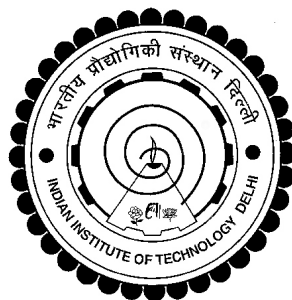


# DESIGN AND CONTROL OF TELE OPERATED MOBILE PLATFORM

AMAREN PRASANNA DAS



DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY DELHI

OCTOBER 2018

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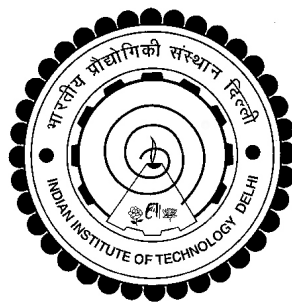
AMAREN P DAS

Department of Mechanical Engineering

*Submitted*

*in fulfillment of the requirements of the degree of Doctor of Philosophy*

*to the*



INDIAN INSTITUTE OF TECHNOLOGY DELHI

OCTOBER 2018



## Certificate

This is to certify that the thesis entitled **DESIGN AND CONTROL OF TELE OPERATED MOBILE PLATFORM**, submitted by **Shri. Amaren Prasanna Das** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Mechanical Engineering, is a record of the original, bona-fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other university or institute for the award of any degree or diploma.

**Dr. S. K. Saha**

Professor

Department of Mechanical Engineering

Indian Institute of Technology Delhi

New Delhi - 110016, India



# Acknowledgements

Amaren P Das

## Abstract

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# Important Symbols and Abbreviations

The important conventions, symbols and abbreviations used in this thesis are listed in the alphabetical order.

Conventions	Description
<i>italic</i>	Scalar quantities in lower-case lightface italic with Latin/Greek letters
<b>bold</b>	Vectors are in lower-case boldface Latin/Greek letters
<b>CAPITAL BOLD</b>	Matrices are in upper-case boldface Latin/Greek letters

Latin letters	Description
$\mathbf{A}_t$	State transition matrix at time $t$
$a_i, \alpha_i, b_i, \theta_i$	Represents four DH parameters namely, Link length, twist angle, joint offset and joint angle, respectively.
$\mathbf{B}_t$	Control input matrix at time $t$
$\mathbf{C}, \mathbf{C}_i$	Matrix of Convective Inertia (MCI) terms associated with the complete system and the $i^{th}$ subsystem, respectively
$f_v$	Viscous friction coefficient at the joint of robotic manipulator
$f_c$	Coulomb friction coefficient at the joint of robotic manipulator
$\mathbf{x}, \hat{\mathbf{x}}_{t t-1}$	State vector and its estimate at time $t$ with data upto time $(t - 1)$
$\mathbf{0}$	Column vector of zeros or null vector with compatible size to the dimensions of the matrices and vectors where it appears
$\mathbf{1}$	Identity matrix of size compatible to the dimensions of the matrices where it appears

---

Greek letters	Description
---------------	-------------

$\alpha_i$	Twist angle of $k^{th}$ link (DH parameter)
$\theta_i, \dot{\theta}_i, \ddot{\theta}_i$	Relative joint displacement of the $i^{th}$ link, its rate, and acceleration, respectively, in a closed-loop system
$\boldsymbol{\theta}_i, \dot{\boldsymbol{\theta}}_i, \ddot{\boldsymbol{\theta}}_i$	Vector of relative joint displacement of the $i^{th}$ link, its rate, and acceleration, respectively, in a closed-loop system
$\mathbf{x}_b$	Vector of base parameters in dynamic model
$\boldsymbol{\lambda}$	Lagrange multiplier representing constraint forces
$\boldsymbol{\omega}_i$	The 3-dimensional angular velocity vector of $i^{th}$ link

---

Acronyms	
----------	--

AMM	Aruco Marker Map
ANOVA	Analysis of Variance
VTM	Velocity Transformation Matrix

---

Functions/Operators	
---------------------	--

$\kappa(\mathbf{M})$	Condition number of matrix $\mathbf{M}$
$\det(\mathbf{M})$	Determinant of matrix $\mathbf{M}$
$\text{diag}(\mathbf{v})$	Element of vector $\mathbf{v}$ at diagonal position of a matrix

# Chapter 1

## Introduction

### 1.1 Research Contributions

The original contributions of the present research are listed below:

- (i) Design of teleoperated mobile platform remote surveillance application.
- (ii) Predictive visual feedback for time delayed teleoperation of mobile platform

### 1.2 Thesis Organization

The thesis contains seven chapters and five appendices. They are organized as follows:

**Chapter 1: Introduction**

**Chapter 2: Literature Review**

**Chapter 3: Design of Mobile Platform**

This chapter highlights the design considerations of tele-operated mobile platform based on environmental conditions mission requirements. This paper presents

the design and control scheme of a mobile manipulator used for radiation survey of Cyclotron vault and cave regions. It discusses the mechanical design for the traction system, the steering gear and the scissor mechanism. Selection of steering system based on terrain condition and power requirement is also discussed.

## **Chapter 4: Mathematica Modelling of Wheeled Mobile Platform**

In this chapter, we present the orthogonal compliment method for deriving the dynamic equation of four wheeled differentially driven platform. The platform has two actuated wheels and two passive wheels. Different types of passive wheels has been studied and there corresponding dynamic equation are presented. The main contribution of this paper is derivation of dynamic equation a differentially driven mobile platform with the most general form of passive wheel.

## **Chapter 5: Control Architecture**

The concept

## **Chapter 6: Simulation Time Delay Tele-operation**

In this chapter,

## **Chapter 7: Predictive Display**

In this chapter

## **Chapter 8: Conclusions**

This chapter summarizes major results of this research work. Limitations along with the future scope from the present experiences are also addressed.

## **Bibliography**

## **Appendix A: Kinematics**

## Appendix B: Dynamics

In this appendix,

### 1.3 Summary



# Chapter 2

## Literature Review

This chapter presents the survey of the literature highlighting design of different mobile robots and there control either used for field operation or for academic research test platforms.

### 2.1 Research Objectives

Based on the surveyed literature the objective of the present research are listed below:

- Identify
- To
- To
- Deve
- Es
- To

### 2.2 Summary

This chapter provided the





# Chapter 3

## Design of Mobile Platform

Most of the mobile robots presented in literature uses differential wheel drive with passive castor, as in, [1], [2] and [3]. The other common methods for locomotion of mobile robots are the omnidirectional wheels [4] and [5], and tracked wheel system [6] and [7]. According to Nagatani [8], a vehicle with Mecanum wheels is susceptible to slippage and same is the case for tracked vehicle, which are inherently skid steered. The slippage of the wheels prevents the most popular dead-reckoning method, using rotary shaft encoders, from being performed well. This chapter discusses the design methodology of a mobile manipulators based on environmental requirements. We also highlights the advantage of Davis steering over castor wheels or other steering methods from the perspective of this mobile manipulator. The Davis steering system modifies the heading of front wheels in a way that, at low speeds, all the wheels are in pure rolling without lateral sliding [9].

### 3.1 Design Overview

The objective of mobile robot is to navigate inside the cyclotron vault and collect radiation intensity data at all the required points decided by the operator. Data is to be collected not only at different planer locations of the floor but also at varying height from the floor. To cater to this operational requirement, a mobile

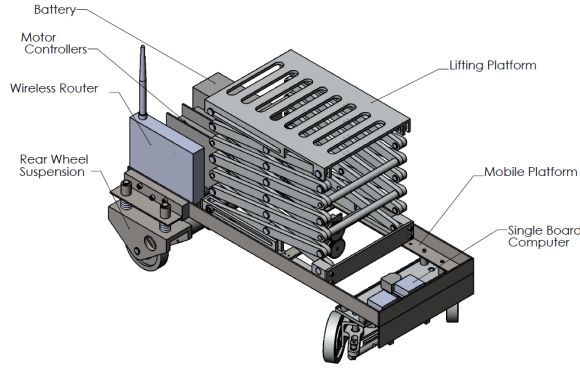


Figure 3.1: 3-D Model of the mobile manipulator

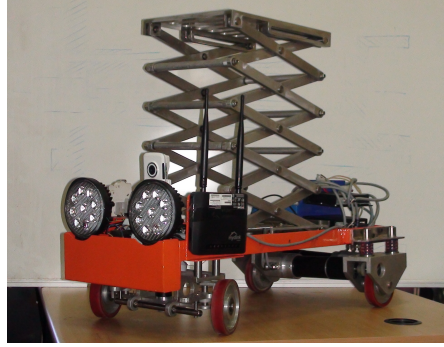


Figure 3.2: Photograph of the actual System

platform with a vertically extendable manipulator arm was developed. Together, they are referred henceforth as mobile-manipulator. The 3-D model of the mobile-manipulator with its major subsystems are shown in Figure 3.1, whereas and the actual system is shown in Figure 3.2.

