Sept 2021

$Q1$. Three people denoted by P_1, P_2, P_3 intended to buy some rolls, buns and cakes from a shop. They pay ₹x_1, ₹x_2, ₹x_3 per unit for rolls, buns and cakes, respectively. P_1 bought 1 unit of rolls, 2 unit of buns, and 3 units of cales, P_2 bought 2 unit of rolls, 4 units of buns, and 5 units of cakes and P_3 bought 1 unit of rolls, 3 unit of buns and 4 units of cakes. The total amount spend by P_1, P_2 and P_3 are ₹12, ₹21 and ₹16 respectively. Answer the subquestions using this given data.

Sub Questions:

Q1) Represent the system of linear equations given in terms of its matrix $Ax = b$,

where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $b = \begin{bmatrix} 12 \\ 21 \\ 16 \end{bmatrix}$. Which one of the following options represent A?

Options:

A. $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 4 & 5 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 1 & 3 & 4 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 4 & 5 \\ 1 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & 5 \\ 1 & 2 & 3 \end{bmatrix}$

Q2) Find the $(2,\ 3) - th$ cofactor of A.

Q3) If $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is the solution of $Ax = b$, then the value of $x_1 + x_2 + x_2$ is

Q4) The determinant of A^T is (A^T denotes the transpose of the matrix A)

Jan 2022

$Q2.$ Suppose the following table represents the number of people (in thousands) travelling to tourists spots from the metro cities in December 2019

To	Manali	Sikkim	Meghalaya
From Chennai	2	1	1
From Kolkata	3	2	1
From Mumbai	1	1	2

Due to covid these numbers got reduced in Decmber 2020 which (in percentage of reduction with respect to Decmeber 2019) is given by:

To	Manali	Sikkim	Meghalaya
From Chennai	50%	50%	25%
From Kolkata	25%	25%	25%
From Mumbai	50%	25%	25%

i.e., the number of tourists from Chennai to Meghalaya in Decmeber 2020 is reduced to 25% of Decmeber 2019 (if 1000 were visiting in December 2019, only 250 are visiting in Decmber 2020) and so on accordig to the table.
Using the given data answer the given sub questions

Sub Questions:

Q1) If $A = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ denotes the column vector representing the number of tourists to Manali (each element represent the number of tourists from diffeerent metro cities Chennai, Kolkata and Mumbai, repectively) in Decmber 2019, and $C = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the column vector representing the number of tourists to Manali in December 202 (where x_1, x_2 and x_3 denote the number of tourists from different metro cities Chennai, Kolkata and Mumbai, respectively) and we have the following equaation $BA = C$, then which of the follwoing matrices can represent B?

Options:

A.
$$\begin{bmatrix} 50 & 50 & 25 \\ 25 & 25 & 25 \\ 50 & 25 & 25 \end{bmatrix}$$

B.
$$\begin{bmatrix} 0.50 & 0.50 & 0.25 \\ 0.25 & 0.25 & 0.25 \\ 0.50 & 0.25 & 0.25 \end{bmatrix}$$

C.
$$\begin{bmatrix} 0.50 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.50 \end{bmatrix}$$

D.
$$\begin{bmatrix} 0.50 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}$$

E. $\begin{bmatrix} 0.50 & 0.25 & 0.50 \end{bmatrix}$

F. $\begin{bmatrix} 0.25 & 0.25 & 0.25 \end{bmatrix}$

Q2) If $A = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$ denotes the row vector representing the numbe of tourists from Kolkata (each element represent the number of tourists from Kolkata to Manali, Sikkim and Meghalaya, resectively) in Decmeber 2019, and $C = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$ be the row vector representing the number of tourists from Kolkata in December 2020 (where x_1, x_2 and x_3 dentoes the number of tourists from Kolkata to Manali, Sikkim and Meghalaya, respectively) and we have the following equation $AB = C$, then which of the following matrices can represent B?

Options:

A.
$$\begin{bmatrix} 50 & 50 & 25 \\ 25 & 25 & 25 \\ 50 & 25 & 25 \end{bmatrix}$$

B.
$$\begin{bmatrix} 0.50 & 0.50 & 0.25 \\ 0.25 & 0.25 & 0.25 \\ 0.50 & 0.25 & 0.25 \end{bmatrix}$$

$$
C. \begin{bmatrix} 0.50 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}
$$

$$
D. \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}
$$

$$
E. \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}
$$

$$
F. \begin{bmatrix} 0.50 \\ 0.25 \\ 0.50 \end{bmatrix}
$$

Q3) Find the total number of tourists (in thousands) to Meghalaya in December 2020 from Chennai, Kolkata and Mumbai?

