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Mechanics of Materials, Eighth Edition

Chapter 2

Stress and Strain – Axial Loading

Contents

Stress & Strain: Axial Loading

- Suitability of a structure or machine may depend on the deformations in the structure as well as the stresses induced under loading. Statics analyses alone are not sufficient.
- Considering structures as deformable allows determination of member forces and reactions which are statically indeterminate.
- Determination of the stress distribution within a member also requires consideration of deformations in the member.
- Chapter 2 is concerned with deformation of a structural member under axial loading. Later chapters will deal with torsional and pure bending loads.

Normal Strain

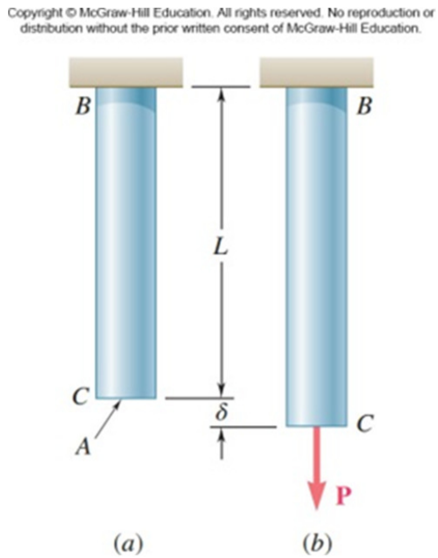


Figure 2.1 Undeformed and deformed axially loaded rod.

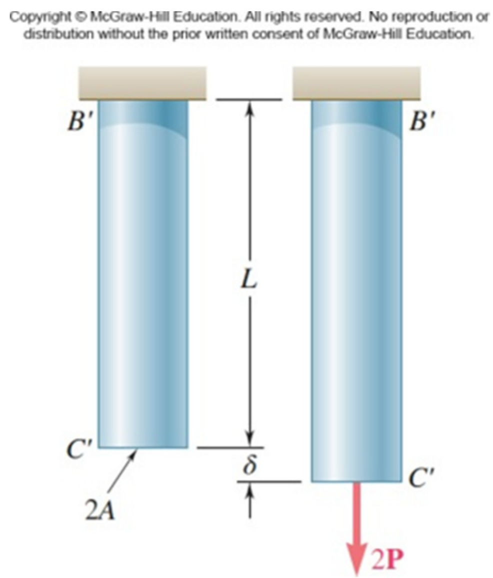


Figure 2.3 Twice the load is required to obtain the same deformation δ when the cross-sectional area is doubled.

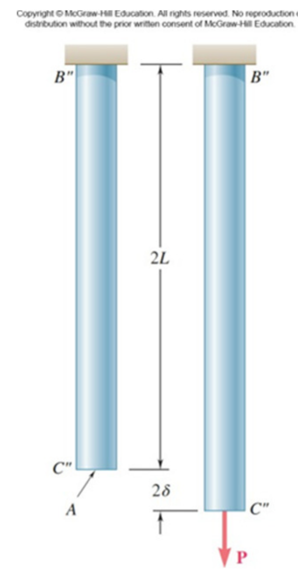


Figure 2.4 The deformation is doubled when the rod length is doubled while keeping the load ***P*** and cross-sectional area A.

$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{normal strain}$$

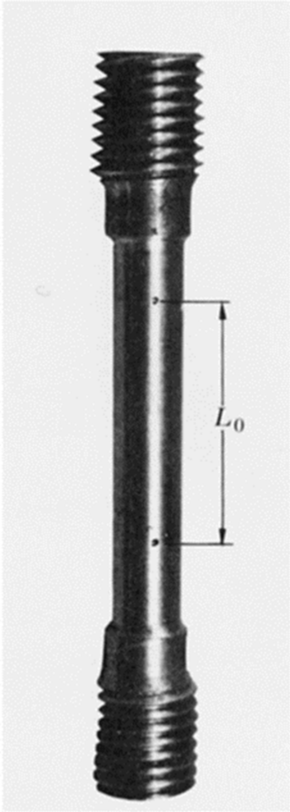
$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$

$$\varepsilon = \frac{\delta}{L}$$

$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

Stress-Strain Test



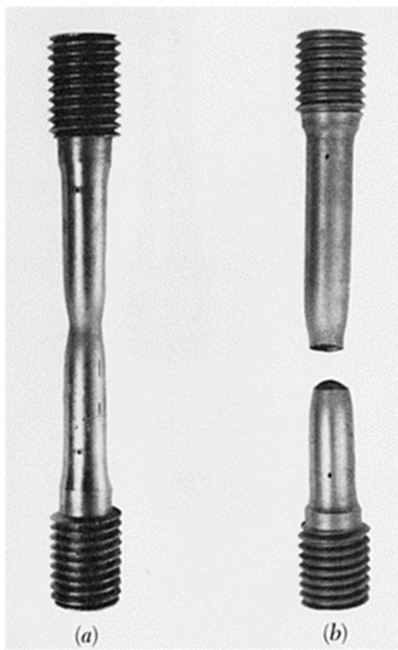
Courtesy of John DeWolf

Courtesy of Tinius Olsen Testing Machine Co., Inc.

Photo 2.1 Elongated tensile-test specimen. Undeformed gage length is L_0.

Photo 2.2 Universal test machine used to test tensile specimens.

Stress-Strain Diagram: Ductile Materials



Courtesy of John DeWolf

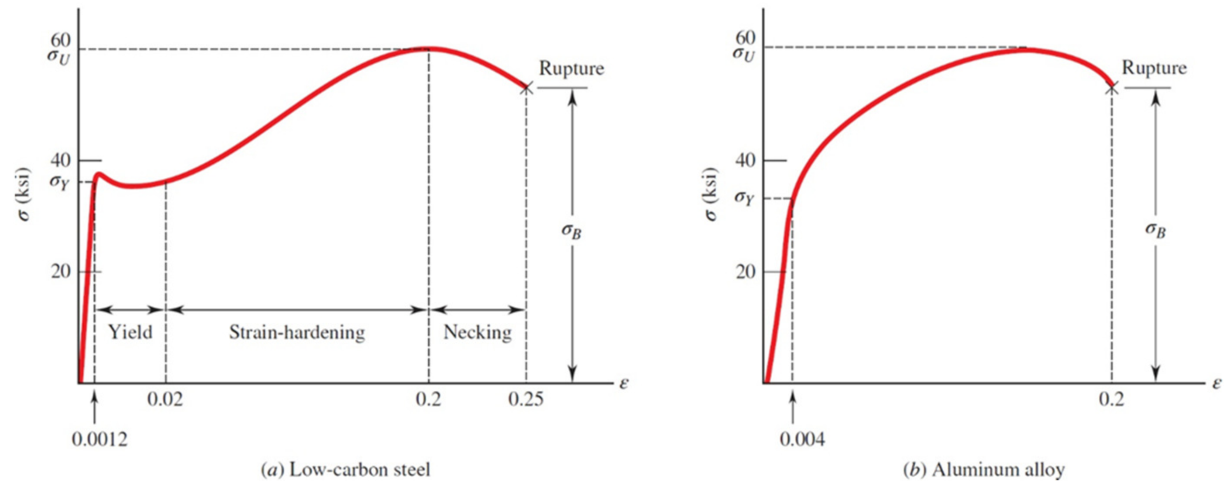
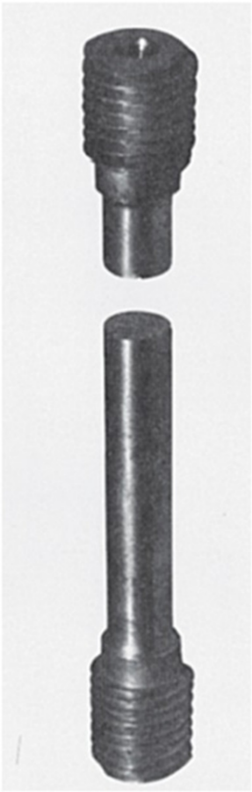


Photo 2.4 Ductile material tested specimens: (a) with cross-section necking, (b) ruptured.

Figure 2.6 Stress-strain diagrams of two typical ductile materials.

Access the text alternative for slide images.

Stress-Strain Diagram: Brittle Materials



Courtesy of John DeWolf

Photo 2.5 Ruptured brittle materials specimen.

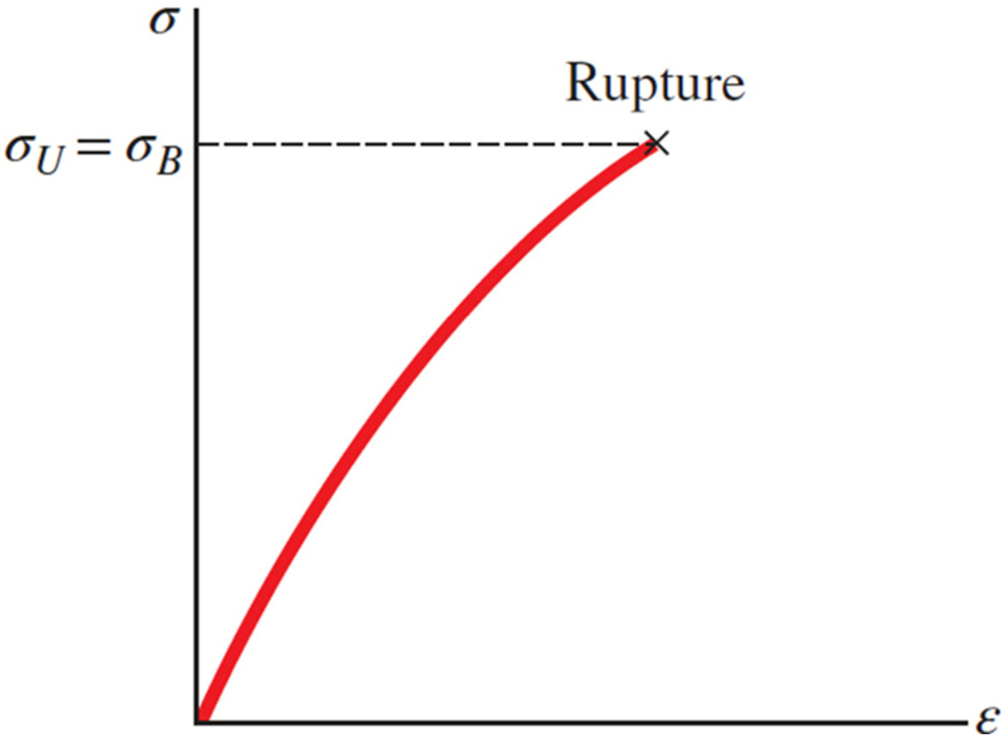


Figure 2.7 Stress-strain diagram for a typical brittle material.

Hooke's Law: Modulus of Elasticity

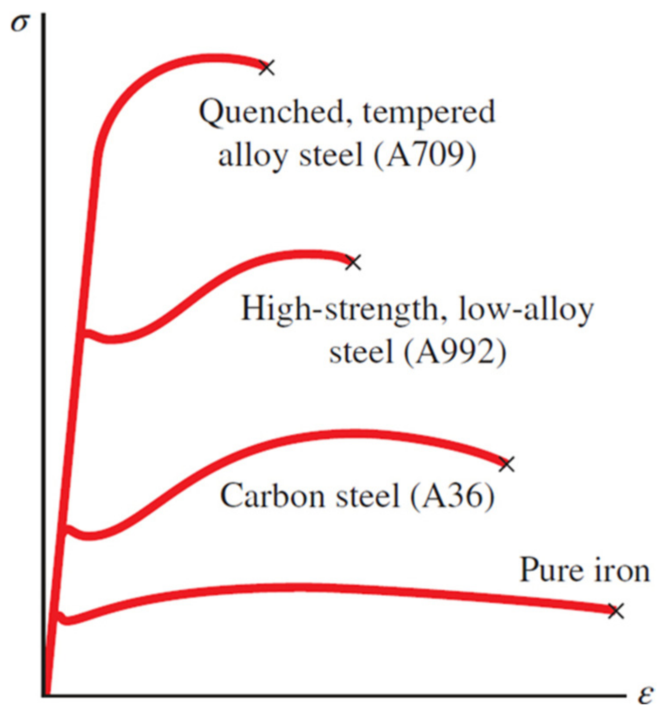


Figure 2.11 Stress-strain diagrams for iron and different grades of steel.

