External priors for the next generation of CMB experiments.

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The next generation of cosmic microwave background (CMB) experiments can dramatically improve what we know about neutrino physics, inflation, and dark energy. Indeed the low level of noise, together with improved angular resolution, will drastically increase the signal to noise of the CMB polarized signal as well as the reconstructed lensing potential of high redshift large scale structure. Projected constraints on cosmological parameters are extremely tight, but these can be improved even further with information from external experiments. Here, we examine quantitatively the extent to which external priors can lead to improvement in projected constraints from a CMB-Stage 4 (S4) experiment on neutrino and dark energy properties. n_s and the baryon fraction will improve the constraint on $N_{\rm eff}$. We find that CMB S4 constraints on neutrino mass will be strongly enhanced by external constraints on the cold dark matter density $\Omega_c h^2$ and the Hubble constant H_0. If the largest scales ($\ell < 50$) will not be measured, an external prior on the primordial amplitude A_s or the optical depth τ will also be important. A CMB constraint on $N_{\rm eff}$ will benefit from an external prior on the spectral index n_s and the baryon energy density $\Omega_b h^2$. Finally, an external prior on H_0 will be crucial to constrain a constant dark energy equation of state(w_0).

I. INTRODUCTION

Since their earliest incarnations, Cosmic Microwave Background (CMB) experiments have been crucial in furthering our understanding of the universe. They will maintain their role also in the near future, thanks to the unprecedented low level of noise and high resolution expected in next generation, dubbed Stage IV (S4) [1]. The recent past is reassuring. Every new generation of satellite experiments improved the level of sensitivity by almost a factor of ten compared to its predecessor, from the first generation instrument COBE to WMAP all the way to the current state of the art represented by Planck [2**?**–5]. These led us from the first detection of anisotropy in the CMB temperature with COBE to a cosmic variance limited measurement of several acoustic peaks with Planck. With the improved sensitivity we have extended our understanding of the universe so that we now have solid evidence for a flat geometry and a well tested model ΛCDM with percent level constraints on its parameters. The same success has characterized ground CMB experiments, where the evolution from [**SD**: need to add at least boomerang, probably lots others if we are going to list a few] [**AM**: it is ok for me to skip the list entirely otherwise feel free to add names I can add references later] DASI [6] to SPT and ACT [7] [8] allowed us to measure the small scale damping tale of the CMB spectrum with increasing accuracy.

This pattern of successes is far from over. The next generation of CMB experiments (S4), now in its planning stage, will potentially measure the E-mode polarization with cosmic variance limited precision together with an order of magnitude improvement in B-mode measurement and lensing reconstruction. As has happened in the past, this new sensitivity together with the progress in the measurement of other cosmological probes will improve our understanding of several areas of astrophysics like dark matter, inflation, dark energy and neutrinos.

The cosmological dependence on neutrinos is two-fold: the number of relativistic species $N_{\rm eff}$ in the early phase of the universe affects the damping tail of the CMB, and sum of the neutrino masses M_ν affects the late-time growth of cosmic structures. Because relativistic species, like neutrinos, are the main drivers of the cosmic expansion in the early Universe their number affects the expansion rate $H(z)$. This rate can be powerfully tested using the CMB, by carefully comparing the sound horizon scale, obtained from the CMB peaks positions, and the Silk damping scale (see [1], [9] and references therein). The future experiments, with their high resolution, will probe deep into the damping tale of the CMB power spectrum and will be sensitive to small variations from the canonical, 3-active neutrino prediction of $N_{\rm eff} = 3.046$. The total mass of neutrinos, on the other hand, has a modest effect on the CMB because, for the range of masses allowed by recent constraints ($M_\nu < 230$ meV from [**?**]), neutrinos are still relativistic at the last scattering surface. However, massive neutrinos alter the growth of the large scale structure responsible for lensing of the CMB photons. Different neutrino masses consequently lead to different CMB lensing spectra. A Stage IV experiment will measure small scale temperature and polarization anisotropies with low noise, dramatically improving the lensing reconstruction. This will turn into a precise constraints of the sum of neutrino masses with a possible hint of the hierarchies of the individual masses. These signatures can be mimicked with

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variations in other cosmic parameters, so an important lingering question is: What external information can be used to improve projected constraints further by breaking degeneracies?

