Sept 2021

$Q1$. Let $v_1 = (1, 0, 3)$, $v_2 = (4, 2, 14)$, $v_3 = (3, 6, 10)$ and $u = (3, -36, 3)$. If $u = av_1 + bv_2 + cv_3$, then find the value of $a - b + c$.

Jan 2022

$Q2$. Consider the matrix

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Which of the following options are true?

Options:

A. $det(A) \neq 0$

B, A is invertible

C. The columns of matrix A are linearly independent

D. $A^{-1} = A^T$

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$Q3$. Consider the set V together with the operations addition and scalar multipliaction defined as follows:

$$V = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$$

$$\text{Addition}: \ (x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2); \\ (x_1, y_1, z_1), (x_2, y_2, z_2) \in V$$

Scalar Multiplication:

$$c(x, y, z) = \begin{cases} (0, 0, 0) & c = 0 \\ \left(cx, cy, \frac{z}{c}\right) & c \neq 0 \end{cases} \quad (x, y, z) \in V, \ c \in \mathbb{R}$$

Consider the following statements

1. There exists an element 0 (called the zero vector of V) in such that $0 + v = v$, $\forall v \in V$.
2. For each v in V, there exists an element v', such that $v' + v = v + v' = 0$
3. For each $v \in V$, $1v = v$
4. For each $v \in V$ and for each pair $a, b \in \mathbb{R}$, $(a + b)v = av + bv$

5. For each $a \in \mathbb{R}$ and for each pair $v_1, v_2 \in V,\ a(v_1 + v_2) = av_1 + av_2$
6. For each $v \in V$ and for each pair $a, b \in \mathbb{R}$, $(ab)v = a(bv)$

Choose the correct statement from the above

Options:

A. 1 is correct
B. 2 is correct
C. 3 is correct
D. 4 is correct
E. 5 is correct
F. 6 is correct

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$Q4$. Consider the following sets with the given operations defined as follows:

$W_1 = \{(x, y, z) \mid x, y, z \in \mathbb{R},\ x + y + z = 1\}$
Addition: $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2);$
$(x_1, y_1, z_1), (x_2, y_2, z_2) \in W_1$
Scalar Multiplication: $c(x, y, z) = (cx, cy, cz);\ (x, y, z) \in W_1,\ c \in \mathbb{R}$

$W_2 = \{(x, y, z) \mid x, y, z \in \mathbb{R},\ x = y - z\}$
Addition: $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2);$
$(x_1, y_1, z_1), (x_2, y_2, z_2) \in W_2$
Scalar Multiplication: $c(x, y, z) = (cx, cy, cz);\ (x, y, z) \in W_2,\ c \in \mathbb{R}$

Based on the above data, answer the given subquestions.
Sub questions:

Q1) Which of the following options are true for W_1?

Options:
A. There does not exist any zero element of W_1

B. W_1 is closed under addition

C. W_1 is a subset of $\mathbb{R}^3$

D. W_1 is not a vector subspace of $\mathbb{R}^3$

Q2) Which of the following options are true for W_2?

Options:

A. There does not exist any zero element of W_2

B. W_2 is closed ubder addition

C. W_2 is closed under scalar multiplication

D. W_2 is a subset of $\mathbb{R}^3$

E. W_2 is a vector subspace of $\mathbb{R}^3$

May 2022

$Q5$. If addition and scalar multiplication on $V = \mathbb{R}^2$ is defined as follows:

Addition: $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2);$
$(x_1, y_1),\ (x_2, y_2) \in V$

Scalar Multiplication: $c(x, y) = (x, cy);\ (x, y) \in V,\ c \in \mathbb{R}$

Consider the following statements

1. There exists an element 0 (called the zero vector of V) in V such that $0 + v = v$, $\forall\ v \in V$
2. There exists a vector v' in V such that $v' + v = v + v' = 0$, $\forall\ v \in V$
3. For each vector $v \in V$, $1v = v$
4. For each vector of $v \in V$ and for each pair $a, b \in \mathbb{R}$, $(a + b)v = av + bv$
5. For each vector of $a \in \mathbb{R}$ and for each pair $v_1, v_2 \in V$, $a(v_1 + v_2) = av_1 + av_2$
6. For each vector of $v \in V$ and for each pair $a, b \in \mathbb{R}$, $(ab)v = a(bv)$

Which of the above statements is not true with respect to the addition and scalar multiplication on $V = \mathbb{R}^2$ defined above? (Enter the serial number of statement which is not true. If statement 2 is not correct then enter 2 as your answer)

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$Q6$. Which of the following subsets are not a vector subspace of $\mathbb{R}^3$ with respect to usual addition and scalar multiplication?

