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Mechanics of Materials, Eighth Edition

Chapter 1

Introduction – Concept of Stress

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Concept of Stress

- The main objective of the study of the mechanics of materials is to provide the future engineer with the means of analyzing and designing various machines and load bearing structures.
- Both the analysis and design of a given structure involve the determination of *stresses* and *deformations*. This chapter is devoted to the concept of stress.

Review of Statics

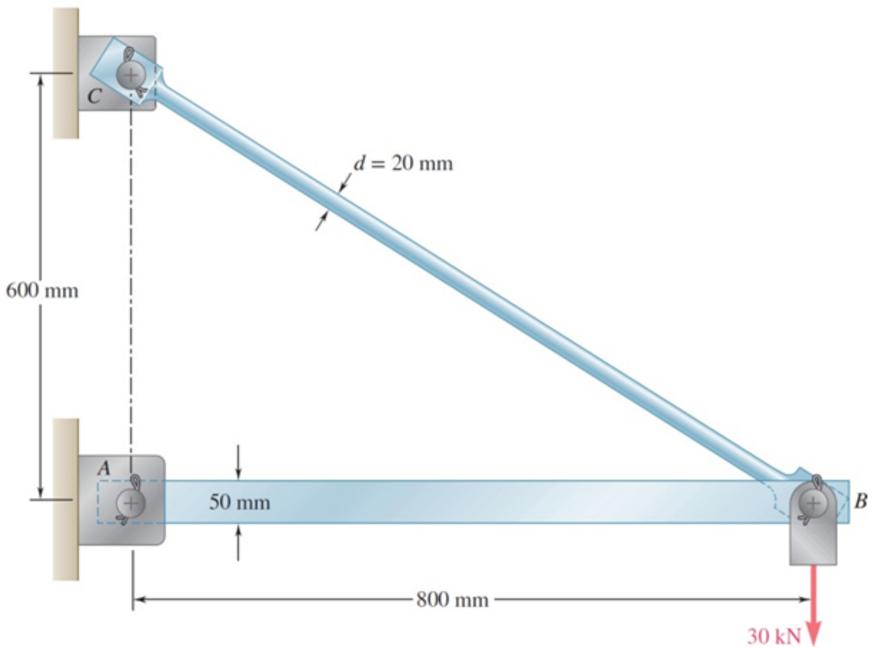

Figure 1.1 Boom used to support a 30-kN load.

- The structure is designed to support a 30 kN load.
- The structure consists of a boom *AB* and rod *BC* joined by pins (zero moment connections) at the junctions and supports.
- Perform a static analysis to determine the reaction forces at the supports and the internal force in each structural member.

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Structure Free-Body Diagram

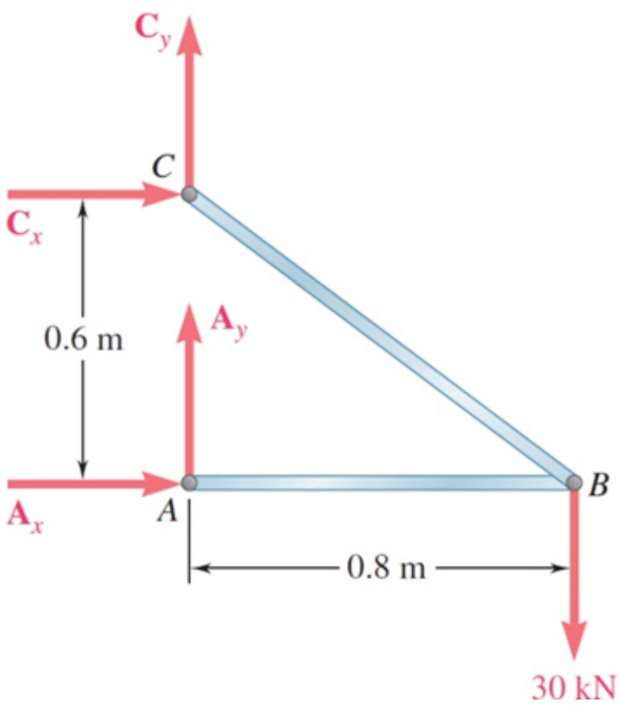

Figure 1.2 Free-body diagram of boom showing Applied load and reaction forces.

- Structure is detached from supports, and the loads and reaction forces are indicated to produce a free-body diagram.
- Conditions for static equilibrium:

$+\circlearrowright \quad M_c = 0$

$\xrightarrow{+} \quad \Sigma F_x = 0$

$+\uparrow \quad \Sigma F_y = 0$

- A_y and C_y cannot be determined from these equations.

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Component Free-Body Diagram

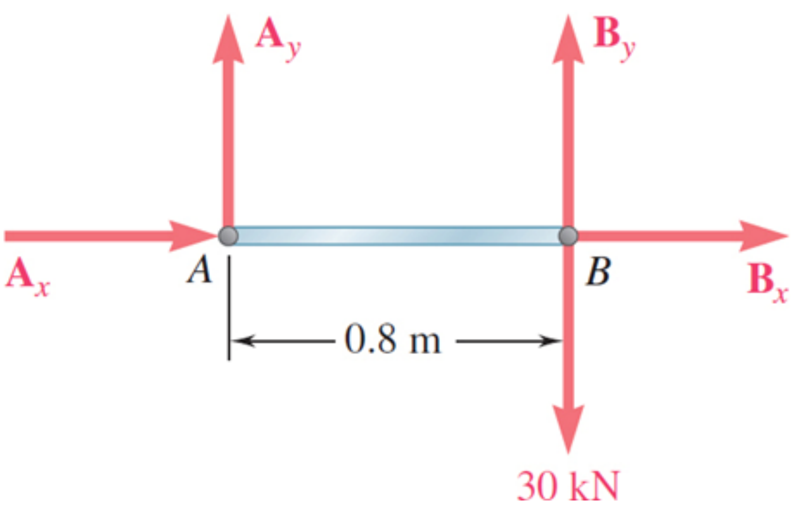


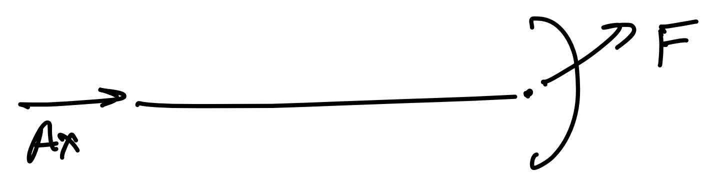
Figure 1.3 Free-body diagram of member *AB* freed from structure.

