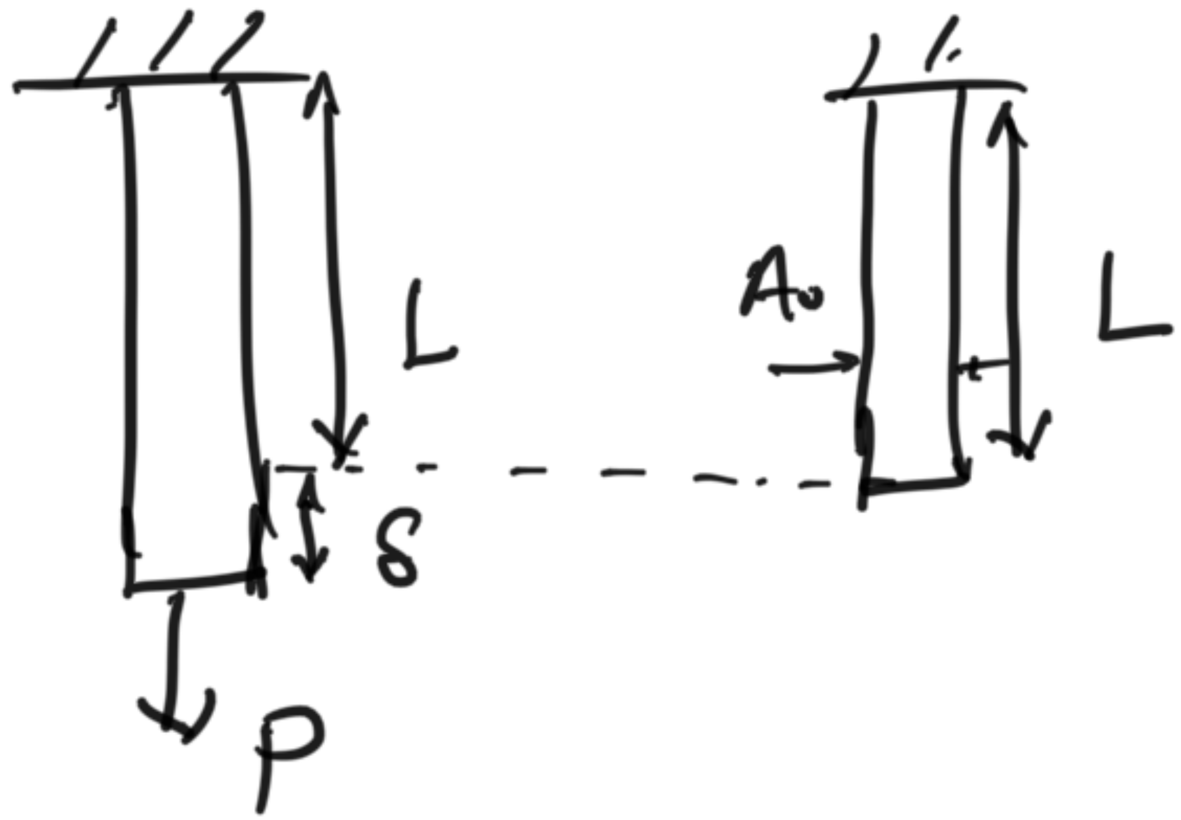
APL 105 Lecture 5

Chapter 2

Normal stresses under axial load ~~&~~



$A_0 =$ Area of cross-section of free member (without load)

