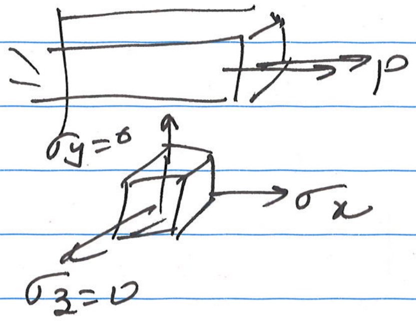
Poisson's Ratio:



$$\sigma_x = \frac{P}{A}, \quad \varepsilon_x = \frac{\sigma_x}{E}$$

Does it mean $\varepsilon_y = \varepsilon_z = 0$

No, because elongation in x accompanied by contraction in y and z. Assuming homogeneous (position) & isotropic (direction) material,

$$\varepsilon_y = \varepsilon_z \neq 0 \quad (-\text{ve generally}).$$

$\varepsilon_y, \varepsilon_z \rightarrow$ lateral strain

$\varepsilon_x \rightarrow$ axial strain

Poisson's ratio $\nu = -\dfrac{\text{lateral strain}}{\text{axial strain}}$

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

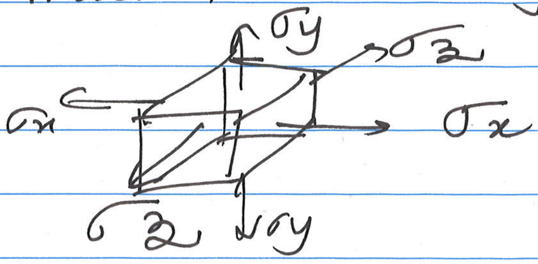
$$\varepsilon_x = \frac{\sigma_x}{E}, \quad \varepsilon_y = \varepsilon_z = -\frac{\nu \sigma_x}{E}$$

(Volume of material cannot conserved)

Some materials $\nu < 0$.

AT Section 2.9 → read yourself

Multiaxial Loading (Generalized Hooke's law)



slender element with axial loads.

$\sigma_x, \sigma_y, \sigma_z \neq 0$ (all different)

Multiaxial loading
isotropic material
Let original cube has side 1
Deformed $(1+\varepsilon_x)$, $(1+\varepsilon_y)$, $(1+\varepsilon_z)$
(Translation not included) only deformation
Superposition
Add all effects
a) stress v/s strain linear
b) Deformation is small and does not affect application of other loads

σ_x: $\frac{\sigma_x}{E}$ in x direction $-\frac{\nu\sigma_x}{E}$ (y, z direction)

σ_y: $\frac{\sigma_y}{E}$ in y direction $-\frac{\nu\sigma_y}{E}$ (x, z)

σ_z: $\frac{\sigma_z}{E}$ in z $-\frac{\nu\sigma_z}{E}$ (x, y)

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$
$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$
$$\varepsilon_z = -\nu\sigma_x/E - \nu\sigma_y/E + \frac{\sigma_z}{E}$$

generalized Hooke's law

Dilitation & Bulk Modulus

$$V = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$$

$\varepsilon_x, \varepsilon_y, \varepsilon_z << 1$

$$V = 1 + \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Change in volume $e = V - 1 = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$$e = \frac{\text{change in Volume}}{\text{Volume}} \rightarrow \text{dilitation}$$

$$e = \frac{\sigma_x + \sigma_y + \sigma_z}{E} - \frac{2\nu(\sigma_x + \sigma_y + \sigma_z)}{E}$$

$$= \frac{(1-2\nu)}{E}(\sigma_x + \sigma_y + \sigma_z)$$

When uniform hydrostatic pressure

$$p = -\frac{(\sigma_x + \sigma_y + \sigma_z)}{3}$$

$$e = -3p\frac{(1-2\nu)}{E}$$

Bulk Modulus $= K = \dfrac{E}{3(1-2\nu)}$

Modulus of compressibility

$$e = -\frac{p}{K}$$

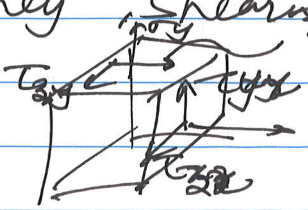
As material subject to p can only reduce

$K > 0 \Rightarrow (1-2\nu) > 0$

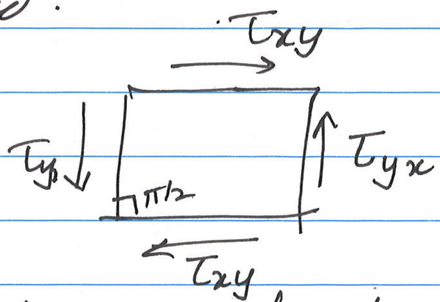
$$0 < \nu < \frac{1}{2}$$

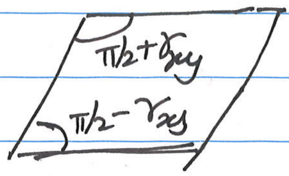
Shearing Strain:

Only shearing stress



$\tau_{yz} \rightarrow$ tends to deform the shape of a cuboidal element into oblique parallelopiped.