- In addition to the complete structure, each component must satisfy the conditions for static equilibrium.
- Consider a free-body diagram of the boom *AB*:

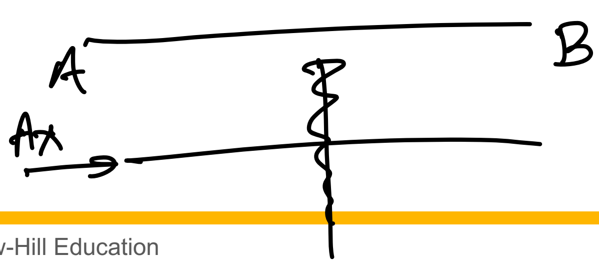
$+\circlearrowleft$ $M_B = 0$

substitute into the structure equilibrium equation

- Results:



Reaction forces are directed along the boom and rod.



Method of Joints

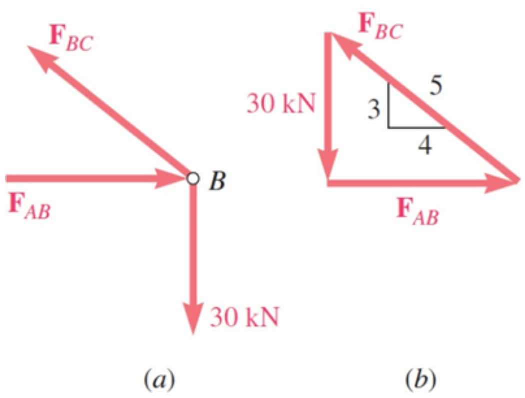


Figure 1.4 Free-body diagram of boom's joint B and associated force triangle.

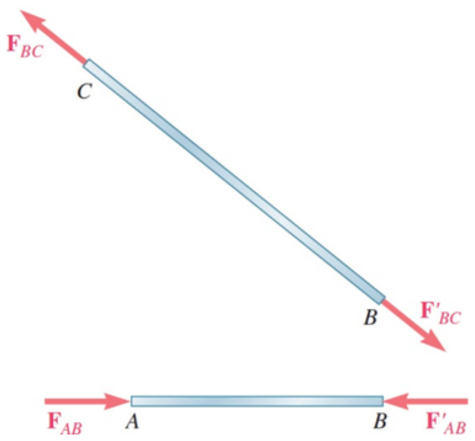
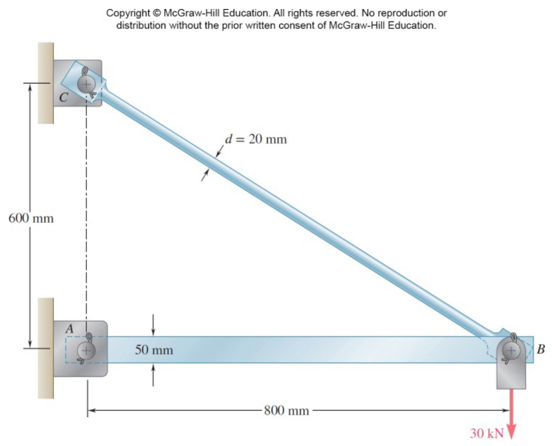


Figure 1.5 Free-body diagrams of two-force members *AB* and *BC*.

- Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:

- The boom and rod are 2-force members, i.e., the members are subjected to only two forces, which are applied at the ends of the members.
- For equilibrium, the forces must be parallel to an axis between the force application points, equal in magnitude and in opposite directions.

Stress Analysis



$d_{BC} = 20$ mm

Figure 1.1 Boom used to support a 30-kN load.

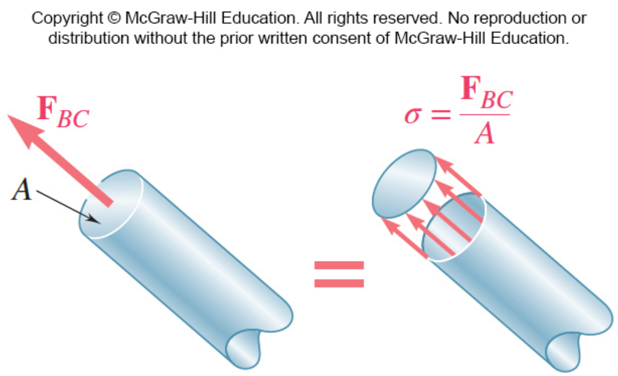


Figure 1.7 Axial force represents the resultant of distributed elementary forces.

Can the structure safely support the 30 kN load if rod *BC* has a diameter of 20 mm?

- From a statics analysis.

  $F_{AB} = 40$ kN (compression)

  $F_{BC} = 50$ kN (tension)

- At any section through member BC, the internal force is 50 kN with a force intensity or stress of,

$$\frac{50\times10^3}{\pi \times \left(\frac{20}{2}\right)^2 mm^2} = +159\ MPa$$

- From the material properties for steel, the allowable stress is,

$$\sigma_{all} = 165 \text{ Mpa}$$

- Conclusion: the strength of member *BC* is adequate.

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Design

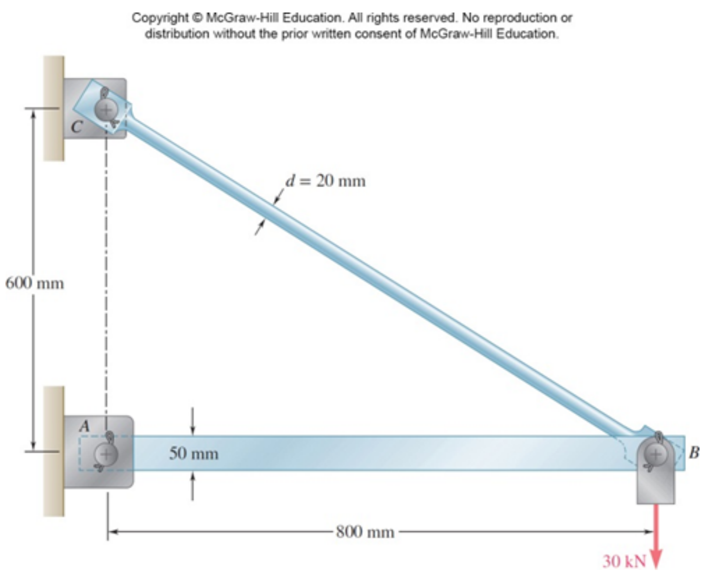

Figure 1.1 Boom used to support a 30-kN load.

