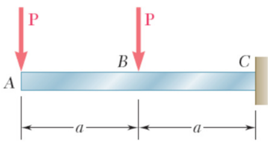
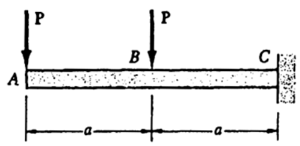
Tutorial and Assignment 6 (Chapter 5)

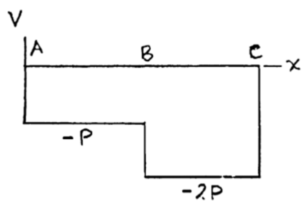


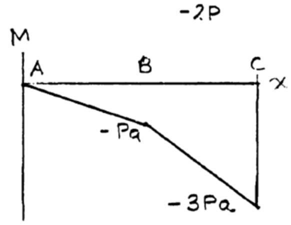
PROBLEM 5.5

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

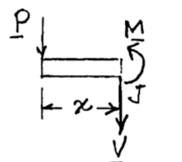
SOLUTION







From *A* to *B*: $0 < x < a$



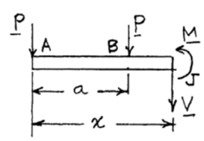
$+\uparrow \Sigma F_y = 0: \quad -P - V = 0$

$V = -P$ ◀

$+\circlearrowleft \Sigma M_J = 0: \quad Px + M = 0$

$M = -Px$ ◀

From *B* to *C*: $a < x \leq 2a$

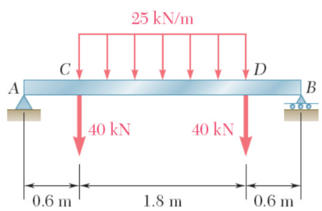


$+\uparrow \Sigma F_y = 0: \quad -P - P - V = 0$

$V = -2P$ ◀

$+\circlearrowleft \Sigma M_J = 0: \quad Px + P(x - a) + M = 0$

$M = -2Px + Pa$ ◀



PROBLEM 5.9

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

The distributed load is replaced with an equivalent concentrated load of 45 kN to compute the reactions.

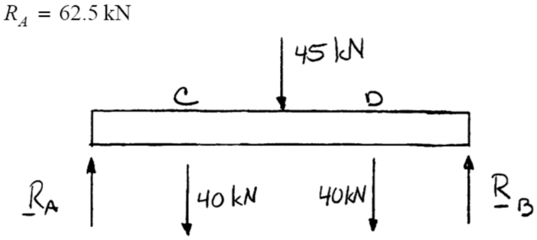
$$(25\ \text{kN/m})(1.8\ \text{m}) = 45\ \text{kN}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(40\ \text{kN})(0.6\ \text{m}) - 45\ \text{kN}(1.5\ \text{m}) - 40\ \text{kN}(2.4\ \text{m}) + R_B(3.0\ \text{m}) = 0$$

$$R_B = 62.5\ \text{kN}$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + 62.5\ \text{kN} - 40\ \text{kN} - 45\ \text{kN} - 40\ \text{kN} = 0$$

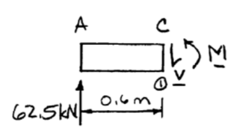
$$R_A = 62.5\ \text{kN}$$



At *C*:

$$+\uparrow \Sigma F_y = 0: \quad V = 62.5\ \text{kN}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad M = (62.5\ \text{kN})(0.6\ \text{m}) = 37.5\ \text{kN} \cdot \text{m}$$



At centerline of the beam:

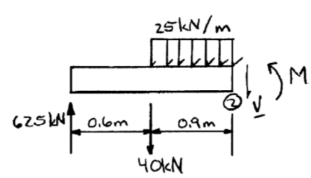
$$+\uparrow \Sigma F_y = 0: \quad 62.5\ \text{kN} - 40\ \text{kN} - (25\ \text{kN/m})(0.9\ \text{m}) - V = 0$$

$$V = 0$$

$$+\circlearrowleft \Sigma M_2 = 0:$$

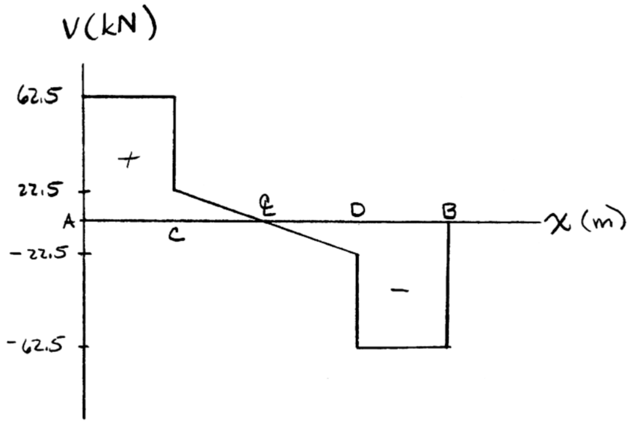
$$M - (62.5\ \text{kN})(1.5\ \text{m}) + (40\ \text{kN})(0.9\ \text{m}) + (25\ \text{kN/m})(0.9\ \text{m})(0.45\ \text{m}) = 0$$

$$M = 47.625\ \text{kN} \cdot \text{m}$$

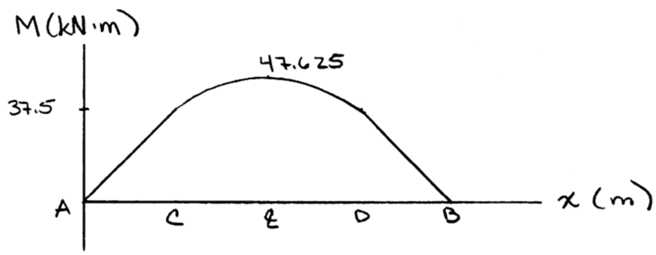


PROBLEM 5.9 *(Continued)*

Shear and bending-moment diagrams:



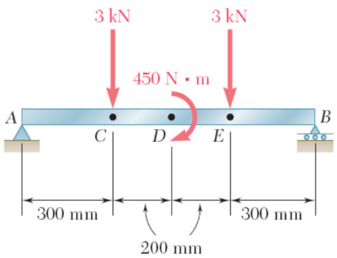
(*a*) $|V|_{max} = 62.5 \text{ kN}$ ◀



(*b*) $|M|_{max} = 47.6 \text{ kN} \cdot \text{m}$ ◀

From A to C and D to B, V is uniform; therefore M is linear.

