Module 4- Digital Electronics

Number System & Representation

Decimal to binary

- $(5)_{10} \longrightarrow (?)_2$.

2	5	
2	2	1
2	1	0
	0	1

$(5)_{10} \longrightarrow (101)_2$.

- $(0.61)_{10} \longrightarrow (\ ?\ )_2$

0.61×2	=	1.22	1
0.22×2	=	0.44	0
0.44×2	=	0.88	0
0.88×2	=	1.76	1
0.76×2	=	1.52	1
0.52×2	=	1.04	1

$(0.61)_{10} \longrightarrow (0.100111)_2$.

- Binary to Decimal

- $(11100)_2 \longrightarrow (?)_{10}$

$\Rightarrow$ 11100 ←

$0\times2^0 + 0\times2^1 + 1\times2^2 + 1\times2^3 + 1\times2^4$

$= 28.$

$(11100)_2 \longrightarrow (28)_{10}.$

- $(0.1101)_2 \longrightarrow (?)_{10}.$

0.1101 →

$1\times2^{-1} + 1\times2^{-2} + 0\times2^{-3} + 1\times2^{-4}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{16}$

$= \frac{8+4+1}{16} = \frac{13}{16}$

$= 0.8125.$

$(0.1101)_2 \longrightarrow (0.8125)_{10}$

- Decimal to octal

- $(246)_{10} \longrightarrow (\,?\,)_8$

$$
\begin{array}{r|ll}
8 & 246 & \\
\hline
8 & 30 & 6 \\
\hline
8 & 3 & 6 \\
\hline
 & 0 & 3
\end{array}
$$

$(246)_{10} \longrightarrow (366)_8$.

- $(0.32)_{10} \longrightarrow (\,?\,)_8$

$0.32 \times 8 = 2.56$ 2

$0.56 \times 8 = 4.48$ 4

$0.48 \times 4 = 1.92$ 1

$0.92 \times 4 = 3.68$ 3.

$(0.32)_{10} \longrightarrow (0.2413)_8$.

- Octal to Decimal

- $(746)_8 \longrightarrow (\,?\,)_{10}$

$\overleftarrow{746} = 6 \times 8^0 + 4 \times 8^1 + 7 \times 8^2.$

$= 486.$

$\Rightarrow (746)_8 \rightarrow (486)_{10}$

- $(0.123)_8 \longrightarrow (?)_{10}$

$1\times 8^{-1} + 2\times 8^{-2} + 3\times 8^{-3}$

$= \cancel{0} \; \frac{1}{8} + \frac{2}{64} + \frac{3}{512}$

$= 0.1621$

$$\boxed{(0.123)_8 \longrightarrow (0.1621)_{10}.}$$

- Decimal to Hexadecimal

- $(2338)_{10} \longrightarrow (?)_{16}.$

16	2338	
16	146	2.
16	9	2.
	0	9

$$\boxed{(2338)_{10} \longrightarrow (922)_{16}.}$$

- $(0.122)_{10} \longrightarrow (?)_{16}.$

$0.122 \times 16 = 1.952$ 1

$.952 \times 16 = 15.232$ $15 \longrightarrow F.$

$0.232 \cancel{15} \times 16 = 3.712$ 3

$0.712 \times 16 = 11.392$ $11 \longrightarrow B.$

$$\boxed{(0.122)_{10} \longrightarrow (0.1F3B)_{16}}$$

Hexadecimal to Decimal

- $(2AF)_{16} \longrightarrow (?)_{10}$

$\longleftarrow$

$F \times 16^0 + A \times 16^1 + 2 \times 16^2$

$15 \times 1 + 10 \times 16 + 2 \times 256$

$\boxed{(2AF)_{16} = (687)_{10}}$

- $(0.F27)_{16} \longrightarrow (?)_{10}.$

$\longrightarrow$

$F \times 16^{-1} + 2 \times 16^{-2} + 7 \times 16^{-3}$

$\frac{15}{16} + \frac{2}{256} + \frac{7}{4096}$

$= 0.94702$

$(0.F27)_{16} \longrightarrow (0.94702)_{10}.$

Binary to Hex

- $(1100110 . 011010)_2 \longrightarrow (\ ?\ )_{16}$

$\longleftarrow \quad \longrightarrow$

$\underbrace{0\,1\,1\,0}_{6}\ \underbrace{0\,1\,1\,0}_{6} \ . \ \underbrace{0\,1\,1\,0}_{6}\ \underbrace{1\,0\,0\,0}_{8}$

$\Rightarrow (66.68)_{10}.$

- Hex to Binary.

