External priors for the next generation of CMB experiments.

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The next generation of cosmic microwave background (CMB) experiments can dramatically improve what we know about neutrino physics, inflation, Dark Matter and Dark Energy. Indeed the low level of noise, together with improved angular resolution, will drastically increase the signal to noise of the CMB polarized signal as well as the reconstructed lensing potential of high redshift large scale structure. Projected constraints on cosmological parameters are extremely tight, but these can be improved even further with information from external experiments. Here, we examine quantitatively with a Fisher matrix approach, the extent to which external priors can lead to improvement in projected constraints from a CMB-Stage 4 experiment. We find that CMB S4 are quite powerful even on .. There are few exceptions.. like w.

I. INTRODUCTION

Since their early stages, Cosmic Microwave Background (CMB) experiments have been crucial in our understanding of the universe and they will maintain their role also in the near future, thanks to the unprecedented low level of noise and high resolution of their next Stage IV (S4) generation. The recent past is indeed reassuring. Every new generation of satellite experiments improved the level of sensitivity by almost a factor of ten compared to its predecessor, from the first generation instrument COBE to WMAP all the way to the current state of the art represented by Planck [1–5]. This lead us from the observation of the first peak of the CMB temperature power spectrum with COBE to a cosmic variance limited measurement of several peaks with Planck. With the improved sensitivity we extend our understanding of the universe scientific from a proof of its almost flat geometry to a well tested model ΛCDM with a percent level constraint on its parameters. The same success characterized ground CMB experiments, where the evolution from DASI [6] to SPT and ACT [7] [8] allowed us to measure the small scale damping tale of the CMB spectrum with increasing accuracy. These series of successes is far from its end. Indeed the next generation of CMB experiments (S4), now in its planning stage, will potentially measure the E-mode polarization with cosmic variance limited precision together with an order of magnitude improvement in B-mode measurement and lensing reconstruction. As it happened in the past, this new sensitivity together with the progress in the measurement of other cosmological probes will improve our understanding of several areas of astrophysics like dark matter, inflation, Dark Energy and neutrinos.

The cosmological dependence to neutrinos is primarily due to their contribution to the number of relativistic species $N_{\rm eff}$ in the early phase of the universe together with the role they play, given their non zero mass M_ν, into the late growth of cosmic structures. Because relativistic species, like neutrinos, are the main drivers of the cosmic expansion in the early Universe their number affects the expansion rate $H(z)$. This rate can be powerfully tested using the CMB, by carefully comparing the sound horizon scale, obtained from the CMB peaks positions, and the Silk damping scale (see [9], [10] and references therein). The future experiments, with their high resolution, will be able to push their measurements deeply into the damping tale of the CMB power spectrum and will unequivocally test the value $N_{\rm eff} = 3.046$ predicted by the standard model . The total mass of neutrinos, on the other hand, has a modest effect on the CMB because, for the range of masses allowed by recent constraints ($M_\nu < 230$ meV from [2]), neutrinos are still relativistic at the last scattering surface. As other low redshift effects however, massive neutrinos modify the CMB by altering the growth of the large scale structure responsible for the the lensing of the CMB photons. Different neutrino masses consequently lead to different CMB lensing. Stage IV experiment will measure small scales temperature and polarization anisotropies with low noise, drastically improving the lensing reconstruction. This will turn into a precise constraints of the sum of neutrino masses with a possible hint of the hierarchies of the individual masses. Quantitatively, CMB alone will constrain the number of relativistic degree of freedom $N_{\rm eff}$ with a 1% precision and the total mass of neutrinos at 60% with a factor of two improvement on M_ν when priors from baryon acoustic oscillations (BAO) data are introduced [11]. The future generation of CMB experiments will be a big step in the understanding of the neutrino sector.

As previously mentioned, CMB is also sensitive to the properties of dark energy (see [12]). Thanks to the current generation of experiments we know with extraordinary accuracy its energy density. The challenge for the next generation is to reveal the nature of this mysterious component. For example, a crucial step to rule out part of the proposed models will be to investigate any

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possible deviations in the equation of state, the ratio of pressure and energy density, from the value $w = -1$ predicted by a cosmological constant. Dark energy affects the CMB because it alters the universe's expansion and it consequently changes the distance to the last scattering surface. Furthermore different DE models lead to a different growth of large scale structure which are tested by CMB lensing. Thanks to this sensitivity to cosmological structures present all the way from us to the last scattering surface, CMB will also be a powerful probe of any time dependence of the dark energy equation of state. However, dark energy properties are strongly degenerate with other geometrical parameters like H_0 and Ω_k. Similarly to the neutrino sector, to get competitive DE constraints, CMB experiments will have to rely on external prior coming from BAO and supernova experiments.

