19MS165

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Q9 $H = \frac{-\hbar^2}{2MR^2} \frac{\partial^2}{\partial \phi^2}$, $H\psi = E\psi$

$\therefore \quad \psi(x) = A \sin\left(\frac{\hbar^2}{E 2MR^2} \phi + \theta\right)$

also we know boundary condition

$$\psi(n) = \psi(2\pi + n)$$

$$\psi'(n) = \psi'(2\pi + n)$$

using these boundary condition

$$\frac{\hbar^2}{2EI} \cdot 2\pi = 2n\pi$$

$$\Rightarrow \quad E_n = \frac{\hbar^2}{2In}$$

~~de~~ now

$$\int_0^{2\pi} \psi^2 d\phi = 1$$

$$1 = A^2 \int_0^{2\pi} \sin^2(k\phi + \theta) \, d\phi \quad , \quad k = 2n\pi$$

$$= A^2 \pi$$

$$\therefore \quad A = \frac{1}{\sqrt{\pi}}$$

$$\therefore \psi_n = \frac{1}{\sqrt{\pi}} \sin(2n\phi + \theta)$$

$$E_n = \frac{\hbar^2}{2 I n}$$

b)

$$U \times \frac{P_\phi}{\hbar} \times U^{-1}$$

$U = e^{-ia\phi}$

$U^{-1} = e^{ia\phi}$

$$U \frac{P_\phi}{\hbar} f(\phi) e^{ia\phi}$$

$$= -i a \left(f'(\phi) + f(\phi) \right) e^{ia\phi}$$

$$= -i \left(f'(\phi) + f(\phi) \right)$$

$$= \left[\frac{P_\phi}{\hbar} + i \right] f(\phi)$$

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Q15 $\cancel{|n\rangle = \sqrt{\frac{2}{h}}}$

Q16 we Know

$$x = \frac{\sqrt{2\hbar m\omega}}{2m\omega} \times (a_+ + a_-)$$

$$p = \frac{\sqrt{2\hbar m\omega}}{2} i \times (a_+ - a_-)$$

i) $$\langle x^2 \rangle = \int_{-\infty}^{\infty} \langle n|x^2|n\rangle \, dx$$

$$= \frac{\hbar}{2m\omega} \times \int_{-\infty}^{\infty} \langle n|(a_+^2 + a_+a_- + a_-a_+ + a_-^2)|n\rangle dx$$

we observe

$$\int_{-\infty}^{\infty} \langle n|a_+^2|n\rangle dx = \sqrt{n+1}\sqrt{n+2}\int_{-\infty}^{\infty} \langle n|\cdot|n+2\rangle dx = 0$$

similarly

$$\int_{-\infty}^{\infty} \langle n|a_-^2|n\rangle dx = \sqrt{n}\sqrt{n-1}\int_{-\infty}^{\infty} \langle n|\cdot|n-2\rangle dx = 0$$

also

$$\int_{-\infty}^{\infty} \langle n|a_+a_-|n\rangle = \sqrt{n}\int_{-\infty}^{\infty} \langle n|a_+|n-1\rangle dx$$

$$= \sqrt{n}\sqrt{n}\int_{-\infty}^{\infty} \langle n|\cdot|n\rangle dx$$

$$= n$$

~~similarly~~

$$\int_{-\infty}^{\infty} \langle n | a_- a_+ | n \rangle = \sqrt{n+1}\sqrt{n+1} \int_{-\infty}^{\infty} \langle n | n \rangle dx$$

$$= n+1$$

$$\therefore \langle x^2 \rangle = \frac{\hbar}{2m\omega} (2n+1)$$

ii) ~~follows~~ in similar fashion

$$\langle p^2 \rangle = -\frac{\hbar m\omega}{2} \int_{-\infty}^{\infty} \langle n | a_+^2 - a_+ a_- - a_- a_+ + a_-^2 | n \rangle dx$$

$$= \frac{\hbar m\omega (2n+1)}{2}$$

iii) now

$$\frac{m\omega^2}{2} \langle x^2 \rangle + \frac{\langle p^2 \rangle}{2m}$$

$$= \frac{m\omega^2}{2} \times \frac{\hbar}{2m\omega} (2n+1) + \frac{\hbar m\omega (2n+1)}{4m}$$

$$= \frac{\hbar\omega}{4} (2n+1) + \frac{\hbar\omega}{4} (2n+1)$$

$$= \frac{\hbar\omega (2n+1)}{2}$$ — Ans

12 $\hat{H} = \frac{p^2}{2m} + V = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V$

$H|n\rangle = E|n\rangle$

$$H x f(x) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(x f(x)\right) + V f(x) x$$

$$= \frac{-\hbar^2}{2m}\frac{\partial}{\partial x}\left(x f'(x) + f(x)\right) + V x f(x)$$

$$= \frac{-\hbar^2}{2m}\left(x f''(x) + 2f'(x)\right)$$
$$+ V x f(x)$$

$$x H f(x) = x\left(-\frac{\hbar^2}{2m} f''(x) + V f(x)\right)$$

$$\therefore [H, x] f = (Hx - xH) f$$

$$= -\hbar^2$$