

Age	Comp ⁿ	Type	Profit
old	yes	slw	Down
old	No	slw	Down
old	No	h/w	Down
mid	yes	slw	Down
mid	yes	h/w	Down
mid	No	h/w	up
new	yes	slw	up
new	No	h/w	up
new	No	slw	up

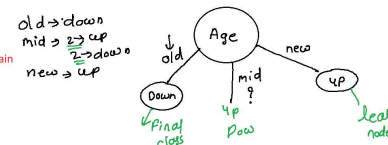
$$(1) \text{ Entropy of class (E)} = - \left[\frac{p}{p+n} \log_2 \left[\frac{p}{p+n} \right] + \frac{n}{p+n} \log_2 \left[\frac{n}{p+n} \right] \right]$$

$$(2) \text{ Information Gain}(p_i, n_i) = - \left[\frac{p_i}{p_i+n_i} \log_2 \left[\frac{p_i}{p_i+n_i} \right] + \frac{n_i}{p_i+n_i} \log_2 \left[\frac{n_i}{p_i+n_i} \right] \right]$$

$$(3) \text{ Entropy of Attribute} = \sum \frac{p_i+n_i}{p+n} \times \text{IG}(p_i, n_i)$$

$$(4) \text{ Gain of Attribute} = \text{Entropy of Class} - \text{Entropy of Attribute}$$

Attribute	Gain
① Age	0.6 ← Highest Gain
② Comp ⁿ	0.1245
③ Type	0



Age	Comp ⁿ	Type	Profit
old	yes	slw	Down
old	No	slw	Down
old	No	h/w	Down
mid	yes	slw	Down
mid	yes	h/w	Down
mid	No	h/w	up
new	yes	slw	up
new	No	h/w	up
new	No	slw	up

P → up = 5
N → Down = 5
Class = Profit

$$\text{Entropy of class} = - \left[\frac{p}{p+n} \log_2 \left[\frac{p}{p+n} \right] + \frac{n}{p+n} \log_2 \left[\frac{n}{p+n} \right] \right]$$

$$= - \left[\frac{5}{10} \log_2 \left[\frac{5}{10} \right] + \frac{5}{10} \log_2 \left[\frac{5}{10} \right] \right]$$

$$= - \left[\frac{5}{10} \log_2 \left[\frac{5}{10} \right] + \frac{5}{10} \log_2 \left[\frac{5}{10} \right] \right] = 1$$

Entropy of class = 1

Entropy of Age

	p _i	n _i	IG(p _i , n _i)
old	0	3	0
mid	2	2	1
new	3	0	0

$$(1) \text{ Entropy of Age} = \sum \frac{p_i+n_i}{p+n} \times \text{IG}(p_i, n_i)$$

$$= \frac{0+3}{5+5} \times 0 + \frac{2+2}{5+5} \times 1 + \frac{3+0}{5+5} \times 0 = \frac{2+2}{5+5} = \frac{4}{10} = 0.4$$

Entropy of Age = 0.4

$$(1) \text{ IG(Old)} = - \frac{0}{5+5} \log_2 \left[\frac{0}{5+5} \right] - \frac{3}{5+5} \log_2 \left[\frac{3}{5+5} \right] = 0$$

$$\text{IG(mid)} = - \frac{2}{5+5} \log_2 \left[\frac{2}{5+5} \right] - \frac{2}{5+5} \log_2 \left[\frac{2}{5+5} \right] = 1$$

$$\text{IG(new)} = - \frac{3}{5+5} \log_2 \left[\frac{3}{5+5} \right] - \frac{0}{5+5} \log_2 \left[\frac{0}{5+5} \right] = 0$$

$$\text{Gain(Age)} = \text{Entropy(Class)} - \text{Entropy(Age)}$$

$$\text{Gain(Age)} = 1 - 0.4 = 0.6$$

Entropy of Competition

	p _i	n _i	IG(p _i , n _i)
Yes	1	3	0.91827
No	4	2	0.91827

$$(1) \text{ IG(Yes)} = - \frac{1}{4} \log_2 \left[\frac{1}{4} \right] - \frac{3}{4} \log_2 \left[\frac{3}{4} \right] = 0.91827$$

$$\text{IG(No)} = - \frac{4}{6} \log_2 \left[\frac{4}{6} \right] - \frac{2}{6} \log_2 \left[\frac{2}{6} \right] = 0.91827$$

$$(1) \text{ Entropy of Competition} = \sum \frac{p_i+n_i}{p+n} \times \text{IG}(p_i, n_i)$$

Entropy of Competition = 0.8754

$$\frac{1+3}{5+5} \times 0.91827 + \frac{4+2}{5+5} \times 0.91827$$

$$\text{Gain(Competition)} = \text{Entropy(Class)} - \text{Entropy(Compe)}$$

$$\text{Gain(Competition)} = 1 - 0.8754 = 0.1245$$

Entropy of Type

	p _i	n _i	IG(p _i , n _i)
slw	3	3	1
h/w	2	2	1

$$(1) \text{ IG(slw)} = 1$$

$$\text{IG(h/w)} = 1$$

$$(1) \text{ Entropy of Type} = \sum \frac{p_i+n_i}{p+n} \times \text{IG}(p_i, n_i)$$

$$\text{Entropy of Type} = 1 = \frac{3+3}{5+5} \times 1 + \frac{2+2}{5+5} \times 1 = \frac{6}{10} + \frac{4}{10} = \frac{10}{10} = 1$$

$$\text{Gain(type)} = \text{Entropy(Class)} - \text{Entropy(type)}$$

$$\text{Gain(type)} = 1 - 1 = 0$$

$$\text{Entropy of Class} = - \left[\frac{2}{4} \log_2 \left[\frac{2}{4} \right] + \frac{2}{4} \log_2 \left[\frac{2}{4} \right] \right] = 1$$

Entropy of Class = 1

Entropy of Competition

	p _i	n _i	IG(p _i , n _i)
Yes	0	2	0
No	2	0	0

$$\text{Entropy of Competition} = \sum \frac{p_i+n_i}{p+n} \times \text{IG}(p_i, n_i)$$

$$= \frac{0+2}{2+2} \times 0 + \frac{2+0}{2+2} \times 0 = 0$$

Entropy of Competition = 0

$$\text{Gain(Competition)} = \text{Entropy(Class)} - \text{Entropy(Compe)}$$

$$\text{Gain(Competition)} = 1 - 0 = 1$$

Entropy of Type

	p _i	n _i	IG(p _i , n _i)
slw	1	1	1
h/w	1	1	1

$$\text{Entropy of Type} = \sum \frac{p_i+n_i}{p+n} \times \text{IG}(p_i, n_i)$$

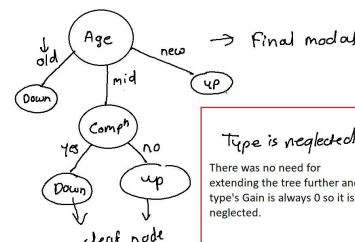
$$= \frac{1+1}{2+2} \times 1 + \frac{1+1}{2+2} \times 1 = \frac{2}{4} + \frac{2}{4} = 1$$

$$\text{Gain(type)} = \text{Entropy(Class)} - \text{Entropy(type)}$$

$$\text{Gain(type)} = 1 - 1 = 0$$

Attribute	Gain
Comp ⁿ	1 ← Highest Gain
Type	0

Compⁿ
yes → Down
no → up



Type is neglected
There was no need for extending the tree further and type's Gain is always 0 so it is neglected.