- Design of new structures requires selection of appropriate materials and component dimensions to meet performance requirements.
- For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (σ_{all}= 100 MPa). What is an appropriate choice for the rod diameter?

$$\sigma_{all} = \frac{P}{A} \Rightarrow A = \frac{P}{\sigma_{all}}$$

$$\Rightarrow \cancel{d}\, r = ? \quad 12.62\,mm$$

- An aluminum rod 26 mm or more in diameter is adequate.

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Axial Loading: Normal Stress

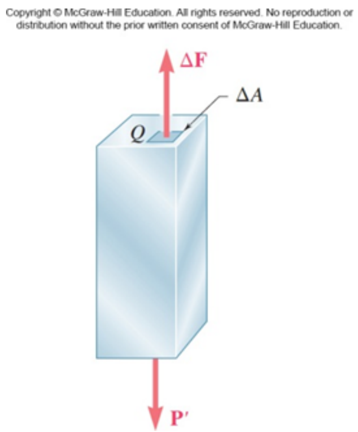

Figure 1.9 Small area ΔA, at an arbitrary cross section point carries/axial ΔF in this member.

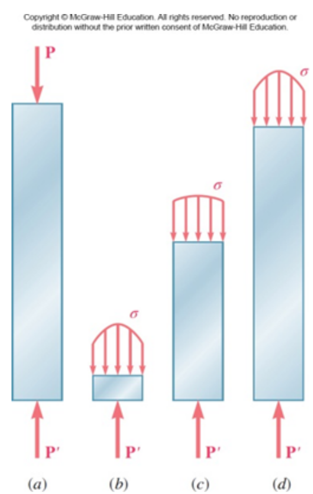

Figure 1.10 Stress distributions at different sections along axially loaded member.

- The resultant of the internal forces for an axially loaded member is *normal* to a section cut perpendicular to the member axis.
- The force intensity on that section is defined as the normal stress.

$$\sigma = \frac{\Delta F}{\Delta A}$$

- The normal stress at a particular point may not be equal to the average stress but the resultant of the stress distribution must satisfy

$$P = \int dF = \int \sigma \, dA$$

- The actual distribution of stresses is statically indeterminate, i.e., can not be found from statics alone.

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Centric & Eccentric Loading

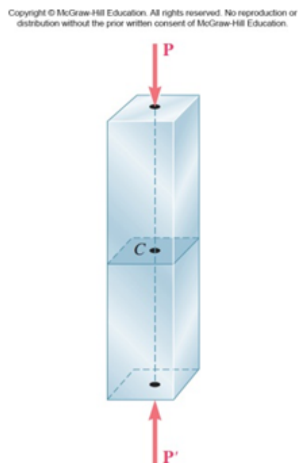

Figure 1.12 Centric loading having resultant forces passing through the centroid of the section.

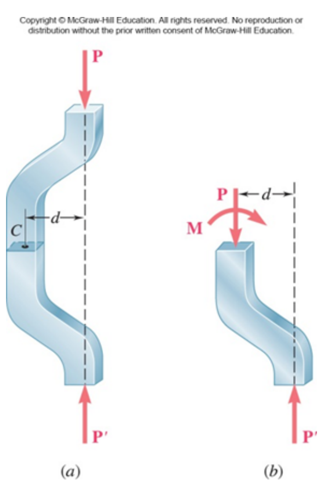

Figure 1.13 An example of simple eccentric loading.

- A uniform distribution of stress in a section infers that the line of action for the resultant of the internal forces passes through the centroid of the section.
- A uniform distribution of stress is only possible if the line of action of the concentrated loads passes through the centroid of the section considered. This is referred to as *centric loading*.
- If a two-force member is *eccentrically loaded*, then the resultant of the stress distribution in a section must yield an axial force and a moment.
- The stress distributions in eccentrically loaded members cannot be uniform or symmetric.

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Shearing Stress

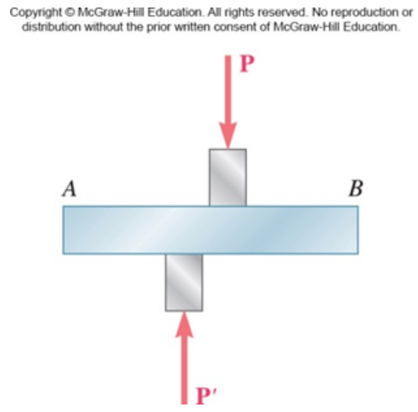

Figure 1.14 Opposing transverse loads creating shear on member *AB*.

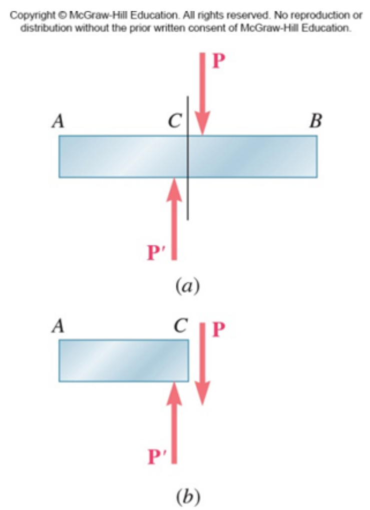

Figure 1.15 This shows the resulting internal shear force on a section between transverse forces.

- Forces P and P′ are applied transversely to the member *AB*.
- Corresponding internal forces act in the plane of section *C* and are called *shearing* forces.
- The resultant of the internal shear force distribution is defined as the *shear* of the section and is equal to the load **P**.
- The corresponding average shear stress is,

$$\tau_{av} = \frac{P}{A}$$

- Shear stress distribution varies from zero at the member surfaces to maximum values that may be much larger than the average value.
- The shear stress distribution cannot be assumed to be uniform.

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Shearing Stress Examples

Single Shear

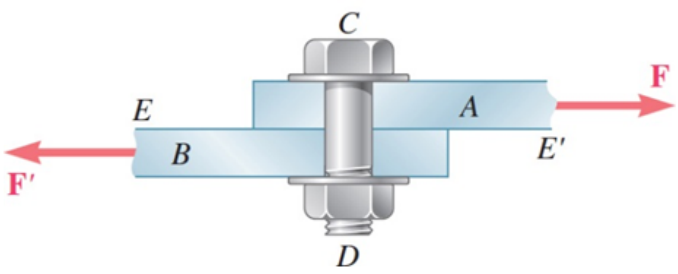

Figure 1.16 Bolt subject to single shear.

