APL 105 Lecture 9 1-2-2024

Bending

Bending of a slender prismatic member (bar)

slender → x dimension >> y and z

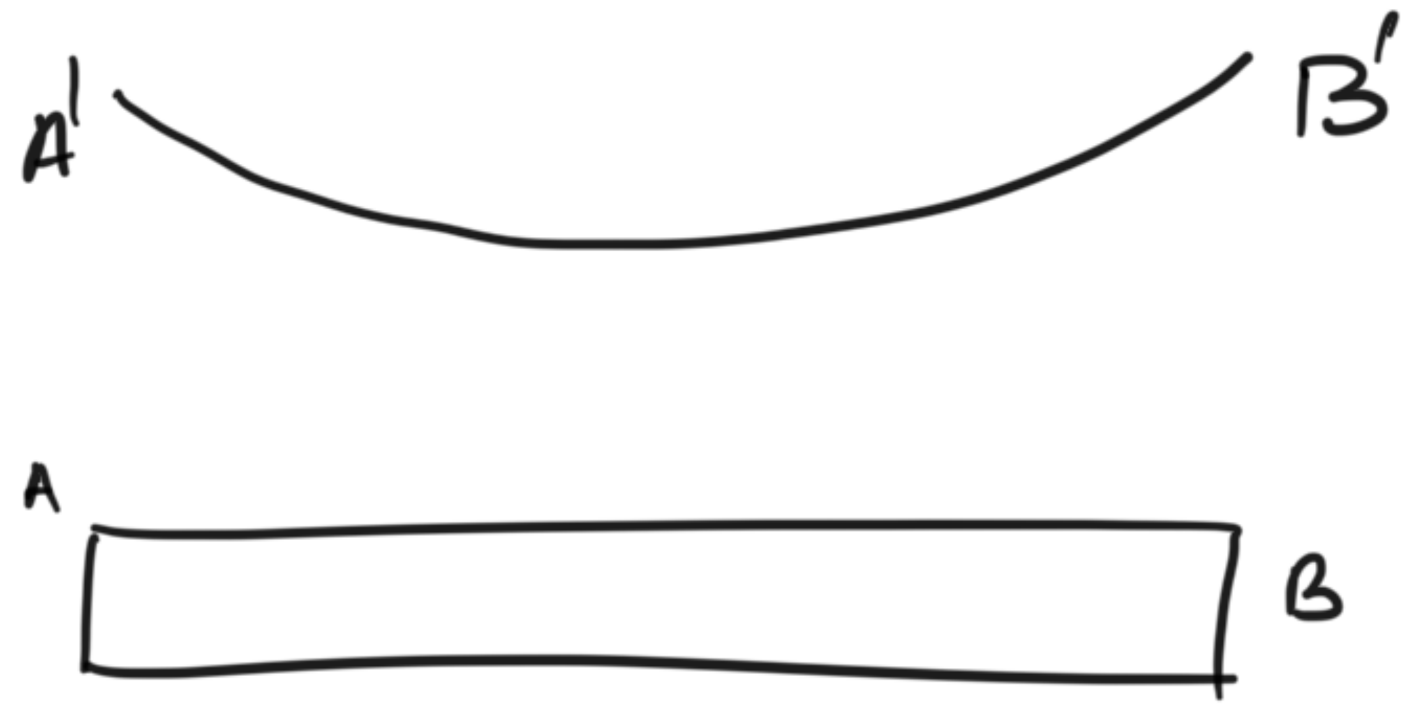
Bending is caused by moments along z and y axis in above configuration. Moment along x axis cause Torsion.

