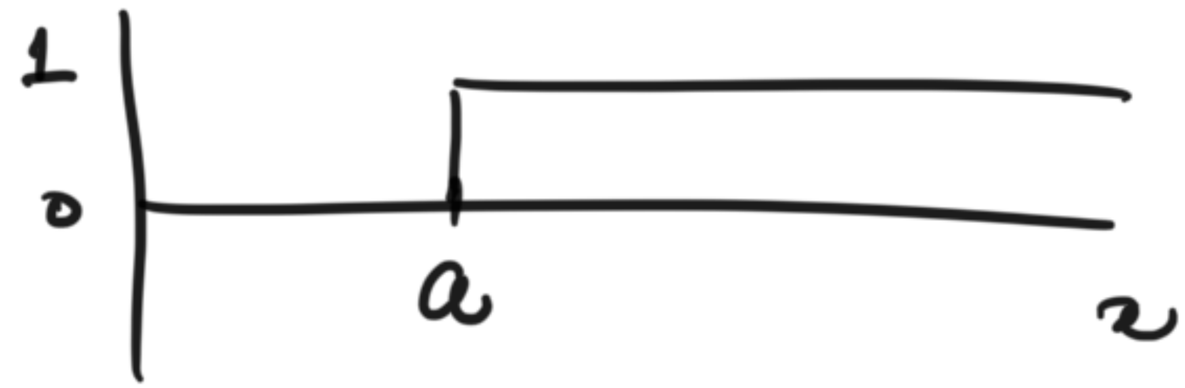
APL 105 Lecture 12 12-2-2024

Singularity functions



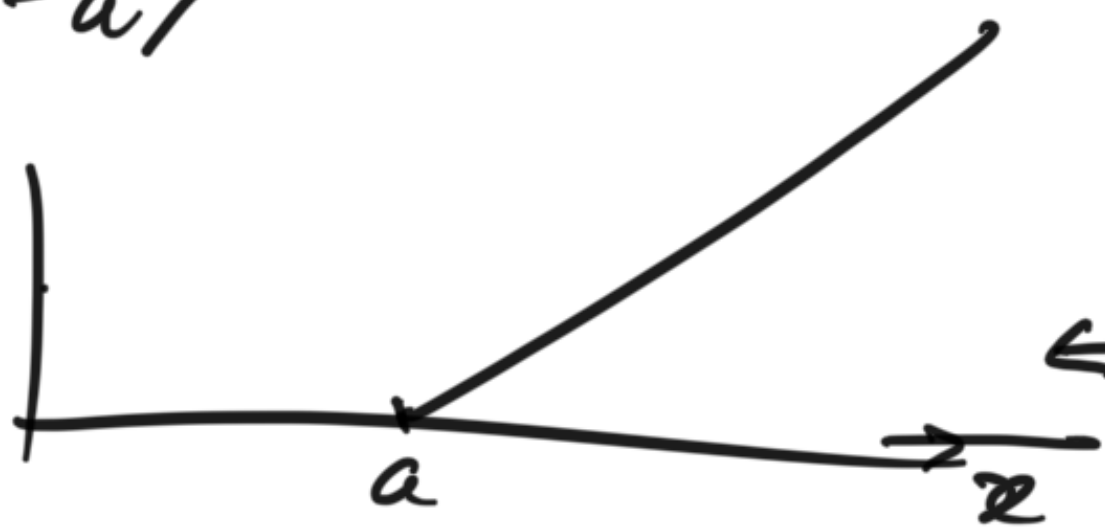
unit step function represent by $\langle x-a \rangle^0$

$f\langle \xi \rangle$ imply that if $\xi < 0$ we ignore or $f=0$
$\xi \geq 0$ then the value is ξ.

