The importance of external priors for the S4 generation of CMB experiments.

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Abstract

The next generation of CMB experiments is expected to drastically improve our understanding of neutrino physics, inflation, Dark Matter and Dark Energy. Indeed they will measure with cosmic variance limited precision the temperature and polarization mode together with a lensing potential reconstruction with signal to noise bigger than one on several scales. Detailed forecasts have been done and will tremendously help to plan and motivate this next generation of CMB experiments. However we think that theoretical priors coming from external non-CMB experiments were not fully analyzed. For this reason in this work we use a fisher-matrix technique to test how improving on external priors can lead to improvement in S4 CMB parameters constraint. We find that..

- We are now using a simplistic lensing noise but I have lensing noise with the usual Hu Okamoto formula. Iterative Seljack not there yet. Needed?

- We are varying τ, n_s, A_s, N_{eff}, H_0, w, $\Omega_\nu h^2$, $\Omega_c h^2$, $\Omega_b h^2$. The neutrino sector consists of one massive neutrino of $m_\nu = 0.083$ eV, which corresponds to $\Omega_\nu h^2 = 0.0009$.

- Next TODO: Check for bugs and errors code (DONE look at Zhen test). PCA? what prior is more important? Full MCMC should not be extremely hard with cosmosis (I implemented a toy likelihood analysis to be improved).

I. INTRODUCTION

Since the first results coming from the COBE experiments great results.

Far from finished S4 experiment will measure

This will give us amazing information on..

Forecast is important [1] ... we think that also a detailed study of priors will be important in planning new experiment.

For this reason we...

This paper is organized as follow:

II. ASSUMPTION AND METHODS

We start this section by introducing the Fisher matrix formalism a simple but powerful technique widely used to predict the constrains on the parameters of the assumed cosmological model with future experiments. We then specify the model we used in this work and the fiducial parameters we assumed. We adopt a standard ΛCDM model with basic possible extensions with parameters taken from recent CMB+BAO+SN fit.

A. Fisher Matrix formalism

The starting point to estimate errors on cosmological parameters is the Bayes theorem that relates the likelihood of measuring a set of data given the parameters of the model to what we want: the posterior probability of those parameters given the data. These are related by the priors probability of the parameters $P(\theta)$ through:

$$\mathcal{L}(\theta|d) \propto \mathcal{L}(d|\theta)P(\theta), \tag{1}$$

where,as usual we have neglect the probability of the data itself. These general framework can be simplified in the context of cosmology by focusing on small perturbations around the maximum of a Gaussian likelihood. Indeed we will assume that the likelihood can be approximated as Gaussian. While this approximate the true likelihood most of the time the opposite as been found in other cases [2]. Even if other methods of analysis have been proposed [3, 4] this is still the standard assumption when future constrains have been forecasted [1]. Secondly we will assume to know the true "fiducial" parameters that maximize

the likelihood $\mathcal{L}(\theta|d)$. A consequence is that we are then able to estimate the uncertainties on those parameters by a simple analysis of the likelihood curvature around the fiducial values. As usual we define the fisher matrix elements as:

$$F_{ij} \equiv - \left\langle \left. \frac{\partial^2 \log \mathcal{L}}{\partial \theta_i \partial \theta_j} \right|_{\boldsymbol{\theta}=\boldsymbol{\theta_0}} \right\rangle, \tag{2}$$

where $\theta_{i,j}$ represents two of the parameters and $\boldsymbol{\theta_0}$ is the parameters array that, by definition, maximize the likelihood. We refer the reader for a detailed explanation of the fact that with the usual definition of power spectrum $< a_{\ell m}^X a_{\ell' m'}^Y >= \delta_{\ell\ell'} \delta_{mm'} C^{X,Y}$ with in our case $X, Y \in \{T, E, \phi\}$ the Fisher matrix can be rewritten as:

$$F_{ij} = \sum_\ell \frac{2\ell+1}{2} f_{sky} \mathrm{Tr} \left(\boldsymbol{C}_\ell^{-1}(\theta) \frac{\partial \boldsymbol{C}_\ell}{\partial \theta_i} \boldsymbol{C}_\ell^{-1}(\theta) \frac{\partial \boldsymbol{C}_\ell}{\partial \theta_j} \right) \tag{3}$$

with $\boldsymbol{C}_\ell$ encapsulating the usual power spectra in a matrix structure:

$$\mathbf{C}_\ell \equiv \begin{pmatrix} C_\ell^{TT} + N_\ell^{TT} & C_\ell^{TE} & C_\ell^{Td} \\ C_\ell^{TE} & C_\ell^{EE} + N_\ell^{EE} & 0 \\ C_\ell^{Td} & 0 & C_\ell^{dd} + N_\ell^{dd} \end{pmatrix}. \tag{4}$$

The power of this method come from the fact that, given the Cramer-Rao inequality we can get the errors on parameter i marginalized over all the other parameters simply inverting the Fisher matrix as:

$$\sigma_i \equiv \sigma(\theta_i) = \sqrt{(\mathbf{F^{-1}})_{ii}}. \tag{5}$$

Approaching the forecast problem using a Fisher matrix introduce some technical difficulties together with the previously mentioned assumption that the likelihood is gaussian. Indeed it is known that the more the paramters used in the analysis the easier it is to face numerical isseus. Fisher matrix are can be ill-conditioned: a small change in the fisher matrix led to a big change in its inverse. Given cramer-rao this can be a problem for error estimation. While we understand this possible concern we carefully compare our results to the literature [5] Furthermore we use the same technique used in [4] to avoid any possible numerical instability in the marginalising procedure. This allow not to invert the entire matrix when we want to marginalize over a set of parameters minimizin numerical issue and conserving parameters degeneracies as much as possible.