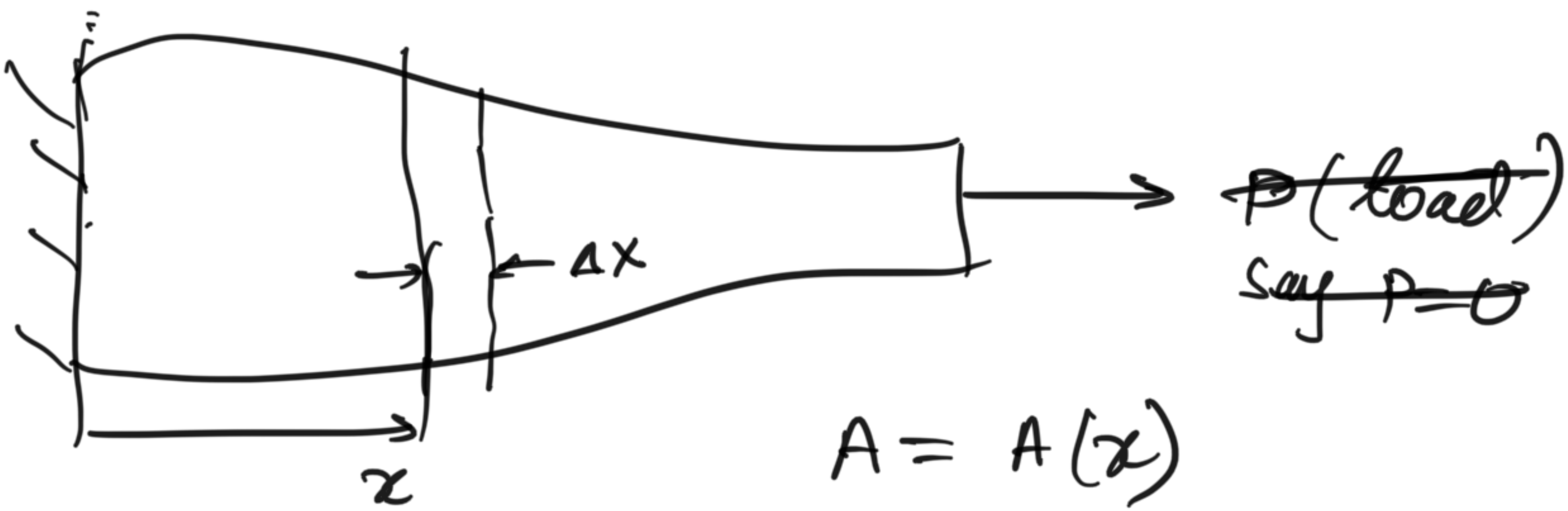
stress $\sigma = \dfrac{P}{A_0}$ ~~engineerin~~ engineering stress / stress

