

**Jan 2022**

Q1. Choose the correct statements for the function

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

**Options:**

A.  $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$

B.  $f$  is not continuous at  $(0, 0)$

C. The directional derivative of  $f$  at  $(0, 0)$  in the direction of  $(1, 1)$  is  $\frac{1}{\sqrt{2}}$

D. The directional derivative of  $f$  at  $(1, 0)$  in the direction of  $(1, 1)$  does not exist.

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Q2. Which of the following options is/are true?

$$f(x, y) = \frac{xy^3}{x^2 + y^6}$$

**Options:**

A.  $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$

B.  $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = \frac{1}{2}$

C.  $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 1$

D.  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  does not exist

E.  $f(x, y)$  is not continuous at  $(0, 0)$

F.  $f(x, y)$  is continuous at  $(0, 0)$

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Q3. The price of product ( $f(x, y)$ ) depends on the price ( $x$ ) of the raw materials and the price ( $y$ ) of transportation of the product the market according to

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$f(x, y) = \begin{cases} x^3 - xy^2 & \text{if } x \neq y \\ x + y & \text{otherwise} \end{cases}$$

Answer the given subquestions using the information above

**Sub Questions:**

**Q1)** What will be the ratio of the price of the raw materials and the price of the transportation ( $x : y$ ) when  $y > x$ , if the rate of change of the price of the product with respect to the price of the raw materials is 0?

(In this context  $x$  and  $y$  both are always positive)

**Options:**

A. 1 : 1

B. 1 : 3

C. 1 :  $\sqrt{3}$

D. The ratio cannot be determined using the given information

**Q2)** Which of the following statements are true?

**Options:**

A.  $f(x, y)$  is a linear function in its domain

B.  $f(cx, cy) = c^3 f(x, y)$  for any real number  $c$ , if  $x \neq y$

C.  $f(x, y)$  is continuous at  $(0, 0)$

D. If the price of raw material and transportation of the product approaches 3 and 2 respectively, then the price of the product approaches 15

E. If the price of raw material and transportation of the product approaches 3 and 2 respectively, then the price of the product approaches 5

**Q3)** If the rate of change of price of the product along the direction of the vector  $(1, 1)$  is  $\frac{1}{\sqrt{2}}[ka^2 + lab + mb^2]$ , when the price of raw material is  $a$  and the price of transportation of the product to the market is  $b$  (where  $a \neq b$ ), then find the value of  $k - l + m$

**Q4)** Which of the following is true for the function:

$$f(x, y) = \frac{x^3 + y^3}{x + y}$$

**Options:**

A.  $\lim_{(x,y) \rightarrow (1,-1)} f(x,y) = 0$

B.  $\lim_{(x,y) \rightarrow (1,-1)} f(x,y) = 3$

C.  $\lim_{(x,y) \rightarrow (1,-1)} f(x,y) = 2$

D.  $\lim_{(x,y) \rightarrow (1,-1)} f(x,y)$  does not exist.

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Q4.

$$u(x, y) = x^2 + e^y$$

and

$$v(x, y) = y^2 + ye^x$$

Consider a function  $g(x, y)$  such that

$$g(x, y) = \begin{cases} u(x, y) & x = y \\ v(x, y) & x \neq y \end{cases}$$

Which of the following option is/are true?

**Options:**

A.  $g(x, y)$  is continuous at origin

B.  $g_{xy}(1, 2) = e$

C.  $g_{xy}(x, y)$  is not continuous at  $(1, 2)$

D.  $g_y(1, 1) = 2 + e$

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Q5. Choose the correct statements for the function

$$f(x, y) = \begin{cases} \frac{x^4 y^2}{x^4 + y^4} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

**Options:**

A.  $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 1$

B.  $f_x(0, 0) = 0$

C. The directional derivative of  $f$  at  $(0, 0)$  in the direction of  $(-1, 1)$  exists.

D.  $f$  is not continuous at  $(0, 0)$

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Q6. Let  $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{Otherwise} \end{cases}$ . Then which of the following statement(s)

is/are true?

**Options :**

- A. The partial derivatives for  $f$  exist everywhere but directional derivatives need not exist at the origin.
- B. All directional derivatives at the origin exist for the function  $f$ .
- C. At  $(0, 0)$ , the directional derivative of  $f$  exists in the direction of  $x$  - axis.
- D. The partial derivatives for  $f$  do not exist at the origin.

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Q7. Let  $f(x, y, z) = x^2y^3 - 3xz$  and  $u = (1, 2, 2)$ . Find the directional derivative of  $f$  in the direction of the vector  $u$  at the point  $(0, 1, -1)$

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Q8. Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  be a function given by

$$f(x, y) = \begin{cases} \frac{(x+y)^2}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Choose the correct option(s) from the following:

**Options:**

- A.  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  exists along the positive  $x$  – axis
- B.  $f(x, y)$  approaches 0 when  $(x, y)$  approaches  $(0, 0)$  along the positive  $y$  – axis
- C.  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  does not exist
- D.  $f_x(0, 0) = 1$
- E. The directional derivative of  $f$  at  $(0, 0)$  in the direction of positive  $x$  – axis does not exist
- F. The directional derivative of  $f$  at  $(0, 0)$  in the direction of positive  $y$  – axis exists

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Q9. Let  $f(x, y) = \begin{cases} \frac{\cos y \sin x}{x} & x \neq 0 \\ \cos y & x = 0 \end{cases}$

Based on the above data, answer the given subquestions

**Sub Questions:**

**Q1)** Find  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$

**Q2)** Is the function continuous at  $(0, 0)$ ? [If the answer is yes, enter 1 and if the answer is no, enter 0]

**Q3)** At how many points does  $f$  have a discontinuity ?

