Sep 2021

$Q1$. Let A be a 6×6 matrix of rank 4. Let S be the affine subspace of $\mathbb{R}^6$ defined by $S = \left\{x \in \mathbb{R}^6 \ : \ Ax = b\right\}$, where b is the 4^{th} column of A. What is the dimension of S ?

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$Q2$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined as $T(x_1, x_2, x_3) = (6x_2, 4x_3, x_1)$. Consider two ordered bases $\beta = \{(1,0,0), (0,1,0), (0,0,1)\}$ and $\gamma = \{(0,0,1), (1,0,0), (0,1,0)\}$ of $\mathbb{R}^3$. Consider the following two matrices:

$$A = \begin{bmatrix} 0 & 6 & 0 \\ 0 & 0 & 4 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 & 6 \\ 4 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Are A and B equivalent

Options:

A. True
B. False

Jan 2022

$Q3$. If A or B is invertible, then AB and BA are similar matrices (i.e., AB is similar to BA).

Options:
A. TRUE
B. FALSE

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$Q4$. Any two scalar matrices are similar.

Options:
A. TRUE
B. FALSE

==

$Q5$. If A is similar to B, then A^k is similar to B^k, for any positive integer k.

Options:
A. TRUE
B. FALSE

==

$Q6$. If A and B are two 3×3 matrices, which are similar to each other. Suppose the homogeneous system of linear equations $Ax = 0$ has a unique solution, then the homogeneous system of linear equations $Bx = 0$ also has a unique solution.

Options:
A. TRUE
B. FALSE

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$Q7$. An inner product on a vector space V is a function $\langle . \ , . \rangle : V \times V \to \mathbb{R}$ satisfying the following conditions:

Condition 1: $\langle v, v \rangle > 0$ for all $v \in V \setminus \{0\}$; $\langle v, v \rangle = 0$ if and only if $v = 0$

Condition 2: $\langle v_1 + v_2, v_3 \rangle = \langle v_1, v_3 \rangle + \langle v_2, v_3 \rangle$

Condition 3: $\langle v_1, v_2 \rangle = \langle v_2, v_1 \rangle$

Condition 4: $\langle cv_1, cv_2 \rangle = c\langle v_1, v_2 \rangle$

Define $V = \mathbb{R}^2$ and the function defined as:

$$\langle . \ , \ . \rangle : \ V \times V \ \to \ \mathbb{R}$$
$$\langle (x_1, y_1) \ , \ (x_2, y_2) \rangle = 2x_1x_2 + 3y_1y_2$$

Which of the above condiitons are satisfied for the above function?

Options:

A. Condition 1
B. Condition 2
C. Condition 3
D. Condition 4

May 2022

$Q8$. An inner product on a vector space V is a function $\langle . \ , \ . \rangle : \ V \times V \to \mathbb{R}$ satisfying the following conditions:

Condition 1: $\langle v, v \rangle > 0$ for all $v \in V \setminus \{0\}$; $\langle v, v \rangle = 0$ if and only if $v = 0$
Condition 2: $\langle v_1 + v_2, v_3 \rangle = \langle v_1, v_3 \rangle + \langle v_2, v_3 \rangle$
Condition 3: $\langle v_1, v_2 \rangle = \langle v_2, v_1 \rangle$
Condition 4: $\langle cv_1, v_2 \rangle = c\langle v_1, v_2 \rangle$

Let $V = \mathbb{R}^2$ and consider the function defined as:

$$\langle . \ , \ . \rangle : \ V \times V \to \mathbb{R}$$
$$\langle (x_1, x_2), \ (y_1, y_2) \rangle = x_1 y_1 - x_1 y_2 - x_2 y_1 + x_2 y_2$$

Which of the following are satisfied by the above function?

Options:

A. Condition 1 is satisfied.
B. Condition 2 is satisfied.
C. Condition 3 is satisfied.
D. Condition 4 is satisfied.

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$Q9$. Let U be a subspace of the vector space $\mathbb{R}^3$ and suppose $\{(1, 0, 1), (0, 1, 2)\}$ is a basis of U. Then which of the following subets of $\mathbb{R}^3$ are appropriate candidates for the affine subspaces of $\mathbb{R}^3$ such that the corresponding vector suspaces is U?

A. $\{(x, y, z) \mid x + 2y + z = 2, \ x, y, z \in \mathbb{R}\}$

B. $\{(x, y, z) \mid x + 2y + z = 1, \ x, y, z \in \mathbb{R}\}$

C. $\{(x, y, z) \mid x - 2y - z = 0, \ x, y, z \in \mathbb{R}\}$

D. $\{(x, y, z) \mid x - 2y - z = 1, \ x, y, z \in \mathbb{R}\}$

E. $\{(x, y, z) \mid x + 2y - z = 2, \ x, y, z \in \mathbb{R}\}$

F. $\{(x, y, z) \mid x + 2y - z = 0,\ x, y, z \in \mathbb{R}\}$

==

$Q10$. Let us consider the follwoing matrices:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Consider the following pairs of matrices:

- **Pair $\mathbf{I}$:** A, B
- **Pair $\mathbf{II}$** : A, C
- **Pair $\mathbf{III}$:** B, C

Choose the correct option from the following.