Priors on cosmological parameters coming from external experiments, the main focus or our work can be easily treated in this contest. For example a 1% prior on H_0 can be obtained

by:

$$F_{H_0 H_0} \rightarrow F_{H_0 H_0} + \frac{1}{(1\% \times H_{0,\text{fid}})^2},$$ (6)

PUT in an appendix?

$$G = F^{\phi\phi} - F^{\phi\psi} U \Lambda^{-1} U^T F^{\phi\psi},$$ (7)

where we define $\phi = \{N_{eff}, ...\}$ and $\psi = \{\Omega_m ... marginal\}$; therefore, $F^{\phi\phi}$ is the block of the total Fisher matrix containing the parameters we want to constrain, whilst $F^{\psi\psi}$ is the nuisance-parameter Fisher sub-matrix. Here, Λ is the diagonal matrix whose elements are the eigenvalues of $F^{\psi\psi}$, whilst U is the orthogonal matrix diagonalising $F^{\psi\psi}$. By using Eq., our marginalising procedure is more stable, since degeneracies in $F^{\phi\phi}$ are properly propagated to G with no instabilities, and we do not even worry about a possibly ill-conditioned $F^{\phi\phi}$ sub-matrix, since we check its stability on the fly by the diagonalisation.
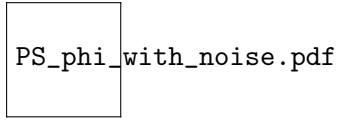

FIG. 1. Lensing potential power spectrum for our fiducial cosmology together with the lensing reconstruction noise N^ϕ used in this work.
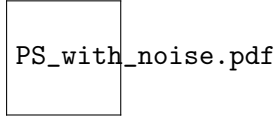

FIG. 2. CMB power spectrum for our fiducial cosmology together with the instrumental noise used in this work. They are basically equivalent to a cosmic variance limited experiment with $l_{max} \sim 3000$

B. CMB S4 experiments: level of noise.

The next generation of CMB experiments defined as S4 will have an extraordinary resolution and depth. We will also assume that for the measurement we use, the temperature and E-mode polarization of the CMB large scale foreground like dust are under control and

negligible. We will deal with the poisson noise in the temperature signal by discarding all the small scales modes with $\ell > \ell_{T,max} = 3000$. The remaining source of noise, the instrumental noise is added to the power spectrum in the usual way:

$$N_\ell^{XX'} = s^2 \exp\left(\ell(\ell+1)\frac{\theta_{\mathrm{FWHM}}^2}{8\log 2}\right), \tag{8}$$

where θ_{FWHM}^2 is the FWHM of the experiment's beam and s represent instrumental the white noise. We decide to use a 1.5 muK and a beam of 1 arcmin. Note that $s \to s \times \sqrt{2}$ in the case of $XX' = \{EE, BB\}$.

Together with E and T we will also consider the information contained in the lensing potential phi reconstructed from CMB experiments. The lensing potential represents the integration along the line of site of the gravitational potential and it leaves its signature in the CMB, both in temperature and polarization, by bending CMB photons. This introduce non gaussianities that couples different modes in the otherwise independent CMB modes and using a quadrature estimator technique it can be reconstructed [6, 7]. For the noise associated to the recosntructed ϕ we follow [6, 7] without using more advanced iterative techniques.

We parametrize our cosmology using a flat $\nu\Lambda$CDM universe. We also allow different Dark Energy framework by introducing the equations of state parameter w as a free parameter.

We use the ΛCDM parameter values from Table 2 of *Planck* best fit value [?], i.e. $\Omega_c h^2 = 0.12029$, $\Omega_b h^2 = 0.022068$, $A_s = 2.215 \times 10^{-9}$ at $k_0 = 0.05$ Mpc^{-1}, $n_s = 0.9624$, $\tau = 0.0925$, and $H_0 = 67.11$ km/s/Mpc. Regarding the ΛCDM extension we chose a neutrino energy densities$\Omega_\nu h^2$=0.0009, which corresponds to $M_\nu \simeq 85$ meV and we also set $w = 1$.

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \tag{9}$$

III. RESULTS:

IV. CONCLUSIONS

We find that...

This means that supernova experiments CMB spatial distortions and large scale structure can/will/should

Future works

TABLE I. Fiducial values used.

H_0
τ
A_s
n_s
N_{eff}

TABLE II. How well we do constrain separate parameters with this data without any external prior?

H_0	M_ν	$\Omega_{bc}h^2$	$\Omega_b h^2$	τ	A_s	n_s
0.80%	40.92%	0.44%	0.11%	3.075%	0.53%	0.18%

TABLE III. Things to notice: this is done with the Zhen test parameters ($\ell < 3000$ and no error on lensing). Now if we trust it CMB alone can get a 0.8% error on H_0 thus I am not surprised if a prior on H would not help. However what do we think about it? is it really CMB better than SN. People will not agree on that. τ will probably improve and also Mν from BAO may help improving parameters.

ACKNOWLEDGMENTS

h0_fisher.pdf

FIG. 3.

FIG. 4.

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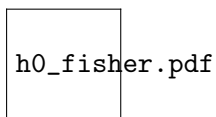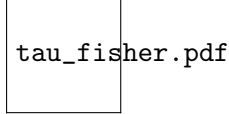
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