The CMB is sensitive also to the properties of dark energy (see [10]). Thanks to the current generation of experiments we know with extraordinary accuracy its energy density. The challenge for the next generation is to reveal the nature of this mysterious component. For example, a crucial step to identifying the mechanism driving cosmic acceleration will be to investigate any possible deviations in the equation of state, the ratio of pressure and energy density, from the value $w = -1$ predicted by a cosmological constant. Dark energy affects the CMB because it alters the universe's expansion and it consequently changes the distance to the last scattering surface. Furthermore different dark energy models lead to different rates of growth of large scale structure which are tested by CMB lensing. Thanks to this sensitivity to cosmological structures present all the way from us to the last scattering surface, the CMB will also be a powerful probe of any time dependence of the dark energy equation of state. However, dark energy properties are strongly degenerate with other geometrical parameters like H_0 and Ω_k. As with the neutrino sector, dark energy constraints from the CMB will be improved by external experiments.

The crucial importance of feedback from external experiments is true not only for the dark energy and neutrino sector. A general thrust in even the current generation of surveys is to combine information from different observations. Here we study the dependance of projected constraints from CMB on the external priors assumed. For all the parameters of the standard ΛCDM with massive neutrinos ($\Omega_c h^2, \Omega_b h^2, A_s, n_s, \tau, H_0, \sum m_\nu$) together with extensions ($N_{\rm eff}$ and w) to the neutrino and dark energy sectors, we quantify the CMB constraints as a function of external priors. This extends the work of Wu *et al.* [11], which worked with a few fixed external priors, by quantifying the extent to which external information will improve the constraining power of a CMB-S4 experiment.

This paper is organized as follow: in §II we introduce the technique and assumptions we use to derive the effect of external priors on the CMB parameter constraints. In §III we will describe our results and we then conclude with a discussion of them in §IV.

II. ASSUMPTIONS AND METHODS

To measure quantitatively the impact of external priors on the CMB ability to constrain cosmological parameters we use a Fisher matrix formalism. In this section we will quickly review the technique and then present the chosen fiducial cosmological model together with the experimental specifications.

As usual, we define the Fisher matrix elements as the curvature of the likelihood:

$$F_{ij} \equiv - \left\langle \frac{\partial^2 \log \mathcal{L}}{\partial\theta_i \partial\theta_j}\bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta_0}} \right\rangle , \quad (1)$$

where $\theta_{i,j}$ represents two of the cosmological parameters and $\boldsymbol{\theta_0}$ is the fiducial values array that, by definition, maximizes the likelihood.

For CMB experiments the Fisher matrix can be related to the power spectrum $\boldsymbol{C}_\ell$ by:

$$F_{ij} = \sum_\ell \frac{2\ell+1}{2} f_{\rm sky} {\rm Tr} \left(\boldsymbol{C}_\ell^{-1}(\theta) \frac{\partial \boldsymbol{C}_\ell}{\partial\theta_i} \boldsymbol{C}_\ell^{-1}(\theta) \frac{\partial \boldsymbol{C}_\ell}{\partial\theta_j} \right) \quad (2)$$

where $f_{\rm sky}$, the fraction of sky covered, is set to 0.75 throughout. In this work we constrain cosmological parameters with CMB temperature and E mode polarization together with the reconstructed lensing potential of large scale structure. Therefore, $\boldsymbol{C}_\ell$ in Eq. (2) is:

$$\mathbf{C}_\ell \equiv \begin{pmatrix} C_\ell^{TT} + N_\ell^{TT} & C_\ell^{TE} & C_\ell^{T\phi} \\ C_\ell^{TE} & C_\ell^{EE} + N_\ell^{EE} & 0 \\ C_\ell^{T\phi} & 0 & C_\ell^{\phi} + N_\ell^{\phi} \end{pmatrix} . \quad (3)$$

