Higher-order topology in condensed matter systems

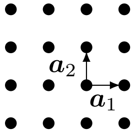
University of Florida, November

Outline

1. Introduction—Band topology
2. Higher-order topology and boundary obstructed topology
3. Chiral Dirac superconductors

Bands structure

Electrons on a lattice can be described by local degrees of freedom,



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- $\boldsymbol{R}$ donate lattice coordinate.
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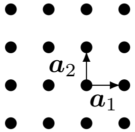

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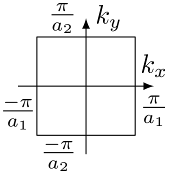
For free electrons,

$$|\boldsymbol{k}\ i\rangle = \sum_{\boldsymbol{R}} e^{i\boldsymbol{k}\cdot\boldsymbol{R}} |\boldsymbol{R}\ i\rangle$$

$$\langle \boldsymbol{k}'\ i|H|\boldsymbol{k}\ j\rangle = \mathcal{H}^{ij}(\boldsymbol{k})\delta_{\boldsymbol{k},\boldsymbol{k}'}$$



- Translational symmetry $\rightarrow$ conservation of crystal momentum.
- Diagonalizing the Hamiltonian we end up with a set of bands.
- We will refer to $\mathcal{H}(\boldsymbol{k})$ as the Hamiltonian.
- We are interested in gapped systems (insulators).

Topology of the occupied bands

Suppose we are studying systems with some symmetry group.
Define an equivalence between $\mathcal{H}^0(\boldsymbol{k})$ and $\mathcal{H}^1(\boldsymbol{k})$:

$$\mathcal{H}^0(\boldsymbol{k}) \approx \mathcal{H}^1(\boldsymbol{k}) \text{ iff } \exists$$

- $\mathcal{H}(\boldsymbol{k}, t)$; $\mathcal{H}(\boldsymbol{k}, 0) = \mathcal{H}^0(\boldsymbol{k})$, $\mathcal{H}(\boldsymbol{k}, 1) = \mathcal{H}^1(\boldsymbol{k})$,
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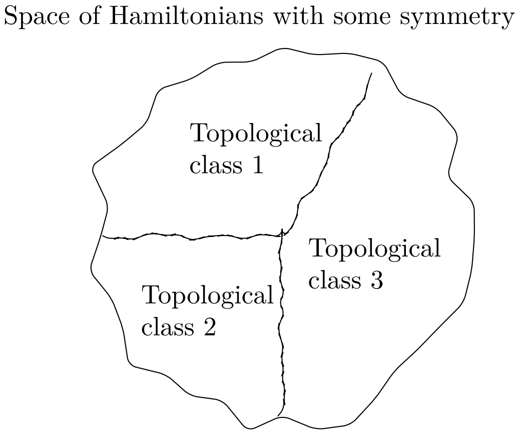
- This can be thought as deforming the occupied (and empty bands) bands from those of $\mathcal{H}^0(\boldsymbol{k})$ to those of $\mathcal{H}^1(\boldsymbol{k})$.
- If this deformation is possible, then the occupied bands of $\mathcal{H}^0(\boldsymbol{k})$ and $\mathcal{H}^1(\boldsymbol{k})$ are topologically equivalent.

Topological classification

- Topological classification require a survey of all possible gapped Hamiltonian and grouping them into equivalence classes under the restrictions of some symmetry group.



- Find topological invariants that differentiate one class from the other.

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Three important internal symmetries:

- Time-reversal symmetry ($\mathcal{T}$)
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Complete topological classification exists for the 10 AZ classes.

