May 2021

$Q1$. Consider the following two statements:

P: If $V = \mathbb{R}^2$, with the operations:
Addition:

$$(x_1, y_1) + (x_2, y_2) = (x_1 x_2, y_1 y_2)\ ;\ (x_1, y_1),\ (x_2, y_2)\ \in V$$

and

Scalar Multiplication:

$$c(x, y) = (cx, cy);\ (x, y) \in V,\ c \in \mathbb{R}$$

is a vector space.

Q: Let V be a vector space w.r.t usual addition and scalar multiplication . If $u, v, w \in V$ are such that $au + bv + cw = 0$ for some scalars $a, b, c \in \mathbb{R}$ and $ac \neq 0$, then $span\{u, v\} = span\{v, w\}$

Consider the following statements:

- **Statement 1:** P is true, but Q is false
- **Statement 2:** Q is true, but P is false
- **Statement 3:** Both P and Q are true
- **Statement 4:** Both P and Q are false

Which one of the above statement is correct? (If Statement 3 is correct, then enter 3 as your answer)

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$Q2$ Match the vector spaces (with the usual scalar multiplication and vector addition as in $M_{2\times 2}(\mathbb{R})$) in column A with their bases in column B in Table: M2Q2F1.

	Vector Space		Basis
a)	$V = \left\{ \begin{bmatrix} x & y \\ y & z \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$	1)	$\left\{ \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 5 & 0 \end{bmatrix} \right\}$

b)	$V = \left\{A \in M_{2\times2}(\mathbb{R}),\ A \text{ is a lower triangular matrix}\right\}$	2)	$\left\{\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 5 & 0 \\ -5 & 0 \end{bmatrix}\right\}$
c)	$V = \left\{\begin{bmatrix} x & y \\ w & z \end{bmatrix} \mid x + w = 0 \text{ and } x, y, z, w \in \mathbb{R}\right\}$	3)	$\left\{\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}\right\}$

Table: M2Q2F1

Using the above table answer the given subquestions

Sub Questions:

Q1) The basis (in Column B) of the vector space in the row a of Column A is

[If the basis of the vector space (a) of Column A has the basis with the serial number 1 in column B
then enter 1 (enter the serial number only) as your answer.]

Q2) The basis (in Column B) of the vector space in the row b of Column A is

[If the basis of the vector space (b) of Column A has the basis with the serial number 1 in column B
then enter 1 (enter the serial number only) as your answer.]

Q3) The basis (in Column B) of the vector space in the row c of Column A is

[If the basis of the vector space (c) of Column A has the basis with the serial number 1 in column B
then enter 1 (enter the serial number only) as your answer.]

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$Q3$. Three people denoted by $P_1,\ P_2, P_3$ intended to buy some rolls, buns and cakes from a shop. They pay ₹x_1, ₹x_2, ₹x_3 per unit for rolls, buns and cakes, respectively. P_1 bought 1 unit of rolls, 2 unit of buns, and 3 units of cales, P_2 bought 2 unit of rolls, 4 units of buns, and 5 units of cakes and P_3 bought 1 unit of rolls, 3 unit of buns and 4 units of cakes. The total amount spend by $P_1,\ P_2$ and P_3 are ₹$12,$ ₹21 and ₹16 respectively. Represent the system of linear equations given in terms of its matrix $Ax = b$, where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $b = \begin{bmatrix} 12 \\ 21 \\ 16 \end{bmatrix}$.

Consider the vector space V spanned by the column vector of A, viewed as vectors in $\mathbb{R}^3$ with usual addition and scalar multiplication. Which of the following options are correct?

Options:

A. $\{(1,2,1),(2,4,3),(3,5,4)\}$ is a linearly independent set in V

B. $\{(1,2,1),(2,4,3),(3,5,4)\}$ is a linearly dependent set in V

C.$\{(1,2,1),(2,4,5),(1,3,4)\}$ is a linearly independent set in V

D.$\{(1,2,1),(2,4,5),(1,3,4)\}$ is not a subset of V

Jan 2022

$Q4$ Consider the vector space $\mathbb{R}^3$ with usual addition and scalar multiplication. Let S be its subset deifned as follows:

$$S = \{(1,0,1),(0,1,1)\}$$

and W_1 and W_2 be its vector subspaces defined as:

$$W_1 = \{(x,y,z) \mid x = z,\ and\ y = z;\ x,y,z \in \mathbb{R}\}$$

$$W_2 = \{(x,y,z) \mid z = x + y;\ x,y,z \in \mathbb{R}\}$$

Answer the given suquestions

Q1) Which of the following opition(s) is(are) true?