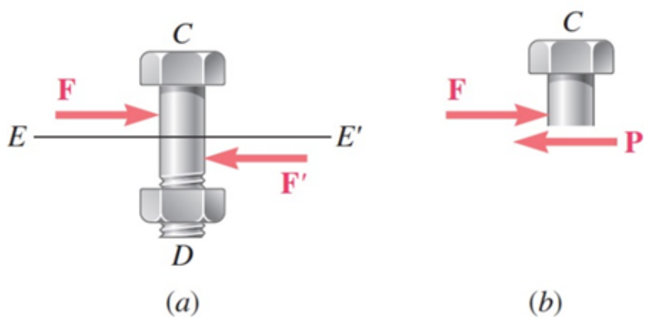

Figure 1.17 (a) Diagram of bolt in single shear; (b) section

Double Shear

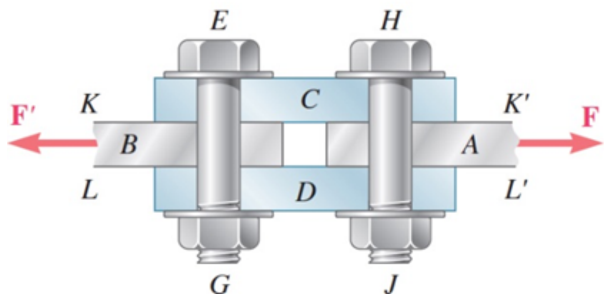

Figure 1.18 Bolt subject to double shear.

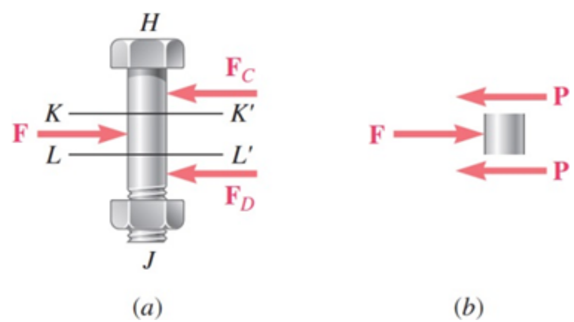

Figure 1.19 (a) Diagram of bolt in double shear; (b) section

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Bearing Stress in Connections

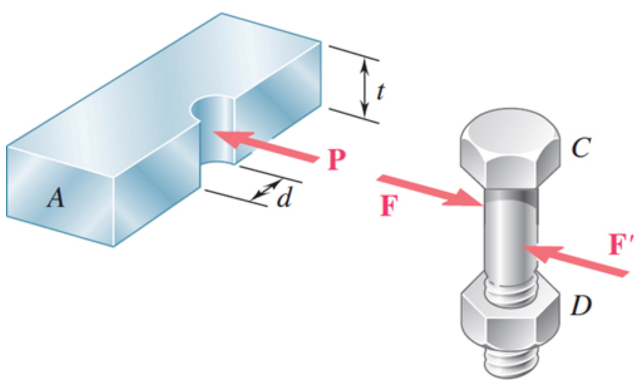

Figure 1.20 Equal and opposite forces between plate and bolt, exerted over bearing surfaces.

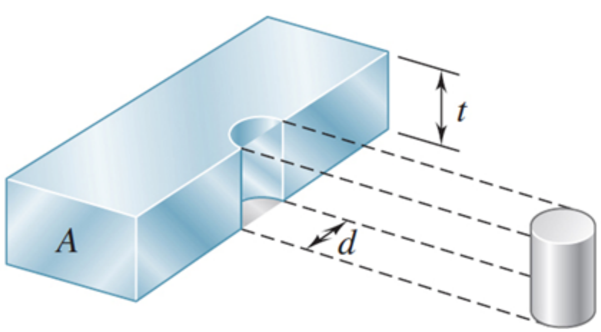

Figure 1.21 Dimensions for calculating bearing stress area.

- Bolts, rivets, and pins create stresses on the points of contact or *bearing surfaces* of the members they connect.
- The resultant of the force distribution on the surface is equal and opposite to the force exerted on the pin.
- Corresponding average force intensity is called the bearing stress,

$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$

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Stress Analysis & Design Example

20mm dia rod BC has flat ends 20mm x 40mm
AB → 30mm x 50mm rectangular section, end B clevis. Connected at B by pin
Load suspended by bracket

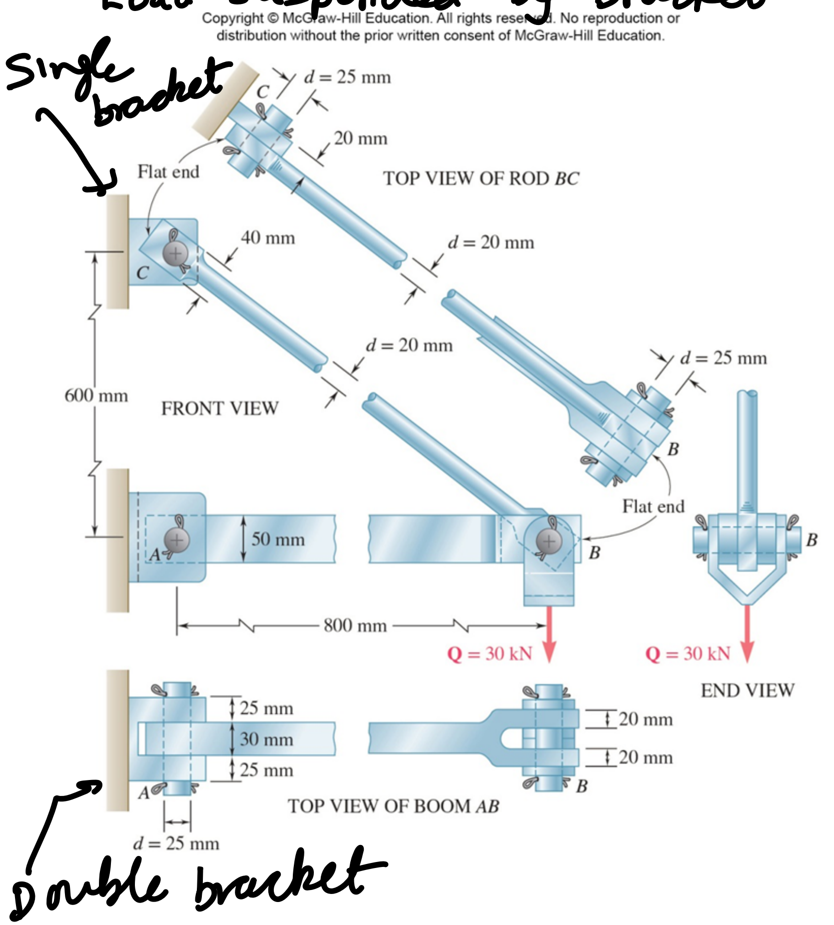

Figure 1.22 Components of boom used to support 30 kN load.