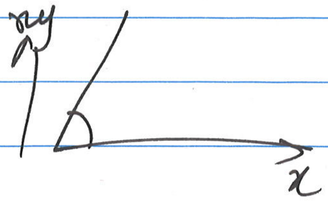


unit cube $\rightarrow$ rhomboid of sides of length 1.

2 angles $\rightarrow$ $\frac{\pi}{2} - \gamma_{xy}$

2 angles $\rightarrow$ $\pi/2 + \gamma_{xy}$ $\qquad \gamma_{xy} \rightarrow$ radians

$\gamma_{xy} \rightarrow$ shearing strain (corresponding to x & y directions)



γ_{xy} positive

(only deformation rotation is not counted)

Plot τ_{xy} v/s γ_{xy} diagram

Same shape, strengths about half of tensile

$$\tau_{xy} = G\,\gamma_{xy} \qquad (G = \text{Shear Modulus}$$
Hooke's law for shear $\qquad$ Modulus of Rigidity)

$\gamma_{xy} \rightarrow$ dimensionless

$G \rightarrow$ units of N/m^2

Generally $G < \frac{1}{2} E$

$G > \frac{1}{3} E$

$$\frac{1}{3}E < G < \frac{1}{2}E$$

Similarly $\tau_{yz} = G\gamma_{yz}$, $\tau_{xz} = G\gamma_{xz}$

Apply superposition (assuming elastic limits)
Hooke's law for isotropic homogeneous material

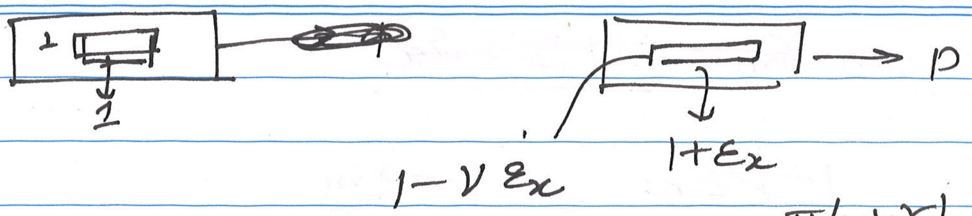
$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

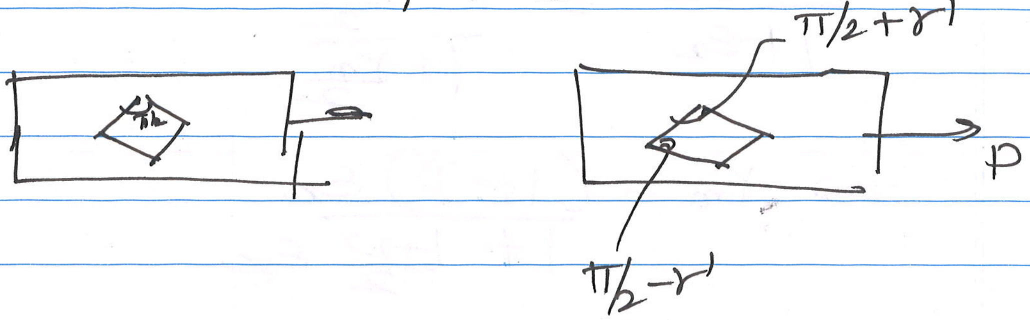
$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

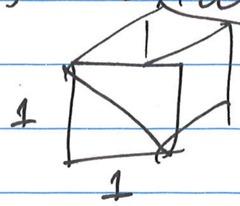
$$\cancel{\tau_{xy}} \quad \gamma_{xy} = \frac{\tau_{xy}}{G}, \quad \gamma_{yz} = \frac{\tau_{yz}}{G}, \quad \gamma_{xz} = \frac{\tau_{xz}}{G}$$

3 quantities E, ν, G

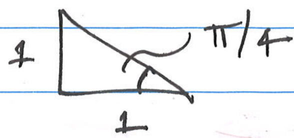




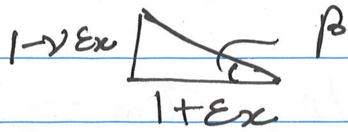
same load P causes elongation/ deformation based on area element.
(Shear strain maximum at an $\angle$ of $45°$ for axial loading).



prismatic elem



After load



$$\beta = \frac{\pi}{4} - \frac{\gamma_m}{2}$$

$$\tan\beta = \frac{\tan\frac{\pi}{4} - \tan\frac{\gamma_m}{2}}{1+\tan\pi/4 \tan\frac{\gamma_m}{2}} = \frac{1-\tan\frac{\gamma_m}{2}}{1+\tan\frac{\gamma_m}{2}}$$