- $(FEA)_{16} \rightarrow (?)_2$.

F → 1111, E → 1110, A → 1010

$$\boxed{(FEA)_{16} \rightarrow (1111\ 1110\ 1010)_2}$$

- $(0.A25)_{16}$

.A25: A → 1010, 2 → 0010, 5 → 0101

$$\Rightarrow \boxed{(0.1010\ 0010\ 0101)_2}$$

- Binary to octal.

$(1001010.01100101)_2 \rightarrow (?)_8$.

← →

001 001 010 . 011 001 010

1 1 2 . 3 1 2

$$\Rightarrow \boxed{(112.312)_8}$$

Scanned with CamScanner

- Octal to Binary.

$(123.567)_8 \longrightarrow (\ ?\ )_2$

1 2 3 . 5 6 7 $\longrightarrow$ $(001\ 010\ 011 . 101\ 110\ 111)_2$.

001 010 011 101 110 111

$\Downarrow$

$\boxed{(1\ 010\ 011\ .101\ 110\ 111)_2}$

- Hex to octal.

(i) Hex $\longrightarrow$ Binary

(ii) Binary $\longrightarrow$ octal.

$(BB.0A)_{16} \longrightarrow (?)_8$

(i) $(1011\ 1011\ .\ 0000\ 1010)_2$

(ii) 010 111 011 . 000 010 100 .

2 7 3 . 0 2 4

$\boxed{(BB.0A)_{16} \longrightarrow (273.024)_8}$

- Octal to Hex

(i) Octal $\rightarrow$ Binary

(ii) Binary $\rightarrow$ Hex

- $(436.71)_8 \longrightarrow (?)_{16}$

(i) 4 3 6 . 7 1

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

100 011 110 111 001

(ii) 0001 0001 1110 . 1110 0100

1 1 E . E 4

$\boxed{(11E.E4)_{16}}$

1's <u>complement</u>.

- Replace '0' by '1' and '1' by '0's.

2's <u>complement</u>.

(i) Find 1^{st} complement

(ii) Add '1' to it.

Scanned with CamScanner

Boolean Algebra.

- ~~Duality~~

$\cancel{\bar{A}\cdot B + A\cdot\bar{B}} \neq \cancel{(\bar{A}+A)\cdot C}$

AND laws.

- $A \cdot 0 = 0$
- $A \cdot 1 = A$
- $A \cdot A = A$
- $A \cdot \bar{A} = 0$

OR laws.

- $A + 0 = A$.
- $A + 1 = 1$
- $A + A = A$
- $A + \bar{A} = 1$

- De Morgan's Theorem.

$\overline{A+B} = \bar{A} \cdot \bar{B}$.

$\overline{AB} = \bar{A} + \bar{B}$

$\Rightarrow (C + AD)$

$(C+A)(C+D)$

- commutative law.

$A + B = B + A$.

$A \cdot B = B \cdot A$

- Associative law.

$$(A+B)+C = A+(B+C)$$

$$A(BCD) = (ABC)D.$$

- Distributive law.

$$A(B+C) = AB+AC.$$

$$A+BC = (A+B)(A+C)$$

$$A+\bar{A}B = (A+\bar{A})(A+B) = 1\cdot(A+B) = A+B.$$

Q. Minimize the following expressions using boolean algebra:

(i) $Y = \bar{A}B + A\bar{B} + AB.$

$= \bar{A}B + A(\bar{B}+B)$

$= \bar{A}B + A\cdot 1$

$= A + \bar{A}B$

$= (A+\bar{A})(A+B)$

$= A+B.$

(ii) $Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$

$= \bar{B}C(\bar{A}+A) + \bar{A}B\bar{C} + A\bar{B}\bar{C}$

$= \bar{B}C + \bar{C}(\bar{A}B + A\bar{B})$

$= \bar{B}C + \bar{C}\bar{A}B + A\bar{B}\bar{\bar{C}}$

$= \bar{B}(C + A\bar{C}) + \bar{A}B\bar{C}.$

$= \bar{B}\left[(C+\bar{C})(C+A)\right] + \bar{A}B\bar{C}$

$= \bar{B}(C+A) + \bar{A}B\bar{C}$

$= \boxed{A\bar{B} + \cancel{A}\bar{B}C + \bar{A}B\bar{C}}$

(iii) $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

$= \bar{B}\bar{C}(\bar{A}+A) + B\bar{C}(\bar{A}+A)$

$= \bar{B}\bar{C} + B\bar{C}$

$= \bar{C}(B+\bar{B})$

$= \bar{C}.$

(iv) $F = \bar{Y}\bar{Z} + \bar{W}\bar{X}\bar{Z} + \bar{W}\bar{X}\bar{Y}\bar{Z} + Y\bar{Z}$