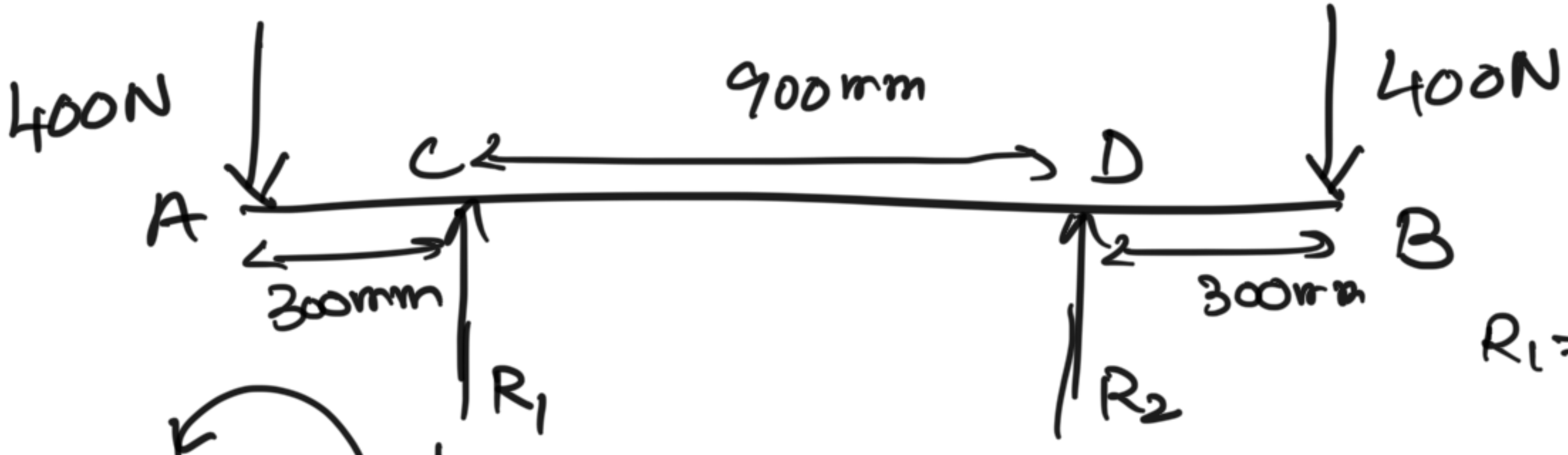
M_z: Bending moment M_z acts on a prismatic member

Along the bar only couple (moment) M and M' act. No force.

