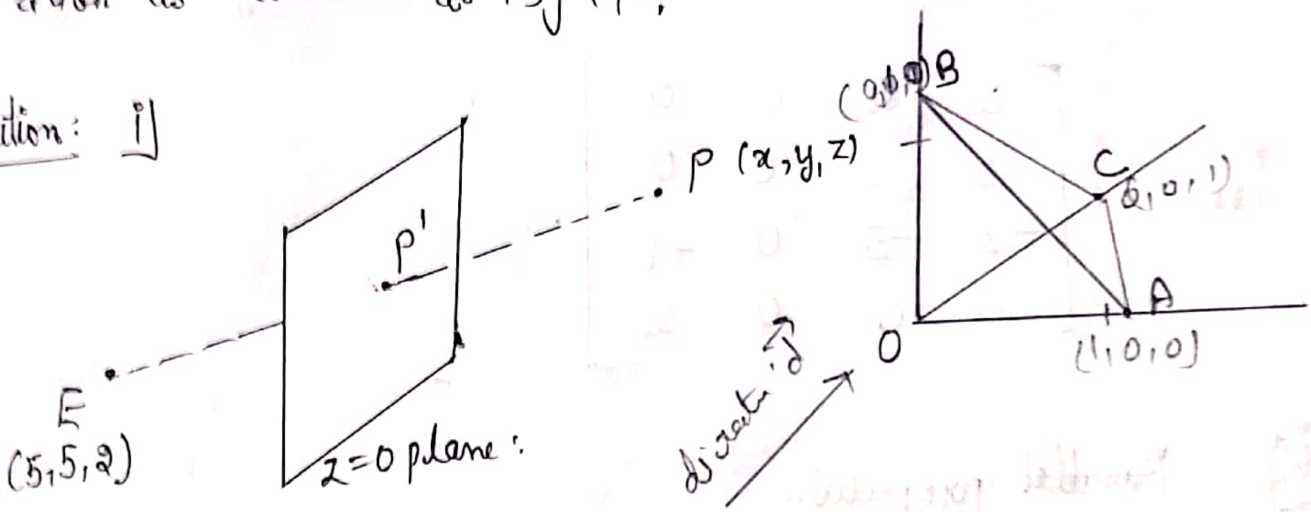
Question-10: Assume that the eye is positioned at (5,5,2). Using back face removal algorithm, find the visible faces of the convex polygonal object defined by the vertices: $O = O(0,0,0)$, $A = A(1,0,0)$, $B = B(0,1,0)$ and $C = C(0,0,1)$ having four faces?

I) Apply perspective projection where the eye is positioned at (5, 5, 2)

II) By denoting the unit vectors along the X-, Y-, Z- axes by i, j and k respectively, apply a parallel projection where the direction of projection d is taken as $d = -2\hat{i}+3\hat{j}+\hat{k}$.

Solution: I)



Let the projection plane is $z = 0$

If the equation of line is $\vec{OP} + t \cdot \vec{EP}$ then.

$\vec{OP'} = \vec{OP} + t \cdot \vec{EP}$

$\langle x', y', z' \rangle = \langle x, y, z \rangle + t \langle (x-5), (y-5), (z-2) \rangle$

$\langle x', y', z' \rangle = \langle x + t(x-5),\ y + t(y-5),\ z + t(z-2) \rangle$

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$\because$ P' lies on z = 0 plane

$\Rightarrow$ $z' = 0$ for.

$z + t(z-2) = 0$

$t = \frac{z}{2-z}$.

then $x' = x + t(x-5) = x + \frac{z(x-5)}{2-z} = \frac{2x-5z}{2-z}$

$y' = y + t(y-5) = y + \frac{z(y-5)}{(2-z)} = \frac{2y-5z}{2-z}$.

$z' = z + t(z-2) = z + \frac{z}{(2-z)} \cdot (z-2) = 0$

hence the projection matrix is: (in homogenous system)

$$P_m = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -5 & -5 & 0 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

ii) ~~Parallel projection~~

Then if $O \to O'$, $A \to A'$, $B \to B'$, $C \to C'$

s.t

$O' = O \cdot P_m = [0,0,0,1] \cdot P_m = [0,0,0,2]$ or $[0,0,0]$ in E.S.

