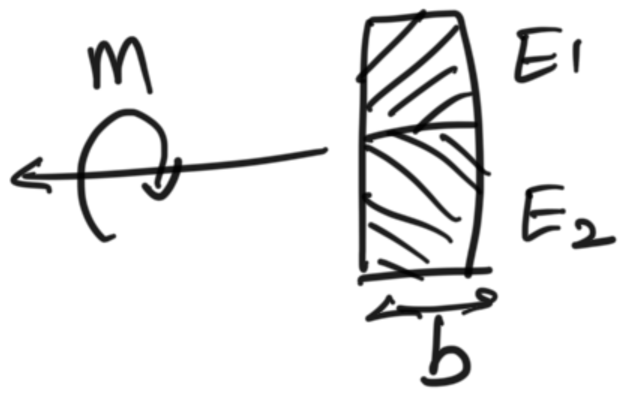
Last class: composite beam in bending



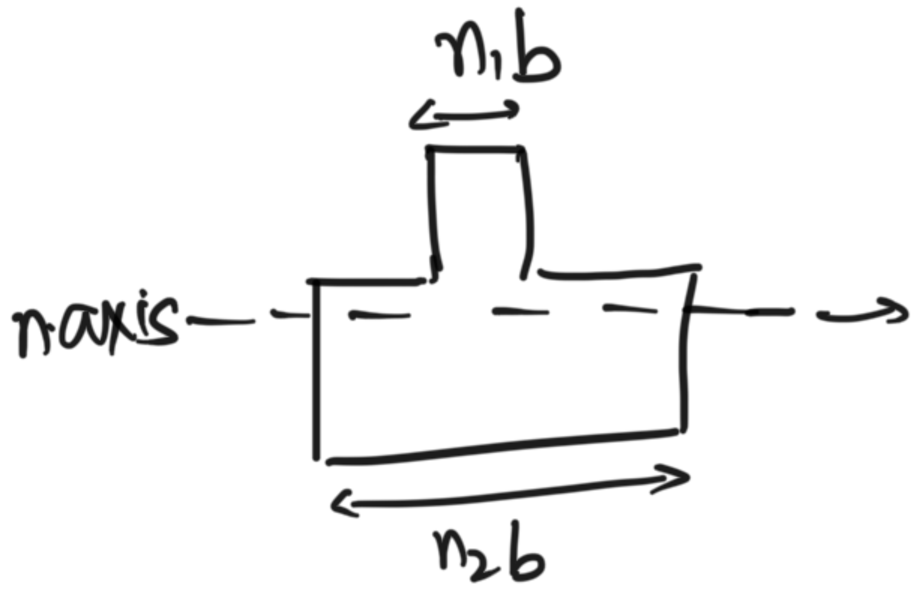
~~E1~~

choose one of the materials as reference (either 1 or 2)

If choose 1, ~~E2~~

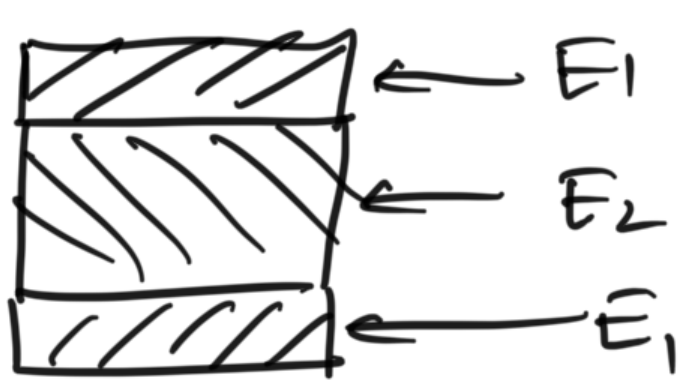
$$n_1 = \frac{E_1}{E_1} = 1$$

$$n_2 = \frac{E_2}{E_1}$$

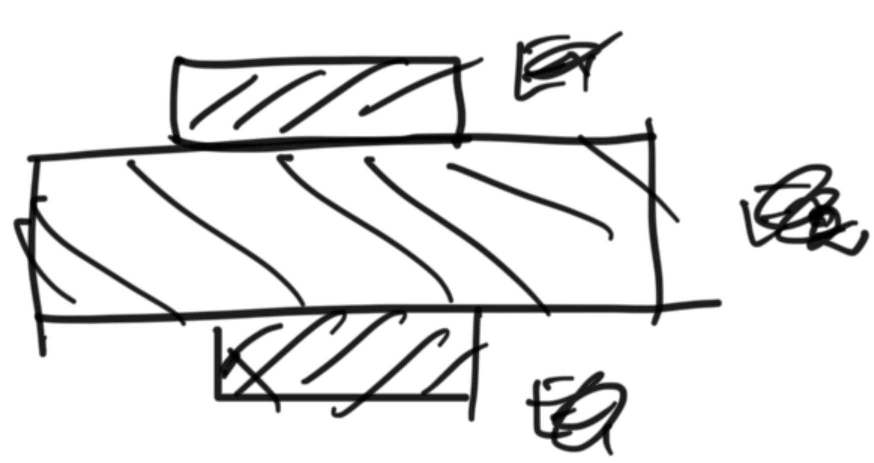


Find neutral axis for transformed section (centroid of transformed area)

