May 2021

$Q1$. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a multivariable saclar continuous function, such that:

$$f_x(x, y) = 2x\, cos(x^2 + y^2) \text{ and } f_y(x, y) = 2y\, cos(x^2 + y^2)$$

Suppose $\lim\limits_{(x,y) \to (0,0)} f(x, y) = 5.$

Answer the subquestions using the given information

Sub Questions:

Q1) Let $L(x, y) = Ax + By + C$ be the linear approximation of the function $z = f(x, y)$ at the point $(0, 0)$, then find the value of $(A + C)(3C - B)$

Q2) Find out the directional derivative of f at the point $(1,\ 1)$ in the direction of the vector $(1,\ 1)$.

Options:

A. $2\sqrt{2}\, cos\, 2$
B. $\sqrt{2}\, cos\, 2$
C. $2\sqrt{2}\, sin\, 2$
D. $\sqrt{2}\, sin\, 2$

==

$Q2$. Suppose the temperature (T) at a point (x, y, z) in $\mathbb{R}^3$ is given by the function

$$T(x, y, z) = e^{x+y+z}$$

Let T_1, T_2 and T_3 be the tangent planes to the function $f(x, y) = \sqrt{9 - x^2 - y^2}$ at the points $(1, 2)$, $(2, 1)$ and $(2, 2)$ respectively. The set of points on each plane T_1, T_2 and T_3 form affine subspaces A_1, A_2 and A_3 respectively, of the vector spaces $\mathbb{R}^3$, with respect to usual addition and scalar multiplication. Let V_1, V_2 and V_3 denote the vector subspaces corresponding to the affine subspaces A_1, A_2 and A_3 respectively.

Answer the subquestion using the given information.

Sub Questions:

Q1) Which of the following denotes the vector subspace $V_1 \cap V_2$?

Options:
A. $\{(x, y, -x-y) \mid x, y \in \mathbb{R}\}$
B. $\{(-2y, -2z, y, z) \mid y, z \in \mathbb{R}\}$
C. $\{(-2z, -2z, 3z) \mid z \in \mathbb{R}\}$
D. $\{(-2z, -z, z) \mid z \in \mathbb{R}\}$

Q2) Which of the following denotes the maximum rate of change of temperature (T) at the point of intersection of the tangent planes $T1,\ T2,$ and $T3$?

Options:
A. $\sqrt{3}e^3$
B. $\sqrt{3}e^{\frac{27}{5}}$
C. $\sqrt{3}e^{\frac{3}{5}}$
D. $3e^{\frac{27}{5}}$

Q3) Let T be a linear transformation between the vector subspaces V_1 and V_3. Suppose the matrix representation of with respect to the ordered bases $\beta = \{(-2,1,0), (-2,0,1)\}$ for V_1 and $\gamma = \{(1,0,-2), (0,1,-2)\}$ for V_3 is given by $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Then which of the following mappings can be an affine mappings f between the affine subspaces A_1 and A_3 corresponding to T?

Options:

A. $f(1+x, 2+y, 2+z) = (2+y, 2, 1+y)$

B. $f(1+x, 2+y, 2+z) = (2+y, 2, 1+2y)$

C. $f(1+x, 2+y, 2+z) = (2+y, 2, 1-2y)$

D. $f(1+x, 2+y, 2+z) = (2+y, 2, 1-y)$

Sept 2021

$Q3$. Consider the functions $t_1,\ t_2 : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$t_1(x,y) = ye^x$$

$$t_2(x,y) = x^2e^y$$

Define a vector valued function as follows:

$$U(x,y) = (t_1 x, y), t_2(x,y)) = \left(ye^x, x^2e^y\right)$$

The tangent plane to the function t_1 at the point $(0,1)$ is given by T_1 and the tangent plane to the function t_2 at the point $(1,0)$ is given by T_2. Let A_1 and A_2 denote the affine subspaces of $\mathbb{R}^3$ formed by the set of points on T_1 and T_2 respectively. Let V_1 and V_2 denote the corresponding vector subspaces associated with A_1 and A_2 respectively.

Sub Questions:

Q1) Which of the following equations represents the tangent plane T_1?