$= \cancel{\bar{Y}\bar{Z}(1+\bar{W}\bar{X}) + \bar{W}\bar{X}\bar{Z}(1+\bar{Y})}$

$= \cancel{\bar{Y}\bar{Z} + \bar{W}\bar{X}\bar{Z}}$

$= \cancel{\bar{Z}(Y}$

$= \bar{Y}\bar{Z}(1+\bar{W}\bar{X}) + \bar{W}\bar{X}\bar{Z} + Y\bar{Z}$

$= \bar{Y}\bar{Z} + \bar{W}\bar{X}\bar{Z} + Y\bar{Z}$

$= \bar{Z}(Y+\bar{Y}) + \bar{W}\bar{X}\bar{Z}$

$= \bar{Z} + \bar{W}\bar{X}\bar{Z}.$

$= \bar{z}(1+\bar{w}\bar{x})$

$\boxed{F = \bar{Z}}$

(v) $F = A + \bar{A}B\bar{C} + \overline{B+C}$

$= A + \bar{A}B\bar{C} + \bar{B}\bar{C}$

$= (A+\bar{A})(A+B\bar{C}) + \bar{B}\bar{C}$

$= A + B\bar{C} + \bar{B}\bar{C}$

$= A + \bar{C}(B+\bar{B})$

$= A + \bar{C}$

(vi) $Y = AB + AC + BC + B.$

$= A(B+C) + B(1+C).$

$= A(B+C) + B.$

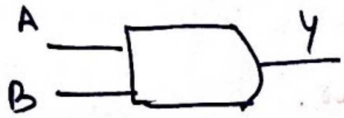
$= AB + AC + B$

$= B(1+A) + AC.$

$= \underline{\underline{B + AC}}.$

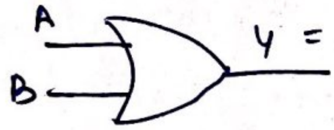
logic gates.

AND



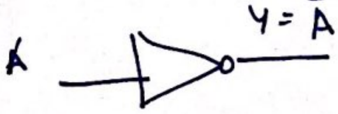
$Y = A \cdot B.$

OR



$Y = A + B$

NOT



$Y = \overline{A}$

XOR.

$Y = \overline{A}B + A\overline{B}$

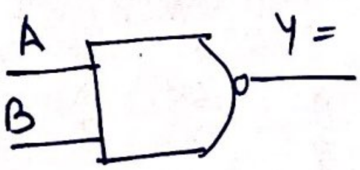
$= A \oplus B.$

XNOR.

$Y = \overline{A}\overline{B} + AB.$

$= \overline{A \oplus B}$

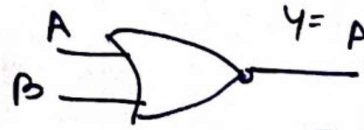
NAND



$Y = \overline{A \cdot B}$

$= \overline{A} + \overline{B}.$

NOR.



$Y = \overline{A+B}$

$= \overline{A} \cdot \overline{B}$

NAND & NOR $\longrightarrow$ universal gates.

K Map.

2 variable K Map.

(i) $Y = A\bar{B} + AB$ (Ans: A)

(ii) $Y = \bar{A}B + AB + A\bar{B}$ (Ans: $A+B$)

(iii) $Y = \Sigma m(0,3)$ (Ans: $\bar{A}\bar{B} + AB$)

(iv) $Y(A,B) = \Sigma m(0,1,2,3)$. (Ans: 1)

3 Variable, 4 Variable.

Q. i) $f(A,B,C) = \Sigma m(0,2,4,6)$

A \ BC	00	01	11	10
0	1_0	$_1$	$_3$	1_2
1	1_4	$_5$	$_7$	1_6

$= \bar{C}$

(ii) $f(A,B,C) = \Sigma m(1,2,4,5)$

A \ BC	00	01	11	10
0	$_0$	1_1	$_3$	1_2
1	1_4	1_5	$_7$	$_6$

$= \bar{B}C + A\bar{B} + \bar{A}B\bar{C}$

Q: $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C$

(Ans: $\bar{A}\bar{B} + \bar{A}\bar{C}$)

(iii) $F(A,B,C,D) = \Sigma m(0,1,2,5,13,15)$

AB \ CD	00	01	11	10
00	1_0	1_1	$_3$	1_2
01	$_4$	1_5	$_7$	$_6$
11	$_{12}$	1_{13}	1_{15}	$_{14}$
10	$_8$	$_9$	$_{11}$	$_{10}$