The terms N_ℓ^X represent the instrumental noise power of the specific experiment and will be described at the end of this section. Note that we are neglecting the term $C_\ell^{E\phi}$ since it contains very little information while adding possible numerical issues [9, 11]. Furthermore, as in [11], we use *unlensed* spectra and gaussian covariances in Eq. (2). [**AM**: My understanding is that people do that: unlensed to avoid double counting in the informations with C^ϕ, Aurelien and Wayne showed that unlensed + gaussian is similar to lensed + non-gaussian how big is the effect i do not know. Aurelien is writing a paper about that] [**SD**: Can we say something more positive about these effects? Why are we allowing ourselves to make this approximation?] Introducing eventual imperfect reconstruction of the unlensed spectra (delensing) and non-gaussian effects described in [12] will go in the direction of degrading the parameters constrained derived in this work.

The projected error on the parameter i, marginalized over all the other parameters, σ_i, is then:

$$\sigma_i \equiv \sigma(\theta_i) \geq \sqrt{(\mathbf{F^{-1}})_{ii}}. \quad (4)$$

We can introduce external priors on cosmological parameters. Indeed we simply need to add to the Fisher matrix elements the external priors (before we perform the matrix inversion of Eq. (4)). For example, a 1% prior on the parameter i can be added by:

$$F_{ii} \rightarrow F_{ii} + \frac{1}{(1\% \times \theta_{i,\rm fid})^2}. \quad (5)$$

We compute the power spectra C_ℓ and the noise power N_ℓ in Eq. (2) using CAMB and the derivatives in Eq. (2) using a 5 points finite difference formula: this high order approach allows us to use larger step-sizes around the fiducial parameters to compute derivatives. As a consequence the differences of power spectra corresponding to different values of the parameters are big enough to ensure numerical accuracy. We also test the robustness of this calculation by changing the derivative steps in the range $2-7\%$ of the correspondent θ_0. The constraints change by at most 10% that should then be considered as a conservative estimate of numerical uncertainties.

Our philosophy is to consider as few extensions as possible. Given the current success of the standard models of particle physics and cosmology, one of the primary goals of CMB-S4 will be to find cracks in these models. As such, the key question is: What information is needed to reliably conclude that an additional parameter is required. The natural first baby step is to include massive neutrinos, which (i) are a small extension to the Standard Model and (ii) are known to exist. So our fiducial cosmology is flat $\nu\Lambda$CDM, with assumed parameters from Table 2 of the *Planck* best fit [13], i.e. $\Omega_c h^2 = 0.12029$, $\Omega_b h^2 = 0.022068$, $A_s = 2.215\times 10^{-9}$ at $k_0 = 0.05\ \mathrm{Mpc}^{-1}$, $n_s = 0.9624$, $\tau = 0.0925$, $H_0 = 67.11$ km/s/Mpc, supplemented by an arbitrary choice of $\sum m_\nu \simeq 85$ meV. We then extend the parameter space by introducing $N_{\rm eff}$ and w as free parameters, in each case keeping the other parameter fixed. The fiducial values of these are $N_{\rm eff} = 3.046$ and a cosmological constant equation of state, $w = -1$.

