Sept 2021

$Q1$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined as $T(x_1, x_2, x_3) = (6x_2, 4x_3, x_1)$.
Consider two ordered bases $\beta = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and
$\gamma = \{(0, 0, 1), (1, 0, 0), (0, 1, 0)\}$ of $\mathbb{R}^3$. Consider the following two matrices:

$$A = \begin{bmatrix} 0 & 6 & 0 \\ 0 & 0 & 4 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 & 6 \\ 4 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Determine whether the statements are true or false in the given subquestions:

Sub Questions:

Q1) Statement 1: A is the matrix representation of T with respect to β for both the domain and codomain.

Options:
A. True
B. False

Q2) Statement 2: B is the matrix representation of with respect to γ for the domain and β for the codomain.

Options:
A. True
B.False

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$Q2$. Let W be the subspace of $\mathbb{R}^3$ spanned by the vectors $\{(7, -7, 0),\ (0, 0, 4),\ (1, -1, 1)\}$.
Let T be a linear transformation from W to $\mathbb{R}^2$ defined as $T(x, y, z) = (x + y, ky - z)$, for all $(x, y, z) \in W$. Answer the given subquestions depending on this given data

Sub Questions:

Q1) Determine whether the given statement is true or false:
If $k = 0$, then $(7, -7, 0)$ is not in $ker(T)$

Options:

A. True
B. False

Q2) Determine whether the given statement is true or false:
There exist real numbers α and β such that $\alpha(7, -7, 0) + \beta(0, 0, 4)$ is in $ker(T)$.

Options:
A. True
B. False

Q3) Choose the correct options.

Options:
A. T is not one-one for any value of k
B. T is one-one when $k = 0$
C. $rank(T) = 1$ for all values of k
D. $nullity(T) = 2$ for some k

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$Q3$. Suppose two publication houses (publication house A and publication house B) have organized a sale of their books. Both of them publish three types of books: novels, poetry collections and collections of short stories. The selling price (in (hundreds) ₹) of these three types of books in publication houses A and B are given as follows:

	Novels	Poetry Collections	Collection of Short Stories
Publication house A	3	4	7
Publication house B	6	12	14

The publication houses announced that in order to avail these special sale prices, customers have to buy equal number of novels, equal number of poetry collection, and equal number of collection of short stories from each of the publication houses (i.e., if a customer buys x

number of novels, y number of poetry collections and z number of collection of short stories from Publication house A; then they have to buy exactly x number of novels, y number of poetry collections and z number of collection of short stories from Publication house B, to avail the benefit of the sale). So there is a map taking the tuple consisting of the number of books of each type bought (Novels, Poetry collections, Collection of short stories) to the prices paid by customers who availed the sale to each of the publication houses, which yields a linear transformation (T) from $\mathbb{R}^3$ to $\mathbb{R}^2$ (where the first and second co-ordinates of the image denotes the prices paid to publication house A and publication house B, respectively). Answer the given subquestions using the above information

Sub Quesions:

Q1) If A is the matrix representation of T with respect to the basis $\{(1,0,0),\ (0,1,0),\ (0,0,1)\}$ for $\mathbb{R}^3$ and to the basis $\{(1,0),(0,1)\}$ for $\mathbb{R}^2$, then A is

Options:

A. $\begin{bmatrix} 3 & 6 \\ 4 & 12 \\ 7 & 14 \end{bmatrix}$

B. $\begin{bmatrix} 3 & 4 & 7 \\ 6 & 12 & 14 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q2) Find the rank of T.

Q3) If $\{(l,m,n)\}$ is a basis of $ket(T)$, then find the value of m if $n = 2$

Q4) If A is the matrix representation of T with respect to the basis

$\{(1,0,1),(0,1,1),(1,1,0)\}$ for $\mathbb{R}^3$ and to the basis $\{(1,0),(0,1)\}$ for $\mathbb{R}^2$, then find the value of the sum of the elements of the bottom most row of the matrix A.

Jan 2022

$Q4$. Anamila, Subhasis and Shreya pool together x, y and z amounts of money (in thousands) respectively, every month. The sum is distributed across three accounts $A_1,\ A_2$ and A_3 as $x+y+z,\ z-2y$ and $2y-z$ respectively. This can be thought of as a linear transformation

$$T:\ \mathbb{R}^3 \to \mathbb{R}^3$$

defined by

$$T(x,y,z) = (x+y+z, z-2y, 2y-z)$$

Note: A negative amount of money signifies the amount withdrawn from the accounts. Answer the subquestions from the information given above.