1. $W = \left\{(x, y, z) \mid x, y, z \in \mathbb{R} \text{ and } x^2 = z^2\right\}$
2. $W = \left\{(x, y, z) \mid x, y, z \in \mathbb{R} \text{ and } x = z\right\}$
3. $W = \left\{(x, y, z) \mid x, y, z \in \mathbb{R} \text{ and } x = y + z\right\}$
4. $W = \left\{(x, y, z) \mid x, y, z \in \mathbb{R} \text{ and } (x + 1) + (y - 1) + z = 0\right\}$

Enter the serial number of the subsets which is not a vector subspace of $\mathbb{R}^3$ with respect to usual addition and scalar multiplication. (If the subset corresponding to serial number 2 is not a subspace, then enter 2 as your answer)

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$Q7$. If S is a (non-empty) linearly independent subset of $\mathbb{R}^3$ (with respect to usual addition and scalar multiplication), then what can be the maximum possible cardinality of S?

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$Q8$. If S is a (non-empty) linearly independent subset of $\mathbb{R}^3$ (with respect to usual addition and scalar multiplication), then what can be the minimum possible cardinality of S?

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$Q9$. Choose the set of correct options.

A. If v and w are linearly independent, then $v + w$ and w are also linearly independent.

B. If $A^n = 0$ for some 2×2 matrix A and some non-zero natural number n, then A must be 0 (zero matrix of order 2)

C. If a homogeneous system of linear equation $Ax = 0$ has some non-zero solution, then it must have infinite number of solutions

D. If $A^3 = I$ for some $n \times n$ matrix A, then it is not necessary that A^3 is an invertible matrix.

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$Q10$. From the list of given terms find out the best possible options for each of the given suquestions:

1) Transpose of a matrix
2) Determinant
3) Closure with respect to addition
4) Closure with respect to scalar multiplication
5) Existence of addititve inverse
6) Commutatuive of addition
7) Associativity of addition
8) Spanning set
9) Linearly independent set
10) Basis

The conditions that need to be checked to identify a subspace W of a vector space V : ________________. (Enter 2 best possible options. Enter only the serial numbers of those options in increasing order without adding any comma or space in between them.) [Suppose your answer is 7 ans 8, then you should enter 78]

Sept 2022

$Q11$. If addition and scalar multiplication on $V = \mathbb{R}^2$ is defined as follows:

Addition: $(x_1, y_1) + (x_2, y_2) = (0, 0)$;
$(x_1, y_1), (x_2, y_2) \in V$
Scalar Multiplication: $c(x, y) = (0, 0);\ (x, y) \in V,\ c \in \mathbb{R}$

Consider the following statements

1. There exists an element 0 (called the zero vector of V) in V such that $0 + v = v,\ \forall\ v \in V$.
2. For each vector of $v \in V$ and for each pair $a, b \in \mathbb{R}$, $(a + b)v = av + bv$
3. For each vector of $a \in \mathbb{R}$ and for each pair $v_1, v_2 \in V,\ a(v_1 + v_2) = av_1 + av_2$
4. For each vector of $v \in V$ and for each pair $a, b \in \mathbb{R}$, $(ab)v = a(bv)$

Which of the above statements is not true with respect to the addition and scalar multiplication on $V = \mathbb{R}^2$ defined above? (Enter the serial number of the statement which is not true. If statement 2 is incorrect, then enter 2 as your answer.)

Jan 2023

$Q12$. For some fixed real number $\alpha \neq 0$, define the set $V_\alpha = \{(x, \alpha, y) \mid x, y \in \mathbb{R}\}$ along with the following operations:

Addiiton: $(x_1, \alpha, y_1) + (x_2, \alpha, y_2) = (x_1 + x_2, \alpha, y_1 + y_2)$
$(x_1, \alpha, y_1), (x_2, \alpha, y_2) \in V_\alpha$
Scalar Multiplication: $c(x, \alpha, y) = (cx, \alpha, cy);\ (x, \alpha, y) \in V,\ c \in \mathbb{R}$

Answer the subquestions with respect to the given information.

Sub Questions:

Q1) Which of the following is(are) true?
Options:

A. V_α is closed under the given addition but not closed uder the given scalar multiplication

B. . V_α is not closed under the given addition but closed uder the given scalar multiplication

C.. V_α is neither closed under the given addition but nor closed uder the given scalar multiplication

D. . V_α is closed under the given addition and also closed uder the given scalar multiplication

Q2) Which of the following is(are) correct?

Options:

A. V_α has no zero element with respect to the given addition

B. $(0, 0, 0)$ is the zero element of V_α with respect to the given addition

C.$(0, \alpha, 0)$ is the zero element of V_α with respect to the given addition

D. (α, α, α) is the zero element of V_α with respect to the given addition

Q3) Consider the element $v = (2, \alpha, 0)$. Which of the following options is(are) correct?

Options:

A. v is not an element of V_α

B. v has no inverse element with respect to the given addition in V_α

C. $(-1)v$ is the inverse element of V_α

D. $(-2, \alpha, 0)$ is the inverse element of V_α

Q4) Choose the correct option(s).