Options:

A. S is linearly independent

B. S spans the vector space $\mathbb{R}^3$ (with ususal addition and scalar multiplication)

C. $span(S)$ is a proper subspace of $\mathbb{R}^3$

Q2) Which of the following option is (are) correct?

Options:

A. S is a basis of W_1

B. S is a basis of W_2

Q3) What is the dimension of W_1?

Q4) What is the dimension of W_2 ?

May 2022

$Q5$. If $W = span\{(1,1,-1),(3,-2,0),(5,0,-2),(0,5,-3)\}$, then find out the dimension of W

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$Q6$. Consider the vector space $\mathbb{R}^3$ with respect to usual addition and scalar multiplication i.e.,

Addition: $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2);$
$$(x_1, y_1, z_1), (x_2, y_2, z_2) \in V$$
Scalar Multiplication: $c(x, y, z) = (cx, cy, cz);\ (x, y, z) \in V,\ c \in \mathbb{R}$

Suppose W_1 and W_2 are two vector subspaces of $\mathbb{R}^3$(with respect to usual addition and scalar multiplication) defined as follows:

$$W_1 = \{(x, y, 0) \mid x, y \in \mathbb{R}\}$$

and

$$W_2 = \{(0, y, 0) \mid y \in \mathbb{R}\}$$

with usual addition and scalar multiplication.

Based on the above data, answer the given sub questions

Q1) What is the dimension of $W_1 \cap W_2$?

Q2) What is the dimension of W_1 ?

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$Q7$. From the list of given terms find out the best possible options for each of the given suquestions:

1) Transpose of a matrix
2) Determinant
3) Closure with respect to addition
4) Closure with respect to scalar multiplication
5) Existence of addititve inverse
6) Commutatuive of addition
7) Associativity of addition
8) Spanning set
9) Linearly independent set
10) Basis

Q1) $S = \{(1,0,1),(0,1,1),(1,1,0)\}$ is a _____________ of $\mathbb{R}^3$.(Enter 3 best possible options. Enter only the serial numbers of those options in increasing order without adding any comma or space in between them.)
[Suppose your answer is 7, 8 and 10, then you should enter 7810]

Q2) A spanning set of $\mathbb{R}^2$ with 2 elements must be a _____(Enter 2 best possible options. Enter only the serial numbers of those options in increasing order without adding any comma or space in between them.)
[Suppose your answer is 7 and 8, then you should enter 78]

Sept 2022

$Q8$. Consider the following two statements:

P: If $V = \mathbb{R}^2$, with the operations:
Addition:

$$(x_1, y_1) + (x_2, y_2) = (x_1x_2, y_1y_2)\ ;\ (x_1, y_1),\ (x_2, y_2)\ \in V$$

and

Scalar Multiplication:

$$c(x, y) = (cx, cy);\ (x, y) \in V,\ c \in \mathbb{R}$$

is a vector space.

Q: Let V be a vector space w.r.t usual addition and scalar multiplication . If $u, v, w \in V$ are such that $au + bv + cw = 0$ for some scalars $a, b, c \in \mathbb{R}$ and $ac \neq 0$, then $span\{u, v\} = span\{v, w\}$

Consider the following statements:

- **Statement 1:** P is true, but Q is false
- **Statement 2:** Q is true, but P is false
- **Statement 3:** Both P and Q are true
- **Statement 4:** Both P and Q are false

Which one of the above statement is correct? (If Statement 3 is correct, then enter 3 as your answer)

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$Q9$. Conisder the following subsets of $\mathbb{R}^3$.

Subset 1) $W = \left\{(x, y, z) \mid x, y, z \in \mathbb{R},\ \text{and } x^2 + z^2 = 0\right\}$
Subset 2) $W = \left\{(x, y, z) \mid x, y, z \in \mathbb{R},\ \text{and } x = z\right\}$
Subset 3) $W = \left\{(x, y, z) \mid x, y, z \in \mathbb{R},\ x = y + z \text{ and } x + z = y\right\}$
Subset 4) $W = \left\{(x, y, z) \mid x, y, z \in \mathbb{R},\ (x + 1) - (y + 1) + z = 0 \text{ and } x + z = y\right\}$

Based on the above data, answer the given subquestions.

Sub Questions:

Q1) Subset 1 is a subspace of dimension ________________. (Enter the numerical value only. Suppose the dimension is 3, then enter 3 as your answer.)

Q2) Subset 2 is a subspace of dimension _______________. (Enter the numerical value only. Suppose the dimension is 3, then enter 3 as your answer.)

Q3) Subset 3 is a subspace of dimension _______________. (Enter the numerical value only. Suppose the dimension is 3, then enter 3 as your answer.)

