

Jan 2022

Q1. Let A be a 3×2 non-zero real matrix.

The maximum value of $\text{nullity}(A)$ is _____.

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Q2. Anamila, Subhasis and Shreya pool together x, y and z amounts of money (in thousands) respectively, every month. The sum is distributed across three accounts A_1, A_2 and A_3 as $x + y + z, z - 2y$ and $2y - z$ respectively. This can be thought of as a linear transformation

$$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

defined by

$$T(x, y, z) = (x + y + z, z - 2y, 2y - z)$$

Note: A negative amount of money signifies the amount withdrawn from the accounts. Answer the subquestions from the information given above.

Sub Questions:

Q1) Which of the following vector spaces consists of vectors which could denote the amount of money deposited by Anamika, Subhasis and Shreya in a particular month such that in that month the amount deposited is 0 in each of the accounts A_1, A_2 and A_3 .

Options:

A. $\text{span}\{(-3t, t, 0), (0, t, 2t) \mid t \in \mathbb{R}\}$

B. $\text{span}\{(-3t, t, 2t) \mid t \in \mathbb{R}\}$

C. $\text{span}\{(3t, -t, 2t) \mid t \in \mathbb{R}\}$

D. $\text{span}\{(3t, -t, 0), (0, -t, 2t) \mid t \in \mathbb{R}\}$

Q2) Find out $\text{nullity}(T)$.

Q3) Find out $\text{rank}(T)$.

Q4) Which of the following options is true?

Options:

- A. T is one to one
- B. T is onto
- C. T is both one to one and onto
- D. T is neither one to one nor onto

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Q3. Which of the following options is/are true?

Options :

A. If A is a non-zero matrix of order 4×3 and rank of A is 3, then the rows of A are linearly independent.

B. If A is a non-zero matrix of order 4×3 and rank of A is 3, then the columns of A are linearly independent.

C. If A is a non-zero matrix of order $m \times (m + 1)$, $m > 1$, then the maximum possible nullity of A is m .

D. If A is a non-zero matrix of order 4×5 and rank of A is 3, then the dimension of the solution space of the homogeneous system $Ax = 0$ is 2.

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Q4. Consider the vector space $V = \left\{ \begin{pmatrix} a & b \\ a & b \end{pmatrix} \mid a, b \in \mathbb{R} \right\}$ and $T: \mathbb{R}^3 \rightarrow V$ defined by

$$T(x, y, z) = \begin{pmatrix} x+y & x+y+z \\ x+y & x+y+z \end{pmatrix}. \text{ Choose the correct option}$$

Options :

- A. T is onto but not one-one
- B. T is one-one but not onto.
- C. T is both one-one and onto
- D. T is neither one-one nor onto.

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Q5. If A is a 2×3 matrix of rank 1, then what is the nullity of AA^T ?

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Q6. Which of the following options is/are true?

Options:

A. There exists an onto linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$

B. There does not exists a one-one linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}$

C. There exists a linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ such that $rank(T) = nullity(T)$

D. There does not exists a linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ such that $rank(T) = nullity(T)$

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Q7. Let V_1 denote the vector space of solution of $AX = 0$, where $A = \begin{pmatrix} 2 & 1 & 4 \\ -1 & 1 & 0 \\ 1 & 2 & 4 \end{pmatrix}$ and

$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$. Let V_2 denote the vector space of solutions of the system $BY = 0$, where

$B = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 2 & 3 \end{pmatrix}$ and $Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$. Answer the given subquestions

Sub Questions:

Q1) What is the nullity of A ?

Q2) What is the rank of B ?

Q3) Which of the following forms a basis β for V_1 ?

Options:

A. $\{(1, 5, 2)\}$

B. $\left\{\left(\frac{-4}{3}, \frac{-4}{3}, 1\right)\right\}$

C. $\left\{\left(\frac{1}{5}, 1, \frac{2}{5}\right)\right\}$

D. $\{(-4, -4, 3)\}$

Q4) Define a linear transformation $T : V_2 \rightarrow \mathbb{R}^2$ by $T(x, y, z) = (x, x + y + z)$. What is the rank of T ?

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Q8. Which of the following functions are linear transformations?

Options:

A. $T: \mathbb{R} \rightarrow \mathbb{R}, T(x) = 2x + 1$

B. $T: \mathbb{R}^4 \rightarrow \mathbb{R}^2, T(x, y, z, w) = (x + y, z + w)$

C. $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3, T(x, y, z) = (-y, -x, 0)$

D. $T: \mathbb{R} \rightarrow \mathbb{R}^2, T(x) = (x + 1, x - 1)$

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Q9. Let T be a linear transformation from \mathbb{R}^2 to \mathbb{R}^3 . $S \subset \mathbb{R}^2$ is a line passing through the origin. Which of the following are possible?

Options:

A. $T(S)$ could be the origin in \mathbb{R}^3

B. $T(S)$ could be a line passing through the origin in \mathbb{R}^3

C. $T(S)$ could be a plane passing through the origin in \mathbb{R}^3

D. $T(S)$ could be \mathbb{R}^3

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Q10. Consider the following system of linear equations:

$$x + 3y - 2z = 0$$

$$y - z = 0$$

$$x + y = 0$$

Let A be the coefficient matrix corresponding to this system

Based on the above data, answer the given subquestions.

Sub questions:

Q1) Which of the following is the nullspace of A ?

Options :

A. $\text{span}\{(-1, 1, 1)\}$

B. $\text{span}\{(1, 1, 0)\}$

C. $\text{span}\{(1, 0, 1), (0, 1, -1)\}$

D. $\text{span}\{(1, 1, 0), (0, 1, -1)\}$

Q2) Let B be a square matrix of order 3. What is the smallest value that the nullity of BA could take?