$Y = \bar{A}\bar{C}D + ABD + \bar{A}\bar{B}\bar{D}$

Q: $F(A,B,C,D) = \Sigma m(1,4,6,12,14,9)$

Ans: $B\bar{D} + \bar{B}\bar{C}D$

(iv) $F(A,B,C,D) = \Sigma m(1,3,7,8,10,12,13,15)$

AB \ CD	00	01	11	10
00	$_{0}$	1 $_{1}$	1 $_{3}$	$_{2}$
01	$_{4}$	$_{5}$	1 $_{7}$	$_{6}$
11	1 $_{12}$	1 $_{13}$	1 $_{15}$	$_{14}$
10	1 $_{8}$	$_{9}$	$_{11}$	1 $_{10}$

Q: $F(A,B,C,D) = \Sigma m(0,8,2,10)$ (Ans: $\bar{B}\bar{D}$)

$= \bar{A}\bar{B}D + BCD + AB\bar{C} + A\bar{B}\bar{D}$

(v). $F(A,B,C,D) = \Sigma m(1,3,5,9,11,13)$

AB \ CD	00	01	11	10
00	$_{0}$	1 $_{1}$	1 $_{3}$	$_{2}$
01	$_{4}$	1 $_{5}$	$_{7}$	$_{6}$
11	$_{12}$	1 $_{13}$	$_{15}$	$_{14}$
10	$_{8}$	1 $_{9}$	1 $_{11}$	$_{10}$

$= \bar{C}D + \cancel{A\bar{B}}\ \bar{B}D.$

Q. (i) $F(A,B,C,D) = \Sigma m(1,4,6,12,14,9)$ (Ans: $B\bar{D} + \bar{B}\bar{C}D$)
(ii) $F(A,B,C,D) = \Sigma m(0,8,2,10)$ (Ans: $\bar{B}\bar{D}$).

(vi) $F(A,B,C,D) = \Sigma m\,(1,5,7,9,11,13,15)$

AB \ CD	00	01	11	10
00	0	1 ₁	3	2
01	4	1 5	1 7	6
11	12	1 13	1 15	14
10	8	1 9	1 11	10

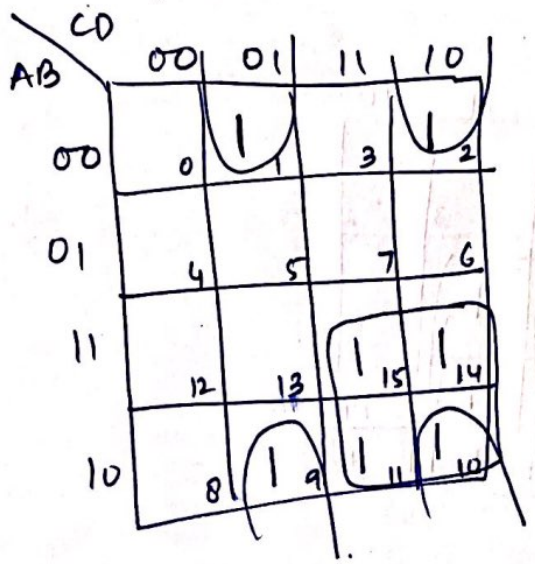
$\bar{C}D + BD + AD.$

(vii) $F(A,B,C,D) = \Sigma m\,(4,5,8,9,11,12,13,15)$

AB \ CD	00	01	11	10
00	0	1	3	2
01	1 4	1 5	7	6
11	1 12	1 13	1 15	14
10	1 8	1 9	1 11	10

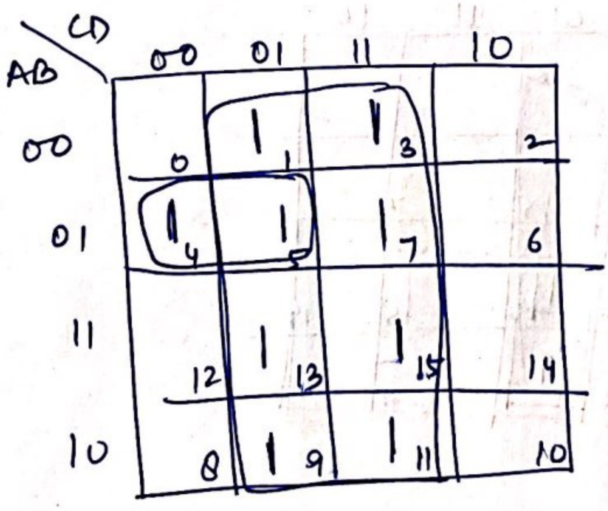
$\cancel{\bar{C}D} + B\bar{C} + A\bar{C} + AD.$

