Basic calculations of Fabric Geometry

Yarn Count

Yarn count represents the coarseness or fineness of yarns.

1. Direct systems (example: Tex, Denier)

2. Indirect systems (example: new English, i.e. Ne, Metric, i.e. Nm)

Yarn Count: Direct systems

Direct systems revolve around expressing the mass of yarn per unit length.

✓Tex: Weight in gm of 1000 m length
✓Denier: Weight in gm of 9000 m length

For example, 10 tex yarn implies that a piece of 1000 m long yarn will have a mass of 10 g.

Similarly, for 10 denier, a piece of 9000 m long yarn will have a mass of 10 g.

As Tex or Denier value increases, yarn becomes coarser

Yarn Count: Indirect systems

In contrast, indirect system expresses the length of yarn per unit mass.

✓English (Ne): Number of hanks of 840 yards length weighing in 1 pound
✓Metric (Nm): Number of hanks of 1000 m length weighing in 1 km.

For example, 10 Ne implies that a 1-pound yarn will have a length of 10 × 840 yards.

As the Ne or Nm value increases, the yarn becomes finer.

Yarn Count

TABLE 1.2 Direct and Indirect Systems of Yarn Count

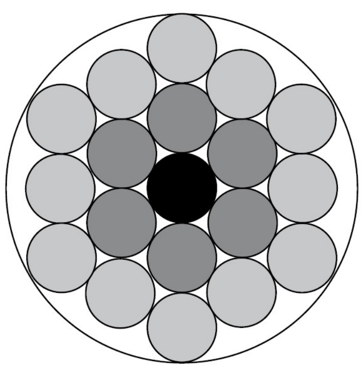
Type	Name	Unit of Mass	Unit of Length
Direct	Tex	Gram	1000 m
	Denier	Gram	9000 m
Indirect	Ne	Pound	Hank (840 yards)
	Metric	Kilogram	Kilometre

The following conversion formula is used to change the yarn count from one system to another.

$$\text{Tex} = \frac{590.5}{\text{Ne}}, \quad \text{Denier} = 9 \times \text{Tex} \quad \text{and} \quad \text{Denier} = \frac{5315}{\text{Ne}}$$

Packing Factor or Packing Coefficient

Packing factor or packing coefficient represents the extent of closeness of fibres within the yarn structure.



For the same yarn linear density, if the fibres are closely packed (high level of twist), then yarn diameter will be less.

$$\text{Packing factor} = \frac{\text{Cumulative area of all fibres}}{\text{Area of yarn cross section}} = \frac{\text{Yarn density}}{\text{Fibre density}}$$

Packing Factor or Packing Coefficient

The equilateral triangle ABC actually indicates the repeat area.

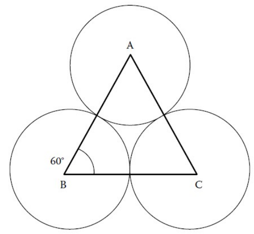
If fibre radius is r_f , then each side of the triangle ABC is having a length of $2r_f$.

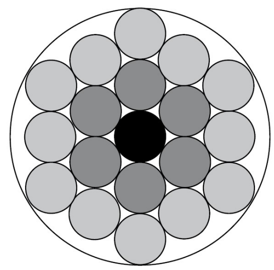
$$\text{The area of triangle ABC} = \frac{\sqrt{3}}{4}\left(2r_f\right)^2 = \sqrt{3}r_f^2$$

$$\text{The total area of fibre inside the triangle ABC} = 3\times\frac{1}{6}\pi r_f^2 = \frac{\pi}{2}r_f^2$$

So maximum possible packing factor

$$= \frac{\text{Total area of fibre inside triangle ABC}}{\text{Area of triangle ABC}} = \frac{(\pi/2)r_f^2}{\sqrt{3}r_f^2} = 0.91$$





Packing Factor or Packing Coefficient

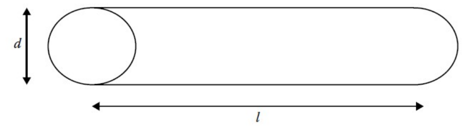
For spun yarns, packing factor generally lies between 0.55 and 0.65.