The environmental condition required that the vehicle be either autonomous or teleoperated. To keep the complexity low it was decided to have wireless teleoperated navigation and control. This gives an operator full flexibility to drive and control the system from a remote station using visual feedback provided by the on board camera. The key parameters of the mobile manipulator are listed in Table 3.1.

Table 3.1: Key parameters and specifications of the mobile manipulator.

Weight	70 Kg	without payload
Payload	10 Kg	—
Footprint	700 mm × 400 mm	-
Height Collapsed	500 mm	Along with Z-Axis
Height Extended	1500 mm	Along with Z-Axis
Steering mechanism	Davis Steering	—
Turning radius	415 mm	-
Ground clarence	45 mm	—
Maximum traction speed	2 m/min	On flat terrain
Ramp climb angle	30°	Checkerboard surface

The mobile-manipulator has a footprint of 700 mm x 400 mm based on the narrow passage which the system has to negotiate. These passages are formed inside the vault area by the pipelines and structural supports of the cyclotron and

its associated equipment. Two DC motors, with speed servo controller, provide the traction to each rear wheels. The two front wheels are inter-connected with Davis steering mechanism [10]. A scissor mechanism provides the vertical motion of the detector.

In order to keep the self weight of the system small all the structural parts are made of aluminum alloy AL6061, apart from the base frame. Stainless Steel (SS304) angle sections is used for the base frame, which give it an excellent strength to weight ratio.

### 3.1.1 Design of the Traction System

Traction is provided by the two rear wheels driven independently. This makes the system over actuated. A mechanical differential connecting the two rear wheels, as used in cars, would over come this. It was not proposed to do so because; this being a teleoperated vehicle operated in environment inaccessible to human, in case one of the wheel loosing contact with ground due to over hand in small pits or while over an obstacle, the simple differential system will keep supplying power to the free hanging wheel. The system will hence get stuck, maybe in an unrecoverable location. Secondly, in case of one of the actuators failing either the traction or the steering motor, still the system can be manoeuvred to a safe location, albeit with dragging of the wheel with stuck actuator.

Each wheel is driven by a Maxon DC RE50 200W Motor through a 26:1 reduction gearbox. The motors are mounted at an offset to the wheel axis for increased ground clearance. See Figure 3.3. Spring suspension is provided at each wheel to ensure sufficient contact force on uneven ground. The diameter of the wheel is 100 mm ( $D_w$ ), which is sufficient to ride over obstacle of height 20 mm (Max). They are made of Aluminum alloy-6061 with 5mm thick molded polyurethane (PU) liner. The PU liner provides large traction on cement flooring while being resistance to wear.

The load distribution was optimized to generate maximum normal reaction,

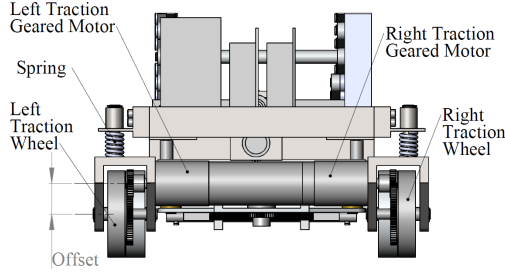


Figure 3.3: Rear Suspension

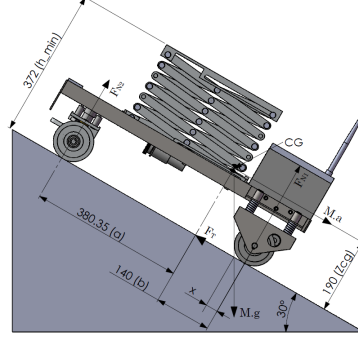


Figure 3.4: Mobile Manipulator on slope

$F_n$ , at the rear wheels without overturning while moving up the ramp of  $30^\circ$ . Maximizing rear wheel reaction by increasing "b" as per Equation 3.1 ensures increased traction,  $F_T = \mu F_N$  ( $\mu$  is the coefficient of friction), but at the same time decreases the stability margin indicated by "X" in Figure 3.4.

$$F_{N2} = \frac{b}{a} F_{N1}, \quad F_{N1} + F_{N2} = mg \cos \theta \quad (3.1)$$

The stability margin  $X$  was fixed as  $30mm$  so as to achieve maximum acceleration of  $0.144g$  over the ramp of  $30^\circ$  without overturning. This was done based on the dynamic stability, Equation 3.2.

$$mgb \cos \theta = (mg \sin \theta + ma)z_{cg}, \quad \Rightarrow g\left(\frac{b}{z_{cg}} \cos \theta - \sin \theta\right) = a \quad (3.2)$$

where  $m$ : mass of vehicle,  $g$ : acceleration due to gravity.

### 3.1.1.1 Selection of Motor and Gearbox

The torque requirement for each rear wheel was calculated based on the static moment balance with the assumption that each rear wheel shares equal load and the total suspended weight is 80 Kg. From the freebody diagram (Figure 3.4), using moment and force balance we get the following:

$$F_{N1} = \frac{aM \cos \theta}{a + b - \mu Z_{cg}}, \quad F_T = \mu F_{N1} \quad (3.3)$$

In order to estimate the traction motor size, we take worst case scenario  $\theta = 0$  and  $\mu = 0.3$ .

$$F_{N1} = 66Kg, \quad F_T = 0.3 * 66 \approx 20Kg$$

Since the traction is provided by the two rear wheels. The torque required per wheel ( $T_w$ ) is given by

$$T_w = (F_T/2)(D_w/2) = (20/2) * 50 = 500Kg - mm \simeq 5Nm \quad (3.4)$$

The motor torque  $T_M$ , required based on the assumption of factor of safety,  $FS = 1.5$  is

$$T_M = (FS) * T_w = 1.5 * 5 = 7.5 \simeq 8Nm \quad (3.5)$$

Assuming the maximum speed,  $V_{ramp} = 1m/s$ , of the mobile manipulator over a ramp, the required power,  $P_M$ , of the traction motor is calculated as,

$$\begin{aligned} \omega_w &= V_{ramp}/(D_w/2) \simeq 200rpm \\ P_m &= \omega_w T_m = 20 * 8 = 160W \end{aligned} \quad (3.6)$$

The nearest Maxon motor available as per the catalogue [11] is 200W, RE50-370354 motor. The nominal speed,  $N_s$  is 5680rpm. Therefore, the gearing ratio required is,  $N_s/\omega_w = 5680/200 \simeq 28.4$ . The nearest gear box available is of ratio 26 : 1, which was chosen.

### 3.1.2 Design of Steering System

This mobile manipulator uses Davis steering mechanism, Figure 3.5, on the front wheels. Caster were not used as they tend to align with small obstacles and thus get stuck. On the other hand tracked wheels have excellent rough terrain capabilities, but is power intensive due to skid steering. Whereas Omnidirectional wheels need complex controller for coordination.

Davis mechanism satisfies the steering equation 3.7, which ensures pure rolling

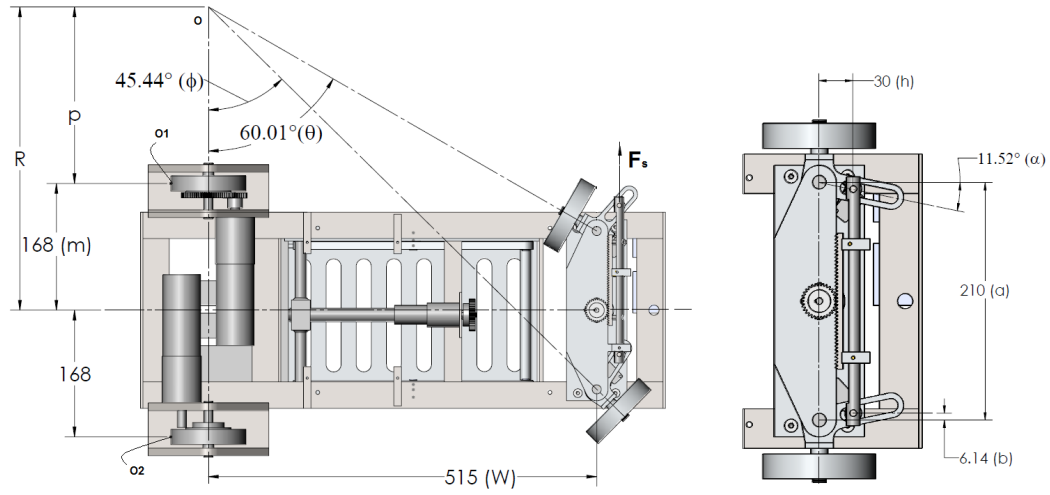


Figure 3.5: Davis steering mechanism

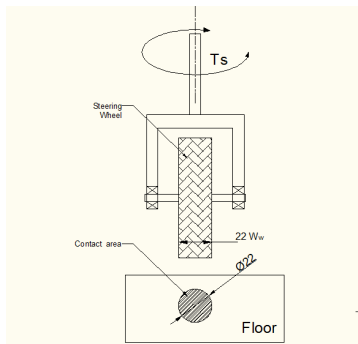


Figure 3.6: Wheel Ground Interaction

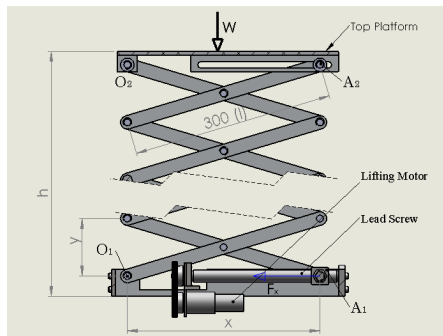


Figure 3.7: Scissor Mechanism

of all wheels over the entire steering range. This makes the system suitable for odometry and energy efficient. The mechanism being positively driven by position controlled servo motor, does not align with the obstacles and thus are able to crossover it. The dimensions of the links used in the steering mechanism is given in Figure 3.5, and are based on the Ackerman steering law given in Equation 3.7

$$\cot \phi - \cot \theta = a/w, \quad \frac{2b}{h} = \frac{a}{w} \quad (3.7)$$

Figure 3.5 shows the extreme values of  $\phi$  and  $\theta$ , limited by mechanical construction. The minimum turning radius can then be calculated as  $a/2 + w \cot \theta = 210/2 + 515 \cot 59.8 = 415mm$ .