$\langle x-a \rangle^n$

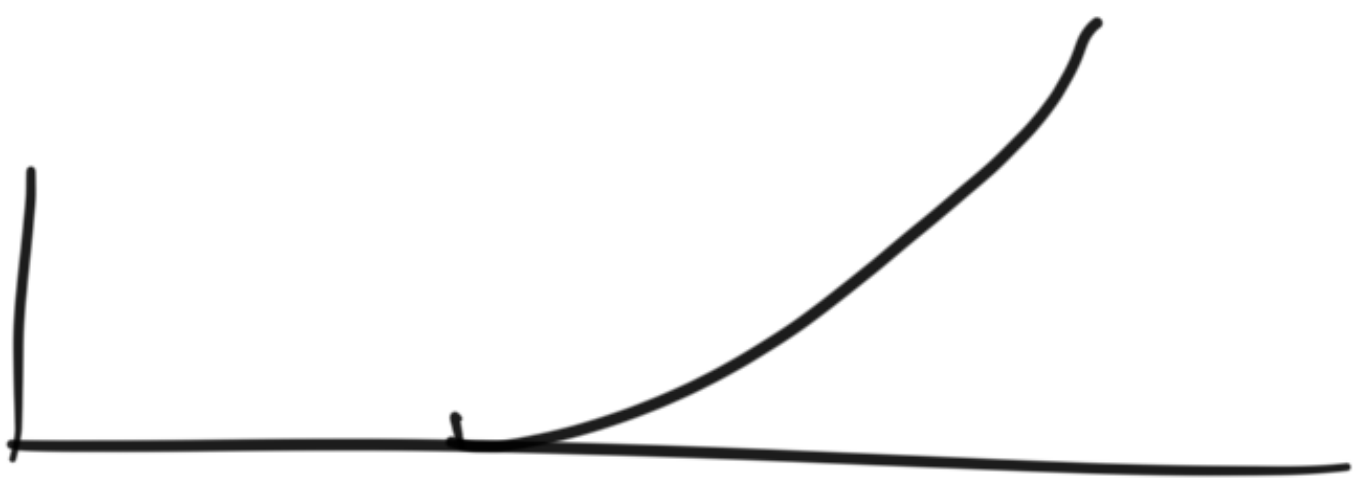
$(x-a)$ if $x \geq a$
0 if $x < 0$

n=1



$\longleftarrow \langle x-a \rangle^1$

n=2



$\langle x-a \rangle^2$

a

$$\langle x-a\rangle^{n} = (x-a)^{n} \quad \text{if } x \geq a$$

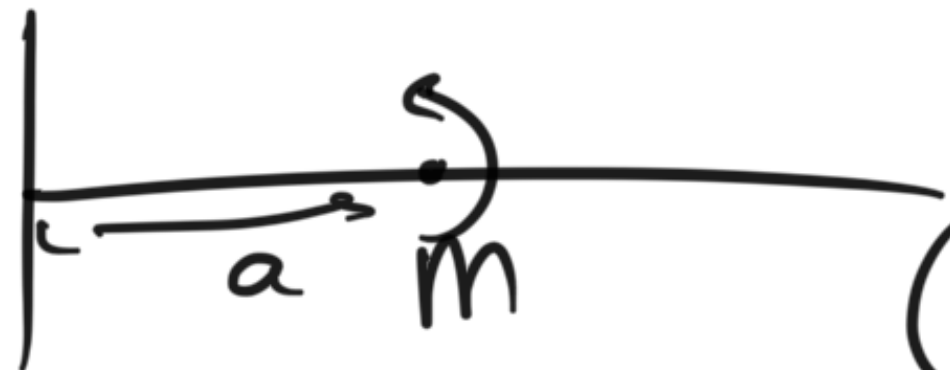
$$= 0 \quad \text{if } x < a$$

$$\int_{-\infty}^{x} \langle x-a\rangle^{n} dx = \frac{\langle x-a\rangle^{n+1}}{n+1}, \quad n \geq 0$$

$\langle x-a\rangle_{-1} =$  impulse function

(unit force) ← loading

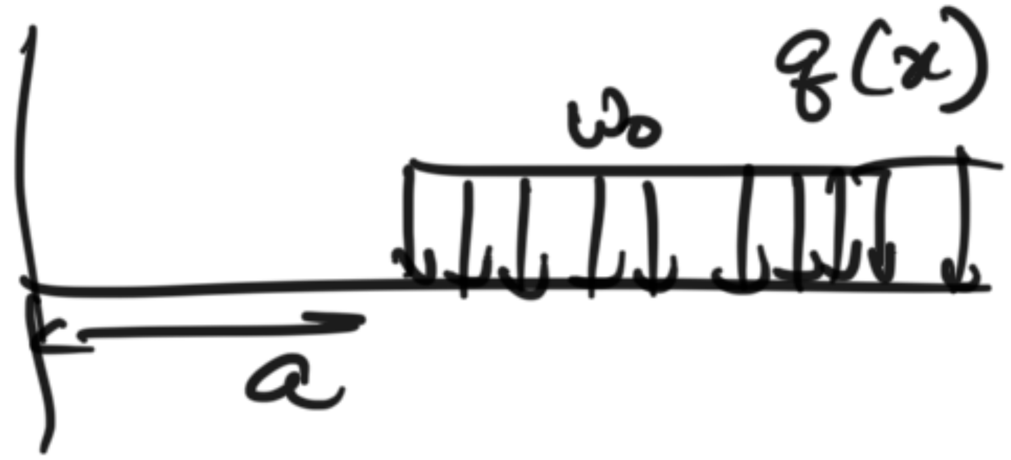
force P represented by $P\langle x-a\rangle_{-1}$ in loading $w(x)$

$\langle x-a\rangle_{-2} =$  (unit moment)

moment M represented by $M\langle x-a\rangle_{-2}$ in loading $w(x)$

$w(x)$

e.g.