strain: $\epsilon = \dfrac{\delta}{L}$ engineering strain / strain

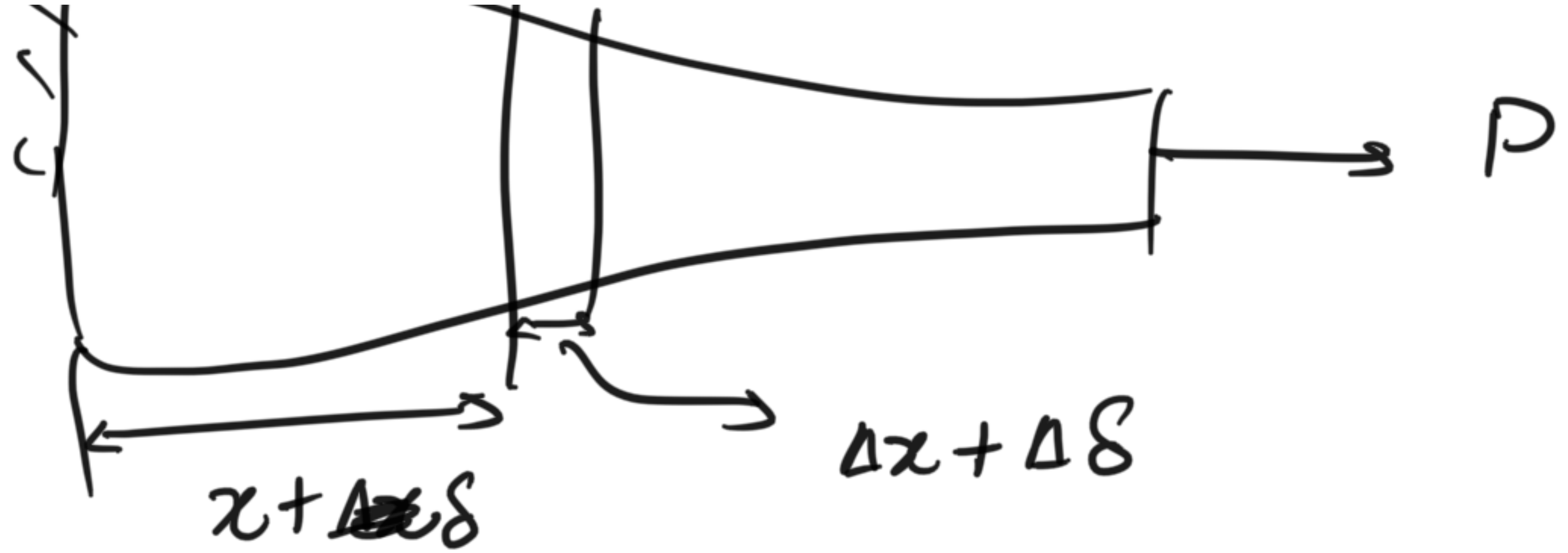
$$\sigma = f(\varepsilon)$$

we plot σ v/s ϵ curve for each material

<u>variable area</u>



$$A = A(x)$$



$$\underset{\text{(in the section } \Delta x)}{\epsilon} = \underset{\Delta x \to 0}{\text{lt}} \frac{\Delta \delta}{\Delta x} = \frac{d\delta}{dx}$$

units of $\epsilon \rightarrow$ dimensionless quantity [0]

