the last of maxima

$$z^* = \sup_{z \in Z} \{z \in Z \mid \mu_{C_k}(z) = hgt(C_k)\}$$



EXAMPLE OF CENTEROID METHOD



ab → $\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}$ $y_1=0$ $x_1=0$
$y_2=0.5$ $x_2=1$

$\dfrac{y-0}{x-0} = \dfrac{0.5}{1-0}$ -

$y=0.5x$ ⇒ 0 to 1 Range

bc = $y=0.5$ 1 to 3.5

cd = $y_1=0.5$ $x_1=3.5$
$y_2=0.8$ $x_2=4$

$y=\frac{3}{5}x-\frac{8}{5}$ Range 3.5 to 4

de $\dfrac{y-0.8}{x-4} = \dfrac{0}{2}$ Range 4 to 6

$y=0.8$

ef $\dfrac{y-0.8}{x-6} = \dfrac{0-0.8}{2}$

$y-0.8 = -0.4x+2.4$

$y=0.4x+3.2$

central extends ...

$$Z^* = \frac{\int \mu_{\bar{z}} . z . dz}{\int \mu_{\bar{z}} . dz}$$

$$= \frac{\int_0^1 0.5z . z dz + \int_1^{3.5} 0.5 z . dz + \int_{3.5}^{4} \left(\frac{3}{5} z - \frac{8}{5}\right) . z . dz + \int_4^6 0.8 z . dz + \int_6^8 (-0.4z + 3.2) . z dz}{\int_0^1 0.5 z dz + \int_1^{3.5} 0.5 dz + \int_{3.5}^{4} \left(\frac{3}{5} z - \frac{8}{5}\right) dz + \int_4^6 0.8 dz + \int_6^8 (-0.4z + 3.2) dz}$$

$Z^* = 4.151$

Fuzzy Control System

The application of fuzzy control extends from individual process control to bio medical instrumentation & various security system

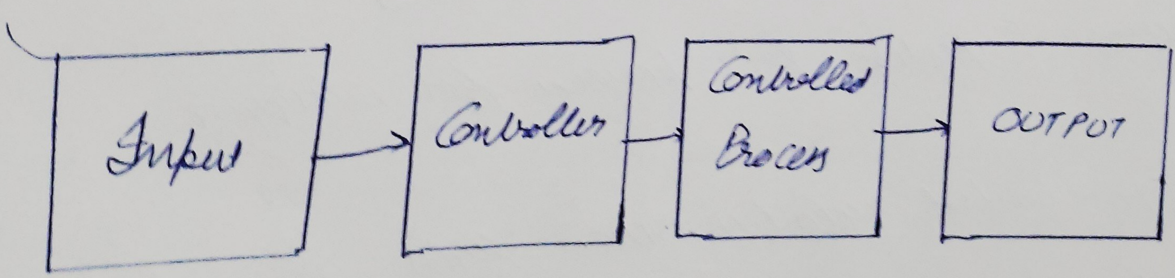
WHAT IS CONTROL SYSTEM?

A control sys. is an arrangement of physical component that is defined to alter another physical sys. so that this sys. will exhibit certain desired characteristics

Types of Control Systems.

1) Open Loop Control System
2) Closed Loop Control System

Open Loop Control System: The input control action is independent of the physical sys output. There is no feedback