- Below the yield stress

  $\sigma = E\,\varepsilon$

  E = Young's Modulus or Modulus of Elasticity

- Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

Elastic vs. Plastic Behavior

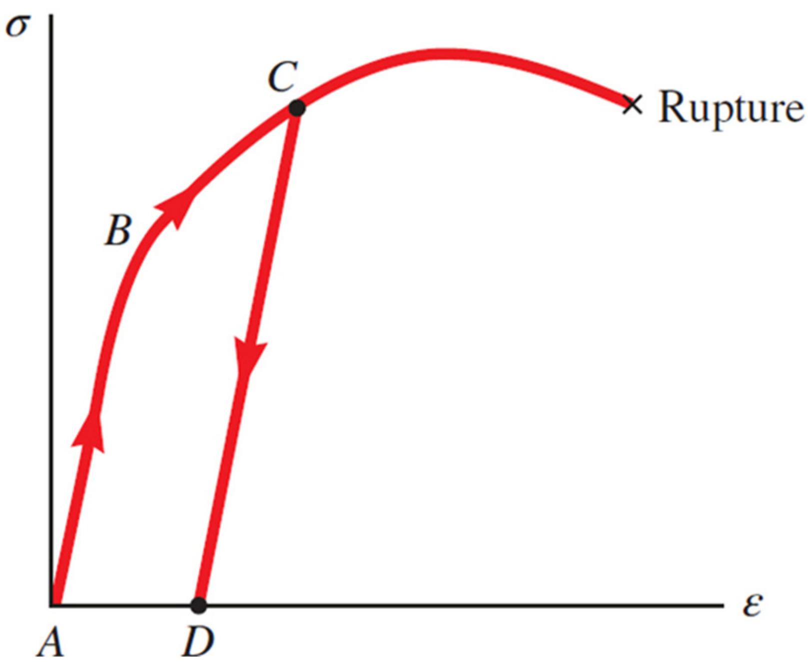


Figure 2.13 Stress-strain response of ductile material load beyond yield and unloaded.

- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*.
- When the strain does not return to zero after the stress is removed, *plastic deformation* of the material has taken place.

Access the text alternative for slide images.

Fatigue

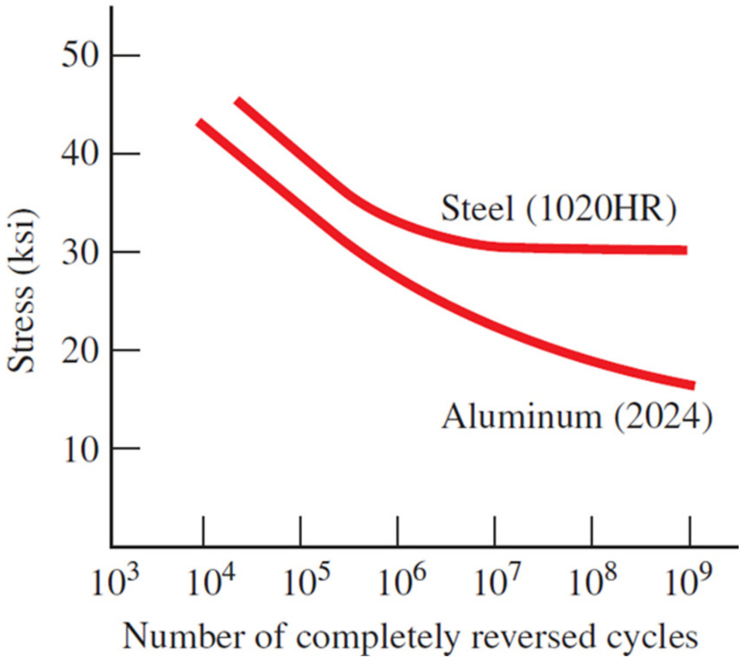


Figure 2.16 Typical σ-n curves.

- Fatigue properties are shown on σ-N diagrams.
- A member may fail due to *fatigue* at stress levels significantly below the ultimate strength if subjected to many loading cycles.
- When the stress is reduced below the *endurance limit*, fatigue failures do not occur for any number of cycles.

Deformations Under Axial Loading

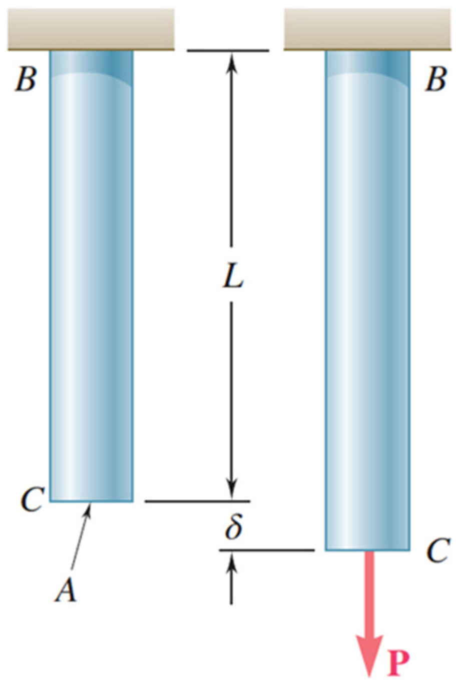


Figure 2.17 Undeformed and deformed axially-loaded rod.

- From Hooke's Law:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

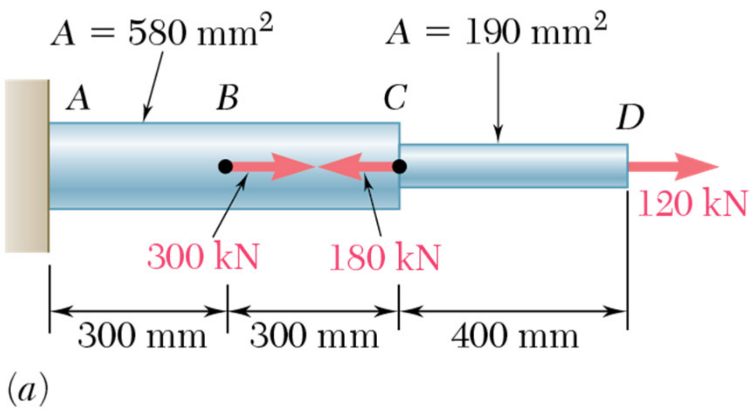
- Equating and solving for the deformation,

$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Concept Application 2.1



$E = 200$ GPa

Determine the deformation of the steel rod shown under the given loads.

SOLUTION:

Divide the rod into components at the load application points.

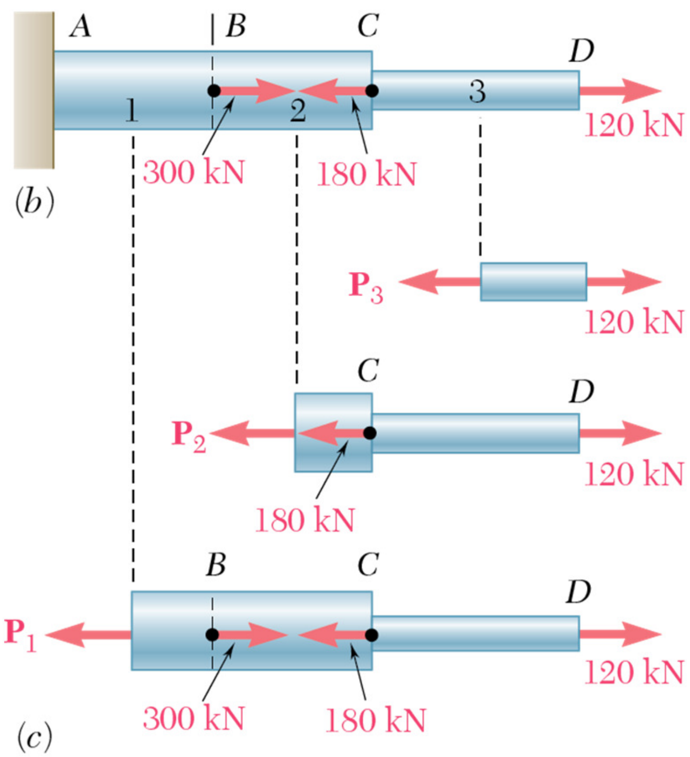
Apply a free-body analysis on each component to determine the internal force

Evaluate the total of the component deflections.

Concept Application 2.1

SOLUTION:

Divide the rod into three components:



$$L_1 = L_2 = 0.3 \text{ m} \qquad L_3 = 0.4 \text{ m}$$

$$A_1 = A_2 = 580 \times 10^{-6} \text{ m}^2 \qquad A_3 = 190 \times 10^{-3} \text{m}^2$$

Apply free-body analysis to each component to determine internal forces,

$$P_1 = 240 \times 10^3 \text{ N}$$

$$P_2 = -60 \times 10^3 \text{ N}$$

$$P_3 = 120 \times 10^3 \text{ N}$$

Evaluate total deflection,

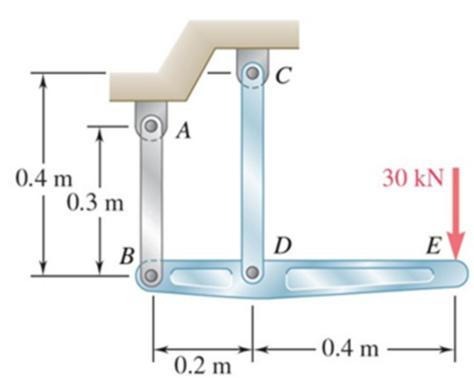
$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$

$$= \frac{1}{29 \times 10^6}\left[\frac{\left(240 \times 10^3\right)(0.3)}{580 \times 10^{-6}} + \frac{\left(-60 \times 10^3\right)(0.3)}{580 \times 10^{-6}} + \frac{\left(120 \times 10^3\right)(0.4)}{190 \times 10^{-6}}\right]$$

$$= 1.729 \times 10^{-3} \text{ m}$$

$$\boxed{\delta = 1.729 \text{ mm}}$$

Sample Problem 2.1 [1]



The rigid bar *BDE* is supported by two links *AB* and *CD*.

Link *AB* is made of aluminum ($E = 70$ GPa) and has a cross-sectional area of 500mm^2. Link *CD* is made of steel ($E = 200$ GPa) and has a cross- sectional area of 600mm^2).

For the 30-kN force shown, determine the deflection (a) of *B*, (b) of *D*, and (c) of *E*.

Solution:

- Apply a free-body analysis to the bar *BDE* to find the forces exerted by links *AB* and *DC*.
- Evaluate the deformation of links *AB* and *DC* or the displacements of *B* and *D*.
- Work out the geometry to find the deflection at E given the deflections at *B* and *D*.

