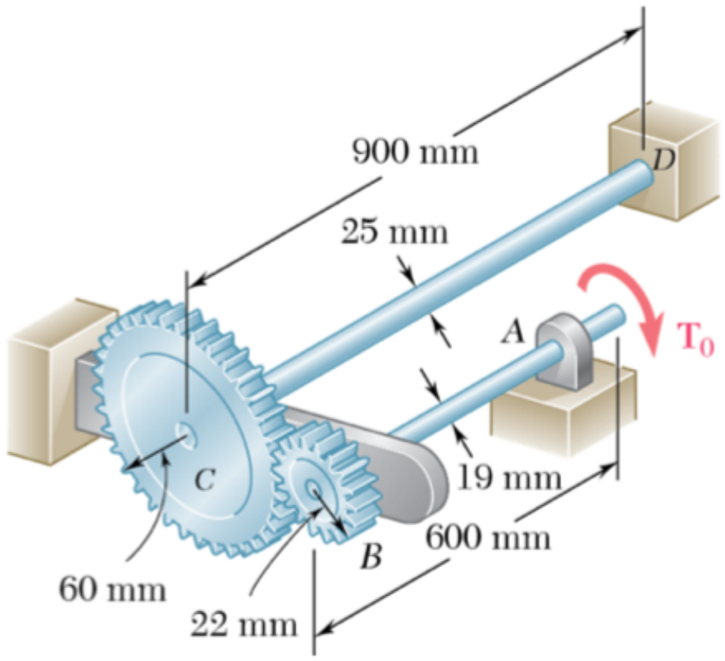
Tutorial 4

Sample Problem 3.4



Two solid steel shafts are connected by gears. Knowing that for each shaft $G = 77$ GPa and that the allowable shearing stress is 55 MPa, determine (*a*) the largest torque T_0 that may be applied to the end of shaft *AB*, (*b*) the corresponding angle through which end *A* of shaft *AB* rotates.

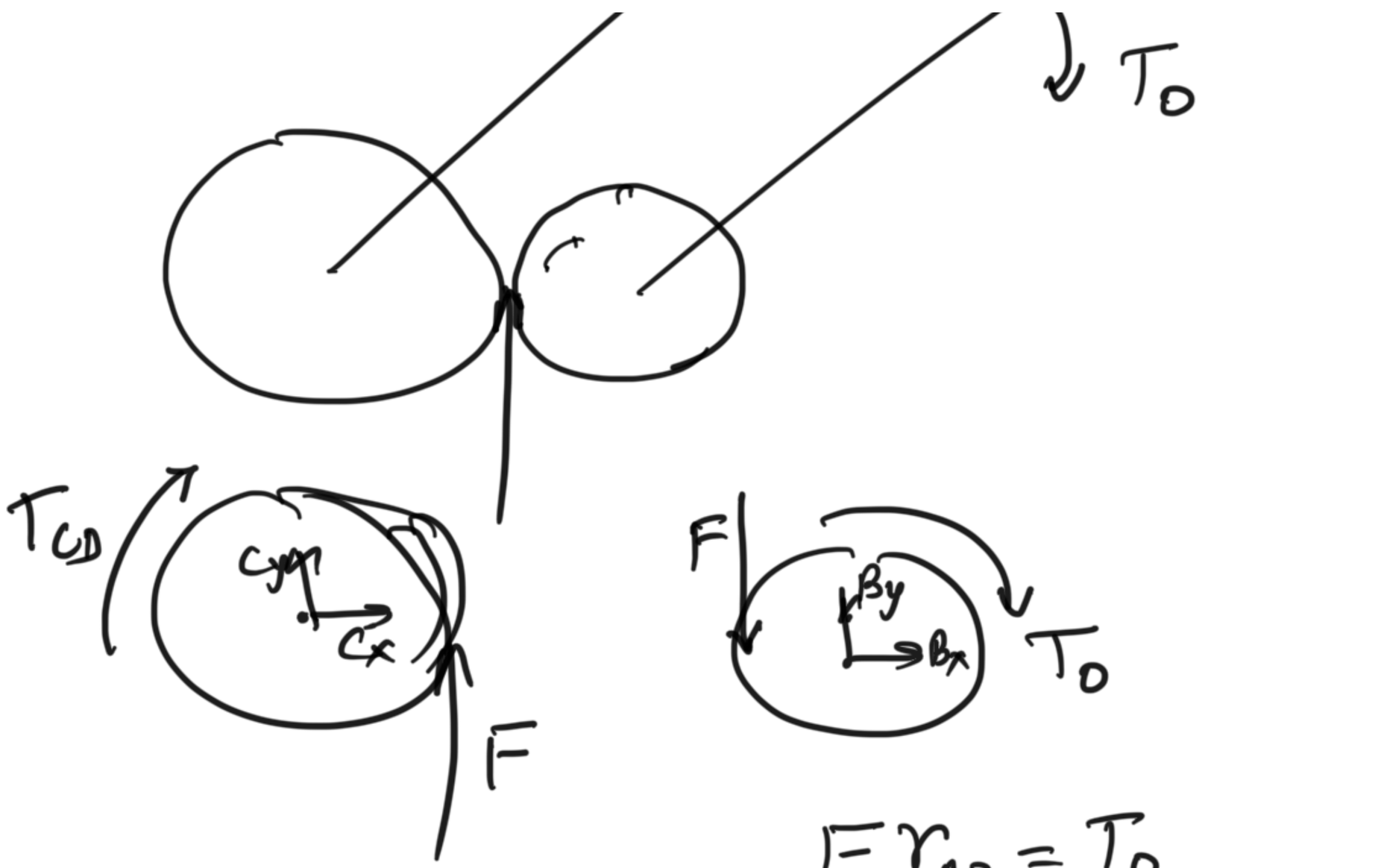
SOLUTION:

Apply a static equilibrium analysis on the two shafts to find a relationship between T_{CD} and T_0.