The crucial importance of feedback from external experiments is true not only for the dark energy and neutrino sector. The future of cosmology will hinge on the ability of combining different probes. In this regard CMB makes no exception: external priors will be fundamental to improve the already tight CMB constraints on cosmological parameters. In particular CMB will really benefit from experiments like large scale structure clustering and weak lensing, BAO targeted experiments, and supernovae. To improve the synergy between different experiments and to guide the plan of future experiments, in this work we study in detail the dependance of CMB future constrains on the external priors assumed. For all the parameters of the standard ΛCDM with massive neutrinos and $w \neq -1$ dark energy model we quantify, using a Fisher matrix approach, the CMB constraints as a function of external priors in each of the cosmological parameters.

This paper is organized as follow: in §II we introduce the technique and assumptions we use to derive the effect of external priors on the CMB parameter constraints. In §III we will describe our results and we then conclude with a discussion of them in §IV.

II. ASSUMPTIONS AND METHODS

In this paper we want to measure quantitatively the effect of external priors on the cosmological parameters constraints derived from S4 CMB experiments. We start this section in section II A by introducing the Fisher matrix formalism, a simple but powerful technique widely used to forecast future experimental constraints. To apply this technique, we need to specify our cosmological model (ΛCDM plus neutrinos and dark energy extensions in our case) as well as a set of fiducial parameters and the specifications of S4 CMB experiments. These will be presented in section II B.

A. Fisher Matrix formalism

To estimate errors on cosmological parameters we follow the Bayes theorem that relates the likelihood to measure a set of data given the parameters of the model $\mathcal{L}(d|\theta)$, to what we want: the posterior probability of those parameters given the data, $\mathcal{L}(\theta|d)$. These two are related by the prior probability of the parameters $P(\theta)$ through:

$$\mathcal{L}(\theta|d) \propto \mathcal{L}(d|\theta)P(\theta), \tag{1}$$

where, as usual, we have neglected the probability of the data $P(d)$. These general framework can be simplified in our case. We will focus not on an entirely general posterior distribution, but we will assume that the likelihoods are gaussian. Furthermore we will not derive the global shape of the likelihood but we will obtain parameters errors by studying small perturbations around its maximum. These are the two basic assumptions of the Fisher-matrix approach.

Regarding the first assumption, we recognize the fact that, even if the gaussian approximation has been shown to be appropriate most of the time, some problems have been found in other cases [13]. Despite this, we do not need the level of accuracy that will require a careful modeling of the likelihoods, because we are looking for the general behavior of parameters errors as a function of external priors. Moreover the gaussian approximation gets better at smaller scales (high ℓ in Fourier space) which, with the exception of the optical depth parameters τ, is where most of the constraining power of the CMB is coming from.

Secondly we will assume to know the true "fiducial" parameters that maximize the likelihood $\mathcal{L}(\theta|d)$ and we will get the errors on those from the likelihood curvature around the fiducial values. Indeed, as usual, we define the fisher matrix elements as the curvature:

$$F_{ij} \equiv -\left\langle \left. \frac{\partial^2 \log \mathcal{L}}{\partial\theta_i \partial\theta_j} \right|_{\boldsymbol{\theta}=\boldsymbol{\theta_0}} \right\rangle, \tag{2}$$

where $\theta_{i,j}$ represents two of the parameters and $\boldsymbol{\theta_0}$ is the parameters values array that, by definition, maximizes the likelihood. With the usual definition [] $< a^X_{\ell m} a^Y_{\ell' m'} >= \delta_{\ell\ell'}\delta_{mm'}C^{X,Y}$, where $a^X_{\ell m}$ represent the spherical harmonics coefficients of the field X, the Fisher matrix of a CMB experiment can be rewritten as:

$$F_{ij} = \sum_\ell \frac{2\ell+1}{2} f_{sky} \mathrm{Tr}\left(\boldsymbol{C}_\ell^{-1}(\theta) \frac{\partial \boldsymbol{C}_\ell}{\partial\theta_i} \boldsymbol{C}_\ell^{-1}(\theta) \frac{\partial \boldsymbol{C}_\ell}{\partial\theta_j} \right). \tag{3}$$