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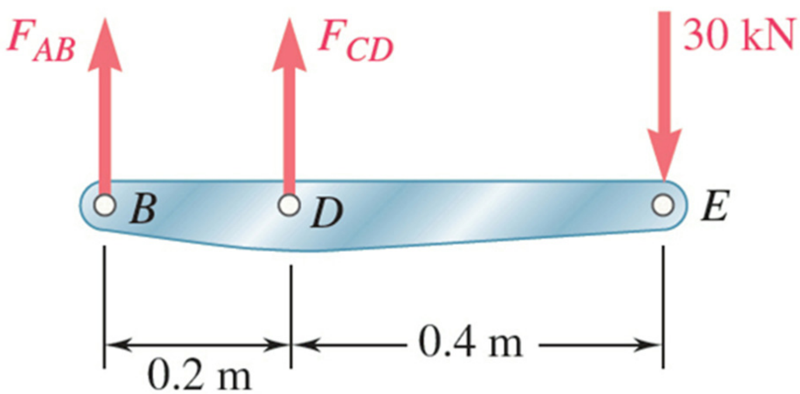
Sample Problem 2.1 [2]

Solution:

Free body: Bar *BDE*



$$+\circlearrowleft \quad \sum M_B = 0$$

$$0 = -(30\,\text{kN})(0.6\,\text{m}) + F_{CD}(0.2\,\text{m})$$

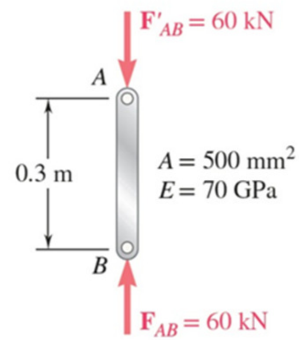
$$F_{CD} = +90\,\text{kN} \quad F_{CD} = 90\,\text{kN}\ \textit{tension}$$

$$+\circlearrowleft \quad \sum \mathrm{M_D} = 0$$

$$0 = -(30\,\text{kN} \times 0.4\,\text{m}) - F_{AB} \times 0.2\,\text{m}$$

$$F_{AB} = -60\,\text{kN} \quad F_{AB} = 60\,\text{kN}\ \textit{compression}$$

Displacement of *B*:



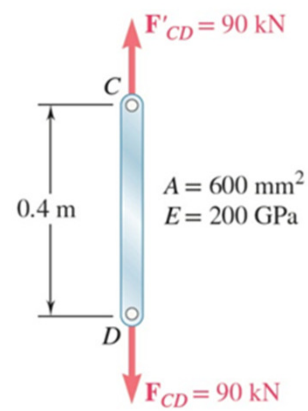
$$\delta_B = \frac{PL}{AE}$$

$$= \frac{(-60 \times 10^3\,\text{N})(0.3\,\text{m})}{(500 \times 10^{-6}\,\text{m}^2)(70 \times 10^9\,\text{Pa})}$$

$$= -514 \times 10^{-6}\,\text{m}$$

$$\boxed{\delta_B = 0.514\ \text{mm} \uparrow}$$

Displacement of *D*:



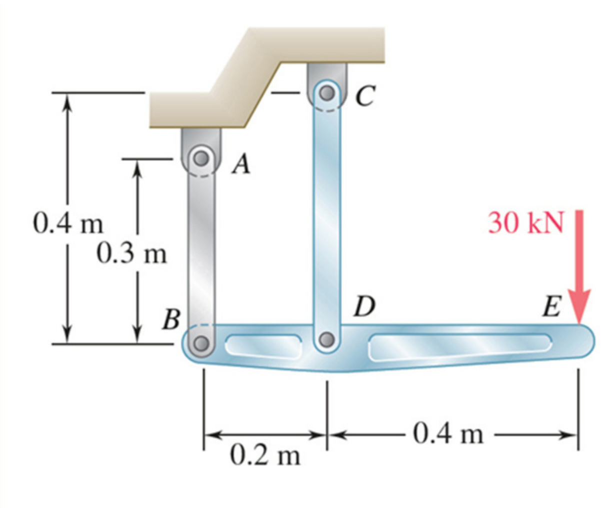
$$\delta_D = \frac{PL}{AE}$$

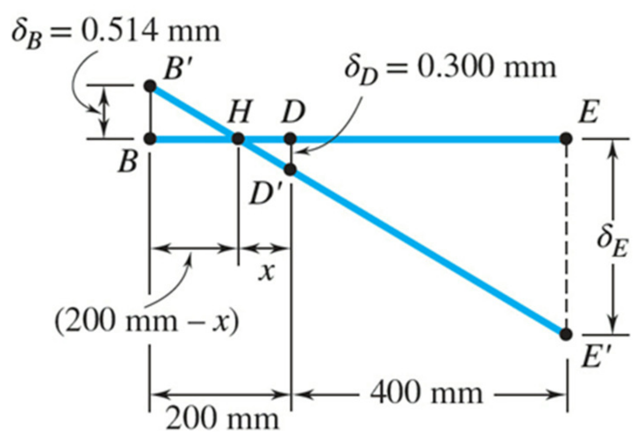
$$= \frac{(90 \times 10^3\,\text{N})(0.4\,\text{m})}{(600 \times 10^{-6}\,\text{m}^2)(200 \times 10^9\,\text{Pa})}$$

$$= 300 \times 10^{-6}\,\text{m}$$

$$\boxed{\delta_D = 0.300\ \text{mm} \downarrow}$$

Sample Problem 2.1 [3]





Deflection of E:

$$\frac{BB'}{DD'} = \frac{BH}{HD}$$

$$\frac{0.514\text{ mm}}{0.300\text{ mm}} = \frac{(200\text{ mm}) - x}{x}$$

$$x = 73.7\text{ mm}$$

$$\frac{EE'}{DD'} = \frac{HE}{HD}$$

$$\frac{\delta_E}{0.300\text{ mm}} = \frac{(400 + 73.7)\text{mm}}{73.7\text{ mm}}$$

$$\delta_E = 1.928\text{ mm}$$

$$\boxed{\delta_E = 1.928\text{ mm}\downarrow}$$

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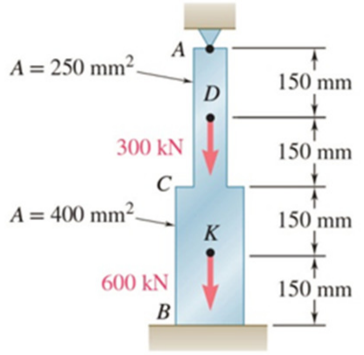
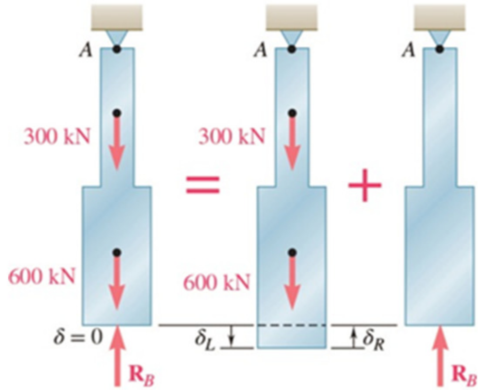
Static Indeterminate Problems



(a)



(b)

Figure 2.23 (a) Restrained axially loaded bar. (b) Reactions will be found by releasing constraint at point B and adding compressive force at point B to enforce zero deformation at point B. (c) Diagram of released structure. (d) Diagram of added reaction force at point B to enforce zero deformation at point B.

- Structures for which internal forces and reactions cannot be determined from statics alone are said to be *statically indeterminate*.
- A structure will be statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium.
- Redundant reactions are replaced with unknown loads which along with the other loads must produce compatible deformations.
- Deformations due to actual loads and redundant reactions are determined separately and then added.

$$\delta = \delta_L + \delta_R = 0$$

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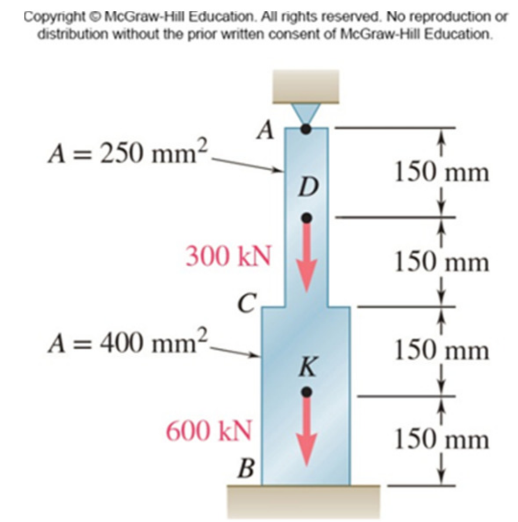
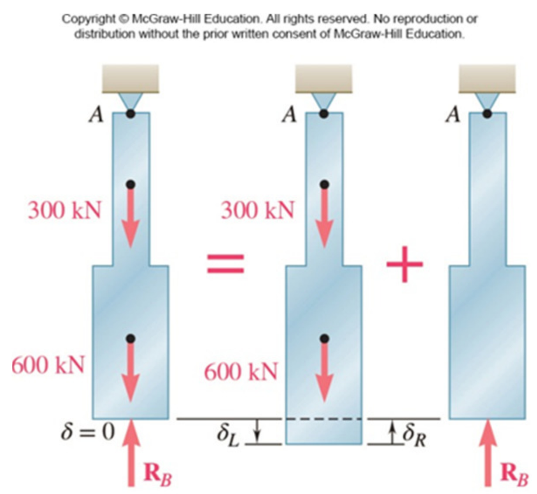
Concept Application 2.4 [1]



(a)



(b)

Determine the reactions at *A* and *B* for the steel bar and loading shown, assuming a close fit at both supports before the loads are applied.

Solution:

- Consider the reaction at *B* as redundant, release the bar from that support, and solve for the displacement at *B* due to the applied loads.
- Solve for the displacement at *B* due to the redundant reaction at $\boldsymbol{R_B}$.
- Require that the displacements due to the loads and due to the redundant reaction be compatible, i.e., require that their sum be zero.
- Solve for the reaction at R_A due to applied loads and the reaction found at R_B.