### 3.1.2.1 Calculation of Steering Torque

The torque required to steer the front wheel is estimated based on a simplified assumption that the wheel deforms under normal load and the contact area thus generated is circular in shape with diameter that of the wheel width,  $W_w$ , as shown in Figure 3.6. In order to estimate the normal reaction on each wheel, we assume that the total weight of 80kg is equally shared by the four wheels. Therefore,  $N_s = 80/4 = 20Kg$ . Next, we apply the uniform pressure formula used to design brakes/clutches, given in Equation 3.8, to find the resistance torque  $T_s$ , between the ground and the wheel.

$$T_s = \frac{N_s \mu}{3} W_w = 0.4Nm \quad (3.8)$$

The resistance torque,  $T_s$ , of both the wheels are balanced by the force  $F_s$  acting on the rack as shown in Figure 3.5. The rack is coupled to the steering motor by a pinion of diameter,  $D_p = 40mm$ . The motor torque,  $T_{ms}$  in Equation 3.9 is calculated with a high factor of safety,  $FS = 3$ . This is because  $T_s$  is estimated based on a simplified model of brake design. The power,  $P_{ms}$  of the steering motor

based on torque  $T_{m_s}$  and the steering speed  $\omega_s$  of 100rpm is

$$T_{m_s} = (FS) \frac{2T_s}{h} \frac{D_p}{2} = 1.6Nm \quad \text{and} \quad P_{m_s} = T_{m_s} * \omega_s = 17W \quad (3.9)$$

Based on the above specifications, a 20W, RE25 DC motor of Maxon make and a gear box GP32 of ratio 159:1 was chosen for the steering mechanism.

### 3.1.3 Design of Scissor Mechanism

The vertical stage was designed to move up to a height of 1.5m from the floor level. This motion was generated using a scissor mechanism, as shown in Figure 3.7. The scissor mechanism has two major advantages over other lifting methods such as telescopic pillar, etc. First, the ratio of height in extended and collapsed condition is very large. In our case it is 3 : 1. Second, the self weight of the mechanism is very low as it is made of rectangular links.

The scissor mechanism, figure 3.7, has 6 stages, where one "X" denotes one stage. The Scissor is connected to the top platform by a pivot joint  $O_2$  and a prismatic joint  $A_2$ . This is coupled to the base frame by pivot joint  $O_1$  and a prismatic joint  $A_1$ . The linear actuation of joint  $A_1$  is provided by a lead screw of pitch ( $P$ ) 1.5 mm and mean diameter ( $d_m$ ) 10mm. This results in vertical motion of the top platform. From geometry we get

$$\begin{aligned} y &= l \sin \theta, \quad x = l \cos \theta, \Rightarrow dy = l \cos \theta d\theta, \quad dx = -l \sin \theta d\theta \\ h &= Ny \rightarrow dh = Ndy \end{aligned} \quad (3.10)$$

Where, N=6 is number of stages. From the principle of virtual work, we get

$$-F_x dx = W dh, \Rightarrow F_x = \frac{WN}{\tan \theta} \quad (3.11)$$

Where,  $F_x$  is the axial force on the prismatic joint,  $A_1$  and W is the payload. From Equation 3.11, it is clear that as  $\theta \rightarrow 0$ , the force  $F_x \rightarrow \infty$ . In the present design



$\theta_{min} = 5^\circ$  and  $\theta_{max} = 45^\circ$ . Therefore, the extended height  $h_{max} = Nl \sin \theta_{max} = 1.3m$  and the collapsed height  $h_{min} = 156mm$ . The maximum force  $F_x = 342Kg$  is required at  $\theta_{min} = 5^\circ$ , assuming  $W = 8kg$  as payload. This was used in formula for screw jack [10] , Equation 3.12, to calculate the motor torque of the scissor mechanism.

$$T_L = \frac{F_x d_m}{2} \left( \frac{p + \pi \mu d_m \sec \alpha}{\pi d_m - \mu p \sec \alpha} \right) = 7.5 Nm \quad (3.12)$$

Where coefficient of friction,  $\mu = 0.1$ , ACME thread angle  $2\alpha = 60^\circ$  and pitch  $p = 1.5mm$  is used. Based on the above specification 10W RE20 DC motor with a gear box of 25:1 ratio is chosen from Maxon motor catalogue [11].

## 3.2 Conclusion

Design calculation for a mobile manipulator are presented in this paper. Different aspects based on the requirements of radiation inspection around cyclotron was taken into account. Advantage of positively steered wheels over caster wheel was highlighted for mobile robots.



## Chapter 4

# Mathematica Modelling of Wheeled Mobile Platform

In the field of mobile robotics extensive research has been carried out. Mobile robots can broadly be divided into three categories-wheeled robots, legged robots [12] and aerial vehicles [13]. There are few mobile robots which use both wheels and legs for locomotion for example the Creadapt [14], in order to take advantage of both modes of locomotion. Among these the most extensively studied are the Wheeled mobile robots (WMR). They have been classified into five generic classes by Champion et.al [15], [16] based on their mobility resulting from the kinematic constraints due to different wheel types. The most common among these are the 3 wheeled differential driven WMR with one castor wheel. Because of its simplicity in modelling, they have been used in most of the control and motion planning algorithms [17], [18] and [19].

In order to develop a model based control algorithm it is imperative to have a good dynamic model of the WMR. These dynamic models are used in simulation software, Software in Loop (SIL) testing and Hardware in Loop testing of controllers. Different methods have been adopted to derive the dynamic model of WMR. A general dynamical model is derived for three-wheel mobile robots with nonholonomic constraints by using a Lagrange formulation by B. d'Andrea-Novell

[20]. Where as Thanjavur and Rajagopalan [21] has used Kane's method and Saha et. al. [22],[3] uses natural orthogonal compliment (NOC) method.

All the papers known to us uses, the standard caster wheel in deriving the dynamic model of the WMR. In this paper we have presented the modelling of Castor wheel in the most general configuration. In the stranded castor wheel the axis of rotation is perpendicular to the line joining the pivot and the axis of rotation as shown in Figure 4.2. Where as in our case they oriented in the most general way as shown in figure Figure 4.1. We have used the Natural Orthogonal Compliment approach as it inherently takes care of the non-holonomic constraint of the wheels. Moreover being a matrix based approach it is convenient for numerical simulations. The paper is divide in to three parts in the first part we briefly introduce the concept of NOC method, in the second part the derivation of dynamic model of WMR is presented, in the last part we discuss few special cases which can be derived from our model.

## 4.1 Natural Orthogonal Compliment (NOC) Method

Let us consider a system  $n$  rigid body, interconnected with different types of joints. Let,  $f_i$  be the net force acting at the center of mass (CM) of the  $i - th$  body and  $n_i$  is the net moment. If  $m_i$  is the mass,  $I_{ci}$  moment of inertia with respect to CM,  $c_i$  position vector of CM and  $\omega_i$  the angular velocity of the same body. Then equation of motion of  $i - th$  rigid body is given by Newton-Euler equations 4.1.

$$f_i = m_i \ddot{c}_i \quad \text{and} \quad n_i = I_{ci} \dot{\omega}_i + \omega_i \times I_{ci} \omega_i \quad (4.1)$$

If we define, twist ( $t$ ) and wrench( $w$ ) as

$$t_i = \begin{pmatrix} \omega_i \\ \dot{c}_i \end{pmatrix} \quad w_i = \begin{pmatrix} n_i \\ f_i \end{pmatrix}$$

