Sept 2021

$Q1$. Choose the correct options.

Options:

A. Product of two orthogonal matrices is orthogonal.

B. Suppose A is a non-zero $m \times n$ matrix such that the vectors in $\mathbb{R}^m$ corresponding to the column of A are mutually orthonormal with respect to the usual inner product of $\mathbb{R}^m$. Then $A^T A = I$, where I is the identity matrix of order n.

C. Sum of two orthognal matrices is orthgonal

D. Suppose A is a non-zero $m \times n$ matrix such that the vectors in $\mathbb{R}^m$ corresponding to the column of A are mutually orthogonal with respect to the usual inner product of $\mathbb{R}^m$. Then AA^T is a diagonal matrix of order m

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$Q2$. Let u and v be two vectors in $\mathbb{R}^2$ with usual inner product (i.e. dot product), such that u is a non-zero scalar multiple of $(2, 3)$ and v is orthogonal to the vector $(2, 3)$ and $u + v = (2, 2)$

Based on the above information answer the given subquestions .

Sub Questions:

Q1) Find the value of $\langle u, v \rangle$

Q2) Find the value of $||u||^2 + ||v||^2$

Jan 2022

$Q3$. Consider the following set $S = \{(1,1,1), (-2,1,1), (0,1,-1)\}$. Which of the following options are true for S ?

Options:

A. The cardinality of S is equal to the number of elements in any basis of $\mathbb{R}^3$.

B. S is a linearly independent set.

C. S spans $\mathbb{R}^3$ (with respect to scalar addition and scalar multilplication).

D. S is a basis of $\mathbb{R}^3$ (with respect to scalar addition and scalar multilplication).

E. S is an orthogonal set with respect to usual inner product i.e. dot product on $\mathbb{R}^3$.

F. S is an orthonormal set with respect to usual inner product i.e. dot product on $\mathbb{R}^3$.

May 2022

$Q4$. Choose the correct options.

Options:

A. The row reduced echeleon form of an $n \times n$ orthogonal matrix is the identity matrix of order n.

B. Suppose that A is a non-zero $m \times n$ matrix such that the vectors in $\mathbb{R}^m$ corresponding to the columns of A are mutually orthonormal with respect to the usual inner product of $\mathbb{R}^m$. Then $A^T A = I$, where I is the identity matrix of order n.

C. The trace of an $n \times n$ orthogonal matrix is 0.

D. Suppose that A is a non-zero $m \times n$ matrix such that the vectors in $\mathbb{R}^m$ corresponding to the columns of A are mutually orthogonal with respect to the usual inner product of $\mathbb{R}^m$. Then $A^T A$ is a diagonal matrix of order m.

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$Q5$. Suppose two publications houses (publication house A and publication house B) have organized a sale of their books. Both of them publish three types of books, novels, poetry collection and collection of short stories. The selling price (in hundreds ₹) of these three types of books in publication house A and B are given as follows:

	Novels	Poetry Collection	Collection of Short Stories
Publication house A	1	2	5
Publication house B	3	3	3

Table: Q2M2T1

The publication houses announced that in order to avail these special sale prices, customers have to buy equal number of novels, equal number number of poetry collection and equal number of collection of short stories from each of the publication houses (i.e. if a customer buys x number of novels, y number of poetry collection and z number of short stories from

Publication house A; then they have to buy exactly x number of novels, y number of poetry collections and z number of collection of short stories from Publication house B, to avail the benefitof the sale). So there is a map taking the tuple consisting of number of books of each type bought (Novels, Poetry collections, Collection of short stories) to the prices paid by customers who availed the sale to each of the publication houses, which yields a linear transformation (T) from $\mathbb{R}^3$ to $\mathbb{R}^2$ (where the first and second co-ordinates of the image denotes the prices paid to publication house A and publication house B, respectively).
A is the matrix representation of T with respect to the basis $\{(1,0,0),(0,1,0),(0,0,1)\}$ for $\mathbb{R}^3$ and to the basis $\{(1,0),(0,1)\}$ for $\mathbb{R}^2$ then,
Answer the subquestions using the above information.

Q1) Let $\beta = \{v_1, v_2\}$ be the orthonormal basis of the row space obtained using the GramSchmidt process (with respect to usual inner product) applied on the ordered basis of the row space given by the first and second row of the matrix A. If

$$v_2 = \frac{1}{\sqrt{195}}(b, c, d)$$

What is the value of $||30v_1||$?