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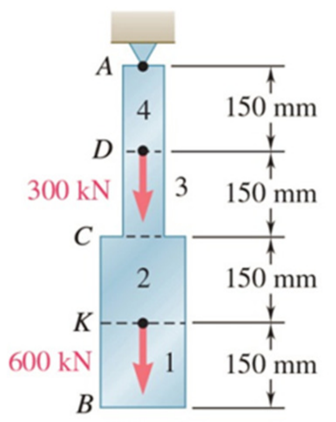
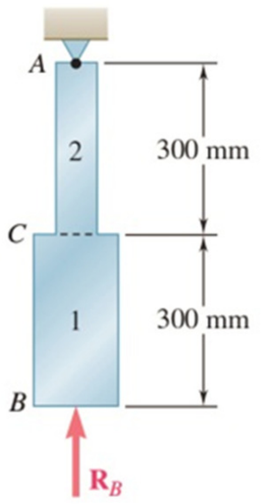
Concept Application 2.4 [2]



(c)



Solution:

- Solve for the displacement at *B* due to the applied loads with the redundant constraint released,

$$P_1 = 0 \quad P_2 = P_3 = 600\times10^3\,\text{N} \quad P_4 = 900\times10^3\,\text{N}$$

$$A_1 = A_2 = 400\times10^{-6}\,\text{m}^2 \quad A_3 = A_4 = 250\times10^{-6}\,\text{m}^2$$

$$L_1 = L_2 = L_3 = L_4 = 0.150\,\text{m}$$

$$\delta_\text{L} = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1.125\times10^9}{E}$$

- Solve for the displacement at *B* due to the redundant constraint,

$$P_1 = P_2 = -R_B$$

$$A_1 = 400\times10^{-6}\,\text{m}^2 \quad A_2 = 250\times10^{-6}\,\text{m}^2$$

$$L_1 = L_2 = 0.300\text{ m}$$

$$\delta_R = \sum_i \frac{P_i L_i}{A_i E_i} = -\frac{\left(1.95\times10^3\right)R_B}{E}$$

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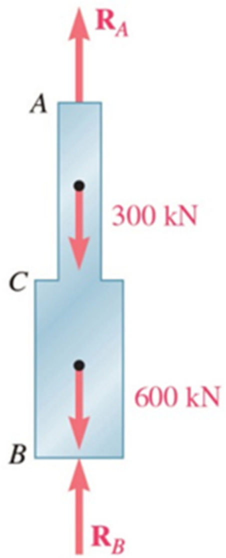
Concept Application 2.4 [3]



(e)

- Require that the displacements due to the loads and due to the redundant reaction be compatible,

$$\delta = \delta_L + \delta_R = 0$$

$$\delta = \frac{1.125\times10^9}{E} - \frac{\left(1.95\times10^3\right)R_B}{E} = 0$$

$$R_B = 577\times10^3\,\text{N} = 577\,\text{kN}$$

- Find the reaction at *A* due to the loads and the reaction at *B*.

$$+\uparrow\ \sum F_y = 0 = R_A - 300\,\text{kN} - 600\,\text{kN} + 577\,\text{kN}$$

$$R_A = 323\,\text{kN}$$

$$\boxed{\begin{aligned} R_A &= 323\,\text{kN} \\ R_B &= 577\,\text{kN} \end{aligned}}$$

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Problems Involving Temperature Change

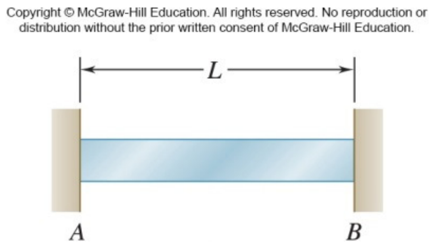


Figure 2.26 (partial)

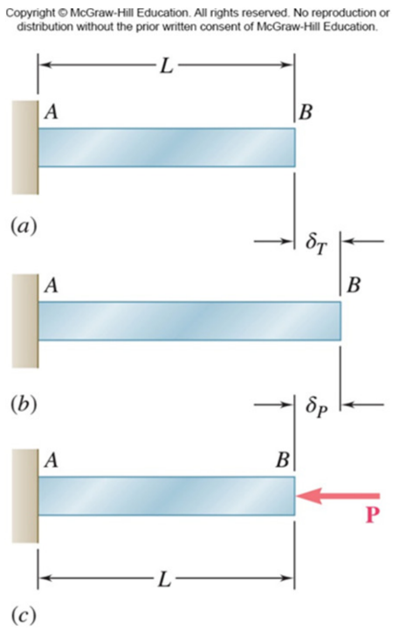


Figure 2.27 Superposition method to find force at point *B* of restrained rod *AB* undergoing thermal expansion. (a) Initial rod length; (b) thermally expanded rod length; (c) force *P* pushes point *B* back to zero deformation.

- A temperature change results in a change in length or *thermal strain*. There is no stress associated with the thermal strain unless the elongation is restrained by the supports.
- Treat the additional support as redundant and apply the principle of superposition.

$$\delta_T = \alpha(\Delta T)L \qquad \delta_P = \frac{PL}{AE}$$

$$\alpha = \text{coefficient of thermal expansion}$$

- The thermal deformation and the deformation from the redundant support must be compatible.

$$\delta = \delta_T + \delta_P = 0 \qquad \alpha(\Delta T)L + \frac{PL}{AE} = 0$$

$$P = -AE\alpha(\Delta T)$$

$$\sigma = \frac{P}{A} = -E\alpha(\Delta T)$$

Poisson's Ratio

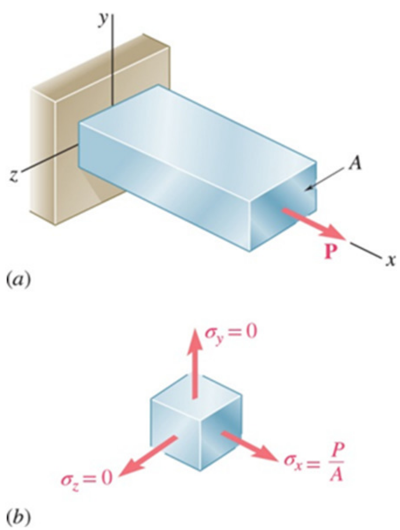


Figure 2.29 A bar in uniaxial tension and a representative stress element.

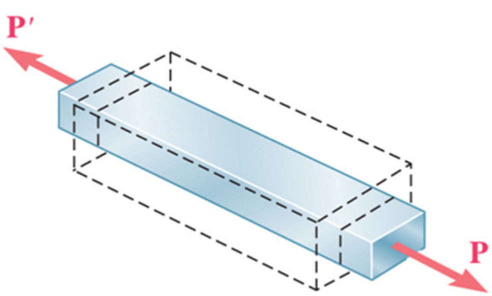


Figure 2.30 Materials undergo transverse contraction when elongated under axial load.

- For a slender bar subjected to axial loading:

$$\varepsilon_x = \frac{\sigma_x}{E} \qquad \sigma_y = \sigma_z = 0$$

- The elongation in the *x*-direction is accompanied by a contraction in the other directions. Assuming that the material is *homogeneous* and *isotropic* (no directional dependence),

$$\varepsilon_y = \varepsilon_z \neq 0$$

- Poisson's ratio is defined as

$$\nu = -\left|\frac{\text{lateral strain}}{\text{axial strain}}\right| = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

Multiaxial Loading: Generalized Hooke's Law

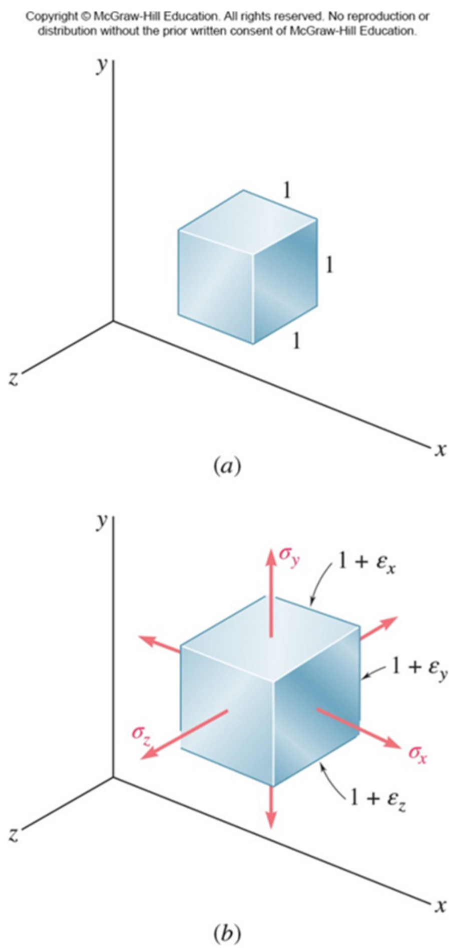


Figure 2.33 Deformation of unit cube under multiaxial loading: (a) unloaded; (b) deformed.

- For an element subjected to multi-axial loading, the normal strain components resulting from the stress components may be determined from the *principle of superposition*. This requires:
    1) Each effect is linearly related to the load that produces it.
    2) The deformation resulting from any given load is small and does not affect the conditions of application of the other loads.
- With these restrictions:

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

Dilatation: Bulk Modulus

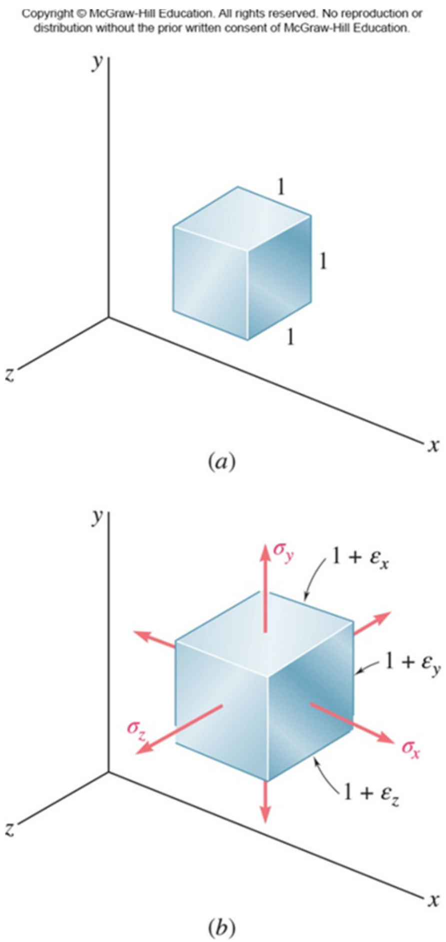


Figure 2.33 Deformation of unit cube under multiaxial loading: (a) unloaded; (b) deformed.

- Relative to the unstressed state, the change in volume is

$$
\begin{aligned}
e &= \nu - 1 = 1 + \left[\varepsilon_x + \varepsilon_y + \varepsilon_z\right] - 1 \\
&= \varepsilon_x + \varepsilon_y + \varepsilon_z \\
&= \frac{1-2\nu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right) \\
&= \text{ dilatation (change in volume per unit volume)}
\end{aligned}
$$

- For element subjected to uniform hydrostatic pressure,

$$e = -\frac{3(1-2\nu)}{E} p = -\frac{p}{k}$$

$$k = \frac{E}{3(1-2\nu)} = \text{bulk modulus or modulus of compression}$$

- Subjected to uniform pressure, dilatation must be negative, therefore

$$0 < \nu < \tfrac{1}{2}$$

Shearing Strain

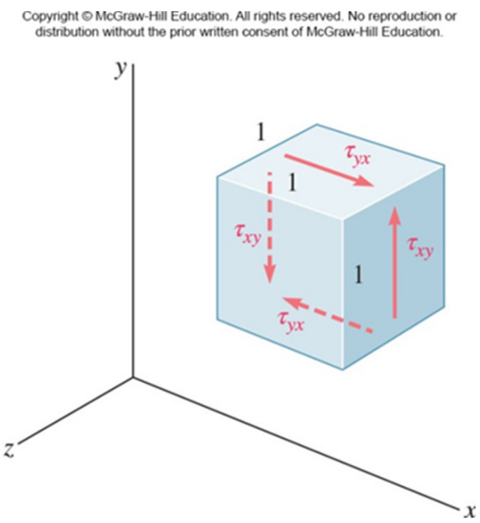


Figure 2.36 Unit cubic element subjected to shearing stress.

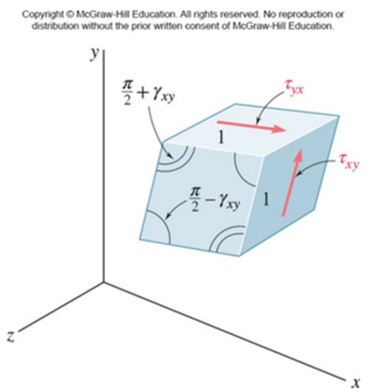


Figure 2.37 Deformation of unit cubic element due to shearing stress.

