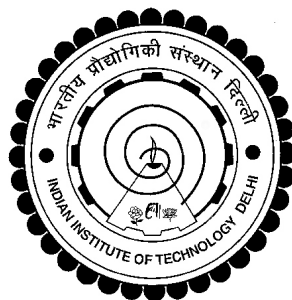


# DESIGN AND CONTROL OF TELEOPERATED MOBILE PLATFORM

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DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY DELHI  
DECEMBER 2018

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# DESIGN AND CONTROL OF TELEOPERATED MOBILE PLATFORM

by

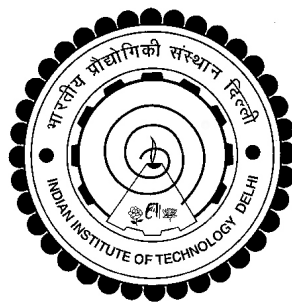
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*Submitted*

*in fulfilment of the requirements of the degree of Doctor of Philosophy*

*to the*



INDIAN INSTITUTE OF TECHNOLOGY DELHI

DECEMBER 2018



# Certificate

This is to certify that the thesis entitled **DESIGN AND CONTROL OF TELE OPERATED MOBILE PLATFORM**, submitted by **Shri. Amaren Prasanna Das** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Mechanical Engineering, is a record of the original, bona-fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other university or institute for the award of any degree or diploma.

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Thanks to every one for your help.

Amaren P Das

## Abstract

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# Chapter 1

## Introduction

The use of robots such as robotic arm figure 1.1 has been used in factories for a long time, basically for repetitive kind of job. Though it started with the intention to reduce human labour, production cost, and increased productivity, with technological development, their scope has expanded beyond manufacturing domain. Robots are now being used for health care, surveillance, exploration, etc as illustrated in figure 1.2 and 1.3. The reduction in development cost has resulted in introduction of robotic systems in entertainment industries and personal care as well. Robots have matured from heavy duty serial linked mechanical arms to a more presentable form such as ASIMO 1.4 by Honda and Aibo by Sony 1.5.

In areas where human access is not preferred or restricted due to risk of life or inhospitable environmental conditions such as which exists in chemical, space or nuclear industries, robotic systems have gained huge popularity in providing services as surveillance, rescue, exploration and remote maintenance. Research in teleoperated and autonomous mobile robotics has been fuelled largely by these requirement. Teleoperated mobile robots are suitable for these applications as the work space required to be covered is very large, and it is essential to maintain physical separation between the robot and its control station. Moreover, the remote environment is in general unknown.

This research too was motivated by a similar requirement for in-situ mea-

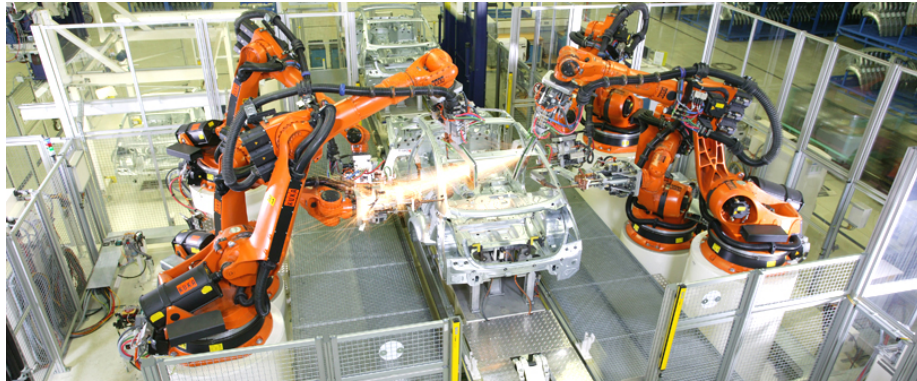


Figure 1.1: Robotics in Factory



Figure 1.2: Health Care

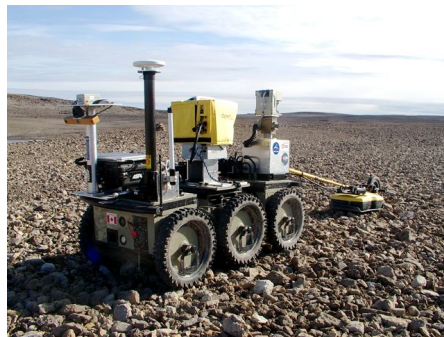


Figure 1.3: Exploration and surveillance



Figure 1.4: Honda Asimo



Figure 1.5: Sony Abio

surement of the radioactive radiation, mostly neutron field, inside the vault and cave areas of K-130, K-500 and Medical Cyclotron operational at VECC, Kolkata, West Bengal. Cyclotrons are used to accelerate charged particle beam to high energy. These are required for experiments in nuclear physics and nuclear medicine. The particles are accelerated to high energy using a high frequency alternating voltage which is applied between two hollow "D"-shaped sheet metal electrodes called "Dees" inside a vacuum chamber. The area surrounding the Dee is called the vault and cave is the areas where beam line (beam of accelerated particles) is available for experimentation. Radiation mapping of these areas are mandatory requirements for getting safety clearance from regulators during commissioning of new units and at regular intervals during operational life of the cyclotron facility. Though there are radiation detectors placed at different locations in these area they can only measure radiation levels at discrete location but can not provide the 3-D radiation map. The advantage of having a radiation map is that it provide detailed input to health physicist of the dose a person may receive and accordingly plan emergency operations. These maps also provide the plant operators with the location of radiation leakage and accordingly tune there system to improve its efficiency.

The challenge faced for in-situ inspection during operation of cyclotron is that the interaction between an accelerated beam of charged particles and the target produce Bremsstrahlung and characteristic x-rays, prompt  $\gamma$ -rays, neutrons and delayed radiation ( $\beta$  and  $\gamma$ ) thus making human presence unacceptable. A tele-operated mobile robot with wireless communication link is the obvious solution. This thesis discusses design analysis and develop of a prototype robot to carry out in-situ measurement and mapping of radiation level.

## 1.1 Research Contributions

The original contributions of the present research are listed below:

- (i) Design of a customized teleoperated mobile robot for remote surveillance and mapping application.
- (ii) Development of kinematic and dynamic model of the robot at hand.
- (iii) Control architecture required for intuitive teleoperation of this mobile robot.
- (iv) Predictive visual feedback control to alleviate problem arising due to time delay.

## 1.2 Thesis Organization

The thesis contains eight chapters and one appendix. They are organized as follows:

### Chapter 1: Introduction

This chapter, discusses the scope of mobile robotics in general and the motivation which led to this research work.

### Chapter 2: Literature Review

This chapter presents the literature review in the following areas: *kinematics and dynamics* of mobile robots, *control of mobile robot*, *control for time delay systems*, *performance of operator under teleoperation*, and *predictive display systems*.

### Chapter 3: Design of a Mobile Robot

This chapter highlights the design considerations of tele-operated mobile robot based on mission requirements and environmental conditions. It will discuss the mechanical design for the traction system, the steering gear and the scissor mechanism. Selection of steering system based on terrain condition and power requirement is also discussed.

### Chapter 4: Mathematical Modelling of Wheeled Mobile Robots

In this chapter, the orthogonal complement method will be presented to derive the

dynamic equations of a four wheeled differentially driven platform. The platform has two actuated wheels and two passive wheels. Different types of passive wheels were studied and corresponding dynamic equations were derived.

### **Chapter 5: Control of mobile manipulator**

The control architecture and the hardware used for tele-operation is presented along with the detailed description of implementation of the controller software at both the remote (mobile robot) and the local station. The experimental results of robots position based on wheel odometry and is torque requirement for few predefined paths are presented. A comparison study with the simulated results based on dynamic analysis is also presented.

### **Chapter 6: Simulation of Tele-operation**

In this chapter, simulation of a teleoperated mobile robot is presented both without time delay and under time delay due to communication link. It is shown via simulation that with increase in time delay teleoperation loop become unstable.

### **Chapter 7: Predictive Display**

In this chapter, we propose a Predictive Display strategy to counter the time delay in video feedback by extrapolating in time the camera view based on the predicted position of the robot at remote location.

### **Chapter 8: Conclusions**

This chapter summarizes major results of this research work. Limitations along with the future scope from the present experiences are also addressed.

### **Bibliography**

### **Appendix A: Measurement of Time Delay in Video Feedback**

Here the experimental set up and methodology used to determine the time delay is presented. In this appendix,



# Chapter 2

## Literature Review

In this chapter we discuss some of the important works published pertaining to the scope of this thesis. Few of the techniques and methods published in these literatures are directly used. The section on *special purpose robot* list literatures which were reviewed to arrive at the over all design of the mobile robot. The dynamic analysis of the mobile platform was based on the work cited in section *Mathamatical modeling of Mobile Robot*. The literatures discussed in *Path tacking algorithm* section was used to arrive at the "human model" proposed in the thesis for simulation of teleoperation. The section on *Teleoperation* discusses literatures in a much broader sense, such as force feedback, haptic interface design etc., than what was adapted in this thesis. This was done for completeness of the subject. Last section on *Predictive Display* though a part of human interface for teleoperation is discussed separately, because it forms one of the major component of the teleoperation system network designed for the mobile robot presented in this thesis.

### 2.1 Special Purpose Mobile Robots

This section discuss some of the special purpose and customized robots built for various typical applications. Design and fabrication of a low cost, solar powered mobile robot for scientific missions on the Antarctic plateau is presented by Ray



[1]. Honeycombe-glassfiber composite is used to provide high strength and low weight. Ohno, et al. [2] developed a robotic control vehicle for measuring the radiation in the Fukushima Daiichi Nuclear Power Plant. Briones [3] presents a vacuum cup based wall climbing robot for inspection of nuclear power plants. Galt [4] has developed eight legged teleoperated mobile robot for use in nuclear industry. Development of magnetic wheel based mobile robot for painting of ship is discussed by Cho [5]. Compliant link based mobile robot was designed and tested by Borenstein[6], in which author claims that due to its unique design, better dead-reckoning accuracy is achieved compared to other contemporary designs. This vehicle has two independent drive units or "trucks" that are free to rotate about a vertical shaft connected to the vehicle body. Each truck comprises two drive motors on a common axes and forms a differential drive system. Mechanical compliance is implemented by means of a linear bearing that allows relative motion between the front and rear truck. Other literatures giving details of mobile robots based on differential wheel, traction belt and omnidirectional wheel are given in the introductory part of chapter 3.

## 2.2 Mathematical Modeling of a Mobile Robot

A very comprehensive list of wheeled mobile robots using different wheel configuration is given by Muir and Neuman [7]. In this work kinematic equation of conventional, omnidirectional and ball wheels is presented. The kinematics of the WMR is derived by combining the kinematic information of individual wheels. Detection of wheel slip based on error in the least square solution is also discussed. Similar issues are addressed by Alexander in [8], the major difference is that it uses physical friction model in their analysis in case rolling constraints are not satisfied in case of over actuated systems. A seminal work by Chaffin [9], gives the structural classification of wheeled mobile robot based on the *degree of mobility*,  $\delta_m$ , and *degree of steerability*,  $\delta_s$ , define there-in based on the number of conventional

fixed wheels and conventional centered orientable wheels. According to them any WMR fall in one of the 5 categories given by  $(\delta_m, \delta_s) \rightarrow (3, 0), (2, 1), (1, 1), (1, 2)$ . The configuration and posture kinematic models of each type is derived. Based on dynamic model, the minimal number of actuators required for full maneuverability of each type is presented. Kinematic analysis of omi-directional over-actuated mobile robot is presented in [10]. Two different method of forward kinematics is discussed along with singularity analysis. Actuator switching scheme based on load distribution to avoid singularity is also described.

Dynamic modeling of mobile manipulator can be categorized as: force based i.e the Newton-Euler (NE) formulation and energy based as Euler-Lagrange (EL). Hoostmans [11] used NE method to arrive at the dynamic model of mobile manipulator that has two link mounted on a mobile platform. Chung [12] used EL method to arrive at the equations of motion for mobile manipulator. Geometric mechanics was used to adapt Luh and walker [13] algorithm by Boyer and Ali [14] for recursive inverse dynamics formulation for wheeled systems.

Orthogonal compliment method utilizes the advantage of NE and EL approach to derive the equations of motion for a multibody dynamical system. It uses the fact that motion can take place only in the null space of the constrain inducing matrix  $A$  defined as  $Ax = 0$ , where  $x$  is a vector independent co-ordinates. The orthogonal compliment of the constraint inducing matrix  $A$  is used to eliminate the non-working constraint forces and moments from the equations of motion. Angeles and Lee [15] used natural orthogonal compliment method to derive the equations of motion for holonomic mechanical system. In this, orthogonal compliment was derived from the velocity constraints naturally, hence the name. This was used by Angeles [16] and Saha in [17],[18] to derive equations of motion for WMR.

## 2.3 Path Tracking Algorithms

Path tracing algorithm for control of mobile robots is used to arrive at the mathematical model of human operator for simulation of tele-operation loop in chapter 6. Geometry based path tracking algorithms are most intuitive and hence suitable for our application. The major algorithms in this category reported in literatures are *pure pursuit* [19], *follow the carot* [20], *vector pursuit* [21], and *follow the past* [22]. In pure-pursuit [19], the steer angle of the robot is set so that the robot moves in circle to reach a *goal point* on the desired path. The goal point is based on the "Look Ahead Distance", which is practically the maximum distance one can see from the current vehicle position. Corrective action is based on position error of the vehicle, orientation error is not taken into account explicitly.

In case of "Follow the Carrot" method [20] the steering angle is set proportional to the *orientation error*. The orientation error is defined as the difference between the current orientation of the vehicle and the orientation required from the present position of the vehicle to reach the goal point on the reference path. The proportionality constant is decided based on trial and error.

The two previous geometric path tracking techniques only generate steering commands based upon the goal point on the reference trajectory to be traced. Hence, the requirement of vehicle posture control for accurate trajectory following remains unsatisfied. Even though, the path orientation and curvature are known at the reference point, an improved path tracking strategy such that the vehicle arrives at the reference point with the correct orientation and curvature is suggested by Wit in [21]. Wit uses Screw theory to find the error between the current Screw and the required screw at the goal position.

Hellstrom [22], has proposed an algorithm which uses the knowledge of previously recorded steer angle, associated with the path traced earlier. In this algorithm, the steer angle of the vehicle is set based on the orientation error, position error and the past recorded steer angle. A recent survey by Paden [23], provides extensive review of other control strategies for path tracking of autonomous un-

manned vehicles such as, those based on Lyapunov method, Model Predictive Controller, adaptive control etc.

## **2.4 Tele-operation**

Tele-operation deals with connecting human operator and the robot in order to reproduce human action at distance. Tele-operation is in general bidirectional or bilateral as the human needs to have a feedback in order to understand the results of his action and to perceive the remote environment. It started with its use in nuclear and space industries [24, 25], but now it is used in underwater exploration, surgery, live-power line maintenance, mining etc. all characterized by reducing the risk to human operators. The two major research areas are the "human interface" and "control" design.

### **2.4.1 Human Interface**

Human interface is a means through which the operator interacts with the remote robot by perceiving the remote environment and sending commands accordingly. Thus, the human interface has two important purpose; one to excite the human senses to show the action of the executed task and to process the human command properly to execute it at the remote end. Force and haptic feedback of remote environment drastically improves operators performance. A serial link haptic device PHANTOM [26] was developed at MIT during 1994, to provide 3-DOF force feedback for haptic(touch feedback) purpose. DELTA Haptice Device described in [27] provide 6-DOF force feedback with moderate force. Clover [28] has reported use of off-the-shelf serial industrial robots for haptic realization of tasks requiring a large workspace and high force capability. Customized 10-DOF haptic device is reported for similar purpose in [29]. Design of 6-DOF parallel mechanism for force feedback is discussed in [30].

Another major form of human interface is the visual feedback, the main chal-

lenge is to provide depth perception of remote environment. Most stereoscopic systems used in telerobotics are based on shutter glasses [31, 32], head-mounted displays [32] or polarized images [33]. Systems based on shutter glasses hide user's eyes alternately in synchronization to screen refreshment, which projects images for left and right eye alternately. A second type of interfaces is based on polarized images. The user is also required to wear glasses that filter the left and right images. The third type of interface are the head mounted display such as "Google cardboard", especially designed to immerse users into virtual environments, where the left and right images are projected to each eye using two separate screens or split screens.

### 2.4.2 Control

Control of tele-operation system deals two issues, *transparency* and *stability*. Transparency deals with what information is to be exchanged between the remote and local station, so that the operator can have a natural feel of the remote environment. A position-position architecture is suggested by Goertz [34], where master position is passed as a command to the slave servo (position) controller, and slave position is returned to the master as a position command. A position-force architecture has been proposed by Flatau [35] in this the master sends the position to the slave and the slave sends back the force felt by it in the remote environment. A general 4-channel architect has been suggested by Lawrence [36], and transparency as measure of performance in teleoperation has been defined and evaluated for different architectures.

An excellent survey article on control of bilateral teleoperation is given by Hokayem & Spong [37]. Few of these are briefly presented here. A teleoperation system, comprised of a master and slave with their corresponding controllers, residing between the human operator and the environment, which can be modeled as a two port network. Passivity based design of stabilizing control using wave-variable concept and scattering theory has been proposed by Anderson and

Spong [38], Rebelo [39] and Anderson and Slotin [40] etc. Port-Hamiltonian based approach has been used in [41, 42]. Design of controller for time delayed systems based on back-stepping method in combination with partial differential heat equation is studied by Kristic [43].

## 2.5 Predictive Display

Delays are inherent in teleoperation over wireless network. Practically much of the delay is due to relay stations and limited network bandwidth. As little as a half second delay in the visual feedback significantly reduces human performance [44]. The operator tends to adopt an inefficient "move then wait and see" policy in order to complete the task.

