APL 105 Lecture 3 8-1-2024

$\vec{f_i}$ → stress vector acting on area (ΔA)

$$= \frac{\vec{f_i}}{\cancel{\Delta A}}$$

$f_i \Delta A$ → total surface force on (ΔA)

Linear function → y is a linear function of x

$$y = \alpha x + \beta$$

y → ~~in~~dependent variable

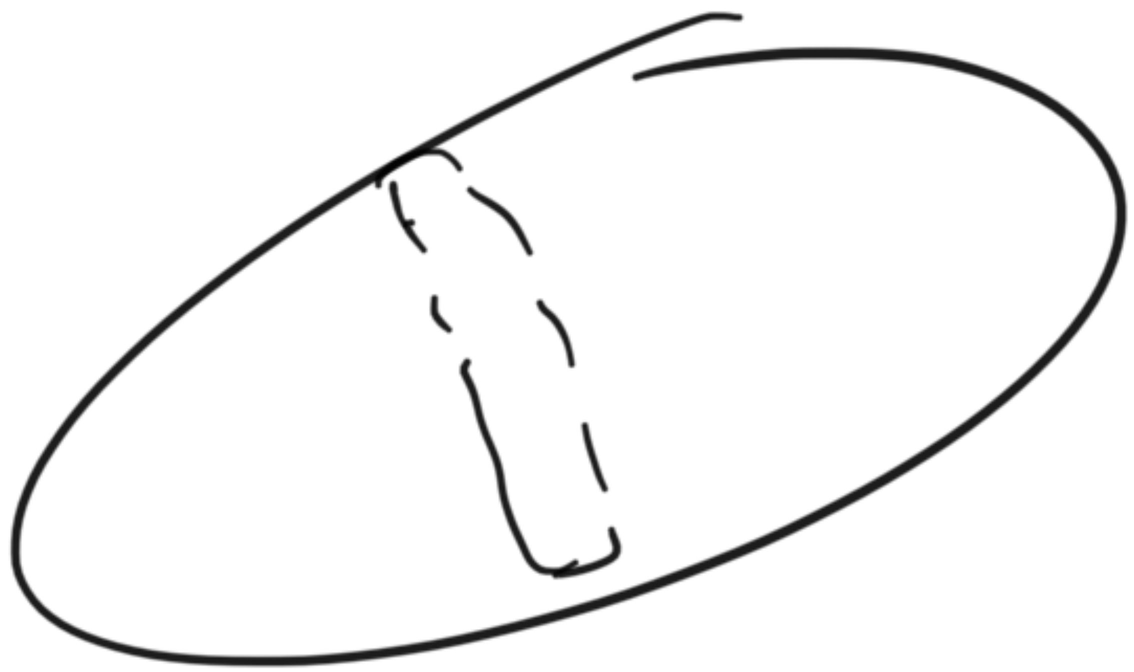
x → independent variable