- A cubic element subjected to only shearing stress will deform into a rhomboid. The corresponding *shearing strain* is quantified in terms of the change in angle between the sides,

$$\tau_{xy} = f(\gamma_{xy})$$

- A plot of shearing stress vs. shearing strain is similar to the previous plots of normal stress vs. normal strain except that the strength values are approximately half. For values of shearing strain that do not exceed the proportional limit,

$$\tau_{xy} = G\gamma_{xy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{zx} = G\gamma_{zx}$$

where G is the *modulus of rigidity* or *shear modulus*.

Access the text alternative for slide images.

Concept Application 2.10

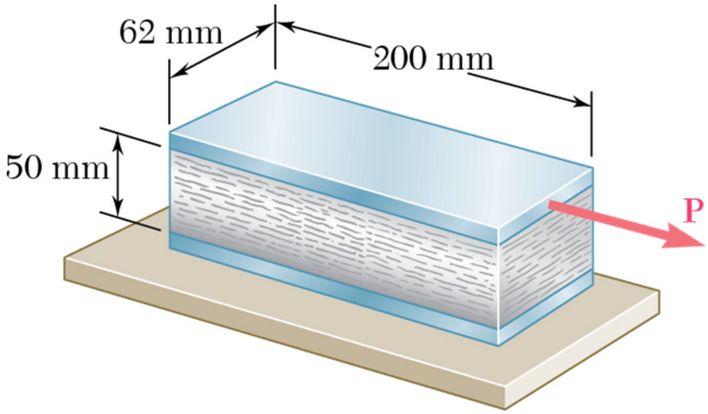


Fig. 2.41(a) Rectangular block loaded in shear.

A rectangular block of material with modulus of rigidity $G = 630$ MPa is bonded to two rigid horizontal plates. The lower plate is fixed, while the upper plate is subjected to a horizontal force **P**. Knowing that the upper plate moves through 1 mm under the action of the force, determine (a) the average shearing strain in the material, and (b) the force **P** exerted on the plate.

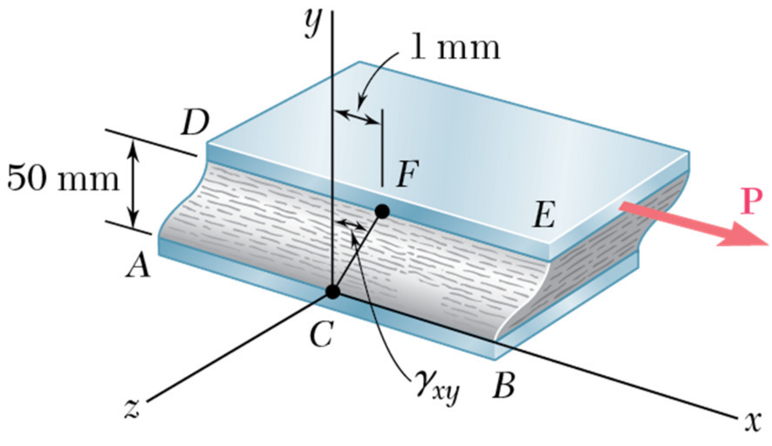
SOLUTION:

Determine the average angular deformation or shearing strain of the block.

Apply Hooke's law for shearing stress and strain to find the corresponding shearing stress.

Use the definition of shearing stress to find the force **P**.

Concept Application 2.10



(b)

Fig. 2.41(b) Deformed block showing the shear strain.

Determine the average angular deformation or shearing strain of the block.

$$\gamma_{xy} \approx \tan \gamma_{xy} = \frac{1\,\text{mm}}{50\,\text{mm}} \qquad \gamma_{xy} = 0.020\,\text{rad}$$

Apply Hooke's law for shearing stress and strain to find the corresponding shearing stress.

$$\tau_{xy} = G\gamma_{xy} = (630\text{ MPa})(0.020\,\text{rad}) = 12.6\,\text{MPa}$$

Use the definition of shearing stress to find the force **P**.

$$P = \tau_{xy} A = (12.6\text{ MPa})(200\,\text{mm})(62\text{ mm}) = 156.2 \times 10^3\,\text{N}$$

$$\boxed{P = 156.2\,\text{kN}}$$

Relation Between *E*, *ν*, and *G*

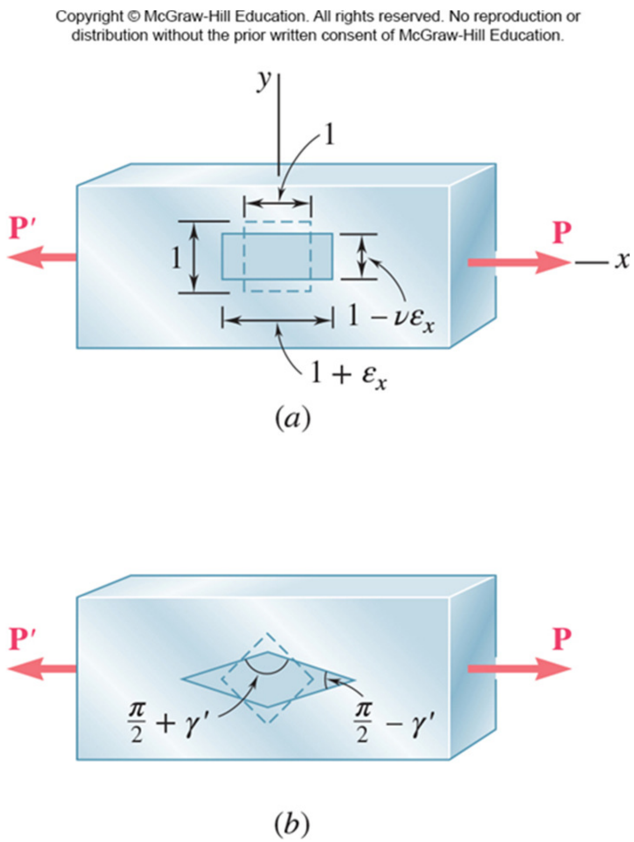


Figure 2.42 Representations of strain in an axially loaded bar: (a) cubic strain element faces aligned with coordinate axes; (b) cubic strain element faces rotated 45° about *z*-axis.

- An axially loaded slender bar will elongate in the *x* direction and contract in both of the transverse *y* and *z* directions.
- An initially cubic element oriented as in Figure 2.42(a) will deform into a rectangular parallelepiped. The axial load produces a normal strain.
- If the cubic element is oriented as in Figure 2.42(b), it will deform into a rhombus. Axial load also results in a shearing strain.
- Components of normal and shearing strain are related,

$$\frac{E}{2G} = (1+\nu) \quad \text{or} \quad G = \frac{E}{2(1+\nu)}$$

Composite Materials

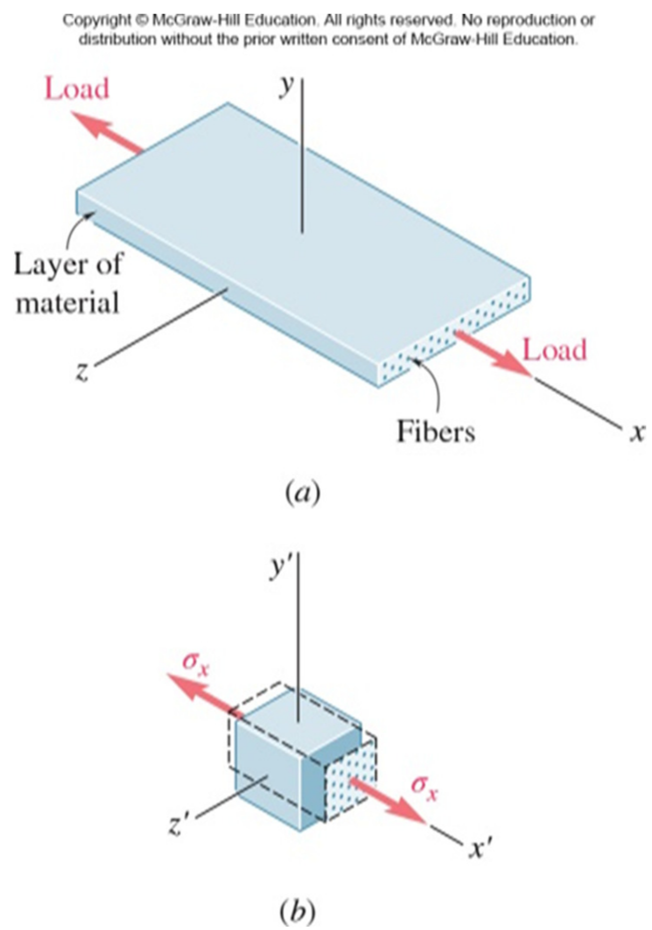


Figure 2.44 Orthotropic fiber-reinforced composite material under uniaxial tensile load.

- Fiber-reinforced composite materials are fabricated by embedding fibers of a strong, still material into a weaker, softer material called a *matrix*.
- Normal stresses and strains are related by Hooke's Law but with directionally dependent moduli of elasticity,

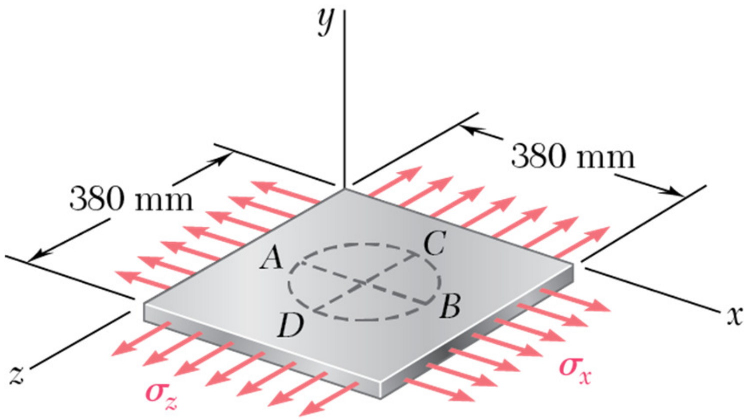
$$E_x = \frac{\sigma_x}{\varepsilon_x} \quad E_y = \frac{\sigma_y}{\varepsilon_y} \quad E_z = \frac{\sigma_z}{\varepsilon_z}$$

- Transverse contractions are related by directionally dependent values of Poisson's ratio, e.g.,

$$\nu_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} \quad \nu_{xz} = -\frac{\varepsilon_z}{\varepsilon_x}$$

- The three components of strain ε_x, ε_y, and ε_z for orthotropic materials can be expressed in terms of normal stress only and do not depend upon any shearing stresses.

Sample Problem 2.5



A circle of diameter d = 225 mm is scribed on an unstressed aluminum plate of thickness t = 18 mm. Forces acting in the plane of the plate later cause normal stresses σ_x = 84 MPa and σ_z = 140 MPa.

For E = 70 GPa and ν = 1/3, determine the change in:

a) the length of diameter *AB*,

b) the length of diameter *CD*,

c) the thickness of the plate, and

d) the volume of the plate.

Sample Problem 2.5

SOLUTION:

Apply the generalized Hooke's Law to find the three components of normal strain.