In this work we will constrain cosmological parameters with the CMB temperature and E mode polarization together with the reconstructed lensing potential. For this reason, $\boldsymbol{C}_\ell$ in Eq. (3) is:

$$\mathbf{C}_\ell \equiv \begin{pmatrix} C_\ell^{TT} + N_\ell^{TT} & C_\ell^{TE} & C_\ell^{Td} \\ C_\ell^{TE} & C_\ell^{EE} + N_\ell^{EE} & 0 \\ C_\ell^{Td} & 0 & C_\ell^{dd} + N_\ell^{dd} \end{pmatrix}. \tag{4}$$

Note that we are neglecting the term $C_\ell^{E\phi}$. As also noticed in previous literature (like [10, 11]) this term contains very little information while adding possible numerical issues. The term N_ℓ^X represents the instrumental noise power of the specific experiment and will be discussed in section II B. The power of the Fisher approach descend from the Cramer-Rao inequality that relates the error on the parameter i, marginalized over all the other parameters, σ_i, to the Fisher matrix as:

$$\sigma_i \equiv \sigma(\theta_i) = \sqrt{(\mathbf{F^{-1}})_{ii}}. \tag{5}$$

Once F_{ij} is computed following Eq. (3) it is straightforwards to get the error σ_i from Eq. (5). Furthermore in this context it is easy to introduce external priors on cosmological parameters. Indeed to introduce priors we simply need to add the Fisher matrixes of the external experiments (before we perform the matrix inversion of Eq. (5)), i.e.:

$$F_{\text{total}} = F_{\text{CMB}} + \sum F_{\text{external}}. \tag{6}$$

In the same way we can add a prior on a single cosmological parameter just by modyfing the correspondent matrix element. For example, a 1% prior on H_0 can be obtained by:

$$F_{H_0H_0} \rightarrow F_{H_0H_0} + \frac{1}{(1\% \times H_{0,\text{fid}})^2}. \tag{7}$$

We chose a Fisher matrix approach because it is able to forecast future experiments performances without generating mock data, together with the ease of including external priors. This technique however introduces also some technical difficulties. Indeed it is known [14] that increasing the number of parameters used in the analysis can lead to numerical issues (see also [15] in the gravitational waves context were several parameters are used). Fisher matrix indeed can become ill-conditioned: a small change in the fisher matrix led to a big change in its inverse. Because we use Eq. (5) this can be a problem for error estimation. Even if other methods have been used [14, 16] Fisher matrices are still the standard method used to forecast future constrains [11]. We carefully try to avoid any possible source of errors in computing the elements of Eq. (3) and in the matrix inversion of Eq. (5). We compute the derivatives in Eq. (3) using a 5 points formula:

$$\left.\frac{\partial C}{\partial \theta}\right|_{\theta_0} \sim \frac{-C(\theta_0+2h)+8C(\theta_0+h)-8C(\theta_0-h)+C(\theta_0-2h)}{12h}. \tag{8}$$

This high order definition allow us to use a bigger gap h around the fiducial parameters θ_0. As a consequence the differences of power spectra corresponding to different values are big enough to make possible numerical accuracy issues in computing C negligible. We also test the robustness of this calculation by changing the gap h in the range $2-7\%$ of the correspondent θ_0 without noticing any significant change in the results. Furthermore we compare our results to similar previous work in the literature [17] obtaining a perfect agreement. Lastly we implement the same technique of [14] to avoid possible numerical instability in the marginalizing procedure. This allow us not to invert the entire matrix when we want to marginalize over a set of parameters, in order to minimize numerical issues and conserve parameters degeneracies as much as possible.

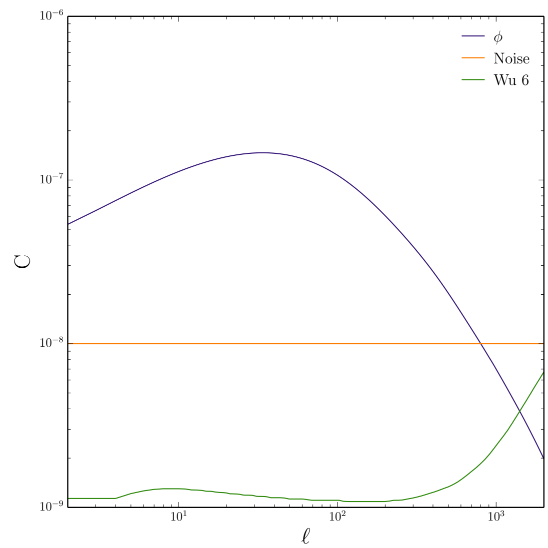

FIG. 1. Lensing potential power spectrum use in this work has a signal to noise bigger than one up to $\ell \simeq 800-1000$. In the figure: the deflection power spectrum for our fiducial cosmology together with the two examples of lensing reconstruction noise N^ϕ used in this work.