The instrumental noise power $N_\ell^{T,E}$ and the lensing reconstruction N_ℓ^{ϕ} in Eq. (2) correspond to the optimistic level for the next generation of CMB experiments (S4) assumed in [1, 9, 11]. For the temperature and E-mode polarization of the CMB, together with the improved depth and resolution we also assume that large scale foregrounds, like dust, are under control or negligible. This allow us to use all the polarization power spectrum multipoles all the way up to $\ell_{\rm E,max} = 5000$. We deal with the poisson noise from point sources in the temperature signal by simply discarding all the small scales modes with $\ell > \ell_{\rm T,max} = 3000$. The remaining source of noise, the instrumental noise, is added to the power spectrum in the usual way:

$$N_\ell^X = s^2 \exp\left(\ell(\ell+1)\frac{\theta^2_{\rm FWHM}}{8\log 2}\right), \tag{6}$$

where $\theta^2_{\rm FWHM}$ is the FWHM of the experiment's beam and s represents the instrumental white noise. We use a level of noise $s = 0.58$ μK-arcmin for $X = T$ and a beam of $\theta_{\rm FWHM} = 1$ arcmin. This corresponds to the $N_{\rm det} = 10^6$ case of [11]. Note that the quoted noise in temperature and we assume that $s \to s \times \sqrt{2}$ in the case of polarization $XX' = \{EE, BB\}$. The noise $N_\ell^{\phi\phi}$ associated with the reconstructed ϕ spectrum is modeled assuming an iterative reconstruction technique [14].

III. RESULTS

In this section we present our main results. We will start from our base $\nu\Lambda$CDM and we then explore extensions to this model ($N_{\rm eff}, w$).

A. Neutrino Masses, $\sum m_\nu$

Small scale structure formation is suppressed if fast-moving neutrinos comprise a significant part of the matter budget. Indeed, below the current free-streaming scale, the matter power power spectrum is suppressed in the presence of massive neutrinos by a factor $\Delta P/P \simeq -8 f_\nu$, where $f_\nu = \Omega_\nu/\Omega_m$ is the contribution of neutrinos to the total matter density. This effect is probed by CMB lensing and, since the active neutrino number densities are known in the standard model, this can be directly transformed into a constraint on the the sum of the neutrino masses $\sum m_\nu$. This is particularly exciting because we now know that neutrinos are massive, with a lower limit $\sum m_\nu > 50$ meV that emerges from oscillation experiments.

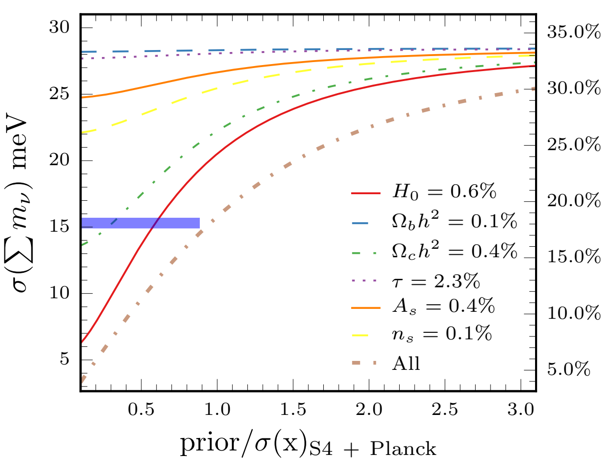

FIG. 1. Constraints on the sum of the neutrino masses as a function of priors on cosmological parameters. Each prior is expressed in units of the error that would be obtained internally from the CMB experiment; e.g., $\sigma(H_0)_{\rm CMB} = 0.6\%$, so the value of unity on the x-axis corresponds to imposing an external prior on H_0 of 0.6%. H_0 and $\Omega_c h^2$ are shown to be the most important external priors to improve neutrino mass constraints. We chose a fiducial value $\sum m_\nu = 85$ meV and we impose a Planck-pol prior . The blue horizontal line correspond to the CMB S4+BAO constraint of [11].