(Suppose your answer is 3000, then enter 3 as your answer)

May 2022

$Q3$. Shubham bought 2 kg of potatoes and c kg of dal and 1 kg of wheat from a shop, and paid ₹80 to the shokeeper. Sushmitha bought 4 kg of potatoes and 1 kg of dal and 2 kg of wheat and paid ₹d to the shopkeeper. Subhasis bought 2 kg of potatoes, 1 kg of dal and 2 kg of wheat and paid ₹80 to the shopkepper. If $x_1(\neq 0)$ represents the price of 1 kg of potato and $x_2(\neq 0)$ represents the price of 1 kg of dal, and $x_3(\neq 0)$ represents the price of 1 kg of wheat, then answer the given sub questions

Sub Questions:

Q1) The matrix representation to find x_1, x_2 and x_3 is

Options:

A. $\begin{bmatrix} 2 & c & 1 \\ 2 & 1 & 2 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 80 \\ d \\ 80 \end{bmatrix}$

A. $\begin{bmatrix} 2 & c & 1 \\ 4 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 80 \\ d \end{bmatrix}$

A. $\begin{bmatrix} 2 & c & 1 \\ 4 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 80 \\ d \\ 80 \end{bmatrix}$

A. $\begin{bmatrix} 4 & 1 & 2 \\ 2 & c & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 80 \\ d \\ 80 \end{bmatrix}$

Q2) If we try to find x_1, x_2 and x_3 using appropriate matrix representation by taking $c = 0.5$ and $d = 100$ then

A. We will find that the price of 1 kg potato is unique

B. We will fail to find the price (as a numerical value) of 1 kg potato

C. We will find that the price of 1 kg potato is ₹20

D. We will find infinitely many values as the price of 1 kg potato

Q3) If we try to find x_1, x_2 and x_3 using appropriate matrix representation by taking $c = 2$ and $d \neq 160$, then which of the following options are true?

A. $x_1 = x_3$

B. $x_1 = x_2$

C. $x_2 = x_3$

D. $x_1 = x_2 = x_3$

Q4) If we try to find x_1, x_2 and x_3 using appropriate matrix representation by taking $c = 3$ and $d = 100$, then which of the following options are true?

A. $x_1 = 12$, $x_2 = 20$ and $x_3 = 10$

B. $x_1 = x_2 = 24$ and $x_3 = 10$

C. $x_1 = 10$, $x_2 = 12$ and $x_3 = 24$

D. $x_1 = x_2 = x_3 = 10$

Sept 2022

$Q4.$ Choose the set of correct options:

Options:

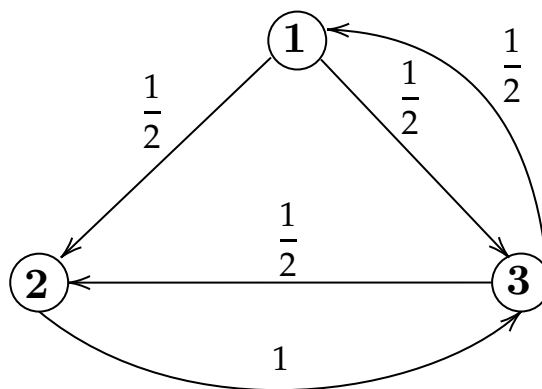A. If there is a sqare matrix A such that $A^2 + A = 0$, then $det(A)$ must be either 0 or -1

B. If u is a solution of the system of linear equation $Ax = c$ and c is a solution of system of linear equation $Ax = b$, then u is a solution of the system of linear equations $A^2x = b$

C. If B is a diagonal matrix of order 3, then $AB - BA = 0$ for all square matrices of order 3

D. If there is an invertible real 3×3 matrix A such that $A\,adj(A) = 3I$, then $det(adj(A))$ must be 9

==

$Q5.$ A system can be in one of 3 possible state at a given time. At the next instant, it changes its state as represented pictorially in the diagram below. The number beside an arrow shows the transition probabilities from the beginning state of the arrow to the ending state of the arrow (e.g. in the diagram M2Q1:1, you can see that there is an arrow starting at state 1 and ending at state 2, with the number $\dfrac{1}{2}$ beside the arrow. It implies that the probability of transition from state 1 to state 2 is $\dfrac{1}{2}$). No arrow from state 2 to state 1 indicates that the direct transition is not possible (equivalently the transition probability is 0). The probability of transition from a state to itself is 0

M2Q1:1

The information in the diagram is represented by the matrix $P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$, where the

$i - j - th$ entry of P denotes the probability of transition from state i to state j. Let the probability that the system is in State 1, State 2 or State 3 initially (i.e. at $t = 0$) be X_0^1, X_0^2 and X_0^3 respectively. This is represented by the initial distribution vector (3×1

matrix) and is denoted by $X_0 = \begin{bmatrix} X_0^1 \\ X_0^2 \\ X_0^3 \end{bmatrix}$. For any positive integer $n,$ the distribution vector

at $t = n$ is denoted by X_n and is given by the equation $P^T X_{n-1} = X_n$.
Answer the given subquestions from the given information.

