External priors for the next generation of CMB experiments.

Alessandro Manzotti[1, 2, *] and Scott Dodelson[1, 2, 3]

[1] *Department of Astronomy & Astrophysics, University of Chicago, Chicago IL 60637*

[2] *Kavli Institute for Cosmological Physics, Enrico Fermi Institute, University of Chicago, Chicago, IL 60637*

[3] *Fermilab Center for Particle Astrophysics, Fermi National Accelerator Laboratory, Batavia, IL 60510-0500*

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The next generation of cosmic microwave background (CMB) experiments can dramatically improve what we know about neutrino physics, inflation, Dark Matter and Dark Energy. Indeed the low level of noise, together with improved angular resolution, will drastically increase the signal to noise of the CMB polarized signal as well as the reconstructed lensing potential of high redshift large scale structure. Projected constraints on cosmological parameters are extremely tight, but these can be improved even further with information from external experiments. Here, we examine quantitatively the extent to which external priors can lead to improvement in projected constraints from a CMB-Stage 4 experiment. We find that, quite surprisingly, CMB S4 are very powerful even on .. There are few exceptions.. like w.

I. INTRODUCTION

Since their early stages, Cosmic Microwave Background (CMB) experiments have been crucial in our understanding of the universe. They will maintain their role also in the near future thanks to the unprecedented level of noise and accuracy of their next generation (S4). The recent past is indeed reassuring. Every new generation of satellite experiments improved the sensitivity by almost a factor of ten compared to its predecessor, from the first generation instrument COBE to WMAP all the way to the current state of the art represented by Planck [1–5]. This lead us from the observation of the first peak of the CMB temperature power spectrum with COBE to a cosmic variance limited measurement of several peaks with Planck. With the improved sensitivity we managed to extend the scientific outcome of these experiments from a measurement of the universe flatness to a percent level constraint on the parameters of the standard ΛCDM model. The same success characterized ground experiments where the progression from DASI [6] to SPT and ACT [7] [8] allowed us to measure the small scale damping tale of the CMB spectrum with increasing accuracy. These series of successes is far from its end. Indeed the next generation of CMB experiments (S4) is now in its planning stage and it will potentially measure the E-mode polarization with cosmic variance limited precision together with an order of magnitude improvement in B-mode measurement and lensing reconstruction. As it happened in the past, this new sensitivity will improve our understanding of several areas of astrophysics like neutrinos, inflation, Dark Energy and Dark matter.

The main effects of neutrinos in the CMB can be parametrize by the number of relativistic degree of freedom in the early universe $N_{\rm eff}$ and the total mass of the three neutrino species M_ν. Indeed the number of relativistic species, like neutrinos, in the early Universe alter the expansion history. This can be probed using the CMB by comparing the sound horizon scale, obtained from the CMB peaks positions, with the silk damping scale taking advantage of the fact that they scale differently with the expansion rate $H(z)$ (see [9], [10] and references therein). This allow CMB experiment to constrain $N_{\rm eff}$. Any deviation from the predicted value $N_{\rm eff} = 3.046$ will be a sign of possible extensions of the Standard Model. The total mass of neutrinos, on the other hand, has a modest effect on the CMB because, for the range of masses allowed by recent constraints ($M_\nu < 230$ meV from [2]), neutrinos are still relativistic at the last scattering surface. As other low redshift effects however, massive neutrinos modify the CMB through the lensing of the CMB photons by large scale structures. Different neutrino masses result into a different growth of structure which consequently lead to different CMB lensing. The lensing potential can be reconstructed from the CMB itself, opening the possibility of constraining the neutrino mass. In the future, according for example to [11], CMB alone will constrain $N_{\rm eff}$ with a 1% precision and the total mass of neutrinos at 60% with a factor of two improvement on M_ν when baryon acoustic oscillations (BAO) data are also used. The future generation of CMB experiments will be a big step in the understanding of the neutrino sector.

As previously mentioned, CMB is also sensitive to the Dark energy properties (see [12]). The current generation of experiments measured with percent accuracy the energy density of Dark Energy. The challenge for future experiments is to identify the nature of this mysterious component. The first step will be to measure any deviation in the equation of state from the one predicted by a cosmological constant model. Furthermore CMB will also be a powerful probe of any time dependence of the dark energy equation of state. Indeed dark energy affect the CMB because due to its effect on the universe's expansion that modifies the distance to the last scattering surface together with a modification of CMB photons energy through the Integrated Sachs Wolfe Effect. Furthermore different DE models lead to a different growth of

* manzotti.alessandro@gmail.com

large scale structure. However dark energy properties are strongly degenerate with other geometrical parameters like H_0 and Ω_k. To get competitive constraints, CMB experiments will have to rely on external prior coming from BAO and supernova experiments.