$$\sigma = \frac{n_i M y}{I}$$ ← w.r.t. transformed section



← In such a case because of configuration transformed section neutral axis will not change



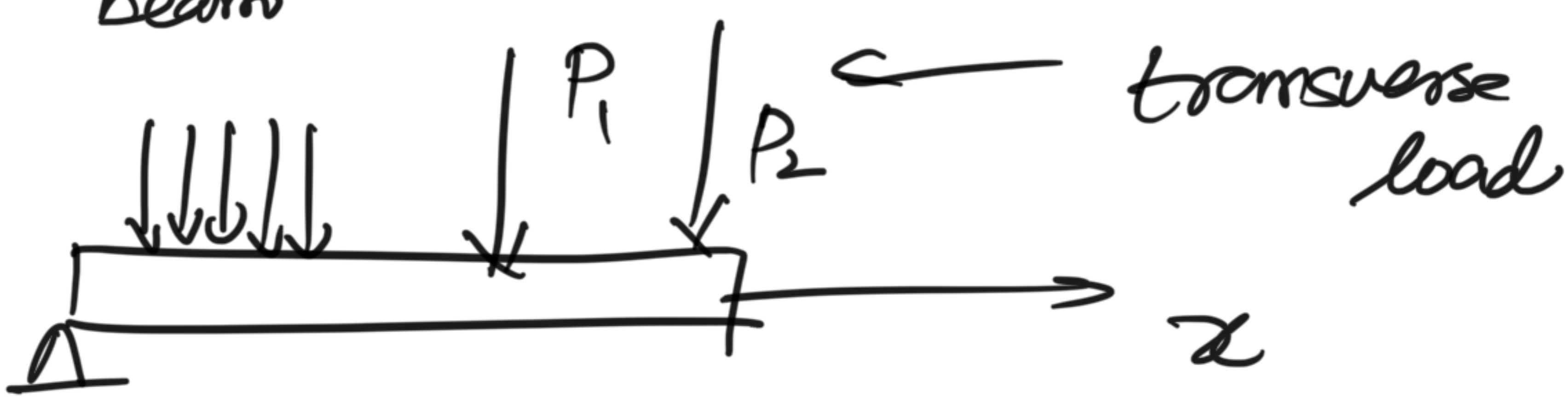
Chapter 5

Design of beams for bending

- Shear Force and Bending Moment Diagrams
- Find relation between applied load and shear force and bending moment at each section of

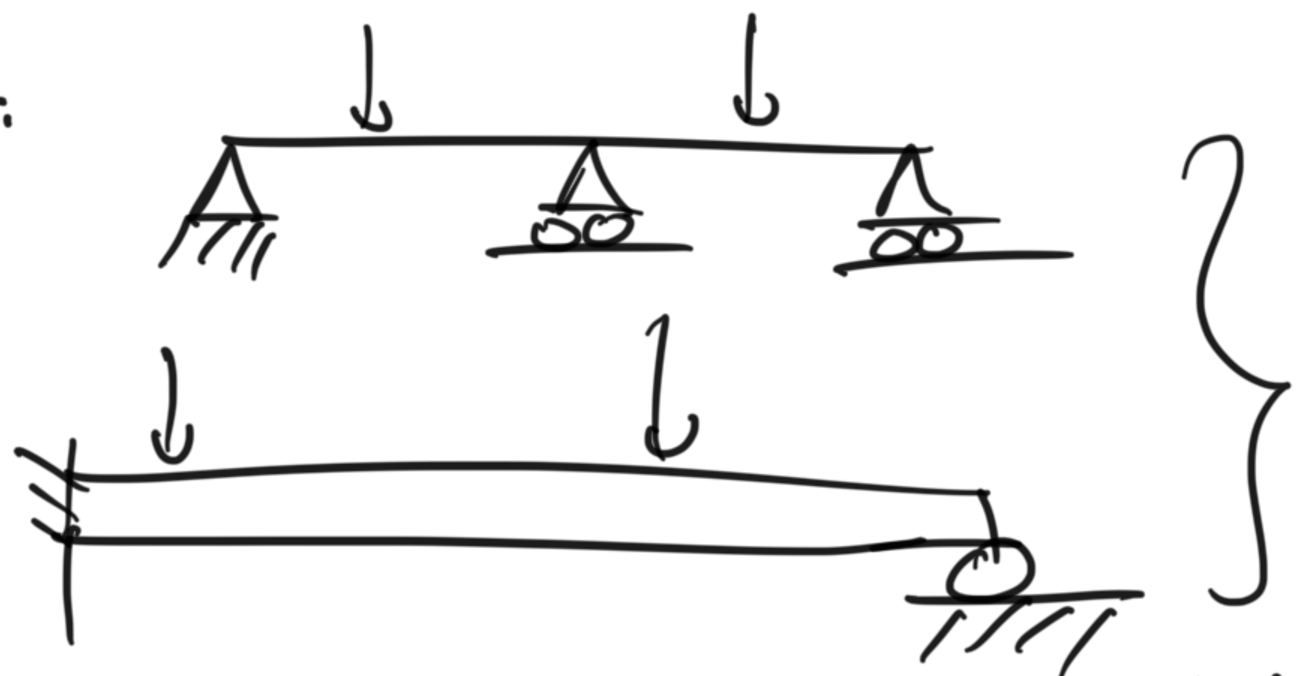
force and bending moment at each section of the beam