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= \frac{1}{70\ \mathrm{GPa}}\left[(84\ \mathrm{MPa}) - 0 - \frac{1}{3}(140\ \mathrm{MPa})\right]$$

$$= +0.533 \times 10^{-3}\ \mathrm{mm/mm}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= -1.067 \times 10^{-3}\ \mathrm{mm/mm}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$= +1.600 \times 10^{-3}\ \mathrm{mm/mm}$$

Evaluate the deformation components.

$$\delta_{B/A} = \varepsilon_x d = \left(+0.533 \times 10^{-3}\ \mathrm{mm/mm}\right)(225\ \mathrm{mm})$$

$$\boxed{\delta_{B/A} = +0.12\ \mathrm{mm}}$$

$$\delta_{C/D} = \varepsilon_z d = \left(+1.600 \times 10^{-3}\ \mathrm{mm/mm}\right)(225\ \mathrm{mm})$$

$$\boxed{\delta_{C/D} = +0.36\ \mathrm{mm}}$$

$$\delta_t = \varepsilon_y t = \left(-1.067 \times 10^{-3}\ \mathrm{mm/mm}\right)(18\ \mathrm{mm})$$

$$\boxed{\delta_t = -0.0192\ \mathrm{mm}}$$

Find the change in volume

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = +1.066 \times 10^{-3}\ \mathrm{mm^3/mm^3}$$

$$\Delta V = eV = 1.066 \times 10^{-3}\left((380\ \mathrm{mm})(380\ \mathrm{mm})(18\ \mathrm{mm})\right)\mathrm{mm^3}$$

$$\boxed{\Delta V = +2770\ \mathrm{mm^3}}$$

Saint-Venant’s Principle

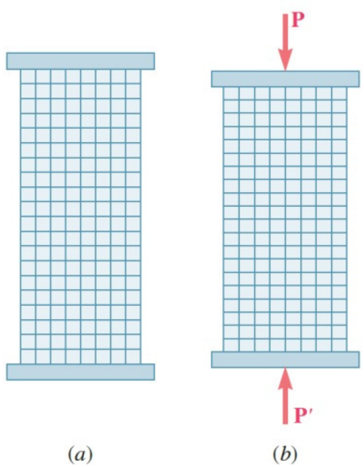


Figure 2.47 Axial load applied by rigid plates to rubber model.

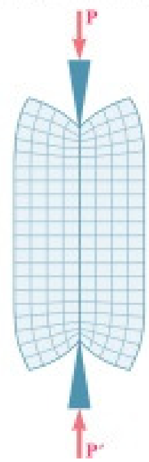


Figure 2.48 Concentrated axial load applied to rubber model.

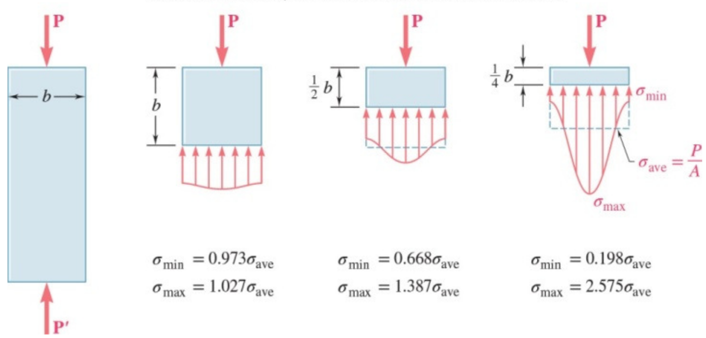


Figure 2.49 Stress distributions in a plate under concentrated axial loads.

- Loads transmitted through rigid plates result in uniform distribution of stress and strain.
- Concentrated loads result in large stresses in the vicinity of the load application point.
- Stress and strain distributions become uniform at a relatively short distance from the load application points.
- **Saint-Venant’s Principle:** Stress distribution may be assumed independent of the mode of load application except in the immediate vicinity of load application points.

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Stress Concentration: Hole

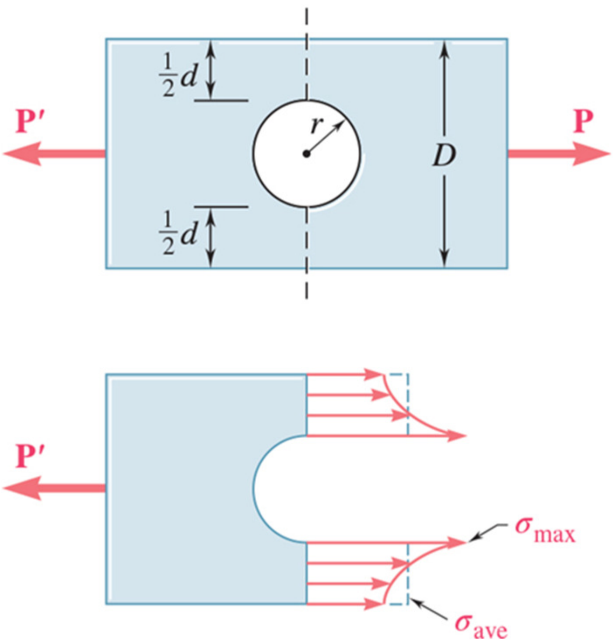
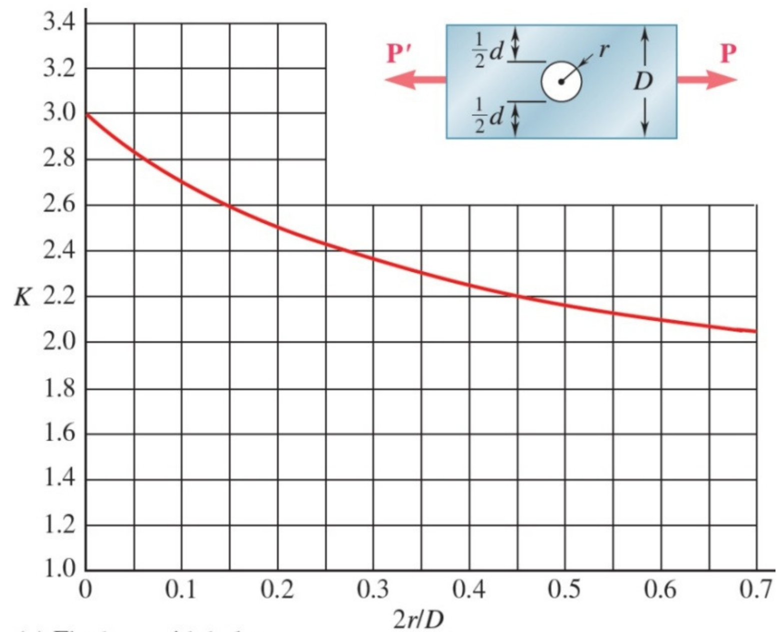


Figure 2.50 Stress distribution near circular hole in flat bar under axial loading.



(*a*) Flat bars with holes

Figure 2.52(a) Stress concentration factors for flat bars under axial loading.

Stress concentration factor

$$K = \frac{\sigma_{\text{max}}}{\sigma_{\text{ave}}}$$

Discontinuities of cross section may result in high localized or *concentrated* stresses.

Access the text alternative for slide images.

Stress Concentration: Fillet

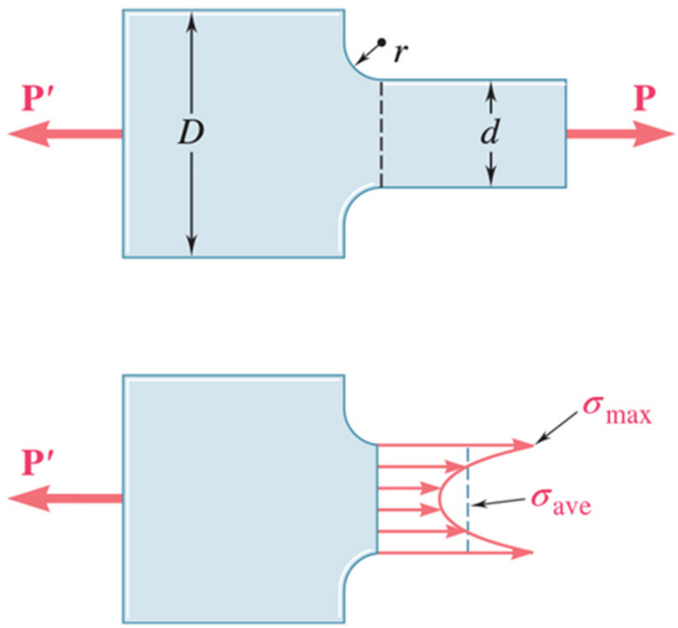


Figure 2.51 Stress distribution near fillets in flat bar under axial loading.

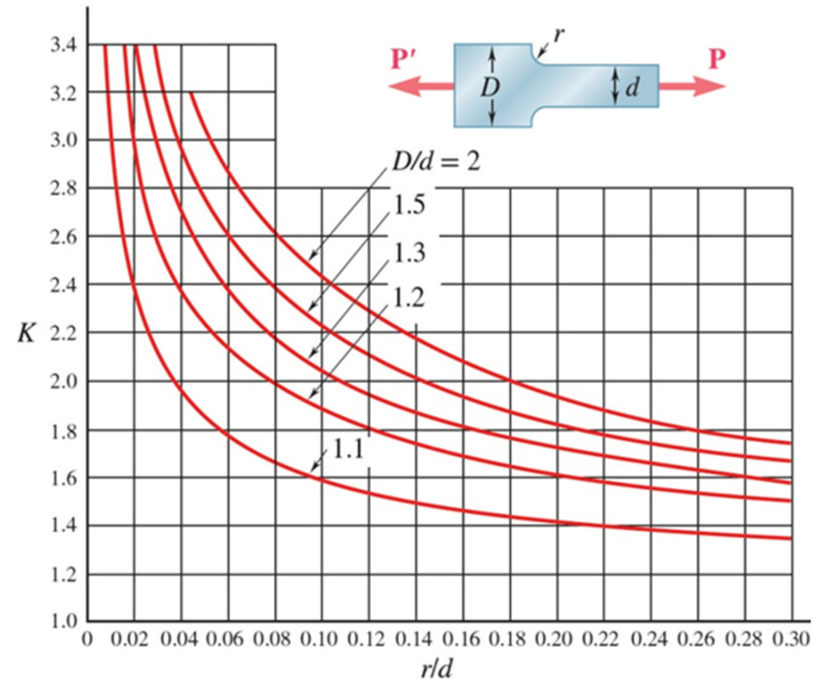


Figure 2.52(b) Stress concentration factors for flat bars under axial loading.

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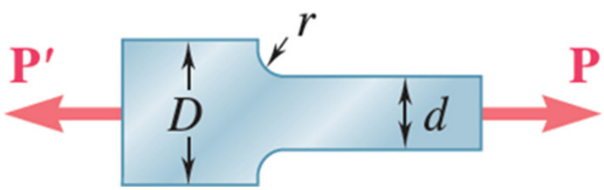
Concept Application 2.12 [1]



Determine the largest axial load **P** that can be safely supported by a flat steel bar consisting of two portions, both 10 mm thick, and respectively 40 and 60 mm wide, connected by fillets of radius r = 8 mm. Assume an allowable normal stress of 165 MPa.

Solution:

- Determine the geometric ratios and find the stress concentration factor from Figure 2.52.
- Find the allowable average normal stress using the material allowable normal stress and the stress concentration factor.
- Apply the definition of normal stress to find the allowable load.

Concept Application 2.12 2

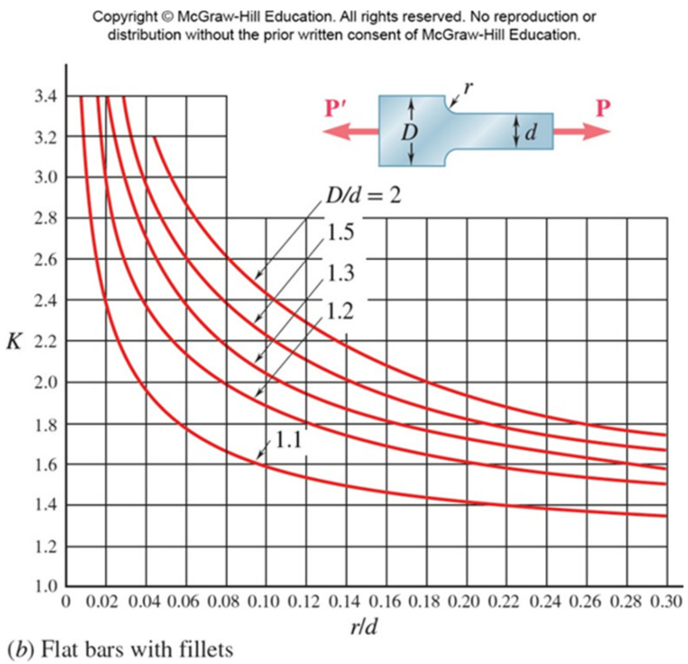


Figure 2.52(b) Stress concentration factors for flat bars under axial loading.