From C to D, V is linear; therefore M is parabolic.



PROBLEM 5.11

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$+\circlearrowleft \Sigma M_B = 0$: $(700)(3) - 450 + (300)(3) - 1000A = 0$

$A = 2.55 \text{ kN} \uparrow$

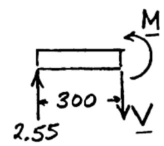
$+\circlearrowleft \Sigma M_A = 0$: $-(300)(3) - 450 - (700)(3) + 1000B = 0$

$B = 3.45 \text{ kN} \uparrow$

At A: $V = 2.55 \text{ kN}$ $M = 0$

A to C: $V = 2.55 \text{ kN}$

At C:



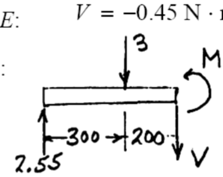
$+\circlearrowleft \Sigma M_C = 0$:

$-(300)(2.55) + M = 0$

$M = 765 \text{ N} \cdot \text{m}$

C to E: $V = -0.45 \text{ N} \cdot \text{m}$

At D:

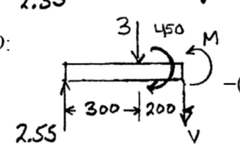


$+\circlearrowleft \Sigma M_D = 0$:

$-(500)(2.55) + (200)(3) + M = 0$

$M = 675 \text{ N} \cdot \text{m}$

At D:



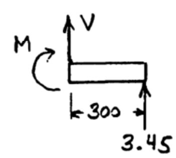
$+\circlearrowleft \Sigma M_D = 0$:

$-(500)(2.55) + (200)(3) - 450 + M = 0$

$M = 1125 \text{ N} \cdot \text{m}$

E to B: $V = -3.45 \text{ kN}$

At E:

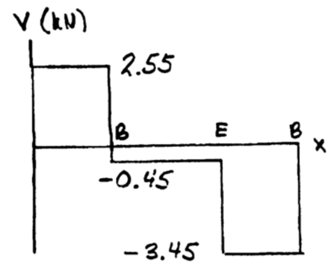


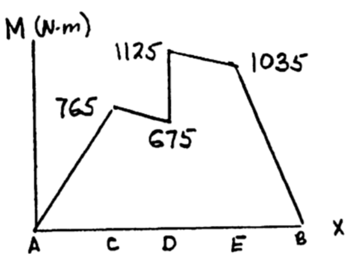
$+\circlearrowleft \Sigma M_E = 0$:

$-M + (300)(3.45) = 0$

$M = 1035 \text{ N} \cdot \text{m}$

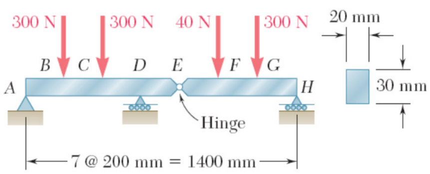
At B: $V = 3.45 \text{ kN},$ $M = 0$





(*a*) $|V|_{max} = 3.45 \text{ kN}$ ◀

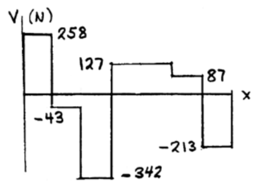
(*b*) $|M|_{max} = 1125 \text{ N} \cdot \text{m}$ ◀

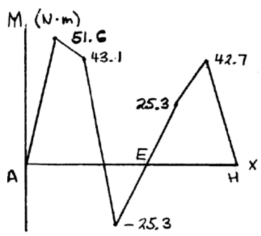


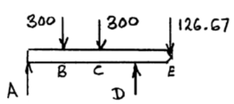
PROBLEM 5.23

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum normal stress due to bending.

SOLUTION



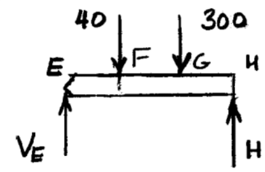




Free body EFGH. Note that $M_E = 0$ due to hinge.

$$+\circlearrowleft \Sigma M_E = 0:\quad 0.6\,H - (0.2)(40) - (0.40)(300) = 0$$

$$H = 213.33 \text{ N}$$



$$+\uparrow \Sigma F_y = 0:\quad V_E - 40 - 300 + 213.33 = 0$$

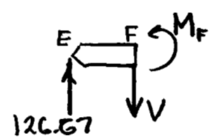
$$V_E = 126.67 \text{ N}$$

Shear:

E to F: $\quad V = 126.67 \text{ N}\cdot\text{m}$

F to G: $\quad V = 86.67 \text{ N}\cdot\text{m}$

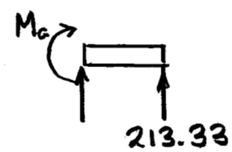
G to H: $\quad V = -213.33 \text{ N}\cdot\text{m}$



Bending moment at F:

$$+\circlearrowleft \Sigma M_F = 0:\quad M_F - (0.2)(126.67) = 0$$

$$M_F = 25.33 \text{ N}\cdot\text{m}$$



Bending moment at G:

$$+\circlearrowleft \Sigma M_G = 0:\quad -M_G + (0.2)(213.33) = 0$$

$$M_G = 42.67 \text{ N}\cdot\text{m}$$

Free body ABCDE.

$$+\circlearrowleft \Sigma M_B = 0:\quad 0.6A + (0.4)(300) + (0.2)(300)$$
$$-(0.2)(126.63) = 0$$

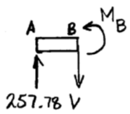
$$A = 257.78 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0:\quad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6D = 0$$

$$D = 468.89 \text{ N}$$

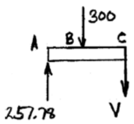
PROBLEM 5.23 *(Continued)*

Bending moment at B.