The wrench  $w_i$  acting on the  $i$  –  $th$  body can be decomposed into  $w_i^w$ , called the *working component* and  $w_i^c$ , the *non-working component*. The working component consists if all the forces and torques, which imparts/extracts energy to/from the system, i.e. motor actuating torque. The non-working component of the wrench consists of the forces and torques which are used to constrain the motion of the body at the joints. Then Newton-Euler Equation (4.1) can be rewritten in a single matrix equation as

$$M_i \dot{t}_i + W_i M_i t_i = w_i^w + w_i^c \quad \because w_i = w_i^w + w_i^c \quad (4.2)$$

$$M_i = \begin{pmatrix} I_{ci} & 0 \\ 0 & m_i \tilde{1} \end{pmatrix}, \quad W_i = \begin{pmatrix} \Omega_i & 0 \\ 0 & 0 \end{pmatrix}, \quad \Omega_i = \omega_i \times \tilde{1} \quad (4.3)$$

Where  $\omega_i \times 1$ , is referred as the cross product matrix of vector  $\omega_i$  and  $\tilde{1}$ , denotes the identity matrix, for details refer to [23],[24]. The equation of all the  $n$  rigid body in the system can be collected and written as a single matrix equation (4.4) referred to as decoupled equation of motion of the system.

$$M \dot{t} + W M t = w^c + w^w \quad (4.4)$$

Where

$$M = \text{diag}[M_1, M_2, \dots M_n], \quad W = \text{diag}[W_1, W_2, \dots W_n], \quad t = [t_1^T, t_2^T, \dots t_n^T]^T$$

$$w^j = [w_1^{jT}, w_2^{jT}, \dots w_n^{jT}]^T, \quad j = c, w$$

The kinematic constraint both holonomic and non-holonomic (i.e. pure rolling) between two bodies  $i$  and  $j$  of a system, give rise to linear homogeneous system of algebraic equations [23] as given below, where  $A_i, A_j$  depend on the DH parameters.

$$A_i t_i + A_j t_j = 0 \quad (4.5)$$

The constraint equations corresponding to all the joints in the system can be written in terms of the *generalized twist vector*  $t$ . Furthermore if,  $\dot{\theta} = (\dot{\theta}_1, \dot{\theta}_2, \dots)$  denote the *independent generalized velocity* of the system, we can then write  $t$  in terms of  $\dot{\theta}$  as  $t = T\dot{\theta}$ . Using the fact, that  $\dot{\theta}$  can take any arbitrary value, we get

$$At = 0, \quad \Rightarrow AT\dot{\theta} = 0 \quad \Rightarrow AT = 0 \quad (4.6)$$

The above equation (4.6) indicates that  $T$  is the orthogonal compliment of  $A$ . Since this relation arises naturally, hence the name *Natural Orthogonal Compliment*. It can be shown [23] that the non-working wrench  $w^c$ , lies in the range space of  $A^T$ . In view of equation 4.6, it can be proved that  $w^c$  lies in the null space of  $T^T$ , therefore

$$T^T w^c = 0 \quad (4.7)$$

To eliminate the non-working forces and moments, i.e  $w^C$  from the uncoupled equation of motion (4.4), we multiply both sides of equation 4.4 by  $T^T$ .

$$T^T M\dot{t} + T^T W M t = T^T w^W, \quad \Rightarrow T^T M T \ddot{\theta} + T^T (M\dot{T} + W M T) \dot{\theta} = T^T w^T \quad (4.8)$$

Equation 4.9 represents the dynamic equation of interconnected  $n$ -body system. This equation can be expressed in terms of the independent generalized velocity  $\dot{\theta}$  and generalized acceleration  $\ddot{\theta}$ , by using the relations  $t = T\dot{\theta}$  and  $\dot{t} = \dot{T}\dot{\theta} + T\ddot{\theta}$  in equation 4.9. thus the final equations of motion can be written as

$$or \quad I(\theta)\ddot{\theta} = C(\theta, \dot{\theta})\dot{\theta} + \tau \quad (4.9)$$

Where

$$I(\theta) \equiv T^T M T, \text{generalized inertia matrix}$$

$$C(\theta, \dot{\theta}) \equiv -T^T (M\dot{T} + W M T) \dot{\theta}, \text{convective inertia matrix}$$

$$\tau \equiv T^T w^T, \text{generalized driving force vectcor}$$

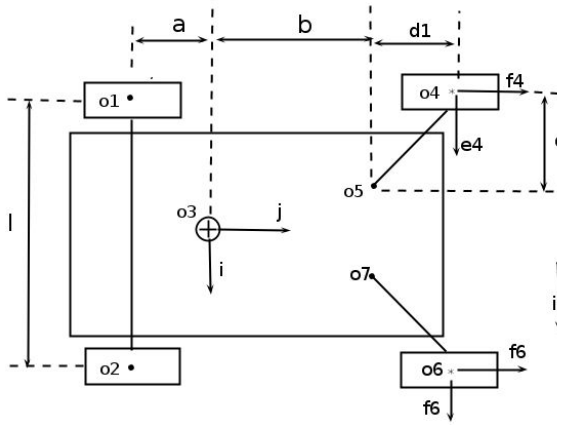


Figure 4.1: WMR-general

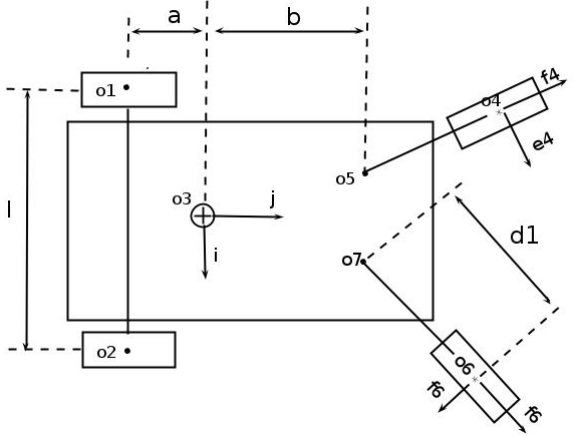


Figure 4.2: WMR-Std. Castor

Figure 4.3: Castor wheel Configuration of WMR

## 4.2 Dynamic Equation of WMR

The dynamic equation of a differentially driven 3 wheeled mobile platform based on Natural orthogonal compliment metohod has been presented by Saha[22]. The vehicle consisted of 2 driven wheel and one standard castor wheel. In general for large vehicles it necessary to have at least four wheels, for the point of stability of the vehicle. Such a vehicle is shown in figure 1. It may be noted that the castor wheels used in this case are the standered castor wheel configuration, where the angle between line  $O_4O_3$  and vector  $e_3$  is 90 deg. This is a special case of the more general configuration of the castor wheel which are shown in figure 2, where angle between line  $O_4O_3$  and vector  $e_3$  is not 90 deg

The vehicle we have considered in this paper is shown in figure 2. It consist two independently driven wheels at the back and two generalized castor wheels at the front. The actuated wheels are labelled as body #1 and #2. The platform is body #3, The first castor wheel and its bracket is labelled as #4 and #5, with the castor pivoted at  $O_4$ . Similarly the second castor is pivoted at  $O_6$ , and its bracket and wheel is labelled as #6 an #7. All the wheels are assumed to be rolling with out slipping.

### 4.2.1 Kinematic analysis

In order to proceed with the kinematic analysis of the vehicle in figure 1, we define a orthogonal triad of vectors  $i, j, k$  at point  $O_3$ , the control point of the platform, as shown in the figure. If  $\dot{\theta}_1$  and  $\dot{\theta}_2$  denotes the rate of rotation of wheel #1 and #2. Then the linear velocity of point  $O_1$  &  $O_2$  under pure rolling condition is given by

$$\dot{O}_i = r\dot{\theta}_j, \quad r = \text{radius of wheel} \quad (4.10)$$

The angular velocity of the platform  $\omega_3$  can be written as

$$\omega_3 = (r/l)(\dot{\theta}_1 - \dot{\theta}_2). \quad (4.11)$$

Further, the velocity of point  $o_3$  can be written as  $\dot{o}_3 = \dot{o}_i + \omega_3 \times (c - o_i)$ ,  $i = 1, 2$ . where,  $o_3$  and  $o_i$  is the position vector of point  $O_3$  and  $O_i$  respectively, with respect to some point fixed to the ground. By eliminate  $\omega_3$  from these two equation, we get

$$\dot{o}_3 = (ar/l)(\dot{\theta}_1 - \dot{\theta}_2) + (r/2)(\dot{\theta}_1 + \dot{\theta}_2) \quad (4.12)$$

Now, the the angular velocity of the drive wheel #1 can be expressed as  $\omega_1 = -\dot{\theta}_1 i + \omega_3 k$ , using equation 4.11, we can write

$$\omega_1 = \begin{pmatrix} -i + (r/l)k & -(r/l)k \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \quad (4.13)$$

Using equation 4.10 and 4.13, we can write the twist for wheel #1 in terms of  $\dot{\theta}_a$ , as

$$t_1 = \begin{pmatrix} \omega_1 \\ \dot{o}_1 \end{pmatrix} = \begin{pmatrix} -i + (r/l)k & -(r/l)k \\ rj & 0 \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \quad (4.14)$$

Similarly, for the other actuated wheel #2, we get



$$t_2 = \begin{pmatrix} \omega_1 \\ \dot{o}_1 \end{pmatrix} = \begin{pmatrix} -i + (r/l)k & -(r/l)k \\ 0 & rj \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \quad (4.15)$$