Q2) Let $\beta = \{v_1, v_2\}$ be the orthonormal basis of the row space obtained using the GramSchmidt process (with respect to usual inner product) applied on the ordered basis of the row space given by the first and second row of the matrix A. If

$$v_2 = \frac{1}{\sqrt{195}}(b, c, d)$$

What is the value of b ?

Q3) Let $\beta = \{v_1, v_2\}$ be the orthonormal basis of the row space obtained using the GramSchmidt process (with respect to usual inner product) applied on the ordered basis of the row space given by the first and second row of the matrix A. If

$$v_2 = \frac{1}{\sqrt{195}}(b, c, d)$$

What is the value of c ?

Q4) Let $\beta = \{v_1, v_2\}$ be the orthonormal basis of the row space obtained using the

GramSchmidt process (with respect to usual inner product) applied on the ordered basis of the row space given by the first and second row of the matrix A. If

$$v_2 = \frac{1}{\sqrt{195}}(b, c, d)$$

What is the value of d ?

Sept 2022

$Q6$. Let W be a proper subspace of an inner product space V , where $dim(V) = 3$ and P_W be the
projection of V on W. Answer the subquestion based on the given data.

Sub Questions:

Q1) If $v \in V$ is vector of norm 5, then the maximum possible norm of the vector $P_W(v)$ is

Q2) Which of the following option is/are true?

Options:

A. Let $v \in V$, then $v - P_W(v)$ is orthogonal to W.

B. If dimension of W is 2, then dimension of the null space of P_W may not be 1.

C. Zero vector is orthogonal to every vector of V.

D. If $v \in W$, then $P_W(v) = v$.

Jan 2023

$Q7$. Let A be an $n \times n$ orthogonal matrix. Choose the correct option(s).

Options :

A. A is invertible.

B. $det(A) = \pm 1$.

C. $det(A)$ may be zero.

D. Nullity of A may be 1

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$Q8$. Le W be the subspace of $\mathbb{R}^3$ with the standard inner product, spanned by the ordered set $\beta = \{(1,-1,0),(0,1,1)\}$

Based on the above data, answer the given subquestions.

Sub Questions:

Q1) If $\left\{\frac{w_1}{||w_1||}, \frac{w_2}{||w_2||}\right\}$ denote the orthonormal basis of W obtained by applying the GramSchmidt process on β, what is $2||w_2||^2$?

Q2) Let P_W denote the projection of $\mathbb{R}^3$ onto W. If $P_w(1,0,1) = (a,b,c)$. What is $a+b+c$?

May 2023

$Q9$. Let A be a 3×3 rotation matrix. Choose the correct option(s)

Options:

A. The rows of A are orthogonal

B. A is an orthogonal matrix

C. The column of A are not orthonormal

D. $det(A) = 0$

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$Q10$. Let W be the subspace of $\mathbb{R}^4$ with standard inner product, spanned by the ordered set $\beta = \{(1, -1, 0, 0),\ (0, 1, 1, 0),\ (0, 1, 1, 0)\}$. Let $\{v_1, v_2\}$ denote the orthonormal basis of W obtained by applying the Gram-Schmidt process on β.
Based on the above data, answer the given subquestions.

Sub Questions:

Q1) Let $P_W : \mathbb{R}^4 \to W$ denote the projection map. What is the nullity of P_W ?

Q2) If $P_W(0, 1, 0, 1) = (a, b, c, d)$, what is $3(a + b + c + d)$?

Sept 2023

$Q11$. Let A be a $n \times n$ orthogonal matrix. Then which of the following statement(s) is/are true?
Options :

A. The rows of A form an orthonormal basis for $\mathbb{R}^n$

B. Suppose T is the linear trasnformation corresponding to A, then $||Tv|| = ||v||$ for any $v \in \mathbb{R}^n$

C. The system of linear equations $Ax = b$ has a unique solution for every $b \in \mathbb{R}^n$

D. The rows of A form an orthogonal basis but not an orthonormla basis for $\mathbb{R}^n$

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$Q12$. Let $W = \{(x, y, z) \, : \, x + 2y - z = 0\}$
If γ is the orthonormal basis of W obtained from the basis $\beta = \{(1, 0, 1), (0, 1, 2)\}$ by using the Gram Schmidt process with respect to the usual inner product and (a, b, c) is the projection of $(1, 3, 1)$ onto W, then what is $a + b + c$?