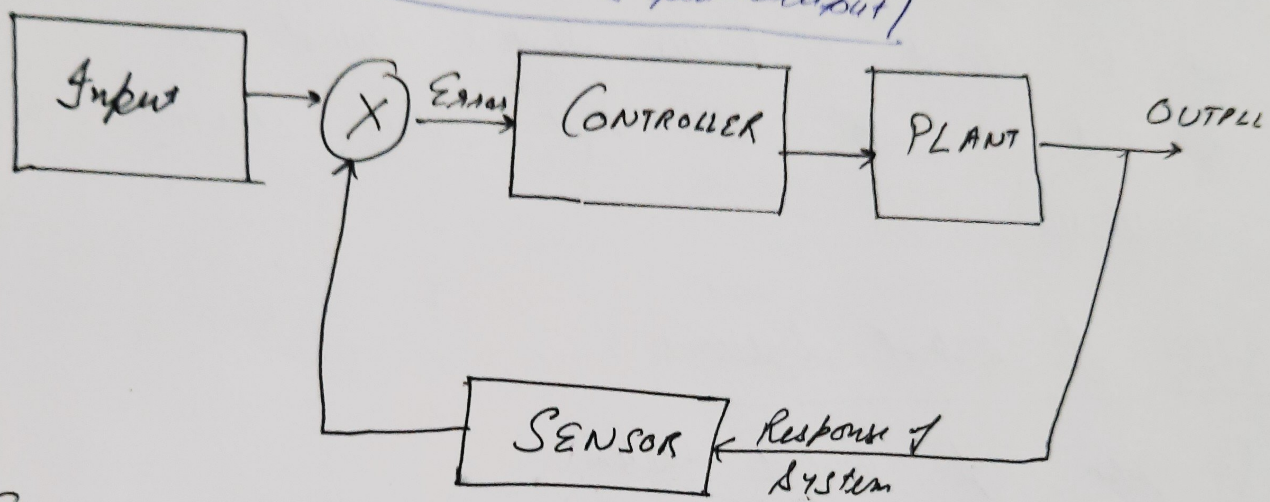


Example: Automatic washing machine

Closed Loop Control System :-

The new output of the sys will depend on the previous output of the sys. The sys. has one or more feedback loop b/w it's output & input

Error Signal : Input-output



Example: Air Conditioner

⇒ WHY USE fuzzy LOGIC IN Control System

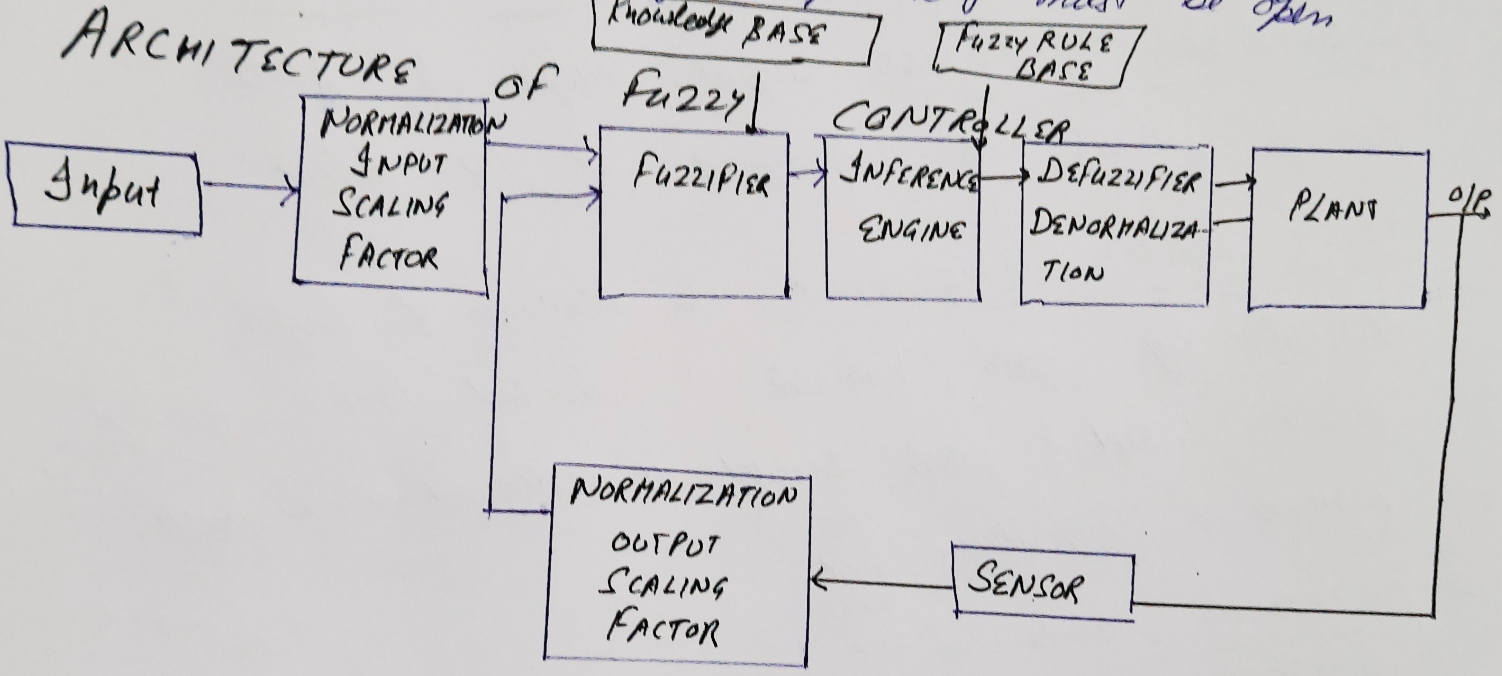
* In traditional control system, we need to know about the model & the objective function that is formulated in a very precise manner
* Utilize human expertise & experience for designing controller
* The fuzzy control rule (IF THEN rule) can be best used in designing controller.