(viii) $f(A,B,C,D) = \Sigma m(1,2,9,10,11,14,15)$



$= AC + \bar{B}C\bar{D} + \bar{B}\bar{C}D.$

(ix) $f(A,B,C,D) = \Sigma m(1,3,4,5,7,9,11,13,15)$



$= \boxed{D + \bar{A}B\bar{C}}$

Q. $f(A,B,C,D) = \Sigma m(1,3,4,5,7,9,11,13,15) + \Sigma d(12,14)$

(Ans: $D + B\bar{C}$)

Q. $F(A,B,C,D) = \Sigma m(1,5,7,9,11,13,15) + \Sigma d(3,2,6,14,10)$

(Ans: $F = D$).

K Map. (5 Variable).

(i) $f(A,B,C,D,E) = \Sigma m(0,1,7,9,11,13,15,16,17,23,25,27)$

BC \ DE	00	01	11	10
00	1_{0}	1_{1}	3	2
01	4	5	1_{7}	6
11	12	1_{13}	1_{15}	14
10	8	1_{9}	1_{11}	10

$\bar{A} = 0$

BC \ DE	00	01	11	10
00	1_{16}	1_{17}	19	18
01	20	21	1_{23}	22
11	28	29	31	30
10	24	1_{25}	1_{27}	26

$A = 1$

$\bar{A}BE + B\bar{C}E + \bar{B}\bar{C}\bar{D} + \bar{B}CDE$

(ii) $f(A,B,C,D,E) = \Sigma m(0,2,5,7,13,15,18,20,21,23,28,29,31)$.

BC \ DE	00	01	11	10
00	1_{0}	1	3	1_{2}
01	4	1_{5}	1_{7}	6
11	12	1_{13}	1_{15}	14
10	8	9	11	10

$\bar{A}$

BC \ DE	00	01	11	10
00	16	17	19	1_{18}
01	1_{20}	1_{21}	1_{23}	22
11	1_{28}	1_{29}	1_{31}	30
10	24	25	27	26

A

$= \boxed{CE + AC\bar{D} + \bar{A}\bar{B}\bar{C}\bar{E} + \bar{B}\bar{C}D\bar{E}}$

(iii). $f = \sum m(1,5,6,7,11,12,13,15)$
(A,B,C,D).

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	1 5	1 7	1 6
11	1 12	1 13	1 15	14
10	8	9	1 11	10

Redundant.

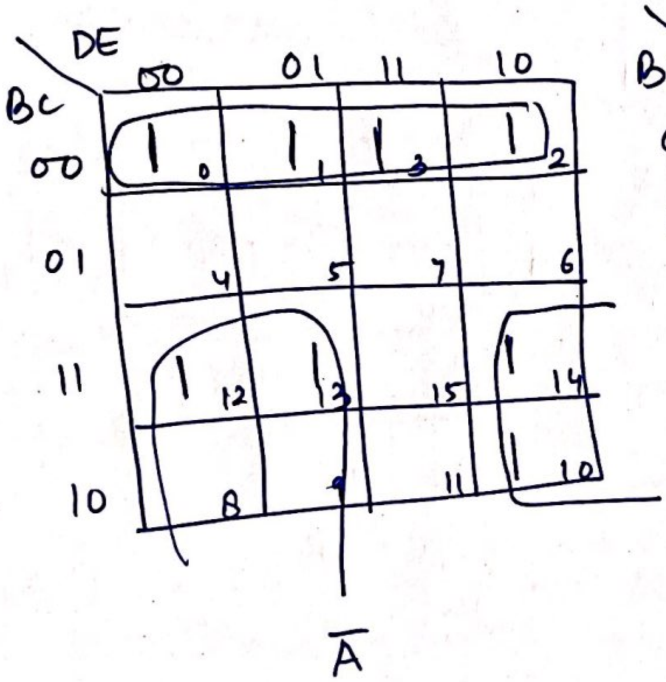
$= \bar{A}BC + \bar{A}\bar{C}D + AB\bar{C} + ACD.$