To calculate the twist,  $t_3$  of the platform, body #3, we use combine equation 4.11 and 4.12, and get

$$t_3 = \begin{pmatrix} \omega_3 \\ \dot{o}_3 \end{pmatrix} = \begin{pmatrix} \rho\delta & -\rho\delta \\ r(\lambda i + (1/2)j) & r(-\lambda i + (1/2)j) \end{pmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} \quad (4.16)$$

where

$$\delta \equiv d/l, \quad \rho \equiv r/d, \quad \lambda \equiv a/l$$

In order to calculate the twist of the caster bracket and the caster wheel, we need to express the unactuated joint rates  $\dot{\psi}_1$  and  $\dot{\phi}_1$ , in terms of actuated angle rate vector  $\dot{\theta}_a$ . Where,  $\dot{\psi}_1$  denotes the rate of rotation of bracket, body #5 about  $O_4$  with respect to the platform, and  $\dot{\phi}_1$  the rate of rotation of caster wheel, body #4 about its axis  $e_3$ , with respect to bracket. The velocity of  $O_5$  can be expressed in two independent forms, one in terms of the velocity of  $o_3$  and the other in terms of the velocity of  $o_5$ , i.e

$$\dot{o}_5 = \dot{o}_4 + \omega_5 \times (d_1 e_4 - d_2 f_4), \quad \dot{o}_5 = \dot{o}_4 + \omega_5 \times (d_1 e_4 - d_2 f_4) \quad (4.17)$$

On equating the above to equations together, and using the rotation matrix between coordiante system  $\{i, j, k\}$  and  $\{e_4, f_4, k\}$ , to express the equation in  $e_4$  &  $f_4$ , we get

$$(-\dot{\phi}_1 r + \dot{\psi}_1 d_1) f_3 + d_3 \dot{\psi}_1 e_3 = \dot{o}_3 + \omega_3 (m \cos \psi_1 - b \sin \psi_1 - d_1) e_4 \quad (4.18)$$

Taking the dot product of the above equation first with  $e_4$  and then with  $f_4$ , and

using equation 4.12 for  $\dot{o}_3$ , we get

$$\begin{pmatrix} d_2 & 0 \\ -d_1 & r \end{pmatrix} \begin{pmatrix} \dot{\psi}_1 \\ \dot{\phi}_1 \end{pmatrix} = \begin{pmatrix} (-ar/l)S_{\psi_1} + (r/2)C_{\psi_1} + \delta_1 & (ar/l)S_{\psi_1} + (r/2)C_{\psi_1} - \delta_1 \\ (ar/l)C_{\psi_1} + (r/2)S_{\psi_1} + \delta_2 & (-ar/l)C_{\psi_1} + (r/2)S_{\psi_1} - \delta_2 \end{pmatrix} \dot{\theta}_a = [F_{ij}]\dot{\theta}_a \quad (4.19)$$

where,

$$\delta_1 = (r/l)(mC_{\psi_1} - bS_{\psi_1} - d_2), \quad \delta_2 = (r/l)(mS_{\psi_1} + bC_{\psi_1} + d_1$$

Similarly for the other caster wheel we get,

$$\begin{pmatrix} d_2 & 0 \\ -d_1 & r \end{pmatrix} \begin{pmatrix} \dot{\psi}_2 \\ \dot{\phi}_2 \end{pmatrix} = \begin{pmatrix} (-ar/l)S_{\psi_2} + (r/2)C_{\psi_2} - \delta_3 & (ar/l)S_{\psi_2} + (r/2)C_{\psi_2} + \delta_3 \\ (ar/l)C_{\psi_2} + (r/2)S_{\psi_2} + \delta_4 & (-ar/l)C_{\psi_2} + (r/2)S_{\psi_2} - \delta_4 \end{pmatrix} \dot{\theta}_a = [G_{ij}]\dot{\theta}_a \quad (4.20)$$

where,

$$\delta_3 = (r/l)(mC_{\psi_2} + bS_{\psi_2} + d_2), \quad \delta_4 = (r/l)(mS_{\psi_2} + bC_{\psi_2} + d_1$$

The angular and the liner velocity of the c.g.of the caster wheel,#4 is written in-terms of the co-ordinate frame fixed to the bracket #5,  $\{e_4, f_4, k\}$ , as

$$\omega_4 = \dot{\phi}_1 e_4 + (\omega_3 + \dot{\psi}_1)k, \quad \dot{o}_4 = \dot{\phi}_1 e_4 \quad (4.21)$$

Using equation 4.19 and 4.11, the twist  $t_4$  can be written as

$$t_4 = \begin{pmatrix} \Theta_4 \\ C_4 \end{pmatrix} \dot{\theta}_a \quad (4.22)$$

where

$$\Theta_4 = [F_{11}e_4 + \bar{F}_{21}k \quad F_{12}e_4 + \bar{F}_{22}k], \quad C_4 = r[-F_{11}f_4 \quad -F_{12}f_4]$$

$$\bar{F}_{21} = F_{21} + \rho\delta, \quad \bar{F}_{22} = F_{22} - \rho\delta$$

The angular and the liner velocity of the c.g.of the caster bracket,#5 is written in-terms of the co-ordinate frame fixed to the bracket, as

$$\omega_4 = \dot{\phi}_1 e_3 + \dot{\psi}_1 k, \quad \dot{o}_4 = \dot{o}_4 + \omega_5 \times [-df_3] \quad (4.23)$$

Now using equations 4.19 & 4.11, the twist  $t_5$  can be written as

$$t_5 = \begin{pmatrix} \Theta_5 \\ C_5 \end{pmatrix} \dot{\theta}_a \quad (4.24)$$

where

$$\Theta_5 = [\bar{F}_{21}k \quad \bar{F}_{22}k], \quad C_5 = d[(1/2)\bar{F}_{21}e_4 - \rho F_{11}f_4 \quad (1/2)\bar{F}_{22}e_4 - \rho F_{12}f_4]$$

In similar manner the twist  $t_6$  &  $t_7$  of the other caster wheel and its bracket can be written as

$$t_6 = \begin{pmatrix} \Theta_6 \\ C_6 \end{pmatrix} \dot{\theta}_a, \quad t_7 = \begin{pmatrix} \Theta_7 \\ C_7 \end{pmatrix} \dot{\theta}_a \quad (4.25)$$

where

$$\Theta_6 = [G_{11}e_6 + \bar{G}_{21}k \quad G_{12}e_6 + \bar{G}_{22}k], \quad C_6 = r[-G_{11}f_6 \quad -G_{12}f_6]$$

$$\Theta_7 = [\bar{G}_{21}k \quad \bar{G}_{22}k], \quad C_7 = d[(1/2)\bar{G}_{21}e_6 - \rho G_{11}f_6 \quad (1/2)\bar{G}_{22}e_6 - \rho G_{12}f_6]$$

$$\bar{G}_{21} = G_{21} + \rho\delta, \quad \bar{G}_{22} = G_{22} - \rho\delta$$

## Dynamic Equations

Based on the twist calculated in terms of the independent actuation vector  $\dot{\theta}_a$ , we now derive the generalized inertia matrix and the Convective inertia term for the coupled equation of motion 4.9.

## Generalized Inertia Matrix, $I$

From the above equations of twist of individual body we get  $t_i = T_i \dot{\theta}_a$ , by definition

$t = [t_1^T, t_2^T, \dots, t_7^T]^T$ . Therefore, we get

$$T = [T_1^T, T_2^T, \dots, T_7^T]^T$$

. Since the matrix  $M$  is block diagonal

$$I = T^T M T = T_1^T M_1 T_1 + T_2^T M_2 T_2 + \dots T_7^T M_7 T_7 \quad (4.26)$$

$$I_m = \sum_{i=1,2} T_i^T M_i T_i = \begin{pmatrix} I_w + (\rho\delta)^2 H + m_w r^2 & -2(\rho\delta)^2 H \\ -2(\rho\delta)^2 H & I + (\rho\delta)^2 H + m_w r^2 \end{pmatrix} \quad \text{where } M_i = \begin{pmatrix} I_w & 0 \\ 0 & m_w \mathbf{1} \end{pmatrix} \quad (4.27)$$

Where  $\tilde{I}_w = \begin{pmatrix} I_w & 0 & 0 \\ 0 & H & \\ 0 & 0 & H \end{pmatrix}$  is the  $3 \times 3$  moment of Inertia matrix expressed in co-ordinate  $\{i, j, i\}$ ,  $m_w$  is the mass of the motorized wheels, and  $\mathbf{1}$  is  $3 \times 3$  identity matrix.