$\delta = 150\, \mu m$

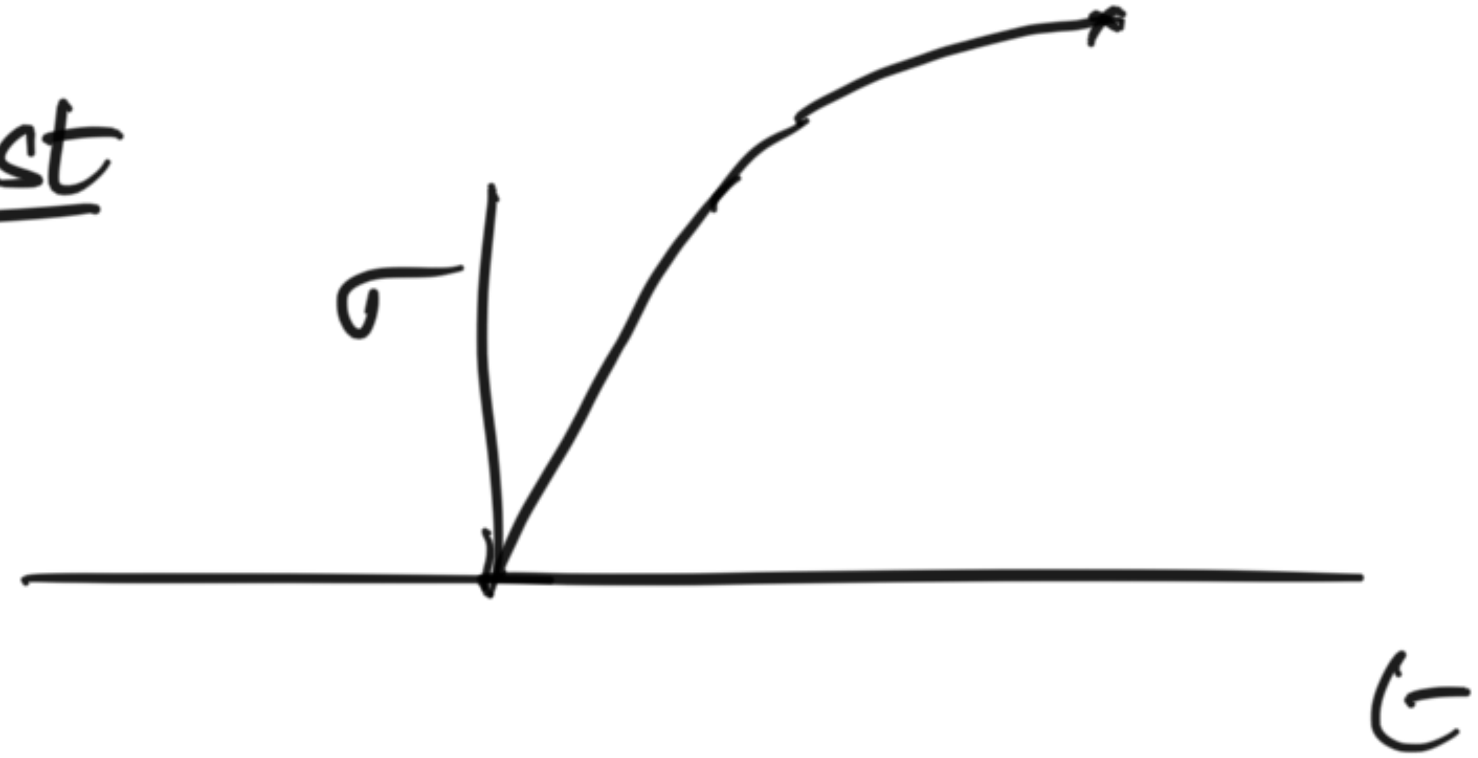
$L = 1\, m$

$$\epsilon = \frac{150 \times 10^{-6}}{1}$$

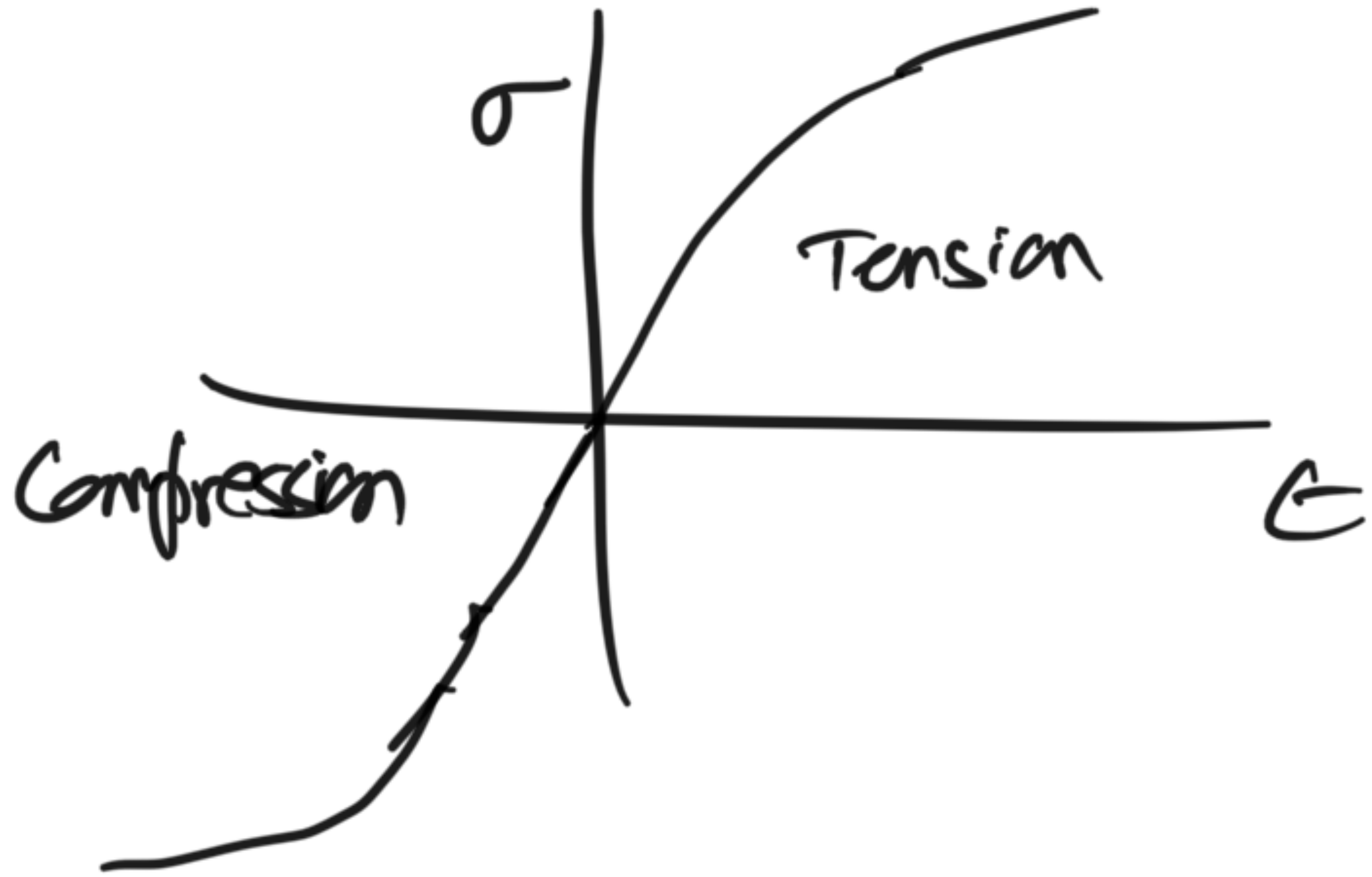
$$= 150\, \mu$$

↑ micro

Tensile test



If material under compression



same slope
as tension

True stress: $\frac{P}{\ }$

$A \leftarrow$ Area (at the load stage)