Options:

A. For each element of $v \in V_\alpha$ and for each pair $a, b \in \mathbb{R}$, $(a + b)v = av + bv$

B. For each vector of $a \in \mathbb{R}$ and for each pair $v_1, v_2 \in V_\alpha$, $a(v_1 + v_2) = av_1 + av_2$

C. For any real number c, we always have $c(0, \alpha, 1) = (0, \alpha, 1)$

D. There does not exist any real number c such that $c(0, \alpha, 1) = (0, \alpha, 1)$

May 2023

$Q13$. Which of the following subsets of $\mathbb{R}^2$ is/are vector spaces with respect to usual addition and usual scalar multiplication ?

Options:

A. $V_1 = \{(x, y) : 2x + 3y = 0\}$

B. $V_2 = \left\{(x, y) \ : \ y^2 = 0, \ x = 2y\right\}$

C. $V_3 = \{(x, y) \ : \ x = 1\}$

D. $V_4 = \{(x, y) \ : \ 2x + 3y - 1 = 0, \ x - y = 0\}$

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$Q14$. Select the true statement(s).

Options :
A. Any subset of a linearly independent set is a linearly independent set.
B. Any superset of a spanning set is a spanning set.
C. Any subset of a basis is a basis.
D. Any superset of a subspace is a subspace.

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$Q15$. Consider the set $W = \left\{A \in M_n(\mathbb{R}) : det\left(A^T\right) = 0\right\}$ with usual addition and usual scalar multiplication of matrices. Which of the following is/are ture?

Options :
A. W is closed under addition.
B. W is closed under scalar multiplication.
C. W is a vector space.
D. W is not a vector space.

Sept 2023

$Q16$. Let $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -6 & 3 & 4 \\ 7 & 2 & -1 \\ 2 & 4 & 2 \end{pmatrix}$.

Let $v_1,\ v_2$ and v_3 be the column vectors of A and let u_1, u_2 and u_3 be the column vectors of B.
Choose the correct option(s) from the following statements.

Options:

A. The vectors $v_1,\ v_2$ and v_3 lie on a plane

B. The vectors u_1, u_2 and u_3 lie on a plane

C. The vectors u_1, u_2 and u_3 are linearly independent

D. The system $Ax = 0$ has a unique solution

E. The system $Bx = 0$ has a unique solution

F. The vector u_3 is a linear combination of u_1 and u_3

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$Q17$. Unless otherwise stated, assume that we consider the usual vector addition and scalar
multiplication. Choose the correct option(s) from the following statements.

Options:

A. The set of vectors $\left\{(a, b) \in \mathbb{R}^2\right\}$ with scalar multiplication defined by $k(a, b) = (0, kb)$ forms a vector space.

B. The set of real numbers with addition defined by $x + y := x - y$ forms a vector space

C. Let $A \in \mathbb{R}^{3\times3}$ be an invertible matrix. The set of all solutions of homogeneous system $AX = 0$ is a vector space of dimension 0

D. Any vector subspace of $\mathbb{R}^2$ with dimenison 1 is of the form $ax + by = 0$ where $a \neq 0$ or $b \neq 0$

E. Any vector subspace of $\mathbb{R}^3$ with dimension 1 is of the form $ax + by = c$ where $a \neq 0$ or $b \neq 0$

F. $V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + b = c + d \right\}$ forms a vector space

G. The set of all $n \times n$ matrices with rank strictly less than n does not form a vector space.

Jan 2024

$Q18$. Let $V = \{(x, y, 5) : \ x, y \in \mathbb{R}\}$. Let us define addition and scalar multiplication as follows:

$Addition : (x_1,\ y_1,\ 5)\ +\ (x_2, y_2, 5) = (x_1 + x_2,\ y_1 + y_2,\ 5);\ (x_1,\ y_1,\ 5), (x_2,\ y_2, 5) \in V$
$Scalar\ multiplication :\ c(x,\ y,\ 5) = (cx,\ cy,\ 5);\ (x,\ y,\ 5)\ \in V,\ c \in \mathbb{R}$

Answer the following questions with respect to the given information.

Sub Questions:

Q1) Is the set V closed under addition?

Options:

A. Yes
B. No

Q2) Is the set V closed under scalar multiplication?

Options:

A. Yes
B. No

Q3) Which of the following is(are) correct?

Options:

A. V has no zero element with respect to the given addition.

B. $(0, 0, 0)$ is the zero element with respect to the given addition.

C. $(0, 0, 5)$ is the zero element with respect to the given addition.

D. For any real number c, we always have $c(0, 1, 5) = (0, 1, 5)$

E. For each element of $v \in V$ and for each pair $a, b \in \mathbb{R}$, $(a + b)v = av + bv$

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$Q19$. Let $u = (1, 2, -1)^T,\ v = (2, 1, 0)^T$ and $w = (-1,\ 4,\ -3)^T$. Let $A \in M_{3\times3}(\mathbb{R})$ such that $Au = u$ and $Av = -v$. If $A^3 w = (a, b, c)^T$, then

Sub Questions:

Q1) a is equal to

Q2) b is equal to

Q3) c is equal to