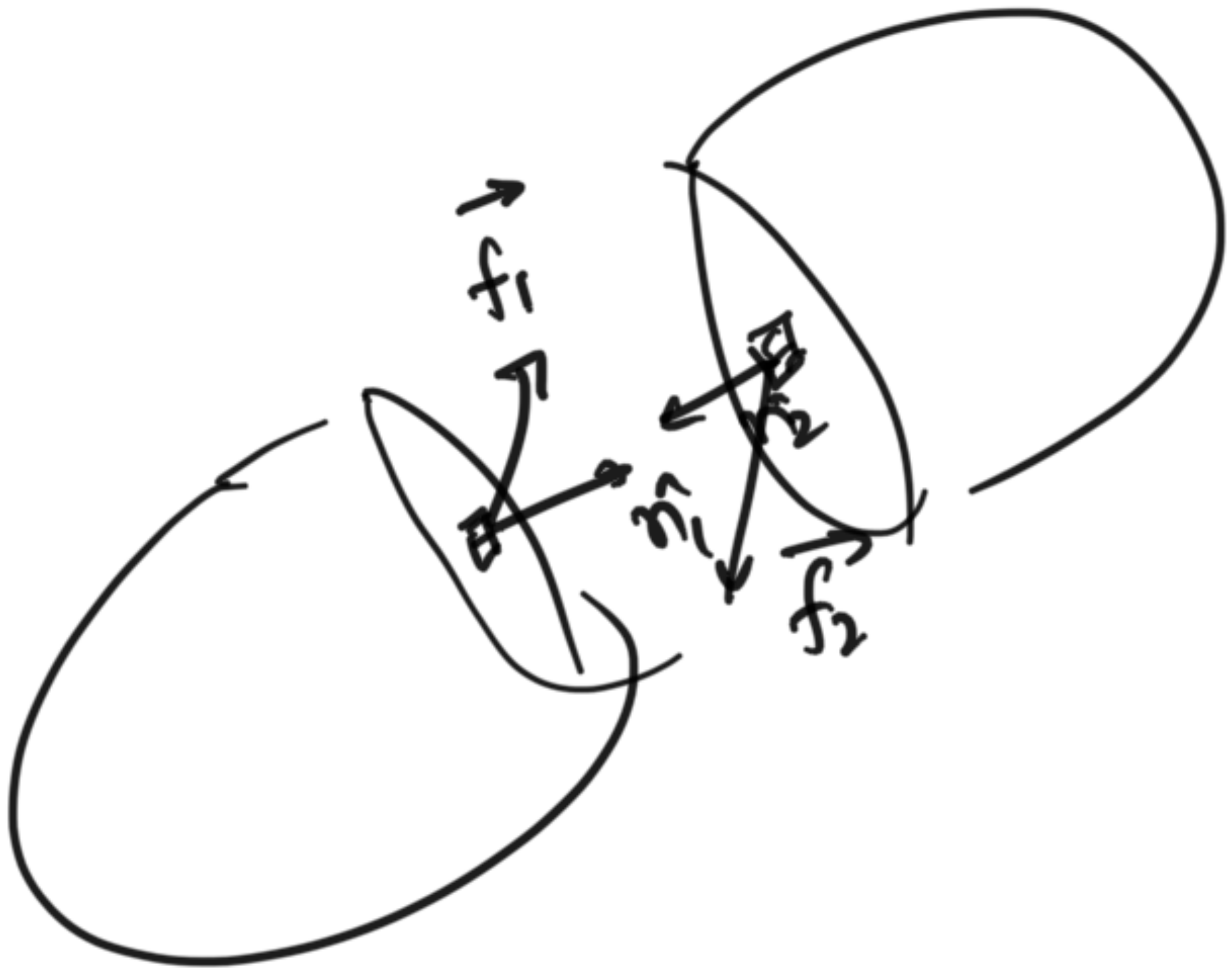
$$y = \alpha x \leftarrow \text{power } 1$$

$\vec{A}$ is a linear function of vector $\vec{B}$

$$A_1 = \lambda_{11} B_1 + \lambda_{12} B_2 + \lambda_{13} B_3$$
$$A_2 = \lambda_{21} B_1 + \lambda_{22} B_2 + \lambda_{23} B_3$$
$$A_3 = \lambda_{31} B_1 + \lambda_{32} B_2 + \lambda_{33} B_3$$

