APL 105 Lecture 7

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y - \nu\sigma_z\right)$$

$$\epsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x - \nu\sigma_z\right)$$

$$\epsilon_z = \frac{1}{E}\left(\sigma_z - \nu\sigma_x - \nu\sigma_y\right)$$

due to axial load

$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \quad \gamma_{yz} = \frac{\tau_{yz}}{G}, \quad \gamma_{xz} = \frac{\tau_{xz}}{G}$$

$$G = \frac{E}{2(1+\nu)}$$

Thermal strain (

$$\epsilon_x^t = \epsilon_y^t = \epsilon_z^t = \alpha(T - T_0)$$

if no strain at T_0 and setup is subject to temperature T.

Equilibrium equations for an isotropic solid and homogeneous

Eqlbm eqns. $\nabla \cdot \tau_{ij} + g_i = 0$ $g_i =$ body force per unit mass

$$
\left.
\begin{aligned}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + g_x &= 0 \\
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + g_y &= 0 \\
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + g_z &= 0
\end{aligned}
\right\} \text{3 eqns.}
$$

Let u, v, w be displacement in x, y, z directions

$$
\left.
\begin{aligned}
&\epsilon_x = \frac{\partial u}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}, \quad \epsilon_z = \frac{\partial w}{\partial z} \\
&\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}, \quad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\
&\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}
\end{aligned}
\right\} \text{6 eqns.}
$$

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha(T - T_0)$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha(T - T_0)$$

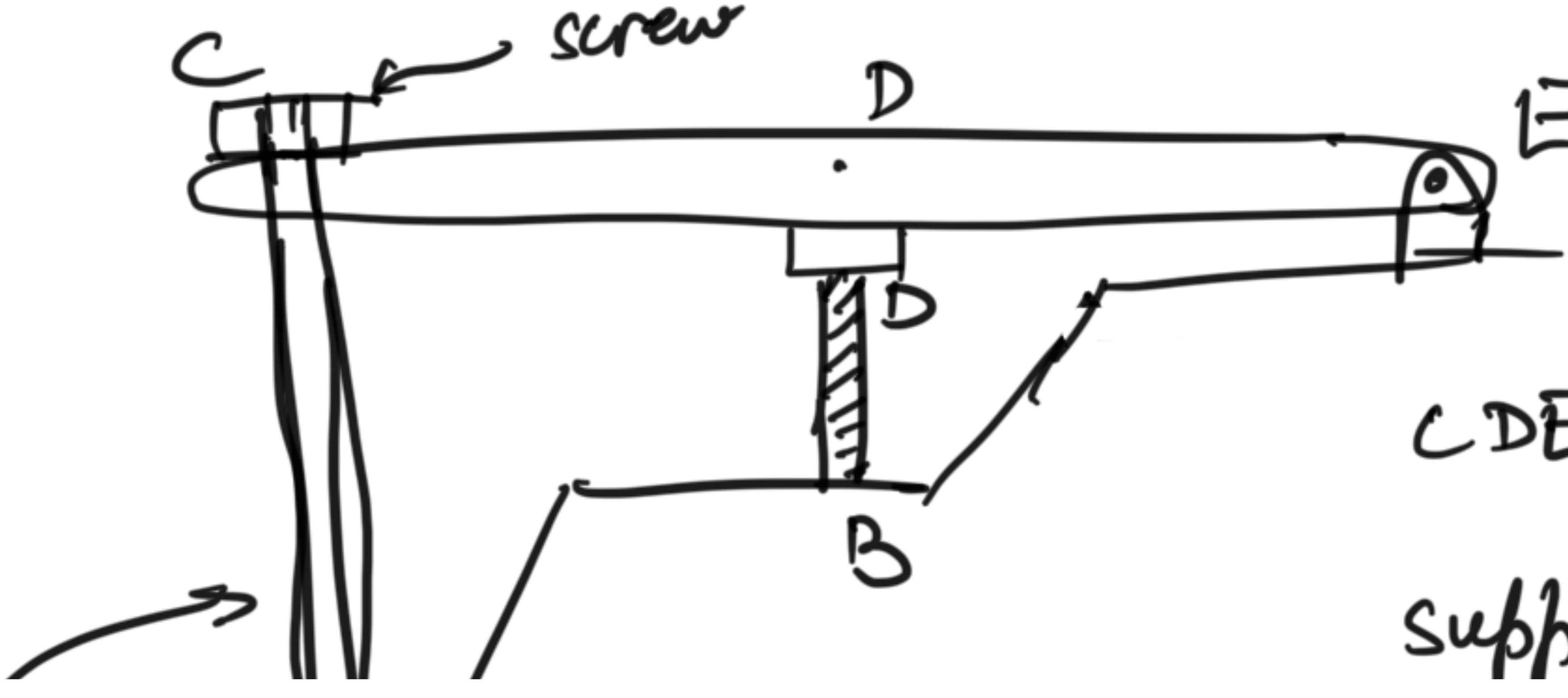
$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] + \alpha(T - T_0)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \quad \gamma_{yz} = \frac{\tau_{yz}}{G}, \quad \gamma_{xz} = \frac{\tau_{xz}}{G}$$

