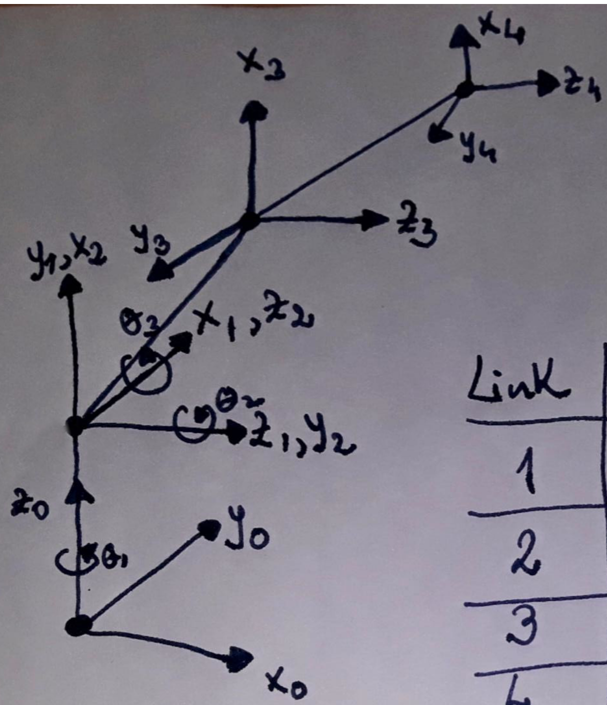
Sphere Follower Robot

Note: we set the configuration in such a way
so that the robot sits up straight when all of $\theta_1, \theta_2, \theta_3, \theta_4$ are 0.
That is, we are using $\theta_1 + 90$ and $\theta_2 + 90$.



Link	α_i	a_i	d_i	θ_i	For θ [illegible] straight
1	90	0	2	θ_1^*	$\theta_1 + 90$
2	90	0	0	θ_2^*	$\theta_2 + 90$
3	-90	3	0	θ_3^*	θ_3
4	0	2	0	θ_4^*	θ_4

First we rotate by z, and then by x when a combination of them is required.

Forward Kinematics

$$Remember : \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b), \sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$$

$$\cos\theta_x = c_x, \sin\theta_x = s_x$$

$$A = T_\theta T_d T_\alpha T_a$$

$$A1 = T_{\theta_1+90}T_{d=2}T_{\alpha=90} = \begin{bmatrix} \cos(\theta_1+\frac{\pi}{2}) & -\sin(\theta_1+\frac{\pi}{2}) & 0 & 0 \\ \sin(\theta_1+\frac{\pi}{2}) & \cos(\theta_1+\frac{\pi}{2}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s_1 & 0 & c_1 & 0 \\ c_1 & 0 & s_1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A2 = T_{\theta_2+90}T_{\alpha=90} = \begin{bmatrix} \cos(\theta_2+\frac{\pi}{2}) & -\sin(\theta_2+\frac{\pi}{2}) & 0 & 0 \\ \sin(\theta_2+\frac{\pi}{2}) & \cos(\theta_2+\frac{\pi}{2}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s_2 & 0 & c_2 & 0 \\ c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A3 = T_{\theta_3}T_{a=3}T_{\alpha=-90} = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_3 & 0 & -s_3 & 3c_3 \\ s_3 & 0 & c_3 & 3s_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A4 = T_{\theta_4}T_{a=2} = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_4 & -s_4 & 0 & 2c_4 \\ s_4 & c_4 & 0 & 2s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A1A2A3 = \begin{bmatrix} -s_1 & 0 & c_1 & 0 \\ c_1 & 0 & s_1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -s_2 & 0 & c_2 & 0 \\ c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & 0 & -s_3 & 3c_3 \\ s_3 & 0 & c_3 & 3s_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_1s_2 & c_1 & -s_1c_2 & 0 \\ -c_1s_2 & s_1 & c_1c_2 & 0 \\ c_2 & 0 & s_2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & 0 & -s_3 & 3c_3 \\ s_3 & 0 & c_3 & 3s_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_1s_2c_3 + c_1s_3 & s_1c_2 & -s_1s_2s_3 + c_1c_3 & 3s_1s_2c_3 + 3c_1s_3 \\ -c_1s_2c_3 + s_1s_3 & -c_1c_2 & c_1s_2s_3 + s_1c_3 & -3c_1s_2c_3 + 3s_1s_3 \\ c_2c_3 & -s_2 & -c_2s_3 & 3c_2c_3 + 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A1A2A3A4 = \begin{bmatrix} s_1s_2c_3 + c_1s_3 & s_1c_2 & -s_1s_2s_3 + c_1c_3 & 3s_1s_2c_3 + 3c_1s_3 \\ -c_1s_2c_3 + s_1s_3 & -c_1c_2 & c_1s_2s_3 + s_1c_3 & -3c_1s_2c_3 + 3s_1s_3 \\ c_2c_3 & -s_2 & -c_2s_3 & 3c_2c_3 + 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & -s_4 & 0 & 2c_4 \\ s_4 & c_4 & 0 & 2s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_1s_2c_3c_4 + c_1s_3c_4 + s_1c_2s_4 & -s_1s_2c_3s_4 - c_1s_3s_4 + s_1c_2c_4 & -s_1s_2s_3 + c_1c_3 & x_e \\ -c_1s_2c_3c_4 + s_1s_3s_4 - c_1c_2s_4 & c_1s_2c_3s_4 - s_1s_3s_4 - c_1c_2c_4 & c_1s_2s_3 + s_1c_3 & y_e \\ c_2c_3c_4 - s_2s_4 & -c_2c_3s_4 - s_2c_4 & -c_2s_3 & z_e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} k_1(q) \\ k_2(q) \\ k_3(q) \end{bmatrix} = \begin{bmatrix} 2s_1s_2c_3c_4 + 2c_1s_3c_4 + 2s_1c_2s_4 + 3s_1s_2c_3 + 3c_1s_3 \\ -2c_1s_2c_3c_4 + 2s_1s_3c_4 - 2c_1c_2s_4 - 3c_1s_2c_3 + 3s_1s_3 \\ 2c_2c_3c_4 - 2s_2s_4 + 3c_2c_3 + 2 \end{bmatrix}$$