To overcome performance deterioration of operator due to time delay in visual feedback two approaches have been reported in literature namely *supervisory control or tele-assistance* and *predictive display*. In *supervisory control* [45, 46, 47] the robot is partly guided by operator by giving the robot intermittent commands to achieve the goal. The drawback of such system is that operator loses direct contact of the task. In predictive display systems, a natural and widely used technique, synthesised view of the remote environment is displayed to the operator based on his movements. It has been used for space teleoperation as early as in 1993, as reported by Sherdan [48], Bejczy [49] and Kim [50]. Whereas the above two used a-prior modeling and calibration of remote environment, Jagersand [51], used delayed visual feedback and operator control signal to build predicated image which was presented to the operator. The system was implemented with a fixed remote environment with a manipulator arm with two wall mounted cameras. An estimation function was proposed  $I_i \approx \phi_k(x_i), i \in 1, ..k$ , which approximates each image  $I_i$  seen so far on the trajectory  $x_1, x_2, ..x_k$ . Camera mounted on manipulator (eye-in-hand) based image predication method has been discussed using uncalibrated monocular camera by Yeres [52] and Deng [53]. Multiple sensors

based dense 3-D map of remote has been reported by Kelly [54] and [55]. While Kelly used fusion of lidar camera data, Burkert used stereo cameras. Hu [56] has used SLAM based Predictive Display (PD) system for telemanipulation of mobile robot. In this approach texture and geometry of the remote site is transmitted instead of video stream. This the author claims reduces bandwidth utilization.

# Chapter 3

## Design of Mobile a Robot

Most of the mobile robots presented in literature uses differential wheel drive with passive castor, as in [57], [58] and [17]. The other common methods for locomotion of mobile robots are the omnidirectional wheels [59] and [60], and tracked wheel system [61] and [62]. According to Nagatani [63], a vehicle with Mecanum wheels is susceptible to slippage and same is the case for tracked vehicle, which are inherently skid steered. The slippage of the wheels prevents the most popular dead-reckoning method using rotary shaft encoders from being performed well. This chapter discusses the design methodology of a novel mobile manipulator based on environmental requirements. We also highlight the advantage of the Davis steering mechanism over castor wheels or other steering methods from the perspective of this mobile manipulator. The Davis steering system modifies the heading of front wheels in a way that, at low speeds, all the wheels are in pure rolling without lateral sliding [64].

### 3.1 Design Overview

The objective of a mobile robot under consideration is to navigate inside the cyclotron vault and collect radiation intensity data at all the required points decided by the operator. Data is to be collected not only at different planer locations of the floor but also at varying height from the floor. To cater to this operational



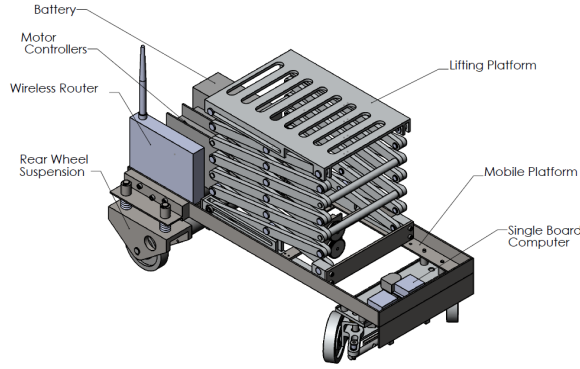


Figure 3.1: 3-D Model of the mobile manipulator

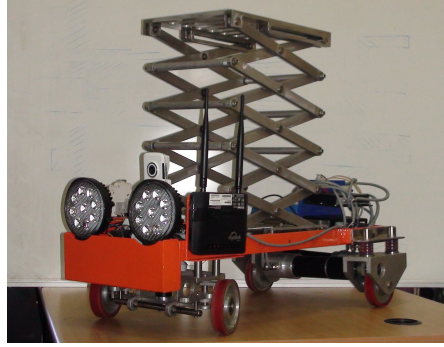


Figure 3.2: Photograph of the actual System

requirement, a mobile platform with a vertically extendable manipulating arm was developed as shown in Figures 3.1 and 3.2. Together, they are referred henceforth as mobile-manipulator. The 3-D model of the mobile-manipulator with its major subsystems are shown in Figure 3.1, whereas and the actual system is shown in Figure 3.2.

The environmental condition required that the vehicle be either autonomous or teleoperated. To keep the complexity low it was decided to have wireless teleoperated navigation and control. This gives an operator full flexibility to drive and control the system from a remote station using visual feedback provided by the on board camera. The key parameters of the mobile manipulator are listed in Table 3.1.

Table 3.1: Key parameters and specifications of the mobile manipulator.

|                        |                 |                      |
|------------------------|-----------------|----------------------|
| Weight                 | 70 Kg           | without payload      |
| Payload                | 10 Kg           | —                    |
| Footprint              | 700 mm × 400 mm | -                    |
| Height Collapsed       | 500 mm          | Along Z-Axis         |
| Height Extended        | 1500 mm         | Along Z-Axis         |
| Steering mechanism     | Davis Steering  | —                    |
| Turning radius         | 415 mm          | -                    |
| Ground clarence        | 45 mm           | —                    |
| Maximum traction speed | 2 m/min         | On flat terrain      |
| Ramp climb angle       | 30°             | Checkerboard surface |

The mobile-manipulator has a footprint of 700 mm x 400 mm based on the narrow passage through which the system has to negotiate. These passages are

formed inside the vault area by the pipelines and structural supports of the cyclotron and its associated equipment. Two DC motors, with speed servo controller, provide the traction to each rear wheels. The two front wheels are inter-connected with a Davis steering mechanism [65]. A scissor mechanism provides the vertical motion of the detector that is mounted on the manipulating arm.

In order to keep the self weight of the system small all the structural parts are made of aluminum alloy AL6061, apart from the base frame. Stainless Steel (SS304) angle sections was used for the base frame, which give it excellent strength to weight ratio.

## 3.2 Design of the Traction System

Traction is provided by the two rear wheels driven independently. This makes the system over actuated. A mechanical differential connecting the two rear wheels, as used in cars, would overcome this. It was not proposed to do so as the proposed vehicle is planned to be teleoperated in a environment inaccessible to humans. This calls for a single-failure-safe design. The proposed design gives two major advantages. Firstly, in case one of the wheel loosing contact with the ground due to overhang in small pits or while over an obstacle, the mechanical differential system would keep supplying power to the free hanging wheel. The system will hence get stuck, maybe in an unrecoverable location. This situation is avoided in the present design as the motor having traction can be independently powered, to move the vehicle. Secondly, using the proposed design, in case of one actuator failure, either the traction or the steering motor, still the vehicle can be manoeuvred to a safe location, albeit with dragging of the wheel with failed actuator.

Each wheel is driven by a Maxon DC RE50 200W Motor through a 26:1 reduction gearbox. The motors are mounted at an offset to the wheel axis for increased ground clearance, as shown in Figure 3.3. Spring suspension is provided at each wheel to ensure sufficient contact force on uneven ground. The diameter

of the wheel is 100 mm ( $D_w$ ), which is sufficient to ride over obstacle of height 20 mm (Max). They are made of Aluminum alloy-6061 with 5mm thick molded polyurethane (PU) liner. The PU liner provides large traction on cement flooring while being resistance to wear.

The load distribution was optimized to generate maximum normal reaction,  $F_n$ , at the rear wheels without overturning while moving up the ramp of  $30^\circ$ . Maximizing rear wheel reaction by increasing "b" as per Equation 3.1 ensures increased traction,  $F_T = \mu F_N$  ( $\mu$  is the coefficient of friction), but at the same time decreases the stability margin indicated by "X" in Figure 3.4. The static moment and force balance yield

$$F_{N2} = \frac{b}{a} F_{N1}, \quad F_{N1} + F_{N2} = mg \cos \theta \quad (3.1)$$

The stability margin  $X$  as shoen in Figure 3.2 was fixed as 30mm so as to achieve maximum acceleration of 0.144g over the ramp of  $30^\circ$  without overturning. This was done based on the condition of dynamic stability given by

$$mgb \cos \theta = (mg \sin \theta + ma)z_{cg}, \quad \Rightarrow g\left(\frac{b}{z_{cg}} \cos \theta - \sin \theta\right) = a \quad (3.2)$$

Where

$F_{N1}$ ,  $F_{N2}$  are normal reaction on the front and rear wheels.

a and b are the distance of the vehicle cg from the rear and front wheels.

$Z_{cg}$  is the height of the cg from the plane containing the contact point of the wheels.

$m$  is mass of the vehicle.

$g$  is acceleration due to gravity.

$\theta$  is the inclination of the traction surface from horizontal.

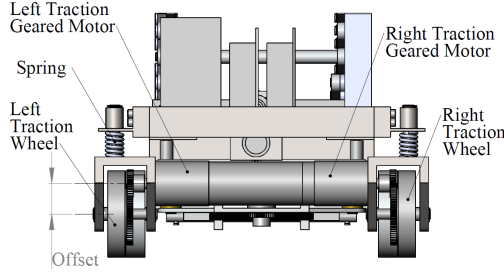


Figure 3.3: Rear Suspension

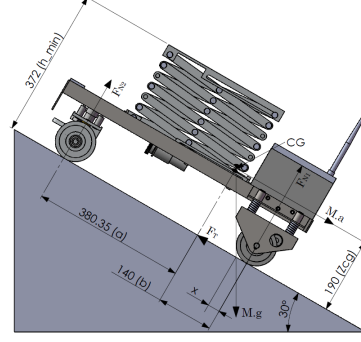


Figure 3.4: Mobile Manipulator on slope

### 3.2.1 Selection of Motor and Gearbox

The torque requirement for each rear wheel was calculated based on the static moment balance with the assumption that each rear wheel shares equal load and the total suspended weight is 80 Kg. From the freebody diagram (Figure 3.4), using moment and force balance we get the following:

$$F_{N1} = \frac{aM \cos \theta}{a + b - \mu Z_{cg}}, \quad F_T = \mu F_{N1} \quad (3.3)$$

In order to estimate the traction motor size, we take worst case scenario of  $\theta = 30^\circ$  and  $\mu = 0.3$ . This leads to

$$F_{N1} = 66Kg, \quad F_T = 0.3 * 66 \approx 20Kg$$

Since the traction is provided by the two rear wheels, the torque required per wheel ( $T_w$ ) is given by

$$T_w = (F_T/2)(D_w/2) = (20/2) * 50 = 500Kg - mm \simeq 5Nm \quad (3.4)$$

The motor torque  $T_M$ , required based on the assumption of factor of safety,  $FS = 1.5$  is

$$T_M = (FS) * T_w = 1.5 * 5 = 7.5 \simeq 8Nm \quad (3.5)$$

Assuming the maximum speed,  $V_{ramp} = 1m/s$ , of the mobile manipulator over a ramp, the required power,  $P_M$ , of the traction motor is calculated as,

$$\begin{aligned}\omega_w &= V_{ramp}/(D_w/2) \simeq 200rpm \\ P_m &= \omega_w T_m = 20 * 8 = 160W\end{aligned}\tag{3.6}$$

The nearest Maxon motor available as per the catalogue [66] is 200W, RE50-370354 motor. The nominal speed,  $N_s$  is 5680rpm. Therefore, the gearing ratio required is,  $N_s/\omega_w = 5680/200 \simeq 28.4$ . The nearest gear box available is of ratio 26 : 1, which was chosen.

### 3.3 Design of Steering System

The design objective of a steering system should ensure rolling motion of all the wheels during every possible maneuvers of the mobile robot. This is important to reduce the friction drag due to sliding motion which reduces the energy efficiency of the mobile robot. In case of four wheeled vehicle with rear wheels fixed and front wheels steered the condition is shown in Figure 3.5, referred in some literature as Ackerman steering conduction.

This mobile manipulator uses Davis steering mechanism, Figure 3.6, on the front wheels. Caster wheels were not used as they tend to align with obstacles and thus get stuck. On the other hand tracked wheels have excellent rough terrain capabilities, but is power intensive due to skid steering. another option was to use Omnidirectional wheels, which need complex controller for coordination and an extra actuator. Moreover, the operation are in general not clean of loose small objects, which may get stuck in between the free rollers of the omnidirectional wheels. This will reduce the efficiency of the vehicle.

Davis mechanism was chosen over Ackerman steering gear as it satisfies the steering condition given by Equation 3.7, which ensures pure rolling of all wheels over the entire steering range. This makes the system suitable for passive wheel

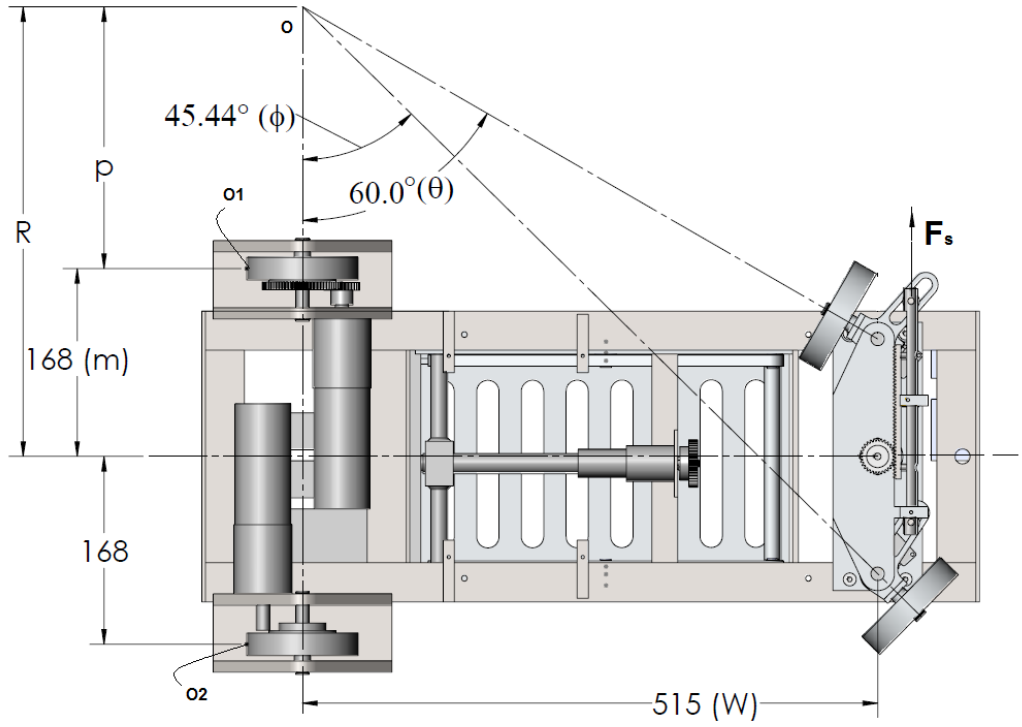


Figure 3.5: Ackerman Steering Condition

odometry and energy efficient. The mechanism being positively driven by position controlled servo motor does not align with the obstacles and thus are able to crossover it. The dimensions of the links used in the steering mechanism is given in Figure 3.6, and are based on the Ackerman steering law given below:

$$\cot \phi - \cot \theta = a/w, \quad \frac{2b}{h} = \frac{a}{w} \quad (3.7)$$

Where  $a$  and  $b$  is limited by the over all size of the vehicle, discussed earlier.

### 3.3.1 Minimum Turning Radius

The mechanical construction of this steering mechanism limits the steering angle. This in turn limits the minimum radius the vehicle can negotiate. Figure 3.5 shows the extreme values of  $\phi$  and  $\theta$ , one side of the steering limits. The **turning radius**  $R$  for a given steer angle  $\theta$  is calculated by the geometry of Figure 3.6 as

$$\tan \theta = \frac{w}{P + m - \frac{a}{2}} \quad \text{and} \quad R = m + p$$

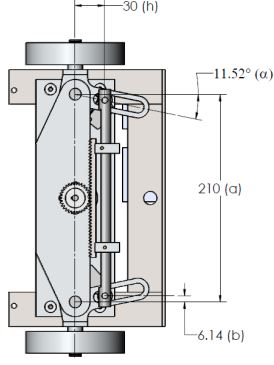


Figure 3.6: Davis Steering Gear

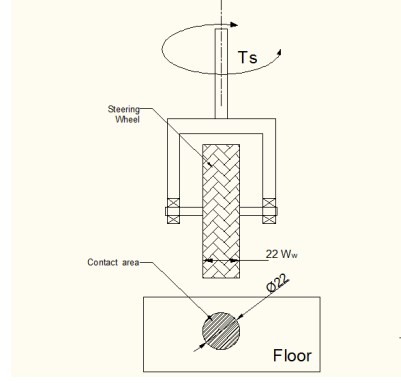


Figure 3.7: Steer Torque

Substituting  $R$  and rearranging the above equation, we get

$$R(\theta) = \frac{a}{2} + w \cot \theta \quad (3.8)$$

The extreme value of  $\theta = 60^\circ$  as shown in Figure 3.5, this gives the *minimum turning radius*  $R_{min}$  as

$$R_{min} = \frac{210}{2} + 515 \times \cot(60^\circ) = 402mm$$

### 3.3.2 Calculation of Steering Torque

The torque required to steer the front wheel is estimated based on a simplified assumption that the wheel deforms under normal load and the contact area thus generated is circular in shape with diameter that of the wheel width,  $W_w$ , as shown in Figure 3.7. In order to estimate the normal reaction on each wheel, we assume that the total weight of 80kg is equally shared by the four wheels. Therefore,  $N_s = 80/4 = 20Kg$ . Next, the uniform pressure formula used for brakes/clutches design was applied to find the resistance torque  $T_s$ , between the ground and the wheel i.e,

$$T_s = \frac{N_s \mu}{3} W_w = 0.4Nm \quad (3.9)$$

The resistance torque,  $T_s$ , of both the wheels are balanced by the force  $F_s$  acting on the rack as shown in Figure 3.6. The rack is coupled to the steering motor by a pinion of diameter,  $D_p = 40mm$ . The motor torque,  $T_{m_s}$  in Equation 3.10 is calculated with a high factor of safety,  $FS = 3$ . This is because  $T_s$  is estimated based on a simplified model of brake design. The power,  $P_{m_s}$  of the steering motor based on torque  $T_{m_s}$  and the steering speed  $\omega_s$  of 100rpm is

$$T_{m_s} = (FS) \frac{2T_s D_p}{h} \frac{1}{2} = 1.6Nm \quad \text{and} \quad P_{m_s} = T_{m_s} * \omega_s = 17W \quad (3.10)$$

Based on the above specifications, a 20W, RE25 DC motor of Maxon make and a gear box GP32 of ratio 159:1 was chosen for the steering mechanism.

### 3.4 Design of Scissor Mechanism for Manipulating Arm

The manipulating arm was designed to move up to a height of 1.5m from the floor level. This motion was generated using a scissor mechanism, as shown in Figure 3.8. The scissor mechanism has two major advantages over other lifting methods such as telescopic pillar, etc. First, the ratio of height in extended and collapsed condition is very large. In our case it is 3 : 1. Second, the self weight of the mechanism is less as it is made of rectangular links.