- Determine the geometric ratios and find the stress concentration factor from Figure 2.52.

$$\frac{D}{d}=\frac{60\text{ mm}}{40\text{ mm}}=1.50 \qquad \frac{r}{d}=\frac{8\,\text{mm}}{40\,\text{mm}}=0.20$$

$$K=1.82$$

- Find the allowable average normal stress using the material allowable normal stress and the stress concentration factor.

$$\sigma_{\text{ave}}=\frac{\sigma_{\text{max}}}{K}=\frac{165\text{ MPa}}{1.82}=90.7\text{ MPa}$$

- Apply the definition of normal stress to find the allowable load.

$$P=A\sigma_{ave}=(40\text{ mm})(10\text{ mm})(90.7\text{ MPa})$$

$$=36.3\times10^{3}\,\text{N}$$

$$\boxed{P=36.3\,\text{kN}}$$

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Elastoplastic Materials

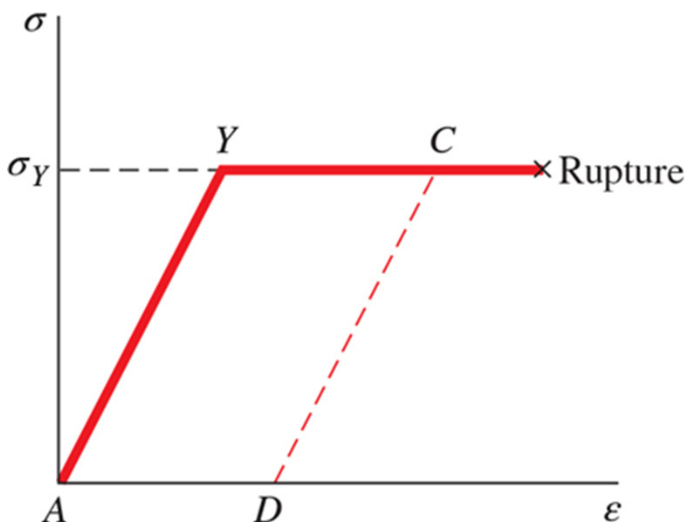

Figure 2.53 Stress-strain diagram for an idealized elastoplastic material.

- Previous analyses based on assumption of linear stress-strain relationship, i.e. stresses below the yield stress.
- Assumption is good for brittle material which rupture without yielding.
- If the yield stress of ductile materials is exceeded, then plastic deformations occur.
- Analysis of plastic deformations is simplified by assuming an idealized *elastoplastic material.*
- Deformations of an elastoplastic material are divided into elastic and plastic ranges.
- Permanent deformations result from loading beyond the yield stress.

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Plastic Deformations

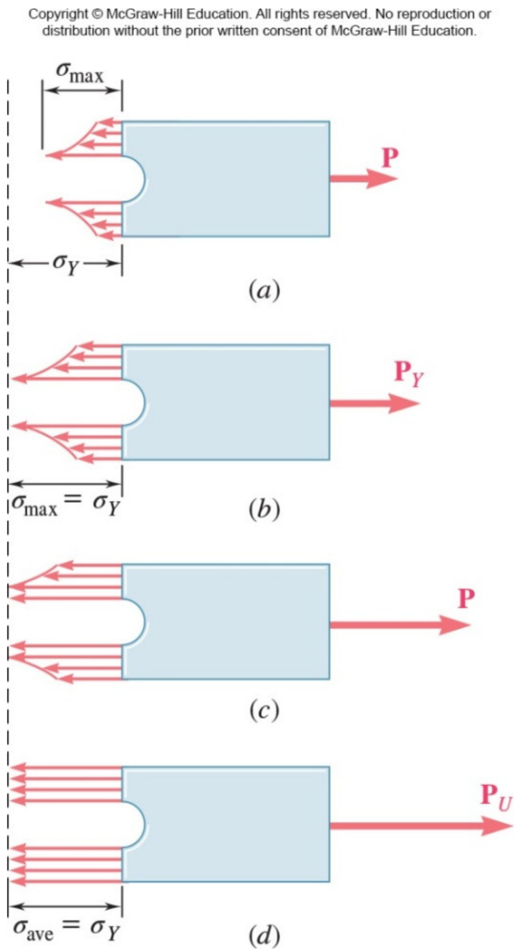

Figure 2.56 Distribution of stresses in elastoplastic material under increasing load.

$$P = \sigma_{\text{ave}} A = \frac{\sigma_{\max} A}{K}$$

$$P_Y = \frac{\sigma_Y A}{K}$$

$$P_U = \sigma_Y A$$
$$= K P_Y$$

- Elastic deformation while maximum stress is less than yield stress.
- Maximum stress is equal to the yield stress at the maximum elastic loading.
- At loadings above the maximum elastic load, a region of plastic deformations develop near the hole.
- As the loading increases, the plastic region expands until the section is at a uniform stress equal to the yield stress.

Residual Stresses

When a single structural element is loaded uniformly beyond its yield stress and then unloaded, it is permanently deformed, but all stresses disappear. It should not be assumed that this will always be the case.

Residual stresses will remain in a structure after loading and unloading if:

- only part of the structure undergoes plastic deformation.
- different parts of the structure undergo different plastic deformations.

Residual stresses also result from the uneven heating or cooling of structures or structural elements.

Concept Applications 2.14, 2.15, 2.16

A cylindrical rod is placed inside a tube of the same length. The ends of the rod and tube are attached to a rigid support on one side and a rigid plate on the other. The load on the rod-tube assembly is increased from zero to 25 kN and decreased back to zero.

a) draw a load-deflection diagram for the rod-tube assembly
b) determine the maximum elongation
c) determine the permanent set
d) calculate the residual stresses in the rod and tube.

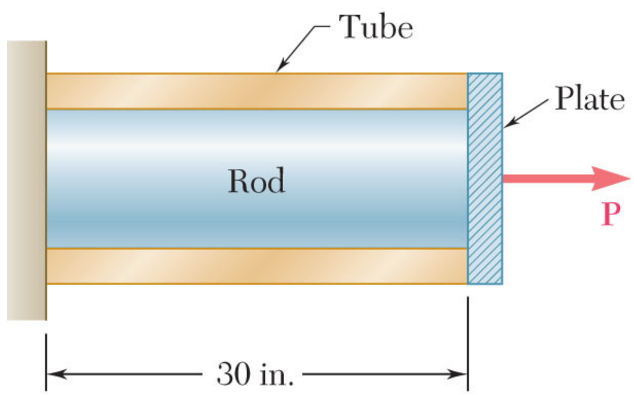

Fig. 2.54(a) Concentric rod-tube assembly axially loaded by rigid plate.

$$A_r = 48\,\text{mm}^2 \qquad A_t = 62\,\text{mm}^2$$

$$E_r = 210\,\text{GPa} \qquad E_t = 105\,\text{GPa}$$

$$(\sigma_r)_Y = 250\,\text{MPa} \qquad (\sigma_t)_Y = 310\,\text{MPa}$$

Concept Applications 2.14, 2.15, 2.16

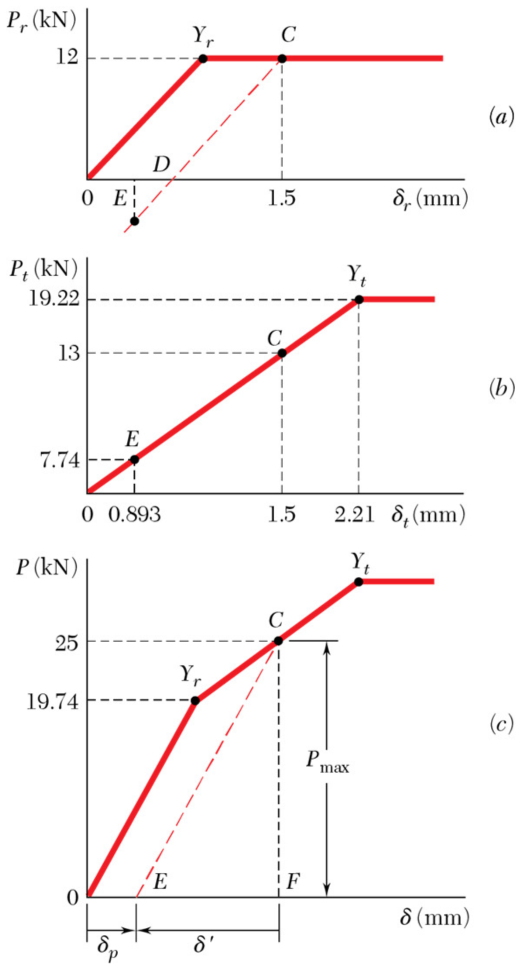

Fig. 2.55 (a) Rod load-deflection response with elastic unloading (red dashed line). (b) Tube load-deflection response. Given loading does not yield tube so unloading is along the original elastic loading line. (c) Combined rod-tube assembly load deflection response with elastic unloading (red dashed line).

a) Draw a load-deflection diagram for the rod-tube assembly

$$(P_r)_Y = (\sigma_r)_Y A_r = \left(250 \times 10^6\,\text{Pa}\right)\left(48 \times 10^{-6}\,\text{m}^2\right)\left(0.075\,\text{in}^2\right) = 12\,\text{kN}$$

$$(\delta_r)_Y = (\varepsilon_r)_Y L = \frac{(\sigma_r)_Y}{E_r} L = \frac{250 \times 10^6\,\text{Pa}}{210 \times 10^6\,\text{Pa}}(0.75\,\text{m})$$

$$= 0.893\,\text{mm}$$

$$(P_t)_Y = (\sigma_t)_Y A_t = \left(310 \times 10^6\,\text{Pa}\right)\left(62 \times 10^{-6}\,\text{m}^2\right) = 19.22\,\text{kN}$$

$$(\delta_t)_Y = (\varepsilon_t)_Y L = \frac{(\sigma_t)_Y}{E_t} L = \frac{310 \times 10^6\,\text{Pa}}{105 \times 10^9\,\text{Pa}}(0.75\,\text{m})$$

$$= 2.21\,\text{mm}$$

$$P = P_r + P_t$$

$$\delta = \delta_r = \delta_t$$

Concept Applications 2.14, 2.15, 2.16

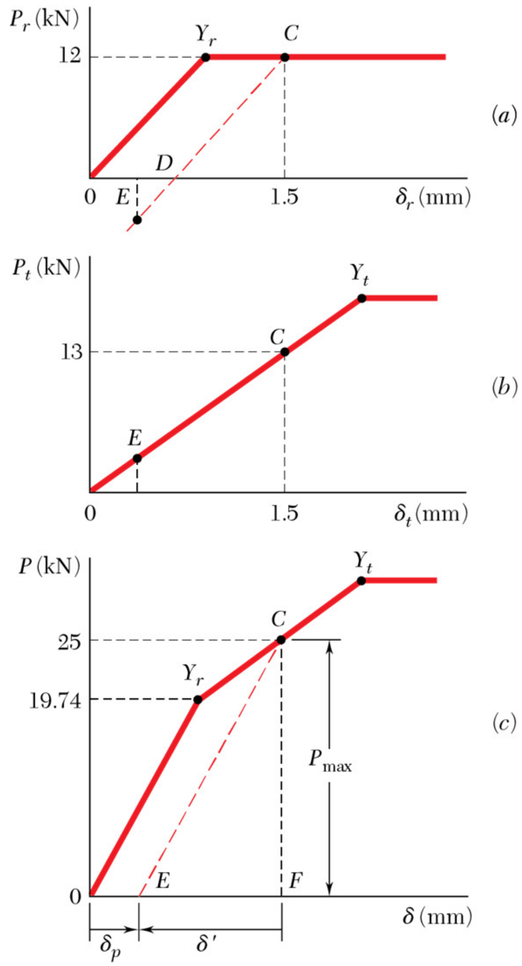

Fig. 2.55 (a) Rod load-deflection response with elastic unloading (red dashed line). (b) Tube load-deflection response. Given loading does not yield tube so unloading is along the original elastic loading line. (c) Combined rod-tube assembly load deflection response with elastic unloading (red dashed line).