— Use section modulus $\left(\frac{I}{c}\right)$ to design the beam



Statically determinate beam (covered in last class)

Beam like:



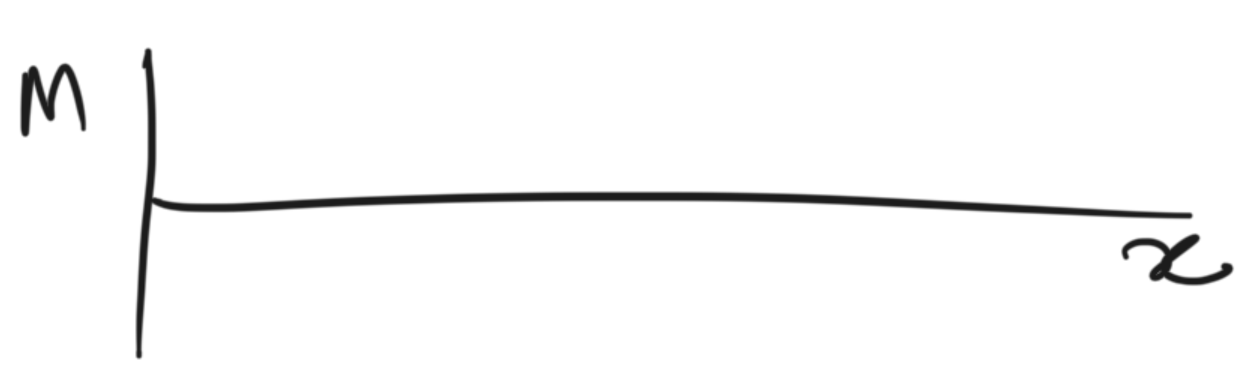
or

statically indeterminate

External transverse load causes shear force and Bending Moment at any section.

Plot shear force as a fn. of x (distance along axis of beam)
Bending moment as a fn. of x

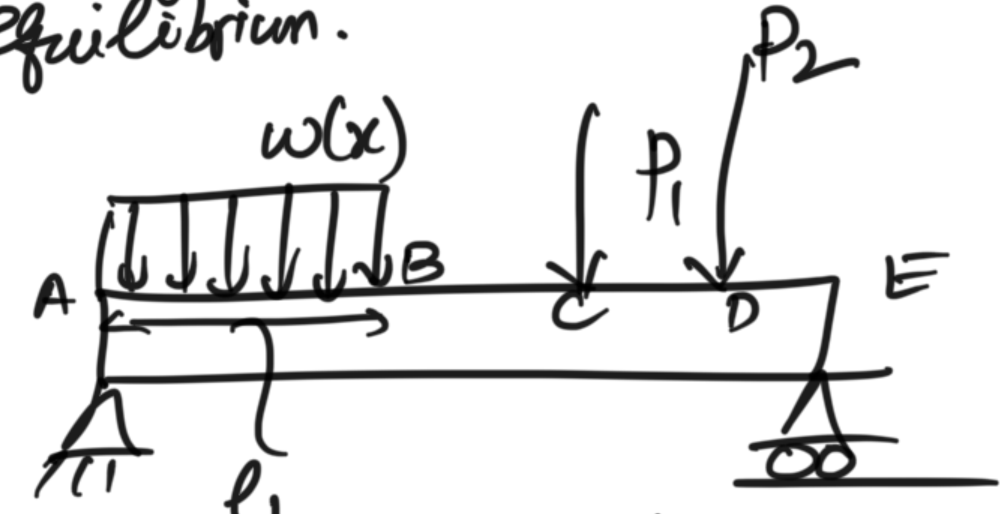
SFD (Shear force diagram)