If the mass of the platform is  $m_p$  and its moment of inertia about vector  $\mathbf{k}$  is  $I_p$ , then [23]

$$I_3 = T_3^T M_3 T_3 = I_p (\rho\delta)^2 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + m_p r^2 \begin{pmatrix} (1/4) + \gamma^2 & (1/4) - \gamma^2 \\ (1/4) - \gamma^2 & (1/4) + \gamma^2 \end{pmatrix} \quad (4.28)$$

Similarly, if  $m_c$  is the mass of the castor wheel and it is assumed to be a solid disk, then the generalized inertia matrix can be written as

$$I_c = \sum_{i=4,6} T_i^T M_i T_i = (m_c r^2 / 4) \begin{pmatrix} 6F_{11}^2 + \bar{F}_{21}^2 & 6F_{11}F_{12} + \bar{F}_{21}\bar{F}_{22} \\ 6F_{11}F_{12} + \bar{F}_{21}\bar{F}_{22} & 6F_{12}^2 + \bar{F}_{22}^2 \end{pmatrix} + \begin{pmatrix} 6G_{11}^2 + \bar{G}_{21}^2 & 6G_{11}G_{12} + \bar{G}_{21}\bar{G}_{22} \\ 6G_{11}G_{12} + \bar{G}_{21}\bar{G}_{22} & 6G_{12}^2 + \bar{G}_{22}^2 \end{pmatrix} \quad (4.29)$$

If the mass of the brackets i.e. body #5 and #7 are small compared to the mass of the castor wheels, then contribution of  $T_5^T M_5 T_5$  and  $T_7^T M_7 T_7$  can be assumed

to be zero.

### Convective Inertia matrix $C$

The Convective inertia term of equation 4.9 can be broken down in to two parts,  $T^T M \dot{T}$  and  $T^T W M T$ . Since the generalized inertia matrix of the motorized wheels and the platform is constant, they do not contribute to either of these terms. Moreover we have considered the mass of the brackets to be zero, so they also do not contribute to convective inertia term. Hence

$$C = T^T M \dot{T} + T^T W M T = \sum_{i=4,6} T_i^T M_i \dot{T}_i + \sum_{i=4,6} T_i^T W_i M_i T_i \quad (4.30)$$

The expression for the first term is found by using equations 4.19,4.20,4.22 and 4.25.  $\dot{F}_{ij}$  and  $\dot{G}_{ij}$  denotes the derivative of the elements of matrix  $F$  and  $G$  defined in 4.19,4.20 . To find  $\dot{T}_4$  &  $\dot{T}_6$  we have used the fact  $\dot{e}_4 = \omega_4 \times e_4$  and  $\dot{e}_6 = \omega_6 \times e_6$ .

$$\begin{aligned} T^T M \dot{T} = (m_c r^2 / 4) & \left[ \begin{pmatrix} 6F_{11}\dot{F}_{11} + \bar{F}_{21}\dot{F}_{21} & 6F_{11}\dot{F}_{12} + \bar{F}_{21}\dot{F}_{22} \\ 6\dot{F}_{11}F_{12} + \dot{F}_{21}\bar{F}_{22} & 6F_{12}^2\bar{F}_{22} \end{pmatrix} \right. \\ & \left. + \begin{pmatrix} 6G_{11}\dot{G}_{11} + \bar{G}_{21}\dot{G}_{21} & 6G_{11}\dot{G}_{12} + \bar{G}_{21}\dot{G}_{22} \\ 6\dot{G}_{11}G_{12} + \dot{G}_{21}\bar{G}_{22} & 6G_{12}^2\bar{G}_{22} \end{pmatrix} \right] \end{aligned} \quad (4.31)$$

The second term of equation 4.31 evaluates to zero, as shown below. Consider castor wheel, body #4. Using equation 4.22 and definition of  $W$  defined in 4.3 we get,

$$T_4^T W_4 M_4 T_4 = [\Theta_4, C_4] \begin{pmatrix} \Omega_4 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} I_4 & 0 \\ 0 & m_4 \mathbf{1} \end{pmatrix} \begin{pmatrix} \Theta_4 \\ C_4 \end{pmatrix} = \Theta_4 \Omega_4 I_4 \Theta_4 \quad (4.32)$$

To evaluate the above equation, we express all the terms in the coordinate

system  $\{\mathbf{e}_4, \mathbf{f}_4, \mathbf{k}\}$  .

$$\Theta_4 = \begin{pmatrix} F_{11} & 0 & F_{12} \\ 0 & 0 & 0 \\ \bar{F}_{21} & 0 & \bar{F}_{22} \end{pmatrix}, \quad \Omega_4 = \begin{pmatrix} 0 & -(\bar{F}_{21}\dot{\theta}_1 + \bar{F}_{22}\dot{\theta}_2) & 0 \\ (\bar{F}_{21}\dot{\theta}_1 + \bar{F}_{22}\dot{\theta}_2) & 0 & -(F_{11}\dot{\theta}_1 + F_{12}\dot{\theta}_2) \\ 0 & F_{11}\dot{\theta}_1 + F_{12}\dot{\theta}_2 & 0 \end{pmatrix}$$

When these expressions are substitute in equation no. 4.32, we get

$$T_4^T W_4 M_4 T_4 = 0, \quad \Rightarrow T^T W M T = 0 \quad (4.33)$$

So the Convective inertia matrix  $C$  of equation 4.9 evaluate to

$$C = (m_c r^2 / 4) \left[ \begin{pmatrix} 6F_{11}\dot{F}_{11} + \bar{F}_{21}\dot{F}_{21} & 6F_{11}\dot{F}_{12} + \bar{F}_{21}\dot{F}_{22} \\ 6F_{11}\dot{F}_{12} + \bar{F}_{21}\dot{F}_{22} & 6F_{12}^2\dot{F}_{22} \end{pmatrix} + \begin{pmatrix} 6G_{11}\dot{G}_{11} + \bar{G}_{21}\dot{G}_{21} & 6G_{11}\dot{G}_{12} + \bar{G}_{21}\dot{G}_{22} \\ 6G_{11}\dot{G}_{12} + \bar{G}_{21}\dot{G}_{22} & 6G_{12}^2\dot{G}_{22} \end{pmatrix} \right] \quad (4.34)$$

All the components of the equation 4.9 have now been evaluated, except the  $\tau$ . These are simply the torque exerted by the actuated wheel. This completes the dynamic model of the WMR with generalized caster wheel configuration.

## 4.3 Special cases

### 4.3.1 Standard caster ( $d_1 = 0$ )

The standard caster wheel configuration can be obtained by setting the value of  $d_1 = 0$ . In such condition the left hand side matrix of equation 4.19 and 4.20 becomes a diagonal matrix. Therefore, the first and second equation of 4.19 and 4.20 gets divided by  $d_2$  and  $r$  respectively. The resulting equations relating the unactuated joint rate to actuated joint rates are similar to those reported by [22],[23].

### 4.3.2 Under Actuated Case ( $d_2 = 0$ )

It can be seen from equation 4.19, that when the *caster offset*  $d_2 = 0$ , the LHS matrix becomes singular. So the unactuated joint rates cannot be determined from  $\dot{\theta}_a$ . It is therefore essential to have proper caster offset in case we need caster like behaviour from a passive wheel.

Another solution is to put an extra actuator to control the bracket motion by controlling  $\psi_i$ . As in the case of ackerman steering mechanism, where the steering wheel controls the orientation of the front passive wheels of a car.

## 4.4 Simulation of The Mobile Platform

The mobile manipulator was modelled as differential drive robot, the front wheels and steering mechanism was not included in the dynamic model as their mass are too small compared to the mobile platform. The weight of a wheel is 300g, the steering mechanism 240g, whereas the weight of platform is 70Kg.

In the simulation the vehicle (point o3), is required to trace a circle of radius 5m and with  $\hat{\mathbf{s}}(t)$  as given below.

$$\beta(t) = \frac{20\pi}{60^3}t^3 + \frac{30\pi}{60^4}t^4 + \frac{2\pi}{60^5}t^5 \quad (4.35)$$

## 4.5 Inverse Dynamics

Using the Inverse Kinematic, the wheel velocity and acceleration are determined using pseudo Inverse from the equation given below. Where  $r$  is radius of the wheel and  $\theta_i$  is the wheel roll angle of wheel #1 and #2. The platform angular velocity is  $\omega$  and the Cartesian velocity of point o3 is given by  $\dot{x}, \dot{y}$  in body coordinate system

The wheel angle, velocity and acceleration are used in the dynamic equation to calculate the torque required by each motor. The results are plotted in figure

2. The size of generalized inertia matrix, and convective term are not reported here as there are size is very large.

The equation of dynamic model is given in equation 4.36, where  $\theta_l, \theta_r$  are the rear left and right wheel role angles,  $t_i^T = [\omega_i, v]$  the twist,  $m_i$  the mass,  $I_i$  the inertia of individual bodies.

$$T^T M T \ddot{\theta}_a = -T^T (M \dot{T} + W M T) \dot{\theta}_a + T^T (w^J + w^G)$$

where  $\theta_a = [\theta_l, \theta_r]^T$ ,  $[t_1, t_2, t_3] = (T \dot{\theta}_a)^T$ ,  $M = \text{diag}(M_1, M_2, M_3)$

$$W = \text{diag}(W_1, W_2, W_3), \quad W_i = \begin{pmatrix} \omega_i \times 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad M_i = \begin{pmatrix} \omega_i \times I_i & 0 \\ 0 & m_i \end{pmatrix} \quad (4.36)$$

$w^G$  and  $w^J$  represents the gravitational force and the external torque/Force applied by motor respectively.