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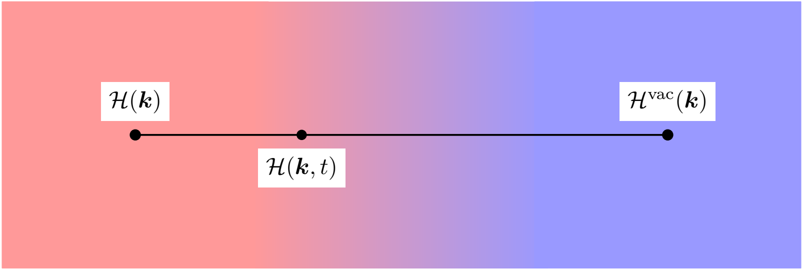
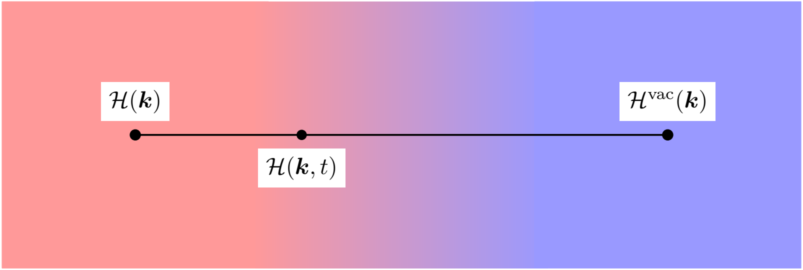
AZ	$\mathcal{T}^2$	$\mathcal{P}^2$	$\mathcal{C}^2$
A	0	0	0
AIII	0	0	1
AI	1	0	0
BDI	1	1	1
D	0	1	0
DIII	-1	1	1
AII	-1	0	0
CII	-1	-1	1
C	0	-1	0
CI	1	-1	1

Gapless boundaries



- A boundary interpolates between a topological system and the vacuum.
- Internal symmetries are not broken on the boundary.

Gapless boundaries



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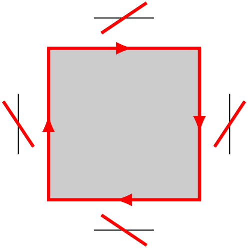
Topological systems protected by internal symmetries *only* will in general have gapless boundaries.

Examples

<u>In 2D:</u>
Chern insulators.
Protecting symmetries: None.
Topological invariant: Chern number.



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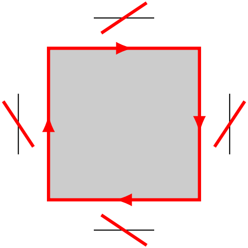

<u>In 1D:</u>
SSH chain.
Protecting symmetry: Chiral symmetry
Topological invariant: Polarization

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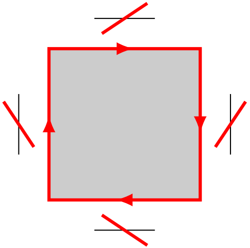
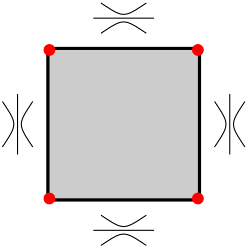
Higher-order topology

Can we find topological systems systems with gapped boundaries?



First-order topology; gapless mode on boundaries of co-dimension 1.



Second-order topology; gapless mode on boundaries of co-dimension 2.

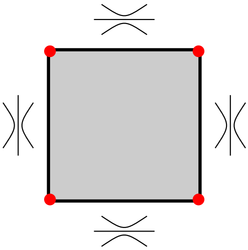
The protecting symmetry must be broken on the boundary. Internal symmetries are not enough.

Spacial symmetries

Spacial symmetries are those that change the position of the particles. $\boldsymbol{R} \to \boldsymbol{R}'$.