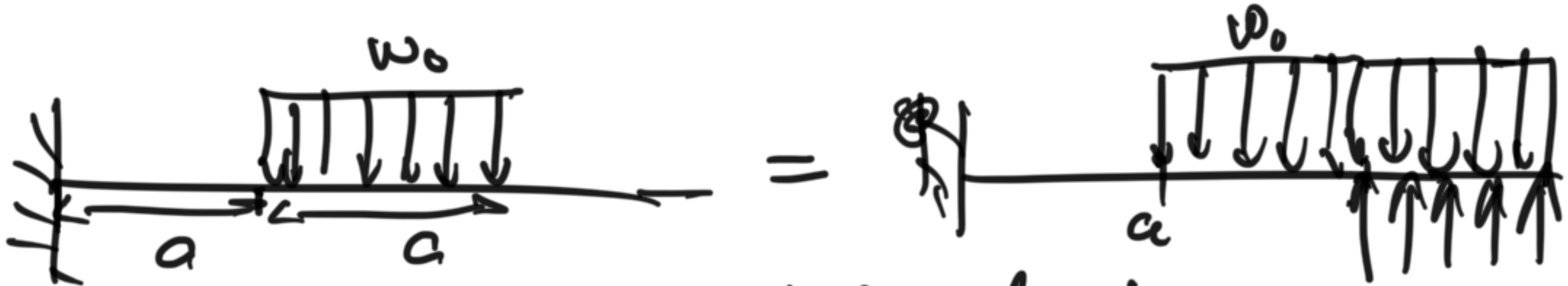


due to $q(x)$

$$w(x) = -q(x)\langle x-a\rangle^0$$

$$w(x) = -w_0\langle x-a\rangle^0$$

$$\frac{dV}{dx} = -w \quad , \quad \frac{dM}{dx} = V$$



$w(x)$ due to w_0

$$= -w_0\langle x-a\rangle^0 + w_0\langle x-2a\rangle^0$$

Use $\langle \quad \rangle$ functions to write expression for $w(x)$, integrate twice to get $M(x)$.

Chapter 9

Deflection of beams

1) Design specifications allows for a maximum value of deflection

2) Indeterminate beams can be solved for using theory of deflections

3) Method of superposition can be used to conviniently solve problems

When a beam undergoes a transverse load

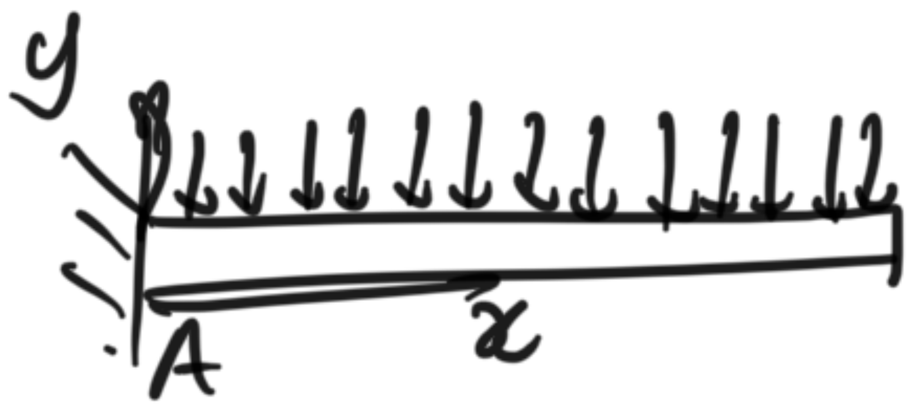
$$\frac{1}{\rho} = \frac{M}{EI}$$

If $M = M(x)$, ρ is also a function of x.

$\rho \rightarrow$ curvature will help us to determine deflection of beam.

~~$\mathcal{J} \equiv \frac{d^2y}{dx^2}$~~ $\mathcal{J}$ related to $\frac{d^2y}{dx^2}$, y = position of beam

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

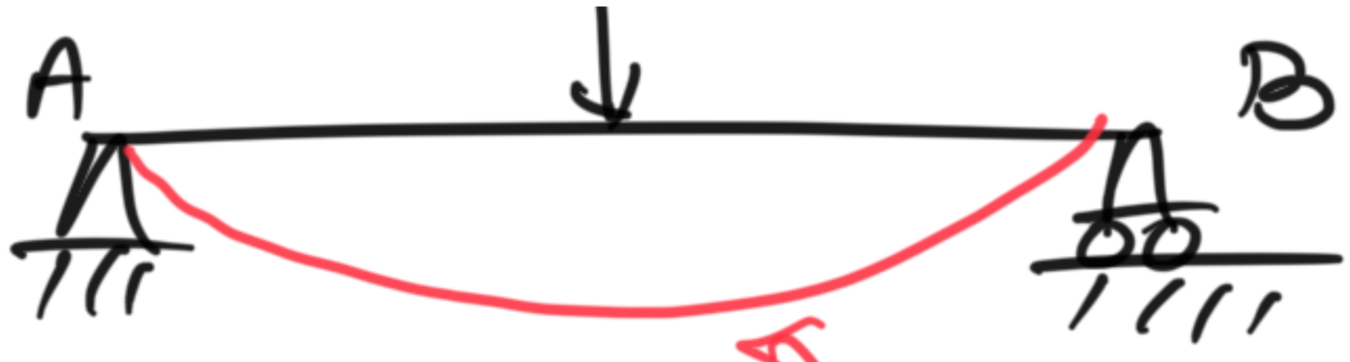


beam shape before load is applied

expected shape of beam after loading

$y_A = 0$

$\frac{dy_A}{dx}$ = slope of deflection $\equiv \theta_A = 0$ (for fixed end - cantilever)