Fig. 1 shows the errors on $\sum m_\nu$ with a CMB S4 + Planck experiment as a function of external priors. Our Planck prior corresponds to the "Planck-pol" specifications of [16], where noise levels were approximated by scaling the current sensitivities according to the Planck Blue Book. Without any external priors (far right in fig-

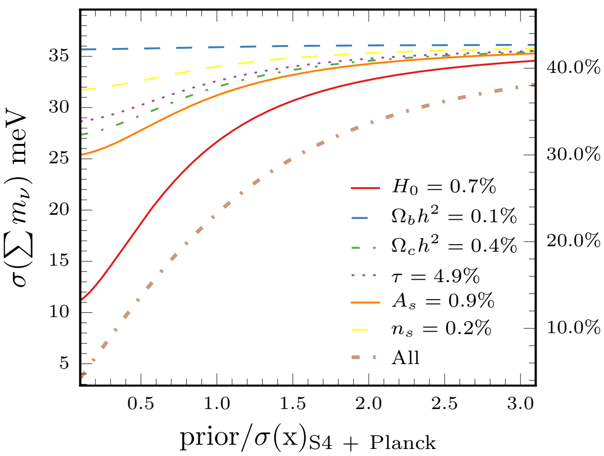

FIG. 2. Some as Fig. 1 but with $\boldsymbol{\ell_{\min} > 50}$. Here A_s and τ priors will also helps because of their degeneracy with $\sum m_\nu$.

ure), upcoming CMB experiments are poised to obtain 1-sigma limits on $\sum m_\nu$ of order 30 meV, corresponding to close to a 2-sigma detection even in the worst case scenario. Adding in external priors helps this significantly though, as discussed, for example, in [1, 15, 16]. There, the prior was described as "DESI BAO", that is a measurement of distances as a function of redshift from the baryon acoustic oscillation feature probed by DESI. Here, the same improvement is parametrized by a strong prior on $\Omega_c h^2$ and H_0, which indeed are constrained by these low-redshift distances. The improvement given by a prior on $\Omega_c h^2$ is also a consequence of the fact that the CMB lensing is sensitive to f_ν; therefore in order to constrain $\Omega_\nu h^2$ from this ratio we need to determine the matter energy density. A prior on H_0 helps with this as well since the effects of external priors are not completely independent. The position of the CMB peaks, well constrained by measurement, is a function $\Omega_c h^{2.93}$ [17] and for this reason too an external prior on H_0 will improve the internal reconstruction of $\Omega_c h^2$. As found in [1, 15, 16], the projected error falls below 20 meV with these priors.

It may be difficult to get the very low-l modes from the ground with CMB-S4, so in Fig. 2 we limit the modes to $\ell_{\min} > 50$. Together with the importance of H_0 and $\Omega_c h^2$ that we also observed in Fig. 1, in this case external priors on A_s and τ will also be helpful. Indeed, as recently pointed out by [16], a degeneracy exists between $\sum m_\nu$ and A_s and, as a consequence, between $\sum m_\nu$ and τ. This can be understood as follows. The CMB lensing reconstruction is noisy at scales larger than the neutrino free streaming scale (see Fig.5 of [1]). For this reason it is hard to measure the unsuppressed lensing CMB power spectra that is then compared to the small scale suppressed one to constrain the values of f_ν. The unsuppressed amplitude of the lensing spectrum is actually better constrained by the primordial amplitude A_s, which is probed by the primordial CMB. Unfortunately, the CMB spectra are sensitive to $A_s \exp^{-2\tau}$, so a measurement of the optical depth is required to infer A_s and therefore to tighten the constraints on $\sum m_\nu$. If CMB S4 will not be designed to measure scales $\ell < 50$ an external prior on the optical depth from future 21 cm surveys [18] or CMB satellite experiments like PIXIE [19] will be needed to fully exploit the CMB constraining power.