Options:

A. $x + y - z = 1$
B. $x + y - z = 0$
C. $x + y + z = 1$
D. $x - y - z = 0$

Q2) If the equation of the tangent plane T_1 and T_2 form a system of linear equations has

Options:

A. A unique solution
B. No solution
C. Infinitely many solutions
D. None of these

Q3) Which of the following sets define the affine space A_2 with respects to the usual addition and scalar multiplication?

Options:

A. $\left\{(x, y, z) \mid 2x + y + z = 1, \text{ where } x, y, z \in \mathbb{R}\right\}$

B. $\left\{(x, y, z) \mid 2x - y - z = 1, \text{ where } x, y, z \in \mathbb{R}\right\}$

C. $\left\{(x, y, z) \mid 2x + y - z = 1, \text{ where } x, y, z \in \mathbb{R}\right\}$

D. $\left\{(x, y, z) \mid -2x + y - z = 1, \text{ where } x, y, z \in \mathbb{R}\right\}$

Q4) If $\frac{1}{\sqrt{a^2+b^2}}(a, b)$ is the direction along which the rate of change of the function t_1 at the point $(0, 1)$ is maximum, then find the value of a.

Q5) If $\frac{1}{\sqrt{a^2+b^2}}(a, b)$ is the direction along which the rate of change of the function t_1 at the point $(0, 1)$ is maximum, then find the value of b.

Q6) If v is the gradient vector of the function t_2 at the point $(1, 0)$, then find the value of $||v||^2$.

Q7) Consider the following sets of vectors:

$$\beta_1 = \{(1, 0, 1), (0, 1, 1)\}$$
$$\beta_2 = \{(1, 0, 2), (0, 1, 1)\}$$
$$\beta_2 = \{(1, 0, 1),\ (1, 1, 0)\}$$
$$\beta_4 = \{(1, 0, 2), (0, 1, 1), (1, 1, 3)\}$$

Which of the following options is (are) true?

Options:

A. β_1 and β_3 both form a basis of V_1

B. β_1 is a basis of V_1 and β_4 is a basis of V_2

C. β_1 is a basis of V_1 and β_2 is a basis of V_2

D. β_3 is a basis of V_1 and β_2 is a basis of V_2.

Q8) U is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ with respect to usual addition and scalar multiplication on $\mathbb{R}^2$.
Is the above statement true or false?

Options:
A. True
B. False

Q9) There are no critical points of the function t_1 in $\mathbb{R}^2$
Is the above statement true or false?

Options:
A. True
B. False

Q10) Consider the linear transformation $T: V_1 \to V_2$, defined by $T(x, y, z) = (x, y, 2x + y)$. Which of the following statements is(are) true?

Options:
A. T is injective
B. T is surjective
C. T is isomorphism
D. There exists a basis γ_1 of V_1 and a basis γ_2 of V_2 such that the matrix representation of T is an identity matrix with respect to γ_1 and γ_2

Jan 2022

$Q4$. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as

$$f(x, y) = x^2y + xy^2$$

The tangent plane to the function f at the point $(1, 1)$ is given by T_1. Let A_1 denote the affine subspace of $\mathbb{R}^3$ formed by the set of points on T_1. Let V_1 denote the corresponding vector subspace associated with A_1.

Consider the affine subspace of $\mathbb{R}^3$ given by

$$A_2 = \left\{(x, y, z) \mid x - y + z = 1, \text{ where } x, y, z \in \mathbb{R}\right\}$$

Let V_2 denote the corresponding vector subspace associated with A_2.

Using the above information answer the given subquestions:

Sub questions:

Q1) Which one of the following represents the equation of the tangent lines to the function f at the point $(1,\ 1)$ in the direction of the unit vector $(1,\ 0)$?

Options:

A. $x(t) = 1 - t,\ y(t) = 1,\ z(t) = 2 + 3t.$
B. $x(t) = 1 + t,\ y(t) = 1,\ z(t) = 2 - 3t.$
C. $x(t) = 1 + t,\ y(t) = 1,\ z(t) = 2 + 3t.$
D. $x(t) = 1 + t,\ y(t) = 0,\ z(t) = 2 + 3t.$

Q2) The unit vector along which the rate of change of the function f at $(1,\ 1)$ is the maximum is

Options:

A. $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

B. $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

B. $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

B. $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Q3) Which of the following sets forms a basis β of V_1?

Options :

A. $\{(1, 0, 1), (0, 1, 1)\}$
B. $\{(-1, 0, -3), (0, 1, 3)\}$
C. $\{(1, 0, -3), (0, 1, 3)\}$
D. $\{(1, 0, -1), (0, 1, -1)\}$

Q4) Which of the following sets forms a basis γ of V_2?