$$A_i = \lambda_{ij} B_j$$





$$\vec{f}(\hat{n}) = -\vec{f}(-\hat{n})$$

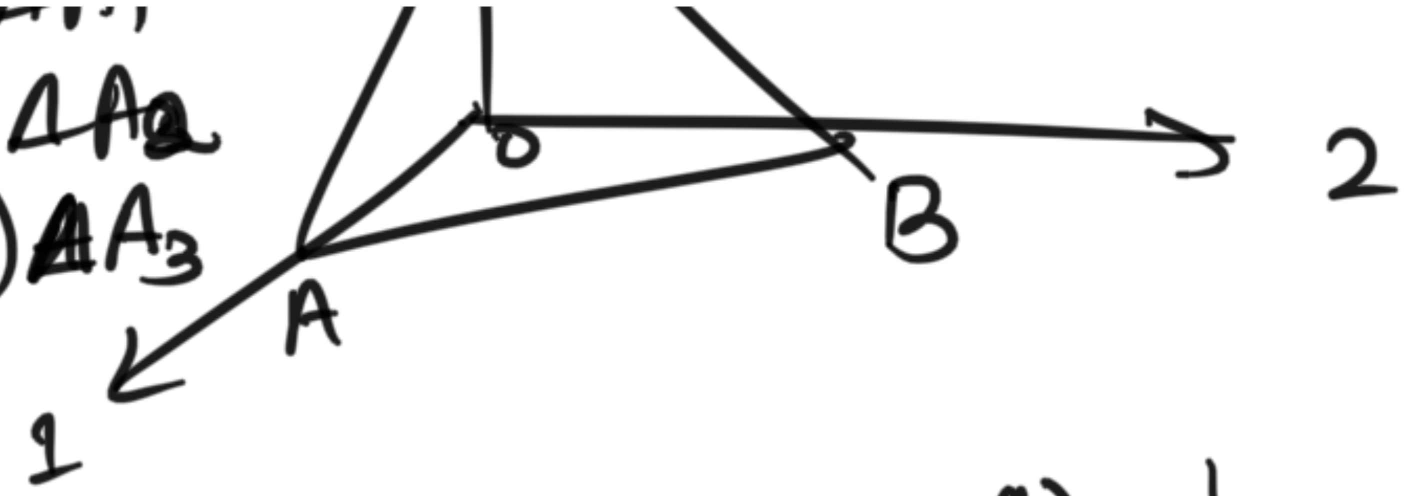
Sum of surface forces

$f_i(\hat{n})\,\Delta A$

$+ f_i(-\hat{e}_1)\,\Delta A_1$

Tetrahedral element OABC.

$$+f_i(-\hat{e}_1)\Delta A_1 + f_i(-\hat{e}_2)\Delta A_2 + f_i(-\hat{e}_3)\Delta A_3$$



Area $BOC = \Delta A_1 (-\hat{e}_1)$

$AOC = \Delta A_2 (-\hat{e}_2)$

$AOB = \Delta A_3 (-\hat{e}_3)$

$ABC = \Delta A (\hat{n})$

$$\Delta A_1 = (\hat{n}\cdot\hat{e}_1)\,\Delta A$$
$$\Delta A_2 = (\hat{n}\cdot\hat{e}_2)\,\Delta A$$
$$\Delta A_3 = (\hat{n}\cdot\hat{e}_3)\,\Delta A$$

On tetrahedral element

Newton's 2^{nd} Law / Euler's 1^{st} axiom

$$\Sigma \vec{F} = m\vec{a}$$

Σ surface force $+ \Sigma$ body force $= \rho \vec{a}\, \delta V$

body forces $\rightarrow \delta^3$

$\rho \vec{a}\, dV \rightarrow \delta^3$

surface force $\rightarrow \delta^2$

In limit $\delta \rightarrow 0$

Σ Surface force $= 0$

$$f_i(\hat{n})\Delta A = \left\{ f_i(\hat{e}_1) n_1 \Delta A + f_i(\hat{e}_2) n_2 \Delta A + f_i(\hat{e}_3) n_3 \Delta A \right\}$$

f_i is a linear function of n

$$f_i = \underbrace{\sigma_{ij}}\, n_j$$

$\sigma_{ij} \rightarrow$ stress tensor

Solid mechanics $\cancel{\sigma_{ii}}$ $\sigma_{\alpha\alpha}$ (no summation in α)

(normal stress)

$\sigma_{\alpha\beta}$ where $\beta \perp \alpha$

(shear stress)

$\sigma_{\alpha\alpha} \longrightarrow$ denoted by σ_α $\longleftarrow$ normal stress

$\sigma_{\alpha\beta}$ $(\beta \perp \alpha)$ $\longrightarrow$ $\tau_{\alpha\beta}$ (shear stress)

~~Infitesmal~~

$$f_i = \sigma_{ij} n_j$$

$\sigma_{ij} \rightarrow 2^{nd}$ order tensor

$$\boxed{\sigma_{ij}' = a_{li}\, a_{mj}\, \sigma_{lm}}$$

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same for solids/ fluids

In the material choose small volume $d\forall$

Σ Forces on volume $= m\vec{a}$

$$\iiint_{\forall} \rho a_i \, d\forall = \text{body force} + \text{surface force}$$

$$= \iiint_{\forall} \rho g_i \, d\forall + \oiint_{A} f_i \, dA$$

$g_i \downarrow$ body force per unit mass

$$\oiint_{A} \sigma_{ij} n_j \, dA$$

$$\iiint_{\forall} \rho a_i \, d\forall = \iiint_{\forall} \rho g_i \, d\forall + \oiint_{A} \sigma_{ij} n_j \, dA$$

$$\iiint_{\forall} \rho a_i \, d\forall = \iiint_{\forall} \rho g_i \, d\forall + \iiint_{\forall} \frac{\partial}{\partial x_j}(\sigma_{ij}) \, d\forall$$

$$\iiint \{ \rho a_i - \rho g_i - \frac{\partial}{\partial x_j}(\sigma_{ij}) \} \, d\forall = 0$$

$$\iiint_V \left\{ \rho a_i - \rho g_i - \frac{\partial}{\partial x_j}(\sigma_{ij}) \right\} dV = 0$$