Sub Questions:

Q1) Suppose at $t = 2$ the distribution vector X_2 is $\begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \\ 2 \\ \frac{2}{3} \end{bmatrix}$. Which of the following are

true?

Options:

A. $X_0 = X_2$
B. $X_0 = X_1$
C. $X_0 \neq X_n$ for some $n \in \mathbb{N}$
D. There are infinitely many vectors, which are possible candidates for X_0.
E. There are infinitely many vectors, which are possible candidates for X_1.

Q2) Suppose at $t = 1$ the distribution vector X_1 is $\begin{bmatrix} \frac{1}{2} \\ 1 \\ \frac{2}{0} \end{bmatrix}$. Which of the following

options is/are true?

Options:

A. The system had positive initial probabilities of being in State 1 or State 2.
B. The system was initially in State 3.
C. The system was initially in State 1.
D. The system had positive initial probabilities of being in State 2 and State 3.

Q3) Choose the set of correct option(s).

Options:

A. Both P and P^2 have the same reduced row echeleon form

B. P is already in reduced row echeleon form

C. $P^2 = \lambda P$ for some real number λ.

D. P^2 is already in reduced row echeleon form.

Jan 2023

Q6. Under which of the following conditions on a and b does the system of linear

equations $\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$ have no solution?

Options:

A. $a = -3,\ b \neq \dfrac{1}{3}$

B. $a = \dfrac{2}{3},\ b \neq \dfrac{1}{3}$

C. $a \neq \dfrac{1}{4},\ b = \dfrac{1}{3}$

D. $a \neq -3,\ b \neq \dfrac{1}{3}$

May 2023

$Q7$. Consider the system of linear equations represented in the matrix form $Ax = b$,

where $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 3 & \alpha \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ 3 \\ \beta \end{pmatrix}$. What is the value of $\alpha + \beta$ if the above system has infinitely may solutions ?

===

$Q8$. Shivani, Shruti and Smriti enjoyed shopping on a Sunday, Shivani bought 2 shirts, a T-shirt and 2 pants, whereas Shruti bought a T-shirt and a pant and Smriti bought 2 shirts and a pants. They paid $Rs.\ 600$, $Rs.\ 400$ and $Rs.\ 300$ respectively. Suppose x_1 is the price of a shirt, x_2 is the price of a T-shirt and x_3 is the price of a pant. Then the above information forms a system of linear equations. If $Ax = b$ is the matrix representation of the above system, where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is the vector that represents the price of a shirt, T-shirt and a pant respectively, answer the given subquestions

Sub Questions:

Q1) Choose the correct option(s):

Options:

A. $A = \begin{pmatrix} 2 & 0 & 2 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}, b = \begin{pmatrix} 600 \\ 400 \\ 300 \end{pmatrix}$

B. $A = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix}, b = \begin{pmatrix} 600 \\ 400 \\ 300 \end{pmatrix}$

C. $A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}, b = \begin{pmatrix} 600 \\ 400 \\ 300 \end{pmatrix}$

D. $A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix}, b = \begin{pmatrix} 600 \\ 400 \\ 300 \end{pmatrix}$

Q2) How many solutions does the given system $Ax = b$ have?

Sept 2023

$Q9$. Consider the system of linear equations $x + ky = 2,\ kx + y = 2$.

Based on the above data, answer the given subquestions.
Sub questions:

    Q1) Find the values of k for which the system of equations has no solution.

    Q2) Find the values of k for which the system of equations has infinitely many solutions.

    Q3) If the system has a unique solution $(a,\ b)$, what is $a - b$?

Jan 2024

$Q10$. Select all the true statements.

Options:

A. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is in reduced row echelon form.

B. The reduced row echelon form of any square, invertible matrix is the identity matrix of same order.

C. Elementary row operations can be performed only on square matrices.

D. If a matrix is in reduced row echelon form, its last row will always be a zero row.