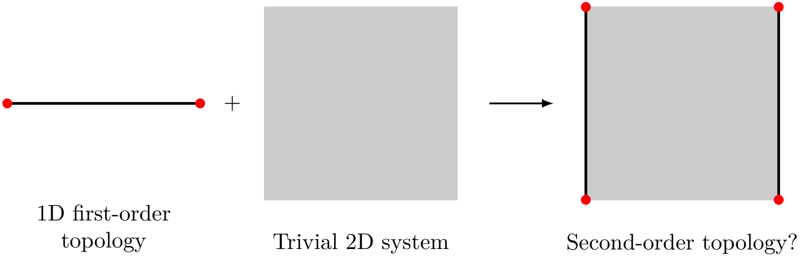
Examples of spacial symmetries:

$$C_4 : (R_x, R_y) \to (-R_y, R_x) \qquad C_2 : (R_x, R_y) \to (-R_x, -R_y)$$



C_4 and C_2 are broken on the boundaries.

A cheap way to get corner modes



- These corner zero modes are not protected by a bulk gap closing.

Higher-order and boundary-obstructed topologies

Second-order topology

Boundary-obstructed topology

- What kinds of system would show such surface signature?
- What kinds of topological invariants we can find?

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Superconductors

The BdG form for superconducting Hamiltonian,

$$H = \frac{1}{2}\int \frac{d\boldsymbol{k}}{(2\pi)^d} \begin{bmatrix}\Psi^\dagger(\boldsymbol{k})\Psi(-\boldsymbol{k})\end{bmatrix}\begin{bmatrix}\mathcal{H}_n(\boldsymbol{k}) & \Delta(\boldsymbol{k}) \\ \Delta^\dagger(\boldsymbol{k}) & -\mathcal{H}_n^*(-\boldsymbol{k})\end{bmatrix}\begin{bmatrix}\Psi(\boldsymbol{k}) \\ \Psi^\dagger(-\boldsymbol{k})\end{bmatrix}$$

- $\Psi^\dagger(\boldsymbol{k}) = (a_1(\boldsymbol{k}), \ldots, a_N(\boldsymbol{k}))$.
- $\mathcal{H}_n(\boldsymbol{k})$ is the normal state Hamiltonian.
- $\Delta(\boldsymbol{k})$ is the superconducting order parameter.
- The BdG Hamiltonian $\mathcal{H}(\boldsymbol{k}) = \begin{bmatrix}\mathcal{H}_n(\boldsymbol{k}) & \Delta(\boldsymbol{k}) \\ \Delta^\dagger(\boldsymbol{k}) & -\mathcal{H}_n^*(-\boldsymbol{k})\end{bmatrix}$ allows us to study the topology of superconductors within the same framework as insulating Hamiltonians.
- BdG Hamiltonians has an *intrinsic* particle-hole symmetry.

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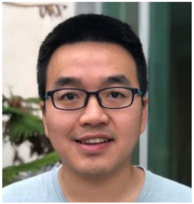

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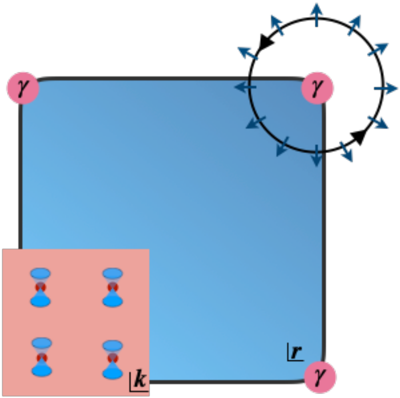
Chiral Dirac superconductors

Can we find a low energy criteria that guarantee the existence of the corner modes?

<u>In 2D</u>

- Dirac points in the normal state.
- A $p + ip$ superconducting order parameter gapping the Dirac points.
- With C_4 symmetry $\rightarrow$ second-order topology
- With C_2 symmetry $\rightarrow$ boundary-obstructed topology.



Summary

- Topological band systems are those that cannot be deformed to the vacuum without:
    1. Closing a gap
    2. Breaking the symmetry
- Topologies protected by internal symmetries alone lead to first-order topology (gapless boundaries).
- Including spacial symmetries can lead to a much richer topological structure.
- Higher-order topologies have gapless modes on boundaries with co-dimension higher than one.
- Boundary-obstructed topologies are only protected by a boundary gap closing.