OPIM 5603 — Statistics in Business Analytics
Fall 2019, University of Connecticut

Homework 3 - v1

Instructions: Please complete the following questions and submit them as an RNotebook (as an `Rmd` file) via the submission link on HuskyCT. You must submit the assignment by the time and due date listed on the course syllabus. Failure to submit a file by the deadline will result in a score of 0 on the assignment.

Set the heading of the RNotebook as an `html_document`, with a table of contents and without numbered sections. Add your name and a date to the header as well. The solution to each problem should be a separate section (specified by #), and each subproblem should be set as a subsection (specified as ##). For example, for Problem 2, you should have a section titled Problem 2, specified by:

```
# Problem 2
```

in your RNotebook. Also, for subproblem b in Problem 2, you should have a subsection, specified by:

```
## Problem 2b
```

As with all course material, the problems appearing in this homework assignment are taken from the instructor's real-world experiences, from other courses taught at the University of Connecticut, and from the sources listed in the course syllabus.

Note that R code submitted should work independent of the data that sits in the data structure. For example, suppose there was a vector `r_vec` with the values $(1, 2, 6)$ and the problem asks for you to create R code to create a vector `answer` which doubles each element of `r_vec`. The answer

$$\texttt{answer} \leftarrow \text{c}(2, 4, 12)$$

would be given no credit. The answer

$$\texttt{answer} \leftarrow 2^{*}\texttt{r\_vec}$$

would be an appropriate answer. If you have any questions, please submit them via email to the instructor and/or the teaching assistant prior to submitting your solution.

Problem 1 (25 points)—Basic Probability

Let A and B be arbitrary events.

a. Is the following statement true or false? If true, explain why, and if false, come up with a counter example. If A and B are independent then A and B' are independent.

b. Suppose $P(A) = \frac{2}{3}$ and $P(B) = \frac{4}{7}$.

   (a) What is the maximum value of $P(A \cap B)$?

   (b) What is the minimum value of $P(A \cap B)$?

   (c) What is the maximum value of $P(A \cap B')$

   (d) What is the minimum value of $P(A \cap B')$

   (e) What is the maximum value of $P(A \cup B)$?

   (f) What is the minimum value of $P(A \cup B)$?

   (g) What is the maximum value of $P(A|B)$?

   (h) What is the minimum value of $P(A|B)$?

Problem 2 (25 points)—Simple Bayes

A laboratory blood test is 95 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1 percent of the healthy persons tested. (That is, if a healthy person is tested, then, with probability 0.01, the test result will imply he has the disease.) If 0.5 percent of the population actually has the disease, what is the probability a person has the disease given that his test result is positive?

Problem 3 (25 points)—Bayes

An aircraft transmitter is a device designed to transmit a signal in the case of a crash. Company A makes 80% of the aircraft transmitters, Company B makes 15% of them, and the Company C makes the other 5%. The transmitters made by Company A have a 3% rate of defect, those made by Company B have a 6.5% rate of defect, and those made by Company C have a 9% rate of defect.

a. If an aircraft transmitter is randomly selected from the general population of all aircraft transmitters, find the probability that it was made by Company A.

b. If a randomly selected aircraft transmitter is tested and is found to be defective, find the probability that it was made by Company A.

Problem 4 (25 points)—Independence

Suppose your firm has three potential investments. The investments are either sucessful or not. Suppose that each investment has probabilty $\frac{1}{2}$ of being successful.

a. What is the probability that the third investment is successful?

b. What is the probability that the third investment is successful, given that the three investments are either all successful or all not successful?

c. What is the probability that the third investment is sucessful, given that two of the three investments is successful?

d. Suppose now that the probability that investment 1 is successful is 0.845, the probability that investment 2 is successful is 0.5505, and the probability that investment 3 is successful is 0.4. Consider these two events: A: two of the three investments are successful, and B: investment 3 is successful. Are these events independent? Why or why not?