At a load of P = 25 kN, the rod has reached the plastic range while the tube is still in the elastic range

$$P_r = (P_r)_Y = 12\,\text{kN}$$

$$P_t = P - P_r = (25 - 12)\,\text{kN} = 13\,\text{kN}$$

$$\sigma_t = \frac{P_t}{A_t} = \frac{13\,\text{kN}}{62\,\text{mm}^2} = 210\,\text{MPa}$$

$$\delta_t = \varepsilon_t L = \frac{\sigma_t}{E_t} L = \frac{210\,\text{MPa}}{105\,\text{GPa}}(0.75\,\text{m})$$

$$\boxed{d_{\max} = d_t = 1.5\,\text{mm}}$$

The rod-tube assembly unloads along a line parallel to $0Y_r$

$$m = \frac{19.74\,\text{kN}}{0.893\,\text{mm}} = 22.1\,\text{kN/mm} = \text{slope}$$

$$d\text{¢} = -\frac{P_{\max}}{m} = -\frac{25\,\text{kN}}{22.1\,\text{kN/mm}} = -1.131\,\text{mm}$$

$$d_p = d_{\max} + d\text{¢} = (1.5 - 1.131)\,\text{mm}$$

$$\boxed{d_p = 0.369\,\text{mm}}$$

Concept Applications 2.14, 2.15, 2.16

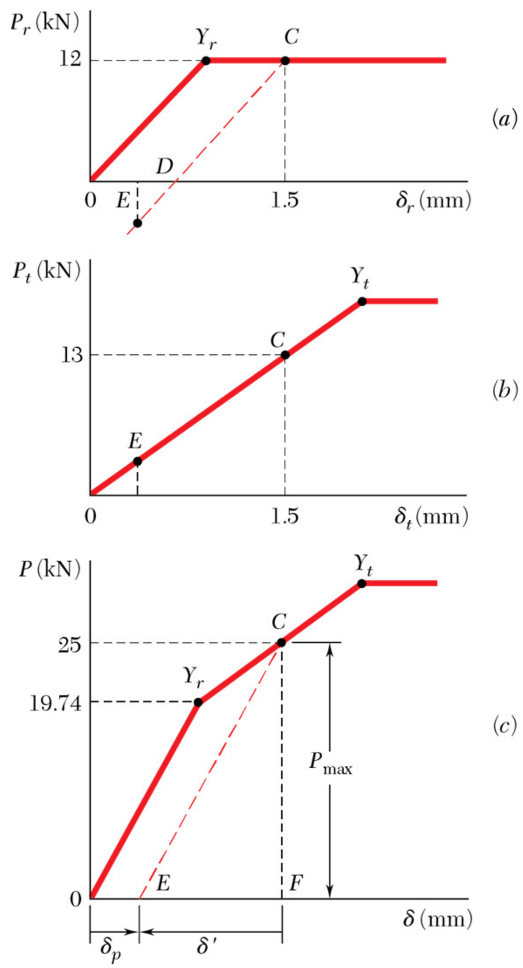

Fig. 2.55 (a) Rod load-deflection response with elastic unloading (red dashed line). (b) Tube load-deflection response. Given loading does not yield tube so unloading is along the original elastic loading line. (c) Combined rod-tube assembly load deflection response with elastic unloading (red dashed line).

Calculate the residual stresses in the rod and tube.

Calculate the reverse stresses in the rod and tube caused by unloading and add them to the maximum stresses.

$$\varepsilon' = \frac{\delta'}{L} = \frac{-1.131\,\text{mm}}{750\,\text{mm}} = -1.508 \times 10^{-3}\,\text{mm/mm}$$

$$\sigma_r' = \varepsilon' E_r = \left(-1.508 \times 10^{-3}\right)\left(210 \times 10^{9}\,\text{Pa}\right) = -316.7\,\text{MPa}$$

$$\sigma_t' = \varepsilon' E_t = \left(-1.508 \times 10^{-3}\right)\left(105 \times 10^{9}\,\text{Pa}\right) = -158.34\,\text{MPa}$$

$$\sigma_{residual,r} = \sigma_r + \sigma_r' = \left(250\,\text{MPa} - 316.7\,\text{MPa}\right) = -66.7\,\text{MPa}$$

$$\sigma_{residual,t} = \sigma_t + \sigma_t' = \left(210\,\text{MPa} - 158.34\,\text{MPa}\right) = 51.7\,\text{MPa}$$

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Accessibility Content: Text Alternatives for Images

Stress-Strain Diagram: Ductile Materials – Text Alternative

Stress-strain diagrams for ductile materials. Y-axis shows stress and x-axis shows strain. Both images show a straight line until they reach the yield strength. The curve after that is concave up for (a) and concave down for (b).

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Elastic vs. Plastic Behavior – Text Alternative

Stress-strain curve for unloading shows that there is a permanent deformation that is reached if a parallel line to the elastic section is drawn from any point inside the plastic deformation section.

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Concept Application 2.1 [1] – Text Alternative

Horizontal bar ABCD has cross-section area of 580 square millimeters from A to C, and 190 square millimeters from C to D. Lengths are 300 millimeters from A to B and B to C. Length CD is 400 millimeters. Bar is fixed in A, and there are three loads: 300 kilonewtons to the right at B, 180 kilonewtons to the left at C, and 120 kilonewtons to the right at D.

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Concept Application 2.1 2 – Text Alternative

Previously described image is divided in three sections, 1, 2, and 3. Section 1 is located between A and B, 2 is located between B and C, and 3 is located between C and D.

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Sample Problem 2.1 [1] – Text Alternative

Horizontal bar BDE is supported by two vertical links AB and CD of lengths 0.3 meters and 0.4 meters respectively. BD is 0.2 meters and DE is 0.4 meters. A vertical force of -30 kilo newtons is located at E.

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Sample Problem 2.1 3 – Text Alternative

Horizontal bar BDE is supported by two vertical links AB and CD of lengths 0.3 meters and 0.4 meters respectively. BD is 0.2 meters and DE is 0.4 meters. A vertical force of -30 kilo newtons is located at E.

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Static Indeterminate Problems – Text Alternative

For Picture (*a*), Vertical rod ADCKB is located between pin A and wall B. All distances between points is 150 millimeters and cross-section areas are 250 millimeters squared from A to C, and 400 millimeters squared from C to B. A vertical load of -300 kilo newtons is located at D and a vertical load of -600 kilo newtons is located at K.

For picture (*b*), Fixture at B is replaced with a reaction force at B, and the figure shows that the addition of the external loads plus the new reaction force is equivalent to the original situation.

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Concept Application 2.4 [1] – Text Alternative

For Picture (*a*), Vertical rod ADCKB is located between pin A and wall B. All distances between points is 150 millimeters and cross-section areas are 250 millimeters squared from A to C, and 400 millimeters squared from C to B. A vertical load of -300 kilo newtons is located at D and a vertical load of -600 kilo newtons is located at K.

For picture (*b*), Fixture at B is replaced with a reaction force at B, and the figure shows that the addition of the external loads plus the new reaction force is equivalent to the original situation.

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Concept Application 2.4 [2] – Text Alternative

Vertical rod ADCKB is hanging from pin A. All distances between points is 150 millimeters and cross-section areas are 250 millimeters squared from A to C, and 400 millimeters squared from C to B. A vertical load of -300 kilonewtons is located at D and a vertical load of -600 kilonewtons is located at K.

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Concept Application 2.4 [3] – Text Alternative

Vertical rod ADCKB is subject to reaction force A and B, at A and B, respectively. A vertical load of -300 kilo newtons is located at D and a vertical load of -600 kilo newtons is located at K.

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Shearing Strain – Text Alternative

For Figure 2.36, A cubic element is subjected to shearing stress: loads parallel to the surface, on the same surface. Loads are compensated by equal loads on perpendicular surfaces to cause deformation and not rotation.

For Figure 2.37, Cubic element deformed by shearing stress shows a deformed rhomboid shape, where the angles formed by the sides can be calculated as Ï€ over 2 Â± shearing strain.

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Concept Application 2.10 [1] – Text Alternative

Rectangular block bonded to two rigid horizontal plates. Horizontal dimensions are 62 and 200 millimeters. Vertical dimension is 50 millimeters. A load P acts parallel to the 200 millimeters dimension.

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Saint-Venant’s Principle – Text Alternative

Stress distributions near concentrated loads show that the distribution is more homogeneous as you move further away from the load (away from the edges where the load is located).

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Stress Concentration: Hole – Text Alternative

Plot for calculating stress concentration factors. The ratio between the diameter of the hole and the distance perpendicular to the load is in the x-axis. Y-axis shows the stress concentration factor, K.

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Stress Concentration: Fillet – Text Alternative

Stress concentration plot for filleted flat bars under axial loading. The ratio of the radius of the fillet to the width of the structure perpendicular to the load is the x-axis. Different curves are shown for different ratios between the thick and thin sections of the bar.

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Concept Application 2.12 [2] – Text Alternative

Stress concentration plot for filleted flat bars under axial loading. The ratio of the radius of the fillet to the width of the structure perpendicular to the load is the x-axis. Different curves are shown for different ratios between the thick and thin sections of the bar.

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Elastoplastic Materials – Text Alternative

Stress-strain diagram for elastic perfectly-plastic materials or "elastoplastic" materials. It shows a horizontal line during the plastic deformation stage instead of the typical curved section before rupture.

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Concept Applications 2.14, 2.15, 2.16 [2]– Text Alternative

Load-deflection curves for the rod, the tube, and them combined. At 1.5 millimeters, the rod is inside the plastic deformation section, while the tube is not. The resulting combined load-deflection curve shows two straight lines with distinct slopes before it reaches a horizontal line at Yield for the tube.

Return to parent-slide containing images.

Concept Applications 2.14, 2.15, 2.16 [3] – Text Alternative

Load-deflection curves for the rod, the tube, and them combined. At 1.5 millimeters, the rod is inside the plastic deformation section, while the tube is not. The resulting combined load-deflection curve shows two straight lines with distinct slopes before it reaches a horizontal line at Yield for the tube.

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Concept Applications 2.14, 2.15, 2.16 [4] – Text Alternative

Load-deflection curves for the rod, the tube, and them combined. At 1.5 millimeters, the rod is inside the plastic deformation section, while the tube is not. The resulting combined load-deflection curve shows two straight lines with distinct slopes before it reaches a horizontal line at Yield for the tube.

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