B. Relativistic Degrees of Freedom, N_{eff}

In the standard cosmology, three active neutrinos are thermally produced in the early universe. Were they to decouple well before the epoch of electron-positron annihilation, their energy density after their decoupling would be equal to $3 \times (7/8) \times (4/11)^{4/3}\,\rho_{\mathrm{cmb}}$, with the first factor capturing the contributions from the 3 active species; the second the difference between fermions and bosons; and the last the relative heating of the photons in the CMB by electron-positron annihilation. However decoupling is not a discrete event and occurs close to the time of electron-positron annihilation, so the neutrinos share a bit in the heating, with the factor of 3 replaced by $N_{\mathrm{eff}} = 3.046$. The additional fraction depends not only on well-known neutrino scattering rates but also finite temperature quantum corrections. Upcoming experiment have the potential to measure this tiny deviation of N_{eff} from 3. This will be an amazing test of our understanding of the Universe when it was about a second old. Furthermore any possible significant deviation from this value could be a hint of a different scenario not predicted by the standard model. As pointed out in [20], tight constraints on N_{eff} will allow us to rule out new particles with couplings that enabled them to thermalize early on but that decoupled when the temperature was above 100 GeV.

Fig. 3 shows projections for how well CMB-S4 will do at measuring N_{eff} as a function of priors on the other seven parameters (the sum of the neutrino masses must be included as a free parameter). With no priors on the other parameters, the projected 1-sigma error is about 0.02, suggesting a 2-sigma detection of the deviation from $N_{\mathrm{eff}} = 3$. The bottom-most curve shows that if all parameters were constrained externally, then the projected error on N_{eff} would improve significantly.

The sensitivity to relativistic degrees of freedom comes from the effect of extra species on the damping tail of the CMB anisotropies [9], both in temperature and polarization, so parameters that also strongly affect this part of the spectrum, like the slope n_s and the baryon density $\Omega_b h^2$, are the most degenerate with N_{eff}. As such, Fig. 3 shows that obtaining external priors on either of these would reduce the errors on N_{eff} to ensure a 3-sigma detection of the partial decoupling prediction.

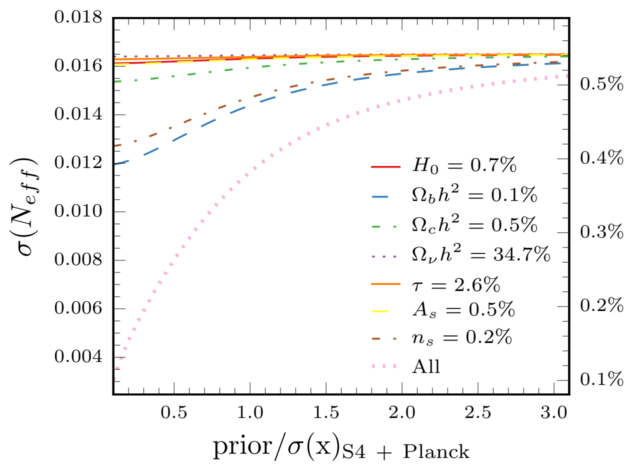

FIG. 3. Projected 1-sigma error on N_{eff} as a function of external priors. The parameters that would be most useful to constrain externally for the purposes of determining N_{eff} are n_s and $\Omega_b h^2$. The bottom-most curve shows the impact of imposing the given prior on *all* the other parameters simultaneously.

C. Dark Energy Equation of State, w

The CMB constrains the late-time dark energy equation of state in two ways. The observed CMB spectra are very sensitive to the distance to the last scattering surface which depends on w. Furthermore CMB lensing probes the growth of structure at late times which is different for different dark energy equations of state.

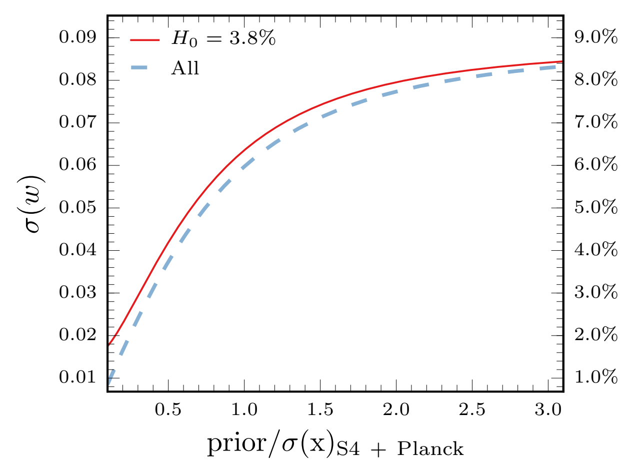

FIG. 4. Constraints on the dark energy equation of state w as a function of external priors on cosmological parameters. An external prior on H_0 will be crucial to improve the constraint.