B. CMB S4 experiments specifications.

In this subsection we describe the assumptions we made in calculating the elements that goes in Eq. (3) and, in particular, the power spectra C_ℓ and the noise power N_ℓ. Firstly we parametrize our cosmology using a flat $\nu\Lambda$CDM universe. We allow a set different Dark Energy models by introducing the equations of state parameter w as a varying parameter. We chose our fiducial parameters following Table 2 of *Planck* best fit [18], i.e. $\Omega_c h^2 = 0.12029$, $\Omega_b h^2 = 0.022068$, $A_s = 2.215 \times 10^{-9}$ at $k_0 = 0.05$ Mpc^{-1}, $n_s = 0.9624$, $\tau = 0.0925$, and $H_0 = 67.11$ km/s/Mpc. Regarding the ΛCDM extension, we chose a neutrino energy densities $\Omega_\nu h^2$=0.0009, which corresponds to $M_\nu \simeq 85$ meV, a standard $N_{\text{eff}} = 3.046$ and a dark energy equation of state, $w = -1$. Given the model, we use CAMB [19] to compute the power spectra C_ℓ at the fiducial values and at those needed to compute derivatives using Eq. (8). Notice that while we vary one

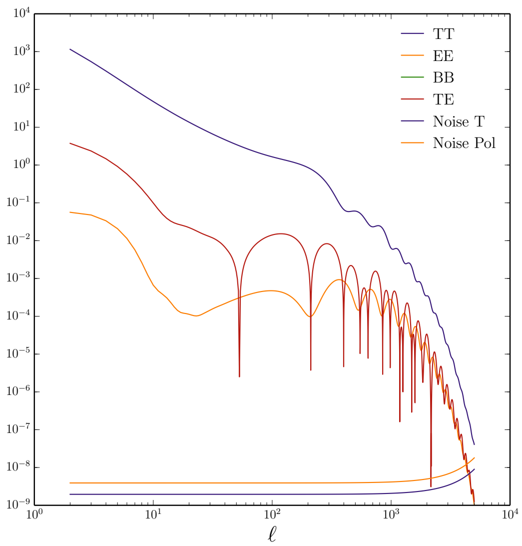

FIG. 2. The next generation $C^{T,E}$ used in this work are almost cosmic variance limited. In the figure: CMB power spectrum for our fiducial cosmology together with the instrumental noise used in this work.

parameter in Eq. (8) we keep all the others fixed with the exception of Ω_Λ which is always changed in order to keep the universe flat ($\Omega_k = 0$).

The instrumental noise power $N_\ell^{T,E}$ and the lensing reconstruction N_ℓ^{ϕ} in Eq. (3) correspond to the expected level for the next generation of CMB experiments (S4). For the temperature and E-mode polarization of the CMB, together with the improved depth and resolution we also assume that large scale foregrounds, like dust, are under control or negligible. We deal with the presence of point sources poisson noise in the temperature signal by simply discarding all the small scales modes with $\ell > \ell_{\mathrm{T,max}} = 3000$. The remaining source of noise, the instrumental noise, is added to the power spectrum in the usual way:

$$N_\ell^X = s^2 \exp\left(\ell(\ell+1)\frac{\theta_{\mathrm{FWHM}}^2}{8\log 2}\right), \tag{9}$$

where θ^2_{FWHM} is the FWHM of the experiment's beam and s represents the instrumental white noise. We decide to use a level of noise $s = 1.5$ μK-arcmin for $X = T$ and a beam of $\theta_{\mathrm{FWHM}} = 1$ arcmin (PRELIMINARY). Note that 1.5 μK-arcmin is the noise in temperature and we need $s \to s \times \sqrt{2}$ in the case of polarization $XX' = \{EE, BB\}$.