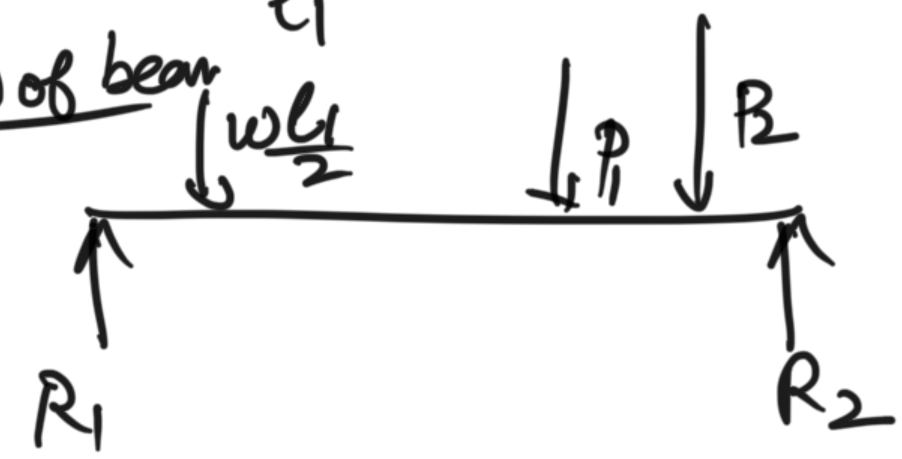
(Bending Moment Diagram)

To ~~f~~ draw these diagrams

a) Finding supports reactions (Force/Moment) by equilibrium.



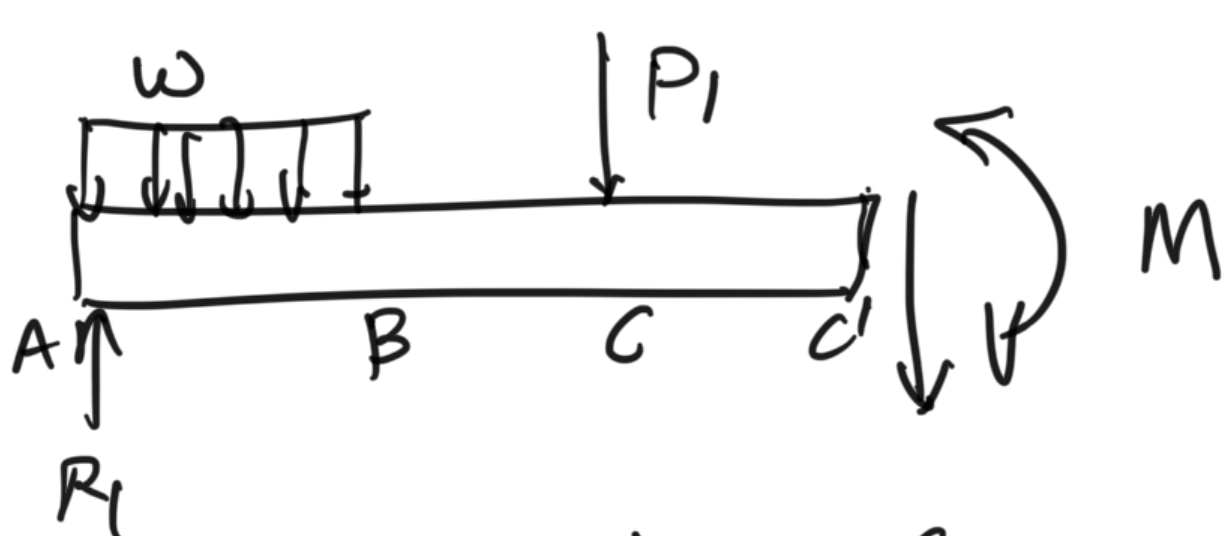
FBD of beam



distributed load replaced by Area under load acting at centroid of the area

$$\left.\begin{array}{l} \Sigma F_y = 0 \\ \Sigma M_2 = 0 \end{array}\right\} \longrightarrow R_1 \,\&\, R_2$$

cut the beam at different x sections.