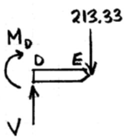


$$+\circlearrowleft \Sigma M_B = 0: \; -(0.2)(257.78) + M_B = 0$$
$$M_B = 51.56 \text{ N} \cdot \text{m}$$

Bending moment at C.



$$+\circlearrowleft \Sigma M_C = 0: \; -(0.4)(257.78) + (0.2)(300) + M_C = 0$$
$$M_C = 43.11 \text{ N} \cdot \text{m}$$



Bending moment at D.

$$+\circlearrowleft \Sigma M_D = 0: \; -M_D - (0.2)(213.33) = 0$$
$$M_D = -25.33 \text{ N} \cdot \text{m}$$

$$\max|M| = 51.56 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

$$S = \frac{1}{6}bh^2 = \frac{1}{6}(20)(30)^2$$
$$= 3 \times 10^3 \text{ mm}^3 = 3 \times 10^{-6} \text{ m}^3$$

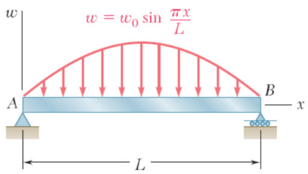
Normal stress:

$$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6 \text{ Pa}$$

$$\sigma = 17.19 \text{ MPa} \quad \blacktriangleleft$$

$$|V|_{\max} = 342 \text{ N} \quad \blacktriangleleft$$

$$|M|_{\max} = 516 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$



PROBLEM 5.52

Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\quad C_2 = 0$

$M = 0$ at $x = L$ $\quad 0 = 0 + C_1 L + 0$

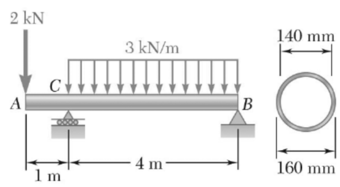
$C_1 = 0$

(*a*) $$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} \blacktriangleleft$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} \blacktriangleleft$$

$$\frac{dM}{dx} = V = 0 \quad \text{at} \quad x = \frac{L}{2}$$

(*b*) $M_{\max} = \dfrac{w_0 L^2}{\pi^2} \sin \dfrac{\pi}{2}$ $$M_{\max} = \frac{w_0 L^2}{\pi^2} \blacktriangleleft$$



PROBLEM 5.54

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

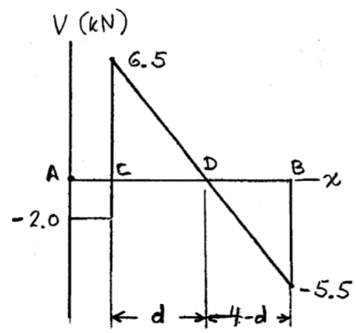
SOLUTION

$$+\circlearrowleft \Sigma M_C = 0:\ (2)(1) - (3)(4)(2) + 4B = 0$$
$$B = 5.5 \text{ kN}$$

$$+\circlearrowleft \Sigma M_B = 0:\ (5)(2) + (3)(4)(2) - 4C = 0$$
$$C = 8.5 \text{ kN}$$

Shear:

A to *C*: $V = -2 \text{ kN}$



C^+ : $V = -2 + 8.5 = 6.5 \text{ kN}$

B: $V = 6.5 - (3)(4) = -5.5 \text{ kN}$

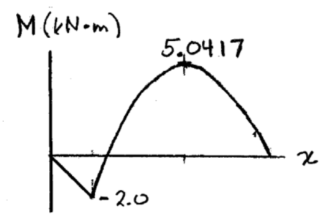
Locate point *D* where $V = 0$.

$$\frac{d}{6.5} = \frac{4-d}{5.5} \qquad 12d = 26$$

$$d = 2.1667 \text{ m} \qquad 4 - d = 3.8333 \text{ m}$$

Areas of the shear diagram:

A to *C*: $\int V dx = (-2.0)(1) = -2.0 \text{ kN} \cdot \text{m}$



C to *D*: $\int V dx = \frac{1}{2}(2.16667)(6.5) = 7.0417 \text{ kN} \cdot \text{m}$

D to *B*: $\int V dx = \frac{1}{2}(3.83333)(-5.5) = -5.0417 \text{ kN} \cdot \text{m}$

Bending moments: $M_A = 0$

$$M_C = 0 - 2.0 = -2.0 \text{ kN} \cdot \text{m}$$
$$M_D = -2.0 + 7.0417 = 5.0417 \text{ kN} \cdot \text{m}$$
$$M_B = 5.0417 - 5.0417 = 0$$

Maximum $|M| = 5.0417 \text{ kN} \cdot \text{m} = 5.0417 \times 10^3 \text{N} \cdot \text{m}$

PROBLEM 5.54 *(Continued)*

For pipe:

$$c_o = \frac{1}{2}d_o = \frac{1}{2}(160) = 80 \text{ mm}, \quad c_i = \frac{1}{2}d_i = \frac{1}{2}(140) = 70 \text{ mm}$$

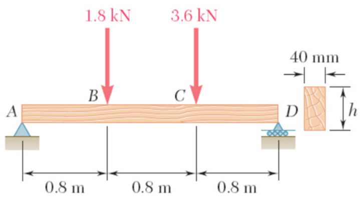
$$I = \frac{\pi}{4}\left(c_o^4 - c_i^4\right) = \frac{\pi}{4}\left[(80)^4 - (70)^4\right] = 13.3125 \times 10^6 \text{ mm}^4$$

$$S = \frac{I}{c_o} = \frac{13.3125 \times 10^6}{80} = 166.406 \times 10^3 \text{ mm}^3 = 166.406 \times 10^{-6} \text{ m}^3$$

