APL105 Lecture 1 11/1/2024

Solid and Fluid Mechanics

Solids, $\underbrace{\text{Liquids, Gases}}_{\text{Fluids}}$

stress $\rightarrow$ Force per unit area

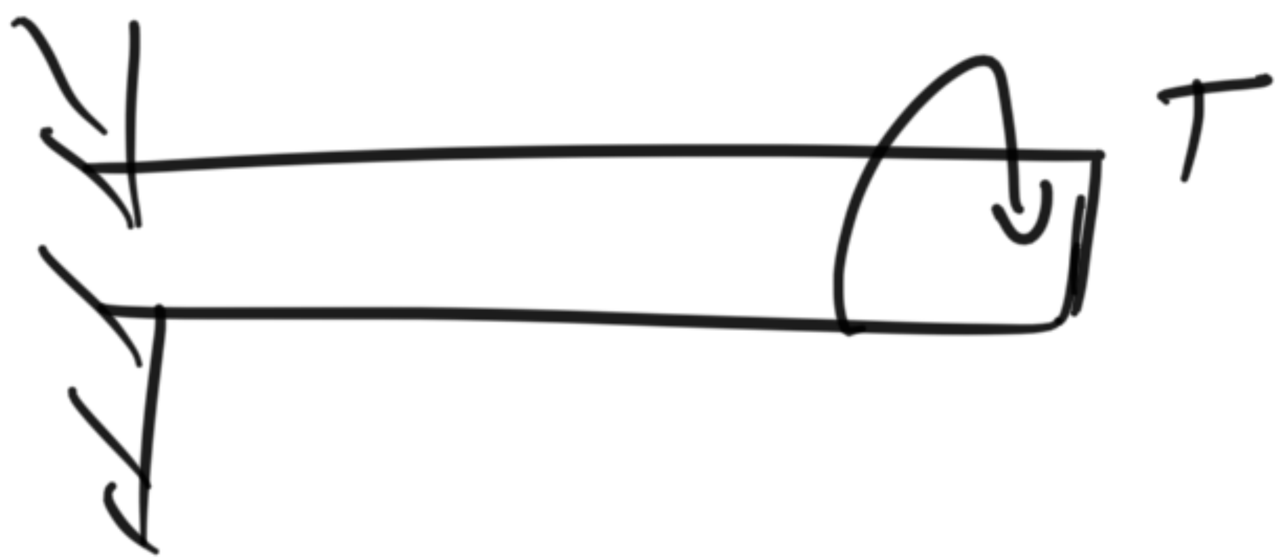
$$\vec{F} = F_n \hat{e}_n + F_{t1} \hat{e}_{t1} + F_{t2} \hat{e}_{t2}$$

F_n (component along $\hat{n}$)

normal force (or Axial force)

$F_{t1}, F_{t2} \rightarrow$ shear force Components

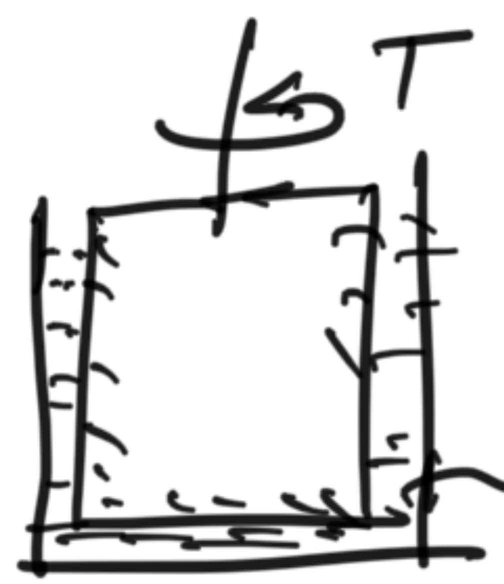
response to applied shear force distinguishes solid and fluid.



Solid

Deformation comes to an equilibrium deformation

Fluids



fluid between 2 containers.

rotate the inner container by applying a Torque T.

Fluid → fluid continues to rotate as long as T is applied.