In general, external priors will be fundamental to improve the already tight CMB constraints on cosmological parameters. For this reason the synergy with other experiments is crucial. In particular CMB will really benefit from experiments like large scale structure clustering and weak lensing, BAO targeted experiments, and supernovae. To improve the synergy between different experiments and to guide the plan of future experiments, in this work we study the dependance of CMB future constrains on the external priors assumed.

This paper is organized as follow: in §II we introduce the technique and assumptions we use to derive the effect of external priors on the CMB parameter constraints. In §III we will describe our results and we then conclude with a discussion in §IV.

II. ASSUMPTIONS AND METHODS

In this paper we want to measure quantitatively the effect of external priors on the cosmological parameters constraints derived from S4 CMB experiments. We start this section in section II A by introducing the Fisher matrix formalism, a simple but powerful technique widely used to forecast future experimental constraints. To apply this technique, we need to specify our cosmological model (ΛCDM plus neutrinos and dark energy extensions in our case) as well as a set of fiducial parameters and the specifications of S4 CMB experiments. These will be presented in section II B.

A. Fisher Matrix formalism

The natural starting point to estimate errors on cosmological parameters is the Bayes theorem that relates the likelihood to measure a set of data given the parameters of the model $\mathcal{L}(d|\theta)$, to what we want: the posterior probability of those parameters given the data, $\mathcal{L}(\theta|d)$. These two are related by the prior probability of the parameters $P(\theta)$ through:

$$\mathcal{L}(\theta|d) \propto \mathcal{L}(d|\theta)P(\theta), \tag{1}$$

where, as usual, we have neglected the probability of the data itself. These general framework can be simplified in our case. We will focus not on an entirely general posterior distribution, but we will assume that the likelihoods are gaussian. Furthermore we will not derive the global shape of the likelihood but we will obtain parameters errors by studying small perturbations around its maximum. These are the two basic assumptions of the Fisher-matrix approach.

Regarding the first assumption, we recognize the fact that, even if the gaussian approximation has been shown to be appropriate most of the time, some problems have been found in other cases [13]. Despite this, we do not need the level of accuracy that will require a careful modeling of the likelihoods, because we are looking for the general behavior of parameters errors as a function of external priors. Moreover the gaussian approximation gets better at smaller scales (high ℓ in Fourier space) which, a part for the exception of the optical depth parameters τ, is where most of the constraining power of the CMB is coming from.

Secondly we will assume to know the true "fiducial" parameters that maximize the likelihood $\mathcal{L}(\theta|d)$ and we will get the errors on those from the likelihood curvature around the fiducial values. Indeed, as usual, we define the fisher matrix elements as the curvature:

$$F_{ij} \equiv -\left\langle \frac{\partial^2 \log \mathcal{L}}{\partial\theta_i \partial\theta_j}\bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta_0}} \right\rangle, \tag{2}$$

where $\theta_{i,j}$ represents two of the parameters and $\boldsymbol{\theta_0}$ is the parameters values array that, by definition, maximize the likelihood. We refer the reader to [] for a detailed proof that with the usual definition of power spectrum $< a^X_{\ell m} a^Y_{\ell' m'} >= \delta_{\ell\ell'}\delta_{mm'}C^{X,Y}$, where $a^X_{\ell m}$ represent the spherical harmonics coefficients of the field X, the Fisher matrix of a CMB experiment can be rewritten as:

$$F_{ij} = \sum_\ell \frac{2\ell+1}{2} f_{sky} \mathrm{Tr}\left(\boldsymbol{C}_\ell^{-1}(\theta) \frac{\partial \boldsymbol{C}_\ell}{\partial\theta_i} \boldsymbol{C}_\ell^{-1}(\theta) \frac{\partial \boldsymbol{C}_\ell}{\partial\theta_j} \right). \tag{3}$$

In this work we will use the CMB temperature and E mode polarization together with the reconstructed lensing potential to constrain parameters. For this reason, $\boldsymbol{C}_\ell$ in Eq. (3) is:

$$\mathbf{C}_\ell \equiv \begin{pmatrix} C_\ell^{TT} + N_\ell^{TT} & C_\ell^{TE} & C_\ell^{Td} \\ C_\ell^{TE} & C_\ell^{EE} + N_\ell^{EE} & 0 \\ C_\ell^{Td} & 0 & C_\ell^{dd} + N_\ell^{dd} \end{pmatrix}. \tag{4}$$