$$M = M' \quad (\text{equilibrium})$$

Pure Bending



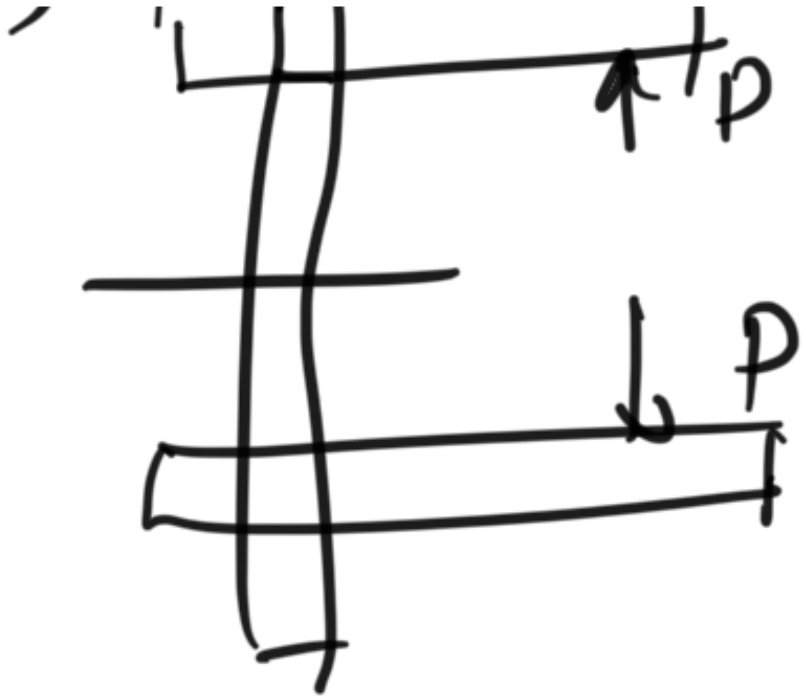


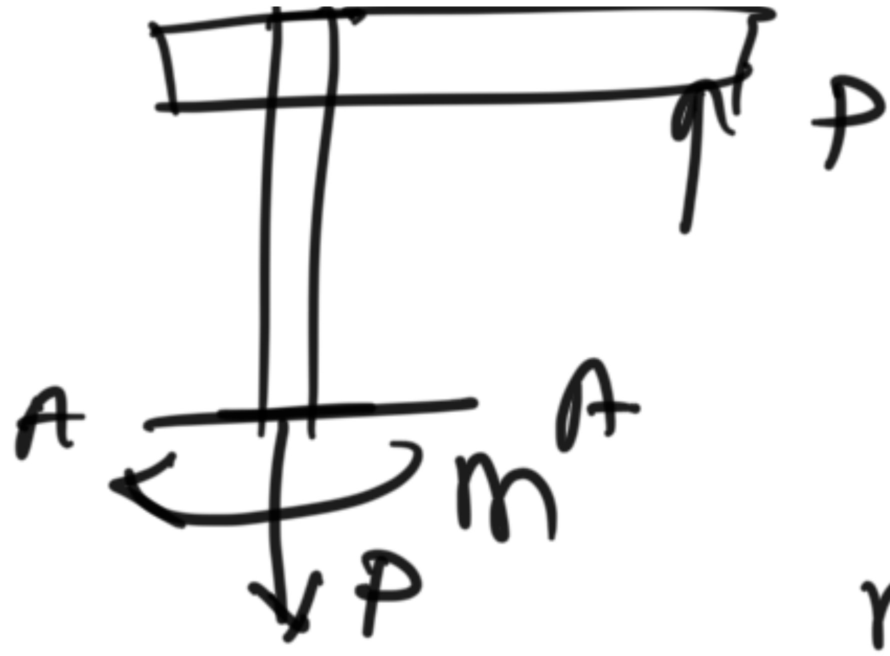
$R_1 = R_2 = 400\,N$

$400 \times 0.3 =$

other modes of bending (not pure bending)

a)