Yarns with lower packing factor are expected to be bulkier and softer.

They can cause higher fabric cover for same fabric construction parameters.

Numerical

Prove that for cotton yarn with packing factor of 0.6, diameter (inch) $\boxed{\dfrac{1}{28\sqrt{Ne}}}$



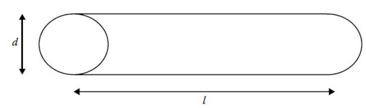
As the yarn count is 'Ne', there will be Ne number of hanks (840 yards) in 1 pound.

Thus length (l) = $Ne \times 840 \times 36$ *inch.*

Density of cotton fibre is 1.51 g/cm^3.

As the packing factor is 0.6, the density of the cotton yarn will be 1.51 × 0.6 = 0.906 g/cm^3.

Numerical



$$\text{The density of cotton yarn} = \frac{0.906 \times 2.54^3}{453.6}\,\text{pound/inch}^3$$
$$= 0.0327\ \text{pound/inch}^3$$

$$\text{The volume of yarn} = \frac{\pi d^2}{4} l\ \text{inch}^3$$

$$\text{Mass of the yarn} = \frac{\pi d^2}{4} l \times 0.0325\ \text{pound}$$
$$= \frac{\pi d^2}{4} \times \text{Ne} \times 840 \times 36 \times 0.0327\ \text{pound}$$

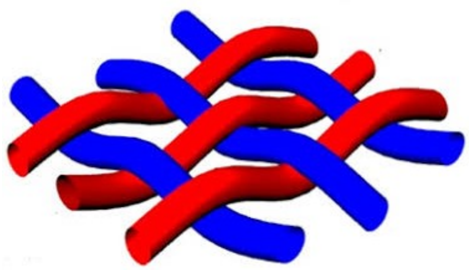
Therefore,

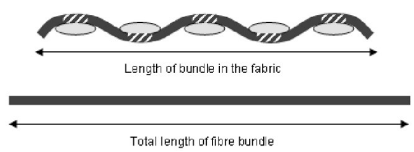
$$d\,(\text{inch}) = \frac{1}{28\sqrt{\text{Ne}}}$$

Crimp

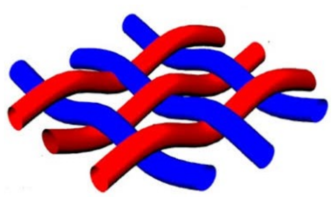
Once the warp and weft are interlaced, both of them assume **wavy or sinusoidal**-like path.

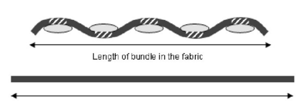
Thus the length of the yarn becomes **more** than that of the fabric within which the former is constrained.





Crimp





Crimp is a measure of the **degree of waviness** present in the yarns inside a woven fabric

Contraction is another measure of yarn waviness

$$\text{Crimp \%} = \frac{\text{Length of yarn} - \text{Length of fabric}}{\text{Length of fabric}} \times 100$$

$$\text{Contraction \%} = \frac{\text{Length of yarn} - \text{Length of fabric}}{\text{Length of yarn}} \times 100$$

Numerical

The length of a fabric is 10 m. The length of a warp yarn, removed from the fabric, in straight condition is 10.8 m. Determine the crimp % and contraction % in warp direction.