Q4) Subset 4 is a subspace of dimension _______________. (Enter the numerical value only. Suppose the dimension is 3, then enter 3 as your answer.)

==

$Q10$.Suppose W_1 and W_2 are subspaces of $\mathbb{R}^3$ defined as follows:

$$W_1 = \{(x, y, x + y) \mid x, y \in \mathbb{R}\}$$

and

$$W_2 = \{(x, y, 0) \mid x, y \in \mathbb{R}\}$$

with usual addition and scalar multiplication, i.e.,

Addition: $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2);$
$(x_1, y_1, z_1),\ (x_2, y_2, z_2) \in V$

Scalar Multiplication: $c(x, y, z) = (cx, cy, cz)\ ;\ (x, y, z) \in V,\ c \in \mathbb{R}$

Based on the above data, answer the given subquestions.

Sub Questions:

Q1) Which of the following options represent $W_1 \cap W_2$? (More than one options may be correct)

Options:
A. $Span\{(1, 1, 0), (1, -1, 0)\}$
B. $Span\{(-1, 1, 0), (1, -1, 0)\}$
C. $Span\{(1, -1, 0)\}$
D. $Span\{(1, 1, 2), (1, 1, 0)\}$

Q2) What is the dimension of $W_1 \cap W_2$?

Q3) Which of the following options is true?

Options:
A. $W_1 \cup W_2$ is a vector space of dimension 3 (with usual addition and scakar multiplication)

B. $W_1 \cup W_2$ is a vector space of dimension 2 (with usual addition and scakar multiplication)
C. $W_1 \cup W_2$ is a vector space of dimension 1 (with usual addition and scakar multiplication)
D. $W_1 \cup W_2$ is not a vector space (with usual addition and scakar multiplication)

Jan 2023

$Q11$. Find out the value of a for which the matrix $\begin{bmatrix} a & 3 \\ 0 & -5 \end{bmatrix}$ will be in the spanning set of the matrices $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ in $M_{2\times2}(\mathbb{R})$ with usual matrix addition and scalar multiplication.

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$Q12$. Let W be the set of 3×3 skew-symmetric real matrices, i.e.

$$W = \left\{A \in M_{3\times3}(\mathbb{R}) \mid A^T = -A\right\}$$

W is a vector subspace of $M_{3\times3}(\mathbb{R})$ with usual matrix addition and sclar multiplication. What is the dimension of W?

==

$Q13$. Two vector subspaces W_1 and W_2 of $\mathbb{R}^3$ are defined as follows:

$$W_1 = \{(x, y, 0) \mid x, y, \in \mathbb{R}\}$$

and

$$W_2 = \{(0, y, z) \mid y, z \in \mathbb{R}\}$$

with usual addition and scalar multiplication, i.e.,

Addition: $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_1, y_1 + y_2, z_1 + z_2);$
$(x_1, y_1, z_1),\ (x_2, y_2, z_2)\ \in \mathbb{R}^3$

Scalar multiplication : $c(x, y, z) = (cx, cy, cz)\ ;\ (x, y, z) \in \mathbb{R}^3, c \in \mathbb{R}$

What is the dimension of $W_1 \cap W_2$?

==

$Q14$. For some fixed real number $\alpha \neq 0$, define the set $V_\alpha = \{(x, \alpha, y) \mid x, y \in \mathbb{R}\}$ along with the following operations:

Addiiton: $(x_1, \alpha, y_1) + (x_2, \alpha, y_2) = (x_1 + x_2, \alpha, y_1 + y_2)$
$(x_1, \alpha, y_1), (x_2, \alpha, y_2) \in V_\alpha$

Scalar Multiplication: $c(x, \alpha, y) = (cx, \alpha, cy);\ (x, \alpha, y) \in V,\ c \in \mathbb{R}$

Consider the subset $S = \{(1, \alpha, 0), (0, \alpha, 1)\}$ of V_α. Which of the following are true ?

Options:

A. S is linearly independent

B. $Span(S) \neq V_\alpha$

C. S forms a basis of V_α

D. $dim(V_\alpha) = 3$

May 2023

$Q15$. What is the dimension of vector spaces for the given subquestions.

Sub Questions:

Q1) $V_1 = \left\{(x, y, z) \in \mathbb{R}^3 : 2x + 3y = 0 = 2z + 3x\right]$ with usual addition and scalar multiplication.

Q2)
$V_2 = \left\{A \in M_3(\mathbb{R}) : \text{sum of diagonal entries of } A \text{ is } 0 \text{ and sum of each row is } 0\right\}$ with usual additon and scalar multiplication of matrices.