Fig. 4 shows that without any priors, CMB experiments will not do much better than current constraints, which hover around 10%. However, an external constraint on the Hubble constant would improve the CMB constraints on w considerably. This is primarily due to the fact that CMB constrains w through a precise measurement of the comoving distance to the last scattering surface. However that can be kept fixed while varying w by accordingly changing the value of H_0 (and Ω_Λ to keep the universe flat). Indeed in a two dimensional (H_0, w) space the CMB constrained approximately lies on the region of constant large scattering surface distance as shown in Fig. 5. Notice that an external prior on H_0 two times more accurate than CMB S4 can improve the CMB constraint almost by a factor of 2 from a 8% error to a 4% level.

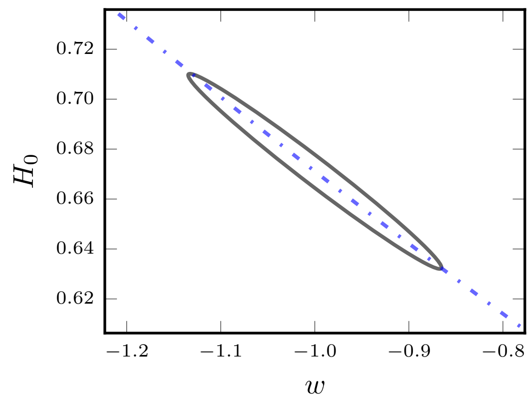

FIG. 5. Two dimensional constraint (CMB S4 + Planck) 1σ ellipse for w, H_0 space. The dashed (red) line correspond to values in the plane corresponding to the same distance to the last scattering surface.

IV. CONCLUSIONS

The design of the next CMB Stage IV experiments represents an exciting challenge. The possible reward is remarkable. We can finally put a tight constraint on the total mass of neutrinos with deeply implications on particle physics like, for example, the solution of the mass hierarchy problem. We will measure with great accuracy the number of relativistic species, both testing standard model prediction and, eventually, probing new physics. The same is true for dark energy. In that context, we will tighten the constraint on w,w_a getting closer to test the nature of the accelerated expansion of the universe. These results, however, will not be possible without an optimal synergy with external experiments observing a variety of others cosmological probes. Indeed this should be taken into account, especially during the CMB Stage IV planning phase. Understanding CMB weaknesses and the impact of external data can have a deep influence in determining the needed experimental specifications. For this reason in this work we explored the impact of external priors on CMB S4 cosmological constraints and, in particular, on the neutrino sector and dark energy parameters. We find:

$\sum m_\nu$: To improve the CMB constraint by a factor of two, low-redshift measurements will be needed. Indeed external prior on $\Omega_c h^2$ and H_0 at the level CMB S4 will reach alone can bring the error on the total mass of neutrinos from 30meV to below 20meV. Furterhemore, if the final design of CMB S4 will not include measurement at large scale ($\ell < 50$) external constraint on the primordial amplitude will be needed. Moreover, an external prior on τ will allow the CMB to constraint A_s internally thus strongly alleviating the problem.

$N_{\rm eff}$: The effective neutrino numbers will benefit from external prior on parameters that affect the shape of the damping tail like the spectral index and the baryons energy density.

w: In this case the improvement will mainly come by external prior on H_0. For example a 2% prior from external experiment will bring the reduce the error on w by a factor of two from 8% to 4%.

Comment on issue and future. This will be even more relevant when we open the parameter space. Will CMB + others be able to discover something CMB can't alone? $N_{\rm eff}$ extensions, mass splitting etc

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