Note that we are neglecting the term $C_\ell^{E\phi}$. As also noticed in previous literature (like [10, 11]) this term contains very little information while adding possible numerical issues. The terms N_ℓ^X represents the instrumental noise power of the specific experiment and will be discussed in section II B. The power of the Fisher approach descend from the Cramer-Rao inequality that relates the errors on parameter i, marginalized over all the other parameters, σ_i, to the Fisher matrix as:

$$\sigma_i \equiv \sigma(\theta_i) = \sqrt{(\mathbf{F^{-1}})_{ii}}. \tag{5}$$

Once F_{ij} is computed following Eq. (3) it is straightforwards to get the error σ_i from Eq. (5). Furthermore in this context it is easy to introduce external priors on cosmological parameters. Indeed to introduce priors we simply need to add the Fisher matrixes of the external experiements before performing the matrix inversion of Eq. (5), i.e.:

$$F_{total} = F_{CMB} + \sum F_{external}. \tag{6}$$

In the same way we can add priors on a single cosmological parameter just by modyfing the correspondent matrix element. For example, a 1% prior on H_0 can be obtained by:

$$F_{H_0 H_0} \to F_{H_0 H_0} + \frac{1}{(1\% \times H_{0,\mathrm{fid}})^2}. \tag{7}$$

We chose a Fisher matrix approach because of its ability to rapidly forecast future experiments performances without generating mock data, together with the ease of including external priors. This technique however introduce some technical difficulties together the previously mentioned assumption that the likelihood is gaussian. Indeed it is known [14] that increasing the number of parameters used in the analysis can lead to numerical issues (see also [15] in the gravitational waves contest were a lot of parameters are used). Fisher matrix indeed can become ill-conditioned: a small change in the fisher matrix led to a big change in its inverse. Because we use Eq. (5) this can be a problem for error estimation. Even if other methods of analysis have been proposed [14, 16] this is still the standard method used to forecast future constrains [11]. We carefully try to avoid any possible source of errors in computing the elements of Eq. (3) and in the matrix inversion of Eq. (5). We compute the derivatives in Eq. (3) using a 5 points formula:

$$\left.\frac{\partial C}{\partial \theta}\right|_{\theta_0} \sim \frac{-C(\theta_0+2h)+8C(\theta_0+h)-8C(\theta_0-h)+C(\theta_0-2h)}{12h}. \tag{8}$$

This high order definition allow us to use a bigger gap h around the fiducial parameters θ_0. As a consequence the differences of power spectra corresponding to different values are big enough to make possible numerical accuracy issues in computing C negligible. We also test the robustness of this calculation by changing the gap h in the range $2-7\%$ of the correspondent θ_0 without noticing any significant change in the results. Furthermore we compare our results to similar previous work in the literature [17] obtaining a perfect agreement. Lastly we implement the same technique of [14] to avoid possible numerical instability in the marginalizing procedure. This allow us not to invert the entire matrix when we want to marginalize over a set of parameters, in order to minimize numerical issues and conserve parameters degeneracies as much as possible.

B. CMB S4 experiments: level of noise.

In this subsection we will describe the assumptions we made in calculating the elements that goes in Eq. (3) and, in particular, the power spectra C and the noise power

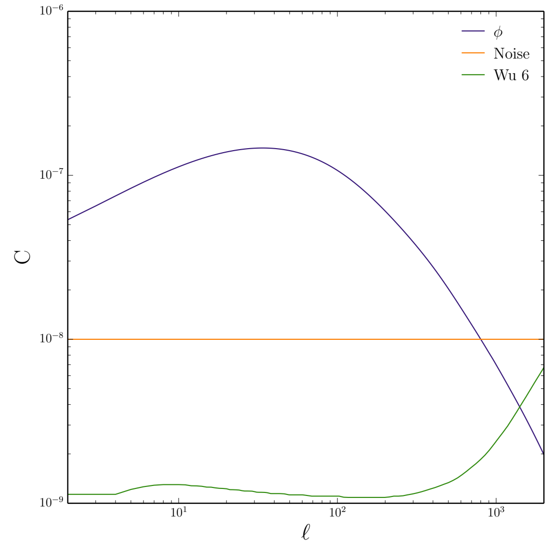

FIG. 1. Lensing potential power spectrum use in this work has a signal to noise bigger than one up to $\ell \simeq 800-1000$. In the figure: the deflection power spectrum for our fiducial cosmology together with the two examples of lensing reconstruction noise N^ϕ used in this work.