- Would like to determine the stresses in the members and connections of the structure shown.
- From a statics analysis:

  F_{AB} = 40 kN (compression)

  F_{BC} = 50 kN (tension)
- Must consider maximum normal stresses in *AB* and *BC*, and the shearing stress and bearing stress at each pinned connection.

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Rod & Boom Normal Stresses

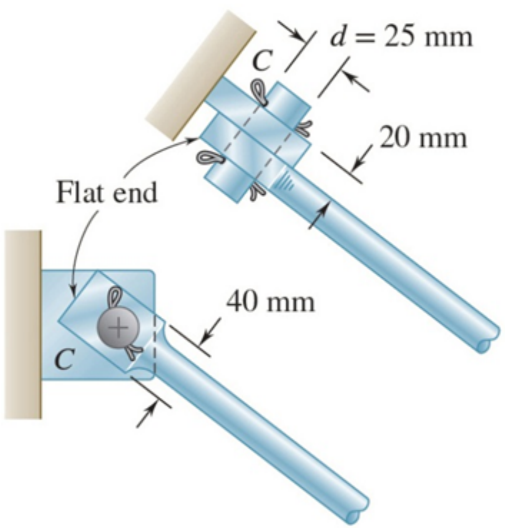

Figure 1.22 (partial)

- The rod is in tension with an axial force of 50 kN.
- At the rod center, the average normal stress in the circular cross-section is $\sigma_{BC} = +159$ MPa.

$$\sigma_{BC} = \frac{50 \times 10^3}{\pi \times (10^{-3})^2} \text{ Pa}$$

- At the flattened rod ends, the smallest cross-sectional area occurs at the pin centerline,

$$\text{Area} = 20\text{mm}\,(40-25)\text{mm}$$
$$= 300 \times 10^{-6}\ \text{m}^2$$
$$\sigma_{BC} = \frac{50 \times 10^3}{300 \times 10^{-6}} = 167 \text{ MPa}$$

- The boom is in compression with an axial force of 40 kN and average normal stress of −26.7 MPa.
- The sections of minimum area at A and B are not under stress, since the boom is in compression, and therefore pushes on the pins.

Pin Shearing Stresses [1]

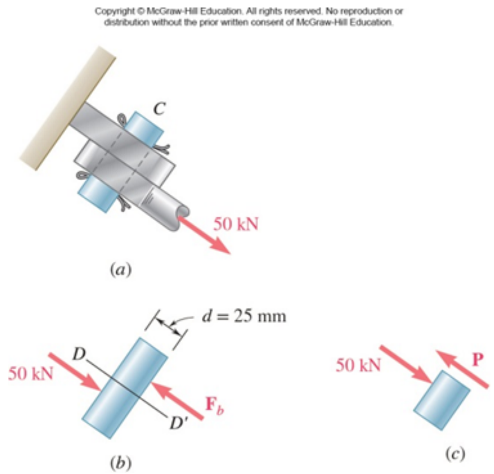

Figure 1.23 Diagrams of the single shear pin at C.

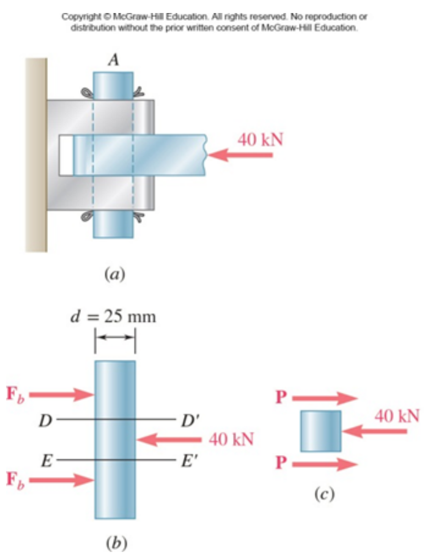

Figure 1.24 Free-body diagrams of the double shear pin at A.

- The cross-sectional area for pins at *A*, *B*, and *C* is,

$$\pi\left(\frac{25\text{ mm}}{2}\right)^2 = 491\times10^{-6}\text{ m}^2$$

- The force on the pin at *C* is equal to the force exerted by the rod *BC* is,

$$\tau_{av} = \frac{50\text{ kN}}{491\times10^{-6}\text{ m}^2} = 102.0\text{ MPa}$$

- The pin at *A* is in double shear with a total force equal to the force exerted by the boom *AB*,

$$\tau_{av} = \frac{20\text{ kN}}{491\times10^{-6}\text{ m}^2} = 40.7\text{ MPa}$$

Pin Shearing Stresses 2

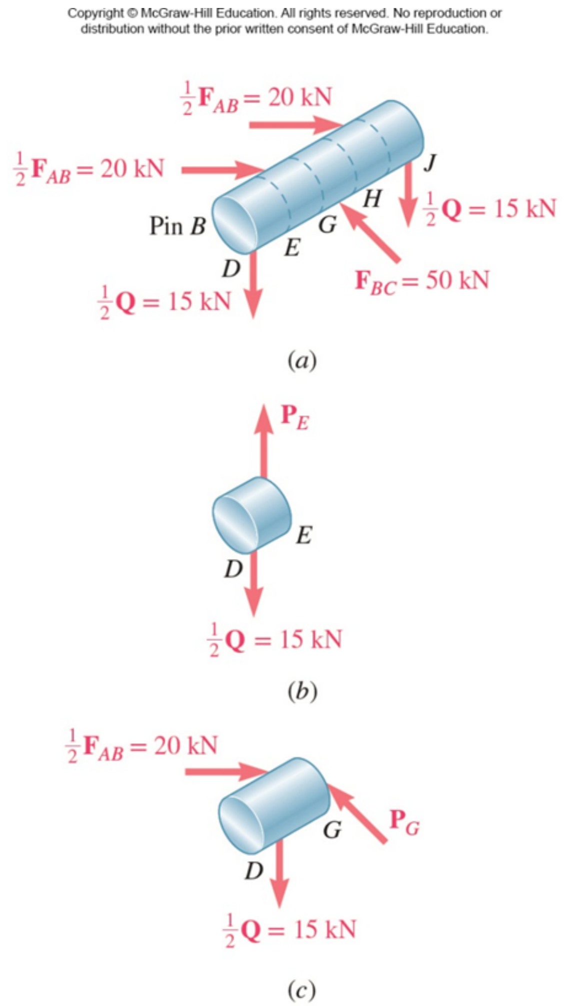

Figure 1.25 Free-body diagrams for various sections at pin *B*.