Options:

A. $\{(1, 0, -1), (0, 1, 1)\}$
B. $\{(1, 0, 3), (0, 1, 3)\}$
C. $\{(1, 0, -3), (0, 1, 3)\}$
D. $\{(1, 0, -1), (0, 1, -1)\}$

Q5) Let T denote the linear transformation from V_1 to V_2 , whose matrix representation with respect to the ordered bases β and γ (mentioned in the previous questions Q3 & Q4), respectively for V_1 and V_2 is the identity matrix, i.e. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Which of the following functions denotes the affine mapping from A_1 to A_2 corresponding to T?

Options:

A. $f(x, y, z) = (-x, -y, x + y + 1)$
B. $f(x, y, z + 4) = (x, y, -x + y + 1)$
C. $f(x, y, z - 4) = (x, y, x + y + 1)$
D. $f(x, y, z - 4) = (-x, y, x + y + 1)$

May 2022

$Q5$: Consider the function $f(x, y) = x^2 + y^2$. Which of the following affine subsace represent the tangent line at the point $(1, 2)$ in the direction of the vector $(1, 1)$?

Options:

A. $\left\{(x, y, z) \in \mathbb{R}^3 \mid \dfrac{x-1}{\frac{1}{\sqrt{2}}} = \dfrac{y-2}{\frac{1}{\sqrt{2}}}\right\}.$

B. $\left\{(x, y, z) \in \mathbb{R}^3 \mid \dfrac{x-1}{\frac{1}{\sqrt{2}}} = \dfrac{y-2}{\frac{1}{\sqrt{2}}} = \dfrac{z-5}{\frac{1}{\sqrt{2}}}\right\}$

C. $\left\{(x, y, z) \in \mathbb{R}^3 \mid \dfrac{x-1}{\frac{1}{\sqrt{2}}} = \dfrac{y-2}{\frac{1}{\sqrt{2}}} = \dfrac{z-5}{\frac{6}{\sqrt{2}}}\right\}.$

D. $\left\{(x, y, z) \in \mathbb{R}^3 \mid \dfrac{x}{\frac{1}{\sqrt{2}}} = \dfrac{y}{\frac{1}{\sqrt{2}}} = \dfrac{z}{\frac{6}{\sqrt{2}}}\right\}$

Sept 2022

$Q6$. Answer the given subquestions about the functions

$$u(x,y) = x^2 + e^y$$

and

$$v(x,y) = y^2 + ye^x$$

Sub Questions:

Q1) If $(s_1,\ s_2)$ is the unit vector along the direction in which the directional derivative of the function $u(x,y)$ at the point $(1,\ 1)$ is the maximum and $(r_1,\ r_2)$ is the unit vector along the direction in which the directional derivative of the function $v(x,\ y)$ at the point $(1,\ 1)$ is the minimum, then which of the following options is/are true?

Options:

A. $s_1 + s_2 = \dfrac{2+e}{\sqrt{4+e^2}}$

B. $s_1 + s_2 = -\dfrac{2+e}{\sqrt{4+e^2}}$

C. $r_1 + r_2 = \dfrac{2+2e}{\sqrt{2e^2+4e+4}}$

D. $r_1 + r_2 = -\dfrac{2+2e}{\sqrt{2e^2+4e+4}}$

Q2) If $L_u(x,\ y)$ is the linear approximation of the function $u(x,\ y)$ at point $(2,\ 3)$ and $L_u(3,\ 4) = (a + be3)$, and if $L_v(x,\ y)$ is the linear approximation of the function $v(x,\ y)$ at point$(1,\ 2)$, and $L_v(3,\ 4) = (c + de)$, then find the value of $(a + c) - (b + d)$, where $a,\ b,\ c,\ d$ are integers.

Q3) Which of the following options is/are true?

Options:

A. $z = 2x + e^2y - 1 - e^2$ is the tangent plane to the function $u(x,y)$ at point $(1,2)$

B. $x(t) = 1 + \frac{1}{\sqrt{2}}t,\ y(t) = 2 + \frac{1}{\sqrt{2}}t,\ z(t) = \left(1 + e^2\right) + \frac{e^2}{\sqrt{2}}t$ is the tangent line of the function $u(x, y)$ at point $(1, 2)$ in the direction of $\frac{1}{\sqrt{2}}(1, 1)$.