The scissor mechanism, Figure 3.8, has 6 stages, where one "X" denotes one stage. The Scissor is connected to the top platform by a pivot joint  $O_2$  and a prismatic joint  $A_2$ . This is coupled to the base frame by pivot joint  $O_1$  and a prismatic joint  $A_1$ . The linear actuation of joint  $A_1$  is provided by a lead screw of pitch ( $P$ ) 1.5 mm and mean diameter ( $d_m$ ) 10mm. This results in vertical motion of the top platform.

The relation between the vertical motion of the platform and the horizontal displacement of point  $A_1$  is given by geometry of the mechanism shown in Figure



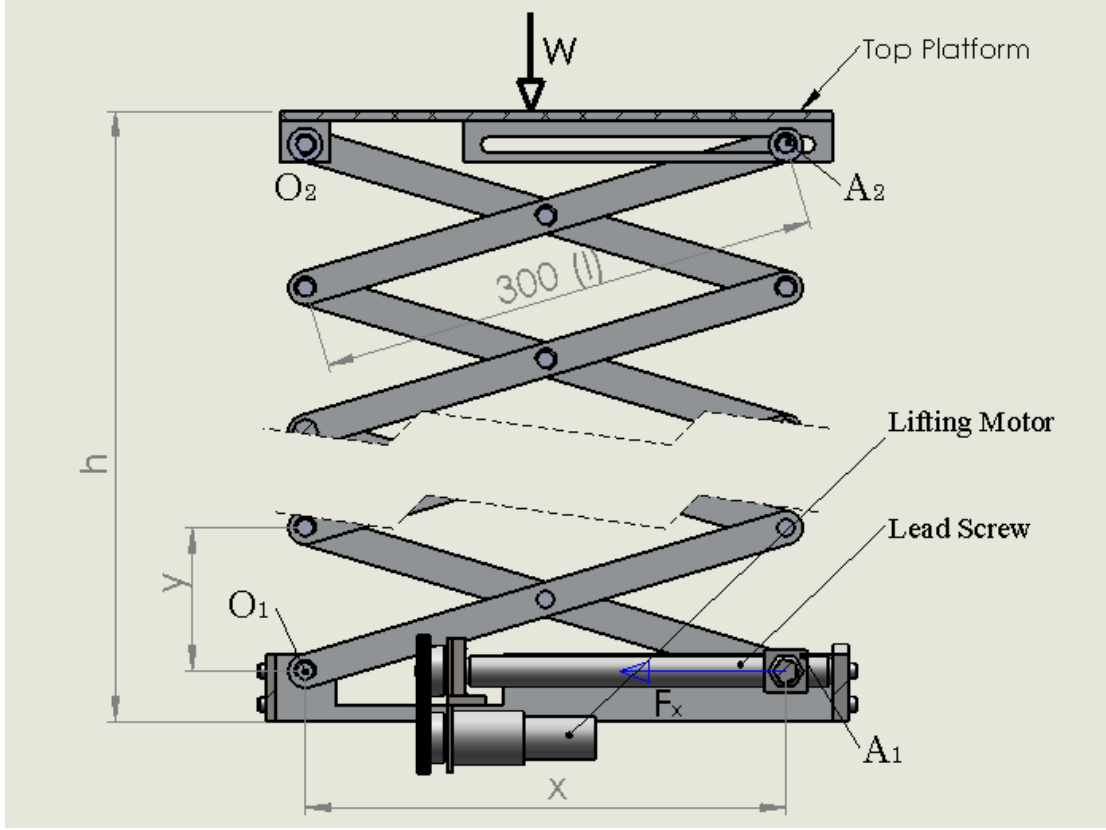


Figure 3.8: Scissor Mechanism

3.8

$$y = l \sin \theta, \quad x = l \cos \theta, \quad \Rightarrow \quad dy = l \cos \theta d\theta, \quad dx = -l \sin \theta d\theta \quad (3.11)$$

$$h = Ny \rightarrow dh = Ndy$$

where  $l$  is the link length,  $\theta$  the angle of the link with horizontal plane and  $h$  the height of platform.

The number of stages used in the scissor mechanism is  $N=6$ . From the principle of virtual work, we get

$$-F_x dx = W dh, \quad \Rightarrow \quad F_x = \frac{WN}{\tan \theta} \quad (3.12)$$

where,  $F_x$  is the axial force on the prismatic joint,  $A_1$  and  $W$  is the payload. From Equation 3.12, it is clear that as  $\theta \rightarrow 0$ , the force  $F_x \rightarrow \infty$ . In the present design,  $\theta_{min} = 5^\circ$  and  $\theta_{max} = 45^\circ$ . Therefore, the extended height  $h_{max} = Nl \sin \theta_{max} = 1.3m$  and the collapsed height  $h_{min} = 156mm$ . Assuming  $W = 8kg$  as payload the maximum force  $F_x = 342Kg$  is required at  $\theta_{min} = 5^\circ$ .

The motor torque required for the scissor mechanism is calculated using screw jack formula given in [65] and here presented as Equation 3.13.

$$T_L = \frac{F_x d_m}{2} \left( \frac{p + \pi \mu d_m \sec \alpha}{\pi d_m - \mu p \sec \alpha} \right) = 7.5 Nm \quad (3.13)$$

where coefficient of friction assumed,  $\mu = 0.1$ , ACME thread angle  $2\alpha = 60^\circ$ , pitch diameter  $d_m = 15mm$  and pitch  $p = 1.5mm$  is used. Based on the above specification 10W RE20 DC motor with a gear box of 25:1 ratio was chosen from Maxon motor catalogue [66].

### 3.5 Summary

Design calculations for the proposed mobile manipulator are presented in this chapter. Different aspects based on the requirements of radiation inspection around cyclotron was taken into account. Advantage of positively steered wheels over caster wheel was highlighted for the proposed mobile robot.



## Chapter 4

# Dynamics of Wheeled Mobile Robots

In the field of mobile robotics, extensive research has been carried out. Mobile robots can broadly be divided into three categories-wheeled robots, legged robots [67], and aerial vehicles [68]. There are few mobile robots which use both wheels and legs for locomotion for example, in Creadapt [69] in order to take advantage of both modes of locomotion. Among these the most extensively studied are the Wheeled Mobile Robots (WMR). They have been classified into five generic classes by Champion et al. [9], [70] based on their mobility resulting from the kinematic constraints due to different wheel types. The most common among these are the 3 wheeled differential drive WMR with one castor wheel. Because of its simplicity in modelling, they have been used in most of the control and motion planning algorithms [71], [72] and [73].

In order to develop a model-based control algorithm it is imperative to have good dynamic model of the WMR. These dynamic models are used in simulation software, Software in Loop (SIL) testing and Hardware in Loop (HIL) testing of the controllers. Different methods have been adopted to derive the dynamic model of WMRs. A general dynamical model was derived for three-wheel mobile robots with nonholonomic constraints by B. d'Andrea-Novell [74] using Lagrange formu-

lation. Alternatively Thanjavur and Rajagopalan [75] has used Kane's method and Saha et al. [18],[17] used Natural Orthogonal Compliment (NOC) method for the same.

All the papers known to us use the standard caster wheel in deriving the dynamic model of the WMR. In this chapter the modelling is carried out considering the Castor wheel in the most general configuration. In the stranded castor wheel configuration the axis of rotation is perpendicular to the line joining the pivot and the axis of rotation, as shown in Figure 4.1 whereas in this study they are oriented in the most general way, as shown in figure Figure 4.2. The concept of the NOC which here inherently takes into account the non-holonomic constraint of the wheels has been used. Moreover, being a matrix based approach, it is convenient for numerical simulations.

## 4.1 Modeling using the Natural Orthogonal Compliment (NOC)

Let us consider a system  $n$  rigid bodies interconnected with different types of joints. Let  $f_i$  be the net force acting at the center of mass (CM) of the  $i - th$  body and  $n_i$  is the net moment. If  $m_i$  is the mass,  $I_{ci}$  is the moment of inertia with respect to the CM,  $c_i$  is the position vector of the CM and  $\omega_i$  is the angular velocity of the same body, then equation of motion of the  $i - th$  rigid body is given by Newton-Euler equations as

$$f_i = m_i \ddot{c}_i \quad \text{and} \quad n_i = I_{ci} \omega_i + \omega_i \times I_{ci} \omega_i \quad (4.1)$$

Let us define twist ( $t$ ) and wrench( $w$ ) as

$$t_i \equiv \begin{pmatrix} \omega_i \\ \dot{c}_i \end{pmatrix} \quad w_i \equiv \begin{pmatrix} n_i \\ f_i \end{pmatrix}$$

Note that the wrench  $w_i$  acting on the  $i - th$  body can be decomposed into  $w_i^w$ , called the *working component* and  $w_i^c$ , the *non-working component*. The working component consists of all the external forces and torques, which imparts/extracts energy to/from the system, e.g. motor actuating torque. The non-working component of the wrench consists of the forces and torques that are used to constrain the motion of the body at the joints. Then, Newton-Euler equation (4.1) can be rewritten in a single matrix equation as

$$M_i \dot{t}_i + W_i M_i t_i = w_i^w + w_i^c \quad \because w_i \equiv w_i^w + w_i^c \quad (4.2)$$

where

$$M_i \equiv \begin{pmatrix} I_{ci} & 0 \\ 0 & m_i \tilde{1} \end{pmatrix}, \quad W_i \equiv \begin{pmatrix} \Omega_i & 0 \\ 0 & 0 \end{pmatrix}, \quad \Omega_i \equiv \omega_i \times \tilde{1} \quad (4.3)$$

in which  $\omega_i \times 1$ , is referred as the crossproduct matrix of vector  $\omega_i$  and  $\tilde{1}$  denotes the identity matrix. For details, refer to [16],[76].

If we define

$$M \equiv \text{diag}[M_1, M_2, \dots, M_n], \quad W \equiv \text{diag}[W_1, W_2, \dots, W_n], \quad t \equiv [t_1^T, t_2^T, \dots, t_n^T]^T$$

and

$$w^j \equiv [w_1^{jT}, w_2^{jT}, \dots, w_n^{jT}]^T, \quad j = c, w$$

Then the equation of all the  $n$  rigid bodies in the system can be collected and written as a single matrix equation as

$$M \dot{t} + W M t = w^c + w^w \quad (4.4)$$

The above equation is referred to as decoupled equations of motion of the system.

The kinematic constraints both holonomic and non-holonomic (e.g. pure rolling) between two bodies  $i$  and  $j$  of a system can be expressed as linear homogeneous

system of algebraic equations [16], namely

$$A_i t_i + A_j t_j = 0 \quad (4.5)$$

where  $A_i, A_j$  depend on the kinematic parameters.

The constraint equations corresponding to all the joints in the system can be written in terms of the *generalized twist vector*  $t$ . Furthermore if,  $\dot{\theta} \equiv (\dot{\theta}_1, \dot{\theta}_2, \dots)^T$  denote the *independent generalized joint rates*, one can then write  $t$  in terms of  $\dot{\theta}$  as  $t = T\dot{\theta}$ . Using the fact, that  $\dot{\theta}$  can take any arbitrary value, we get

$$At = 0, \quad \Rightarrow AT\dot{\theta} = 0 \quad \Rightarrow AT = 0 \quad (4.6)$$

The above equation (4.6) indicates that  $T$  is the orthogonal compliment of  $A$ . Since this relation arises naturally, hence the name *Natural Orthogonal Complement*. It can be shown [16] that the non-working wrench  $w^c$  lies in the range space of  $A^T$ . In view of equation 4.6, it can also be proved that  $w^c$  lies in the null space of  $T^T$ , therefore

$$T^T w^c = 0 \quad (4.7)$$

To eliminate the non-working forces and moments, i.e.  $w^C$  from the uncoupled equation of motion (4.4), we multiply both sides of the equation by  $T^T$ ,

$$T^T M \dot{t} + T^T W M t = T^T w^W, \quad \Rightarrow T^T M T \ddot{\theta} + T^T (M \dot{T} + W M T) \dot{\theta} = T^T w^T \quad (4.8)$$

Equation 4.8 represents the dynamic equation of interconnected  $n$ -body system. This equation is expressed in terms of the independent generalized joint rates  $\dot{\theta}$  and cosponsoring acceleration  $\ddot{\theta}$ . Further. using the relations  $t = T\dot{\theta}$  and  $\dot{t} = \dot{T}\dot{\theta} + T\ddot{\theta}$  in equation 4.8 the final equations of motion can be written as

$$I(\theta)\ddot{\theta} = C(\theta, \dot{\theta})\dot{\theta} + \tau \quad (4.9)$$

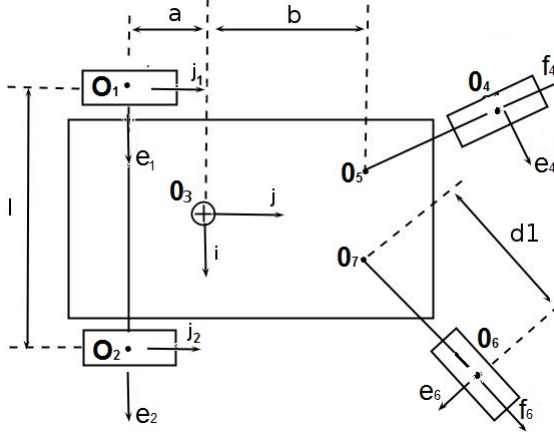


Figure 4.1: WMR-Std. Castor

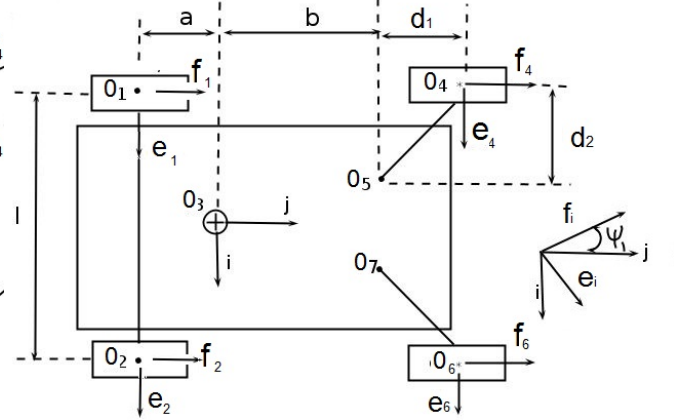


Figure 4.2: WMR-general

Figure 4.3: Castor wheel configuration of a WMR

where

$$I(\theta) \equiv T^T M T \quad : \text{generalized inertia matrix}$$

$$C(\theta, \dot{\theta}) \equiv -T^T (M \dot{T} + W M T) \dot{\theta} \quad : \text{generalized matrix of convective inertia terms}$$

$$\tau \equiv T^T w^T \quad : \text{generalized vector of driving forces}$$

## 4.2 Dynamic Equation of WMR

The dynamic equation of a differentially driven 3 wheeled mobile robot based on the Natural Orthogonal Compliment (NOC) has been presented by Saha [18]. The vehicle consisted of 2 driven wheel and one standard castor wheels. In general, for large vehicles, it necessary to have at least four wheels from the point of stability of the vehicle. Such a vehicle is shown in figure 4.1. It may be noted that the castor wheels used in this case are the standard castor wheel configuration, where the angle between line  $O_4O_5$  and vector  $e_4$  is 90 deg. Which is a special case of the more general configuration of the castor wheels shown in figure 4.2, where angle between line  $O_4O_5$  and vector  $e_4$  is not 90 deg

The vehicle considered for analysis in this chapter is shown in figure 4.2. It consists of two independently driven wheels at the back and two generalized castor



wheels at the front. The actuated wheels are labelled as body #1 and #2. The platform is body #3. The first caster wheel and its bracket is labelled as #4 and #5 respectively, with the castor pivoted at  $O_5$ . Similarly the second castor is pivoted at  $O_7$ , and its bracket and wheel is labelled as #6 and #7 respectively, all the wheels are assumed to be rolling without slipping.

