Jan 2022

$Q1$. From the list of given terms find out the best possible options for each of the given subquestions:

1) Rank 2) Limit 3) Determinant 4) Closure with respect to addition and scalar multiplication 5) Existence of zero element 6) Existence of additive inverse 7) Communitative of addition 8) Associativity of addition 9) Elements 10) Local maxima 11) Local minima 12) Saddle points 13) Gradient 14) Directional Derivative 15) Partial Derivative 16) Set of orthonoraml vectors 17) Standard ordered basis 18) Affine spaces

Similar matrices have the same ___________. (Enter 2 best possible options. Enter only the serial
numbers of those options in increasing order without adding any comma or space in between them.) [Suppose your answer is 7, 14 and 17, then you should enter 71417]

May 2022

$Q2$. An inner prdouct on a vector space V is a function $\langle \cdot\ ,\ \cdot \rangle : V \times V \to \mathbb{R}$ satisfying the following conditions:

Condition 1: $\langle v, v \rangle > 0$ for all $v \in V \setminus \{0\}$; $\langle v, v \rangle = 0$ if and only if $v = 0$
Conditon 2: $\langle v_1 + v_2, v_3 \rangle = \langle v_1, v_3 \rangle + \langle v_2, v_3 \rangle$
Condition 3: $\langle v_1, v_2 \rangle = \langle v_2, v_1 \rangle$
Condition 4: $\langle cv_1, v_2 \rangle = c\langle v_1, v_2 \rangle$

Let $V = \mathbb{R}^2$ and consider the function defined as:

$$\langle \cdot, \cdot \rangle : V \times V \to \mathbb{R}$$
$$\langle (x_1, x_2), (y_1, y_2) \rangle = x_1y_1 - 4x_2y_2$$

Which of the following are satisfied by the above function?

Options:

A. Condition 1 is satisfied
B. Condition 2 is satisfied.
C. Condition 3 is satisfied.
D. Condition 4 is satisfied.

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$Q3$. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ -1 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 3 & 4 \\ 0 & 2 & 1 \\ -1 & 1 & 3 \end{pmatrix}$. Choose the correct option(s).

Options:

A. A is not equivalent to
B. A is equivalent to B
C. A is similiar to B
D. A is not similiar to B

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$Q4$. Which of the following option is/are true?

Options:

A. If L and L' are two affine subspaces of $\mathbb{R}^3$, then $L \cap L'$ is an affine subspace of $\mathbb{R}^3$

B. Let $Ax = b$ be a system of linear equations with infinitely many solutions and let P be an invertible matrix such that product PA is well defined. Then the system of linear equations $PAx = b$ has infinitely many solutions.

C. Let A and B be two matrices such that reduced echeleon form of A and B are the same. Then $rank(A) > rank(B)$.

D. Let $Ax = 0$ be a system of linear equations such that reduced echeleon form of the cofficient matrix A is the identity matrix. Then the system $Ax = 0$ has infinitely many solutions.

Sept 2022

$Q5$. Consider the affine subspace $L_1 = \left\{(x, y, z) \in \mathbb{R}^3 : x + y + 2z = 1\right\}$ and $L_2 = \left\{(x, y, z) \in \mathbb{R}^3 : z = 3\right\}$. Let W_1 and W_2 be the subspces corresponding to L_1 and L_2 respectively. Choose the correct options:

Options:

A. $W_1 = span\{(1, -1, 0), (0, 1, 2)\}$
B. $W_2 = span\{(1, 0, 0), (0, 1, 0)\}$
C. $W_1 \cap W_2$ represents a line passing through the origin in $\mathbb{R}^3$.
D. $W_1 \cap W_2$ represents a plane passing through the origin in $\mathbb{R}^3$.

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$Q6$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x, y) = (x + y, x - y)$. Choose the correct option(s) about T.
Let A be the matrix representation of T with respect to a basis $\{(1, 1), (1, 2)\}$ for the domain and $\{(1, 1), (1, -1)\}$ for the co-domain and B be the matrix representation of T with respect to the standard ordered basis for $\mathbb{R}^2$ for both the domain and co-domain. Choose the correct option(s)

Options:
A. Rank of the matrix A is 1.
B. A and B are equivalent matrices.
C. Both the systems $Ax = 0$ and $Bx = 0$ have a unique solution.
D. A and B are similar matrices.

Jan 2023

$Q7$. Let V_1, V_2 and V_3 represent the subspaces $\{(x,y,z)\ :\ 2x+y+z=0\}$; $\{(x,y,z)\ :\ x+2y+z=0\}$ and $\{(x,y,z)\ :\ x+y+2z=0\}$, respectively of $\mathbb{R}^3$. Let $A_i,\ i=1,2,3$ be the affine space corresponding to $V_i,\ i=1,2,3$. Suppose A_i contains $(1,1,-2)$, A_2 contains $(-2,1,1)$ and A_3 contains $(1,-2,1)$. If (x_1,x_2,x_3) is the point of intersection of A_1, A_2 and A_3, find $x_1+2x_2+x_3$

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$Q8$. Let U denote the set of all 2×2 upper triangular matrices. Consider an ordered basis $\beta=\left\{\begin{pmatrix}1&0\\0&0\end{pmatrix},\begin{pmatrix}0&1\\0&0\end{pmatrix},\begin{pmatrix}0&0\\0&1\end{pmatrix}\right\}$. Let $T:U\to\mathbb{R}^2$ be a linear transfromation defined as $T\begin{pmatrix}a&b\\0&c\end{pmatrix}=(a+b,c)$. Let matrix A be the ,atrix representation of T with respect to the ordered basis β for U and the standard basis for the co-domain $\mathbb{R}^2$.
Let B be the matrix representation of T with respect to the ordered basis β for the domain and an ordered basis $\{(1,0),(1,1)\}$ for the oo-domain $\mathbb{R}^2$. Which of the following is/are true?

