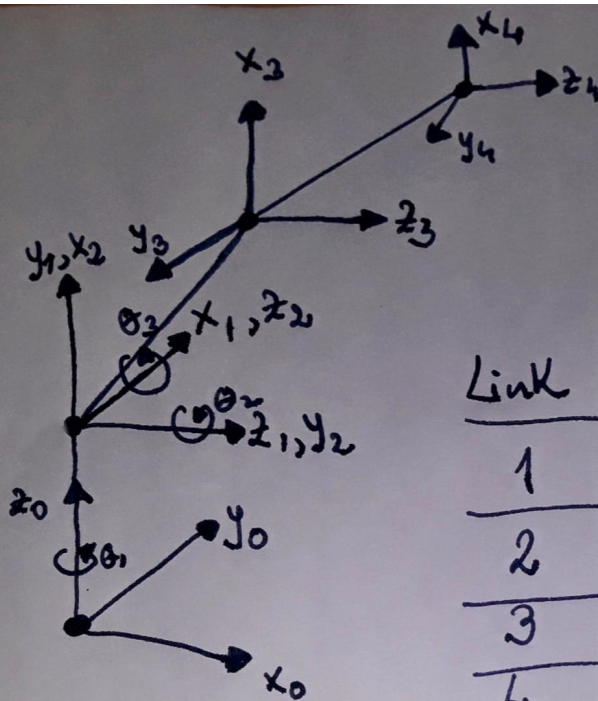


Sphere Follower Robot

Note: we set the configuration in such a way so that the robot sits up straight when all of $\theta_1, \theta_2, \theta_3, \theta_4$ are 0. That is, we are using $\theta_1 + 90$ and $\theta_2 + 90$.



Link	α_i	a_i	d_i	θ_i	For θ_0 straight
1	90	0	2	θ_1^*	$\theta_1 + 90$
2	90	0	0	θ_2^*	$\theta_2 + 90$
3	-90	3	0	θ_3^*	θ_3
4	0	2	0	θ_4^*	θ_4

First we rotate by z , and then by x when a combination of them is required.

Forward Kinematics

Remember : $\cos(a + b) = \cos(a) \cos(b) - \sin(a) \sin(b)$, $\sin(a + b) = \sin(a) \cos(b) + \sin(b) \cos(a)$

$$\cos \theta_x = c_x, \sin \theta_x = s_x$$

$$A = T_\theta T_d T_\alpha T_a$$

$$A1 = T_{\theta_1+90} T_{d=2} T_{\alpha=90} = \begin{bmatrix} \cos(\theta_1 + \frac{\pi}{2}) & -\sin(\theta_1 + \frac{\pi}{2}) & 0 & 0 \\ \sin(\theta_1 + \frac{\pi}{2}) & \cos(\theta_1 + \frac{\pi}{2}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s_1 & 0 & c_1 & 0 \\ c_1 & 0 & s_1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A2 = T_{\theta_2+90} T_{\alpha=90} = \begin{bmatrix} \cos(\theta_2 + \frac{\pi}{2}) & -\sin(\theta_2 + \frac{\pi}{2}) & 0 & 0 \\ \sin(\theta_2 + \frac{\pi}{2}) & \cos(\theta_2 + \frac{\pi}{2}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s_2 & 0 & c_2 & 0 \\ c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A3 = T_{\theta_3} T_{a=3} T_{\alpha=-90} = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_3 & 0 & -s_3 & 3c_3 \\ s_3 & 0 & c_3 & 3s_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A4 = T_{\theta_4} T_{a=2} = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_4 & -s_4 & 0 & 2c_4 \\ s_4 & c_4 & 0 & 2s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A1A2A3 = \begin{bmatrix} -s_1 & 0 & c_1 & 0 \\ c_1 & 0 & s_1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -s_2 & 0 & c_2 & 0 \\ c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & 0 & -s_3 & 3c_3 \\ s_3 & 0 & c_3 & 3s_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_1 s_2 & c_1 & -s_1 c_2 & 0 \\ -c_1 s_2 & s_1 & c_1 c_2 & 0 \\ c_2 & 0 & s_2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & 0 & -s_3 & 3c_3 \\ s_3 & 0 & c_3 & 3s_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_1 s_2 c_3 + c_1 s_3 & s_1 c_2 & -s_1 s_2 s_3 + c_1 c_3 & 3s_1 s_2 c_3 + 3c_1 s_3 \\ -c_1 s_2 c_3 + s_1 s_3 & -c_1 c_2 & c_1 s_2 s_3 + s_1 c_3 & -3c_1 s_2 c_3 + 3s_1 s_3 \\ c_2 c_3 & -s_2 & -c_2 s_3 & 3c_2 c_3 + 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A1A2A3A4 = \begin{bmatrix} s_1 s_2 c_3 + c_1 s_3 & s_1 c_2 & -s_1 s_2 s_3 + c_1 c_3 & 3s_1 s_2 c_3 + 3c_1 s_3 \\ -c_1 s_2 c_3 + s_1 s_3 & -c_1 c_2 & c_1 s_2 s_3 + s_1 c_3 & -3c_1 s_2 c_3 + 3s_1 s_3 \\ c_2 c_3 & -s_2 & -c_2 s_3 & 3c_2 c_3 + 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & -s_4 & 0 & 2c_4 \\ s_4 & c_4 & 0 & 2s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_1 s_2 c_3 c_4 + c_1 s_3 c_4 + s_1 c_2 s_4 & -s_1 s_2 c_3 s_4 - c_1 s_3 s_4 + s_1 c_2 c_4 & -s_1 s_2 s_3 + c_1 c_3 & x_e \\ -c_1 s_2 c_3 c_4 + s_1 s_3 s_4 - c_1 c_2 s_4 & c_1 s_2 c_3 s_4 - s_1 s_3 s_4 - c_1 c_2 c_4 & c_1 s_2 s_3 + s_1 c_3 & y_e \\ c_2 c_3 c_4 - s_2 s_4 & -c_2 c_3 s_4 - s_2 c_4 & -c_2 s_3 & z_e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} k_1(q) \\ k_2(q) \\ k_3(q) \end{bmatrix} = \begin{bmatrix} 2s_1 s_2 c_3 c_4 + 2c_1 s_3 c_4 + 2s_1 c_2 s_4 + 3s_1 s_2 c_3 + 3c_1 s_3 \\ -2c_1 s_2 c_3 c_4 + 2s_1 s_3 c_4 - 2c_1 c_2 s_4 - 3c_1 s_2 c_3 + 3s_1 s_3 \\ 2c_2 c_3 c_4 - 2s_2 s_4 + 3c_2 c_3 + 2 \end{bmatrix}$$