shape after deflection

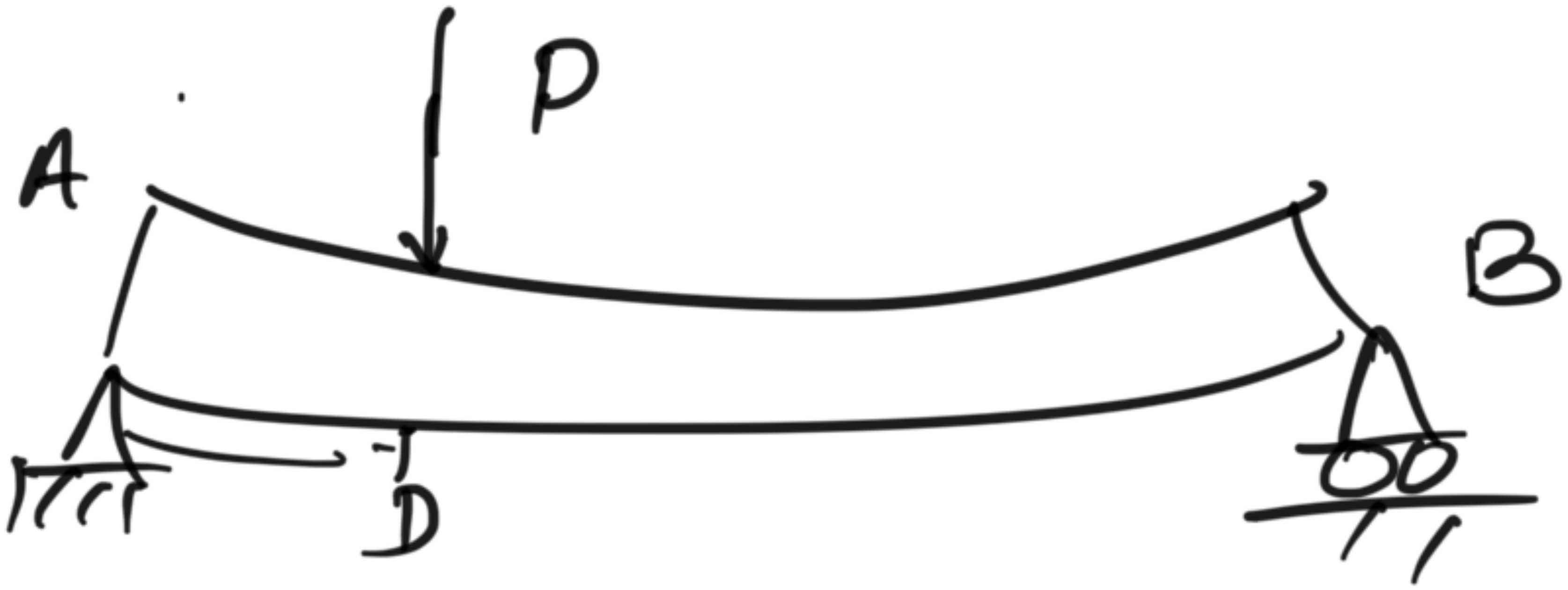
$y_A = y_B = 0$

(even after deflection)

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

2nd order ODE

integrate twice.



Y obtained after successive integration.

deflected beam due to load P

$y_1(x)$ be deflection from A to D. ← based on $M(x)$ from A to D

$y_2(x)$ be deflection from D to B ← based on

$M(x)$ from DB B.

integrate 2nd order ODE for ~~θ <~~ refer AD $\rightarrow$ 2 constants

refer DB $\rightarrow$ 2 constants.