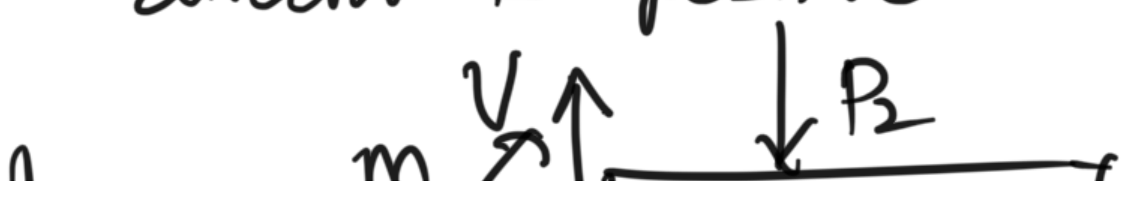
cut x between C & D

$V \rightarrow$ shear force

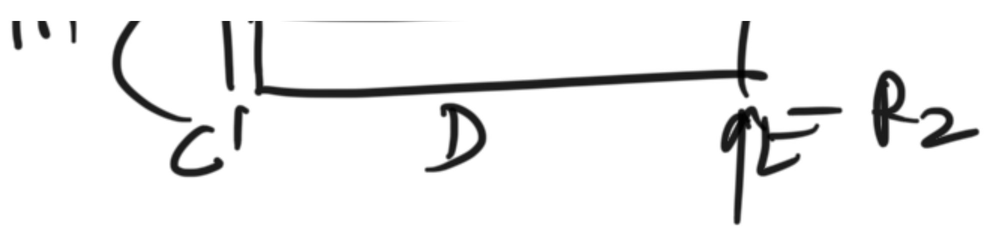
$M \rightarrow$ bending moment

sign convention: on a + face, shear force pointing in −ve y direction $\rightarrow$ is +ve

on a + face, Bending Moment in CCW direction is positive.



(V & M +ve)



M causes normal stress
V cause shear stress

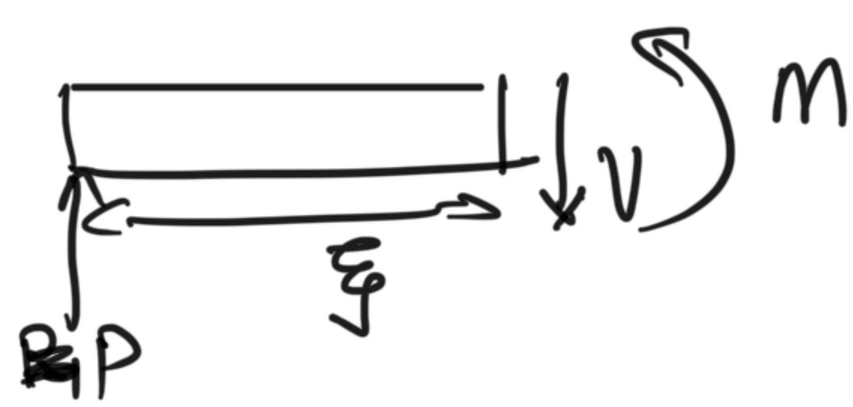
Normally design based on maximum normal stress

$$\sigma_m = \frac{|M|\, c}{I} = \frac{|M|}{S}$$ c ← maximum value of distance from neutral axis.

$$\sigma_x = -\frac{My}{I}$$ ← I is moment of inertia wrt centroidal axis ⊥ plane.
$y \rightarrow$ distance from neutral axis

Since $\sigma \propto |M|$, maximum σ occurs where M is ~~maxim~~ maximum. So we plot the BMD.