and not $\frac{P}{A_0}$

True strain: $\epsilon = \sum \Delta\epsilon = \sum \frac{\Delta L}{L}$

$$= \int_{L_0}^{L} \frac{dL}{L} = \ln \frac{L}{L_0}$$

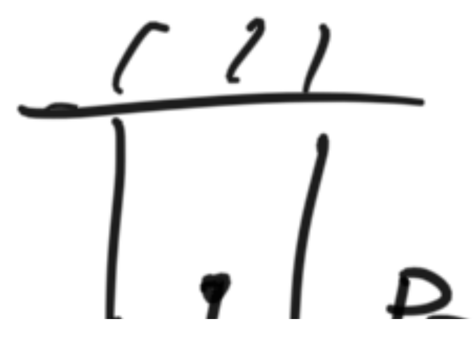
Hooke's Law

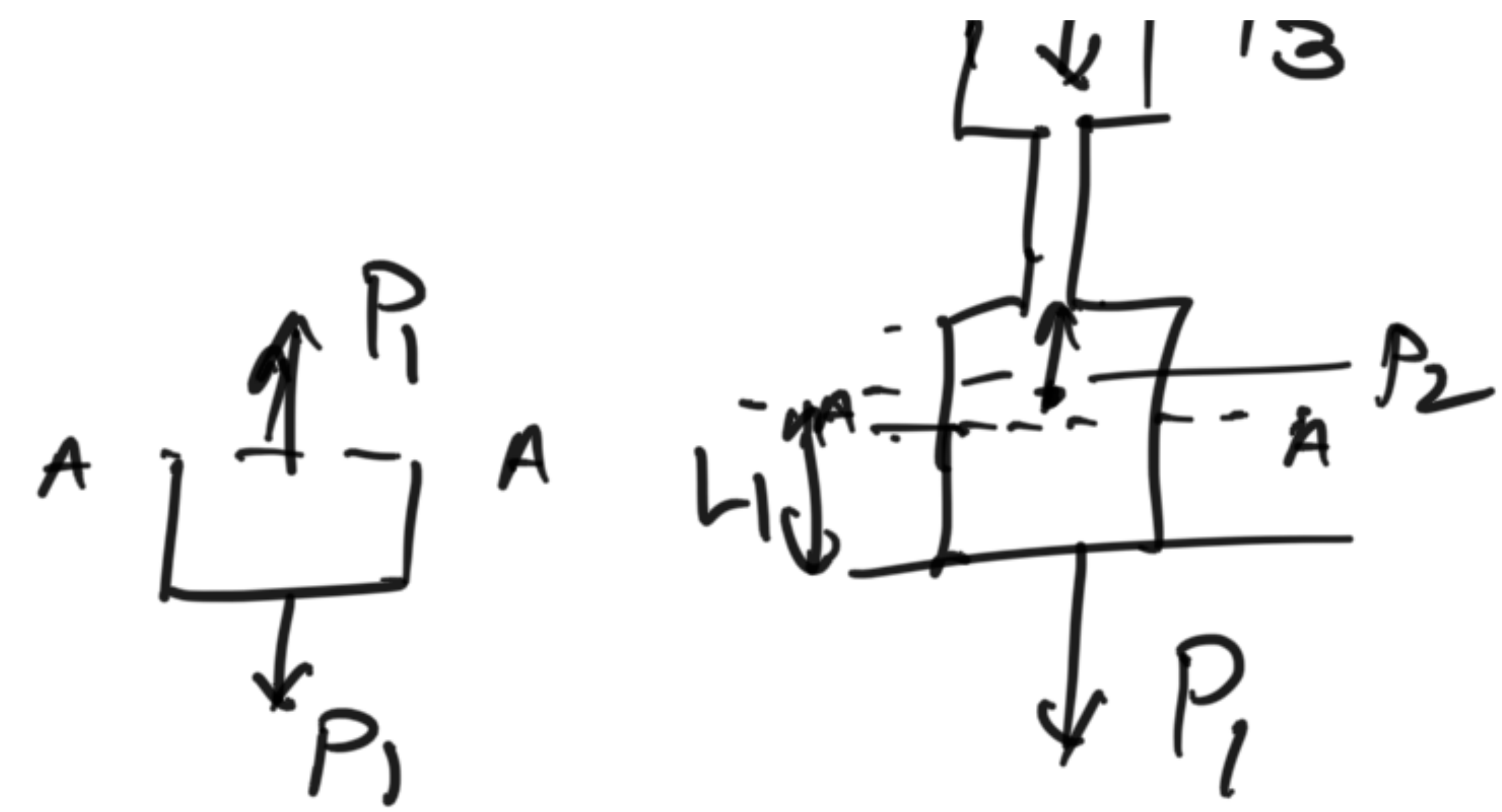
Modulus of Elasticity, E defined by

$\sigma = E\epsilon$ $\qquad$ E = modulus of elasticity or Young's modulus

$$\epsilon = \frac{\sigma}{E} \rightarrow \frac{\delta}{L} = \frac{P}{AE}$$

$$\delta = \frac{PL}{AE}$$