Normal stress:

$$\sigma = \frac{M}{S} = \frac{5.0417 \times 10^3}{166.406 \times 10^{-6}} = 30.3 \times 10^6 \text{ Pa}$$

$\sigma = 30.3$ MPa ◀



PROBLEM 5.65

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

SOLUTION

Reactions:

$$+\circlearrowleft \Sigma M_D = 0: \quad -2.4A + (1.6)(1.8) + (0.8)(3.6) = 0$$

$A = 2.4$ kN

$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.8)(1.8) - (1.6)(3.6) + 2.4D = 0$$

$D = 3$ kN

Construct shear and bending moment diagrams:

$$|M|_{\text{max}} = 2.4 \text{ kN}\cdot\text{m} = 2.4\times10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa}$$

$$= 12\times10^6 \text{ Pa}$$

$$S_{\text{min}} = \frac{|M|_{\text{max}}}{\sigma_{\text{all}}} = \frac{2.4\times10^3}{12\times10^6}$$

$$= 200\times10^{-6} \text{ m}^3$$

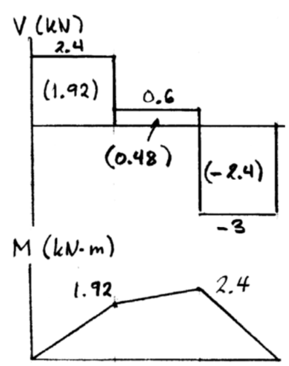
$$= 200\times10^3 \text{ mm}^3$$

$$S = \frac{1}{6}bh^2 = \frac{1}{6}(40)h^2$$

$$= 200\times10^3$$

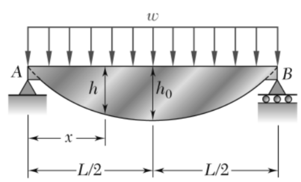
$$h^2 = \frac{(6)(200\times10^3)}{40}$$

$$= 30\times10^3 \text{ mm}^2$$



$h = 173.2$ mm ◀

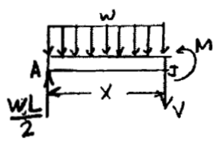
PROBLEM 5.126



The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (b) Determine the maximum allowable load if $L = 0.9$ m, $h_0 = 300$ mm, $b = 30$ mm, and $\sigma_{all} = 165$ MPa.

SOLUTION

$$+\uparrow \Sigma F_y = 0 \quad R_A + R_B - w, L = 0 \quad R_A = R_B = \frac{wL}{2}$$



$$+\circlearrowleft \Sigma M_J = 0 \quad \frac{wL}{2}x - wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L - x)$$

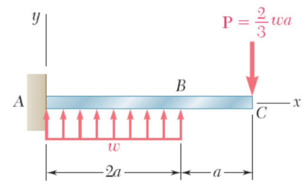
$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$$

For a rectangular cross section $\quad S = \frac{1}{6}bh^2$

Equating $\quad \frac{1}{6}bh^2 = \frac{wx(L-x)}{2\sigma_{all}} \qquad h = \left\{\frac{3wx(L-x)}{\sigma_{all}b}\right\}^{1/2}$ ◀

(a) At $x = \frac{L}{2}$ $\quad h = h_0 = \left\{\frac{3wL^2}{4\sigma_{all}b}\right\}^{1/2} \qquad h = h_0\left[\frac{x}{L}\left(1 - \frac{x}{L}\right)\right]^{1/2}$ ◀

(b) Solving for w $\quad w = \frac{4\sigma_{all}bh_0^2}{3L^2} = \frac{(4)(165\times10^6)(0.03)(0.3)^2}{(3)(0.9)^2} = 733.3 \text{ kN/m}$ ◀



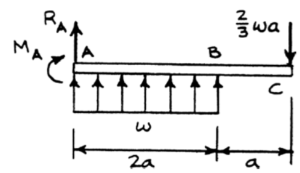
PROBLEM 9.5

For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.

SOLUTION

Using *ABC* as a free body,

FBD *ABC*:



$$+\uparrow \Sigma F_y = 0: \quad R_A + 2wa - \frac{2}{3}wa = 0$$

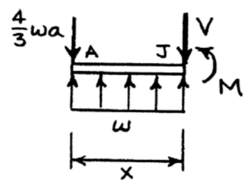
$$R_A = -\frac{4}{3}wa = \frac{4}{3}wa \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + (2wa)(a) - \left(\frac{2}{3}wa\right)(3a) = 0$$

$$M_A = 0$$

Using *AJ* as a free body,

FBD *AJ*:



$$+\circlearrowleft \Sigma M_J = 0: \quad M + \left(\frac{4}{3}wa\right)(x) - (wx)\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI\frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{2}{3}wax^2 + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right]: \quad 0 = 0 - 0 + C_1 \quad \therefore \quad C_1 = 0$$

$$EIy = \frac{1}{24}wx^4 - \frac{2}{9}wax^3 + C_2$$

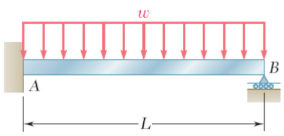
$$[x = 0, y = 0]: \quad 0 = 0 - 0 + C_2 \quad \therefore \quad C_2 = 0$$

(*a*) Elastic curve over *AB*. $\qquad y = \frac{w}{72EI}(3x^4 - 16ax^3)$ ◀

$$\frac{dy}{dx} = \frac{w}{6EI}(x^3 - 4ax^2)$$

(*b*) y at $x = 2a$. $\qquad y_B = -\frac{10wa^4}{9EI}$ $\qquad y_B = \frac{10wa^4}{9EI} \downarrow$ ◀

(*c*) $\frac{dy}{dx}$ at $x = 2a$. $\qquad \left(\frac{dy}{dx}\right)_B = -\frac{4wa^3}{3EI}$ $\qquad \theta_B = \frac{4wa^3}{3EI}$ ◀



PROBLEM 9.19

For the beam and loading shown, determine the reaction at the roller support.