Options:

A. A is equivalent to B.
B. Rank of B is 3.
C. Nullity of B is 1.
D. A is not equivalent to B.

May 2023

$Q9$. Let $W = \{(x, y, z) \in \mathbb{R}^3 : 2x - y - z = 3\}$. Then which of the following affine subspaces of $\mathbb{R}^3$ is/are same as subspaces W?

Options:

A. $(6, 0, 0) + U$, where $U = \{(x, y, z) \in \mathbb{R}^3 : 2x = y + z\}$

B. $(0, -3, 0) + U$, where $U = \{(x, y, z) \in \mathbb{R}^3 : y = 2x - z\}$

C. $(0, 0, -3) + U$, where $U = \{(x, y, z) \in \mathbb{R}^3 : z = 2x - y\}$

D. $(0, -3, 0) + U$, where $U = \{(x, y, z) \in \mathbb{R}^3 : 2x = y - z\}$

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$Q10$. Which of the following statement(s) about similar matrices is/are true?

Options:

A. Let $T : V \to W$ be a linear transformation. If A and B are two diffferent matrices corresponding to T, then A and B must be similiar.

B. If A and B are $n \times n$ matrices such that $det(A) = det(B) = 1$, then A and B are similiar matrices.

C. If A is similiar to identity matrix, the A is a scalar matrix.

D. If A is invertible, then AB and BA are similiar.

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$Q11$. Conisder the following matrices

$A_1 = \begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}$, $A_2 = \begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix}$, $A_3 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, and $A_4 = \begin{pmatrix} -5 & -3 \\ 6 & 5 \end{pmatrix}$

Choose the correct option(s) from the following:

Options:

A. A_1 and A_2 are similiar matrices.

B. A_1 and A_3 are equivalent but not similiar matrices.

C. $A_4 = P^{-1}A_2P$ where $P = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$.

D. A_2 and A_4 are equivalent but not similiar matrices.

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$Q12$. Let L_U denote the affine subspace with the associated subspace U of $\mathbb{R}^3$.

	Affine Subsoace L_U		Associated subsoace U		Dimension of L_U
(i)	$\{(x, y, z) \in \mathbb{R}^3 : x - y + z = 2,$ $x + z = 1\}$	(1)	$span\{(1, 0, -1), (2, 1, 0)\}$	(a)	2
(ii)	$\left\{(x, y, z) \in \mathbb{R}^3 : x - 2y + z = 3\right\}$	(2)	$span\{2, -1, 3\}$	(b)	1
(iii)	$\{(x - 1, y + 1, z) \in \mathbb{R}^3 :$ $x + 2y = 0,\ 3y + z = 0\}$	(3)	Nullspace of $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$	(c)	1

Table: M2ES2

Based on the above data, answer the given subquestions.

Sub Questions:

Q1) Choose the correct option to match the affine subspace of Row 1 with the associated subspaces and dimension.

Options:

A. $(i) \to (3) \to (b)$
B. $(i) \to (2) \to (a)$
C. $(i) \to (2) \to (c)$
D. $(i) \to (1) \to (a)$

Q2) Choose the correct option to match the affine subspace of Row 2 with the associated subspaces and dimension.

Options:

A. $(ii) \to (3) \to (a)$
B. $(ii) \to (1) \to (a)$
C. $(ii) \to (2) \to (c)$
D. $(ii) \to (1) \to (b)$

Q3) Choose the correct option to match the affine subspace of Row 3 with the associated subspaces and dimension.

Options:

A. $(iii) \to (3) \to (a)$
B. $(iii) \to (2) \to (c)$
C. $(iii) \to (2) \to (a)$
D. $(iii) \to (3) \to (b)$

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$Q13.$ Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ and $A = \begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ be the matrix corresponding to the linear transformation T with respect to the bases $\beta = \{(1,0,0), (0,1,0), (0,0,1)\}$ and $\gamma = \{(1,1,0),\ (0,1,1), (1,0,1)\}$.

Which of the following statement(s) about T is/are true?

Options:

A. T is one-one but not onto.
B. T is neither one-one nor onto.
C. Matrices corresponding to T with respect to any basis has determinant zero.
D. There exist a basis β of $\mathbb{R}^3$ such that the matrix corresponding to T with respect to β for both domain and co-domain is invertible

Sept 2023

$Q14$. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is an isomorphism. Let A be the matrix representation of T with respect to the standard ordered basis $\mathcal{B} = \{e_1, e_2, e_3\}$ for both domain and co-domain and B be the matrix representation of T with respect to the basis $\mathcal{B}' = \{e_1, e_1 - e_2, e_2 - e_3\}$ for both domain and co-domain.
Choose the correct option(s) from the following:

Options:

A. A^{-1} and B^{-1} are equivalent but not similair matrices

B. The matrix A is invertible but B is not invertible

C. A^{-1} and B^{-1} are similiar matrices

D. AB is similiar to BA