The torque required at wheels for the vehicle to move up a spiral ramp of slope  $10^\circ$  and radius 5m is given figure 4.4. The numerical values used for the forward dynamic simulation is given in table 4.1.

Table 4.1: Dynamic parameters used in forward dynamic simulation.

Part Name	Mass Property	Value
Rear Wheels	mass	300g
	Moment of Inertia	$\text{diag}(242, 242, 465)\text{kg } mm^2$
Base Frame	mass	70Kg
	Moment Of Inertia	$\begin{pmatrix} 1.18 & 0.01 & -0.05 \\ 0.01 & 1.28 & 0.08 \\ -0.05 & 0.08 & 0.53 \end{pmatrix} Kg - m^2$

## 4.6 Conclusion

We have presented the dynamic equation of the most general form of caster wheel configuration. Even though we have considered here only two caster wheel the formulation can be extended to any number of caster wheels, in such case we have to find equation 4.19 for each caster wheel system, and then proceed mechanical as to calculate the corresponding twists. We have also shown that the dynamic of



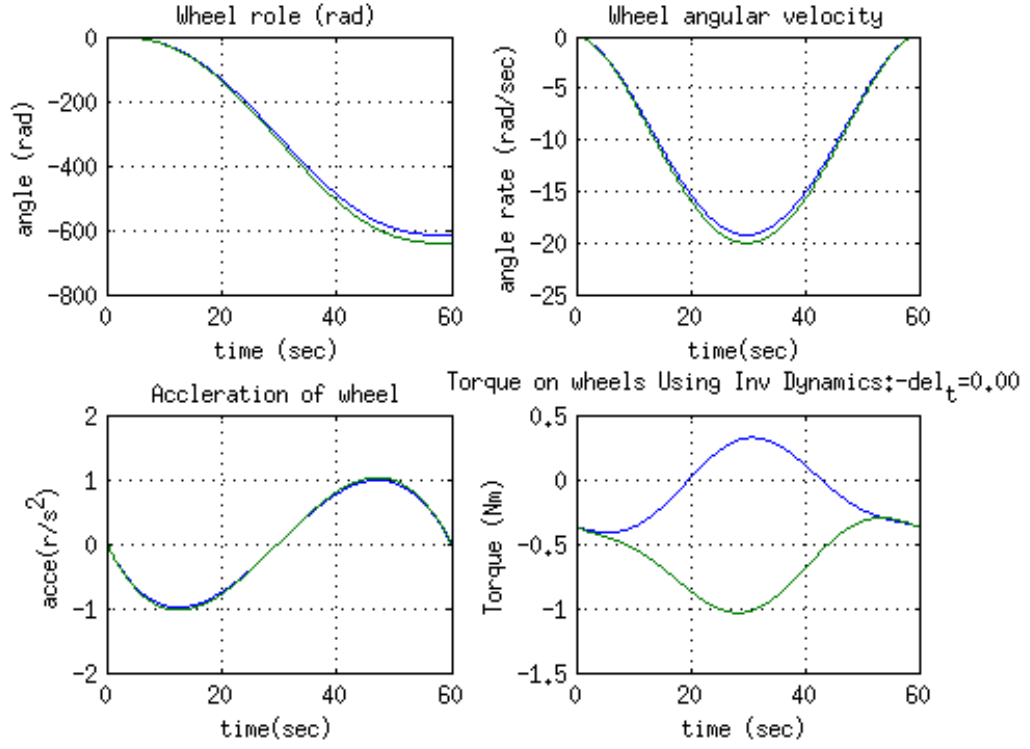


Figure 4.4: Inverse Dynamics of Mobile Robot moving up on  $15^\circ$  ramp

standard caster wheels is a special case of our general case with  $d_1 = 0$ . We have also proved why a caster needs a non-zero caster offset distance, and the need of extra actuators in case  $d_2 = 0$ .

## 4.7 Summary

In this chapter,



# Chapter 5

## Control

In this chapter we present the control architecture of the mobile robot.

### 5.1 Control Architecture and Hardware

The mobile manipulator is planned to be teleoperated over a wireless network. The control block diagram and architecture are shown in Figure 5.1. It has a remote control station which is the interface for the operator and a local controller on the mobile manipulator. They communicate over a wifi network. The remote station send data packet every 50 millisecond to the mobile manipulator. The commanded velocity, the steer angle, the z position of the platform and the state of the detector and headlamps constitutes the data packet sent by the remote station. The on

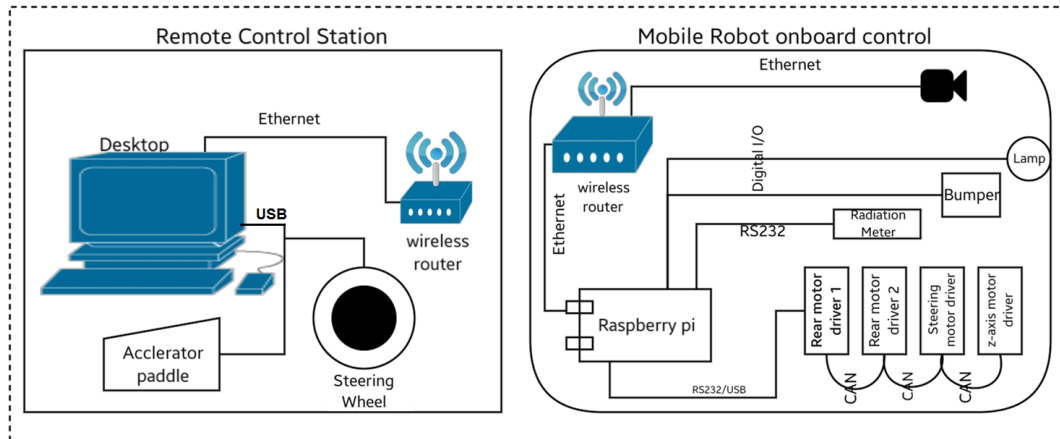


Figure 5.1: Control Architecture Block Diagram

board control of the mobile robot replies with a data packet consisting of the  $X$ ,  $Y$  position and orientation of the robot, the current steer angle, angular velocities of each wheels, the  $z$  position of the top platform and bumper status.

### **5.1.1 Local Onboard Controller**

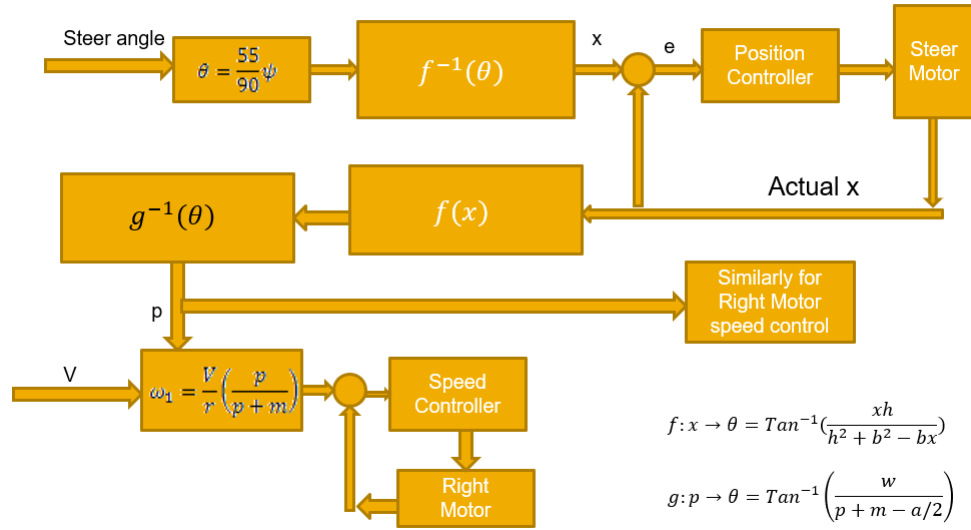
The on-board computer receives the command from the remote station and controls the robot hardware through Robot Operating System (ROS). The computer is daisy chained to the four Maxon EPOS2 motor controllers/drivers. The communication between the onboard computer and the first Maxon controllers is over usb/RS232 interface using Maxon's proprietorial protocol [29]. The first controller serves as CAN master for the rest of the controllers. The rear wheel motor drivers are configured in velocity servo loop. The steering and the  $z$ -axis motors drivers are configured in position control loop. The camera mounted on the mobile robot and Raspberry Pi is connected over Ethernet via a wireless hub.

### **5.1.2 The Remote Control Station**

The local station consists of a desktop computer running Windows XP. A Steering wheel and two foot switches are connected to the desktop. The steering wheel is used for turning mobile robot. One of the two foot switch acts as an accelerator to set the mean velocity the other is used to brake the vehicle.

The screen of the desktop displays the video streaming from the mobile robot on board camera. A graphical user interface (GUI) also displays the robot's parameters such as current steer angle, velocity of each rear wheel and the position of the  $z$ -axis. Buttons on the GUI operates the  $z$ -axis, head lamps, etc.