Jacobian Matrix (velocity of end effector knowing angle velocity)

Note: Here $q_1 = \theta_1$, $q_2 = \theta_2$, $q_3 = \theta_3$, $q_4 = \theta_4$.

$$\dot{\vec{x_e}} = J(q)\dot{\vec{q}} = \begin{bmatrix} \frac{\partial k_1(q)}{\partial q_1} & \frac{\partial k_1(q)}{\partial q_2} & \frac{\partial k_1(q)}{\partial q_3} & \frac{\partial k_1(q)}{\partial q_4} \\ \frac{\partial k_2(q)}{\partial q_1} & \frac{\partial k_2(q)}{\partial q_2} & \frac{\partial k_2(q)}{\partial q_3} & \frac{\partial k_2(q)}{\partial q_4} \\ \frac{\partial k_3(q)}{\partial q_1} & \frac{\partial k_3(q)}{\partial q_2} & \frac{\partial k_3(q)}{\partial q_3} & \frac{\partial k_3(q)}{\partial q_4} \end{bmatrix} \begin{bmatrix} \frac{\partial q_1}{\partial t} \\ \frac{\partial q_2}{\partial t} \\ \frac{\partial q_3}{\partial t} \\ \frac{\partial q_4}{\partial t} \end{bmatrix}$$

$$= \begin{bmatrix} 2c_1s_2c_3c_4 - 2s_1s_3c_4 + 2c_1c_2s_4 + 3c_1s_2c_3 - 3s_1s_3 & 2s_1c_2c_3c_4 - 2s_1s_2s_4 + 3s_1c_2c_3 & -2s_1s_2s_3c_4 + 2c_1c_3c_4 - 3s_1s_2s_3 + 3c_1c_3 & -2s_1s_2c_3s_4 - 2c_1s_3s_4 + 2s_1c_2c_4 \\ 2s_1s_2c_3c_4 + 2c_1s_3c_4 + 2s_1c_2s_4 + 3s_1s_2c_3 + 3c_1s_3 & -2c_1c_2c_3c_4 + 2c_1s_2s_4 - 3c_1c_2c_3 & 2c_1s_2s_3c_4 + 2s_1c_3c_4 + 3c_1s_2s_3 + 3s_1c_3 & 2c_1s_2c_3s_4 - 2s_1s_3s_4 - 2c_1c_2c_4 \\ 0 & -2s_2c_3c_4 - 2c_2s_4 - 3s_2c_3 & -2c_2s_3c_4 - 3c_2s_3 & -2c_2c_3s_4 - 2s_2c_4 \end{bmatrix} \begin{bmatrix} \frac{\partial q_1}{\partial t} \\ \frac{\partial q_2}{\partial t} \\ \frac{\partial q_3}{\partial t} \\ \frac{\partial q_4}{\partial t} \end{bmatrix} \tag{1}$$

Inverse Jacobian Matrix (velocity of angles knowing end effector velocity)

Change of angles with respect to time is $\dot{\vec{q}}$.
Desired change of direction with respect to time is $\dot{\vec{x_e}} \approx \vec{x_{des}} - \vec{x_e}$.

$$=> \dot{\vec{q}} = J^{-1}(q)\dot{\vec{x_e}} \approx J^{-1}(q)(\vec{x_{des}} - \vec{x_e})$$