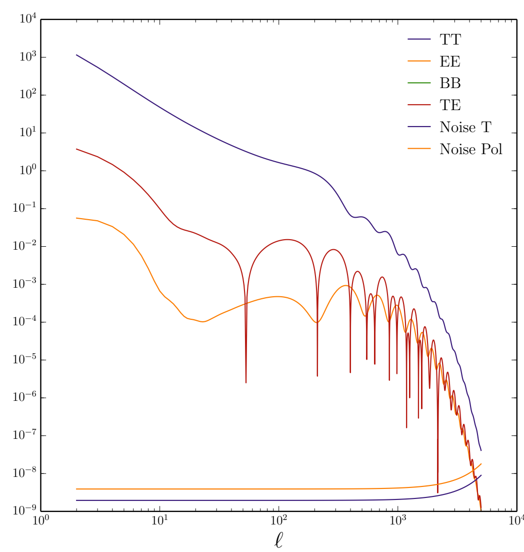

FIG. 2. The next generation $C^{T,E}$ used in this work are almost cosmic variance limited. In the figure: CMB power spectrum for our fiducial cosmology together with the instrumental noise used in this work.

N. The first step is the cosmological model we used to compute $C^{X,Y}$ with $X, Y \in \{T, E, \phi\}$. We parametrize our cosmology using a flat $\nu\Lambda$CDM universe. We allow a set different Dark Energy model by introducing the equations of state parameter w as a varying parameter. We chose our fiducial parameters following Table 2 of *Planck* best fit [18], i.e. $\Omega_c h^2 = 0.12029$, $\Omega_b h^2 = 0.022068$, $A_s = 2.215 \times 10^{-9}$ at $k_0 = 0.05$ Mpc^{-1}, $n_s = 0.9624$, $\tau = 0.0925$, and $H_0 = 67.11$ km/s/Mpc. Regarding the ΛCDM extension, we chose a neutrino energy densities $\Omega_\nu h^2$=0.0009, which corresponds to $M_\nu \simeq 85$ meV a standard $N_{\rm eff} = 3.046$ and a dark energy equation of state, $w = -1$. Given the model, we use the CAMB [19] software to compute the power spectra C_ℓ at the fiducial values and at those needed to compute derivatives using Eq. (8). Notice that while we vary one parameter in Eq. (8) we keep all the others fixed with the exception of Ω_Λ which is always changed in order to keep the universe flat ($\Omega_{\rm k} = 0$).

We chose the second element of Eq. (3), the noise power in the CMB $N_\ell^{T,E}$ and in the lensing reconstruction N_ℓ^{ϕ} with the next generation of CMB experiments (S4) in mind. For the temperature and E-mode polarization of the CMB, together with the an improved depth and resolution we will also assume that large scale foregrounds, like dust, are under control or negligible. We will deal with the presence of point sources poisson noise in the temperature signal by simply discarding all the small scales modes with $\ell > \ell_{\rm T,max} = 3000$. The remaining source of noise, the instrumental noise is added to the power spectrum in the usual way:

$$N_\ell^X = s^2 \exp\left(\ell(\ell+1)\frac{\theta_{\rm FWHM}^2}{8\log 2}\right), \tag{9}$$

where $\theta_{\rm FWHM}^2$ is the FWHM of the experiment's beam and s represent the instrumental white noise. We decide to use a level of noise $s = 1.5$ μK-arcmin for $X = T$ and a beam of $\theta_{\rm FWHM} = 1$ arcmin (PRELIMINARY). Note that 1.5 μK-arcmin is the noise in temperature and we need $s \to s \times \sqrt{2}$ in the case of polarization $XX' = \{EE, BB\}$.

Together with E and T we will use the information contained in the lensing potential ϕ as it is reconstructed from CMB experiments. The lensing potential represents the integration along the line of site of the gravitational potential and it leaves its signature in the CMB, both in temperature and polarization, by bending the trajectory of CMB photons. This introduce non gaussianities that couple different otherwise independent CMB modes and it can be reconstructed using a quadratic estimator technique [20, 21]. For the noise $N_\ell^{\phi\phi}$ associated to the reconstructed ϕ we follow the non-iterative technique of [20, 21].