Fluid → substance which ~~continues to~~ deforms

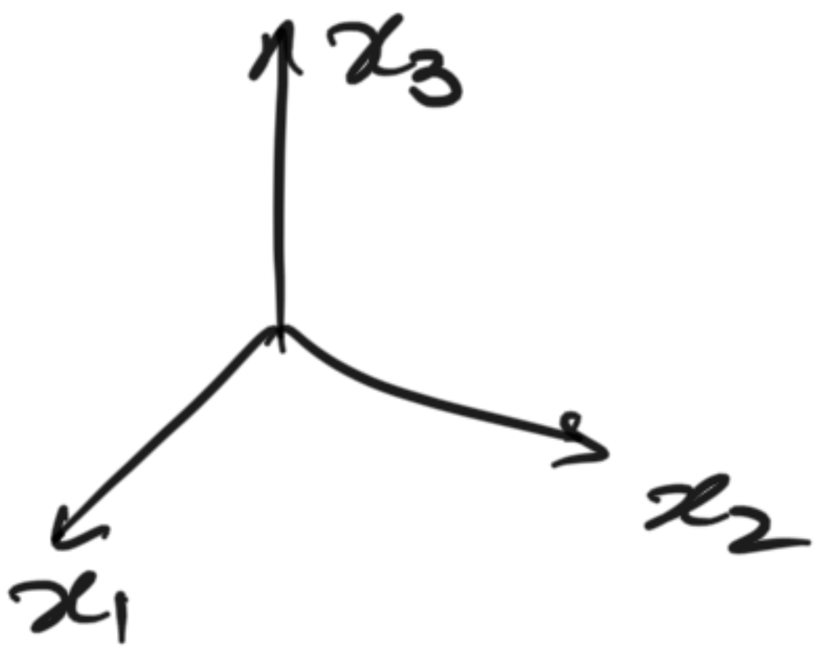
Fluid → substance which ~~continues to~~ deforms continously under the action of a shear force (however small it may be)

However, fluid offers resistance to the applied shear force and there is an equilibrium velocity which is attained. (Resistance offered by fluid is due to property called Viscosity).

Introduction to tensor notation

Cartesian tensors

axes:

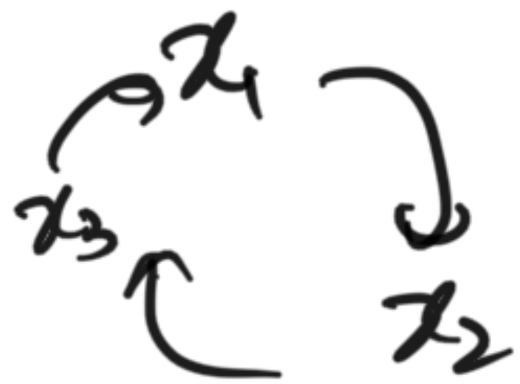


3 mutually $\perp$ directions x_1, x_2, x_3 (fixed in reference frame)

$x_1 \perp x_2 \perp x_3$.

Right handed sense:

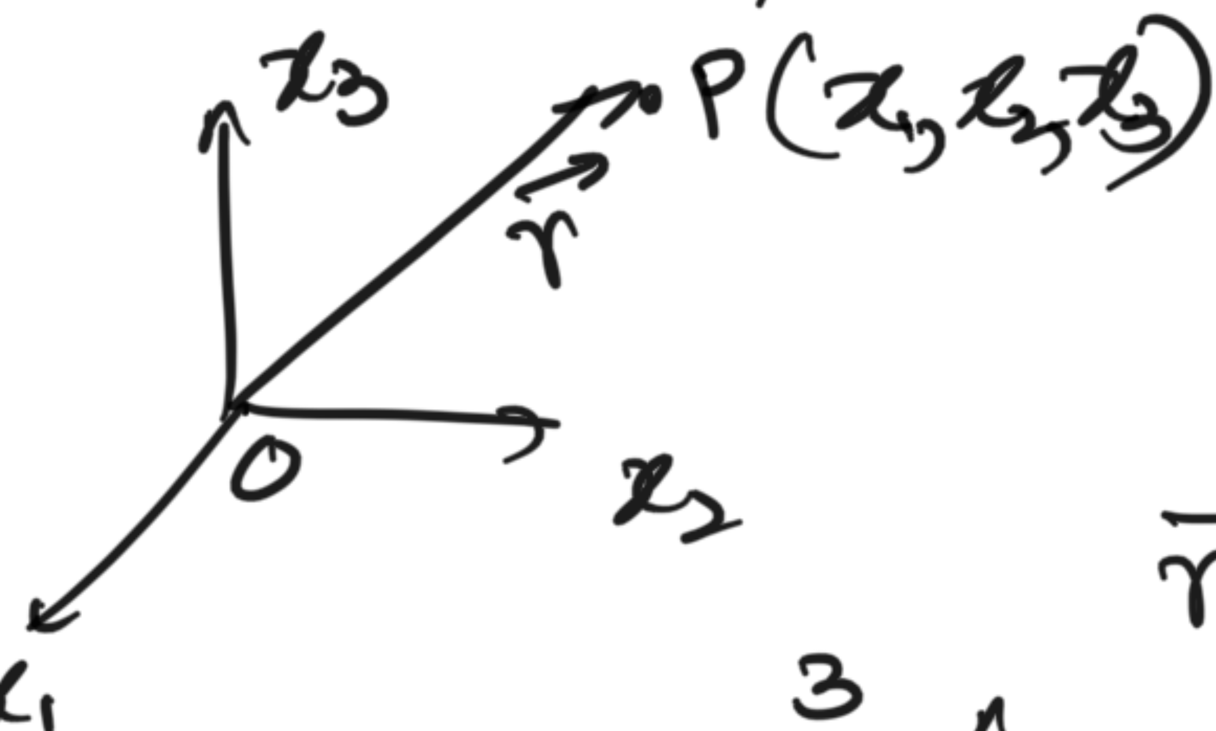
right hand, rotate fingers from x_1 to x_2 → thumb points in x_3



$\hat{e}_1, \hat{e}_2, \hat{e}_3$ → unit vectors along x_1, x_2, x_3 (respectively)

$$\left.\begin{aligned} \hat{e}_1 \cdot \hat{e}_2 &= 0 \\ \hat{e}_1 \cdot \hat{e}_3 &= 0 \\ \hat{e}_1 \cdot \hat{e}_1 &= 1 \end{aligned}\right\} \quad \hat{e}_i \cdot \hat{e}_j = \underbrace{\delta_{ij}}_{\text{Kronecker delta}}$$

$$\delta_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



P, position vector $\vec{OP} = \vec{r}$

$$\vec{r} = x_1 \hat{e}_1 + x_2 \hat{e}_2 + x_3 \hat{e}_3$$

$$\vec{r} = \sum^{3} \hat{e}_i x_i$$

$$r = \sum_{i=1} e_i x_i$$

Introduce summation convention, where if an index is repeated, then Σ term is not written, it is implied.

$$\vec{r} = \hat{e}_i x_i$$

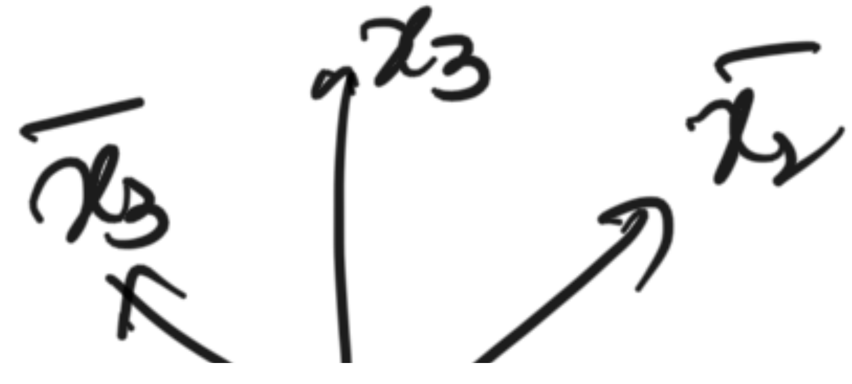
j^{th} component of vector $\vec{r} = x_j$