- Divide the pin at *B* into sections to determine the section with the largest shear force,

$$P_E = 15 \text{ kN}$$

$$P_G = 25 \text{ kN} \quad \text{(largest)}$$

- Evaluate the corresponding average shearing stress,

$$\tau_{av} = \frac{P_G}{A} = \frac{25\,\text{kN}}{491 \times 10^{-6}\,\text{mm}^2} = 50.9\,\text{MPa}$$

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Pin Bearing Stresses

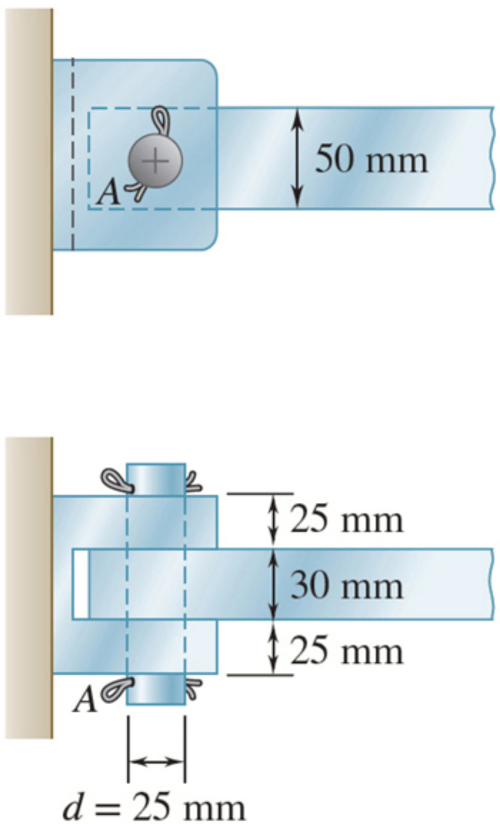

Figure 1.22 (partial)

To determine the bearing stress at *A* in the boom *AB*, we have $t = 30$ mm and $d = 25$ mm,

$$\sigma_b = \frac{P}{td} = \frac{40\,kN}{30 \times 25} = 53.3\,MPa$$

To determine the bearing stress at *A* in the bracket, we have $t = 2(25 \text{ mm}) = 50$ mm and $d = 25$ mm,

$$\sigma_b = \frac{40\,kN}{(50mm)(25mm)} = 32.0\,MPa$$

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Stress in Two Force Members

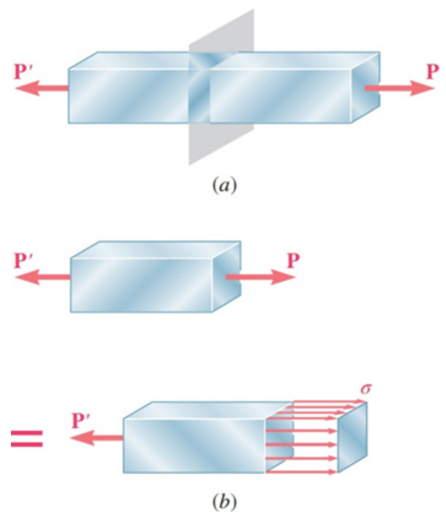

Figure 1.26 Axial forces on a two-force member. (a) Section plane perpendicular to member away from load application. (b) Equivalent force diagram models of resultant force acting at centroid and uniform normal stress.

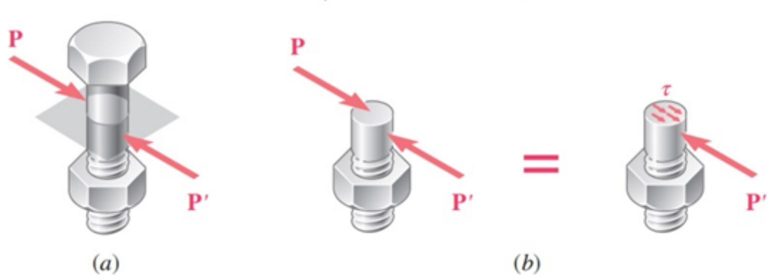

Figure 1.27 (a) Diagram of a bolt from a single shear joint with a section plane normal to the bolt. (b) Equivalent force diagram model of the resultant force acting at the section centroid and the uniform average shear stress.

- Axial forces on a two force member result in only normal stresses on a plane cut perpendicular to the member axis.
- Transverse forces on bolts and pins result in only shear stresses on the plane perpendicular to bolt or pin axis.
- Axial or transverse forces may produce both normal and shear stresses with respect to a plane other than one cut perpendicular to the member axis.

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Stress on an Oblique Plane

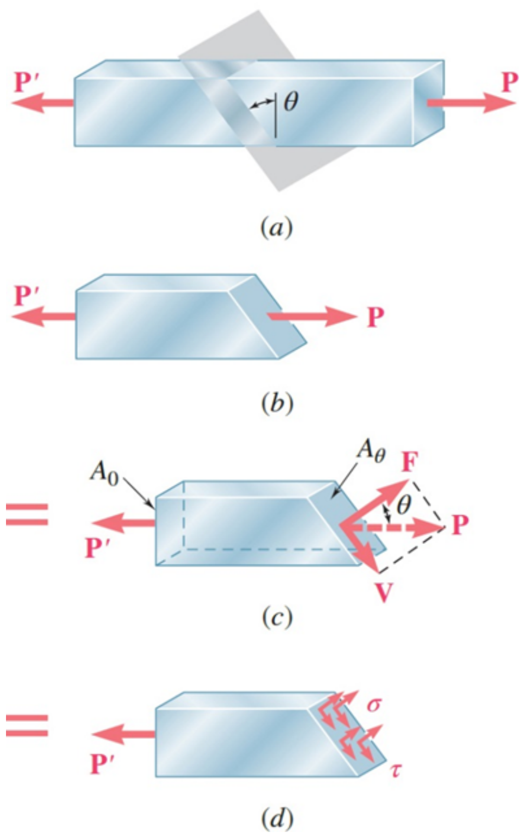

Figure 1.28 Oblique section through a two-force member. (a) Section plane made at an angle θ to the member normal plane, (b) Free-body diagram of left section with internal resultant force **P**. (c) Free-body diagram of resultant force resolved into components **F** and **V** along the section plane's normal and tangential directions, respectively. (d) Free-body diagram with equivalent as normal stress, σ, and shearing stress, *τ*.