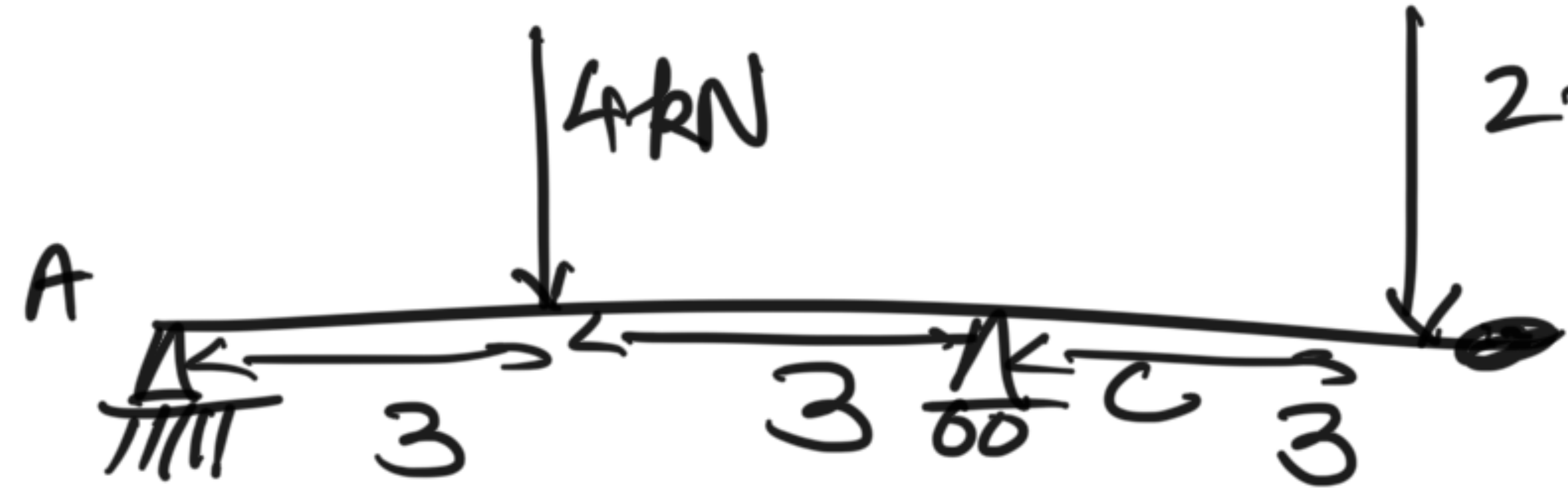
$y_A = 0 \qquad y_B = 0$

At D, deflection & slope are equal

$$y_1(D) = y_2(D)$$

$$\theta_1(D) = \theta_2(D) \qquad \text{where } \theta = \frac{dy}{dx}$$

Eg.



FBD

statically determinate

A_y C_y

$$\left(\begin{array}{l} \sum F_y = 0 \\ \sum M_{A \text{ or } C} = 0 \end{array} \right)$$