#### 4.2.1 Kinematic analysis

In order to proceed with the kinematic analysis of the vehicle in figure 4.2, we define a orthogonal triad of vectors  $i, j, k$  at point  $O_3$ , the control point of the platform, as shown in the figure. If  $\dot{\theta}_1$  and  $\dot{\theta}_2$ <sup>1</sup> denote the rates of rotation of wheel #1 and #2 then the linear velocity of points  $O_1$  &  $O_2$  under pure rolling condition is given by

$$\dot{\mathbf{o}}_i = r\dot{\theta}_i \mathbf{j}, \quad r = \text{radius of wheel} \quad (4.10)$$

The angular velocity of the platform  $\omega_3$  can be written as

$$\omega_3 = (r/l)(\dot{\theta}_1 - \dot{\theta}_2) \mathbf{k} \quad (4.11)$$

Further, the velocity of point  $O_3$  can be written as  $\dot{\mathbf{o}}_3 = \dot{\mathbf{o}}_i + \omega_3 \times (\mathbf{c} - \mathbf{o}_i)$ ,  $i = 1, 2$ . Where  $\mathbf{o}_3$  and  $\mathbf{o}_i$  is the position vector of points  $O_3$  and  $O_i$  respectively, with respect to some point fixed to the ground. By eliminate  $\omega_3$  we get the following:

$$\dot{\mathbf{o}}_3 = [(ar/l)(\dot{\theta}_1 - \dot{\theta}_2) + (r/2)(\dot{\theta}_1 + \dot{\theta}_2)] \mathbf{i} \quad (4.12)$$

Now, the angular velocity of the drive wheel #1 can be expressed as  $\omega_1 = -\dot{\theta}_1 \mathbf{i} + \omega_3 \mathbf{k}$ . Using equation 4.11, the same can be rewritten as

$$\omega_1 = \begin{pmatrix} -\dot{\theta}_1 + (r/l)\dot{\theta}_2 & -(r/l)\dot{\theta}_2 \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \quad (4.13)$$

---

<sup>1</sup> positive when pointing along  $i$

Based on equations 4.10 and 4.13, we can write the twist for wheel #1 in terms of  $\dot{\theta}_a \equiv (\dot{\theta}_1, \dot{\theta}_2)^T$ , as

$$t_1 = \begin{pmatrix} \omega_1 \\ \dot{o}_1 \end{pmatrix} = \begin{pmatrix} -i + (r/l)k & -(r/l)k \\ rj & 0 \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \quad (4.14)$$

Similarly, for the other actuated wheel #2, one get

$$t_2 = \begin{pmatrix} \omega_1 \\ \dot{o}_1 \end{pmatrix} = \begin{pmatrix} -i + (r/l)k & -(r/l)k \\ 0 & rj \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \quad (4.15)$$

To calculate the twist,  $t_3$  of the platform body #3, equation 4.11 and 4.12 are combined to get

$$t_3 = \begin{pmatrix} \omega_3 \\ \dot{o}_3 \end{pmatrix} = \begin{pmatrix} \rho\delta & -\rho\delta \\ r(\lambda i + (1/2)j) & r(-\lambda i + (1/2)j) \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \quad (4.16)$$

where

$$\delta \equiv d/l, \quad \rho \equiv r/d, \quad \lambda \equiv a/l$$

From the above three equations we get  $T_1 = \begin{pmatrix} -i + (r/l)k & -(r/l)k \\ rj & 0 \end{pmatrix}$ ,  $T_2 = \begin{pmatrix} -i + (r/l)k & -(r/l)k \\ 0 & rj \end{pmatrix}$ ,  $T_3 = \begin{pmatrix} \rho\delta & -\rho\delta \\ r(\lambda i + (1/2)j) & r(-\lambda i + (1/2)j) \end{pmatrix}$

In order to calculate the twist of the caster bracket and the caster wheel, we need to express the unactuated joint rates,  $\dot{\psi}_1$  and  $\dot{\phi}_1$ , in terms of the actuated joint rate vector  $\dot{\theta}_a$ . note here that  $\dot{\psi}_1$  denotes the rate of rotation of bracket body #5 about  $O_5$  with respect to the platform, and  $\dot{\phi}_1$  the rate of rotation of caster wheel body #4 about its axis  $e_4$  with respect to bracket. The velocity of  $O_5$  can be expressed in two independent forms, namely, one in terms of the velocity of  $O_3$  and the other one in terms of the velocity of  $O_5$ , i.e.,

$$\dot{o}_5 = \dot{o}_4 + \omega_5 \times (d_1 e_4 - d_2 f_4), \quad \dot{o}_5 = \dot{o}_4 + \omega_5 \times (d_1 e_4 - d_2 f_4) \quad (4.17)$$

On equating the above two equations together, and using the rotation matrix between coordiante system  $\{i, j, k\}$  and  $\{e_4, f_4, k\}$ , to express the equation in  $e_4$  and  $f_4$ , we get

$$(-\dot{\phi}_1 r + \dot{\psi}_1 d_1) f_3 + d_3 \dot{\psi}_1 e_3 = \dot{o}_3 + \omega_3 (m \cos \psi_1 - b \sin \psi_1 - d_1) e_4 \quad (4.18)$$

Taking the dot product of the above equation first with  $e_4$  and then with  $f_4$ , and using equation 4.12 for  $\dot{o}_3$ , we get

$$\begin{aligned} \begin{pmatrix} d_2 & 0 \\ -d_1 & r \end{pmatrix} \begin{pmatrix} \dot{\psi}_1 \\ \dot{\phi}_1 \end{pmatrix} &= \begin{pmatrix} (-ar/l)S_{\psi_1} + (r/2)C_{\psi_1} + \delta_1 & (ar/l)S_{\psi_1} + (r/2)C_{\psi_1} - \delta_1 \\ (ar/l)C_{\psi_1} + (r/2)S_{\psi_1} + \delta_2 & (-ar/l)C_{\psi_1} + (r/2)S_{\psi_1} - \delta_2 \end{pmatrix} \dot{\theta}_a \\ &= [F_{ij}] \dot{\theta}_a \end{aligned} \quad (4.19)$$

where,

$$\delta_1 = (r/l)(mC_{\psi_1} - bS_{\psi_1} - d_2), \quad \delta_2 = (r/l)(mS_{\psi_1} + bC_{\psi_1} + d_1)$$

Similarly, for the other caster wheel we get,

$$\begin{aligned} \begin{pmatrix} d_2 & 0 \\ -d_1 & r \end{pmatrix} \begin{pmatrix} \dot{\psi}_2 \\ \dot{\phi}_2 \end{pmatrix} &= \begin{pmatrix} (-ar/l)S_{\psi_2} + (r/2)C_{\psi_2} - \delta_3 & (ar/l)S_{\psi_2} + (r/2)C_{\psi_2} + \delta_3 \\ (ar/l)C_{\psi_2} + (r/2)S_{\psi_2} + \delta_4 & (-ar/l)C_{\psi_2} + (r/2)S_{\psi_2} - \delta_4 \end{pmatrix} \dot{\theta}_a \\ &= [G_{ij}] \dot{\theta}_a \end{aligned} \quad (4.20)$$

where,

$$\delta_3 = (r/l)(mC_{\psi_2} + bS_{\psi_2} + d_2), \quad \delta_4 = (r/l)(mS_{\psi_2} + bC_{\psi_2} + d_1)$$

The angular and the liner velocity of the CM of the caster wheel, #4 is written in-terms of the co-ordinate frame fixed to the bracket #5, i.e.  $\{e_4, f_4, k\}$ , as

$$\omega_4 = \dot{\phi}_1 e_4 + (\omega_3 + \dot{\psi}_1) k, \quad \dot{o}_4 = \dot{\phi}_1 e_4 \quad (4.21)$$

Using equations 4.19 and 4.11, the twist  $t_4$  can be written as

$$t_4 = \begin{pmatrix} \Theta_4 \\ C_4 \end{pmatrix} \dot{\theta}_a \quad (4.22)$$

Using the definition of  $F(i, j)$  in equation 4.19,  $\Theta_4$  and  $C_4$  can be written as

$$\Theta_4 = [F_{11}e_4 + \bar{F}_{21}k \quad F_{12}e_4 + \bar{F}_{22}k], \quad C_4 = r[-F_{11}f_4 \quad -F_{12}f_4]$$

$$\bar{F}_{21} = F_{21} + \rho\delta, \quad \bar{F}_{22} = F_{22} - \rho\delta$$

The angular and the liner velocity of the CM of the caster bracket#5 expressed in the co-ordinate frame fixed to the bracket is given by

$$\omega_4 = \dot{\phi}_1 e_3 + \dot{\psi}_1 k, \quad \dot{o}_4 = \dot{o}_4 + \omega_5 \times [-df_3] \quad (4.23)$$

Using equations 4.11 & 4.20, the twist  $t_5$  can be written as

$$t_5 = \begin{pmatrix} \Theta_5 \\ C_5 \end{pmatrix} \dot{\theta}_a \quad (4.24)$$

where

$$\Theta_5 \equiv [\bar{F}_{21}k \quad \bar{F}_{22}k], \quad C_5 \equiv d[(1/2)\bar{F}_{21}e_4 - \rho F_{11}f_4 \quad (1/2)\bar{F}_{22}e_4 - \rho F_{12}f_4]$$

In a similar manner, the twist  $t_6$  and  $t_7$  of the other caster wheel and its bracket can be written as

$$t_6 = \begin{pmatrix} \Theta_6 \\ C_6 \end{pmatrix} \dot{\theta}_a, \quad t_7 = \begin{pmatrix} \Theta_7 \\ C_7 \end{pmatrix} \dot{\theta}_a \quad (4.25)$$

Using  $G(i, j)$  defined in equation 4.20,  $\Theta_6$ ,  $C_6$ ,  $\Theta_7$  and  $C_7$  can be written as

$$\Theta_6 \equiv [G_{11}e_6 + \bar{G}_{21}k \quad G_{12}e_6 + \bar{G}_{22}k], \quad C_6 \equiv r[-G_{11}f_6 \quad -G_{12}f_6]$$

$$\Theta_7 \equiv [\bar{G}_{21}k \quad \bar{G}_{22}k], \quad C_7 \equiv d[(1/2)\bar{G}_{21}e_6 - \rho G_{11}f_6 \quad (1/2)\bar{G}_{22}e_6 - \rho G_{12}f_6]$$

$$\bar{G}_{21} \equiv G_{21} + \rho\delta, \quad \bar{G}_{22} \equiv G_{22} - \rho\delta$$

## 4.2.2 Dynamic equations

Based on the twist calculated in terms of the independent joint rate vector  $\dot{\theta}_a$ , we now derive the generalized inertia matrix and the matrix of convective inertia term for the coupled equation of motion 4.9.

### 4.2.2.1 Generalized Inertia Matrix, $I$

The above equations gives the twist of individual body, i.e  $t_i = T_i\dot{\theta}_a$ . As defined earlier  $t = [t_1^T, t_2^T, \dots, t_7^T]^T$  and  $t = T\theta$  we get

$$T = [T_1^T, T_2^T, \dots, T_7^T]^T$$

Since the matrix  $M$  is block diagonal, the inertia matrix of the full system is given by

$$I = T^T M T = T_1^T M_1 T_1 + T_2^T M_2 T_2 + \dots T_7^T M_7 T_7 \quad (4.26)$$

We define the contribution of rear wheels alone to the inertia matrix by  $I_m$  as

$$I_m = \sum_{i=1,2} T_i^T M_i T_i$$

then

$$I_m = \begin{pmatrix} I_w + (\rho\delta)^2 H + m_w r^2 & -2(\rho\delta)^2 H \\ -2(\rho\delta)^2 H & I + (\rho\delta)^2 H + m_w r^2 \end{pmatrix} \quad (4.27)$$

where  $M_i \equiv \begin{pmatrix} \tilde{I}_w & 0 \\ 0 & m_w \mathbf{1} \end{pmatrix}$  and  $\tilde{I}_w \equiv \begin{pmatrix} I_w & 0 & 0 \\ 0 & H \\ 0 & 0 & H \end{pmatrix}$ . Matrix  $\tilde{I}_w$  is the  $3 \times 3$  moment

of inertia matrix of the wheel in co-ordinate frame  $\{i, j, k\}$ , mass of the motorized wheels is  $m_w$  and  $m_b \mathbf{1}$  is the  $3 \times 3$  identity matrix. If the mass of the platform is

$m_p$  and its moment of inertia about vector  $k$  is  $I_p$ , then as derived in [16]

$$I_3 = T_3^T M_3 T_3 = I_p(\rho\delta)^2 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + m_p r^2 \begin{pmatrix} (1/4) + \gamma^2 & (1/4) - \gamma^2 \\ (1/4) - \gamma^2 & (1/4) + \gamma^2 \end{pmatrix} \quad (4.28)$$

Similarly, if  $m_c$  is the mass of the castor wheel and it is assumed to be a solid disk, then the generalized inertia matrix can be written as

$$I_c = \sum_{i=4,6} T_i^T M_i T_i = (m_c r^2 / 4) \begin{pmatrix} 6F_{11}^2 + \bar{F}_{21}^2 & 6F_{11}F_{12} + \bar{F}_{21}\bar{F}_{22} \\ 6F_{11}F_{12} + \bar{F}_{21}\bar{F}_{22} & 6F_{12}^2 + \bar{F}_{22}^2 \end{pmatrix} + \begin{pmatrix} 6G_{11}^2 + \bar{G}_{21}^2 & 6G_{11}G_{12} + \bar{G}_{21}\bar{G}_{22} \\ 6G_{11}G_{12} + \bar{G}_{21}\bar{G}_{22} & 6G_{12}^2 + \bar{G}_{22}^2 \end{pmatrix} \quad (4.29)$$

If the mass of the brackets, i.e., body #5 and #7, are small compared to the mass of the castor wheels, then the contribution of  $T_5^T M_5 T_5$  and  $T_7^T M_7 T_7$  can be neglected.

#### 4.2.2.2 Matrix of Convective Inertia term $C$

The matrix of convective inertia terms of equation 4.9 can be broken down into two parts,  $T^T M \dot{T}$  and  $T^T W M T$ . As given in equations 4.14, 4.15 and 4.16  $T_1, T_2, T_3$  of the rear wheels and the platform is constant. Therefore  $T^T M \dot{T} = 0$ . The generalized inertia matrix too is constant for the rear wheels and the platform, so the vector  $I_i \omega_i$  is parallel to vector  $\omega_i, i = 1..3 \rightarrow \omega \times I \omega = 0 \rightarrow T^T W M T = 0$ . This shows that contribution of the rear wheels and the platform to the convective inertia term is zero. Moreover we have considered the mass of the brackets to be zero, so they also do not contribute to the convective inertia term. Hence

$$C = T^T M \dot{T} + T^T W M T = \sum_{i=4,6} T_i^T M_i \dot{T}_i + \sum_{i=4,6} T_i^T W_i M_i T_i \quad (4.30)$$

The expression for the first term is found by using equations 4.19, 4.20, 4.22 and 4.25. The terms  $\dot{F}_{ij}$  and  $\dot{G}_{ij}$  denote the derivatives of the elements of the matrix  $F$

and  $G$  defined in 4.19, 4.20 . To find  $\dot{T}_4$  and  $\dot{T}_6$  we have used the fact  $\dot{e}_4 = \omega_4 \times e_4$  and  $\dot{e}_4 = \omega_6 \times e_6$ .

$$T^T M \dot{T} = (m_c r^2 / 4) \left[ \begin{pmatrix} 6F_{11}\dot{F}_{11} + \bar{F}_{21}\dot{F}_{21} & 6F_{11}\dot{F}_{12} + \bar{F}_{21}\dot{F}_{22} \\ 6\dot{F}_{11}F_{12} + \dot{F}_{21}\bar{F}_{22} & 6F_{12}^2\bar{F}_{22} \end{pmatrix} + \begin{pmatrix} 6G_{11}\dot{G}_{11} + \bar{G}_{21}\dot{G}_{21} & 6G_{11}\dot{G}_{12} + \bar{G}_{21}\dot{G}_{22} \\ 6\dot{G}_{11}G_{12} + \dot{G}_{21}\bar{G}_{22} & 6G_{12}^2\bar{G}_{22} \end{pmatrix} \right] \quad (4.31)$$

The second term of equation 4.30 i.e,  $\sum_{i=4,6} T_i^T W_i M_i T_i$  evaluates to zero, as shown below. Consider castor wheel (body #4). Using equation 4.22 and  $W$  defined in the equation 4.3 we get,

$$T_4^T W_4 M_4 T_4 = [\Theta_4, C_4] \begin{pmatrix} \Omega_4 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} I_4 & 0 \\ 0 & m_4 \mathbf{1} \end{pmatrix} \begin{pmatrix} \Theta_4 \\ C_4 \end{pmatrix} = \Theta_4 \Omega_4 I_4 \Theta_4 \quad (4.32)$$

To evaluate the above equation, we express all the terms in the coordinate system  $\{e_4, f_4, k\}$ . Moreover,  $\omega_4 = \Theta_4 \dot{\theta}_a$ , using definition of  $\Theta_4$  from equation 4.22 we get  $\omega_4 = (F_{11}\dot{\theta}_1 + F_{12}\dot{\theta}_2)e_4 + (F_{21}\dot{\theta}_1 + F_{22}\dot{\theta}_2)k$  and the cros product matrix of  $\omega_4$  as

$$\Omega_4 \equiv \begin{pmatrix} 0 & -(\bar{F}_{21}\dot{\theta}_1 + \bar{F}_{22}\dot{\theta}_2) & 0 \\ (\bar{F}_{21}\dot{\theta}_1 + \bar{F}_{22}\dot{\theta}_2) & 0 & -(F_{11}\dot{\theta}_1 + F_{12}\dot{\theta}_2) \\ 0 & (F_{11}\dot{\theta}_1 + F_{12}\dot{\theta}_2) & 0 \end{pmatrix}$$

When the above expressions are substituted in equation 4.32, we get

$$T_4^T W_4 M_4 T_4 = 0, \quad \Rightarrow T^T W M T = 0 \quad (4.33)$$

So the matrix of convective inertia term  $C$  of equation 4.9 is evaluated as

$$C = (m_c r^2 / 4) \left[ \begin{pmatrix} 6F_{11}\dot{F}_{11} + \bar{F}_{21}\dot{F}_{21} & 6F_{11}\dot{F}_{12} + \bar{F}_{21}\dot{F}_{22} \\ 6\dot{F}_{11}F_{12} + \dot{F}_{21}\bar{F}_{22} & 6F_{12}^2\bar{F}_{22} \end{pmatrix} + \begin{pmatrix} 6G_{11}\dot{G}_{11} + \bar{G}_{21}\dot{G}_{21} & 6G_{11}\dot{G}_{12} + \bar{G}_{21}\dot{G}_{22} \\ 6\dot{G}_{11}G_{12} + \dot{G}_{21}\bar{G}_{22} & 6G_{12}^2\bar{G}_{22} \end{pmatrix} \right] \quad (4.34)$$

All the components of equation 4.9 have now been evaluated, except the  $\tau$ . These are simply the torques exerted by the actuated wheels. This completes the dynamic model of the WMR with generalized caster wheel configuration.

### 4.3 Special cases

#### 4.3.1 Standard caster ( $d_1 = 0$ )

The standard caster wheel configuration can be obtained by setting the value of  $d_1 = 0$ . In such condition, the left hand side matrix of equations 4.19 and 4.20 become a diagonal matrix. Therefore, the first and second equation of 4.19 and 4.20 get divided by  $d_2$  and  $r$ , respectively. The resulting equations relating the unactuated joint rate to actuated joint rates are similar to those reported by [18],[16].

#### 4.3.2 Under Actuated Case ( $d_2 = 0$ )

It can be seen from equation 4.19 that when the *caster offset*  $d_2 = 0$ , the LHS matrix becomes singular. So the unactuated joint rates cannot be determined from  $\dot{\theta}_a$ . It is therefore essential to have proper caster offset in case we need caster like behaviour from a passive wheel.

Another solution is to put an extra actuator to control the bracket motion by controlling  $\psi_i$ . As in the case of Ackerman steering mechanism, where the steering wheel controls the orientation of the front passive wheels of a car.



## 4.4 Simulation

The mobile manipulator proposed in chapter 3 was modelled as a differential drive robot. The front wheels and steering mechanism were not included in the dynamic model as their masses are too small compared to the platform. The weight of each wheel is 0.3Kg, the steering mechanism 0.24Kg, whereas the weight of the platform is 70Kg.