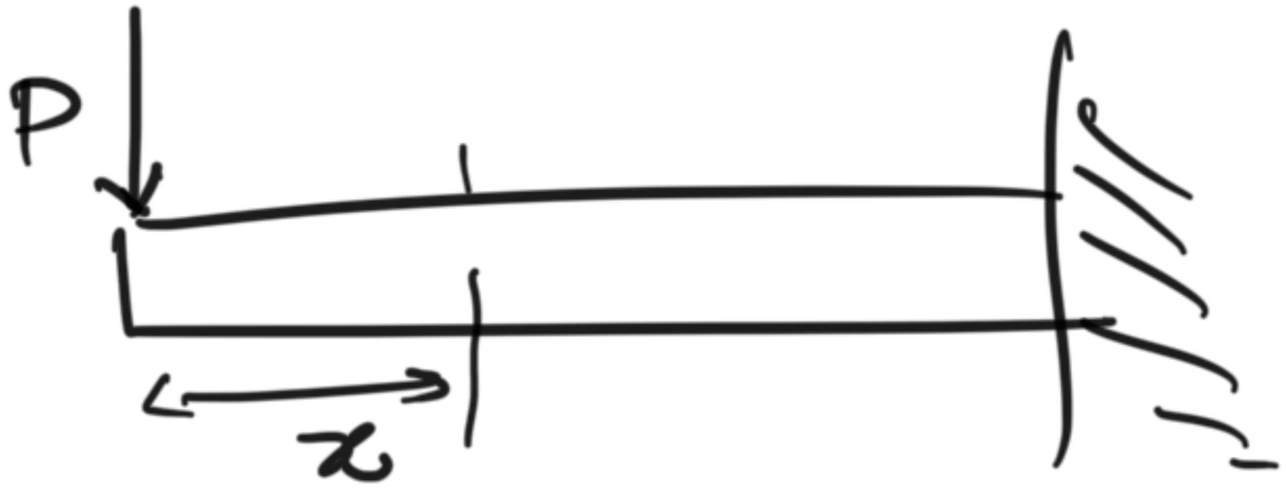


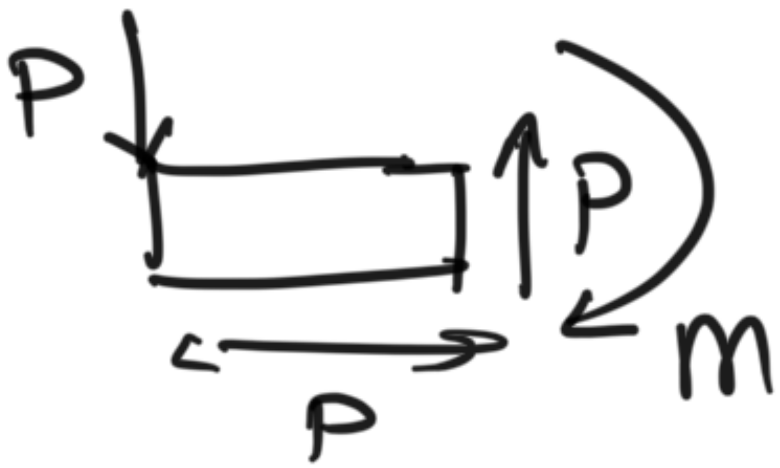


Force / Moment at AA

$$M = Pd$$

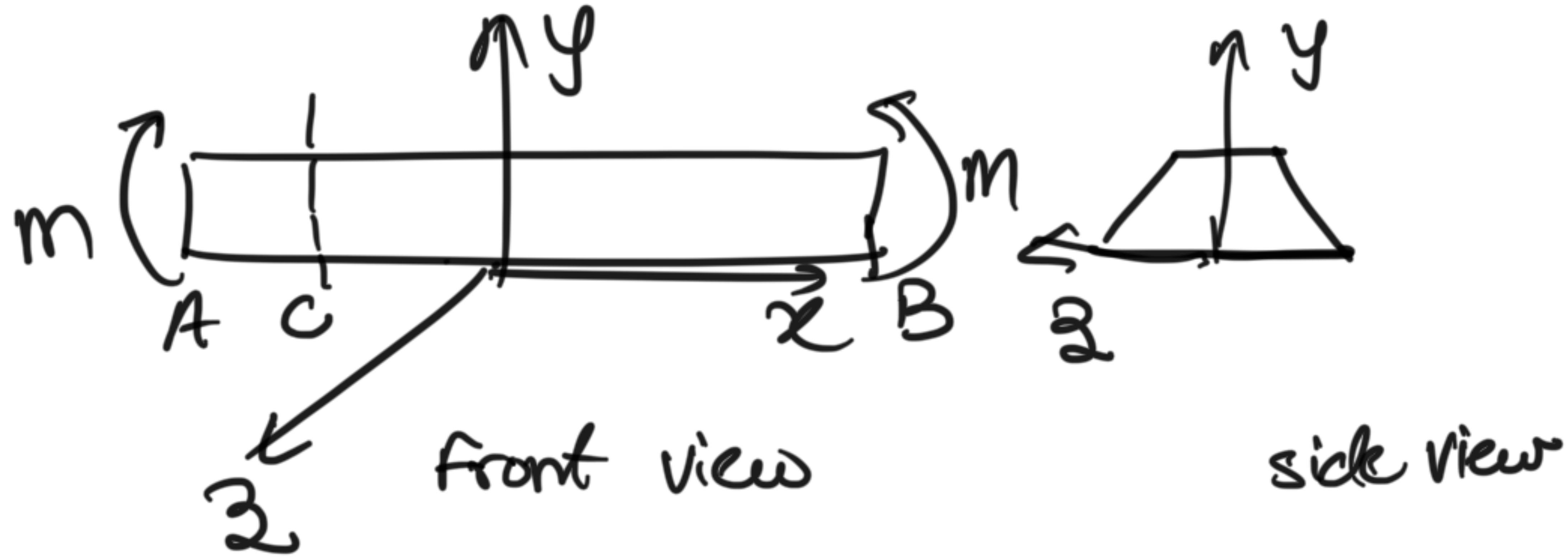
b) Beam in transverse loading





$$M = Px$$

