May 2021

$Q1$. Choose the set of correct options.

Options:

A. For any non-zero inner product space V, there exists an orthonormal basis of V.

B. If u and v are two orthogonal vectors in $\mathbb{R}^2$ with respect to the usual inner product, then $T(u)$ and $T(v)$ must be orthogonal vectors fro any linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$.

C. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such tha $T = cI$ for some real number c (i.e., $T(v) = cI(v) = cv$ for all $v \in \mathbb{R}^2$), then the images of two non-zero orthogonal vectors must be orthogonal.

D. Let u and v be two vectors of an inner product space V. If $u + v$ and $u - v$ are orthogonal to each other, then $||u|| = ||v||$.

E. Let A be non-zero $m \times n$ matrix such that the vectors in $\mathbb{R}^m$ corresponding to each column of A are mutually orthogonal with respect to the usual inner dot product of $\mathbb{R}^m$. Then $AA^T = I$, where A^T denotes the transpose of A and I denotes the $m \times m$ identity matrix .

Sept 2021

$Q2$. Consider the given statements and answer the subquestions:

Q1) A and B aret two orthogonal matrices of order 4×4, then A and B are equivalent.

Options:
A. True
B. False

Q2) If A is an orthogonal matrix of order 2 with $Ax = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, then $x_1 + x_2$ always represents the sum of entries of the second row of A.

Options:
A. True
B. False

Jan 2022

$Q3$. Choose the set of correct options.

A. Consider the linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ such that $T(v_1) = w_1$ and $T(v_2) = w_2$. If w_1 and w_2 are linearly independent, then v_1 and v_2 are linearly independent.

B. Consider the linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ such that $T(v_1) = w_1$ and $T(v_2) = w_2$, where $\mathbb{R}^m$ and $\mathbb{R}^n$ are inner product spaces. If w_1 and w_2 are orthogonal, then v_1 and v_2 are orthogonal.

C. Let u and v be two vectors of an inner product space V. If $u + v$ and $u - v$ are orthogonal to each other, then $||u|| = ||v||$.

D. Let A be an $n \times n$ matrix. The null space of A is the orthogonal complement of the row space of A when both are viewed as subspaces of $\mathbb{R}^n$ with the usual inner product.

May 2022

$Q4$. Consider the matrix $A = \begin{bmatrix} x & -1 & 2 \\ y & 1 & 2 \\ z & 3 & 4 \end{bmatrix}$.

Let W be a subspace of $\mathbb{R}^3$ (with inner product as the dot product) such that $W = \{(x, y, z) \mid det(A) = 0\}$. If β is an orthonormal basis of W, then find the cardinality of the set β.

==

$Q5$. Consider $V = \mathbb{R}^3$ with the inner product as the the dot product and $W = \{(x, y, z) \mid x = 0,\ y = z\}$ is a subspace of V. Let $P_W : V \to W$ be a projection on W. Answer the given subquestions:

Q1) Find the dimension of the image space P_W.

Q2) Find the dimension of the null space of P_W.

Q3) If $v \in W$ is such that $||v|| = 2$, then find $||P_W(v)||$

Q4) Let A be the matrix representation of P_W with respect to some orthonomral bases β and γ for V and W, respectively. Then find the dimension of the null space A^2.

==

$Q6$. Let $A = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 1 & 2 \\ 1 & 4 & -1 \end{pmatrix}$. Let B be the matrix whose rows are obtained by normalizing the rows of A. Answer the given subquestions about matrix B

Sub Questions:

Q1) Choose the correct options about the matrix B.

Options:

A. A and B have the same reduced row echeleon form

B. $BB^T x = 0$ has infinitely many solutions

C. B is an orthogonal matrix

D, The columns of B are not orthonormal

Q2) What is $z_2 + 2z_3$, where $z = (z_1, z_2, z_3)^T$ is the solution of $B_z = (1, -2, 1)^T$ ontained using the Cramer's rule?

Sept 2022

$Q7$. Let A be a 3×3 orthogonal matrix. Which of the following options is/are true?