In simulation the vehicle, (point  $O_3$ ), is required to trace a circle of radius 5m. As shown in the figure 4.4.  $\beta$  is the angle between the line joining point  $O_3$  and the origin  $O$  with respect to  $X - axis$ . The function  $\beta(t)$  is defined such that the full circle is completed in 60Sec.

$$\beta(t) = \frac{20\pi}{60^3}t^3 - \frac{30\pi}{60^4}t^4 + \frac{12\pi}{60^5}t^5 \quad (4.35)$$

Moreover the velocity and acceleration of the robot is zero at the beginning ( $t=0$ ) and end of travel ( $t=60$ ). The initial pose of the vehicle is parallel to the  $Y - axis$  i.e.  $\beta = 0$  shown by dotted line in figure 4.4.

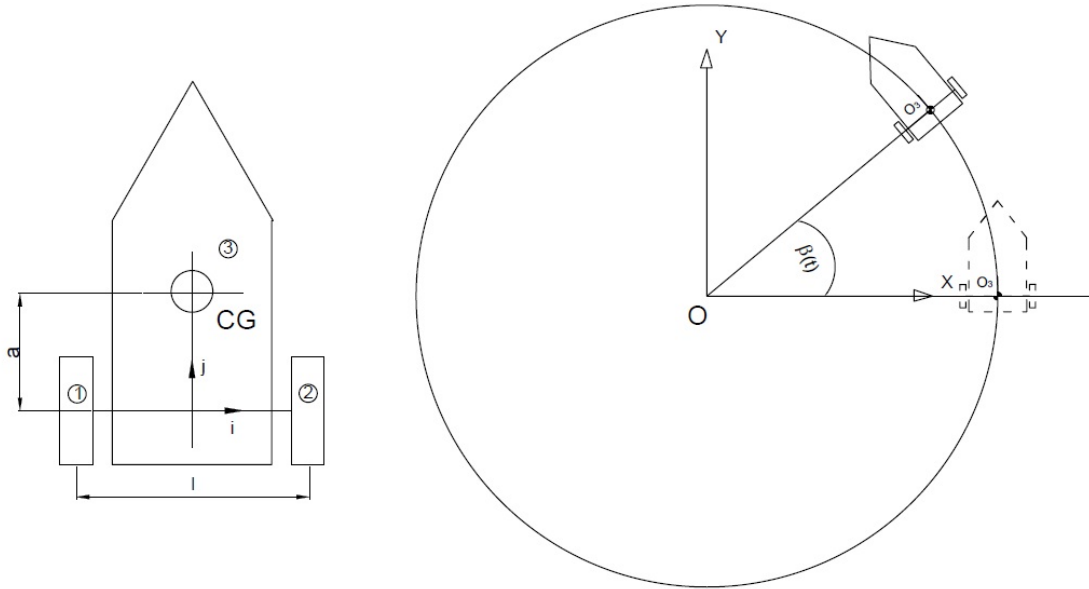


Figure 4.4: Path Traced by Robot

#### 4.4.1 Inverse Dynamics

Using the model inverse kinematic, the wheel velocity and acceleration are determined using pseudo Inverse. The wheel angle, velocity and acceleration were used in the dynamics equation to calculate the torque required by each motor. The results are plotted in figure 4.5.

The equation of dynamic model is given in equation 4.36, where the actuated joints are  $\theta_a(t) = (\theta_l, \theta_r)$  are the rear left and right wheel rotation angles. The dynamic and kinematic parameters used in the simulation is listed in table 4.1. The rear wheels ( $i = 1, 2$ ) and the platform  $i = 3$ , twist  $t_i^T = T_i \theta_a$ , is given in equations 4.14, 4.15 and 4.16. The dynamic equation is of the vehicle on slope is given as

$$\begin{aligned}
T^T M T \ddot{\theta}_a &= -T^T (M \dot{T} + W M T) \dot{\theta}_a + T^T (w^J + w^G) \\
\text{where } T &= (T_1^T T_2^T T_3^T)^T, \quad M = \text{diag}(M_1, M_2, M_3) \\
T^T M T &= I_1 + I_2 + I_m + I_3 = I_m + I_3 \\
W &= \text{diag}(W_1, W_2, W_3), \quad w^G = g \sin \alpha, \quad \alpha = 10^\circ
\end{aligned} \tag{4.36}$$

$w^G$ ,  $w^J$  and  $\alpha$  represent the gravitational force acting along the inclined the external torque/Force applied by a motor and the inclination of the plane to horizontal respectively. The generalized inertia matrix  $I_m$  and  $I_3$  are given in 4.27 and 4.28. As stated earlier the convective inertia term is zeros as the inertia matrices are constant. The motor torque are then given a

$$T_1^T w_1^j = \tau_1, \quad T_2^T w_2^j = \tau_2 \tag{4.37}$$

$$T_3^T w^g = \begin{pmatrix} \rho \delta k & r(\lambda i + (1/2)j) \\ -\rho \delta k & r(-\lambda i + (1/2)j) \end{pmatrix} \begin{pmatrix} 0 \\ -j g \sin \alpha \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} r g \sin \alpha \\ -\frac{1}{2} r g \sin \alpha \end{pmatrix} \tag{4.38}$$

The torques required at the wheels of the vehicle to move up a spiral ramp of slope  $10^\circ$  and radius 5m is given figure 4.5.

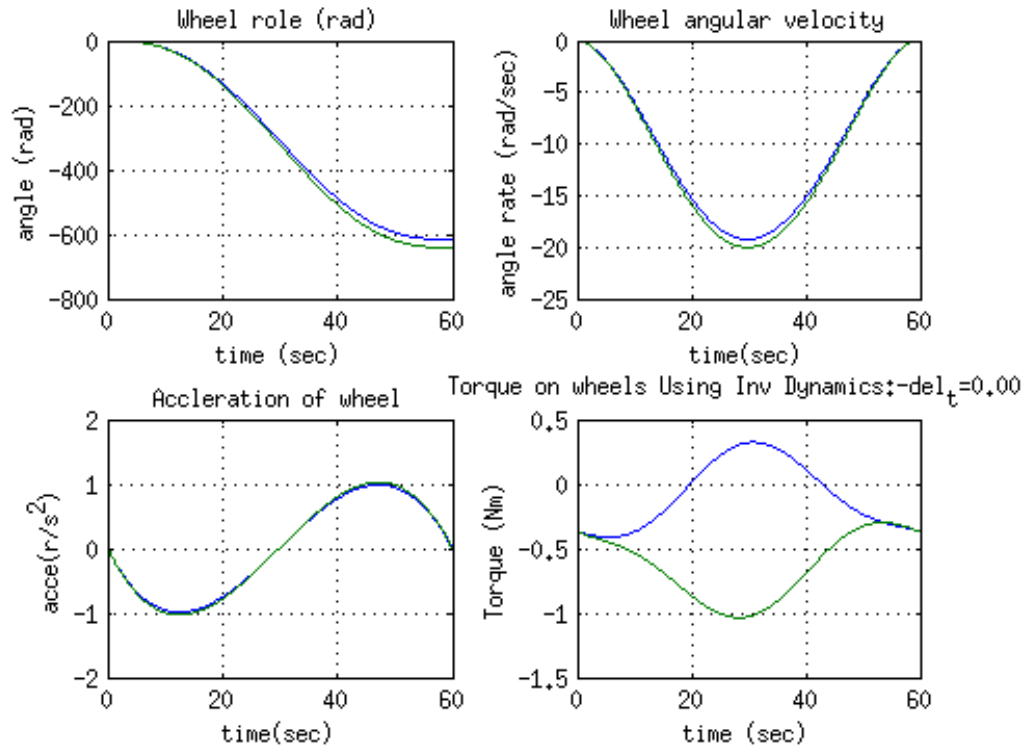


Figure 4.5: Inverse dynamics of the mobile robot

Table 4.1: Dynamic & kinematic parameters

| <i>Part Name</i> | <i>Property</i>   | <i>Value</i>  |
|------------------|-------------------|---|
| Rear Wheels      | mass              | 300g  |
|                  | Moment of Inertia | diag(242, 242, 465)kg $mm^2$  |
| Base Frame       | mass              | 70Kg  |
|                  | Moment Of Inertia | $\begin{pmatrix} 1.18 & 0.01 & -0.05 \\ 0.01 & 1.28 & 0.08 \\ -0.05 & 0.08 & 0.53 \end{pmatrix} Kg - m^2$ |
| l                | length            | 400mm   |
| r                | wheel radius      | 100mm   |
| a                | see figure        |   |

## 4.5 Summary

In this chapter the dynamic equation was derived for the most general form of caster wheel configuration. Even though only two caster wheel configuration was considered the same formulation can be extended to any number of caster wheels. It is shown that the dynamics of standard caster wheels is a special case of the general case with  $d_1 = 0$ . It is also proven why a caster needs non-zero caster offset. There is a need of extra actuator in case  $d_2 = 0$ .



# Chapter 5

## Control of Mobile Manipulator

In this chapter, the control architecture of the teleoperated mobile robot is presented. The user interface for teleoperation is discussed. The control algorithm running on the mobile robot and the hardware used for the control of traction and steering is discussed. The protocol used for communication between the robot and the user interface is also described in detail.

### 5.1 Control Architecture and Hardware

The mobile manipulator explained in chapter 3 was planned to be teleoperated over a wireless network. The control block diagram and architecture are shown in Figure 5.1. It has a remote control station which is the interface for the operator

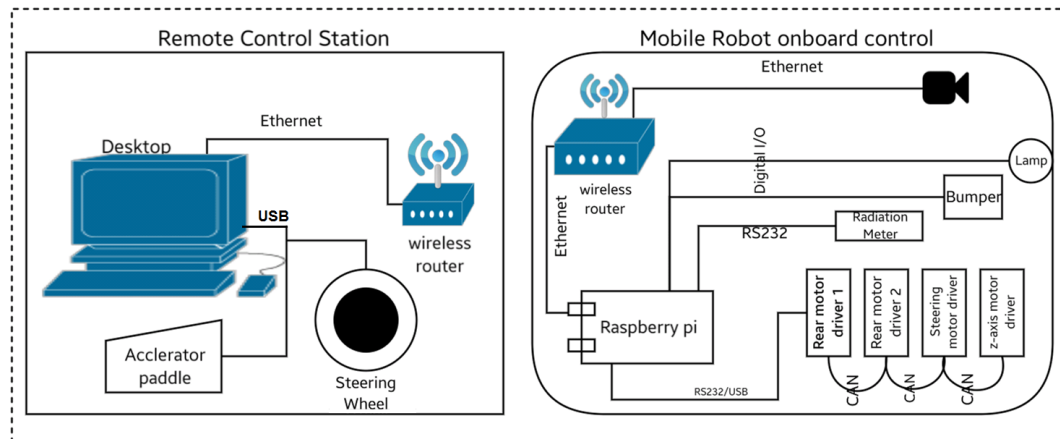
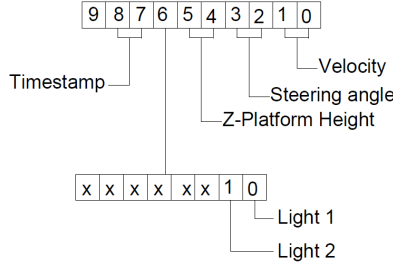


Figure 5.1: Control architecture

PC to Raspberri Pi



PI to PC

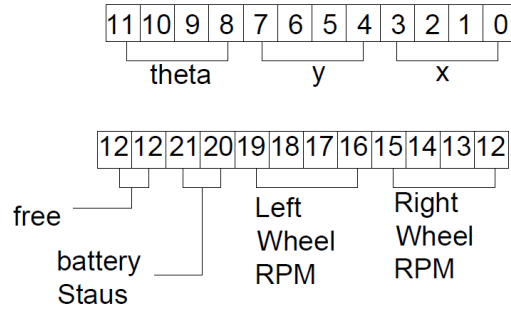


Figure 5.2: Data from PC to the robot    Figure 5.3: Data from the robot to PC

and a local onboard controller of the mobile robot. They communicate over a dedicated wireless network. The remote station sends data packet every in 50 millisecond (20Hz) to the mobile robot. The commanded velocity, steer angle, z position of the platform, and state of the detector and headlamps constitute the data packet sent by the remote station as shown in Figure 5.2. The onboard controller of the mobile robot replies with a data packet consisting of the  $X$ ,  $Y$  position and orientation  $\theta$  of the robot, the current steer angle, angular velocities of each wheel, the z position of the top platform, battery voltage and current of each motor. They are indicated in Figure 5.3.

### 5.1.1 Local Onboard Controller

The onboard computer which is Raspberry Pi running raspian (linux) os receives command from the remote station and controls the robot hardware through customized c++ application. The Raspberry Pi is daisy chained to the four Maxon Make, EPOS2 motor controllers/drivers. The communication between the onboard computer and the first Maxon controller is over usb/RS232 interface using Maxon's proprietorial protocol [77]. The first controller serves as CAN master for the rest of the controllers. The rear wheel motor drivers were configured in velocity servo loop. The drivers for steering and the z-axis motors were configured in position control loop. The camera mounted on the mobile robot and Raspberry Pi were connected over Ethernet via a wireless hub. The wiring digram of the

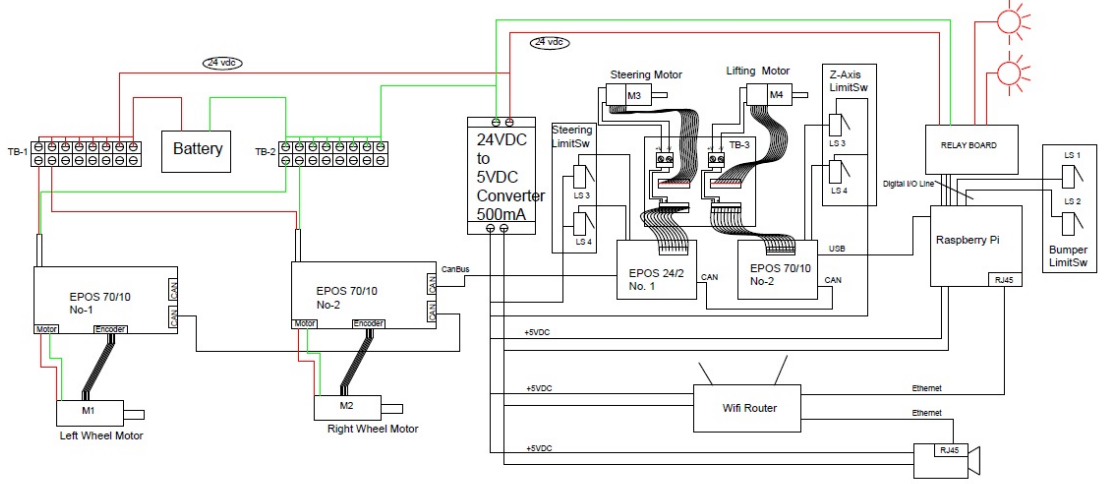


Figure 5.4: Wiring diagram of the WMR

robot is given in Figure 5.4. Since the onboard camera is connected to the wireless network directly, it does not interfere with the command loop between the Raspberri Pi and the PC.

The mobile robot is teleoperated using position-speed command as in [78]. The workspace for the mobile robot was assumed infinite compared to the input device. In case of manipulators position-position control approach is generally used with scaling. In this case, the mixed approach was used. The steering angle was controlled in position-position mode whereas the mobile robot's speed was controlled by foot paddle's position, i.e. in position-velocity mode. This can be given by the following equation:

$$\begin{pmatrix} V \\ \theta_s \end{pmatrix} = \begin{pmatrix} K_v & 0 \\ 0 & K_s \end{pmatrix} \begin{pmatrix} \tilde{X}_p \\ \tilde{\Theta}_s \end{pmatrix} \quad (5.1)$$

where  $\tilde{X}_p$  is the displacement of paddle,  $\tilde{\theta}_s$  is the twist of steering wheel,  $K_v$  and  $K_s$  are the proportionality constants,  $V$  is the velocity of point  $O_3$  Figure 5.10 and  $\theta_s$  is the displacement of the steer motor. These proportionality constants are derived as below based on extreme limits and listed in Table 5.1.



Table 5.1: Proportionality constant table

| Robot parameters | range           | Joy-Stick Parameter | range                     | parameter Value |
|------------------|-----------------|---------------------|---------------------------|-----------------|
| $\theta_s$       | -60 to +60      | $\Theta_s$          | $-90^\circ$ to $90^\circ$ | $K_s = 2/3$     |
| $V$              | 0 to +60 mm/sec | $\tilde{X}_p$       | 0 to 30mm                 | $K_v = 2$       |

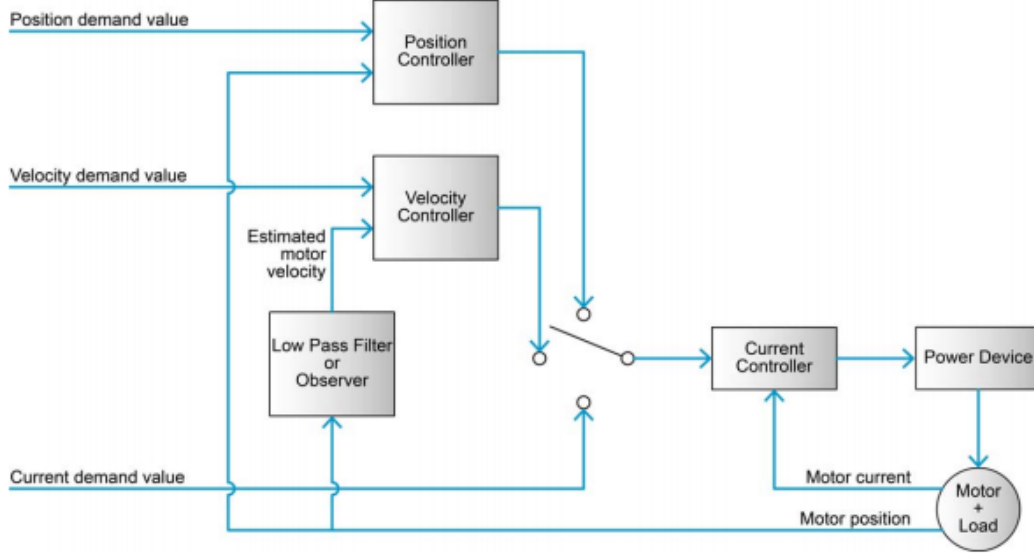


Figure 5.5: Block diagram of EPOS4 controller

### 5.1.2 Details of the Motor Controller

Three EPOS2 controllers from Maxon Motors were used to control the mobile robot's rear wheel velocity and the steering gears position. Each controller can control one motor. The overall architecture of the controller as given in [79], [80] is shown in Figure 5.5. The controller can be configured in either current, position or velocity control mode. The inner most current loop controls the torque of the motor. The current feedback loop shown in Figure 5.6 is a Proportional Integrator (PI) controller, running at 25KHz and the transfer function of the PI block is given as 5.2.

$$C(s) = K_p + \frac{K_I}{s} \quad (5.2)$$

where  $K_p$  and  $K_I$  are the proportional and integral gains.

