Jan 2022

$Q1$. The number of local minima of the function $f(x,y) = 2x^2 - 4xy + y^4 + 2$ is

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$Q2$. A potter wants to make a rectangular box of volume 17 cubic units such that its total surface area is minimum. He successfully creates one of dimensions $x,\ y,$ and z units. Find the value of $x^3 + z^3$

[Note: If the dimensions of a rectangular box are $l,\ b,$ and h units, then volume of the box $V = lbh$ cubic units and the total surface area of the rectangular box is $S = 2(lb + bh + lh)$ square units.]

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$Q3$. From the list of given terms find out the best possible options for each of the given subquestions:

1) Rank
2) Limit
3) Determinant
4) Closure with respect to addition and scalar multiplication
5) Existence of zero element
6) Existence of additive inverse
7) Communitative of addition
8) Associativity of addition
9) Elements
10) Local maxima
11) Local minima
12) Saddle points
13) Gradient
14) Directional Derivative
15) Partial Derivative
16) Set of orthonoraml vectors
17) Standard ordered basis
18) Affine spaces

Critical points can be __________. (Enter 3 best possible options. Enter only the serial numbers of
those options in increasing order without adding any comma or space in between them.)
[Suppose your answer is 7, 14 and 17, then you should enter 71417]

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$Q4$. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as

$$f(x, y) = x^2y + xy^2$$

The tangent plane to the function f at the point $(1, 1)$ is given by T_1. Let A_1 denote the affine subspace of $\mathbb{R}^3$ formed by the set of points on T_1. Let V_1 denote the corresponding vector subspace associated with A_1.

Let A denote the Hessian matrix of the function f at the point $(1, 1)$

Consider the affine subspace of $\mathbb{R}^3$ given by

$$A_2 = \left\{(x, y, z) \mid x - y + z = 1, \text{ where } x, y, z \in \mathbb{R}\right\}$$

Let V_2 denote the corresponding vector subspace associated with A_2.

Using the above information answer the given subquestions:

Sub questions:

Q1) The number of critical points of the function f is _______________.

Q2) What is the rank of A?

May 2022

$Q5$. Consider a matrix $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that $f(a, b) = det(A)$. Answer the given subquestions:

Sub Questions:

Q1) Find the number of critical points of the function $f(a, b)$.

Q2) Which of the following options is/are true?

Options:

A. A is not a symmetric matrix for all $a, b \in \mathbb{R}$

B. A^2 is a symmetric matrix for all $a, b \in \mathbb{R}$

C. If (β, γ) is a critical point of $f(a, b)$, then the matrix $A = \begin{bmatrix} \beta & \gamma \\ \gamma & \beta \end{bmatrix}$ satisfies that, $A^2 = A$.

D. If (β, γ) is a critical point of $f(a, b)$, then the matrix $A = \begin{bmatrix} \beta & \gamma \\ \gamma & \beta \end{bmatrix}$ satisfies that, $rank(A) = 1$

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$Q6$. If a, b and c are three positive numbers which satisfies the following two properties:

- The sum of $a,\ b$ and c is 27
- The sum of the squares of a, b and c is minimum among the sum of squares of any such positive numbers which sum upto 27

Find the value of $a - b + c$.

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$Q7$. Consider the function $f(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x$. Answer the given subquestions:

Sub Questions:

Q1) Find the number of local maxima using the Hessian test.

Q2) Find the number of local minima using the Hessian test.

Q3) Find the number of saddle points using the Hessian test.

Q4) Find the number of points at which the Hessian test is indeterminate.

Sept 2022

$Q8$. Consider the function $f(x, y, z) = x^2 + y^2 + z^2 + xy + yz + kzx$. Use this information to answer the given subquestions

Sub Questions:

Q1) Which of the following matrices represent the Hessian matrix of the function at any critical point?

Options:

A. $\begin{bmatrix} 2 & 1 & k \\ 1 & k & 1 \\ k & 1 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 1 & k \\ 1 & 2 & 1 \\ k & 1 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 1 & k \\ 1 & 2 & 1 \\ 2 & 1 & k \end{bmatrix}$

D. $\begin{bmatrix} k & 1 & 2 \\ 1 & k & 1 \\ 2 & 1 & k \end{bmatrix}$

Q2) Which of the following options is/are true?

Options :

A. If $k = 2$, then the Hessian test is inconclusive.

B. If $k = 1$, then the function attains a value at the critical point which is locally maximum.

C. If $k = 3$, then the function attains a value at the critical point which is locally minimum.

D.There is at least one value of k for which the critical point is a saddle point.

Q3) Which of the following options is/are true?

