OPIM 5603 — Statistics in Business Analytics
Fall 2019, University of Connecticut

Homework 4 - v1

Instructions: Please complete the following questions and submit them as an RNotebook (as an `Rmd` file) via the submission link on HuskyCT. You must submit the assignment by the time and due date listed on the course syllabus. Failure to submit a file by the deadline will result in a score of 0 on the assignment.

Set the heading of the RNotebook as an `html_document`, with a table of contents and without numbered sections. Add your name and a date to the header as well. The solution to each problem should be a separate section (specified by #), and each subproblem should be set as a subsection (specified as ##). For example, for Problem 2, you should have a section titled Problem 2, specified by:

```
# Problem 2
```

in your RNotebook. Also, for subproblem b in Problem 2, you should have a subsection, specified by:

```
## Problem 2b
```

As with all course material, the problems appearing in this homework assignment are taken from the instructor's real-world experiences, from other courses taught at the University of Connecticut, and from the sources listed in the course syllabus.

Note that R code submitted should work independent of the data that sits in the data structure. For example, suppose there was a vector `r_vec` with the values $(1, 2, 6)$ and the problem asks for you to create R code to create a vector `answer` which doubles each element of `r_vec`. The answer

$$\texttt{answer} \leftarrow \text{c}(2, 4, 12)$$

would be given no credit. The answer

$$\texttt{answer} \leftarrow 2^{*}\texttt{r\_vec}$$

would be an appropriate answer. If you have any questions, please submit them via email to the instructor and/or the teaching assistant prior to submitting your solution.

Problem 1 (35 points)—Binomial Distribution

Eventbrite handles the sales of VIP tickets to a partciular sporting event every year. Based on historical data they estimate that 8% of the visitors do not show up and thereby seats are left unoccupied. Eventbrite is considering overbooking for the actual capacity of 300 VIP seats in order to fill all the seats and increase their revenue from pre-sold ticket. Suppose they decide to sell 325 tickets.

a. What is the expected number of attendees that show up?

b. What is the probability that all seats are occupied?

c. What is the probability that more people show up than there is space for?

d. Given that at least one of the seats is vacant, what is the probability that attendees for at least 90% of the VIP capacity showed up?

e. Suppose that all of the seats are occupied. What is the probability that exactly 300 people arrived?

f. Suppose that VIP tickets sell for \$250 and Eventbrite has to pay every person that shows up and doesn't have a seat \$300 on top of refunding the \$250. How many VIP tickets should Eventbrite offer to maximize revenue?

Problem 2 (35 points)—Poisson Distribution

A bank has two branches in a small town, and estimates that on average three customers every 15 minutes show up to the first branch and two customers every 15 minutes show up to the second branch.

a. What is the expected number of customers that show up to the first bank in a standard business day (9am-5pm)?

b. What is the probability that more than 100 customers show up to the first branch in a normal business day?

c. What is the probabilty that greater than or equal to 3 but less than 10 customers show up to the first branch in two hours?

d. Challenge: What is the probabilty that at least 7 people show up to either branch within 15 minutes?

Problem 3 (30 points)—`R` programming

In class we explored how to calculate probabilities associated with events on binomial random variables. In this problem we will replicate one of the functions and add a few more.

a. Careate a function called `my_dbinom` which takes three required arguments. The arguments are `x`, `n`, and `pi`. The function should return the probability that a random variable that is binomially distributed equals `x` given that the number of independent trials is `n` and that the success probability is `pi`.

b. Write a function that calculates the expectation of a binomial random variable, which takes the two parameters of the binomial distribution as its arguments. Make the argument associated with the success probability an optional argument with default value 0.5.

c. Write a function that calculates the variance of a binomial random variable, which takes the two parameters of the binomial distribution as its arguments. Make the argument associated with the success probability an optional argument with default value 0.5.