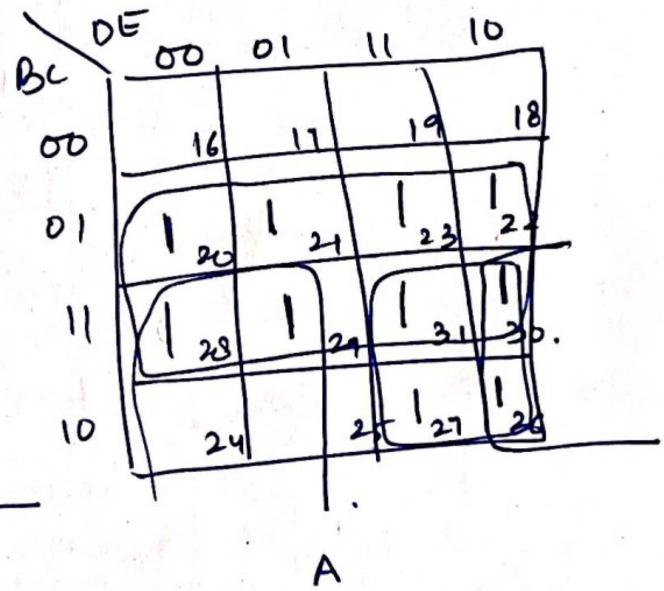
(iv). $f(A,B,C,D) = \sum m(0,1,2,5,13,15)$

AB \ CD	00	01	11	10
00	1 0	1 1	3	1 2
01	4	1 5	7	6
11	12	1 13	1 15	14
10	8	9	11	10

$= \bar{A}\bar{B}\bar{D} + \bar{A}\bar{C}D + ABD.$

(v) $F(A,B,C,D,E) = \Sigma m\,(0,1,2,3,10,12,13,14,$
$20,21,22,23,26,27,28,29,$
$30,31)$





$AC + BC\bar{D} + BD\bar{E} + \bar{A}\bar{B}\bar{C}$

$+ ABD.$

K Map (6 variable).

$A'B'$

CD \ EF	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$A'B$

CD \ EF	00	01	11	10
00	16	17	19	18
01	20	21	23	22
11	28	29	31	30
10	24	25	27	26

AB'