Which of the following matrices is the matrix representation of T with respect to ordered basis $\{(1,0,0),(1,1,0),(1,1,1)\}$ of the domain and standard ordered basis for $\mathbb{R}^3$ for co-domain?

Options:

A. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 2 & -1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & -2 & 2 \\ 3 & -1 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{bmatrix}$

May 2022

$Q5$. Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ defined as $T(x, y, z) = (x + y - z, y + z)$. Let A be the matrix representation of T with respect to the basis $\beta = \{(1, 1, 0), (0, 1, 1), (1, 0, 1)\}$ for the domain and the basis $\gamma = \{(1, 1), (1, 0)\}$ for the co-domain.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

Let $S = \{(x, y, z) \mid x = mz,\ y = nz\ ;\ x, y, z \in \mathbb{R}\}$ be the null space of the T. Answer the subquestions based on the given data

Sub Questions:

Q1) What is the value of $d - a$?

Q2) What is the value of $e - b$?

Q3) What is the value of $f - c$?

Q4) What is the value of m?

Q5) What is the value of n ?

Q6) Find out the nullity of T.

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$Q6$. Suppose two publications houses (publication house A and publication house B) have organized a sale of their books. Both of them publish three types of books, novels, poetry collection and collection of short stories. The selling price (in hundreds ₹) of these three types of books in publication house A and B are given as follows:

	Novels	Poetry Collection	Collection of Short Stories
Publication house A	1	2	5
Publication house B	3	3	3

Table: Q2M2T1

The publication houses announced that in order to avail these special sale prices, customers have to buy equal number of novels, equal number number of poetry collection and equal number of collection of short stories from each of the publication houses (i.e. if a customer buys x number of novels, y number of poetry collection and z number of short stories from Publication house A; then they have to buy exactly x number of novels, y number of poetry collections and z number of collection of short stories from Publication house B, to avail the benefitof the sale). So there is a map taking the tuple consisting of number of books of each type bought (Novels, Poetry collections, Collection of short stories) to the prices paid by customers who availed the sale to each of the publication houses, which yields a linear transformation (T) from $\mathbb{R}^3$ to $\mathbb{R}^2$ (where the first and second co-ordinates of the image denotes the prices paid to publication house A and publication house B, respectively). Answer the subquestions using the above information.

Q1) If A is the matrix representation of T with respect to the basis $\{(1,0,0),(0,1,0),(0,0,1)\}$ for $\mathbb{R}^3$ and to the basis $\{(1,0),(0,1)\}$ for $\mathbb{R}^2$, then A is

A. $\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 5 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 2 & 5 \\ 3 & 3 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 3 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Q2) We apply the sequence of row operations on A as follows:

- **Step 1:** $R_2 - 3R_1$
- **Step 2:** $-\frac{1}{3}R_2$
- **Step 3:** $R_1 - 2R_2$

Applying this row operation in the given order, the matrix B is derived. Let

$$B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

What is the value of a?

Q3) We apply the sequence of row operations on A as follows:

- **Step 1:** $R_2 - 3R_1$
- **Step 2:** $-\frac{1}{3}R_2$
- **Step 3:** $R_1 - 2R_2$

Applying this row operation in the given order, the matrix B is derived. Let

$$B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

What is the value of d?

Q4) We apply the sequence of row operations on A as follows:

- **Step 1:** $R_2 - 3R_1$
- **Step 2:** $-\frac{1}{3}R_2$
- **Step 3:** $R_1 - 2R_2$

Applying this row operation in the given order, the matrix B is derived. Let

$$B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

What is the value of e?

Q5) If $\{(l,\ m,\ n)\}$ is a basis of $ker(T)$, then find the value of l if n is 1.

Q6) If $\{(l,\ m,\ n)\}$ is a basis of $ker(T)$, then find the value of m if n is 1.

Sept 2022

$Q7$. Consider two linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(x, y, z) = (x + y, y + z)$ and $S: \mathbb{R}^2 \to \mathbb{R}^3$ such that $S(x, y) = (x, y, x + y)$. Let $\beta = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ be an ordered basis for $\mathbb{R}^3$ and $\gamma = \{(1, 0), (0, 1)\}$ be an ordered basis for $\mathbb{R}^2$. Answer the subquestions based on the given data.

