Problem Sheet 1

Q1. The $\bar{E}$ coordinate system is obtained from the E system by a counter-clockwise rotation of 90° about the x_3 axis. The E^* coordinate system is obtained by a reflection of the x_3 axis about the x_1-x_2 plane.

a) Sketch the E, $\bar{E}$ and the E^* coordinate systems.

b) Write $\bar{e}_i$ and e_i^* in terms of e_i.

c) Determine the direction cosines, $\bar{a_{ij}}$ ($\equiv e_i.\bar{e}_j$) and a_{ij}^* ($\equiv e_i.e_j^*$).

d) Verfiy that $\bar{a_{ik}}\ \bar{a_{jk}} = \delta_{ij}$ and $a_{ik}{}^*\ a_{jk}{}^* = \delta_{ij}$

e) Determine the determinants of $\bar{a_{ij}}$ and a_{ij}^*.

Q2. With ϕ being a scalar, $\mathbf{u}$ a vector, and $\omega \equiv \nabla \times \mathbf{u}$, show using the cartesian tensor notation (no other method accepted).

a) $\nabla.\omega = 0$.

b) $\nabla \times (\nabla\phi) = 0$.

c) $\nabla \times (\nabla \times \mathbf{u}) = \nabla(\nabla.\mathbf{u})$ - $\nabla^2\mathbf{u}$.

d) $\mathbf{u} \times \omega = 1/2\ \nabla(\mathbf{u}.\mathbf{u})$ - $(\mathbf{u}.\nabla)\mathbf{u}$.

Q3. Let $r = (x_k x_k)^{1/2}$ denote the distance of a point x from the origin. Using the cartesian tensor notation find:

a) $\nabla r = \partial r/\partial x_i$.

b) $\nabla(1/r) = \partial(1/r)/\partial x_i$.

c) $\nabla^2(1/r) = \partial^2(1/r)/\partial x_i \partial x_i$.

Q4. Show that δ_{ij} is an isotropic tensor. What is its order?