Symmetric members in pure bending



front view   side view

xy plane of symmetry.

section passing through C

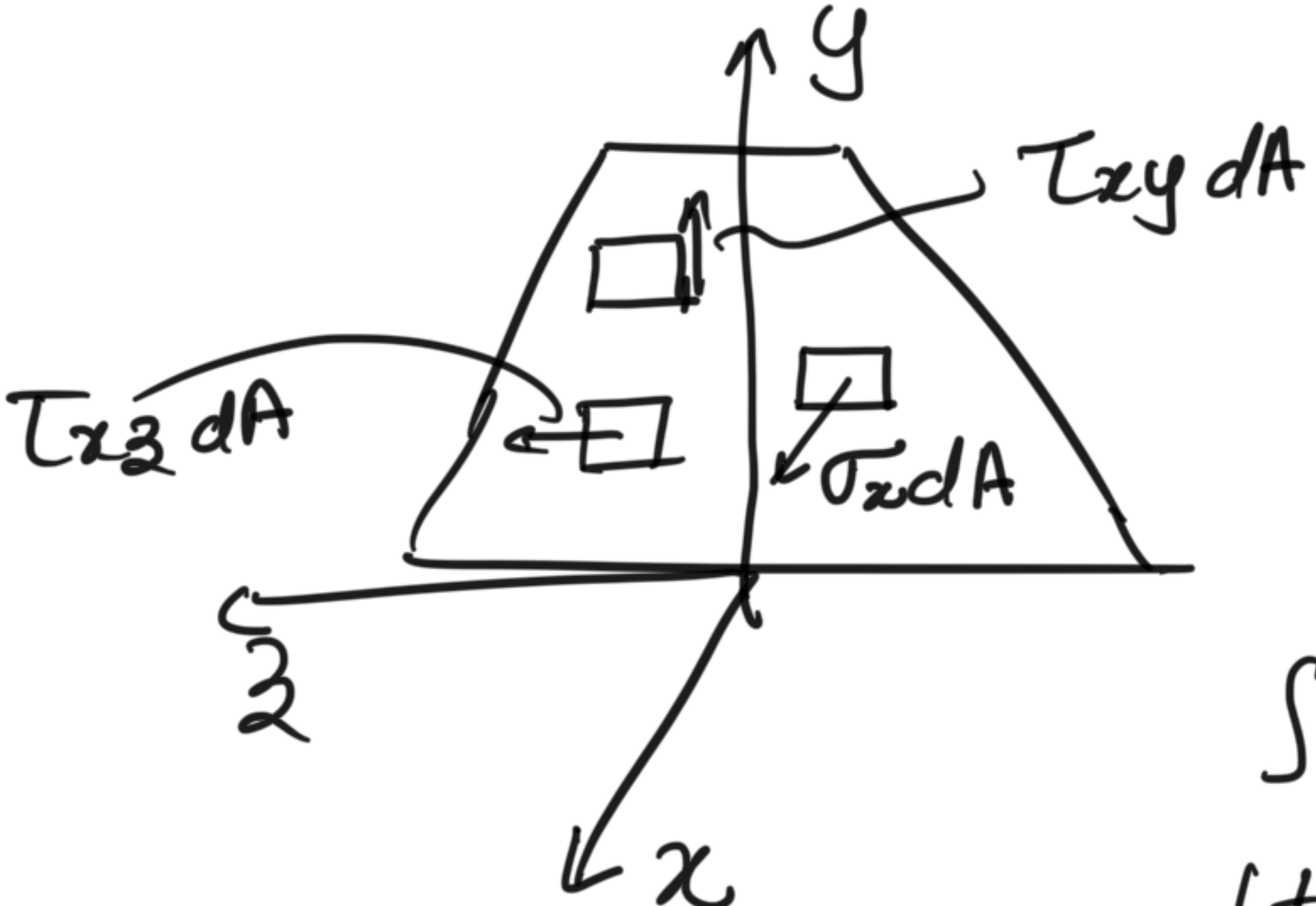
Loading on AC
(internal resultant force system at C
= M)