Options:

A. Equations $f_x = 0,\ f_y = 0,\ f_z = 0$ forms a system of linear equations.

B. For no value of k the Hessian matrix is a symmetrix matrix

C. For any value of k, the Hessian matrix has rank 2

D. If $k = 2$, then the nullity of the Hessian Matrix is 1.

Q4) Let H denote the matrix obtained by putting $k = 1$ in the Hessian matrix. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation $T(x, y, z) = (a_1x + b_1y + c_1z,\ a_2x + b_2y + c_2z,\ a_3x + b_3y + c_3z)$. If H is the matrix representation of w.r.t the standard ordered basis $\{e_1, e_2, e_3\}$ fot both domain and codomain, then find the value of $(a_1 + a_2) - (b_2 + b_3) + (c_1 + c_3)$

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$Q9$. Consider the plane $x + y + z = 1$ in $\mathbb{R}^3$. Let (a, b, c) be the point on the plane such that the distance of the point from the origin is the least.
Use the information to answer the given subquestions:

Sub Questions:

Q1) Find the value of $48(a\ +\ b)$.

Q2) If d is the minimum distance from the origin, then find the value of $27d^2$

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$Q10$. Consider the function $f(a,b) = b^3 + \int_0^a \left(3x^2 + 3b^2 - 15\right) dx$.

Based on the above data , Answer the given subquestions

Sub Questions:

Q1) Find the number of critical points.

Q2) Let $p_1 = \left(0, \sqrt{2}\right)$, $p_2 = \left(-\sqrt{5}, 0\right)$, $p_3 = (-1, 2)$, $p_4 = (1, -2)$ and $p_5 = (2, -1)$. Amswer the following questions with respect to these points

Write down the values of i in increasing order for which $p_i,\ i = 1, 2, 3, 4, 5$ is a point of local minima according to the Hessian test e.g. if p_1 and p_3 are points of local minima, your answer should be 13. If there is no such i, enter 0 as your answer.

Q3) Let $p_1 = \left(0, \sqrt{2}\right)$, $p_2 = \left(-\sqrt{5}, 0\right)$, $p_3 = (-1, 2)$, $p_4 = (1, -2)$ and $p_5 = (2, -1)$. Amswer the following questions with respect to these points

Write down the values of i in increasing order for which $p_i,\ i = 1, 2, 3, 4, 5$ is a saddle point according to the Hessian test e.g. if p_1 and p_3 are points of local minima, your answer should be 13. If there is no such i, enter 0 as your answer.

Jan 2023

$Q11$. What is the maximum product of three non-negative numbers whose sum is 6?

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$Q12$. Let $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$

Based on the above data, answer the given subquestions.

Sub Questions:

Q1) The number of critical points of $f(x, y)$ is

Q2) The number of saddle points of $f(x, y)$ is

Q3) The number of local maxima of $f(x, y)$ is

Q4) The number of local minima of $f(x, y)$ is

Q5) iF $H(x, y)$ denotes the Hessian matrix of $f(x, y)$, find the determinant of $H(1, 0)$.

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$Q13$. From the list of given terms find out the best possible options.

1) Rank 2
2) Non-zero nullity
3) Non -zero determinant
4) Closure with respect to addition and scalar multiplication
5) Existence of addititve inverse
6) Existence of additive inverse
7) Commutative of addition
8) Associativity of addition
9) Elements
10) Global maxima
11) Global minima
12) A critical points
13) Gradient exists
14) Directional derivative exist in any direction
15) Partial derivatives exist
16) Orthonormal columns
17) Standard ordered basis
18) Affine subspaces
19) Limit exists

A local maxima is/can be _______________ . (Enter 2 best possible options. Enter only the serial
numbers of those options in increasing order without adding any comma or space in between them.) [Suppose your answer is 7 and 17, then you should enter 717]

Sept 2023

$Q14$. Let $f(x, y) = x^2 - y^2 + 4$ on the disc $S = \left\{(x, y) : x^2 + y^2 \leqslant 1\right\}$. Choose the correct option(s) from the following:

Options:

A. The absolute maximum for f occurs at two points and is equal to 5

B. The absoluete maximum for f occurs only at the point $(1, 0)$

C. The absoluete minimum for f occurs only at $(0, 1)$

D. The absoluete minimum for f occurs at two points on the boundary of the disc S

E. $(0, 0)$ is a saddle point of f

F. There are no saddle point for f

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$Q15$. Let $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 5$. Use the Hessian test to answer the given sub questions

Sub Questions:

Q1) Find the number of local maxima.

Q2) Find the number of local minima.

Q3) Find the number of saddle points.

Q4) If there are no local maxima, enter the answer 100. Else, let (a, b) be a local maxima such that it is farthest from the origin, and if there are more than one such points, then it has the largest $x - coordinate$ amongst them. Find $f(a, b)$

Q5) If there are no local minima, enter the answer 100. Else, let (a, b) be a local minima such that it is farthest from the origin, and if there are more than one such points, then it has the largest $x - coordinate$ amongst them. Find $f(a, b)$