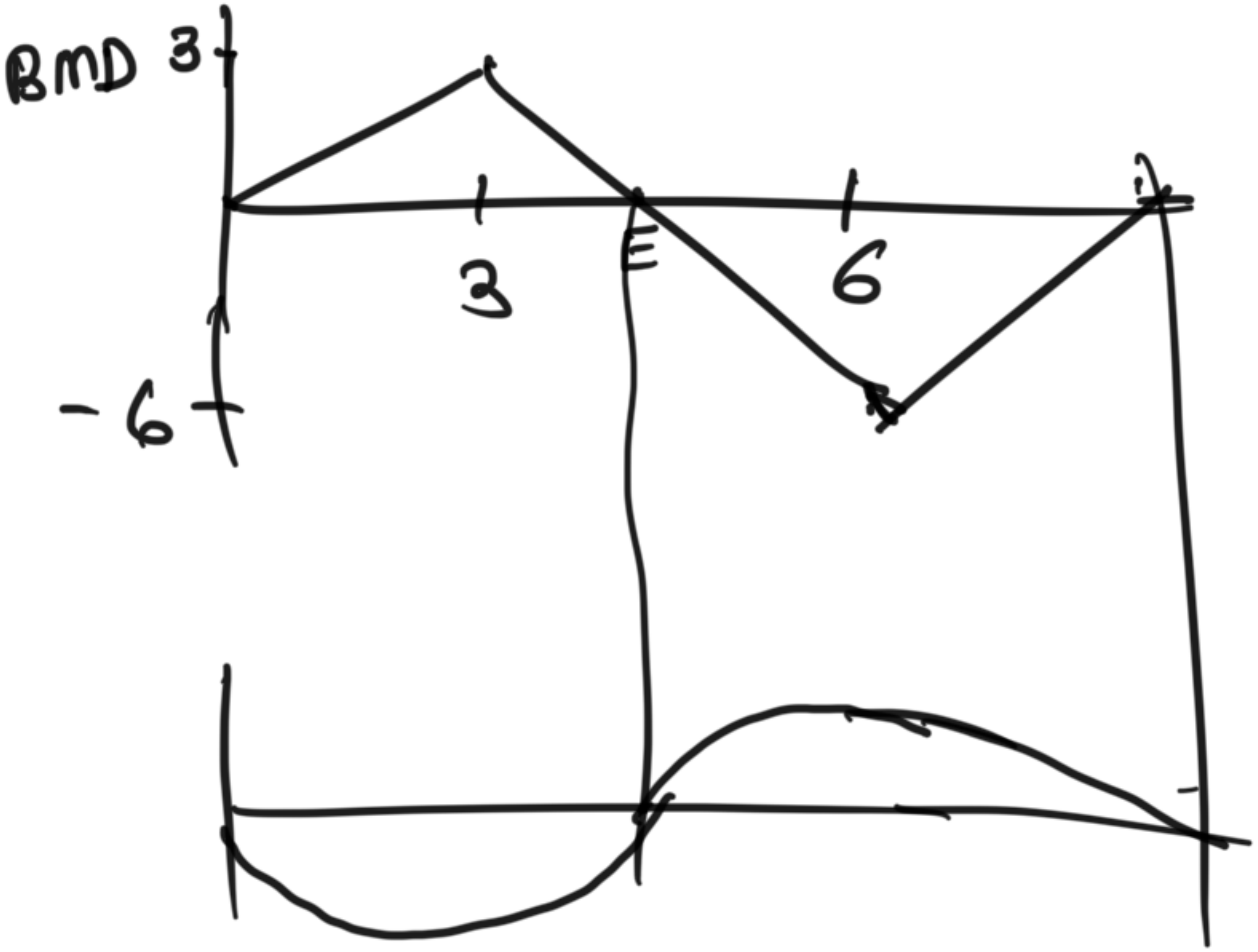
$A_y = 1$ kN, $C_y = 5$ kN

$0 < x < 3$

$m = 1x$

$3 < x < 6$

$$m = x - 4(x-3)$$
$$= x - 4x + 12$$
$$= 12 - 3x$$



expected shape of deflected Beam

expected shape of beam based on BMD

$m > 0$ +ve curvature concave upwards

<u>Calculus:</u> $$\frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

$y(x)$ for beam $\rightarrow$ Elastic curve of beam

$\frac{dy}{dx} \ll 1 \implies$ denominator can be ~~wr~~ written as

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} \approx 1$$

$$\frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

$$EI\frac{d^2y}{dx^2} = M(x)$$

x

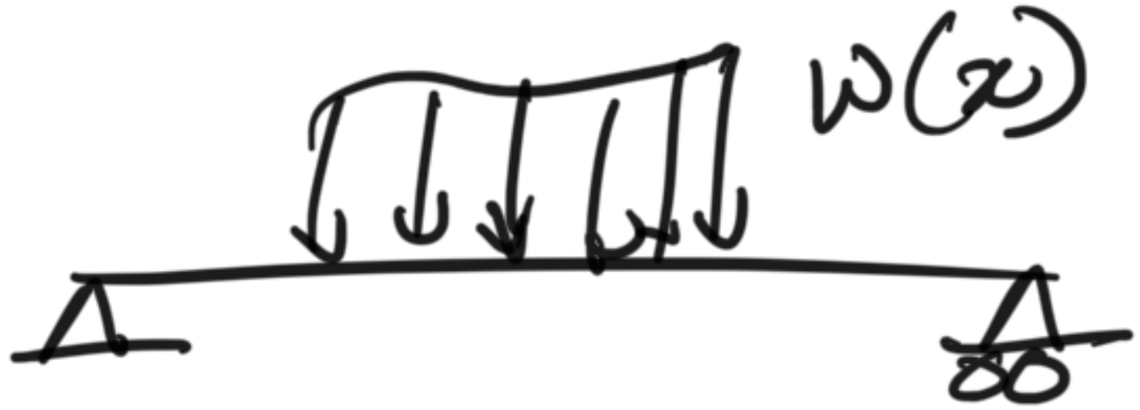
$$EI \frac{dy}{dx} = \int_0^x M(x)\,dx + C_1$$

$$EI\,y(x) = \left[\int_0^x \left\{ \int_0^x M(x)\,dx + C_1 \right\} dx \right.$$

$$\frac{dy}{dx} = \theta(x) \rightarrow \text{slope (radians)}$$

tangent to the elastic curve

$$\frac{dy}{dx} = \tan\theta \approx \theta \quad (\text{if } \theta \text{ is small.}$$



$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

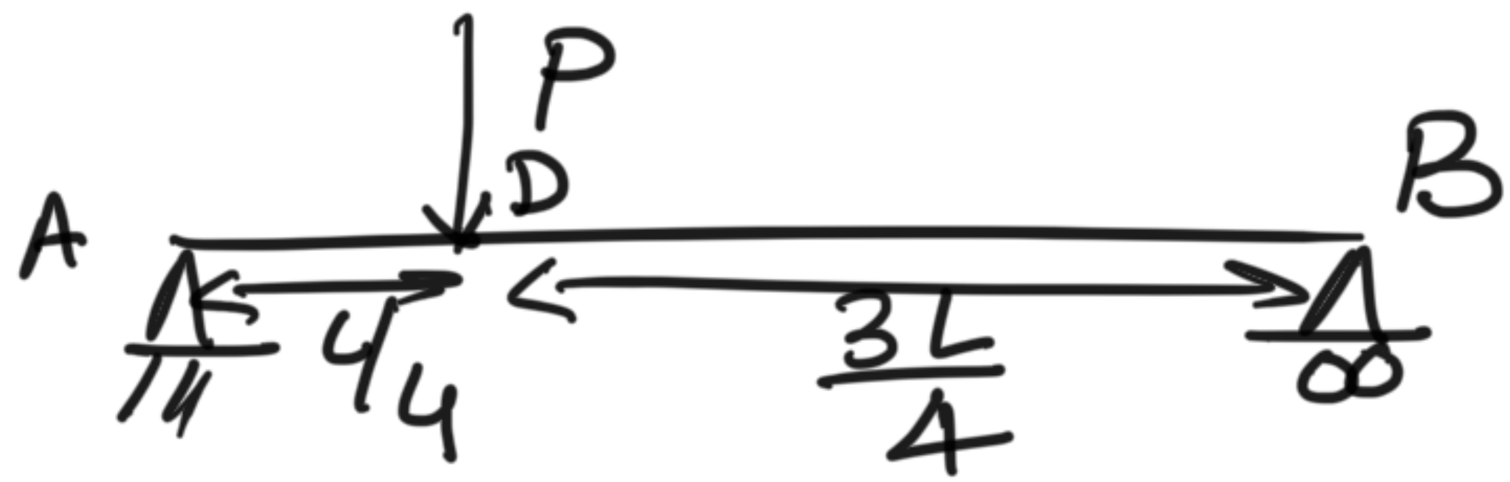
$$\frac{dM}{dx} = V \Rightarrow \frac{d^3y}{dx^3} = \frac{V(x)}{EI}$$