Options:

A. Only the matrices in Pair I are similar matrices.
B. All the pairs consist of similar matrices.
C.Only the matrices in Pair III are similar matrices.
D. None of these pairs consist of similar matrices.

==

$Q11$. A norm on a vector space V is a function

$$\begin{aligned} \|\cdot\| :\ & V \to \mathbb{R} \\ & x \to \|x\| \end{aligned}$$

satisfying the follwoing conditions:

Condition 1: $\|x + y\| \leqslant \|x\| + \|y\|$ for all $x, y \in V$
Condition 2: $\|cx\| = |c|\,\|x\|$ for all $c \in \mathbb{R}$ and for all $x \in V$
Condition 3: $\|x\| \geqslant 0$ for all $x \in V$; $\|x\| = 0$ if and only if $x = 0$

Consider a function $\|\cdot\| : \mathbb{R}^3 \to \mathbb{R}$ defined as

$$\|(x_1, x_2, x_3)\| = |x_1 + x_2 + x_3|$$

on the vector space $\mathbb{R}^3$.
Which of the following are satisfied by the above function?

Options:
A. Condition 1 is satisfied.
B. Condition 2 is satisfied.
C. Condition 3 is satisfied.
D. None of these conditions are satisfied.

Sept 2022

$Q12$. Which of the following options is/are true?
Options:

A. Every matrix is similiar to itself
B. If A is similiar to , then A^{-1} is similiar to B^{-1}
C. $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is similiar to $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$
D. If A is similiar to $B + C$, then $rank(A) = rank(B) = rank(C)$

==

$Q13$. Consider $V = \mathbb{R}^2$ with respect to the inner product defined as
$\langle x_1, x_2), (y_1, y_2)\rangle = x_1y_1 - (x_1y_2 + x_2y_1) + 2x_2y_2$, for all $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$.
Answer the subquestions based on the given data

Sub Questions:

Q1) Find $||(1, 3)||^2$

Q2) Which of the following is/are unit vectors in V ?
Options:

A. $(1, 1)$

B. $\frac{1}{2}(2, 2)$

C. $\frac{1}{\sqrt{13}}(2, 3)$

D.$(0, 1)$
E. None of these

==

$Q14$. The teacher asked Soumya and Sohini to consider an affine space each Soumya considered the affine subspace L and Sohini considered the affine subspace L' of $\mathbb{R}^3$, where $L = U$ and $L' = (2, 0, 1) + U'$, for some vector subspaces $U = Span\{(2, 0, 1), (1, 1, 0), (0, 1, 0)\}$ and $U' = Span\{(1, 0, 1), (0, 1, 1)\}$ of $\mathbb{R}^3$. Suppose there is a linear transformation $T : U \rightarrow U'$ such that $(0, 1, 0) \in ker(T)$, $T(2, 0, 1) = (0, 1, 1)$ and $T(1, 1, 0) = (1, 0, 1)$. An affine mapping $f : L \rightarrow L'$ is obtained by defining $f(u) = (2, 0, 1) + T(u)$, for all $u \in U$. By using the above information answer the given subquestions:

Sub Questions:

Q1) Which of the following affine subspaces was considered by Soumya?

Options:

A. $L = \{(x, y, z) \mid x - y - 2z = 0\}$
B. $L = \{(x, y, z) \mid x + y - z = 1\}$
C. $L = \{(x, y, z) \mid x + y - z = 0\}$
D. $L = \mathbb{R}^3$

Q2) Which of the following affine subspaces was considered by Sohini?

Options:

A. $L = \{(x, y, z) \mid x - y - 2z = 0\}$
B. $L = \{(x, y, z) \mid x + y - z = 1\}$
C. $L = \{(x, y, z) \mid x + y - z = 0\}$
D. $L = \mathbb{R}^3$

Q3) Which of the following functions represents f correctly?

Options:

A. $f(x, y, z) = (x - 2z + 2, z, x - z + 1)$
B. $f(x, y, z) = \left(x - 2z + 2, \frac{x}{2}, x - z + 1\right)$
C. $f(x, y, z) = (x - 2z, z, x - z)$

D. It cannot be determined from the given information

Jan 2023

$Q15$. An inner product on a vector space V is a function $\langle \cdot, \ \cdot \rangle : V \times V \to \mathbb{R}$ satisfying the following conditions:

Condition 1: $\langle v, v \rangle > 0$ for all $v \in V \setminus \{0\}$; $\langle v, v \rangle = 0$ if and only if $v = 0$
Condition 2: $\langle v_1 + v_2, v_3 \rangle = \langle v_1, v_3 \rangle + \langle v_2, v_3 \rangle$
Condition 3: $\langle v_1, v_2 \rangle = \langle v_2, v_1 \rangle$
Condition 4: $\langle cv_1, v_2 \rangle = c\langle v_1, v_2 \rangle$

Let $V = \mathbb{R}^2$ and consider the function defined as:

$$\langle \cdot \ , \cdot \rangle : V \times V \to \mathbb{R}$$
$$\langle (x_1, x_2), (y_1, y_2) \rangle = x_1y_1 - x_2y_1 + x_2y_2$$

Which of the following is/are satisfied by the above functions?