The rear motor controllers are configured in the velocity control mode. The block diagram of the velocity loop is given in Figure 5.7. The velocity controller is a PI controller with velocity and acceleration feed-forward. The transfer function

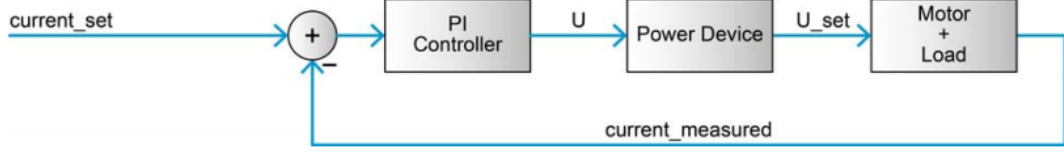


Figure 5.6: Current control block

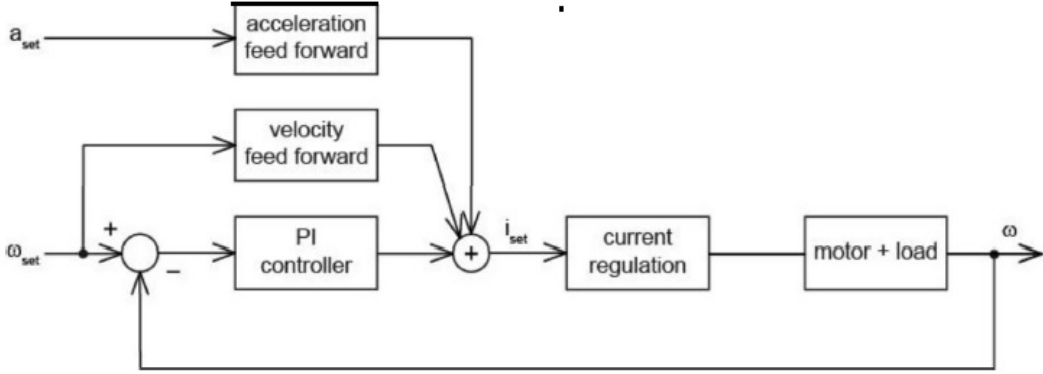


Figure 5.7: Velocity control block

$V(s)$  of the velocity loop PI block is given by

$$V(s) = K_{p\omega} + \frac{K_{I\omega}}{s} \quad (5.3)$$

where  $K_{p\omega}$  and  $K_{I\omega}$  are proportional and integral gains for velocity, respectively. The sampling rate of the velocity loop is 2.5 KHz. The feedforward acceleration and velocity was used to compensate for the known inertial load and viscous frictional load [80] respectively. Velocity is estimated from differentiation of the position data, the low-pass filter in Figure 5.5 eliminates noise due to differentiation. The transfer function  $H(s)$  for the low-pass filter is given by

$$H(s) = \frac{1}{1 + \frac{K_{p\omega}}{48K_{I\omega}}} \quad (5.4)$$

The gain values used for each rear motor in the velocity control mode is listed in Table 5.2. No acceleration or velocity feedforward was used. These parameters were determined by auto tuning.

The steering motor was in position control mode. The block diagram is shown

Table 5.2: Parameters for left and right rear wheel motor controllers

| <i>Gain</i><br><i>Parameter</i> | <i>Right motor</i><br><i>Value</i> | <i>Left motor</i><br><i>Value</i> | <i>Unit</i>          |
|---------------------------------|------------------------------------|-----------------------------------|----------------------|
| $K_p$                           | 300                                | 230                               | $\frac{mV}{A}$       |
| $K_I$                           | 100                                | 53                                | $\frac{mV}{A.mS}$    |
| $K_{P\omega}$                   | 1000                               | 5182                              | $\frac{rad}{mA.sec}$ |
| $K_{I\omega}$                   | 100                                | 425                               | $\frac{mA}{rad}$     |

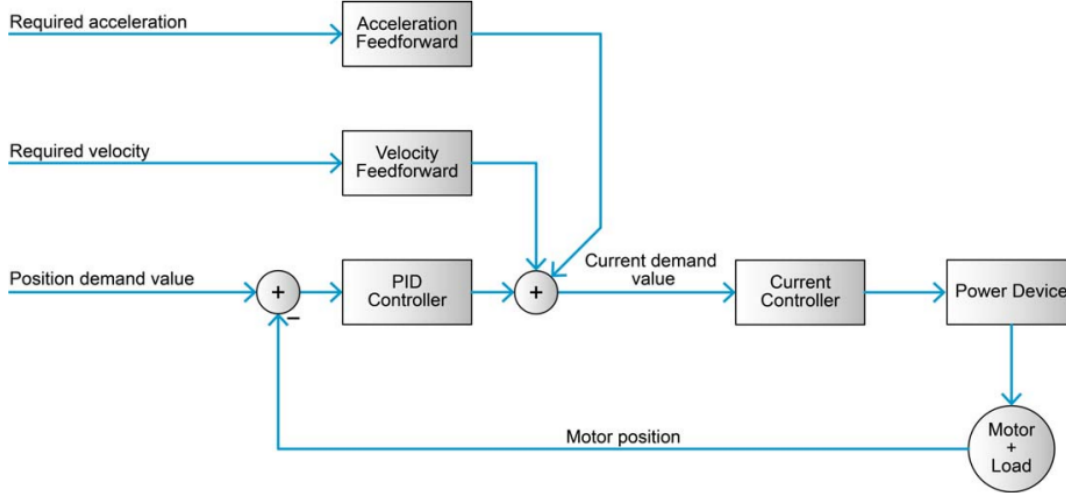


Figure 5.8: Position control block

in Figure 5.8. It is a PID controller with transfer function given as

$$P(s) = K_{PP} + K_{IP}s + \frac{K_{DPS}}{1 + \frac{K_{DP}}{10K_{PP}}s} \quad (5.5)$$

where  $K_{PP}$ ,  $K_{IP}$  and  $K_{DP}$  are position proportional, integral and derivative gains respectively. The velocity feed-forward  $F_{\omega P}$  and acceleration feed-forward  $F_{\alpha P}$  were used in position control loop to take care of viscous friction and known inertial load. The gains for controller were decided using auto tuning software provided by Maxon Motors. The values are reported in Table 5.3.

## 5.2 The control algorithm for the WMR

This section discusses in detail the algorithm running on the onboard controller. Command received by the controller was parsed to extract the velocity and the

Table 5.3: Parameters of steering motor controller

| Gain Parameter | Value | Unit                         |
|----------------|-------|------------------------------|
| $K_p$          | 537   | $\frac{mV}{A}$               |
| $K_I$          | 307   | $\frac{mV}{mA \cdot sec}$    |
| $K_{PP}$       | 128   | $\frac{rad}{mA}$             |
| $K_{IP}$       | 663   | $\frac{rad}{mA \cdot sec}$   |
| $K_{ID}$       | 200   | $\frac{rad}{mA \cdot sec^2}$ |
| $F_{\omega P}$ | 0     | $\frac{rad}{mA \cdot sec}$   |
| $F_{\alpha P}$ | 54    | $\frac{rad}{mA \cdot sec^2}$ |

steer angle information. They were suitably scaled to get command velocity  $V$  in mm/sec, and steering angle  $\theta_s$  in radians. It may be noted that the velocity  $V$  corresponds to the velocity of point  $O_r$  the reference point of the mobile robot. Next the set point for each motor was calculated and sent to individual drive. The algorithm is listed below and the block diagram for the same in Figure 5.9

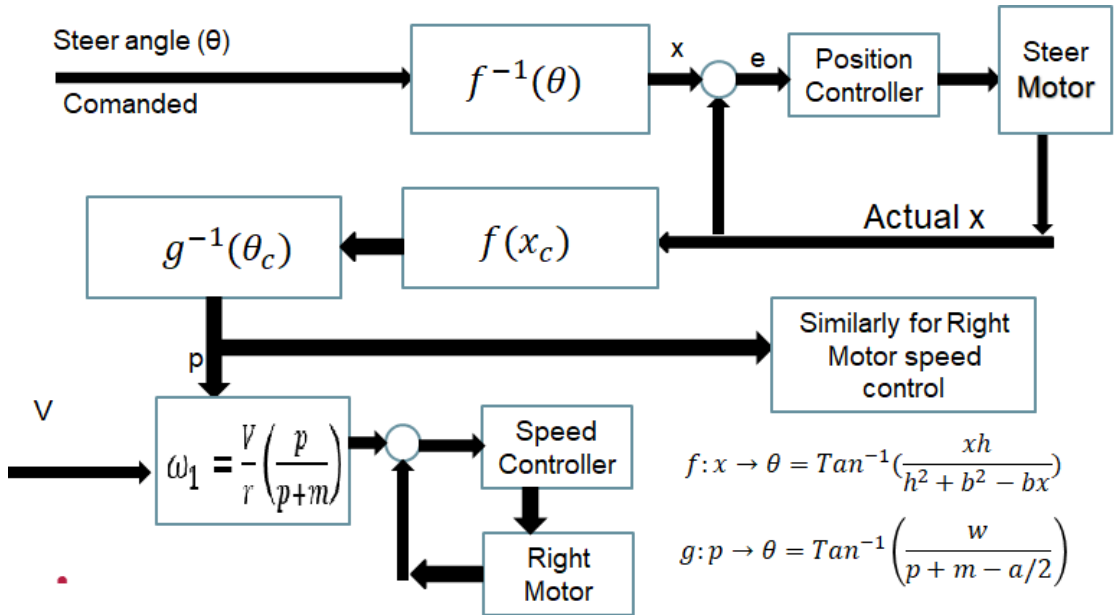


Figure 5.9: Block diagram of WMR controler

1. Calculate the setpoint for steering motor  $\theta_{SM}$  based on  $\theta_s$ .
2. Read the current steering angle  $\phi_{ic}$ .
3. Calculate the velocity setpoints  $\omega_i$  and  $\omega_o$  of rear wheels based on the  $V$  and  $\phi_{ic}$ .

4. Command setpoints  $\omega_i$ ,  $\omega_o$  and  $\theta_{SM}$  to each motor.

The above loop is repeated every 50 mSec.

It may be noted that the steer angle command received from the control station is not directly sent to the steer motor as set point after suitable scaling. The steer set point is based on the current rear wheel velocities. This is important as the response time of the motors are different. The above methodology helps minimize the deviation from the Ackerman steering condition even during transit condition, particularly, in case of large change in commanded  $v$  and  $\theta_s$ . Each block in Figure 5.9 is discussed next.

It may be noted that the  $\theta_s$  always refers to the steer angle of the inner front wheel  $\phi_i$  and  $V$  refers to  $O_r$  as shown in figure 5.10, i.e.

$$\theta_s = \phi_i \quad (5.6)$$

The set point of the steering motor at the output of gear box  $\theta_{SM}$  is given by equations 5.7 and 5.8 based on the geometry of Davis steering gear [65].

$$\tan \phi_i = \frac{xh}{h^2 + b^2 - bx}$$

or

$$f(\phi_i) : x = \frac{\tan \phi_i (h^2 + b^2)}{h + b \tan \phi_i} \quad (5.7)$$

where  $x$  is the displacement of the rack and  $h$  and  $b$  are link lengths. The rack is connected to the steering motor by pinion of pcd  $D_p$  (30mm) as shown in Figure 5.11. Therefore, the steering motor angle,  $\theta_{Sm}$ , is given below as

$$H(x) : \theta_{Sm} = x \frac{360}{\pi D_p} \quad (5.8)$$

Next, the equation relating current steer angle  $\phi_i$  and rear wheel set point

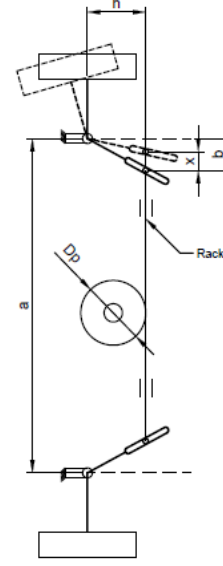
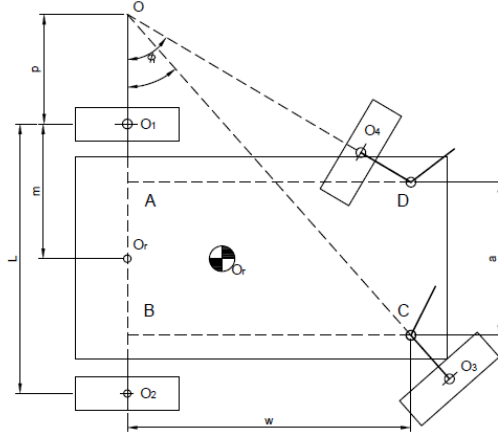


Figure 5.10: Ackerman Steering Condition      Figure 5.11: Davis Steering Gear

velocities  $\omega_{RS}$  and  $\omega_{LS}$  is presented. From the geometry of Figure 5.10, one gets

$$\tan \phi_i = \frac{\bar{BD}}{\bar{OB}} = \frac{w}{p + m - a/2}$$

$$g : \phi_i \rightarrow p, \quad p = \frac{w}{\tan \phi_c} - m + \frac{a}{2} \quad (5.9)$$

Now using Equations 4.10 and 4.11 in equations relating the right and left wheel velocities to the WMR platform angular velocity  $\omega_3$  and velocity,  $V$ , of the reference point  $O_r$ , presented below

$$\dot{O}_r = \dot{O}_i + \omega_3 \times (O_r - O_i)$$

$$\dot{O}_r = \dot{O}_o + \omega_3 \times (O_r - O_o)$$

We get the velocity of each rear wheel as

$$\omega_i = \frac{Vp}{r(p + m)}$$

$$\omega_o = \frac{V(p + 2m)}{r(p + m)}$$

Using the above equations and Equation 5.9, the setpoints for the rear wheels are

as follows

$$\omega_i = \frac{V \left( \frac{w}{\tan \theta_o} - m + \frac{a}{2} \right)}{r \frac{w}{\tan \theta_o} - m + \frac{a}{2} + m} \quad (5.10)$$

$$\omega_o = \frac{V \left( \frac{w}{\tan \theta_o} - m + \frac{a}{2} \right) + 2m}{r \frac{w}{\tan \theta_o} - m + \frac{a}{2} + m} \quad (5.11)$$

### 5.2.1 Wheel Odometry

The dead reckoning odometry can be performed based on either the differential drive or Bicycle model. In the present case, we use the differential drive model was used/ Where the rear wheel velocities were used to determine the position and orientation of the mobile robot. The position here means the position of the reference point  $O_r$ . The steps are followes.

1. calculate  $V$  and  $\omega_3$  from current wheel velocities  $\omega_1$  and  $\omega_2$  using equations 4.11 and 4.12 with  $a = 0$ .
2. integrate  $V$  and  $\omega_3$  over time step.

If  $x(t)$ ,  $y(t)$  are the coordinate of  $O_r$  and  $\beta(t)$  be the orientation of the robot with some global coordinate system, the kinematic model of the differential wheel robots is given by [9]

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\beta} \end{pmatrix} = \begin{pmatrix} \cos \beta & 0 \\ \sin \beta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} V \\ \omega_3 \end{pmatrix} \quad (5.12)$$

Equation 5.12 is numerically integrated for time  $\delta t$  using the following expressions:

$$x(i+1) = x(i) + \delta t V(i) \cos \beta(i); \quad (5.13)$$

$$y(i+1) = y(i) + \delta t V(i) \sin \beta(i); \quad (5.14)$$

$$\beta(i+1) = \beta(i) + \delta t \omega_3(i); \quad (5.15)$$

where  $i$  is at time step  $t_i$ . Next, the actual odometric results for the vehicle moving in a circle and a straight line are presented.