- Pass a section through the member forming an angle θ with the normal plane.
- From equilibrium conditions, the distributed forces (stresses) on the plane must be equivalent to the force *P*.
- Resolve P into components normal and tangential to the oblique section,

$$F = P\cos\theta \qquad A_\theta = \frac{A_0}{\cos\theta}$$
$$V = P\sin\theta$$

- The average normal and shear stresses on the oblique plane are,

$$\sigma = \frac{P\cos\theta}{A_0/\cos\theta} = \frac{P}{A_0}\cos^2\theta$$

$$\tau = \frac{P\sin\theta}{A_0/\cos\theta} = \frac{P}{A_0}\sin\theta\cos\theta$$

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Maximum Stresses

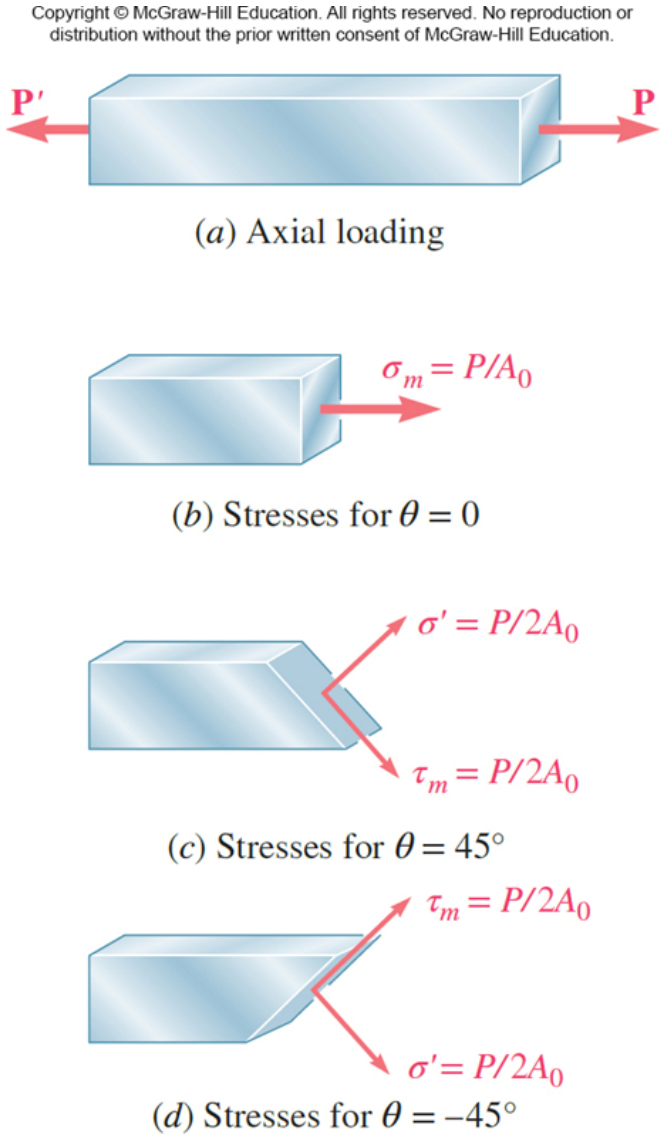

Figure 1.29 Selected stress results for axial loading.

- Normal and shearing stresses on an oblique plane,
- The maximum normal stress occurs when the reference plane is perpendicular to the member axis,

  $\theta = 0° \Rightarrow \sigma_m = P/A_0$
- The maximum shear stress occurs for a plane at $\pm 45°$ with respect to the axis,

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Stress Under General Loadings

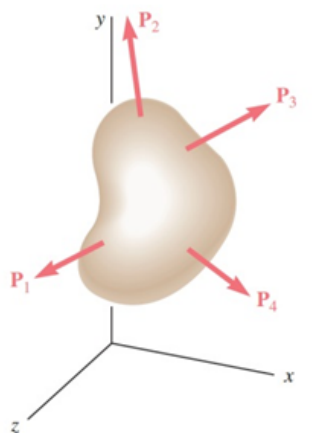

Figure 1.30 Multiple loads on a general body.

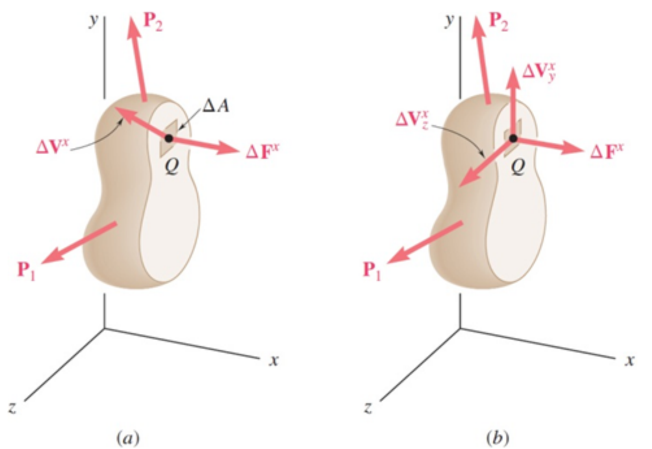

Figure 1.31 (a) Resultant shear and normal forces, acting on small area ΔA at point Q. (b) Forces on ΔA resolved into force in coordinate directions.

- A member subjected to a general combination of loads is cut into two segments by a plane passing through *Q.*
- The distribution of internal stress components may be defined as,
- For equilibrium, an equal and opposite internal force and stress distribution must be exerted on the other segment of the member.