$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

~~uniform~~ ~~+~~ single load P on variable area $A(x)$

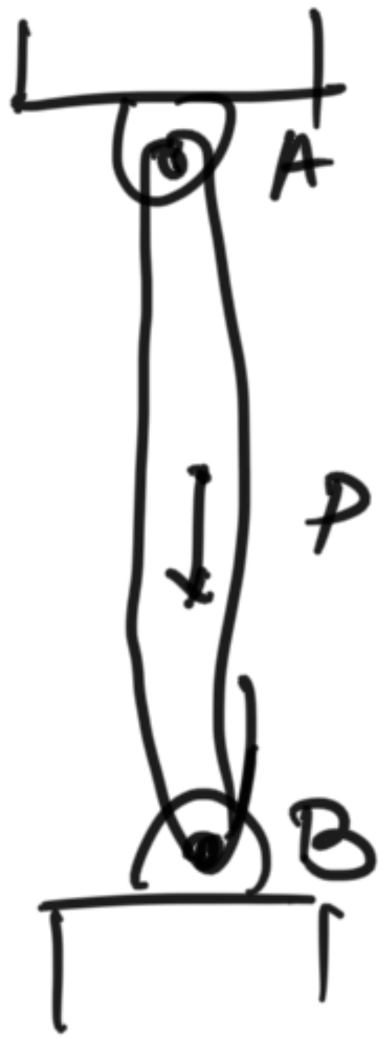
$$\delta = \int \frac{P\,dx}{AE}$$

statically determinate v/s statically indeterminate

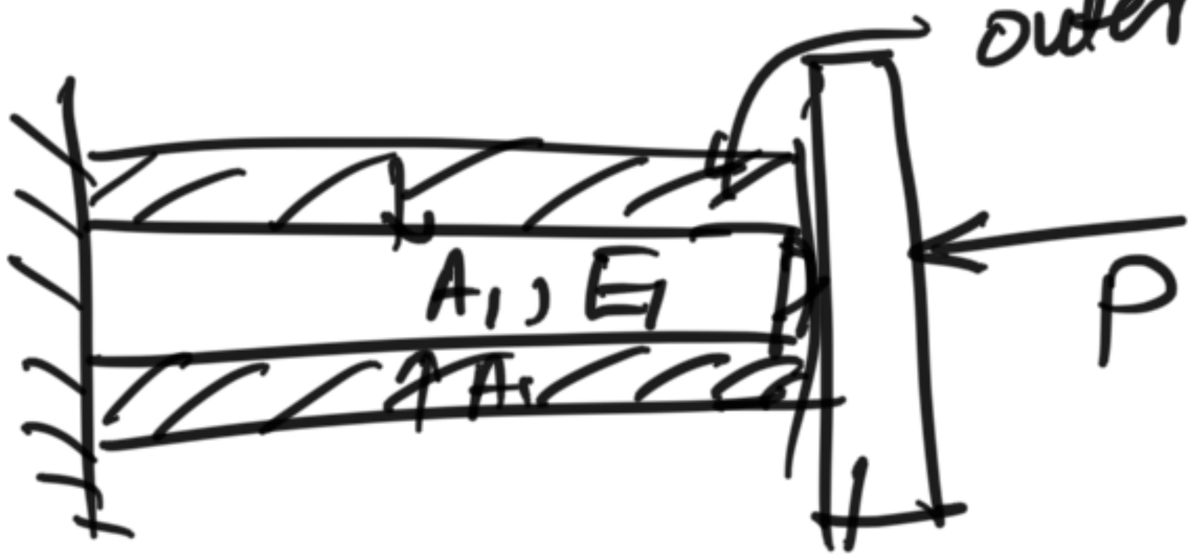
& we can solve for unknown reactions using statics equations

No. of equations not enough to solve the unknown reactions

v/s



statically indeterminate problems are solved using compatibility of deformations.