$[x = 0,\ y = 0]$ $\qquad$ $[x = L,\ y = 0]$

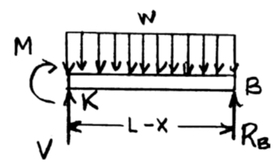
$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown above.

Using free body *KB*,



$$+\circlearrowleft M_K = 0: \quad R_B(L-x) - w(L-x)\left(\frac{L-x}{2}\right) - M = 0$$

$$M = R_B(L-x) - \frac{1}{2}w(L-x)^2$$

$$EI\frac{d^2y}{dx^2} = R_B(L-x) - \frac{1}{2}w(L-x)^2$$

$$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L-x)^2 + \frac{1}{6}w(L-x)^3 + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]: \quad 0 = -\frac{1}{2}R_BL^2 + \frac{1}{6}wL^3 + C_1$$

$$C_1 = \frac{1}{2}R_BL^2 - \frac{1}{6}wL^3$$

$$EI_y = \frac{1}{6}R_B(L-x)^3 - \frac{1}{24}w(L-x)^4 + C_1x + C_2$$

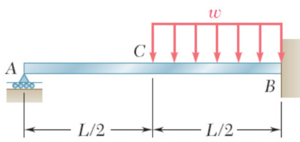
$$[x = 0,\ y = 0]: \quad 0 = \frac{1}{6}R_BL^3 - \frac{1}{24}wL^3 + C_2$$

$$C_2 = -\frac{1}{6}R_BL^3 + \frac{1}{24}wL^4$$

$$[x = L,\ y = 0]: \quad 0 = 0 - 0 + C_1L + C_2$$

$$\frac{1}{2}R_BL^3 - \frac{1}{6}wL^4 - \frac{1}{6}R_BL^3 + \frac{1}{24}wL^4 = 0$$

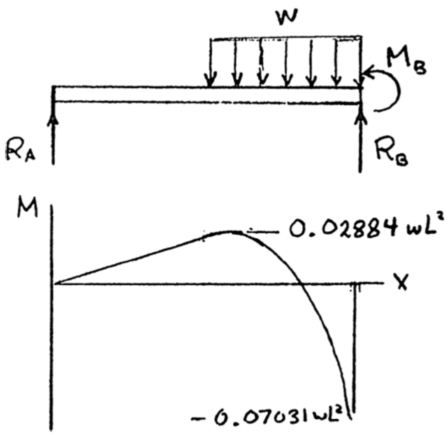
$$R_B = \frac{3}{8}wL \uparrow \blacktriangleleft$$



PROBLEM 9.28

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION



Reactions are statically indeterminate.

$$0 < x < \frac{L}{2}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x \qquad (1)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1 \qquad (2)$$

$$EIy = \frac{1}{6}R_A x^3 + C_1 x + C_2 \qquad (3)$$

$$\frac{L}{2} < x < L$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2}w\left(x - \frac{L}{2}\right)^2 \qquad (4)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}w\left(x - \frac{L}{2}\right)^3 + C_3 \qquad (5)$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}w\left(x - \frac{L}{2}\right)^4 + C_3 x + C_4 \qquad (6)$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 + 0 + C_2$ $\quad C_2 = 0$

$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$ $\quad \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + C_1 = \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + 0 + C_3$ $\quad C_1 = C_3$

$\left[x = \frac{L}{2},\ y = y\right]$ $\quad \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} + \cancel{C_1\frac{L}{2}} + C_2 = \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} - 0 + \cancel{C_3\frac{L}{2}} + C_4$ $\quad C_2 = C_4 = 0$

$\left[x = L,\ \frac{dy}{dx} = 0\right]$ $\quad \frac{1}{2}R_x L^2 - \frac{1}{6}w\left(\frac{L}{2}\right)^3 + C_3 = 0$ $\quad C_3 = \frac{1}{48}wL^3 - \frac{1}{2}R_A L^2$

$[x = L,\ y = 0]$ $\quad \frac{1}{6}R_A L^2 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + \left(\frac{1}{48}wL^3 - \frac{1}{2}R_A L^2\right)L + 0 = 0$

PROBLEM 9.28 *(Continued)*

$$\left(\frac{1}{2}-\frac{1}{6}\right)R_A L^3 = \left(\frac{1}{48}-\frac{1}{384}\right)wL^4 \qquad \frac{1}{3}R_A = \frac{7}{384}wL \qquad R_A = \frac{7}{128}wL \uparrow \blacktriangleleft$$

From (1), with $x = \dfrac{L}{2}$, $\quad M_C = R_A\left(\dfrac{L}{2}\right) = \dfrac{7}{256}wL^2 \qquad M_C = 0.0273wL^2 \blacktriangleleft$

From (4), with $x = L$, $\quad M_B = R_A L - \dfrac{1}{2}w\left(\dfrac{L}{2}\right)^2 = \left(\dfrac{7}{128}-\dfrac{1}{8}\right)wL - \dfrac{9}{128}wL^2$

$$M_B = -\,0.0703wL \blacktriangleleft$$

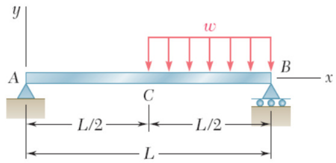
Location of maximum positive M:

$$\frac{L}{2} < x < L \qquad V_m = R_A - w\left(x_m - \frac{L}{2}\right) = 0 \qquad x_m - \frac{L}{2} = \frac{R_A}{w} = \frac{7}{128}L$$

$$x_m = \frac{L}{2} + \frac{7}{128}L = \frac{71}{128}L$$

From (4), with $x = x_m$, $\quad M_m = R_A x_m - \dfrac{1}{2}w\left(x_m - \dfrac{L}{2}\right)^2$

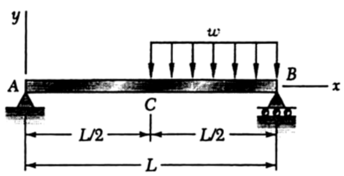
$$= \left(\frac{7}{128}wL\right)\left(\frac{71}{128}L\right) - \frac{1}{2}w\left(\frac{7}{128}L\right)^2 \qquad M_m = 0.0288wL^2 \blacktriangleleft$$



PROBLEM 9.35

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.

