May 2021

$Q1$. Match the vector subspace of $\mathbb{R}^3$ (with the usual scalar multiplication and vector addition) in column A with their bases in column B and the dimension of the vector space in column C in Table: M2ES2

	Vector Space (Column A)		Basis (Column B)		Dimension of the vector space (Column C)
a)	$W = \{(x, y, z) \mid x - y + z = 0, x + y = 0, x, y, z \in \mathbb{R}\}$	1)	$\{(0, 1, 0), (0, 0, 1)\}$	i)	1
b)	$W = \{(x, y, z) \mid x = 0,\ x, y, z \in \mathbb{R}\}$	2)	$\{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$	ii)	3
c)	$W = span\{(1, 2, 1), (2, 1, 3), (0, 6, -2)\}$	3)	$\{(-1, 1, 2)\}$	iii)	2
d)	$W = span\{(1, 2, 1), (2, 1, 3), (3, 3, 3)\}$	4)	$\{(1, 5, 0), (0, -3.1)\}$	iv)	2

Table: M2ES2

Options:

A. a $\longrightarrow$ 1 $\longrightarrow$ iii, b $\longrightarrow$ 3 $\longrightarrow$ i, c $\longrightarrow$ 4 $\longrightarrow$ iv, d $\longrightarrow$ 2 $\longrightarrow$ ii

B.a $\longrightarrow$ 3 $\longrightarrow$ i, b $\longrightarrow$ 1 $\longrightarrow$ iii, c $\longrightarrow$ 4 $\longrightarrow$ iv, d $\longrightarrow$ 2 $\longrightarrow$ ii

C. a $\longrightarrow$ 3 $\longrightarrow$ i, b $\longrightarrow$ 1 $\longrightarrow$ iii, c $\longrightarrow$ 2 $\longrightarrow$ ii, d $\longrightarrow$ 4 $\longrightarrow$ iv

D. a $\longrightarrow$ 4 $\longrightarrow$ iv, b $\longrightarrow$ 1 $\longrightarrow$ iii, c $\longrightarrow$ 3 $\longrightarrow$ i, d $\longrightarrow$ 2 $\longrightarrow$ ii

Jan 2022

$Q2$: Match the vector subspaces of $\mathbb{R}^3$ (with the usual scalar multiplication and vector addition) in column A with their bases in column B and the dimension of vector spaces in column C in Table: M2D1

	Vector Space (Column A)		Basis (Column B)		Dimension of the vector space (Column C)
a)	$W = \{(x, y, z) \mid x - 3y + z - 0, x, y, z \in \mathbb{R}\}$	1)	$\{(1, 3, 0), (3, 1, 0)\}$	i)	2
b)	$W = \{(x, y, z) \mid z = 0,\ x, y \in \mathbb{R}\}$	2)	$\{(1, 0, 0), (1, 0, 1), (0, 1, 1)\}$	ii)	2
c)	$W = span\{(1.1.1), (1, 0, 3), (2, 2, 3)\}$	3)	$\{(2, 0, -1), (-3, -1, 0)\}$	ii)	3

Table: M2D1

Options:

A. a → 3 → i.
B. b → 2 → iii.
C. c → 2 → i.
D. a → 1 → ii.
E. b → 1 → i.
F. c → 2 → iii.

Sept 2022

$Q3$. Which of the following subspaces have a basis whose cardinality is 2?

Options:

A. $\{A \in \mathcal{M}_{3\times 3}(\mathbb{R})\ :\ A \textit{ is a diagonal matrix}\}$.
B. $\{A \in \mathcal{M}_{3\times 3}(\mathbb{R}):\ A \textit{ is a diagonal matrix and the sum of the diagonal entries of } A \textit{ is } 0\}$
C. $span\{(1,1,0,0),(1,1,1,0),(4,4,1,0),(-1,-1,5,0)\}$
D. $span\{(1,3,5),(2,6,10),(-3,-9,-15)\}$

Jan 2023

$Q4$. Consider the subspace $W = \{(x, y, z, w) \mid x + y = z,\ z + w = x - y\}$ of $\mathbb{R}^4$.
Which of the following is a basis β for W ?