Together with E and T we will use the information contained in the lensing potential ϕ as it is reconstructed from the CMB. The lensing potential represents the integration along the line of site of the gravitational potential and it leaves its signature in the CMB, both in temperature and polarization, by bending the trajectory of CMB photons. This introduces non gaussianities that couple different, otherwise independent, CMB modes and it can be reconstructed using a quadratic estimator technique [20, 21]. For the noise $N_\ell^{\phi\phi}$ associated to the reconstructed ϕ we follow the non-iterative technique of [20, 21].

III. RESULTS

Our main results are shown in Fig. 4 Fig. 3, Fig. 5

H_0	M_ν	$\Omega_{bc}h^2$	$\Omega_b h^2$	τ	A_s	n_s
0.80%	40.92%	0.44%	0.11%	3.075%	0.53%	0.18%

TABLE I. How well we do constrain separate parameters with this data without any external prior? Things to notice: this is done with the Zhen test parameters ($\ell < 3000$ and no error on lensing). Now if we trust it CMB alone can get a 0.8% error on H_0 thus I am not surprised if a prior on H would not help. However what do we think about it? is it really CMB better than SN. People will not agree on that. τ will probably improve and also Mν from BAO may help improving parameters.

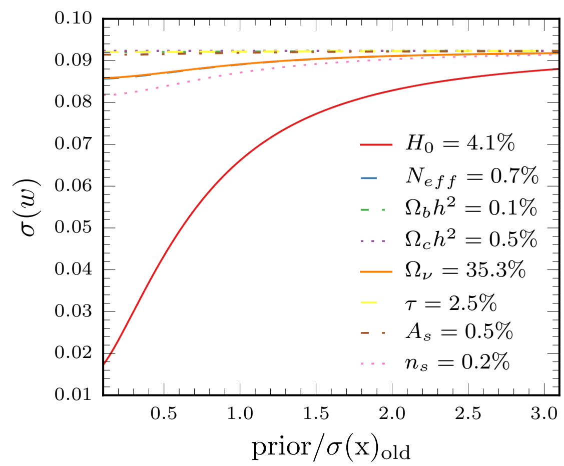

FIG. 3. w costraints will be dramatically improved by H_0 external priors. In the figure: on the y axis we have the relative error on w as a function of the prior we impose on the other parameters. These priors on the x-axis are measured relative to the error on that parameters from CMB Stage 4 experiments alone.

IV. CONCLUSIONS

–We find that...

–This means that supernova experiments CMB spatial distortions and large scale structure can/will/should

–Future works

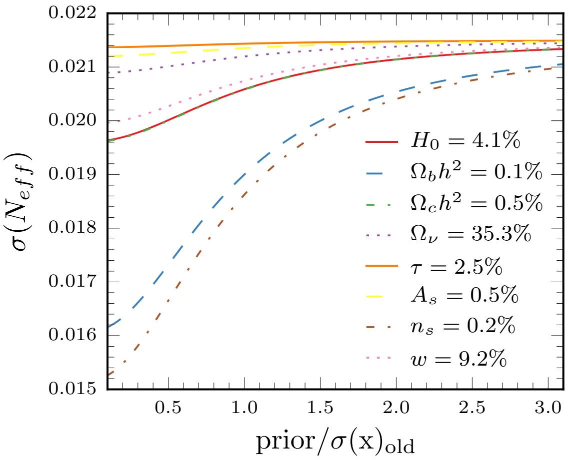

FIG. 4. N_{eff} costraints mainly benefit from Ω_b and n_s external priors.

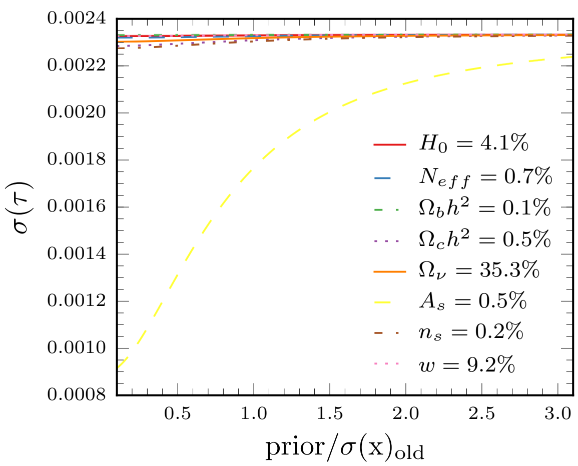

FIG. 5. τ constraints are limited by the well known degeneracies with A_s. An external prior on A_s already tightly constrained from CMB alone can still be beneficial.

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