$A' = [1,0,0,1] \cdot P_m = [2,0,0,2] = [1,0,0]$ in Euclid

$B' = [0,1,0,1] \cdot P_m = [0,2,0,2] = [0,1,0]$

$C' = [0,0,1,1] \cdot P_m = [-5,-5,0,1] = [-5,-5,0]$ in Euclid

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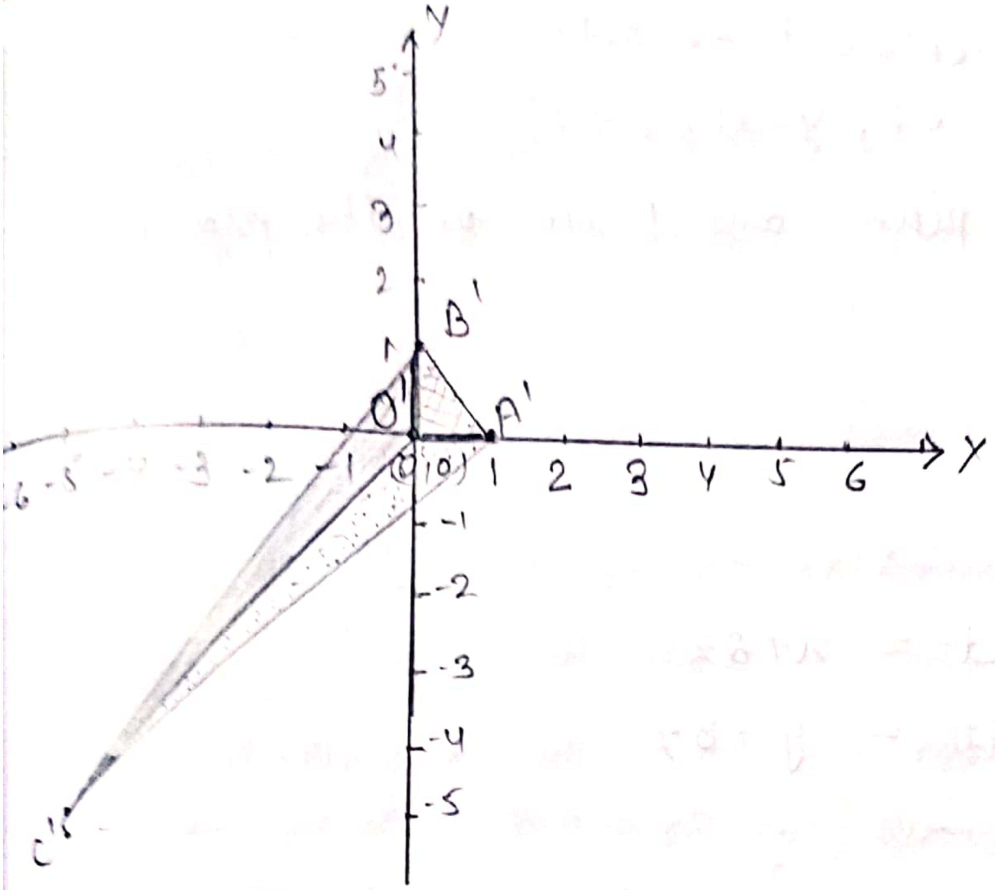
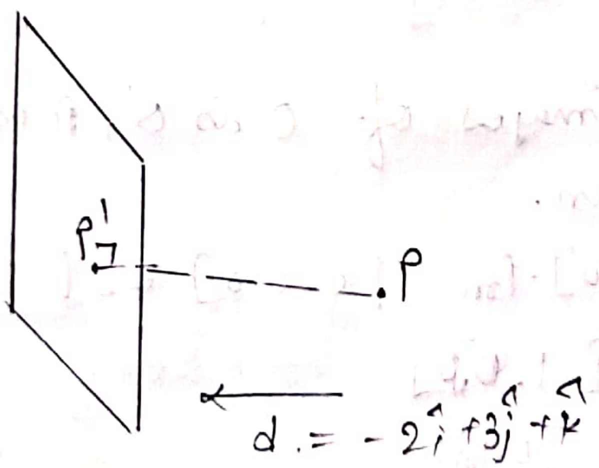


Here visible faces are $O'A'B'O'$, $O'A'C'O'$, $O'B'C'O'$.
Back face $A'B'C'$ is not visible.

ii] <u>Parallel projection:</u>

$$\vec{d} = -2\hat{i} + 3\hat{j} + \hat{k}$$



if the equation of line segment PP' is $\vec{OP} + t\cdot\vec{d}$

then $\vec{OP'} = \vec{OP} + t\cdot\vec{d}$

$\vec{OP'} = \langle x, y, z \rangle + t\langle -2, 3, 1 \rangle$

$\langle x', y', z' \rangle = \langle x - 2t, y + 3t, z + t \rangle$

$\because$ $z = 0$ is plane and P' lies on the plane then

$z' = 0$

$z + t = 0$

$t = -z$

then

$x' = x - 2t = x + 2z$

$y' = y + 3t = y - 3z$

$z' = 0$

Hence the projection matrix would be: [In Euclidean system].

$$P_m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -3 & 0 \end{bmatrix}$$

Now if projected images of O is O', A is A', B is B', and C is C' then.