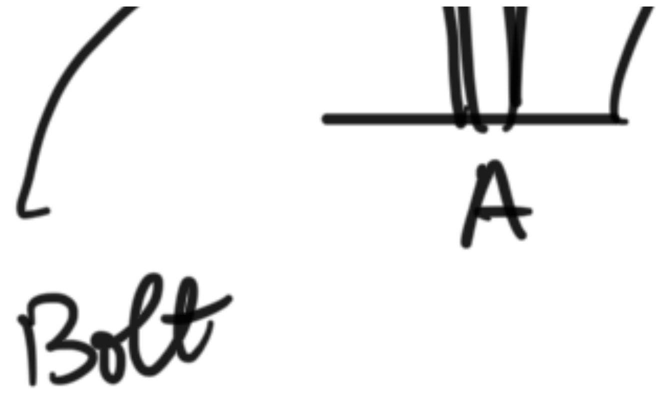
6 eqns.

<u>Elastic Limit</u>

<u>Sample Problem 2.4</u>



CDE is a rigid bar supported to a pin at E



rests on a 30 mm dia. brass cylinder BD

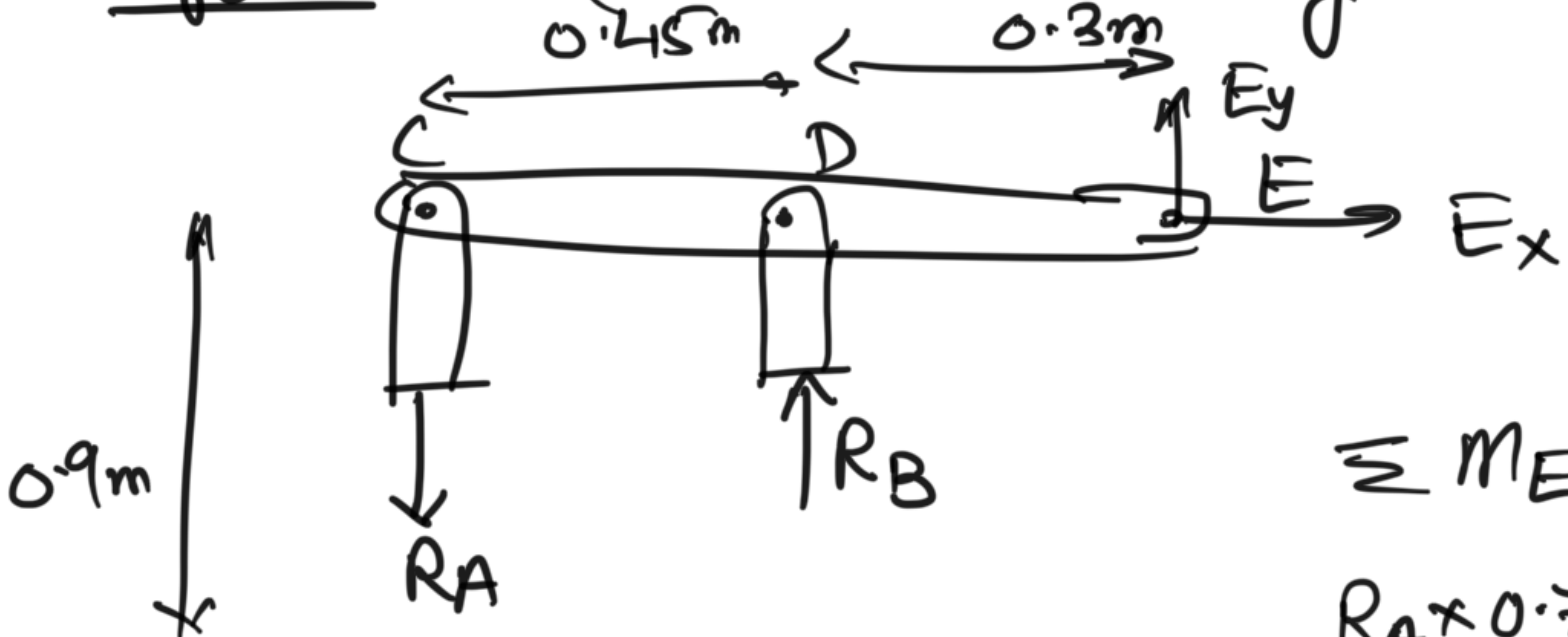
AC steel
$E = 200$ GPa
$\alpha = 11.7 \times 10^{-6} /{}^\circ C$

BD (brass)
$E = 115$ GPa
$\alpha = 20.9 \times 10^{-6} /{}^\circ C$

22 mm dia. steel bolt AC which passes through ~~rod~~ a hole in the bar and is snugly fit when temp. = 20°C

Brass cylinder heated to 50°C while steel rod is kept at 20°C.