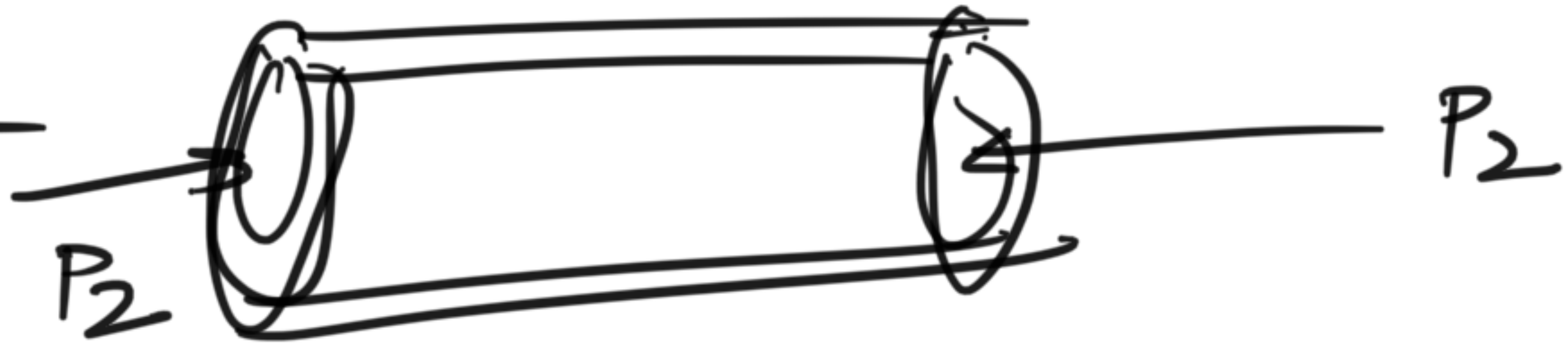


rod of area A_1 & E_1

kept in a cylinderical tube of area A_2, E_2.

FBD of body 1

FBD of 2



$$P_1 + P_2 = P$$

But $\delta_1 = \delta_2$

$$P_1 L \quad = P_2 L \quad \longrightarrow \quad \text{find } P_1 \text{ \& } P_2$$