Apply a kinematic analysis to relate the angular rotations of the gears.

Find the maximum allowable torque on each shaft – choose the smallest.

Find the corresponding angle of twist for each shaft and the net angular rotation of end *A*.



$F r_{AB} = T_O$

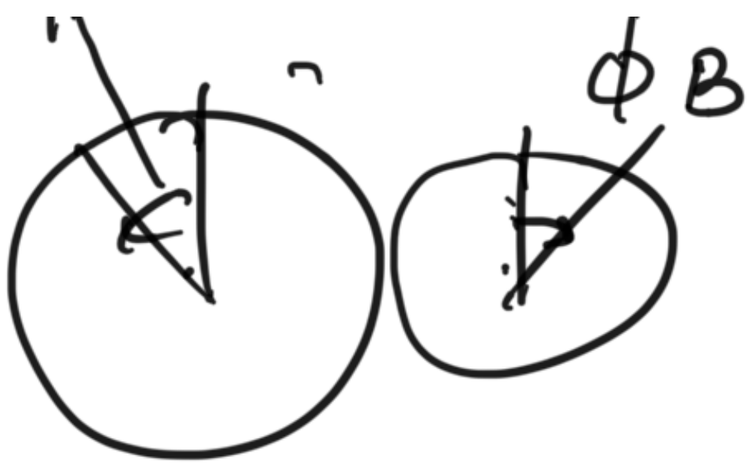
$F r_{CD} = T_{CD}$

T_{CD} is ~~a gear~~ ~~inter~~ the torque in gear resisting the motion

$F(22\,mm) = T_O$

$F(60\,mm) = T_{CD}$

Φ_C



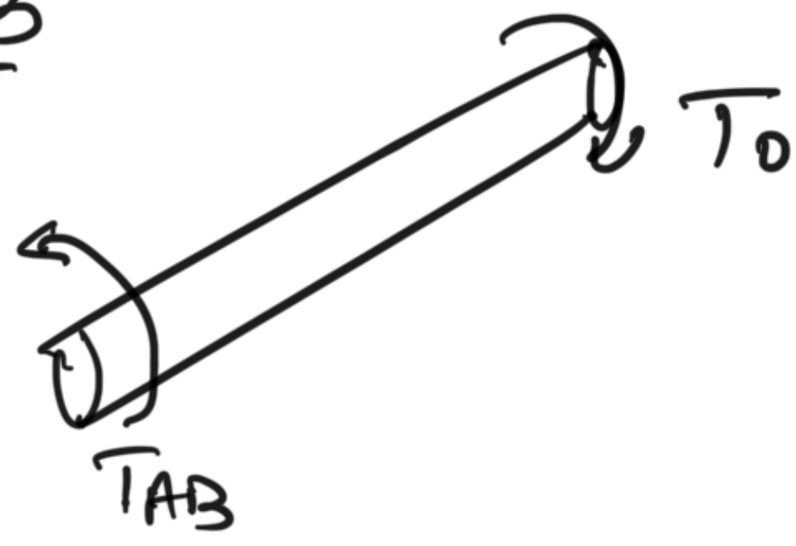
$$\frac{T_{CD}}{60} = \frac{T_0}{22}$$

$$\Rightarrow T_{CD} = 2.73\,T_0$$

$$r_{C}\,\phi_C = r_{B}\,\phi_B \Longrightarrow \phi_B = \phi_C \times \frac{60}{22} = 2.73\phi_C$$

(Gear C) (Gear B)

Shaft AB



$$T_{AB} = T_0$$

$$c = 9.5\,mm$$

$$\tau = \frac{T_{AB}\,C}{J} \Rightarrow 55\times10^{6} = \frac{T_0\,(9.5\times10^{-3})}{\frac{\pi}{2}(9.5\times10^{-3})^4}$$

$$T_0 = 74.1\ N.m$$

$$\Rightarrow \quad T_{CD} = 2.73\, T_0$$
$$= 202.83 \ N\text{-}m$$

$$\tau_{CD} = \frac{T_{CD}\, C}{J} = \frac{202.3 \times (12.5\times10^{-3})}{\frac{\pi}{2}(12.5\times10^{-3})^4}$$

$$= 1\cancel{03}\times10^{6} \ 65.9 \ MPa$$

exceeds τ_{max}.

So we have to rework starting from rod CD.

$$T_{CD} = 2.73 T_0$$

$$\tau_{max} = 55\times10^{6} = \frac{2.73 \times T_0 \,(12.5\times10^{-3})}{\frac{\pi}{2}(12.5\times10^{-3})^4}$$

$T_o = 61.8 \text{ N.m}$ ⟶

$T_{CD} = 168.7 \text{ N.m}$

end D is fixed

$$\phi_{C/D} = \frac{TL}{GJ} = \frac{168.7 \times 0.9}{(77\times10^9)\left(\frac{\pi}{2}\right)(0.0125)^4}$$

$$= 0.0514 \text{ rad}$$

shaft AB = $T_{AB} = T_o = 61.8$

$$\phi_{A/B} = \frac{T_o L}{JG} = \frac{(61.8)(0.6)}{(77\times10^9)\left(\frac{\pi}{2}\right)(0.0095\times10^{-3})^4}$$

$$= 0.0376 \text{ rad}$$

$\phi_B = 2.73\phi_C = (2.73)(0.0514)$ rad

=

$\phi_A = \phi_B + \phi_{A/B}$

$\varphi_A - \varphi_B$, $\varphi_{A/B} =$

$\pi\ rad = 180^\circ$