SOLUTION



$$\frac{dV}{dx} = -w\left\langle x - \frac{L}{2}\right\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\left\langle x - \frac{L}{2}\right\rangle^1$$

$$M = \cancel{M_A} + R_A X - \frac{1}{2}w\left\langle x - \frac{L}{2}\right\rangle^2$$

$[x = 0, M = 0]$ $[x = L, M = 0]$

$$[x = L, M = 0] \quad R_A L - \frac{1}{2}w\left(\frac{L}{2}\right)^2 = 0$$

$[x = 0, y = 0]$ $[x = L, y = 0]$

$$R_A = \frac{1}{8}wL$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{8}wLx - \frac{1}{2}w\left\langle x - \frac{L}{2}\right\rangle^2$$

$$EI\frac{dy}{dx} = \frac{1}{16}wLx^2 - \frac{1}{6}w\left\langle x - \frac{L}{2}\right\rangle^3 + C_1$$

$$EIy = \frac{1}{48}wLx^3 - \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 + C_1x + C_2$$

$$[x = 0, y = 0] \qquad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$$

$$[x = L, y = 0] \qquad \frac{1}{48}wL^4 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + C_1L + 0 = 0$$

$$C_1 = -\left(\frac{1}{48} - \frac{1}{24}\cdot\frac{1}{16}\right)wL^3 = -\frac{7}{384}wL^3$$

(*a*) Elastic curve.

$$EIy = \frac{1}{48}wLx^3 - \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 - \frac{7}{384}wL^3x$$

$$y = \frac{w}{EI}\left\{\frac{1}{48}Lx^3 - \frac{1}{24}\left\langle x - \frac{L}{2}\right\rangle^4 - \frac{7}{384}L^3x\right\} \quad ◀$$

$$\frac{dy}{dx} = \frac{w}{EI}\left\{\frac{1}{16}Lx^2 - \frac{1}{6}\left\langle x - \frac{L}{2}\right\rangle^3 - \frac{7}{384}L^3\right\}$$

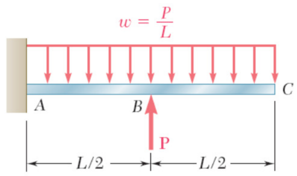
PROBLEM 9.35 *(Continued)*

(*b*) Slope at *A*. ($x = 0$ in slope equation)

$$\theta_A = -\frac{7}{384}\frac{wL^3}{EI} \blacktriangleleft$$

(*c*) Deflection at *C*. $\left(x = \frac{L}{2} \text{ in deflection equation}\right)$

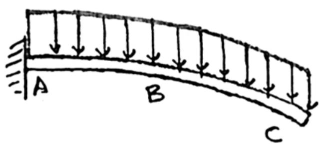
$$y_C = \frac{wL^4}{EI}\left\{\frac{1}{48}\cdot\frac{1}{8} - \frac{7}{384}\cdot\frac{1}{2}\right\} = \left(\frac{1}{384} - \frac{7}{768}\right)\frac{wL^4}{EI} = -\frac{5}{768}\frac{wL^4}{EI} \blacktriangleleft$$



PROBLEM 9.66

For the cantilever beam and loading shown, determine the slope and deflection at the end.

SOLUTION

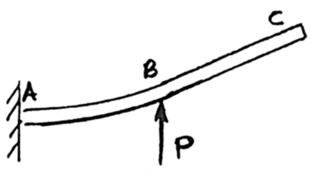


Loading I: Uniformly distributed downward loading with $w = P/L$.

Case 2 of Appendix D.

$$\theta'_C = -\frac{(P/L)L^3}{6EI} = -\frac{1}{6}\frac{PL^2}{EI}$$

$$y'_C = -\frac{(P/L)L^4}{8EI} = -\frac{1}{8}\frac{PL^3}{EI}$$



Loading II: Upward concentrated load at P at point B.

Case 1 of Appendix D applied to portion AB.

$$\theta''_B = \frac{P(L/2)^2}{2EI} = \frac{1}{8}\frac{PL^2}{EI}$$

$$y''_B = \frac{P(L/2)^3}{3EI} = \frac{1}{24}\frac{PL^3}{EI}$$

Portion BC remains straight.

$$\theta''_C = \theta''_B = \frac{1}{8}\frac{PL^2}{EI}$$

$$y''_C = y''_B + \frac{L}{2}\theta''_B = \frac{1}{24}\frac{PL^3}{EI} + \frac{1}{16}\frac{PL^3}{EI} = \frac{5}{48}\frac{PL^3}{EI}$$

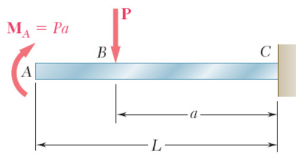
By superposition,

$$\theta_C = \theta'_C + \theta''_C = -\frac{1}{6}\frac{PL^2}{EI} + \frac{1}{8}\frac{PL^2}{EI} = -\frac{1}{24}\frac{PL^2}{EI}$$

$$\theta_C = \frac{PL^2}{24EI} \;\measuredangle \;◀$$

$$y_C = y'_C + y''_C = -\frac{1}{8}\frac{PL^3}{EI} + \frac{5}{48}\frac{PL^2}{EI} = -\frac{1}{48}\frac{PL^3}{EI}$$

$$y_C = \frac{PL^3}{48EI} \downarrow \;◀$$



PROBLEM 9.68

For the cantilever beam and loading shown, determine the slope and deflection at the free end.

SOLUTION

<u>Loading I</u>: M_A at A. Case 3 of appendix D.