Bending is as shown

M is +ve.

(on a positive face M is CCW, then M is positive)

+ve M Bends the bar concave upwards



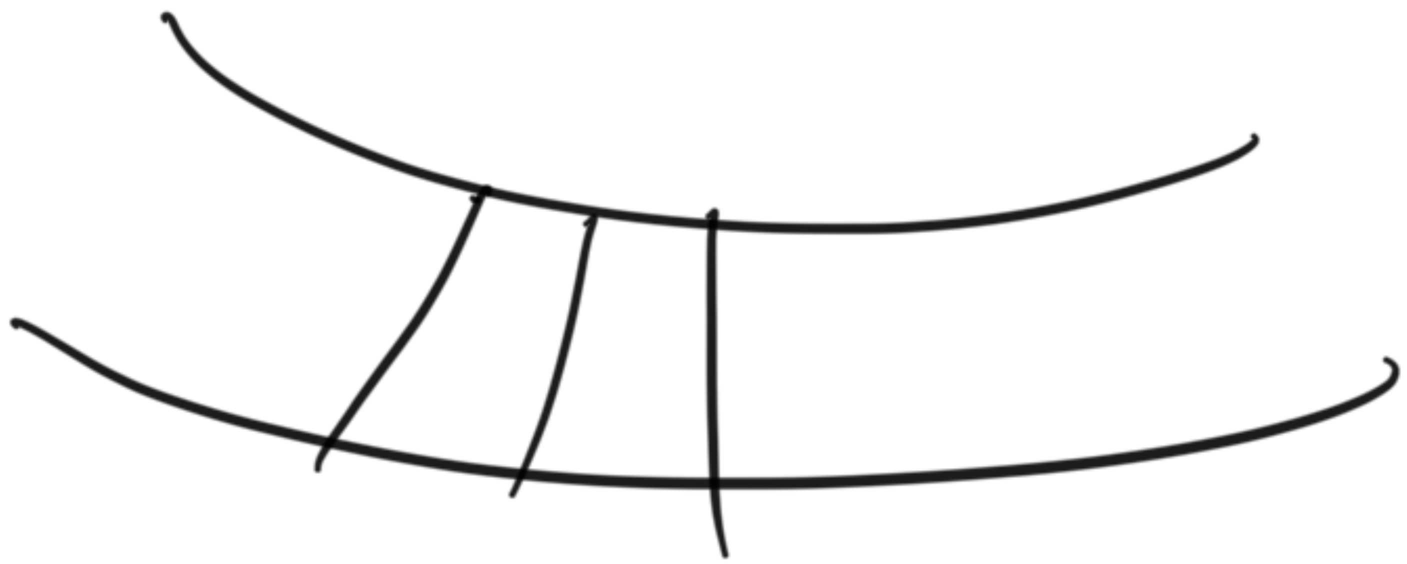
$$\int \sigma_x \, dA = F_x$$

(tensile σ_x

$$\int y \, \sigma_x \, dA = -M$$

$$\int \tau_{xy} \, dA = F_y = 0$$

$$\int \tau_{xz} \, dA = 0$$

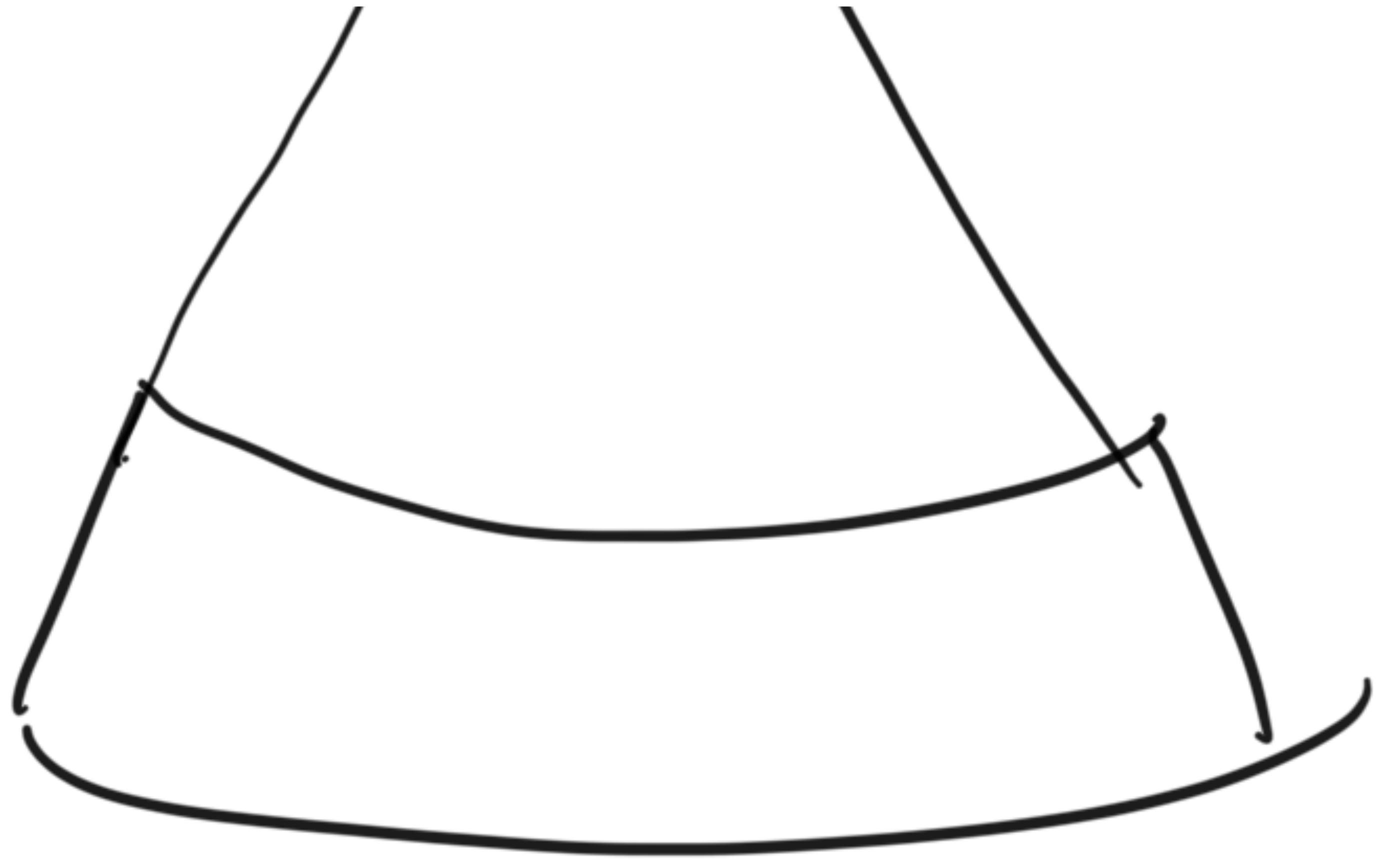


couple M acts on plane of symmetry.

Member will bend but stay symmetric w.r.t. xy plane. So the member bends uniformly.

Any cross section $\perp$ axis remains planar and the plane passes through C, centre of curvature.