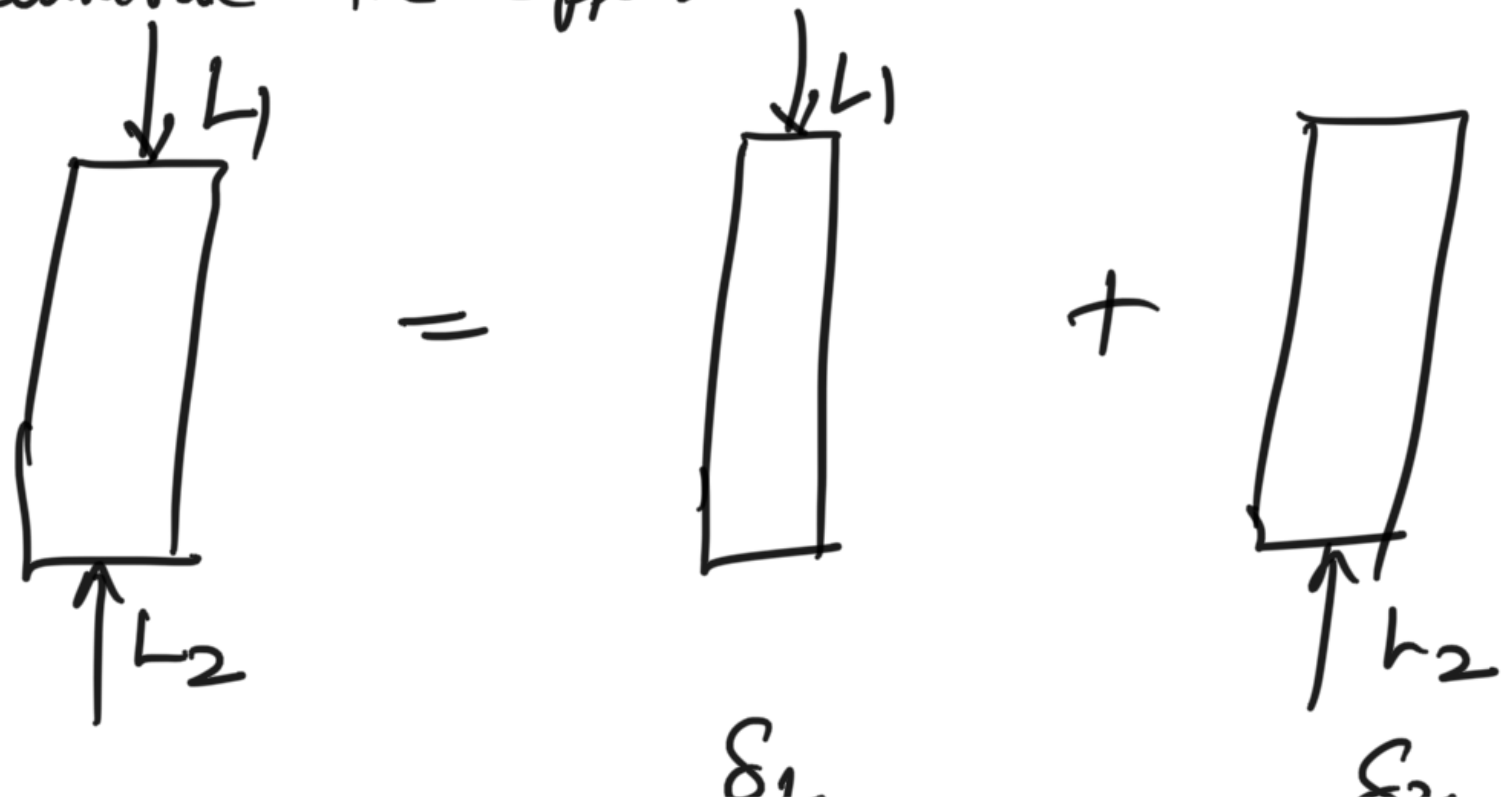
A_1E_1 A_2E_2

because
$P_1+P_2 = P$

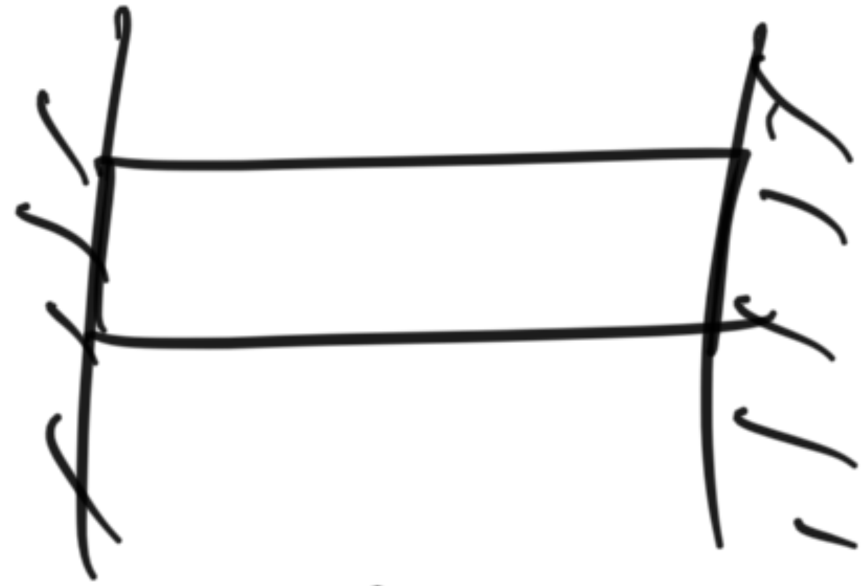
<u>Method of superposition</u> (statically ~~indeterminate~~ indeterminate problems)

In such problems there are more supports than those needed for equilibrium

Designate one load as redundant and eliminate the support



Temperature change



length change due to Temperature change, $\delta_T = \alpha(\Delta T) L$

coefficient of thermal expansion

$$\varepsilon_T = \alpha(\Delta T)$$

no stress associated with ε_T

when heated by ΔT

$$\delta_P = \frac{PL}{AE} \qquad \delta_T = (\alpha \Delta T) L$$

$$\delta_P + \delta_T = 0 \implies \alpha(\Delta T) L + \frac{PL}{AE} = 0$$

$$P = -AE\alpha(\Delta t)$$