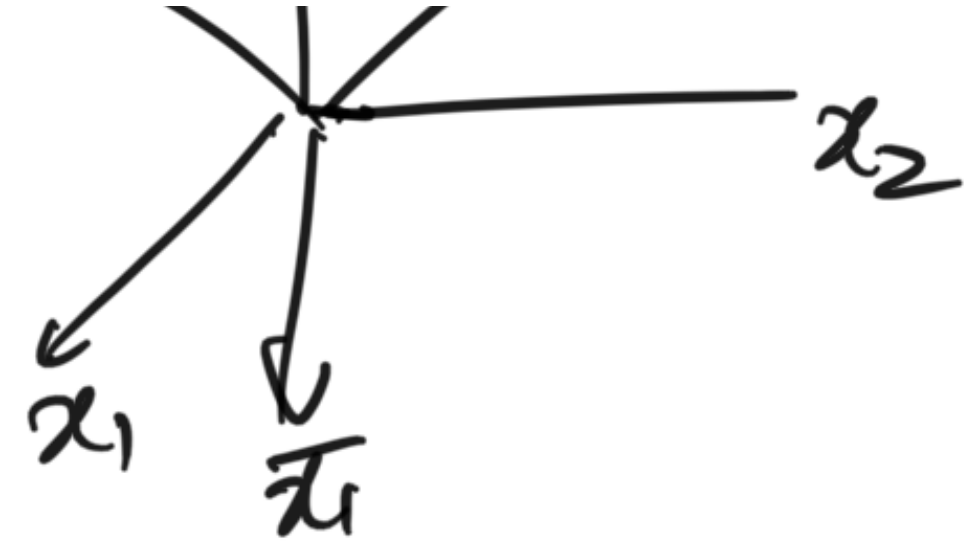
$$x_j = \hat{e}_j \cdot \vec{r} = \hat{e}_j \cdot \hat{e}_i x_i = \delta_{ji} x_i$$

$$x_j = \delta_{ji} x_i$$

$$\delta_{lm} a_l = a_m$$

$$\delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33} = 3 \quad (\text{not } 1)$$





$\underline{E} \to x_1, x_2, x_3$ axes

rotate/reflect axes keeping origin same

new system $\bar{E}$ axes $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$

$\hat{\bar{e}}_1, \hat{\bar{e}}_2, \hat{\bar{e}}_3$ (new unit vectors)

$$\bar{e}_i \cdot \bar{e}_j = \delta_{ij}$$

$$\vec{r} = \bar{e}_j \bar{x}_j$$

$\vec{r}$ stays same (components change)

E_i (old axes) $\qquad \bar{E}_j$ (new axes)

related by direction cosines

$a_{ij} = e_i \cdot \bar{e}_j$ (cosine ~~of~~ of angle between

i axis and $\bar{j}$ axis)

$$\vec{r} = x_i \hat{e}_i = \bar{x}_j \hat{\bar{e}}_j$$

$$\hat{e}_k \cdot \vec{r} = x_k = \underbrace{\hat{e}_k \cdot \bar{x}_j \hat{\bar{e}}_j} = a_{kj} \bar{x}_j$$

$$\boxed{x_k = a_{kj} \bar{x}_j}$$

$$\vec{r} = x_i \hat{e}_i$$

dot product with $\hat{\bar{e}}_k$

$$\hat{\bar{e}}_k \cdot \vec{r} = \bar{x}_k = \underbrace{\hat{\bar{e}}_k \cdot \hat{e}_i}\, x_i$$

$$\boxed{\bar{x}_k = a_{ik} x_i}$$

$$x_j = a_{jk} \bar{x}_k = a_{jk} a_{ik} x_i$$

$$\delta_{ij} x_i = a_{jk} a_{ik} x_i$$

$$\Rightarrow \boxed{a_{jk} a_{ik} = \delta_{ij}}$$

$$\frac{\partial \bar{x}_k}{\partial x_j} = \frac{\partial}{\partial x_j}(x_i a_{ik}) = a_{ik}\frac{\partial x_i}{\partial x_j} = a_{ik}\delta_{ij}$$

$$= a_{jk}$$

$\underset{\sim}{a_{jk}}$ $\quad j \rightarrow$ old axes

$\quad k \rightarrow$ new axes

scalar function $\phi \longrightarrow \dfrac{\partial \phi}{\partial x_i} = g_i$

gradient of scalar ϕ