$$\text{Crimp \%} = \frac{L_{yarn} - L_{fabric}}{L_{fabric}} \times 100$$

where
L_{yarn} is the length of warp yarn removed from the fabric
L_{fabric} is the length of fabric

$$\text{Crimp} = \frac{(1.08-1)}{1} \times 100 = 8\%$$

$$\text{Contraction} = \frac{L_{yarn} - L_{fabric}}{L_{yarn}} \times 100$$

$$= \frac{1.08-1}{1.08} \times 100 = 7.41\%$$

Fractional Cover and Cover Factor

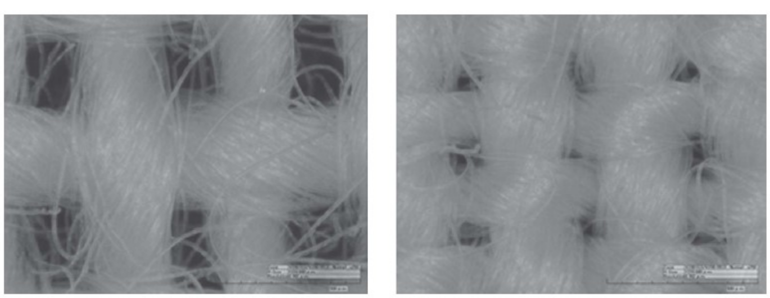
Fractional cover is the **ratio of the area covered by the yarns to the total area of the fabric**.

It influences the following properties of woven fabrics:

✓Air permeability

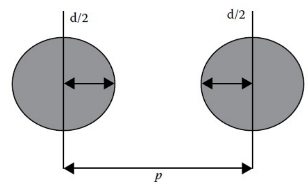
✓Moisture vapour permeability

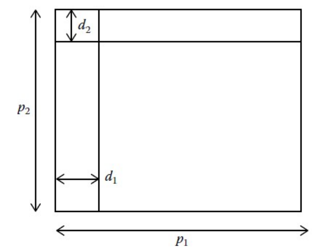
✓Ultraviolet or any other types of radiation protection



Fractional Cover and Cover Factor

If diameter of warp yarn is d1 inch and spacing, that is gap between the two consecutive ends is p1 inch, **then fractional cover for warp (k1) is d1/p1**.





Now, for cotton yarns, having packing factor of 0.6, the relationship between yarn diameter (d) in inch and yarn count (Ne) is as follows:

$$d = \frac{1}{28\sqrt{\mathrm{Ne}}}$$

Fractional Cover and Cover Factor

The relationship between end spacing (p1) and **ends per inch** (n1)

$$n_1 = \frac{1}{p_1}$$

After rearranging, the fractional cover for warp

$$k_1 = \frac{n_1}{28\sqrt{\mathrm{Ne}_1}}$$

Similarly, for fractional cover for weft (*k2*)

$$k_2 = \frac{n_2}{28\sqrt{\mathrm{Ne}_2}}$$

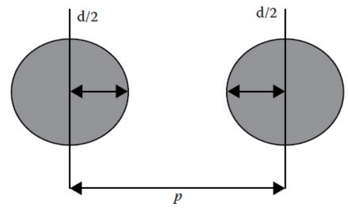
Cover factor by multiplying fractional cover with 28.

$$\text{Warp cover factor} = k_w = 28 \times k_1 = \frac{n_1}{\sqrt{\mathrm{Ne}_1}}$$

Numerical

A cotton fabric is made from 20 Ne warp and ends per inch is 50. Determine the warp cover factor.

$$\text{End spacing}\,(p) = \frac{1}{50''} = 0.02 \text{ inch}$$



Spacing between two ends.

$$\text{Warp yarn diameter}\,(d) = \frac{1}{28\sqrt{\text{Ne}}} \text{ inch}$$

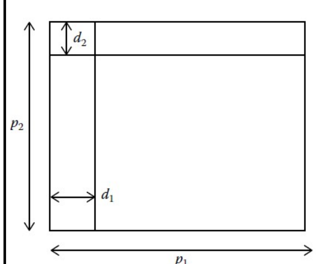
$$= \frac{1}{28\sqrt{20}} = 0.008 \text{ inch}$$

$$\text{Warp cover factor} = 28 \times \frac{d}{p}$$

$$= 28 \times \frac{0.008}{0.02} = 11.2$$

Numerical

Show that the expression for fabric cover factor is $\boxed{K_1 + K_2 - \frac{K_1 K_2}{28}}$, where K1 is warp cover factor and K2 is weft cover factor.