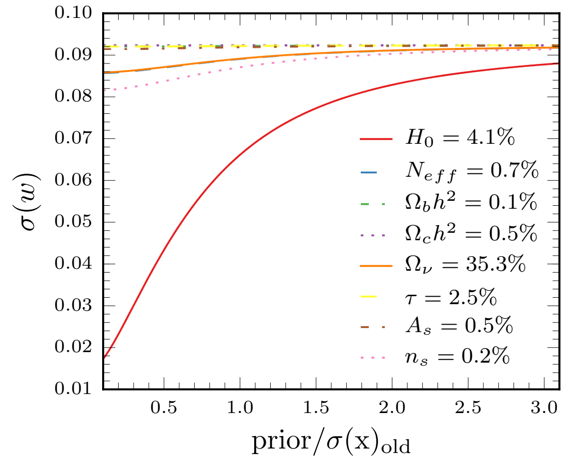

FIG. 3.

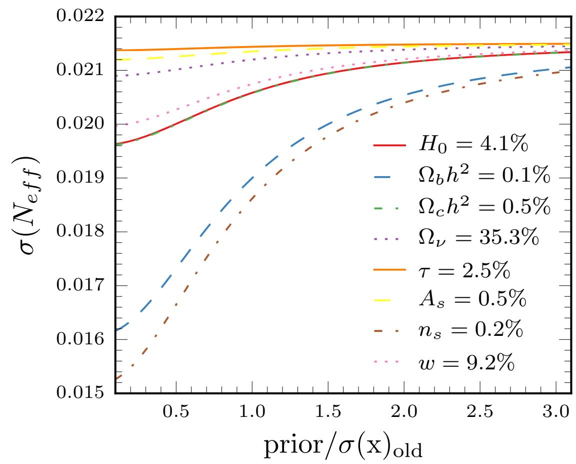

FIG. 4.

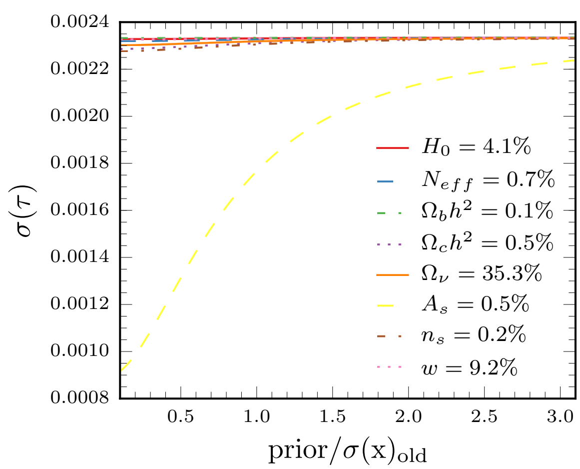

FIG. 5.

H_0	M_ν	$\Omega_{bc}h^2$	$\Omega_b h^2$	τ	A_s	n_s
0.80%	40.92%	0.44%	0.11%	3.075%	0.53%	0.18%

TABLE I. How well we do constrain separate parameters with this data without any external prior? Things to notice: this is done with the Zhen test parameters ($\ell < 3000$ and no error on lensing). Now if we trust it CMB alone can get a 0.8% error on H_0 thus I am not surprised if a prior on H would not help. However what do we think about it? is it really CMB better than SN. People will not agree on that. τ will probably improve and also Mν from BAO may help improving parameters.

III. RESULTS

IV. CONCLUSIONS

–We find that...

–This means that supernova experiments CMB spatial distortions and large scale structure can/will/should

–Future works

Appendix A: Marginalization , not needed probably

$$G = F^{\phi\phi} - F^{\phi\psi} U \Lambda^{-1} U^T F^{\phi\psi}, \tag{A1}$$

where we define $\phi = \{N_{eff}, ...\}$ and $\psi = \{\Omega_m ... marginal\}$; therefore, $F^{\phi\phi}$ is the block of the total Fisher matrix containing the parameters we want to constrain, whilst $F^{\psi\psi}$ is the nuisance-parameter Fisher submatrix. Here, Λ is the diagonal matrix whose elements are the eigenvalues of $F^{\psi\psi}$, whilst U is the orthogonal matrix diagonalising $F^{\psi\psi}$. By using Eq., our marginalizing procedure is more stable, since degeneracies in $F^{\phi\phi}$ are properly propagated to G with no instabilities, and we do not even worry about a possibly ill-conditioned $F^{\phi\phi}$ sub-matrix, since we check its stability on the fly by the diagonalisation.

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