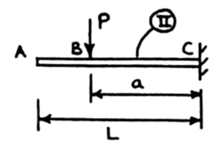
$$\theta'_A = -\frac{M_A L}{EI} \qquad y'_A = \frac{M_A L^2}{2EI}$$

with

$$M_A = Pa$$

$$\theta'_A = -\frac{PaL}{EI}$$

$$y'_A = \frac{PaL^2}{2EI}$$



<u>Loading II</u>: P downward at B. Case 1 of appendix D applied to portion BC.

$$\theta''_B = \frac{Pa^2}{2EI} \qquad y''_B = -\frac{Pa^3}{3EI}$$

AB remains straight.

$$\theta''_A = \theta''_B = \frac{Pa^2}{2EI}$$

$$y''_A = y''_B - (L-a)\theta''_B$$

$$= -\frac{Pa^3}{3EI} - (L-a)\frac{Pa^2}{2EI} = -\frac{Pa^2L}{2EI} + \frac{Pa^3}{6EI}$$

By superposition,

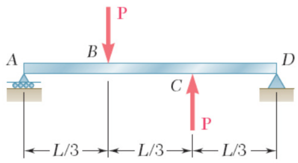
$$\theta_A = \theta'_A + \theta''_A = -\frac{PaL}{EI} + \frac{Pa^2}{2EI} = -\frac{Pa}{2EI}(2L-a)$$

$$\frac{Pa}{2EI}(2L-a) \blacktriangleleft$$

$$y_A = y'_A + y''_A = \frac{PaL^2}{2EI} - \frac{Pa^2L}{2EI} + \frac{Pa^3}{6EI}$$

$$= \frac{Pa}{6EI}(3L^2 - 3aL + a^2)$$

$$\frac{Pa}{6EI}(3L^2 - 3aL + a^2) \uparrow \blacktriangleleft$$



PROBLEM 9.72

For the beam and loading shown, determine (*a*) the deflection at point *C*, (*b*) the slope at end *A*.

SOLUTION

<u>Loading I</u>: Downward load P at B.

Use Case 5 of Appendix D with

$$P = P, \quad a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad L = L, \quad x = \frac{2L}{3}$$

For $x < a$, given elastic curve is $\quad y = \frac{Pb}{EIL}[x^3 - (L^2 - b^2)x]$

To obtain elastic curve for $x > a$, replace x by $L - x$ and interchange a and b to get

$$y = \frac{Pa}{6EIL}[(L-x)^3 - (L^2 - a^2)(L-x)] \text{ with } x = \frac{2L}{3} \text{ at point } C.$$

$$y_C = \frac{P(L/3)}{6EIL}\left[\left(\frac{L}{3}\right)^3 - \left(L^2 - \left(\frac{L}{3}\right)^2\right)\left(\frac{L}{3}\right)\right] = -\frac{7}{486}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(2L/3)[L^2 - (2L/3)^2]}{6EIL} = -\frac{5}{81}\frac{PL^2}{EI}$$

<u>Loading II</u>: Upward load at C. Use Case 5 of Appendix D with

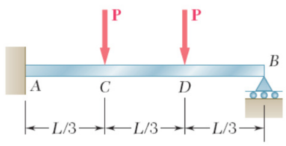
$$P = -P, \quad a = \frac{2L}{3}, \quad b = \frac{L}{3}, \quad L = L, \quad x = a = \frac{2L}{3}$$

$$y_C = -\frac{(-P)(2L/3)^2(L/3)^2}{3EIL} = \frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{(-P)(L/3)(L^2 - (L/3)^2)}{6EIL} = \frac{4}{81}\frac{PL^2}{EI}$$

(*a*) <u>Deflection at *C*.</u> $\quad y_C = -\frac{7}{486}\frac{PL^3}{EI} + \frac{4}{243}\frac{PL^3}{EI}$ $\qquad y_C = \frac{1}{486}\frac{PL^3}{EI} \uparrow$ ◀

(*b*) <u>Slope at *A*.</u> $\quad \theta_A = -\frac{5}{81}\frac{PL^2}{EI} + \frac{4}{81}\frac{PL^2}{EI}$ $\qquad \theta_A = \frac{1}{81}\frac{PL^2}{EI}$ ◀

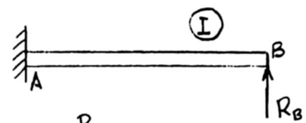


PROBLEM 9.79

For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.

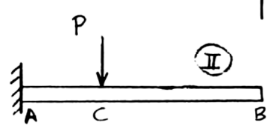
SOLUTION

Consider R_B as redundant and replace loading system by I, II and III.



<u>Loading I</u>: Case 1 of Appendix *D* applied to *AB*.

$$(y_B)_I = \frac{R_B L^3}{3EI}$$



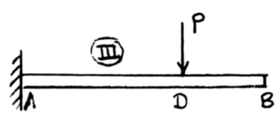
<u>Loading II</u>: Case 1 applied to portion *AC*.

$$(\theta_C)_{II} = -\frac{P(L/3)^2}{2EI} = -\frac{1}{18}\frac{PL^2}{EI}$$

$$(y_C)_{II} = -\frac{P(L/3)^3}{3EI} = -\frac{1}{81}\frac{PL^3}{EI}$$

Portion *CB* remains straight.