Options:

A. The rows of A form a basis for $\mathbb{R}^3$.

B. The columns of form an orthogonal set of vectors of the inner product space $\mathbb{R}^3$ w.r.t the dot product.

C. The rank of A is 2.

D. The system of linear equations $Ax = b$, where $b \neq 0$ has infinitely many solutions.

==

$Q8$. Consider $V = \mathbb{R}^3$ with inner product as the dot product and $W = \{(x, y, z) \mid x = y\}$ is a subspace of V. If (a, b, c) is the projection of $(1, 2, 3)$ onto W, then what is $a - b + c$?

Jan 2023

$Q9$. If A is a 3×3 orthogonal matrix with positive determinant, what is the determinant of $3A$?

==

$Q10$. Consider the subspace $W = \{(x, y, z, w) \mid x + y = z,\ z + w = x - y\}$ of $\mathbb{R}^4$. Let $\beta = \{(1, 0, 1, 0), (0, 1, 1, -2)\}$ be a basis of W. If γ is the orthonormal basis of W from β obtained using the GramSchmidt process with respect to the usual inner product, and (a, b, c, d) is the projection of $\left(0, \sqrt{\frac{11}{2}}, 0, \sqrt{\frac{11}{2}}\right)$ onto W, then what is $\sqrt{22}(a + b + c + d)$?

===

$Q11$. From the list of given terms find out the best possible options.

1) Rank 2
2) Non-zero nullity
3) Non -zero determinant
4) Closure with respect to addition and scalar multiplication
5) Existence of addititve inverse
6) Existence of additive inverse
7) Commutative of addition
8) Associativity of addition
9) Elements
10) Global maxima
11) Global minima
12) A critical points
13) Gradient exists
14) Directional derivative exist in any direction
15) Partial derivatives exist
16) Orthonormal columns
17) Standard ordered basis
18) Affine subspaces
19) Limit exists

Orthogonal matrix of order 3 has ______________ . (Enter 2 best possible options. Enter only the
serial numbers of those options in increasing order without adding any comma or space in between them.) [Suppose your answer is 7 and 17, then you should enter 717]

May 2023

$Q12$. Let $U = \{(x, y, z) \in \mathbb{R}^3 : x = y = z\}$ and $V = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$. Let P_U and P_V be the projection on the spaces U and V respectively. Which of the following statement(s) is/are true>

Options:

A. $\{(-1, 1, 0),\ (-1, -1, 2)\}$ is a basis for the range space of P_U

B. $\{(-1, 1, 0), (-1, -1, 2)\}$ is a basis for the null space of P_U

C. $\{(1, 1, 1)\}$ is a basis for the range space of P_V

D. $\{(1, 1, 1)\}$ is a basis for the null space of P_V

==

$Q13$. Let A be an orthogonal matrix. Then the sum of squares of every elements of every row is:

==

$Q14$. Let u, v be vectors in $\mathbb{R}^3$ such that $u + v$ and $u - v$ are orthogonal. If $u = (1, -2, 2)$ then $||v|| =$

Sept 2023

$Q15$. Let $W = \{(x, y, z) \in \mathbb{R}^3 : x - 2y = z\}$ be a subspace of the inner product space $\mathbb{R}^3$ with respect to the dot product.

Based on the above data, answer the given subquestions:

Sub Questions:

Q1) Find $dim(W^{\perp})$

Q2) If (a, b, c) is a vector in $W^{\perp}$, then $2a + b$ equals

Q3) Choose the correct option(s) from the following:

Options :

A. If P_W represents the projection from $\mathbb{R}^3$ to W, then $W^{\perp}$ is the nullspace of P_W.

B. The range of P_W is given by $span\{(1, 0, 1), (0, 1, 2)\}$.

C. If $v \in span\{(1, 0, 1), (0, 1, -2)\}$, then $(I - P_W)(v) = 0$ where I is the identity transformation on $\mathbb{R}^3$.

D. If S is a subspace of W, then $S^{\perp} \subseteq W^{\perp}$.