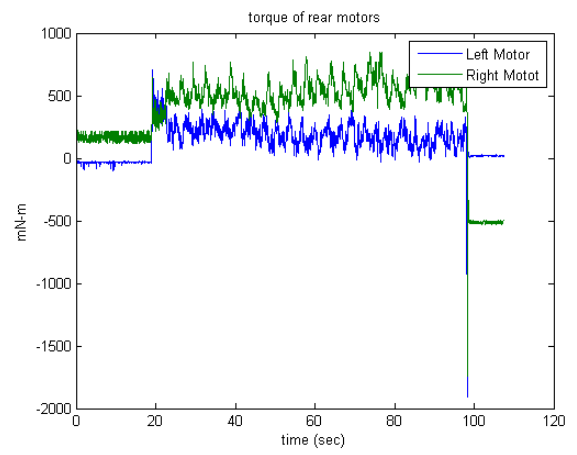


Figure 5.12: Tracing a line

Figure 5.13: Motor torque

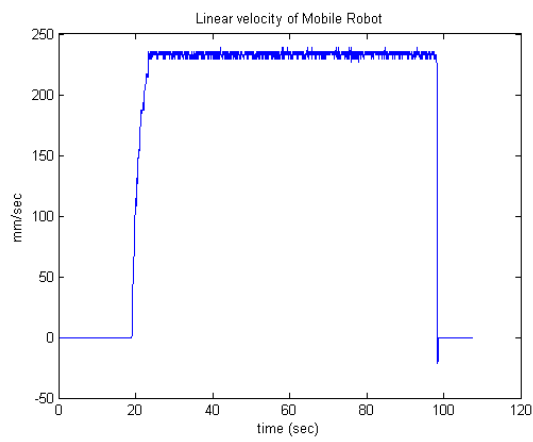


Figure 5.14: Linear velocity

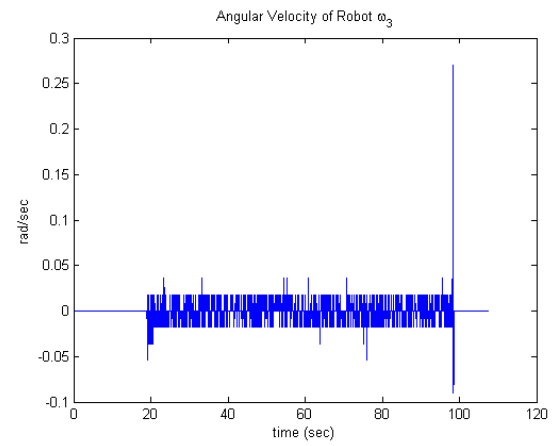


Figure 5.15: Angular velocity



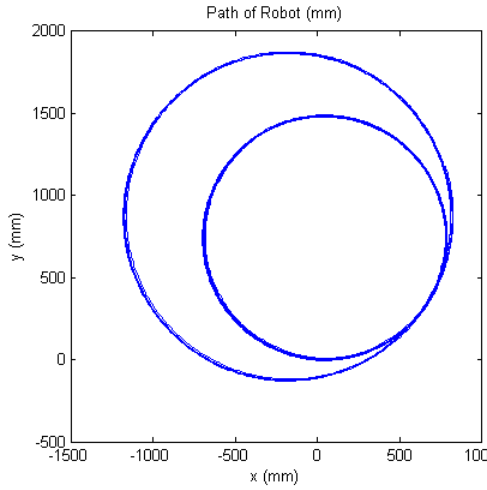


Figure 5.16: Tracing a circle

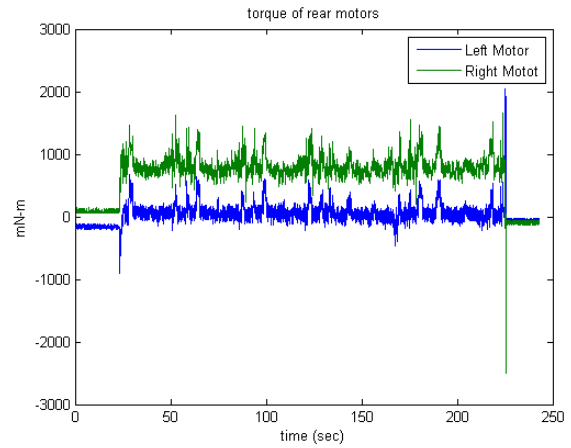


Figure 5.17: Motor torque

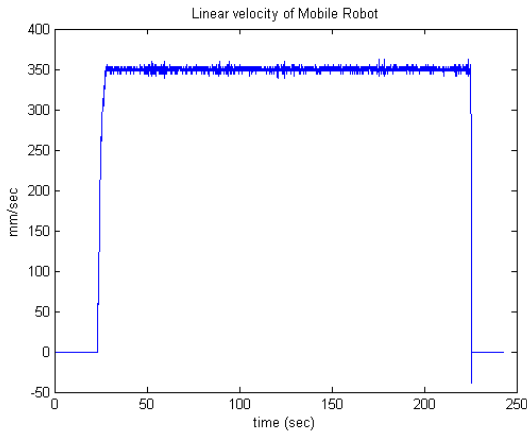


Figure 5.18: Linear velocity

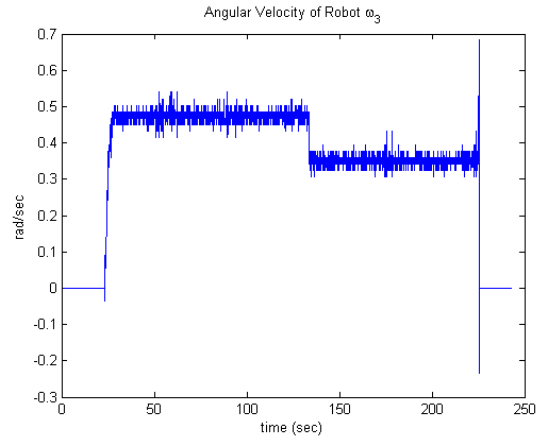


Figure 5.19: Angular velocity

As seen in the graph of Figure 5.12 there is a lateral shift in the robots path calculated using odometry. There is a linear shift too, which can be observed due to longer path calculated by odometry. The lateral shift of 200 mm and a linear shift of 300 mm for 15000 mm long path was calculated. This clearly indicates slip in the wheels. The torque curves also show that one wheel is more loaded than the other; this is expected as the battery weight was on one side of the robot.

### 5.2.2 The Remote Control Station

The operator controls the vehicle from a local station away from the robot over a wireless network. The control station consists of a desktop computer running Windows XP. A steering wheel and two foot switches are connected to the desktop.

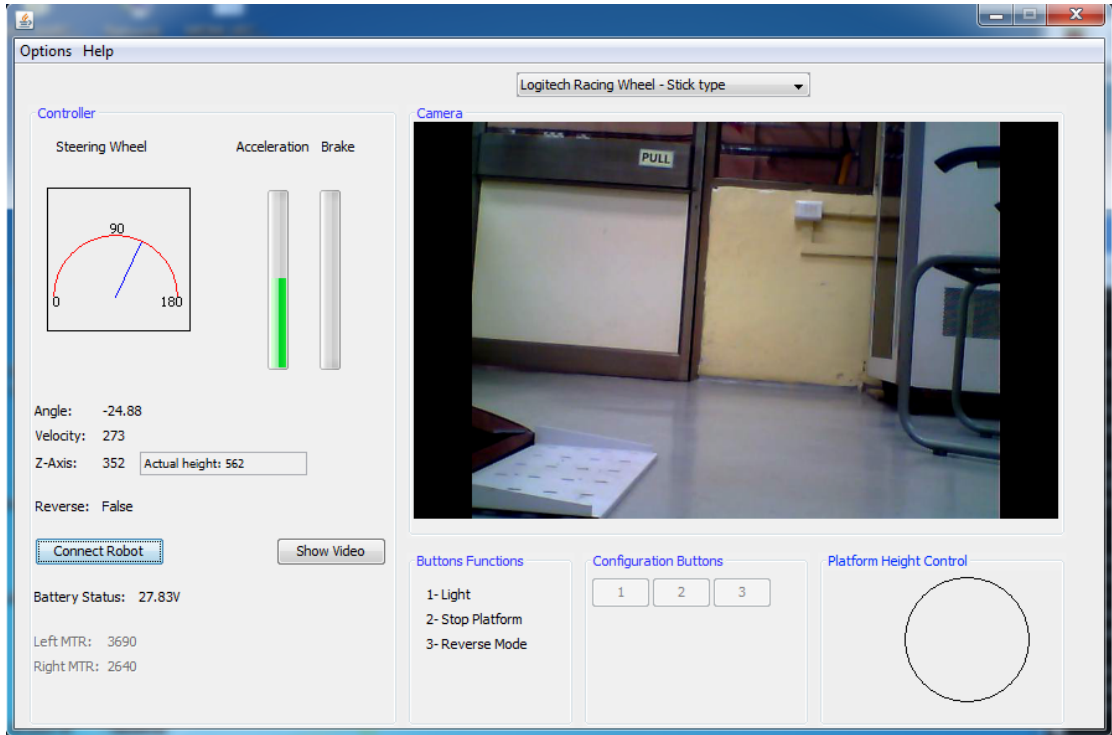


Figure 5.20: User interface for teleoperation

The steering wheel sets the steering of the remote mobile robot and the footpadel is used to set the velocity  $V$  of the robot. A push button in the steering wheel is used to reverse the direction of motion of the robot.

The screen of the desktop displays video streaming from the mobile robot's on board camera. A graphical user interface (GUI) shown in Figure 5.20 displays the robot's parameters such as current steer angle, velocity of each rear wheels and the position of the z-axis. Buttons on the GUI operates the z-axis, head lamps, etc.

### 5.3 Summary

In this chapter, the control architecture of the mobile robot has been presented. The algorithm used to move the robot was discussed in detail. The odometry used for pose estimation of the robot was also presented with the experimental results.

## 5.4 note

Include legends in graphs

increase font in figure 5.10

Remove jitter in odometry add new result.

# Chapter 6

## Simulation Time Delay

### Tele-operation

In this chapter simulation of a teleoperated mobile platform is presented. In teleoperation the human operators observe a remote scene through camera/s, and manipulating the local steering wheel and accelerator paddle as shown in figure 6.1 . The command is transmitted to the mobile robot over wireless network. The operator response is based on the latest feedback images from the cameras. In general there is time lag when communication take place over wireless network. The time lag deteriorates the human performance as discussed in [44] and references there in. The this chapter simulation of teleoperation both for delayless and delay transmission network are presented.

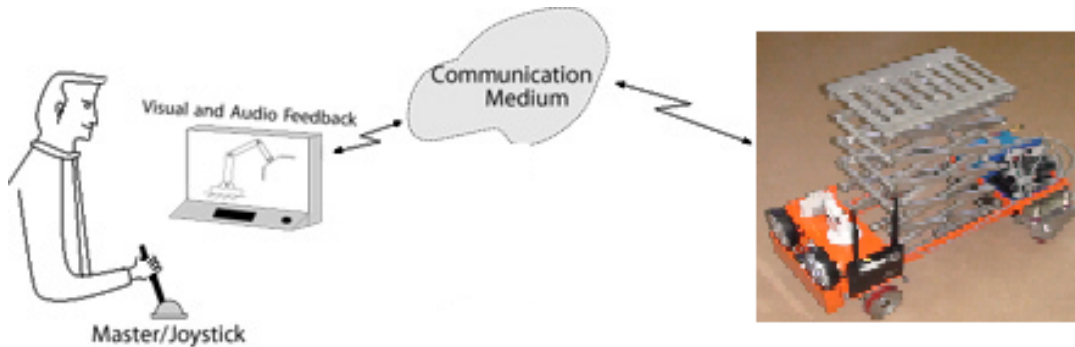


Figure 6.1: Teleoperation Architecture

## 6.1 Modeling of mobile platform

The standard kinematic model as described in [9] of the mobile platform is used for the simulation. This use of kinematic is justified as the vehicle is expected to move at relatively slow speed and model is simple. Inputs to the model are left and right rear wheel velocities. The front wheels are steered to satisfy the Ackerman condition as presented in \*\*\*\* and are assumed to attain the desired angle instantaneously. Therefore the robot can be treated as differential drive robot. The kinematic model of the platform is presented below

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} r_w/2 & r_w/2 \\ 1/b & -1/b \end{pmatrix} \begin{pmatrix} \dot{\phi}_L \\ \dot{\phi}_R \end{pmatrix} \quad (6.1)$$

Where ,  $b$  is the distance between the rear wheels,  $r_w$  wheel radius.  $\dot{\phi}_R$  and  $\dot{\phi}_L$  are the left and right wheel rotational velocity.

The operator station sends the command  $u_1$  and  $u_2$  over the wireless network, in general it will be delayed by  $\delta$  time. These commands are interpreted by the robot controller as the left and right wheel velocities. Therefore by taking the time delay into consideration we can write

$$\begin{pmatrix} \dot{\phi}_R(t) \\ \dot{\phi}_L(t) \end{pmatrix} = \begin{pmatrix} u_r(t - \delta) \\ u_l(t - \delta) \end{pmatrix} \quad (6.2)$$

The control inputs to the mobile robot  $u_1$  and  $u_2$  are generate by the operator based on the visual data available to him. We next present the model of the human operator to be used for simulation the complete loop.

## 6.2 Modeling the human operator

In order to simulate the teleoperation loop we need a mathematical model of human operator. The mathematical modelling of the operator's action is modelled

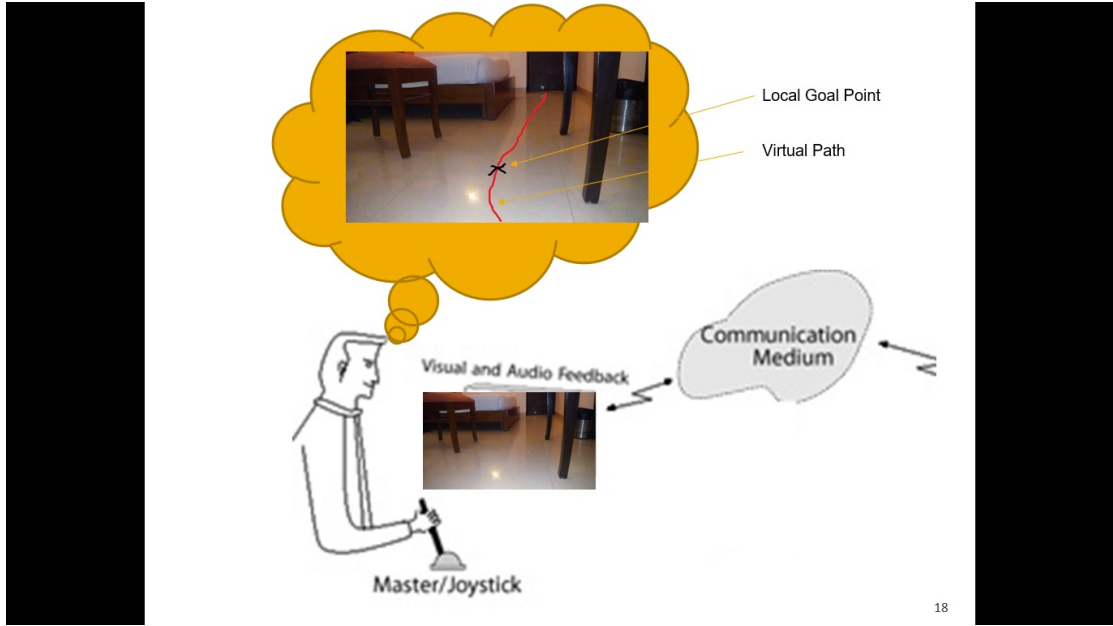


Figure 6.2: Assumed driving strategy

assuming a car driving metaphor. The video feedback, which the he receives of the remote environment, give him the idea of the vehicles position and the tentative next goal point ( $p$ ) based on a lookahead distance ( $l$ ). He then constructs a virtual path mentally and tries to manoeuvre or steers the robot to follow that path as shown in figure 6.2. As he moves forward the goal point keeps changing until he reaches the desired location. This methodology of path tracing is known as pure pursuit [19] or following the carrot strategy.

The mathematical model for the pure pursuit method of path following can be derived as given in figure 6.3. As shown in figure 6.3, the origin of the coordinate system is at point  $o$ , the middle of rear axis of the robot. Since the differential drive robot can move only about a circle with center lying on the line along its rear axis, a arc  $op$  of radius  $r$ , is drawn with center  $o1$  and passing through  $o$  and  $p$ . Where  $p$  is a point on the path to be traced by the robot. The linear distance  $l$  between the points  $o$  and  $p$  is called the look ahead distance. This distance in the case of a teleoperated robot will depend on the field of view of the remote location camera and the obstacles in present in the remote environment.

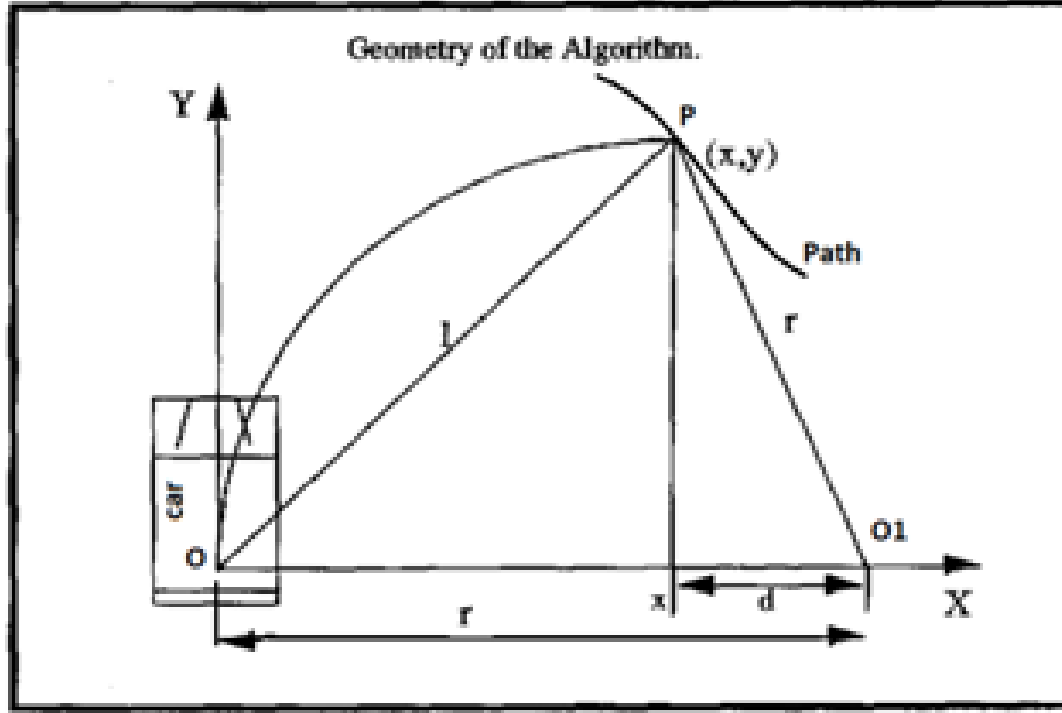


Figure 6.3: Geometry of Pure Pursuit

If  $(x, y)$  is the coordinate of point  $p$  in  $X - Y$  coordinate system, then

$$x^2 + y^2 = l^2, \quad d = r - x \quad (6.3)$$

Similarly, from triangle  $p, x, o1$  we get

$$d^2 + y^2 = r^2 \Rightarrow (r - x)^2 + y^2 = r^2 \Rightarrow x^2 + y^2 - 2rx = 0 \quad (6.4)$$

Replacing  $x^2$  and  $y^2$  in equation 6.4 with 6.3 we get

$$2rx = l^2 \Rightarrow r = \frac{l^2}{2x} \quad (6.5)$$

Once, the radius  $r$  the desired linear velocity  $v$  of robot is known the angular velocity of the vehicle is  $\dot{\theta} = -v/r$ . The rear wheel desired velocities  $\dot{\phi}_d(t)$  and  $\dot{\phi}(t)$  can be calculated from equation 6.1. Where  $\dot{y} = v$  and to match the orientation of vehicle with that in figure 6.3 we set  $\theta = 90^\circ$ . We get

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ r_w/2 & r_w/2 \\ b/2 & -b/2 \end{pmatrix} \begin{pmatrix} \dot{\phi}_L \\ \dot{\phi}_R \end{pmatrix} \Rightarrow \begin{pmatrix} \dot{\phi}_L \\ \dot{\phi}_R \end{pmatrix} = \begin{pmatrix} 1/r_w & 1/r_w \\ 1/b & -1/b \end{pmatrix} \begin{pmatrix} v \\ \dot{\theta} \end{pmatrix} \quad (6.6)$$

The operator station sends  $v$  and  $\dot{\theta}$  as the command over the communication network to the robot. Therefore

$$\begin{pmatrix} u_r(t) \\ u_1(t) \end{pmatrix} = \begin{pmatrix} 1/r_w & 1/r_w \\ 1/b & -1/b \end{pmatrix} \begin{pmatrix} v \\ \dot{\theta} \end{pmatrix} \quad (6.7)$$

## 6.3 Simulation and Results

The teleoperation loop consists the operator model described in section 6.2 at one end of the communication link and Mobile robot model described in 6.1 on the other link. As shown in figure 6.4 there will a delay in both direction of communication. In real system the video image is streamed by the robot. Due to large quantity of data and limited bandwidth the delay  $h_1 \gg h_2$ . The amount of delay  $h_1$  was measured and it was found around 0.5sec as described in appendix A. The command sent by the operator is  $v$  and  $\dot{\theta}$  which is few bites only. Therefore in simulation  $h_2 = 0$  is assumed.