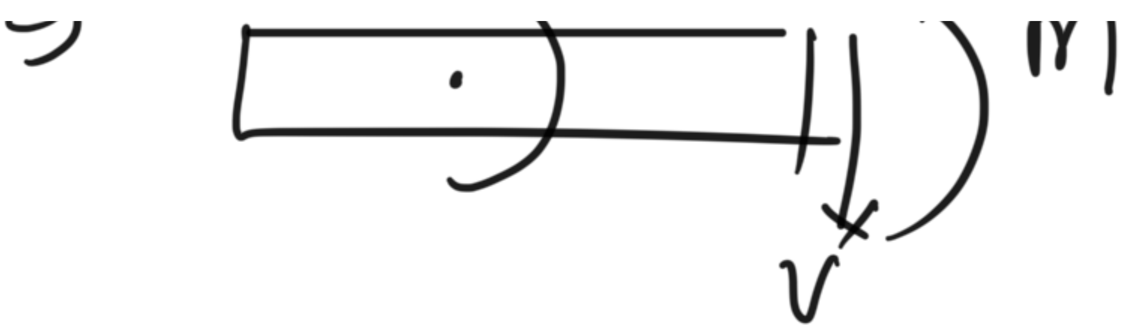
<u>Rules:</u> if start from left



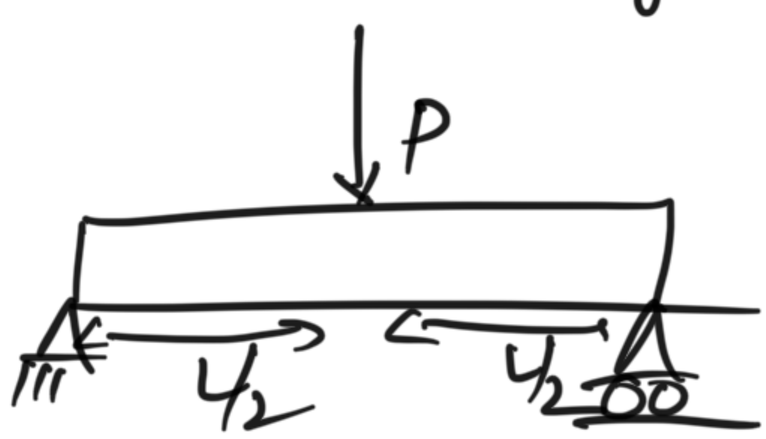
a) an upward force of P induces a V equal to P on all sections to its right. (V +ve if downwards)

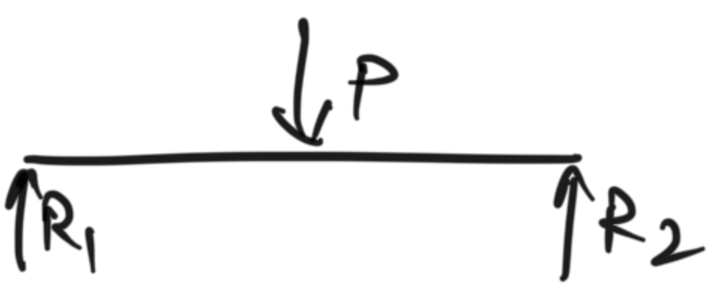
b) An upward force of P induces a bending moment $= P\xi$ on all sections to its right
where ξ = distance between the cut section & line of action of P.

c)



A concentrated moment M_1 (CCW) induce a bending moment $= -M_1$ on all sections to its right.
No explicit change in value of V.



