

**Jan 2022**

Q1. Let  $A$  be a  $3 \times 2$  non-zero real matrix.

The maximum value of  $\text{nullity}(A)$  is \_\_\_\_\_.

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Q2. Anamila, Subhasis and Shreya pool together  $x, y$  and  $z$  amounts of money (in thousands) respectively, every month. The sum is distributed across three accounts  $A_1, A_2$  and  $A_3$  as  $x + y + z, z - 2y$  and  $2y - z$  respectively. This can be thought of as a linear transformation

$$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

defined by

$$T(x, y, z) = (x + y + z, z - 2y, 2y - z)$$

Note: A negative amount of money signifies the amount withdrawn from the accounts. Answer the subquestions from the information given above.

### Sub Questions:

**Q1)** Which of the following vector spaces consists of vectors which could denote the amount of money deposited by Anamika, Subhasis and Shreya in a particular month such that in that month the amount deposited is 0 in each of the accounts A1 , A2 and A3.

### Options:

A.  $\text{span}\{(-3t, t, 0), (0, t, 2t) \mid t \in \mathbb{R}\}$

B.  $\text{span}\{(-3t, t, 2t) \mid t \in \mathbb{R}\}$

C.  $\text{span}\{(3t, -t, 2t) \mid t \in \mathbb{R}\}$

D.  $\text{span}\{(3t, -t, 0), (0, -t, 2t) \mid t \in \mathbb{R}\}$

**Q2)** Find out  $\text{nullity}(T)$ .

**Q3)** Find out  $\text{rank}(T)$ .

**Q4)** Which of the following options is true?

**Options:**

- A.  $T$  is one to one
- B.  $T$  is onto
- C.  $T$  is both one to one and onto
- D.  $T$  is neither one to one nor onto

**Sept 2022**

Q3. Which of the following options is/are true?

**Options :**

- A. If  $A$  is a non-zero matrix of order  $4 \times 3$  and rank of  $A$  is 3, then the rows of  $A$  are linearly independent.
- B. If  $A$  is a non-zero matrix of order  $4 \times 3$  and rank of  $A$  is 3, then the columns of  $A$  are linearly independent.
- C. If  $A$  is a non-zero matrix of order  $m \times (m + 1)$ ,  $m > 1$ , then the maximum possible nullity of  $A$  is  $m$ .
- D. If  $A$  is a non-zero matrix of order  $4 \times 5$  and rank of  $A$  is 3, then the dimension of the solution space of the homogeneous system  $Ax = 0$  is 2.

**Jan 2023**

Q4. Consider the vector space  $V = \left\{ \begin{pmatrix} a & b \\ a & b \end{pmatrix} \mid a, b \in \mathbb{R} \right\}$  and  $T: \mathbb{R}^3 \rightarrow V$  defined by

$$T(x, y, z) = \begin{pmatrix} x + y & x + y + z \\ x + y & x + y + z \end{pmatrix}. \text{ Choose the correct option}$$

**Options :**

- A.  $T$  is onto but not one-one
- B.  $T$  is one-one but not onto.
- C.  $T$  is both one-one and onto
- D.  $T$  is neither one-one nor onto.

**May 2023**

Q5. If  $A$  is a  $2 \times 3$  matrix of rank 1, then what is the nullity of  $AA^T$  ?

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Q6. Which of the following options is/are true?

**Options:**

- A. There exists an onto linear transformation  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$
  - B. There does not exist a one-one linear transformation  $T: \mathbb{R}^3 \rightarrow \mathbb{R}$
  - C. There exists a linear transformation  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  such that  $\text{rank}(T) = \text{nullity}(T)$
  - D. There does not exist a linear transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  such that  $\text{rank}(T) = \text{nullity}(T)$
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Q7. Let  $V_1$  denote the vector space of solution of  $AX = 0$ , where  $A = \begin{pmatrix} 2 & 1 & 4 \\ -1 & 1 & 0 \\ 1 & 2 & 4 \end{pmatrix}$  and

$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ . Let  $V_2$  denote the vector space of solutions of the system  $BY = 0$ , where

$B = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 2 & 3 \end{pmatrix}$  and  $Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ . Answer the given subquestions

**Sub Questions:**

**Q1)** What is the nullity of  $A$ ?

**Q2)** What is the rank of  $B$ ?

**Q3)** Which of the following forms a basis  $\beta$  for  $V_1$ ?

**Options:**

A.  $\{(1, 5, 2)\}$

B.  $\left\{\left(\frac{-4}{3}, \frac{-4}{3}, 1\right)\right\}$

C.  $\left\{\left(\frac{1}{5}, 1, \frac{2}{5}\right)\right\}$

D.  $\{(-4, -4, 3)\}$

**Q4)** Define a linear transformation  $T: V_2 \rightarrow \mathbb{R}^2$  by  $T(x, y, z) = (x, x + y + z)$ . What is the rank of  $T$ ?

**Sept 2023**

*Q8.* Which of the following functions are linear transformations?

**Options:**

- A.  $T: \mathbb{R} \rightarrow \mathbb{R}$ ,  $T(x) = 2x + 1$
  - B.  $T: \mathbb{R}^4 \rightarrow \mathbb{R}^2$ ,  $T(x, y, z, w) = (x + y, z + w)$
  - C.  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ,  $T(x, y, z) = (-y, -x, 0)$
  - D.  $T: \mathbb{R} \rightarrow \mathbb{R}^2$ ,  $T(x) = (x + 1, x - 1)$
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*Q9.* Let  $T$  be a linear transformation from  $\mathbb{R}^2$  to  $\mathbb{R}^3$ .  $S \subset \mathbb{R}^2$  is a line passing through the origin. Which of the following are possible?

**Options:**

- A.  $T(S)$  could be the origin in  $\mathbb{R}^3$
  - B.  $T(S)$  could be a line passing through the origin in  $\mathbb{R}^3$
  - C.  $T(S)$  could be a plane passing through the origin in  $\mathbb{R}^3$
  - D.  $T(S)$  could be  $\mathbb{R}^3$
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*Q10.* Consider the following system of linear equations:

$$\begin{aligned}x + 3y - 2z &= 0 \\y - z &= 0 \\x + y &= 0\end{aligned}$$

Let  $A$  be the coefficient matrix corresponding to this system

Based on the above data, answer the given subquestions.

**Sub questions:**

**Q1)** Which of the following is the nullspace of  $A$ ?

**Options :**

- A.  $\text{span}\{(-1, 1, 1)\}$
- B.  $\text{span}\{(1, 1, 0)\}$
- C.  $\text{span}\{(1, 0, 1), (0, 1, -1)\}$
- D.  $\text{span}\{(1, 1, 0), (0, 1, -1)\}$

**Q2)** Let  $B$  be a square matrix of order 3. What is the smallest value that the nullity of  $BA$  could take?