$$\text{Fractional cover} = \frac{\text{Area covered by the yarns within the repeat}}{\text{Area of the repeat}}$$

$$= \frac{d_1 p_2 + d_2 p_1 - d_1 d_2}{p_1 p_2}$$

$$= \frac{d_1}{p_1} + \frac{d_2}{p_2} - \frac{d_1 d_2}{p_1 p_2}$$

$$\text{Cover factor} = 28 \times \text{Fractional cover}$$

$$= 28\frac{d_1}{p_1} + 28\frac{d_2}{p_2} - 28\frac{d_1 d_2}{p_1 p_2}$$

$$\text{Fabric cover factor}\,(K_f) = K_1 + K_2 - \frac{K_1 K_2}{28}$$

Porosity

Porosity is a measure of **presence of void or air** inside the fabric or fibrous assemblies.

It indicates the percentage of volume of fabric that has been occupied by the air.

For example, woven and knitted fabrics can have typical porosity of 70%–80% and 80%–90%, respectively.

Porosity influences the **thermal conductivity** of the fabric or fibrous assemblies.

Porosity

Let Fabric areal density or gram per square meter (GSM) or (G) in g/m^2, Thickness of fabric (T) in m, Density of fibre (ρ) in g/m^3 and Porosity (P) in %.

$$\text{The mass of 1 m}^3 \text{ fabric will be} = \frac{\text{Areal density}}{\text{Thickness}} = \frac{G}{T}\ \text{g}$$

Mass of 1 m^3 fabric = Volume occupied by fibre in 1 m^3 fabric × density of fibre

$$\left(1 - \frac{P}{100}\right) \times \rho\ \text{g.}$$

$$\text{Therefore, } \left(1 - \frac{P}{1000}\right) \times \rho = \frac{G}{T}$$

$$\text{So porosity}(\%) = \left(1 - \frac{G}{T \times \rho}\right) \times 100$$

Thermal conductivity

Air is a poor conductor of heat, and its thermal conductivity (K_{air}) is 0.025W/mK

On the other hand, thermal conductivity of cotton fibre is around 0.24W/m K, which is approximately 10 times more than that of air

$$\text{Thermal conductivity of fabric} = \left(1 - \frac{P}{100}\right) K_{fibre} + \frac{P}{100} \times K_{air}$$

Therefore, **higher porosity implies lower thermal conductivity** of the fabric and vice versa.

Numerical

Calculate the porosity and thermal conductivity of a needle-punched nonwoven fabric made of 100% polypropylene fibres (density is 0.9 g/cm^3) having the thickness of 3 mm. The areal density of the nonwoven fabric is 300 g/m^2 and the thermal conductivity of polypropylene fibre is 0.12 W/m K.

$$\text{Then porosity}(\%) = \left(1 - \frac{G}{T \times \rho}\right) \times 100$$

$$\text{Porosity} = \left(1 - \frac{300}{0.003 \times 0.9 \times 10^6}\right) \times 100$$
$$= (1 - 0.111) \times 100 = 88.89\%$$

$$\text{Thermal conductivity of fabric} = \left(1 - \frac{P}{100}\right) K_{fibre} + \frac{P}{100} \times K_{air}$$
$$= \left(1 - \frac{88.89}{100}\right) 0.12 + \frac{88.89}{100} \times 0.025$$
$$= 0.036 \text{ W/m K}$$

Areal Density

Areal density is expressed by the mass of the fabric per unit area (g/m^2), which is popularly called GSM.