### 6.3.1 The Simulation Algorithm

- Convert the path from global coordinate system (CS) to Robot Local CS
- With a given look ahead distance (l) search for a point on the path
  - If point is found goto step 3
  - If not found increase look ahead distance
- Determine the turning radius (r) using equation 6.5



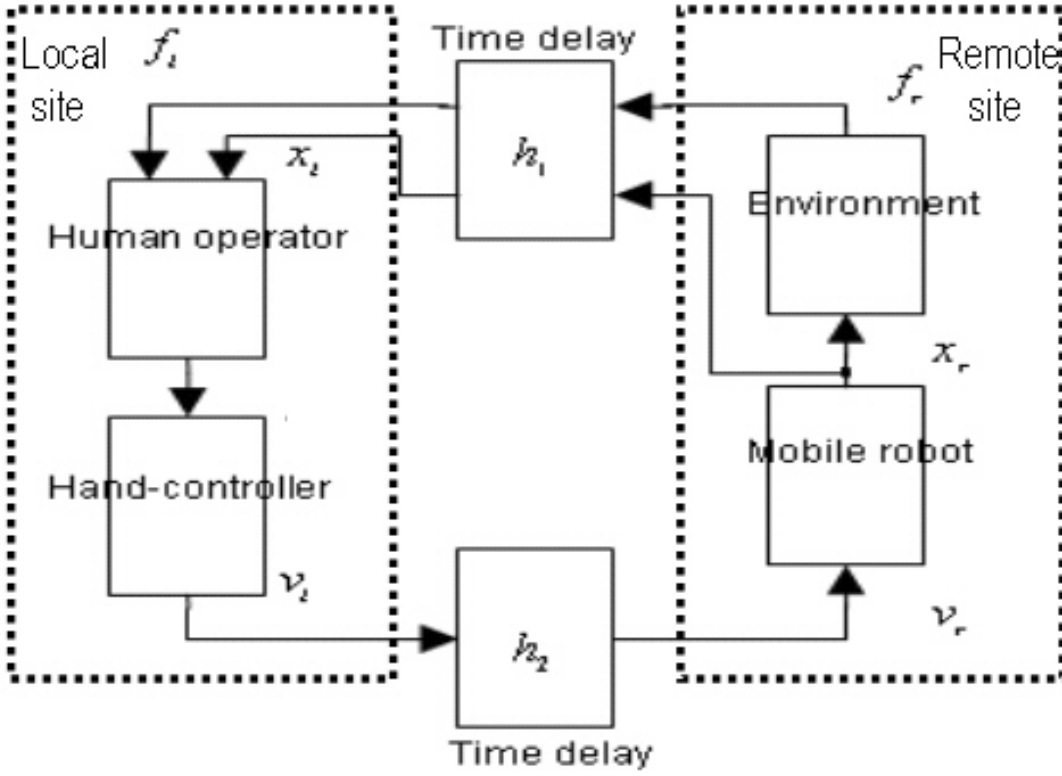


Figure 6.4: Teleoperation block diagram

- Calculate the command to the robot from equation 6.7. Note that these commands are based on the old visual data the operator see.
- Solve delayed differential equation in Matlab using `dde24`.

Simulation was carried using Matlab. Delay differential equation solver `dde24` was used to solve equation 6.1 and 6.2 with delayed inputs  $u_r$  and  $u_l$ . The path tracked was a circle of radius 5m centred at origin of the global coordinate system. The human action was modelled with look ahead distance of 0.5m and linear velocity of 0.5m/s. The initial position of the robot was (4.5,0.0).

The performance of the system with zero delay, i.e.  $h_1 = h_2 = 0$  is shown in figure 6.5. Figure 6.6 and 6.7 shows the plot robot motion under delay of 0.5 and 0.8 sec. It is seen that oscillation becomes visible at 0.5sec delay and with 0.8 sec delay the system is on the verge of instability.

It was also observed that the with large vehicle velocity,  $v$  and large look ahead distance,  $l$  the instability commences with smaller time delay,  $\delta$  in equation 6.2.

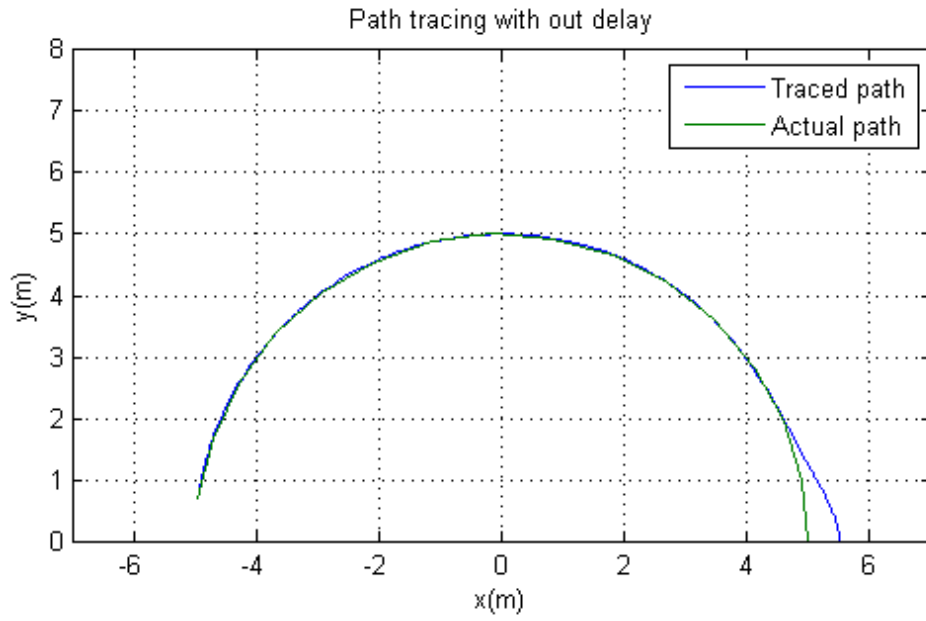


Figure 6.5: Simulation with no time delay in either direction

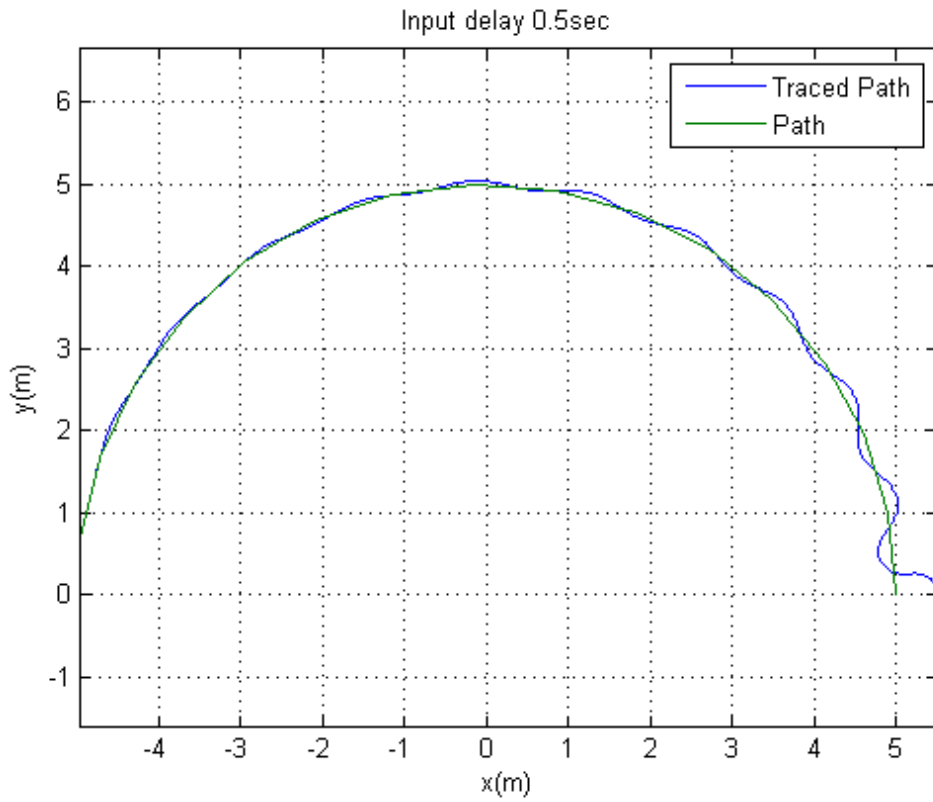


Figure 6.6: Simulation with time delay  $h_1 = .5sec$  and  $h_2 = 0$

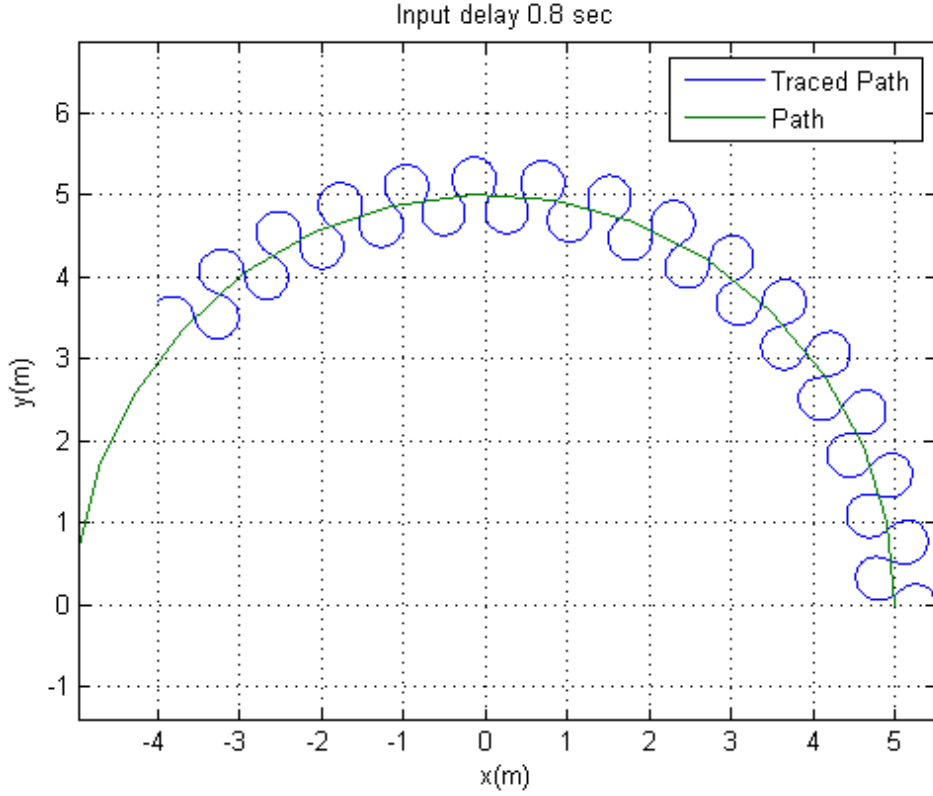


Figure 6.7: Simulation with time delay  $h_1 = .8sec$  and  $h_2 = 0$

## 6.4 Summary

In this chapter, simulation study of a teleoperated mobile robot was presented. A mathematical model was presented of the human operator action while driving the robot base on visual feed back. It was shown in by the simulation results that the behaviour of the system deteriorates with increase in delay in communication between the local and remote station. With large time delay the system becomes unstable.

In the next section we propose a predictive model based feedback control which is used to stabilize robot motion under time delay teleoperation.

# Chapter 7

## Predictive Display

In the last chapter it was shown using simulation results that time delay between the remote and local station leads the system towards instability. In this chapter we use predictive display to alleviate the above problem.

Predictive display has been defined as using the computer for extrapolating the display forward in time [81]. In this a local model of the remote scene is used to predict and render the remote scene in response to operator command. It replaces the delayed video feedback with extrapolated synthesised image of the remote environment and local enables the operator to perform the task normally.

### 7.1 Remote Scene Extrapolation

The visual data present in the current frame is the view that the robot has seen  $h_1$  second earlier. In order to predict the current scene that the robot might be seeing we need to estimate the current position of the robot. Once the current position of the robot is known we re-construct a view from the old scene by moving the view point to the current position of the robot. To explain this further and to use it in the teleoperation simulation model discussed in chapter 6

#### 7.1.1 here



# Chapter 8

## Conclusions

### 8.1 Thesis Summary

This thesis describes the design and development of a customized mobile manipulator developed at Bhabha Atomic Research Centre, Mumbai, India for mapping radiation in different areas of Cyclotron building where human access is restricted during Cyclotron operation. The custom mobile manipulator is a four wheeled mobile robot with a vertical lifting platform on which the radiation detector is mounted. The mobile platform has two rear wheels individual coupled to two motors and the front two wheels are mutually connected using motorized Davis steering mechanism.

The first chapter describes the various types of mobile manipulator reported in literature and different areas in which mobile manipulators have been successfully used. The chapter also describes the motivation and requirement of a customized mobile manipulator. Second chapter gives the literature review related to design, mathematical modeling and teleoperation setup of mobile robot which forms the complete system and is the scope of this thesis.

In chapter 3 the mechanical design, selection of motors and optimization of steering mechanism to achieve minimum turning radius within the required space is discussed. The optimal location of CG for the mobile manipulators so as to

provide maximum traction as well as stability over  $30^\circ$  ramp is described. It also discusses different lifting platform for vertical motion required by the sensor for scanning. Detail design of scissor based lifting platform and its advantage with respect to the system requirements are discussed.

The kinematic and dynamic modeling of the customized mobile manipulator is presented in chapter 4. Natural orthogonal compliment method is used to derive the dynamic model of the wheeled platform. This analysis highlights the effect of caster offsets on the maneuverability of a mobile platform. The need of having a positive drive for certain castor offsets are also brought in the analysis. This chapter also include the dynamic modeling and simulation of steering mechanism and lifting platform used in the mobile robot.

Control structure for intuitive teleoperation of the mobile robot is discussed in chapter 5. This chapter discuss the user interface at the operator side, the hardware and software required for teleoperation. The control algorithm running on the robot on-board computer is described. The uniqueness of the algorithm used is that it synchronizes all the three motors; two in velocity control mode and one in position control mode (steering), in spite of difference in response time, so as to always satisfy the wheel rolling condition given by Ackerman relation. The mobile robot uses encoder based dead reckoning for odometry is also describe and the actual results are presented.

The mobile robot is controlled by a operated physically separated from the mobile manipulator over a dedicated wireless network based on the video of remote environment sent by the robot on-board camera. The limited bandwidth results in the delay of the video. Chapter 6 deals with simulation of teleoperation of mobile robot under time delay. A mathematical model of the human operator based on pure pursuit algorithm is presented. The simulation studies predicts instability with increase in time delay and linear velocity of the mobile platform. These studies qualitative matches with the operator behaviour under similar condition.

The time delay in the video feedback reduces the efficiency of the operator was

highlighted above. In order to overcome this predictive display has been used time delayed teleoperation. In chapter 7 we present a new methodology for predictive display based on the 3-d model generated using the RGB-Depth data provided by Kinect Sensor mounted on the mobile robot and the predicted position of the robot based on dynamic model of the robot presented in in chapter 4. This novel method does not need to have the 3-D model of the remote environment known before hand as reported in literature earlier. This flexibility gives us to tele-operate the mobile robot in unknown environment.

## **8.2 Current status**

A prototype mobile manipulator and teleopration system was manufactured and assembled at BARC, Mumbai, India based on the analysis and algorithms presented in this thesis. The robot has been lab tested and the video of the same can be accessed on at the following url. <http://xxxx.com>. The system will shortly be deployed at Variable energy cyclotron centre at Kolkata, India.

## **8.3 Future Scope of the work**

The testing and evaluation of teleoperation system during actual trial opens up further scope of improvement. Few of these are listed below

- (i) Use of inertial sensors such as gyroscope and accelerometer on the mobile robot in combination with wheel odometer for more accurate pose estimation.
- (ii) Pose estimation using encoder mounted on front passive wheels for slip detection.
- (iii) Semi-automated navigation using intermediate goal point.
- (iv) Texture mapping of predicted of visual image instead 3-D point cloud model currently used.



- (v) Some kind of force feedback to the operator for estimation of obstacle location in the remote environment.





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# **Appendix A**

## **Simulation Time Delay**

## **Tele-operation**

### **A.1 Measurment of Time Delay in Video Feed-back**

The experimental method used to find the delay in video stream coming from the remote station is described here.



# Appendix B

## Dynamics

### B.1 Inertia Dyad

The dyadic product takes two vector and returns a second-order tensor. If  $\mathbf{a}$  and  $\mathbf{b}$  are two vectors expressed as  $\mathbf{a} \equiv a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$  and  $\mathbf{b} \equiv b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ . Then the dyadic product of  $\mathbf{a}$  and  $\mathbf{b}$  are defined as:



# Publications from the Thesis

Research papers published/presented/under preparation are listed below

1. Hayat, A.A., Abhishek, V., Saha, S.K., Dynamic Identification of Manipulator: Comparison between CAD and Actual Parameters, *17th National and 2nd International Conference on Machine and Mechanisms, iNaCoMM 2015*, IIT Kanpur, Dec. 16–19, 2015.
2. Udai, A. D., Hayat, A. A., Saha, S. K., “Parallel Active/Passive Force Control of Industrial Robots with Joint Compliance,” *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2014)*, Chicago, Illinois, Sept. 14–18 , 2014.





## Brief Bio-data of the Author