Since V is arbitrary, $\{ \quad \} = 0$

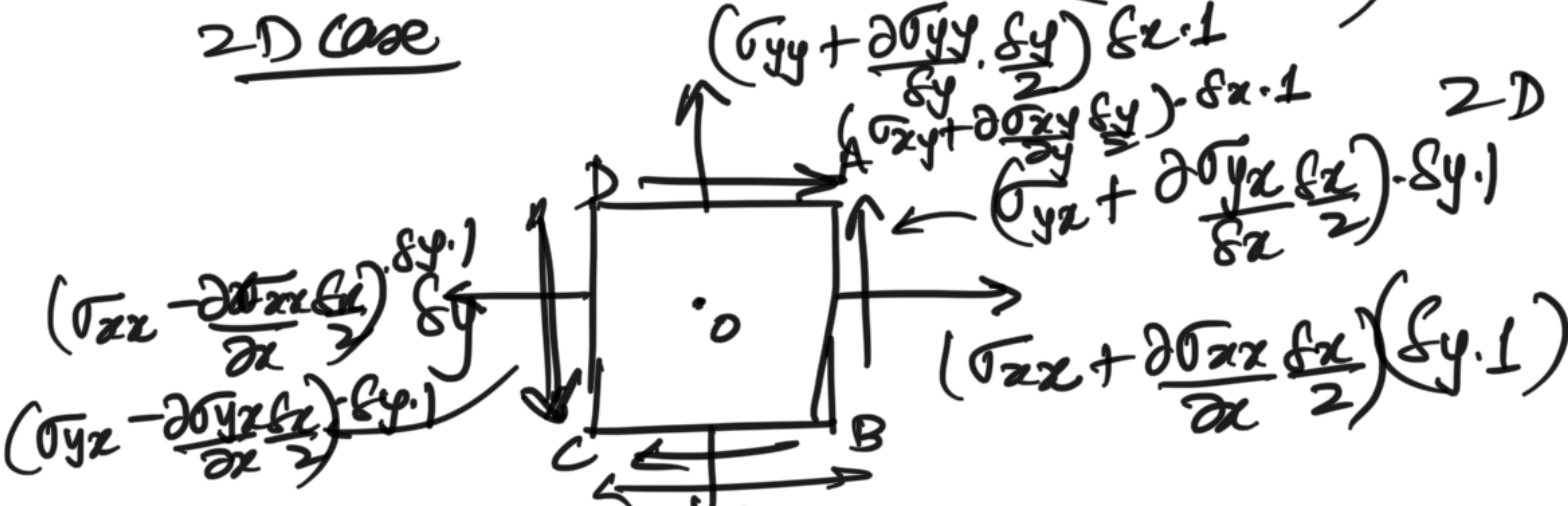
integrand

$$\Rightarrow \boxed{\rho a_i = \rho g_i + \frac{\partial}{\partial x_j}(\sigma_{ij})} \quad \text{Cauchy's equation}$$

Euler's 2nd axiom or $(T = I\alpha)$

<u>2D case</u>

2D element $(x\text{-}y)$ notation



All faces

Stress at O ($\sigma_{xx}, \sigma_{xy}, \sigma_{yx}, \sigma_{yy}$)

Stress on AB:

$$\hat{i}: \left\{ \sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} \frac{\delta x}{2} \right\}$$

Taylor series

$$\hat{j}: \left\{ \sigma_{yx} + \frac{\partial \sigma_{yx}}{\partial x} \frac{\delta x}{2} \right\}$$

$$\Sigma M_o = \bar{I} \alpha$$

$$\Rightarrow \quad (\sigma_{yx} - \sigma_{xy}) = 0$$

$$\boxed{\sigma_{xy} = \sigma_{yx}}$$

COMPLIMENTARITY OF SHEAR STRESS

$$\boxed{\sigma_{ij} = \sigma_{ji}}$$

3D version