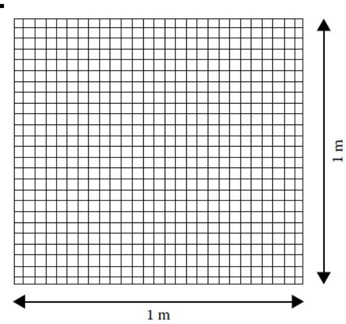
TABLE 1.3 Cover, Areal Density, Thickness and Porosity Values of Fabrics

Fabric Type	Cover (%)	Areal Density (g/m^2)	Thickness (mm)	Porosity (%)
Polyester-cotton	78.4	122	0.298	72.9
Polyester-cotton	84.6	136	0.307	70.7
100% cotton	88.2	135	0.402	77.8
100% cotton	92.9	155	0.424	75.8
100% cotton	95.2	171	0.442	74.4

Areal Density

It depend on

- ✓Warp yarn count (tex): T1
- ✓Weft yarn count (tex): T2
- ✓Ends per unit length (EPcm): N1
- ✓Picks per unit length (PPcm): N2
- ✓Crimp % in warp: C1
- ✓Crimp % in weft: C2



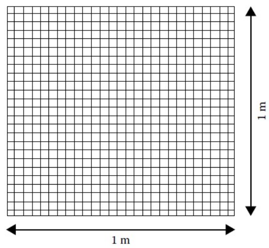
Let us consider a piece of fabric having dimensions of 1 m × 1 m.

The number of ends per cm is N1.

So, the total number of ends in the given fabric is 100 N1.

The projected length of one end is 1 m when incorporated in the fabric. However, the end has some crimp in it.

Areal Density



$$\text{Straightened length of one end} = 1 \times \left(1 + \frac{C_1}{100}\right) \text{ m}$$

Total length of ends (warp)

$$= \text{Total number of ends} \times \text{straightened length of one end}$$

$$= 100N_1 \times \left(1 + \frac{C_1}{100}\right) \text{ m.}$$

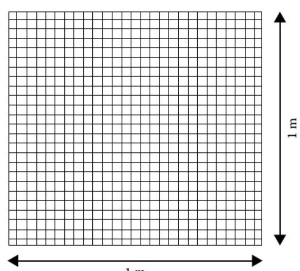
$$\text{Mass of warp yarns (g)} = \frac{\text{Total length of ends in m}}{1000} \times \text{tex of warp}$$

$$= \frac{100N_1 \times \left(1 + \frac{C_1}{100}\right)}{1000} \times T_1 = \frac{N_1 T_1}{10}\left(1 + \frac{C_1}{100}\right)$$

Similarly

$$\text{The mass of weft yarns (g)} = \frac{N_2 T_2}{10}\left(1 + \frac{C_2}{100}\right)$$

Areal Density



$$\text{Total mass of the fabric (g)} = \text{Mass of warp yarns (g)} + \text{Mass of weft yarns (g)}$$

$$= \frac{N_1 T_1}{10}\left(1 + \frac{C_1}{100}\right) + \frac{N_2 T_2}{10}\left(1 + \frac{C_2}{100}\right)$$

$$= \frac{1}{10}\left[N_1 T_1 \left(1 + \frac{C_1}{100}\right) + N_2 T_2 \left(1 + \frac{C_2}{100}\right)\right]$$

$$\text{Areal density of fabric or GSM} = \frac{1}{10}\left[N_1 T_1 \left(1 + \frac{C_1}{100}\right) + N_2 T_2 \left(1 + \frac{C_2}{100}\right)\right]$$

Numerical

Calculate the areal density (g/m²) of the cotton fabric having the following specifications: Warp and weft count: 22s × 18s; 25 ends per cm × 16 picks per cm; warp crimp is 6.5% and weft crimp is 8.5%.

$$\text{Warp count is } 22\text{ Ne} = \frac{590.5}{22}\text{ tex} = 26.84\text{ tex } (T_1)$$

$$\text{Weft count is } 18\text{ Ne} = \frac{590.5}{18} = 32.81\text{ tex } (T_2)$$

$$\text{Areal density of fabric} = \frac{1}{10}\left[N_1T_1\left(1+\frac{C_1}{100}\right)+N_2T_2\left(1+\frac{C_2}{100}\right)\right]$$

$$= \frac{1}{10}\left[25\times 26.84\left(1+\frac{6.5}{100}\right)+16\times 32.81\left(1+\frac{8.5}{100}\right)\right]$$

$$= 128.42\text{ g/m}^2$$