Note: ALL The necessary parameter used in fuzzy controller are defined by membership function

Assumptions in Fuzzy Logic Controller design:

1 The plant is observable & Controllable
2 Existance of a Knowledge Body
3 Existance of a Solution
4 Good Enough solution is enough
5 Range of Precision
6 Issue regarding stability & optimality must be open

ARCHITECTURE OF FUZZY CONTROLLER



* **Normalized I/P O/P Scaling factor:** It is used for scaling & Normalizing I/P & O/P Variable between [0,1] and [-1,1] interval

* **FUZZIFIER:** Convert crisp Value to fuzzy Value

* **Fuzzy Knowledge Base:** Store Knowledge about all I/P & O/P fuzzy relationship. It also has membership function which define I/P Variable to the fuzzy rule base & O/P Variable to the plant that is under Control

Fuzzy Rule Base: Store the knowledge / Logic about the operation of the process

Inference Logic: Simulates human decision for performing approximate reasoning to get the desired output

Defuzzifier: Convert fuzzy value into Crisp Value

→ STEPS FOR DESIGNING FUZZY LOGIC CONTROLLER

1 **Identification of Variable**: Input, output & State Variable must be identified of the plant

2 **FUZZY SUBSET CONFIGURATION**: The Universe of Info. spanned by each Variable is divided into a no of fuzzy subset & each subset is assigned a Linguistic Variable eg. Temprature — Hot, Cold, Warm, Cool

3 **OBTAINING Membership function** → Obtain membership function for each fuzzy subset

4 **Fuzzy RULE BASE CONFIGURATION**: Formulate fuzzy rule base by assigning relationship b/w fuzzy I/P & O/P

5 **Normalizing & Scaling factors**: Appropriate scaling factor for I/P & O/P Variable must be Chosen to normalize Variable b/w [0,1] and [-1,1] Intervals

6 FUZZIFICATION: This Process is done with the help of fuzzifier

7 **Identification of o/p**: Identify the output from each rule using fuzzy approximation reasoning & Combine the fuzzy output obtain from each rule

8 **Defuzzification**: Initiate defuzzification to crisp set

ADVATAGES & DISADVANTAGES OF FUZZY CONTROL SYSTEM

ADVANTAGES	DISADVANTAGES
1 Cheaper	1 Require Lot's of data to be applied
2 Robust	2 Need high human expertise
3 Customizable	3 Need regular updating of the Rule
4 Emulate human deductive thinking	
5 Reliability & Efficiency	

APPLICATION of Fuzzy Logic SYSTEM

Traffic Control

Aircraft flight Control

Steam engine

Elevator Control