C. $z = 2ex + (4 + e)y + 4 - 3e$ is the tangent plane to the function $v(x, y)$ at point $(1, 2)$

D. $x(t) = 1 + \frac{1}{\sqrt{2}}t,\ y(t) = 2 + \frac{1}{\sqrt{2}}t,\ z(t) = (4 + 2e) + \left(2\sqrt{2} + \frac{3e}{\sqrt{2}}\right)t$ is the tangent line of the function $v(x, y)$ at point $(1, 2)$ in the direction of $\frac{1}{\sqrt{2}}(1, 1)$.

Jan 2023

$Q7$. Consider the function $f(x,y) = x^3 + y^3$. Which of the following affine subspaces represents the tangent line at the point $(1,1)$ in the direction of the vector $(1,1)$?

Options:

A. $\left\{(x,y,z) \in \mathbb{R}^3 \mid \frac{x-1}{\frac{1}{\sqrt{2}}} = \frac{y-1}{\frac{1}{\sqrt{2}}} = \frac{z-1}{\frac{1}{\sqrt{2}}}\right\}$

B. $\left\{(x,y,z) \in \mathbb{R}^3 \mid \frac{x-1}{\frac{1}{\sqrt{2}}} = \frac{y-1}{\frac{1}{\sqrt{2}}} = \frac{z-2}{\frac{1}{\sqrt{2}}}\right\}$

C. $\left\{(x,y,z) \in \mathbb{R}^3 \mid \frac{x-1}{\frac{1}{\sqrt{2}}} = \frac{y-1}{\frac{1}{\sqrt{2}}} = \frac{z-2}{\frac{6}{\sqrt{2}}}\right\}$

D. $\left\{(x,y,z) \in \mathbb{R}^3 \mid \frac{x}{\frac{1}{\sqrt{2}}} = \frac{y}{\frac{1}{\sqrt{2}}} = \frac{z}{\frac{1}{\sqrt{2}}}\right\}$

==

$Q8$. From the list of given terms find out the best possible options.

1) Rank 2 2) Non-zero nullity 3) Non -zero determinant 4) Closure with respect to addition and scalar multiplication 5) Existence of addititve inverse 6) Existence of additive inverse 7) Commutative of addition 8) Associativity of addition 9) Elements 10) Global maxima 11) Global minima 12) A critical points 13) Gradient exists 14) Directional derivative exist in any direction 15) Partial derivatives exist 16) Orthonormal columns 17) Standard ordered basis 18) Affine subspaces 19) Limit exists

If the tangent plane exists for a function at a point then at that point ________________ .
(Enter 4 best possible options. Enter only the serial numbers of those options in increasing order without
adding any comma or space in between them.) [Suppose your answer is 7, 14, 15 and 17, then you
should enter 7141517]

==

$Q9$. Match the equation of the surface in Column A with the tangent plane at the point $(1, 1, 2)$ in column B and the vector subspace corresponding to the affine subspace (of $\mathbb{R}^3$) formed by the tangnet plane, in Column C.

	Equation of the surface		Equation of the tangent plane at $(1, 1, 2)$		Vector subspace corresponding to the affine subspace formed by tangent plane

$i)$	$z = x^2 + y^2$	$a)$	$3x + 3y + 2z = 10$	$1)$	$\left\{(x, y, z) \mid x + y = \frac{z}{2},\ x, y, z \in \mathbb{R}\right\}$
$ii)$	$x^2 + y^2 + z^2 = 6$	$b)$	$z = 2x + 2y - 2$	$2)$	$\left\{(x, y, z) \mid x + y = -\frac{2}{5}z,\ x, y, z \in \mathbb{R}\right\}$
$iii)$	$xy + yz + zx = 5$	$c)$	$x + y + 2z = 6$	$3)$	$\{(x, y, z) \mid 2z = -x - y\ ,\ x, y, z \in \mathbb{R}\}$

Table: M2ES1

Choose the correct option from the following:

Options:

A. $i) \rightarrow b \rightarrow 1,\ ii \rightarrow a \rightarrow 2,\ iii) \rightarrow c \rightarrow 3$

B. $i) \rightarrow a \rightarrow 2,\ ii \rightarrow c \rightarrow 1,\ iii) \rightarrow b \rightarrow 3$

C. $i) \rightarrow b \rightarrow 1,\ ii \rightarrow c \rightarrow 3,\ iii) \rightarrow a \rightarrow 2$

D. $i) \rightarrow b \rightarrow 1,\ ii \rightarrow c \rightarrow 2,\ iii) \rightarrow a \rightarrow 3$

May 2023

$Q10$. A skier is on a mountain with equation $z = 20 - 0.4x^2 - 0.3y^2$ where z denotes the height. THe skier is located at the point with $xy - cordinates$ $(1, -1)$ and wants to ski downhil along the steepest possible path. In which direction should the skier begin skiing ?

Options :

A. $(0.8,\ 0.6)$
B. $(-0.8,\ 0.6)$
C. $(-0.8,\ -0.6)$
D. $(0.8,\ -0.6)$

==

$Q11$. Let $f(x, y) = sinx\ cosy$
Based on the above data, answer the given subquestions.

Sub Questions:

Q1) Choose the correct option for the parametric equation of the line tangent to f at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ in the direction of $x - axis$.

Options:

A. $x(t) = \frac{\pi}{2}, y(t) = \frac{\pi}{2} - t, z(t) = 0$

B. $x(t) = \frac{\pi}{2} + t, y(t) = \frac{\pi}{2}, z(t) = 0$

C. $x(t) = \frac{\pi}{2} + t, y(t) = \frac{\pi}{2}, z(t) = t$

Q2) Choose the correct option for the parametric equation of the line tangent to f at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ in the direction of $y - axis$.

Options:

A. $x(t) = \frac{\pi}{2}, y(t) = \frac{\pi}{2} + t, z(t) = -t$

B. $x(t) = \frac{\pi}{2} + t, y(t) = \frac{\pi}{2}, z(t) = t$

C. $x(t) = \frac{\pi}{2}, y(t) = \frac{\pi}{2} - t, z(t) = 0$

Q3) Choose the correct option for the parametric equation of the line tangent to f at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ in the direction of $(-1, 1)$

Options:

A. $x(t) = \frac{\pi}{2} + \frac{t}{\sqrt{2}}, y(t) = \frac{\pi}{2} - \frac{t}{\sqrt{2}}, z(t) = -\frac{1}{\sqrt{2}}$

B. $x(t) = \frac{\pi}{2} - \frac{t}{\sqrt{2}}, y(t) = \frac{\pi}{2} + \frac{t}{\sqrt{2}}, z(t) = \frac{1}{\sqrt{2}}$

C. $x(t) = \frac{\pi}{2} - \frac{t}{\sqrt{2}}, y(t) = \frac{\pi}{2} + \frac{t}{\sqrt{2}}, z(t) = -\frac{t}{\sqrt{2}}$

Sept 2023

$Q12$. Let $f(x,y) = x^3 + 2x^2y + y^2 - 2x + y + 1$. Choose the correct option(s) from the following:

Options:

A. The tangent plane of f at $(1,-1)$ is given by $z = 3x - y - 2$

B. The tangent plane of f at $(1,-1)$ is given by $3x - y + z - 2 = 0$

C. The best linear approximation for f close to the point $(1,1)$ is given by $L_f(x,y) = 5x + 5y - 6$

D.The best linear approximation for f close to the point $(1,1)$ is given by $L_f(x,y) = x - 5y + 6$

E. If T_1 is the tangent plane of f at $(1,-1)$ and T_2 is the tangent plane of f at $(1,1)$, then T_1 and T_2 intersects at infinitely many points

F. If T_1 is the tangent plane of f at $(1,-1)$ and T_2 is the tangent plane of f at $(1,1)$, then T_1 and T_2 intersects at only one point.

==

$Q13$. Let $f(x,y) = 2x^2 + \frac{3y^2}{2}$. If $u = (a,b)$ is the direction in which

Based on the above data, answer the given subquestions:

Sub Questions:

Q1) f increases most rapidly at the point $(1,1)$, find $5(a+b)$

Q2) f decreases most rapidly at the point $(1,1)$, find $5(a+b)$

Q3) there is no chane in f at the point $(1,1)$, find $5|a+b|$