Options:

A. $\beta = \{(1, 1, 1, -2)\}$
B. $\beta = \{(1, 0, 1, 0), (0, 1, 1, -2)\}$
C. $\beta = \{(1, 0, 1, 0), (0, 1, 1, -2), (1, 1, 1, -2)\}$
D. $\beta = \{(1, 0, 1, 0),\ (0, 1, 0, 1), (1, 1, 1, -2)\}$

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$Q5$. Consider the system of linear equations formed by the equations in column B of Table M2ES3 and let A be its coefficient matrix. Answer the releated subquestions

	Equation of the surface		Equation of the tangent plane at $(1, 1, 2)$		Vector subspace corresponding to the affine subspace formed by tangent plane
$i)$	$z = x^2 + y^2$	$a)$	$3x + 3y + 2z = 10$	1)	$\left\{(x, y, z) \mid x + y = \frac{z}{2},\ x, y, z \in \mathbb{R}\right\}$
$ii)$	$x^2 + y^2 + z^2 = 6$	$b)$	$z = 2x + 2y - 2$	2)	$\left\{(x, y, z) \mid x + y = -\frac{2}{5}z,\ x, y, z \in \mathbb{R}\right\}$
$iii)$	$xy + yz + zx = 5$	$c)$	$x + y + 2z = 6$	3)	$\{(x, y, z) \mid 2z = -x - y\ ,\ x, y, z \in \mathbb{R}\}$

Sub Questions:

Q1) Find $rank(A)$

Q2) Which of the following denotes the column space of A?(More than one option may be correct)

Options:

A. $Span\{(3,2,1)\}$
B. $Span\{(2,-1,2)\}$
C. $Span\{(3,2,1),\ (2,-1,2)\}$
D. $Span\{(3,2,1),\ (2,-1,2),\ (1,3,-1)\}$

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$Q6$. Which of the following statements is/are true for a system of linear equations $Ax = b$?

Options:

A. If x_1 and x_2 are solutions of the system, then for any $\alpha, \beta \in \mathbb{R}$, $\alpha x_1 + \beta x_2$ is also a solution if and only if $b = 0$

B. If x_1 and x_2 are solutions of the linear equation, then $\alpha x_1 + \beta x_2$ is also a solution for any b in the column space of A

C. The system $Ax = 0$ has a unique solution when the number of equations is less than the number of variables.

D. If the system $Ax = 0$ has a unique solution, then the columns of A are linearly independent

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$Q7$. Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $E = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ and $F = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Then which of the following statement(s) is/are true?

Options:

A. $span\{B, C, D\} = M_2(\mathbb{R})$

B. $span\{A, B, D\} = span\{A, F\}$

C. $span\{A, E, F\} = \left\{A \in M_2(\mathbb{R}) \ : \ A \text{ is symmetric i.e., } A^T = A\right\}$

D. The set of all 2×2 diagonal matrices are spanned by the matrices A and E

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$Q8$. Choose the correct statement

Options:

A. If A and B are two matrices such that AB and BA are well defined, then both A and B are square matrices

B. If A is a square matrix of order 2 such that $A^2 = I$, then A has to be equal to I

C. A and B are square matrices of order n. If B is not invertible, then AB is not invertible

D. A and B are square matrices of order n and $k \leqslant n$. If $rank(AB) = rank(B) = k$, then $rank(A) < k$

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$Q9$. Let V be a vector space and v_1, v_2 , v_3 be three disctinct element in V. If every $v \in V$ can be expressed as a unique linear combination of these three vectors, which of the following statement is true? Here $dim(V)$ refers to the dimension of V

Options:

A. $dim(V) = 3$

B. We cannot comment on the exact value of $dim(V)$. It lies in this range; $1 \leqslant dim(V) \leqslant 3$

C. We cannot comment on the exact value of $dim(V)$. It lies in this range; $0 \leqslant dim(V) \leqslant 3$

D. We cannot pass any judgement whatsoever on the value of $dim(V)$ with the given data

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$Q10$. Consider the subspace $W = \{(x, y, z, w) \mid x + z = y,\ y + w = z - x\}$ of $\mathbb{R}^4$

Based on the above data, answer the given subquestions

Sub Questions:

Q1) Which of the following are bases for W ?
Choose the correct options

Options:

A. $\beta = \{(-1, -1, 0, 2)\}$

B. $\beta = \{(-1, -1, 0, 2),\ (0, 1, 1, 0)\}$

C. $\beta = \{(1, 1, 0, -2),\ (1, 0, -1, -2)\}$

D. $\beta = \{(1, 1, 0, -2),\ (0, 1, 1, 0),\ (2, 1, -1, -4)\}$

E. $\beta = \{(-1, -1, 0, 2),\ (2, 1, -1, -4),\ (0, 1, 1, 0)\}$

Q2) What is the dimension of W?