

Homework 6

Friday, November 21, 2025

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$$1) \quad C_L = C_0 \cdot V_{out}$$

$$a) \quad E_{V_{DD}} = \int_{V_{out}=0}^{V_{DD}} V_{DD} C_L dV_{out} \rightarrow \int_{V_{out}=0}^{V_{DD}} V_{DD} (C_0 \cdot V_{out}) dV_{out}$$

$$E_{V_{DD}} = V_{DD} C_0 \int_0^{V_{DD}} V_{out} dV_{out} = V_{DD} C_0 \cdot \left(\frac{1}{2} V_{out}^2 \Big|_0^{V_{DD}} \right)$$

$$E_{V_{DD}} = \frac{1}{2} C_0 V_{DD}^3$$

$$b) \quad E_C = \int_0^{V_{DD}} C_L V_{out} dV_{out} = \int_0^{V_{DD}} C_0 V_{out} \cdot V_{out} dV_{out}$$

$$= C_0 \int_0^{V_{DD}} V_{out}^2 dV_{out} = C_0 \left(\frac{1}{3} V_{out}^3 \Big|_0^{V_{DD}} \right)$$

$$= \frac{1}{3} C_0 V_{DD}^3$$

$$c) \quad E_{V_{DD}} - E_C = \frac{1}{2} C_0 V_{DD}^3 - \frac{1}{3} C_0 V_{DD}^3$$

$$E_{diss} = \frac{1}{6} C_0 V_{DD}^3$$

$$2) \quad 10 \text{ M gates}, \text{ each gate is } 20 \text{ aF} = 20 \times 10^{-18}$$

$$V_{DD} = 1.2 \text{ V} \quad \frac{I_{off}^n + I_{off}^p}{2} = 1 \text{ nA}$$

$$\alpha = 0.05 \quad f = 1 \text{ GHz}$$

$$P_{avg} = C_L V_{DD}^2 \alpha f + V_{DD} \left(\frac{I_{off}^n + I_{off}^p}{2} \right)$$

$$= 20 \times 10^{-18} (1.2^2) (0.05) (1 \times 10^9)$$

$$+ 1.2 (1 \times 10^{-9}) = 2.64 \times 10^{-9}$$

$$(10 \times 10^6) (2.64 \times 10^{-9}) = 0.0264 \text{ W} = \boxed{26.4 \text{ mW}}$$

$$3) \quad t_{switch} = 1 \text{ ps}$$

$$V_{DD} = 1 \text{ V}$$

$$I_{sc} = 10 \text{ nA}$$

$$\frac{\text{Energy}}{f} = \frac{0.1230 - 0.063}{1} = \frac{0.06 \text{ nW}}{\text{GHz}}$$

$$E_{cycle} = 0.06 \times \frac{10^{-6} \text{ J/s}}{10^9 / \text{s}} = 60 \times 10^{-18} \text{ J}$$

$$P_{static} = F(0) = 0.0330 \text{ nW} - (0.06 \frac{\text{nW}}{\text{GHz}} \times 0.5 \text{ GHz})$$

$$= 0.0330 - 0.03 = 0.0030 \text{ nW}$$

$$= 3 \text{ nW}$$

$$P_{static} = V_{DD} * I_{Leak}$$

$$3 \text{ nW} = 1 \text{ V} * I_{Leak}$$

$$I_{Leak} = 3 \text{ nA}$$

$$E_{sc} = 1 \text{ V} \cdot 10 \text{ nA} \cdot 1 \text{ ps}$$

$$= 1 (10 \times 10^{-6}) (1 \times 10^{-12})$$

$$= 10 \times 10^{-18} \text{ J} = 10 \text{ aJ}$$

$$E_{cycle} = E_{dynamic} + E_{static}$$

$$60 = E_D + 10$$

$$E_{dynamic} = 50 \text{ aJ}$$

$$E_{dynamic} = C_L V_{DD}^2$$

$$50 \text{ aJ} = C_L (1)^2$$

$$C_L = 50 \text{ aF}$$