γ_m small so $\tan\gamma \to \gamma \Rightarrow \tan\beta = \dfrac{1-\frac{\gamma_m}{2}}{1+\frac{\gamma_m}{2}}$

$$\tan\beta = \frac{1-\nu\varepsilon_x}{1+\varepsilon_x}$$

P.T.O.

$$\Rightarrow \cancel{\gamma_m = (1+\nu)\varepsilon_x}$$

$$\frac{1-\nu\varepsilon_x}{1+\varepsilon_x} = \frac{1-\frac{\gamma_m}{2}}{1+\frac{\gamma_m}{2}}$$

$$\Rightarrow \gamma_m = \frac{(1+\nu)\varepsilon_x}{1+\frac{1-\nu}{2}\varepsilon_x}$$

$\varepsilon_x << 1$

$\gamma_m = (1+\nu)\varepsilon_x$

$\gamma_m = \frac{\tau_m}{G}$, axial load $\varepsilon_x = \frac{\sigma_x}{E}$

$\cancel{\frac{\tau_m}{G} =}$

$$\frac{\tau_m}{G} = (1+\nu)\frac{\sigma_x}{E}$$

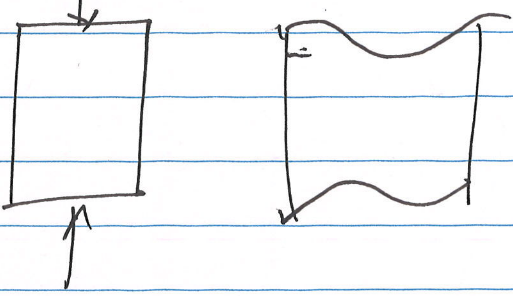
$$\frac{E}{G} = (1+\nu)\frac{\sigma_x}{\tau_m}$$

$\sigma_x = P/A$, $\tau_m = \frac{P}{2A} \Rightarrow \frac{\sigma_x}{\tau_m} = 2$

$$\frac{E}{G} = (1+\nu)2$$

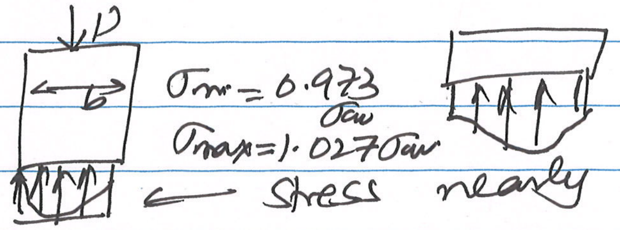
$$\boxed{\frac{E}{2G} = 1+\nu \quad \text{or} \quad G = \frac{E}{2(1+\nu)}}$$

Saint Venant Principle



Deformation maximum close to load

$\Rightarrow$ stress maximum.



$\sigma_{min} = 0.973\,\sigma_{av}$

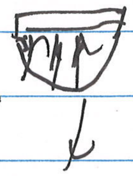
$\sigma_{max} = 1.027\,\sigma_{av}$

$\leftarrow$ stress nearly uniform

section area

$\sigma_{max} = 1.387\,\sigma_{av}$

$\sigma_{min} = 0.668\,\sigma_{av}$



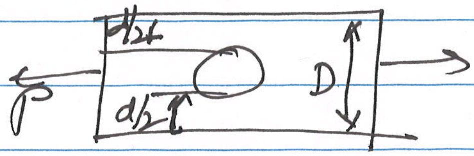
$\sigma_{min} = 0.198\,\sigma_{av}$

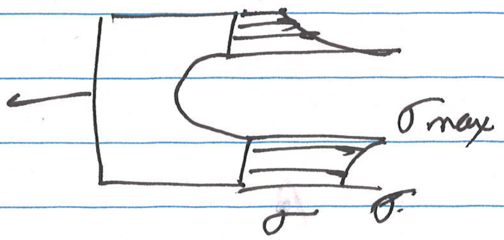
$\sigma_{max} = 2.575\,\sigma_{av}$

distance $> b$ assume σ = uniform

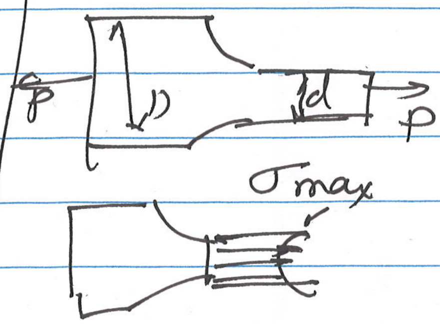
Saint Venant's Principle) French

Stress concentration (qualitative)





σ_{av}.



$$K = \frac{\sigma_{max}}{\sigma_{av}}$$

$\swarrow$ stress concentration factor

K dependson $\frac{D}{d}$