$O' = O \cdot P_m = [0, 0, 0] \cdot P_m = [0, 0, 0]$ or [

$A' = [1, 0, 0] \cdot P_m = [1, 0, 0]$

$B' = [0, 1, 0] \cdot P_m = [0, 1, 0]$

$C' = [0, 0, 1] \cdot P_m = [2, -3, 0]$

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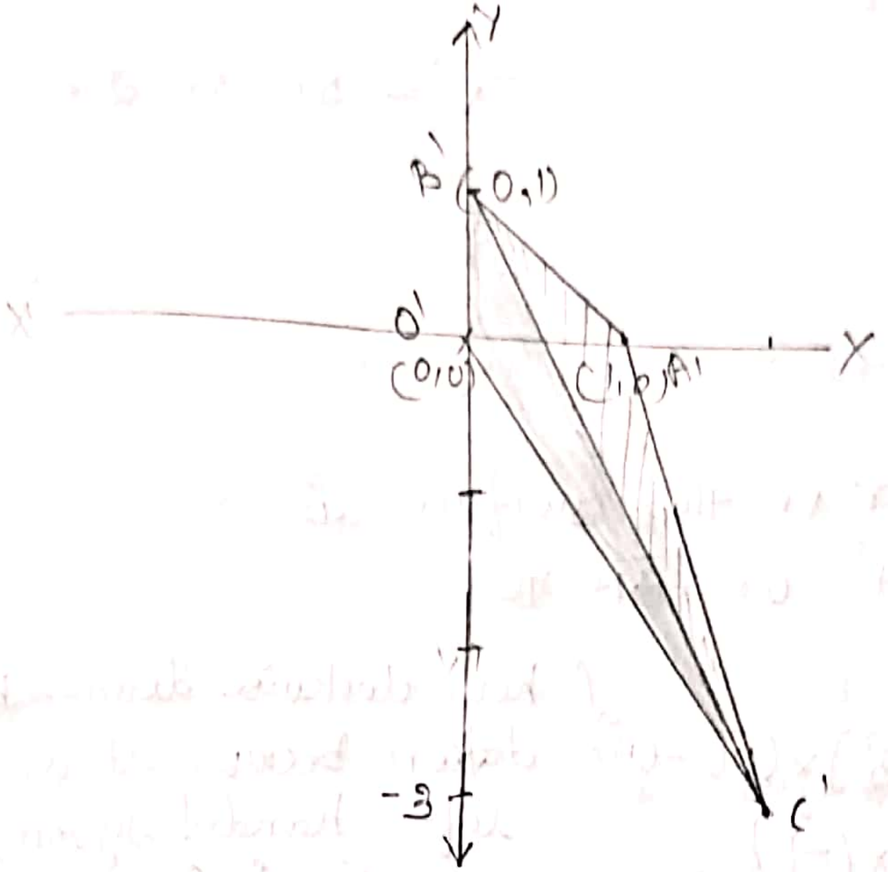
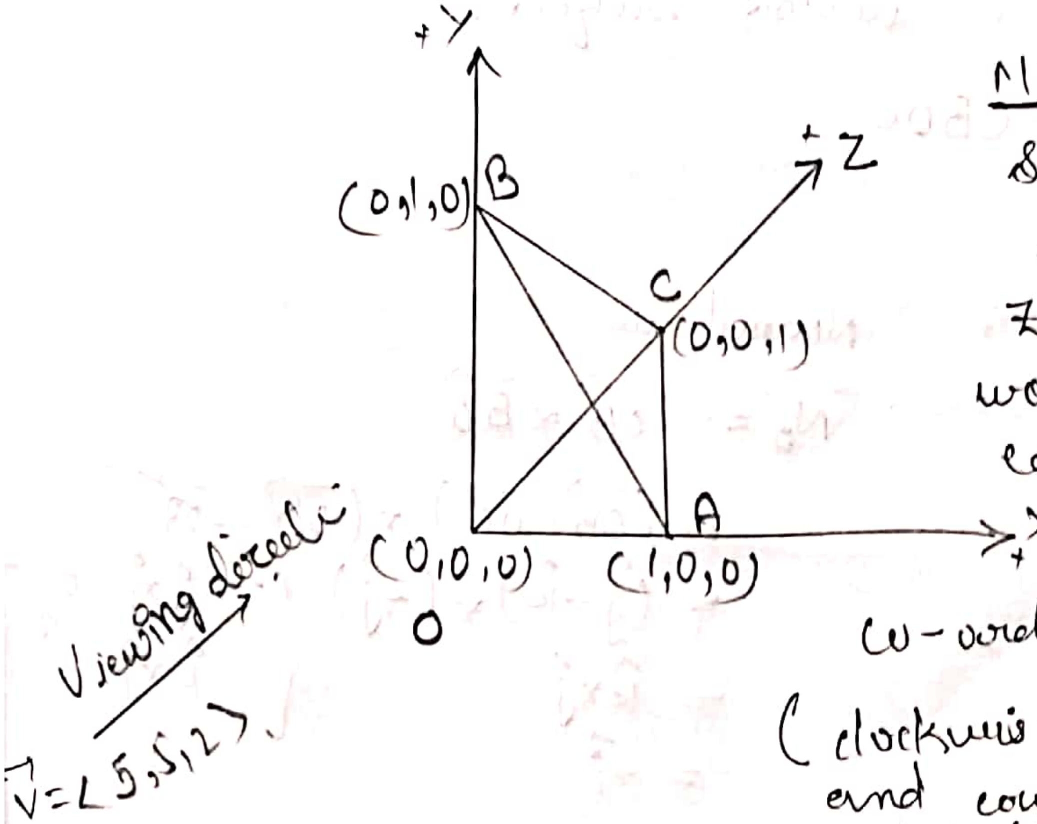


