Super Sample Lensing

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We treat the effects of CMB lensing of the power spectrum by modes that are larger than the survey area.

Following Takada & Hu, this effect is described by squeezed trispectrum configurations. Let us start with the flat-sky trispectrum of the lensed temperature field

$$\langle \Theta(\mathbf{l}_1)\Theta(\mathbf{l}_2)\Theta(\mathbf{l}_3)\Theta(\mathbf{l}_4)\rangle_c = (2\pi)^2\delta(\mathbf{l}_1+\mathbf{l}_2+\mathbf{l}_3+\mathbf{l}_4)T(\mathbf{l}_1,\mathbf{l}_2,\mathbf{l}_3,\mathbf{l}_4). \tag{1}$$

To leading order in the lenses, the lensing trispectrum is given by

$$T(\mathbf{l}_1,\mathbf{l}_2,\mathbf{l}_3,\mathbf{l}_4) = C_{l_1}C_{l_3}C^{\phi\phi}_{|\mathbf{l}_1+\mathbf{l}_2|}\left[(\mathbf{l}_1+\mathbf{l}_2)\cdot\mathbf{l}_1\right]\left[(\mathbf{l}_3+\mathbf{l}_4)\cdot\mathbf{l}_3\right] + \text{perm} \tag{2}$$

where perm means all permutations of the $\mathbf{l}_i$ and the delta function condition sets $\mathbf{L} \equiv \mathbf{l}_1 + \mathbf{l}_2 = -(\mathbf{l}_3+\mathbf{l}_4)$. The covariance between binned estimators of the power spectrum in shells around l and l' is

$$\mathrm{Cov}[C_l, C_{l'}] = \frac{1}{A^2}\int\frac{d^2l}{A_l}\int\frac{d^2l'}{A_l'}\int\frac{d^2L}{(2\pi)^2}|W(L)|^2 T(l,-l+L,l',-l'-L) + \text{subsurvey terms} \tag{3}$$

where A is the survey area, $A_l = 2\pi l dl$ is the shell area, W is the transform of the sky mask assumed to be 1 in the measured region and 0 in the unmeasured. Note that the window connects multipoles that are separated by less than its fundamental mode and so the covariance no longer depends only on degenerate quadrilaterals with $(\mathbf{l},-\mathbf{l},\mathbf{l}',-\mathbf{l}')$.

Now let us evaluate the squeezed limit. In this case we are interested in lens modes $L \ll l_1, l_3$. We therefore expand

$$\begin{aligned} l_2 &= l_1 - \frac{\mathbf{l}_1\cdot(\mathbf{l}_1+\mathbf{l}_2)}{l_1} \\ l_4 &= l_3 - \frac{\mathbf{l}_3\cdot(\mathbf{l}_3+\mathbf{l}_4)}{l_3} \end{aligned} \tag{4}$$

so that

$$\begin{aligned} C_{l_2} &= C_{l_1}\left[1 - \frac{\partial \ln C_l}{\partial \ln l}\Big|_{l_1}\frac{\mathbf{l}_1\cdot(\mathbf{l}_1+\mathbf{l}_2)}{l_1^2}\right] \\ C_{l_4} &= C_{l_3}\left[1 - \frac{\partial \ln C_l}{\partial \ln l}\Big|_{l_3}\frac{\mathbf{l}_3\cdot(\mathbf{l}_3+\mathbf{l}_4)}{l_3^2}\right] \end{aligned} \tag{5}$$

and rewrite

$$\begin{aligned}(\mathbf{l}_1+\mathbf{l}_2)\cdot\mathbf{l}_2 &= -(\mathbf{l}_1+\mathbf{l}_2)\cdot\mathbf{l}_1 + (\mathbf{l}_1+\mathbf{l}_2)\cdot(\mathbf{l}_1+\mathbf{l}_2) \\ (\mathbf{l}_3+\mathbf{l}_4)\cdot\mathbf{l}_4 &= -(\mathbf{l}_3+\mathbf{l}_4)\cdot\mathbf{l}_3 + (\mathbf{l}_3+\mathbf{l}_4)\cdot(\mathbf{l}_3+\mathbf{l}_4) \end{aligned} \tag{6}$$

Averaging over the orientation of the modes only quadratic terms in the dot products survive giving

$$\begin{aligned} T(\mathbf{l}_1,\mathbf{l}_2,\mathbf{l}_3,\mathbf{l}_4) &= \frac{1}{4} C_{l_1} C_{l_3} L^4 C_L^{\phi\phi} \frac{\partial \ln C_l}{\partial \ln l}\Big|_{l_1} \frac{\partial \ln C_l}{\partial \ln l}\Big|_{l_3} + C_{l_1} C_{l_3} L^4 C_L^{\phi\phi} \\ &= \frac{1}{4} C_{l_1} C_{l_3} L^4 C_L^{\phi\phi} \frac{\partial \ln l^2 C_l}{\partial \ln l}\Big|_{l_1} \frac{\partial \ln l^2 C_l}{\partial \ln l}\Big|_{l_3} \end{aligned} \tag{7}$$

We can now interpret this expression physically: the factor $L^4 C_L^{\phi\phi}/4$ is the convergence power spectrum in the flat sky approximation. The derivatives of $l^2 C_l$ represent the DC mode of the convergence shifting the angular scale of the dimensionless temperature power spectrum. In other words each realization of a finite survey has a net mean convergence which shifts the whole power spectrum in angular scale from the ensemble mean. This simple effect masquerades as a covariance between modes since all the multipoles below the survey shift in a correlated way.

Thus the trispectrum model for the super sample mode covariance is

$$\mathrm{Cov}[C_l, C_{l'}] = C_l C_{l'} \frac{\partial \ln l'^2 C_{l'}}{\partial \ln l'} \frac{\partial \ln l^2 C_l}{\partial \ln l} \sigma_\kappa^2 + \text{subsurvey terms} \tag{8}$$

where σ_κ^2 is the variance of the convergence field in the survey mask

$$\sigma_\kappa^2 = \frac{1}{A^2} \int \frac{d^2 L}{(2\pi)^2} \frac{L^4}{4} C_L^{\phi\phi} |W(L)|^2 \tag{9}$$

where the factors of A come from convention for the normalization of W such that $\lim_{L\to 0} W(L)/A = 1$.

To see if this is an important effect, we can compare this to the precision with which a parameter that similarly shifts the angular scale of the power spectrum can be measured with Gaussian statistics. Since θ_* serves this purpose for the acoustic peaks, we can compare to that as a proxy. Alternately, we can construct an artificial parameter

$$C_l = C_l^{\rm fid} \left[1 + \frac{\partial \ln l^2 C_l^{\rm fid}}{\partial \ln l} s \right], \tag{10}$$

do a Fisher estimate of σ_s^2 (no other parameters margignalized) and compare it to σ_κ^2. Preliminary estimates suggest that it is marginally important at best.