CD \ EF	00	01	11	10
00	32	33	35	34
01	36	37	39	38
11	44	45	47	46
10	40	41	43	42

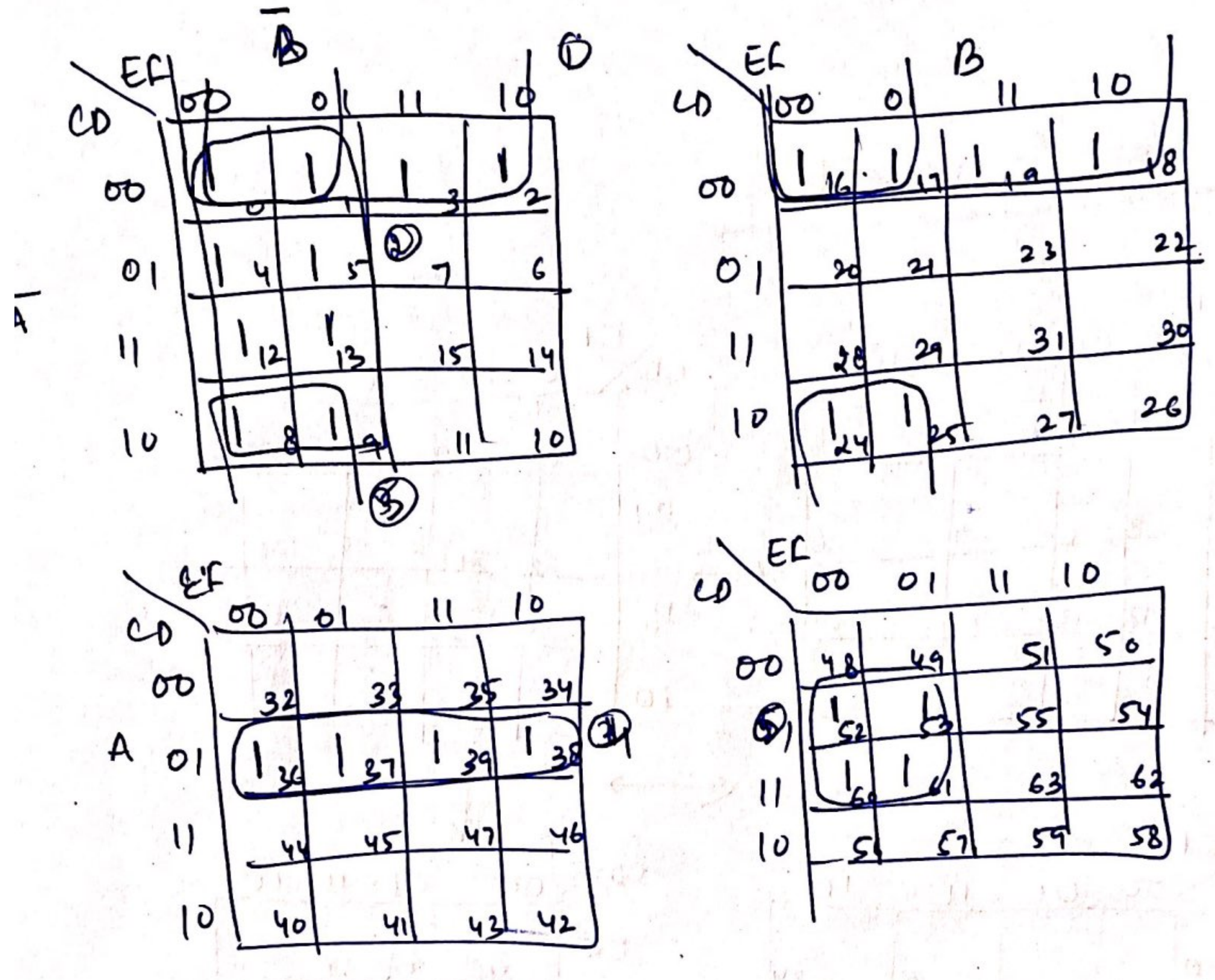
AB

CD \ EF	00	01	11	10
00	48	49	51	50
01	52	53	55	54
11	60	61	63	62
10	56	57	59	58

- first check the corners of all K Maps.

(i) $Y = \Sigma m(0, 1, 2, 3, 4, 5, 8, 9, 12, 13, 16, 17, 18, 19, 24, 25, 36, 37, 38, 39, 52, 53, 60, 61)$