Sub Questions:

Q1) If A is the matrix representation of $S \ o \ T$ (the transformation defined by $(S \ o \ T)(x, y, z) = S(T(x, y, z))$ with respect to the ordered basis β for both the domain and codomain and order of A is $m \times n$, then find the value of $m + n$.

Q2) If $K = \{(x, y, z) \mid ax + by = 0,\ cy + dz = 0\}$ is the null space of $S \ o \ T$, then find the value of $\left(\frac{a}{b}\right) - 2\left(\frac{c}{d}\right)$

Q3) Rank of $S \ o \ T$ is

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$Q8$. Consider a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ such that the matrix representation of T is $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 3 \end{bmatrix}$ with respect to the ordered bases $\beta = \{(1, 0, 0), (0, 1, 0), (1, 1, 1)\}$ and $\gamma = \{(1, 0),\ (1, 1)\}$ for the domain and codomain respectively. Answer the subquestions based on the given data

Sub Questions:

Q1: Nullity of the matrix A is

Q2) Which of the following option is true?

Options :

A. T is one-one.

B. T is onto.
C. T is an isomorphism.
D. T is neither one-one nor onto.

Q3) If $T(x, y, z) = (mx + ny + sz, px + qy + rz)$, then find the value of $(m + n + s) - 3(p + q + r)$

Jan 2023

$Q9$. Let $V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_{2\times2}(\mathbb{R}) \ : \ a + b = c + d \right\}$ and $T : V \to \mathbb{R}^2$ be a linear transformation. If T is onto, what is the dimension of the kernel of T ?

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$Q10$. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation defined by $T(x, y) = (x + y, x - y, 3x + y)$. If $A = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$ denote the matrix of with respect to $\{(1, 1),\ (1, -1)\}$ for $\mathbb{R}^2$ and $\{(1, 1, 1),\ (1, 1, 0),\ (-1, 0, 0)\}$ for $\mathbb{R}^3$, then what is $a + d + e$?

May 2023

$Q11$. Consider the vector space $V = \left\{ \begin{pmatrix} a & b \\ c & a \end{pmatrix} \mid c = a + b,\ a, b, c \in \mathbb{R} \right\}$ and $T : V \to \mathbb{R}^4$ defined by $T(A) = (a, b, c, a + b - c)$. Choose the correct option(s)

Options:

A. T is onto but not one-one

B. T is one-one but not onto

C. Nullspace of T is a 2 dimensional subspace of V

D. Range of T is a 2 dimensional subspace of $\mathbb{R}^4$.

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$Q12$. Let V_1 denote the vector space of solution of $AX = 0$, where $A = \begin{pmatrix} 2 & 1 & 4 \\ -1 & 1 & 0 \\ 1 & 2 & 4 \end{pmatrix}$ and $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$. Let V_2 denote the vector space of solutions of the system $BY = 0$, where $B = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 2 & 3 \end{pmatrix}$ and $Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$.

Let $S : V_1 \to V_2$ be a linear transformation. If $m \times n$ is the order of the matrix D of the linear transformation S with respect to some ordered basis α_1 for V_1 and an ordered basis α_2 for V_2, what is $2m - 3n$?

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$Q13$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by $T(x, y, z) = (x + y + z, x - y - z, x)$

If $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ denotes the matrix of T with respect to $\{(1,1,1),(1,1,0),(1,0,0)\}$ for domain and co-domain, then what is $2b + 2e + 2h$?

Sept 2023

$Q14$. Consider a vector space V with bases $\beta = \{v_1, v_2\}$ and $\gamma = \{v_1 + v_2, v_1 - v_2\}$. T is a linear transformation from V to itself such that $T(v_1) = v_1 + 2v_2$ and $T(v_2) = 2v_1 - v_2$

Based on the above data, answer the given subquestions.

Sub questions:

Q1) Find the matrix corresponding to T if γ is used as the basis for both the domain and co-domain

Options:

A. $\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

C. $\begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 1.5 & 0.5 \\ -0.5 & 1.5 \end{bmatrix}$

Q2) Is T an isomorphism?

Options:

A. Yes

B. No