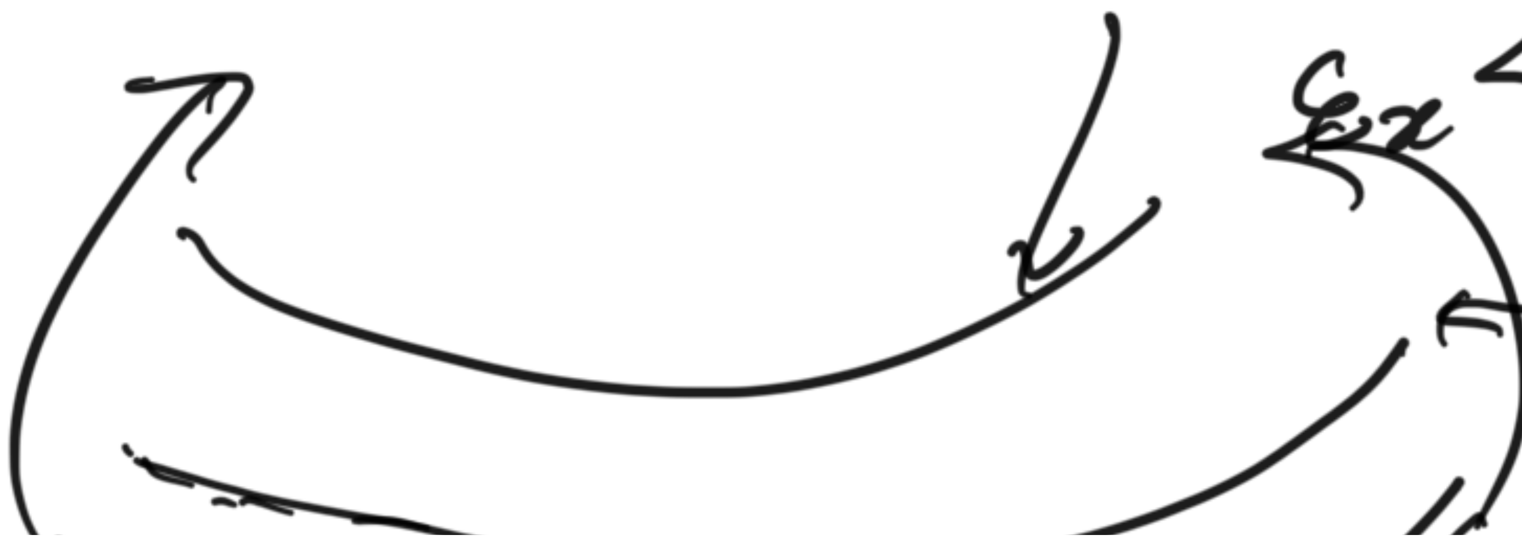
straight member



AB bends to the shape of a circular arc under pure bending.

On surface of member, $\sigma_y, \sigma_z, \tau_{yz} = 0$

Only non zero element of stress $= \sigma_x$
all other stresses for a symmetric beam in pure bending are equal to zero $(\sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{xz} = 0)$



