

Jan 2022

Q1. Choose the correct statements for the function

$$f(x, y) = \begin{cases} \frac{x^2y}{x^4 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Options:

A. $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$

B. f is not continuous at $(0, 0)$

C. The directional derivative of f at $(0, 0)$ in the direction of $(1, 1)$ is $\frac{1}{\sqrt{2}}$

D. The directional derivative of f at $(1, 0)$ in the direction of $(1, 1)$ does not exist.

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Q2. Which of the following options is/are true?

$$f(x, y) = \frac{xy^3}{x^2 + y^6}$$

Options:

- A. $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$
- B. $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = \frac{1}{2}$
- C. $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 1$
- D. $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist
- E. $f(x, y)$ is not continuous at $(0, 0)$
- F. $f(x, y)$ is continuous at $(0, 0)$

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Q3. The price of product ($f(x, y)$) depends on the price (x) of the raw materials and the price (y) of transportation of the product to the market according to

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$f(x, y) = \begin{cases} x^3 - xy^2 & \text{if } x \neq y \\ x + y & \text{otherwise} \end{cases}$$

Answer the given subquestions using the information above

Sub Questions:

Q1) What will be the ratio of the price of the raw materials and the price of the transportation ($x : y$) when $y > x$, if the rate of change of the price of the product with respect to the price of the raw materials is 0?

(In this context x and y both are always positive)

Options:

- A. 1 : 1
- B. 1 : 3
- C. 1 : $\sqrt{3}$
- D. The ratio cannot be determined using the given information

Q2) Which of the following statements are true?

Options:

- A. $f(x, y)$ is a linear function in its domain
- B. $f(cx, cy) = c^3 f(x, y)$ for any real number c , if $x \neq y$
- C. $f(x, y)$ is continuous at $(0, 0)$
- D. If the price of raw material and transportation of the product approaches 3 and 2 respectively, then the price of the product approaches 15
- E. If the price of raw material and transportation of the product approaches 3 and 2 respectively, then the price of the product approaches 5

Q3) If the rate of change of price of the product along the direction of the vector $(1, 1)$ is $\frac{1}{\sqrt{2}}[ka^2 + lab + mb^2]$, when the price of raw material is a and the price of transportation of the product to the market is b (where $a \neq b$), then find the value of $k - l + m$

Q4) Which of the following is true for the function:

$$f(x, y) = \frac{x^3 + y^3}{x + y}$$

Options:

A. $\lim_{(x,y) \rightarrow (1,-1)} f(x, y) = 0$

B. $\lim_{(x,y) \rightarrow (1,-1)} f(x, y) = 3$

C. $\lim_{(x,y) \rightarrow (1,-1)} f(x, y) = 2$

D. $\lim_{(x,y) \rightarrow (1,-1)} f(x, y)$ does not exist.

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Q4.

$$u(x, y) = x^2 + e^y$$

and

$$v(x, y) = y^2 + ye^x$$

Consider a function $g(x, y)$ such that

$$g(x, y) = \begin{cases} u(x, y) & x = y \\ v(x, y) & x \neq y \end{cases}$$

Which of the following option is/are true?

Options:

A. $g(x, y)$ is continuous at origin

B. $g_{xy}(1, 2) = e$

C. $g_{xy}(x, y)$ is not continuous at $(1, 2)$

D. $g_y(1, 1) = 2 + e$

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Q5. Choose the correct statements for the function

$$f(x, y) = \begin{cases} \frac{x^4 y^2}{x^4 + y^4} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Options:

- A. $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 1$
- B. $f_x(0, 0) = 0$
- C. The directional derivative of f at $(0, 0)$ in the direction of $(-1, 1)$ exists.
- D. f is not continuous at $(0, 0)$

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Q6. Let $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{Otherwise} \end{cases}$. Then which of the following statement(s) is/are true?

Options :

- A. The partial derivatives for f exist everywhere but directional derivatives need not exist at the origin.
 - B. All directional derivatives at the origin exist for the function f .
 - C. At $(0, 0)$, the directional derivative of f exists in the direction of $x - axis$.
 - D. The partial derivatives for f do not exist at the origin.
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Q7. Let $f(x, y, z) = x^2y^3 - 3xz$ and $u = (1, 2, 2)$. Find the directional derivative of f in the direction of the vector u at the point $(0, 1, -1)$

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Q8. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a function given by

$$f(x, y) = \begin{cases} \frac{(x+y)^2}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Choose the correct option(s) from the following:

Options:

- A. $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exists along the positive $x - axis$
 - B. $f(x, y)$ approaches 0 when (x, y) approaches $(0, 0)$ along the positive $y - axis$
 - C. $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist
 - D. $f_x(0, 0) = 1$
 - E. The directional derivative of f at $(0, 0)$ in the direction of positive $x - axis$ does not exist
 - F. The directional derivative of f at $(0, 0)$ in the direction of positive $y - axis$ exists
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Q9. Let $f(x, y) = \begin{cases} \frac{\cos y \sin x}{x} & x \neq 0 \\ \cos y & x = 0 \end{cases}$

Based on the above data, answer the given subquestions

Sub Questions:

Q1) Find $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$

Q2) Is the function continuous at $(0, 0)$? [If the answer is yes, enter 1 and if the answer is no, enter 0]

Q3) At how many points does f have a discontinuity ?