Eqbm. (~~in the~~ assuming no strains)



$L_{BD} = 0.3$ m

$\Sigma M_E = 0$

$R_A \times 0.75 = R_B (0.3)$

$R_A = 0.4 R_B$

Remove R_B (redundant)

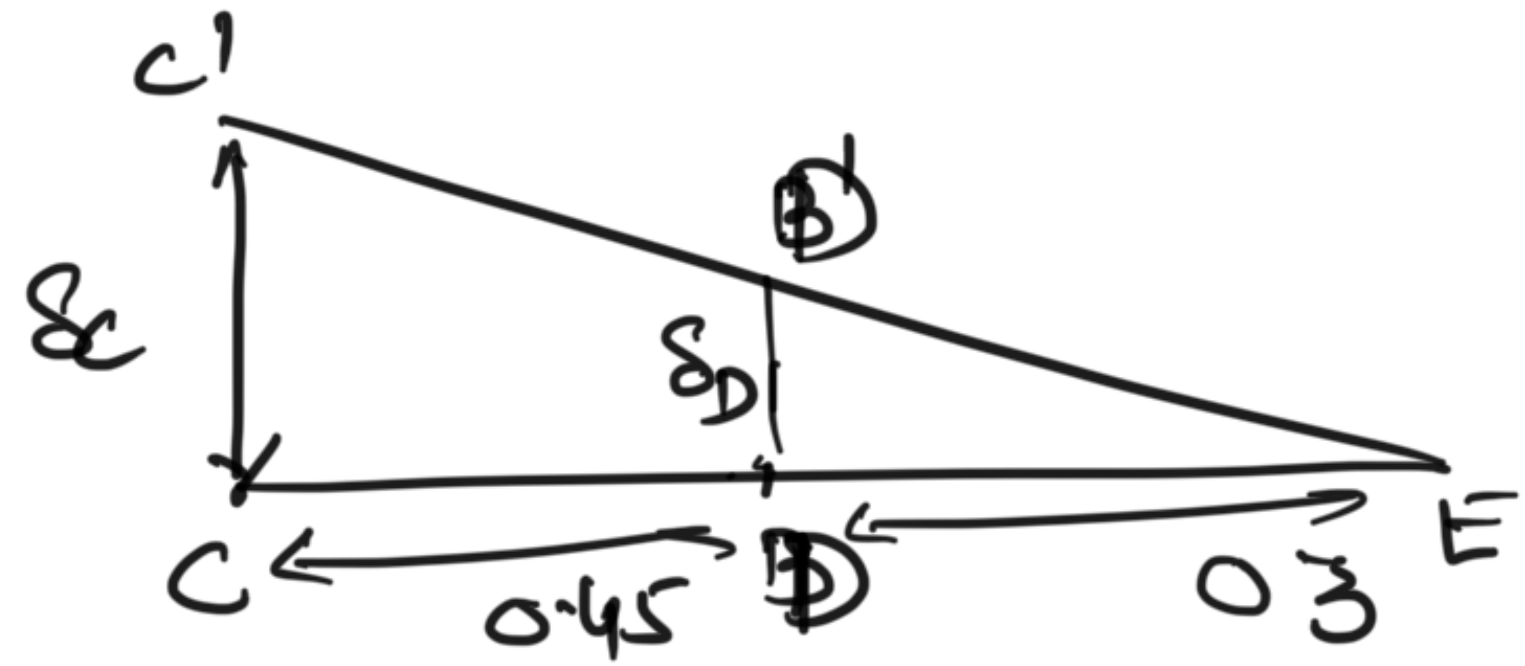
<u>find δ_{BD}</u>

δ_T in rod BD

Let R_B cause a deflection + δ_1

$$\delta_T = L(\Delta T)\alpha = 0.3(30^\circ C)(20.9 \times 10^{-6}/^\circ C)$$

$$= 188.1 \times 10^{-6} \text{ mm}$$



$$\frac{\delta_D}{\delta_C} = \frac{0.3}{0.75} \Rightarrow \delta_D = 0.4\,\delta_C$$

δ_C

$$\delta_C = \frac{R_A L}{A_s E_s} = \frac{R_A (0.9)}{\frac{\pi}{4}(0.022)^2 \times (200 \times 10^9)} = 11.84 \times 10^{-9} R_A \uparrow$$

$$\delta_D = 0.4\,\delta_C = 4.74 \times 10^{-9} R_A \uparrow$$

$$\delta_{B/D} = \frac{R_B L}{A_B E_B} = \frac{R_B (0.3)}{\frac{\pi (0.03)^2}{4} \times (105 \times 10^9)} = 4.04 \times 10^{-9} R_B \uparrow$$

δ_1 (net compresion at B) $= \delta_D + \delta_{B/D}$

$$= 4.74 \times 10^{-9} R_A + 4.04 \times 10^{-9} R_B$$

$$= 5.94 \times 10^{-9} R_B \uparrow$$

$\delta_T = \delta_1 \implies$ R_B can be found