$\epsilon_x < 0 \Rightarrow \sigma_x < 0$

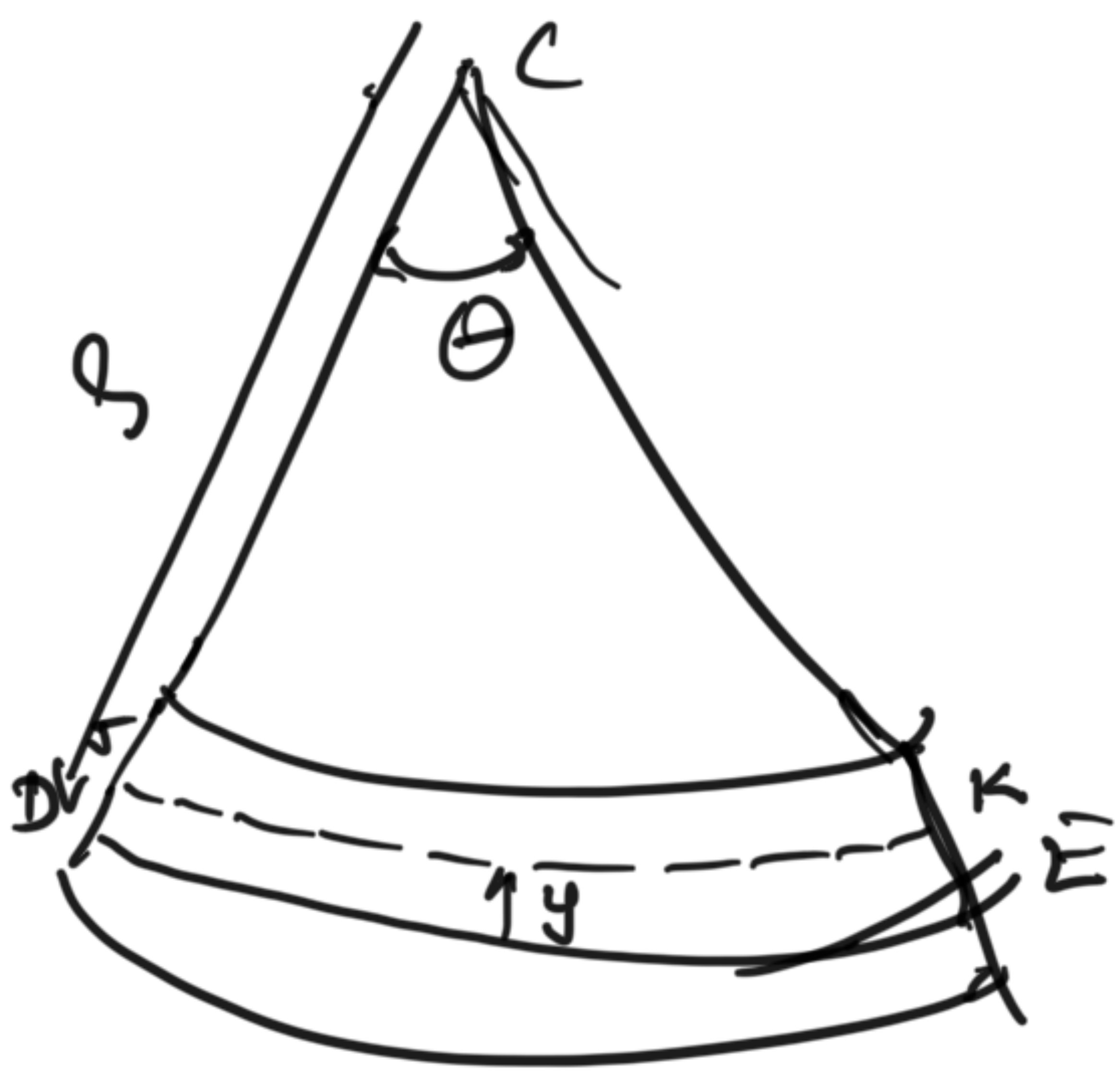
$\epsilon_x = 0$

$\varepsilon_x > 0$
$\Rightarrow \sigma_x > 0$

state of uniaxial stress (but not uniform)

There must be a surface $\parallel$ x axis where $\varepsilon_x = 0$

NEUTRAL SURFACE ($\sigma_x = 0$)



$\leftarrow$ neutral surface

Arc DE intersects a transverse section along a straight line $\parallel$ z axis $\rightarrow$ neutral axis.

C is centre of curvature. $\rho \rightarrow$ distance from C to D.

Arc Length $DE = \rho\theta = L$

arc JK located y above neutral axis,

$$JK = (\rho - y)\theta = L'$$

$$\delta = L' - L = (\rho - y)\theta - \rho\theta = -y\theta$$

$$\epsilon_x = -\frac{y\theta}{L} = -\frac{y\theta}{\rho\theta} = -\frac{y}{\rho}$$

$-$ve sign because bending moment is +ve & beam concave upwards.

ϵ_x varies linearly with y.

$|\epsilon_x|$ maximum when y is largest.

Denote largest y as c

$$\varepsilon_m = \frac{c}{\rho} \Rightarrow \frac{1}{\rho} = \frac{\varepsilon_m}{c}$$

$$\varepsilon_x = -\frac{y}{\rho} \Rightarrow \varepsilon_x = -\frac{y}{c}\varepsilon_m$$

Elastic range $\sigma_x = E\varepsilon_x$

$$E\varepsilon_x = -\frac{y}{c}(E\varepsilon_m)$$

$$\sigma_x = -\frac{y}{c}\sigma_m$$

σ_m = absolute maximum value of stress.

stress varies linearly with distance from neutral axis.

$$\int \sigma_x \, dA = 0 \Rightarrow \int -\frac{y}{c}\sigma_m \, dA = 0$$

$$\Rightarrow -\frac{\sigma_m}{c} \underbrace{\int y \, dA}= 0$$

Neutral axis passes through centroid of cross section for a) elastic range
b) pure bending

$$\int -y \, \sigma_x \, dA = M$$

if z axis coincides with neutral axis

$$\int -y \left(-\frac{y}{c} \right) \sigma_m \, dA = M$$

$$\frac{\sigma_m}{c} \int y^2 \, dA = M$$

$\int y^2 dA \rightarrow$ second moment of inertia

$$= I$$

$$\left.\begin{aligned} \sigma_m &= \cancel{R}\,\frac{Mc}{I} \\ \sigma_x &= -\frac{My}{I} \end{aligned}\right\}$$

Elastic flexural formulae

σ_x compressive above Neutral axis

σ_x tensile below neutral axis

$$\frac{I}{c} = S \quad (\text{section modulus})$$