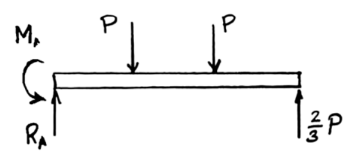
$$(y_B)_{II} = (Y_C)_{II} + \frac{2L}{3}(\theta_C)_{II} = -\frac{4}{81}\frac{PL^3}{EI}$$



<u>Loading III</u>: Case 1 applied to portion *AD*.

$$(\theta_D)_{III} = \frac{P(2L/3)^2}{2EI} = -\frac{2}{9}\frac{PL^2}{EI}$$

$$(y_D)_{III} = \frac{P(2L/3)^3}{3EI} = -\frac{8}{81}\frac{PL^3}{EI}$$



Portion *DB* remains straight.

$$(y_C)_{III} = (y_D)_{III} + \frac{L}{3}(\theta_D)_{III} = -\frac{14}{81}\frac{PL^3}{EI}$$

Superposition and constraint:

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$\frac{1}{3}R_B L^3 - \frac{4}{81}\frac{PL^3}{EI} - \frac{14}{81}\frac{PL^3}{EI} = \frac{1}{3}\frac{R_B L^3}{EI} - \frac{2}{9}\frac{PL^3}{EI} = 0$$

(*b*) $\mathbf{R_B} = \frac{2}{3}P \uparrow$ ◀

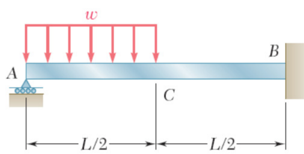
<u>Statics</u>:

$$+\uparrow \Sigma F_y = 0: \quad R_A - P - P + \frac{2}{3}P = 0$$

(*a*) $\mathbf{R_A} = \frac{4}{3}P \uparrow$ ◀

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - P\left(\frac{L}{3}\right) - P\left(\frac{2L}{3}\right) + \left(\frac{2}{3}P\right)(L) = 0$$

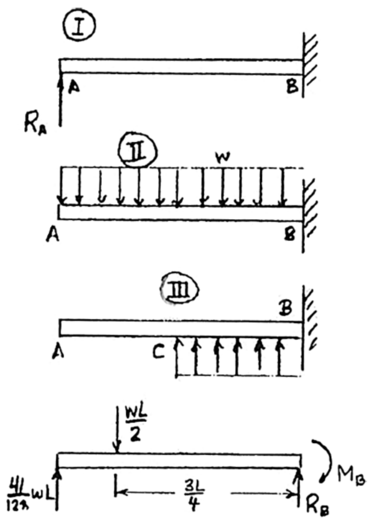
$\mathbf{M_A} = \frac{1}{3}PL \circlearrowleft$ ◀



PROBLEM 9.80

For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.

SOLUTION



Beam is indeterminate to first degree. Consider R_A as redundant and replace the given loading by loadings I, II, and III.

Loading I: Case 1 of Appendix *D*.

$$(y_A)_I = \frac{R_A L^3}{3EI}$$

Loading II: Case 2 of Appendix *D*.

$$(y_A)_{II} = -\frac{wL^4}{8EI}$$

Loading III: Case 2 of Appendix *D* (portion *CB*).

$$(\theta_C)_{III} = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48}\frac{wL^3}{EI}$$

$$(y_C)_{III} = \frac{w(L/2)^4}{8EI} = \frac{1}{128}\frac{wL^4}{EI}$$

Portion *AC* remains straight.

$$(y_A)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{7}{384}\frac{wL^4}{EI}$$

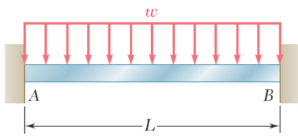
Superposition and constraint: $y_A = (y_A)_I + (y_A)_{II} + (y_A)_{III} = 0$

(*a*) $\frac{1}{3}\frac{R_A L^3}{3EI} - \frac{1}{8}\frac{wL^4}{EI} + \frac{7}{384}\frac{wL^4}{EI} = \frac{1}{3}\frac{R_A L^3}{EI} - \frac{41}{384}\frac{wL^4}{EI} = 0$ $\qquad R_A = \frac{41}{128}wL \uparrow$ ◀

Statics:

(*b*) $+\uparrow \Sigma F_y = 0: \quad \frac{41}{128}wL - \frac{1}{2}wL + R_B = 0$ $\qquad R_B = \frac{23}{128}wL \uparrow$ ◀

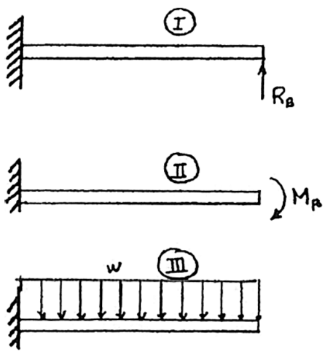
$+\circlearrowleft \Sigma M_B = 0: \quad -\left(\frac{41}{128}wL\right)L - \left(\frac{1}{2}wL\right)\left(\frac{3L}{4}\right) - M_B = 0$ $\qquad M_B = \frac{7}{128}wL^2 \circlearrowright$ ◀



PROBLEM 9.84

For the beam shown, determine the reaction at B.

SOLUTION



Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.

Loading I: Case 1 of Appendix D.

$$(y_B)_I = \frac{R_B L^3}{3EI} \qquad (\theta_B)_I = \frac{R_B L^2}{2EI}$$

Loading II: Case 3 of Appendix D.

$$(y_B)_{II} = -\frac{M_B L^2}{2EI} \qquad (\theta_B)_{II} = -\frac{M_B L}{EI}$$

Loading III: Case 2 of Appendix D.

$$(y_B)_{III} = -\frac{wL^4}{8EI} \qquad (\theta_B)_{III} = -\frac{wL^3}{6EI}$$

Superposition and constraint:

$$y_B = (y_B)_{\mathrm{I}} + (y_B)_{\mathrm{II}} + (y_B)_{\mathrm{III}} = 0$$

$$\frac{L^3}{3EI} R_B - \frac{L^2}{2EI} M_B - \frac{wL^4}{8EI} = 0 \qquad (1)$$

$$\theta_B = (\theta_B)_{\mathrm{I}} + (\theta_B)_{\mathrm{II}} + (\theta_B)_{\mathrm{III}} = 0$$

$$\frac{L^2}{2EI} R_B - \frac{L}{EI} M_B - \frac{wL^3}{6EI} = 0 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$R_B = \frac{1}{2} wL \uparrow \blacktriangleleft$$

$$M_B = \frac{1}{12} wL^2 \; \circlearrowright \blacktriangleleft$$