Jacobian Matrix (velocity of end effector knowing angle velocity)

Note: Here $q_1 = \theta_1$, $q_2 = \theta_2$, $q_3 = \theta_3$, $q_4 = \theta_4$.

$$\dot{\vec{x}}_e = J(q)\dot{\vec{q}} = \begin{bmatrix} \frac{\partial k_1(q)}{\partial q_1} & \frac{\partial k_1(q)}{\partial q_2} & \frac{\partial k_1(q)}{\partial q_3} & \frac{\partial k_1(q)}{\partial q_4} \\ \frac{\partial k_2(q)}{\partial q_1} & \frac{\partial k_2(q)}{\partial q_2} & \frac{\partial k_2(q)}{\partial q_3} & \frac{\partial k_2(q)}{\partial q_4} \\ \frac{\partial k_3(q)}{\partial q_1} & \frac{\partial k_3(q)}{\partial q_2} & \frac{\partial k_3(q)}{\partial q_3} & \frac{\partial k_3(q)}{\partial q_4} \end{bmatrix} \begin{bmatrix} \frac{\partial q_1}{\partial t} \\ \frac{\partial q_2}{\partial t} \\ \frac{\partial q_3}{\partial t} \\ \frac{\partial q_4}{\partial t} \end{bmatrix}$$

$$= \begin{bmatrix} 2c_1s_2c_3c_4 - 2s_1s_3c_4 + 2c_1c_2s_4 + 3c_1s_2c_3 - 3s_1s_3 & 2s_1c_2c_3c_4 - 2s_1s_2s_4 + 3s_1c_2c_3 \\ 2s_1s_2c_3c_4 + 2c_1s_3c_4 + 2s_1c_2s_4 + 3s_1s_2c_3 + 3c_1s_3 & -2c_1c_2c_3c_4 + 2c_1s_2s_4 - 3c_1c_2c_3 \\ 0 & -2s_2c_3c_4 - 2c_2s_4 - 3s_2c_3 \\ -2s_1s_2s_3c_4 + 2c_1c_3c_4 - 3s_1s_2s_3 + 3c_1c_3 & -2s_1s_2c_3s_4 - 2c_1s_3s_4 + 2s_1c_2c_4 \\ 2c_1s_2s_3c_4 + 2s_1c_3c_4 + 3c_1s_2s_3 + 3s_1c_3 & 2c_1s_2c_3s_4 - 2s_1s_3s_4 - 2c_1c_2c_4 \\ -2c_2s_3c_4 - 3c_2s_3 & -2c_2c_3s_4 - 2s_2c_4 \end{bmatrix} \begin{bmatrix} \frac{\partial q_1}{\partial t} \\ \frac{\partial q_2}{\partial t} \\ \frac{\partial q_3}{\partial t} \\ \frac{\partial q_4}{\partial t} \end{bmatrix} \quad (1)$$

Inverse Jacobian Matrix (velocity of angles knowing end effector velocity)

Change of angles with respect to time is $\dot{\vec{q}}$.

Desired change of direction with respect to time is $\dot{\vec{x}}_e \approx \vec{x}_{des} - \vec{x}_e$.

$$\Rightarrow \dot{\vec{q}} = J^{-1}(q)\dot{\vec{x}}_e \approx J^{-1}(q)(\vec{x}_{des} - \vec{x}_e)$$