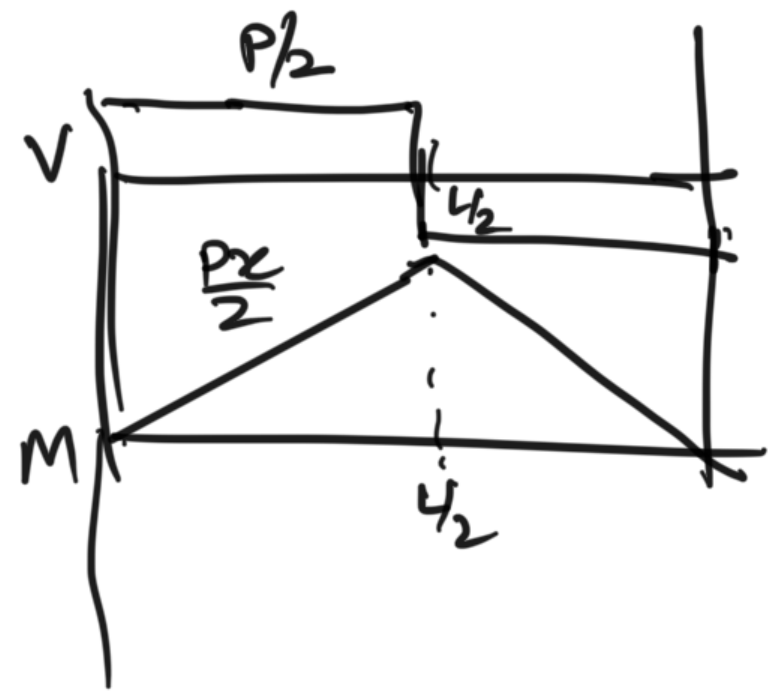


$R_1 = R_2 = \frac{P}{2}$

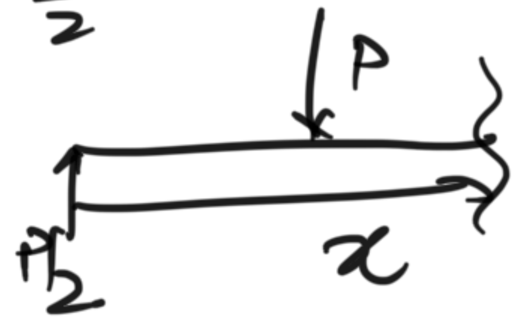
$0 < x < \frac{L}{2}$

$V = R_1 = \frac{P}{2}$

$M = \frac{P}{2}x$

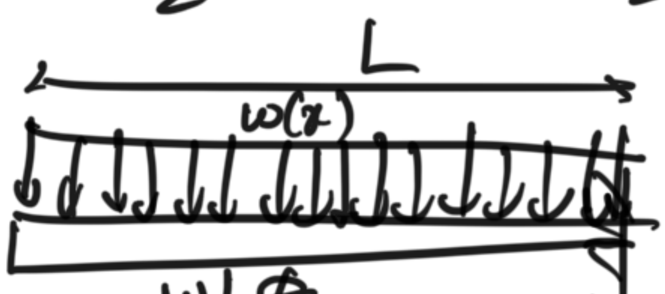


$\frac{L}{2} < x < L$



$V = \frac{P}{2} - P = -P/2$

$M = \frac{P}{2}x - P\left(x - \frac{L}{2}\right) = \frac{P}{2}(L - x)$



$w(x) =$ UDL
uniformly distributed load

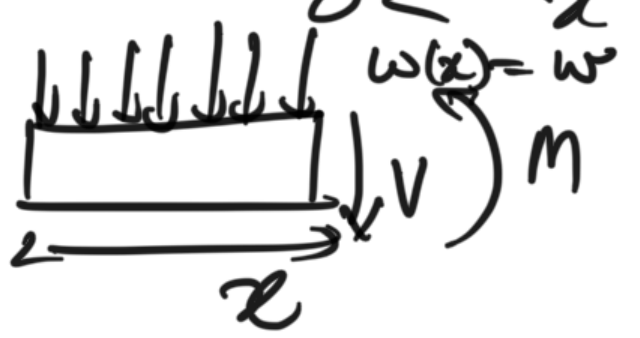
$R = WL$

$R_1 = wL$

$$M = -\frac{wL^2}{2}$$

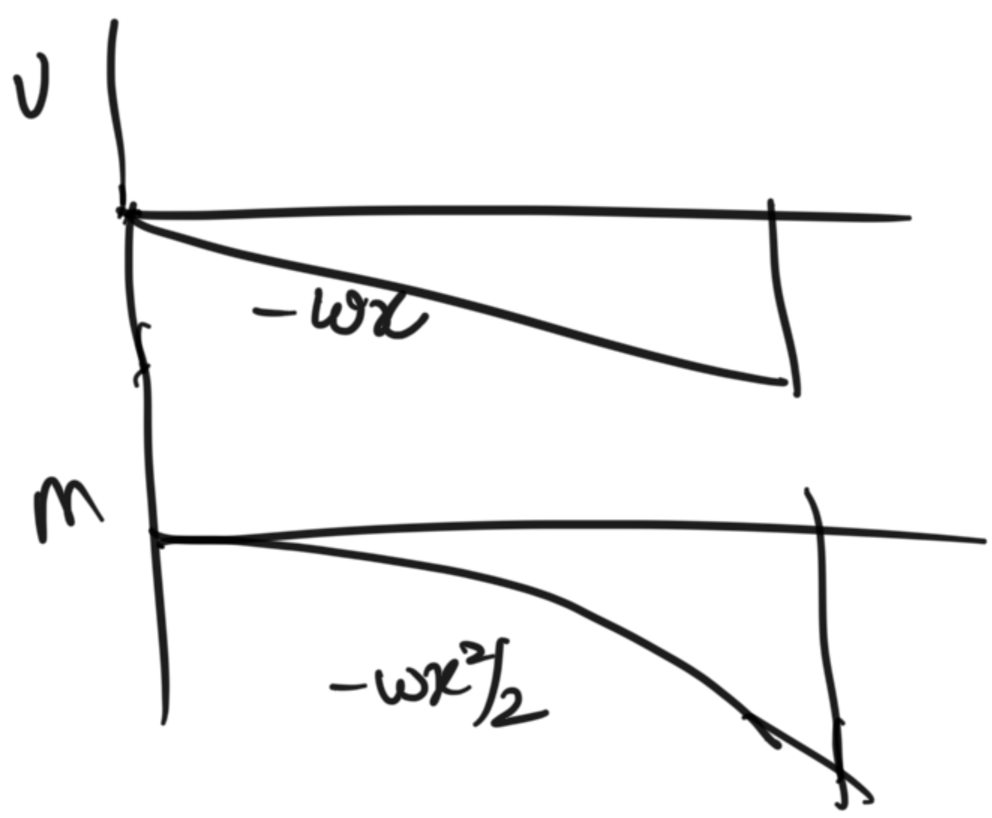
SFD & BMD

$0 < x < L$



$$V = -wx$$

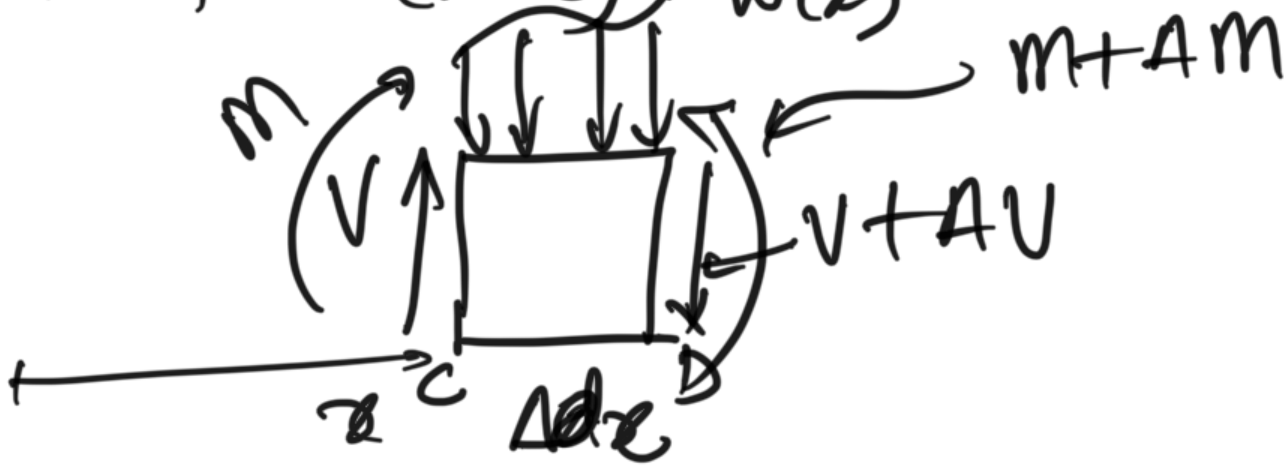
$$M = -wx\left(\frac{x}{2}\right)$$



Relation between distributed load $w(x)$, shear (V) and Bending Moment M.

$w(x)$ = distributed load

thin axial length dx of beam at a section x from $(x=0)$



(V & M on left & right faces = V and V+dV
M and M+dM

~~FBD~~ eqns. of eqbm.

$$V - (V + \Delta V) - w(\Delta x) = 0$$

$$\Rightarrow \quad \Delta V = -w\,\Delta x \Rightarrow \frac{\Delta V}{\Delta x} = -w$$

$Lt\ \Delta x \to 0$

$$\frac{dV}{dx} = -w$$

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx$$

area under the loading curve

(no concentrated load between C & D)

Take moments about D:

$$(M + \Delta M) - M + w(\Delta x)(\beta \Delta x) - V\Delta x = 0$$

~~$Lt\ \Delta x \to 0$~~

$$\Delta M + w\beta\frac{(\Delta x)^2}{2} - V(\Delta x) = 0$$

$Lt\ \Delta x \to 0$ & $(\Delta x)^2 \to$ terms ignored

$$\frac{dM}{dx} = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V\,dx$$

(area under SFD gives moment)

$$V = 0 \Rightarrow \frac{dM}{dx} = 0$$ (local maxima or minima force)

M

Find maximum $|M|$ from BMD

$$\sigma_m = \frac{|M_{max}|}{S}, \quad \text{where } S = \frac{I}{c}$$

(eg $S = \frac{bh^2}{6}$ for rectangular beam)

$$\sigma_{max} \leq \sigma_{allowable}$$

To design a beam

i) determine σ_{all} for material selected from properties of material. ($\frac{\sigma_u}{\text{factor of safety}}$ may be used)

Assume σ_{all} is same in T and C

ii) Draw SFD & BMD corresponding to the loading Condition. Use this to find $|M|_{max}$.

iii) determine value of $S_{min} = \frac{|M|_{max}}{\sigma_{all}}$

For eg rectangular beam of timber of depth h and width b, or ratio $\frac{h}{b}$ ~~sets~~ specified.

$$S = \frac{bh^2}{6} > S_{min}.$$

For example problem See

Video on Engineering Mechanics youtube

site of lectures of APL100 at

Sanghi Labs ? ITD

1. youtube.com/watch?v=ty6R1POijg

2. youtube.com/watch?v=5N8SkMh6ld8