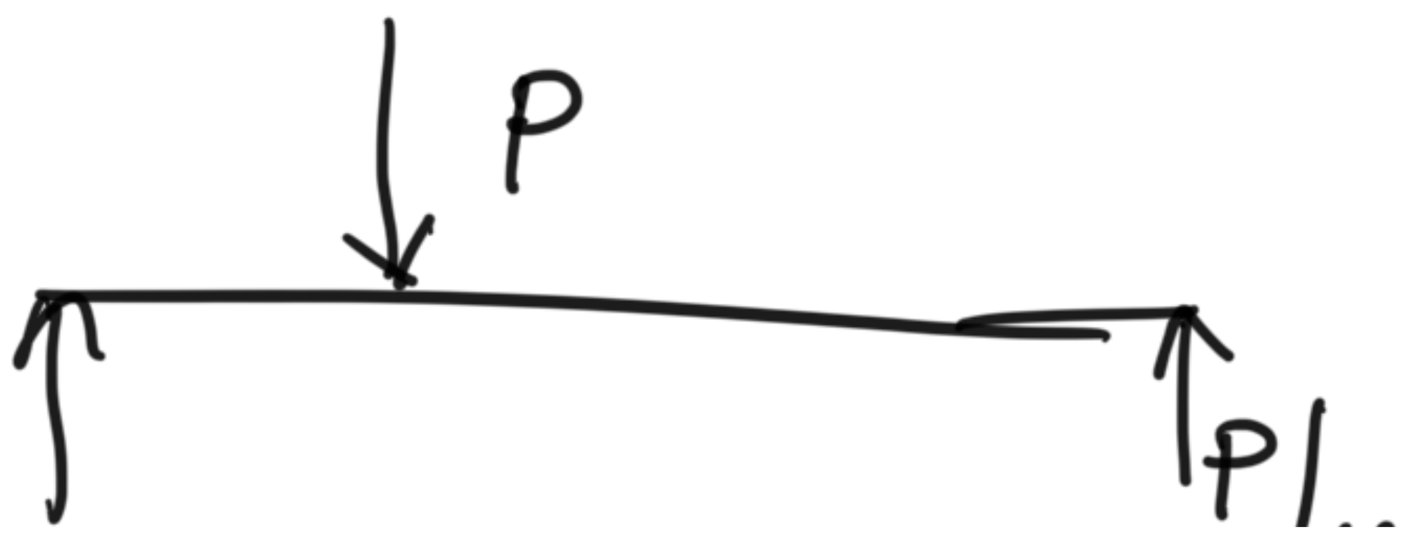
$$\frac{dV}{dx} = -w \Rightarrow \frac{d^4y}{dx^4} = -\frac{w(x)}{EI}$$

At free end, $V = 0$

$M = 0$

On simply supported end $M = 0$

Eg. 



Eqbm.