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State of Stress [1]

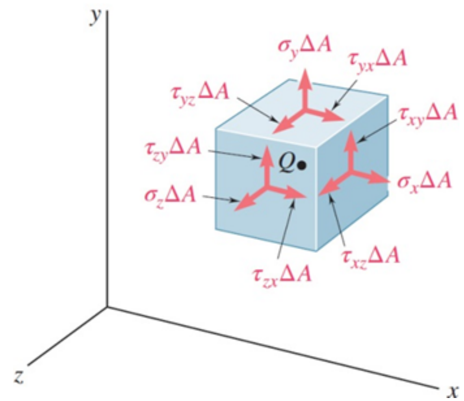

Figure 1.35 Positive resultant forces on a small element at point *Q* resulting from a state of general stress.

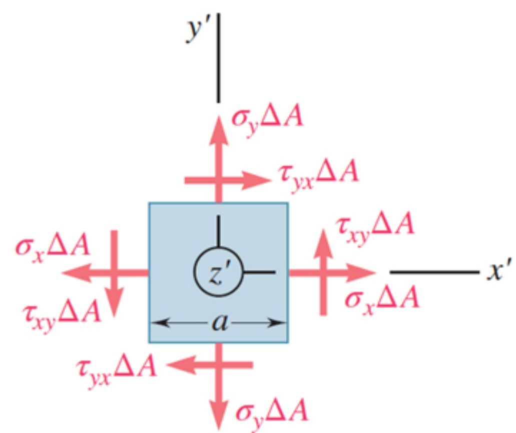

Figure 1.36 Free-body diagram of small element at *Q* viewed on projected plane perpendicular to

Resultant forces on positive and negative (not shown) act through the thus do not contribute to the moment about that axis.

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State of Stress 2

- Stress components are defined for the planes cut parallel to the x, y and z axes. For equilibrium, equal and opposite stresses are exerted on the hidden planes.
- The combination of forces generated by the stresses must satisfy the conditions for equilibrium:

- Consider the moments about the z axis:

- Only six components of stress are required to define the complete state of stress.

Factor of Safety

Structural members or machines must be designed such that the working stresses are less than the ultimate strength of the material.

Factor of safety considerations:

- uncertainty in material properties.
- uncertainty of loadings.
- uncertainty of analyses.
- number of loading cycles.
- types of failure.
- maintenance requirements and deterioration effects.
- importance of member to integrity of whole structure.
- risk to life and property.
- influence on machine function.

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Accessibility Content: Text Alternatives for Images

Review of Statics – Text Alternative

Horizontal boom AB is supported by a pin at the wall at A, and joined by a pin to rod CB at B. Rod CB is attached to the same wall through a pin at C (A and C have the same x coordinate and are 600 millimeters apart).

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Structure Free-Body Diagram – Text Alternative

Free body diagram of Figure 1.1 (described before). Horizontal and vertical reaction forces are shown at A and C, and the downward 30 kilonewtons force at B are shown.

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Stress Analysis – Text Alternative

Horizontal boom AB is supported by a pin at the wall at A, and joined by a pin to rod CB at B. Rod CB is attached to the same wall through a pin at C (A and C have the same x coordinate and are 600 millimeters apart).

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Design – Text Alternative

Horizontal boom AB is supported by a pin at the wall at A, and joined by a pin to rod CB at B. Rod CB is attached to the same wall through a pin at C (A and C have the same x coordinate and are 600 millimeters apart).

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Axial Loading: Normal Stress – Text Alternative

Stress profile on a cut of a beam near the external, punctual force shows higher values near the location of the load. Stress profile on a cut far away from the external, punctual force is closer to a uniform distribution.

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Centric & Eccentric Loading – Text Alternative

Cross section in the middle of a beam subjected to compression shows point C, in the center of the cross section; same axis as that on which the external loads are located.

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Shearing Stress – Text Alternative

An element AB is subjected to shear. This is achieved by using two blocks, one on the bottom going up, one on the top going down. The blocks are placed such that the right side of the bottom block shares the same plane as the left side of the top block. The shear will occur at exactly the shared plane.

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Shearing Stress Examples – Text Alternative

Members A and B are joined by a bolt. If the members are subjected to opposing forces, the bolt would be undergoing shear exactly on the plane where the members meet.

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Bearing Stress in Connections – Text Alternative

The dimensions for calculating the bearing stress area are shown. Specifically, these are the diameter of the bolt and the thickness of the plate, which would reconstruct a projected rectangle of the bolt on the plate, viewed from the side.

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Stress Analysis & Design Example – Text Alternative

Structure described in Figure 1.1 with added details: Single shear pin is located at C. Double shear pin is located at A, and double shear yolk is translating the 30 kilonewtons load to point B. Dimensions for pins are also included.

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Pin Shearing Stresses 2 – Text Alternative

Three-dimensional free-body diagram of pin B shows the 30 kilonewtons load divided into two 15 kilonewtons loads going down, a 50 kilonewtons load in the BC direction, and two 20 kilonewtons horizontal loads that add up to the 40 kilonewtons known load AB.

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Pin Bearing Stresses – Text Alternative

Dimensions of pin and plates at A. Member AB is 50 millimeters tall and 30 millimeters thick. Double shear plate through which pin A passes is 25 millimeters on each side. Pin A is 25 millimeters in diameter.

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Stress in Two Force Members – Text Alternative [1]

Cross-section plane is shown in the center of a beam subjected to tension by loads P and .'. The internal load P at the cut is shown to be equivalent to a stress sigma affecting the cross-section area.

Advance to rest of text alternative.

Stress in Two Force Members – Text Alternative [2]

A cut at the center of a single shear bolt is shown to illustrate that the shear force P is causing a shearing stress Tau on the surface of the cut, represented by various vectors, parallel to the surface area of the cut.

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Stress on an Oblique Plane – Text Alternative

A slanted cut inside a member subjected to tension is shown. The internal load P is resolved into components F and V, perpendicular and parallel to the slanted plane of the cut, respectively. Forces are then represented by normal and shearing stresses, respectively.

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Maximum Stresses – Text Alternative

Angles of 0°, 45°, and 45° are used for the previous figure, to show that the shearing and normal stresses are equal to P over 2 times the perpendicular area, A not.

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Stress Under General Loadings – Text Alternative

A random cut inside the potato, on a plane parallel to the y z plane shows vectors of forces that are either parallel or perpendicular to the surface of the cut. They are labelled V and F, as usual.

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State of Stress [1] – Text Alternative

A 2D side view of a small cube element shows the shear and normal stresses on the 4 sides of the observed square: The figure points out the conventional notation: first subscript of the shear stress refers to the plane on which the stress acts, and the second, the direction of the vector used to represent the stress.

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