5) LETS START BY INTRODUCING THE SLACK VARIABLES

$$\min_{f} \left[\frac{1}{m} \sum_i (1 - f(x_i))_{\geq 0} + \frac{1}{2C} \|f\|^2 \right] \to$$

$$① \min_{f;\, \xi_{i..m}} \frac{1}{2C} \|f\|^2 + \frac{1}{m} \sum_i \xi_i \quad \text{s.t} \quad y_i f(x_i) \geq 1 - \xi_i \quad \xi_i > 0$$

NOW BECAUSE OF THE REPRESENTER THEOREM WE KNOW THAT THE SOLUTION WILL BE OF THE FORM $f(x) = \sum_i \gamma_i \, k(x_i; x)$

SO (MULTIPLYING ALL BY C, STILL THE SAME MAX)

$$② \min \frac{1}{2\cancel{C}} \sum_i \sum_j \gamma_i \gamma_j k(x_i; x_j) + \frac{C}{m} \sum_i \xi_i \quad \text{s.t} \quad y_i \sum_i \gamma_i k(x_i; x) \geq 1 - \xi_i \quad \xi_i > 0$$

THIS IS BECAUSE $\langle f; f \rangle_H = \langle \sum \gamma_i K_{x_i}; \sum \gamma_j K_{x_j} \rangle = \sum_i \sum_j \gamma_i \gamma_j \langle K_{x_i}; K_{x_j} \rangle_H$

$$= \sum_i \sum_j K(x_i x_j) \to$$

$\boxed{\text{WHERE WE USE LINEARITY AND THE DEFINITION OF A RKHS}}$

FROM ② USING LAGRANGIAN MULTIPLIERS (β, α)

$$\mathcal{L}(\beta; \xi; \alpha; \gamma) = \frac{C}{m} \sum_i \xi_i + \frac{1}{2} \gamma^t K \gamma - \sum \alpha_i y_i \left(\sum_j \gamma_j k(x_i x_j) - 1 + \xi_i \right) - \sum_i \beta_i \xi_i$$

WITH $\beta > 0$

DUAL IS $\arg\max_{\alpha; \beta > 0} \inf_{\gamma \xi} \mathcal{L}(\beta; \xi; \alpha \gamma)$

1) $\dfrac{\partial \mathcal{L}}{\partial \gamma} = 0 \to \gamma_i = \alpha_i y_i$

2) $\dfrac{\partial L}{\partial \xi_i} = 0 \quad \dfrac{C}{m} - \alpha_i - \beta_i = 0 \to 0 \leq \alpha_i \leq C/m$

PLAGGING $\beta_i = \frac{1}{m} - \alpha_i$ IN $\mathcal{L}$ WE HAVE

$$\arg\max \inf_{\gamma} \mathcal{L}(\gamma; \alpha) - \frac{1}{2} \gamma^t K \gamma + \sum_i \alpha_i \left(1 - y_i \sum_j k(x_i; x_j) \gamma_j \right)$$

INDEED THE ξ_i FACTOR CANCELS

NOW WE PLUG IN $\gamma_i = \alpha_i Y_i$ AND WE GET FINALLY

$$\boxed{\arg\max_{\alpha \geq 0} L(\alpha) = \sum_i \alpha_i - \frac{1}{2} \underbrace{\sum \alpha_i Y_i K(x_i ; x_J) \alpha_J Y_J}}$$

$$\frac{1}{2} \alpha^t \operatorname{diag}(Y) K \operatorname{diag}(Y) \alpha$$