Here visible faces are O'B'C'O' and A'B'C'A'.

First Part:

Back-face removal Algorithm:



Note:
Since the viewing direction is positive Z axis hence we would do all the calculation under left-handed co-ordinate system (clockwise is positive and counter-clockwise is negative)

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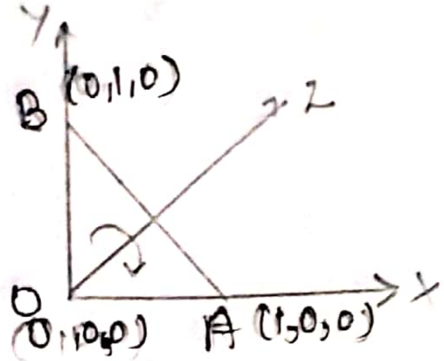
Step 1: Face OABO

$$\vec{V} = 5\hat{i} + 5\hat{j} + 2\hat{k}$$



Then normal vector on this surface is

$$\vec{N_1} = \vec{OB} \times \vec{BA} \text{ or } \vec{BA} \times \vec{AO}$$

$= \vec{BA} \times \vec{AO}$ (here clockwise direction has taken because it is left handed system)

$= (\vec{OA} - \vec{OB}) \times (\vec{OO} - \vec{OA})$

$= (\hat{i} - \hat{j}) \times (-\hat{i})$

$= \hat{j} \times \hat{i}$ $\{\because \hat{i} \times \hat{i} = 0,\ \hat{j} \times \hat{i} = -\hat{k}\}$

$= -\hat{k}$

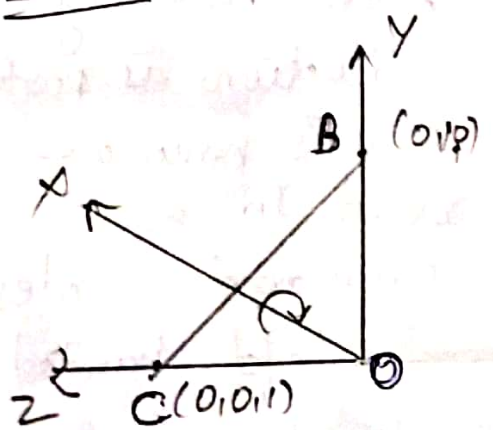
Now,

$$\vec{V} \cdot \vec{N_1} = (5\hat{i} + 5\hat{j} + 2\hat{k}) \cdot (-\hat{k})$$

$$\vec{V} \cdot \vec{N_1} = -2 < 0$$

Hence this is a visible surface..