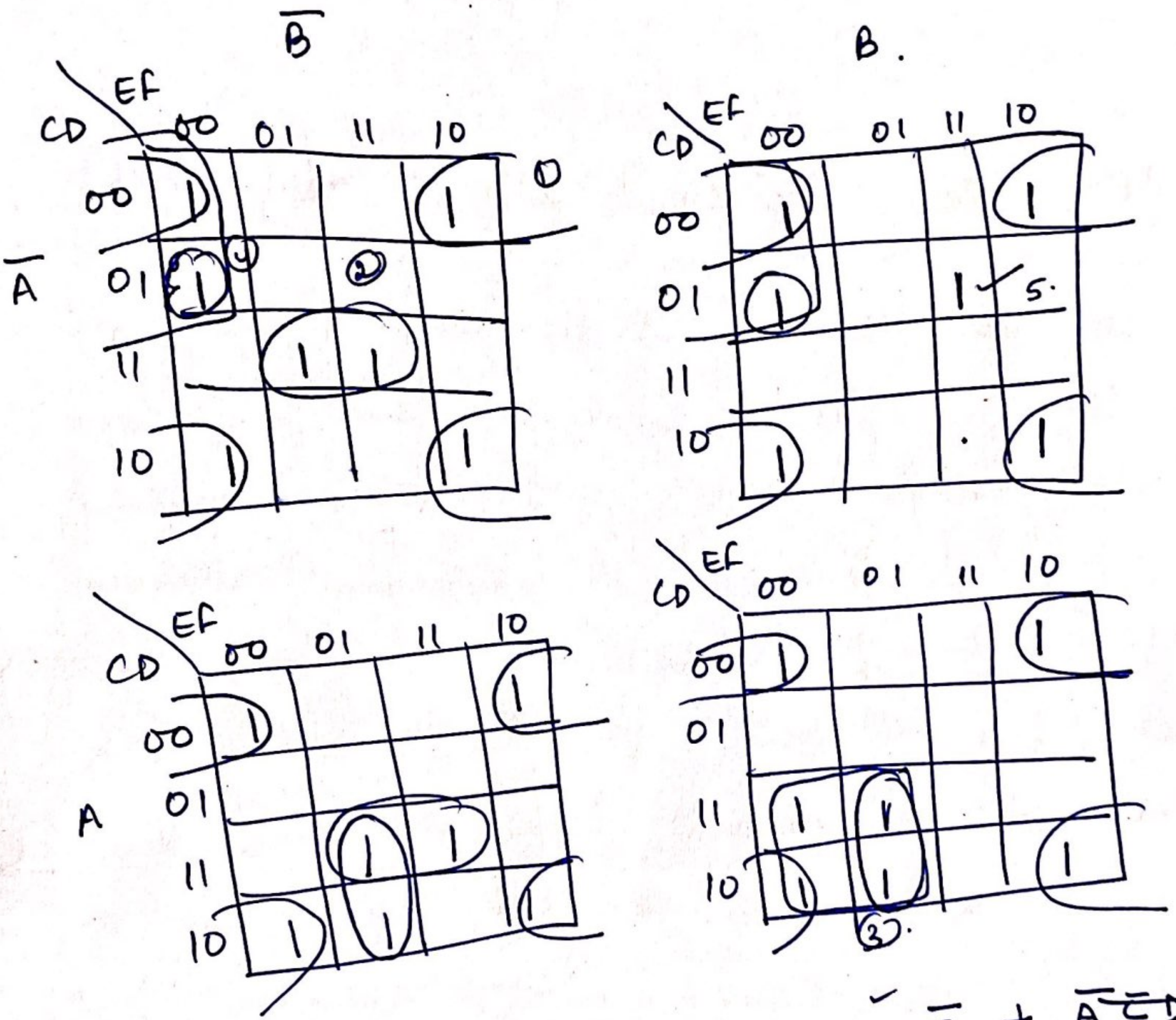
Scanned with CamScanner



$= \bar{A}\bar{B}E + \bar{A}\bar{C}\bar{D} + A\bar{B}\bar{C}D + ABD\bar{E}$

$+ \cancel{\bar{A}\bar{C}\bar{D}} \cdot \bar{A}\bar{D}\bar{E}$

(ii) $Y = \Sigma(0, 2, 4, 8, 10, 13, 15, 16, 18, 20, 23, 24, 26, 32, 34, 40, 41, 42, 45, 47, 48, 50, 56, 57, 58, 60, 61)$



$$= \bar{D}\bar{F} + \bar{B}CDF + ABC\bar{E} + \cancel{\bar{A}\bar{C}D\bar{E}\bar{F}}$$
$$\bar{A}C\bar{E}\bar{F}$$
$$+ \bar{A}B\bar{C}DEF.$$
$$+ AC\bar{E}F.$$

Introduction to IC technology : SSI, MSI, LSI

⇒

The first integrated circuits contained only a few
transistors and so were called "Small scale
integration" (SSI). They used circuits containing
transistors numbering in the tens. They were
very crucial in development of early computers.
SSI was followed by introduction of the
devices which contained hundreds of transistors
on each chip and so were called "Medium-
Scale Integration (MSI)".
MSI were attractive economically. Next development
was of Large Scale Integration (LSI). The
development of LSI was driven by economic
factors and each chip comprised of
tens of thousands of transistors. It was in 1970s when
LSI started getting manufactured in huge quantities.

⇒ An integrated circuit or monolithic integrated circuit
(IC, chip or microchip) is a set of electronic
circuits on one small flat piece of semiconductor
material that is normally Silicon.

The integration of large no. of tiny MOS transistors into a small chip results in circuits that are orders of magnitude smaller, faster and less expensive than those constructed of discrete electronic components. The IC's mass production capability, reliability and building-block approach to integrated circuit design has ensured the rapid adoption of standardized IC's in place of designs using discrete transistors.

IC's are now used in virtually all electronic equipments.

Advantages of IC's over discrete circuits.

↔ cost and performance.

⇒ cost is low because the chips, with all their components are printed as a unit by photolithography rather than being constructed one transistor at a time.

Performance is high because The IC's components switch quickly and consume comparatively little power because of their small size and proximity.

The main disadvantage of IC's is the high cost to design them and fabricate the required photomasks.

		Year	Transistor count	Logic gates no.
SSI	Small scale Integration	1964	1 to 10	1 – 12
MSI	Medium scale Integration	1968	10-500	13 - 99
LSI	Large scale Integration	1971	500 - 20,000	100 - 9999
VLSI	very large scale Integration	1980	20,000-1,00,0000	10,000 – 99,999

VLSI (Very Large Scale Integration)

The final step in the development process, starting in the 1980s and continuing through the present is, "very large scale integration" (VLSI). The development started with hundreds of thousands of transistors in the early 1980s. As of 2016, transistor counts continue to grow beyond ten billion transistors per chip.

Multiple developments were required to achieve this increased density. Manufacturers moved to smaller MOSFET design rules and cleaner fabrication facilities so that they could make chips with more transistors and maintain adequate yield. Electronic design tools improved enough to make it practical to finish these designs in a reasonable time. The more energy efficient CMOS replaced NMOS and PMOS, avoiding a prohibitive increase in power consumption.

Modern VLSI devices contain so many transistors, layers, interconnections and other features that is no longer feasible to check the masks or do the original design by hand. Instead, engineers use EDA tools to perform most functional verification work.