Figure 5.2: Local Controller Block Digram



## 5.2 Admissible path

## 5.3 Trajectory Tacking

## 5.4 Summary

In this chapter,



# Chapter 6

## Simulation Time Delay Tele-operation

In tele-operation tasks, human operators observe a remote scene through cameras, while tele-manipulating the mobile robot. The operator response is based on the latest feedback images from the cameras.

### 6.1 Tele operation architecture

In this study standard kinematic model of the mobile platform is used. The inputs to the model are the left and right rear wheel velocities. Based on this velocity these velocities the front wheels are steered to satisfy the Ackerman condition. The kinematic model of the platform is presented below

Where  $b$  = Distance between the rear wheels,  $V_L$  and  $V_R$  are the left and right wheel velocities. Since, the commands  $u_1$  and  $u_2$  reaches the remote platform instantly.

## **6.2 Modeling the human operator**

## **6.3 Simulation Results**

## **6.4 Predictive Model based Feedback**

### **6.4.1 using only dynamic model of vehicle**

### **6.4.2 Kalman Filter with odometer data and Accelerometer?**

## **6.5 Summary**

In this chapter,



# Chapter 7

## Predictive Display

This chapter includes the thesis summary, major contributions and future scope of the work.



# Chapter 8

## Conclusions

This chapter includes the thesis summary, major contributions and future scope of the work.

### 8.1 Thesis Summary

### 8.2 Future Scope of the work

The thesis opens up the following future directions:

- (i) The
- (ii) The
- (iii) The
- (iv) Implementing



# Bibliography

- [1] Y. Yamamoto and X. Yun, “Coordinating locomotion and manipulation of a mobile manipulator,” in *Decision and Control, 1992., Proceedings of the 31st IEEE Conference on*, pp. 2643–2648, IEEE, 1992.
- [2] S. Rajendran and et.al., “Mobile robot for reactor vessel inspection,” in *National Conference on Advanced Manufacturing and Robotics*, pp. 2527–2532, CMERI, Durgapur, WB., 2004.
- [3] S. K. Saha and J. Angeles, “Kinematics and dynamics of a three-wheeled 2-dof agv,” in *Robotics and Automation, 1989. Proceedings., 1989 IEEE International Conference on*, pp. 1572–1577, IEEE, 1989.
- [4] F. G. Pin and S. M. Killough, “A new family of omnidirectional and holonomic wheeled platforms for mobile robots,” *IEEE transactions on robotics and automation*, vol. 10, no. 4, pp. 480–489, 1994.
- [5] J. E. M. Salih, M. Rizon, S. Yaacob, A. H. Adom, and M. R. Mamat, “Designing omni-directional mobile robot with mecanum wheel,” *American Journal of Applied Sciences*, vol. 3, no. 5, pp. 1831–1835, 2006.
- [6] J. Suthakorn, S. S. H. Shah, S. Jantarajit, W. Onprasert, W. Saensupo, S. Saeung, S. Nakdhamabhorn, V. Sa-Ing, and S. Reaungamornrat, “On the design and development of a rough terrain robot for rescue missions,” in *Robotics and Biomimetics, 2008. ROBIO 2008. IEEE International Conference on*, pp. 1830–1835, IEEE, 2009.
- [7] M. Guarnieri, R. Debenest, T. Inoh, E. Fukushima, and S. Hirose, “Development of helios vii: an arm-equipped tracked vehicle for search and rescue operations,” in *Intelligent Robots and Systems, 2004.(IROS 2004). Proceedings. 2004 IEEE/RSJ International Conference on*, vol. 1, pp. 39–45, IEEE, 2004.
- [8] K. Nagatani, S. Tachibana, M. Sofne, and Y. Tanaka, “Improvement of odometry for omnidirectional vehicle using optical flow information,” in *Intelligent Robots and Systems, 2000.(IROS 2000). Proceedings. 2000 IEEE/RSJ International Conference on*, vol. 1, pp. 468–473, IEEE, 2000.
- [9] J. Y. Wong, *Theory of ground vehicles*. John Wiley & Sons, 2008.
- [10] T. Bevan, *Theory of Machines, Third Edition*. Delhi, India: CBS Publishers & Distributors, 1984.

- [11] *Motor catalogue 2014*. Switzerland: Maxon Motors, 2017.
- [12] J. T. Machado and M. F. Silva, “An overview of legged robots,” in *International symposium on mathematical methods in engineering*, MME Press Ankara, Turkey, 2006.
- [13] K. P. Valavanis and G. J. Vachtsevanos, *Handbook of unmanned aerial vehicles*. Springer Publishing Company, Incorporated, 2014.
- [14] J.-B. Mouret, *Evolutionary Adaptation in Natural and Artificial Systems*. PhD thesis, Université Pierre et Marie Curie, 2015.
- [15] G. Campion, G. Bastin, and B. Dandrea-Novel, “Structural properties and classification of kinematic and dynamic models of wheeled mobile robots,” *IEEE transactions on robotics and automation*, vol. 12, no. 1, pp. 47–62, 1996.
- [16] G. Campion and W. Chung, “Wheeled robots,” in *Springer Handbook of Robotics*, pp. 391–410, Springer, 2008.
- [17] R. M. DeSantis, “Modeling and path-tracking control of a mobile wheeled robot with a differential drive,” *Robotica*, vol. 13, no. 04, pp. 401–410, 1995.
- [18] K. C. Koh and H. S. Cho, “A smooth path tracking algorithm for wheeled mobile robots with dynamic constraints,” *Journal of intelligent & robotic systems*, vol. 24, no. 4, pp. 367–385, 1999.
- [19] B. d’Andréa Novel, G. Campion, and G. Bastin, “Control of nonholonomic wheeled mobile robots by state feedback linearization,” *The International journal of robotics research*, vol. 14, no. 6, pp. 543–559, 1995.
- [20] B. d’Andrea Novel, G. Bastin, and G. Campion, “Modelling and control of non-holonomic wheeled mobile robots,” in *Robotics and Automation, 1991. Proceedings., 1991 IEEE International Conference on*, pp. 1130–1135, IEEE, 1991.
- [21] K. Thanjavur and R. Rajagopalan, “Ease of dynamic modelling of wheeled mobile robots (wmrs) using kane’s approach,” in *Robotics and Automation, 1997. Proceedings., 1997 IEEE International Conference on*, vol. 4, pp. 2926–2931, IEEE, 1997.
- [22] S. K. Saha and J. Angeles, “Dynamics of nonholonomic mechanical systems using a natural orthogonal complement,” *Journal of Applied Mechanics*, vol. 58, no. 1, pp. 238–243, 1991.
- [23] J. Angeles, *Fundamentals of robotic mechanical systems: theory, methods, and algorithms*, vol. 124. Springer Science & Business Media, 2013.
- [24] S. Saha, *Introduction to robotics*. Tata McGraw-Hill, New Delhi, 2010.
- [25] S. K. Saha, *Introduction to Robotics*. New Delhi, India: Tata McGraw Hill Education Pvt. Ltd., 2010.

- [26] S. K. Saha, “Dynamics of serial multibody systems using the decoupled natural orthogonal complement matrices,” *Journal of applied mechanics*, vol. 66, no. 4, pp. 986–996, 1999.
- [27] S. K. Saha and W. O. Schiehlen, “Recursive kinematics and dynamics for parallel structured closed-loop multibody systems,” *Mechanics of Structures and Machines*, vol. 29, no. 2, pp. 143–175, 2001.
- [28] J. A. S K Saha, “Dynamics of nonholonomic mechanical system using natural orthogonal complement,” *Transactions of ASME*, vol. 58, March, 1991.
- [29] *EPOS Application Note: RS232 to CANopen Getway*. Maxon Motors, 2012.





# Appendix A

## Kinematics

### A.1 Coordinate Frames and DH Parameters

In order to model, i.e., specify the configuration and dimension of a spatial mechanism like a robot manipulator, the DH parameters and the corresponding coordinate frames are defined here.



# Appendix B

## Dynamics

### B.1 Inertia Dyad

The dyadic product takes two vector and returns a second-order tensor. If  $\mathbf{a}$  and  $\mathbf{b}$  are two vectors expressed as  $\mathbf{a} \equiv a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$  and  $\mathbf{b} \equiv b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ . Then the dyadic product of  $\mathbf{a}$  and  $\mathbf{b}$  are defined as:



# Publications from the Thesis

Research papers published/presented/under preparation are listed below

1. Hayat, A.A., Abhishek, V., Saha, S.K., Dynamic Identification of Manipulator: Comparison between CAD and Actual Parameters, *17th National and 2nd International Conference on Machine and Mechanisms, iNaCoMM 2015*, IIT Kanpur, Dec. 16–19, 2015.
2. Udai, A. D., Hayat, A. A., Saha, S. K., “Parallel Active/Passive Force Control of Industrial Robots with Joint Compliance,” *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2014)*, Chicago, Illinois, Sept. 14–18 , 2014.



## Brief Bio-data of the Author