Sept 2021

$Q1$. Consider the vector space V of all upper triangular matrices of order 2 with respect to the usual addition and sclar multiplication in matrices. Let $S : V \to V$ be a linear transformation defined by $S(A) = -2A$. Let $\beta = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ be an ordered basis of V and B be the matrix representation of S with respect to the ordered basis β for both domain and codomain. Use this information to answer the given subquestions

Sub Questions:

Q1) Which of the following options are true?

Options:
A. Order of matrix B is 3×3
B. Order of matrix B is 2×2
C. Order of matrix B is 3×1
D. B is a scalar matrix
E. B is a symmetric matrix

Q2) Find the rank of S.

Q3) Find the nullity of the matrix B.

Jan 2022

$Q2$.Supposw $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation defiend by

$$T(x, y, z) = (x + y + z, x + 2y, y + z)$$

Let A denote the matrix representation of T with respect to ordered bases $\beta = \{(1,1,0),(2,3,4),(1,-1,1)\}$ for the domain and $\gamma = \{(0,1,0),(0,0,1),(1,0,0)\}$ for the co-domain. Then the sum of the elements in the first row of A is

May 2022

$Q3$. Consider the matrix $A = \begin{bmatrix} x & -1 & 2 \\ y & 1 & 2 \\ z & 3 & 4 \end{bmatrix}$.

Let $T : \mathbb{R}^3 \to \mathbb{R}$ be a linear transformation such that $T(x, y, z) = det(A)$. Let B be the matrix representation of T with respect to some ordered bases for the domain and co-domain. Let $m \times n$ be the order of the matrix B and r be the nullity of B. Then find the value of $m + n + r$.

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$Q4$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by
$T(x, y, z) = (x + 2y, z - 3y, x - y + z)$.
Choose the correct option(s).

Options:

A. $\{(-4, 2, 6)\}$ is a basis for the kernel of T

B. $\{(0, 1, 3),\ (-2, 1, 0)\}$ is a basis for the kernel of T

C. There exists an isomorphism from the range of T to $\mathbb{R}^2$

D. There exists an isomorphism from the range of T to $\mathbb{R}$

Sept 2022

$Q5.$ Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x, y) = (x + y, x - y)$. Choose the correct option(s) about T.

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the matrix representation of T with respect to a basis $\{(1, 1), (1, 2)\}$ for the domain and $\{(1, 1),\ (1, -1)\}$ for the co-domain, then what is $a + d$?

Jan 2023

$Q6$. Let U denote the set of all 2×2 upper triangular matrices. Consider an ordered basis $\beta = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$. Let $T : U \to \mathbb{R}^2$ be a linear transfromation defined as $T\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = (a + b, c)$. Let matrix A be the ,atrix representation of T with respect to the ordered basis β for U and the standard basis for the co-domain $\mathbb{R}^2$.

Which of the following matrix is A?

Options:

A. $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

B. $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

C. $A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$

D. $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$

May 2023

$Q7$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ and $A = \begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ be the matrix corresponding to the linear transformation T with respect to the bases $\beta = \{(1,0,0),(0,1,0),(0,0,1)\}$ and $\gamma = \{(1,1,0),\ (0,1,1),(1,0,1)\}$. Using the above information answer the given subquestions

Sub Questions:

Q1) Choose the appropriate linear transformation T for the matrix A.

Options:

A. $T(x,y,z) = (x+y, y+z, z-x)$

B. $T(x,y,z) = (x, y-z, z+x)$

C. $T(x,y,z) = (x+y, y-z, z+x)$

D. $T(x,y,z) = (x+y, -z, z+x)$

Q2) Let B be the matrix corresponding to the linear transformation T with respect to the basis $\{(1,\ 0,\ 0),\ (0,\ 1,\ 0),\ (0,\ 0,\ 1)\}$ for both domain and codomain.

Which of the following statement(s) is/are true?

Options:

A. The column space of B is spanned by $\{(1,0,1),(1,1,0)\}$

B. The nullspace of is spanned by $\{(-1,0,0),(0,1,1)\}$

C. $rank(B) = 2$ and $nullity(B) = 1$

D. $rank(B) = 1$ and $nullity(B) = 2$

Q3) Let B be the matrix corresponding to the linear transformation T with respect to the basis $\{(1,\ 0,\ 0),\ (0,\ 1,\ 0),\ (0,\ 0,\ 1)\}$ for both domain and codomain.

The trace of B is

Q4) Let B be the matrix corresponding to the linear transformation T with respect to the basis $\{(1,\ 0,\ 0),\ (0,\ 1,\ 0),\ (0,\ 0,\ 1)\}$ for both domain and codomain.

The determinant of B is

Sept 2023

$Q8$. Suppose $T:\mathbb{R}^3 \to \mathbb{R}^3$ is an isomorphism. Let A be the matrix representation of T with respect to the standard ordered basis $\mathcal{B} = \{e_1, e_2, e_3\}$ for both domain and co-domain and B be the matrix representation of T with respect to the basis $\mathcal{B}' = \{e_1, e_1 - e_2, e_2 - e_3\}$ for both domain and co-domain

Sub Questions:

Q1) Find $Rank(A)$

Q2) Find the dimension of the solution space of the homogeneous equation $Bx = 0$