Options:

A. Condition 1 is satisfied.
B. Condition 2 is satisfied.
C. Condition 3 is satisfied.
D. Condition 4 is satisfied.

==

$Q16$. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation defined by $T(x, y) = (x + y, x - y, 3x + y)$.
If $A = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$ denote the matrix of with respect to $\{(1, 1), \ (1, -1)\}$ for $\mathbb{R}^2$ and $\{(1, 1, 1), \ (1, 1, 0), \ (-1, 0, 0)\}$ for $\mathbb{R}^3$. Let B denote the matrix of T with respect to the standard ordered bases for $\mathbb{R}^2$ and $\mathbb{R}^3$. Choose the correct options

Options:

A. A is equivalent to B.
B. A is not equivalent to B.
C.There exist two invertible matrices C and D such that $BD \ = \ CA$.
D.There are no matrices C and D such that $BD \ = \ CA$.

==

$Q17$. Consider the system of linear equations $Ax = b$, where $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Let L denote the set of all solutions of the above system. Clearly, L forms an affine space. Let W denote the subspace corresponding to L. Answer the given sub questions

Sub Questions:

Q1) What is the nullity of A?

Q2) What is the dimension of L ?

Q3) Define $T : W \to \mathbb{R}^2$ by $T(x, y, z) = (0, x - z)$, what is the rank of T ?

Q4) If the $m \times n$ matrix B is the matrix of T with respect to some basis for W and the standard basis for $\mathbb{R}^2$, then what is $m + n$?

May 2023

$Q18$. An inner product on a vector space V is a function $\langle \cdot , \cdot \rangle : V \times V \to \mathbb{R}$ satisfying the following conditions:

Condition 1: $\langle v, v \rangle > 0$ for all $v \in V \setminus \{0\}$; $\langle v, v \rangle = 0$ if and only if $v = 0$
Condition 2: $\langle v_1 + v_2, v_3 \rangle = \langle v_1, v_3 \rangle + \langle v_2, v_3 \rangle$, $\forall\, v_1, v_2, v_3 \in V$
Condition 3: $\langle v_1, v_2 \rangle = \langle v_2, v_1 \rangle$, $\forall\, v_1, v_2 \in V$
Condition 4: $\langle cv_1, v_2 \rangle = c\langle v_1, v_2 \rangle$, $\forall\, v_1, v_2 \in V$

Let $V = \mathbb{R}^2$ and consider the function defined as:

$$\langle \cdot , \cdot \rangle : V \times V \to \mathbb{R}$$
$$\langle (x_1, x_2), (y_1, y_2) \rangle = x_1y_1 - x_2y_1 - x_2y_2$$

Which of the following is/are satisfied by the above functions?

Options:

A. Condition 1 is satisfied.
B. Condition 2 is satisfied.
C. Condition 3 is satisfied.
D. Condition 4 is satisfied.

==

$Q19$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by
$T(x, y, z) = (x + y + z, x - y - z, x)$.
If $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ denotes the matrix of T with respect to $\{(1,1,1), (1,1,0), (1,0,0)\}$ for domain and co-domain and let B denote the matrix of T with respect to the standard ordered basis for both domain and co-domain. Choose the correct option(s)

Options:

A. A is similiar to B
B. A is not similiar to B
C. $det(A) = det(B) = 0$
D. $det(A) = det(B) = 2$

Sep 2023

$Q20$. Let A and be $n \times n$ matrices. Which of the following statement(s) is/are true?

Options

A. If A and B are similiar, then the nullity of A and nullity of B are equal

B. Let A and B be similiar matrices. Then the homogeneous system of linear equations $Ax = 0$ has a unique solution if and only if the homogeneous system of linear equation $Bx = 0$ has a unique solution

C. If A^k and B^k are similiar for some positive integer $k,$ then A and B are similiar

D. If A and are similiar matrices where A is a scalar matrix, then $A = B$

==

$Q21$. Let $L = \{(x, y) \; : \; y = x + 1\}$ and $L' = \{(x, x + z - 2, z) : x, z \in \mathbb{R}\}$

Based on the above data, answer the given subquestions.
Sub questions:

Q1) Choose the correct option from the following.

A. The subspace associated with the affine space L is the line $y = x + 1$

B. The subspace associated with the affine space L is given by $\{(x, x) : x \in \mathbb{R}\}$

C. The subspace associated with the affine space L' is given by $\{(x, y, z) \; : \; x - y + z = 0\}$

D. The subspace associated with the affine space L' is the $xz - plane$

Q2) If the dimension of L is m and the dimension of L' is n, then $m + n$ is