May 2021

$Q1:$ Consider the system of linear equations given by $Ax = b$, where $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 4 & 5 \\ 7 & 8 & 19 \end{bmatrix}$,

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ 11 \\ 34 \end{bmatrix}$. The row echeleon form of the matrix A is given by

$R = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & \frac{-1}{4} \\ 0 & 0 & 0 \end{bmatrix}$.

Choose the set of correct options.

Options:

A. The system of linear equations $Ax = 0$, where A and x are given, has trivial solution only.

B. $Nullity(A) = 1$, and $Rank(A) = 2$

C. $Nullity(A) = 2$, and $Rank(A) = 2$

D. $Nullity(A) = 0$, and $Rank(A) = 3$

E. The solution space of the system of linear equations $Rx = 0$ is the same as the nullspace of A

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$Q2$ Choose the correct options from the following:

Options:

A. Suppose W is a 3-dimensional subspace of $\mathbb{R}^4$. Then every basis of $\mathbb{R}^4$ can be reduced to a basis for W by removing one vector.

B. If the columns of a matrix are linearly independent, then rows are also linearly independent.

C. If A is an $m \times n$ matrix, then the sum of dimension of column space and dimension of nullspace of A is n.

D. If A is an $m \times n$ matrix, then the sum of dimension of column space and dimension of nullspace of A is m.

E. If the columns of a matrix A are linearly independent, then nullity of A is 0.

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$Q3$. Consider the matrix $A = \begin{bmatrix} x & -1 & 2 \\ y & 1 & 2 \\ z & 3 & 4 \end{bmatrix}$.

Let $T : \mathbb{R}^3 \to \mathbb{R}$ be a linear transformation such that $T(x, y, z) = det(A)$. Then which of the following options is/are true?

Options:

A. T is injective.
B. T is surjective.
C. T is an isomorphism.
D. T is neither injective not surjective.

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$Q4$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by
$T(x, y, z) = (x + 2y, z - 3y, x - y + z)$.
Based on the above data, answer the given subquestions.

Q1) Find the nullity of T.

Q2) Find the rank of T.

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$Q5$. Which of the following options is/are true?

Options:

A. If a system of linear equations $Ax = b$ has a solution, then b is a linear combination of the columns of the coefficient matrix A

B. Consider a system of linear equations $Ax = b$. If b is a linear combination of the columns of the coefficeint matrix A, then the system $Ax = b$ has no solution

C. If the number of variables is more than the number of equations in a system of linear equation $Ax = 0$, then it has unique solution.

D. The kernel of the linear transformation $S : \mathbb{R}^3 \to \mathbb{R}^3$ such that $S(v) = Av^T$, where $v = (x, y, z)$, is the solution space of the system of linear equation $Av^T = 0$

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$Q6$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x, y) = (x + y, x - y)$. Choose the correct option(s) about T

Options:

A. T is one-one
B. T is onto
C. The range of T is a one-dimensional subspace of $\mathbb{R}^3$
D. The nullity of T is 1.

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$Q7$. Let U denote the set of all 2×2 upper triangular matrices. Consider an ordered basis $\beta = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$. Let $T : U \to \mathbb{R}^2$ be a linear transfromation defined as $T\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = (a+b, c)$.

Which of the following is/are true about T ?

Options:

A. Rank of T is 3.

B. T is onto.

C. Nullity of T is 1.

D.T is one-one.

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$Q8$. Consider the system of linear equations formed by the equations in column B of Table M2ES3 and let A be its coefficient matrix. Answer the releated subquestions

	Equation of the surface		Equation of the tangent plane at $(1,1,2)$		Vector subspace corresponding to the affine subspace formed by tangent plane
$i)$	$z = x^2 + y^2$	$a)$	$3x + 3y + 2z = 10$	1)	$\left\{(x,y,z) \mid x + y = \frac{z}{2},\ x,y,z \in \mathbb{R}\right\}$
$ii)$	$x^2 + y^2 + z^2 = 6$	$b)$	$z = 2x + 2y - 2$	2)	$\left\{(x,y,z) \mid x + y = -\frac{2}{5}z,\ x,y,z \in \mathbb{R}\right\}$

$iii)$	$xy + yz + zx = 5$	$c)$	$x + y + 2z = 6$	3)	$\{(x, y, z) \mid 2z = -x - y\ ,\ x, y, z \in \mathbb{R}\}$

Sub Questions:

Q1) Which of the following denotes the solution space of this system of linear equations?
Options:

A. $\{(x, y, z) \mid x + y = 0,\ x, y \in \mathbb{R}\}$
B. $\{(x, 2 - x, 2) \mid x \in \mathbb{R}\}$
C. $\{(x, 2, 2) \mid x \in \mathbb{R}\}$
D. $\{(2, 0, 2)\}$

Q2) Which of the following sets denotes $nullspace(A)$?

Options:

A. $\{(x, y, 0) \mid x - y = o,\ x, y, \in \mathbb{R}\}$
B. $\{(x, -x, 0) \mid x \in \mathbb{R}\}$
C. $\{(x, 0, 0) \mid x \in \mathbb{R}\}$
D. $\{(0, 0, 0)\}$

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$Q9$. Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear trasnformation defined as follows:

$$T(x, y, z) = (x + z, z - y, x - y + 2z)$$

Based on the above data, answer the given subquestions:

Sub Questions:

Q1) Find the nullity of T

Q2) Find the rank of T

Q3) Choose the correct option

Options:

A. T is an isomorphism
B. T is one-one but not onto
C. T is onto but not one-one
D. T is neither one-one nor onto