Step 2: Face CBOC



Normal is:

$$\vec{N_2} = \vec{CB} \times \vec{BO}$$

$= (\vec{OB} - \vec{OC}) \times (\vec{OO} - \vec{OB})$

$= (\hat{j} - \hat{k}) \times (-\hat{j})$ $\{\because \hat{j} \times \hat{j} = 0,\ \hat{k} \times \hat{j} = -\hat{i}\}$

$= \hat{k} \times \hat{j}$

$= -\hat{i}$

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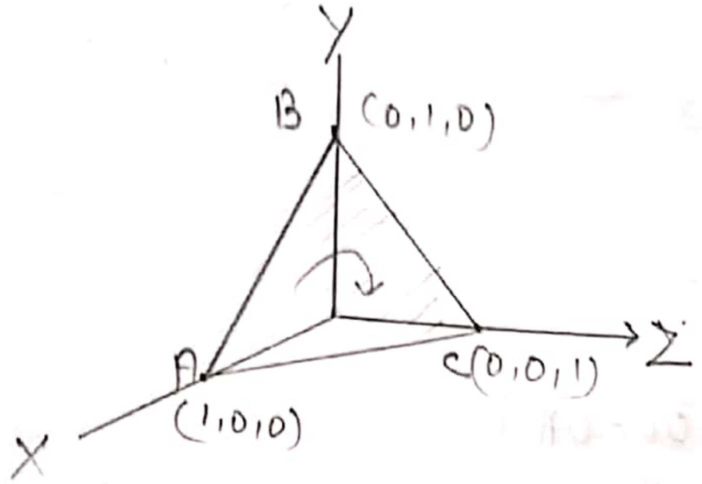
$\Rightarrow \vec{V} \cdot \vec{N_2} = (5\hat{i} + 5\hat{j} + 2\hat{k}) \cdot (-\hat{i})$
$= -5 < 0$

Hence this is also a visible face.

Step 3: Face BCAB



Normal vector is :

$\vec{N_3} = \vec{BC} \times \vec{CA}$ (take in clockwise direction)

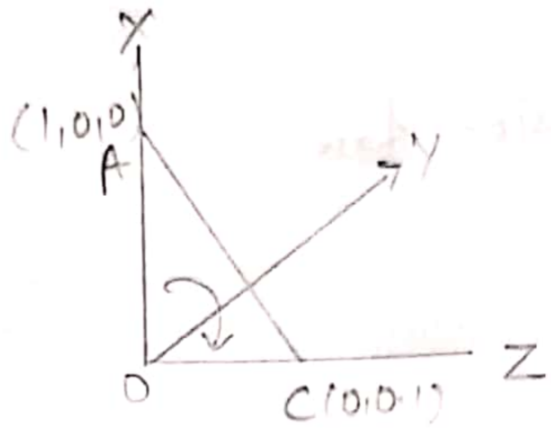
$= (-\hat{j} + \hat{k}) \times (\hat{i} - \hat{k})$
$= -\hat{j} \times \hat{i} + \hat{j} \times \hat{k} + \hat{k} \times \hat{i} - \hat{k} \times \hat{k}$
$= -(-\hat{k}) + \hat{i} + \hat{j} - 0$
$= \hat{i} + \hat{j} + \hat{k}$

$\because \left\{ \begin{array}{l} \hat{j} \times \hat{i} = -\hat{k} \\ \hat{j} \times \hat{k} = \hat{i} \\ \hat{k} \times \hat{i} = \hat{j} \\ \hat{k} \times \hat{k} = 0 \end{array} \right\}$

$\Rightarrow \vec{V} \cdot \vec{N_3} = (5\hat{i} + 5\hat{j} + 2\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$
$= 5 + 5 + 2$
$= 12 > 0$

Hence this is a back face ~~and~~ won't be visible ~~after~~ the projection.

Step 4: Face OACO



Normal vector is:

$$\vec{N_4} = \vec{OA} \times \vec{AC}$$

$$= (\hat{i}) \times (\vec{OC} - \vec{OA})$$

$$= \hat{i} \times (-\hat{i} + \hat{k})$$

$$= -\hat{i} \times \hat{i} + \hat{i} \times \hat{k}$$

$$= 0 + (-\hat{j})$$

$$= -\hat{j}$$

$\because \hat{i} \times \hat{i} = 0$

$\hat{i} \times \hat{j} = -\hat{j}$

~~hence~~ $\Rightarrow \vec{N_4} \cdot \vec{V} = (5\hat{i} + 5\hat{j} + 2\hat{k}) \cdot (-\hat{j})$

$$= -5 < 0$$

Hence this is a visible face:

Here overall there are only three visible named. OABO, OACO, CBOC.

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