$3P/4$ 14

A to D: $x < \frac{L}{4} \Rightarrow M = \frac{3P}{4}x$

$$EI\frac{d^2y_1}{dx^2} = \frac{3P}{4}x$$

$$EIy_1 = \frac{Px^3}{8} + C_1x + C_2$$

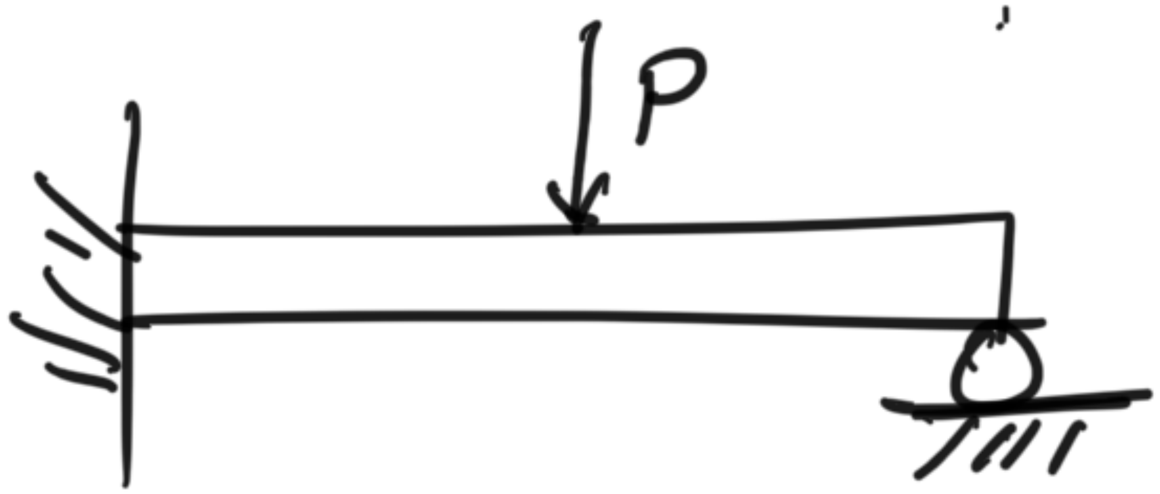
from D to B:

$$M(x) = \frac{3Px}{4} - P\left(x - \frac{L}{4}\right)$$

$$= \frac{P}{4}(L-x) = EI\frac{d^2}{dx^2}$$

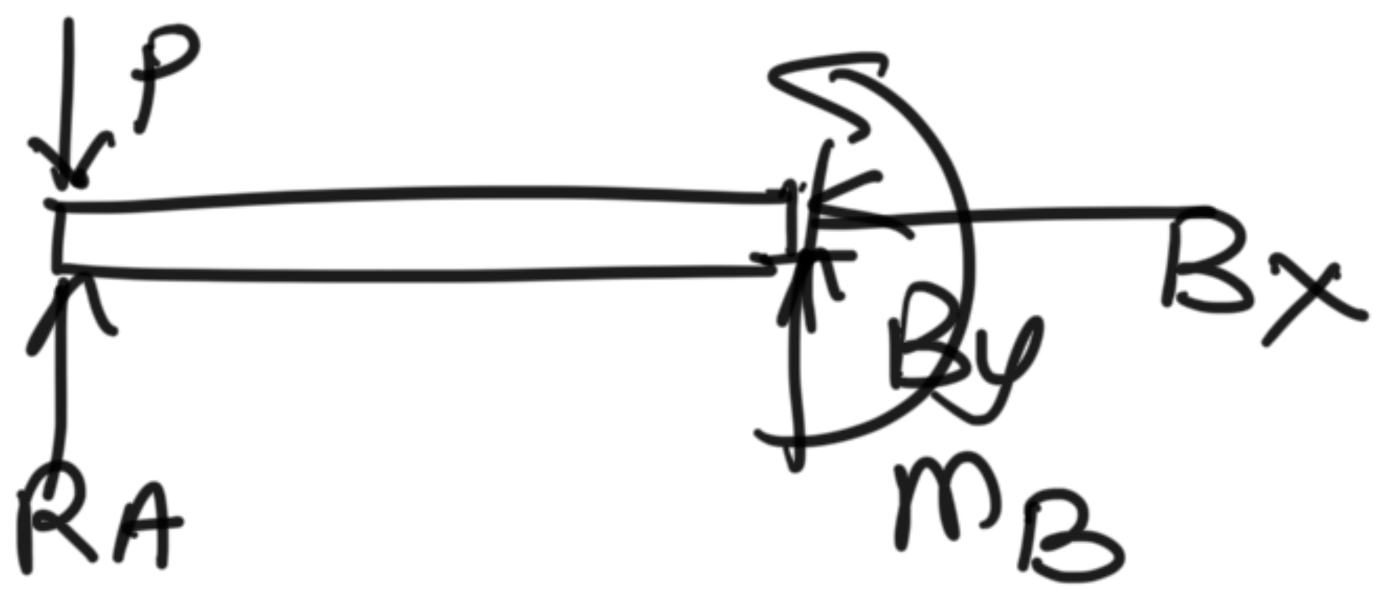
$$y_2 = -\frac{Px^3}{24} + \frac{PL}{8}x^2 + C_3x + C_4$$

statically indeterminate problem



eg. of statically indeterminate problem.

eg.2



$M = (R_A - P)\,x$

$$M = (R_A - P)\, x$$

$$EI \frac{d^2y}{dx^2} = (R_A - P)\, x$$

$$EI\, y = (R_A - P)\frac{x^3}{6} + C_1 x + C_2$$

$$\left.\begin{aligned} y(0) &= 0, \\ y(L) &= 0 \\ \theta(L) &= 0 \end{aligned}\right\} \text{get } R_A,\ C_1, C_2$$