==

$Q16$. Shivani, Shruti and Smriti enjoyed shopping on a Sunday, Shivani bought 2 shirts, a T-shirt and 2 pants, whereas Shruti bought a T-shirt and a pant and Smriti bought 2 shirts and a pants. They paid $Rs.\ 600$, $Rs.\ 400$ and $Rs.\ 300$ respectively. Suppose x_1 is the price of a shirt, x_2 is the price of a T-shirt and x_3 is the price of a pant. Then the above information forms a system of linear equations. If $Ax = b$ is the matrix representation of the above system, where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is the vector that represents the price of a shirt, T-shirt and a pant respectively, answer the given subquestions

Sub Questions:

Q1) Consider the set S of solutions of the system $Ax = 0$, where A is as given. Clearly, S is a vector space with respect to usual addition and scalar multiplication. What is the dimension of S ?

Q2) Which of the following forms a basis for S ?

Options:

A. $\left\{\left(\frac{1}{2}, 1, -1\right), (0, 1, -1)\right\}$

B. $\left\{\left(\frac{1}{2}, 1, -1\right)\right\}$

C. $\left\{\left(\frac{1}{2}, 1, 1\right), (0, 1, -1)\right\}$

D. $\left\{\left(\frac{1}{2}, 1, 1\right)\right\}$

Q3) What is the rank of A ?

Sept 2023

$Q17$. Let $W_1 = \left\{(x, y, z) \in \mathbb{R}^3 : 2x - y + z = 0\right\}$ and

$W_2 = \left\{(x, y, z) \in \mathbb{R}^3 : 2x - y + z = 0,\ 2x + y - 3z = 0\right\}$ and W_3 be the $xy - plane$.

Based on the above data, answer the given subquestions.

Sub questions:

Q1) Choose the correct option(s) from the following statements.

A. The set $\{(1, 0, -2), (0, 1, 1)\}$ forms a basis for W_1

B. W_2 is the set of all solutions of the system $AX = 0$ where $A = \begin{pmatrix} 2 & 2 \\ -1 & 1 \\ 1 & -3 \end{pmatrix}$

C. The intersection of W_1 and W_3 is the line $y = 2x$

D. The intersection of W_1 and W_3 is spanned by the vector $(2, 1, 0)$

E. W_2 is the straight line in $\mathbb{R}^3$ passing through the origin and the vector $(1, 4, 2)$

Q2) Find $dim(W_1 + W_2)$.

==

$Q18$. Let $W = \left\{A \in \mathbb{R}^{2\times 2} : A = -A^T\right\}$

Based on the above data, answer the given subquestions.

Sub questions:

Q1) Let $A \in W$ be a non-zero matrix. Then rank of A is

Q2) What is the dimension of the vector space W?

Q3) Which of the following sets form a basis for W?

Options:

A. $S = \left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right\}$

B. $S = \left\{ \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} \right\}$

C. $S = \left\{ \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right\}$

D. $S = \left\{ \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$

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$Q19$. Let $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$. What is the rank of A?

Jan 2024

$Q20$. Consider the following subsets of $M_{3\times3}(\mathbb{R})$.

Sub Questions:

Q1) $W_1 = \left\{ \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} : a, b, c \in \mathbb{R} \; such \; that \; a + b + c = 1 \right\}$.

If W_1 is a subspace, find the dimension else write the answer as 0

Q2) $W_2 = \left\{ \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} : a, b, c \in \mathbb{R} \; such \; that \; a = b = c \right\}$ If W_2 is a subspace, find the dimension else write the answer as 0.

Q3) $W_3 = \left\{ \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$ If W_3 is a subspace, find the dimension else write the answer as 0.

==

$Q21$. Consider the vectors $v_1 = (1, -1, 0),\ v_2 = (2, 3, -1)$ and $v_3 = (a, b, c)$ in $\mathbb{R}^3$. Choose
the correct options from the following.

Options:

A. If $a = 5,\ b = 0,\ c = -1,$ then the set $\{v_1, v_2, v_3\}$ forms a basis for $\mathbb{R}^3$.

B. If $a = 5,\ b = 0,\ c = -1,$ then the set $\{v_1, v_2, v_3\}$ are linearly dependent.

C.If $a = 5, b = 0,\ c = -1$ and A is the matrix with $v_1, v_2 \; and \; v_3$ as its columns, then $rank(A) = 3$.

D. If $a = 2,\ b = 3,\ c = 1$, then the subspace spanned by the vectors $\{v_1,\ v_2,\ v_3\}$ has dimension 3.

E. If $a = 2,\ b = 3,\ c = 1$ and A is the matrix with $v_1,\ v_2,\ and \; v_3$ as its columns, then A is invertible.